

Master Math: Basic Math and Pre-Algebra

Including Everything
From Basic Arithmetic,
Fractions and Decimals
to Logarithms,
Roots and Radicals

DEBRA ANNE ROSS

Master Math: Basic Math and Pre-Algebra

By
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Table of Contents

.....

Introduction	1
Chapter 1 Numbers and Their Operations	5
1.1. Digits and the base ten system	6
1.2. Whole numbers	7
1.3. Addition of whole numbers	7
1.4. Subtraction of whole numbers	15
1.5. Multiplication of whole numbers	22
1.6. Division of whole numbers	35
1.7. Divisibility, remainders, factors and multiples	43
1.8. Integers	45
1.9. Even and odd integers	46
1.10. Zero	47
1.11. Addition and subtraction of negative and positive integers	48
1.12. Multiplication and division of negative and positive integers	51
1.13. The real number line	52
1.14. Absolute value	53
1.15. Prime numbers	54

1.16. Rational vs. irrational numbers	54
1.17. Complex numbers	55
1.18. Rounding, truncating and estimating numbers	59
1.19. Inequalities, $>$, $<$, \geq , \leq	62
1.20. Factorial	66

Chapter 2 Fractions 67

2.1. Definitions	67
2.2. Multiplying fractions	68
2.3. Adding and subtracting fractions with common denominators	69
2.4. Adding and subtracting fractions with different denominators	70
2.5. Dividing fractions	76
2.6. Reducing fractions	77
2.7. Complex fractions, mixed numbers and improper fractions	78
2.8. Adding and subtracting mixed numbers	82
2.9. Comparing fractions: Which is larger or smaller?	83

Chapter 3 Decimals 87

3.1. Definitions	87
3.2. Adding and subtracting decimals	90
3.3. Multiplying decimals	93
3.4. Dividing decimals	95
3.5. Rounding decimals	97
3.6. Comparing the size of decimals	98
3.7. Decimals and money	99

Chapter 4 Percentages	101
4.1. Definitions	101
4.2. Figuring out the percents of numbers	102
4.3. Adding, subtracting, multiplying and dividing percents	105
4.4. Percent increase and decrease (percent change)	107
4.5. Simple and compound interest	109
 Chapter 5 Converting Percentages, Fractions and Decimals	 111
5.1. Converting fractions to percents	111
5.2. Converting percents to fractions	112
5.3. Converting fractions to decimals	113
5.4. Converting decimals to fractions	113
5.5. Converting percents to decimals	115
5.6. Converting decimals to percents	117
 Chapter 6 Ratios, Proportions and Variation	 118
6.1. Definitions	118
6.2. Comparing ratios to fractions and percents	120
6.3. Variation and proportion	121

Chapter 7 Powers and Exponents	123
7.1. Definition of the exponent or power	123
7.2. Negative exponents	124
7.3. Multiplying exponents with the same base	125
7.4. Multiplying exponents with different bases	126
7.5. Dividing exponents with the same base	126
7.6. Dividing exponents with different bases	128
7.7. Raising a power of a base to a power	128
7.8. Distributing exponents into parenthesis	129
7.9. Addition of exponents	130
7.10. Subtraction of exponents	131
7.11. Exponents involving fractions	131
Chapter 8 Logarithms	133
8.1. Definition of the logarithm	133
8.2. Common (base ten) and natural logarithm	135
8.3. Solving equations with logarithms or exponents	136
8.4. Exponential form and logarithmic form	138
8.5. Laws of logarithms: addition, subtraction, multiplication, division, power and radical	139
8.6. Examples: the Richter scale, pH and radiometric dating	140

Chapter 9 Roots and Radicals	145
9.1. Definitions	146
9.2. The square root	147
9.3. Estimating square roots	149
9.4. The cubed root	152
9.5. The fourth and fifth roots	153
9.6. Simplifying radicals by factoring	154
9.7. Multiplying radicals	155
9.8. Dividing radicals	155
9.9. Radicals involving fractions	156
9.10. Rationalizing the denominator	156
9.11. Addition of radicals	157
9.12. Subtraction of radicals	158
 Chapter 10 Important Statistical Quantities	 159
10.1. Average (arithmetic mean)	159
10.2. Median	161
10.3. Mode	163
10.4. Probability	164
10.5. Standard deviation, variance, histograms and distributions	164
 Index	 170
 Appendix Tables of Contents of Second and Third Books in the <i>Master Math</i> Series	 173

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Introduction

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Basic Math and Pre-Algebra is the first of three books in the *Master Math* series. The second and third books are entitled *Algebra* and *Pre-Calculus and Geometry*. The *Master Math* series presents the general principles of mathematics from grade school through college including arithmetic, algebra, geometry, trigonometry, pre-calculus and introductory calculus.

Basic Math and Pre-Algebra is a comprehensive arithmetic book that explains the subject matter in a way that makes sense to the reader. It begins with the most basic fundamental principles and progresses through more advanced topics to prepare a student for algebra. *Basic Math and Pre-Algebra* explains the principles and operations of arithmetic, provides step-by-step procedures and solutions and presents examples and applications.

Basic Math and Pre-Algebra is a reference book for grade school and middle school students that explains and clarifies the arithmetic principles they are learning in school. It is also a comprehensive reference source for more advanced students learning algebra

Basic Math and Pre-Algebra

and pre-calculus. *Basic Math and Pre-Algebra* is invaluable for students of all ages, parents, tutors and anyone needing a basic arithmetic reference source.

The information provided in each book and in the series as a whole is progressive in difficulty and builds on itself, which allows the reader to gain perspective on the connected nature of mathematics. The skills required to understand every topic presented are explained in an earlier chapter or book within the series. Each of the three books contains a complete table of contents, a comprehensive index, and the tables of contents of the other two books in the series so that specific subjects, principles and formulas can be easily found. The books are written in a simple style that facilitates understanding and easy referencing of sought-after principles, definitions and explanations.

Basic Math and Pre-Algebra and the *Master Math* series are not replacements for textbooks but rather reference books providing explanations and perspective. The *Master Math* series would have been invaluable to me during my entire education from grade school through graduate school. There is no other source that provides the breadth and depth of the *Master Math* series in a single book or series.

Finally, mathematics is a language—the universal language. A person struggling with mathematics should approach it in the same fashion he or she would approach learning any other language. If someone moves to a foreign country, he or she does not expect to know the language automatically. It takes practice and contact with a language in order to master it.

Introduction

After a short time in the foreign country he or she would not say, "I do not know this language well yet. I must not have an aptitude for it." Yet many people have this attitude toward mathematics. If time is spent learning and practicing the principles, mathematics will become familiar and understandable. Don't give up.

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Chapter 1

Numbers and Their Operations

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- 1.1 Digits and the base ten system
- 1.2 Whole numbers
- 1.3 Addition of whole numbers
- 1.4 Subtraction of whole numbers
- 1.5 Multiplication of whole numbers
- 1.6 Division of whole numbers
- 1.7 Divisibility, remainders, factors and multiples
- 1.8 Integers
- 1.9 Even and odd integers
- 1.10 Zero
- 1.11 Addition and subtraction of negative and positive integers
- 1.12 Multiplication and division of negative and positive integers
- 1.13 The real number line
- 1.14 Absolute value
- 1.15 Prime numbers
- 1.16 Rational vs. irrational numbers
- 1.17 Complex numbers
- 1.18 Rounding, truncating and estimating numbers
- 1.19 Inequalities, $>$, $<$, \geq , \leq
- 1.20 Factorial

1.1. Digits and the base ten system

The system of numbers used for counting is based on groups of ten and is called the base ten system.

- Numbers are made up of digits that each correspond to a value. For example, the number 5,639,248 or five million, six hundred thirty-nine thousand, two hundred forty-eight represents:

5 millions, 6 hundred thousands, 3 ten thousands, 9 thousands, 2 hundreds, 4 tens, and 8 ones

This number can also be written:

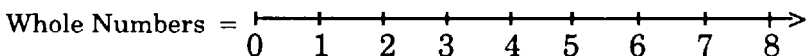
$$5,000,000 + 600,000 + 30,000 + 9,000 + 200 + 40 + 8$$

- The ones digit indicates the number of ones, (1).
- The tens digit indicates the number of tens, (10).
- The hundreds digit indicates the number of hundreds, (100).
- The thousands digit indicates the number of thousands, (1,000).
- The ten thousands digit indicates the number of tens of thousands, (10,000).
- The hundred thousands digit indicates the number of hundreds of thousands, (100,000).
- The millions digit indicates the number of millions, (1,000,000).

1.2. Whole numbers

Whole numbers include zero and the counting numbers greater than zero.

- Negative numbers and numbers in the form of fractions, decimals, percents or exponents are *not* whole numbers.
- Whole numbers are depicted on the number line and include zero and numbers to the right of zero.



- Whole numbers can be written in the form of a set.

Whole Numbers = $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, \dots\}$

1.3. Addition of whole numbers

This section includes definitions, a detailed explanation of addition, and a detailed description of the standard addition technique.

- The symbol for addition is $+$.
- Addition is written $6 + 2$.
- The numbers to be added are called *addends*.

Basic Math and Pre-Algebra

- The answer obtained in addition is called the *sum*.
- The symbol for what the sum is equal to is the equal sign $=$.
- Note: The symbol for not equal is \neq .
- For any number n , the following is true: $n + 0 = n$
(Note that letters are often used to represent numbers.)

A Detailed Explanation of Addition

- Consider the following examples:

$$5 + 3 = 8$$

$$34 + 76 = 110$$

$$439 + 278 = 717$$

$$5 + 3 + 2 = 10$$

$$33 + 28 + 56 = 117$$

$$456 + 235 + 649 = 1,340$$

- To add numbers that have two or more digits, it is easier to write the numbers in a column format with ones, tens, hundreds, etc., aligned, then add each column.

- Example: Add $5 + 3$.

$$\begin{array}{r} 5 \text{ ones} \\ + 3 \text{ ones} \\ \hline 8 \text{ ones} \end{array}$$

Numbers and Their Operations

- Example: Add $4 + 6$.

$$\begin{array}{r} 4 \text{ ones} \\ +6 \text{ ones} \\ \hline 10 \text{ ones} \end{array}$$

- Example: Add $7 + 0$.

$$\begin{array}{r} 7 \text{ ones} \\ +0 \text{ ones} \\ \hline 7 \text{ ones} \end{array}$$

- Example: Add $7 + 8$.

$$\begin{array}{r} 7 \text{ ones} \\ +8 \text{ ones} \\ \hline 15 \text{ ones} \end{array}$$

Note that 15 ones is equivalent to 1 ten and 5 ones.

- Example: Add $21 + 32$.

$$\begin{array}{r} 21 \\ +32 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 2 \text{ tens} + 1 \text{ ones} \\ +3 \text{ tens} + 2 \text{ ones} \\ \hline 5 \text{ tens} + 3 \text{ ones} \end{array}$$

$$= 50 + 3 = 53$$

Basic Math and Pre-Algebra

- Example: Add $34 + 76$.

$$\begin{array}{r} 34 \\ +76 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 3 \text{ tens} + 4 \text{ ones} \\ +7 \text{ tens} + 6 \text{ ones} \\ \hline 10 \text{ tens} + 10 \text{ ones} \\ = 100 + 10 = 110 \end{array}$$

This equation can also be rewritten as:

$$\begin{array}{r} 30 + 4 \\ +70 + 6 \\ \hline 100 + 10 \\ = 110 \end{array}$$

- Example: Add $7 + 28$.

$$\begin{array}{r} 7 \\ +28 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 0 \text{ tens} + 7 \text{ ones} \\ + 2 \text{ tens} + 8 \text{ ones} \\ \hline 2 \text{ tens} + 15 \text{ ones} \end{array}$$

$$= 20 + 15 = 35$$

• **Example:** Add $439 + 278$.

$$\begin{array}{r} 439 \\ + 278 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 4 \text{ hundreds} + 3 \text{ tens} + 9 \text{ ones} \\ + 2 \text{ hundreds} + 7 \text{ tens} + 8 \text{ ones} \\ \hline 6 \text{ hundreds} + 10 \text{ tens} + 17 \text{ ones} \end{array}$$

$$= 600 + 100 + 17 = 717$$

This equation can also be rewritten as:

$$\begin{array}{r} 400 + 30 + 9 \\ + 200 + 70 + 8 \\ \hline 600 + 100 + 17 \end{array}$$

$$= 717$$

A Detailed Description of the Standard Addition Technique

- An alternative to separating the ones, tens, hundreds, etc., is to align the proper digits in columns (ones over ones, tens over tens, etc.). Then add each column, beginning with the ones, and carry-over digits to the left. (Carry over the tens generated in the ones' column into the tens' column, carry over the hundreds generated in the tens' column into the hundreds' column, carry over the thousands generated in the hundreds' column into the thousands' columns, and so on.)

- Example: Add $35 + 46$.

$$\begin{array}{r} 35 \\ +46 \\ \hline ? \end{array}$$

First add $5 + 6 = 11$, and carry over the 10 into the tens' column.

$$\begin{array}{r} 1 \\ 35 \\ + 46 \\ \hline 1 \end{array}$$

Add the tens' column.

Numbers and Their Operations

$$\begin{array}{r} 1 \\ 35 \\ + 46 \\ \hline 81 \end{array}$$

- Example: Add $56 + 789$.

$$\begin{array}{r} 56 \\ + 789 \\ \hline ? \end{array} \quad \text{Switching order:} \quad \begin{array}{r} 789 \\ + 56 \\ \hline ? \end{array}$$

Add $9 + 6 = 15$, and carry over the 10 into the tens' column.

$$\begin{array}{r} 1 \\ 789 \\ + 56 \\ \hline 5 \end{array}$$

Add the tens' column, $1 + 8 + 5 = 14$ tens, and carry over 100 to the hundreds' column.

$$\begin{array}{r} 11 \\ 789 \\ + 56 \\ \hline 45 \end{array}$$

Add the hundreds' column, $1 + 7 = 8$ hundreds.

Basic Math and Pre-Algebra

$$\begin{array}{r} 11 \\ 789 \\ + 56 \\ \hline 845 \end{array}$$

- Example: Add $5 + 49 + 387 + 928$.

$$\begin{array}{r} 928 \\ 387 \\ 49 \\ + 5 \\ \hline ? \end{array}$$

Add the ones, $8 + 7 + 9 + 5 = 29$, and carry over the 20 into the tens' column.

$$\begin{array}{r} 2 \\ 928 \\ 387 \\ 49 \\ + 5 \\ \hline 9 \end{array}$$

Add the tens, $2 + 2 + 8 + 4 = 16$ tens, and carry over the 100 into the hundreds' column.

$$\begin{array}{r} 12 \\ 928 \\ 387 \\ 49 \\ + 5 \\ \hline 69 \end{array}$$

Add the hundreds, $1 + 9 + 3 = 13$ hundreds.

$$\begin{array}{r} 12 \\ 928 \\ 387 \\ 49 \\ + 5 \\ \hline 1369 \end{array}$$

1.4. Subtraction of whole numbers

This section includes definitions, a detailed explanation of subtraction, and a detailed description of the standard subtraction technique.

- Subtraction is the process of finding the difference between two numbers.
- The symbol for subtraction is $-$.
- Subtraction is written $6 - 2$.

Basic Math and Pre-Algebra

- The number to be subtracted from is called the *minuend*.
- The number to be subtracted is the *subtrahend*.
- The answer obtained in subtraction is called the *difference*.
- Subtraction is the reverse of addition.

If $2 + 3 = 5$, then $5 - 2 = 3$ or $5 - 3 = 2$.

- For any number n , the following is true: $n - 0 = n$
(Note that letters are often used to represent numbers.)
- To subtract more than two numbers, subtract the first two, then subtract the third number from the difference of the first two, and so on. Subtraction must be performed in the order that the numbers are listed.

A Detailed Explanation of Subtraction

- Consider the examples:

$$6 - 2 = 4$$

$$26 - 8 = 18$$

$$10,322 - 899 = 9,423$$

- To subtract numbers with two or more digits, it is easier to write the numbers in a column format with ones, tens, hundreds, etc., aligned, then subtract each column beginning with the ones. To subtract a digit in

Numbers and Their Operations

a given column from a smaller digit in the same column, borrow from the next larger column.

- Example: Subtract $6 - 2$.

$$\begin{array}{r} 6 \text{ ones} \\ - 2 \text{ ones} \\ \hline 4 \text{ ones} \end{array}$$

- Example: Subtract $26 - 8$.

$$\begin{array}{r} 26 \\ - 8 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 2 \text{ tens} + 6 \text{ ones} \\ - 0 \text{ tens} + 8 \text{ ones} \\ \hline ? \end{array}$$

Borrow 10 from the tens' column.

$$\begin{array}{r} 1 \text{ tens} + 16 \text{ ones} \\ - 0 \text{ tens} + 8 \text{ ones} \\ \hline ? \end{array}$$

Subtract the ones, $16 - 8 = 8$.

Basic Math and Pre-Algebra

$$\begin{array}{r} 1 \text{ tens} + 16 \text{ ones} \\ - 0 \text{ tens} + 8 \text{ ones} \\ \hline 8 \text{ ones} \end{array}$$

Subtract the tens, $1 - 0 = 1$ ten.

$$\begin{array}{r} 1 \text{ tens} + 16 \text{ ones} \\ - 0 \text{ tens} + 8 \text{ ones} \\ \hline 1 \text{ tens} + 8 \text{ ones} \end{array}$$
$$= 10 + 8 = 18$$

- Example: Subtract $10,322 - 899$.

$$\begin{array}{r} 10,322 \\ - 899 \\ \hline ? \end{array}$$

$$\begin{array}{r} 1 \text{ ten thousand} + 0 \text{ thousands} + 3 \text{ hundreds} + 2 \text{ tens} + 2 \text{ ones} \\ - 0 \text{ ten thousand} + 0 \text{ thousands} + 8 \text{ hundreds} + 9 \text{ tens} + 9 \text{ ones} \\ \hline ? \end{array}$$

To subtract the ones, borrow 10 from the tens' column, then subtract $12 - 9 = 3$.

$$\begin{array}{r} 1 \text{ ten thousand} + 0 \text{ thousands} + 3 \text{ hundreds} + 1 \text{ tens} + 12 \text{ ones} \\ - 0 \text{ ten thousand} + 0 \text{ thousands} + 8 \text{ hundreds} + 9 \text{ tens} + 9 \text{ ones} \\ \hline 3 \text{ ones} \end{array}$$

To subtract the tens, borrow 100 from the hundreds' column, then subtract $110 - 90 = 20$.

Numbers and Their Operations

$$\begin{array}{r} 1 \text{ ten thousand} + 0 \text{ thousands} + 2 \text{ hundreds} + 11 \text{ tens} + 12 \text{ ones} \\ - 0 \text{ ten thousand} + 0 \text{ thousands} + 8 \text{ hundreds} + 9 \text{ tens} + 9 \text{ ones} \\ \hline 2 \text{ tens} + 3 \text{ ones} \end{array}$$

To subtract the hundreds, first borrow 10,000 from the ten thousands' column into the thousands' column, then borrow 1,000 from the thousands' column.

$$\begin{array}{r} 0 \text{ ten thousand} + 9 \text{ thousands} + 12 \text{ hundreds} + 11 \text{ tens} + 12 \text{ ones} \\ - 0 \text{ ten thousand} + 0 \text{ thousands} + 8 \text{ hundreds} + 9 \text{ tens} + 9 \text{ ones} \\ \hline 2 \text{ tens} + 3 \text{ ones} \end{array}$$

Next, subtract the hundreds, $1,200 - 800 = 400$.

Then, subtract the thousands, $9,000 - 0 = 9,000$.

Then, subtract the ten thousands, $0 - 0 = 0$.

$$\begin{array}{r} 0 \text{ ten thousand} + 9 \text{ thousands} + 12 \text{ hundreds} + 11 \text{ tens} + 12 \text{ ones} \\ - 0 \text{ ten thousand} + 0 \text{ thousands} + 8 \text{ hundreds} + 9 \text{ tens} + 9 \text{ ones} \\ \hline 9 \text{ thousands} + 4 \text{ hundreds} + 2 \text{ tens} + 3 \text{ ones} \end{array}$$

$$= 9,000 + 400 + 20 + 3 = 9,423$$

A Detailed Description of the Standard Subtraction Technique

- An alternative to separating the ones, tens, hundreds, etc., is to align the proper digits in columns and borrow (if needed) from the column to the left, which is 10 times larger.

Basic Math and Pre-Algebra

- Example: Subtract $26 - 8$.

$$\begin{array}{r} 26 \\ - 8 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 1\{16\} \\ - 8 \\ \hline ? \end{array}$$

Subtract the ones, $16 - 8 = 8$.

$$\begin{array}{r} 1\{16\} \\ - 8 \\ \hline 8 \end{array}$$

Subtract the tens, $1 - 0 = 1$ ten.

$$\begin{array}{r} 1\{16\} \\ - 8 \\ \hline 1 \quad 8 \end{array}$$

Therefore, $26 - 8 = 18$.

To check the subtraction results, add the difference to the number that was subtracted.

Does $18 + 8 = 26$? Yes.

Numbers and Their Operations

- **Example:** Subtract $10,322 - 899$.

$$\begin{array}{r} 10,322 \\ - \quad 899 \\ \hline \end{array}$$

?

Borrow from the tens' column and subtract the ones.

$$\begin{array}{r} 10,31\{12\} \\ - \quad 89\ 9 \\ \hline \end{array}$$

3

Borrow from the hundreds' column and subtract the tens.

$$\begin{array}{r} 10,2\{11\}\{12\} \\ - \quad 8\ 9\ 9 \\ \hline \end{array}$$

2\ 3

Borrow from the thousands' column and subtract hundreds.

$$\begin{array}{r} 9\{12\}\{11\}\{12\} \\ - \quad 8\ 9\ 9 \\ \hline \end{array}$$

4\ 2\ 3

Subtract the thousands.

$$\begin{array}{r} 9\{12\}\{11\}\{12\} \\ - \quad 8\ 9\ 9 \\ \hline \end{array}$$

9\ 4\ 2\ 3

or 9,423

Basic Math and Pre-Algebra

To check the subtraction results, add the difference to the number that was subtracted.

$$\begin{array}{r} 9423 \\ + 899 \\ \hline 10,322 \end{array}$$

- Note that if a larger number is subtracted from a smaller number, a negative number results. For example, $5 - 8 = -3$ and $40 - 50 = -10$. See Section 1.11 “Addition and subtraction of negative and positive integers.”

1.5. Multiplication of whole numbers

This section includes definitions, an explanation of multiplication, and a detailed description of the standard multiplication technique.

- Multiplication is a shortcut for addition.
- The symbols for multiplication are \times , $*$, \bullet , $()()$.
- The numbers to be multiplied are called the *multiplicand* (the first number) and the *multiplier* (the second number).
- The answer obtained in multiplication is called the *product*.

- Multiplication is written using the following symbols:

$$6 \times 2, 6 * 2, 6 \bullet 2, (6)(2)$$

- If you add 12 twelves, the answer is 144.

$$12+12+12+12+12+12+12+12+12+12+12+12 = 144$$

Adding 12 twelves is the same as multiplying twelve by twelve

$$(12)(12) = 144$$

- What is 3 times 2, (3)(2)? It is the value of 3 two times, or $3 + 3 = 6$.

- Equivalently, what is 2 times 3, (2)(3)? It is the value of 2 three times or, $2+2+2 = 6$.

- What is 2 times 4 times 3?

$$(2)(4)(3) = (2)(4 \text{ three times})$$

$$= (2)(4+4+4) = (2)(12) = 24$$

Also, $(2)(12) = 2$ twelve times

$$= 2+2+2+2+2+2+2+2+2+2+2+2 = 24$$

Basic Math and Pre-Algebra

- Equivalently, 2 times 4 times 3 is:

$$(2)(4)(3) = (4 \text{ two times})(3) = (4+4)(3)$$

$$= (8)(3) = 8 \text{ three times} = 8 + 8 + 8 = 24$$

- The order in which numbers are multiplied does not affect the result, just as the order in which numbers are added does not affect the result.

- If a number is multiplied by zero, that number is multiplied zero times and equals zero.

- For any number n , the following is true: $n \times 0 = 0$
(Note that letters are often used to represent numbers.)

An Explanation of Multiplication

- Consider the following examples:

$$28 * 7 = ?$$

$$6,846 * 412 = ?$$

$$10 * 10 = ?$$

$$10 * 100 = ?$$

$$10 * 1,000 = ?$$

- To multiply numbers with two or more digits, it is easier to write the numbers in a column format. Then multiply each digit in the multiplicand (top number), beginning with the ones' digit, by each digit in the multiplier (bottom number), beginning with the ones' digit.

- Example: Multiply $28 * 7 = ?$.

$$\begin{array}{r} 28 \\ \times 7 \\ \hline ? \end{array}$$

This equation can be rewritten as:

$$\begin{array}{r} 2 \text{ tens} + 8 \text{ ones} \\ \times 0 \text{ tens} + 7 \text{ ones} \\ \hline ? \end{array}$$

Multiply the 8 ones by the 7 ones to obtain 56 ones.

$$\begin{array}{r} 2 \text{ tens} + 8 \text{ ones} \\ \times 0 \text{ tens} + 7 \text{ ones} \\ \hline 56 \text{ ones} \end{array}$$

Multiply the 2 tens (20) by the 7 ones to obtain 14 tens (140).

$$\begin{array}{r} 2 \text{ tens} + 8 \text{ ones} \\ \times 0 \text{ tens} + 7 \text{ ones} \\ \hline 14 \text{ tens} + 56 \text{ ones} \end{array}$$

$$= 140 + 56 = 196$$

A Detailed Description of the Standard Multiplication Technique

- To multiply numbers, multiply each digit in the multiplier with each digit in the multiplicand (carrying-over as in addition) to create partial products, then add the partial products.

Basic Math and Pre-Algebra

- Example: Multiply $28 * 7 = ?$.

$$\begin{array}{r} 28 \\ \times 7 \\ \hline ? \end{array}$$

Multiply $8 * 7 = 56$, and carry over the 5 tens.

$$\begin{array}{r} 5 \\ 28 \\ \times 7 \\ \hline 6 \end{array}$$

Multiply $2 * 7 = 14$ tens, and add the 5 tens resulting in 19 tens.

$$\begin{array}{r} 5 \\ 28 \\ \times 7 \\ \hline 196 \end{array}$$

- Example: Multiply $6,846 * 412 = ?$.

$$\begin{array}{r} 6846 \\ \times 412 \\ \hline ? \end{array}$$

Because there is more than one digit in the multiplier, multiply each digit in the multiplier separately to create partial products. Then, add partial products.

Note that alignment is important! Each partial product must be aligned with the right end of the multiplier digit.

First, multiply the ones' digit in the multiplier with each digit in the multiplicand beginning with the ones' digit, $6 * 2 = 12$, carry over the 1 ten.

$$\begin{array}{r} 1 \\ 6846 \\ \times 412 \\ \hline 2 \end{array}$$

Multiply the ones' digit in the multiplier with the tens' digit in the multiplicand, $4 * 2 = 8$, then add the 1 ten that was carried over (there is nothing new to carry).

$$\begin{array}{r} 1 \\ 6846 \\ \times 412 \\ \hline 92 \end{array}$$

Multiply the ones' digit in the multiplier with the hundreds' digit in the multiplicand, $8 * 2 = 16$, (nothing carried over to add), carry over the 1 thousand.

$$\begin{array}{r} 1 \quad 1 \\ 6846 \\ \times 412 \\ \hline 692 \end{array}$$

Basic Math and Pre-Algebra

Multiply the ones' digit in the multiplier with the thousands' digit in the multiplicand, $6 * 2 = 12$, add the 1 thousand that was carried.

$$\begin{array}{r} 1 \ 1 \\ 6846 \\ \times 412 \\ \hline 13692 \end{array}$$

Resulting in the partial product aligned with the right end of the ones' digit of the multiplier.

Next, multiply the tens' digit in the multiplier with the ones' digit in the multiplicand, $6 * 1 = 6$, (nothing to carry).

$$\begin{array}{r} 6846 \\ \times 412 \\ \hline 13692 \\ 6 \end{array}$$

Multiply the tens' digit in the multiplier with the tens' digit in the multiplicand, $4 * 1 = 4$, (nothing to add or carry).

$$\begin{array}{r} 6846 \\ \times 412 \\ \hline 13692 \\ 46 \end{array}$$

Numbers and Their Operations

Multiply the tens' digit in the multiplier with the hundreds' digit in the multiplicand, $8 * 1 = 8$, (nothing to add or carry).

$$\begin{array}{r} 6846 \\ \times 412 \\ \hline 13692 \\ 846 \end{array}$$

Multiply the tens' digit in the multiplier with the thousands' digit in the multiplicand, $6 * 1 = 6$, (nothing carried over to add).

$$\begin{array}{r} 6846 \\ \times 412 \\ \hline 13692 \\ 6846 \end{array}$$

Resulting in the partial product aligned with the right end of the tens' digit of the multiplier. (Each partial product must be aligned with the right end of the multiplier digit.)

Next, multiply the hundreds' digit in the multiplier with the ones' digit in the multiplicand, $6 * 4 = 24$, and carry over the 2 to the tens' column.

Basic Math and Pre-Algebra

$$\begin{array}{r} 2 \\ 6846 \\ \times 412 \\ \hline 13692 \\ 6846 \\ 4 \end{array}$$

Multiply the hundreds' digit in the multiplier with the tens' digit in the multiplicand, $4 * 4 = 16$, add the 2 over the tens' column, and carry over the 1 to the hundreds' column.

$$\begin{array}{r} 12 \\ 6846 \\ \times 412 \\ \hline 13692 \\ 6846 \\ 84 \end{array}$$

Multiply the hundreds' digit in the multiplier with the hundreds' digit in the multiplicand, $8 * 4 = 32$, add the 1 over the hundreds' column, and carry over the 3 to the thousands' column.

$$\begin{array}{r} 312 \\ 6846 \\ \times 412 \\ \hline 13692 \\ 6846 \\ 384 \end{array}$$

Multiply the hundreds' digit in the multiplier with the thousands' digit in the multiplicand, $6 * 4 = 24$, and add the 3 over the thousands' column.

$$\begin{array}{r} 312 \\ 6846 \\ \times \quad 412 \\ \hline 13692 \\ 6846 \\ 27384 \end{array}$$

Resulting in the partial product aligned with the right end of the hundreds' digit of the multiplier. (Each partial product is aligned with the right end of the multiplier digit.)

Next, add the three partial products for a total product.

$$\begin{array}{r} 6846 \\ \times \quad 412 \\ \hline 13692 \\ 6846 \\ \underline{27384} \\ 2820552 \end{array}$$

Therefore, $6,846 * 412 = 2,820,552$.

Basic Math and Pre-Algebra

- Example: Zeros in the multiplier.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline ? \end{array}$$

Multiply $3 * 0 = 0$.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 0 \end{array}$$

Multiply $0 * 0 = 0$.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 00 \end{array}$$

Multiply $6 * 0 = 0$.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \end{array}$$

Resulting in the first partial product.

Multiply $3 * 2 = 6$.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \\ 6 \end{array}$$

Multiply $0 * 2 = 0$.

Numbers and Their Operations

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \\ 06 \end{array}$$

Multiply $6 * 2 = 12$.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \\ 1206 \end{array}$$

Resulting in the second partial product.

Next, add the partial products.

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \\ 1206 \\ \hline 12060 \end{array}$$

Therefore, $603 * 20 = 12,060$.

Note: When there is a zero in the multiplier, a single zero can be inserted directly below the zero in the multiplier, instead of a row of zeros.

Basic Math and Pre-Algebra

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 000 \\ 1206 \\ \hline \end{array}$$

Is equivalent to:

$$\begin{array}{r} 603 \\ \times 20 \\ \hline 12060 \end{array}$$

• Examples of multiplying by numbers containing zeros:

$$\begin{aligned} 10 * 10 &= 100 \\ 10 * 100 &= 1,000 \\ 10 * 1,000 &= 10,000 \\ 10 * 10,000 &= 100,000 \end{aligned}$$

$$\begin{aligned} 5 * 10 &= 50 \\ 5 * 100 &= 500 \\ 5 * 1,000 &= 5,000 \\ 5 * 10,000 &= 50,000 \\ 5 * 100,000 &= 500,000 \end{aligned}$$

Where,

Multiplying by 10 inserts 1 zero
Multiplying by 100 inserts 2 zeros
Multiplying by 1,000 inserts 3 zeros
Multiplying by 10,000 inserts 4 zeros
Etc.

1.6. Division of whole numbers

This section provides definitions and describes in detail the long division format.

- Division evaluates how many times one number is present in another number.
- The symbols for division are \div , $/$, $\overline{)}$.
- The number that gets divided is called the *dividend*.
- The number that does the dividing (or divides into the dividend) is called the *divisor*.
- The answer obtained after division is called the *quotient*.
- Division is written $6 \div 2$, $\frac{6}{2}$, $(6)/(2)$, $2\overline{)6}$.
- Division is the inverse of multiplication.

$$2 * 3 = 6, 6 \div 2 = 3, 6 \div 3 = 2$$

- Six divides by two three times or by three two times.

$$6 \div 2 = 3, 6 \div 3 = 2$$

$$6 = 3 + 3 = 2 + 2 + 2$$

Basic Math and Pre-Algebra

- Because $6 + 6 + 6 + 6 = 24$, there are four sixes in 24.

$$6 * 4 = 24 \text{ or } 24 \div 6 = 4$$

- For any number n , the following is true: $n \div 0 =$ Undefined. (Note that letters are often used to represent numbers.)

- For any number n , the following is true: $0 \div n = 0$

A Description of the Long Division Technique

- Example: $4,628 \div 5 = ?$.

This division problem is easier to solve by writing it in a long division format.

$$5 \overline{)4628}$$

Divide the divisor (5) into the left digit(s) of the dividend. To do this, choose the smallest part of the dividend the divisor will divide in to. Because 5 does not divide into 4, the next smallest part of the dividend is 46. Estimate how many times 5 will divide into 46. First try 9. What is $5 * 9$? $5 * 9 = 45$, which is 1 less than 46. Write the 9 over the right end of the number it will divide into (46), and place 45 under the 46. (See Section 1.18 on rounding, truncating and estimating for assistance on estimating.)

Numbers and Their Operations

$$\begin{array}{r} 9? \\ 5 \overline{)4628} \\ \underline{45} \end{array}$$

Subtract 46 - 45.

$$\begin{array}{r} 9? \\ 5 \overline{)4628} \\ \underline{45} \\ 01 \end{array}$$

Bring down the next digit (2) to obtain the next number (12) to divide the 5 into.

$$\begin{array}{r} 9? \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \end{array}$$

Estimate the most number of times 5 will divide into 12. The estimate is 2. Because, $5 * 2 = 10$, (with a remainder of 2), 5 will divide into 12 two times. Write the 2 over the right end of the number it will divide into (12), and place 10 under 12.

Basic Math and Pre-Algebra

$$\begin{array}{r} 92? \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \\ \underline{10} \end{array}$$

Subtract 12 - 10.

$$\begin{array}{r} 92? \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \\ \underline{10} \\ 2 \end{array}$$

Bring down the next digit (8) to obtain the next number (28) to divide 5 into.

$$\begin{array}{r} 92? \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \\ \underline{10} \\ 28 \end{array}$$

Estimate the most number of times 5 will divide into 28. The estimate is 5. Because $5 * 5 = 25$, (with a remainder of 3), 5 will divide into 28 five times. Write the 5 over the right end of the number it will divide into (28), and place 25 under 28.

$$\begin{array}{r} 925 \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \\ \underline{10} \\ 28 \\ \underline{25} \end{array}$$

Subtract $28 - 25$.

$$\begin{array}{r} 925 \\ 5 \overline{)4628} \\ \underline{45} \\ 012 \\ \underline{10} \\ 28 \\ \underline{25} \\ 3 \end{array}$$

There are no more numbers to bring down, therefore the division is complete, and 3 is the final remainder.

Therefore, $4,628 \div 5 = 925$ plus a remainder of 3.

To check division, multiply the quotient by the divisor and add the remainder to obtain the dividend.

Basic Math and Pre-Algebra

$$\begin{array}{r} 925 \\ \times \quad 5 \\ \hline 4625 \end{array}$$

Add the remainder.

$$\begin{array}{r} 4625 \\ + \quad 3 \\ \hline 4628 \end{array}$$

- Example: $160,476 \div 364 = ?$.

Arrange in the long division format.

$$\begin{array}{r} ? \\ 364 \overline{)160476} \end{array}$$

Divide the divisor (364) into the left digits of the dividend. To do this, choose the smallest part of the dividend the divisor will divide in to. 364 will divide into 1,604. Estimate the most number of times 364 will divide into 1,604. The estimate is 4. $364 * 4 = 1,456$ with a remainder of 148. Write the 4 over the right end of the number it will divide into (1,604), and place 1,456 under the 1,604.

$$\begin{array}{r} 4?? \\ 364 \overline{)160476} \\ 1456 \end{array}$$

Subtract 1,604 - 1,456.

Numbers and Their Operations

$$\begin{array}{r} 4?? \\ 364 \overline{)160476} \\ \underline{1456} \\ 0148 \end{array}$$

Bring down the next digit (7) to obtain the next number (1,487) to divide 364 into.

$$\begin{array}{r} 4?? \\ 364 \overline{)160476} \\ \underline{1456} \\ 01487 \end{array}$$

Estimate the most number of times 364 will divide into 1,487. The estimate is 4. $364 * 4 = 1,456$ with a remainder of 31. Write the 4 over the end of the number it will divide in to (1,487), and place 1,456 under the 1,487.

$$\begin{array}{r} 44? \\ 364 \overline{)160476} \\ \underline{1456} \\ 01487 \\ \underline{1456} \end{array}$$

Subtract $1,487 - 1,456$.

Basic Math and Pre-Algebra

$$\begin{array}{r} 44? \\ 364 \overline{)160476} \\ \underline{1456} \\ 01487 \\ \underline{1456} \\ 31 \end{array}$$

Bring down the next digit (6) to obtain the next number (316) to divide 364 into.

$$\begin{array}{r} 44? \\ 364 \overline{)160476} \\ \underline{1456} \\ 01487 \\ \underline{1456} \\ 316 \end{array}$$

Because 364 will not divide into 316, place a zero over the right end of 316.

$$\begin{array}{r} 440 \\ 364 \overline{)160476} \\ \underline{1456} \\ 01487 \\ \underline{1456} \\ 316 \end{array}$$

316 becomes the remainder. Therefore, $160,476 \div 364 = 440$ plus a remainder of 316.

To check division, multiply the quotient by the divisor and add the remainder to obtain the dividend.

$$\begin{array}{r} 364 \\ \times 440 \\ \hline 14560 \\ 1456 \\ \hline 160160 \end{array} \qquad \begin{array}{r} 160160 \\ + 316 \\ \hline 160476 \end{array}$$

Add the remainder.

1.7. Divisibility, remainders, factors and multiples

This section defines and gives examples of divisibility, remainders, factors and multiples.

- The *divisibility* of a number is determined by how many times that number can be divided evenly by another number. For example, 6 is divisible by 2 because 2 divides into 6 a total of 3 times with no remainder. An example of a number that is not divisible by 2 is 5, because 2 divides into 5 a total of 2 times with 1 remaining.
- The *remainder* is the number left over when a number cannot be divided evenly by another number.

Basic Math and Pre-Algebra

- A smaller number is a *factor* of a larger number if the smaller number can be divided into the larger number without producing a remainder. Numbers that are multiplied together to produce a product are factors of that product.

- The factors of 6 are 6 and 1, or 2 and 3.

$$6 \div 6 = 1, \quad 6 \div 1 = 6, \quad 6 \div 2 = 3, \quad 6 \div 3 = 2,$$

$$6 \times 1 = 6, \quad 2 \times 3 = 6$$

5 is not a factor of 6 because $6 \div 5 = 1$ with a remainder of 1.

- The factors of 10 are 2 and 5, or 10 and 1, where:

$$10 = (2)(5), \quad 10 = (10)(1)$$

- If a and b represent numbers,

$$6ab = (2)(3)(a)(b)$$

Where 2, 3, a and b are factors of 6ab.

- A *multiple* of a number is any number that results after that number is multiplied with any number. Multiples of zero do not exist because any number multiplied by zero, including zero, results in zero. It is possible to create infinite multiples of numbers by simply multiplying them with other numbers.

- The following are examples of multiples of the number 3:

$$3 * 5 = 15$$

$$3 * 2 = 6$$

$$3 * 4 = 12$$

$$3 * 20 = 60$$

Where 15, 6, 12 and 60 are some of the multiples of 3.

1.8. Integers

Integers include positive numbers and zero (whole numbers) and also negative numbers.

- The set of all integers is represented as follows:

Integers = {...-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, ...}

- Numbers that are *not* integers include numbers in the form of fractions, decimals, percents or exponents.
- Consecutive integers are integers that are arranged in an increasing order according to their size from the smallest to the largest without any integers missing in between.

- The following are examples of consecutive integers:

$\{-10, -9, -8, -7\}$

$\{0, 1, 2, 3, 4\}$

$\{-2, -1, 0, 1, 2, 3, 4, 5\}$

$\{99, 100, 101, 102, 103, 104, 105\}$

1.9. Even and odd integers

This section defines and provides examples of even integers and odd integers.

- Even integers are integers that can be divided evenly by 2.

Even Integers = $\{...-6, -4, -2, 0, 2, 4, 6, 8, ...\}$

- Zero is an even integer.
- Odd integers are integers that cannot be divided evenly by 2, and therefore are not even.

Odd Integers = $\{...-7, -5, -3, -1, 1, 3, 5, 7, ...\}$

- Fractions are neither even nor odd. If division of two numbers yields a fraction the result is not odd or even.
- It is possible to immediately determine if a large number is even or odd by observing whether the digit in the ones position is even or odd.
- Consecutive even integers are even integers that are arranged in an increasing order according to their size without any integers missing in between.
- The following is an example of consecutive even integers: $\{-10, -8, -6, -4, -2, 0, 2, 4, 6\}$
- Consecutive odd integers are odd integers that are arranged in an increasing order according to their size without any integers missing in between.

- The following is an example of consecutive odd integers: $\{-3, -1, 1, 3, 5\}$

- Adding and subtracting even and odd integers:

even + even = even, $4 + 6 = 10$, $0 + 6 = 6$

even + odd = odd, $4 + 5 = 9$, $0 + 5 = 5$

odd + odd = even, $3 + 5 = 8$, $1 + 5 = 6$

even - even = even, $6 - 4 = 2$, $4 - 6 = -2$

even - odd = odd, $6 - 3 = 3$, $4 - 3 = 1$

odd - odd = even, $5 - 3 = 2$, $1 - 3 = -2$

- Multiplying even and odd integers:

even * even = even, $4 * 6 = 24$, $0 * 6 = 0$

even * odd = even, $4 * 5 = 20$, $0 * 5 = 0$

odd * odd = odd, $5 * 5 = 25$, $1 * 5 = 5$

1.10. Zero

Zero is both an integer and a whole number.

- Zero is even.

- Zero is not a positive or negative number.

- If zero is added to or subtracted from any number the value of that number is not changed and the result is that number:

$$n + 0 = n \text{ and } n - 0 = n$$

Basic Math and Pre-Algebra

- If zero is multiplied by any number the result is zero:

$$n * 0 = 0$$

- Dividing by zero is “undefined”:

$$n \div 0 = \text{undefined}$$

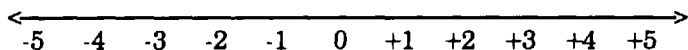
- Zero divided by a number (represented by letter n) is zero:

$$0 \div n = 0$$

1.11. Addition and subtraction of negative and positive integers

This section describes addition and subtraction of negative integers and positive integers.

- Remember the number line.

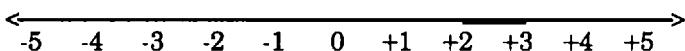


- When numbers are added and subtracted, think of moving along the number line.
- Begin with the first number and move to the right or left depending on the sign of the second number and whether it is being added or subtracted to the first number.

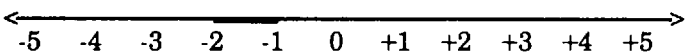
Numbers and Their Operations

- To add a positive number, begin at the first number and move to the right the value of the second number.

$$2 + 1 = 3$$

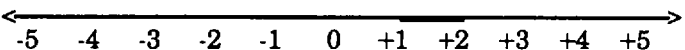


$$-2 + 1 = -1$$

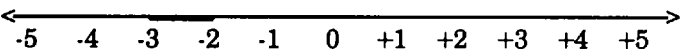


- To add a negative number (subtract), begin at the first number and move to the left the value of the second number.

$$2 + -1 = 1$$

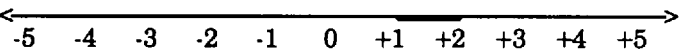


$$-2 + -1 = -3$$



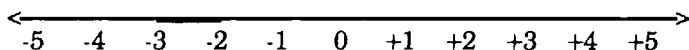
- To subtract a positive number, begin at the first number and move to the left the value of the second number.

$$2 - 1 = 1$$



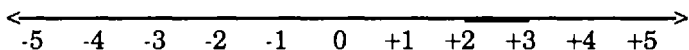
Basic Math and Pre-Algebra

$$-2 - 1 = -3$$

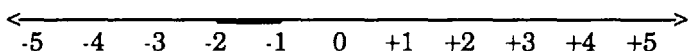


- To subtract a negative number, begin at the first number and move to the right the value of the second number, (the two negatives cancel each other out).

$$2 - -1 = 2 + 1 = 3$$



$$-2 - -1 = -2 + 1 = -1$$



- If a and b represent numbers,

$$a + + b = a + b$$

$$a + - b = a - b$$

$$a - + b = a - b$$

$$a - - b = a + b$$

1.12. Multiplication and division of negative and positive integers

If a negative number and a positive number are multiplied or divided, the result will be negative. If two negative numbers are multiplied or divided, the negative signs will cancel each other and the resulting number will be positive.

• The following rules apply to multiplication and division of positive and negative numbers, (a and b represent numbers):

positive * positive = positive, $3 * 2 = 6$, $a * b = ab$

negative * negative = positive, $-3 * -2 = 6$, $-a * -b = ab$

positive * negative = negative, $3 * -2 = -6$, $a * -b = -ab$

negative * positive = negative, $-3 * 2 = -6$, $-a * b = -ab$

positive ÷ positive = positive, $6/2 = 3$, $a/b = a/b$

negative ÷ negative = positive, $-6/-2 = 3$, $-a/-b = a/b$

positive ÷ negative = negative, $6/-2 = -3$, $a/-b = -a/b$

negative ÷ positive = negative, $-6/2 = -3$, $-a/b = -a/b$

• A summary of the multiplication rules are:

$$(+)(+) = +$$

$$(-)(-) = +$$

$$(+)(-) = -$$

$$(-)(+) = -$$

Basic Math and Pre-Algebra

- A summary of the division rules are:

$$(+)\div(+)=+$$

$$(-)\div(-)=+$$

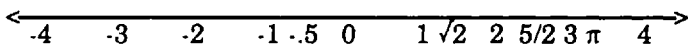
$$(+)\div(-)=-$$

$$(-)\div(+)= -$$

1.13. The real number line

Real numbers include whole numbers, integers, fractions, decimals, rational numbers and irrational numbers.

- Real numbers can be expressed as the sum of a decimal and an integer.
- All real numbers except zero are either positive or negative.
- All real numbers correspond to points on the real number line and all points on the number line correspond to real numbers.
- The real number line reaches from negative infinity $(-\infty)$ to positive infinity $(+\infty)$.

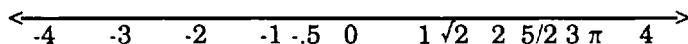


Real numbers include -0.5 , $\sqrt{2}$, $5/2$ and π . ($\pi=3.14$)
All numbers to the left of zero are negative.
All numbers to the right of zero are positive.

- The distance between zero and a number on the number line is called the absolute value or the magnitude of the number.

1.14. Absolute value

The absolute value is the distance between zero and the number on the number line.



- The absolute value is always positive or zero, never negative.
- The symbol for absolute value of a number represented by n is $|n|$.
- Positive 4 and negative 4 have the same absolute value.

$$|4| = 4 \quad \text{and} \quad |-4| = 4$$

- Properties of absolute value are (x and y represent numbers):

$$|x| \geq 0$$

$$|x - y| = |y - x|$$

$$|x| |y| = |xy|$$

$$|x + y| \leq |x| + |y|$$

(See Section 1.19 for description of \leq and \geq .)

1.15. Prime numbers

A prime number is a number that can only be divided evenly (not producing a remainder) by itself and by 1.

- For example, 7 can only be divided evenly by 7 and by 1.
- Examples of prime numbers are:
 $\{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
- Zero and 1 are *not* prime numbers.
- The only even prime number is 2.

1.16. Rational vs. irrational numbers

In this section, rational numbers and irrational numbers are defined.

- A number is a *rational number* if it can be expressed in the form of a fraction, x/y , and the denominator is not zero.
- For example, 2 is a rational number because it can be expressed as the fraction $2/1$.
- Every integer can be expressed as a fraction and is a rational number.

(Integer)/1 or $n/1$

Where n = any integer.

- Whole numbers are included in the set of integers, and whole numbers are rational numbers.
- A number is an *irrational number* if it is not a rational number and therefore cannot be expressed in the form of a fraction.
- Examples of irrational numbers are numbers that possess endless non-repeating digits to the right of the decimal point, such as,

$$\pi = 3.1415\dots, \text{ and } \sqrt{2} = 1.414\dots$$

1.17. Complex numbers

In this section, complex numbers, real numbers and imaginary numbers are defined, and addition, subtraction, multiplication and division of complex numbers are described.

- *Complex numbers* are numbers involving $\sqrt{-1}$, (see chapter on roots and radicals for a description of $\sqrt{}$). There is no number that when squared equals -1. By definition $(\sqrt{-1})^2$ should equal -1, but it does not, therefore the symbol i is introduced, such that:

$$\sqrt{-x} = i\sqrt{x} \quad \text{Where } x \text{ is a positive number and } (i)^2 = -1.$$

Basic Math and Pre-Algebra

- For example, evaluate $(\sqrt{-5})^2$.

$$(\sqrt{-5})^2 = (i\sqrt{5})(i\sqrt{5}) = (i)^2 (\sqrt{5})(\sqrt{5})$$

$$= (i)^2 \sqrt{(5)(5)} = (-1)(5) = -5$$

- Complex numbers are numbers involving i and are generally in the form:

$x + iy$ Where x and y are *real numbers*.

- In the expression, $x + iy$, x is called the real part and iy is called the imaginary part.

- A real number multiplied by i forms an *imaginary number*.

$$(\text{real number})(i) = \text{imaginary number}$$

- A real number added to an imaginary number forms a *complex number*.

$$(\text{real number}) + (\text{real number})(i) = \text{complex number}$$

- To add or subtract complex numbers, add or subtract the real parts and the imaginary parts separately.

- Example: Add $(5 + 4i) + (3 + 2i)$.

$$(5 + 4i) + (3 + 2i) = (5 + 3) + (4i + 2i) = 8 + 6i$$

- Example: Subtract $(5 + 4i) - (3 + 2i)$.

$$(5 + 4i) - (3 + 2i) = (5 - 3) + (4i - 2i) = 2 + 2i$$

- Complex numbers are multiplied as ordinary binomials, and $(i)^2$ is replaced by -1 . (See Chapter 5, “Polynomials,” Section 5.4 for multiplication of polynomials in the second book of the *Master Math* series entitled *Algebra*.) To multiply binomials, each term in the first binomial is multiplied by each term in the second binomial, and like terms are combined (added).

- Example: Multiply $(5 + 4i) \times (3 + 2i)$.

$$(5 + 4i)(3 + 2i)$$

$$= (5)(3) + (5)(2i) + (4i)(3) + (4i)(2i)$$

$$= 15 + 10i + 12i + 8(i)^2 = 15 + 22i + 8(-1)$$

$$= 15 + 22i - 8 = (15 - 8) + 22i = 7 + 22i$$

- To divide complex numbers, first multiply the numerator and denominator by what is called the *complex conjugate* of the denominator. The complex conjugate of $(3 + 2i)$ is $(3 - 2i)$, and the complex conjugate of $(3 - 2i)$ is $(3 + 2i)$. The product of a complex number and its conjugate is a real number. Remember to replace $(i)^2$ by -1 . (See chapters 5 entitled “Polynomials” and 6 entitled “Algebraic Fractions with Polynomial Fractions” for division and multiplication of polynomials and polynomial fractions in the second book of the *Master Math* series entitled *Algebra*.)

Basic Math and Pre-Algebra

• **Example:** Divide $(5 + 4i) \div (3 + 2i)$, (note that $(3 - 2i)$ is the complex conjugate of the denominator).

$$\begin{aligned}\frac{(5 + 4i)}{(3 + 2i)} &= \frac{(5 + 4i)(3 - 2i)}{(3 + 2i)(3 - 2i)} \\&= \frac{(5)(3) + (5)(-2i) + (4i)(3) + (4i)(-2i)}{(3)(3) + (3)(-2i) + (2i)(3) + (2i)(-2i)} \\&= \frac{15 + -10i + 12i + -8i^2}{9 + -6i + 6i + -4i^2} \\&= \frac{(15 + -8(i)^2) + (-10i + 12i)}{(9 + -4(i)^2) + (-6i + 6i)} \\&= \frac{(15 + -8(-1)) + 2i}{(9 + -4(-1)) + 0} = \frac{15 + 8 + 2i}{9 + 4 + 0} \\&= \frac{23 + 2i}{13} = \frac{23}{13} + \frac{2i}{13}\end{aligned}$$

Note that the answer is not in the form $(23+2i)/(13)$ because this is a complex number.

1.18. Rounding, truncating and estimating numbers

Quickly estimating the answer to a problem can be achieved by rounding the numbers that are to be added, subtracted, multiplied or divided, then adding, subtracting, multiplying or dividing the rounded numbers. Numbers can be rounded to the nearest ten, hundred, thousand, million, etc., depending on their size.

- For example, estimate the sum of $(38 + 43)$ by rounding.

Round 38 to the nearest ten. 40

Round 43 to the nearest ten. 40

The estimated sum is $40 + 40 = 80$

Compare the estimate with the actual sum.

$$38 + 43 = 81$$

80 is a good estimate of the actual value, 81.

- To round a number to the nearest ten, hundred, thousand, etc., the last retained digit should either be increased by one or left unchanged according to the following rules:

If the left most digit to be dropped is less than 5, leave the last retained digit unchanged.

If the left most digit to be dropped is greater than 5, increase the last retained digit by one.

Basic Math and Pre-Algebra

If the left most digit to be dropped is equal to 5, leave the last retained digit unchanged if it is even or increase the last retained digit by one if it is odd.

- **Example:** Round the following numbers to the nearest ten.

11 rounds to 10

65 rounds to 60

538 rounds to 540

65,236 rounds to 65,240

- **Example:** Round the following numbers to the nearest hundred.

238 rounds to 200

650 rounds to 600

9,436 rounds to 9,400

750 rounds to 800

740 rounds to 700

- **Example:** Round the following numbers to the nearest tenth. (See Chapter 3, "Decimals.")

5.01 rounds to 5.0

8.38 rounds to 8.4

9.55 rounds to 9.6

9.65 rounds to 9.6

Numbers and Their Operations

- **Example:** Estimate the difference of $(956 - 838)$ by rounding to the nearest hundred and to the nearest ten.

Round to the nearest hundred.

956 rounds to 1,000

838 rounds to 800

Subtract rounded numbers. $1,000 - 800 = 200$

Round to the nearest ten.

956 rounds to 960

838 rounds to 840

Subtract rounded numbers. $960 - 840 = 120$

Compare with the actual values. $956 - 838 = 118$

Rounding to the nearest hundred did not produce as good of an estimate as rounding to the nearest ten.

- **Example:** Estimate the product of (54×2) by rounding.

Round 54 to 50

Multiply rounded numbers. $50 \times 2 = 100$

Compare with the actual numbers. $54 \times 2 = 108$

- **Example:** Estimate the quotient of $(43 \div 2)$ by rounding.

Round 43 to 40

Divide rounded numbers. $40 \div 2 = 20$

Compare with the actual numbers. $43 \div 2 = 21.5$

Basic Math and Pre-Algebra

- **Example:** Estimate the sum of $(56 + 68 + 43)$ by rounding.

Round each number and add the estimate.

$$60 + 70 + 40 = 170$$

Compare with the actual numbers.

$$56 + 68 + 43 = 167$$

- Truncating rounds down to the smaller ten, hundred, thousand, etc. For example,

43 truncates to 40

768 truncates to 760

1.19. Inequalities, $>$, $<$, \geq , \leq

Inequalities are represented by the symbols for greater than and less than, and describe expressions in which the value on one side of the symbol is greater than the value on the other side of the symbol.

- The symbol for greater than is $>$
- The symbol for less than is $<$
- The symbol for greater than or equal to is \geq
- The symbol for less than or equal to is \leq
- If a and b are numbers,

$a < b$ represents a is less than b , or a is to the left of b on the number line.

Numbers and Their Operations

$a > b$ represents a is greater than b , or a is to the right of b on the number line.

$a \leq b$ represents a is less than or equal to b .

$a \geq b$ represents a is greater than or equal to b .

$a < c < b$ represents a is less than c and c is less than b .

$a > c > b$ represents a is greater than c and c is greater than b .

- **Examples:**

$$2 < 4$$

$$4 > 2$$

$$5 < 8$$

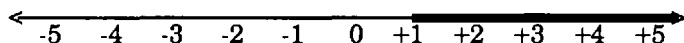
$$8 > 5$$

$$2 < 5 < 8$$

$$5 \leq 8$$

$$5 \leq 5$$

- The number line below describes $x > 1$.



- If c is a positive number, then $c > 0$. For example, if $c = 5$, then $5 > 0$.

- If d is a negative number, then $d < 0$. For example, if $d = -5$, then $-5 < 0$.

- **Example:** If a , b and c represent numbers, and

If $a < b$

Then $a + c < b + c$

Also $a - c < b - c$

Basic Math and Pre-Algebra

- Example: If $a = 2$, $b = 3$ and $c = 4$, and

Because $2 < 3$

Then $2 + 4 < 3 + 4$

Equivalently $6 < 7$

Also $2 - 4 < 3 - 4$

Equivalently $-2 < -1$

(Remember -2 is to the left of -1 on the number line.)

- Example: If a , b and c represent numbers, and

If $a > b$

And if $c > 0$

Then $a + c > b + c$

Also $a - c > b - c$

If $a < b$

And if $c > 0$

Then $ac < bc$

If $a > b$

And if $c > 0$

Then $ac > bc$

- Example: If $a = 2$, $b = 3$ and $c = 4$, and

Because $2 < 3$

Then $(2)(4) < (3)(4)$

Equivalently $8 < 12$

- Multiplying or dividing by a negative number causes the inequality sign to reverse.

- Example: If a , b and d represent numbers, and

If $d < 0$

And if $a < b$

Then $ad > bd$

(If a negative number is multiplied, the inequality reverses.)

- Example: If $a = 2$, $b = 3$ and $d = -4$, and

Because $-4 < 0$

And $2 < 3$

Then $(2)(-4) > (3)(-4)$

Equivalently $-8 > -12$

(Remember, -12 is to the left of -8 on the number line.)

- Example: If a , b and d represent numbers, and

If $d < 0$

And if $a > b$

Then $a + d < b + d$

(If a negative number is divided, the inequality reverses.)

- Example: If $a = 4$, $b = 6$ and $d = -2$, and

Because $-2 < 0$

And $4 < 6$

Then $4 + -2 > 6 + -2$

Equivalently $-2 > -3$

(Remember, -2 is to the right of -3 on the number line.)

- In summary, for an inequality:

If a number is added to both sides, the inequality remains unchanged.

If a number is subtracted from both sides, the inequality remains unchanged.

If a positive number is multiplied, the inequality sign remains unchanged.

If a positive number is divided, the inequality sign remains unchanged.

If a negative number is multiplied, the inequality sign reverses.

If a negative number is divided, the inequality sign reverses.

1.20. Factorial

The factorial of a positive integer is the product of that integer and each consecutive positive integer from one to that number.

- The symbol for factorial is “!” (the explanation point).

- Examples of the factorial of numbers are:

$$5! = 1 * 2 * 3 * 4 * 5 = 120$$

$$7! = 1 * 2 * 3 * 4 * 5 * 6 * 7 = 5,040$$

$$3! = 1 * 2 * 3 = 6$$

$$n! = 1 * 2 * 3 * \dots n$$

$$0! = 1 \text{ by definition.}$$

Chapter 2

Fractions

2.1 Definitions

2.2 Multiplying fractions

2.3 Adding and subtracting fractions with common denominators

2.4 Adding and subtracting fractions with different denominators

2.5 Dividing fractions

2.6 Reducing fractions

2.7 Complex fractions, mixed numbers and improper fractions

2.8 Adding and subtracting mixed numbers

2.9 Comparing fractions: Which is larger or smaller?

2.1. Definitions

A fraction is a number that is expressed in the form a/b or $\frac{a}{b}$.

- A fraction can also be defined as:

Basic Math and Pre-Algebra

$\frac{\text{part}}{\text{whole}}$, $\frac{\text{section}}{\text{whole}}$ or $\frac{\text{some}}{\text{all}}$

- If a pie is cut into 8 pieces, then 2 pieces are a fraction of the whole pie.

$$\frac{2 \text{ pieces}}{\text{whole pie}} = \frac{2 \text{ pieces}}{8 \text{ pieces in whole pie}}$$

- The top number is the *numerator*, and the bottom number is the *denominator*.

$$\frac{\text{Numerator}}{\text{Denominator}} = \text{Numerator} \div \text{Denominator}$$

- Fractions express division. $\frac{5}{8} = 5 \div 8$
- Equivalent fractions are fractions that have equal value. The following are equivalent fractions:

$$1/2 = 2/4 = 4/8 = 8/16 = 16/32 = 32/64$$

$$2/3 = 4/6 = 8/12 = 16/24 = 32/48$$

$$3/5 = 6/10 = 12/20 = 24/40 = 48/80$$

2.2. Multiplying fractions

To multiply fractions, multiply the numerators, multiply the denominators and place the product of the numerators over the product of the denominators.

- For example,

$$\frac{9}{2} \times \frac{5}{8} \times \frac{3}{4} = \frac{9 \times 5 \times 3}{2 \times 8 \times 4} = \frac{135}{64}$$

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12} = \frac{1}{2}$$

- If the numerators are less than the denominators, then after multiplying, the value of the product will be less than the values of the original fractions. This is true because multiplying a number or fraction by a fraction is equivalent to taking a fraction of the first number or fraction.

- For example: If you have a piece of a pie (which means you have a fraction of a pie), then you eat a fraction of the piece, you have eaten less than a piece. In other words, you have eaten a fraction of a fraction.

2.3. Adding and subtracting fractions with common denominators

To add or subtract two or more fractions that have the same denominators, simply add or subtract the numerators and place the sum or difference over the *common denominator*.

• Examples:

$$\frac{9}{8} + \frac{5}{8} + \frac{3}{8} = \frac{9 + 5 + 3}{8} = \frac{17}{8}$$

$$\frac{9}{8} - \frac{5}{8} - \frac{3}{8} = \frac{9 - 5 - 3}{8} = \frac{1}{8}$$

2.4. Adding and subtracting fractions with different denominators

To add or subtract two or more fractions that have different denominators, a common denominator must be found. Then, each original fraction must be multiplied by multiplying-fractions that have their numerator equal to their denominator, so that new equivalent fractions are created that have the same common denominator. These fractions can then be added or subtracted as shown above. The procedure is:

1. Find a common denominator for two or more fractions by calculating multiples of each denominator until one number is obtained that is a multiple of each denominator. This is called a *common multiple*. For example, if there are two denominators, 4 and 6, the multiples of 4 are: 4, 8, 12, 16, ..., and the multiples of 6 are: 6, 12, 18, 24, ... The smallest number that is a multiple of both 4 and 6 is 12. Therefore, 12 is called the lowest common multiple and it is also the *lowest common denominator* for 4 and 6.

2. After the lowest common denominator is found, transform the original fractions into fractions with common denominators so that they can be added or subtracted. To do this, multiply each original fraction by a *multiplying-fraction* that has its numerator equal to its denominator, so as to create *new equivalent fractions* that have the same common denominators. By having the numerator equal to the denominator in these multiplying-fractions, the value of each of the original fractions remains unchanged. For example, $(1/2) \times (2/2) = (2/4)$, where $1/2$ is the original fraction, $2/2$ is the multiplying-fraction and $2/4$ is the resulting equivalent fraction with its value equal to $1/2$.

To determine each multiplying-fraction, compare the new common denominator with the original denominators of each fraction, then create multiplying-fractions for each original fraction such that when the original denominator is multiplied with the multiplying-fraction's denominator, the resulting denominator is the common denominator.

3. After the equivalent fractions with their common denominators have been obtained, add or subtract the numerators and place the sum or difference over the common denominator.
4. Reduce the resulting fraction if it is possible, (see section below on reducing fractions).

Basic Math and Pre-Algebra

- For example, add $1/4 + 1/6 = ?$.

First, find the lowest common denominator.

Multiples of 4: 4, 8, 12, 16, ...

Multiples of 6: 6, 12, 18, ...

The lowest common multiple and lowest common denominator is 12.

Next, multiply each original fraction by a multiplying-fraction (with its numerator equal to its denominator) such that each equivalent fraction will have a denominator of 12.

For the first fraction, because $4 \times 3 = 12$,

$$1/4 + ?? = ?/12$$

$$1/4 \times 3/3 = 3/12$$

$3/12$ is equivalent to $1/4$.

For the second fraction, because $6 \times 2 = 12$,

$$1/6 + ?? = ?/12$$

$$1/6 \times 2/2 = 2/12$$

$2/12$ is equivalent to $1/6$.

After the equivalent fractions with their common denominators have been obtained, add the numerators and place the sum over the common denominator.

$$3/12 + 2/12 = 5/12$$

- Example: Add

$$\frac{9}{2} + \frac{5}{5} + \frac{3}{10} = \frac{?}{?}$$

First, find the lowest common denominator.

Multiples of 2: 2, 4, 6, 8, 10, ...

Multiples of 5: 5, 10, 15, 20, ...

Multiples of 10: 10, 20, ...

The lowest common multiple is 10, therefore the lowest common denominator is 10. (Notice that the smallest number that 2, 5, and 10 will all divide into is 10.) (Also, the factors of 10 are: (2)(5) and (1)(10).)

Next, multiply each original fraction by a multiplying-fraction (with its numerator equal to its denominator) such that each equivalent fraction will have a denominator of 10.

For the first fraction, because $2 \times 5 = 10$:

$$\frac{9}{2} \times \frac{5}{5} = \frac{45}{10}$$

For the second fraction, because $5 \times 2 = 10$:

$$\frac{5}{5} \times \frac{2}{2} = \frac{10}{10}$$

For the third fraction, because $10 \times 1 = 10$:

Basic Math and Pre-Algebra

$$\frac{3}{10} \times \frac{1}{1} = \frac{3}{10}$$

Next, add the new equivalent fractions with their common denominators (add the numerators and place the sum over the common denominator).

$$\frac{45}{10} + \frac{10}{10} + \frac{3}{10} = \frac{45 + 10 + 3}{10} = \frac{58}{10} = 29/5$$

• Example: Subtract $9/2 - 5/8 - 3/4$.

$$\frac{9}{2} - \frac{5}{8} - \frac{3}{4} = \frac{?}{?}$$

Find the lowest common denominator.

Multiples of 2: 2, 4, 6, 8, 10, ...

Multiples of 8: 8, 16, ...

Multiples of 4: 4, 8, 12, ...

The lowest common multiple is 8, therefore the lowest common denominator is 8.

To find the multiplying-fractions, consider the denominators:

$$2 \times 4 = 8$$

$$8 \times 1 = 8$$

$$4 \times 2 = 8$$

The multiplying-fractions are:

$$\frac{4}{4}, \frac{1}{1}, \text{ and } \frac{2}{2}$$

Therefore, the equation is:

$$\frac{9 \times 4}{2 \times 4} - \frac{5 \times 1}{8 \times 1} - \frac{3 \times 2}{4 \times 2} =$$

$$\frac{36}{8} - \frac{5}{8} - \frac{6}{8} = \frac{36 - 5 - 6}{8} = \frac{25}{8}$$

• An alternative method for adding and subtracting two fractions is as follows:

1. Multiply the two denominators to find a common denominator. The result will be a common multiple of each, but not necessarily the lowest common multiple.

Find the new numerator of the first fraction by multiplying the numerator of the first fraction with the original denominator of the second fraction.

Find the new numerator of the second fraction by multiplying the numerator of the second fraction with the original denominator of the first fraction.

2. Then, add or subtract the new numerators and place the result over the common denominator.
3. Reduce the resulting fraction if it is possible (see section below on reducing fractions).

Basic Math and Pre-Algebra

- For example, add $1/2 + 3/4$.

Multiply denominators. $2 \times 4 = 8$

The new common denominator is 8.

The first new numerator is $1 \times 4 = 4$.

The second new numerator is $3 \times 2 = 6$.

The new equivalent fractions are:

$$\frac{4}{8} + \frac{6}{8} = \frac{10}{8}$$

To reduce $10/8$, divide 2 into both the numerator and the denominator.

$$\frac{10 / 2}{8 / 2} = \frac{5}{4}$$

2.5. Dividing fractions

To divide fractions, turn the second fraction upside down to invert the numerator and denominator. (An inverted fraction is called a reciprocal.) Then, multiply the first fraction with the reciprocal fraction.

$$\frac{3}{4} \div \frac{5}{8} = \frac{3}{4} \times \frac{8}{5} = \frac{24}{20} = \frac{6}{5}$$

Where $8/5$ is the reciprocal of $5/8$.

- Multiplying the reciprocal is equivalent to dividing because:

Fractions

$$\frac{3}{4} \div \frac{5}{8} = \frac{3 / 4}{5 / 8} = \frac{3}{4 \times 5 / 8} = \frac{3 \times 8}{4 \times 5}$$

$$\frac{a}{b / c} = \frac{a \times c}{b} \quad a, b \text{ and } c \text{ represent numbers.}$$

$$\frac{a / d}{b / c} = \frac{a \times c}{b \times d} \quad a, b, c \text{ and } d \text{ represent numbers.}$$

2.6. Reducing fractions

Reducing fractions is an important step in the process of problem-solving, and can be performed before, during or after the primary operations are completed. After fractions have been added, subtracted, multiplied or divided, the resulting fractions may be large. Large fractions can often be reduced by dividing-out or canceling factors that are contained in both the numerator and denominator.

- For example, reduce $25/10$.

Factor 25 and 10.

$$25 = 5 \times 5$$

$$10 = 5 \times 2$$

There is a factor of 5 contained in both the numerator and denominator that can be canceled.

$$\frac{25}{10} = \frac{5 \times 5}{2 \times 5} = \frac{5}{2}$$

- Example: Reduce 30/40.

Write the fraction in factored form and cancel the factors common to both the numerator and denominator.

$$\frac{30}{40} = \frac{3 \times 2 \times 5}{2 \times 2 \times 2 \times 5} = \frac{3}{4}$$

Note that by inspection a 10 could have been canceled from both the numerator and the denominator.

2.7. Complex fractions, mixed numbers and improper fractions

In this section, improper fractions, mixed numbers and complex fractions are defined, and methods for converting improper fractions to mixed numbers and converting mixed numbers to improper fractions are described.

- Every integer can be expressed as a fraction. For example, $6 = 6/1$, $23 = 23/1$.
- *Improper fractions* are fractions with their numerators larger than their denominators. Examples of improper fractions are:

$$\frac{13}{2}, \frac{6}{1}, \frac{12}{2}$$

- If a value is represented by an integer and a fraction, it is called a *mixed number*. A mixed number always has an integer and a fraction. The following are examples of mixed numbers:

$$6\frac{1}{2}, \quad 25\frac{3}{4}$$

- An improper fraction can be expressed as a mixed number by dividing the numerator with the denominator. For example:

$$\frac{4}{3} = 1\frac{1}{3}$$

Where $\frac{4}{3}$ is the improper fraction and $1\frac{1}{3}$ is the mixed number.

- When performing calculations, it is generally easier to work with improper fractions than with mixed numbers.
- Mixed numbers are easily converted to improper fractions. Following are two methods used to convert mixed numbers into improper fractions.

Method 1

1. Multiply the integer and the denominator.
2. Add the numerator to the result, which results in a new numerator.

Basic Math and Pre-Algebra

3. Place the new numerator over the original denominator.

- For example, convert $6 \frac{1}{2}$ to an improper fraction.

Multiply the integer and the denominator. $(6)(2) = 12$
Add the numerator to the result to obtain the new numerator. $1 + 12 = 13$

Place the new numerator over the denominator to obtain the improper fraction. $13/2$

Method 2

1. Find the common denominator.

2. Add the fractions.

- For example, convert $6 \frac{1}{2}$ to an improper fraction.

$$6\frac{1}{2} = 6 + \frac{1}{2} = \frac{6}{1} + \frac{1}{2}$$

The lowest common denominator is 2. Create equivalent fractions with common denominators of 2 and add. ($1/2$ does not need to be multiplied.)

$$\frac{6}{1} \times \frac{2}{2} + \frac{1}{2} = \frac{12}{2} + \frac{1}{2} = \frac{13}{2}$$

Therefore, $6 \frac{1}{2} = 13/2$.

- Improper fractions are easily converted to mixed numbers by dividing the numerator with the denominator.

Fractions

- For example, convert $25/4$ to a mixed fraction. Divide the numerator by the denominator. $25 \div 4 = 6$ plus a remainder of 1. Place the remainder over the divisor resulting in $6 \frac{1}{4}$.

To check the result, multiply the integer and the denominator. $6 \times 4 = 24$

Add the numerator and place the result over the denominator. $24 + 1 = 25$

$25/4$ is the original improper fraction.

- A *complex fraction* has a fraction in the numerator or in the denominator or in both so that there is one or more fractions within a fraction.
- The following are examples of complex fractions (x and y represent numbers):

$$\frac{1/3}{4}, \frac{3}{4/5}, \frac{x+2/y}{5}, \frac{3}{x/y}$$

- Complex fractions can be simplified by performing the indicated division of the fractions or sub-fractions. For example, simplify the following (remember when dividing fractions, multiply the reciprocal).

$$\frac{1/3}{4} = \frac{1}{3} \div 4 = \frac{1}{3} \div \frac{4}{1} = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

$$\frac{3}{4/5} = \frac{3}{1} \div \frac{4}{5} = \frac{3}{1} \times \frac{5}{4} = \frac{15}{4}$$

2.8. Adding and subtracting mixed numbers

In this section, addition and subtraction of mixed numbers are described.

- To add or subtract mixed numbers:
 1. First convert each fraction into improper fractions.
 2. Find a common denominator.
 3. Add or subtract the numerators.
 4. Then place the result over the common denominator.
- For example, add $6 \frac{1}{2} + 25 \frac{3}{4}$.

Convert to improper fractions (see previous section for method).

Convert the first fraction.

$$6 \times 2 = 12$$

$$12 + 1 = 13$$

$$6 \frac{1}{2} = \frac{13}{2}$$

Convert the second fraction.

$$25 \times 4 = 100$$

$$100 + 3 = 103$$

$$25 \frac{3}{4} = \frac{103}{4}$$

Find a common denominator for the two improper fractions, $13/2$ and $103/4$.

4 is a multiple of both 2 and 4.

Multiply each fraction by a fraction with its numerator equal to its denominator such that the result is a common denominator of 4 in each fraction.

Add the numerators and place the result over the common denominator.

$$\frac{13 \times 2}{2 \times 2} + \frac{103 \times 1}{4 \times 1}$$
$$= \frac{26}{4} + \frac{103}{4} = \frac{26 + 103}{4} = \frac{129}{4}$$

Therefore, $6 \frac{1}{2} + 25 \frac{3}{4} = 129/4$.

2.9. Comparing fractions: Which is larger or smaller?

In this section, methods for comparing the value of fractions are described.

- Two fractions can be directly compared if they have the same denominator. For example, which of the following fractions is larger?

$3/4$ or $1/2$?

Basic Math and Pre-Algebra

Because 4 is a common multiple of both fractions, it is also a common denominator. To obtain a common denominator, multiply $\frac{3}{4}$ by $\frac{1}{1}$ and $\frac{1}{2}$ by $\frac{2}{2}$.

$$\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$$

$$\frac{3}{4} \times \frac{1}{1} = \frac{3}{4}$$

The two fractions become:

$\frac{3}{4}$ and $\frac{2}{4}$

Because $3 > 2$,

$$\frac{3}{4} > \frac{2}{4}$$

• To compare several fractions, rather than converting all of the fractions to the same denominator, it may be easier to compare two at a time. Which of the following are larger?

$\frac{1}{8}$ or $\frac{3}{4}$ or $\frac{5}{6}$?

First, compare:

$\frac{1}{8}$ and $\frac{3}{4}$

The common denominator is 8.

To obtain a common denominator, multiply $\frac{3}{4}$ by $\frac{2}{2}$.

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

The two fractions become:

Fractions

$\frac{6}{8}$ and $\frac{1}{8}$

Because $6 > 1$,

$\frac{6}{8} > \frac{1}{8}$

Therefore, $\frac{3}{4} > \frac{1}{8}$.

Next, compare:

$\frac{3}{4}$ and $\frac{5}{6}$

The common denominator is 12, because 12 is a multiple of both 4 and 6.

Multiply each fraction by a multiplying-fraction so that the new denominators will be 12.

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

$$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

Because $10 > 9$,

$\frac{10}{12} > \frac{9}{12}$

Therefore, $\frac{5}{6} > \frac{3}{4}$

Comparing all three fractions, we determined that:

$\frac{5}{6} > \frac{3}{4}$ and $\frac{3}{4} > \frac{1}{8}$

Therefore, $\frac{5}{6} > \frac{3}{4} > \frac{1}{8}$

Basic Math and Pre-Algebra

- To quickly assess whether one fraction is larger or smaller than another fraction, it may be useful to determine whether each fraction is larger or smaller than $1/2$.
- The smaller the numerator is in comparison to the denominator, the smaller will be the value of fraction.

$$5/10 > 5/100 > 5/1000 > 5/10,000 > 5/100,000 > 5/1,000,000$$

$58/59$ is almost 1.

$99/985,236$ is approximately one-ten thousandth.

Chapter 3

Decimals

3.1 Definitions

3.2 Adding and subtracting decimals

3.3 Multiplying decimals

3.4 Dividing decimals

3.5 Rounding decimals

3.6 Comparing the size of decimals

3.7 Decimals and money

3.1. Definitions

The decimal system and decimals are based on tenths or the number 10.

- The digits to the right of the decimal point are called *decimal fractions*.
- A decimal can be expressed in the form of a fraction, and a fraction can be expressed in the form of a decimal.

Basic Math and Pre-Algebra

- Decimals that do not have a digit to the left of the decimal point are written 0.95 or .95. Inserting the zero before the decimal point prevents the viewer from mistaking .95 for 95.

- The digits to the right of the decimal point correspond to tenths, hundredths, thousandths, ten thousandths, hundred thousandths, etc. For example, the digits in the number 48.35679 correspond to:

4 tens, 8 ones . 3 tenths, 5 hundredths, 6 thousandths, 7 ten thousandths, 9 hundred thousandths

- The decimal point is always present after the ones digit even though it may not be written. For example:

$$7 = 7.0 = 7.00 = 7.000 = 7.0000000$$

$$29 = 29.00 = 29.0 = 29.000000000$$

$$-36 = -36.0 = -36.00 = -36.00000$$

- The following are examples of the equivalent forms of decimals and fractions:

$$0.1 = 1/10$$

$$0.01 = 1/100$$

$$0.001 = 1/1,000$$

$$0.0001 = 1/10,000$$

$$0.00001 = 1/100,000$$

- Decimals have equivalent forms as fractions. For example, the decimal 0.5 is equal to the fraction 5/10. This can be proven by dividing the fraction 5/10 or $5 \div 10$.

Decimals

Using the long division format:

$$10 \overline{)5}$$

Insert the decimal point and a zero in the tenth position.

$$10 \overline{)5.0}$$

10 divides into 5.0, 0.5 times, then multiply $10 \times 0.5 = 5.0$.

$$\begin{array}{r} .5 \\ 10 \overline{)5.0} \\ \underline{5.0} \\ 00 \end{array}$$

Therefore, 0.5 is equal to $5/10$.

• In general, a fraction can be transformed into its decimal equivalent by dividing the numerator by the denominator. (See Section 3.4, "Dividing decimals.")

$$\begin{array}{r} 1/2 = 2 \overline{)1} = 2 \overline{)1.0} = 2 \overline{)1.0} \\ \underline{1.0} \\ 00 \end{array}$$

Therefore, $1/2 = 0.5$.

Basic Math and Pre-Algebra

• Proper names that correspond to decimal fractions are used to describe measurements or quantities in various units such as grams, moles, seconds, liters, meters, etc. For example, if a scientist is measuring extremely small amounts of a chemical in grams, proper names for the following decimal quantities are:

0.1	$= 10^{-1}$	= 100 milligrams
0.01	$= 10^{-2}$	= 10 milligrams
0.001	$= 10^{-3}$	= 1 milligram
0.0001	$= 10^{-4}$	= 100 micrograms
0.00001	$= 10^{-5}$	= 10 micrograms
0.000001	$= 10^{-6}$	= 1 microgram
0.000000001	$= 10^{-9}$	= 1 nanogram
0.0000000000001	$= 10^{-12}$	= 1 picogram
0.0000000000000001	$= 10^{-15}$	= 1 femtogram

3.2. Adding and subtracting decimals

To add or subtract decimals, align the decimal points, then proceed with the addition or subtraction (as if the decimal points are not there).

• To add decimals, arrange in column format, add each column beginning with the right column, and carry over digits to the next larger column as necessary.

Decimals

- Example: Add $389.32 + 65.2 + 2 + 0.05$.

Arrange in column format, fill in zeros, and add the hundredths' column.

$$\begin{array}{r} 389.32 \\ 65.20 \\ 2.00 \\ + .05 \\ \hline 7 \end{array}$$

Add the tenths' column.

$$\begin{array}{r} 389.32 \\ 65.20 \\ 2.00 \\ + .05 \\ \hline .57 \end{array}$$

Add the ones' column, and carry over the ten.

$$\begin{array}{r} 1 \\ 389.32 \\ 65.20 \\ 2.00 \\ + .05 \\ \hline 6.57 \end{array}$$

Add the tens' column, and carry over the hundred.

Basic Math and Pre-Algebra

$$\begin{array}{r} 11 \\ 389.32 \\ 65.20 \\ 2.00 \\ + \quad .05 \\ \hline 56.57 \end{array}$$

Add the hundreds' column.

$$\begin{array}{r} 11 \\ 389.32 \\ 65.20 \\ 2.00 \\ + \quad .05 \\ \hline 456.57 \end{array}$$

Therefore, $389.32 + 65.2 + 2 + 0.05 = 456.57$.

- To subtract decimals, arrange in column format, subtract each column beginning with the right column and borrow when necessary from the next larger column. If there are more than two numbers, subtract the first two, then subtract the third from the difference, and so on.

- Example: Subtract $23 - 3.89$.

Arrange in column format and subtract the hundredths' column.

$$\begin{array}{r} 23.00 \\ - 3.89 \\ \hline ? \end{array}$$

To borrow from the tenths' column, first borrow from the ones' column. Subtract the hundredths and tenths.

$$\begin{array}{r} 22.\{9\}\{10\} \\ - 3.89 \\ \hline .11 \end{array}$$

To subtract the ones' column, first borrow from the tens'. Subtract the ones and tens.

$$\begin{array}{r} \{1\}\{12\}.\{9\}\{10\} \\ - 3.89 \\ \hline 19.11 \end{array}$$

Therefore, $23 - 3.89 = 19.11$.

3.3. Multiplying decimals

To multiply decimals, ignore the decimal points and multiply the numbers, then place the decimal point in the product so that there are the same number of digits to the right of the decimal point in the product as there are in the all of the numbers that were multiplied combined.

Basic Math and Pre-Algebra

- For example, if 2.2 and 0.3 are multiplied, because there are two digits to the right of the decimal points in 2.2 and 0.3 (one for each number), there must be two digits to the right of the decimal point in the product. Therefore, $2.2 \times 0.3 = 0.66$.

- Example: Multiply 35.268 and 2.5.

There are a total number of four digits to the right of the decimal points in these two numbers, therefore the product will have four digits to the right of the decimal point.

Multiply the 5 in the multiplier with each number in the multiplicand beginning at the right.

$$\begin{array}{r} 35.268 \\ \times \quad 2.5 \\ \hline 176340 \end{array}$$

Multiply the 2 in the multiplier with each number in the multiplicand beginning at the right.

$$\begin{array}{r} 35.268 \\ \times \quad 2.5 \\ \hline 176340 \\ 70536 \\ \hline \end{array}$$

Add the partial products.

$$\begin{array}{r} 35.268 \\ \times \quad 2.5 \\ \hline 176340 \\ 70536 \\ \hline 881700 \end{array}$$

Place the decimal point four digits from the right.
Therefore, $35.268 \times 2.5 = 88.1700$.

3.4. Dividing decimals

To divide decimals, arrange the numbers into the long division format, move the decimal point in the divisor to the right until there are no digits to the right of the decimal point, move the decimal point in the dividend the same number of places to the right so that the overall value of the division is unchanged (zeros may be inserted in the dividend as required), then divide using long division. The decimal point will be placed in the quotient above where it moved to in the dividend.

- Example: $10 \div 0.5 = ?$

Arrange the numbers into the long division format.

Basic Math and Pre-Algebra

$$\begin{array}{r} ? \\ 0.5 \overline{)10} \end{array}$$

Move the decimal point in the divisor and the dividend to the right until there are no digits to the right of the decimal point in the divisor (insert zeros as required).

$$\begin{array}{r} ? \\ 5. \overline{)100.} \end{array}$$

Divide 5 into 10.

$$\begin{array}{r} 2? \\ 5. \overline{)100.} \\ \underline{10} \end{array}$$

Subtract $10 - 10 = 0$, and bring down the last 0.

$$\begin{array}{r} 2? \\ 5. \overline{)100.} \\ \underline{10} \\ 000 \end{array}$$

There is nothing for 5 to divide in to, so place a 0 above the last 0 in the dividend.

$$\begin{array}{r} 20. \\ 5. \overline{)100.} \\ \underline{10} \\ 000 \end{array}$$

Therefore, $10 \div 0.5 = 20$.

3.5. Rounding decimals

To round decimals, the last retained digit should either be increased by one or left unchanged according to the following rules:

If the left most digit to be dropped is less than 5, leave the last retained digit unchanged.

If the left most digit to be dropped is greater than 5, increase the last retained digit by one.

If the left most digit to be dropped is equal to 5, leave the last retained digit unchanged if it is even or increase the last retained digit by one if it is odd.

- For example, round the following to the nearest integer.

3.4 rounds to 3.

2.5 rounds to 2.

45.64 rounds to 46.

- Decimals may also be rounded to the nearest tenth, hundredth, thousandth, etc., depending on how many decimal places there are and the accuracy required.

- For example, round the following as specified.

45.689 rounded to the nearest tenth is 45.7.

1.9654 rounded to the nearest hundredth is 1.96.

1.545454 rounded to the nearest thousandth is 1.545.

Basic Math and Pre-Algebra

- When solving complex mathematical or engineering problems, it is important to retain the same number of “significant digits” in the intermediate and final results. The number of decimal places in the resulting numbers should not exceed the number of decimal places in the initial numbers because the resulting numbers cannot be known with greater accuracy than the original numbers. Therefore, depending on the least number of significant digits in the initial numbers, rounding intermediate and final results will be required to maintain the decimal places to ones, tenths, hundredths, thousandths, etc.
- For example, if 45.689 and 1.9654 are added, the accuracy of the result cannot be greater than three decimal places.

$$45.689 + 1.9654 = 47.6544 = 47.654$$

3.6. Comparing the size of decimals

To compare the value of two or more decimals to determine which decimal is a larger or smaller, the following procedure can be applied:

1. Place the decimals in a column.
2. Align the decimal points.
3. Fill in zeros to the right so that both decimals have the same number of digits to the right of the decimal point.

4. The larger decimal will have the largest digit in the greatest column (the farthest to the left).

- Example: Compare 0.00025 and 0.000098.

Place the decimals in a column, align the decimal points, and fill in zeros.

0.000250

0.000098

The 2 in the ten-thousandths place is greater than the 9 in the hundred-thousandths place. Therefore:

0.000250 is larger than 0.000098.

3.7. Decimals and money

There is a relationship between money and decimals because the money system is based on decimals. Dollars are represented by whole numbers and cents are represented by tenths and hundredths of a dollar.

- For example, $\$25.99 = 25$ dollars and 99 cents.
- The following are equivalent: Ninety-nine hundredths of a dollar, 99 cents, $99/100$ of a dollar, and 0.99 dollars.

Basic Math and Pre-Algebra

- Examples:

10 cents is one tenth or $1/10$ of a dollar or 0.1 dollars.

70 cents is seven tenths or $7/10$ of a dollar or 0.7 dollars.

1 cent is one hundredth or $1/100$ of a dollar or 0.01 dollars.

6 cents is six hundredths or $6/100$ of a dollar or 0.06 dollars.

- The amount, \$0.50 is $1/2$ of a dollar. Similarly, the decimal 0.5 equals the fraction $1/2$.

- The word cent refers to one hundred, (remember a *century* has 100 years), and in money, cents refers to the number of hundredths.

Chapter 4

Percentages

4.1 Definitions

4.2 Figuring out the percents of numbers

4.3 Adding, subtracting, multiplying and dividing percents

4.4 Percent increase and decrease (percent change)

4.5 Simple and compound interest

4.1. Definitions

Percent is defined as a rate or proportion per hundred, one one-hundredth, or $1/100$.

- The symbol for a percent is %.
- A percent is a form of a fraction with its denominator equal to 100. To remember that *percent* is per one-hundred, think of a *century*, which has 100 years.
- A percent can be converted to a fraction or decimal. A percent can be reduced and manipulated like a fraction or decimal.

Basic Math and Pre-Algebra

- For example, seventy-five percent = $75\% = 75/100 = 0.75$.

- Examples of common percents are:

$$1/4 = 0.25 = 25\%$$

$$1/2 = 0.50 = 50\%$$

$$3/4 = 0.75 = 75\%$$

$$1/3 = 0.33... = 33\frac{1}{3}\%$$

$$2/3 = 0.66... = 66\frac{2}{3}\%$$

$$1/5 = 0.20 = 20\%$$

$$1/100 = 0.01 = 1\%$$

$$1/50 = 0.02 = 2\%$$

$$1/25 = 0.04 = 4\%$$

$$1/10 = 0.10 = 10\%$$

$$2/5 = 0.40 = 40\%$$

$$3/5 = 0.60 = 60\%$$

$$4/5 = 0.80 = 80\%$$

$$1 = 1.00 = 100\%$$

$$6/5 = 1.20 = 120\%$$

$$5/4 = 1.25 = 125\%$$

$$3/2 = 1.50 = 150\%$$

4.2. Figuring out the percents of numbers

Percents of a number can be determined using the definition of a percent and multiplying the number by the amount of the percent and $1/100$. The following examples represent percents of the number nine:

Percentages

$$1\% \text{ of } 9 = 1 \times \frac{1}{100} \text{ of } 9$$

$$= (1)\left(\frac{1}{100}\right)(9) = (9)\left(\frac{1}{100}\right) = \frac{9}{100} = 0.09$$

$$10\% \text{ of } 9 = 10 \times \frac{1}{100} \text{ of } 9$$

$$= (10)\left(\frac{1}{100}\right)(9) = (90)\left(\frac{1}{100}\right) = \frac{90}{100} = 0.9$$

$$100\% \text{ of } 9 = 100 \times \frac{1}{100} \text{ of } 9$$

$$= (100)\left(\frac{1}{100}\right)(9) = (900)\left(\frac{1}{100}\right) = \frac{900}{100} = 9$$

$$1,000\% \text{ of } 9 = 1,000 \times \frac{1}{100} \text{ of } 9$$

$$= (1,000)\left(\frac{1}{100}\right)(9) = (9,000)\left(\frac{1}{100}\right) = \frac{9,000}{100} = 90$$

Basic Math and Pre-Algebra

- Percents of a number can also be determined by moving the decimal point according to the percent amount. The following examples represent percents of the number nine:

1% of 9. Move the decimal to the left two digits. $= 0.09$

10% of 9. Move the decimal to the left one digit. $= 0.9$

100% of 9. The decimal does not move. $= 9$

1000% of 9. Move the decimal to the right one digit.
 $= 90$

10,000% of 9. Move the decimal to the right two digits.
 $= 900$

100,000% of 9. Move the decimal to the right three digits. $= 9,000$

- The following are examples of percents of numbers:

What is 6 percent of 30?

$$6 \times \frac{1}{100} \text{ of } 30 = 6 \times \frac{1}{100} \times 30$$

$$= \frac{6}{100} \times 30 = \frac{180}{100} = 1.80$$

What is 5 percent of 20?

$$5 \times \frac{1}{100} \text{ of } 20 = 5 \times \frac{1}{100} \times 20 = \frac{100}{100} = 1.00$$

What is 20 percent of 80?

$$20 \times \frac{1}{100} \text{ of } 80 = 20 \times \frac{1}{100} \times 80 = \frac{1600}{100} = 16$$

- The examples above can also be solved using an alternative method.

What is 6 percent of 30?

Because 1% of 30 is 0.30, and 6% is 6 times 1%,
then $0.30 \times 6 = 1.80$.

What is 5 percent of 20?

Because 10% of 20 is 2, and 5% is half of 10%,
then half of 2 is 1.

What is 20 percent of 80?

Because 10% of 80 is 8, and 20% is 2 times 10%,
then $8 \times 2 = 16$.

4.3. Adding, subtracting, multiplying and dividing percents

To add or subtract percents, simply add or subtract as with integers but keep track of the percent. For example, add or subtract the following:

$$26\% + 4\% = 30\%$$

$$158\% - 6\% = 152\%$$

$$3\% + 16\% + 4\% = 23\%$$

- For example, if you have 50% of your assets in stocks and 20% of your assets in bonds, then 70% of your assets are in stocks and bonds.

$$50\% \text{ in stocks} + 20\% \text{ in bonds} = 70\% \text{ in stocks and bonds}$$

Basic Math and Pre-Algebra

- To multiply and divide percents:
 1. Convert percents to decimal form by dividing by 100 (move decimal two places to left).
 2. Multiply or divide the decimals (as described in Chapter 3).
 3. Convert back to percent by multiplying by 100 (move decimal point two places to right).

- Multiply or divide the following examples:

$$25\% \times 5\% = 0.25 \times 0.05 = 0.0125$$

$$\text{Convert to \%: } 0.0125 \times 100 = 1.25\%$$

$$75\% \div 100\% = 0.75 \div 1.00 = 0.75$$

$$\text{Convert to \%: } 0.75 \times 100 = 75\%$$

$$80\% \div 25\% = 0.8/0.25 = 3.2$$

$$\text{Convert to \%: } 3.2 \times 100 = 320\%$$

$$6\% \text{ of } 30 = 6\% \times 30 = 0.06 \times 30 = 1.80$$

$$5\% \text{ of } 20 = 5\% \times 20 = 0.05 \times 20 = 1$$

$$20\% \text{ of } 80 = 20\% \times 80 = 0.2 \times 80 = 16$$

$$20\% \text{ of } 70\% \text{ of something} = 0.2 \times 0.7 = 0.14 = 14\% \text{ of something}$$

$$5 \text{ is what percent of } 20? \quad 5/20 = 0.25 = 25\%$$

4.4. Percent increase and decrease (percent change)

In this section, percent change and discount are discussed.

- To determine the percent that a number has increased or decreased, the following equation is used:

$$\frac{\text{amount of increase or decrease (change)}}{\text{original amount}} = \frac{?}{100}$$

This equation can also be written:

$$\frac{\text{amount of increase or decrease (change)}}{\text{original amount}} (100) = \text{percent change}$$

- **Example:** If the price of a house is discounted from \$250,000 to \$200,000, what is the percent decrease or discount in the price?

$$\frac{\$250,000 - \$200,000}{\$250,000} (100) = \frac{\$50,000}{\$250,000} (100)$$

$$= (0.2)(100) = 20\%$$

Basic Math and Pre-Algebra

• **Example:** If the owners of a house are willing to discount the price of \$150,000 by 6%, how much will the house cost? (See Chapter 3, “Solving Simple Algebraic Equations,” in *Algebra*, the second book of the *Master Math* series, for explanations of techniques used below.)

$$\frac{\$150,000 - \text{discount price}}{\$150,000} = \frac{6\%}{100}$$

$$\frac{\$150,000 - \text{discount price}}{\$150,000} = 0.06$$

Multiply both sides by \$150,000.

$$\$150,000 - \text{discount price} = (0.06)(\$150,000)$$

$$\$150,000 - \text{discount price} = \$9,000$$

Subtract \$9,000 from both sides, and add discount price to both sides.

$$\$150,000 - \$9,000 = \text{discount price}$$

$$\$141,000 = \text{discount price}$$

To verify the result, use the discount price and calculate the percent discount given (6%).

$$\frac{\$150,000 - \$141,000}{\$150,000} (100) = ? = 6\%$$

4.5. Simple and compound interest

Simple interest is generally computed annually, therefore the time it will take to earn a certain percent on money invested in a simple interest account is a year.

- **Example:** How much simple interest will \$5,000 earn in a year and in 6 months at a rate of 4%?

$\$5,000 \times 4\% = (\$5,000)(0.04) = \$200$ will be earned in a year.

In the first 6 months, \$5,000 will earn:

$$\$200 \times (6 \text{ months}) / (12 \text{ months}) = \$200 \times 1/2 = \$100$$

Therefore, in 6 months the \$5,000 becomes \$5,100 and in one year the \$5,000 becomes \$5,200.

- *Compound interest* is compounded periodically during the year. To determine the compound interest, divide the interest rate by the number of times it is compounded in the year and apply it during each period.

- For example, if the interest rate is 4% and it is compounded semiannually, then 2% is applied to the principle every 6 months. If 4% is compounded quarterly then 1% is applied to the principle every 3 months or four times a year.

Basic Math and Pre-Algebra

- Example: How much interest will \$5,000 earn in six months and in a year at 4% interest compounded semi-annually?

Divide the interest by the number of times it is compounded. $4\% \div 2 = 2\%$

Apply 2% interest for the first 6-month period.
 $\$5,000 \times 2\% = (\$5,000)(0.02) = \$100$ earned in 6 months.

The second 6-month period begins with \$5,100.

Apply 2% interest again for the second 6-month period.
 $\$5,100 \times 2\% = (\$5,100)(0.02) = \$102$ earned in the second 6-month period.

In 6 months the \$5,000 becomes \$5,100
And in one year the \$5,000 becomes \$5,202

- Note that compound interest earns slightly more than simple interest for the same rate.

Chapter 5

Converting Percentages, Fractions and Decimals

- 5.1 Converting fractions to percents
- 5.2 Converting percents to fractions
- 5.3 Converting fractions to decimals
- 5.4 Converting decimals to fractions
- 5.5 Converting percents to decimals
- 5.6 Converting decimals to percents

5.1. Converting fractions to percents

To convert a fraction to a percent, divide the numerator by the denominator, then multiply by 100.

- For example, convert $\frac{3}{5}$ into a percent.

Divide the numerator by the denominator using long division.

$$\begin{array}{r} 5 \overline{)3.0} \quad ? \\ 5 \overline{)3.0} \quad .6 \\ \underline{3.0} \\ 0.0 \end{array}$$

Multiply by 100. $0.6 \times 100 = 60\%$

Therefore, $3/5 = 60\%$.

- A fraction can also be converted to a percent by determining an equivalent form of the fraction that has a denominator of 100.
- For example, convert $3/5$ into a percent.

$$\frac{3}{5} = \frac{6}{10} = \frac{60}{100} = 60\%$$

5.2. Converting percents to fractions

To convert a percent to a fraction, divide by 100.

- For example, convert 60% into a fraction.

Divide by 100.

$$60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

Therefore, $60\% = 3/5$

Basic Math and Pre-Algebra

$$0.02 = \frac{2}{100} = \frac{1}{50}$$

$$0.5 = \frac{5}{10} = \frac{1}{2}$$

$$0.6 = \frac{6}{10} = \frac{3}{5}$$

$$0.0056 = \frac{56}{10,000} = \frac{7}{1,250}$$

$$0.446 = \frac{446}{1,000} = \frac{223}{500}$$

To check the results, divide each fraction.

- Remember, 0.6 is 6 tenths, 0.06 is 6 hundredths, and so on.

- Consider the following decimals in their fractional form.

$$0.1 = 1/10$$

$$0.01 = 1/100$$

$$0.001 = 1/1,000$$

$$0.0001 = 1/10,000$$

$$0.00001 = 1/100,000$$

$$0.000001 = 1/1,000,000$$

5.5. Converting percents to decimals

To convert a percent to a decimal, divide by 100.

- Note that dividing by 100 is equivalent to moving the decimal point two places to the left.

- Example: Convert 25% to its decimal form.

Divide 25 by 100.

$$25\% = \frac{25}{100} = 0.25$$

- The above example is verified using long division.

$$25\% = \frac{25}{100} = 25 \div 100 = ?$$

$$100 \overline{) 25}^?$$

Insert the decimal point and a zero after 25, divide 100 into 25.0, multiply the result by 100, and subtract.

$$\begin{array}{r} .2 \\ 100 \overline{) 25.0} \\ \underline{20.0} \end{array}$$

Basic Math and Pre-Algebra

$$\begin{array}{r} .2 \\ 100 \overline{) 25.0} \\ \underline{20.0} \\ 05.0 \end{array}$$

Insert another zero, divide 100 into 5.00, multiply the result by 100, and subtract.

$$\begin{array}{r} .25 \\ 100 \overline{) 25.00} \\ \underline{20.0} \\ 05.00 \\ \underline{05.00} \end{array}$$

$$\begin{array}{r} .25 \\ 100 \overline{) 25.00} \\ \underline{20.0} \\ 05.00 \\ \underline{05.00} \\ 00.00 \end{array}$$

Therefore, 25% is equivalent to 0.25.

5.6. Converting decimals to percents

To convert a decimal to a percent, multiply by 100.

- Note that multiplying by 100 is equivalent to moving the decimal point two places to the right.
- Example: Convert the decimal 0.25 to its percent form.

Multiply 0.25 by 100.

$$0.25 \times 100 = 25\%$$

- Two final notes:

If two decimals are multiplied, the product will have the same number of digits to the right of its decimal point as in the multiplicand and multiplier combined.

When converting between fractions, percents and decimals, think about whether the result seems reasonable. For example, if the fraction $\frac{3}{4}$ is converted to a percent and the result is 0.0075%, it does not seem reasonable that 0.0075% could be equivalent to three-fourths. Instead, 75% does seem reasonable.

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Chapter 6

Ratios, Proportions and Variation

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6.1 Definitions

6.2 Comparing ratios to fractions and percents

6.3 Variation and proportion

6.1. Definitions

In this section, ratios and proportions are defined.

- *Ratios* depict the relation between two similar values with respect to the number of times the first contains the second. A ratio represents a comparison between two quantities. For example, if the ratio between apples and oranges in a fruit bowl is 3 to 2, then for every 3 apples there are 2 oranges.

- Ratios are written 3 to 2, 3:2 and $3/2$.

- The difference between ratios and fractions is that fractions represent part-per-whole, and ratios represent part-per-part.

- *Proportions* are a comparative relation between the size, quantity, etc., of objects or values. Mathematical proportions represent two equal ratios. For example, the following are equivalent:

$1/2 = 2/4$, $1:2 = 2:4$, $1/2$ is proportional to $2/4$, $1:2::2:4$

- Note that in a proportion, the product of the “extreme” outer numbers equals the product of the “mean” middle numbers. $1:2::2:4$ can be written $1 \times 4 = 2 \times 2$.

- When equivalent fractions, such as $1/2 = 2/4$, are cross-multiplied (by multiplying the opposite numerators with the opposite denominators), the two products are always equal. This principle is useful when determining the quantity of one of the items when the ratio is known.

- For example, how many apples are there in a mixture containing 10 oranges, if the ratio is 3 apples to 2 oranges? To solve this, set up the proportion:

$$\frac{3 \text{ apples}}{2 \text{ oranges}} = \frac{? \text{ apples}}{10 \text{ oranges}}$$

Using cross-multiplication the equation becomes:

$$3 \times 10 = 2 \times ?$$

$$30 = 2 \times ?$$

Divide both sides by 2.

$$30/2 = 1 \times ?$$

$$15 = ?$$

Therefore, there are 15 apples in this mixture.

6.2. Comparing ratios to fractions and percents

The difference between ratios and fractions is that fractions represent part-per-whole, and ratios represent part-per-part.

- Ratios can be expressed in the form of a fraction (or in the form of division).
- The definition of a ratio differs from that of a fraction but all the rules for manipulating fractions apply to ratios.
- The ratio 1 to 2 (or 1:2 or $1/2$) can be expressed as 50% or 0.50.
- Example: If Tom ate 3 pies for every 5 pies Ted ate, what is the ratio of pies eaten by Tom and Ted?

Tom : Ted
(3 pies) / (5 pies)

Tom ate $3/5$ the number of pies that Ted ate.

Tom ate $3/5 = 0.60 = 60\%$ of the number of pies that Ted ate.

- If a ratio of two values is known, the percent that one of the values is of the total can be determined by adding the parts to get the whole, placing the one value over the whole, and multiplying by 100.

- For example, what percent of the total number of pies did Tom and Ted eat?

The ratio is (3 pies) to (5 pies), which represents part/part.

And the percent of the total number is (part/whole)(100).

If the total number or whole is 3 pies + 5 pies = 8 pies.

Therefore, the percent of the total number of pies Tom ate is $3 \text{ pies} / 8 \text{ pies} = 3/8 = 0.375$ and $0.375 \times 100 = 37.5\%$

Also, the percent of the total number of pies Ted ate is $5 \text{ pies} / 8 \text{ pies} = 5/8 = 0.625$ and $0.625 \times 100 = 62.5\%$

6.3. Variation and proportion

Variation describes how one quantity varies or changes as another quantity varies or changes.

- If as one number increases then decreases, another number increases then decreases simultaneously, the numbers are in *direct variation* or *direct proportion* to each other. The equation for direct variation or direct proportion is:

$$y = kx$$

Basic Math and Pre-Algebra

The k represents some constant number, and x and y represent numbers that vary directly. As x increases, y increases. As x decreases, y decreases.

- If as one number increases then decreases, another number decreases then increases simultaneously, the numbers are in *indirect variation* and are said to be *inversely proportional*. The equation for indirect variation representing inversely proportional quantities is:

$xy = k$ which can be rearranged as $y = k/x$

The k represents some constant number, and x and y represent numbers that vary indirectly. As x increases, y decreases. As x decreases, y increases.

Chapter 7

Powers and Exponents

- 7.1 Definition of the exponent or power
- 7.2 Negative exponents
- 7.3 Multiplying exponents with the same base
- 7.4 Multiplying exponents with different bases
- 7.5 Dividing exponents with the same base
- 7.6 Dividing exponents with different bases
- 7.7 Raising a power of a base to a power
- 7.8 Distributing exponents into parenthesis
- 7.9 Addition of exponents
- 7.10 Subtraction of exponents
- 7.11 Exponents involving fractions

7.1. Definition of the exponent or power

An exponent represents a number that is multiplied by itself the number of times as the exponent or power defines.

Basic Math and Pre-Algebra

- The following exponent represents 2 raised to the 6th power:

$$2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$$

2 is called the *base* and 6 is the *exponent* or *power*.

- Examples of exponents are 2^6 , 25^7 and 219^{65} .
- If a positive whole number is raised to a power greater than one, the result is a larger number.
- If a number (except zero) is raised to the 1st power, the result is the number itself. $3^1 = 3$.
- If a number is raised to the zero power, the result is 1.

$$3^0 = 1$$

$$a^0 = 1$$

a = any number except zero

$$0^0 = \text{undefined}$$

7.2. Negative exponents

In this section, negative numbers raised to even and odd powers and positive and negative numbers raised to negative powers are presented.

- If a negative number is raised to a positive even power, the result is a positive number. For example:

$$(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2) = 64$$

- If a negative number is raised to a positive odd power, the results is a negative number. For example:

$$(-2)^5 = (-2)(-2)(-2)(-2)(-2) = -32$$

- If a positive or negative number is raised to a negative power it can be equivalently written, one-over that number raised to the positive power. For example:

$$2^{-6} = \frac{1}{2^6} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{64}$$

$$(-2)^{-6} = \frac{1}{(-2)^6} = \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} \times \frac{1}{-2} = \frac{1}{64}$$

7.3. Multiplying exponents with the same base

To multiply exponents with the same base, simply add the powers.

- For example:

$$3^2 \times 3^4 = 3^{2+4} = 3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$$

This example can be equivalently written:

$$(3 \times 3) \times (3 \times 3 \times 3 \times 3) = 9 \times 81 = 729$$

- The following is one of the laws of exponents (a, b and c represent numbers):

$$a^b \times a^c = a^{b+c}$$

7.4. Multiplying exponents with different bases

To multiply exponents with different bases, each exponent must be expressed individually, then multiplied. The powers cannot be added directly.

- For example:

$$3^2 \times 2^4 = (3 \times 3) \times (2 \times 2 \times 2 \times 2) = 9 \times 16 = 144$$

$$a^b \times d^c = a^b d^c \quad \text{Where } a, b, c \text{ and } d \text{ represent numbers.}$$

- To multiply exponents with different bases but the same power, the following law of exponents applies (a , b and c represent numbers):

$$a^c \times b^c = (a \times b)^c$$

Which can also be written:

$$a^c b^c = (ab)^c$$

7.5. Dividing exponents with the same base

To divide exponents with the same base, subtract the powers.

- For example:

$$3^4 \div 3^2 = (3^4)/(3^2) = 3^{4-2} = 3^2 = 3 \times 3 = 9$$

This example can be equivalently written:

$$3^4 \div 3^2 = \frac{3^4}{3^2} = \frac{3 \times 3 \times 3 \times 3}{3 \times 3} = \frac{81}{9} = 9$$

- Example: Divide $3^2 \div 3^4$.

$$3^2 \div 3^4 = (3^2)/(3^4) = 3^{2-4} = 3^{-2} = 1/3^2 = 1/9$$

- Consider the following examples (a, b and c represent numbers):

$$a^3b^2/a^2b = (aaabb)/(aab) = a^{3-2}b^{2-1} = a^1b^1 = ab$$

$$a^2b/a^3b^2 = (aab)/(aaabb)$$

$$= a^{2-3}b^{1-2} = a^{-1}b^{-1} = (ab)^{-1} = 1/(ab)$$

$$a^b \div a^c = (a^b)/(a^c) = a^{b-c}$$

$$a^b \div a^b = (a^b)/(a^b) = a^{b-b} = a^0 = 1$$

$$(a+b)^b \div (a+b)^b = (a+b)^b/(a+b)^b = (a+b)^{b-b} = (a+b)^0 = 1$$

- A base with an exponent of zero is equal to 1.

$$54^0 = 1$$

$$a^0 = 1 \quad \text{Where } a \text{ represents a number.}$$

$$0^0 \text{ is undefined.}$$

7.6. Dividing exponents with different bases

To divide exponents with different bases, each exponent must be expressed individually, then divided. The powers cannot be subtracted directly.

- For example:

$$\frac{3^3}{2^2} = \frac{3 \times 3 \times 3}{2 \times 2} = \frac{27}{4} = 6.75$$

7.7. Raising a power of a base to a power

To express an exponent raised to a power, multiply the powers.

- For example:

$$(3^2)^3 = 3^{2 \times 3} = 3^6 = (3 \times 3 \times 3 \times 3 \times 3 \times 3) = 729$$

Equivalently:

$$(3^2)^3 = (3^2)(3^2)(3^2) = (3 \times 3)(3 \times 3)(3 \times 3) = 9 \times 9 \times 9 = 729$$

Equivalently:

$$(3^2)^3 = (3 \times 3)^3 = (9)^3 = (9) \times (9) \times (9) = 729$$

- Example: Evaluate $(-y^2)^2$, (y represents a number and $*$ represents the multiplication sign).

$$(-y^2)^2 = (-y^2) * (-y^2) = (-1 * y * y) * (-1 * y * y)$$

$$= 1 * y * y * y * y = y^4$$

Equivalently:

$$(-y^2)^2 = (-1 * y^2)^2 = (-1)^2 * (y^2)^2 = 1 * y^{2*2} = 1 * y^4 = y^4$$

- The following is one of the laws of exponents, (a , b and c represent numbers):

$$(a^b)^c = a^{b * c} = a^{bc}$$

7.8. Distributing exponents into parenthesis

If numbers or variables inside parenthesis are multiplied or divided and have an exponent outside the parenthesis, the exponent can be *distributed* to each number or variable inside the parenthesis.

- For example:

$$(3 * 2)^2 = (3^2 * 2^2) = 9 * 4 = 36$$

Basic Math and Pre-Algebra

- Consider the following examples (a, b and c represent numbers):

$$(ab)^2 = (a^2 \times b^2) = a^2b^2$$

$$(ab)^c = a^cb^c$$

$$(2b)^2 = (2^2 \times b^2) = 4b^2$$

$$(2/3)^2 = (2^2/3^2) = 4/9$$

$$(a/b)^2 = (a^2/b^2) = a^2/b^2$$

- If numbers or variables inside parenthesis are added or subtracted, the exponent outside cannot be *distributed* as above. Instead:

$$(a + b)^2 = (a + b) \times (a + b)$$

$$= a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

(See multiplication of binomials in Chapter 5 of the second book in the *Master Math* series, *Algebra*.)

7.9. Addition of exponents

To add exponents whether the base is the same or different, express each exponent individually, then add. The powers cannot be added directly.

- For example:

$$3^2 + 2^4 = (3 \times 3) + (2 \times 2 \times 2 \times 2) = 9 + 16 = 25$$

$$3^2 + 3^4 = (3 \times 3) + (3 \times 3 \times 3 \times 3) = 9 + 81 = 90$$

7.10. Subtraction of exponents

To subtract exponents whether the base is the same or different, express each exponent individually, then subtract. The powers cannot be subtracted directly.

- For example:

$$2^4 - 3^2 = (2 \times 2 \times 2 \times 2) - (3 \times 3) = 16 - 9 = 7$$

$$3^4 - 3^2 = (3 \times 3 \times 3 \times 3) - (3 \times 3) = 81 - 9 = 72$$

7.11. Exponents involving fractions

To express fractions containing exponents, apply the principles in the preceding sections in this chapter to each expression within the fraction. For example, if parenthesis surround numbers or variables multiplied together, the exponent can be distributed to each number or variable within the parenthesis.

Basic Math and Pre-Algebra

- **Example:** Express the following fraction.

$$\frac{(a^b)^c}{(2b)^2} = \frac{a^{b \times c}}{(2^2 \times b^2)} = \frac{a^{bc}}{4b^2}$$

- A number raised to a negative power is equivalent to one-over that number.

$$3^{-1} = \frac{1}{3} \quad \text{and} \quad 3^{-6} = \frac{1}{3^6}$$

- If a positive fraction is raised to a power, and the numerator is smaller than the denominator, the value of the resulting fraction is less.

$$(1/2)^2 = (1/2)(1/2) = 1/4 \quad \text{Where } 1/4 < 1/2.$$

- If a positive fraction is raised to a power, and the numerator is larger than the denominator, the value of the resulting fraction is greater.

$$(5/2)^2 = (5/2)(5/2) = 25/4 \quad \text{Where, } 25/4 > 5/2.$$

$$\text{Remember, } 5/2 = 10/4.$$

- Consider the following examples with fractional exponents:

$$3^{1/2} = \sqrt{3}$$

$$3^{2/3} = (3^{1/3})^2 = (3^2)^{1/3}$$

Chapter 8

Logarithms

- 8.1 Definition of the logarithm
- 8.2 Common (base ten) and natural logarithm
- 8.3 Solving equations with logarithms or exponents
- 8.4 Exponential form and logarithmic form
- 8.5 Laws of logarithms: addition, subtraction, multiplication, division, power and radical
- 8.6 Examples: the Richter scale, pH and radiometric dating

8.1. Definition of the logarithm

The logarithm is the exponent of the power to which a base number must be raised to equal a given number.

- A logarithm is the inverse of an exponent.
- Each exponent has an inverse logarithm.

Basic Math and Pre-Algebra

- Consider the following exponential equation (x and y represent numbers):

$$x = a^y$$

Where a is an integer and is the base of the exponent.

The inverse of this equation is:

$$y = \log_a x$$

Where a is the base of the logarithm.

The inverse is obtained by taking the base a logarithm of both sides of the exponential equation.

$$x = a^y$$

$$\log_a(x) = \log_a(a^y)$$

Because $\log_a(a)$ cancels, the equation becomes:

$$\log_a(x) = y$$

- Example: Convert the exponential equation $2^4 = 16$ to a logarithmic equation.

Because the exponent has a power of 2, take the base 2 logarithm of both sides.

$$\log_2(2^4) = \log_2(16)$$

Because $\log_2(2)$ cancels, the equation becomes:

$$4 = \log_2(16)$$

8.2. Common (base ten) and natural logarithm

The two most common logarithms are the common logarithm, also called the base ten logarithm, and the natural logarithm.

- The common or base ten logarithm is written:

$$\log_{10}x \text{ or simply } \log x$$

- The natural logarithm is written:

$$\log_e x \text{ or } \ln x$$

Where $e = 2.71828182846$.

- The relationship between the common logarithm and the natural logarithm can be written:

$$\log_e x = (2.3026)\log_{10}x$$

Or equivalently:

$$\ln x = (2.3026)\log x$$

8.3. Solving equations with logarithms or exponents

To solve logarithmic equations, take the inverse or exponent.

- Example: Solve the equation $y = \log x$ for x .

Because the base is 10, raise both sides of the equation by base 10.

$$10^y = 10^{(\log x)}$$

Because $10^{(\log_{10})}$ cancels itself, $10^{(\log x)} = x$

The equation becomes:

$$10^y = x$$

- Example: Solve the equation $y = \ln x$ for x .

Because the base is e , raise both sides of the equation by e to isolate x .

$$e^y = e^{(\ln x)}$$

Because e^{\ln} cancels itself, $e^{(\ln x)} = x$

The equation becomes:

$$e^y = x$$

Logarithms

- To solve exponential equations, take the inverse or logarithm.

- Example: Solve the equation $y = 10^x$ for x .

Because the base is 10, take \log_{10} of both sides of the equation.

$$\log y = \log(10^x)$$

Because $\log_{10}(10)$ cancels itself, $\log(10^x) = x$

The equation becomes:

$$\log y = x$$

- Example: Solve the equation $y = e^x$ for x .

Because the base is e , take \ln of both sides of the equation.

$$\ln y = \ln(e^x)$$

Because $\ln(e)$ cancels itself, $\ln(e^x) = x$

The equation becomes:

$$\ln y = x$$

8.4. Exponential form and logarithmic form

In this section, examples of computed logarithms and exponents are presented.

Exponential Form

$$10^0 = 1$$

$$10^1 = 10$$

$$10^2 = 100$$

$$10^3 = 1,000$$

$$10^4 = 10,000$$

$$10^5 = 100,000$$

$$e^1 = 2.71828182846$$

$$e^\infty = \infty$$

$$e^{-\infty} = 0$$

$$e^0 = 1$$

$$e^{2.30258509299} = 10$$

Logarithmic Form

$$\log 1 = 0$$

$$\log 10 = 1$$

$$\log 100 = 2$$

$$\log 1,000 = 3$$

$$\log 10,000 = 4$$

$$\log 100,000 = 5$$

$$\ln (2.71828182846) = \ln e = 1$$

$$\ln \infty = \infty$$

$$\ln 0 = -\infty$$

$$\ln 1 = 0$$

$$\ln 10 = 2.30258509299$$

- To approximate or compute numerical values of logarithms or exponents, refer to tables contained in mathematical handbooks and selected algebra books, or use the function keys of technical calculators.

8.5. Laws of logarithms: addition, subtraction, multiplication, division, power and radical

The following are laws of logarithms
(a represents any base):

$$\log_a(x*y) = \log_a x + \log_a y$$

$$\log_a(x/y) = \log_a x - \log_a y$$

$$\log_a(x^n) = (n)\log_a x$$

$$\log_a(\sqrt[n]{x}) = (1/n)\log_a x \quad \text{Where } \sqrt[n]{x} = x^{1/n}.$$

Also, note that: $\log_b a = 1/\log_a b$

Where a and b represent numbers.

Because *a* represents any base, these laws apply to base *e* (natural logarithms) or base ten (common logarithms). Proofs of the laws of logarithms can be found in selected algebra books.

8.6. Examples: the Richter scale, pH and radiometric dating

In this section, applications of logarithmic and exponential equations for the Richter scale, the pH scale and radioactive dating are presented.

- The Richter scale is a logarithmic scale that ranges from 1 to 10 and is used for indicating the intensity of an earthquake. The Richter scale was originated in 1931 by K. Wadati in Japan and was developed by Charles Richter in California in 1935. The Richter scale is a quantitative scale that measures wave amplitudes generated from ground shaking using a seismograph. Richter defined the magnitude of a local earthquake as the logarithm to base ten of the maximum seismic-wave amplitude (in thousandths of a millimeter) recorded on a standard seismograph at a distance of 100 kilometers from the earthquake epicenter. Using the logarithmic scale, for each one unit increase in magnitude, the amplitude of the earthquake waves increase ten times.

- pH expresses the acidity or basicity of a solution in terms of the pH (potency of hydrogen) scale. pH is defined as the negative of the logarithm of the hydrogen (or hydronium) ion concentration in an aqueous (water is the solvent) solution.

$$\text{pH} = -\log[\text{H}^+] \quad \text{or} \quad \text{pH} = -\log[\text{H}_3\text{O}^+]$$

Logarithms

For example, pure water that has a neutral pH of 7 has a hydrogen ion concentration of 1×10^{-7} M or 10^{-7} M (M is moles per liter). The logarithm of 10 raised to the power -7 is equal to -7.

$$\text{pH} = -\log[\text{H}^+] = -\log[10^{-7}] = -(-7) = 7$$

In an acidic solution, the H^+ concentration is greater than 10^{-7} M and less than the OH^- concentration. An acidic solution with 1×10^{-4} M of H^+ has a pH of 4.

$$\text{pH} = -\log[\text{H}^+] = -\log[10^{-4}] = -(-4) = 4$$

In a basic solution, the H^+ concentration is less than 10^{-7} M and less than the OH^- concentration. A basic solution with 3.5×10^{-10} M of H^+ has a pH of 9.46.

$$\text{pH} = -\log[\text{H}^+] = -\log[3.5 \times 10^{-10}]$$

$$= -(\log[3.5] + \log[10^{-10}])$$

$$= -(0.54 - 10) = -(-9.46) = 9.46$$

- Radioactive substances are subject to exponential decay. The decay of radioactive substances is used to estimate the age of objects. For example, carbon-14 is used to estimate the age of plants and animals that were once alive by comparing the ratio $\text{C}^{14}/\text{C}^{12}$. (C^{12} is the more common form of carbon.) Carbon-14 is produced in the atmosphere when high-energy neutrons (produced from cosmic rays colliding with

Basic Math and Pre-Algebra

atmospheric molecules) collide with the nuclei of nitrogen-14 in the air. The carbon-14 then reacts with oxygen to form carbon-14-carbon dioxide.

The carbon-14-carbon dioxide is incorporated into plants through photosynthesis and into animals through ingestion of plants. While a plant or animal is alive, the content of carbon-14 in its system is assumed to remain constant in small but measurable quantities. After the plant or animal has died, carbon-14 can no longer be incorporated by photosynthesis or ingestion, and the carbon-14 that is present begins to decay back to nitrogen-14 by emission of beta radiation (electrons).

The half-life of carbon-14 is 5,730 years, therefore after 5,730 years half of the original amount of carbon-14 in a given sample will have decayed back to nitrogen-14. (Note that radioactive dating techniques assume: (a) the rate of radioactive decay has remained constant over time; (b) the concentration of the radioactive substance that is naturally abundant today is equivalent to the concentration that was naturally abundant in the past; and (c) the substance did not interact with the environment so that there were no original atoms or decay products added or removed.)

The equation for the mass of a substance at any time, t , can be derived as follows. If the half-life of a radioactive substance is $t_{1/2}$ and its initial mass is c ,

Logarithms

then, in $t_{1/2}$ units of time (years), the mass will reduce to $c/2$. The mass is given by $y(t_{1/2}) = ce^{-kt_{1/2}}$, therefore:

$$ce^{-kt_{1/2}} = c/2$$

Canceling the c 's and taking the natural logarithm of both sides, the equation becomes:

$$\ln(e^{-kt_{1/2}}) = \ln(1/2)$$

Because $\ln e$ cancels itself, and because $\ln(1/2) = -\ln 2$, the equation becomes:

$$-kt_{1/2} = -\ln 2$$

Canceling the negative signs on both sides, and dividing both sides by k , the equation becomes:

$$t_{1/2} = (\ln 2)/k = (0.693)/k$$

So:

$$t_{1/2} = (0.693)/k$$

Multiplying both sides by k , and then dividing both sides by $t_{1/2}$, the equation becomes:

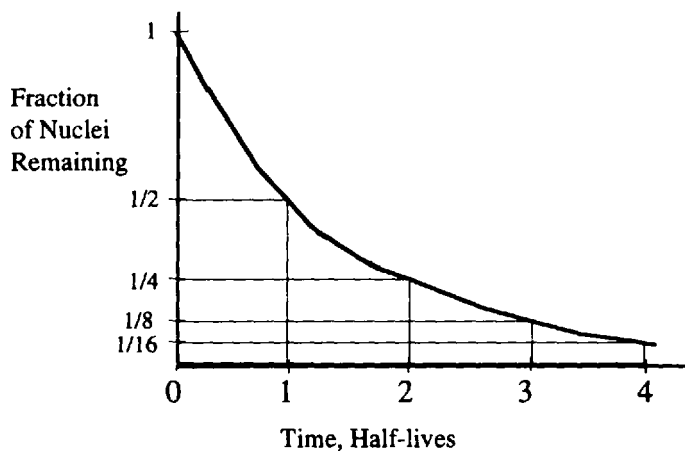
$$k = (0.693)/t_{1/2}$$

Therefore, the mass of a substance at any time t is:

$$y(t) = ce^{-kt} = ce^{-(0.693)(t/t_{1/2})}$$

(See Chapter 3 of *Algebra*, the second book in the *Master Math* series, for an explanation of solving equations using algebraic techniques.)

Basic Math and Pre-Algebra



The number of nuclei remaining is halved during each half-life. The greatest decrease occurs during the first half-life and the amount of decrease lessens with each succeeding half-life.

Chapter 9

Roots and Radicals

- 9.1 Definitions
- 9.2 The square root
- 9.3 Estimating square roots
- 9.4 The cubed root
- 9.5 The fourth and fifth roots
- 9.6 Simplifying radicals by factoring
- 9.7 Multiplying radicals
- 9.8 Dividing radicals
- 9.9 Radicals involving fractions
- 9.10 Rationalizing the denominator
- 9.11 Addition of radicals
- 9.12 Subtraction of radicals

9.1. Definitions

A *radical* is written using the radical symbol, $\sqrt{}$. The number that the *root* is calculated for is inside the radical symbol. The word radical is derived from the Latin word *rad*, which means root.

- The number inside the radical symbol is called the *radicand*. In the case of \sqrt{x} , x represents the radicand.
- Different *roots* of numbers exist. For example, for a number represented by x , possible roots are the square root, cubed root, fourth root, etc., represented by $\sqrt[2]{x}$, $\sqrt[3]{x}$, $\sqrt[4]{x}$, etc., respectively.
- An equivalent form of writing roots is $x^{1/2}$, $x^{1/3}$, $x^{1/4}$, where x represents any number. For example:

$$\sqrt[2]{x} = x^{1/2}$$

$$\sqrt[n]{x} = x^{1/n}$$

$$\sqrt[2]{45x^2} = (45x^2)^{1/2}$$

9.2. The square root

The square root of a number, is equal to a number that, when squared, equals the original number.

- For example, the square root of 4 is equal to 2, such that 2^2 equals 4. Also, $(-2)^2$ is equal to 4, so the square root of 4 could also be -2.
- If a number has two equal factors, then it has a square root. For example, 4 has factors 2×2 .
- A *perfect square* is a number that can be expressed as the product of two equal numbers or factors. The following are perfect squares:

1, 4, 9, 16, 25,

because they can be expressed as:

1×1 , 2×2 , 3×3 , 4×4 , and 5×5 respectively.

- The square root of 25 is ± 5 , because $5^2 = (5)(5)$ or $(-5)(-5) = 25$.
- The *radical symbol*, $\sqrt{\quad}$, represents the positive square root.

Basic Math and Pre-Algebra

- It is not possible to find a perfect square root of a negative number such as $\sqrt{-4}$ because when two identical numbers are multiplied together the result is always a positive number.

$$2 \times 2 = 4 \quad \text{and} \quad -2 \times -2 = 4$$

(Note: See Section 1.17, "Complex numbers," for a discussion of $\sqrt{-1}$.)

- $\sqrt[2]{x}$ is commonly written \sqrt{x} , with the "2" not displayed.
- The following are examples of square roots:

$$\sqrt{25} = \sqrt{5 \times 5} = \sqrt{5^2} = \pm 5$$

$$\sqrt{36} = \sqrt{6 \times 6} = \sqrt{6^2} = \pm 6$$

$$\sqrt{9x^2} = \sqrt{(3x)(3x)} = \sqrt{(3x)^2} = \pm 3x$$

$$\sqrt{a^2} = \sqrt{(a)(a)} = \sqrt{(a)^2} = \pm a$$

$$\sqrt{0} = 0$$

$$\sqrt{1} = \pm 1$$

9.3. Estimating square roots

To determine the square root of a number that is *not* a *perfect square*:

1. Determine what two perfect squares the number is between.
 2. Estimate the square root of the number to one decimal point accuracy (0.1).
 3. Divide your estimate into the number (and evaluate to the nearest thousandth).
 4. Find the average of the estimate and the quotient.
 5. Check how good the estimate is by squaring the result.
- A technical calculator can also be used to estimate square roots.
 - Example: Estimate the square root of 12.

The two perfect squares this number is between are 9 and 16.

Therefore, the $\sqrt{12}$ is between $\sqrt{9}$ and $\sqrt{16}$.

Because $\sqrt{9} = 3$ and $\sqrt{16} = 4$, the $\sqrt{12}$ must be between 3 and 4.

Basic Math and Pre-Algebra

Estimate the square root of the number to one decimal point accuracy (0.1).

Because 12 is 3 away from 9 and 4 away from 16, the estimate is 3.4.

Divide the estimate into 12 and evaluate it to the nearest thousandth.

Using a calculator $12 \div 3.4 = 3.529$.

Alternatively, using long division:

$$3.4 \overline{)12}$$

Move the decimal point to the right in both numbers so there is no decimal in the divisor.

$$34. \overline{)120.00}$$

34 will divide into 120 three times. Multiply 3 by 34 and subtract the result from 120.

$$\begin{array}{r} 3.??? \\ 34. \overline{)120.000} \\ \underline{102} \\ 18.0 \end{array}$$

Bring down the zero, divide 34 into 18.0, multiply the result by 34, and subtract that result from 18.0.

$$\begin{array}{r}
 3.5?? \\
 34. \overline{)120.000} \\
 \underline{102} \\
 18.0 \\
 \underline{17.0} \\
 1.00
 \end{array}$$

Bring down the zero, divide 34 into 1.00, multiply the result by 34, and subtract that result from 1.00.

$$\begin{array}{r}
 3.52? \\
 34. \overline{)120.000} \\
 \underline{102} \\
 18.0 \\
 \underline{17.0} \\
 1.00 \\
 \underline{.68} \\
 .32
 \end{array}$$

Bring down the zero, and divide 34 into 0.320.

$$\begin{array}{r}
 3.529 \\
 34. \overline{)120.000} \\
 \underline{102} \\
 18.0 \\
 \underline{17.0} \\
 1.00 \\
 \underline{.68} \\
 .320 \\
 \underline{.306} \\
 .014
 \end{array}$$

Find the average of the estimate and the quotient.

$$\frac{3.4 + 3.529}{2} = \frac{6.929}{2} = 3.47$$

Check how good the result is by squaring it:

$$(3.47)^2 = 12.04.$$

This process can be repeated to better the approximation by dividing 3.47 into 12 and proceeding as above.

9.4. The cubed root

The cubed root of a positive number is the number that, when multiplied by itself three times, equals that number.

- For example, the cubed root of 8 written, $\sqrt[3]{8}$ is equal to 2.

This is true because, $2^3 = (2)(2)(2) = 8$

Also:

$$\sqrt[3]{8} = \sqrt[3]{(2)(2)(2)} = \sqrt[3]{2^3} = 2$$

If a represents a number then:

$$\sqrt[3]{(a)(a)(a)} = \sqrt[3]{a^3} = a$$

9.5. The fourth and fifth roots

The fourth root of a positive number equals a number that when multiplied by itself four times equals the number.

- For example:

$$\sqrt[4]{16} = \sqrt[4]{(2)(2)(2)(2)} = \sqrt[4]{2^4} = 2$$

$$\sqrt[4]{a^4} = \sqrt[4]{(a)(a)(a)(a)} = a$$

Where a represents a number.

- The fifth root of a positive number equals a number that when multiplied by itself five times equals the number.

- For example:

$$\sqrt[5]{32} = \sqrt[5]{(2)(2)(2)(2)(2)} = \sqrt[5]{2^5} = 2$$

9.6. Simplifying radicals by factoring

To simplify radicals by factoring, factor the radicand, look for perfect squares for square roots, or cubes for cubed roots, etc., and if multiple factors exist, bring each root out of the radical.

- For example, simplify the following by factoring and reducing the radicals.

$$\sqrt{36} = \sqrt{6 \times 6} = 6$$

$$\sqrt{32} = \sqrt{8 \times 4} = \sqrt{2 \times 2 \times 2 \times 2 \times 2} = 2 \times 2 \sqrt{2} = 4\sqrt{2}$$

$$\sqrt{42} = \sqrt{6 \times 7} = \sqrt{2 \times 3 \times 7}$$

There are no squared factors, so this radical cannot be reduced further.

$$\sqrt{a^2 b^3} = \sqrt{a a b b b} = ab\sqrt{b}$$

$$\sqrt{45x^5} = \sqrt{(9)(5)(x^2)(x^2)(x)}$$

$$= \sqrt{(3^2)(x^2)(x^2)(5)(x)} = 3(x)(x)\sqrt{(5)(x)} = 3x^2\sqrt{5x}$$

$$\sqrt[3]{8} = \sqrt[3]{(2)(2)(2)} = \sqrt[3]{2^3} = 2$$

$$\sqrt[4]{16} = \sqrt[4]{(2)(2)(2)(2)} = \sqrt[4]{2^4} = 2$$

9.7. Multiplying radicals

To multiply radicals, combine the numbers under one radical symbol, multiply and simplify.

- For example, multiply the following radicals:

$$\sqrt{3}\sqrt{6} = \sqrt{3 \times 6} = \sqrt{3 \times 3 \times 2} = 3\sqrt{2}$$

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

9.8. Dividing radicals

To divide radicals, combine the numbers under one radical symbol, divide and simplify.

- For example, divide the following radicals:

$$\sqrt{3} \div \sqrt{6} = \frac{\sqrt{3}}{\sqrt{6}} = \sqrt{\frac{3}{6}} = \sqrt{\frac{3}{3 \times 2}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

(Note that the 3 in the numerator canceled with the 3 in the denominator.)

$$\sqrt{8} \div \sqrt{6} = \frac{\sqrt{8}}{\sqrt{6}} = \sqrt{\frac{8}{6}} = \sqrt{\frac{(2)(2)(2)}{(3)(2)}} = (2)\sqrt{\frac{1}{3}} = \frac{2}{\sqrt{3}}$$

$$\sqrt{a} \div \sqrt{b} = \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

9.9. Radicals involving fractions

To simplify a radical containing fractions or a fraction containing radicals, combine the radicands in each radical under one radical symbol, divide and simplify.

- For example, simplify the following radicals:

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

$$\sqrt{\frac{9}{36}} = \sqrt{\frac{3 \times 3}{6 \times 6}} = \frac{3}{6} = \frac{1}{2}$$

9.10. Rationalizing the denominator

Rationalizing the denominator of a fraction containing radicals creates a fraction without any radicals in the denominator.

- To rationalize the denominator, simplify the radical in the denominator, multiply both the numerator and the denominator by the radical in the denominator (that will make the denominator's radicand a perfect square), then simplify the fraction.

- For example, rationalize the denominator for $1/\sqrt{8}$.

First simplify the denominator.

$$\frac{1}{\sqrt{8}} = \frac{1}{\sqrt{2 \times 2 \times 2}} = \frac{1}{2\sqrt{2}}$$

Multiply the numerator and denominator by $\sqrt{2}$.

$$\frac{(1)\sqrt{2}}{2\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{2\sqrt{2 \times 2}} = \frac{\sqrt{2}}{2 \times 2} = \frac{\sqrt{2}}{4}$$

9.11. Addition of radicals

To add radicals, express each radical individually, then add. The radicands cannot be combined under one radical symbol.

- For example, add the following radicals:

$$\sqrt{8} + \sqrt{12} = 2\sqrt{2} + 2\sqrt{3}$$

$$\sqrt{4} + \sqrt{6} = 2 + \sqrt{6}$$

$$\sqrt{36} + \sqrt{25} = 6 + 5 = 11$$

$$3\sqrt{6} + 4\sqrt{6} + 5\sqrt{6} = (3+4+5)\sqrt{6} = 12\sqrt{6}$$

$$\begin{aligned} & 3\sqrt{5} + 4\sqrt{6} + 5\sqrt{5} + 7\sqrt{6} \\ &= (3+5)\sqrt{5} + (4+7)\sqrt{6} = 8\sqrt{5} + 11\sqrt{6} \end{aligned}$$

(Note that $\sqrt{6}$ and $\sqrt{5}$ were factored out of each term in the above two examples.)

- Adding and subtracting radicals is similar to combining like terms in algebraic equations.

9.12. Subtraction of radicals

To subtract radicals, express each radical individually, then subtract. The radicands cannot be combined under one radical symbol.

- For example, subtract the following radicals:

$$\sqrt{36} - \sqrt{25} = 6 - 5 = 1$$

$$\sqrt{4} - \sqrt{6} = 2 - \sqrt{6}$$

$$5\sqrt{6} - 4\sqrt{6} = (5-4)\sqrt{6} = 1\sqrt{6} = \sqrt{6}$$

$$7\sqrt{6} - 4\sqrt{6} - 5\sqrt{5} = (7-4)\sqrt{6} - 5\sqrt{5} = 3\sqrt{6} - 5\sqrt{5}$$

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Chapter 10

Important Statistical Quantities

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10.1 Average (arithmetic mean)

10.2 Median

10.3 Mode

10.4 Probability

10.5 Standard deviation, variance, histograms and distributions

10.1. Average (arithmetic mean)

In this section the average or arithmetic mean and the weighted average are defined.

- The *average* and the *arithmetic mean* are equivalent.
- To find the average of a group of numbers, add all of the numbers together and divide the sum by how many numbers there are.

$$\text{Average} = \frac{\text{total sum of the numbers}}{\text{how many numbers there are}}$$

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- Example: What is the average of {5, 6, 9, 25}?

The total sum is:

$$5 + 6 + 9 + 25 = 45$$

Divide the sum by how many numbers there are, or 4.

$$45 \div 4 = 11.25$$

- Example: What is the average of {9, 6, 6, 9, 25, 25}?

The total sum is:

$$9 + 6 + 6 + 9 + 25 + 25 = 80$$

Divide the sum by how many numbers there are, or 6.

$$80 \div 6 = 13.333333....$$

Which rounds to 13.33.

- To solve a problem using the *weighted average*, each quantity is weighted (multiplied) by the number of items having that quantity. After multiplying each quantity by the number of items having that quantity, add the products and divide the sum by the total number of items.

- For example, what is the weighted average of the following: If you purchase 5 shrubs and two cost \$2 and three cost \$7, what is the average cost per shrub?

To solve this problem as a weighted average, the two costs are weighted (multiplied) by the number of items and the products are added.

$$2(\$2) + 3(\$7) = \$25$$

$$\$4 + \$21 = \$25$$

Then, divide the sum by the total number of shrubs.

$$\$25 \div 5 = \$5$$

Therefore, the average cost per shrub is \$5.

This problem can also be solved as a standard average.

The total cost is:

$$\$2 + \$2 + \$7 + \$7 + \$7 = \$25$$

Divide the sum by the total number of shrubs, or 5.

$$\$25 \div 5 = \$5$$

Therefore, the average cost per shrub is \$5.

10.2. Median

The median is the value of the number in the middle of a sequential list of numbers. If there are two numbers in the middle, the median is the value of their average. There must be an equal number of numbers above and below the median.

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- To find the median of a group of numbers, list the numbers sequentially; the median is either the number in the middle, or the average of the two numbers in the middle.

- To find the median value of an odd number of numbers, list the numbers in an ordered sequence from least in value to greatest; the median is the number in the middle. For example:

Find median of {5, 7, 3, 10, 1}.

List in order. 1, 3, 5, 7, 10

The middle number, or median, is 5.

- To find the median value of an even number of numbers, list the numbers in an ordered sequence from least in value to greatest; the median is the average of the two middle numbers. For example:

Find median of {5, 7, 3, 12, 10, 1}.

List in order. 1, 3, 5, 7, 10, 12

The middle two numbers are 5 and 7.

The average of the middle two numbers is:

$$\frac{5 + 7}{2} = \frac{12}{2} = 6$$

Therefore, the median is 6.

10.3. Mode

The mode of a list of numbers is the number that occurs most frequently.

- **Example:** Find the mode of:

{3, 6, 29, 8, 6, 4, 3, 85, 2, 8, 35, 3, 8, 9}.

It is easier to list the numbers sequentially.

2, 3, 3, 3, 4, 6, 6, 8, 8, 8, 9, 29, 35, 85

The numbers 3 and 8 both occur three times, which is the greatest number of times.

Therefore, 3 and 8 are both modes.

- As shown in this example, there can be more than one mode.
- If no number occurs more than once, then the mode does not exist for that list.

10.4. Probability

Probability defines the chances of some certain event occurring.

- The probability, p , is generally expressed as a fraction.

$$\text{Probability} = \frac{\text{number of possible ways}}{\text{total number of ways}}$$

- For example, if a cube has one yellow face and five blue faces, what is the probability that the yellow face will land on top if the cube is tossed in the air?

The answer is: $p = 1/6$

10.5. Standard deviation, variance, histograms and distributions

In this section, standard deviation, variance, histograms and distributions are defined and explained.

Standard Deviation and Variance

- Consider experimental data that has been collected where a measurement, x_1 , is made of a quantity x . If other measurements are made, we expect random error within our measurement that will be distributed around the correct value. The standard deviation and the variance represent the uncertainties associated with experimental attempts to determine the true value of some quantity.

- The symbol for standard deviation is sigma σ and the symbol for variance is σ^2 .

- The *standard deviation* is a measure of the dispersion in a frequency distribution. It is equal to the square root of the mean of the squares of the deviations from the arithmetic mean of the distribution. The *variance* is the square of the standard deviation. The standard deviation is the square root of the variance.

The standard deviation: $\sigma = \pm \sqrt{\sigma^2}$

Where σ^2 is the variance.

$$\begin{aligned}\sigma^2 &= \text{Lim}_{N \rightarrow \infty} \left[\frac{1}{N} \sum (x_i - \mu)^2 \right] \\ &= \text{Lim}_{N \rightarrow \infty} \left(\frac{1}{N} \sum x_i^2 \right) - \mu^2 = \langle x^2 \rangle - \mu^2\end{aligned}$$

Where:

$\langle x \rangle$ is a weighted average over x of all possible values of x .

x is the quantity that is being measured.

x_i represents the measurements made.

N is the number of measurements made and $i=1$ is the first measurement.

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The sum (\sum) of all measurements from $i=1$ to $i=N$ is given by:

$$\sum_{i=1}^N (x_i) = x_1 + x_2 + x_3 + x_4 + \dots + x_N = \sum (x_i)$$

The $\text{Lim}_{N \rightarrow \infty}$ represents the Limit as the number of measurements get larger and larger, (see Chapter 5, "Limits," in the third book of the *Master Math* series, *Pre-Calculus and Geometry*). μ represents the average of all possible measurements.

$$\mu = \text{Lim}_{N \rightarrow \infty} \left(\frac{1}{N} \sum x_i \right)$$

The average is the sum of the values of the measurements divided by the total number of measurements, N .

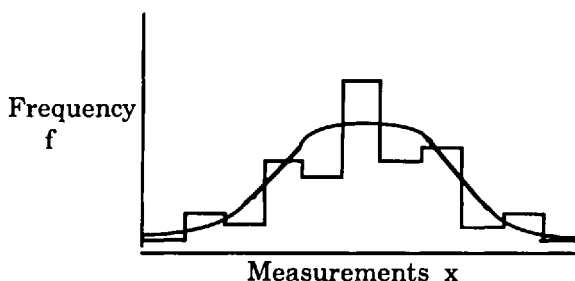
• The variance (and from that the standard deviation) can be calculated using the equation:

$$\begin{aligned} \sigma^2 &= \frac{1}{N-1} \left[\sum x_i^2 - (x_{\text{ave}}) \sum x_i \right] \\ &= \frac{1}{N-1} \left[\sum x_i^2 - N (x_{\text{ave}})^2 \right] \end{aligned}$$

x_{ave} represents the average of the measurements (the experimental mean).

The Histogram, Binomial Distribution, Gaussian Distribution and Poisson Distribution

- If 100 measurements of the length of a football field are made, the measurements can be graphed in terms of a frequency distribution indicating the number of times each length was measured. A mean of the data points can be calculated and a standard deviation estimated. A *histogram graph* is used to depict the number of measurements recorded for each length.



The curve represents an estimate distribution assuming infinite measurements could be made.

- The *binomial distribution* describes the probability of observing x for n attempts where only two mutually exclusive outcomes are possible. The probability for success in each attempt is p . A *binomial experiment* is an experiment with n repeated attempts where: a) the attempts are independent; b) each attempt results in only two possible outcomes, “success” and “failure”; and c) the probability, p , of a success on each attempt, remains constant. x is the number of attempts that

Basic Math and Pre-Algebra

result in success and has a binomial distribution with parameters p and $n = 1, 2, 3, \dots$. The probability function is:

$$P_B = \frac{n!}{x! (n-x)!} p^x (1-p)^{n-x}$$

Where $\mu = np$ and $\sigma^2 = np(1-p)$.

(μ represents the mean and σ represents the standard deviation for the distribution of the hypothetical infinite population of all possible observations.)

The total number of different sequences of attempts that contain x successes and $n-x$ failures is:

$$\frac{n!}{x! (n-x)!}$$

- The *Gaussian distribution* or normal error distribution, is a bell-shaped curve and describes an expected distribution of random observations for a given experiment. In fact, it seems to describe the distribution of estimations of the parameters of most probability distributions. The Gaussian distribution is an approximation of the binomial distribution for the case where the number of possible different observations is large and the probability of success for each measurement is large. The Gaussian distribution is useful for smooth symmetrical distributions with large n and infinite p .

$$P_G = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

(Note that exp means raise to the exponent e.)

• The *Poisson distribution* represents an approximation to the binomial distribution for the case when the average number of successes is smaller than the possible number. The Poisson distribution is useful for describing small samples with large numbers of events that are difficult to observe.

$$P_P = \frac{\mu^x}{x!} \exp[-\mu]$$

For $\sigma^2 = \mu$

Index

- absolute value 53
- add exponents 130
- add or subtract decimals 90
- add or subtract percents 105
- add or subtract two or more fractions that have different denominators 70
- add or subtract two or more fractions that have the same denominators 69
- add radicals 157
- addends 7
- adding and subtracting even and odd integers 47
- addition 7, 48
- addition and subtraction of mixed numbers 82
- addition, subtraction, multiplication and division of complex numbers 55
- arithmetic mean 159
- average 159

- base 124
- base number 133
- base of the logarithm 134
- base ten logarithm 135

- base ten system 6
- binomial distribution 167

- cents 99
- common denominator 70
- common logarithm 135
- common multiple 70
- compare the value of two or more decimals 98
- compare the value of fractions 83
- complex conjugate 57
- complex fractions 81
- complex numbers 55
- compound interest 109
- consecutive even integers 46
- consecutive integers 45
- consecutive odd integers 46
- convert decimal to fraction 113
- convert decimal to percent 117
- convert fraction to decimal 113
- convert fraction to percent 111
- convert percent to decimal 115
- convert percent to fraction 112
- converting improper fractions to mixed numbers 78

- converting mixed numbers to
 - improper fractions 78
- counting numbers 7
- cubed root 152
- decimal fractions 87, 90
- decimal point 87, 103
- decimal system 87
- decimals 52, 87
- decimals have equivalent
 - forms as fractions 88
- denominator 68
- difference 16
- digits 6
- discount 107
- divide decimals 95
- divide exponents with
 - different bases 128
- divide exponents with the
 - same base 127
- divide fractions 76
- divide radicals 155
- dividend 35
- dividing by zero 48
- divisibility 43
- division 35
- divisor 35
- equivalent fractions 68
- estimating 59
- even and odd powers 124
- even integers 46
- exponent 123, 133
- exponent outside the
 - parenthesis 129
- exponent raised to a
 - power 128
- exponential equation 133, 137
- exponents 138
- factorial 66
- factoring and reducing
 - radicals 154
- factors 43
- fifth root 153
- fourth root 153
- fraction 54, 67
- fraction containing radicals 156
- fractions 52
- fractions containing exponents
 - 131
- Gaussian distribution 168
- greater than 62
- greater than or equal to 62
- histograms 164
- hundredths 88, 99
- i* 55
- imaginary numbers 55
- improper fractions 78
- inequalities 62
- integers 45, 47, 52, 55
- inversely proportional 122
- inverted fraction 76
- irrational numbers 52, 54
- laws of exponents 126, 129
- laws of logarithms 139
- less than 62
- less than or equal to 62
- logarithm 133, 138
- logarithmic equation 134, 136
- logarithmic scale 140
- long division 35
- median 161
- minuend 16
- mixed number 79
- mode 163
- money and decimals 99
- multiples 43
- multiplicand 22
- multiplication 22
- multiplier 22

multiply and divide
 percents 106
 multiply decimals 93
 multiply exponents with
 different bases 126
 multiply exponents with
 the same base 125
 multiply fractions 68
 multiply radicals 155
 multiplying even and odd
 integers 47

 natural logarithm 135
 negative integers 48
 negative numbers 45
 negative numbers
 multiplied or divided 51
 negative powers 124, 132
 number line 48, 63
 numerator 68

 odd integers 46

 percent 101, 102, 121
 percent converted to a
 fraction or decimal 101
 percent change 107
 percents of a number 103
 perfect square 147, 149
 pH scale 140
 Poisson distribution 169
 positive integers 48
 positive numbers 45
 power 123, 133
 prime number 54
 probability 164
 product 23
 proportion 122
 proportions 119

 quotient 35

 radical 146
 radical containing fractions 156

 radical symbol 147
 radicals by factoring 154
 radicand 146
 radioactive dating 140
 ratio 118
 rational numbers 52, 54
 rationalize denominator 156
 ratios and fractions 120
 real number line 52
 real numbers 52, 55
 reciprocal 76
 reducing fractions 77
 remainder 37, 43
 Richter scale 140
 root 146
 round decimals 97
 rounding 59

 simple interest 109
 square root 147
 square root, cubed root, fourth
 root 146
 standard deviation 164
 subtract a negative number 50
 subtract exponents 131
 subtract more than two
 numbers 16
 subtract radicals 158
 subtraction 15, 48
 subtrahend 16
 sum 8

 tenths 87, 99
 thousandths 88
 truncating 62

 variance 164
 variation 122

 weighted average 159
 whole numbers 7, 47, 52, 55

 zero 46, 47
 zero power 124