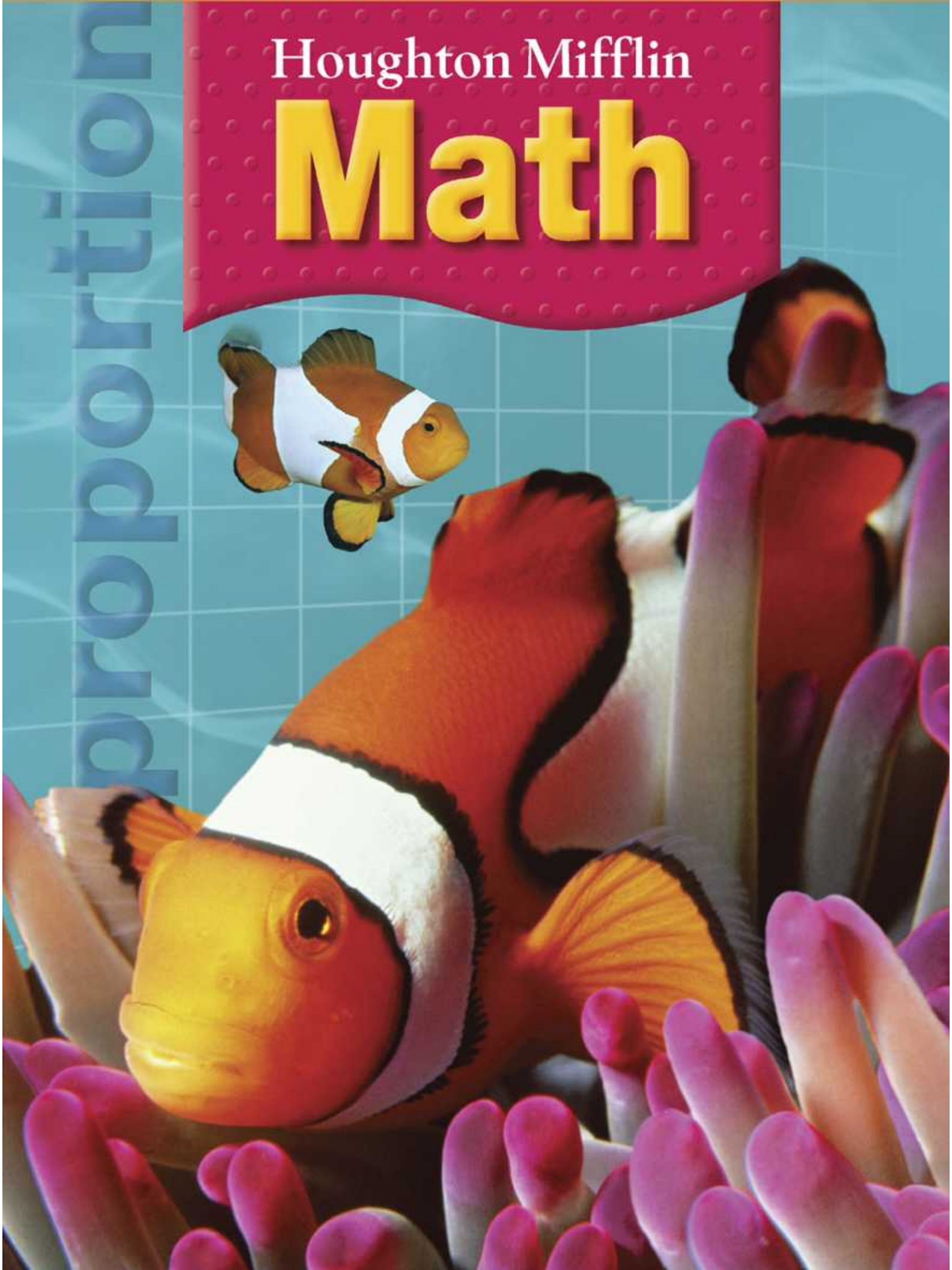
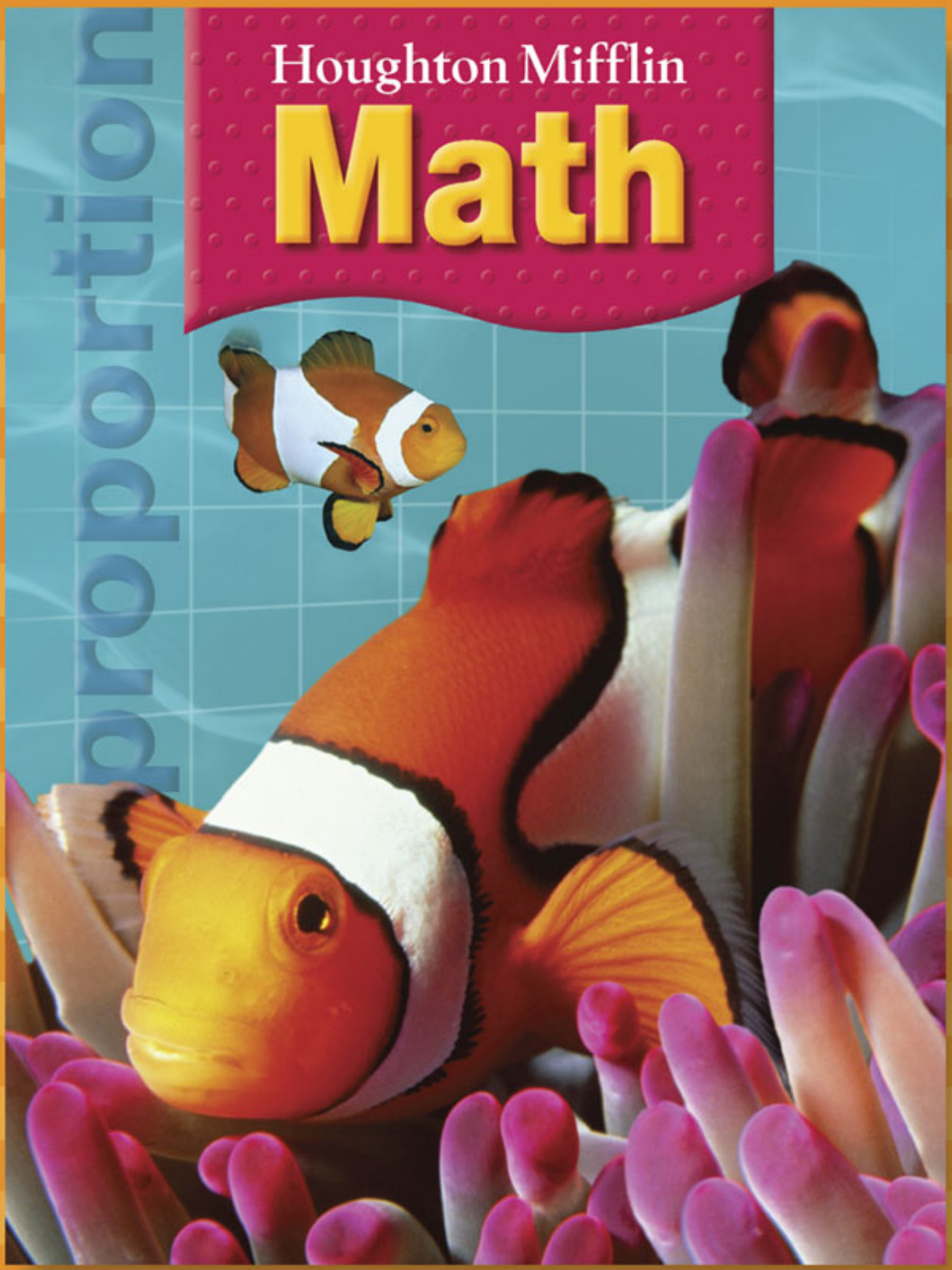
Houghton Mifflin

Math





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Back to School

Welcome!

Scientists, athletes, artists, and health care workers all use math every day—and you will too. This year in math you'll learn about numbers, patterns, shapes, and different ways to measure. You'll use the mathematics you know to solve problems and describe objects and patterns you see. You can get started by finding out about yourself as a mathematician and about the other students in your class.



Activity

Real Life Connection

Introducing Me, the Mathematician

About Me

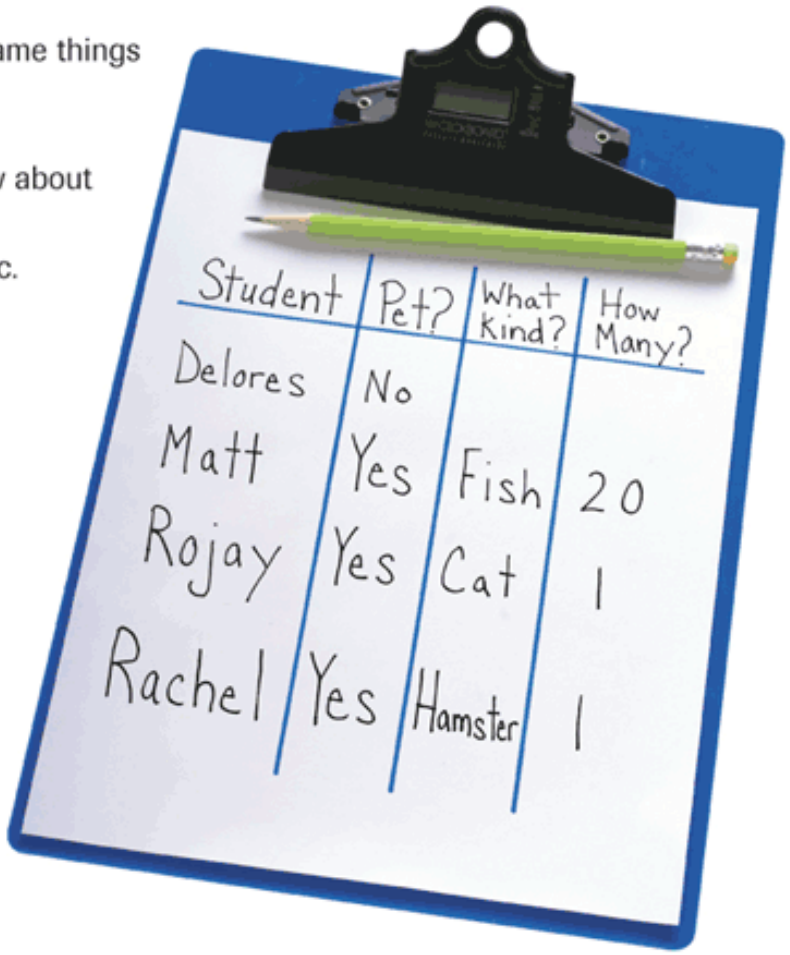
Write your math autobiography by answering these questions. You can write about other experiences as well, as long as they tell about you as a math student.

- Tell about a time you first remember doing math, even if you were very young.
- What are you good at in math?
- What would you like to improve or know more about?
- What is one thing you would like your teacher to know about you and mathematics?

About My Class

Many of your classmates may like the same things that you like. Other classmates may like totally different things.

- Think of one topic you'd like to know about your classmates.
- Write a survey question for your topic.
- Conduct your survey among your classmates.
- Make a graph to show your results.
- Write a paragraph, and use numbers to describe your results. Include predictions about your survey results if you surveyed 100 students.



Back to School

Lesson

Problem Solving and Numbers

Objective Review basic number and problem-solving skills.

Review and Remember

This year, you will learn about integers, rational numbers, and equations. In this lesson, you will use what you know about fractions and decimals to solve problems and find new information.

Suppose you are going to make your family's special meatloaf for a Back-to-School potluck dinner. You need to make 6 times as much meatloaf as the recipe shows.

How much turkey will you need? How much beef?

Mighty Meatloaf

Serves 4

- $\frac{1}{2}$ pound ground turkey
- $1\frac{1}{4}$ pounds ground beef
- $\frac{1}{4}$ pound thinly sliced cooked ham
- 2 eggs
- 2 cups bread crumbs
- 1 medium onion diced
- $1\frac{1}{2}$ teaspoons salt
- $\frac{1}{2}$ teaspoon pepper
- 1 teaspoon parsley



Turkey: $6 \times \frac{1}{2} = 3$

Beef: $6 \times 1\frac{1}{4} = \frac{6}{1} \times \frac{5}{4} = \frac{6 \times 5}{1 \times 4} = \frac{30}{4}$ or $7\frac{2}{4}$ or $7\frac{1}{2}$

Solution: You will need 3 pounds of ground turkey and $7\frac{1}{2}$ pounds of ground beef.

Check: Think: $\frac{1}{2} = 0.5$; $6 \times 0.5 = 3$ ✔

Think: $6 \times 1\frac{1}{4} = (6 \times 1) + (6 \times \frac{1}{4})$

$= 6 + \frac{6}{4} = 6 + \frac{4}{4} + \frac{2}{4}$

$= 6 + 1 + \frac{1}{2} = 7\frac{1}{2}$ ✔

Guided Practice

Find how much of each ingredient you need for 6 meatloaves. Write fractions in simplest form. Label your answers.

1. Ham **2.** Eggs **3.** Bread crumbs

4. Onion **5.** Parsley **6.** Salt

Ask Yourself

- Is my answer reasonable?

Explain Your Thinking Choose one exercise from above. Explain how you know your answer is reasonable.

Practice and Problem Solving

Multiply. Write each answer in simplest form.

7. $\frac{2}{3} \times \frac{4}{5}$ **8.** $\frac{3}{10} \times \frac{1}{3}$ **9.** $\frac{7}{8} \times \frac{2}{9}$ **10.** $\frac{5}{6} \times 8$

11. $\frac{7}{8} \times 12$ **12.** $3\frac{1}{4} \times 6$ **13.** $1\frac{1}{5} \times 4\frac{1}{6}$ **14.** $2\frac{1}{2} \times 1\frac{3}{10}$

You have learned many problem-solving strategies. Solve each problem. Choose a strategy and a computation method that works best for the problem. Explain your choices.

15. Use the recipe shown on page B2. What is the total amount of meat you will need for one meatloaf? for six meatloaves?

16. **Multistep** Casey made macaroni and cheese for the potluck dinner. She bought $2\frac{1}{2}$ pounds of cheese but used just $\frac{2}{3}$ of it. How many pounds of cheese did she have left?

17. Seventy-six people attended the potluck dinner. There were 18 fewer students than adults. How many students went to the dinner?

18. Marty set up tables for the 76 people at the dinner. Eight people can sit at each table. They also need 3 tables for food and 2 tables for displays. How many tables did Marty set up?

You Choose

Strategy

- Make a Drawing
- Guess and Check
- Make a Table
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

At the end of some lessons, there are problems like those you might find on a standardized test.

Mixed Review and Test Prep

Open Response

Multiply or divide. (Grade 5)

19. 14.8×3 **20.** 17.2×2.5

21. 0.3×0.2 **22.** $16.8 \div 3$

23. $9.8 \div 7$ **24.** $20.64 \div 0.2$

Multiple Choice

25. The bill for the meal is \$30. A tip of \$6 is left. If five people share the cost equally, how much should each person pay? (Grade 5)

A \$5 **C** \$6.20

B \$6 **D** \$7.20

Back to School

Hands On

Lesson

Measurement

Objective Review basic measurement skills needed to start sixth grade.

Materials

rulers
inch ruler
centimeter ruler

Work Together

You can use math to describe your classroom and the objects in it. First, practice measuring in customary and metric units.

Work with a partner. Make a chart like the one below to record your work.

STEP 1

Estimate the length and width of a book to the nearest half-inch. Record your estimate.

	Book	Estimate	Measurement in inches
1.	Length		
2.	Width		
3.	Area		

STEP 2

Find the zero mark on your ruler. Work together to measure the length and width of the book. Remember to round each measurement to the nearest half-inch.

- How close are your measurements to your estimates?

STEP 3

Now estimate the area of the book and record your estimates. Then use your measurements to find the actual area of the book.

- How do you find the area of a rectangle?
- How do you label area?
- How close was your estimate to the actual area of your book?

On Your Own

Repeat the experiment from page B4, this time measuring your desk or table.

- Estimate the length, width, and area. Record your estimates.
- Measure the desk or table. Record your actual measurements.
- Calculate the area.

	Desk or table	Estimate	Measurement in inches
1.	Length		
2.	Width		
3.	Area		

Now measure the same book and desk (or table) to the nearest centimeter. Then find the area.

	Book	Estimate	Measurement in centimeters
4.	Length		
5.	Width		
6.	Area		

	Desk or table	Estimate	Measurement in centimeters
7.	Length		
8.	Width		
9.	Area		

10. Perimeter is the distance around an object or figure. What is the perimeter of your desk or table? Explain how you found your answer.

11. If you were going to cover your desk with fabric, would you need to know the area or perimeter? How much fabric would you need?

Talk About It • Write About It

12. You are explaining to a third grader how to choose the correct measuring instrument for a project. For which types of projects would you use a ruler? A yardstick?

13. Precision is determined by the unit of measure that you use. A smaller unit of measure produces a more precise measurement than a larger unit. Of all the measurements you made in this lesson, which is the most precise?

Science Connection

Meteorological Measurement

Materials
centimeter grid paper
ruler

Meteorologists use statistical data to record the average temperature and precipitation. Analysis of this data helps them forecast the weather.

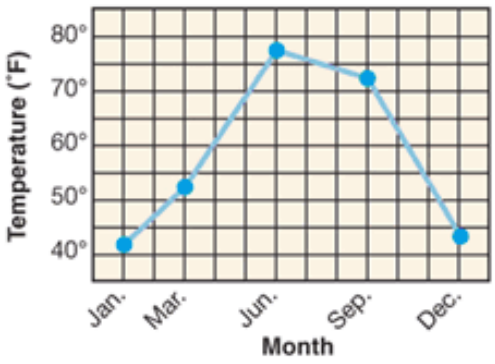
These tables show the average temperature and precipitation in four states at significant months.

Average Temperature (˚F)				
Month	GA	MS	NM	OH
January	45.2	43.8	36.5	25.5
March	56.1	55.7	48.2	38.2
June	77.8	78.0	74.2	64.0
September	74.5	75.0	68.6	63.6
December	47.6	46.6	37.2	30.8

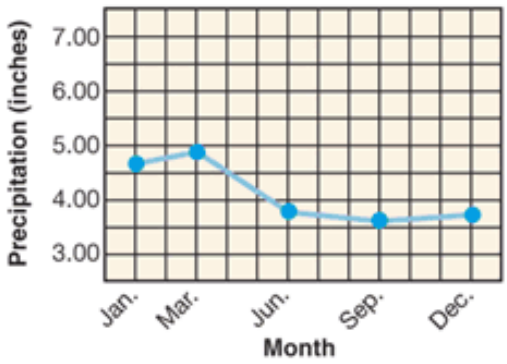
Average Precipitation (inches)				
Month	GA	MS	NM	OH
January	4.66	5.58	0.39	2.43
March	4.88	6.32	0.53	3.04
June	4.05	4.21	1.49	3.99
September	3.77	3.41	1.54	3.28
December	3.64	5.59	0.47	3.00

Meteorological data may also be displayed in line graphs like those shown below.

Average Temperature for Athens, GA



Average Precipitation for Athens, GA



1. Examine the two line graphs above. Describe what happens to the average temperature and the average precipitation in Athens, GA in one year.

2. Use the tables on this page to create line graphs like the ones above. On one of the graphs, select two of the other three states listed and plot their average temperature. On the other graph, plot the average precipitation of the same two states.

UNIT 1

Whole Numbers, Decimals, and Algebra

CHAPTER 1

Place Value, Addition, and Subtraction

page 2



CHAPTER 2

Multiply and Divide Whole Numbers

page 30



CHAPTER 3

Number Theory

page 56



Reading Mathematics

Reviewing Vocabulary

Here are some math vocabulary words that you should know.

base of a power	the number that is used as a factor when evaluating powers: 10^3 ← exponent, ↑ base
exponent	the number that shows how many times a base is used as a factor
power of ten	a number that can be written as a product of tens
multiple	the product of a given number and any whole number
Addition Properties	properties such as the Associative, Commutative, and Identity Properties that help you find the value of addition expressions
Multiplication Properties	properties such as the Associative, Commutative, Identity, Zero, and Distributive Properties that help you find the value of multiplication expressions



Reading Words and Symbols

There are different ways to represent the same idea. These phrases and the model at right all represent the same expression.

- a number increased by 7
- the sum of a number and 7
- *x* plus 7
- $x + 7$



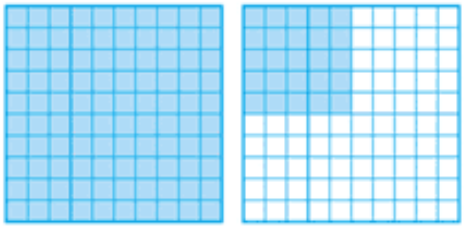
Write a mathematical expression for each phrase.

1. the product of 6 and a number
2. a number decreased by 6
3. a number divided by 6
4. 6 increased by a number

Reading Test Questions

Choose the correct answer for each.

5. Which decimal represents the shaded part of the grids?



a. 2.75

b. 2.25

c. 1.75

d. 1.25

Represents means "stands for" or "names."

6. Which of the following is equivalent to the expression 10^2?

a. 20

b. 102

c. 100

d. 1,000

Equivalent means "has the same value as" or "is equal to."

7. Which digit is in the hundredths place? 1,234.579

a. 1

b. 2

c. 7

d. 9

A **digit** is any of the numerals 0–9.

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

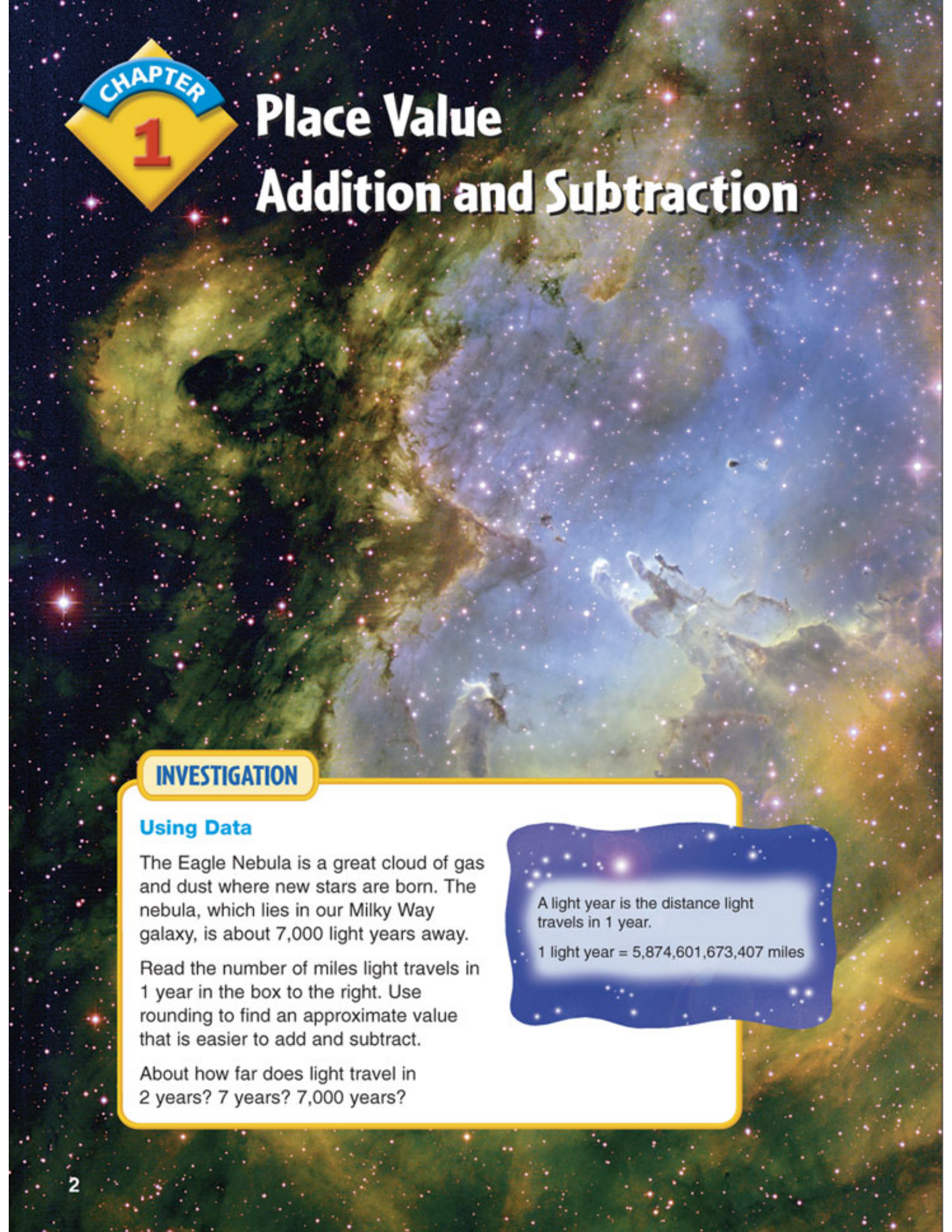
- **compatible numbers**
- **millionth**
- **prime factorization**
- **greatest common factor (GCF)**
- **least common multiple (LCM)**

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "The Festival of Murals" on pages 634–635. Then work with a partner to answer the questions about the story.



CHAPTER 1

Place Value Addition and Subtraction

INVESTIGATION

Using Data

The Eagle Nebula is a great cloud of gas and dust where new stars are born. The nebula, which lies in our Milky Way galaxy, is about 7,000 light years away.

Read the number of miles light travels in 1 year in the box to the right. Use rounding to find an approximate value that is easier to add and subtract.

About how far does light travel in 2 years? 7 years? 7,000 years?

A light year is the distance light travels in 1 year.

1 light year = 5,874,601,673,407 miles

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

Vocabulary

periods

estimate

hundredths

thousandths

1. When a number is written is standard form, commas are used to separate the _____.
2. A digit three places to the right of the decimal point is in the _____ place.
3. You should _____ to tell whether your computed answer is reasonable.

CONCEPTS AND SKILLS

Write the value of the underlined digit.

4. 35<u>6</u>,032 **5.** 907.2<u>1</u>8 **6.** <u>4</u>,345,622 **7.** 0.<u>0</u>653

Order each set of numbers from least to greatest.

8. 4,821 4,128 4,218 **9.** 158 1,580 185 **10.** 0.64 0.46 0.04

Round each number to the underlined place.

11. 1,<u>6</u>43,523 **12.** 157,<u>0</u>58,104 **13.** <u>9</u>6,526

14. 5.<u>3</u>9 **15.** 0.<u>0</u>55 **16.** 12<u>5</u>.862

Find each sum or difference.

17. 369,054 + 640,951 **18.** 7,043 − 2,961 **19.** 4.06 + 2.8

Write About It

20. Sandy said that the more digits a number has to the right of the decimal point, the greater the number. Is Sandy correct? Explain.

Facts Practice, See page 654.

Lesson 1

Place Value

Objective Read, write, and understand whole numbers and decimals.

Vocabulary
- ten thousandth
- hundred thousandth
- millionth



Learn About It

At times, Pluto is at a distance from the Sun of about 5,913,603,800 kilometers. Twenty times the circumference of Earth is only about 0.000136 of the distance from Pluto to the Sun.

The value of each digit in a number depends on its place in the number. In what ways can you read and write the two numbers mentioned above?

Read and Write Numbers

Billions			,	Millions			,	Thousands			,	Ones			.	Decimals					
hundred billions	ten billions	billions		hundred millions	ten millions	millions		hundred thousands	ten thousands	thousands		hundreds	tens	ones		tenths	hundredths	thousandths	ten thousandths	hundred thousandths	millionths
		5	,	9	1	3	,	6	0	3	,	8	0	0	.						
			,				,				,			0	.	0	0	0	1	3	6

Whole Numbers

Standard form: 5,913,603,800

Word form: five billion, nine hundred thirteen million, six hundred three thousand, eight hundred

Short word form: 5 billion, 913 million, 603 thousand, 8 hundred

Expanded form: 5,000,000,000 + 900,000,000 + 10,000,000 + 3,000,000 + 600,000 + 3,000 + 800

Decimals

Standard form: 0.000136

Word form: one hundred thirty-six millionths

Short word form: 136 millionths

Expanded form: 0.0001 + 0.00003 + 0.000006

Another Example Hundred Billions

Standard form: 469,072,580,000

Word form: four hundred sixty-nine billion, seventy-two million, five hundred eighty thousand

Short word form: 469 billion, 72 million, 580 thousand

Expanded form: 400,000,000,000 + 60,000,000,000 + 9,000,000,000 + 70,000,000 + 2,000,000 + 500,000 + 80,000

Guided Practice

Write the value of each underlined digit.

1. 4̲26,378,203 **2.** 53.0̲671

3. 39,628,47̲0 **4.** 3.6512̲7

Explain Your Thinking How is zero used as a placeholder to tell the difference between the numbers 1.607 and 1.67?

Ask Yourself

- In what place is each digit?

Practice and Problem Solving

Write each number in word form and in expanded form.

5. 151,072,830 **6.** 736.083 **7.** 0.59811 **8.** 784,210,520,000

Write each number in standard form.

9. 300,000,000,000 + 20,000,000,000 + 50,000,000 + 80,000 + 1,000 + 40

10. eighty-six and twenty-one ten thousandths

Write the numbers that are 1,000 more and 1,000 less than each given number.

11. 38,756 **12.** 1,053,022 **13.** 430,833,002 **14.** 2,909,882,004

Data **Use the table for Problems 15–17.**

15. Write Mercury's length of day and length of year in word form and in expanded form.

16. Analyze There are 1,000 meters in 1 kilometer. If you express the diameter of Mercury in meters, what will be the value of the first digit?

17. Reasoning About how many birthdays would you have in an Earth year if you were living on Mercury? Explain.

Mercury Data

Average distance from the Sun	57,894,376 km
Planet diameter	4,878 km
Length of day	58.6461 Earth days
Length of year	87.9694 Earth days

Mixed Review and Test Prep

Open Response

Add or subtract. (Grade 5)

18. 1,246 + 987 **19.** 9,007 − 4,888

20. 15,779 + 48,037 **21.** 50,000 − 37,266

22. 63,002 − 5,823 **23.** 99,999 + 77,777

Multiple Choice

24. The average distance between the Earth and the Sun is about 92,950,000 miles. What is the value of the digit 2? (Ch. 1, Lesson 1)

A 2,000 **C** 2,000,000

B 20,000 **D** 20,000,000

Extra Practice See page 29, Set A.

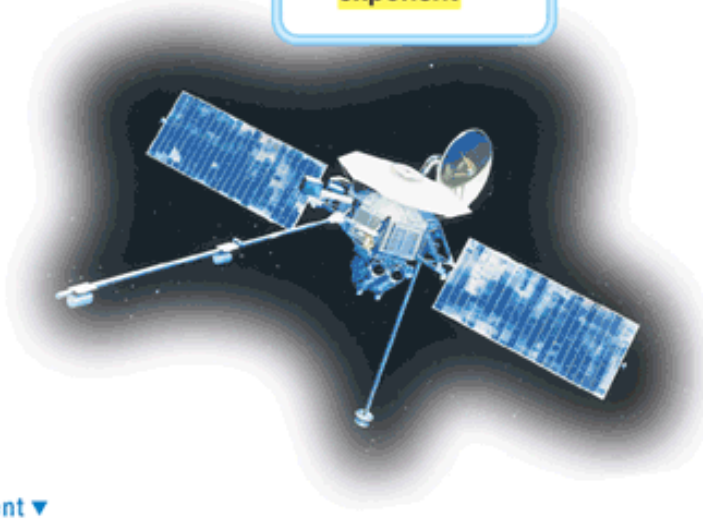
Lesson 2

Audio Tutor 1/1 Listen and Understand

Place Value and Powers of Ten

Objective Relate powers of 10 to place value.

Vocabulary
- power of 10
- base
- factor
- exponent



Learn About It

The product found by multiplying the number 10 by itself one or more times is a **power of 10**. You use exponents to write very large and very small numbers as powers of 10. Use 10 as the **base** and the number of times that 10 is a **factor** as the **exponent**.

exponent ▼
10^4
◀base

To read 10^4, say "ten to the fourth power."

$10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

exponent ▼
10^{-4}
◀base

To read 10^{-4}, say "ten to the negative fourth power."

$10^{-4} = \frac{1}{10 \times 10 \times 10 \times 10} = \frac{1}{10{,}000} = 0.0001$

Study the table below.

Powers of 10

Power of 10	Standard Form	Fractional Form	Place Value
10^4	10,000	$\frac{10{,}000}{1}$	Ten thousands
10^3	1,000	$\frac{1{,}000}{1}$	Thousands
10^2	100	$\frac{100}{1}$	Hundreds
10^1	10	$\frac{10}{1}$	Tens
10^0	1	$\frac{1}{1}$	Ones
10^{-1}	0.1	$\frac{1}{10}$	Tenths
10^{-2}	0.01	$\frac{1}{100}$	Hundredths
10^{-3}	0.001	$\frac{1}{1{,}000}$	Thousandths
10^{-4}	0.0001	$\frac{1}{10{,}000}$	Ten thousandths

Look at the exponents.

The exponents are positive for numbers greater than 1 and negative for numbers less than 1. The exponent for 1 is zero.

Look at the pattern of zeros.

For numbers greater than 1, the exponent tells the number of zeros in the numerator. For numbers less than 1, the exponent tells the number of zeros in the denominator.

You can write whole numbers and decimals in expanded form, using powers of 10.

$3{,}654 = (3 \times 10^3) + (6 \times 10^2) + (5 \times 10^1) + (4 \times 10^0)$

$0.25 = (2 \times 10^{-1}) + (5 \times 10^{-2})$

Extra Help at eduplace.com/map

Guided Practice

Write each number in expanded form, using powers of 10.

1. 2,000,000 **2.** 0.0321 **3.** 125,317 **4.** 5.212

5. Write $(5 \times 10^{-1}) + (8 \times 10^{-2}) + (3 \times 10^{-3})$ in standard form.

Ask Yourself
- What place value do I start with?
- Should I use a positive or negative exponent?

Explain Your Thinking If the exponent is ^{-}n, where n is positive, how many places will follow the decimal point?

Practice and Problem Solving

Write each number in expanded form, using powers of 10.

6. 3,000 **7.** 3.29 **8.** 26,000,000 **9.** 16.5

10. 0.0024 **11.** 2,350,000,000 **12.** 0.00071 **13.** 7.315

Write each number in standard form.

14. $(3 \times 10^4) + (4 \times 10^3)$ **15.** $(6 \times 10^{-1}) + (7 \times 10^{-2})$ **16.** $(8 \times 10^0) + (4 \times 10^{-1})$

17. $(2 \times 10^4) + (3 \times 10^3) + (5 \times 10^1)$ **18.** $(4 \times 10^{-1}) + (9 \times 10^{-2}) + (5 \times 10^{-3}) + (8 \times 10^{-4})$

Algebra • Patterns

Find a pattern in each sequence of numbers. Use your pattern to find the two missing terms.

19. $10^2, 10^4, 10^6, \blacksquare, \blacksquare, 10^{12}$ **20.** $3^{-4}, 4^{-5}, 5^{-6}, \blacksquare, \blacksquare, 8^{-9}$ **21.** $1^{-3}, 3^{-5}, 5^{-7}, \blacksquare, \blacksquare, 11^{-13}$

Solve.

22. The *Cassini* spacecraft is traveling almost 1,000,000,000 miles to arrive at Saturn in 2004. Write this distance in expanded form using powers of 10.

23. Neptune is about $(2 \times 10^9) + (8 \times 10^8)$ miles from the Sun. Pluto is about $(3 \times 10^9) + (6 \times 10^8)$ miles distant. Write these numbers in standard form.

24. *Mariner 10* passed within 705 kilometers of Mercury. Write this number in expanded form using powers of 10.

25. Explain How could you use mental math to write 100,000,000,000,000 as a power of 10?

Mixed Review and Test Prep

Open Response

Divide. (Grade 5)

26. $8\overline{)92}$ **27.** $7\overline{)601}$ **28.** $6\overline{)1{,}183}$

29. $2\overline{)3{,}059}$ **30.** $9\overline{)7{,}318}$ **31.** $3\overline{)1{,}045}$

32. Mercury is about $(3 \times 10^7) + (5 \times 10^6) + (9 \times 10^5) + (8 \times 10^4)$ miles from the Sun. Write this distance in standard form. (Ch. 1, Lesson 2)

Extra Practice See page 29, Set A.

Lesson 3

Compare and Order Numbers

Vocabulary
number line

Objective Compare and order whole numbers and decimals.

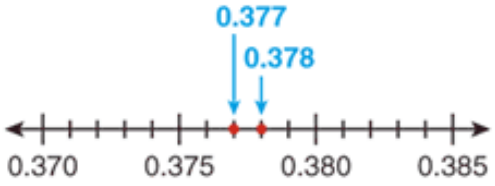
Learn About It

If you lived on Mercury, your weight would only be 0.378 of your weight on Earth. If you lived on Mars, your weight would be 0.377 of your weight on Earth. On which planet would you weigh more?

Compare 0.378 and 0.377.

Different Ways to Compare 0.378 and 0.377

Way 1 You can use a number line.



Since 0.378 is to the *right* of 0.377, $0.378 > 0.377$.

Way 2 You can compare digits.

Align the decimal points. Compare digits from left to right until they are different.

same different

0.378

0.377

$8 > 7$, so $0.378 > 0.377$.

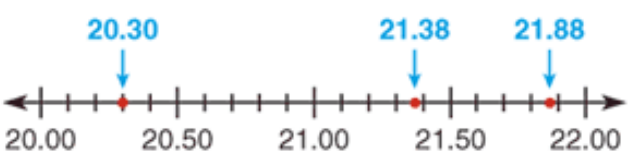
Solution: Since $0.378 > 0.377$, your weight on Mercury would be greater than your weight on Mars.

The planet Mars is known as "the red planet." Why do you think this is so?

Another Example

Order 21.38, 20.30, and 21.88 from least to greatest.

Use a Number Line



$20.30 < 21.38 < 21.88$

Compare Digits

First difference

21.38
20.30
21.88

Second difference

Guided Practice

Ask Yourself
- Did I align the decimal points correctly?
- Where do I start comparing digits?

Compare. Write >, <, or = for each ●.

1. 1,463 ● 1,436 **2.** 2,164 ● 2.164 **3.** 0.351 ● 0.3510

Order from greatest to least. Use a number line.

4. 1.53 15.3 1.3 **5.** 0.404 0.440 0.044 0.40 **6.** 258 825 285 852

Explain Your Thinking Why should you align the decimal points when comparing decimals?

Practice and Problem Solving

Compare. Write >, <, or = for each ●.

7. 8.7 ● 8.70 **8.** 0.557 ● 0.65 **9.** 10.1 ● 1.01 **10.** 216.7 ● 217.6

11. 43.75 ● 47.35 **12.** 99.9 ● 9.99 **13.** 3.203 ● 3.023 **14.** 0.003 ● 0.0030

Order from least to greatest. Use a number line.

15. 5.3 3.6 6.4 **16.** 43.16 43.02 43.05 **17.** 0.11 0.1 0.01

18. 97.5 9.75 9.075 90.7 **19.** 0.0625 0.002 0.027 0.207

Solve.

20. Europa and Ganymede are two moons of Jupiter. Your weight on Europa would be 0.1336 of your weight on Earth. On Ganymede, you would weigh 0.1448 of your Earth weight. On which moon would you weigh more?

21. **Explain** Scientists have given Earth a mass of 1, so that masses can be easily compared. The mass of our moon is 0.0123, while the mass of Pluto is 0.0021. Which has the greater mass, the Moon or Pluto? Show your work.

22. One day on Jupiter lasts 9.92 hours, on Neptune 16.11 hours, on Saturn 10.65 hours, and on Uranus 17.23 hours. Order the planets from shortest to longest day.

23. **What's Wrong?** Decide if this statement is true or false: Andrew says that 7.85 is greater than 7.9 because 85 is greater than 9. Explain your reasoning.

Mixed Review and Test Prep

Open Response

Multiply. (Grade 5)

24. 572 × 8 **25.** 66 × 9 **26.** 4 × 2,607

27. 3,315 × 6 **28.** 905 × 7 **29.** 5,724 × 9

Multiple Choice

30. Which number is between 3.18 and 3.1811? (Ch. 1, Lesson 3)

A 3.1081 **C** 3.1801

B 3.1181 **D** 3.1811

Extra Practice See page 29, Set B.

Lesson 4

Round and Estimate Numbers

Objective Round and estimate whole numbers and decimals.

Learn About It

Each planet takes a different amount of time to complete one orbit about the Sun. Since Venus is closer to the Sun than is Earth, Venus will complete one orbit in less time. It takes about 0.615 Earth year for Venus to complete one orbit about the Sun. How can you round the number 0.615 to the nearest tenth?

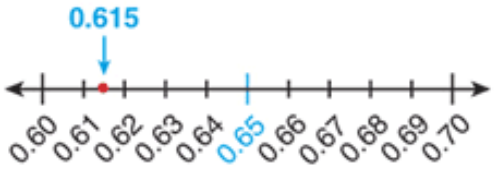
Rounding Rules

- Find the place to which you will round.
- If the digit to its right is 5 or greater, increase the rounding-place digit by one.
- If the digit to its right is less than 5, do not change the rounding-place digit.
- For a whole number, change all digits to the right of the rounded digit to zero.
- For a decimal, drop the digits to the right of the rounded digit.

Different Ways to Round 0.615 to the Nearest Tenth

Way 1 Use a Number Line

STEP 1 Place 0.615 on a number line. Find the halfway point between 0.6 and 7. This is 0.65.



STEP 2 Round the number up if it is equal to or greater than the halfway number. Round the number down if it is less than the halfway number.

$0.615 < 0.65$

So, 0.615 rounds down to 0.6.

Way 2 Use Rounding Rules

STEP 1 Find the tenths place.

0.615

↑

The tenths digit is 6.

STEP 2 Look at the digit to its right. This digit is in the hundredths place. Round.

0.615 rounds to 0.6

↑

$1 < 5$, so the rounding-place digit does not change.

Solution: The number 0.615 rounded to the nearest tenth is 0.6.

Another Example

Whole Numbers

Round 1,328,940 to the nearest ten thousand.

1,328,940 $8 > 5$, so increase 2 to 3.

The number 1,328,940 rounds to 1,330,000.

A rounded number is not exact—it is an estimate. Knowing about rounding and estimating can help you check your answers when you add and subtract whole numbers or decimals, even when using a calculator or computer.

Estimate the sum and difference of 5.53 and 2.32.

Different Ways to Estimate Sums and Differences

Way 1 Use Front-End Estimation

- Add the digits in the greatest place.

$$\begin{array}{r} 5.53 \\ +\ 2.32 \\ \hline \end{array}$$

Estimate: 7

- Adjust. Look at the digits in the next greatest place.

$$\begin{array}{r} 5.53 \\ +\ 2.32 \\ \hline \end{array}$$

Adjusted Estimate: 7.8

- Subtract the digits in the greatest place.

$$\begin{array}{r} 5.53 \\ -\ 2.32 \\ \hline \end{array}$$

Estimate: 3

- Adjust by subtracting the digits in the next greatest place.

$$\begin{array}{r} 5.53 \\ -\ 2.32 \\ \hline \end{array}$$

Adjusted Estimate: 3.2

Way 2 Use Rounding

- Round to the nearest whole number. Then add.

$$\begin{array}{rcl} 5.53 & \rightarrow & 6 \\ +\ 2.32 & \rightarrow & +\ 2 \\ \hline \end{array}$$

Estimate: 8

- Round to the nearest whole number. Subtract.

$$\begin{array}{rcl} 5.53 & \rightarrow & 6 \\ -\ 2.32 & \rightarrow & -\ 2 \\ \hline \end{array}$$

Estimate: 4

Note that the first front-end estimate, 7, is an *underestimate,* while the rounded estimate, 8, is an *overestimate.* The actual sum, 7.85, is between the underestimate and the overestimate.

This radar image lets you see the surface of Venus through the thick clouds that cover the planet.

Guided Practice

Round each number to the underlined place.

1. 0.1282 **2.** 1.95 **3.** 100,617.4 **4.** 57,497

Estimate each sum or difference. Identify the method you used.

5. 326 + 124 **6.** 8.39 − 3.72 **7.** 358 + 415 + 527 **8.** 874.31 − 426.95

Ask Yourself

- What digit is to the right of the underlined digit?
- Have I adjusted my front-end estimate to make it more precise?

Explain Your Thinking If you are rounding to a place in which the digit is zero, what should you do?

Go On

Practice and Problem Solving

Round each number to the underlined place.

9. 0.2$\underline{8}$8

10. 0.524$\underline{1}$7

11. 0.$\underline{1}$8475

12. 4$\underline{3}$6.546

13. 52,4$\underline{1}$7

14. 19$\underline{9}$,756

Estimate each sum or difference. Identify the method you used.

15. 1.54 + 1.2 + 2.87

16. 11.54 − 1.42

17. 0.12 + 4.89 + 0.11

18. 83.05 − 12.98

19. 5.23 + 2.69 + 0.56

20. 15.63 − 8.12

21. 56.23 − 12.11

22. 6.54 + 12.14 + 7.45

Estimate the range within which the actual answer falls.

23. 2.53 + 5.45

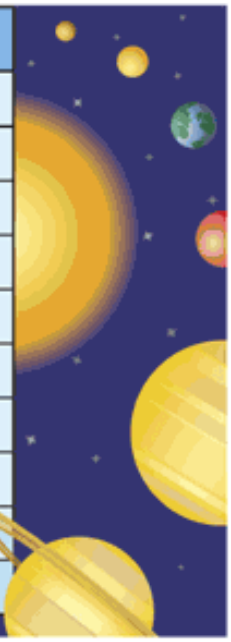
24. 8.56 − 3.41

25. 5.21 + 2.56

26. 7.85 − 4.46

Data **Use the table for Problems 27–30.**

Time to Orbit the Sun	
Planet	**Earth Years**
Mercury	0.241
Venus	0.615
Earth	1.000
Mars	1.881
Jupiter	11.862
Saturn	29.457
Uranus	84.011
Neptune	164.79
Pluto	247.65



27. To the nearest tenth, about how much longer does Mars take to orbit the Sun once than does Venus?

28. To the nearest hundredth, about how much longer does Venus take to orbit the Sun once than does Mercury?

29. To the nearest tenth, how much longer does Neptune take to orbit the Sun once than does Saturn?

30. To the nearest whole number, how many Earth years does it take Pluto to complete one orbit about the Sun?

Solve.

31. Would $5.00 be enough money to buy a pencil for $0.30, an eraser for $0.52, a notebook for $2.99, and a protractor for $1.99? Explain.

32. **What's Wrong?** Nestor says that $10^3 + 10^0 = 1{,}000$ because 10^0 equals zero. Explain Nestor's error. Then find the correct answer.

33. **Write About It** Which estimation method results in an answer that is closer to the exact sum of 7.83 and 6.5, front-end estimation or rounding to the nearest whole number? Write a paragraph to support your reasoning.

34. **Represent** A bookcase has four shelves. Each shelf is $\frac{1}{2}$-inch thick. One shelf is used for the top of the bookcase, and one is used as bottom. The space between shelves is 12 inches. Find the total height of the bookcase. Draw a diagram to help.

Extra Practice See page 29, Set C.

Problem Solving

Number Sense

Roman Numerals

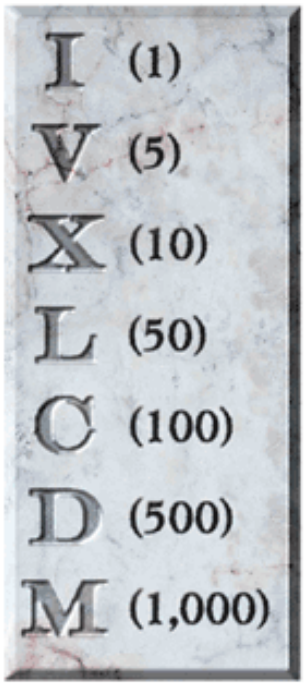
The Romans created a simple number system that does not use place value. The Roman system used repetition. When the numerals are arranged from greatest to least, add the values of each to find the total value.

XXXII = 10 + 10 + 10 + 1 + 1 = 32 CLV = 155

When a lesser numeral is written to the left of a greater numeral, the lesser numeral is subtracted from the greater one.

IX = (10 − 1) = 9 CDL = (500 − 100) + 50 = 450

XXIV = 10 + 10 + (5 − 1) = 24



Write the equivalent base-10 numeral for each.

1. XCIV **2.** DX **3.** LXVII

4. XCIX **5.** DV **6.** MMMDLVI

7. MMMDCXLIV **8.** MCMXXXIV

Quick Check

Check your understanding of Lessons 1–4.

1. Write the number two hundred forty-three ten thousandths in standard form and in expanded form, using powers of 10. (Lessons 1 and 2)

Order from least to greatest. (Lesson 3)

2. 250,413.7 250.413 25.062 250.96

3. Round 4.5$\underline{8}$7 to the underlined place. (Lesson 4)

Estimate each sum or difference. (Lesson 4)

4. 538 + 651 + 277 **5.** 47.18 − 18.45

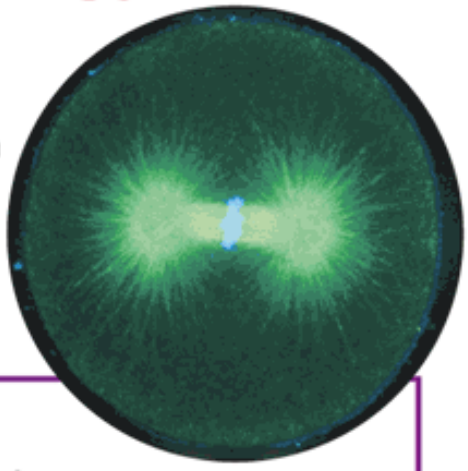
Lesson 5

Audio Tutor 1/2 Listen and Understand

Problem-Solving Strategy

Find a Pattern

Objective Find a pattern to help you solve a problem.



Problem Living cells, like those of bacteria, reproduce through a process called cell division. One cell divides to make two new cells. The two new cells in turn divide to make two cells each, in a continuous process. If this pattern continues, how many cells will there be after the sixth division?

UNDERSTAND

What is the question?

How many cells will there be after the sixth division?

What do you know?

- Each cell divides to make two new cells.
- This pattern of cell division continues.

PLAN

Look for a pattern in the number of cells after each division.
Use the pattern to determine the number of cells after the sixth division.

SOLVE

- List what you know. Look for a pattern in the number of cells.

Division:	0	1	2	3	4	5	6
Number of cells:	1	2	4	8	?	?	?

×2 ×2 ×2 ← The rule is "multiply by 2."

- Use the rule to extend the pattern.

Division:	0	1	2	3	4	5	6
Number of cells:	1	2	4	8	16	32	64

×2 ×2 ×2 ×2 ×2 ×2

Solution: There will be 64 cells after the sixth division.

LOOK BACK

Look back at the problem. Does the solution make sense?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

Ask Yourself

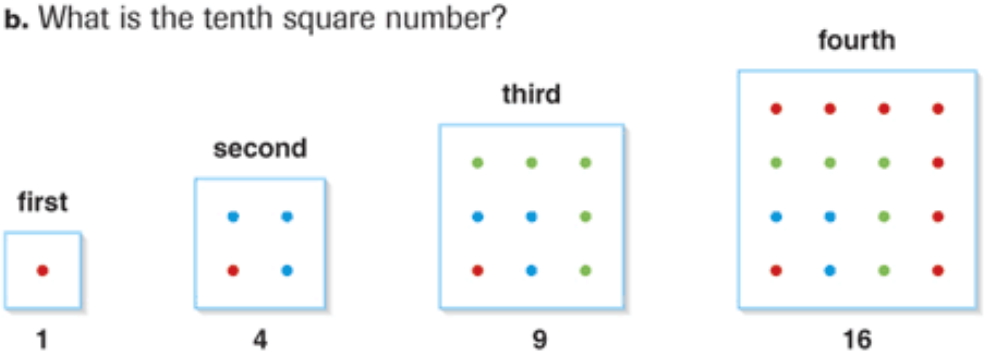
- **UNDERSTAND** — What facts do I know?
- **PLAN** — Can I find a pattern?
- **SOLVE** — How can I use the pattern to help solve the problem?
- **LOOK BACK** — Does the solution make sense?

1. A population of bacteria grown in a petri dish reached 4,096 in number on Day 12. If the population doubles each day, how many bacteria were there in the petri dish on Day 7?

2. Look at the diagram below. Square numbers are numbers that form a square pattern.

   a. What is the fifth square number?

   b. What is the tenth square number?



Independent Practice

Find a pattern to solve each problem.

3. For a science project, Steve recorded the times that sunset occurred over a one-week period, as shown in the table at right. According to this pattern, at what time will the Sun set on Saturday?

Sunset Times

Day of the Week	Sun.	Mon.	Tue.	Wed.	Thur.	Fri.	Sat.
Time	7:29	7:30	7:32	7:33	7:35	7:36	?

4. A certain bacterial population doubles in size every 6 minutes. Some bacteria are placed on a microscope slide, and in 2 hours the slide is covered. When was the slide half-covered?

5. **Create and Solve** Create a pattern that uses one or more geometric figures. Draw the first three designs in the pattern. Ask a classmate to draw the next two designs in the pattern.

6. Dana is designing a new letterhead graphic for the Natural Science club at her school. The design is made up of a pattern of dots and lines. Draw the next three likely figures in the pattern.



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Adrienne made this number pattern and challenged her brother to find the next two numbers.

6, 8, 12, 14, 18, 20, 24

If the pattern continues, what will the next two numbers likely be?

8. Your mother is planting flowers along the edges of a rectangular flowerbed. She will place the plants 6 inches apart. The perimeter of the garden measures 84 inches. How many plants will your mother need?

9. **Analyze** Samantha is 2 inches shorter than Isobel, who is 3 inches taller than Marlee. Marlee is 6 inches shorter than Noah. If Samantha is 60 inches tall, how tall is Noah?

You Choose

Strategy

- Draw a Picture
- Find a Pattern
- Guess and Check
- Make an Organized List
- Use Logical Reasoning

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data Use the graph to solve Problems 10–13.

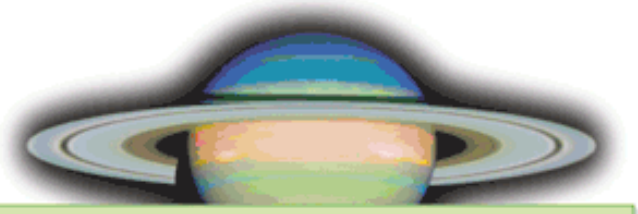
The bar graph on the right shows the average orbital speeds in miles per hour of the planets orbiting the Sun. For example, the graph shows that Earth has an average orbital speed of about 66,500 miles per hour.

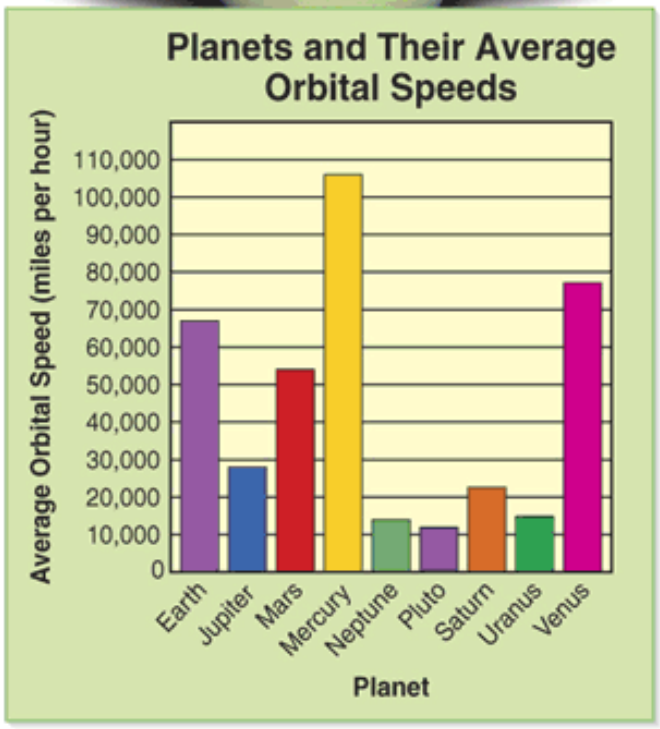
10. Which two planets have the highest average orbital speeds?

11. Approximately how much greater is the average orbital speed of Mars than that of Neptune?

12. How many planets have average orbital speeds less than 30,000 miles per hour?

13. Planets that are farther from the Sun have lower orbital speeds. List the four planets that are farthest from the Sun.





Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer. If the correct answer is not here, choose NH.

1. A baby's heart beats about 120 times per minute. How many times does a baby's heart beat in one week?

A 10,080 **C** 172,800

B 120,960 **D** 1,209,600

(Grade 5)

2. Anthony, Bob, and Carlos live in Houston, San Francisco, and Boston. Bob is the brother of the man who lives in San Francisco. Either Anthony or Carlos lives in Houston. Anthony is an only child. Which city does Anthony live in?

F Houston **H** San Francisco

G Boston **J** NH

(Grade 5)

Open Response

Solve each problem.

3. In one week, a tomato plant grew from 69 centimeters tall to 79 centimeters tall. If the plant continues to grow the same amount each week, how tall will it be 4 weeks after it was first measured?

Represent Support your solution with a picture.

(Chapter 1, Lesson 5)

4. On opening day, a new music store gave a poster to every 15th customer and a coupon for $10 off music purchases to every 18th customer. Which customer would be the first to receive both a poster and a coupon?

Explain Show how you decided.

(Grade 5)

Extended Response

5. The list shows last season's batting averages for four members of a softball team.

Season Batting Averages

Player	Average
Ada	0.201
Bernard	0.211
Cory	0.194
Denise	0.207

a. Which player has the highest average? the lowest?

b. What is the difference between the highest average and the lowest average?

c. Suppose you are also on the team. Your average is greater than Denise's but less than Bernard's. List three possible batting averages that you could have.

(Chapter 1)



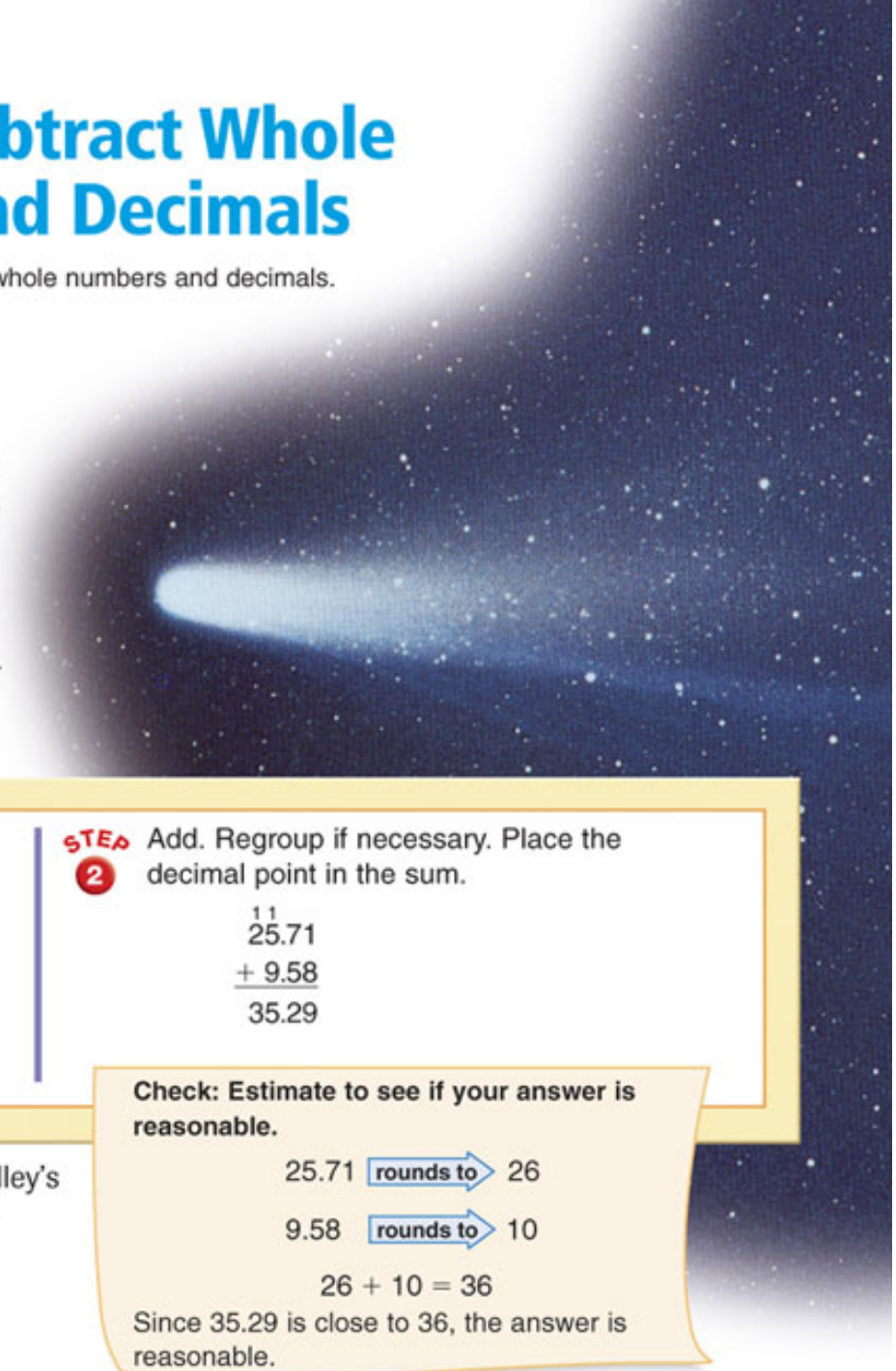
Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 6

Add and Subtract Whole Numbers and Decimals

Objective Add and subtract whole numbers and decimals.



Learn About It

When Halley's comet is at its maximum distance from the Sun, it is about 25.71 AU (astronomical units) farther from the Sun than the mean distance of Saturn from the Sun. Saturn's mean distance is about 9.58 AU. Find the maximum distance of Halley's comet from the Sun.

Add. 25.71 + 9.58 = ■

STEP 1 Align the decimal points.

$$\begin{array}{r} 25.71 \\ +\ 9.58 \\ \hline \end{array}$$

STEP 2 Add. Regroup if necessary. Place the decimal point in the sum.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 25.71 \\ +\ 9.58 \\ \hline 35.29 \end{array}$$

Solution: The maximum distance of Halley's comet from the Sun is about 35.29 AU.

Check: Estimate to see if your answer is reasonable.

25.71 rounds to 26

9.58 rounds to 10

26 + 10 = 36

Since 35.29 is close to 36, the answer is reasonable.

Other Examples

A. Add Whole Numbers

829,035 + 20,318 + 362

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 829{,}035 \\ 20{,}318 \\ +\ \ 362 \\ \hline 849{,}715 \end{array}$$

Align the digits by place value.

B. Add Three Decimals

0.3517 + 0.483 + 0.1924

$$\begin{array}{r} {\scriptstyle 1\ 2\ 1} \\ 0.3517 \\ 0.4830 \\ +\ 0.1924 \\ \hline 1.0271 \end{array}$$

Use zero as a placeholder.

Align the decimal points.

C. Subtract Decimals

2,300.61 − 584.1069

$$\begin{array}{r} {\scriptstyle 1\ 12\ 9\ 10\ \ 0\ 9\ 10} \\ 2{,}3\not{0}\not{0}.6\not{1}\not{0} \\ -\ \ 584.1069 \\ \hline 1{,}716.5031 \end{array}$$

Use two zeros as placeholders.

Guided Practice

Find each sum or difference.

1. 545.29 − 82.5727 **2.** 1,452,009 + 7,639,823

3. 85.36 + 14.2 + 5 **4.** 35.2611 − 3.6

Ask Yourself

- Did I align the digits by place value?
- Did I place a decimal point in the answer in problems involving decimals?

Explain Your Thinking Why is it important to align the decimal points when adding or subtracting?

Practice and Problem Solving

Find each sum or difference.

5. 53.86 + 7.4763 **6.** 941.8 − 7.3533 **7.** 5.36 − 2.2564 **8.** 7,852,012 + 999

9. 325.1652 + 76.4 **10.** 4,527,113 − 558.9 **11.** 8,795.3599 − 800 **12.** $3,806,321 − $342

Algebra • Patterns

Find the next two likely numbers in each sequence.

13. 2.3, 3.4, 4.5, . . . **14.** 1.9, 1.6, 1.3, . . . **15.** 80.1, 85.3, 90.5, . . . **16.** 0.02, 0.04, 0.08, . . .

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Tell which method you chose.

17. Mercury's mean distance from the Sun is 0.387 AU. Pluto's is about 39.24 AU. About how much farther away from the Sun is Pluto than Mercury?

18. An astronomy club wants to buy six new eyepieces for their telescope. Four of the lenses cost $53.98 each. Two lenses cost $21 each. What is the total cost?

19. Predict Halley's comet passes by Earth every 76 years. The two most recent visits were in 1910 and in 1986. When will Halley's comet appear twice in the same century next time?

20. The orbit of the Moon about Earth is not circular. During 2002, the distance between the Moon and Earth varied between 252,715 miles and 221,766 miles. Find the difference in distance.

Mixed Review and Test Prep

Open Response

Find each sum or difference. (Grade 5)

21. 8,343 − 3,389 **22.** 6,329 + 77,334

23. 68,004 + 53,291 **24.** 22,300 − 18,525

25. Which two of the following numbers have a sum between 9.5 and 10? (Ch. 1, Lesson 6)

8.089 1.005 1.493 1.918

Extra Practice See page 29, Set D.

Hands On Lesson 7

Audio Tutor 1/3 Listen and Understand

Variables and Expressions

Objective Model and use variables in algebraic expressions.

Vocabulary
- algebraic expression
- term

Learn About It

A numerical expression contains only numbers and operation symbols. An **algebraic expression** contains at least one variable. It may also contain numbers and operation symbols.

Phrase: the sum of 6 and a number n
Algebraic expression: $6 + n$

In the algebraic expression above, 6 and the variable n are called the **terms** of the expression.

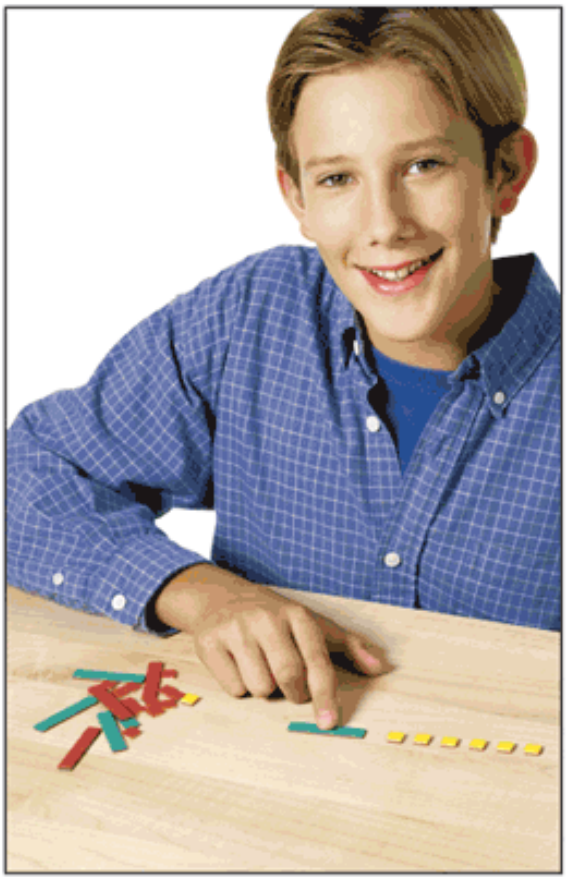
You can use algebra tiles to model both numerical and algebraic expressions.



This 1-by-1 square tile represents the number 1.



This 1-by-x rectangular tile represents the variable x.



Modeling With Algebra Tiles

▶ **Beige tiles represent ones and green tiles represent variables.**

$2 + 3$



$3 + x$



▶ **Red tiles show subtraction.**

$x - 3$

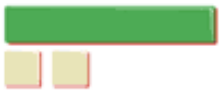


$6 - 2x$



Guided Practice

Write a numerical or algebraic expression for each model.

1. 

2. 

3. 

Ask Yourself

- How many *x*-tiles are there?
- Does the model show addition or subtraction?

Explain Your Thinking You can write $9 \times n$ as $9n$. Can you write 9×7 as 97? Explain.

Practice and Problem Solving

Use algebra tiles to model each algebraic expression. Then write each expression in words.

4. $x + x + 5$ **5.** $x - 4$ **6.** $8 - x$ **7.** $2x + 4$

Write a numerical expression to represent each phrase.

8. 8 increased by 20

9. 7 less than 100 plus 12.6

10. 15 more than 60 decreased by 25

11. 12 less than 30 increased by 10.37

Solve. Choose the expression that correctly represents each situation.

12. A club paid \$125 to rent a documentary on constellations. The club raised T dollars from ticket sales. Which expression shows how much money the club had left after paying the rental fee?

a. $125 + T$ **b.** $125 - T$ **c.** $T - 125$

13. On one night, 312 people attended a movie. On the following night, M people attended the same movie. How many people altogether attended the movie on the two nights?

a. $312 + M$ **b.** $M - 312$ **c.** $312 - M$

Mixed Review and Test Prep

Open Response

Divide. (Grade 5)

14. $7\overline{)82}$ **15.** $6\overline{)512}$ **16.** $4\overline{)1{,}264}$

17. $3\overline{)3{,}029}$ **18.** $8\overline{)7{,}226}$ **19.** $5\overline{)1{,}035}$

20. $9\overline{)5{,}811}$ **21.** $2\overline{)4{,}631}$ **22.** $4\overline{)3{,}138}$

Multiple Choice

23. Choose the expression that correctly represents the following statement:

"The star chart costs 8 dollars less than the poster P plus 2 dollars postage." (Ch. 1, Lesson 7)

A $(8 - P) + 2$ **C** $(P - 8) + 2$

B $8 - (P + 2)$ **D** $P - (8 + 2)$

Extra Practice See page 29, Set E.

Lesson 8

Audio Tutor 1/4 Listen and Understand

Use Addition Properties

Objective Evaluate expressions, using addition properties.

Vocabulary
evaluate

Learn About It

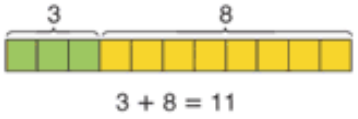
Knowing the addition properties can help you **evaluate**, or find the value of, numerical and algebraic expressions.

Properties of Addition

Commutative Property

The order in which numbers are added does not affect their sum.

$3 + 8 = 8 + 3$



$3 + 8 = 11$



$8 + 3 = 11$

The Commutative Property allows you to add two or more numbers in any order.

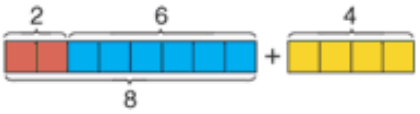
All these sums are the same.

$2 + 3 + 5$ $\quad 3 + 2 + 5$ $\quad 5 + 2 + 3$
$2 + 5 + 3$ $\quad 3 + 5 + 2$ $\quad 5 + 3 + 2$

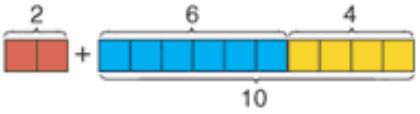
Associative Property

The way in which numbers are grouped does not affect their sum.

$(2 + 6) + 4 = 2 + (6 + 4)$



$(2 + 6) + 4 = 8 + 4 = 12$



$2 + (6 + 4) = 2 + 10 = 12$

The Associative Property allows you to group three or more numbers in any way to add them.

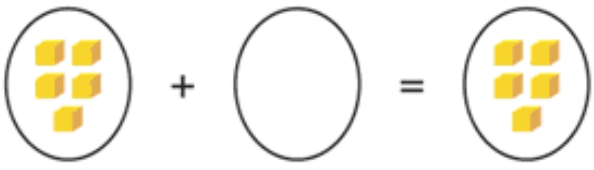
All these sums are the same.

$[(2 + 3) + 4] + 5$ $\quad 2 + [3 + (4 + 5)]$
$2 + [(3 + 4) + 5]$ $\quad [2 + (3 + 4)] + 5$

Identity Property

The sum of any number and zero is that number.

$5 + 0 = 5$



5 0 5

The properties of addition also can be shown using algebraic expressions.

Commutative Property	Associative Property	Identity Property
For any addends a and b,	For any addends a, b, and c,	For any addend a,
$a + b = b + a$	$(a + b) + c = a + (b + c)$	$a + 0 = a$

To evaluate an algebraic expression, substitute a number for the variable. Then use the addition properties.

Evaluate, given $p = 10$ and $r = 35$.

$97 + p + 3 + 25 + r$

$97 + 10 + 3 + 25 + 35$ ← Substitute for p and r.

$97 + 3 + 25 + 35 + 10$ ← Commutative Property

$(97 + 3) + (25 + 35) + 10$ ← Associative Property

$100 + 60 + 10$

170

Remember
You can use the properties to arrange numbers so that they are easy to add mentally.

Guided Practice

Complete. Identify the property you used.

1. ■ + 7 = 7 + 3

2. (3 + 2) + 5 = 3 + (■ + 5)

3. 0 + 3,250 = ■

4. (18 + 3) + 9 = ■ + (18 + 3)

Ask Yourself
- Did I arrange the numbers so they were easier to add?
- Did I identify which properties I used?

Evaluate. Identify the property or properties you used.

5. 47 + (89 + 13)

6. 43.785 + 0

7. (9 + 12.5) + 17.5

8. $p + 0$

9. $r + (r + 8)$, given $r = 5$

10. $4.5 + m + 6$, given $m = 5$

Explain Your Thinking How can using the commutative and associative properties help you use mental math to add numbers?

Practice and Problem Solving

Evaluate. Identify the property or properties you used.

11. 3 + 65 + 12

12. 78 + 10 + 12

13. 39 + 0 + 9.72

14. 6.25 + 89 + 3.75

15. 13 + 32 + a, given $a = 7$

16. 85 + (b + 6), given $b = 20$

17. $x + 43 + 3$, given $x = 57$

18. $66 + s + 14$, given $s = 3$

19. (c + 8) + 12, given $c = 45$

Go On

Use >, <, or = to make each statement true.

20. $18.4 + 0 \bigcirc 18.4$

21. $178 + a + 17 \bigcirc 178 + a + 16$

22. $12.5 + (55 + 23) \bigcirc (12.5 + 23) + 45$

23. $10 + 6 \bigcirc 10 + 15$

24. $(b + c) + (d + e) \bigcirc (b + d) + (c + e)$

25. $67 + c \bigcirc 60 + 7 + c$

26. Explain how $26 + 32 + 3 + 4 + 18$ can be rearranged to make the numbers easy to add.

27. **Analyze** Why can there not be a Commutative Property for subtraction? Use an example to explain.

Use addition properties to show that the expressions are equal. At each step, indicate what property you used.

28. $(s + 63) + 19$ and $(63 + 19) + s$

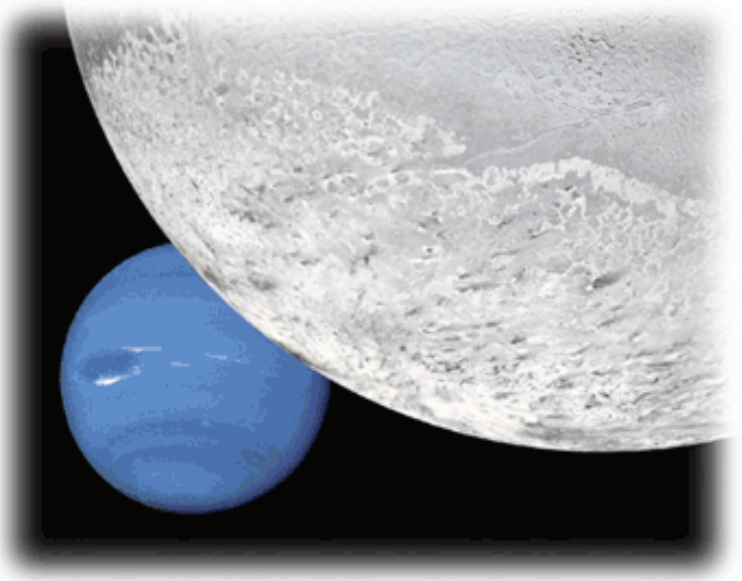
29. $p + (18 + 0)$ and $18 + p$

Solve.

30. Neptune has 11 moons. Jupiter has 61 moons. Pluto has 1 moon. How many moons do Pluto and Neptune have in all?

31. Jupiter has 61 moons, more than any other planet in the Solar System. Saturn has 31 moons. Earth has 1 moon. How many moons do the three planets have in all?

32. Nancy needs to be at school at 8:00 A.M. It takes her 0.75 hour to shower and get dressed, 0.5 hour to eat breakfast, and 15 minutes to walk to school. What time does Nancy need to get up?



The planet Neptune, seen behind its moon Triton.

33. **Write About It** While doing a puzzle, you come across this notation.

9 ✪ 7 ✇ 2	5 ✇ 8 ✪ 3
8 ✪ 6 ✇ 2	1 ✇ 1 ✪ 0

Explain what you think the symbols ✪ and ✇ mean.

Mixed Review and Test Prep

Open Response

Multiply. (Grade 5)

34. 99×5 **35.** $3{,}450 \times 3$ **36.** $8{,}456 \times 9$

37. $4{,}297 \times 4$ **38.** 925×4 **39.** $7{,}915 \times 2$

40. 895×8 **41.** $9{,}086 \times 7$ **42.** $1{,}259 \times 6$

43. Do the two expressions have the same value? (Ch. 1, Lesson 8)

$68 + 34$ and $(68 + 30) + 4$

Explain how you know.

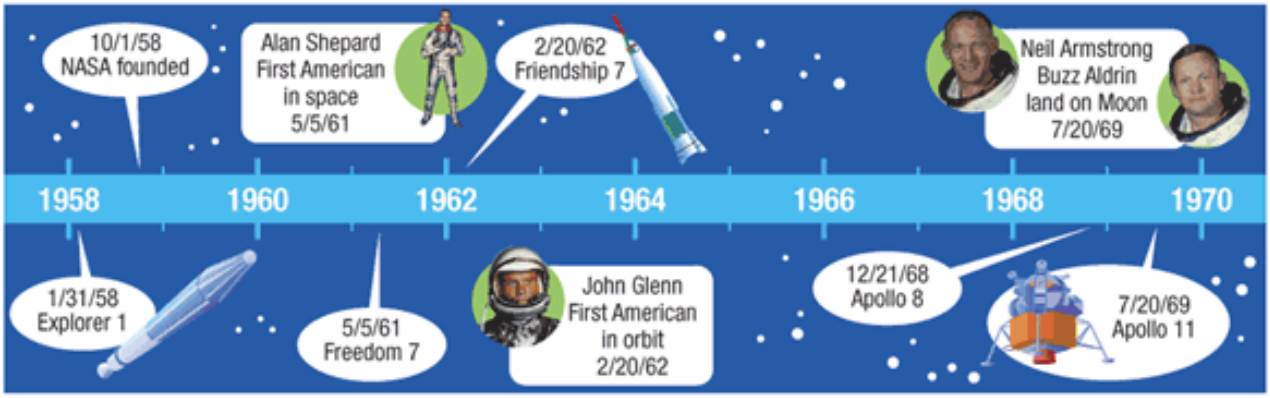
Extra Practice See page 29, Set E.

Problem Solving

Social Studies Connection

Using a Time Line

A *time line* is a graph that is used to show how events are related in time. The dates of important events in the U.S. space program, leading up to the landing on the Moon, are shown in the time line below.



1. Which two of the events happened in 1958?
2. About how many years does this time line cover?
3. **You Decide** Think about important events in your life, such as your birth, the births of your siblings, when you started school, and so on. Create your own personal time line.

Math Reasoning

Miles and Miles

Use the signpost.



1. How far, and in what direction, is Kent from York and York from Essex?
2. Two of the cities are 78.1 miles apart. Which cities are they?

Brain Teaser

Which one of the tickets does not belong?



Education Place

Check out **eduplace.com/map** for more brain teasers.

Lesson 9

Problem-Solving Decision
Estimated or Exact Answer?

Objective Review how to decide whether an estimated or exact answer is needed.



When you solve a problem, you need to decide if you need an exact answer or whether an estimate will be enough.

Problem During a visit to the National Air and Space Museum in Washington, D.C., Alex wants to buy a book on space exploration for $12.95, a space shuttle model kit for $15.95, and a mouse pad for $4.95. If Alex has $32, does he have enough money to buy all the items?

Ask Yourself

Look at the information in the problem. Will an estimate be enough?

Use mental math to estimate the cost.

$12.95 rounds to $13

$15.95 rounds to $16

$ 4.95 rounds to $ 5

Add. $13 + $16 + $5 = $34.

The estimated total cost is more than $32.

Look at the question being asked. Is an exact answer needed?

Compute the answer.

$$\begin{array}{r} \$12.95 \\ \$15.95 \\ +\ \$\ 4.95 \\ \hline \$33.85 \end{array}$$

Compare. $33.85 > $32

Solution: Alex does not have enough money to buy all three items.
Would you have used an estimate or an exact answer? Why?

Try These

Solve. Explain whether you used an estimate or an exact answer to solve each problem.

1. Alex and his family are spending 5 nights in Washington. Their hotel costs $189.95 per night. About how much should the family budget for hotel costs?
2. In the museum gift shop, small model rockets sell for $3.29 each. If you have $10.00, how much money would you have left after buying 3 model rockets?
3. Mikhala buys 6 large seashells in a nature store. Each seashell costs $2.35. If Mikhala gives the cashier $20.00, how much change should she receive?
4. Fred buys a bookmark for each of his 8 friends. The bookmarks cost $1.98 each. About how much money does Fred need to buy the bookmarks?

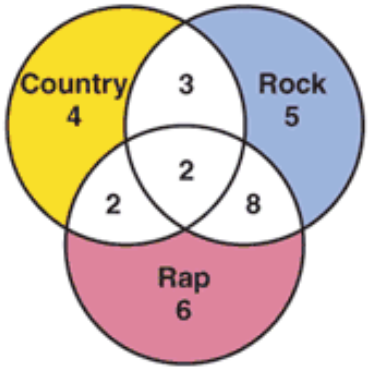
Math Challenge

Using Venn Diagrams

Activity

You know that one way to show collections of people (or objects) and their relationship to the whole group is to use a Venn diagram.

Favorite Music Types



1. Eleven students liked country, while 18 students liked rap. How many students liked rock?
2. How many students liked only rap? How many students liked both rock and country?
3. How many students liked all three types of music?
4. What was the total number of students surveyed?

Work in groups to survey the students at your school about their music preferences. Draw your own Venn diagram to show the results.

Using the Venn diagram that you drew, answer the following.

5. How many categories did you use in the survey?
6. How many students were represented where all categories overlapped?
7. What was the total number of students surveyed?
8. Which category did the students surveyed like the most? The least?
9. **Analyze** Do you think that your findings are representative of the student body as a whole? Why or why not?



Chapter Review/Test

VOCABULARY

1. A(n) ____ is the product found by multiplying the number 10 by itself one or more times.
2. When 10 is the base, a(n) ____ tells the number of times 10 is used as a factor.
3. Every ____ contains at least one variable.
4. The fourth place to the right of the decimal point is the ____ place.

Vocabulary

- exponent
- power of 10
- thousandths
- ten-thousandths
- algebraic expression

CONCEPTS AND SKILLS

Write the value of each underlined digit. Then write each number in expanded form, using powers of 10. (Lessons 1–2, pp. 4–7)

5. 5.2$\underline{7}$ **6.** $\underline{2}$5,341,000 **7.** 0.000$\underline{9}$2

Compare. Write >, <, or = for each ●. (Lesson 3, pp. 8–9)

8. 9.2 ● 9.23 **9.** 0.035 ● 0.053 **10.** 42.6 ● 4.26

Find each sum or difference. Estimate first.
(Lesson 4, pp. 10–12, Lesson 6, pp. 18–19)

11. 64.75 − 9.836 **12.** 4,032,098 + 999 **13.** 37.1 − 9.42

Write a numerical expression to represent each phrase. (Lesson 7, pp. 20–21)

14. 89 less than 127 **15.** 21 increased by x decreased by 8

Evaluate. Identify the property or properties you used. (Lesson 8, pp. 22–24)

16. 72 + 10 + 18 **17.** 41 + 0 + 19.56 **18.** n + 45 + 4, given n = 8

PROBLEM SOLVING

Solve. (Lesson 5, pp. 14–16, Lesson 9, p. 26)

19. Abner created a pattern of letters: ZA, YB, XC, WD, VE, ...
If the pattern continues, what will the next three pairs of letters be?

20. Silk flowers cost $3.79 each. If Rachel has $30, does she have enough to buy 8 flowers? Explain whether you used an estimate or an exact answer to solve.

Write About It

Show You Understand

Explain the relationship between the exponent in a power of 10 and the number of zeros in the related number. Include both positive and negative exponents.

Extra Practice

Set A (Lessons 1–2, pp. 4–7)

Write each number in word form and in expanded form, using powers of 10.

1. 201,457,060 **2.** 348.162 **3.** 5.00688 **4.** 4,325,000,000

Write each number in standard form.

5. 600,000,000,000 + 70,000,000,000 + 300,000,000 + 10,000,000 + 60

6. seventy-two and four hundred thirty-six thousandths

Set B (Lesson 3, pp. 8–9)

Compare. Write >, <, or = for each ⬭.

1. 2.315 ⬭ 2.351 **2.** 9.30 ⬭ 9.3 **3.** 7.053 ⬭ 7.035

Order from least to greatest. Use a number line.

4. 3.9 0.39 4.1 **5.** 1.702 1.72 1.027 **6.** 5.046 5.64 5.406 5.064

Set C (Lesson 4, pp. 10–12)

Round each number to the underlined place.

1. 0.47 **2.** 4,339 **3.** 101,129 **4.** 9.4688

Estimate each sum or difference. Identify the method you used.

5. 7.8 + 1.4 **6.** 36.42 − 1.44 **7.** 2,219 + 631 **8.** 312 − 287.6

Set D (Lesson 6, pp. 18–19)

Find each sum or difference.

1. 19.084 + 30.6 **2.** 426.3 − 9.07 **3.** 2,065,316 + 794 **4.** 327.85 − 9.76

Set E (Lessons 7–8, pp. 20–24)

Write an expression to represent each phrase.

1. 12 decreased by 8 **2.** 6 plus n decreased by 3 **3.** x less than 7 increased by 5

Evaluate. Identify the property or properties you used.

4. 46 + 0 + 12.4 **5.** 24 + 18 + a, given $a = 6$ **6.** b + 27 + 9, given $b = 23$

CHAPTER 2

Multiply and Divide Whole Numbers

INVESTIGATION

Using Data

The Chinese calendar is made up of five cycles of 12 years each. A different animal represents each year. When is the next year of the Rat? Use the pattern in the diagram to show what animal represents each year from 1990 to 2020. What animal will represent the year that you expect to graduate from high school?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. The answer in division is called the _____.
2. The number sentence $5 \times 1 = 5$ illustrates the _____ Property of Multiplication.
3. When two or more factors are multiplied, the resulting answer is called the _____.

Vocabulary

- divisor
- product
- Identity
- quotient
- Commutative

CONCEPTS AND SKILLS

Estimate. Tell which method you used.

4. 7×49
5. 9×152
6. $8 \times \$7.51$
7. $6 \times 4{,}679$

Find each product.

8. 5×98
9. 4×721
10. 45×56
11. 39×73

Find each quotient and remainder.

12. $562 \div 9$
13. $4{,}329 \div 8$
14. $985 \div 32$
15. $788 \div 45$

Use properties to complete. Name each property.

16. $12 \times (7 \times 5) = (\blacksquare \times 5) \times 7$
17. $26 \times 54 = 54 \times \blacksquare$
18. $245 \times \blacksquare = 0$
19. $(34 + 6) \times \blacksquare = 34 \times 17 + 6 \times 17$



20. Miguel needs to spend 12 hours next week at piano practice to get ready for a recital. Mathematically, does it make a difference whether he practices 3 hours on each of 4 days or 4 hours on each of 3 days? Draw a diagram to explain.

Facts Practice, See page 657.

Lesson 1

Audio Tutor 1/5 Listen and Understand

Estimate Products and Quotients

Objective Estimate products and quotients of whole numbers.

Vocabulary
- overestimate
- underestimate
- compatible numbers

Learn About It

When you estimate products, **overestimates** help you know that you have more than enough while **underestimates** are less than actual products.

Juneteenth is a celebration that commemorates the end of slavery in Texas on June 19, 1865. Ella needs about 1,500 bottles of strawberry soda for a Juneteenth picnic. Will 75 cartons with 24 bottles in each carton give Ella the 1,500 bottles of strawberry soda that she needs?

Different Ways to Estimate 75 × 24

Way 1

Round both factors to the nearest 10.

$$\begin{array}{rcr} 75 & \rightarrow & 80 \\ \times\ 24 & \rightarrow & \times\ 20 \\ & & 1{,}600 \end{array}$$

Way 2

Round both factors up to get an overestimate.

$$\begin{array}{rcr} 75 & \rightarrow & 80 \\ \times\ 24 & \rightarrow & \times\ 30 \\ & & 2{,}400 \end{array}$$

Way 3

Use front-end estimation to get an underestimate.

$$\begin{array}{rcr} 75 & \rightarrow & 70 \\ \times\ 24 & \rightarrow & \times\ 20 \\ & & 1{,}400 \end{array}$$

Solution: The exact product will be between the overestimate 2,400 and the underestimate 1,400. Rounding both factors to the nearest 10 gives an estimate closer to the exact product. Since 1,600 > 1,500, Ella will have enough.



Strawberry soda and barbecued meats are popular foods found at Juneteenth picnics.

You can use compatible numbers to estimate quotients. **Compatible numbers** are numbers that are easy to divide and are close to the original numbers.

Another Example

Estimate Quotients

Estimate 3,718 ÷ 43.

Use compatible numbers.

$$\begin{array}{r} 90 \\ 40\overline{)3{,}600} \end{array} \qquad \begin{array}{r} ? \\ 43\overline{)3{,}718} \end{array} \qquad \begin{array}{r} 80 \\ 50\overline{)4{,}000} \end{array}$$

Both 80 and 90 are reasonable estimates. The exact quotient will be between the underestimate of 80 and the overestimate of 90.

Guided Practice

Estimate each product or quotient. Identify the method you used.

1. 46×88 **2.** 34×319 **3.** 689×943

4. $7\overline{)477}$ **5.** $12\overline{)634}$ **6.** $45\overline{)2{,}856}$

Ask Yourself

- Did I round the numbers correctly?
- Are the compatible numbers I chose close to the actual numbers?

Explain Your Thinking Do you think your estimate for Exercise 6 is an overestimate or an underestimate?

Practice and Problem Solving

Find an overestimate and an underestimate. Then tell which you think will be closer to the actual product. Check your work.

7. 39×37 **8.** $59 \times 2{,}854$ **9.** 515×309

Estimate using compatible numbers.

10. $39\overline{)897}$ **11.** $15\overline{)4{,}910}$ **12.** $33\overline{)6{,}745}$

Solve.

13. Twenty-six families each bring a dozen cookies to share at the Juneteenth picnic. What is a reasonable estimate of how many cookies that is in all?

14. Ella orders 1,944 rolls for barbecue sandwiches for the picnic. The rolls come on trays with 36 rolls on a tray. About how many trays of rolls does Ella order?

15. **Predict** Which estimate for 73×456 will be closer to the exact product? Use a calculator to check.

a. 70×500 **c.** 100×456

b. 80×500 **d.** 100×500

16. Which digit belongs in the box to make the equation true?

$\square 3 \times 98 = 5{,}194$

Mixed Review and Test Prep

Open Response

Write a number that is 1,000 more than and a number that is 1,000 less than each given number. (Ch. 1, Lesson 1)

17. 6,754 **18.** 19,208

19. 40,016 **20.** 99,650

21. 200,574 **22.** 1,404,036

Multiple Choice

23. If the quotient is 84 and the dividend is 1,932, which could be the divisor? (Ch. 2, Lesson 1)

A 230 **C** 23

B 32 **D** 19

Extra Practice See page 55, Set A.

Lesson 2

Multiply Whole Numbers

Objective Multiply whole numbers.



The idea of celebrating Fourth of July with fireworks was suggested by John Adams.

Learn About It

The Fireworks Committee in a city has planned a 25-minute display to celebrate Fourth of July. The plan calls for an average of 96 fireworks per minute. How many fireworks will be used in all?

Estimate the product first.

96×25 rounds to 100×25 or 2,500

Since one factor is rounded up, the product is an overestimate.

Find 96×25.

STEP 1 Multiply by the ones digit. Regroup if necessary.

$$\begin{array}{r} {\scriptstyle 3}\\ 96\\ \times\ 25\\ \hline 480 \end{array} \leftarrow 96 \times 5$$

STEP 2 Multiply by the tens digit. Regroup if necessary.

$$\begin{array}{r} {\scriptstyle 1}\\ 96\\ \times\ 25\\ \hline 480\\ 1920 \end{array} \leftarrow 96 \times 20$$

STEP 3 Add the partial products.

$$\begin{array}{r} {\scriptstyle 1}\\ 96\\ \times\ 25\\ \hline 480\\ +\ 1920\\ \hline 2{,}400 \end{array}$$

Check. Compare your estimate to your answer. Since 2,400 is close to 2,500, the answer is reasonable.

Solution: The display will use 2,400 fireworks.

Other Examples

A. Multiply by a Three-Digit Number

$$\begin{array}{rl} 831 & \\ \times\ 749 & \\ \hline 7\,479 & \leftarrow 831 \times 9 \\ 33\,240 & \leftarrow 831 \times 40 \\ +\ 581\,700 & \leftarrow 831 \times 700 \\ \hline 622{,}419 & \end{array}$$

B. Use Expanded Form

$$\begin{aligned} 831 \times 749 &= 831 \times (700 + 40 + 9) \\ &= (831 \times 700) + (831 \times 40) + (831 \times 9) \\ &= 581{,}700 + 33{,}240 + 7{,}479 \\ &= 622{,}419 \end{aligned}$$

Guided Practice

Find each product. Estimate to see if your answer is reasonable.

1. $\begin{array}{r} 56 \\ \times\ 27 \\ \hline \end{array}$ **2.** $\begin{array}{r} 4{,}902 \\ \times\ 23 \\ \hline \end{array}$ **3.** $\begin{array}{r} 181 \\ \times\ 413 \\ \hline \end{array}$

4. 22×28 **5.** 45×714 **6.** 53×820

Ask Yourself

- Did I estimate to see if my answer is reasonable?
- Did I use zeros as placeholders in the partial products?

Explain Your Thinking When a factor has two or more digits, why do you place zeros in the partial products?

Practice and Problem Solving

Find each product. Estimate to see if your answer is reasonable.

7. $\begin{array}{r} 78 \\ \times\ 38 \\ \hline \end{array}$ **8.** $\begin{array}{r} 67 \\ \times\ 22 \\ \hline \end{array}$ **9.** $\begin{array}{r} 59 \\ \times\ 78 \\ \hline \end{array}$ **10.** $\begin{array}{r} 804 \\ \times\ 19 \\ \hline \end{array}$ **11.** $\begin{array}{r} 658 \\ \times\ 53 \\ \hline \end{array}$

12. 395×46 **13.** $2{,}027 \times 23$ **14.** $3{,}708 \times 329$ **15.** 843×796

Mental Math **Find each product, using mental math.**

16. $\begin{array}{r} 21 \\ \times\ 20 \\ \hline \end{array}$ **17.** $\begin{array}{r} 820 \\ \times\ 300 \\ \hline \end{array}$ **18.** $\begin{array}{r} 3{,}020 \\ \times\ 40 \\ \hline \end{array}$ **19.** $\begin{array}{r} 7{,}500 \\ \times\ 20 \\ \hline \end{array}$ **20.** $\begin{array}{r} 440 \\ \times\ 200 \\ \hline \end{array}$

Solve.

21. A marching band with 35 rows of musicians led the Fourth of July parade. If there were 12 musicians in each row, how many members were in the marching band?

22. Suppose the band in Problem 21 had 42 rows. How many more members would there be in the band? How did you find your answer?

23. **What's Wrong?** Sasha multiplied 425×553 and got 5,525. What did she do wrong? What should the answer be?

Mixed Review and Test Prep

Open Response

Identify the place value of the digit 4 in each number. (Ch. 1, Lesson 1)

24. 1,246,318 **25.** 64,382,995

26. 723,409,356 **27.** 42,577,800

28. 92,364,126 **29.** 1,982,416

30. For the Fourth of July concert, 176 rows of 36 seats were set up. For extra seating, 48 rows of 24 seats each were set up. How many seats were set up in all?

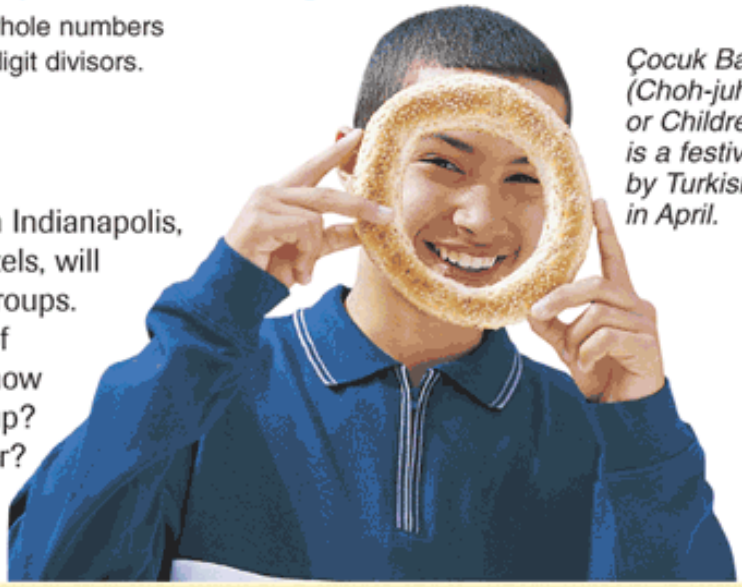
Explain how you got your answer. (Ch. 2, Lesson 2)

Extra Practice See page 55, Set B.

Lesson 3

Divide by One-Digit Divisors

Objective Divide whole numbers by one-digit divisors.



Çocuk Bayrami (Choh-juhk Bai-RAM-uh), or Children's Day, is a festival celebrated by Turkish communities in April.

Learn About It

At the Children's Day festival in Indianapolis, 1,483 *simit,* or soft, round pretzels, will be served to each of 7 lunch groups. If there are the same number of children in each lunch group, how many children are in each group? How many simit will be left over?

Find 1,483 ÷ 7.

STEP 1 There are not enough thousands to divide.

```
   200
7)1,400
```

Place the first digit in the hundreds place.

STEP 2 Divide the hundreds.

```
   2
7)1,483
- 1 4      ← Multiply. 2 × 7 = 14
  0         Subtract. 14 − 14 = 0
            Compare. 0 < 7
```

STEP 3 Bring down the tens digit. Divide the tens.

```
   21
7)1,483
- 1 4
   08
  − 7      ← Multiply. 1 × 7 = 7
   1         Subtract. 8 − 7 = 1
             Compare. 1 < 7
```

STEP 4 Bring down the ones digit. Write the remainder.

```
   211 R6
7)1,483
- 1 4
   08
  − 7
    13     ← Multiply. 1 × 7 = 7
  − 7        Subtract. 13 − 7 = 6
     6       Compare. 6 < 7
```

Solution: There are 211 children in each group. Six simit will be left over.

Check your work.
Multiplication and division are inverse operations. Multiply to check division. Then add any remainder.

$211 \times 7 = 1{,}477$

$1{,}477 + 6 = 1{,}483$

Another Example

Zero in the Quotient

```
  502 R3
6)3,015
- 30
   015   ← Bring down the 5.
  − 12
     3
```

1 < 6. Write 0 in the quotient.
Then continue to divide.

Guided Practice

Ask Yourself
- Where should I place the first digit in the quotient?
- Did I check my answer?

Divide. Then check your work.

1. $6\overline{)396}$
2. $7\overline{)641}$
3. $2\overline{)1,423}$
4. $505 \div 3$
5. $2{,}785 \div 9$
6. $14{,}876 \div 9$

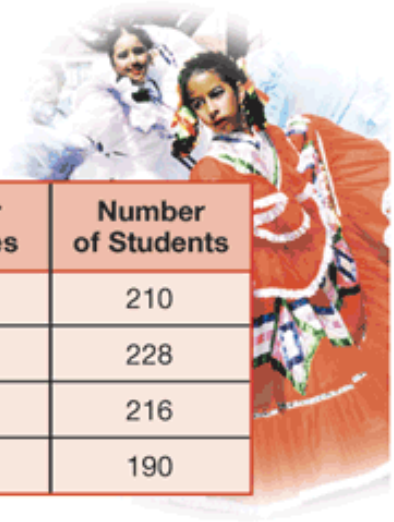
Explain Your Thinking After subtracting, what does it mean if the difference is greater than the divisor?

Practice and Problem Solving

Divide. Then check your work.

7. $8\overline{)367}$
8. $2\overline{)902}$
9. $4\overline{)326}$
10. $6\overline{)708}$
11. $44{,}432 \div 4$
12. $35{,}012 \div 3$
13. $28{,}752 \div 6$
14. $10{,}800 \div 4$

Data **School children participated in a children's festival. Use the table for Problems 15 and 16.**



Grade	Number of Classes	Number of Students
1	6	210
2	7	228
3	6	216
4	5	190

15. The second graders formed 6 equal rings to participate in dances at the festival. How many children were in each ring?
16. **Analyze** In each grade, each class has about the same number of children. Which grade has the greatest number of children in each class?
17. At the festival, 292 children were sitting at tables, seating 8 or 12. All seats were filled. There were 15 tables with 12 seats. How many tables with 8 seats were there?

Mixed Review and Test Prep

Open Response

Add or subtract. (Ch. 1, Lesson 6)

18. $6.4 + 2.8$
19. $0.09 + 1.254 + 0.4$
20. $3 - 1.28$
21. $0.26 - 0.085$
22. $8.6 + 2.4$
23. $22.04 + 1.16$
24. $10 - 0.9$
25. $2.01 - 1.02$

Multiple Choice

26. On Saturday morning, the odometer in Mr. Webb's car read 52,688 miles. On Sunday evening it read 53,166 miles. If he drove the same distance both days, how far did Mr. Webb drive each day? (Ch. 2, Lesson 3)

A 234 miles **C** 244 miles

B 239 miles **D** 478 miles

Extra Practice See page 55, Set C.

Lesson 4

Divide by Two-Digit Divisors

Vocabulary
quotient

Objective Divide whole numbers by two-digit divisors.

Learn About It

Each March people flock to Miami, Florida, for El Festival de la Ocho or Calle Ocho.

Lillian works for a company that sponsors the festival. She is busy organizing souvenir key chains in batches of 24. Lillian has 1,460 key chains. How many batches of 24 key chains can she make?



Calle Ocho is the largest Hispanic cultural celebration in the United States. The festival is sponsored by over 100 major corporations.

Divide. 1,460 ÷ 24 = *n*.

STEP 1 Estimate the **quotient**, using compatible numbers.

Think:

$$\begin{array}{r} 70 \\ 20\overline{)1{,}400} \end{array} \qquad \begin{array}{r} ? \\ 24\overline{)1{,}460} \end{array} \qquad \begin{array}{r} 50 \\ 30\overline{)1{,}500} \end{array}$$

The quotient is between 50 and 70.

STEP 2 Place the first digit in the quotient in the tens place. Try 7.

$$\begin{array}{r} 7 \\ 24\overline{)1{,}460} \\ -\ 1\,68 \end{array} \leftarrow \text{Since } 168 > 146, \text{ try } 6.$$

STEP 3 Divide the tens.

$$\begin{array}{r} 6 \\ 24\overline{)1{,}460} \\ -\ 1\,44 \\ \hline 2 \end{array}$$

Multiply. $6 \times 24 = 144$
Subtract. $146 - 144 = 2$
Compare. $2 < 24$

STEP 4 Bring down the ones digit. Divide the ones. Write any remainder.

$$\begin{array}{r} 60\ \text{R20} \\ 24\overline{)1{,}460} \\ -\ 1\,44\downarrow \\ \hline 20 \\ -\ 0 \\ \hline 20 \end{array}$$

Since $20 < 24$, write 0 in the quotient.

Multiply. $0 \times 24 = 0$
Subtract. $20 - 0 = 20$
Compare. $20 < 24$

Check your work.
Multiply. Then add.

$60 \times 24 = 1{,}440$
$1{,}440 + 20 = 1{,}460$

Solution: Lillian can make 60 batches of key chains. There will be 20 key chains left over.

Another Example

First Estimate of the Quotient Is Too Small

Find $16\overline{)8{,}608}$.

Think
$20\overline{)8{,}000}$
400

$$\begin{array}{r} 4 \\ 16\overline{)8{,}608} \\ -\ 6\,4 \\ \hline 2\,2 \end{array}$$
← 22 > 16, so try 5.

$$\begin{array}{r} 538 \\ 16\overline{)8{,}608} \\ -\ 8\,0 \\ \hline 60 \\ 48 \\ \hline 128 \\ 128 \\ \hline 0 \end{array}$$

Guided Practice

Estimate the quotients, using compatible numbers. Write the compatible numbers you used.

1. 168 ÷ 18 **2.** 4,115 ÷ 62 **3.** $25\overline{)14{,}883}$

Ask Yourself
- Where should I place the first digit in the quotient?
- Do I need to adjust my first estimate?

Divide. Then use estimation or multiplication to check your answer.

4. $13\overline{)1{,}562}$ **5.** $54\overline{)2{,}978}$ **6.** 789 ÷ 32 **7.** 14,883 ÷ 25

Explain Your Thinking How do you decide whether or not the digit you try in the quotient is correct?

Practice and Problem Solving

Estimate the quotients, using compatible numbers. Write the compatible numbers you used.

8. $10\overline{)194}$ **9.** $58\overline{)1{,}200}$ **10.** 7,103 ÷ 43 **11.** 84,013 ÷ 93

Divide. Then use estimation or multiplication to check your answer.

12. $13\overline{)303}$ **13.** $17\overline{)565}$ **14.** 862 ÷ 82 **15.** 2,006 ÷ 33

16. 6,523 ÷ 33 **17.** 7,694 ÷ 95 **18.** 11,686 ÷ 11 **19.** 12,500 ÷ 25

Mental Math **Divide, using mental math.**

20. $12\overline{)600}$ **21.** $70\overline{)840}$ **22.** 8,808 ÷ 44 **23.** 75,500 ÷ 25

Go On

Calculator Place the numbers 31, 42, 53, and 64 in the boxes to make each division sentence true. In each case the remainder is 0.

24. $\square\overline{)1{,}643}$ **25.** $\square\overline{)2{,}688}$ **26.** $\square\overline{)1{,}984}$ **27.** $\square\overline{)2{,}226}$

Algebra • Functions

Find each missing value of *y*.

28.

Rule: $y = x \div 33$				
x	396	660	1,056	2,574
y	■	■	■	■

29.

Rule: $y = 3{,}384 \div x$				
x	2	4	8	9
y	■	■	■	■

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Data

Use the table for Problems 30–32. Then tell which method you chose.

Kind of Souvenir	Number of Each
Pen	1,032
Magnet	1,440
Key Chain	900



30. The table shows the number of souvenirs Lillian has on display at the Calle Ocho Festival. Lillian puts rubber bands around each dozen pens. How many rubber bands does she use?

31. Analyze Souvenir magnets come in boxes of 5 dozen. So far, Lillian has unpacked 600 magnets. How many more boxes of magnets does Lillian have to unpack for her display?

32. You Decide Choose the number of each kind of souvenir Lillian should use for a Festival souvenir bag. Then find how many such bags Lillian can make. Explain how you arrived at your answer.

33. Ana visited the Calle Ocho with her friends. She spent $8.75 on food, $12 on a T-shirt, and $4.95 on a poster. How much money did Ana spend?

34. Mental Math The numbers 29, 31, 28, 32, 33, and 30 cluster around 30.

$29 + 31 + 28 + 32 + 33 + 30 = ?$

How can you use multiplication to estimate the sum?

Extra Practice See page 55, Set C.

Game

Activity

Fan Tan: A Chinese Game of Division

2–4 players

What You'll Need • game board (Learning Tool 30) • beans • chopstick • score sheet (Learning Tool 12)

How to Play



1. Give each player a number 0 to 3. Place the game board on a desk or tabletop. Players sit by their given numbers.
2. One player places a handful of beans in a pile on the game board.
3. Another player uses the chopstick to divide the beans into four equal piles. How many beans are left over?
4. If your given number equals the number of beans left over, you earn one point. Record the result of each round on the score sheet. Keep playing until one player has 5 points.

Quick Check

Check your understanding of Lessons 1–4.

Estimate the product or quotient. (Lesson 1)

1. 54×126 **2.** $38 \times 2{,}893$ **3.** $404 \div 16$ **4.** $3{,}536 \div 62$

Multiply. Check your answer. (Lesson 2)

5. 45×16 **6.** $38 \times 2{,}341$ **7.** 326×403

Divide. Estimate to see if your answer is reasonable. (Lessons 3 and 4)

8. $7\overline{)54}$ **9.** $92\overline{)8{,}385}$ **10.** $27\overline{)13{,}708}$

Lesson 5

Problem-Solving Strategy

Work Backward

Objective Solve a problem by working backward.

Problem Bao and his brother sold moon cakes, or Bánh Nuong *(Bang Neung)*. Bao sold 10 of his boxes of moon cakes. Each brother started with the same number of boxes to sell. They started with 864 moon cakes, packed in 24 boxes of equal size. How many moon cakes does Bao have left to sell?



The Vietnamese Mid-Autumn Moon Festival is called Tet Trung Thu (Ted Jung Too).

UNDERSTAND

What facts do you know?

- Bao sold 10 of his boxes.
- Each brother started with the same number of boxes.
- They started with 864 moon cakes, packed in 24 equal boxes.

PLAN

You can start with what you know and work backward to solve the problem.

Because inverse operations "undo" each other, use inverse operations to work backward.

Addition and subtraction are inverse operations. Multiplication and division are inverse operations.

SOLVE

Start with the total number of moon cakes. Then work backward.

- Divide to find the number of moon cakes in each box.
  $864 \div 24 = 36$
- Divide by 2 to find how many boxes each brother started with.
  $24 \div 2 = 12$
- Subtract to find the number of boxes Bao has left.
  $12 - 10 = 2$
- Multiply to find the number of cakes in 2 boxes.
  $2 \times 36 = 72$

Solution: Bao has 72 more moon cakes to sell.

LOOK BACK

Look back at the problem.

Did you answer the question that was asked?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Linh sold 3 times as many lanterns in the first hour as in the second. She sold 84 more lanterns in the second hour than in the third. She sold her last 29 lanterns in the third hour. How many lanterns did Linh sell in all?

   Hint Should you start with the number she sold in the first hour or in the third hour?

2. A food stall sold twice as many moon cakes as lotus seed paste cakes. It sold three times as many lotus seed paste cakes as mung bean cakes. If it sold 72 mung bean cakes, how many cakes did the stall sell?

Ask Yourself

UNDERSTAND
- What facts do I know?

PLAN
- Can I solve the problem by working backward?

SOLVE
- Did I start with the facts I know?
- Did I need to use inverse operations?
- Did I work backward through the facts in the problem?

LOOK BACK
- Did I answer the question?

Independent Practice

Work backward to solve each problem.

3. A vendor at the Tet Trung Thu festival has 12 more star lanterns than fish lanterns. He has twice as many fish lanterns as butterfly lanterns. How many star lanterns does the vendor have if he has 25 butterfly lanterns?

4. Team *A* has half as many dancers as Team *B*. Team *B* had 16 more dancers than Teams *C* and *D* combined. Team *C* has 3 times as many dancers as Team *D*. Team *D* has 6 dancers. How many dancers are on Team *A*?

5. In the parade there were four times as many drummers as there were unicorn dancers. There were twice as many cymbal players as drummers. If there were 56 cymbal players, how many people were in the parade in all?

6. At a Tet Trung Thu festival, 3 time slots are used for traditional storytelling and songs. If the festival lasts 4 hours and each time slot lasts 30 minutes, how many hours and minutes are left for other activities?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. At the festival, Vân displays toy boats in rows, with one row above the other. Each row has one boat less than the row below it. The top row has 1 boat, and the bottom row has 6 boats. How many boats are on display?

8. One mask for the festival costs $14, another kind costs $18, and a third kind costs $9. If Aimee started with $38 and has $2 left, which two masks did she buy?

9. **Analyze** Sue bought four boxes of candles at $3.95 each and a lantern for $10.95. Gary bought three moon masks at $7.50 each and a toy boat for $8.95. Did Sue or Gary spend more money? How much more?

You Choose

Strategy
- Find a Pattern
- Guess and Check
- Solve a Simpler Problem
- Use Logical Reasoning
- Write an Equation

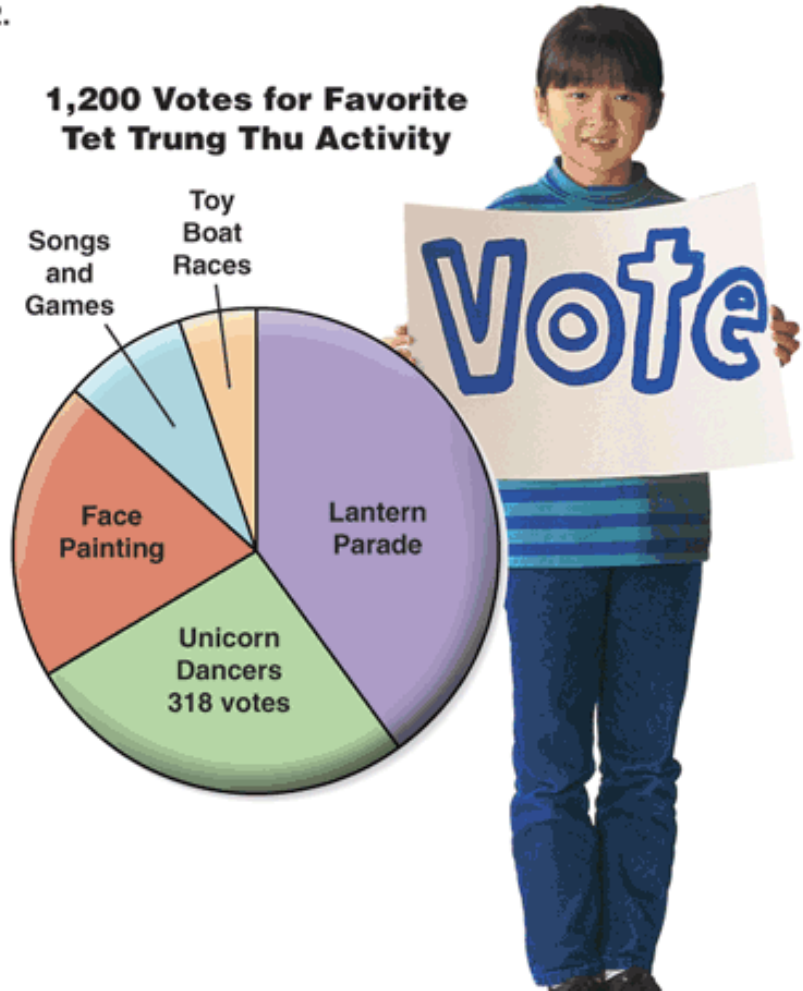
Computation Method
- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the graph to solve Problems 10–12.**

10. **Reasoning** The number of students who voted for the most popular activity is 402 less than the total number of votes minus the number of votes for the Unicorn Dancers. What is the most popular activity? How many votes did it get?

11. Face painting received half the number of votes as did the most popular activity. How many votes is that?

12. **Predict** Suppose all 3,600 students in this school voted, and the results were similar to these. Would you expect that the Unicorn Dancers would get about 1,000 votes? Why or why not?



Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. The table below shows Penny's bank balance for the past 6 weeks. If the pattern continues, what will her balance be in Week 8?

Week	Balance
1	$112
2	$142
3	$135
4	$165
5	$158
6	$188

A $175 **C** $211

B $204 **D** $241

(Chapter 1, Lesson 5)

2. Evaluate $a + b$ for $a = 2.5$ and $b = 0.08$.

F 1.7 **H** 2.42

G 2.3 **J** 2.58

(Chapter 1, Lesson 8)

Open Response

Solve each problem.

3. A principal put 228 sixth-graders into groups of 38 for their homerooms. How many homerooms are there?

Represent Write an equation showing your work.

(Chapter 2, Lesson 4)

4. Amber is in a movie that is 2 hours 32 minutes long. Her scene begins 44 minutes after the start of the movie. After her scene, 1 hour 36 minutes remain. How long is her scene? Explain how you decided.

(Chapter 2, Lesson 5)

Constructed Response

5. You are in charge of renting tables for a school dinner. There will be 1,518 students and 132 adults attending. Students and adults will sit at different tables.

a. Decide whether you will rent tables that seat 6 people each or 4 people each. How many tables will you need?

b. Will any table have empty seats? Explain.

c. It costs less to rent tables that seat 10 people. Can you take advantage of the special price and also have full tables?

d. There are also tables that seat 24 people. If you decide to have students and adults sit together, could you fill all these tables? Explain.

(Chapter 2)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 6

Audio Tutor 1/6 Listen and Understand

Use Multiplication Properties



Objective Use properties of multiplication to evaluate expressions.

Learn About It

In Tennessee, National Arbor Day is observed on the first Friday in March. Sixth-graders in Memphis, Tennessee, planted saplings, or baby trees. They planted 5 rows of 6 saplings and 5 rows of 4 saplings. How many saplings did they plant?

Find $(5 \times 6) + (5 \times 4)$.

Properties of Multiplication

Commutative Property
Changing the order of factors does not change their product.

$a \times b = b \times a$
Example: $3 \times 20 = 20 \times 3$

Associative Property
Changing the grouping of factors does not change their product.

$a \times (b \times c) = (a \times b) \times c$
Example: $3 \times (4 \times 9) = (3 \times 4) \times 9$

Identity Property
The product of any number and 1 is that number.

$f \times 1 = f$ and $1 \times f = f$
Example: $45 \times 1 = 45$ $1 \times 45 = 45$

Zero Property
The product of any number and 0 is 0.

$g \times 0 = 0$ and $0 \times g = 0$
Example: $15 \times 0 = 0$ $0 \times 15 = 0$

Distributive Property
The product of a factor and a sum (or difference) equals the sum (or difference) of the products.

$a \times (b + c) = (a \times b) + (a \times c)$
Example: $8 \times (20 + 4) = (8 \times 20) + (8 \times 4)$

Solution: Use the distributive property to make the problem easier to solve: $(5 \times 6) + (5 \times 4) = 5 \times (6 + 4) = 5 \times 10 = 50$.
The sixth-graders planted 50 saplings.

Other Examples

A. Use Properties to Evaluate Expressions

Evaluate $30 \times n \times 4$, given $n = 5$.
$30 \times 5 \times 4$ ← Substitute.
$30 \times (5 \times 4)$ ← Associative Property
30×20 ← Multiply.
600

B. Different Ways to Write Multiplication

Here are four ways to write 5 times n.
$5 \times n$ $5 \cdot n$ $5(n)$ $5n$

Extra Help at eduplace.com/map

Guided Practice

Ask Yourself

- Did I use multiplication properties to help me do mental math?

Use properties to complete. Identify each property.

1. $(3 \times 12) \times 30 = 3 \times (12 \times \square)$
2. $a \times (65 + 3) = (a \times \square) + (a \times 3)$
3. $785 \times \square = 785$
4. $43 \times 0 = \square$

Given $n = 12$, evaluate each expression.

5. $1 \cdot (n \cdot 4)$
6. $(2 \times n) - (2 \times 5)$
7. $4n \times 0$

Explain Your Thinking How can you evaluate the expression $a \times b \times c \times 0$ without substituting or multiplying?

Practice and Problem Solving

Use properties to complete. Identify each property.

8. $(24 \times 16) + (\square \times 4) = 24\ (16 + 4)$
9. $4 \times (16 \times 1) = 4 \times \square$
10. $3 \times 24 \times 2 = 3 \times \square \times 24$
11. $(15 \times 4) \times 2 = \square \times (4 \times 2)$

Evaluate each expression, given $p = 3.5$, $q = 24$, and $s = 10$.

12. $(s \times 1) \div 2$
13. $5(p + 2.5)$
14. $(q \times 6) - (q \times 3)$
15. $(0 \times p) + qs$

Compare. Write >, <, or = for each ⬭.

16. $(6 \times 5) + 30 \text{ ⬭ } (6 \times 30) + (6 \times 5)$
17. $(18 \cdot 20) \cdot 5 \text{ ⬭ } 18 \cdot (20 \cdot 5)$
18. $1.4(a + b) \text{ ⬭ } 1.4a + 1.4b$
19. $(a \times 6) - (a \times b) \text{ ⬭ } a(6 - b)$

Solve.

20. **What's Wrong?** Mrs. Walters wrote the expression $3x$ on the board. Mike evaluated the expression for $x = 4$ this way.

$$3x = 34$$

What did Mike do wrong? What should the value be?

21. **Represent** Grade 6 planted 10 times the number of trees that Grade 4 planted. Grade 4 planted twice as many trees as Grade 1 did. Grade 1 planted n trees. Which expression below represents the total number of trees planted? Explain.

22. **Reasoning** A number n is twice the product of 10 and 3. Write an expression for n. Then find the value of n.

$10 \times (2 \times n)$ or $n + 2n + 20n$

Mixed Review and Test Prep

Open Response

Evaluate each expression, given $a = 9$, $b = 2.5$, and $c = 2$. (Ch. 1, Lesson 8)

23. $b + c$
24. $a - b$
25. $a - (b + c)$
26. $(a + b) - c$

27. What number completes the following equation? (Ch. 2, Lesson 6)

$(6 \times 17) + (6 \times 3) = 6\ (\square + 3)$

Show how you know that this number will make the equation true.

Extra Practice See page 55, Set D.

Lesson 7

Audio Tutor 1/7 Listen and Understand

Order of Operations

Objective Evaluate expressions, using the order of operations.

Vocabulary

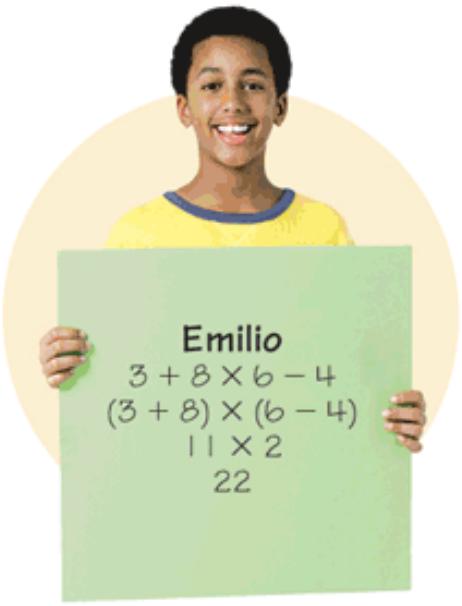
evaluate

Learn About It

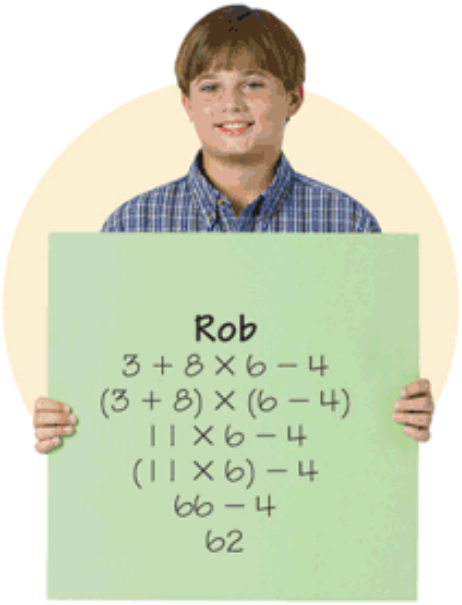
When evaluating expressions, it is important to use the order of operations correctly. Emilio, Inez, and Rob evaluated the expression $3 + 8 \times 6 - 4$, but they each got a different result. Which student used the order of operations correctly?

Order of Operations

1. Always do operations in parentheses first.
2. Rewrite any exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.







Solution: Inez used the order of operations correctly. First she multiplied. Then she added and subtracted from left to right.

One easy way to remember the order of operations is to use a sentence like this.

Please	Excuse	My	Dear	Aunt	Sally.
↑	↑	↑	↑	↑	↑
Parentheses	Exponents	Multiplication	Division	Addition	Subtraction

Other Examples

A. Two Operations in Parentheses

Evaluate $4 + (6 + 12 \div 3)$.

$4 + (6 + 12 \div 3)$ ← Divide within parentheses.
$4 + (6 + 4)$ ← Add within parentheses.
$4 + 10$ ← Add.
14

B. Exponents and Parentheses

Evaluate $(4 \times 3^2) \times 6 \div 2^2$.

$(4 \times 3^2) \times 6 \div 2^2$ ← Rewrite exponent in parentheses.
$(4 \times 9) \times 6 \div 2^2$ ← Multiply within parentheses.
$36 \times 6 \div 2^2$ ← Rewrite exponent.
$36 \times 6 \div 4$ ← Multiply.
$216 \div 4$ ← Divide.
54

Guided Practice

Ask Yourself
- Did I use the order of operations correctly?
- Did I indicate the operations I used at each step?

Evaluate each expression. Indicate the order of operations you performed.

1. $7 + 3 \times 6$ **2.** $6 - 3 \times 4 \div 4$ **3.** $3^2 + (4 - 1)^2 \times 5$

Evaluate each expression, given $a = 2$, $b = 3$, and $c = 5$.

4. $36 - (a + b \times c)$ **5.** $a^2 + b^2 \times c^2$ **6.** $60 \times a^2 \div 2 \times b \times c$

Explain Your Thinking When evaluating expressions, why is it important to have rules for the order of operations?

Practice and Problem Solving

Evaluate. Indicate the order of operations you performed.

7. $4^2 \div 8 + 4$ **8.** $(32 - 27)^2 \times 2$ **9.** $3 \times (2 + 8)^2 - 5 \times (4 - 3)$

Evaluate each expression, given $m = 1$, $s = 4$, and $t = 10$.

10. $t \div (s + m) \times t^2$ **11.** $(s^2 - m \times t)^2 + 41$ **12.** $40 \times t - (s \div m + 11 \times 6)$

Insert parentheses to make each pair of expressions equal.

13. $3 + 4 \times 6 = 42$ **14.** $2 + 3^2 = 25$ **15.** $4 \div 4 \times 4 \div 4 = 1$

16. $4 + 4 \div 4 + 4 = 1$ **17.** $4 + 4 \div 4 + 4 = 9$ **18.** $3 \times 8 + 6^2 - 8 \div 2 = 26$

Data **Use the table for Problems 19 and 20.**

Food	Price
Soft Vegetable Taco	$2
Beef and Bean Burrito	$5
Chicken Fajitas	$4
Veggie Nachos	$3



19. How much less does it cost to buy 3 tacos than 2 burritos?

20. You buy 4 tacos, 2 fajitas, and 2 nachos. How much do you spend? Write an expression, then evaluate.

21. Create and Solve Write an expression that contains parentheses, exponents, and at least two operations. Challenge a classmate to evaluate the expression.

Mixed Review and Test Prep

Open Response

Add or subtract. (Ch. 1, Lesson 6)

22. $6.8 + 2.45 + 0.15$

23. $9.23 + 0.8$

24. $5 - 2.4$

25. $0.7 - 0.07$

Multiple Choice

26. Use the order of operations. Which of these expressions evaluates to 10? (Ch. 2, Lesson 7)

A $4 + 6^2 \div 4$ **C** $(4 + 6)^2 \div 4$

B $4 + (6^2 \div 4)$ **D** $(4 + 6^2) \div 4$

Extra Practice See page 55, Set D.

Lesson 8

Use Mental Math to Solve Equations

Objective Solve equations by using mental math.

Vocabulary
equation
solution



Learn About It

An **equation** is a statement that uses an equals sign to show that two expressions are equal. If the equation contains a variable, then a value for the variable that makes the equation true is a **solution** of the equation.

Each year Native Americans from across the Americas gather to celebrate and share their cultures. At these gatherings, called powwows, traditional dances are performed. In a group of 25 dancers, 9 dancers are ready to perform. How many dancers are not ready?

Different Ways to Represent the Problem

Way 1 Use a model.

Total number of dancers: 25	
Dancers who are ready: 9	Dancers who are not ready: ?

What number should I subtract to get 9?

Try 16. $25 - 16 = 9$ It checks.

So 16 dancers are not ready.

Way 2 Use an equation.

Let n represent the dancers who are not ready.

$$9 + n = 25$$

Use mental math to solve.
9 plus what number equals 25?

Try substituting 16 for n.

$9 + 16 = 25$ It checks.
$n = 16$

Solution: Sixteen dancers are not ready.

Other Examples

A. Use Fact Families

Solve $n - 8 = 37$.
Write the fact family.

$n - 8 = 37$
$n - 37 = 8$
$8 + 37 = n$
$37 + 8 = n$

Try $n = 45$.
$45 - 8 = 37$ It checks.
$n = 45$

B. Use Properties

Solve $43 \times n = 43$.
Use the identity property of multiplication.

Try $n = 1$.
$43 \times 1 = 43$ It checks.
$n = 1$

C. Use Patterns

Solve $n \div 40 = 100$.
Use division by multiples of 10.
Think: $40 \div 40 = 1$
$400 \div 40 = 10$
$4{,}000 \div 40 = 100$

Try $n = 4{,}000$.
$4{,}000 \div 40 = 100$ It checks.
$n = 4{,}000$

Guided Practice

Use mental math to solve each equation.

1. $16 + n = 23$ **2.** $a \times 3 = 36$ **3.** $10{,}000 \div b = 1{,}000$

4. $27 - x = 22$ **5.** $m + 18 = 24$ **6.** $12 \times p = 60$

Ask Yourself

- Did I substitute my value for the variable to check that the equation is true?

Explain Your Thinking How did you solve the equation in Exercise 6? Is there another way you could have used? Explain.

Practice and Problem Solving

Use mental math to solve each equation.

7. $n + 18 = 20$ **8.** $s \div 3 = 2$ **9.** $29 - y = 2$ **10.** $17 \times c = 34$

11. $2{,}000 \div a = 100$ **12.** $f - 9 = 4$ **13.** $m \times 6 = 60$ **14.** $14 + t = 21$

15. $b \times 8 = 0$ **16.** $42 - x = 34$ **17.** $25 - u = 9$ **18.** $39 \div k = 13$

Determine whether each solution is reasonable. Write *solution* or *not a solution.* Explain your reasoning.

19. $120 - n = 50$; $n = 40$

20. $423 + n = 500$; $n = 223$

21. $125 - n = 99$; $n = 26$

22. $540 \div n = 60$; $n = 90$

23. $34n = 102$; $n = 3$

24. $240 \div 48 = n$; $n = 5$

Solve.

25. This year, 56 more Native Americans came to the powwow than last year. If the total number this year was 278, how many Native Americans came last year? Use a model or an equation to solve.

26. Tickets to the powwow are \$6 for adults and \$3 for children. A family bought two tickets for adults and some children's tickets. If the family spent \$24 on tickets, how many children's tickets did they buy?

27. Write About It How would you explain to a younger student how to use mental math to solve equations?

28. The height of a triangle is 3 times the length of its base. If the base is 6 centimeters, what is the triangle's height?

Mixed Review and Test Prep

Open Response

Evaluate each expression, given $a = 9$, $b = 3.8$, and $c = 5.6$. (Ch. 1, Lesson 8)

29. $a + b + c$ **30.** $a - c$

31. $(b + c) - a$ **32.** $c - b$

33. Explain how you would find and check the solution to this equation. (Ch. 2, Lesson 8)

$8 \times n = 96$

Extra Practice See page 55, Set E.

Lesson 9

Problem-Solving Decision

Reasonable Answers

Objective Decide whether or not an answer to a problem is reasonable.



After you solve a problem, look back and decide whether or not the answer is reasonable.

Problem On April 19, Middlesex County Colonial Minutemen will give 5 performances at a Patriots Day celebration. Each performance includes 24 minutes of poems and stories, and a 30-minute re-enactment on Lexington Green. How long will the 5 performances take in all?

The Battle of Lexington began at dawn on April 19, 1775. It was the first battle of the American Revolution.

Ask Yourself

Did I interpret the problem correctly?	Are my calculations accurate?	Did I answer the question?
$24 + 30 = 54$ min	$24 + 30 = 54$ min $54 \times 5 = 270$ min $270 \div 6 = 45$ h	$24 + 30 = 54$ min $54 \times 5 = 270$ min $270 \div 60 = 4$ R30 4 R 30 = 4 h 30 min
54 min is unreasonable. This is the time of one performance. There will be 5 performances.	45 h is unreasonable. Each performance lasts less than 1 h. So five performances should take less than 5 h.	4 h 30 min is a reasonable answer.

Solution: The 5 performances will take 4 hours 30 minutes in all.

Try These

Determine whether each answer is reasonable. Explain why or why not.

1. There were 187 people at the first performance, 234 at the second, 212 at the third, 176 at the fourth, and 205 at the fifth. Amy estimated that about 1,000 people attended the 5 performances.
2. Small banners cost \$15 each and big banners cost \$24 each. There are 2 boxes of each size banner with 12 banners in each box. Joe said that the cost for all the banners in the boxes is \$78.
3. The 216 Minutemen will take buses from Lexington to Concord to perform. Each bus holds 48 Minutemen. Fran divided and said that they need 4 R24 buses.
4. The local cable station made a video of the performance. One copy of the tape costs \$7.65. Tim gave the station \$10 and expected to get \$3.35 back.

Calculator Connection

Order of Operations

Try this problem on your calculator.

$16 + 24 \div 4$

Following the order of operations rules, the answer is 22.

$16 + 24 \div 4 = 16 + 6 = 22$

If your calculator displays 10, then it computes in the order the numbers are entered. It doesn't have a built-in order of operations feature. So when you enter the expression, you'll need to enter it using the order of operations.





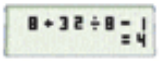
Press: 2 4 ÷ 4 and then + 1 6 =

Suppose you are using a simple calculator. What key sequence would you need to use in order to evaluate the expression correctly?

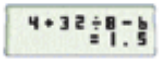
1. $5 + 12 \times 6$ **2.** $2(6 - 3) \div 3$ **3.** $6 + 64 \div 8 - 5$

Five students are using calculators but are not sure whether they have the built-in order of operations feature. Below are the expressions and the results from the calculator. Whose calculator has the feature and whose does not?

4. Beth $8 + 32 \div 8 - 1$



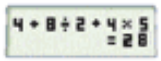
5. Terri $4 + 32 \div 8 - 6$



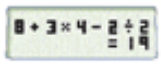
6. Amal $15 + 16 \div 8 \times 2$



7. Kynesha $4 + 8 \div 2 + 4 \times 5$



8. Thien $8 + 3 \times 4 - 2 \div 2$



9. **Analyze** Use a calculator to evaluate the expression $3 + 4 \times 5$. If you do not follow the order of operations, would you get the same answer? Explain how you can tell.

Chapter Review/Test

VOCABULARY

1. In division, the number you divide by is called the _____.
2. Another name for a number sentence is a(n) _____.
3. _____ numbers are easy to divide and close to the original numbers.
4. A(n) _____ is a symbol, usually a letter, that represents a number.

Vocabulary

divisor
equation
solution
variable
compatible

CONCEPTS AND SKILLS

Find each product. Estimate to check your answer.
(Lessons 1–2, pp. 32–35)

5. $21 \times 6{,}308$ **6.** 11×629 **7.** $69 \times 4{,}304$ **8.** 378×299

Divide. Use estimation or multiplication to check. (Lessons 3–4, pp. 36–40)

9. $691 \div 5$ **10.** $15\overline{)1{,}415}$ **11.** $835 \div 13$ **12.** $93\overline{)63{,}231}$

Use properties to complete. Name each property. (Lesson 6, pp. 46–47)

13. $(9 \times t) \times 75 = \blacksquare \times (t \times 75)$

14. $12 \times (15 + m) = (12 \times 15) + (\blacksquare \times m)$

15. $118{,}677 \times \blacksquare = 0$

16. $\blacksquare \times 35{,}902 = 35{,}902$

Evaluate each expression, given $a = 8$, $b = 10$, and $c = 25$. (Lesson 7, pp. 48–49)

17. $450 - (a + b \times c)$ **18.** $a^2 \div 4 \times b$ **19.** $a \times b \times c - b \times c$

Use mental math to solve each equation. (Lesson 8, pp. 50–51)

20. $150 - n = 50$ **21.** $x \div 6 = 20$ **22.** $b \times 8 = 72$ **23.** $f + 30 = 90$

PROBLEM SOLVING

Solve. (Lesson 5, pp. 42–44; Lesson 9, p. 52)

24. There were half as many blue jays as chickadees at a bird feeder. There were 15 more chickadees than finches. If there were 18 blue jays, how many birds were at the feeder?

25. A minivan holds 7 passengers and 1 driver. There are 56 adults going to a baseball game. Mr. Jackson says that they will need at least 8 minivans. Is he correct? Explain.

Write About It

Show You Understand

Sue evaluated the expression

$$3^2 \times 4 + 7 + (4 + 2)^2.$$

She said the answer was 79. Was she correct? Explain how Sue evaluated the expression.

Extra Practice

Set A (Lesson 1, pp. 32–33)

Estimate each product and quotient. Identify the method you used.

1. 71×98 **2.** 56×377 **3.** 862×312 **4.** $21 \times 4{,}421$

5. $22\overline{)8{,}834}$ **6.** $36\overline{)675}$ **7.** $18\overline{)579}$ **8.** $54\overline{)6{,}797}$

Set B (Lesson 2, pp. 34–35)

Find each product. Estimate to see if your answer is reasonable.

1. $\begin{array}{r} 98 \\ \times\ 36 \\ \hline \end{array}$ **2.** $\begin{array}{r} 82 \\ \times\ 54 \\ \hline \end{array}$ **3.** $\begin{array}{r} 705 \\ \times\ 62 \\ \hline \end{array}$ **4.** $\begin{array}{r} 6{,}473 \\ \times\ 15 \\ \hline \end{array}$

5. $3{,}002 \times 37$ **6.** 409×67 **7.** 274×877 **8.** 201×203

Set C (Lessons 3–4, pp. 36–40)

Divide. Then use estimation or multiplication to check your answer.

1. $7\overline{)693}$ **2.** $6{,}325 \div 8$ **3.** $4\overline{)9{,}209}$ **4.** $53{,}118 \div 9$

5. $159 \div 25$ **6.** $9{,}506 \div 49$ **7.** $17{,}632 \div 58$ **8.** $648 \div 81$

9. $12\overline{)7{,}197}$ **10.** $77\overline{)84{,}315}$ **11.** $19\overline{)5{,}904}$ **12.** $31\overline{)6{,}882}$

Set D (Lessons 6–7, pp. 46–49)

Use properties to complete. Identify each property.

1. $(4 \times 8) \times 5 = 4 \times (\blacksquare \times 5)$ **2.** $3(5 + 9) = 3(5) + \blacksquare(9)$ **3.** $72 \times \blacksquare = 0$

4. $6 \times (10 \times 8) = (10 \times 8) \times \blacksquare$ **5.** $8(3 + 5) = 8(\blacksquare) + \blacksquare\,(5)$ **6.** $\blacksquare \times 1 = 2{,}002$

Evaluate. At each step, indicate what operation you performed.

7. $10(5 + 6) \times 4$ **8.** $(2 \times 4)^2 - 20 \times 0.5$ **9.** $6^2 - 6 \div 2 + 40$

Set E (Lesson 8, pp. 50–51)

Use mental math to solve each equation.

1. $75 - n = 50$ **2.** $x \div 7 = 10$ **3.** $b \times 4 = 120$ **4.** $63 \div m = 9$

5. $15 \times p = 30$ **6.** $87 - p = 7$ **7.** $20 + f = 29$ **8.** $k - 25 = 100$

Extra Practice at **eduplace.com/map**

CHAPTER 3

Number Theory



INVESTIGATION

Using Data

The graceful candle dance of Malaysia is a delight to watch. Dancers balance a lit candle in each hand. The dancers are to give a performance.

How many different ways can you arrange the dancers in equal groups so exactly 24 lit candles are used? Design a chart like the one to the right to show your answer.

Number of Groups	Dancers per Group

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A _____ is a whole number with exactly two factors–1 and itself.
2. A number that is a multiple of two or more numbers is a _____ of those numbers.
3. A _____ is a whole number with more than two factors.

Vocabulary

- prime number
- common factor
- common multiple
- composite number

CONCEPTS AND SKILLS

List the factors of each number.

4. 8	**5.** 81	**6.** 17	**7.** 24
8. 42	**9.** 36	**10.** 11	**11.** 120

List the first five multiples of each number.

12. 5	**13.** 9	**14.** 4	**15.** 11
16. 7	**17.** 15	**18.** 12	**19.** 14

Write *true* or *false* for each statement.

20. The product of 3×4 is 12, so 3 and 4 are factors of 12.

21. The numbers 3 and 4 are the only factors of 12.

22. Every number except 0 is divisible by itself and 1.

23. A factor is always greater than the number itself.

24. One multiple of a number is the number itself.



25. Three friends had 100 buttons for an art project. Jodi used 25 buttons. Chris used 32 buttons. After Matt used buttons, there were 17 buttons left. How many buttons did Matt use?

Facts Practice, See page 655.

Lesson 1

Divisibility

Objective Test numbers for divisibility by 2, 3, 4, 5, 6, 9, and 10.



Learn About It

The 90 members of a school marching band need to arrange themselves in rows with an equal number of marchers in each row. They tried 2, 3, 4, 5, 6, 9, and 10 marchers in each row. Which number of rows would have no marchers left over?

Knowing divisibility rules can help you to decide how many marchers could be in each row with no marchers left over.

Divisibility Rules

Rule	Marchers per row	Works with 90 performers?	How do you know?
2: Last digit is even.	2	Yes	0 is even.
3: Sum of the digits is divisible by 3.	3	Yes	$9 + 0 = 9$ $9 \div 3 = 3$
4: Number made up of last two digits is divisible by 4.	4	No	$90 \div 4 \rightarrow 22$ R2
5: Last digit is 5 or 0.	5	Yes	Last digit is 0.
6: It is divisible by both 2 and 3.	6	Yes	90 is divisible by both 2 and 3.
9: Sum of the digits is divisible by 9.	9	Yes	$9 + 0 = 9$ $9 \div 9 = 1$
10: Last digit is 0.	10	Yes	Last digit is 0.

Solution: The marchers can arrange themselves in rows of 2, 3, 5, 6, 9, or 10 marchers each with no marchers left over.

Guided Practice

Determine whether the first number listed is divisible by the second number. Write *yes* or *no*.

1. 124 2 **2.** 97 2 **3.** 135 5 **4.** 250 5

5. 24 2 **6.** 32 6 **7.** 15 6 **8.** 36 6

Ask Yourself

- Did I check the last digit to test for divisibility?
- Did I add digits to test for divisibility?

Explain Your Thinking Why does the divisibility rule for 6 include testing for divisibility by both 2 and 3?

Practice and Problem Solving

Determine whether the first number listed is divisible by the second number.

9. 256 4 **10.** 426 6 **11.** 2,745 5 **12.** 916 6 **13.** 603 10

14. 9,231 3 **15.** 25,001 3 **16.** 991 9 **17.** 734 4 **18.** 9,801 9

Copy and complete this table. Write *yes* or *no*.

		48	72	321	576	724	1,273	3,732
19.	Divisible by 3?							
20.	Divisible by 4?							
21.	Divisible by 12?							

22. **Analyze** Look back at Exercises 19–21. Use your results to write a rule for divisibility by 12.

For each number, find the value of the missing digit that will make the number divisible by 9.

23. 8■2 **24.** 3,62■ **25.** 127,■75 **26.** 1,456,■28

Solve.

27. A band director wants to arrange the band members in groups of 4, 5, 6, and 8. What is the least number of band members that can form groups of 4, 5, 6, and 8?

28. **What's Wrong?** Hector invented this divisibility test for 8: If a number is divisible by both 2 and 4, it is divisible by 8. Use an example to show that Hector's test is not always true.

29. **Mental Math** Which four-digit numbers made up of the digits 3, 4, 5, and 6 in any order are divisible by 4?

30. **You Decide** Create a rule for divisibility by 15. Give examples to support your rule.

Mixed Review and Test Prep

Open Response

Order each set of numbers from least to greatest. (Ch. 1 Lesson 3)

31. 197; 179; 791

32. 5,371; 5,317; 5,703

33. 875; 871; 879; 897

34. 3,050; 3,005; 3,505; 3,055

Multiple Choice

35. Jackie has 96 pencils that she wants to divide equally among her friends. Which number of friends could *not* receive an equal number of pencils? (Ch. 3, Lesson 1)

A 3 **C** 6

B 4 **D** 9

Extra Practice See page 75, Set A.

Lesson 2

Prime and Composite Numbers

Vocabulary
prime number
composite number

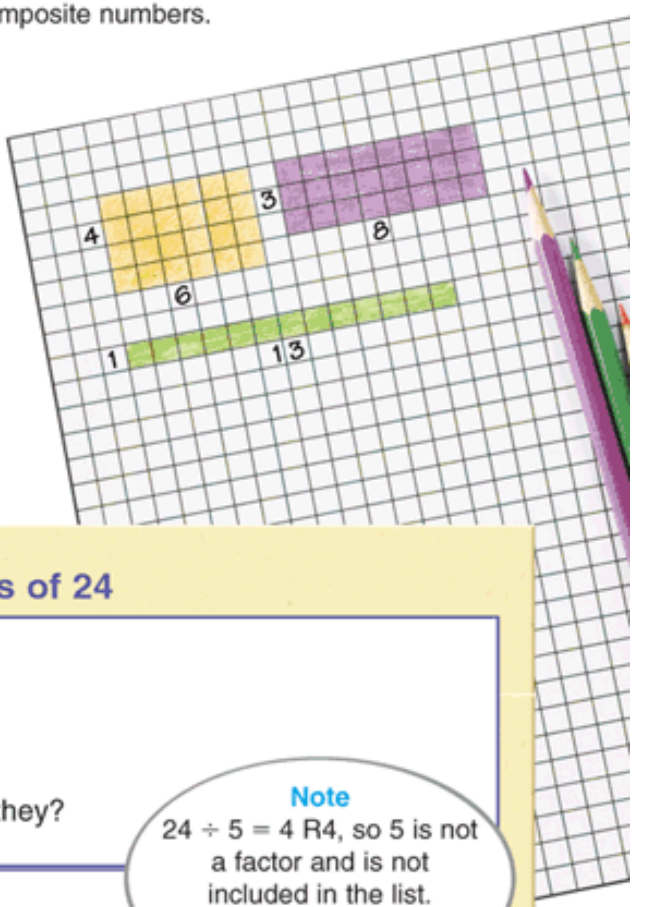
Objective Distinguish between prime numbers and composite numbers.

Learn About It

A **prime number** is a whole number with exactly two factors, itself and 1. The number 13 is prime because its only factors are the numbers 1 and 13.

A **composite number** is a whole number that has more than two factors. The number 16 is composite because the whole numbers 1, 2, 4, 8, and 16 are factors, or divisors, of 16.

The number 1 is neither a prime nor a composite number.



Is the number 24 prime or composite?

Different Ways to Find the Factors of 24

Way 1 Use Rectangular Arrays

Two rectangular arrays for 24 are shown above.
They show that 24 has the factors 4 and 6 and 3 and 8.

- There are two other rectangular arrays for 24. What are they?

Note
$24 \div 5 = 4$ R4, so 5 is not a factor and is not included in the list. The same is true for 7, 9, 10, and 11.

Way 2 Use Division

Begin by trying to divide 24 by each number, starting with 1. List the results that show pairs of factors. When a factor is repeated, you can stop.

$24 \div 1 = 24$, so 1 and 24 are factors.
$24 \div 2 = 12$, so 2 and 12 are factors.
$24 \div 3 = 8$, so 3 and 8 are factors.
$24 \div 4 = 6$, so 4 and 6 are factors.

Solution: Since 24 has the factors 1, 2, 3, 4, 6, 8, 12, and 24, the number 24 is a composite number.

Guided Practice

Use arrays or division. List all factors of each number. Then identify each number as *prime* or *composite*.

1. 14 **2.** 21 **3.** 32 **4.** 50

5. 11 **6.** 36 **7.** 48 **8.** 51

Ask Yourself
- Did I list 1 and the number itself as factors?
- Did I divide and then use the divisor and quotient as factors?

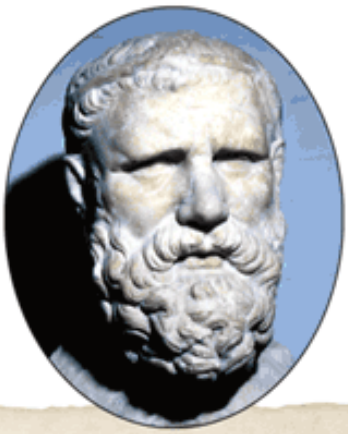
Explain Your Thinking Explain why the number 1 is defined as neither prime nor composite.

Practice and Problem Solving

List all the factors of each number. Then identify each number as *prime* or *composite*.

9. 18	**10.** 30	**11.** 31	**12.** 25	**13.** 86	**14.** 105
15. 17	**16.** 45	**17.** 54	**18.** 63	**19.** 64	**20.** 43

Copy the table below on 10 × 10 grid paper and continue the numbering to 100. Use the table for Problems 21–25.



In ancient Greece, a mathematician named Eratosthenes became interested in prime and composite numbers. He developed a method, known as the Sieve of Eratosthenes, for finding prime numbers.

21. Follow his steps to decide whether 53 is a prime number.

- **a.** Cross out 1 since 1 is not prime.
- **b.** Circle 2. Then cross out all multiples of 2.
- **c.** Circle 3. Then cross out all multiples of 3.
- **d.** Go to the next number that is not crossed out. Circle it. Then cross out its multiples.
- **e.** Repeat step **d** until all the numbers are either circled or crossed out.

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

22. Using your copy of the Sieve of Eratosthenes, write an ordered list of all the prime numbers between 1 and 100.

23. **Explain** Which has more prime numbers, the interval from 1 to 50 or the interval from 50 to 100?

24. **Reasoning** Could you extend the Sieve of Eratosthenes to find prime numbers greater than 100? Demonstrate that this can be done.

25. **Analyze** The numbers 11 and 13 are called *twin primes* because they are consecutive numbers in the list of primes. What are some other twin primes?

26. Christian Goldbach (1690–1764) held that every even number greater than 2 could be written as the sum of two primes. Write the even numbers from 11 through 20 as the sum of two primes.

27. Goldbach also conjectured that every odd number greater than 5 can be written as the sum of three prime numbers. Write the odd numbers from 11 through 20 as the sum of three primes.

Mixed Review and Test Prep

Open Response

Find each product. Estimate to check if your answer is reasonable. (Ch. 2, Lesson 2)

28. 596 × 9

29. 813 × 39

30. 158 × 82

31. 31,099 × 6

32. Which of the factors of 60 are prime numbers?

Explain how you know. (Ch. 3, Lesson 2)

Extra Practice See page 75, Set B.

Lesson 3

Audio Tutor 1/8 Listen and Understand

Prime Factorization

Objective Express composite numbers as products of prime numbers in exponent form.

Vocabulary
- prime factorization
- exponent
- base

Learn About It

Every composite number can be expressed as a product of prime numbers. This is called **prime factorization**. The factors in a prime factorization often are written from least to greatest.

Example: The prime factorization of 12 is $2 \times 2 \times 3$.

You can use a factor tree to find the prime factorization of a number. What is the prime factorization of the number 48?

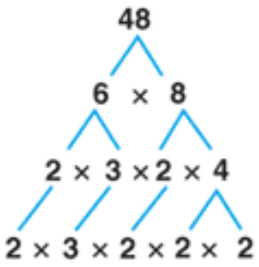
Find the prime factorization of 48.

STEP 1 Start with two factors (other than 1) that have a product of 48. There is more than one way to do this.

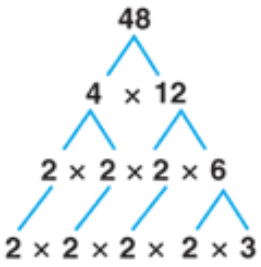
STEP 2 If a factor is composite, find its factors.

STEP 3 Check your work. Check that all factors are prime numbers.

Multiply to check your computations.



all prime factors



all prime factors

A factor tree is complete when each branch ends with a prime number.

$2 \times 3 \times 2 \times 2 \times 2 = 48$ $2 \times 2 \times 2 \times 2 \times 3 = 48$

Solution: The prime factorization of 48 is $2 \times 2 \times 2 \times 2 \times 3$.

In Chapter 1, you learned that you can use exponents to show repeated factors.

Example: $81 = 3 \times 3 \times 3 \times 3 = 3^4$

3^4 is read as "three to the fourth power."

The **exponent** (4) tells how many times the **base** (3) is used as a factor.



Extra Help at eduplace.com/map

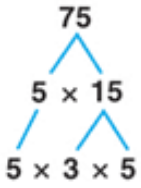
You can use exponents to write the prime factorization of a composite number like 48.

$$2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1 = 2^4 \times 3$$

The exponent 1 in 3^1 shows that the factor 3 is used only once. It is not necessary to write this exponent.

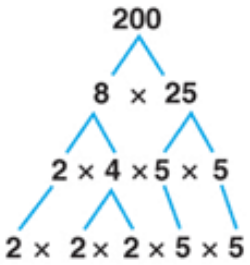
Other Examples

A. The Prime Factorization of 75



The prime factorization of 75 is $3 \times 5 \times 5$ or 3×5^2.

B. The Prime Factorization of 200



The prime factorization of 200 is $2 \times 2 \times 2 \times 5 \times 5$ or $2^3 \times 5^2$.

Guided Practice

Ask Yourself

- Did I count the number of times a factor was used?
- Does each branch in the factor tree end with a prime?

Write each, using exponents.

1. $2 \times 2 \times 5 \times 5$ **2.** $2 \times 2 \times 3 \times 5 \times 5$

Re-write each as an expression with no exponents. Then evaluate.

3. 2^3 **4.** 3^5 **5.** $3^2 \times 7^2$ **6.** $2^3 \times 5^2 \times 7$

Draw a factor tree to show the prime factorization of each. Then rewrite, using exponents.

7. 16 **8.** 39 **9.** 625 **10.** 2,500

Explain Your Thinking Do you think that there can be more than one prime factorization for any whole number greater than 1? Use an example to explain.

Practice and Problem Solving

Write each, using exponents.

11. $3 \times 3 \times 7$ **12.** $4 \times 4 \times 9$ **13.** $5 \times 5 \times 8 \times 8$ **14.** $6 \times 6 \times 6 \times 7$

Write each as a product of factors. Then evaluate.

15. $4^3 \times 5$ **16.** $2^5 \times 3^2$ **17.** $5^2 \times 7$ **18.** 9^3

Go On

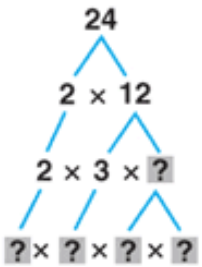
Draw a factor tree to show the prime factorization of each. Then rewrite, using exponents. If the number is prime, write *prime*.

19. 14 **20.** 22 **21.** 26 **22.** 49 **23.** 37

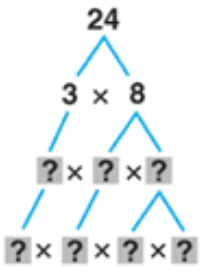
24. 42 **25.** 91 **26.** 97 **27.** 108 **28.** 1,440

Complete each factor tree for the number 24.

29.



30.



31.



Algebra • Expressions

Evaluate each expression, given $a = 2$, $b = 3$, and $c = 4$.

32. 2×3^a **33.** $6 \times c^a$ **34.** $2^a \times a^5$ **35.** $a^5 \times c^2$ **36.** $a^b \times b^a$

Data

Use the diagram for Problems 37 and 38.

37. Name the number represented by the factor tree.



38. Write About It Could this be the factor tree for any other number? Explain why or why not.

39. Predict Write each number in this list, using an exponent of 2. If this pattern continues, what will the tenth number likely be?

4, 9, 16, 25, 36, 49

40. List the prime factors of 36 and 72. Which prime factors do the two numbers have in common?

41. Represent Use the numbers 2, 2, 3, 3, 3, and 13 as the bottom of the branches of a factor tree. Work backward to create two different factor trees with the same number at the top.

Use the list of numbers at the right for Problems 42 and 43.

2, 5, 6, 9, 10, 15, 18, 20, 24, 27, 30, 42, 48, 50, 60

42. Which numbers have only 2s and 3s as prime factors?

43. Which numbers have only 2s and 5s as prime factors?

Extra Practice See page 75, Set C.

Problem Solving

Number Sense

Greek *Arithmetike*

The ancient Greeks, including the mathematician Pythagoras, were fascinated by the relationship between musical harmonies and numerical properties.



A number is **perfect** if the sum of its factors, except for the number itself, equals the number.

Factors of 6: 1, 2, 3
$1 + 2 + 3 = 6$
6 is a perfect number.

A number is **deficient** if the sum of its factors is less than the number. A number is **abundant** if the sum of its factors is greater than the number.

1. Show that the number 8 is deficient while the number 18 is abundant.

Classify each composite number as perfect, deficient, or abundant.

2. 10 **3.** 12 **4.** 28 **5.** 30 **6.** 42 **7.** 50

8. Explain Collin says that if the sum of all the factors of a number is twice the number, it is a perfect number. Is he correct?

Quick Check

Check your understanding of Lessons 1–3.

Determine whether the first number is divisible by the second number. (Lesson 1)

1. 5,010 6 **2.** 709 9 **3.** 1,110 5

List all the factors of each number. Then identify each number as *prime* or *composite*. (Lesson 2)

4. 28 **5.** 82 **6.** 19

Draw a factor tree to show the prime factorization of each. Then rewrite, using exponents. If the number is prime, write *prime*. (Lesson 3)

7. 27 **8.** 40 **9.** 59 **10.** 242

Lesson 4

Audio Tutor 1/9 Listen and Understand

Greatest Common Factor

Objective Find the greatest common factor of two or more numbers.

Vocabulary

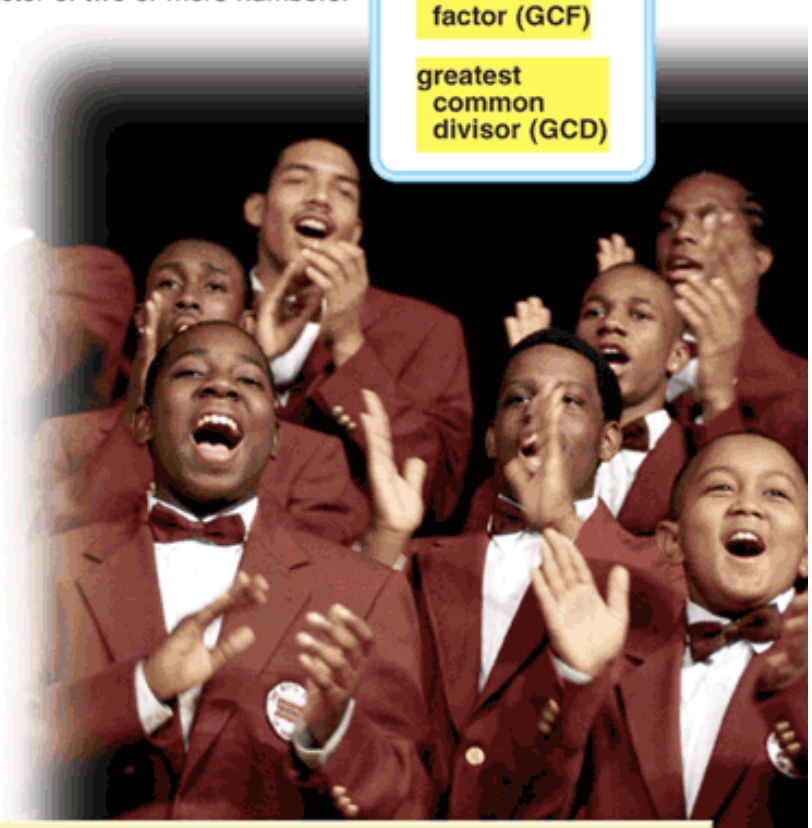
- greatest common factor (GCF)
- greatest common divisor (GCD)

Learn About It

A chorus is made up of 24 sixth graders and 30 seventh graders. Each row should have the same number of students with sixth graders and seventh graders in separate rows. What is the greatest number of students that can be in a row?

Find the greatest number that is a factor of both 24 and 30. This number is called the **greatest common factor (GCF)**. The greatest common factor is sometimes called the **greatest common divisor (GCD)**.

Find the greatest common factor of 24 and 30.



The Boys Choir of Harlem

Different Ways to Find the GCF of 24 and 30.

Way 1 You can make a list.

STEP 1 List all the factors of both numbers.

24: 1, 2, 3, 4, 6, 8, 12, 24

30: 1, 2, 3, 5, 6, 10, 15, 30

STEP 2 Circle the common factors.

A *common factor* is a factor that appears in both sets of factors.

24: (1), (2), (3), 4, (6), 8, 12, 24

30: (1), (2), (3), 5, (6), 10, 15, 30

The greatest common factor circled is the GCF. The GCF of 24 and 30 is 6.

Way 2 You can use the prime factorization.

STEP 1 Write the prime factorization of both numbers. Circle common prime factors.

$24 = (2) \times 2 \times 2 \times (3)$

$30 = (2) \times (3) \times 5$

STEP 2 Then find the product of the common factors.

$GCF = 2 \times 3 = 6$

Solution: The GCF of 24 and 30 is 6. So the greatest number of students in each row is 6.

Other Examples

A. The GCF is One of the Numbers

$12 = 2 \times 2 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

GCF of 12, 48 and 72: $2 \times 2 \times 3 = 12$

B. Use Exponents

$54 = 2 \times 3 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$

Common prime factors: 2 and 3^2
GCF of 54 and 36: $2 \times 3^2 = 18$

Guided Practice

Find the GCF of each pair of numbers by listing the factors of each.

1. 6, 15 **2.** 8, 12 **3.** 20, 30 **4.** 25, 35

Find the GCF for each pair of numbers, using prime factorization.

5. 8, 16 **6.** 18, 20 **7.** 11, 22 **8.** 7, 18

Ask Yourself

- Did I use only prime numbers as factors?
- Did I multiply all the common prime factors?

Explain Your Thinking Why can the greatest common factor also be called the greatest common divisor?

Practice and Problem Solving

Find the GCF of each pair of numbers by listing the factors of each.

9. 22, 23 **10.** 14, 35 **11.** 24, 40 **12.** 40, 50

Find the GCF of each pair of numbers, using prime factorization.

13. 49, 56 **14.** 41, 82 **15.** 99, 121 **16.** 36, 57

Find the GCF of each group of numbers. Use either a list of factors or prime factorization.

17. 300, 144 **18.** 84, 120 **19.** 8, 28, 52 **20.** 12, 18, 42

Reasoning **In each exercise, *n* represents a counting number. Write *true* or *false* to describe each statement. Explain your answers.**

21. The GCF of the expressions $4n$ and $6n$ is $2n$.

22. The GCF of the expressions n and $n + 5$ is n.

23. The GCF of the expressions n and n^2 is n.

24. The GCF of the expressions $2n$ and $2n + 1$ is $2n$.

Go On

Algebra • Expressions

In each of the following expressions, n represents a counting number. Find the GCF of each pair of expressions.

25. $6n^3$ and $9n^3$

26. $9n^3$ and $9n^4$

27. $12n^4$ and $16n^4$

28. $6n^3$ and $6n^4$

Data

Solve. Use the table below for Problems 32 and 33.

29. The GCF of two numbers is 15. The greater number is 75. What are the possible values of the lesser number?

30. Explain The prime factorizations of two numbers are $2^3 \times 3^2$ and 2×3^3. Explain how to use the exponents to decide which factors should be part of the GCF.

31. Write About It What is the GCF of any two prime numbers? What is the GCF of any composite number and any prime number? Use examples to explain.

32. A dance group featured 16 dancers dressed in white, 24 in red, and 40 in blue. Each row had dancers wearing the same color. What is the greatest number of dancers in a row?

33. Analyze There were 15 fewer singers and 9 fewer dancers in last year's show. Each scene had the same number of performers. Each performer appeared in just one scene. What was the greatest number of performers in each group?



Variety Show Performers

Type	Number
Singers	75
Dancers	45

Mixed Review and Test Prep

Open Response

Divide. Then check your work. (Ch. 2, Lesson 4)

34. $16\overline{)538}$

35. $33\overline{)659}$

36. $862 \div 86$

37. $3{,}004 \div 26$

38. $4{,}503 \div 57$

39. $7{,}125 \div 35$

40. In a parade, 18 boys and 24 girls are marching in separate rows. Each row requires the same number of students. What is the greatest number of students that can be in a row? Explain. (Ch. 3, Lesson 4)

Extra Practice See page 75, Set D.

Problem Solving

Number Sense

Finding the GCF

Here are two ways to find the greatest common factor of large numbers. Find the greatest common factor (GCF) of 294 and 378.

You can find the GCF by dividing by common prime factors.

STEP 1 Divide both numbers by the smallest common prime divisor.

$$\begin{array}{r} 147 \\ 2\overline{)294} \end{array} \qquad \begin{array}{r} 189 \\ 2\overline{)378} \end{array}$$

STEP 2 Divide both quotients by their smallest common prime divisor.

$$\begin{array}{r} 49 \\ 3\overline{)147} \end{array} \qquad \begin{array}{r} 63 \\ 3\overline{)189} \end{array}$$

STEP 3 49 and 63 are not both divisible by 2, 3, or 5, but they are both divisible by 7.

$$\begin{array}{r} 7 \\ 7\overline{)49} \end{array} \qquad \begin{array}{r} 9 \\ 7\overline{)63} \end{array}$$

STEP 4 No prime number divides both 7 and 9. The GCF is the product of all the common prime divisors.

$$2 \times 3 \times 7$$

Solution: The GCF of 294 and 378 is $2 \times 3 \times 7 = 42$.

You can also find the GCF without finding common factors.

STEP 1 Divide the larger number by the smaller number.

$$294\overline{)378}$$

$$\begin{array}{r} 1 \text{ R}84 \\ 294\overline{)378} \\ -294 \\ \hline 84 \end{array}$$

STEP 2 Note the remainder. Divide the previous divisor by the remainder.

$$\begin{array}{r} 3 \text{ R}42 \\ 84\overline{)294} \\ -252 \\ \hline 42 \end{array}$$

STEP 3 Note the remainder. Divide the previous divisor by the remainder.

When the remainder is 0, the final divisor is the GCF.

$$\begin{array}{r} 2 \\ 42\overline{)84} \\ -84 \\ \hline 0 \end{array}$$

Solution: The largest number that divides both 294 and 378 is 42.

Explain How do these methods differ from the ways you have been finding the GCF? How are they similar?

Lesson 5

Audio Tutor 1/10 Listen and Understand

Least Common Multiple

Objective Find the least common multiple of two or more numbers.

Vocabulary
least common multiple (LCM)

Learn About It



On the first day of the theater school, both students and teachers performed. After that, the students performed every fourth day and the teachers every sixth day. If the school lasts 14 days, will the students and teachers perform again on the same day?

You can find the answer by finding the least common multiple (LCM) of 4 and 6. The **least common multiple (LCM)** is the least number that is a multiple of two or more numbers.

Different Ways to Find the LCM of 4 and 6.

Way 1 Make a list.

STEP 1 List some multiples of 4 and 6. Circle the *common multiples*, the numbers that appear in both lists of multiples.

4: 4, 8, (12), 16, 20, (24), 28
6: 6, (12), 18, (24), 30, 36

STEP 2 Identify the least common multiple.

The least common multiple is 12. The LCM of 4 and 6 is 12.

Way 2 Use prime factorization.

STEP 1 Write the prime factorization of 4 and 6, using exponents.

$4 = 2 \times 2 = 2^2$
$6 = 2 \times 3$

STEP 2 For each of the two different prime factors, circle the greatest power.

$2 \times 2 = (2^2)$
$2 \times (3)$

STEP 3 The product of the greatest powers of the prime factors is the LCM.

$LCM = 2^2 \times 3 = 2 \times 2 \times 3 = 12$

Solution: Yes. During a 14-day period, students and teachers will perform together again on Day 12.

Guided Practice

Use a list or prime factorization to find the LCM of each pair of numbers.

1. 4, 5 **2.** 16, 36 **3.** 5, 17 **4.** 4, 42

Ask Yourself
- Did I circle the greatest power of each different prime factor?

Explain Your Thinking If the LCM of two numbers is equal to the product of the two numbers, what must be true about their prime factorizations?

Practice and Problem Solving

Write the first three common multiples for each pair of numbers.

5. 4 and 6 **6.** 8 and 9 **7.** 5 and 10 **8.** 2 and 11 **9.** 3 and 12

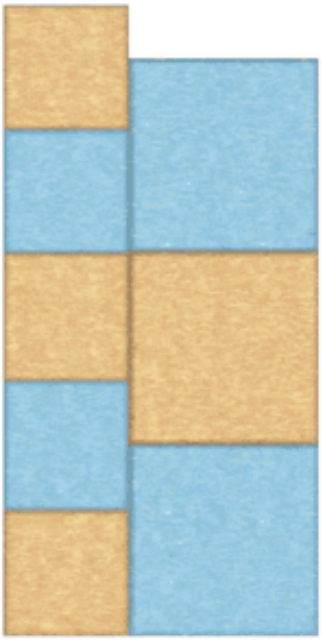
Use a list or prime factorization to find the LCM of each pair or group of numbers.

10. 6, 14 **11.** 16, 6 **12.** 9, 12 **13.** 11, 13 **14.** 17, 51

15. 12, 20 **16.** 11, 4 **17.** 28, 42 **18.** 5, 6, 10 **19.** 12, 16, 24

Solve.

20. Piano classes start at 9:00 A.M. and are given every 60 minutes. Violin classes also start at 9:00 A.M. and are given every 45 minutes. What is the next time at which a piano lesson and a violin lesson will start at the same time?



21. **Explain** When is the LCM of two numbers the same as one of the numbers? Use an example to explain.

22. **Analyze** Howard and Claudia are placing square carpeting tiles in a wide hallway in the theater. The tiles on one side have 16-inch sides, and the tiles on the other side have 24-inch sides.

a. How many of each tile will they have to buy for a 12-foot-long hallway?

b. How many times will edges of each kind of tile line up?

Mixed Review and Test Prep

Open Response

Divide. Then check your work. (Ch. 2, Lesson 4)

23. $17\overline{)478}$ **24.** $66\overline{)794}$

25. $751 \div 74$ **26.** $5{,}007 \div 34$

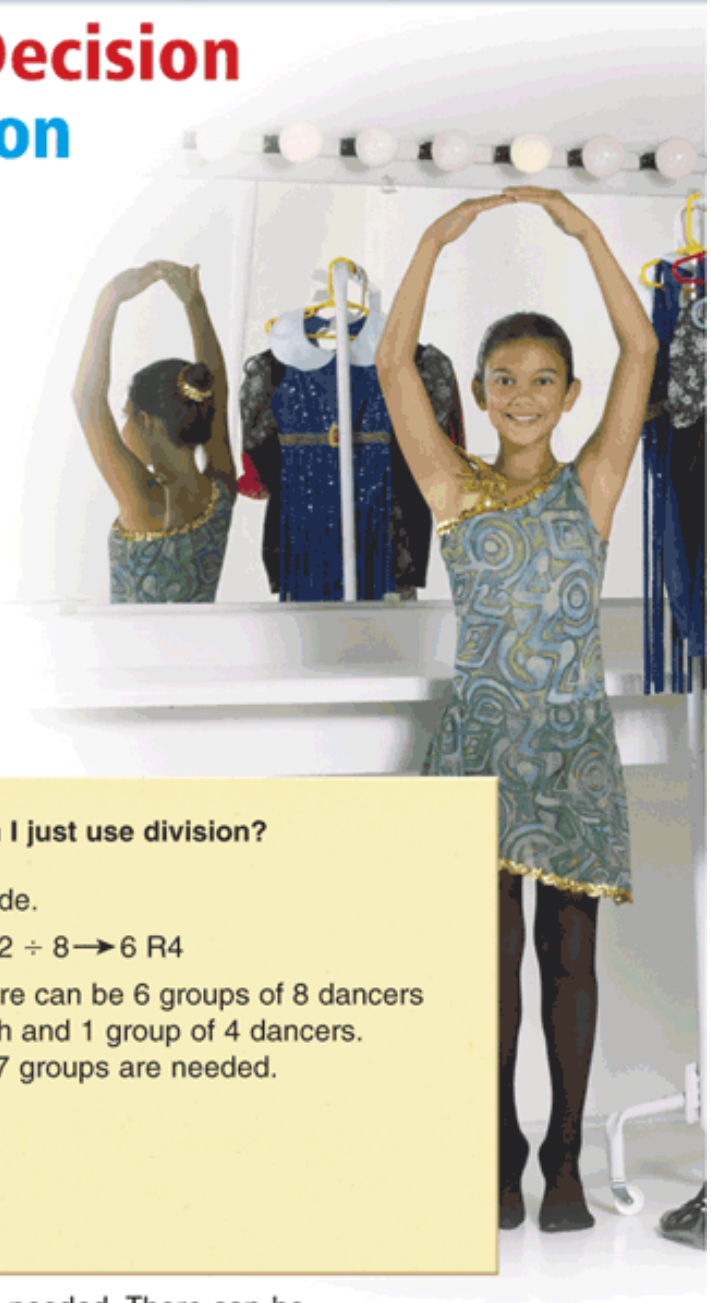
27. The LCM of two numbers is $2^3 \times 3^3 \times 5$. Find two pairs of numbers that fit this description. Explain how you found your answer. (Ch. 3, Lesson 5)

Extra Practice See page 75, Set E.

Lesson 6

Problem-Solving Decision
Explain Your Solution

Objective Review how to support your solution to a problem.



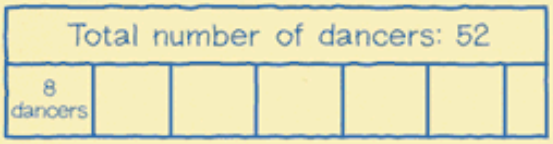
You can use a diagram, a sentence, or a combination of methods to explain your solution.

Problem Tina is a student at dance academy. For a dance performance, 52 students will perform in groups of at most 8 students. Tina says that at least 7 groups of dancers are needed. Is she correct? How could you explain the solution?

Ask Yourself **How can I explain my solution?**

Can I use a diagram to explain?

You can draw a model of the information.



$8 \times 6 = 48$. $48 < 52$, 6 groups are too few.

$8 \times 7 = 56$. $56 > 52$, 7 groups is enough.

Can I just use division?

Divide.

$52 \div 8 \rightarrow 6$ R4

There can be 6 groups of 8 dancers each and 1 group of 4 dancers. So 7 groups are needed.

Solution: Tina is right. At least 7 groups of dancers are needed. There can be 6 groups of 8 dancers each and 1 group of the 4 dancers left over.

Try These

Solve. Explain how you found your solution.

1. Alma and Bert were both at the library on Tuesday, April 2. Alma goes to the library every 4 days, and Bert goes every 5 days. What is the next date on which they were both at the library?
2. Lina has 24 square ceramic tiles, each with 1-inch sides. She wants to arrange them in a rectangular design on a tabletop. What are the possible dimensions of the design?
3. A florist divided 86 roses equally into 7 vases. She placed as many roses as possible in each vase. How many roses did the florist have left?
4. Kelvin has 60 square tiles, each with 4-inch sides. He wants to arrange them in a rectangular array on a wall. What are the possible dimensions of the array?

Calculator Connection

Tic-Tac-Toe

Activity

The answer to a division problem can be shown as either a whole number with a remainder or as a decimal.

Find 45 ÷ 6.

Your calculator can show the answer in either form:

Answer With a Remainder



Answer as a Decimal



The quotient and remainder 7 R3 and the quotient 7.5 represent the same number.

- Work with a partner. Draw a tic-tac-toe board. Choose X or O.
- Players take turns choosing an expression from the list below and picking a square on the board. Decide who goes first.
- The player evaluates the expression, using either the Int÷ or the ÷ key.
- The other player decides which expression the first player picked.
- If that player is correct, he or she marks X or O on the board. If not, the other player does.

67 ÷ 5	39 ÷ 2	75 ÷ 6	93 ÷ 4	43 ÷ 4
486 ÷ 8	453 ÷ 5	349 ÷ 5	261 ÷ 5	586 ÷ 4

Analyze How can you change an answer from decimal form to remainder form?

- Return to the list of division problems above. Use your calculator to solve these problems. Change the answers to each of the problems to the opposite form of the answers that were originally given. For example, if the original answer given was an answer with a remainder, now change that answer to an answer as a decimal.

Chapter Review/Test

VOCABULARY

1. Showing a composite number as a product of prime numbers is called _____ .
2. A(n) _____ tells how many times the base is used as a factor.
3. The _____ is the greatest whole number that is a shared factor of two or more numbers.
4. The _____ is the smallest number that is a multiple of two or more numbers.

Vocabulary

- exponent
- prime number
- least common multiple (LCM)
- greatest common factor (GCF)
- prime factorization
- composite number

CONCEPTS AND SKILLS

Determine whether the first number listed is divisible by the second number. (Lesson 1, pp. 58–59)

5. 486 9 **6.** 2,206 2 **7.** 42,242 4 **8.** 948 6

9. 3,334 3 **10.** 7,220 5 **11.** 567 9 **12.** 58,342 4

List all the factors of each number. Then identify each number as *prime* or *composite*. (Lesson 2, pp. 60–61)

13. 65 **14.** 147 **15.** 23 **16.** 88 **17.** 79

Draw a factor tree to show the prime factorization of each. Then rewrite, using exponents. (Lesson 3, pp. 62–64)

18. 84 **19.** 90 **20.** 56 **21.** 162

Find the GCF of each group of numbers. (Lesson 4, pp. 66–68)

22. 45, 50 **23.** 72, 90, 144 **24.** 14, 35 **25.** 6, 9, 18 **26.** 16, 32, 56

Find the LCM of each group of numbers. (Lesson 5, pp. 70–71)

27. 29, 87 **28.** 2, 5, 8 **29.** 6, 14 **30.** 10, 90 **31.** 3, 4, 7

PROBLEM SOLVING

Solve. Explain your solution. (Lesson 6, p. 72)

32. Bus 1 and Bus 2 both leave the station at 8:00 A.M. Bus 1 leaves the station every 20 minutes. Bus 2 leaves every 25 minutes. When will they leave the station at the same time again?



Show You Understand

Prime numbers can never end in 0, 2, 4, 5, 6, or 8. Explain why.

Extra Practice

Set A (Lesson 1, pp. 58–59)

Determine whether the first number listed is divisible by the second number.

1. 657 3	**2.** 559 5	**3.** 1,230 10	**4.** 392 2	**5.** 2,115 9
6. 7,853 2	**7.** 1,290 6	**8.** 5,001 10	**9.** 2,528 4	**10.** 64,260 3

Set B (Lesson 2, pp. 60–61)

List all the factors of each. Then identify each number as prime or composite.

1. 55	**2.** 26	**3.** 29	**4.** 99	**5.** 49
6. 56	**7.** 37	**8.** 84	**9.** 34	**10.** 153

Set C (Lesson 3, pp. 62–64)

Draw a factor tree to show the prime factorization of each number. Then rewrite, using exponents. If the number is prime, write *prime*.

1. 18	**2.** 25	**3.** 36	**4.** 19	**5.** 72
6. 48	**7.** 225	**8.** 41	**9.** 126	**10.** 64

Set D (Lesson 4, pp. 66–68)

Find the GCF of each group of numbers. Use either a list of factors or prime factorization.

1. 16, 144	**2.** 51, 34	**3.** 61, 183	**4.** 47, 94	**5.** 24, 39, 87
6. 110, 88, 44	**7.** 35, 40, 50	**8.** 12, 16, 22	**9.** 21, 35, 49	**10.** 18, 36, 54

Set E (Lesson 5, pp. 70–71)

Use a list or prime factorization to find the LCM of each pair or group of numbers.

1. 6, 15	**2.** 8, 12	**3.** 4, 14	**4.** 9, 15	**5.** 12, 30
6. 10, 160	**7.** 19, 57	**8.** 5, 16	**9.** 3, 8, 12	**10.** 3, 11, 66

Extra Practice at **eduplace.com/map**

WEEKLY WR READER®
Science Connection

Distances in Space

Orbiting 375 miles above Earth, the Hubble Space Telescope uses powerful optics to take pictures of distant stars and galaxies.

Since these objects are so far away, Earth miles are too small to express the distances. Instead, astronomers use much larger units of measure: *astronomical units, light years,* and *parsecs.*



Gomez's Hamburger: 6,500 light years

Problem Solving

Calculator **Use the units at right and a calculator to solve Problems 1–6.**

1. **Estimate** About how many miles equal 1 parsec?
2. At a distance of 24,000 parsecs, the Sagittarius Dwarf Elliptical Galaxy is the closest galaxy to our Milky Way. About how many light years is that?
3. Gomez's Hamburger is too big to bite into! This object is about 6,500 light years away. About how many parsecs is that?
4. How many miles can light travel in one minute?
5. How many whole minutes does it take light to travel at least one billion miles?
6. List these objects in order from nearest to most distant. Round to the nearest whole unit.

An *astronomical unit* (AU) is equal to the mean distance from the Sun to the Earth. 1 AU is about 93,000,000 miles.

A *light year* (ly) is the distance light travels in one year. 1 light year is about 6 trillion miles.

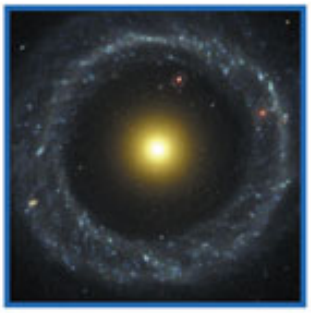
A *parsec* (pc) is a little more than 3 light years. Our Milky Way galaxy is about 45,000 parsecs across.



Crab Nebula: 4,000 light years



Andromeda Galaxy: 70,000 parsecs



Hoag's Object: 600 million light years



Hubble 5 Nebula: 2,200 light years

Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic.

Enrichment: Number Systems

BASE TWO



Computers use the binary system to operate. They "read" the digits 1 and 0 as the commands on *and* off.

The **binary system** uses only the digits 0 and 1. It is also known as *base two*.

In the base-two system, each place has a value 2 times greater than the place to its right. The binary system also uses 0 as a placeholder.

Look at the base-two place-value chart below. Notice that the expanded form of the numbers is written by using powers of two.

Base Two				
sixteens 2^4	eights 2^3	fours 2^2	twos 2^1	ones 2^0
				1
			1	0
			1	1
		1	0	0
		1	0	1

$\mathbf{1}_{two} = (1 \times 2^0) = \mathbf{1}_{ten}$

$\mathbf{10}_{two} = (1 \times 2^1) + (0 \times 2^0) = \mathbf{2}_{ten}$

$\mathbf{11}_{two} = (1 \times 2^1) + (1 \times 2^0) = \mathbf{3}_{ten}$

$\mathbf{100}_{two} = (1 \times 2^2) + (0 \times 2^1) + (0 \times 2^0) = \mathbf{4}_{ten}$

$\mathbf{101}_{two} = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = \mathbf{5}_{ten}$

Try These!

Write each number in base ten.

1. 1001_{two}
2. 10000_{two}
3. 10101_{two}
4. 11111_{two}

Write each number in base two.

5. 7
6. 10
7. 14
8. 28

9. Write your age in base two.

10. **Challenge** Suppose that an ancient civilization built a base-five number system, using the 5 fingers on a hand. 2_{ten} would be 2 fingers (2_{five}) while 8_{ten} would be 1 hand and 3 fingers (13_{five}). Explain how you would find a base-five numeral for 13_{ten} and a base-ten numeral for 44_{five}.



Break It Down



Many calculators can't perform operations with very large numbers because the answer can't fit in the display. To check your calculator, find 1,234,567 × 89.

If your display shows Error or E, then it cannot handle large numbers. So you must use number properties to break down the problem.

Find 1,234,567 × 89.

- Use the **Distributive Property**:

$$89(1{,}234{,}567)$$
$$89(1{,}234{,}000 + 567)$$
$$(89 \times 1{,}234{,}000) + (89 \times 567)$$

- Use a calculator.

$$(89 \times 1{,}234 \times 1{,}000) + (89 \times 567)$$

- Use mental math.

$$(109{,}826 \times 1{,}000) + (50{,}463)$$
$$109{,}826{,}000 + 50{,}463$$

- Use paper and pencil.

$$\begin{array}{r} 109{,}826{,}000 \\ +\quad 50{,}463 \\ \hline 109{,}876{,}463 \end{array}$$

Solution: 1,234,567 × 89 = 109,786,463

Solve, using a calculator, mental math, and paper and pencil. Show your work. Estimate to check.

1. 1,682,532 × 431
hundred million

2. 323,323,323 × 16
ten thousand

3. 88,888 × 2,456
hundred thousand

4. 104,101 × 101,101
million

5. 14,982 × 63,423
ten million

6. 5,999,871 × 84
hundred million

7. 26,104,810 × 53
billion

8. 345,614 × 721
ten million

9. Place values are given below each of Exercises 1–8. Write the digit in the answer with this place value.

10. Enter the digits you wrote, in order, into your calculator. Flip your calculator upside down to solve the riddle below.

What can a turtle never run out of?

Unit 1 Test

VOCABULARY Open Response

1. Numbers that are easy to divide and are close to the original dividend and divisor are called ____.
2. Another name for ____ is the greatest common divisor.
3. The fifth place to the right of the decimal point is the ____ place.

Vocabulary

- millionths
- compatible numbers
- hundred thousandths
- greatest common factor (GCF)
- prime factorization

CONCEPTS AND SKILLS Open Response

Write each number in standard form. (Chapter 1)

4. $(3 \times 10^4) + (1 \times 10^0) + (4 \times 10^{-1})$
5. $(6 \times 10^{-1}) + (5 \times 10^{-3}) + (2 \times 10^{-5})$

Order from least to greatest. (Chapter 1)

6. 86.4 8.64 8.064 8.06
7. 0.0736 0.003 0.038 0.308

Estimate, then find each sum or difference. (Chapter 1)

8. $469 + 526 + 638$
9. $1.63 + 2.3 + 3.98$
10. $67.63 - 9.06$
11. $74.05 - 13.89$

Estimate, then find each product or quotient. (Chapter 2)

12. 97×508
13. 616×309
14. $8\overline{)6{,}397}$
15. $37\overline{)12{,}396}$

Evaluate each expression, given $a = 1$, $b = 3$, and $c = 12$. (Chapter 2)

16. $c \div (a + b) \times b^2$
17. $a \times b \times c - b \div a$

Use mental math to solve each equation. (Chapter 2)

18. $a + 27 = 30$
19. $42 \div n = 14$
20. $d \times 15 = 0$
21. $k - 7 = 8$

Find each prime factorization. Then rewrite, using exponents. If the number is prime, write *prime*. (Chapter 3)

22. 46 **23.** 54 **24.** 67 **25.** 45

Find the GCF and LCM of each pair or group of numbers. (Chapter 3)

26. 8, 12 **27.** 9, 18 **28.** 15, 18 **29.** 12, 18, 24

PROBLEM SOLVING Open Response

Solve.

30. Use the table at right. If Newtown's population continues to grow in the same pattern, what will its population be in 2020?

31. Nico has $15. Does he have enough to buy 4 pairs of socks that cost $3.89 a pair including tax?

32. Gladys wants to use 30 square paving bricks for a rectangular patio. Each brick has 18-inch sides. List all possible dimensions of the patio.

33. Ed has 18 more coins than Fran does. Fran has three times as many coins as Gil. Gil has 15 coins. How many coins does Ed have?

Population of Newtown

Year	Population
1960	425
1970	850
1980	1,700
1990	3,400
2000	6,800

Performance Assessment

Extended Response



Task You have been put in charge of buying CDs, videos, and DVDs for your school for the entire year! Use the prices above and the information at right.

How many of each item should you buy so that the sum of your purchases is within your budget? Explain your thinking.

Information You Need

- You must spend at least $2,325 but no more than $2,350.
- You must buy at least 10 of each item.
- The number of videos must be twice the number of DVDs.
- The number of CDs must be 4 more than the number of videos.

Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

Sometimes a test has true/false statements. You can use numbers to test each choice.

Look at the example below.

Which of the following statements is false?

A The product of two even numbers is even.

B The sum of two odd numbers is even.

C Any number divisible by 12 is also divisible by 9.

D If the product of two whole numbers is odd, then the factors are both odd.

THINK

Use numbers to test each answer choice.

Statement *A*: $2 \times 4 = 8$

Statement *B*: $7 + 11 = 18$

Statement *C*: $24 \div 9 \rightarrow 2$ R6

Statement *D*: $15 = 3 \times 5$

Since 24 is divisible by 12, but not by 9, answer choice *C* is false.

Multiple Choice

1. The phrase "5 less than a number plus 7" can also be written as

A $5 + 7 - n$ **C** $5 - (n + 7)$

B $(n + 7) + 5$ **D** $(n + 7) - 5$

(Chapter 1, Lesson 7)

2. For whole numbers, when a dividend is greater than the divisor, which phrase is true?

F quotient is less than dividend

G remainder is greater than divisor

H remainder is greater than quotient

J divisor and remainder are equal

(Chapter 2, Lesson 3)

3. Which statement about the number 30 is false?

A It is divisible by 3.

B It is divisible by 5.

C It is divisible by 8.

D It is divisible by 15.

(Chapter 3, Lesson 1)

4. Which statement is true about the GCF of two numbers?

F It is greater than both numbers.

G It is the same as the LCM.

H It is always greater than 1.

J It is equal to or less than the lesser number.

(Chapter 3, Lesson 4)

For Test-Taking Tips, See page 648.

Open Response

5. These are the heights in meters of four soccer players:

1.58 1.6 1.55 1.49

What is the height of the second tallest player?

(Chapter 1, Lesson 3)

6. This year, Wendy finished a race in 35.56 seconds. This was 2.72 seconds less than her time in last year's race. What was her time in last year's race?

(Chapter 1, Lesson 6)

7. Burger World is open from 12 noon until midnight. During that time it served 1,476 burgers. On average, how many burgers did it serve per hour?

(Chapter 2, Lesson 4)

8. Eighteen brass players and 24 woodwind players march in separate rows. If each row has the same number of players, what is the greatest number of players that can be in a row?

(Chapter 3, Lesson 4)

9. Kevin and Sarah checked their e-mail today. Kevin checks his e-mail every 4 days, and Sarah checks hers every 3 days. When will both students check their e-mail on the same day again?

(Chapter 3, Lesson 5)

Extended Response

10. Below are two units that can be used to measure very short lengths of time.

millisecond: one thousandth of a second microsecond: one millionth of a second

A Express 3 milliseconds as a decimal part of a second and as a power of 10.

B Express 3 microseconds as a decimal part of a second and as a power of 10.

C How many milliseconds are in ten seconds? How do you know?

D How many microseconds are in one millisecond? Explain how you found your answer.

E Write 230,000 microseconds as a number of seconds. How could you use a place-value chart to justify your answer?

(Chapter 1, Lessons 1 and 2)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 1

Look back at the big ideas and vocabulary in this unit.

Big Ideas

Each decimal place in a number can be written by using a power of 10.

You can find the greatest common factor (GCF) and the least common multiple (LCM) by listing the factors or by using the prime factorization of the numbers.

Key Vocabulary

- power of ten
- greatest common factor (GCF)
- least common multiple (LCM)
- prime factorization

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how to estimate this quotient: 62,954 ÷ 29
2. Explain how to test the number 55,242 for divisibility by 6.
3. Explain how to write 30.8502 in expanded form, using powers of 10.
4. Explain how to find the least common multiple of 25 and 30.
5. **Write About It** Look in newspapers, almanacs, science books, or on the Internet for examples of very large and very small numbers in decimal form. Explain how the numbers are used.



UNIT 2

Fractions and Decimals

CHAPTER 4

Fraction and Decimal Concepts

page 88



CHAPTER 5

Add and Subtract Fractions

page 106



CHAPTER 6

Multiply and Divide Fractions

page 124



CHAPTER 7

Multiply and Divide Decimals

page 148

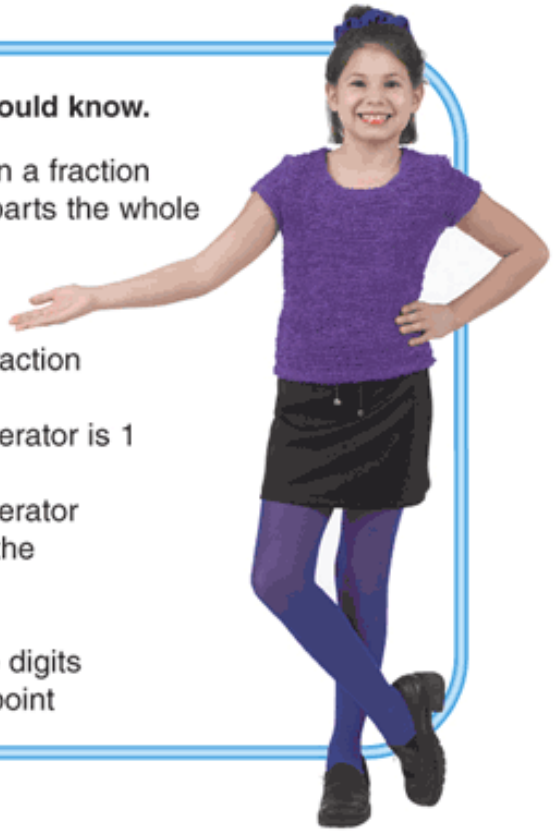


Reading Mathematics

Reviewing Vocabulary

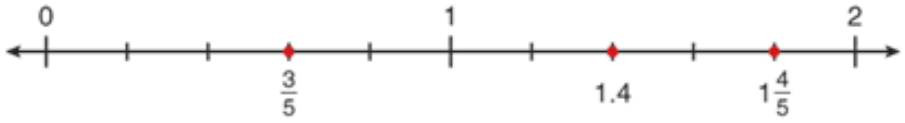
Here are some math vocabulary words that you should know.

denominator	the number below the bar in a fraction that tells how many equal parts the whole is divided into
mixed number	a number that is the sum of a whole number and a fraction
unit fraction	a fraction in which the numerator is 1
improper fraction	a fraction in which the numerator is greater than or equal to the denominator
decimal	a number with one or more digits to the right of the decimal point



Reading Words and Symbols

You can use a number line to represent fractions, mixed numbers, and decimals.



Draw a number line from 0 to 3 and mark each fourth. Then place each number on your number line.

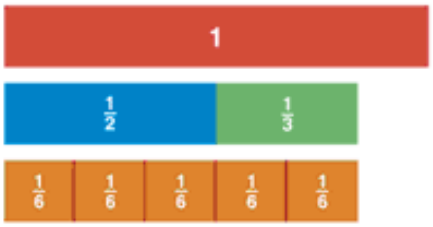
1. $\frac{3}{4}$ **2.** 0.25 **3.** $1\frac{3}{4}$ **4.** 2.50 **5.** $\frac{11}{4}$

6. Look back at Exercises 1–5. How does the number line help you decide which is greater, 2.50 or $\frac{11}{4}$?

Reading Test Questions

Choose the correct answer for each.

7. What fraction operation does the model show?



a. $\frac{1}{2} - \frac{1}{3}$ **c.** $\frac{1}{2} \times \frac{1}{3}$

b. $\frac{1}{2} + \frac{1}{3}$ **d.** $\frac{1}{2} \div \frac{1}{3}$

A **model** is a way to use a picture, drawing, base-ten blocks, or fraction strips to represent a number.

8. How could you rename $4\frac{3}{9}$ before subtracting?

$$\begin{array}{r} 4\frac{3}{9} \\ -\ 1\frac{8}{9} \\ \hline \end{array}$$

a. $3\frac{9}{3}$ **c.** $4\frac{9}{3}$

b. $3\frac{12}{9}$ **d.** $4\frac{12}{9}$

To **rename** a number means to represent it in a different way. For example, $1 = \frac{9}{9}$.

9. Solve for n, using mental math.

$$0.45 \times 10 = n$$

a. 10.45 **c.** 4.5

b. 5.4 **d.** 0.54

Mental math is a computation you can do in your head without using paper and pencil.

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

- **least common denominator**
- **multiplicative inverse**
- **reciprocal**
- **invert**
- **terminating decimal**
- **repeating decimal**

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "Tusks, Teeth, and Fractions" on pages 636-637. Then work with a partner to answer the questions about the story.

CHAPTER 4

Fraction and Decimal Concepts

INVESTIGATION

Using Data

Firefighters must have good judgment, be team players, and be able to work in hazardous situations. They also need to be in excellent physical shape.

The circle graph shows one firefighters' exercise program. The graph is separated into three specific exercise activities. If weightlifting is $\frac{1}{3}$ of his program, what fraction of the whole is represented by "Other"? How did you find your answer?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

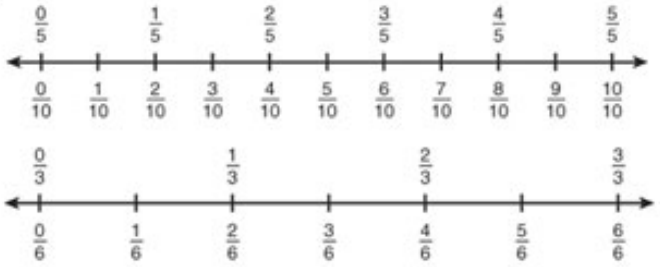
1. In a fraction, the _____ tells how many equal parts of the whole.
2. A(n) _____ is a fraction that has a numerator that is greater than or equal to its denominator.
3. A(n) _____ consists of a whole number and a fraction.

Vocabulary

- equivalent fraction
- numerator
- denominator
- mixed number
- improper fraction

CONCEPTS AND SKILLS

Use the number lines to find the equivalent fractions.



4. $\frac{4}{5} = \frac{\square}{10}$
5. $\frac{\square}{10} = \frac{3}{5}$
6. $\frac{2}{3} = \frac{\square}{6}$
7. $\frac{2}{6} = \frac{\square}{3}$

Compare. Write >, <, or = for each ●.

8. $\frac{1}{3}$ ● $\frac{1}{4}$
9. 0.4 ● 0.04
10. 2% ● 5%
11. 0.09 ● 0.1
12. 15% ● 50%
13. $\frac{1}{5}$ ● $\frac{1}{6}$
14. 0.50 ● 0.07
15. $\frac{2}{11}$ ● $\frac{3}{11}$

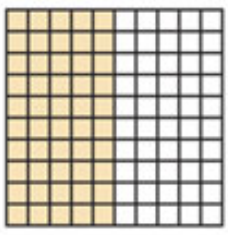
Match each number with the corresponding words.

16. 20%
17. 0.2
18. $\frac{1}{5}$
19. $\frac{20}{100}$

a. twenty hundredths
b. twenty percent
c. one divided by five
d. two tenths

Write About It

20. Does the picture at right show all of the following: 50%, 0.5, $\frac{1}{2}$, and $\frac{50}{100}$? Explain.



Facts Practice, See page 658.

Lesson 1

Equivalent Fractions

Objective Find equivalent fractions and write fractions in simplest form.

Vocabulary
- equivalent fractions
- simplest form



Learn About It

Equivalent fractions represent the same number. For example, the fractions $\frac{1}{1}$, $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ are equivalent because they are all names for the number 1.

Sports doctors have found that as many as 18 out of every 40 soccer injuries occur in players younger than 15. What is another way to write $\frac{18}{40}$?

Different Ways to Find Fractions Equivalent to $\frac{18}{40}$

Way 1 You can multiply.

Multiply the numerator and the denominator by the same number.

$$\frac{18}{40} = \frac{18 \times 2}{40 \times 2} = \frac{36}{80}$$

Multiplying any number by $\frac{2}{2}$, or 1, does not change its value.

$$\frac{18}{40} = \frac{18}{40} \times \frac{2}{2} = \frac{36}{80}$$

Way 2 You can divide.

Divide the numerator and the denominator by the same non-zero number.

$$\frac{18}{40} = \frac{18 \div 2}{40 \div 2} = \frac{9}{20}$$

Dividing any number by $\frac{2}{2}$, or 1, does not change its value.

$$\frac{18}{40} = \frac{18}{40} \div \frac{2}{2} = \frac{9}{20}$$

Solution: $\frac{36}{80}$ and $\frac{9}{20}$ are both equivalent fractions for $\frac{18}{40}$.

The fraction $\frac{9}{20}$ is in **simplest form** because the only common factor of its numerator and its denominator is 1.

Other Examples

A. Using a Given Denominator

$\frac{3}{7} = \frac{\blacksquare}{21}$ Think: $7 \times 3 = 21$

$\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$

So, $\frac{3}{7} = \frac{9}{21}$.

B. Simplify Improper Fractions

$\frac{6}{4}$ Think: The GCF of 6 and 4 is 2.

$\frac{6}{4} = \frac{6 \div 2}{4 \div 2} = \frac{3}{2}$

So, $\frac{6}{4} = \frac{3}{2}$.

C. Simplify Mixed Numbers

$3\frac{12}{15}$ Think: The GCF of 12 and 15 is 3.

$\frac{12}{15} = \frac{12 \div 3}{15 \div 3} = \frac{4}{5}$

So, $3\frac{12}{15} = 3\frac{4}{5}$.

Guided Practice

Write two equivalent fractions for each. Use multiplication to write one and division to write the other.

1. $\frac{10}{12}$ **2.** $\frac{15}{30}$ **3.** $\frac{22}{24}$ **4.** $\frac{30}{40}$

Ask Yourself

- Did I multiply or divide the numerator and denominator by the same number?

Explain Your Thinking Are any of the fractions you wrote in Exercises 1–4 in simplest form? How do you know?

Practice and Problem Solving

Write two equivalent fractions for each. Use multiplication to write one and division to write the other.

5. $\frac{5}{10}$ **6.** $\frac{15}{21}$ **7.** $\frac{12}{36}$ **8.** $\frac{20}{30}$

Write each fraction or mixed number in simplest form.

9. $\frac{4}{8}$ **10.** $\frac{14}{18}$ **11.** $\frac{7}{21}$ **12.** $\frac{16}{12}$

13. $\frac{12}{24}$ **14.** $1\frac{22}{48}$ **15.** $\frac{16}{36}$ **16.** $2\frac{14}{20}$

Algebra • Variables **Write the missing numerator or denominator.**

17. $\frac{8}{20} = \frac{\square}{5}$ **18.** $\frac{10}{15} = \frac{2}{\square}$ **19.** $\frac{7}{42} = \frac{\square}{6}$ **20.** $\frac{49}{63} = \frac{7}{\square}$

Solve.

21. Wrist injuries account for 27 out of every 75 shoulder, arm, and hand injuries among snowboarders. Write a fraction in simplest form to express the number of wrist injuries compared to all shoulder, arm, and hand injuries.

22. Reasoning Among snowboarders, 11 out of every 20 chest injuries are rib fractures. Among skiers, 41 out of 100 chest injuries are rib fractures. In which of the two sports are rib fractures more common? Explain.

23. Starting at 11 A.M., the temperature rose by 2°F every hour for 5 hours. Then it dropped 3°F every hour for the next 5 hours. If it was 67°F at 2 P.M., when was the temperature the lowest? the highest?

24. Analyze At which two times during the day in Problem 23 was the temperature the same? Between which two times was there a difference in temperature of 11 degrees?

Mixed Review and Test Prep

Open Response

Write each improper fraction as a mixed number. (Grade 5)

25. $\frac{9}{8}$ **26.** $\frac{20}{8}$ **27.** $\frac{14}{3}$ **28.** $\frac{36}{6}$

Multiple Choice

29. Which fraction is *not* equivalent to $\frac{16}{32}$? (Ch. 4, Lesson 1)

A $\frac{4}{16}$ **B** $\frac{1}{2}$ **C** $\frac{13}{26}$ **D** $\frac{20}{40}$

Extra Practice See page 105, Set A.

Lesson 2

Relate Fractions, Mixed Numbers, and Decimals

Objective Convert between fractions, mixed numbers, and decimals.



The stock ticker, invented in 1866, got its name from the clicking sound it made printing stock prices on a paper tape.

Learn About It

Stockbrokers used to buy and sell shares of stock with prices quoted in fractions of a dollar. Today, stock prices are given in decimal form. Suppose the price of a stock was quoted as $\$22\frac{1}{4}$. Since $\frac{1}{4}$ dollar is the same as 25 cents, or $\frac{25}{100}$ of a dollar, you can write the price as \$22.25 in decimal form.

Fractions and decimals are related because they can be used to express the same number.

Writing a Fraction as a Decimal

To write a fraction with a denominator of 10 or 100 as a decimal, read the word name and place the decimal point.

Fraction	Word Form	Decimal
$\frac{48}{100}$	forty-eight hundredths	0.48
$3\frac{4}{10}$	three *and* four tenths	3.4

Writing a Decimal as a Fraction

To write a decimal as a fraction, write a fraction with the decimal-place name as the denominator. Simplify the fraction.

Decimal	Word Form	Fraction
0.48	forty-eight hundredths	$\frac{48}{100} = \frac{12}{25}$
3.4	three *and* four tenths	$3\frac{4}{10} = 3\frac{2}{5}$

Guided Practice

For each fraction or mixed number, write the equivalent decimal.

1. $\frac{3}{10}$ **2.** $\frac{9}{100}$ **3.** $2\frac{37}{100}$ **4.** $\frac{15}{1{,}000}$

For each decimal, write the equivalent fraction or mixed number.

5. 0.7 **6.** 0.23 **7.** 5.75 **8.** 0.18

Ask Yourself

- Did I use a power of ten as the denominator?
- Did I find an equivalent fraction to write as a decimal?

Explain Your Thinking When writing a fraction in decimal form, why might you first find an equivalent fraction whose denominator is a power of ten?

Practice and Problem Solving

Write an equivalent decimal for each.

9. $\frac{1}{10}$ **10.** $3\frac{18}{100}$ **11.** $\frac{10}{10}$ **12.** $4\frac{25}{100}$ **13.** $\frac{62}{1{,}000}$

Write two equivalent fractions or mixed numbers for each.

14. 0.1 **15.** 0.05 **16.** 2.8 **17.** 1.004 **18.** 1.48

19. 0.28 **20.** 1.75 **21.** 0.6 **22.** 0.13 **23.** 14.55

Write as a decimal and as a fraction or mixed number.

24. five tenths **25.** fifty hundredths

26. two and two tenths **27.** four and eight hundredths

Algebra • Equations

Use mental math to find the value of *n* that makes each equation true.

28. $\frac{1}{2} \times \frac{n}{n} = \frac{5}{10}$ **29.** $0.5 \times n = 5$ **30.** $\frac{1}{8} \times \frac{n}{n} = \frac{125}{1{,}000}$ **31.** $1.2 \times n = 120$

32. $\frac{3}{4} \times \frac{n}{n} = \frac{750}{1{,}000}$ **33.** $0.9 \times n = 900$ **34.** $\frac{2}{5} \times \frac{n}{n} = \frac{4{,}000}{10{,}000}$ **35.** $0.047 \times n = 4.7$

Solve.

36. The price of a share of stock rose 75 cents in one day of trading. Write 75 cents as a decimal and as a fraction in simplest form.

37. A stockbroker charged 5 cents for each share of stock he bought or sold. Express 5 cents in decimal form and as a fraction of a dollar.

38. Write About It Write a fraction that is equivalent to 0.0625. How is the number of places in a decimal related to the number of zeros in the denominator of its fraction equivalent?

39. What's Wrong? Danielle got 16 hits out of 40 times at bat. She says this is a 0.350 batting average. Is she correct? Explain how you would find her correct batting average.

Mixed Review and Test Prep

Open Response

Compare. Use >, <, or =. (Ch. 1, Lesson 3)

40. 0.167 ● 0.16 **41.** 2.62 ● 2.623

Order from least to greatest.

42. 9.4, 9.7, 9.42 **43.** 1.6, 1.74, 1.30

44. 5.43, 5.42, 5.51 **45.** 0.723, 0.732, 0.712

46. What are the next five likely numbers in this pattern?

$$0.1, \frac{1}{10}, 0.2, \frac{1}{5}, 0.3$$

Explain how you found your answer. (Ch. 4 Lesson 2)

Extra Practice See page 105, Set B.

Lesson 3

Audio Tutor 1/11 Listen and Understand

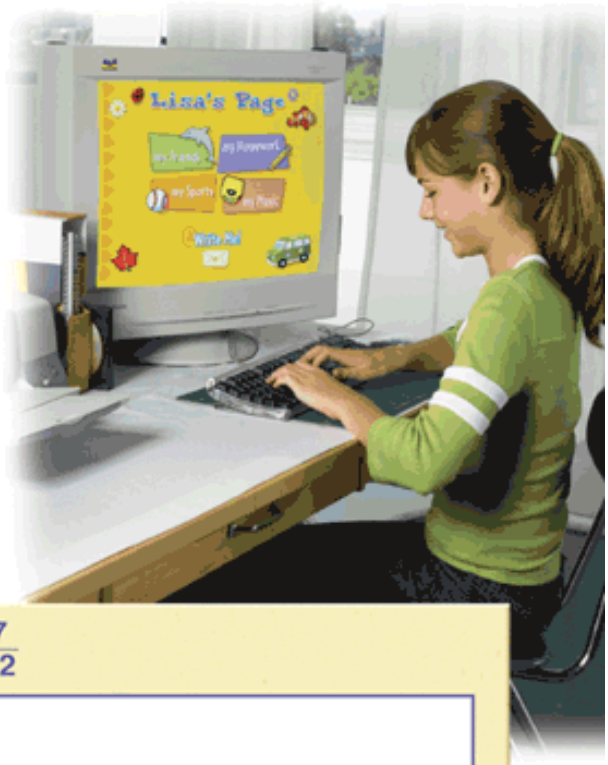
Compare and Order Fractions and Decimals

Objective Compare and order fractions, mixed numbers, and decimals.

Vocabulary

- common denominator
- least common denominator (LCD)

Learn About It



Lisa attached a counter to the e-mail link on her home page. The first day, 5 out of 8 visitors to her site sent e-mail. The second day, 7 out of 12 visitors sent e-mail. On which day did more of the visitors send e-mail?

Two fractions can be compared easily if they have the same denominator, known as a **common denominator**. For example, the fractions $\frac{2}{4}$ and $\frac{3}{4}$ have 4 as a common denominator. The **least common denominator (LCD)** of two fractions is the least common multiple of their denominators.

Compare the fractions $\frac{5}{8}$ and $\frac{7}{12}$.

Different Ways to Compare $\frac{5}{8}$ and $\frac{7}{12}$

Way 1 Use any common denominator.

STEP 1 Multiply to find a common denominator.

$8 \times 12 = 96$

Use 96 as the common denominator.

STEP 2 Write equivalent fractions.

$$\frac{5}{8} = \frac{5 \times 12}{8 \times 12} = \frac{60}{96}$$

$$\frac{7}{12} = \frac{7 \times 8}{12 \times 8} = \frac{56}{96}$$

STEP 3 Compare the numerators.

$\frac{60}{96} > \frac{56}{96}$, so $\frac{5}{8} > \frac{7}{12}$

Way 2 Use the least common multiple as the LCD.

STEP 1 Find the LCM of the denominators 8 and 12.

$8 = 2^3$ $\quad 12 = 2^2 \times 3$

The LCM is $2^3 \times 3$, or 24.

STEP 2 Write equivalent fractions.

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{7}{12} = \frac{7 \times 2}{12 \times 2} = \frac{14}{24}$$

STEP 3 Compare the numerators.

$\frac{15}{24} > \frac{14}{24}$, so $\frac{5}{8} > \frac{7}{12}$

Solution: Since $\frac{5}{8} > \frac{7}{12}$, more of the visitors sent e-mail on the second day.

Extra Help at eduplace.com/map

To compare and order decimals, fractions, and mixed numbers, first express the numbers in the same form.

Compare 1.6 and $1\frac{4}{5}$.

STEP 1 Write 1.6 as a mixed number with 10 as the denominator.

$$1.6 = 1\frac{6}{10}$$

STEP 2 Change $1\frac{4}{5}$ to an equivalent mixed number with 10 as the denominator.

$$1\frac{4}{5} = 1\frac{8}{10}$$

STEP 3 Compare the mixed numbers.

$$1\frac{6}{10} < 1\frac{8}{10},$$

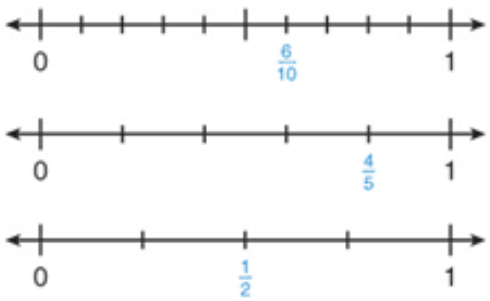
so $1.6 < 1\frac{4}{5}$

Order 0.6, $\frac{4}{5}$, and $\frac{1}{2}$ from least to greatest.

STEP 1 Draw three number lines of equal length. Write each number as a fraction.

$$0.6 = \frac{6}{10}$$

Mark the locations of the three fractions.



STEP 2 Compare.

The number farthest to the left on the number line is the least. The number farthest to the right is the greatest.

$$\frac{1}{2} < \frac{6}{10} < \frac{4}{5}, \text{ so } \frac{1}{2} < 0.6 < \frac{4}{5}$$

How would you order these numbers without using a number line?

Guided Practice

Compare. Write >, <, or = for each ●.

1. $\frac{3}{4}$ ● $\frac{5}{6}$
2. $\frac{2}{3}$ ● $\frac{3}{4}$
3. $2\frac{4}{9}$ ● $2\frac{1}{2}$
4. $3\frac{1}{2}$ ● $3\frac{1}{3}$
5. 0.5 ● $\frac{3}{4}$
6. 1.8 ● $1\frac{1}{2}$

Compare. Write in order from least to greatest.

7. $\frac{2}{3}$, $1\frac{1}{3}$, 0.75, $1\frac{1}{2}$
8. $1\frac{1}{3}$, 0.6, $\frac{5}{10}$, 0.55

Ask Yourself

- Did I compare the numerators after finding the LCD?
- Did I find an equivalent fraction with 10 or 100 in the denominator?

Explain Your Thinking When is it better to use the LCD to compare two fractions rather than any common multiple of the denominators?

Go On

Practice and Problem Solving

Compare. Write >, <, or = for each ●.

9. $\frac{5}{6}$ ● $\frac{4}{7}$

10. $\frac{5}{8}$ ● $\frac{7}{10}$

11. $\frac{2}{7}$ ● 0.4

12. $9\frac{1}{8}$ ● $9\frac{2}{11}$

13. 0.6 ● $\frac{2}{3}$

14. $3\frac{3}{5}$ ● 3.6

15. $\frac{5}{8}$ ● 0.25

16. $1\frac{4}{8}$ ● 1.5

17. $\frac{1}{2}$ ● 0.05

18. $\frac{5}{7}$ ● 0.7

19. $\frac{7}{10}$ ● 0.75

20. $2\frac{2}{3}$ ● 2.67

Compare. Write in order from least to greatest.

21. $\frac{1}{3}, \frac{7}{12}, \frac{5}{6}$

22. $\frac{2}{9}, \frac{1}{6}, \frac{2}{3}$

23. $\frac{5}{9}$, 0.5, $\frac{11}{18}$

24. $1\frac{4}{5}, 1\frac{1}{2}$, 1.7, $\frac{13}{16}$

25. $\frac{3}{4}, \frac{5}{7}$, 0.65, 0.85

26. $\frac{5}{10}$, 0.6, 0.55, $\frac{9}{14}$

Algebra • Expressions

Compare. Write >, <, or = for each ●.

27. $\frac{x}{8}$ ● $\frac{4}{6}$, given $x = 3$

28. $\frac{3}{5}$ ● $\frac{x}{5}$, given $x = 7$

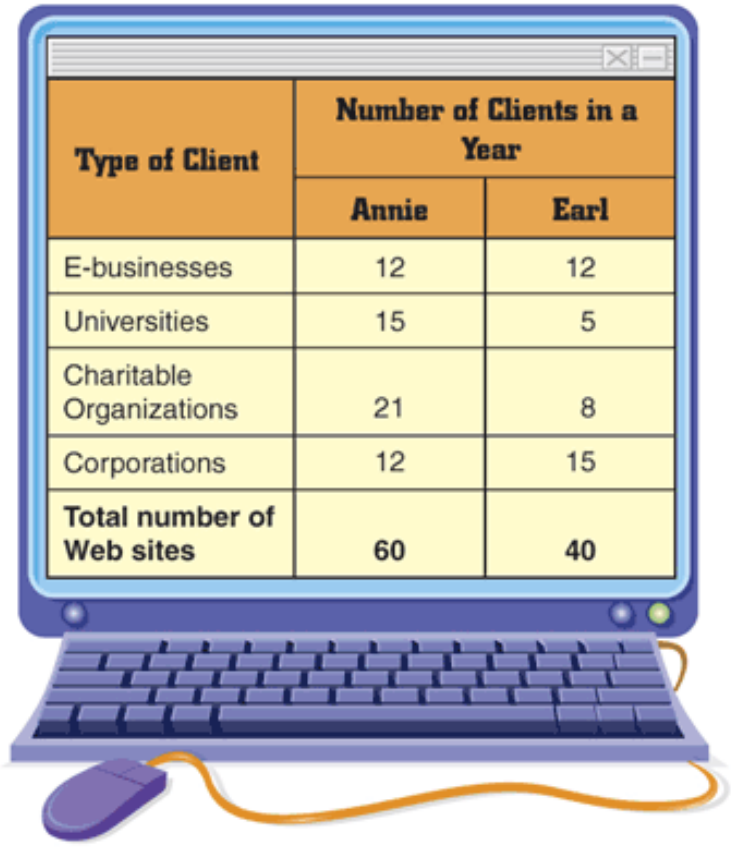
29. $\frac{x}{6}$ ● $\frac{5}{y}$, given $x = 3$, $y = 10$

30. $\frac{9}{x}$ ● $\frac{y}{11}$, given $x = 12$, $y = 9$

Data

Annie and Earl are both Web designers. The table below shows the number of Web sites each has designed in a year. Use the table for Problems 31–34.

Type of Client	Number of Clients in a Year	
	Annie	Earl
E-businesses	12	12
Universities	15	5
Charitable Organizations	21	8
Corporations	12	15
Total number of Web sites	**60**	**40**



31. What fraction in simplest form expresses the portion of Annie's Web-site designs that were for e-businesses? of Earl's Web-site designs?

32. Which represents a greater portion of the total number of sites that each person designed, the sites Earl designed for corporations or the sites Annie designed for universities?

33. **Explain** For which designer did charitable organizations make up the greater portion of the total number of clients? Tell how you can use equivalent fractions to find out.

34. **Reasoning** How many more e-business clients would Annie need in order to have e-business clients make up the same portion of her total number of clients as they do for Earl?

Extra Practice See page 105, Set C.

Game

Activity

Comparing Fractions

2 Players

What You'll Need • spinner with the numbers 1, 3, 4, 6, 8, and 9 in six equal regions (Learning Tool 34)

How to Play



1. Each player forms a fraction by spinning the spinner twice. The smaller number becomes the numerator.
2. Players record their fractions.
3. Players use mental math to decide which fraction is greater and use $>$ or $<$ to record their predictions.
4. Players check their predictions.
5. Repeat Steps 1–4 nine times or until one player has had the greater fraction 5 times. (A longer game could be any odd number of rounds.)



Quick Check

Check your understanding of Lessons 1–3.

Write each in simplest form. Then write as a decimal. (Lesson 1)

1. $\frac{12}{48}$ **2.** $5\frac{5}{25}$ **3.** $\frac{4}{80}$

Write a fraction for each decimal. (Lesson 2)

4. 0.36 **5.** 0.12 **6.** 0.08

Compare. Write in order from least to greatest. (Lesson 3)

7. $\frac{5}{8}$, $\frac{11}{22}$, 0.55, 0.58 **8.** $\frac{7}{15}$, 0.7, 0.65, $\frac{15}{30}$ **9.** $\frac{10}{3}$, $3\frac{2}{9}$, 3.3

Solve. (Lesson 3)

10. Tina has read $\frac{2}{7}$ of the books in the school library. Nikki has read $\frac{3}{9}$ of the books in the same library. Who has read the greater fraction of books?

Hands On Lesson 4

Relate Decimals and Percents

Vocabulary
percent

Objective Write percents as decimals and decimals as percents.

Learn About It

A nationwide survey of 600 students conducted by Junior Achievement showed that over 78 percent of the students want to own their own business someday.

The symbol % is used to show a percent. You can write seventy-eight percent as 78%.

Percent means "per hundred," or "out of one hundred."

Seventy-eight percent is another way to say "78 hundredths," or 0.78.



Junior Achievement is an organization dedicated to educating young people about business and economics.

Try this activity to see how percents are related to decimals.

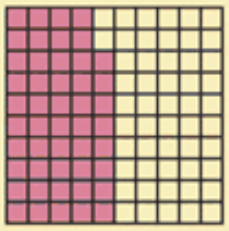
Materials: Decimal/Percent models (Learning Tool 1), colored pencils

STEP 1 On one of the grids on Learning Tool 1, shade an area that represents 0.48.

- How many squares on the grid are shaded?
- What percent of the 100 squares are shaded?

STEP 2 Use another grid. Shade an area that represents 0.09.

- What percent of the grid is shaded?



Equivalent Decimals and Percents

Writing an Equivalent Decimal

To write a percent as a decimal, remove the percent symbol and move the decimal point two places to the left.

$67\% = 0.67. = 0.67$

$8\% = 0.08. = 0.08$

Writing an Equivalent Percent

To write a decimal as a percent, move the decimal point two places to the right and write a percent symbol.

$0.96 = 0.96, = 96\%$

$0.05 = 0.05, = 5\%$

Guided Practice

Ask Yourself
- How do I write a decimal as a percent?
- How do I write a percent as a decimal?

Write each decimal as a percent.

1. 0.65 **2.** 0.07 **3.** 0.34 **4.** 0.01

Write each percent as a decimal.

5. 22% **6.** 2% **7.** 42% **8.** 5%

Explain Your Thinking Why must you write a zero in the tenths place when writing a decimal for percents less than 10%?

Practice and Problem Solving

Write each decimal as a percent.

9. 0.31 **10.** 0.18 **11.** 0.09 **12.** 0.44 **13.** 0.06

Write each percent as a decimal.

14. 16% **15.** 1% **16.** 88% **17.** 0% **18.** 72%

Data **Use the table for Problems 19–21.**

A polltaker asked students what they think is the main reason people do not start businesses.

Main Reason People Do Not Start Businesses	Percent
Not motivated enough	14%
Afraid to fail	39%
Lack of start-up money	28%
Other	19%

19. What reason was given by 14 out of every 100 students as the main reason people do not start businesses?

20. What reason was given by 39 out of every 100 students as the main reason people do not start businesses?

21. What decimal expresses the percent of students who think lack of start-up money is the main reason for not starting a business?

22. Of the sixth-graders Jim surveyed at his school, 0.67 said they wanted to start a business someday. Express the decimal 0.67 as a percent.

Mixed Review and Test Prep

Open Response

Find each quotient. (Ch. 2, Lesson 3)

23. 3,947 ÷ 5 **24.** 3,321 ÷ 6

25. 6,482 ÷ 7 **26.** 4,893 ÷ 8

27. 2,843 ÷ 9 **28.** 1,104 ÷ 4

Multiple Choice

29. Which decimal expresses 48%? (Ch. 4, Lesson 4)

A 4.8 **C** 0.048

B 0.48 **D** 0.0048

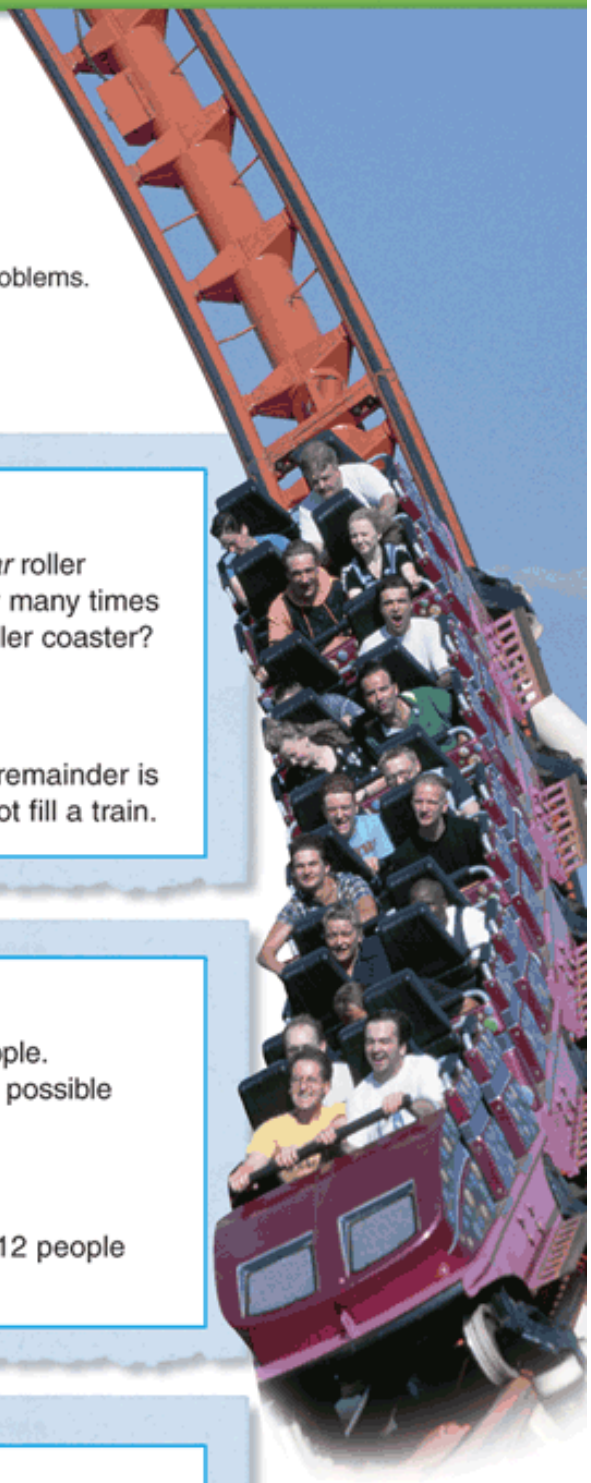
Extra Practice See page 105, Set D.

Lesson 5

Problem-Solving Application

Interpret Remainders

Objective Decide how to use remainders when solving problems.



When you solve a problem that has a remainder other than zero, you need to decide how to interpret that remainder.

Sometimes you ignore the remainder.

The engineers who designed the *Outer Limits: Flight of Fear* roller coaster in Virginia designed its train to hold 24 people. How many times can the train be filled completely if 2,000 people ride the roller coaster?

$$2{,}000 \div 24 \rightarrow 83 \text{ R}8$$

Solution: The train will be filled completely 83 times. The remainder is not part of the solution, because 8 people will not fill a train.

Sometimes the remainder is the solution.

The *Mantis* roller coaster in Ohio has trains that hold 32 people. If 3,500 people ride the roller coaster and as many trains as possible are completely filled, how many will ride in the last train?

$$3{,}500 \div 32 \rightarrow 109 \text{ R}12$$

Solution: After as many trains as possible are filled, only 12 people will be left to ride in the last train.

Sometimes the remainder affects the solution.

For the roller coaster *Superman: The Escape* in California, designers made trains that carry 15 passengers. How many trains are needed so that everyone in a 100-person tour group can ride the roller coaster?

$$100 \div 15 \rightarrow 6 \text{ R}10$$

Solution: The group needs at least 7 trains, since 6 trains would only carry 90 people.

Look Back Why does thinking about the question in each situation help you decide what to do with the remainder?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

Ask Yourself

- What does the question ask me to find?
- Is the remainder the solution, or does it affect the solution?
- Does my answer make sense?

1. A roller coaster car can carry 12 passengers. How many cars are needed to carry 145 passengers?

2. Anna and three friends want to share 15 ride tickets equally. Anna will give any leftover tickets to her brother. How many tickets will Anna's brother get?

   Hint What does the question tell you about the remainder?

Independent Practice

Solve. Explain how you used the remainder.

3. A school bus can hold 44 students. How many school buses are needed to take 120 students to the amusement park?

4. **Reasoning** How many students will ride on each of the buses in Problem 3 if two of the buses have 6 empty seats each? Explain your reasoning.

5. **Analyze** Five teachers and 68 students go on a thrill ride. Each car can hold 20 people. One teacher rides in each filled car.

   a. How many cars will be filled?

   b. How many teachers and students ride in the car that is not filled?

6. A teacher took 52 photos during the visit to the amusement park. She wants to place all the photos in an album. Each album page will hold 8 photos. What is the least number of pages the teacher needs for all her photos?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. At a beach party, $\frac{3}{4}$ of the people are adults. Of the people who are not adults, $\frac{1}{2}$ are boys and 6 are girls. How many people are at the beach party?

8. Tim, Roz, and Sylvia collected 57 rocks together. They gave 15 rocks to Jan and divided the rest among themselves. How many rocks does each have now?

9. Ty has 15 seashells. Mike has 3 times as many seashells as Ty. Amy has 8 more seashells than Mike. Jeff also has seashells. The four friends have a total of 167 seashells. How many seashells does Jeff have?

You Choose

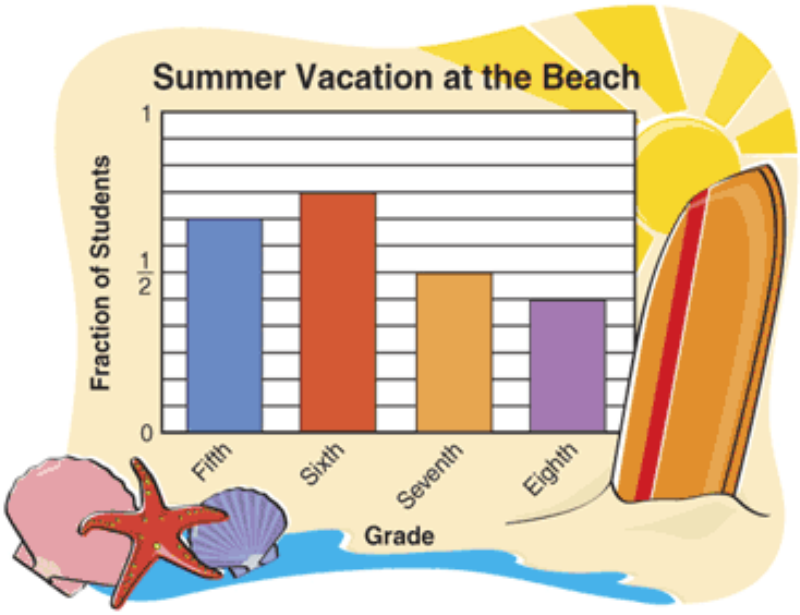
Strategy

- Draw a Diagram
- Make an Organized List
- Work Backward
- Use Logical Reasoning
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the bar graph for Problems 10 and 11. Solve. Then tell which method you chose.**



10. The bar graph on the right shows the fractions of students in each grade who took their summer vacation at the beach. In which grade do more than $\frac{2}{3}$ of the students spend summer at the beach?

11. **Analyze** What fraction of students in each grade did not vacation at the beach? How can you use the information on the bar graph to find the answer without subtracting?

12. Sarah sells necklaces made from seashells. At the end of one day, she sold 12 necklaces and collected $54. If each necklace sold for the same price, what was the price of each necklace?

13. One weekend, Alexia and her friends built 9 sandcastles. They worked on the sandcastles for 6 hours. On average, how many sandcastles did Alexia and her friends build in 1 hour?

Social Studies Connection

State the Letter

There are 50 states in the United States.

Over 50% of the state names begin with one of only five letters of the alphabet.



Look at the map of the United States.

1. Identify the 5 letters.
2. For each letter, identify the fraction of all states whose name begins with that letter.
3. What fraction of all 50 states has a name that begins with any one of the 5 letters?
4. How many states begin with the letter W? What percentage is this of all states?
5. How many states have a name that begins with a vowel?
6. What states have a name composed of two words? What percentage of these states names begin with the letter N?

Chapter Review/Test

VOCABULARY

1. _____ means "per hundred" or "out of a hundred."
2. A(n) _____ is the least common multiple of the denominators of two or more fractions.
3. Fractions that represent the same number are _____.
4. Any common multiple of the denominators of two or more factions is a _____.

Vocabulary

- percent
- simplest form
- common denominator
- equivalent fractions
- least common denominator (LCD)

CONCEPTS AND SKILLS

Write each fraction in simplest form. Then write another equivalent fraction for each. (Lesson 1, pp. 90–91)

5. $\frac{10}{20}$ **6.** $\frac{30}{48}$ **7.** $\frac{18}{27}$ **8.** $\frac{22}{44}$

Write an equivalent decimal for each. (Lesson 2, pp. 92–93)

9. $\frac{65}{1{,}000}$ **10.** $1\frac{5}{10}$ **11.** $2\frac{42}{100}$ **12.** $\frac{37}{100}$ **13.** $\frac{9}{10}$

Write two equivalent fractions or mixed numbers for each. (Lesson 2, pp. 92–93)

14. 5.75 **15.** 0.025 **16.** 2.4 **17.** 0.82 **18.** 0.9

Compare. Write in order from least to greatest. (Lesson 3, pp. 94–96)

19. $\frac{2}{3}$, $\frac{2}{5}$, 0.50, 0.70 **20.** $\frac{4}{7}$, 0.45, $\frac{1}{2}$ **21.** $\frac{1}{4}$, $\frac{2}{9}$, 0.21, 0.30

Write each decimal as a percent. (Lesson 4, pp. 98–99)

22. 0.52 **23.** 0.04 **24.** 0.33 **25.** 0.08 **26.** 0.91

Write each percent as a decimal. (Lesson 4, pp. 98–99)

27. 2% **28.** 71% **29.** 6% **30.** 76% **31.** 23%

PROBLEM SOLVING

Solve. Explain how you used the remainder. (Lesson 5, pp. 100–102)

32. Mr. Mason has a box of 260 sandwiches for 110 campers. He gives each camper an equal number of sandwiches. The leftover sandwiches go to the counselors. How many do the counselors get?

Write About It

Show You Understand

Rosa can tell that $\frac{1}{2} > \frac{1}{3}$, $\frac{3}{4} > \frac{3}{5}$, and $\frac{7}{9} > \frac{7}{10}$. How does she know this without using a common denominator?

Extra Practice

Set A (Lesson 1, pp. 90–91)

Write two equivalent fractions for each. Use multiplication to write one and division to write the other.

1. $\frac{4}{6}$ 2. $\frac{4}{16}$ 3. $\frac{25}{50}$ 4. $\frac{28}{32}$ 5. $\frac{30}{33}$

Write each fraction or mixed number in simplest form.

6. $\frac{3}{21}$ 7. $\frac{4}{20}$ 8. $\frac{6}{18}$ 9. $\frac{12}{10}$ 10. $3\frac{3}{24}$

Set B (Lesson 2, pp. 92–93)

Write an equivalent decimal for each.

1. $\frac{35}{100}$ 2. $1\frac{3}{10}$ 3. $\frac{100}{100}$ 4. $5\frac{12}{100}$ 5. $\frac{44}{1{,}000}$

Write two equivalent fractions or mixed numbers for each.

6. 0.8 7. 0.17 8. 0.36 9. 2.5 10. 11.25

Set C (Lesson 3, pp. 94–96)

Compare. Write >, <, or = for each ●.

1. $\frac{4}{9}$ ● $\frac{3}{7}$ 2. $8\frac{1}{4}$ ● 8.25 3. $\frac{5}{6}$ ● 0.4 4. $3\frac{2}{5}$ ● 3.45

5. $\frac{5}{12}$ ● $\frac{3}{8}$ 6. $2\frac{3}{7}$ ● 2.44 7. 1.6 ● $1\frac{6}{10}$ 8. $\frac{7}{9}$ ● 0.75

Compare. Write in order from least to greatest.

9. $\frac{1}{5}, \frac{1}{2}, \frac{3}{10}$ 10. $\frac{3}{4}, \frac{7}{8}, \frac{5}{6}$ 11. $\frac{1}{3}$, 0.3, $\frac{2}{9}$

12. $\frac{1}{2}, \frac{3}{7}$, 0.75, 0.55 13. $\frac{2}{5}, \frac{1}{3}$, 0.5, 0.44 14. $\frac{5}{6}, \frac{4}{7}$, 0.8, 0.72

Set D (Lesson 4, pp. 98–99)

Write each decimal as a percent.

1. 0.45 2. 0.09 3. 0.82 4. 0.03 5. 0.99

Write each percent as a decimal.

6. 4% 7. 63% 8. 8% 9. 77% 10. 81%

CHAPTER 5

Add and Subtract Fractions

INVESTIGATION

Using Data

Most of the Earth's active volcanoes are located in "The Ring of Fire," which encircles the Pacific Ocean. Combine two regions together. What fraction of active volcanoes are found in the regions that you combined?

List some regions of the world that would be included in the category "Other."



ACTIVE VOLCANOS

Region	Fraction of Total
Asia and Pacific	$\frac{9}{25}$
Americas	$\frac{47}{100}$
Europe	$\frac{7}{50}$
Other	$\frac{3}{100}$

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

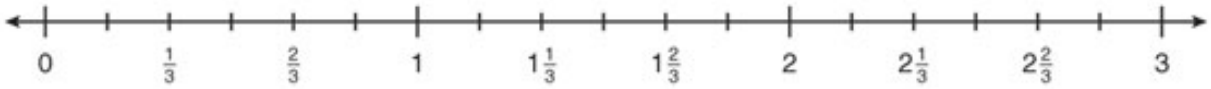
1. A number that is a multiple of two or more numbers is a(n) _____.
2. A number to be added in an addition expression is a(n) _____.
3. In the fraction $\frac{5}{8}$, the _____ tells you that there are 8 parts in the whole.

Vocabulary

- addend
- numerator
- denominator
- common multiple
- common denominator

CONCEPTS AND SKILLS

Use the number line to help you add or subtract.



4. $\frac{2}{3} + \frac{2}{3}$
5. $1\frac{1}{3} + 1\frac{1}{3}$
6. $\frac{2}{3} + 2\frac{1}{3}$
7. $1\frac{1}{3} + \frac{1}{6}$
8. $1\frac{2}{3} - \frac{1}{3}$
9. $2\frac{1}{3} - \frac{2}{3}$
10. $3 - \frac{2}{3}$
11. $2\frac{1}{3} - \frac{1}{6}$

Choose a common denominator for each pair of fractions.

12. $\frac{1}{10}, \frac{1}{5}$ — 5, 8, 10
13. $\frac{2}{3}, \frac{1}{2}$ — 3, 6, 9
14. $\frac{1}{4}, \frac{1}{8}$ — 2, 4, 8
15. $\frac{1}{3}, \frac{1}{6}$ — 3, 6, 9
16. $\frac{1}{3}, \frac{1}{4}$ — 8, 12, 15
17. $\frac{1}{3}, \frac{1}{5}$ — 5, 10, 15
18. $\frac{1}{2}, \frac{1}{6}$ — 4, 6, 8
19. $\frac{1}{4}, \frac{1}{12}$ — 12, 15, 16

Write About It

20. How can you use the drawing to show that $\frac{2}{8} = \frac{1}{4}$? What other things does the drawing tell you about equivalent fractions?





Facts Practice, See page 659.

Lesson 1

Estimate Sums and Differences

Objective Estimate sums and differences of fractions and mixed numbers.

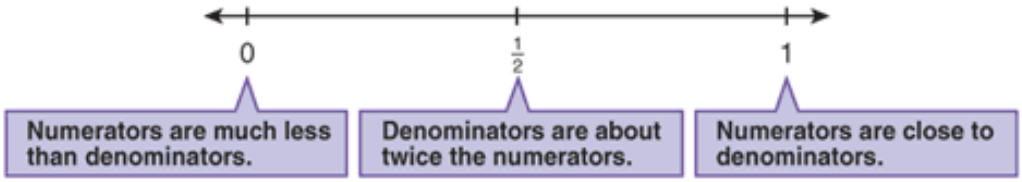


The wettest 24 hours in Tallahassee, Florida, occurred when slightly more than $10\frac{1}{10}$ inches of rain fell.

Learn About It

Suppose that during four consecutive 6-hour periods a town receives $\frac{1}{10}$ inches, $1\frac{3}{8}$ inches, $3\frac{3}{10}$ inches, and $3\frac{7}{8}$ inches of rain. Is the total amount greater than or less than Tallahassee's record rainfall of $10\frac{1}{10}$ inches?

To estimate whether a fraction is close to 0, $\frac{1}{2}$, or 1, compare its numerator and denominator.



Estimate the sum of $\frac{1}{10}$, $1\frac{3}{8}$, $3\frac{3}{10}$, and $3\frac{7}{8}$.

STEP 1 Round each fraction or mixed number.

$\frac{1}{10}$ rounds to 0; $1\frac{3}{8}$ rounds to $1\frac{1}{2}$

$3\frac{3}{10}$ rounds to $3\frac{1}{2}$; $3\frac{7}{8}$ rounds to 4

STEP 2 Add the estimates.

$0 + 1\frac{1}{2} + 3\frac{1}{2} + 4 = 9$

Solution: The total is about an inch less than Tallahassee's record rainfall.

▶ You can also estimate differences of fractions and mixed numbers.

Estimate the difference of $3\frac{3}{10}$ and $1\frac{3}{8}$.

STEP 1 Round each fraction or mixed number.

$3\frac{3}{10}$ rounds to $3\frac{1}{2}$

$1\frac{3}{8}$ rounds to $1\frac{1}{2}$

STEP 2 Subtract the estimates.

$3\frac{1}{2} - 1\frac{1}{2} = 2$

Solution: $3\frac{3}{10} - 1\frac{3}{8}$ is about 2.

Guided Practice

Estimate each sum or difference.

1. $\frac{4}{5} + \frac{1}{8}$ **2.** $4\frac{2}{3} + 7\frac{11}{16}$ **3.** $3\frac{4}{7} - \frac{3}{8}$ **4.** $9\frac{1}{3} - 5\frac{7}{10}$

Ask Yourself

- Did I compare the numerator and the denominator to round a fraction?

Explain Your Thinking Find a range of estimates for Exercise 2. Why is it useful to find a range of sums?

Practice and Problem Solving

Estimate each sum or difference.

5. $\frac{3}{5} + \frac{1}{7}$ **6.** $6\frac{2}{3} + \frac{13}{16}$ **7.** $8\frac{4}{7} - 2\frac{3}{8}$ **8.** $4\frac{1}{3} - 2\frac{7}{10}$

9. $25\frac{5}{6} - 8\frac{11}{15}$ **10.** $8\frac{1}{6} - 5\frac{1}{8}$ **11.** $3\frac{5}{9} + 7\frac{1}{8} + 2\frac{9}{10}$ **12.** $10\frac{5}{11} + 23\frac{7}{8} + 8\frac{1}{10}$

Estimate the sum or difference. Write $<$ or $>$ for each ●.

13. $7\frac{1}{3} - 2\frac{4}{5}$ ● $2\frac{1}{7} + 3\frac{1}{2}$ **14.** $8\frac{7}{16} + 2\frac{5}{11}$ ● $30\frac{5}{8} - 18\frac{1}{5}$

Solve.

15. In Tallahassee, July is normally the wettest month with about $8\frac{4}{5}$ inches of rain on average. October is the driest month with an average of $2\frac{9}{10}$ inches of rain. On average, about how much more rain falls in July than in October?

16. Donita made snacks for an emergency kit. She mixed $1\frac{3}{16}$ pounds of raisins, $\frac{7}{8}$ pound of peanuts, and $\frac{3}{8}$ pound of sunflower seeds. About how many more pounds of ingredients does she need to make 5 pounds of snacks altogether?

17. Reasoning In the emergency kit, there is a board game that can be played by 4 players. How many different combinations of 5 students can play?

18. What's Wrong? Yukio says that the numbers $4\frac{1}{2}$ and 4.2 are equal to each other. Explain why he is incorrect. Then give the correct answer.

Mixed Review and Test Prep

Open Response

Solve. (Ch. 4, Lesson 1)

19. $\frac{8}{28} = \frac{\square}{7}$ **20.** $\frac{40}{88} = \frac{5}{\square}$

21. $\frac{2}{3} = \frac{6}{\square}$ **22.** $\frac{7}{8} = \frac{\square}{32}$

23. Side-by-side posters along an 8-foot wall are $2\frac{1}{4}$, $1\frac{3}{8}$, and $2\frac{4}{5}$ feet wide. About how much wall space is left? Explain. (Ch. 5, Lesson 1)

Extra Practice See page 123, Set A.

Lesson 2

Add and Subtract With Like Denominators

Objective Add and subtract fractions and mixed numbers with like denominators.



Learn About It

In 2001, Buffalo, New York, had almost 7 feet of snow from a single storm. If $\frac{3}{8}$ foot of snow fell the first hour and $\frac{7}{8}$ foot fell the second hour, how much snow fell in the first two hours?

About how much more snow fell in the second hour than in the first hour?

Find $\frac{3}{8} + \frac{7}{8}$.

To add fractions with like denominators, add the numerators. Keep the same denominator.

$$\frac{3}{8} + \frac{7}{8} = \frac{(3+7)}{8} = \frac{10}{8}$$

$$\frac{10}{8} = \frac{8}{8} + \frac{2}{8} = 1\frac{2}{8} = 1\frac{1}{4} \leftarrow \text{simplest form}$$

Solution: In the first two hours, $1\frac{1}{4}$ feet of snow fell in all.

Find $\frac{7}{8} - \frac{3}{8}$.

To subtract fractions with like denominators, subtract the numerators. Keep the same denominator.

$$\frac{7}{8} - \frac{3}{8} = \frac{7-3}{8} = \frac{4}{8} = \frac{1}{2} \leftarrow \text{simplest form}$$

Solution: In the second hour, $\frac{1}{2}$ foot more snow fell than in the first hour.

Other Examples

A. Adding Mixed Numbers

Sometimes you need to rename the sum to simplify the answer.

$$\begin{array}{r} 2\frac{5}{12} \\ +\ 3\frac{11}{12} \\ \hline 5\frac{16}{12} \end{array}$$

- Add the fractions.
- Add the whole numbers.
- Simplify the answer.

$$5\frac{16}{12} = 5 + \frac{12}{12} + \frac{4}{12} = 6\frac{4}{12} = 6\frac{1}{3}$$

B. Subtracting Mixed Numbers

Sometimes you need to rename the numerator first in order to subtract.

$$\begin{array}{rcl} 5\frac{1}{6} & = & 4\frac{7}{6} \\ -\ 2\frac{5}{6} & = & -\ 2\frac{5}{6} \\ \hline & & 2\frac{2}{6} = 2\frac{1}{3} \end{array}$$

- Cannot subtract $\frac{5}{6}$ from $\frac{1}{6}$; rename the first number.
- Subtract the fractions and the whole numbers.
- Simplify the answer.

Guided Practice

Add or subtract. Write your answer in simplest form.

1. $\frac{5}{8} + \frac{1}{8}$ **2.** $\frac{8}{15} - \frac{4}{15}$ **3.** $1\frac{1}{4} + 2\frac{3}{4}$ **4.** $6\frac{9}{16} - 6\frac{5}{16}$

5. $7\frac{3}{8} + 1\frac{7}{8}$ **6.** $4\frac{1}{4} - 2\frac{3}{4}$ **7.** $\frac{15}{12} - \frac{11}{12}$ **8.** $5\frac{3}{7} - 2\frac{6}{7}$

Ask Yourself

- Do I need to rename to subtract?
- Did I simplify the answer?

Explain Your Thinking Why do you add the numerators but not the denominators when the fractions have like denominators?

Practice and Problem Solving

Add or subtract. Write your answer in simplest form.

9. $\frac{3}{7} + \frac{2}{7}$ **10.** $\frac{9}{16} + \frac{5}{16}$ **11.** $\frac{7}{12} - \frac{5}{12}$ **12.** $2\frac{3}{4} + 1\frac{1}{4}$ **13.** $3\frac{5}{9} - 1\frac{2}{9}$

14. $4\frac{3}{5} + 3\frac{1}{5}$ **15.** $5\frac{11}{14} - 5\frac{9}{14}$ **16.** $5\frac{5}{8} + 5\frac{5}{8}$ **17.** $10\frac{1}{9} - \frac{5}{9}$ **18.** $7\frac{3}{8} - 1\frac{7}{8}$

Mental Math **Add or subtract. Count on or count back to make the next whole.**

19. $6\frac{3}{4} + \frac{3}{4} = 6\frac{3}{4} + \frac{1}{4} + \frac{2}{4} = \blacksquare$

20. $6\frac{1}{5} - \frac{3}{5} = 6\frac{1}{5} - \frac{1}{5} - \frac{2}{5} = \blacksquare$

21. $3\frac{9}{16} + \frac{13}{16}$ **22.** $1\frac{3}{8} + \frac{7}{8}$ **23.** $3\frac{1}{12} - \frac{9}{12}$ **24.** $4\frac{2}{9} - \frac{5}{9}$

Solve.

25. A storm dropped $1\frac{5}{12}$ feet of snow in its first hour and $\frac{7}{12}$ foot more in its second hour. How much snow fell during the two hours?

26. **Represent** Draw a number line from 0 to 2 and mark it in eighths. Show how you would represent the subtraction problem $1\frac{3}{8} - \frac{7}{8}$ on the number line.

Mixed Review and Test Prep

Open Response

Compare. Write > or <. (Ch. 4, Lesson 3)

27. $\frac{8}{15} \bigcirc \frac{1}{2}$ **28.** $\frac{7}{16} \bigcirc \frac{8}{18}$

29. $4\frac{1}{12} \bigcirc 4\frac{1}{10}$ **30.** $2\frac{2}{3} \bigcirc \frac{22}{3}$

31. $\frac{4}{7} \bigcirc \frac{5}{6}$ **32.** $0.25 \bigcirc \frac{3}{4}$

Multiple Choice

33. A $5\frac{1}{4}$-foot-tall snowdrift thawed by $\frac{1}{4}$ foot on Day 1 and twice that amount on Day 2. How tall was the snowdrift by the end of Day 2? (Ch. 5, Lesson 2)

A 6 feet **C** $4\frac{3}{4}$ feet

B 5 feet **D** $4\frac{1}{2}$ feet

Extra Practice See page 123, Set B.

Lesson 3

Audio Tutor 1/12 Listen and Understand

Add With Unlike Denominators

Vocabulary
least common multiple (LCM)

Objective Add fractions and mixed numbers with unlike denominators.

Learn About It

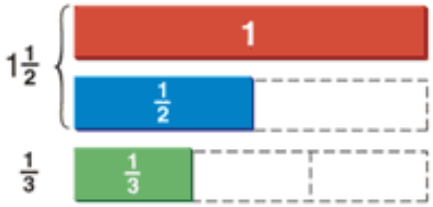
Severe earthquakes can last more than a minute and cause great damage. Suppose the earth trembled for $1\frac{1}{2}$ minutes and then shook harder for $\frac{1}{3}$ minute more. How long did the earthquake last altogether?

Add. $1\frac{1}{2} + \frac{1}{3} = n$

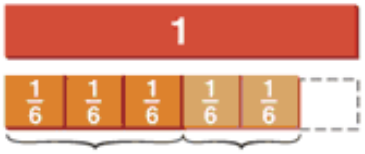
Different Ways to Add Fractions and Mixed Numbers

Way 1 You can use fraction strips.

STEP 1 Use fraction strips to model the numbers.



STEP 2 Use equivalent strips. Then count the shaded parts.



$\frac{1}{2} = \frac{3}{6}$ $\frac{1}{3} = \frac{2}{6}$

$1 + \frac{3}{6} + \frac{2}{6} = 1\frac{5}{6}$

Way 2 You can use any common denominator.

STEP 1 Write equivalent fractions with a common denominator.

$2 \times 3 = 6$

common denominator

$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

STEP 2 Add the numerators. Write the common denominator. Simplify.

$$\begin{array}{r} 1\frac{1}{2} = 1\frac{3}{6} \\ +\frac{1}{3} = +\frac{2}{6} \\ \hline 1\frac{5}{6} \end{array}$$

Solution: The earthquake lasted for $1\frac{5}{6}$ minutes altogether.

One of the biggest earthquakes in North America occurred in San Francisco, California, in 1906.



Extra Help at eduplace.com/map

You can also use the **least common multiple (LCM)** of the denominators to add fractions and mixed numbers with unlike denominators. Often the LCM is less than the product of the denominators. Using the LCM may make calculating easier.

Find $\frac{3}{8} + \frac{5}{6}$.

STEP 1 Find the least common multiple (LCM) of the denominators.

Multiples of 8: 8, 16, (24), 32, …
Multiples of 6: 6, 12, 18, (24), 30, …
The LCM is 24.

STEP 2 Write equivalent fractions, using the LCM. Add the numerators. Simplify.

$$\frac{3 \times 3}{8 \times 3} = \frac{9}{24}$$

$$\frac{5 \times 4}{6 \times 4} = \frac{20}{24}$$

$$\frac{9}{24} + \frac{20}{24} = \frac{29}{24} = \frac{24}{24} + \frac{5}{24} = 1\frac{5}{24}$$

Solution: $\frac{3}{8} + \frac{5}{6} = 1\frac{5}{24}$

Other Examples

A. Two Mixed Numbers

$$\begin{aligned} 2\tfrac{3}{5} &= 2\tfrac{9}{15} \\ +\,4\tfrac{2}{3} &= +\,4\tfrac{10}{15} \end{aligned}$$

$$6\tfrac{19}{15} = 6 + \tfrac{15}{15} + \tfrac{4}{15} = 7\tfrac{4}{15}$$

B. More Than Two Addends

$$\begin{aligned} 5\tfrac{3}{4} &\quad 5\tfrac{45}{60} \\ \tfrac{4}{5} &\quad \tfrac{48}{60} \\ +\,3\tfrac{1}{6} &\quad +\,3\tfrac{10}{60} \end{aligned}$$

The LCM of 4, 5, and 6 is $2 \times 2 \times 3 \times 5$, or 60.

$$8\tfrac{103}{60} = 8 + \tfrac{60}{60} + \tfrac{43}{60} = 9\tfrac{43}{60}$$

Guided Practice

Find each sum. Write your answer in simplest form.

1. $\frac{3}{5} + \frac{4}{15}$
2. $\frac{5}{6} + \frac{1}{4}$
3. $\frac{11}{14} + \frac{3}{4} + \frac{1}{2}$
4. $1\frac{2}{3} + \frac{1}{9}$
5. $6\frac{3}{5} + 4\frac{2}{3}$
6. $\frac{1}{6} + \frac{2}{5} + \frac{1}{3}$
7. $5\frac{7}{10} + \frac{4}{5}$
8. $17\frac{3}{4} + 2\frac{2}{3}$
9. $3\frac{1}{9} + \frac{3}{18}$

Ask Yourself

- Did I write equivalent fractions?
- Did I use the LCM of the denominators?
- Did I simplify the sum?

Explain Your Thinking When adding $\frac{5}{6} + \frac{1}{4}$, is it easier to use the LCM or 24 (the product of the denominators)?

Go On

Practice and Problem Solving

Use mental math to find each sum.

10. $\frac{1}{2} + \frac{1}{4}$

11. $\frac{1}{8} + \frac{7}{16}$

12. $2\frac{7}{8} + 1\frac{1}{2} + \frac{1}{8}$

13. $7\frac{2}{3} + \frac{1}{2} + \frac{1}{3}$

14. $\frac{3}{4} + \frac{3}{4} + \frac{1}{2}$

15. $\frac{1}{5} + \frac{3}{5} + \frac{1}{5} + \frac{3}{10}$

Find each sum. Write your answer in simplest form.

16. $\frac{1}{3} + \frac{1}{2}$

17. $3\frac{2}{5} + 1\frac{2}{3}$

18. $\frac{11}{15} + \frac{2}{3}$

19. $\frac{1}{2} + \frac{6}{7}$

20. $\frac{2}{9} + 6\frac{11}{12}$

21. $9\frac{3}{7} + \frac{5}{21}$

22. $2\frac{1}{3} + \frac{1}{6}$

23. $4\frac{4}{5} + \frac{7}{20}$

24. $6\frac{2}{3} + 5\frac{1}{2}$

25. $5\frac{3}{5} + 4\frac{2}{5}$

26. $\frac{9}{10} + \frac{5}{6} + \frac{2}{3}$

27. $\frac{8}{15} + \frac{1}{6} + \frac{1}{2}$

Algebra • Properties

Use addition properties to find the value of *n*.

28. $\frac{3}{4} + n = \frac{3}{4}$

29. $\frac{7}{8} + \frac{5}{6} = \frac{5}{6} + n$

30. $\frac{3}{10} + \frac{7}{10} + n = 1$

31. $\frac{1}{12} + \frac{5}{12} + n = 1 - \frac{1}{2}$

Solve.

32. Estimate In the 1906 earthquake, the crack was about 290 miles long. In the 1989 Loma Prieta earthquake, the crack was about 25 miles long. About how many times longer was the 1906 crack?

33. Hanh found a spot where an earthquake had opened a crack $8\frac{1}{2}$ inches wide. If another quake increases the opening by $1\frac{7}{8}$ inches, how wide would the crack be?

34. Represent Dave says that when one denominator is a multiple of the other, you can use the larger denominator as the LCM. Sketch a diagram to show why Dave's observation is correct.

35. Use the information on the trail sign below. Leo and his grandmother go on a hike. They walk for $\frac{1}{3}$ hour, rest for $\frac{1}{4}$ hour, and then walk for $\frac{1}{6}$ hour. Did they finish in the typical time? Explain.



Extra Practice-See page 123, Set C.

Game

Activity

Spin for the Sum

2 Players

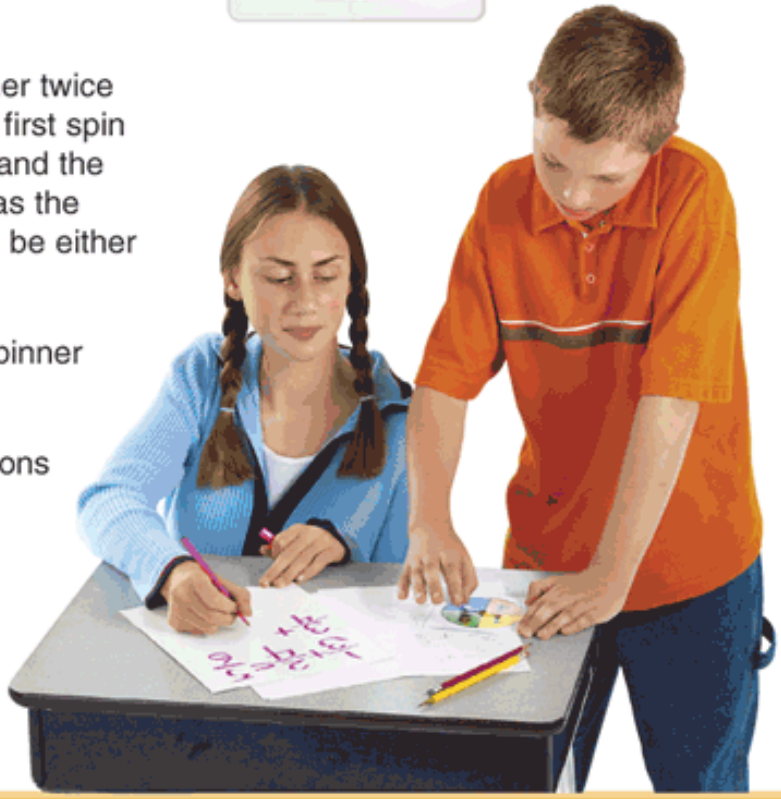
What You'll Need • spinner with four equal regions, labeled 1, 2, 3, and 4 (Learning Tool 35); paper and pencil



How to Play

1. The first player spins the spinner twice and uses the number from the first spin as the numerator of a fraction and the number from the second spin as the denominator. (The fraction can be either proper or improper.)
2. The second player spins the spinner twice to write another fraction.
3. Both players add the two fractions and compare their answers. A player earns one point for each correct sum.

Players repeat Steps 1–3. The first player to reach a total of 5 points wins.



Quick Check

Check your understanding of Lessons 1–3.

Estimate each sum or difference. Then add or subtract. Write your answer in simplest form. (Lessons 1–3)

1. $\frac{2}{9} + 4\frac{1}{9}$ **2.** $6\frac{3}{5} + 4\frac{2}{3}$ **3.** $4\frac{2}{5} - 3\frac{1}{5}$ **4.** $7\frac{2}{5} - 4\frac{3}{5}$

Which is the sum in simplest form? (Lesson 3)

5. $\frac{1}{4} + \frac{3}{4}$

a. 1 **b.** 2 **c.** 4

6. $1\frac{4}{9} + \frac{6}{9}$

a. 2 **b.** $2\frac{1}{9}$ **c.** 4

7. $1\frac{1}{5} + 3\frac{1}{5}$

a. 3 **b.** $4\frac{2}{5}$ **c.** $6\frac{1}{3}$

Lesson 4

Audio Tutor 1/13 Listen and Understand

Subtract With Unlike Denominators

Objective Subtract fractions and mixed numbers with unlike denominators.



Learn About It

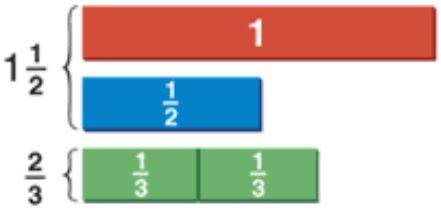
When Mount St. Helens erupted in 1980, it sent volcanic ash many miles into the air. One city in Washington State was covered by $1\frac{1}{2}$ inches of ash. Another city farther away was covered by $\frac{2}{3}$ inch. How much more ash settled on the first city than on the second?

Subtract. $1\frac{1}{2} - \frac{2}{3} = n$

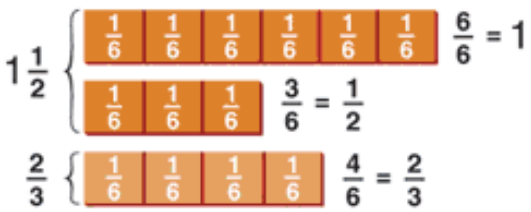
Different Ways to Subtract Fractions and Mixed Numbers

Way 1 You can use fraction strips.

STEP 1 Use fraction strips to model the numbers.



STEP 2 Use equivalent strips to show the subtraction.



$$1\frac{1}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$$

Way 2 You can use a common denominator.

STEP 1 Find a common denominator. Write equivalent fractions.

$$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$$

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

STEP 2 Rename $1\frac{3}{6}$ as $\frac{9}{6}$.

$$1\frac{3}{6} = \frac{6}{6} + \frac{3}{6} = \frac{9}{6}$$

$$-\frac{4}{6} \qquad -\frac{4}{6}$$

STEP 3 Subtract the fractions. Simplify.

$$\frac{9}{6}$$

$$-\frac{4}{6}$$

$$\frac{5}{6}$$

Solution: The first city had $\frac{5}{6}$ inch more ash.

Just as with whole numbers, you may need to rename before you subtract mixed numbers.

Smoke jumpers fight fires caused by hot lava and must be able to run $1\frac{1}{2}$ miles in less than $10\frac{1}{2}$ minutes. Ted can run this distance in $12\frac{1}{4}$ minutes. By how much time does he have to improve his run to become a smoke jumper?

Find $12\frac{1}{4} - 10\frac{1}{2}$.

STEP 1 Write equivalent fractions, using the LCM.

The LCM of 4 and 2 is 4.

$$\begin{array}{rl} 12\frac{1}{4} = & 12\frac{1}{4} \\ -\,10\frac{1}{2} = & -\,10\frac{2}{4} \end{array}$$

STEP 2 Since $\frac{2}{4}$ cannot be subtracted from $\frac{1}{4}$, you need to rename.

$$12\frac{1}{4} = 11 + \frac{4}{4} + \frac{1}{4}$$
$$= 11\frac{5}{4}$$

STEP 3 Subtract the fractions. Simplify.

$$\begin{array}{r} 11\frac{5}{4} \\ -\,10\frac{2}{4} \\ \hline 1\frac{3}{4} \end{array}$$

Solution: Ted has to improve his run by $1\frac{3}{4}$ minutes.

Other Examples

A. Subtract From a Whole Number.

Rename the whole number first.

$$\begin{array}{rl} 12 = & 11 + \frac{3}{3} = 11\frac{3}{3} \\ -\,7\frac{2}{3} & -\,7\frac{2}{3} \\ \hline & 4\frac{1}{3} \end{array}$$

B. Compatible Denominators

Write an equivalent fraction, using the greater denominator.

$$\begin{array}{rl} 8\frac{2}{3} = & 8\frac{4}{6} \\ -\,3\frac{1}{6} = & -\,3\frac{1}{6} \\ \hline & 5\frac{3}{6} = 5\frac{1}{2} \end{array}$$

Guided Practice

Find each difference. Write your answer in simplest form.

1. $\frac{3}{5} - \frac{4}{15}$ **2.** $\frac{2}{3} - \frac{1}{2}$ **3.** $4\frac{7}{8} - 2\frac{3}{4}$ **4.** $4\frac{1}{5} - 1\frac{2}{3}$

Ask Yourself

- Do I have to rename in order to subtract?
- Can I simplify the answer?

Explain Your Thinking Why do you have to rename fractions in some examples but not in others?

Go On

Practice and Problem Solving

Find each difference. Write your answer in simplest form.

5. $9\frac{5}{6} - \frac{1}{3}$ **6.** $7\frac{1}{3} - \frac{5}{6}$ **7.** $\frac{3}{8} - \frac{1}{6}$ **8.** $\frac{3}{4} - \frac{3}{8}$

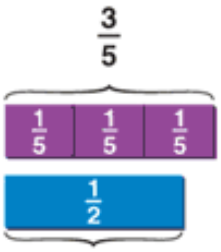
9. $2\frac{1}{2} - 1\frac{2}{5}$ **10.** $\frac{5}{6} - \frac{1}{2}$ **11.** $2\frac{1}{3} - \frac{2}{3}$ **12.** $2\frac{5}{9} - \frac{2}{3}$

13. $3\frac{5}{7} - \frac{1}{2}$ **14.** $10 - 3\frac{2}{5}$ **15.** $3 - 1\frac{5}{7}$ **16.** $5\frac{5}{12} - 1\frac{7}{8}$

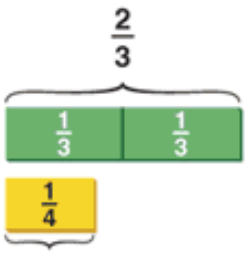
17. $4\frac{3}{7} - \frac{3}{7}$ **18.** $5\frac{7}{8} - 4$ **19.** $6\frac{7}{9} - 3\frac{3}{4}$ **20.** $6\frac{2}{5} - \frac{3}{10}$

Write and solve a subtraction problem for each model.

21.



22.



Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Data **Use the recipe for Problems 23–25. Then tell which method you chose.**

23. For a science fair, Dan made a clay model of a volcano. The recipe made $2\frac{2}{3}$ cups of clay. Dan used $1\frac{7}{8}$ cups for his model. How much of the clay was left?

24. Suppose the 2-cup glass measuring cup is filled to $\frac{1}{4}$ cup. How much more boiling water is needed to make the recipe?

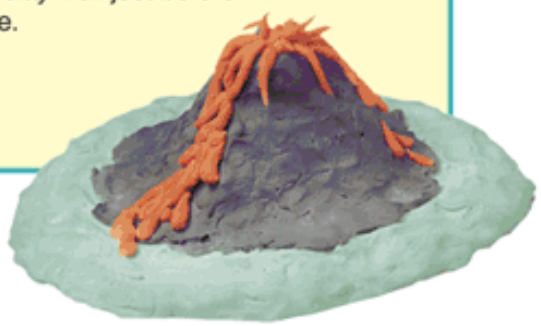
25. **Analyze** Jessica has $1\frac{1}{2}$ cups of salt, 10 cups of flour, and 1 cup of vegetable oil. How many batches of this clay recipe can she make? Explain how you found your answer.

Modeling Clay Recipe

Measure into a large bowl:
3 cups flour
$\frac{1}{2}$ cup salt
$\frac{1}{4}$ cup vegetable oil

In a 2-cup glass measuring cup, add in order:
1 tablespoon alum (in spice section)
food coloring
boiling water to equal 2 cups liquid total

Add liquid to flour mixture. Stir until well blended. Clay will be quite moist, but do not add more flour. Clay will thicken overnight. Store in an airtight container. Knead clay well just before first use.



Algebra • Equations

Use mental math and the properties for addition to find the value of *n* in each equation.

26. $n - \frac{4}{9} = \frac{7}{9}$

27. $\frac{7}{8} + n = 1\frac{1}{8}$

28. $n + 1\frac{1}{3} = 3$

29. $n - 1\frac{3}{5} = 5$

30. $n - \frac{4}{5} = \frac{1}{10}$

31. $12\frac{1}{2} - n = 6$

32. $5\frac{1}{4} - n = 5\frac{1}{4}$

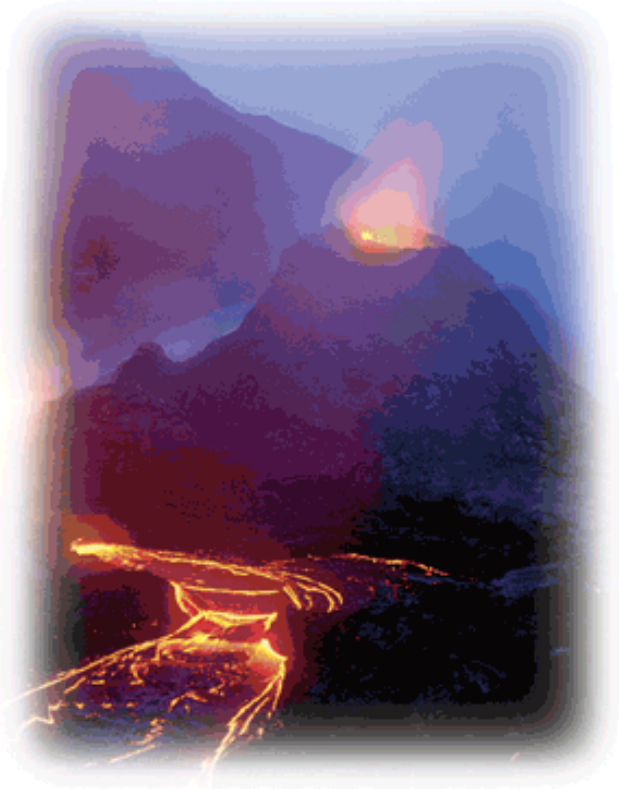
33. $15 - n = 7\frac{3}{4}$

Solve.

34. You Decide Each day, volcano walkers train by doing 100 push-ups and sit-ups and then hiking $1\frac{3}{8}$ miles uphill. Suppose a volcano walker has hiked less than a mile. Choose a distance less than a mile. How much is left of the uphill hike?

35. Write About It The first part of a lecture on volcanoes lasts $\frac{1}{2}$ hour. The next two parts each last $\frac{1}{4}$ hour. There are breaks of 5 minutes between each part. If you must leave after 55 minutes, how much of the lecture will you miss? Explain.

36. Nathan is preparing a poster on volcanoes. Text will take up $\frac{7}{8}$ of the poster. Art and photos will take up $\frac{1}{6}$ of the poster. Will the text, art, and photos fit on the poster? Show your work.



PU' U O'O Vent, Kilauea, Volcanoes National Park, Hawaii

Mixed Review and Test Prep

Open Response

Compare. Write the numbers in order from least to greatest. (Ch. 4, Lesson 3)

37. $\frac{1}{6}, \frac{1}{8}, \frac{1}{4}$

38. $\frac{2}{3}, \frac{5}{6}, \frac{5}{9}$

39. $\frac{1}{2}, \frac{7}{12}, \frac{3}{4}$

40. $1\frac{1}{25}, 2, \frac{25}{2}$

Multiple Choice

41. Luis unscrambled three volcano names in $1\frac{1}{10}$ minutes. Sandy took $\frac{2}{3}$ minute for the first one and $\frac{1}{4}$ minute for each of the two others. How many minutes faster was Luis? (Ch. 5, Lesson 4)

A $1\frac{1}{6}$ **B** $\frac{1}{4}$ **C** $\frac{5}{12}$ **D** $\frac{1}{15}$

Extra Practice See page 123, Set D.

Lesson 5

Problem-Solving Decision

Multistep Problems

Objective Review how to solve problems that have more than one step.

Before you solve a problem that has more than one step, decide what operations to use for each step.



Problem One rainy day Jamie, Cora, and Mavis watched three movies in a row. Between movies they made two batches of popcorn: $9\frac{1}{2}$ cups and $8\frac{2}{3}$ cups. If each girl ate $5\frac{3}{4}$ cups of popcorn, how much popcorn was left?

Ask Yourself

What do I need to do first?

The girls made two batches.



$9\frac{1}{2}$ cups $8\frac{2}{3}$ cups

Add to find the cups made.

$$9\frac{1}{2} + 8\frac{2}{3} = 9\frac{3}{6} + 8\frac{4}{6}$$
$$= 17\frac{7}{6} = 18\frac{1}{6}$$

What do I need to do next?

Each girl ate $5\frac{3}{4}$ cups.



$5\frac{3}{4}$ cups $5\frac{3}{4}$ cups $5\frac{3}{4}$ cups

Add to find the cups eaten.

$$5\frac{3}{4} + 5\frac{3}{4} + 5\frac{3}{4} = 15\frac{9}{4}$$
$$= 17\frac{1}{4}$$

What do I need to do to find the solution?

Subtract to find the cups left.

$$18\frac{1}{6} = 18\frac{2}{12} = 17\frac{14}{12}$$
$$-\,17\frac{1}{4} = -\,17\frac{3}{12} = -\,17\frac{3}{12}$$
$$\frac{11}{12}$$

Solution: There were $\frac{11}{12}$ of a cup left.

Try These

Solve. Explain the steps you need.
Is your answer reasonable? Why or why not?

1. If you want to frame a movie poster that is $24\frac{1}{2}$ inches wide and $30\frac{1}{3}$ inches high, is 10 feet of framing lumber enough to build the frame?

2. On a cruise, Emily spent $\frac{1}{8}$ of the day playing volleyball, $\frac{1}{12}$ watching entertainment, $\frac{1}{6}$ swimming, and $\frac{1}{4}$ at meals. How much of the day was left?

3. While in Illinois, the Franklin family visited the World's Biggest Catsup Bottle, a 170-foot water tower. They bought two souvenir mugs for $4.48 each. How much change should they receive from a $10 bill?

4. Nancy is making a display case with five shelves for her bedroom. Each shelf is $2\frac{1}{2}$ inches longer than the one above it. The top shelf is 10 inches long. How long is the bottom shelf?

5. Jeff had $\frac{1}{2}$ ft of rope. Mark had 9 in. of rope. Peg had more rope than Mark. Cal had twice as much rope as Peg. Who had the most rope?

6. A carpenter is making a window frame. Two sides are $2\frac{1}{2}$ feet long, and two sides are $1\frac{1}{8}$ feet long. What is the total length of wood the carpenter needs to make the frame?

Problem Solving

Calculator Connection
Secret Message

Find $2\frac{5}{6} + 2\frac{1}{2}$.

To find the sum on your calculator, press:



To simplify the answer, press: Simp 2 Enter



Use your calculator to solve. Write each answer in simplest form. Then match the letters to the answers below to decode the secret message.

1. $81\frac{4}{5} + \frac{2}{10}$ (U)
2. $6\frac{4}{12} - \frac{1}{3}$ (H)
3. $19\frac{7}{8} - 7\frac{3}{8}$ (E)
4. $11\frac{6}{8} + 2\frac{4}{6}$ (A)
5. $5\frac{10}{15} - 3\frac{1}{3}$ (S)
6. $1\frac{1}{6} + 3\frac{2}{5}$ (M)
7. $4\frac{3}{12} + 16\frac{2}{4}$ (R)
8. $60\frac{3}{6} - 3\frac{1}{3}$ (T)
9. $7\frac{2}{14} + \frac{1}{7}$ (L)

___	___	___	___		___	___	___	___	___ !
$4\frac{17}{30}$	$14\frac{5}{12}$	$57\frac{1}{6}$	6		$20\frac{3}{4}$	82	$7\frac{2}{7}$	$12\frac{1}{2}$	$2\frac{1}{3}$

Chapter Review/Test

VOCABULARY

1. The smallest number that is a multiple of two or more numbers is the _____.
2. The _____ is the number above the bar in a fraction.
3. In a fraction, the _____ tells how many parts are in the whole.
4. Any common multiple of the denominators of two or more fractions is a _____.

Vocabulary

numerator
denominator
common multiple
common denominator
least common multiple

CONCEPTS AND SKILLS

Estimate each sum or difference. (Lesson 1, pp. 108–109)

5. $1\frac{3}{4} + 3\frac{7}{8}$
6. $\frac{3}{5} + \frac{3}{10}$
7. $4\frac{4}{11} - 1\frac{1}{3}$
8. $9\frac{5}{6} - 3\frac{2}{7}$

Estimate the sum or difference. Write < or > for each ●. (Lesson 1, pp. 108–109)

9. $4\frac{1}{3} + 2\frac{7}{10}$ ● $15\frac{1}{4} - 8\frac{7}{8}$
10. $7\frac{5}{6} - 2\frac{2}{5}$ ● $5\frac{1}{9} + \frac{7}{8}$

Add or subtract. Write your answer in simplest form. (Lesson 2, pp. 110–111)

11. $\frac{5}{7} + \frac{1}{7}$
12. $4\frac{5}{6} + 2\frac{5}{6}$
13. $7\frac{2}{9} - 3\frac{7}{9}$
14. $6\frac{3}{8} + 4\frac{7}{8}$
15. $\frac{9}{11} - \frac{4}{11}$

Find each sum or difference. Write your answer in simplest form.
(Lessons 3–4, pp. 112–114, 116–119)

16. $2\frac{1}{3} + 3\frac{4}{9}$
17. $6\frac{3}{8} + 2\frac{3}{12}$
18. $2\frac{3}{11} + 5\frac{21}{22}$
19. $8\frac{5}{9} + 2\frac{5}{6}$
20. $\frac{4}{5} + 5\frac{2}{3}$
21. $4\frac{3}{10} + 3\frac{3}{4}$
22. $3\frac{2}{3} + \frac{5}{6} + 2\frac{1}{2}$
23. $\frac{1}{4} + \frac{1}{3} + \frac{7}{8}$
24. $8 - 4\frac{5}{9}$
25. $\frac{9}{10} - \frac{2}{5}$
26. $12\frac{1}{4} - 9\frac{7}{12}$
27. $3\frac{3}{5} - 1\frac{3}{4}$
28. $6\frac{8}{9} - 1\frac{1}{6}$
29. $2\frac{1}{7} - \frac{5}{7}$
30. $4\frac{2}{3} - 2\frac{5}{12}$
31. $\frac{8}{11} - \frac{4}{9}$

PROBLEM SOLVING

Solve. (Lesson 5, p. 120)

32. Miguel buys 4 juice box packs at $3.75 per pack. He gives the clerk $20. How much change should he get back?
33. Mr. Lee pays $10.20 for a dozen boxes of pretzels. He sells the pretzels in his neighborhood store for $1.25 per box. How much profit does he make on $2\frac{1}{2}$ dozen boxes?

Write About It

Show You Understand

How do you add fractions if you use the least common multiple as the common denominator?

Extra Practice

Set A (Lesson 1, pp. 108–109)

Estimate each sum or difference.

1. $\frac{4}{9} + \frac{1}{11}$
2. $2\frac{5}{8} + 3\frac{1}{3}$
3. $6\frac{3}{8} - 2\frac{1}{3}$
4. $5\frac{1}{4} - 3\frac{7}{9}$
5. $27\frac{4}{5} - 12\frac{7}{8}$
6. $10\frac{2}{3} - 4\frac{1}{7}$
7. $\frac{3}{7} + 5\frac{8}{9} + \frac{1}{8}$
8. $28\frac{1}{6} + \frac{4}{7} + \frac{1}{3}$

Estimate the sum or difference. Write < or > for each ●.

9. $9\frac{6}{7} - 1\frac{1}{4}$ ● $6\frac{5}{8} + 1\frac{2}{5}$
10. $7\frac{1}{6} + 5\frac{4}{11}$ ● $26\frac{1}{7} - 13\frac{14}{15}$

Set B (Lesson 2, pp. 110–111)

Add or subtract. Write your answer in simplest form.

1. $\frac{4}{9} + \frac{3}{9}$
2. $\frac{8}{10} - \frac{3}{10}$
3. $\frac{7}{15} + \frac{5}{15}$
4. $2\frac{2}{5} + 3\frac{3}{5}$
5. $4\frac{3}{8} - 3\frac{1}{8}$
6. $4\frac{7}{12} - \frac{5}{12}$
7. $6\frac{4}{7} + 6\frac{4}{7}$
8. $12\frac{1}{6} - 2\frac{5}{6}$
9. $4\frac{5}{9} + 5\frac{7}{9}$
10. $6\frac{3}{4} + 4\frac{3}{4}$

Set C (Lesson 3, pp. 112–115)

Find each sum. Write your answer in simplest form.

1. $\frac{1}{4} + \frac{1}{8}$
2. $\frac{1}{3} + 3\frac{5}{6}$
3. $7\frac{1}{2} + 2\frac{5}{12}$
4. $\frac{3}{7} + 4\frac{2}{3}$
5. $5\frac{5}{8} + 1\frac{11}{24}$
6. $3\frac{5}{9} + 4\frac{5}{6}$
7. $7\frac{3}{4} + 4\frac{5}{12}$
8. $3\frac{9}{20} + 2\frac{5}{8}$
9. $\frac{4}{5} + \frac{1}{2} + \frac{3}{4}$
10. $3\frac{1}{6} + \frac{2}{3} + 1\frac{1}{2}$
11. $3\frac{2}{9} + \frac{2}{3}$
12. $1\frac{1}{2} + \frac{3}{7}$
13. $8\frac{3}{10} + \frac{2}{15}$
14. $\frac{3}{4} + 2\frac{2}{5}$
15. $\frac{7}{9} + \frac{7}{10} + \frac{3}{5}$

Set D (Lesson 4, pp. 116–119)

Find each difference. Write your answer in simplest form.

1. $\frac{7}{8} - \frac{1}{4}$
2. $\frac{11}{12} - \frac{2}{3}$
3. $5\frac{1}{4} - \frac{3}{8}$
4. $2\frac{1}{6} - \frac{5}{12}$
5. $7\frac{2}{9} - 2\frac{1}{3}$
6. $4\frac{5}{6} - 2\frac{3}{4}$
7. $9 - 2\frac{3}{8}$
8. $4\frac{4}{5} - \frac{3}{10}$
9. $5\frac{3}{7} - 3\frac{2}{3}$
10. $11\frac{1}{8} - 2\frac{5}{12}$
11. $1\frac{2}{3} - \frac{3}{4}$
12. $\frac{17}{20} - \frac{4}{5}$
13. $8\frac{5}{12} - \frac{2}{9}$
14. $3\frac{1}{4} - \frac{2}{5}$
15. $4 - \frac{5}{6}$

CHAPTER 6

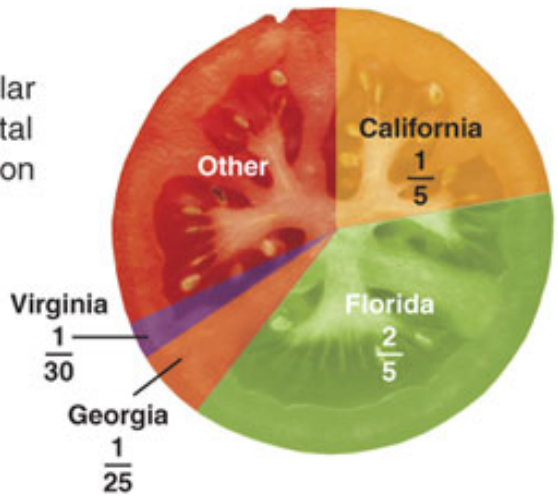
Multiply and Divide Fractions

INVESTIGATION

Using Data

The tomato is the most popular vegetable in the U.S.! The total fresh market tomato production in the U.S. is about 3,700,000,000 lb per year.

Explain how you could estimate how many pounds of tomatoes are produced in the southern United States each year.



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. The _____ states when evaluating expressions, you should do operations in parentheses first.
2. Any two numbers whose product is 1 are called _____.
3. A _____ number has only two factors—itself and 1.

Vocabulary

- prime
- compatible
- reciprocals
- order of operations
- multiplication properties

CONCEPTS AND SKILLS

Round each to the nearest whole number.

4. $2\frac{3}{4}$
5. $4\frac{3}{8}$
6. $1\frac{4}{7}$

Write each in simplest form.

7. $\frac{4}{16}$
8. $3\frac{8}{22}$
9. $\frac{14}{5}$
10. $\frac{28}{12}$

Write each as an improper fraction.

11. $2\frac{3}{7}$
12. $9\frac{1}{4}$
13. $7\frac{8}{9}$

Write the prime factorization of each number.

14. 12
15. 15
16. 24

Evaluate each expression, given $a = 2$, $b = 5$, and $c = 3$.

17. $a \times 4 - b$
18. $(7 - b) + (c \times a)$
19. $(a - a) \times b \times c$



20. Explain how you can tell that $\frac{12}{25} < \frac{7}{13}$ without using equivalent fractions.

Facts Practice, See page 661.

Lesson 1

Audio Tutor 1/14 Listen and Understand

Estimate Fraction Products and Quotients

Vocabulary
factor

Objective Estimate products and quotients of fractions and mixed numbers.

Learn About It

A recipe for Mexican burritos calls for $1\frac{3}{4}$ cups shredded cheddar cheese. You want to make $3\frac{1}{2}$ times the recipe to serve at a party. Estimate how much cheddar cheese you will need.

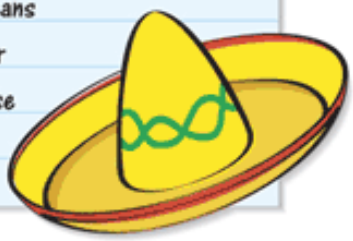
To estimate the product of two mixed numbers, you can round each **factor** to its nearest whole number.

Speedy Microwave Bean Burritos

Prep time: 5 minutes — Serves: 4

Cook: 1 to 2 minutes 40 seconds

4 (10-inch) flour tortillas
1 (16-ounce) can refried beans
$1\frac{3}{4}$ cups shredded Cheddar or Monterey Jack cheese
pickled jalapeño slices



Estimate $3\frac{1}{2} \times 1\frac{3}{4}$.

STEP 1 Round each factor to the nearest whole number.

Since $3\frac{1}{2}$ is halfway between 3 and 4, round $3\frac{1}{2}$ to 4.

$3\frac{1}{2}$ rounds to 4

Since $1\frac{3}{4}$ is closer to 2 than to 1, round $1\frac{3}{4}$ to 2.

$1\frac{3}{4}$ rounds to 2

STEP 2 Multiply the rounded factors.

$$3\tfrac{1}{2} \times 1\tfrac{3}{4}$$
$$\downarrow \quad \downarrow$$
$$4 \times 2 = 8$$

$3\frac{1}{2} \times 1\frac{3}{4}$ is about 8

Solution: You will need about 8 cups of cheddar cheese to make $3\frac{1}{2}$ times the recipe.

Other Examples

A. Round to Estimate Quotient

Estimate $11\frac{7}{8} \div 3\frac{9}{10}$.

$11\frac{7}{8}$ rounds to 12

$3\frac{9}{10}$ rounds to 4

$12 \div 4 = 3$

$11\frac{7}{8} \div 3\frac{9}{10}$ is about 3.

B. Use Compatible Numbers

Estimate $\frac{3}{8}$ of 35.

Look for a multiple of 8 that is close to 35. 32, or 8×4, is close to 35.

$\frac{1}{8}$ of $32 = 4$

$\frac{3}{8}$ of $32 = 3 \times 4 = 12$

$\frac{3}{8}$ of 35 is about 12.

Extra Help at eduplace.com/map

Guided Practice

Estimate the products and quotients. Identify the method you used to estimate.

1. $1\frac{7}{8} \times 2\frac{1}{6}$ **2.** $10\frac{1}{5} \div 2\frac{6}{7}$ **3.** $\frac{1}{6}$ of 50

4. $13\frac{4}{5} \div 4\frac{2}{3}$ **5.** $\frac{3}{5}$ of 88 **6.** $12\frac{3}{10} \times 5\frac{1}{11}$

Ask Yourself
- Would rounding to the nearest whole number help?
- Would compatible numbers help?

Explain Your Thinking Do you think your estimate for Exercise 4 is an underestimate or an overestimate?

Practice and Problem Solving

Estimate each product. Identify the method you used to estimate.

7. $3\frac{1}{4} \times 5\frac{5}{8}$ **8.** $\frac{1}{6}$ of 128 **9.** $4\frac{3}{4} \times 3\frac{1}{6}$ **10.** $\frac{2}{3}$ of 89

11. $8\frac{2}{3} \times 1\frac{1}{5}$ **12.** $\frac{2}{5}$ of 232 **13.** $5\frac{2}{11} \times 8\frac{1}{10}$ **14.** $\frac{3}{4}$ of 413

Estimate each quotient.

15. $5\frac{2}{3} \div 1\frac{4}{5}$ **16.** $13\frac{1}{8} \div 6\frac{1}{3}$ **17.** $10\frac{2}{5} \div 5\frac{2}{7}$ **18.** $8\frac{2}{3} \div 3\frac{3}{10}$

19. $19\frac{1}{8} \div 8\frac{5}{6}$ **20.** $50\frac{1}{2} \div 6\frac{7}{8}$ **21.** $80\frac{1}{4} \div 8\frac{6}{7}$ **22.** $41\frac{1}{5} \div 5\frac{8}{11}$

Solve. Is your answer reasonable? Why or why not?

23. A burrito recipe also calls for $1\frac{7}{8}$ cups of refried beans. Estimate how many cups of refried beans you need if you want to triple the recipe.

24. Explain A recipe for beef taco salad calls for $1\frac{1}{4}$ cups of chunky red salsa. Explain how to estimate the number of cups of salsa needed to make $4\frac{1}{2}$ times the recipe.

25. Reasoning A Mexican costume requires $3\frac{7}{8}$ yards of material. You have $15\frac{1}{4}$ yards of material. Do you have enough material to make five costumes? Explain.

26. Mrs. McDonnell wants to store a set of 23 cookbooks on a shelf that is $42\frac{1}{2}$ inches long. Each book is $1\frac{7}{8}$ inches thick. Will all of the books fit on the shelf? Explain.

Mixed Review and Test Prep

Open Response

Write an equivalent percent or decimal for each. (Ch. 4, Lesson 4)

27. 56% **28.** 6.08 **29.** 7%

30. 26.1% **31.** 0.09 **32.** 3.2

Multiple Choice

33. Jason travels $2\frac{1}{2}$ miles each day on his paper route. About how many miles does he travel in 6 days? (Ch. 6, Lesson 1)

A 18 **B** 15 **C** 14 **D** 12

Extra Practice See page 147, Set A.

Lesson 2

Multiply Fractions

Objective Multiply fractions.

Learn About It



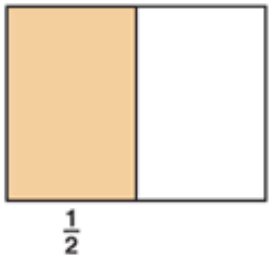
In the United States, the most popular meat pizza is pepperoni. Suppose one half of a large rectangular pan pizza is pepperoni. Yoshi and two friends each eat $\frac{1}{3}$ of the pepperoni piece. What fraction of the whole pizza does each child eat?

Multiply. $\frac{1}{3} \times \frac{1}{2} = \blacksquare$

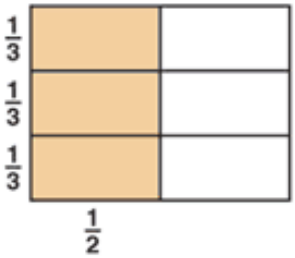
Different Ways to Multiply $\frac{1}{3} \times \frac{1}{2}$

Way 1 Use a model.

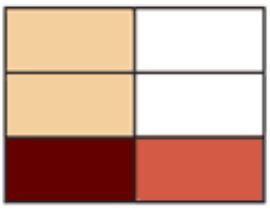
STEP 1 Fold a sheet of paper in half. Shade $\frac{1}{2}$.



STEP 2 Fold the paper into thirds widthwise. Unfold. There are 6 congruent rectangles.



STEP 3 Shade $\frac{1}{3}$ using a different color. There is one rectangle with both shadings.



$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$

Way 2 Multiply the numerators and the denominators.

STEP 1 Multiply the numerators to find the numerator of the product.

$$\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{\blacksquare \times \blacksquare} = \frac{1}{\blacksquare}$$

STEP 2 Multiply the denominators to find the denominator of the product.

$$\frac{1}{3} \times \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$$

Solution: Each child ate $\frac{1}{6}$ of the pizza.

Guided Practice

Multiply. Write each product in simplest form.

1. $\frac{1}{3} \times \frac{1}{4}$ **2.** $3 \times \frac{4}{7}$ **3.** $\frac{5}{6} \times \frac{1}{3}$ **4.** $5 \times \frac{3}{5}$

5. $\frac{1}{2} \times \frac{4}{5}$ **6.** $\frac{7}{8} \times 7$ **7.** $2 \times \frac{4}{9}$ **8.** $\frac{2}{3} \times 9$

Ask Yourself

- How can I rewrite a whole number as a fraction?

Explain Your Thinking Would it be helpful to rewrite the numerators and denominators as products of prime factors first?

Practice and Problem Solving

Multiply. Write each product in simplest form.

9. $\frac{2}{3} \times \frac{4}{7}$ **10.** $\frac{9}{10} \times 3$ **11.** $\frac{5}{8} \times 16$ **12.** $\frac{7}{8} \times \frac{7}{12}$ **13.** $7 \times \frac{1}{2}$

14. $\frac{3}{11} \times 2$ **15.** $\frac{5}{12} \times \frac{3}{16}$ **16.** $\frac{3}{16} \times \frac{4}{5}$ **17.** $\frac{4}{10} \times \frac{5}{8}$ **18.** $5 \times \frac{7}{10}$

Algebra • Expressions **Write each expression as a single fraction.**

19. $\frac{1}{a} \times \frac{1}{e}$ **20.** $\frac{m}{n} \times \frac{c}{s}$ **21.** $\frac{3}{a} \times e$ **22.** $\frac{a}{b} \times \frac{2}{c}$

Solve.

23. Ray orders $\frac{1}{2}$ of a large rectangular sausage pan pizza with pepperoni on $\frac{2}{3}$ of it. What fraction of the pizza has both sausage and pepperoni?

24. Marita purchased a CD during a "$\frac{1}{4}$ off sale." The original price of the CD was $12. Use mental math to find the sale price of the CD.

25. Write About It A town plans to use $\frac{1}{2}$ of a square park for a recreation area. A soccer field will take up $\frac{3}{5}$ of this area. Use words and diagrams to explain what fraction of the park will be taken up by the soccer field.

26. Represent Linda tiles $\frac{3}{4}$ of a wall with yellow and green tiles. Two-thirds of that area is tiled with yellow tiles. Use paper folding to show the fraction of Linda's wall that is covered with yellow tiles.

Mixed Review and Test Prep

Open Response

Find each sum or difference. Write your answers in simplest form. (Ch. 5, Lesson 2)

27. $\frac{2}{9} + \frac{4}{9}$ **28.** $7\frac{2}{3} - 4\frac{1}{3}$

29. $6\frac{3}{4} + 8\frac{3}{4}$ **30.** $9\frac{11}{12} - 6\frac{1}{12}$

31. Martin delivers 42 newspapers each day. He delivers $\frac{1}{3}$ of the papers to houses on his street. How many does he deliver on his street? Explain. (Ch. 6, Lesson 2)

Extra Practice See page 147, Set B.

Lesson 3

Audio Tutor 1/15 Listen and Understand

Multiply Mixed Numbers

Objective Multiply mixed numbers.



Learn About It

Hummus is a Middle Eastern dipping sauce that is often served with pita bread. A recipe for hummus calls for $3\frac{3}{4}$ cups of chickpeas. Kamini wants to make $2\frac{2}{3}$ times the recipe to serve at a family reunion. How many cups of chickpeas does she need?

Multiply. $3\frac{3}{4} \times 2\frac{2}{3} = \blacksquare$

STEP 1 Rename $3\frac{3}{4}$ and $2\frac{2}{3}$ as improper fractions.

$$3\frac{3}{4} = \frac{15}{4} \leftarrow (4 \times 3) + 3$$

$$2\frac{2}{3} = \frac{8}{3} \leftarrow (3 \times 2) + 2$$

STEP 2 Multiply. Write the product in simplest form.

$$\frac{15}{4} \times \frac{8}{3} = \frac{\overset{1}{\cancel{3}} \times 5 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}}}$$

$$= \frac{5 \times 2}{1} = \frac{10}{1} = 10$$

Solution: Kamini will need 10 cups of chickpeas.

Other Examples

A. Mixed Numbers and Fractions

$$\frac{3}{4} \times 4\frac{2}{5} = \frac{3}{4} \times \frac{22}{5}$$

$$= \frac{3 \times \overset{1}{\cancel{2}} \times 11}{2 \times \underset{1}{\cancel{2}} \times 5} = \frac{3 \times 11}{2 \times 5} = \frac{33}{10} = 3\frac{3}{10}$$

B. Mixed Numbers and Whole Numbers

$$2\frac{1}{3} \times 6 = \frac{7}{3} \times \frac{6}{1} = \frac{7 \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{3}} \times 1}$$

$$= \frac{7 \times 2}{1} = \frac{14}{1} = 14$$

Guided Practice

Multiply. Write each product in simplest form.

1. $\frac{1}{2} \times 2\frac{3}{4}$ **2.** $5\frac{4}{7} \times 1\frac{2}{3}$ **3.** $6 \times 2\frac{1}{2}$

4. $4\frac{5}{6} \times 2\frac{1}{4}$ **5.** $3 \times 2\frac{2}{5}$ **6.** $2\frac{2}{3} \times 4\frac{1}{2}$

Ask Yourself
- Can I use prime factorization to simplify the product?
- Did I cancel all common factors?

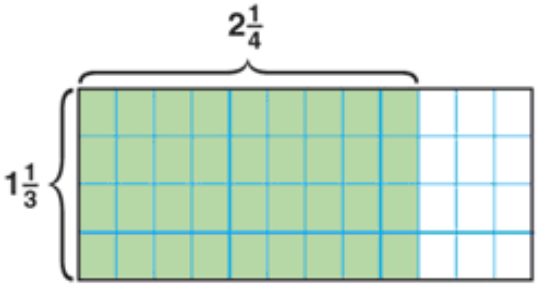
Explain Your Thinking Does finding $(5 \times 1) + (\frac{4}{7} \times \frac{2}{3})$ help you find the product in Exercise 2?

Practice and Problem Solving

Multiply. Write each product in simplest form.

7. $5\frac{1}{3} \times 2\frac{1}{4}$
8. $\frac{3}{11} \times \frac{4}{9}$
9. $\frac{7}{10} \times 5$
10. $\frac{2}{5} \times 3\frac{1}{3}$
11. $\frac{3}{7} \times 2\frac{5}{8}$
12. $1\frac{1}{8} \times 2\frac{2}{3}$
13. $\frac{3}{4} \times \frac{8}{12}$
14. $4\frac{1}{2} \times \frac{3}{4}$
15. $5 \times 3\frac{1}{5}$
16. $\frac{2}{7} \times 2\frac{5}{8}$
17. $2\frac{2}{5} \times 6\frac{2}{3}$
18. $9\frac{3}{5} \times 3\frac{3}{4}$

19. **Analyze** The diagram at right shows a model for multiplying $1\frac{1}{3} \times 2\frac{1}{4}$. Describe how the distributive property is used.



20. **Represent.** Draw a model for multiplying $3\frac{1}{2}$ by 2.

Data **Use the recipe at right for Problems 21 and 22.**

Garlicky Hummus

A Middle Eastern Dipping Sauce

Prep time: 10 minutes Makes: $2\frac{1}{4}$ cups

- 2 (15-ounce) cans chickpeas (garbanzo beans), drained
- $\frac{1}{4}$ cup tahini (Middle Eastern sesame seed paste)
- 2 tablespoons extra-virgin olive oil
- $1\frac{1}{2}$ tablespoons fresh lemon juice
- 2 large garlic cloves, crushed
- $\frac{1}{2}$ teaspoon salt
- Chopped parsley and paprika
- Pita bread



21. The hummus recipe also calls for $1\frac{1}{2}$ tablespoons of fresh lemon juice. How much lemon juice does Kamini need if she wants to triple the recipe?

22. Suppose you want to make twice as much hummus. Rewrite the recipe.

23. **Reasoning** A recipe for 12 bran muffins calls for $1\frac{1}{4}$ cups of wheat flour. Harold wants to make 48 muffins. He has about 4 cups of wheat flour. Does Harold have enough flour to make 48 muffins? Explain.

24. **Create and Solve** Write a word problem that can be solved by multiplying the mixed numbers $2\frac{1}{2}$ and $3\frac{1}{4}$. Have a classmate solve your problem.

Mixed Review and Test Prep

Open Response

Find each sum or difference. Write your answers in simplest form. (Ch. 5, Lessons 3–4)

25. $\frac{1}{8} + \frac{2}{3}$
26. $1\frac{3}{5} - \frac{1}{4}$
27. $\frac{9}{10} - \frac{5}{6}$
28. $\frac{4}{7} + \frac{7}{8}$
29. $5\frac{5}{6} + 7\frac{1}{4}$
30. $3\frac{7}{12} - 1\frac{1}{2}$

Multiple Choice

31. Dana needs $2\frac{1}{8}$ yards of fabric to make a costume. How much fabric will she need to make 4 costumes? (Ch. 6, Lesson 3)

A $1\frac{1}{4}$ yd **C** $7\frac{1}{4}$ yd

B $6\frac{1}{2}$ yd **D** $8\frac{1}{2}$ yd

Extra Practice See page 147, Set B.

Lesson 4

Divide Fractions

Objective Divide fractions.

Vocabulary

- multiplicative inverse
- reciprocal
- invert

Learn About It

Find the products. What pattern do you see?

$3 \times \frac{1}{3} = \blacksquare$ $\quad \frac{5}{4} \times \frac{4}{5} = \blacksquare$ $\quad 2\frac{1}{2} \times \frac{2}{5} = \blacksquare$

The products are all equal to 1.

Any two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals** of each other. For example,

3 and $\frac{1}{3}$ are reciprocals.

$2\frac{1}{2}$ and $\frac{2}{5}$ are reciprocals.

One way to find the reciprocal of a fraction is to **invert** or interchange the numerator and denominator:

$\frac{5}{4}$ is the reciprocal of $\frac{4}{5}$.

Marco is preparing Buffalo chicken wings. He has $\frac{3}{4}$ cup of hot sauce. Each chicken wing needs $\frac{3}{16}$ cup of hot sauce. How many chicken wings can Marco top with hot sauce?

Buffalo chicken wings are often served with celery sticks and blue-cheese dressing.



Divide. $\frac{3}{4} \div \frac{3}{16} = \blacksquare$

Different Ways to Divide $\frac{3}{4}$ by $\frac{3}{16}$

Way 1 You can multiply by the reciprocal.

Since reciprocals are multiplicative inverses, you can use reciprocals to divide fractions.

STEP 1 Rewrite as a multiplication problem, using the reciprocal of the divisor.

$$\frac{3}{4} \div \frac{3}{16}$$

$$= \frac{3}{4} \times \frac{16}{3}$$

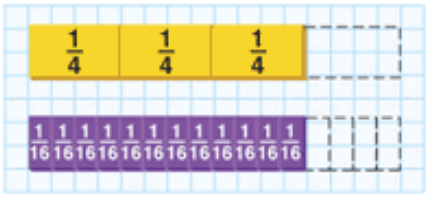
STEP 2 Simplify by using prime factorization. Then multiply.

$$\frac{3}{4} \times \frac{16}{3}$$

$$= \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{2}} \times 2 \times 2}{2 \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}}} = \frac{4}{1} = 4$$

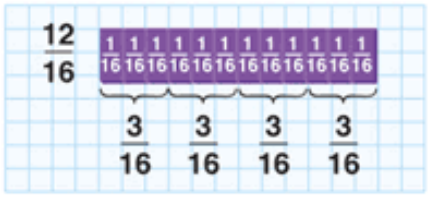
Way 2 You can use models.

STEP 1 Use grid paper to draw fraction models to model $\frac{3}{4}$ in 16ths.



- How many $\frac{3}{16}$ths are in $\frac{3}{4}$?

STEP 2 Count the number of groups of $\frac{3}{16}$.



There are 4 groups of $\frac{3}{16}$ in $\frac{3}{4}$.

$\frac{12}{16} \div \frac{3}{16} = 4$

Solution: Marco can top 4 chicken wings with hot sauce.

Other Examples

A. Whole Number and a Fraction

$$3 \div \frac{3}{5}$$
$$= \frac{3}{1} \div \frac{3}{5}$$
$$= \frac{\overset{1}{\cancel{3}}}{1} \times \frac{5}{\underset{1}{\cancel{3}}} = \frac{1}{1} \times \frac{5}{1} = \frac{5}{1} = 5$$

B. Fraction and a Whole Number

$$\frac{1}{2} \div 10$$
$$= \frac{1}{2} \div \frac{10}{1}$$
$$= \frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$$

Guided Practice

Ask Yourself
- Did I use the reciprocal of the divisor?
- Did I multiply by the reciprocal?

Find the reciprocal of each number.

1. $\frac{11}{5}$ **2.** $2\frac{1}{4}$ **3.** 9 **4.** $8\frac{2}{3}$ **5.** $\frac{1}{7}$

Find each quotient. Write each quotient in simplest form.

6. $\frac{2}{3} \div \frac{3}{4}$ **7.** $\frac{7}{3} \div \frac{5}{6}$ **8.** $6 \div \frac{3}{4}$ **9.** $\frac{2}{3} \div 10$

10. $\frac{5}{9} \div \frac{7}{3}$ **11.** $\frac{1}{5} \div \frac{2}{3}$ **12.** $\frac{3}{2} \div \frac{7}{8}$ **13.** $\frac{7}{9} \div 14$

Explain Your Thinking Why can you use multiplication to check a division problem?

Go On

Practice and Problem Solving

Find the reciprocal of each number.

14. $\frac{1}{6}$ **15.** $\frac{7}{5}$ **16.** $\frac{3}{4}$ **17.** 3 **18.** $9\frac{7}{8}$

Find each quotient. Write each quotient in simplest form.

19. $\frac{3}{11} \div \frac{6}{7}$ **20.** $\frac{8}{9} \div \frac{4}{3}$ **21.** $\frac{2}{7} \div \frac{3}{2}$ **22.** $\frac{4}{7} \div 8$

23. $7 \div \frac{3}{2}$ **24.** $\frac{5}{4} \div 9$ **25.** $\frac{7}{8} \div \frac{21}{40}$ **26.** $10 \div \frac{5}{6}$

27. What can you say about the reciprocal of a proper fraction?

28. Does 0 have a reciprocal? Is there a number that gives a product of 1 when it is multiplied by 0? Explain.

Algebra • Equations **Find a value for *n* that makes each equation true.**

29. $\frac{3}{4} \times n = 1$ **30.** $14 \div n = 56$ **31.** $\frac{15}{16} \div \frac{3}{4} = n$ **32.** $n \div \frac{2}{3} = 6$

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Data **Use the recipe to solve Problems 33–37. Then tell which method you chose.**

33. Benicio has 7 cups of recycled paper scraps. How many batches of homemade paper can he make?

34. Represent Jana measures out $2\frac{2}{5}$ cups of water. Draw a diagram to help you find how many batches of paper Jana can make.

35. Mark has $\frac{2}{3}$ cup of recycled paper scraps. Can he make 4 batches of paper?

36. Write About It Another way to state the paper-making instructions is to say "Use 4 parts water to 1 part paper scraps." Why is this true?

37. Create and Solve Write a division word problem involving the recipe for homemade paper. Then write an explanation that tells how to solve your problem.



Extra Practice See page 147, Set C.

Number Sense

Use Equivalent Fractions to Divide

Activity

Before dividing two fractions, you can rewrite them as equivalent fractions with a common denominator. Then divide the numerators to divide the fractions.

Divide. $\frac{3}{4} \div \frac{2}{5} = \blacksquare$

STEP 1 Rewrite the fractions as equivalent fractions with a common denominator.

$$\frac{3}{4} \div \frac{2}{5} = \frac{15}{20} \div \frac{8}{20}$$

STEP 2 Divide the numerators.

$$15 \div 8 = \frac{15}{8} = 1\frac{7}{8}$$

Write each quotient in simplest form.

1. $\frac{7}{12} \div \frac{1}{3}$
2. $\frac{4}{5} \div \frac{1}{10}$
3. $\frac{3}{7} \div \frac{5}{14}$
4. $\frac{5}{8} \div \frac{1}{4}$
5. $\frac{5}{6} \div \frac{5}{12}$

6. **Analyze** How is the method related to the use of reciprocals to divide?

Quick Check

Check your understanding of Lessons 1–4.

Estimate each product or quotient. Identify the method you used to estimate. (Lesson 1)

1. $2\frac{1}{4} \times 3\frac{7}{8}$
2. $\frac{1}{4}$ of 139
3. $5\frac{5}{6} \div 1\frac{4}{5}$

Multiply. Write each product in simplest form. (Lessons 2–3)

4. $\frac{1}{2} \times \frac{2}{3}$
5. $\frac{3}{5} \times 4\frac{5}{6}$
6. $5\frac{4}{9} \times 1\frac{2}{7}$
7. $1\frac{1}{2} \times 2\frac{1}{3}$

Divide. Write each quotient in simplest form. (Lesson 4)

8. $\frac{2}{3} \div \frac{1}{2}$
9. $\frac{9}{10} \div \frac{2}{5}$
10. $12 \div \frac{3}{4}$
11. $\frac{3}{11} \div \frac{1}{6}$

Lesson 5

Audio Tutor 1/16 Listen and Understand

Divide Mixed Numbers

Objective Divide mixed numbers.

Learn About It

Natalia combined raisins, sunflower seeds, peanuts, walnuts, and coconut to make $6\frac{1}{4}$ pounds of trail mix for a hiking trip. How many $1\frac{1}{4}$-pound bags of trail mix can Natalia make?



Trail mix may also include pumpkin seeds, dried apricots and figs, banana chips, and assorted nuts.

Divide. $6\frac{1}{4} \div 1\frac{1}{4} = \blacksquare$

STEP 1 Write the mixed numbers as improper fractions.

$$6\frac{1}{4} \div 1\frac{1}{4} = \frac{25}{4} \div \frac{5}{4}$$

STEP 2 Rewrite using the reciprocal of the divisor.

$$\frac{25}{4} \div \frac{5}{4} = \frac{25}{4} \times \frac{4}{5}$$

STEP 3 Simplify by using prime factorization. Then multiply.

$$\frac{25}{4} \times \frac{4}{5} = \frac{5 \times \cancel{5}^{1} \times \cancel{2}^{1} \times \cancel{2}^{1}}{\underset{1}{2} \times \underset{1}{2} \times \underset{1}{5}} = 5$$

Solution: Natalia can make five $1\frac{1}{4}$-pound bags of trail mix.

Other Examples

A. Mixed Numbers and Fractions

$$4\frac{3}{8} \div \frac{5}{6} = \frac{35}{8} \div \frac{5}{6}$$

$$= \frac{35}{8} \times \frac{6}{5} = \frac{\cancel{5}^{1} \times 7 \times \cancel{2}^{1} \times 3}{2 \times 2 \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{5}}}$$

$$= \frac{21}{4} = 5\frac{1}{4}$$

B. Divide a Whole Number by a Mixed Number

$$3 \div 6\frac{3}{5} = \frac{3}{1} \div \frac{33}{5}$$

$$= \frac{3}{1} \times \frac{5}{33} = \frac{\cancel{3}^{1} \times 5}{1 \times \underset{1}{\cancel{3}} \times 11}$$

$$= \frac{5}{11}$$

Guided Practice

Divide. Express each quotient in simplest form.

1. $4\frac{2}{3} \div 6$
2. $1\frac{1}{2} \div \frac{5}{6}$
3. $2\frac{3}{5} \div 1\frac{7}{10}$
4. $4\frac{8}{9} \div 2\frac{2}{3}$
5. $3\frac{1}{3} \div 6\frac{2}{3}$
6. $2\frac{5}{12} \div 3\frac{1}{3}$

Ask Yourself

- Did I change the mixed numbers to improper fractions?
- Did I invert the divisor?

Explain Your Thinking How is multiplying by a proper fraction related to dividing by a mixed number?

Practice and Problem Solving

Divide. Express each quotient in simplest form.

7. $2\frac{1}{2} \div 3$

8. $10 \div 2\frac{3}{4}$

9. $3 \div 2\frac{3}{8}$

10. $4\frac{5}{7} \div 3\frac{2}{3}$

11. $\frac{1}{2} \div 3\frac{4}{5}$

12. $3\frac{3}{4} \div 2$

13. $2\frac{4}{5} \div \frac{7}{8}$

14. $2\frac{1}{3} \div 1\frac{2}{5}$

15. $3\frac{2}{3} \div 2\frac{1}{2}$

16. $\frac{2}{3} \div 4\frac{1}{6}$

17. $\frac{3}{4} \div 2\frac{1}{3}$

18. $6\frac{3}{4} \div 2\frac{1}{4}$

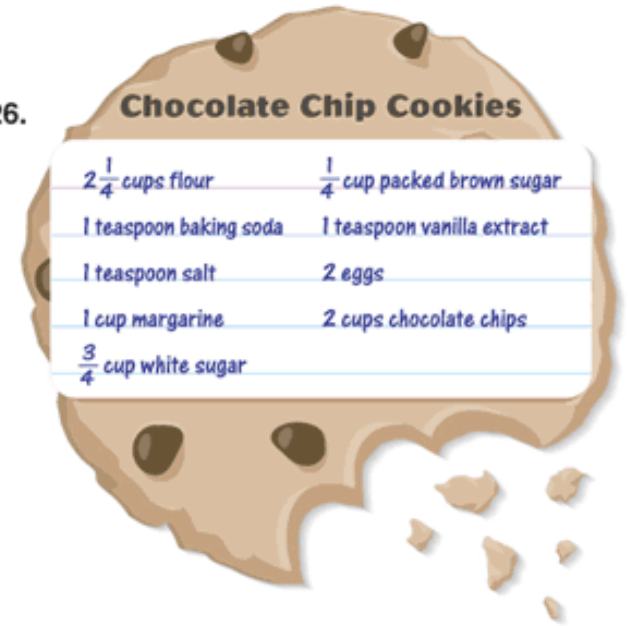
19. $3\frac{4}{5} \div 8\frac{1}{5}$

20. $2\frac{7}{8} \div 1\frac{1}{2}$

21. $1\frac{3}{4} \div 1\frac{3}{4}$

22. $4\frac{2}{3} \div 1\frac{5}{6}$

Data **Use the recipe at the right for Problems 24–26.**



23. Natalia made $5\frac{1}{2}$ pounds of trail mix to share with friends during a charity walk. Does Natalia have enough trail mix to fill seven $\frac{3}{4}$-pound bags? Explain.

24. **Predict** The recipe on the right makes 1 batch of cookies. Suppose you changed the recipe to include a 2-pound bag of flour (1 cup $= \frac{1}{2}$ pound). Would you have enough flour to make 4 batches? Explain.

25. **Write About It** Suppose you changed the recipe to include a 4-pound bag of white sugar (1 cup $= \frac{1}{2}$ pound). How many batches of cookies could you make? Explain how you solved the problem.

26. **What's Wrong?** Leroy decided to change the recipe to include a 3-pound bag of brown sugar (1 cup $= \frac{1}{2}$ pound). Leroy divided 3 by $\frac{1}{2}$ and found that he could make 6 batches. What mistake did Leroy make? Explain.

Mixed Review and Test Prep

Open Response

Write an equivalent percent or decimal for each. (Ch. 4, Lesson 4)

27. 38%

28. 2.48

29. 88%

30. 46.5%

31. 0.07

32. 6.9

33. 0.135

34. 303%

35. 0.002

36. Niko divided $4\frac{1}{2}$ pounds of fruit salad into $1\frac{1}{2}$-pound containers. How many containers did he use? (Ch. 6, Lesson 5)

Explain how you found your answer.

Extra Practice See page 147, Set D.

Lesson 6

Audio Tutor 1/17 Listen and Understand

Problem-Solving Strategy

Solve a Simpler Problem

Objective Solve a problem by first solving a simpler problem.

Problem Six fractoids, creatures from a far away galaxy, are playing in a video-game tournament. Each fractoid plays one match against each of the other players. How many matches must be played?



UNDERSTAND

This is what you know:

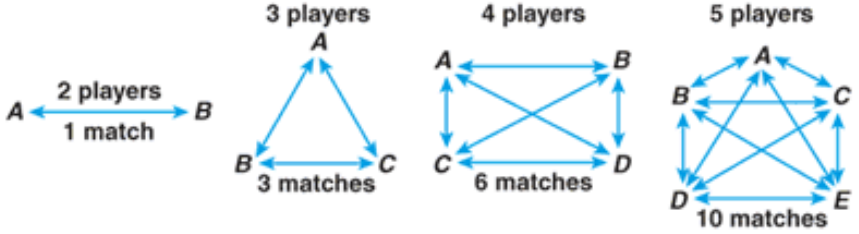
- Two fractoids play in each match.
- Each fractoid plays against each of the other five players once.

PLAN

You can solve a simpler problem to help you solve this more complicated problem.

SOLVE

- Solve the problem for fewer players and look for a pattern.



- Look for a pattern.

Number of Players	2	3	4	5	6
Number of Matches	1	3	6	10	15

Difference pattern: 2 3 4 5

- The difference increases by 1 each time. Continue the pattern.

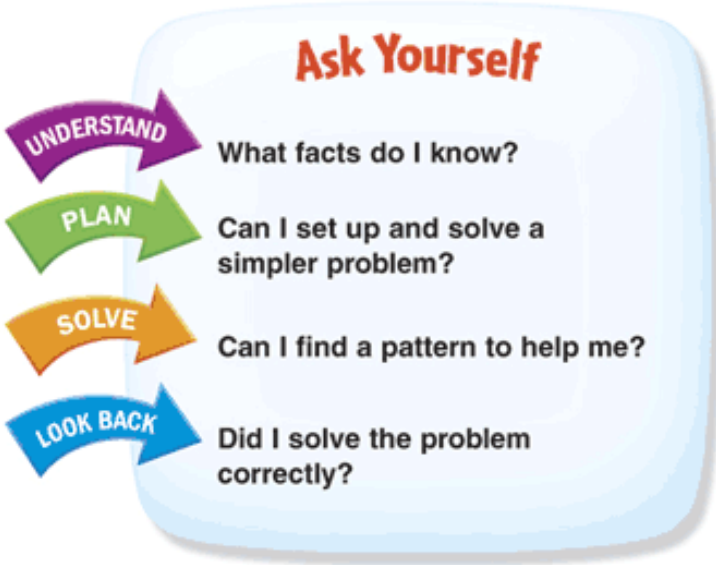
Solution: Fifteen matches must be played.

LOOK BACK

Look back at the problem. Does the solution make sense?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.



1. One fractoid was challenged to find the sum of all odd numbers from 1 through 49. After finding the sum of the numbers from 1 through 19, he gave up. What is the sum of all the odd numbers from 1 through 49?

   Hint Find the sum of the first 2, 3, 4, and 5 odd numbers and look for a pattern.

2. **Explain** A line of 1,500 fractoids is waiting to see a museum exhibit about planet Earth. Every 20 minutes, a guard allows 55 fractoids to enter. The exhibit is open for 8 hours daily. Will all 1,500 fractoids get in?

Independent Practice

Use the Solve-a-Simpler-Problem strategy to solve each problem.

3. Fractoids live in condo units that are shaped like equilateral triangles. The side of each triangle is $3\frac{1}{2}$ meters. If 42 triangles are attached in a row, what is the combined length of all of their sides?



4. The condos on Galaxy Avenue are numbered from 1 to 140. How many addresses contain the digit 6 at least once?

5. **Challenge** One fractoid needs your help with solving the following brain teaser.
   Is the sum of $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \ldots$ greater than 1, equal to 1, or less than 1? Justify your reasoning.

6. The national flag of the fractoids is shown at right. It has an octagon (eight-sided figure) with diagonals that extend from each vertex (corner) to every other vertex. Find the number of diagonals in the octagon without making a drawing.



Go On

Mixed Problem Solving

Solve each problem. Tell what strategy you used.

7. Emilio and Rose sell half of their book collection at a garage sale. They donate $\frac{1}{5}$ of the remaining books to a resale shop and keep 40 books. How many books did they have to start with?

8. Roscoe works as a cake decorator in a bakery. If it takes him $1\frac{1}{4}$ hours to decorate a two-layer birthday cake, how many birthday cakes can he decorate in a 40-hour week?

9. In Mr. Grant's cooking class, $\frac{1}{4}$ of the students are making appetizers. One half is making main courses. The remaining 10 students are making desserts. How many students are making appetizers?

You Choose

Strategy

- Draw a Diagram
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

10. While weighing fruit, Michelle finds that 6 apples weigh the same as 2 grapefruits and 2 kiwis. A grapefruit weighs the same as 8 kiwis. How many kiwis weigh the same as one apple?

Data **Use the circle graph on the right to solve Problems 11–15. Then tell which method you chose.**

11. What bakery item has the highest average monthly sales?

12. Suppose the average total sales are $36,000. How much money do sales of bread and rolls bring in each month?

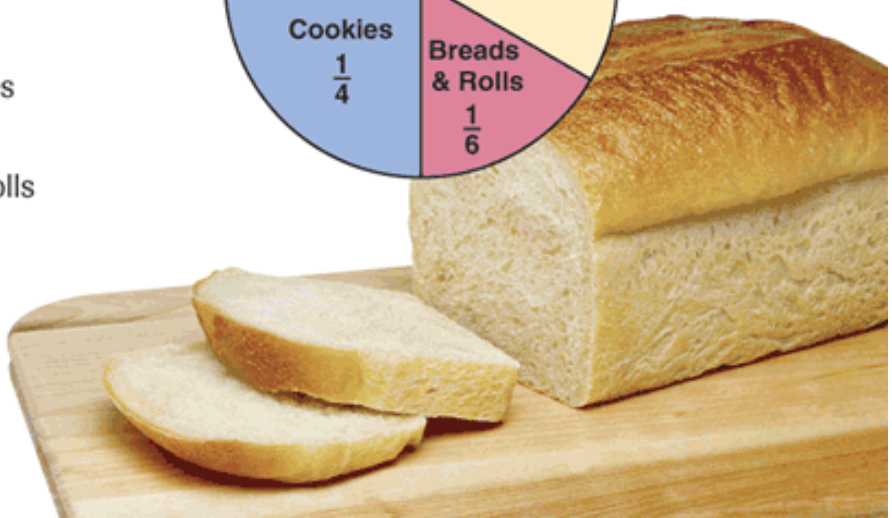
13. How much more money do pie sales bring in than cake sales?

14. Last month, cake sales increased $1\frac{1}{4}$ times. How much money did cake sales account for last month?

15. One third of the sales of breads and rolls is from rolls. What fraction of the total sales does the sale of rolls make up?

Average Monthly Sales at Bakery





Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. For practice, the speech teacher asks each of the 10 speech-club members to call every other member and give a short speech over the phone. What is the least number of phone calls needed?

A 10 **B** 21 **C** 36 **D** 45

(Chapter 6, Lesson 6)

2. Mr. Lee teaches music every fifth school day and art every seventh school day. Today, he teaches both art and music. In how many school days will Mr. Lee teach both art and music on the same day again?

F 12 **G** 20 **H** 35 **J** 42

(Chapter 3, Lesson 5)

Open Response

Solve each problem.

3. When Lisa was in Canada, one Canadian dollar was worth about $0.652 United States dollars. Lisa estimated that one Canadian dollar was worth about 65 U.S. cents. Is her estimate correct?

Explain How did you solve the problem?

(Chapter 1, Lesson 4)

4. Joe is selling tickets to a concert. He sold $\frac{1}{2}$ of the tickets on Monday and sold 15 tickets on Tuesday. On Wednesday he sold 3 times as many tickets as on Tuesday. He has 10 tickets left. How many tickets did Joe have to start with?

(Chapter 2, Lesson 5)

Constructed Response

5. Liliana, Casey, and Shang want to make a large batch of trail mix that is similar to a small bag that they bought. To discover the ingredients and proportions in the small bag, they counted every item in the bag.

Trail Mix Ingredients
10 sunflower seeds
30 raisins
15 fruit chews
45 oat cereal pieces
45 corn cereal pieces

a. Liliana wants to make certain that the large batch has the same fractional parts as the small bag. What fractions of each ingredient make up the whole? Show your answers in simplest form.

b. If the students already have 200 sunflower seeds, how much of each other ingredient will they need if they use all of the sunflower seeds?

c. **Analyze** If they decide to add 15 apricot pieces to the small bag, how will it change the fractional parts of the trail mix?

(Chapter 6)



Education Place
See **eduplace.com/map** for more Test-Taking Tips.

Lesson 7

Evaluate Expressions With Fractions

Objective Evaluate expressions containing fractions and mixed numbers.

Vocabulary
- evaluate
- simplest form

Learn About It

In Chapter 2, you learned about the multiplication properties for whole numbers. You can use the properties along with the order of operations to **evaluate** expressions with fractions and mixed numbers.

Order of Operations

1. Always do operations in parentheses first.
2. Rewrite any exponents.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

Evaluate $(1\frac{3}{4} + 2\frac{1}{2}) \times 2 + 1\frac{1}{2}$.

STEP 1 Work within the parentheses first.

$$(1\tfrac{3}{4} + 2\tfrac{1}{2}) \times 2 + 1\tfrac{1}{2}$$
$$= (\tfrac{7}{4} + \tfrac{5}{2}) \times 2 + \tfrac{3}{2}$$
$$= (\tfrac{7}{4} + \tfrac{10}{4}) \times 2 + \tfrac{3}{2}$$
$$= \tfrac{17}{4} \times 2 + \tfrac{3}{2}$$

STEP 2 Evaluate, using the correct order of operations. Multiply.

$$\tfrac{17}{4} \times 2 + \tfrac{3}{2}$$
$$= \tfrac{34}{4} + \tfrac{3}{2}$$

STEP 3 Add. Write the answer in **simplest form**.

$$\tfrac{34}{4} + \tfrac{3}{2} = \tfrac{34}{4} + \tfrac{6}{4}$$
$$= \tfrac{40}{4}$$
$$= 10$$

Sometimes you can use properties to rearrange the terms to make an expression easier to evaluate.

Other Examples

A. Use Properties

Evaluate $(\frac{3}{5} \times 7\frac{1}{8}) \times 5$.

$$(\tfrac{3}{5} \times 7\tfrac{1}{8}) \times 5$$
$$= (7\tfrac{1}{8} \times \tfrac{3}{5}) \times 5 \quad \leftarrow \text{Commutative Property}$$
$$= 7\tfrac{1}{8} \times (\tfrac{3}{5} \times 5) \quad \leftarrow \text{Associative Property}$$
$$= 7\tfrac{1}{8} \times 3$$
$$= (7 + \tfrac{1}{8}) \times 3 \quad \leftarrow \text{Distributive Property}$$
$$= 21 + \tfrac{3}{8}$$
$$= 21\tfrac{3}{8}$$

B. Substitute

Evaluate $b \times (\frac{4}{5} + \frac{1}{2})$, given $b = \frac{1}{8}$.

Substitute $\frac{1}{8}$ for b.

$$b \times (\tfrac{4}{5} + \tfrac{1}{2})$$
$$= \tfrac{1}{8} \times (\tfrac{4}{5} + \tfrac{1}{2})$$
$$= \tfrac{1}{8} \times (\tfrac{8}{10} + \tfrac{5}{10})$$
$$= \tfrac{1}{8} \times \tfrac{13}{10}$$
$$= \tfrac{13}{80}$$

C. Evaluate, given $a = \frac{1}{2}$, $b = \frac{3}{4}$, $c = \frac{2}{3}$.

$a + (8 \times b) + c$

$\frac{1}{2} + (8 \times \frac{3}{4}) + \frac{2}{3}$ ← Substitute for *a*, *b*, and *c*

$\frac{1}{2} + 6 + \frac{2}{3}$ ← Simplify within parentheses

$\frac{3}{6} + 6 + \frac{4}{6}$ ← Find the common denominator

$6\frac{7}{6}$ or $7\frac{1}{6}$ ← Add and write in simplest form

Guided Practice

Evaluate each expression.

1. $\frac{3}{4} + \frac{1}{8} - \frac{3}{10}$

2. $3\frac{1}{4} \times (\frac{3}{8} + \frac{1}{2})$

Evaluate each expression, given $a = \frac{1}{2}$.

3. $(a + a + a) + 8\frac{2}{3}$

4. $6\frac{2}{3} \times a + \frac{1}{3} \times a$

Ask Yourself

- Did I perform the operations in the correct order?
- Can I use properties to simplify expressions?

Explain Your Thinking What multiplication properties did you use to evaluate the expression in Exercise 4?

Practice and Problem Solving

Evaluate each expression. Identify any properties you used.

5. $\frac{3}{4} \times 1\frac{1}{6} - \frac{1}{8}$

6. $\frac{1}{2} + \frac{2}{5} \times 8\frac{1}{2}$

7. $3\frac{1}{3} \cdot \frac{1}{5} - \frac{1}{4}$

8. $1\frac{3}{7} \times 1\frac{1}{5} + 1\frac{1}{2}$

9. $\frac{1}{6} - \frac{1}{8} \cdot \frac{5}{6} + \frac{1}{3}$

10. $2\frac{2}{3} \times 2\frac{1}{2} - 3\frac{1}{6}$

11. $\frac{5}{9} \times \frac{2}{7} \times \frac{9}{10}$

12. $\frac{3}{2} \times 1\frac{7}{8} \times \frac{2}{3}$

13. $6\frac{2}{3} \times \frac{1}{2} \times 3$

14. $2\frac{1}{5} \times 3 + 4 \times 2\frac{1}{4}$

15. $12\frac{1}{2} \times \frac{1}{2} \times (2 + 1)$

16. $10 - \frac{2}{5} \times 1\frac{1}{4}$

Evaluate each expression, given $a = \frac{2}{3}$, $b = \frac{1}{8}$, and $c = 1\frac{1}{4}$.

17. $a \times 3\frac{1}{6} - a \times \frac{7}{6}$

18. $4\frac{2}{3} + (b + 2\frac{7}{8})$

19. $(\frac{7}{11} - \frac{7}{11}) \times \frac{2}{5} \times c$

20. $(10 \times a) + (2 \times a)$

21. $3a + b \times c$

22. $3 \times (a + b) \times c$

23. How could you use the Distributive Property to evaluate the expression in Exercise 20?

Go On

Decide which expression represents each problem.

24. The average person in the United States consumes over 11 pounds of chocolate in one year. It takes about 400 cocoa beans to make one pound of chocolate. About how many beans are needed to make n pounds of chocolate?

a. $400 \times n$ **b.** $400 \div n$

25. A 1-ounce piece of chocolate may have 6 grams of fat. You have already eaten 23 grams of fat today. If you eat a chocolate bar that weighs n ounces, how many grams of fat will you have eaten altogether?

a. $23 + 6 \div n$ **b.** $23 + n \times 6$



cacao beans

Chocolate Chip Pancakes

(Makes 10 four-inch pancakes)

Dry Ingredients

- $1\frac{1}{2}$ c cake flour
- $2\frac{1}{2}$ tbsp sugar
- $\frac{1}{2}$ tsp salt
- $\frac{3}{4}$ tsp baking soda
- $1\frac{3}{4}$ tsp baking powder
- 1 c chocolate chips

Liquid Ingredients

- $1\frac{1}{4}$ c buttermilk
- $\frac{1}{4}$ c water
- 2 tbsp melted butter
- $\frac{1}{4}$ c egg whites



Solve. Use the recipe for Problems 26–28.

26. The recipe makes 10 4-inch pancakes. Jon wants to make only half that many pancakes. How much baking soda will Jon need? Express your answer as a fraction.

27. **Analyze** Jon wants to make pancakes for himself and 4 friends. He estimates that each person will eat four 4-inch pancakes. How many pancakes will he need to make?

28. **Explain** A beef burger has 636 calories. A turkey burger has $\frac{2}{3}$ of that amount. A serving of French fries has 540 calories. Potato wedges have $\frac{1}{2}$ of that amount. Does a turkey burger with potato wedges have less than 600 calories?

Extra Practice See page 147, Set E.

Mixed Review and Test Prep

Open Response

Find the GCF of each pair of numbers. (Ch. 3, Lesson 4)

29. 12, 16

30. 20, 24

31. 16, 17

32. 24, 36

Multiple Choice

33. What is 36.7886 rounded to the nearest thousandth? (Ch. 1, Lesson 4)

A 36.80 **C** 36.789

B 36.79 **D** 36.788

Problem Solving

Social Studies Connection

Calculation Comparisons

The Swiss Alps are one of the most popular hiking destinations in the world. If you want to buy a pair of hiking boots in Zürich, you need to know your shoe size in the Continental system.

In the Continental system, each larger size shoe is $\frac{2}{3}$ cm longer than the previous size. Children's sizes range from size 15, which is 10 cm long, through size 30. Adults' shoe sizes range from size 31 through size 45.

To find the Continental size, divide the length in centimeters by $\frac{2}{3}$.



Write whether each size is a child's shoe size or an adult's shoe size.

1. 14 cm **2.** 26 cm

3. 18 cm **4.** 30 cm

Chapter Review/Test

VOCABULARY

1. If $a \times b = 1$, b is called the _____ of a.
2. To _____ a fraction, you reverse the numerator and the denominator.
3. When you use _____ , you factor a number into its prime factors only.

Vocabulary

- divide
- invert
- simplest form
- prime factorization
- multiplicative inverse

CONCEPTS AND SKILLS

Estimate each product or quotient. (Lesson 1, pp. 126–127)

4. $4\frac{3}{4} \times 3\frac{1}{8}$
5. $11\frac{2}{3} \div 2\frac{1}{5}$
6. $\frac{1}{2}$ of 118
7. $45\frac{1}{8} \div 8\frac{3}{4}$

Multiply. Write each product in simplest form. (Lessons 2–3, pp. 128–131)

8. $\frac{1}{2} \times \frac{5}{8}$
9. $3 \times \frac{7}{15}$
10. $2\frac{3}{8} \times 1\frac{1}{2}$
11. $9\frac{1}{9} \times \frac{1}{4}$
12. $9 \times 3\frac{2}{3}$

Divide. Write each quotient in simplest form. (Lessons 4–5, pp. 132–134, 136–137)

13. $\frac{11}{12} \div \frac{5}{6}$
14. $6 \div \frac{7}{8}$
15. $\frac{1}{2} \div 5$
16. $1\frac{1}{2} \div 4$
17. $12\frac{2}{5} \div 5\frac{1}{6}$
18. $30 \div 1\frac{3}{10}$
19. $3\frac{1}{4} \div \frac{3}{7}$
20. $11\frac{1}{2} \div 3$

Evaluate each expression, given $a = \frac{3}{4}$, $b = \frac{1}{8}$, and $c = 2\frac{1}{3}$. (Lessons 7, pp. 142–144)

21. $(a + c) \div \frac{3}{8}$
22. $(2\frac{3}{5} \times b) + 1\frac{3}{5}$
23. $(a + \frac{3}{8}) \times 3 - c$

PROBLEM SOLVING

Solve. Use the Solve-a-Simpler-Problem strategy. (Lesson 6, pp. 138–140)

24. Kate's mom is decorating for a party. She uses eight $8\frac{1}{2}$-foot pink streamers and four $11\frac{1}{3}$-foot green streamers. How many feet of streamers will she use in all?
25. To fence the length of his front yard, Mr. Brown places fence posts at the beginning and end of each 4 feet. If the front yard is 60 feet long, how many fence posts will he need?

Write About It

Show You Understand

A pancake recipe calls for $1\frac{1}{2}$ cups flour and $1\frac{3}{4}$ teaspoons baking powder. The recipe makes 10 pancakes. Explain how you could rewrite the recipe to make 25 pancakes.

Extra Practice

Set A (Lesson 1, pp. 126–127)

Estimate each product or quotient.

1. $5\frac{1}{7} \times 8\frac{1}{6}$
2. $14\frac{4}{5} \div 3\frac{1}{3}$
3. $9\frac{1}{11} \times 8\frac{1}{9}$
4. $47\frac{5}{6} \div 6\frac{3}{7}$
5. $2\frac{1}{8} \times 11\frac{7}{8}$
6. $56\frac{1}{6} \div 6\frac{3}{4}$
7. $71\frac{1}{2} \div 9\frac{1}{4}$
8. $\frac{3}{5}$ of 49

Set B (Lessons 2–3, pp. 128–131)

Multiply. Write each product in simplest form.

1. $\frac{3}{10} \times \frac{4}{5}$
2. $18 \times \frac{2}{3}$
3. $\frac{5}{12} \times 3$
4. $\frac{4}{9} \times \frac{3}{4}$
5. $\frac{1}{8} \times \frac{5}{6}$
6. $\frac{2}{6} \times \frac{5}{8}$
7. $4 \times 2\frac{1}{6}$
8. $\frac{3}{5} \times 1\frac{3}{12}$
9. $7\frac{2}{5} \times 5\frac{2}{4}$
10. $8\frac{5}{8} \times 1\frac{1}{3}$
11. $10\frac{2}{5} \times \frac{5}{4}$
12. $3\frac{5}{9} \times 1\frac{3}{8}$

Set C (Lesson 4, pp. 132–134)

Find the reciprocal of each number.

1. $\frac{5}{6}$
2. $3\frac{1}{3}$
3. 4
4. $\frac{8}{7}$
5. $2\frac{3}{5}$

Divide. Write each quotient in simplest form.

6. $\frac{5}{16} \div \frac{1}{8}$
7. $10 \div \frac{5}{9}$
8. $\frac{1}{2} \div 10$
9. $\frac{1}{20} \div \frac{4}{5}$
10. $\frac{1}{7} \div \frac{2}{3}$
11. $\frac{1}{6} \div 5$
12. $3 \div \frac{2}{3}$
13. $\frac{1}{2} \div \frac{3}{4}$

Set D (Lesson 5, pp. 136–137)

Divide. Write each quotient in simplest form.

1. $1\frac{1}{2} \div 4$
2. $8\frac{7}{8} \div \frac{1}{16}$
3. $2\frac{4}{7} \div 7\frac{1}{5}$
4. $12\frac{2}{5} \div 5\frac{1}{6}$
5. $6\frac{7}{8} \div 2\frac{1}{2}$
6. $\frac{2}{3} \div 1\frac{2}{5}$
7. $4 \div 2\frac{5}{8}$
8. $5\frac{1}{5} \div 3\frac{1}{3}$

Set E (Lesson 7, pp. 142–144)

Evaluate each expression, given $a = \frac{1}{10}$, $b = \frac{3}{4}$, and $c = 2\frac{1}{2}$.

1. $4 \times b + \frac{4}{5}$
2. $(c \times 1\frac{1}{2}) + (\frac{3}{10} - a)$
3. $5 - (a \times 2\frac{1}{4}) + \frac{2}{5}$
4. $(3 \times c) \times (1 - b)$
5. $a \times (b \times c)$
6. $c \times (b + \frac{7}{8})$

Extra Practice at **eduplace.com/map**

CHAPTER 7

Multiply and Divide Decimals

INVESTIGATION

Using Data

How many whole $5 bills will fit on a sheet of 8.5-in. × 11-in. paper without overlap? Explain.

A $5 bill is 0.0043 inch thick. In inches, how tall would a stack of $1,000,000 in $5 bills be?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. You can _____ to help you place the decimal point in a quotient or a product.
2. In 10^3, the 3 is a(n) _____.
3. You _____ numbers to help you estimate when you multiply or divide decimals.

Vocabulary

round
factor
estimate
exponent

CONCEPTS AND SKILLS

Find each product or quotient.

4. $\frac{2}{10} \times \frac{3}{10}$
5. $\frac{12}{100} \times \frac{4}{10}$
6. $\frac{9}{10} \div \frac{3}{10}$
7. $\frac{25}{100} \div \frac{5}{10}$
8. 0.2×0.3
9. 0.12×0.4
10. $0.9 \div 0.3$
11. $0.25 \div 0.5$

Multiply by using patterns.

12. $6 \times 7 = \blacksquare$
    $6 \times 0.7 = \blacksquare$
    $6 \times 0.07 = \blacksquare$
13. $8 \times 20 = \blacksquare$
    $0.8 \times 20 = \blacksquare$
    $0.08 \times 20 = \blacksquare$
14. $15 \times 15 = \blacksquare$
    $15 \times 1.5 = \blacksquare$
    $1.5 \times 1.5 = \blacksquare$

Divide and check.

15. $5.4 \div 6$
16. $7.2 \div 9$
17. $0.35 \div 5$
18. $3.33 \div 3$
19. $14.4 \div 12$

Write About It

20. Which do you think will be greater, the product 0.94×90 or the product 9.4×9? Explain how you could tell without multiplying.

Facts Practice, See page 660.

Hands On Lesson 1

Explore Multiplication and Division With Decimals

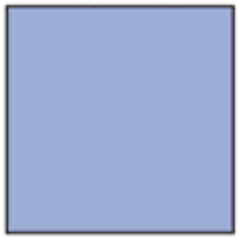
Materials
Learning Tool 1
or grid paper
colored pencils
scissors

Objective Use a model to multiply and divide decimals.

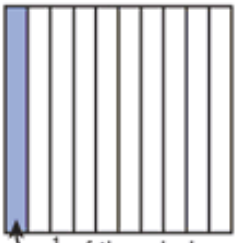
Work Together

You can shade decimal models or use grid paper to model multiplication and division with decimals.

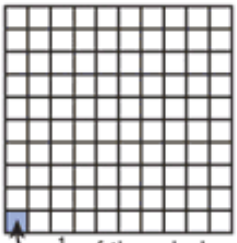
Start with a single square.
The whole square represents 1.



Divide each side into 10 parts.



$\frac{1}{10}$ of the whole square, or 0.1.

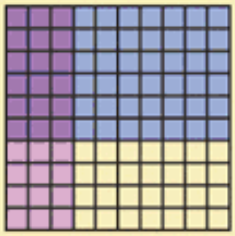


$\frac{1}{100}$ of the whole square, or 0.01.

Model 0.3 × 0.6. Use hundredths decimal models.

STEP 1 Shade 3 columns (or $\frac{30}{100}$) of the 10 × 10 decimal model red to represent the first factor.

- Why do 3 columns represent 0.3?



STEP 2 Shade 6 rows (or $\frac{60}{100}$) of the 10 × 10 decimal model blue to represent the second factor.

- Why do 6 rows represent 0.6?
- What are the dimensions of the purple-shaded rectangle?

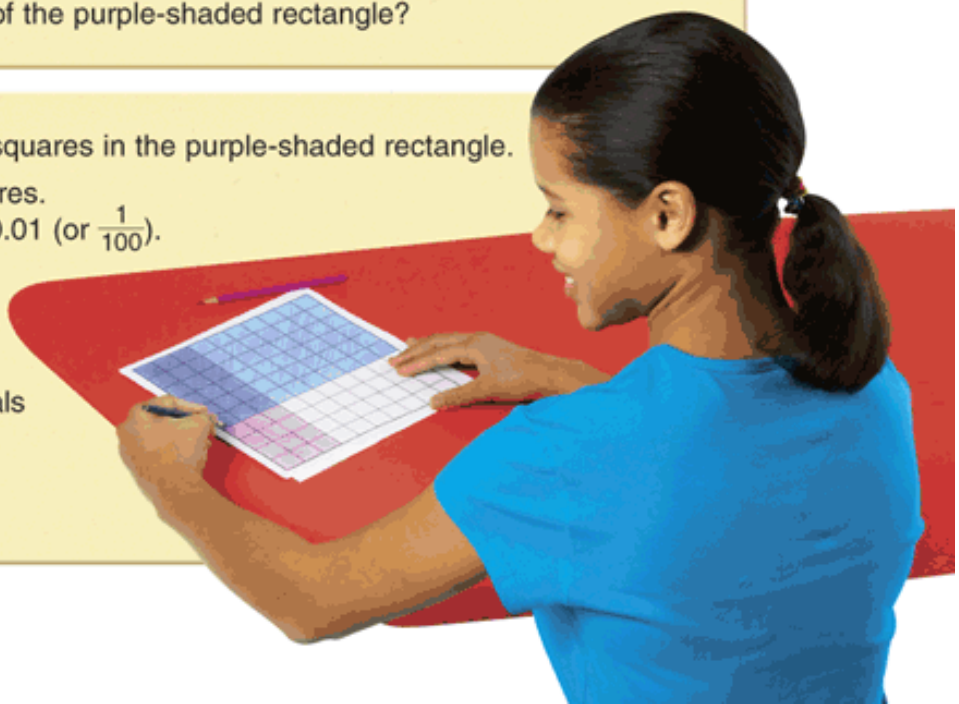
STEP 3 Count the number of small squares in the purple-shaded rectangle.

There are 18 purple squares.
Each square represents 0.01 (or $\frac{1}{100}$).

So, $0.3 \times 0.6 = 0.18$.

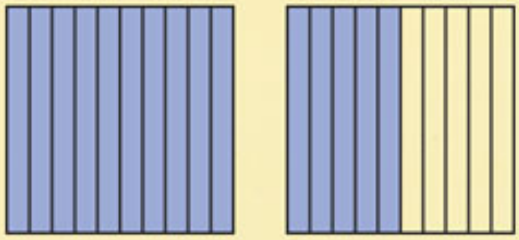
You can check your answer by multiplying fractions that are equivalent to the decimals 0.3 and 0.6.

$\frac{3}{10} \times \frac{6}{10} = \frac{18}{100}$



Model 1.5 ÷ 0.3. Use tenths decimal models.

STEP 1 Represent 1.5 by shading 1 whole tenths decimal model and 5 columns of a second tenths decimal model.



STEP 2 Cut the shaded portions of both models into strips, each representing 0.3.

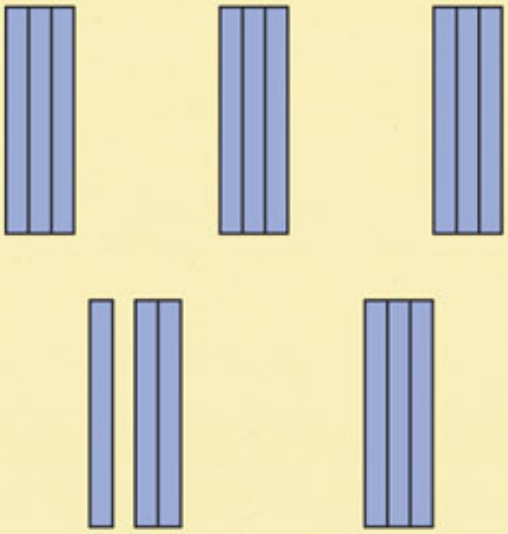
- How many strips representing 0.3 are there?
- What is 1.5 ÷ 0.3?

You can check the answer by multiplying fractions that are equivalent to the decimals.

$$1\frac{5}{10} \div \frac{3}{10} = \frac{15}{10} \times \frac{10}{3} = \frac{15 \times 10}{10 \times 3}$$

$$= \frac{\overset{5}{\cancel{15}} \times \overset{1}{\cancel{10}}}{\underset{1}{\cancel{10}} \times \underset{1}{\cancel{3}}}$$

$$= \frac{5}{1} = 5$$



On Your Own

Find each product or quotient. Use decimal models.

1. 0.5 × 0.9 **2.** 0.7 × 0.7 **3.** 1.2 ÷ 0.4 **4.** 0.56 ÷ 0.8

Talk About It • Write About It

You learned to use models to find decimal products and quotients.

5. What do you notice about the product of two numbers that are tenths?

6. Can you show the product 0.8 × 0.25 by using only a 10 × 10 decimal model?

Lesson 2

Multiply Decimals

Objective Multiply decimals by whole numbers and by decimals.

Learn About It

Multiplying decimals is much like multiplying whole numbers. The number of decimal places in the product equals the sum of the number of decimal places in the factors.

Spain, along with most other countries in Europe, uses the euro, €, as its currency. Peter wants to buy a souvenir book of Spain that costs €24. To find the cost of the book in U.S. dollars, multiply the euro price by the euro-to-U.S. dollar exchange rate of €1 = $0.991.

CURRENCY EXCHANGE

Country & Currency	Rate of Exchange for Euro (as of July 2002)
U.S. Dollar	0.9912
Canada Dollar	1.5091
Brazil Real	2.8106

Find 24 × 0.991.

STEP 1 Estimate first.

0.991 rounds to 1
× 24 × 24
24

So the cost should be about 24 U.S. dollars.

STEP 2 Multiply the factors as if there were no decimal point.

991
× 24
3964
1982
23784

STEP 3 Place the decimal point in the product.

0.991 ← 3 decimal places
× 24 ← + 0 decimal places
23.784 ← 3 decimal places

Solution: The book costs about $23.78 U.S. dollars. $23.78 is close to the estimate of $24.00, so the answer is reasonable.

▶ When both factors are decimals, you may need to write zeros in order to place the decimal point in the product.

Find 0.998 × 0.013.

STEP 1 Multiply.

0.998
× 0.013
2 994
9 980
12 974

STEP 2 Place the decimal point in the product.

0.998 ← 3 decimal places
× 0.013 ← + 3 decimal places
0.012974 ← 6 decimal places

Solution: 0.998 × 0.013 = 0.012974.

Guided Practice

Find each product. Estimate first to help you place the decimal point.

1. 2.5×2 **2.** $\$4.10 \times 6$ **3.** 5.16×15

4. 2.2×2.3 **5.** 5.4×1.02 **6.** $\$0.33 \times 4.5$

Ask Yourself

- Should the answer be greater than or less than the whole-number factor?
- How many decimal places should the product have?

Explain Your Thinking What is the product of 1.525×0.004? How do you know where to place the decimal point?

Practice and Problem Solving

Find each product. Estimate first.

7. 2.8×3 **8.** 7.7×7 **9.** 4.6×12 **10.** 4.3×52

11. $\$7.01 \times 53$ **12.** $\$3.70 \times 19$ **13.** 0.22×33 **14.** 0.52×58

15. $\begin{array}{r} 4.7 \\ \times\ 0.2 \\ \hline \end{array}$ **16.** $\begin{array}{r} 8.9 \\ \times\ 1.6 \\ \hline \end{array}$ **17.** $\begin{array}{r} 18.3 \\ \times\ 8.62 \\ \hline \end{array}$ **18.** $\begin{array}{r} 14.2 \\ \times\ 34.5 \\ \hline \end{array}$

19. 561.2×2.15 **20.** 4.215×4.22 **21.** 321.7×46.5 **22.** 16.3×245.2

Place a decimal point in the product to make each equation true. Use estimation to help you.

23. $17.5 \times 5 = 875$ **24.** $10.32 \times 6 = 6192$ **25.** $27.2 \times 8 = 2176$

Algebra • Equations

Find *n* for each equation. Use mental math.

26. $0.014 \times 1{,}000 = n$ **27.** $1.25 \times 100 = n$ **28.** $n \times 10 = 280$

29. $4.2567 \times n = 42{,}567$ **30.** $0.6 = n \times 10$ **31.** $4.28 = n \times 107$

Data

Use the table at right for Problems 32 and 33.

32. It costs money to make money. Which United States coin costs the most to make? About how many cents does it cost to make that coin?

33. Compare the value of each coin to the cost of making that coin. Which coin is the most expensive to produce compared to its value?

Total Expense for Making One Coin

Penny	\$0.0082
Nickel	\$0.0316
Dime	\$0.0192
Quarter	\$0.0438

Go On

Use number sense to tell whether each statement is *true* or *false*. Explain your answer.

34. The product of 9 and 0.75 is less than 9.

35. The product 9×9 is greater than the product 0.9×100.

36. The product of 2 and 0.388 is less than 0.6.

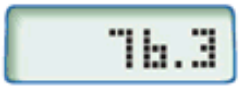
37. The product of two decimals that are less than 1 is always less than 1.

Calculator **Decide whether each calculator display is correct. Explain your answer.**

38. 18×1.5



39. 10.9×7



40. 12.5×9.5

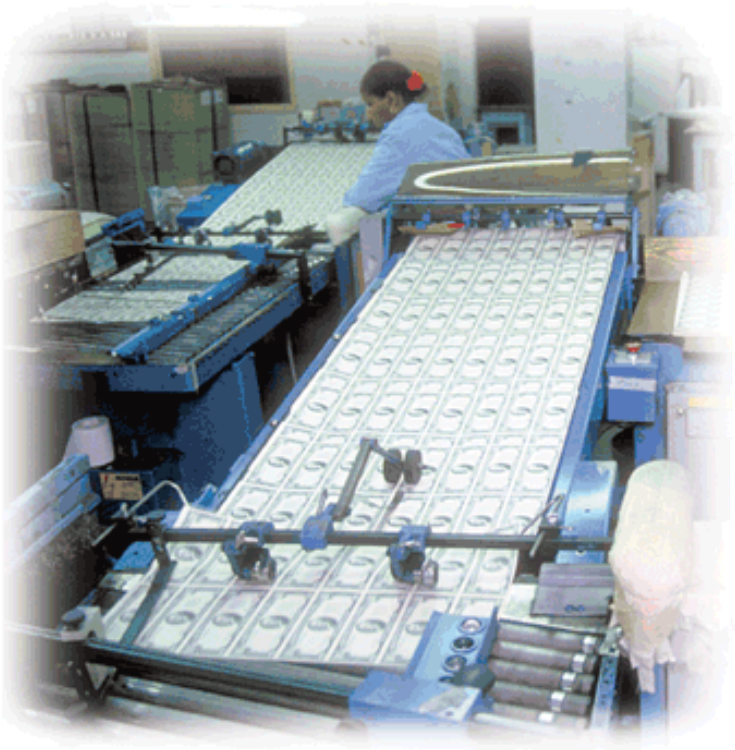


41. 12×1.2



Solve.

42. Dollar bills are printed in sheets of 32 bills. An individual dollar bill is 6.14 inches long and 2.61 inches wide. What is the length and width of a sheet of 32 bills?



43. **Explain** A visitor to the United States wants to buy a pair of jeans that cost \$39. She has a travelers check worth 75 Brazilian real. Does she have enough money to buy the jeans if 1 real is worth \$0.353 U.S. dollars?

44. **Reasoning** Santos bought some postcards and some souvenir pins. The postcards cost \$0.60 each and the souvenir pins cost \$0.45 each. If he spent \$6.00 for 12 items, how many of each did he buy?

Mixed Review and Test Prep

Open Response

Identify the place value of the underlined digit. (Ch. 1, Lesson 1)

45. 67.0$\underline{8}$2

46. 0.00$\underline{3}$68

47. 247.$\underline{0}$4763

48. 9.389$\underline{5}$36

49. 0.12121$\underline{2}$

50. 43.400$\underline{2}$3

Multiple Choice

51. Pam bought 4.5 pounds of potatoes. If potatoes cost \$0.33 a pound, how much will Pam pay? (Ch. 7, Lesson 2)

A \$14.85 **C** \$1.49

B \$1.59 **D** \$1.39

Extra Practice See page 171, Set A.

Activity

Real World Connection

Playing the Market

A stock market is a place where people buy and sell shares of stock in different companies.

When you own shares of stock in a company, you own a part of, or are investing in, the company.

1. Valerie bought 3 shares of stock in Company *A* at $12.50 a share. The share price then increased by an average of $0.75 each month for 6 months. How much is Valerie's stock worth at the end of 6 months?
2. Pick a stock from the stock tables in a newspaper. (Think of a restaurant chain where you eat, a store where you shop, or a company where a relative works). Follow the closing price of your stock for one week. How much did its price change?



The New York Stock Exchange trading floor

Logical Thinking

Greatest and Least

Use the numbers 3, 4, 6, and 7 to fill in the boxes. Build the greatest product. Then build the least product.

□ . □

× □ . □

Repeat, using the numbers 1, 5, 8, and 9.

Make up your own greatest and least puzzle!

Brain Teaser

Gene wrote a number on a piece of paper.

Gene says: "The reciprocal of my number is an improper fraction greater than 1."

- Sue says: "Gene's number could be a mixed number."
- Jerry says: "The number could be a proper fraction."

Which statement is correct? Explain and give examples to show your thinking.

Education Place

Check out **eduplace.com/map** for more brain teasers.

Lesson 3

Audio Tutor 1/18 Listen and Understand

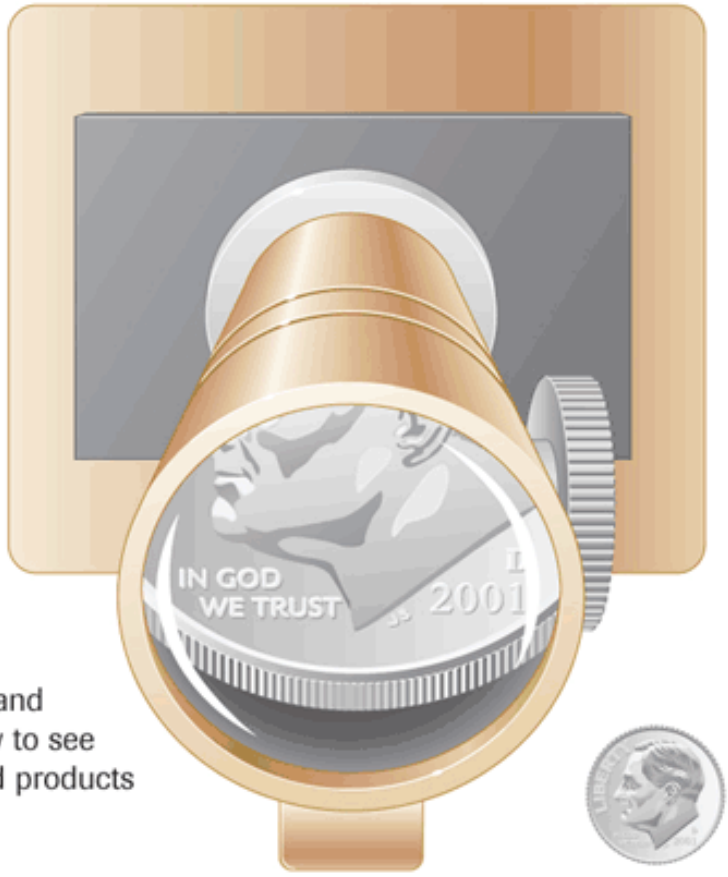
Multiply and Divide by Powers of Ten

Vocabulary
power of 10

Objective Use patterns to multiply and divide by powers of 10.

Learn About It

Lenses and microscopes are used to enlarge images of small objects. Lenses and microscopes usually magnify the object by some **power of 10**.



Recall that you can use exponents to write powers of 10:

$$10 = 10^1$$

$$100 = 10 \times 10 = 10^2$$

$$1{,}000 = 10 \times 10 \times 10 = 10^3$$

$$\frac{1}{1{,}000} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = 10^{-3}$$

You can use powers-of-10 patterns to multiply and divide with decimals. Look at the patterns below to see how the value of the exponent can help you find products and quotients.

Multiply by Powers of 10

Think: How does the exponent help me place the decimal point in the product?

$4 \times 10^1 = 40$	$752 \times 10^1 = 7{,}520$	$5.0724 \times 10^1 = 50.724$
$4 \times 10^2 = 400$	$752 \times 10^2 = 75{,}200$	$5.0724 \times 10^2 = 507.24$
$4 \times 10^3 = 4{,}000$	$752 \times 10^3 = 752{,}000$	$5.0724 \times 10^3 = 5{,}072.4$

Divide by Powers of 10

Think: How does the exponent help me place the decimal point in the quotient?

$7{,}000 \div 10^1 = 700$	$305{,}000 \div 10^1 = 30{,}500$	$83.036 \div 10^1 = 8.3036$
$7{,}000 \div 10^2 = 70$	$305{,}000 \div 10^2 = 3{,}050$	$83.036 \div 10^2 = 0.83036$
$7{,}000 \div 10^3 = 7$	$305{,}000 \div 10^3 = 305$	$83.036 \div 10^3 = 0.083036$

Another Example

Patterns With Zero and Negative Exponents

Remember: You can write a negative power of 10 as a decimal or as a fraction.

$36 \times 10^{0} = 36 \times 1 = 36$

$36 \times 10^{-1} = 36 \times 0.1 = 3.6$

$36 \times 10^{-2} = 36 \times 0.01 = 0.36$

$36 \times 10^{-3} = 36 \times 0.001 = 0.036$

$36 \times 10^{0} = 36 \times \frac{1}{1} = 36$

$36 \times 10^{-1} = 36 \times \frac{1}{10} = \frac{36}{10} = 3\frac{6}{10}$

$36 \times 10^{-2} = 36 \times \frac{1}{100} = \frac{36}{100}$

$36 \times 10^{-3} = 36 \times \frac{1}{1{,}000} = \frac{36}{1{,}000}$

Guided Practice

Ask Yourself

- Which way should the decimal point be moved?
- How many zeros do I need in my answer?

Use patterns to help you find each product or quotient.

1. $4.0813 \times 10^{1} = 40.813$

$4.0813 \times 10^{2} = \blacksquare$

$4.0813 \times 10^{3} = \blacksquare$

2. $51.045 \div 10^{1} = 5.1045$

$51.045 \div 10^{2} = \blacksquare$

$51.045 \div 10^{3} = \blacksquare$

Find each product or quotient, using mental math.

3. 5.06×10^{3}

4. $57{,}000 \times 10^{-1}$

5. $4{,}800 \div 10^{1}$

Explain Your Thinking When you multiply by a negative power of 10, which way is the decimal point in the product moved? Why?

Practice and Problem Solving

Use patterns to help you find each product or quotient.

6. $3.4872 \times 10^{1} = 34.872$

$3.4872 \times 10^{2} = \blacksquare$

$3.4872 \times 10^{3} = \blacksquare$

7. $4.951 \div 10^{1} = 0.4951$

$4.951 \div 10^{2} = \blacksquare$

$4.951 \div 10^{3} = \blacksquare$

8. $17 \times 10^{-1} = 1.7$

$17 \times 10^{-2} = \blacksquare$

$17 \times 10^{-3} = \blacksquare$

9. $42 \times 10^{-1} = 4.2$

$42 \times 10^{-2} = \blacksquare$

$42 \times 10^{-3} = \blacksquare$

10. $51.16 \div 10^{1} = 5.116$

$51.16 \div 10^{2} = \blacksquare$

$51.16 \div 10^{3} = \blacksquare$

11. $9 \div 10^{1} = 0.9$

$9 \div 10^{2} = \blacksquare$

$9 \div 10^{3} = \blacksquare$

Go On

Find each product or quotient, using mental math.

12. $1{,}500 \div 10^2$ **13.** $13{,}100 \div 10^2$ **14.** $93.1 \div 10^3$ **15.** $95{,}700 \div 10^1$

16. $4{,}295 \times 10^3$ **17.** $7{,}200 \times 10^{-3}$ **18.** $4{,}002.6 \times 10^3$ **19.** $8{,}000 \times 10^{-2}$

Algebra • Functions

Copy. Then complete each table by following the rule.

Rule: Multiply by 10^2

	Input	Output
20.	32	
21.		0.008
22.	1.08	

Rule: Divide by 10^3

	Input	Output
23.	25,953	
24.		0.016
25.	320	

Rule: Multiply by 10^{-2}

	Input	Output
26.		0.20
27.	45,046	
28.		0.2222

Data

Use the table for Problems 29–32.

29. How tall would a stack consisting of 10^3, or one thousand, pennies be?

30. **Estimate** About how many one-dollar bills would you have to stack to make up the thickness of a one-dollar coin?

31. Would a stack of $1,000,000 in one-dollar bills be shorter than or taller than you are? About how tall would the stack be? (Hint: 1 foot is about 300 millimeters.)

32. Suppose there is a single stack of dimes that is exactly as tall as you are. How much is it worth?

How Thick is Our Money?	
Penny	1.55 millimeters
Dime	1.35 millimeters
Dollar (coin)	2.00 millimeters
Dollar (bill)	0.010922 millimeters



33. **Write About It** The size of an object on a microscope slide is 2 millimeters. An enlarged image of the object is 2,000 millimeters. Use powers of ten to explain how the two sizes are related.

Extra Practice See page 171, Set B.

Number Sense

Use Properties of Powers to Multiply

Activity

What happens when you multiply numbers with exponents? Try this.

- Start with the expression: $10^2 \times 10^2$
- Rename the expression as repeated multiplication: $10 \times 10 \times 10 \times 10$
- Rename the multiplication expression with a single exponent: 10^4

1. Try it yourself. Copy and complete this table

Multiplication with Exponents	Repeated Multiplication	Single Exponent
$10^2 \times 10^2$	$10 \times 10 \times 10 \times 10$	10^4
$10^1 \times 10^3$		
$10^2 \times 10^4$		
$10^2 \times 10^5$		
$10^1 \times 10^4$		

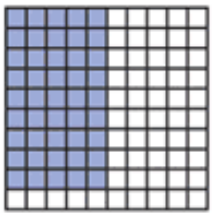
2. Compare the exponents in the first and last column of your table. What pattern do you see? Can you write a rule for multiplying numbers with exponents?

Quick Check

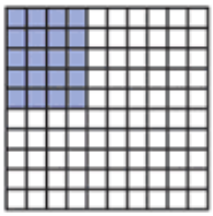
Check your understanding of Lessons 1–3.

Write a multiplication sentence and a related division sentence that could be modeled by each grid. (Lesson 1)

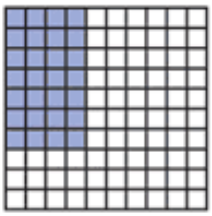
1.



2.



3.



Find each product. Estimate first. (Lesson 2)

4. 4.5×2 **5** $\$5.72 \times 3$ **6.** 17.4×9.48 **7.** 1.231×0.004

Find each product or quotient, using mental math. (Lesson 3)

8. $1{,}200 \div 10^2$ **9.** $95{,}700 \div 10^1$ **10.** $18{,}000 \div 10^0$

Lesson 4

Divide Decimals by Whole Numbers

Objective Divide decimals by whole numbers.

Learn About It

The U. S. Mint sells sets of specially minted coins called proof sets to collectors. While visiting the mint, Tanya bought 7 proof sets. She spent a total of $139.65. If each proof set cost the same, how much did each proof set cost?



Find $139.65 ÷ 7.

STEP 1 Estimate, using compatible numbers.

$$7\overline{)139.65}$$

is about

$$7\overline{)140}\quad 20$$

STEP 2 Divide as if the decimal point weren't there.

$$\begin{array}{r} 19\ 95 \\ 7\overline{)139.65} \\ -\ 7 \\ \hline 69 \\ -\ 63 \\ \hline 66 \\ -\ 63 \\ \hline 35 \\ -\ 35 \\ \hline 0 \end{array}$$

STEP 3 Place the decimal point in the quotient directly above the decimal point in the dividend.

$$\begin{array}{r} 19.95 \\ 7\overline{)139.65} \\ -\ 7 \\ \hline 69 \\ -\ 63 \\ \hline 66 \\ -\ 63 \\ \hline 35 \\ -\ 35 \\ \hline 0 \end{array}$$

Check your work.

$$\begin{array}{r} \$19.95 \\ \times\quad 7 \\ \hline \$139.65 \end{array}$$

Solution: Each proof set cost $19.95. Since $19.95 is close to the estimate of $20, the solution is reasonable.

Other Examples

A. Zeros in the Dividend

Find 34.8 ÷ 8.

$$\begin{array}{r} 4.35 \\ 8\overline{)34.80} \\ -\ 32 \\ \hline 2\ 8 \\ -\ 2\ 4 \\ \hline 40 \\ -\ 40 \\ \hline 0 \end{array}$$

← Write a zero and bring down to find the quotient to more decimal places.

B. Divide Whole Numbers

Find 60 ÷ 16.

$$\begin{array}{r} 3.75 \\ 16\overline{)60.00} \\ -\ 48 \\ \hline 12\ 0 \\ -\ 11\ 2 \\ \hline 80 \\ -\ 80 \\ \hline 0 \end{array}$$

Guided Practice

Divide. Then check your work.

1. $5\overline{)5.5}$ **2.** $4\overline{)30.4}$ **3.** $8\overline{)\$3.12}$ **4.** $6\overline{)12.24}$

5. $33.201 \div 9$ **6.** $112.5 \div 75$ **7.** $8.551 \div 17$

Ask Yourself

- Where should I place the decimal point in the quotient?
- When should I write a zero in the dividend and continue dividing?

Explain Your Thinking Why don't you write remainders as whole numbers when you divide decimals?

Practice and Problem Solving

Divide. Then check your work.

8. $2\overline{)3.6}$ **9.** $7\overline{)0.49}$ **10.** $3\overline{)60.15}$ **11.** $6\overline{)7.2}$

12. $8\overline{)17.92}$ **13.** $8\overline{)8.84}$ **14.** $5\overline{)7.53}$ **15.** $4\overline{)\$63.04}$

16. $5.125 \div 25$ **17.** $0.98 \div 56$ **18.** $\$52.19 \div 17$ **19.** $597.54 \div 20$

Solve.

20. A coin museum has 31 coins that were made by the U.S. Mint between 1796 and 1830. When the coins were new, their total value was \$77.50. If each coin had the same value, what was the value of each coin?

21. **Analyze** Alexander buys a Liberty coin set by mail order. The set costs \$18.41. Shipping and handling is \$3.55. If Alexander pays the total amount in 4 equal payments, how large is each payment?

22. Ludmilla bought a used mountain bike for \$149. She made a down payment of \$50. If she pays \$9 each week, how long will it take Ludmilla to pay the amount remaining?

23. **You Decide** This pattern is formed by dividing the previous term by 2.

25.6, 12.8, 6.4, 3.2, . . .

Start with 25. Divide by a whole number to make a different pattern.

Mixed Review and Test Prep

Open Response

Write each fraction or mixed number as a decimal. (Ch. 4, Lesson 2)

24. $\frac{3}{4}$ **25.** $\frac{1}{5}$

26. $\frac{1}{4}$ **27.** $1\frac{3}{5}$

28. $2\frac{7}{10}$ **29.** $7\frac{4}{5}$

30. A tennis team raised \$577.28 to buy new uniforms. If there are 8 students on the tennis team, how much money can the tennis team spend on each student's uniform?

Explain how you got your answer.

Extra Practice See page 171, Set C.

Lesson 5

Audio Tutor 1/19 Listen and Understand

Divide Decimals by Decimals

Objective Divide one decimal by another decimal.



Learn About It

Sometimes you need to divide a decimal by another decimal.

Towards the end of a coin-collectors' fair, a coin dealer was closing his stand and selling bags of coins for $0.75. Marta has $6.30. How many bags of coins can she buy?

Find 6.30 ÷ 0.75.



STEP 1 Estimate first.

6.30 ÷ 0.75 rounds to 6 ÷ 1 = 6

The quotient is about 6.

STEP 2 Change the divisor to a whole number. Multiply both the divisor and the dividend by a power of 10.

$0.75\overline{)6.30}$

Multiply both numbers by 10^2, or 100.

STEP 3 Place the decimal point in the quotient. Divide as you would with whole numbers.

$$\begin{array}{r} 8. \\ 75\overline{)630} \\ -\,600 \\ \hline 30 \end{array}$$

← decimal point

STEP 4 Place a zero next to the dividend and continue to divide.

$$\begin{array}{r} 8.4 \\ 75\overline{)630.0} \\ -\,600 \\ \hline 30\,0 \\ -\,30\,0 \\ \hline 0 \end{array}$$

Check.
0.75 × 8.4 = 6.30

Solution: Since Marta cannot buy 0.4 of a bag, disregard the decimal part of the quotient. Marta can buy 8 bags.

Another Example

Place Two or More Zeros in the Dividend

Find 34.2 ÷ 2.4.
Estimate first.

34 ÷ 2 = 17

The quotient of 34.2 ÷ 2.4 is about 17.

Then divide.

$$\begin{array}{r} 14.25 \\ 24\overline{)342.00} \\ -\,24 \\ \hline 102 \\ -\,96 \\ \hline 60 \\ -\,48 \\ \hline 120 \\ -\,120 \\ \hline 0 \end{array}$$

Check.
2.4 × 14.25 = 34.2

Extra Help at eduplace.com/map

Guided Practice

Ask Yourself
- Should I write zeros in the dividend? If so, how many?
- Did I multiply by the correct power of 10?

Divide. Then check your work.

1. $0.44\overline{)4.62}$ **2.** $0.03\overline{)0.24}$ **3.** $3.4\overline{)8.84}$ **4.** $1.6\overline{)8.92}$

5. $6.5 \div 3.2$ **6.** $61.32 \div 1.4$ **7.** $1.222 \div 0.52$

Explain Your Thinking When multiplying to change the divisor to a whole number, how do you know which power of 10 to use?

Practice and Problem Solving

Divide. Then check your work.

8. $4.4\overline{)46.2}$ **9.** $0.05\overline{)4.0}$ **10.** $0.7\overline{)21.28}$ **11.** $0.03\overline{)1.215}$

12. $9.5\overline{)2.299}$ **13.** $4.2\overline{)8.19}$ **14.** $3.5\overline{)4.508}$ **15.** $0.0369 \div 0.9$

16. $22.68 \div 6.3$ **17.** $7.221 \div 0.08$ **18.** $59.04 \div 4.5$ **19.** $6.7204 \div 0.16$

Algebra • Expressions

Evaluate each expression, given $a = 2.7$, $b = 11.34$, and $c = 0.03$.

20. $a \div c$ **21.** $b \div c$ **22.** $b \div a$ **23.** $b \div (a \div c)$

Solve.

24. Inez keeps quarters, dimes, and nickels in a jar. The value of the nickels is $4.65. How many nickels does she have?

25. Analyze A coin dealer sells coin albums at 4 for $10.00 or 10 for $18.00. How much will Liam save per album if he chooses the better deal?

26. What's Wrong? Sammy claims that $63.41 \div 3.4 = 1.865$. Is he correct? Use estimation to explain your answer.

27. Is the quotient of $3.52 \div 0.25$ the same as the quotient of $35.2 \div 2.5$? Explain.



Mixed Review and Test Prep

Open Response

Write each decimal as a fraction or mixed number in simplest form. (Ch. 4, Lesson 2)

28. 0.4 **29.** 0.55

30. 0.125 **31.** 0.17

32. 0.101 **33.** 2.02

Multiple Choice

34. Freda has a piece of wire that is 4.5 meters long. She wants to cut it into pieces that are 0.25 meters long. How many pieces can she cut? (Ch. 7, Lesson 5)

A 1.8 **C** 16

B 8 **D** 18

Extra Practice See page 171, Set D.

Lesson 6

Audio Tutor 1/20 Listen and Understand

Terminating and Repeating Decimals

Objective Express fractions as either terminating or repeating decimals.

Vocabulary
terminating decimal
repeating decimal

Learn About It

In a recent year, the Denver Mint produced $\frac{21}{40}$ of all U. S. coins made that year. The Philadelphia Mint produced $\frac{3}{11}$ of all coins minted. Which mint produced more coins?



To solve the problem, you can compare fractions by changing each fraction to a decimal. To change a fraction to a decimal, divide the numerator by the denominator.

Write $\frac{21}{40}$ as a decimal.

$\frac{21}{40} = 21 \div 40$

$$\begin{array}{r} 0.525 \\ 40\overline{)21.000} \\ -\,20\,0 \\ \hline 1\,00 \\ -80 \\ \hline 200 \\ -200 \\ \hline 0 \end{array}$$

You can end the division when there is a remainder of zero. This decimal is called a **terminating decimal**.

$\frac{21}{40} = 0.525$

Write $\frac{3}{11}$ as a decimal.

$\frac{3}{11} = 3 \div 11$

$$\begin{array}{r} 0.2727\ldots \\ 11\overline{)3.0000\ldots} \\ -\,2\,2 \\ \hline 80 \\ -\,77 \\ \hline 30 \\ -\,22 \\ \hline 80 \\ -\,77 \\ \hline 3 \end{array}$$

The division does not end with a remainder of zero. The digits 2 and 7 repeat in the quotient. This decimal is called a **repeating decimal**.

A bar is usually placed above the repeating digits.

$\frac{3}{11} = 0.\overline{27}$

Solution: Compare. $0.525 > 0.\overline{27}$, so $\frac{21}{40} > \frac{3}{11}$. The Denver Mint produced more coins than the Philadelphia Mint.

Guided Practice

Write each fraction as a decimal. If the decimal is a repeating decimal, use a bar to show repeating digits.

1. $\frac{5}{12}$ **2.** $\frac{4}{6}$ **3.** $\frac{3}{4}$ **4.** $\frac{1}{11}$

5. $\frac{2}{9}$ **6.** $\frac{4}{5}$ **7.** $\frac{10}{90}$ **8.** $\frac{4}{25}$

Ask Yourself

- Did I stop dividing too soon?
- Did I place a bar over all the repeating digits?

Explain Your Thinking Do $0.23\overline{888}$, $0.2\overline{388}$, and $0.23\overline{8}$ name different repeating decimals? Explain.

Practice and Problem Solving

Write each fraction as a decimal. Use a bar as needed.

9. $\frac{3}{5}$ **10.** $\frac{3}{10}$ **11.** $\frac{9}{11}$ **12.** $\frac{5}{6}$ **13.** $\frac{1}{3}$ **14.** $\frac{3}{8}$

15. $\frac{7}{11}$ **16.** $\frac{1}{2}$ **17.** $\frac{2}{3}$ **18.** $\frac{7}{8}$ **19.** $\frac{1}{12}$ **20.** $\frac{1}{8}$

21. $\frac{5}{9}$ **22.** $\frac{1}{6}$ **23.** $\frac{6}{8}$ **24.** $\frac{1}{4}$ **25.** $\frac{5}{8}$ **26.** $\frac{4}{11}$

Solve.

27. The Denver mint is expected to produce about $\frac{11}{19}$ of all the 50-cent coins that will be made in the year 2005. Write this fraction as a decimal, then as a percent.

28. **Reasoning** About $\frac{6}{11}$ of the students visiting the mint bought souvenirs. Write this fraction as a decimal rounded to the nearest hundredth.

29. Suppose the sales tax in a state is 0.075 per dollar spent. Victor wants to buy a coin set that costs $18.95. What is the amount of tax that Victor will have to pay?

30. Magda collects state quarters. Of her quarters, $0.\overline{6}$ were minted in Philadelphia. The others were minted in Denver. If she has 18 quarters in all, how many were minted in Denver?

Mixed Review and Test Prep

Open Response

Find each product or quotient. (Ch. 6, Lessons 2 and 4)

31. $\frac{5}{8} \times \frac{4}{9}$ **32.** $25 \div \frac{5}{6}$

33. $28 \times \frac{3}{4}$ **34.** $6 \div \frac{3}{10}$

35. $\frac{5}{9} \times 27$ **36.** $\frac{2}{3} \div \frac{3}{4}$

37. Predict which denominators between 1 and 20 will most likely result in terminating decimals. Explain your prediction. (Ch. 7, Lesson 6)

Extra Practice See page 171, Set E.

Lesson 7

Problem-Solving Application

Use Decimals

Objective Use decimals to solve problems.

You can use decimals to solve a variety of problems.

Problem Randy orders chimichangas. Becky and CJ each have chicken and rice. All three have quesadillas and baked apples. They decide to divide the bill equally. If CJ has a $5 coupon that can be applied to the total bill, how much will each person pay?



Find the total cost of the meals. Subtract the value of the coupon.

- Multiply the cost of each item by the number of orders.
- Then add to find the total cost.

Chimichangas		Chicken and Rice		Quesadillas		Baked Apples		
$(1 \times \$5.95)$	+	$(2 \times \$6.99)$	+	$(3 \times \$2.25)$	+	$(3 \times \$2.45)$		
$= \$5.95$	+	$\$13.98$	+	$\$6.75$	+	$\$7.35$	$=$	$\$34.03$

Total cost

- Subtract the value of the coupon.

$$\begin{array}{r} \$34.03 \\ -\ 5.00 \\ \hline \$29.03 \end{array}$$

Divide to find each person's share.

Use a calculator. 29.03 ÷ 3 Enter =



Solution: Since you are working with money, round the quotient to the nearest cent. Each person will pay $9.68.

Look Back Does your answer seem reasonable?

Guided Practice

Use the menu on page 166 for Problem 1.

1. Ty, Lauren, and Raj also eat at the restaurant. Ty orders quesadillas, beef fajitas, and flan. Lauren and Raj each have chicken fingers and coleslaw. How much change should they receive if they give the server $30?

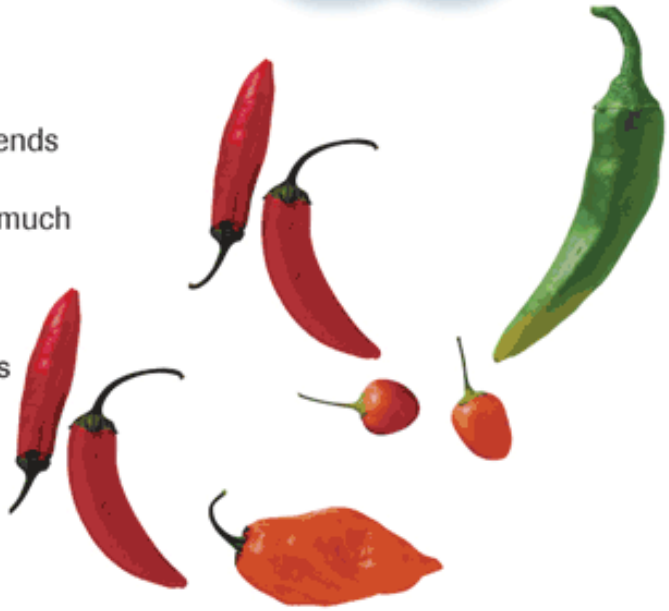
Hint What operation should you use to find out the amount of change?

2. As a special deal, Lauren and two of her friends share one health club membership. The membership costs $66.15 per month. How much does each person save on the cost of membership per month?

Hint This problem is different from the ones you've solved so far because it does not involve multiplication.

Ask Yourself

- What does the question ask me to find?
- What operations will I have to use?
- Does my answer make sense?



Independent Practice

Use order of operations with decimals to solve each problem.

3. At a coin show, Al paid $3.75 for a penny, $9.60 for a nickel, and $5.50 for a dime. How much change should he receive from $20?

4. Cora and Meda split the cost of a sandwich that cost $5.50 and a bottle of spring water that cost $1.98. How much did each person pay?

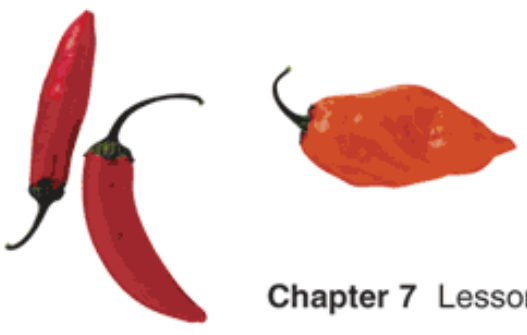
5. Floyd belongs to a mail-order music club. He ordered 3 cassette tapes at $7.99 each and 3 CDs at $10.99 each. The shipping and handling cost was $5.95. What was Floyd's total cost?

6. Over a 4-month period, the amount in the class treasury increased from $70 to $96.40. If the amount of increase was the same each month, how much was in the treasury after the first month?

7. The Miller family paid for their new computer in 4 equal payments. The first 3 payments amounted to $706.50 in all. What was the total cost of the computer?

8. Harry buys 3 shirts at $11.99 each and 2 pairs of pants at $17.99 each. How much change should he receive if he gives the sales clerk four 20-dollar bills?

9. **Create and Solve** Use the menu on page 166 to write a real-life problem involving decimals. The problem should take more than one step to solve. Give your problem to a classmate to solve.



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

10. Here are the amounts that Sue had in her bank account in April, May, June, and July:

$175, $205, $235, $265

If this pattern started in January, how much was in her account in February?

11. While at the gym, Jeanne spent $\frac{1}{4}$ hour in the pool, $\frac{1}{2}$ hour on a treadmill, and $\frac{3}{4}$ hour in the weight room. She left the gym at 7 P.M. At what time did she arrive?

12. Felicia spent $36 on refreshments for a party. If the food cost twice as much as the drinks, how much did Felicia spend on food and on drinks?

13. **Explain** A quarter is 1.75 millimeters thick. If you could make a single stack of 1,000 quarters, what would be the height of the stack? Show your work.

14. A recipe for a batch of cookies requires $\frac{2}{3}$ cup of sugar. If you wanted to triple the recipe, how many cups of sugar would you need?

15. The Nelson family owes a balance of $3,550 on their car. They make monthly payments of $89.50. About how many months will it take the Nelsons to pay off the car?

16. Ron's body temperatures each morning for a week were: 98.8°F, 98.9°F, 99.4°F, 98.6°F, 99.1°F, 98.7°F, and 99.5°F. What was Ron's average morning body temperature that week?

17. A store sold 15 CD players for a total of $1,477.50 plus tax. The total tax was $88.65. What was the price of each CD player, including tax?

18. On Friday, Delia worked for $2\frac{1}{2}$ hours and Ed for $1\frac{1}{2}$ hours. On Saturday, Delia worked for $1\frac{3}{4}$ hours and Ed for 2 hours. Who worked longer and by how much?

You Choose

Strategy

- Find a Pattern
- Make a Table
- Make an Organized List
- Use Logical Reasoning
- Work Backward

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Real World Connection
Buying Multiple Items

Activity

Cat food is on sale at 3 cans for $1.00. How much would 4 cans of cat food cost?

The exact price for each can is $33\frac{1}{3}$ cents.
The receipt shows how the market computed the price for 4 cans. It rounded the price of the first can up to the next cent, $0.34.



Julie's Market
Customer Receipt Copy
PET FOOD

1 FANCY TURKEY	
1 @ 3/$1.00	.34
1 FANCY FISH	
1 @ 3/$1.00	.33
1 FANCY BEEF	
1 @ 3/$1.00	.33
1 FANCY CHICKEN	
1 @ 3/$1.00	.34
PET FOOD SUBTOTAL:	**$1.34**

1. Use the market's method to find the cost of 5 cans of cat food.

2. The regular price per can of cat food is 42 cents. How much money is saved if you purchase 8 cans at the sale price?

3. Price Saver Market advertises the same cat food in their flyer. The price listed is 4 cans for a dollar. How much money would you save on the cans if you bought 6 cans at Price Saver Market?

4. Rolls are on sale at 6 for $0.99. How might the market charge you for 2 rolls? 3 rolls? 4 rolls? 8 rolls?

5. If you also bought 6 rolls at the sale price listed in Problem 4 and 6 cans of cat food at the sale price, how much would you pay?

6. Get some cash register receipts. Check how the store computed the prices for multiple items. How did they calculate the price per item?

Chapter Review/Test

VOCABULARY

1. A(n) _____ is a decimal quotient that contains a repeating block of digits.
2. A(n) _____ can be written by using exponents.
3. A(n) _____ is a decimal quotient with a remainder of zero.

Vocabulary

decimal
power of 10
terminating decimal
repeating decimal

CONCEPTS AND SKILLS

Find each product. Estimate first. (Lesson 2, pp. 152–154)

4. $\$8.13 \times 42$ **5.** 13.6×715.6 **6.** 51.7×4.32 **7.** 8.5×3.8 **8.** 18.4×0.7

Find each product or quotient, using mental math. (Lesson 3, pp. 156–158)

9. $8{,}490 \div 10^3$ **10.** $74{,}300 \times 10^{-1}$ **11.** $67.8 \div 10^2$ **12.** 49.17×10^0

Divide. Then check your work. (Lessons 4–5, pp. 160–163)

13. $1.4\overline{)4.2}$ **14.** $6\overline{)0.81}$ **15.** $2.4\overline{)6.024}$ **16.** $3.4\overline{)\$640.22}$ **17.** $12\overline{)146.4}$

18. $41.7 \div 15$ **19.** $55.44 \div 1.2$ **20.** $41.08 \div 5.2$ **21.** $0.195 \div 0.15$

Write each fraction as a decimal. Use a bar as needed. (Lesson 6, pp. 164–165)

22. $\frac{5}{11}$ **23.** $\frac{9}{20}$ **24.** $\frac{2}{11}$ **25.** $\frac{11}{12}$ **26.** $\frac{7}{9}$

Write each fraction as a decimal, then as a percent. (Lesson 6, pp. 164–165)

27. $\frac{3}{5}$ **28.** $\frac{6}{25}$ **29.** $\frac{8}{20}$ **30.** $\frac{5}{50}$ **31.** $\frac{7}{10}$

PROBLEM SOLVING

Solve. (Lesson 7, pp. 166–168)

32. A relay race is 3.2 miles long. There are 4 runners on each team. If each runner runs $\frac{1}{4}$ of the course, how many miles will each person run?

33. The tennis team is having 8 tennis rackets restrung. The total cost is \$319.60. What is the average cost per racket for restringing?

Write About It

Show You Understand

Which way does the decimal point move when you are dividing by a power of 10? Explain.

Extra Practice

Set A (Lesson 2, pp. 152–154)

Find each product. Estimate first.

1. 1.9×6 **2.** $\$3.20 \times 91$ **3.** 62.4×0.3 **4.** 654.2×7.55

5. $\begin{array}{r} 12.5 \\ \times\ 2.7 \\ \hline \end{array}$ **6.** $\begin{array}{r} \$6.03 \\ \times\ 0.41 \\ \hline \end{array}$ **7.** $\begin{array}{r} 7.1 \\ \times\ 0.254 \\ \hline \end{array}$ **8.** $\begin{array}{r} 7.82 \\ \times\ 0.21 \\ \hline \end{array}$

Set B (Lesson 3, pp. 156–158)

Use patterns to help you find each product or quotient.

1. $6.7134 \times 10^1 = 67.134$
$6.7134 \times 10^2 = \blacksquare$
$6.7134 \times 10^3 = \blacksquare$

2. $93 \times 10^{-1} = 9.3$
$93 \times 10^{-2} = \blacksquare$
$93 \times 10^{-3} = \blacksquare$

3. $25.83 \div 10^1 = 2.583$
$25.83 \div 10^2 = \blacksquare$
$25.83 \div 10^3 = \blacksquare$

Find each product or quotient, using mental math.

4. $3{,}600 \div 10^2$ **5.** $42{,}900 \times 10^{-3}$ **6.** $87.5 \div 10^0$ **7.** 1.004×10^2

Set C (Lesson 4, pp. 160–161)

Divide. Then check your work.

1. $6\overline{)2.4}$ **2.** $7\overline{)9.66}$ **3.** $8\overline{)32.24}$ **4.** $3\overline{)\$21.60}$

5. $215.82 \div 33$ **6.** $0.46 \div 4$ **7.** $923.3 \div 25$ **8.** $15{,}225 \div 42$

Set D (Lesson 5, pp.162–163)

Divide. Then check your work.

1. $0.5\overline{)2.05}$ **2.** $1.2\overline{)24.48}$ **3.** $3.2\overline{)77.28}$ **4.** $1.8\overline{)806.94}$

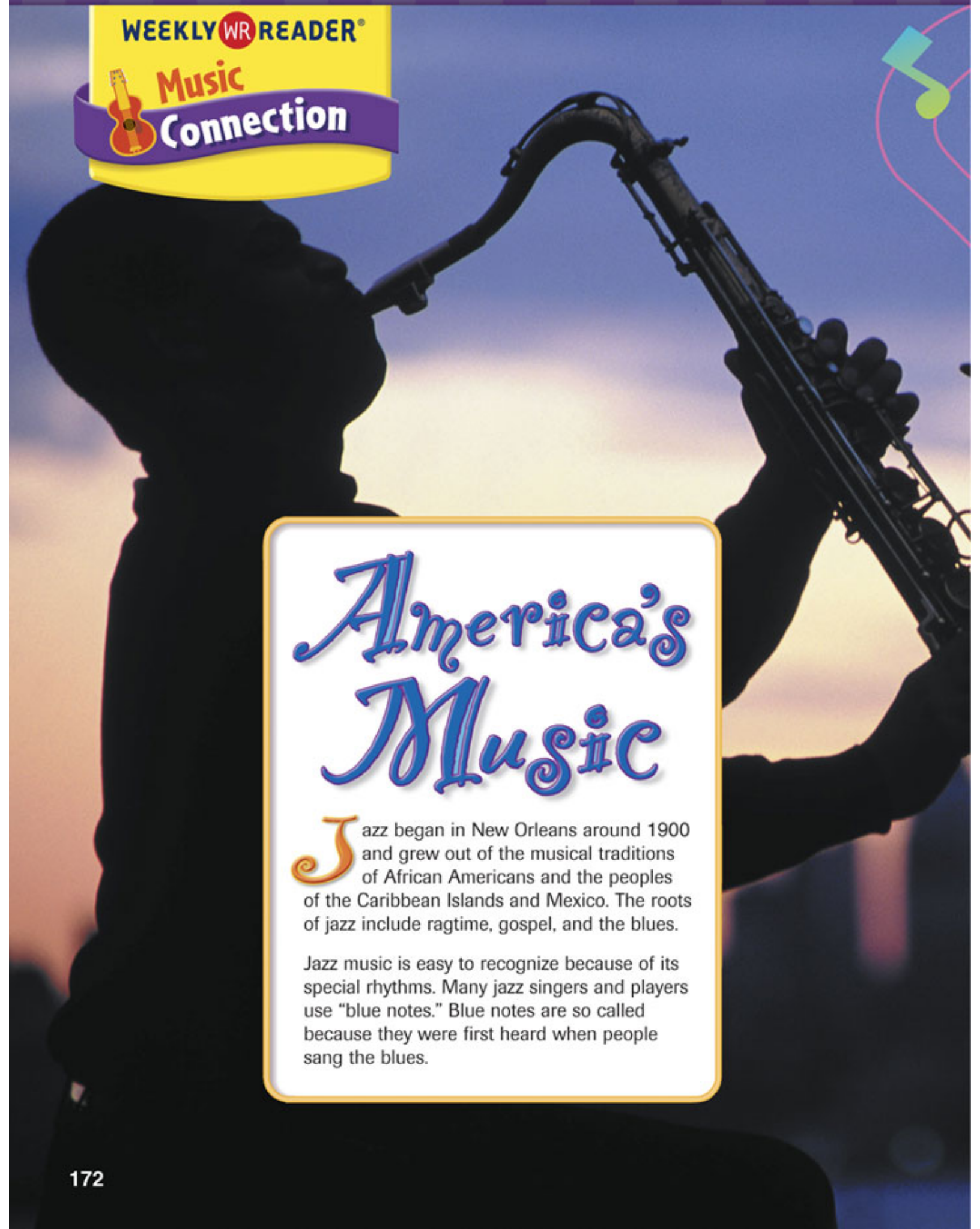
5. $931.3 \div 6.7$ **6.** $7.844 \div 0.74$ **7.** $2.5688 \div 1.3$ **8.** $1.960 \div 0.32$

Set E (Lesson 6, pp. 164–165)

Write each fraction as a decimal. Use a bar as needed.

1. $\frac{7}{8}$ **2.** $\frac{7}{9}$ **3.** $\frac{2}{5}$ **4.** $\frac{1}{7}$ **5.** $\frac{8}{9}$

6. $\frac{17}{20}$ **7.** $\frac{18}{25}$ **8.** $\frac{1}{11}$ **9.** $\frac{1}{25}$ **10.** $\frac{5}{16}$



America's Music

Jazz began in New Orleans around 1900 and grew out of the musical traditions of African Americans and the peoples of the Caribbean Islands and Mexico. The roots of jazz include ragtime, gospel, and the blues.

Jazz music is easy to recognize because of its special rhythms. Many jazz singers and players use "blue notes." Blue notes are so called because they were first heard when people sang the blues.

Problem Solving

Use the information at right to solve Problems 1–5.

1. Express each of the notes in the table as a fraction and as a decimal. Order them from least to greatest.

2. The first line of a jazz song has no whole notes. It has one half note and 6 eighth notes. The rest are quarter notes. How many quarter notes are there, if the first line is equivalent to 4 whole notes?

3. How many quarter notes have the same value as a dotted half note?

4. How many eighth notes have the same value as a dotted quarter note?

5. Write a division sentence that shows how many dotted quarter notes there would be in 6 whole notes.

6. Lakeisha's solo in a concert is 15 measures long. There are no blue notes in 13 of the measures. Write a decimal to show the part of her solo that contains blue notes.

The length of musical notes are given in fractions. To show the lengths of musical notes, different symbols are used.

Name	Note
Whole Note	𝅝
Half Note	𝅗𝅥
Quarter Note	♩
Eighth Note	♪
Dotted Whole Note	𝅝. = 𝅝 + 𝅗𝅥

A dot to the right of a note gives it a value $1\frac{1}{2}$ times its normal value.

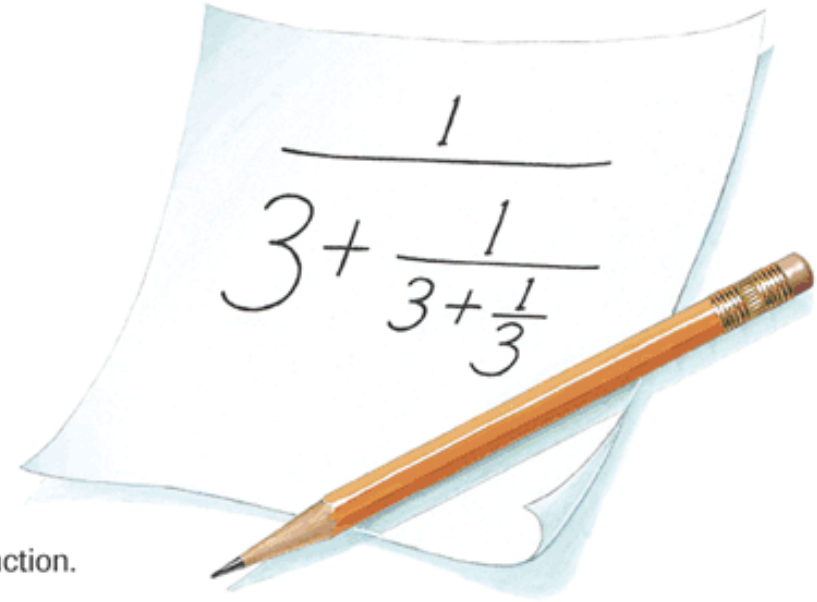


Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic.

Enrichment: Number Sense

CONTINUED FRACTIONS



A fraction that has another fraction in the denominator is called a **continued fraction**.

You can use what you know about operations with fractions to help you simplify a continued fraction.

Start from the bottom and work backward.

$3 + \frac{1}{3} = \frac{10}{3}$, so $\dfrac{1}{3 + \dfrac{1}{3 + \frac{1}{3}}} = \dfrac{1}{3 + \frac{1}{\frac{10}{3}}}$

But $\dfrac{1}{\frac{10}{3}} = \dfrac{3}{10}$, so $\dfrac{1}{3 + \frac{1}{\frac{10}{3}}} = \dfrac{1}{3 + \frac{3}{10}} = \dfrac{1}{3\frac{3}{10}} = \dfrac{1}{\frac{33}{10}}$

Since $\dfrac{1}{\frac{33}{10}} = \frac{10}{33}$, the continued fraction simplifies to $\frac{10}{33}$.

Try These!

Simplify each continued fraction.

1. $\dfrac{1}{2 + \frac{1}{2 + \frac{1}{2}}}$
2. $\dfrac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{4}}}}$
3. $5 + \dfrac{1}{1 + \frac{1}{5}}$
4. $2 + \dfrac{1}{2 + \frac{1}{2 + \frac{1}{2}}}$

5. **Challenge** You can write any fraction as a continued fraction by using the opposite steps you used to simplify a continued fraction. For example,

$$\frac{7}{16} = \frac{1}{\frac{16}{7}} = \frac{1}{2 + \frac{2}{7}} = \frac{1}{2 + \frac{1}{\frac{7}{2}}} = \frac{1}{2 + \frac{1}{3 + \frac{1}{2}}}$$

Use the example to help you write $\frac{8}{11}$ as a continued fraction.

Technology Time

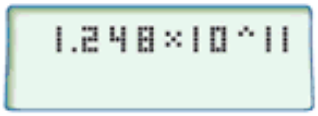
Mighty and Tiny



You may have noticed that your calculator cannot display very large or very small numbers in the usual manner.

Press:





Your calculator uses scientific notation to shorten very large or very small numbers. Scientific notation is a way of writing numbers as a product of a number between 1 and 10 and a power of 10.

Numbers in scientific notation can be written in standard form.

Positive Exponents	Negative Exponents
Write 1.248×10^5 in standard form.	**Write 1.248×10^{-5} in standard form.**
• Raise 10 to the power of 5. $1.248 \times 10 \times 10 \times 10 \times 10 \times 10$	• Raise 10 to the power of $^-5$. $1.248 \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$ $1.248 \times \frac{1}{100{,}000}$
• Multiply. $1.248 \times 100{,}000 = 124{,}800$	• Multiply. $1.248 \times 0.00001 = 0.00001248$

Use your calculator to solve. To answer the riddle, write the letter found next to each problem above the exponent found in that problem's answer.

1. $8{,}989{,}898{,}989 \times 89$	R	**2.** $0.0000010689 \div 999{,}986$	W
3. $0.000010056 \div 498{,}623$	G	**4.** $987{,}654{,}321 \times 4{,}321$	V
5. $525{,}852{,}585{,}258 \times 5{,}258$	E	**6.** $0.0000010302 \div 998{,}986{,}591$	O
7. $0.0000002 \div 898{,}763$	S	**8.** $789{,}987{,}789{,}987 \times 789$	P
9. $0.0000000089 \div 99{,}999$	I	**10.** $159{,}753{,}159{,}753 \times 159$	!

Riddle: How are calculators and exponents alike?

THEY BOTH ____ ____ ____ ____ YOU ____ ____ ____ ____ ____ ____ ____

$^-11$ $^-14$ 12 15 14 $^-15$ $^-12$ 15 11 $^-13$ 13

Unit 2 Test

VOCABULARY Open Response

1. To ____ a fraction means to switch the numerator and the denominator.
2. When you divide the numerator of a fraction by the denominator and have a remainder of zero, the decimal is a(n) ____.
3. Another name for the reciprocal of a fraction is ____.
4. A(n) ____ is a quotient with a sequence of digits that appear over and over.

Vocabulary

- invert
- reciprocal
- terminating decimal
- multiplicative inverse
- repeating decimal

CONCEPTS AND SKILLS Open Response

Compare. Write in order from least to greatest. (Chapter 4)

5. 124.124 $\quad \frac{400}{3} \quad$ 1241.24 $\quad 124\frac{3}{5}$

Write each decimal as a percent. (Chapter 4)

6. 0.3 **7.** 0.25 **8.** 0.7

Estimate. Then add or subtract. Write your answers in simplest form. (Chapter 5)

9. $2\frac{4}{5} + \frac{2}{5}$ **10.** $3\frac{1}{8} - 1\frac{3}{8}$ **11.** $\frac{1}{2} + \frac{1}{6}$

12. $4\frac{3}{4} + 2\frac{2}{3}$ **13.** $9 - 6\frac{3}{7}$ **14.** $\frac{3}{8} - \frac{1}{6}$

Estimate. Then multiply or divide. Write your answers in simplest form. (Chapter 6)

15. $\frac{1}{3} \times \frac{2}{5}$ **16.** $5 \div \frac{5}{8}$ **17.** $2\frac{1}{4} \times \frac{5}{6}$

18. $7\frac{4}{5} \div 2\frac{1}{2}$ **19.** $2 \times 3\frac{1}{2}$ **20.** $4\frac{2}{3} \div 6$

Evaluate each expression, given $a = \frac{1}{3}$, $b = \frac{1}{2}$, and $c = 1\frac{1}{2}$. Identify any properties you used. (Chapter 6)

21. $a + 2\frac{1}{3} - c$ **22.** $10 - 2\frac{3}{4} + (b \times c)$ **23.** $\frac{3}{4} \times (a + 4\frac{2}{3})$

Estimate. Then multiply or divide. (Chapter 7)

24. 8.02×59

25. $4{,}352 \times 0.04$

26. $8.6 \div 10^1$

27. $3.4\overline{)8.84}$

28. $2.40\overline{)\$32.16}$

29. $5{,}000.5 \div 10^3$

PROBLEM SOLVING Open Response

Solve.

30. On average, a sea turtle lays 10^2 eggs during one visit to the beach. If 32 turtles visited the beach to lay eggs each week for 1.5 months, about how many eggs did they lay in all?

31. Enrique's hamster cage is $3\frac{1}{3}$ feet long, his mouse cage is $2\frac{1}{2}$ feet long, and his rabbit cage is $4\frac{3}{4}$ feet long. How much longer is the longest cage than each of the other two?

32. Denny studied magic tricks for 21 months before he performed in his first magic show. Use a decimal to express in years the length of time he studied.

33. It costs Amy \$0.42 cents per cookie to make a batch of 108 cookies. If she sells a bag of 6 cookies for \$3.75 each, what is Amy's profit per batch?

Performance Assessment

Constructed Response

Task You are in charge of buying muffins for a Tuesday morning class meeting. Use the information on the sign.

You want to buy enough muffins for your class so that every student and your teacher each get one. Which special should you buy if you want to spend the least amount of money? Explain your thinking.



Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

When reading word problems on a test, circle important words to help you remember what you are being asked to find.

A pitcher holds 10 cups of lemonade. Jasmine must put exactly $1\frac{1}{2}$ cups of lemonade in each glass. How much lemonade is left over?

A 1 cup

B $2\frac{1}{2}$ cups

C $6\frac{2}{3}$ cups

D 9 cups

THINK

Circle the words "left over." Then find out how many whole glasses she filled.

$10 \div 1\frac{1}{2} = 6\frac{2}{3}$ glasses

She filled 6 glasses.

6 glasses $\times$ $1\frac{1}{2}$ cups = 9 cups

Jasmine used 9 cups of lemonade and has 1 cup left.

Multiple Choice

1. Marissa places 52 cards into several stacks. One stack has 8 cards in it. She divides the rest of the cards equally so that each stack has an odd number of cards in it. How many stacks are there?

A 2

B 5

C 8

D 10

(Chapter 1, Lesson 7)

2. Monarch butterflies migrate a distance of 2,000 miles. They fly about 80 miles a day. How many days does it take the butterflies to fly half the distance?

F 12.5

G 18.5

H 14

J 25

(Chapter 2, Lesson 4)

3. Rosa makes trail mix using $2\frac{1}{3}$ cups of peanuts and $1\frac{1}{2}$ cups of raisins. She eats $\frac{1}{4}$ cup of the trail mix. How many cups are left?

A $3\frac{1}{2}$

B $3\frac{7}{12}$

C $3\frac{3}{4}$

D $3\frac{9}{10}$

(Chapter 5, Lesson 4)

4. Kendra bought a box of greeting cards for $12.25. Each card cost $0.49. How many cards are in 4 boxes?

F 25

G 50

H 75

J 100

(Chapter 7, Lesson 5)

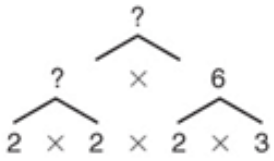
For Test-Taking Tips, See page 648.

Open Response

5. A new stadium ballpark will cost the state of Minnesota $330 million. Write this amount in standard form.

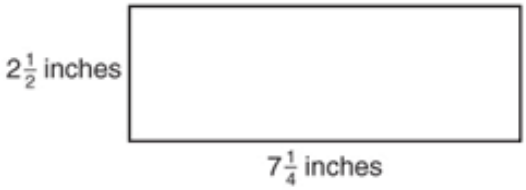
(Chapter 1, Lesson 1)

6. For what number does this factor tree show the prime factorization?



(Chapter 3, Lesson 3)

7. Find the perimeter of the rectangle.



(Chapter 5, Lesson 3)

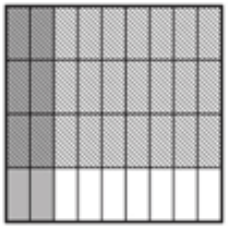
8. It takes the planet Venus about $7\frac{1}{2}$ Earth months to complete one orbit about the Sun. How many Earth days is that? (Use 1 month = 30 days.)

(Chapter 6, Lesson 3)

Extended Response

9. The model shows a way to find the product of two fractions.

$$\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \frac{1}{6}$$



A Use the model to find the product of $\frac{3}{4}$ and $\frac{7}{9}$.

B Draw a different model to find the product of $\frac{3}{8}$ and $\frac{2}{3}$.

(Chapter 6, Lesson 2)

10. You can use addition properties to find the sum of these addends in different ways.

$$\begin{array}{r} 275 \\ 116 \\ 3.57 \\ 0 \\ +\ 225 \\ \hline \end{array}$$

A Find the sum, using the commutative, the associative, and the identity properties.

B Describe each property and explain how you used it to find the sum.

(Chapter 1, Lesson 8)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 2

Look back at the big ideas and vocabulary in this unit.

Big Ideas

A number line can help you compare and order fractions, mixed numbers, and decimals.

Addition and multiplication properties can help you evaluate expressions.

You can use patterns and mental math to multiply or divide by powers of ten.

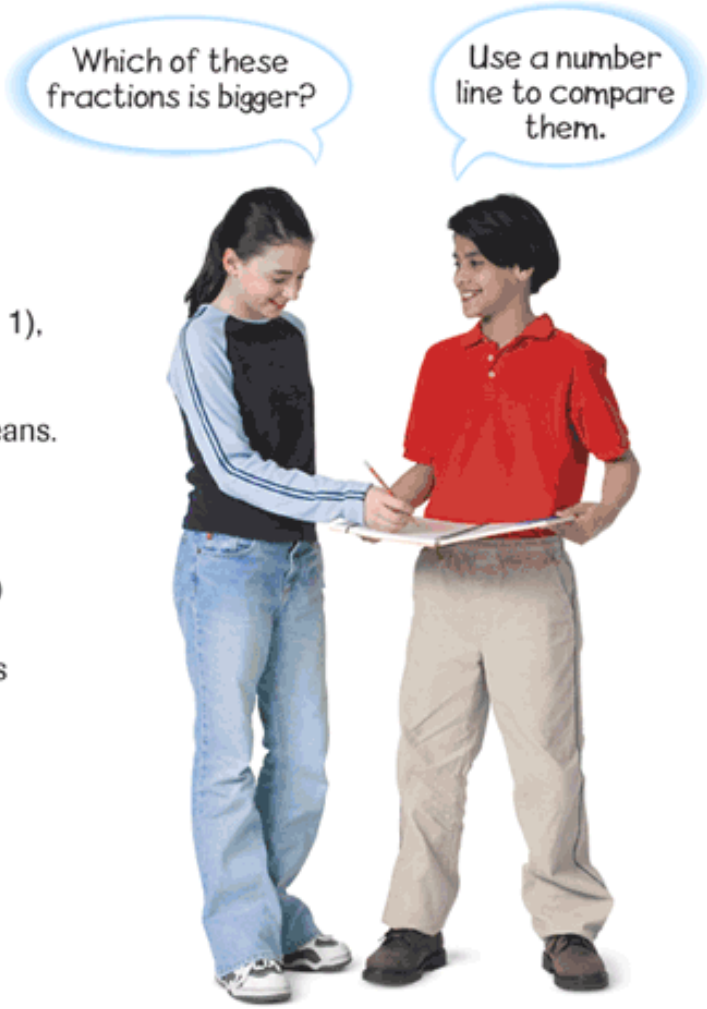
Key Vocabulary

property
evaluate
expression

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain which property is shown by the following number sentence.
   $3\frac{1}{5} + 4 = 4 + \frac{16}{5}$
2. Explain how to evaluate $14 + n \times (m - 1)$, given $m = 3\frac{1}{2}$ and $n = \frac{1}{3}$.
3. Explain how to read $0.\overline{69}$ and what it means.
4. Explain how you can use mental math to divide 27,032 by 10^5.
5. **Write About It** Suppose you have $100 to decorate a room. Look for flyers from home-decorating stores. Find out what is on sale. Calculate how much you would spend on paint, wood, fabric, and so on. Will you have money left for posters?



UNIT 3

Measurement/ Data and Graphing

CHAPTER 8

Customary and Metric Units of Measure

page 184



CHAPTER 9

Collect and Analyze Data

page 208



CHAPTER 10

Graph Data

page 232



Reading Mathematics

Reviewing Vocabulary



Here are some math vocabulary words that you should know.

mass	the amount of matter in an object
capacity	the amount a container can hold
square unit	a unit used to measure area
data	a set of numbers or pieces of information
line graph	a graph that uses a broken line to show changes in data
bar graph	a graph that uses rectangular bars to show information

Reading Words and Symbols

When you read mathematics, sometimes you read only words, sometimes you read words and symbols, and sometimes you read only symbols.

Units of measure are often abbreviated, or shortened, to save space.

Examples: 7 *ft* 4 *in.* is read as *seven feet four inches.*
4 *m* is read as *four meters.*

Write each, using words.

1. 1 yd 3 ft **2.** 7 lb 11 oz **3.** 1 gal 3 qt

Write each, using numbers and abbreviated units.

4. three liters **5.** forty-six kilometers **6.** one hundred nine grams

Reading Test Questions

Choose the correct answer for each.

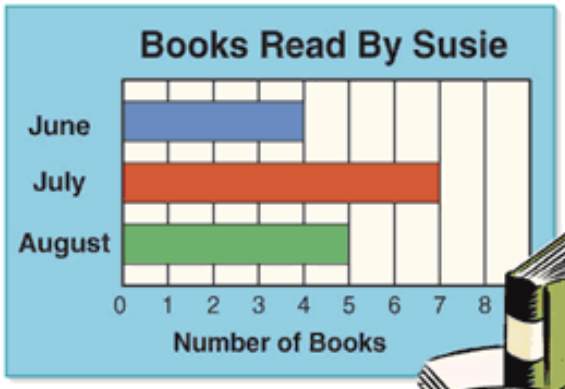
7. What conclusion can you draw from the bar graph?

a. Susie found it too hot to read in August.

b. Susie reads books only in the summer.

c. Susie read 16 books last summer.

d. Susie read fewer books in July than in June.



A **conclusion** is a summary you reach by analyzing information.

8. Which tool would you use to measure an amount of milk?

a. scale **c.** clock

b. yardstick **d.** measuring cup

A **tool** is an instrument used for doing some work, such as taking measurements.

9. Which is a reasonable unit of measure for the width of a house?

a. liter **c.** grams

b. gallons **d.** feet

Reasonable means "makes sense."

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

- sample
- cluster
- outlier
- histogram
- frequency table
- box-and-whisker plot
- measures of central tendency

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "Skateboard and Scooter Alert!" on page 638. Then work with a partner to answer the questions about the story.

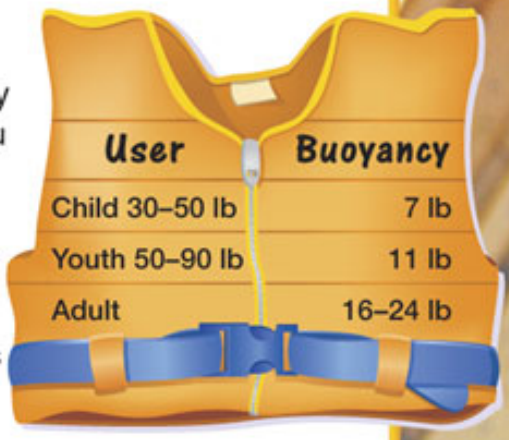
CHAPTER 8

Customary and Metric Units of Measure

INVESTIGATION

Using Data

As in any water sports, it is absolutely essential to wear a life vest when you go canoeing. Life vests provide "lift," or buoyancy to help you stay afloat. The buoyancy of water reduces body weight by about 90%.



Five people together weighing 540 lbs go on a canoe trip. What are some possible combinations of children, youths, and adults wearing life vests on the trip? Can you estimate the total buoyancy of these life vests? Explain.

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

Vocabulary

area
length
weight
capacity

1. A milliliter is a unit of measure of _____.
2. _____ is measured in square units.
3. An ounce is a unit of measure of _____.
4. You can use a yardstick to measure _____.

CONCEPTS AND SKILLS

Match each item at left with the appropriate unit at right.

5. the weight of a large dog	gallon
6. the length of a paper clip	mile
7. the height of a person	pound
8. the amount of water in a fish tank	meter
9. the distance across your town	ounce
10. the weight of an apple	centimeter

Copy and complete.

11. 5 yd = ■ ft **12.** 36 in. = ■ ft **13.** 6 lb = ■ oz

14. 2 qt = ■ pt **15.** 1 km = ■ m **16.** 4,000 lb = ■ T

17. 16 c = ■ gal **18.** 50 dL = ■ L **19.** 3,000 g = ■ kg

Write About It

20. Can you draw two squares of different sizes so that each has a perimeter of 16 inches? Explain.

Facts Practice, See Page 655.

Hands On
Lesson 1

Measurement Concepts

Objective Estimate and then use nonstandard units to measure length, weight, and capacity.

Materials
paper clips
metal or wood cubes
a balance
big and small glasses
3 different-sized containers
fishbowl

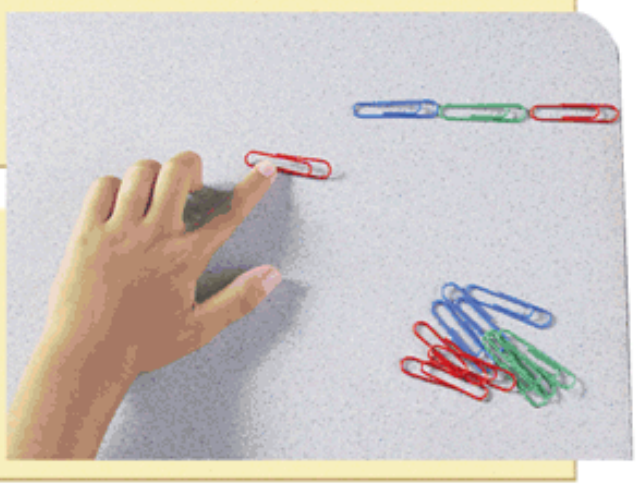
Work Together

When you measure an object, you are comparing the measure of the object to the unit of measure chosen. Your unit of measure can be nonstandard or standard.

Find the length of your desk.

STEP 1 Estimate how many paper clips would fit across the length of your desk.

- What is your estimate?



STEP 2 Use paper clips to measure the length of the desk.

- What is the length of your desk?
- How does your estimate compare with your measure?

STEP 3 Now measure your desk using the long edge of your math book.

- Do you need more paper clips or more books to measure the length of your desk? Explain.

Find the weight of one of your shoes.

STEP 1 Place one of your shoes in one pan of the balance. Estimate how many cubes you need to place in the other pan to make both pans balance.

- What is your estimate?

STEP 2 Place cubes in the other pan of the balance until both pans balance.

- What does your shoe weigh?
- If you used cubes of two different weights, would your shoe weigh the same? Explain.



Find the capacity of the fishbowl.

STEP 1 Estimate how many small glasses of water the fishbowl can hold. Use the smaller glass to fill the fishbowl.

- How many small glasses does the fishbowl hold?

STEP 2 Estimate how many big glasses of water the fishbowl can hold. Use the bigger glass to fill it.

- How many big glasses does the fishbowl hold?



You can use familiar objects to help you estimate a measurement. For example, the doorway to a house is about 1 yard or 1 meter wide; a large paper clip has a width of about 1 centimeter; 5 grapes weigh about 1 ounce; a quart container of juice holds about 1 liter.

On Your Own

1. Estimate and then measure with paper clips the width of the door to your classroom.
2. Estimate and then measure with paper clips the width of a bookcase in your classroom.
3. Estimate how many cubes it will take to balance one of your books. Then use cubes to measure the weight of the book.
4. Estimate how many cubes it will take to balance a big glass. Then use cubes to measure the weight of the glass.
5. Estimate and then measure to find how many small glasses of water the medium-sized container can hold.
6. Estimate and then measure to find how many big glasses of water the biggest container can hold.

Talk About It • Write About It

7. Why do you think standard units of measure, such as the inch, were invented?
8. Name some standard units of length, weight, and capacity that you know.
9. Why is it better to use a large measurement unit to measure large items and a small measurement unit to measure small items?

Hands On Lesson 2

Customary Units of Measure

Objective Change between customary units of length, capacity, and weight.

Vocabulary
precision
accuracy

Materials
a yardstick



Learn About It

The foot, pound, and quart are units of length, weight, and capacity in the customary system of measure. Other units are shown in the table below.

The **precision** of a measurement depends on the units of measure you use. The smaller the unit of measure you use, the greater the precision. The **accuracy** of a measurement is its closeness to the true measure.

Try this activity to estimate and measure the length of your classroom.

STEP 1 Estimate the length of your classroom in yards. Then use a yardstick to measure the length of your classroom to the nearest yard.

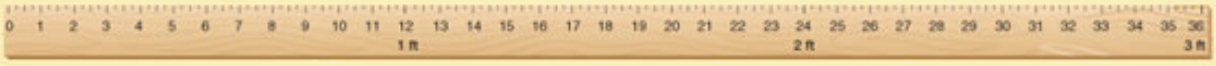
- Which is more accurate, the estimated length or the measured length? Explain.

STEP 2 Measure the length of the classroom to the nearest foot, then to the nearest inch.

- Which measurement—nearest yard, nearest foot, or nearest inch—is the most precise? Why?

Customary Units of Measure

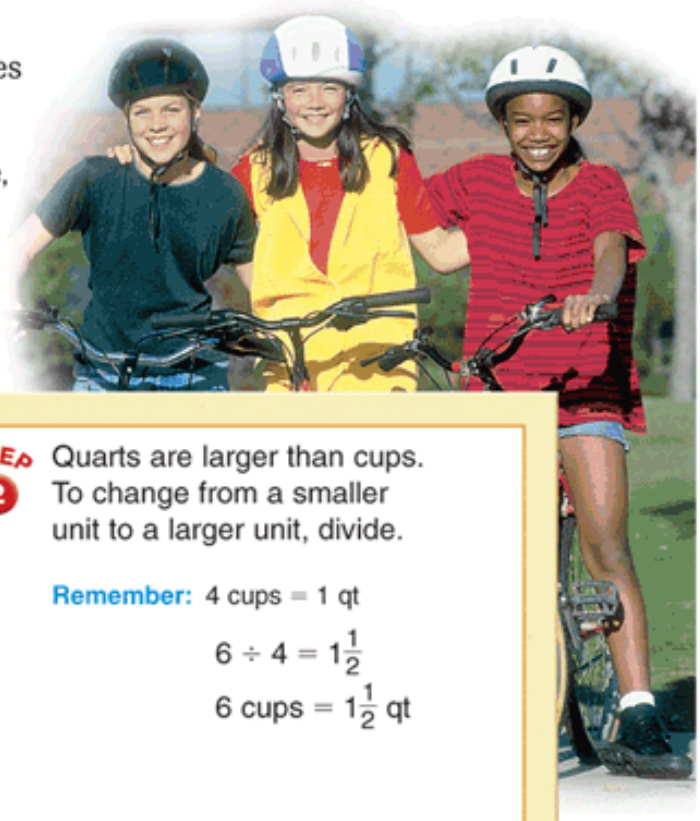
Length	
1 foot (ft)	= 12 inches (in.)
1 yard (yd)	= 3 feet (ft)
1 mile (mi)	= 5,280 feet (ft)
Capacity	
1 cup (c)	= 8 fluid ounces (fl oz)
1 pint (pt)	= 2 cups (c)
1 pint (pt)	= 16 fluid ounces (fl oz)
1 quart (qt)	= 2 pints (pt)
1 gallon (gal)	= 4 quarts (qt)
Weight	
1 pound (lb)	= 16 ounces (oz)
1 ton (T)	= 2,000 pounds (lb)



A yardstick can be used to measure in yards, in feet, or in inches.

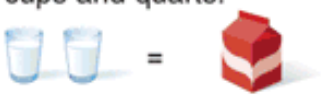
You can use the information in the table of measures to change from one unit to another.

The Bicycle Club is planning an all-day trip. Mr. Lee, the club's advisor, recommends that each student should carry about 6 cups of water. How many quarts of water does each student need?



Calculate. 6 cups = ■ qt

STEP 1 Write the equality that relates cups and quarts.



Remember: 2 cups = 1 pt



Remember: 1 qt = 2 pt = 4 c

STEP 2 Quarts are larger than cups. To change from a smaller unit to a larger unit, divide.

Remember: 4 cups = 1 qt

$6 \div 4 = 1\frac{1}{2}$

6 cups = $1\frac{1}{2}$ qt

Solution: Each student needs about $1\frac{1}{2}$ quarts of water.

Other Examples

A. Change Weights

To change from a larger unit to a smaller unit, multiply.

$1\frac{3}{4}$ tons = ■ pounds

Remember: 1 T = 2,000 lb

$1\frac{3}{4} \times 2{,}000 = 3{,}500$

$1\frac{3}{4}$ T = 3,500 lb

B. Change Lengths

Sometimes when you divide, you need to write the remainder, using the smaller unit.

42 inches = ■ feet ■ inches

Remember: 12 in. = 1 ft

$42 \div 12 = 3$ R6

42 in. = 3 ft 6 in.

Guided Practice

Ask Yourself
- Which equality should I use?
- Am I changing to a larger unit or a smaller unit?

Determine whether there will be more or fewer of the new units. Then find the equivalent measure.

1. 5 yd = ■ ft **2.** 8 c = ■ pt **3.** 40 oz = ■ lb ■ oz

4. 5 qt = ■ pt **5.** 48 in. = ■ ft **6.** $2\frac{1}{2}$ T = ■ lb

Which measurement is more precise?

7. 2 ft or 24 in. **8.** 8 qt or 2 gal **9.** 3 T or 6,000 lb

Explain Your Thinking How do you know whether to multiply or divide when changing from one unit to another?

Go On

Practice and Problem Solving

Determine whether there will be more or fewer of the new units. Then find the equivalent measure.

10. 5 c = ■ fl oz

11. $4\frac{1}{2}$ ft = ■ in.

12. 2 qt = ■ gal

13. 3 pt = ■ qt

14. 8 lb = ■ oz

15. $18\frac{1}{3}$ yd = ■ ft

16. 40 in. = ■ yd ■ in.

17. 6 pt = ■ gal

18. 52 oz = ■ lb ■ oz

19. 3,520 yd = ■ ft

20. $5\frac{1}{4}$ mi = ■ yd

21. 15,840 ft = ■ mi

22. 3,000 lb = ■ T

23. $\frac{3}{4}$ yd = ■ in.

24. $\frac{3}{4}$ c = ■ fl oz

Choose the most reasonable measure for each.
Write *in., ft, yd, mi, oz, lb, T, fl oz, c, pt, qt,* or *gal.*

25. a glass of milk

26. the distance to another state

27. the water in a fish tank

28. your body weight

29. your height

30. a handful of flour

31. a truckload of bricks

32. a pitcher of juice

33. the height of a building

Which measurement is more precise?

34. 48 fl oz or 1.5 qt

35. 2 T or 4,000 lb

36. 4 yd or 3 yd 2 ft 10 in.

Solve.

37. Before the ride, Geoff drank two 8-fluid-ounce glasses of water. After the ride, he drank two more glasses of water. If Geoff drank 48 fluid ounces of water during the ride, how many cups of water did he drink the day of the ride?

38. **Estimate** Tina made $5\frac{1}{4}$ pounds of trail mix for her friends who were going on the ride starting at 11:00 A.M. the next day. She divided the mix into 10-ounce servings. About how many servings did Tina make?

39. **Explain** During a hike, Imelda walked an average of 3 miles per hour. About how many yards per minute did she walk? Explain how you found your answer.

40. **Create and Solve** Write a word problem that includes customary length, weight, or capacity measures. Your problem should require at least two steps to solve. Give it to a classmate to solve.



41. **Explain** How would you find the number of inches in one million miles?

42. **You Decide** To measure the length of small objects, you can use fractional parts of an inch like half inch, quarter inch, eighth inch, and sixteenth inch. Measure some small objects in your classroom. How accurate are your measurements? How do you know?



Algebra • Expressions

Write a multiplication or division expression to complete each sentence.

43. If there are 12 inches in 1 foot, there are __?__ inches in *f* feet.

44. If there are 36 inches in 1 yard, there are __?__ yards in *i* inches.

45. There are __?__ pounds in *o* ounces.

46. There are __?__ tons in *p* pounds.

47. There are __?__ pints in *c* cups.

48. There are __?__ quarts in *g* gallons.

Use the recipe for Problems 49–52.

49. You are making Spanish gazpacho soup for a club lunch. The recipe calls for 4 pints of tomato juice, but your measuring cup only holds one cup. How many cups of tomato juice do you need?

50. How many fluid ounces of tomato juice are in the gazpacho recipe?

51. About how many gallons will the gazpacho recipe make?

52. **Measurement** Suppose you wanted to make a gazpacho recipe that served 96 people. How many quarts of tomato juice would you need? How many ounces of cold water would you need?



Mixed Review and Test Prep

Open Response

Order the money amounts from least to greatest. (Grade 4)

53. $3.50, $0.35, $35, $3.05

54. $2.46, $2.64, $4.62, $4.26

55. $9, $8.88, $9.89, $9.98

56. You need to pour all the liquids from the gazpacho recipe above into a large pot. What does the capacity of the pot need to be in quarts?

Explain how you got your answer. (Ch. 8, Lesson 2)

Extra Practice See page 207, Set A.

Lesson 3

Compute With Measures

Objective Compute with measures that involve different units.



Tents	50 lb 2 oz
Sleeping bags	9 lb 11 oz
Cooking gear	32 lb 8 oz
Extra clothing	74 lb 6 oz
Lanterns	21 lb 4 oz

Learn About It

Sometimes you need to add and subtract measures to solve problems.

Mary Jo and her friends are going on a camping trip. The chart shows the weights of items to be loaded on a truck. How much do the cooking gear, extra clothing, and sleeping bags weigh in all?

Add. 32 lb 8 oz + 74 lb 6 oz + 9 lb 11 oz = ■

STEP 1 Add the measures.

$$\begin{array}{r} 32 \text{ lb } 8 \text{ oz} \\ 74 \text{ lb } 6 \text{ oz} \\ +\ 9 \text{ lb } 11 \text{ oz} \\ \hline 115 \text{ lb } 25 \text{ oz} \end{array}$$

STEP 2 Simplify the sum.

Remember: 16 oz = 1 lb

$$\begin{aligned} 115 \text{ lb } 25 \text{ oz} &= 115 \text{ lb} + 16 \text{ oz} + 9 \text{ oz} \\ &= 115 \text{ lb} + 1 \text{ lb} + 9 \text{ oz} \\ &= 116 \text{ lb } 9 \text{ oz} \end{aligned}$$

Solution: The cooking gear, extra clothing, and sleeping bags weigh 116 pounds 9 ounces in all.

How much more do the tents weigh than the lanterns?

Subtract. 50 lb 2 oz − 21 lb 4 oz = ■

STEP 1 Write the measures.

$$\begin{array}{r} 50 \text{ lb } 2 \text{ oz} \\ -\ 21 \text{ lb } 4 \text{ oz} \\ \hline \end{array}$$

STEP 2 Rename 1 lb as 16 oz. (2 oz < 4 oz)

$$\begin{array}{r} {}^{49}\cancel{50} \text{ lb } {}^{18}\cancel{2} \text{ oz} \\ -\ 21 \text{ lb } 4 \text{ oz} \\ \hline \end{array}$$

STEP 3 Subtract.

$$\begin{array}{r} {}^{49}\cancel{50} \text{ lb } {}^{18}\cancel{2} \text{ oz} \\ -\ 21 \text{ lb } 4 \text{ oz} \\ \hline 28 \text{ lb } 14 \text{ oz} \end{array}$$

Solution: The tents weigh 28 lb 14 oz more than the lanterns.

Guided Practice

Find the sum or difference.

1. 5 ft 7 in. + 4 ft 6 in.

2. 1 gal 3 qt + 3 gal 1 qt

3. 5 yd 1 ft − 2 yd 2 ft

4. 3 h 30 min − 2 h 30 min

Ask Yourself

- Do I have to simplify the sum?
- Do I have to rename in order to subtract?

Explain Your Thinking ▶ When do you have to simplify a sum?

Extra Help at eduplace.com/map

Practice and Problem Solving

Find the sum or difference.

5. 3 mi 300 yd − 1 mi 700 yd

6. 8 lb 14 oz + $9\frac{5}{8}$ lb

7. 9 qt 20 fl oz − 8 qt 14 fl oz

8. 5 pt 6 fl oz + 1 pt 10 fl oz

9. $15\frac{3}{4}$ ft − $11\frac{5}{6}$ ft

10. 5 min 45 s + 3 min 18 s

11.
$$\begin{array}{r} 2 \text{ yd } 1 \text{ ft } 7 \text{ in.} \\ +\ 1 \text{ yd } 2 \text{ ft } 11 \text{ in.} \\ \hline \end{array}$$

12.
$$\begin{array}{r} 2 \text{ h } 30 \text{ min } 12 \text{ s} \\ -\ \ 45 \text{ min } 30 \text{ s} \\ \hline \end{array}$$

13.
$$\begin{array}{r} 3 \text{ gal} \\ -\ 1 \text{ gal } 1 \text{ qt } 1 \text{ pt} \\ \hline \end{array}$$

Solve.

14. What's Wrong? Tess claims that if she walks 1,500 yards per day, she will walk a total of about 6 miles in one week. Is she right or wrong? Explain.

15. Estimate Use the chart on page 192. Suppose you also bring about 25 pounds of supplies. Will the total weight of gear, clothing, and supplies be more than 210 pounds? How do you know?



Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Tell which method you chose.

16. The campers arrived at the campground at 11:30 A.M. They ate lunch and then started to set up camp at 1:15 P.M. They finished at 3:30 P.M. How long did it take them to set up camp?

17. Each of 12 campers drinks 8 fluid ounces of water with each of the 3 meals they eat per day. How many quarts and ounces of water do they drink with their meals each day?

18. Analyze The campers have spools of string that are 24 yards 2 feet long. How many spools would make 1 mile of string?

19. A fish fillet weighs between 5 ounces and 6 ounces. You have 4 pounds of fish. Do you have enough fish for 12 fillets?

Mixed Review and Test Prep

Open Response

Write as decimals. Round each answer to the nearest tenth. (Ch. 7, Lesson 6)

20. $\frac{7}{11}$

21. $\frac{16}{21}$

22. $\frac{24}{42}$

23. $\frac{38}{76}$

24. $\frac{44}{15}$

25. $\frac{193}{278}$

Multiple Choice

26. After you saw two pieces of wood off a board measuring 9 ft 3 in., you have 9 in. of board left. One piece of wood is 2 ft 8 in. long. How long is the other piece? (Ch. 8, Lesson 3)

A 3 ft 7 in.

C 5 ft 10 in.

B 5 ft 6 in.

D 7 ft 4 in.

Extra Practice See page 207, Set B.

Hands On Lesson 4

Metric Units of Measure

Objective Use powers of 10 to change from one metric unit to another.

Vocabulary
metric system



Learn About It

A hiking-boot lace is about 1 meter long and has a mass of about 4 grams. A water bottle holds about 1 liter of water.

The meter, gram, and liter are the basic units of length, mass, and capacity in the **metric system**. The prefix (*milli-, centi-, deci-, kilo-*) of a metric unit tells you how the unit is related to the basic unit.

Try this activity to learn about metric measures.
Materials: metric rulers, a pan balance, metric beakers

STEP 1 Select five objects from the classroom to measure. Find as many ways as possible to measure the objects in terms of length, mass, or capacity.

- Estimate the measurement of each object.

STEP 2 Choose a measuring tool and find the length, mass, or capacity of the object. Share your results with your classmates.

- Did you get the same results as other students?
- How accurate were your estimates?

The metric system is based on powers of 10. You can use what you know about multiplying and dividing by powers of 10 to change from one metric unit to another.

Metric Units of Length

1 millimeter (mm) = 0.001 meter (m)
1 centimeter (cm) = 0.01 meter
1 decimeter (dm) = 0.1 meter
1 kilometer (km) = 1,000 meters

Changing Metric Units of Length

Multiply to change a larger unit to a smaller unit.

0.23 km = ■ m

Remember: 1 km = 1,000 m

$0.23 \times 1{,}000 = 230$
0.23 km = 230 m

Divide to change a smaller unit to a larger unit.

1.3 cm = ■ m

Remember: 100 cm = 1 m

$1.3 \div 100 = 0.013$
1.3 cm = 0.013 m

Changing metric units of mass and capacity follows the same pattern.

Changing Metric Units of Mass

573 mg = ■ g

Grams are larger than milligrams, so divide.

Remember: 1 g = 1,000 mg

573 ÷ 1,000 = 0.573

573 mg = 0.573 g

Metric Units of Mass
1 gram (g) = 1,000 milligrams (mg)
1 kilogram (kg) = 1,000 grams (g)
1 metric ton (t) = 1,000 kilograms (kg)



Changing Metric Units of Capacity

1.08 L = ■ mL

Milliliters are smaller than liters, so multiply.

Remember: 1 L = 1,000 mL

1.08 × 1,000 = 1,080

1.08 L = 1,080 mL

Metric Units of Capacity
1 liter (L) = 1,000 milliliters (mL)
1 deciliter (dL) = 100 milliliters
1 milliliter (mL) = 1 cubic centimeter (cc)



Guided Practice

Ask Yourself

- Am I changing to a smaller unit or a larger unit?
- Should I multiply or divide?

Complete.

1. 15 m = ■ cm
2. 642 cm = ■ dm
3. 0.09 g = ■ mg
4. 1,200 kg = ■ t
5. 16 dL = ■ mL
6. 2.6 L = ■ mL

Which measurement is more precise?

7. 27 dL or 2,700 mL
8. 531 cm or 53.1 dm
9. Which has the greater mass, a 2,200-kilogram car or a 2.5-metric ton shipment of computers? Explain.

Explain Your Thinking Why do you multiply when changing from a larger unit to a smaller unit and divide when changing from a smaller unit to a larger unit?

Go On

Practice and Problem Solving

Complete.

10. If 1 cm = 0.01 m, then 40 cm = ■ m.

11. If 1 kg = 1,000 g, then 1.5 kg = ■ g.

12. 86 m = ■ cm

13. 629 dL = ■ mL

14. 43 mg = ■ g

15. 58.4 kg = ■ g

16. 0.549 km = ■ cm

17. 678 mL = ■ L

18. 8.41 t = ■ kg

19. 533.2 km = ■ m

20. 0.6 m = ■ mm

Choose the most likely measurement.

21. Is the length of a car closer to 1 meter or 4 meters?

22. Does an aquarium hold 25 milliliters of water or 25 liters of water?

23. Would a dictionary have a mass of 2 kilograms or 200 milligrams?

24. Is the amount of water a sink can hold closer to 35 liters or 35 deciliters?

25. Which is the shorter hiking path, one that is 100 meters long or one that is 500 decimeters long? Explain.

26. Which holds more, a fish tank with a capacity of 450 deciliters or a fuel tank with a capacity of 50 liters? Explain.

Algebra • Functions Copy and complete each table.

	m	cm
27.	45	■
28.	■	0.005
29.	■	862.3
30.	*a*	■

	g	kg
31.	15,753	■
32.	■	0.022
33.	■	5.3
34.	*b*	■

	mL	L
35.	■	0.23
36.	45,777	■
37.	42.8	■
38.	■	*c*

Solve.

39. The Nature Club is setting up a hiking trail. The students place markers about 1,800 decimeters apart. How many kilometers apart are the markers?

40. **Analyze** To prepare for a hike, Carlos ran 10 times around a 400-meter track. Kim ran 1 kilometer more than one half the distance Carlos ran. How far did Kim run?

41. Dan is on a 3-day hike. The first day he hikes $\frac{1}{3}$ of the trail. The next day he hikes $\frac{1}{3}$ of the trail plus 2.4 kilometers. The last day he hikes 5.6 kilometers to the end of the trail. How long is the trail?

42. For an all-day hike, Judy brought 2 liters of water, a 750-milliliter bottle of sports drink, and three 2-deciliter cartons of juice. She had 5 deciliters of water left after the hike. How much fluid did Judy drink?

Extra Practice See page 207, Set C.

Problem Solving

Real World Connection

Using A Calendar

The Lake City Summer Camp is open from July 5 through August 31.



JULY	M	T	W	T	F	S	S
					1	2	3
	4	5	6	7	8	9	10
	11	12	13	14	15	16	17
	18	19	20	21	22	23	24
	25	26	27	28	29	30	31

AUGUST	M	T	W	T	F	S	S
	1	2	3	4	5	6	7
	8	9	10	11	12	13	14
	15	16	17	18	19	20	21
	22	23	24	25	26	27	28
	29	30	31				

1. Use the calendar to count the number of complete weeks. Count the number of days after the last complete week. Include the last day. How many weeks and days is the camp open?

2. Camp staff have to arrive 10 days before the camp's opening and stay 3 days after its closing. On what dates does the camp staff arrive and leave? How long does the staff stay?

3. Suppose the camp opened on June 22. On what day of the week would June 22 fall?

WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

1. Estimate how many pens would fit across your desk. Then measure. (Lesson 1)

Complete. (Lesson 2)

2. 3 c = ■ fl oz
3. 80 oz = ■ lb
4. If 3 ft = 36 in., then 1.5 ft = ■ in.

Which measurement is more precise? (Lesson 2)

5. 36 in. or 3 ft
6. 2 qt or 64 fl oz

Find the sum or difference. (Lesson 3)

7. 3 lb 10 oz + 2 lb 12 oz
8. 2 ft 5 in. − 8 in.

Complete. (Lesson 4)

9. 125 cm = ■ m
10. 13 dL = ■ L

Lesson 5

Audio Tutor 1/21 Listen and Understand

Relate Customary and Metric Units

Objective Convert between customary and metric units of measure.

Learn About It

The metric system is the official system of measures in most countries in the world. If a measurement from another country is given in metric units, you want to know the equivalent measurement in customary units.

The Tour de France is a 23-day bicycle race held every year in France. In 2001, the total course length was about 3,500 kilometers. About how many miles is that?



These tables show approximate equivalences between some customary and metric units of measure.

Length		Capacity	Weight or Mass
1 in. ≈ 2.5 cm	1 cm ≈ 0.4 in.	1 fl oz ≈ 29.6 mL	1 oz ≈ 28.3 g
1 ft ≈ 30.5 cm	1 m ≈ 3.28 ft	1 mL ≈ 0.033 fl oz	1 lb ≈ 454 g
1 yd ≈ 0.9 m	1 m ≈ 1.09 yd	1 qt ≈ 0.95 L	1 kg ≈ 2.2 lb
1 mi ≈ 1.6 km	1 km ≈ 0.62 mi	1 L ≈ 1.06 qt	1 T ≈ 0.9 t
			1 t ≈ 1.1 T

The symbol ≈ is read as "is approximately equal to."

Complete. 3,500 km ≈ ■ mi

STEP 1 Use the tables. Find how many miles are in 1 kilometer.

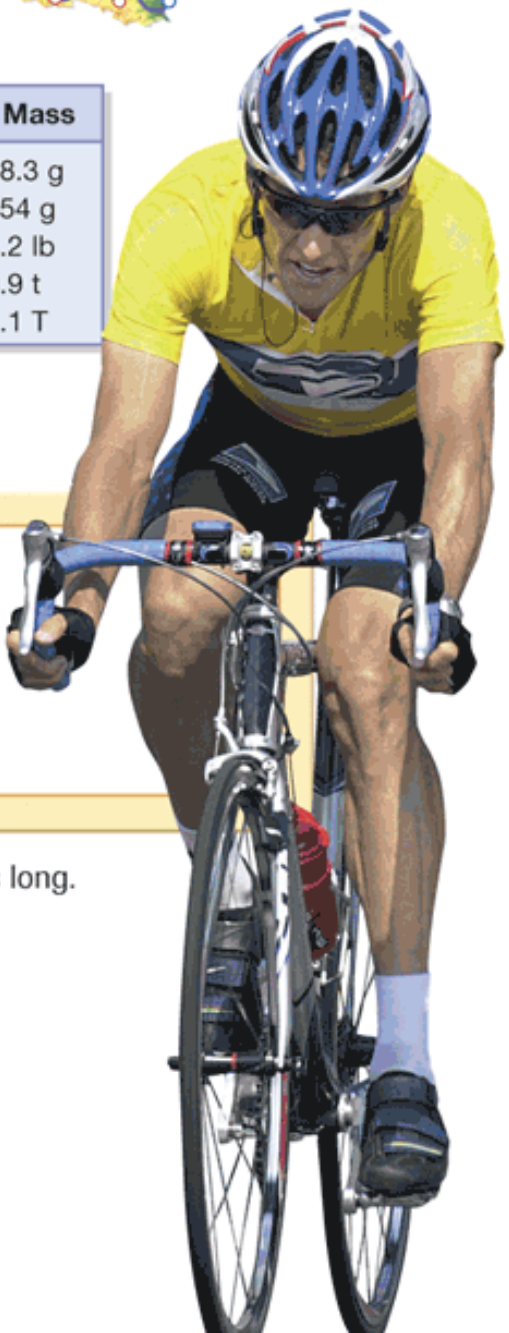
1 km ≈ 0.62 mi

STEP 2 Multiply to find the number of miles in 3,500 km.

3,500 × 0.62 = 2,170

3,500 km ≈ 2,200 mi

Solution: In 2001, the Tour de France course was about 2,200 miles long.



Other Examples

2 gal ≈ ■ L

Remember: 2 gal = 8 qt

1 qt ≈ 0.95 L

8 × 0.95 = 7.6

2 gal ≈ 8 L

18,000 kg ≈ ■ T

Remember: 18,000 kg = 18 t

1 t ≈ 1.1 T

18 × 1.1 = 19.8

18,000 kg ≈ 20 T

Guided Practice

Complete.

1. 10 yd ≈ ■ m **2.** 100 qt ≈ ■ L **3.** 10 lb ≈ ■ g

Ask Yourself

- Which unit is given?
- Which equivalence should I use?

Explain Your Thinking When you convert centimeters to inches, do you get more or fewer inches than centimeters?

Practice and Problem Solving

Complete.

4. 20 mi ≈ ■ km **5.** 30 L ≈ ■ qt **6.** 50 T ≈ ■ t **7.** 4 kg ≈ ■ lb

8. 4 in. ≈ ■ cm **9.** 20 cm ≈ ■ in. **10.** 10 m ≈ ■ ft **11.** 10 oz ≈ ■ g

Solve. Use the conversion table on page 198.

12. A city recycled 100,000 kilograms of metal. About how many pounds of metal did the city recycle?

13. A sign along a road read 100 kilometers to the next rest stop. About how many miles away is the next rest stop?

14. A bottle of spring water contains 300 milliliters. About how many ounces does the bottle contain?

15. **Analyze** One liter of water has a mass of 1 kilogram. One gallon of water weighs about 8.3 pounds. Which is lighter, 4 liters of water or 1 gallon of water? How much lighter?



16. Suppose the first four stages of a bicycle race are 6.3 kilometers, 10.25 kilometers, 15.6 kilometers, and 12.5 kilometers. How many miles is that in all?

17. Roadrace bicycles weigh about 20 pounds. Track bicycles are about 5 to 8 pounds lighter. Estimate the mass of a typical track bicycle in kilograms.

18. **Calculator** In the 2001 Tour de France race, the highest point on the course was 2,115 meters above sea level. About how many feet is that?

19. It is recommended that you drink eight glasses of water a day. If each glass of water holds 0.5 pint, how many liters of water would you need to drink?

Mixed Review and Test Prep

Open Response

Estimate each sum or difference. (Grade 4)

20. \$2.97 + \$7.04 + \$8.95 **21.** \$9.95 − \$2.99

22. \$514 + \$899 + \$688 **23.** \$802 − \$394

24. The distance around a bicycle course is 2 kilometers. How many full laps do you need to ride to travel at least 2 miles? (Ch. 8, Lesson 5)

Explain how you know.

Extra Practice See page 207, Set D.

Hands On Lesson 6

Perimeter and Area

Objective Explore perimeter and area.

Vocabulary
- perimeter
- area

Work Together

Materials
- grid paper (Learning Tool 15)
- string
- tape
- ruler
- unit squares

The distance around a closed figure is called the **perimeter**. Perimeter is measured in units of length. The **area** of a closed figure is the number of square units that cover the figure completely without overlapping.

You can estimate the perimeter of irregular figures by using string and a ruler.

Estimate the perimeter of your hand.

STEP 1 Place your hand on a sheet of grid paper. Trace the outline of your hand.

STEP 2 Place the string along the tracing. You can use tape to hold the string in place. Mark off the length of string used. Measure the length of the string to find the perimeter of your hand.

- What is the perimeter of your hand?

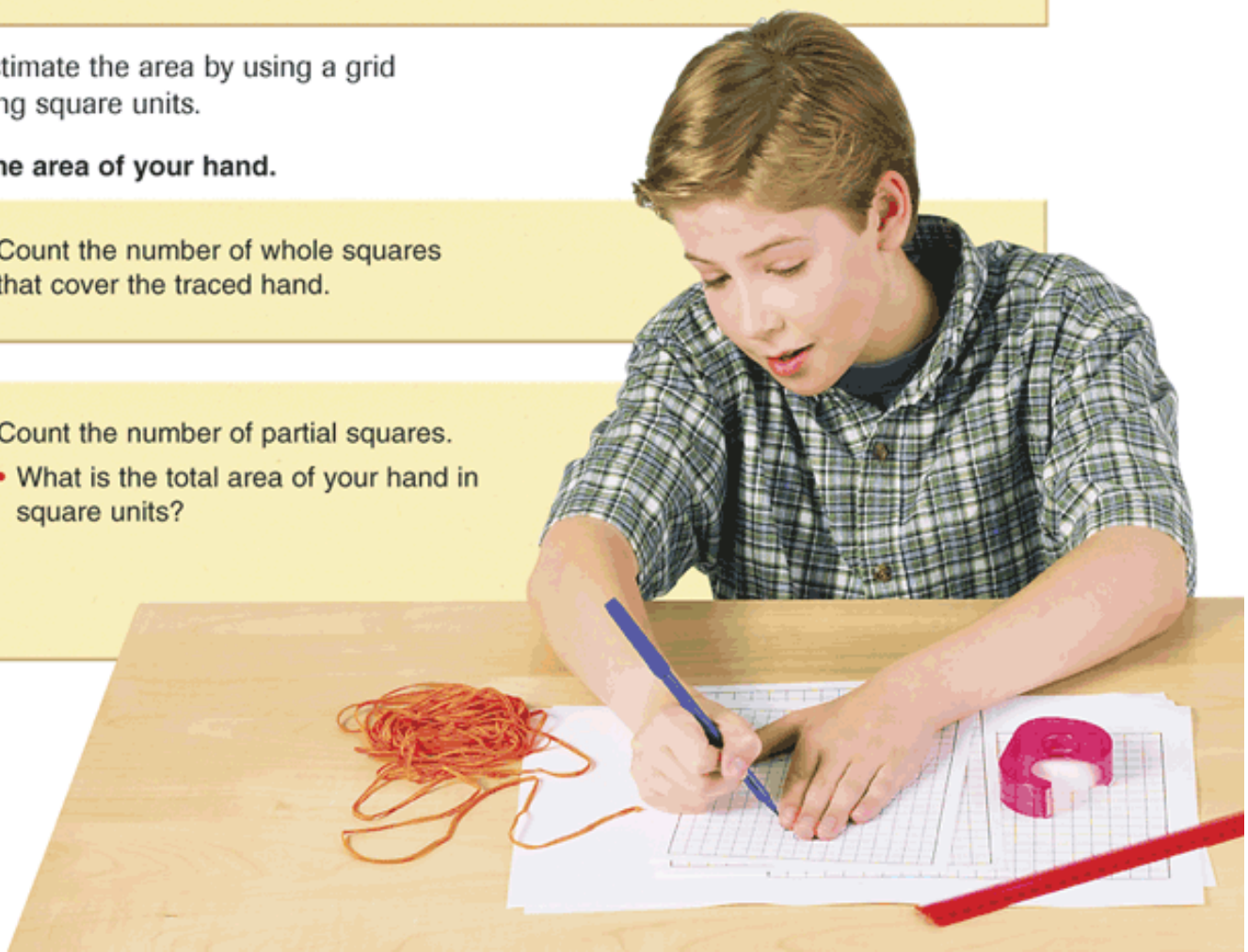
You can estimate the area by using a grid and counting square units.

Estimate the area of your hand.

STEP 1 Count the number of whole squares that cover the traced hand.

STEP 2 Count the number of partial squares.

- What is the total area of your hand in square units?

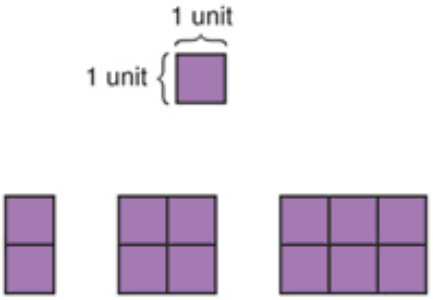


On Your Own

1. Trace the outline of your shoe on grid paper. Estimate the perimeter and the area.

2. Draw an irregular closed figure on grid paper. Estimate the perimeter and the area.

This pattern of figures made with unit squares was created by adding two squares at a time. Use the pattern for Exercise 3.



3. **Predict** Record the areas and the perimeters for the three figures at right. Then predict the perimeter of a figure made with a total of 50 unit squares.

Use unit squares to help you solve each problem.

4. Find all possible figures that can be formed by using 5 unit squares. Make sure that the unit squares in your figures share at least one full side with another unit square. Then find the perimeter of each figure.

5. What is the greatest possible perimeter for a figure made up of 8 unit squares? the least possible perimeter?

6. What is the greatest possible perimeter for a figure made up of 9 unit squares? the least possible perimeter?

7. Find all rectangles that have a perimeter of 18. Record the length and width in a table as shown on the right. Extend the table. As the length increases by 1 unit, what happens to the width? What do you notice about the areas?

Rectangles With Perimeter of 18 Units

Length	Width	Area
1	?	?
2	?	?
3	?	?

Talk About It • Write About It

You have learned how area and perimeter change as the shapes and sizes of figures are changed.

8. Is it possible for two figures to have the same perimeters but different areas? Give examples.

9. Is it possible for two figures to have the same areas but different perimeters? Give examples.

Lesson 7

Audio Tutor 1/22 Listen and Understand

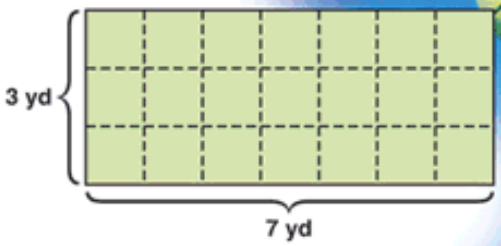
Perimeter and Area of Rectangles and Squares

Objective Find the perimeter and area of a rectangle and a square.



Learn About It

A community group proposed a plan to set aside a rectangular plot of land for a ramp in the skate park. How can you find the perimeter and area of this rectangle?



▶ Perimeter is the distance around a closed figure.

Different Ways to Find the Perimeter of a Rectangle

Way 1 Add the sides of the figure.

7 yd + 3 yd + 7 yd + 3 yd = 20 yd

Way 2 Use the formula.

$P = 2l + 2w$
$= (2 \times 7) + (2 \times 3)$
$= 14 \text{ yd} + 6 \text{ yd}$
$= 20 \text{ yd}$

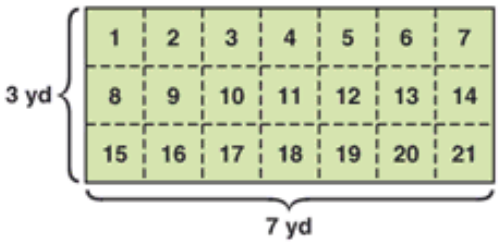
l = length
w = width
P = perimeter

Solution: The perimeter of the rectangle is 20 yards.

▶ Area is the number of square units that cover a closed figure without overlapping.

Different Ways to Find the Area of a Rectangle

Way 1 Count the number of 1-yard squares.



Way 2 Use the formula.

$A = l \times w$
$= 7 \text{ yd} \times 3 \text{ yd}$
$= 21 \text{ yd}^2$

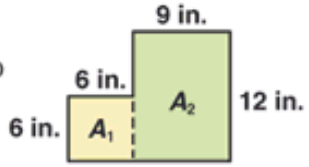
l = length
w = width
A = area

Solution: The area of the rectangle is 21 square yards.

Another Example

Area of Complex Figures

Separate the figure into two rectangles.



$A_1 = 6 \times 6 = 36 \text{ in.}^2$
$A_2 = 12 \times 9 = 108 \text{ in.}^2$
$A_1 + A_2 = 36 \text{ in.}^2 + 108 \text{ in.}^2$
Total area $= 144 \text{ in.}^2$

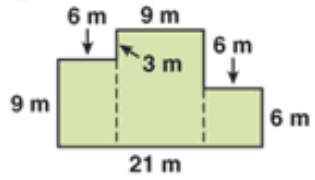
Guided Practice

Find the perimeter and the area of each figure.

1. $l = 7$ m, $w = 6$ m

2. $l = \frac{3}{4}$ in., $w = \frac{3}{4}$ in.

3.



Ask Yourself

- Did I use the correct formulas for perimeter and area?

Explain Your Thinking What is the difference between the perimeter of a figure and the area of a figure?

Practice and Problem Solving

Find the perimeter and the area of each figure.

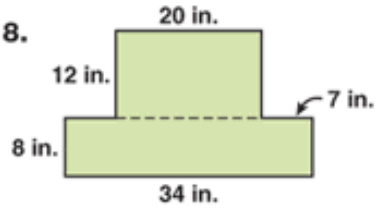
4. $l = 11$ ft, $w = 9$ ft

5. $l = 2.7$ km, $w = 4$ km

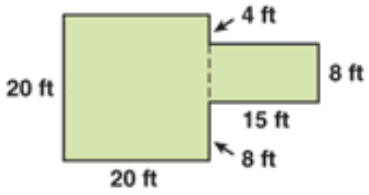
6. $l = \frac{8}{5}$ yd, $w = \frac{3}{5}$ yd

7. $l = 1.5$ mi, $w = 0.5$ mi

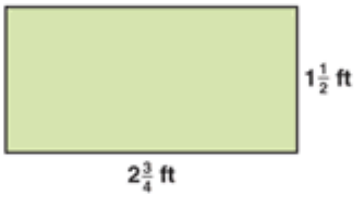
8.



9.



10.



Use the chart on the right for Exercises 11–13.

11. 72 ft^2 = ■ yd^2

12. 72 $in.^2$ = ■ ft^2

13. 1 yd^2 = ■ $in.^2$

1 ft^2 = 144 $in.^2$
1 yd^2 = 9 ft^2

Solve.

14. The floor of a square porch has an area of 64 square feet. What is the perimeter of the porch?

15. Reasoning The length of a sign is twice its width. The area of the sign is 72 square feet. Find the length and width of the sign.

16. Analyze A rectangle has a length that is 3 times its width. How does its area compare with the area of a rectangle whose length and width are three times as large?

17. A square is a rectangle whose length and width are equal. Let $s = l = w$ be the side of a square. Use s to write formulas for the area and perimeter of a square.

Mixed Review and Test Prep

Open Response

Find each product or quotient. (Grade 5)

18. 250 × 1,000

19. 1,900 ÷ 10

20. 3.584 × 100

21. 96 ÷ 1,000

Multiple Choice

22. What is the area of a square mirror if its perimeter is 48 cm? (Ch. 8, Lesson 7)

A 24 cm^2

B 48 cm^2

C 96 cm^2

D 144 cm^2

Extra Practice See page 207, Set E.

Lesson 8

Problem-Solving Decision

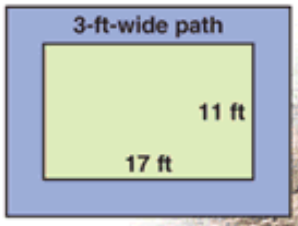
Identify Relevant Information

Objective Review how to identify relevant information.



A problem may contain too much information, too little information, or hidden information. You need to think carefully about what the problem asks.

Problem A rectangular flower garden is 17 feet long, 11 feet wide, and 3 feet above the ground. There is a paved path 3 feet wide around the garden. What is the outer perimeter of the path?



Ask Yourself

Do I have too much information?

To find the perimeter of the outer path, I need to know the length and width of the outer rectangle.

I do not need to know that the garden is 3 feet off the ground.

Is any information missing?

From the diagram I can see that the outer length of the garden and path is:

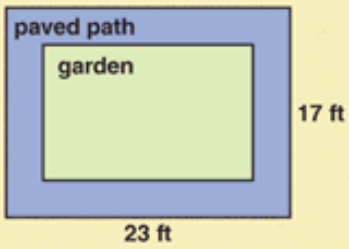
3 ft + 17 ft + 3 ft = 23 ft

Similarly, I can find the outer width of the garden and path:

3 ft + 11 ft + 3 ft = 17 ft

Is there enough information?

I now have enough information to use the formula $P = 2l + 2w$.



$P = 2(23) + 2(17) = 80$

Solution: The outer perimeter of the path is 80 feet.

Try These

Solve. If there is not enough information to solve, tell what information is missing.

1. Kenesha's backyard is a square that measures 30 feet on a side. She plans to cover the yard with sod. How many carloads of sod will she have to buy?
2. The rug in Mr. Turner's office has an area of 88 square feet. The rug is 11 feet long and is $\frac{1}{2}$ inch thick. What is the perimeter of the rug in feet?
3. The area of a field is 800 square meters. Fertilizer comes in 25-kilogram sacks. How many sacks will be needed to fertilize the entire field?
4. A rectangular mural is 6 feet high and covers 30 square feet of wall space. If the artist increases its width by 3 feet, what will be the area of the enlarged mural?

Problem Solving

Math Challenge

Land Measures

Small pieces of land are sometimes measured in square yards. Very large pieces of land are measured in square miles. Another commonly used area unit for measuring land is the *acre*. Central Park in New York City measures 843 acres.

The chart below shows how acres, square yards, and square miles are related.

1 acre (ac)	= 4,840 square yards (yd^2)
1 square mile (mi^2)	= 3,097,600 yd^2
1 mi^2	= 640 ac



Complete.

1. 25 ac = ■ yd^2

2. 320 ac = ■ mi^2

3. 1,280 ac = ■ mi^2

4. $\frac{1}{4}$ ac = ■ yd^2

5. $\frac{1}{4}$ ac = ■ mi^2

6. $\frac{1}{4}$ mi^2 = ■ yd^2

7. 1 mi^2 = ■ yd^2

8. 5 ac = ■ yd^2

9. 64 mi^2 = ■ yd^2

10. $\frac{1}{2}$ mi^2 = ■ yd^2

11. 2 ac = ■ yd^2

12. 15 mi^2 = ■ yd^2

13. A piece of land measures 21 acres. How many square miles is this?

14. Yellowstone National Park measures 2,219,791 acres. What is the area of Yellowstone National Park in square miles?

Chapter Review/Test

VOCABULARY

1. The _____ is the distance around a figure.
2. The smaller the unit of measure you use, the greater the _____.
3. The number of square units in a region is its _____.

Vocabulary

area
accuracy
perimeter
precision

CONCEPTS AND SKILLS

Determine whether there will be more or fewer of the new units. Then find the equivalent measure. (Lessons 1, 2, and 4, pp. 186–191, 194–196)

4. $3\frac{1}{2}$ yd = ■ in.
5. 8 c = ■ fl oz
6. $3\frac{1}{4}$ lb = ■ oz
7. 150 cm = ■ m
8. 789 mL = ■ L
9. 70.3 g = ■ kg

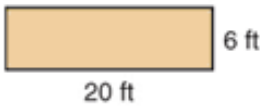
Find the sum or difference. (Lesson 3, pp. 192–193)

10. 7 yd 2 ft + 3 yd 2 ft
11. 10 mi − 1,695 ft
12. 5 lb 10 oz + 6 lbs 7 oz
13. 12 min 38 s − 6 min 45 s
14. 4 gal 2 qt 1 pt + 3 gal 3 qt 1 pt
15. 3 pt − 14 fl oz

Complete. Use the table on page 198. (Lesson 5, pp. 198–199)

16. 3 m ≈ ■ ft
17. 7 qt ≈ ■ L
18. 8 in. ≈ ■ cm
19. 345 km ≈ ■ mi
20. 440 lb ≈ ■ kg
21. 10 oz ≈ ■ g

Find the perimeter and the area of each figure. (Lessons 6–7, pp. 200–203)

22. $l = 12$ mi, $w = 24$ mi
23. $l = 3.4$ cm, $w = 3.4$ cm
24. 

PROBLEM SOLVING

Solve. If there is not enough information to solve, tell what information is missing. (Lesson 8, p. 204)

25. Mervin wants to put a fence around his rectangular garden. The garden is 6 yards wide. If fencing costs $0.25 per foot, how much will Mervin spend?

Write About It

Show You Understand

Rectangle *A* is 9 times longer than it is wide. Rectangle *B* is 4 times longer than it is wide. Both rectangles have an area of 36 square units. Which one has the greater perimeter?

Draw a diagram to explain.

Extra Practice

Set A (Lesson 2, pp. 188–191)

Determine whether there will be more or fewer of the new units. Then find the equivalent measure.

1. 3 c = ■ fl oz
2. 6,120 ft = ■ yd
3. 8 qt = ■ gal
4. $3\frac{1}{2}$ qt = ■ pt
5. 60 oz = ■ lb ■ oz
6. 108 in. = ■ yd
7. 40 fl oz = ■ pt
8. $2\frac{1}{4}$ mi = ■ ft
9. 9,600 lb = ■ T

Set B (Lesson 3, pp. 192–193)

Find the sum or difference.

1. 9 lb 7 oz − 3 lb 9 oz
2. $7\frac{1}{3}$ ft + $13\frac{5}{6}$ ft
3. 10 min − 3 min 14 s
4. 2 mi 450 ft + 6 mi 780 ft
5. 7 qt 10 fl oz − 3 qt 2 fl oz
6. 8 ft 5 in. + 9 ft 9 in.
7. 7 yd 1 ft 7 in. − 1 yd 2 ft 10 in.
8. 6 mi 1,500 yd + 4 mi 500 yd
9. 4 gal 3 qt − 2 gal 2 qt 1 pt

Set C (Lesson 4, pp. 194–196)

Complete.

1. 425 g = ■ mg
2. 108 cm = ■ m
3. 120 mL = ■ L
4. 4.5 m = ■ dm
5. 12 g = ■ kg
6. 570 m = ■ km
7. 55 L = ■ dL
8. 3.2 t = ■ kg
9. 0.18 dL = ■ mL

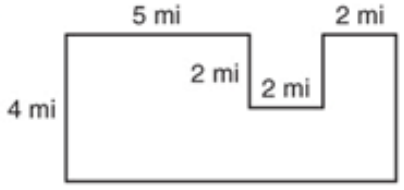
Set D (Lesson 5, pp. 198–199)

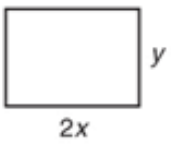
Complete. Use the table on page 198.

1. 100 kg ≈ ■ lb
2. 40 cm ≈ ■ in.
3. 30 mi ≈ ■ km
4. 5 t ≈ ■ T
5. 3 L ≈ ■ qt
6. 7 m ≈ ■ ft

Set E (Lesson 7, pp. 202–203)

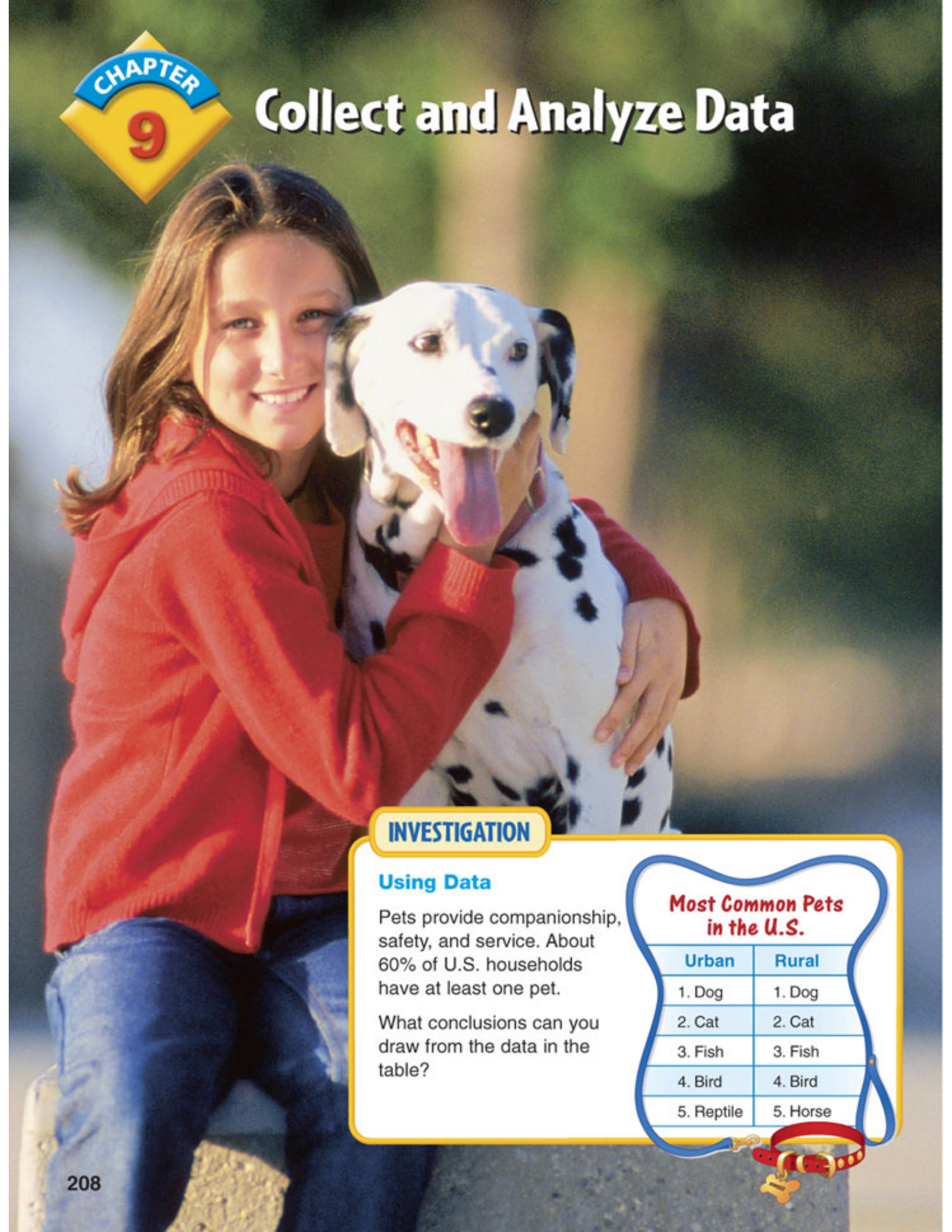
Find the perimeter and the area of each figure.

1. l = 13 in.
   w = 5 in.
2. l = 4.5 mm
   w = 4.5 mm
3. 

4. 

CHAPTER 9

Collect and Analyze Data



INVESTIGATION

Using Data

Pets provide companionship, safety, and service. About 60% of U.S. households have at least one pet.

What conclusions can you draw from the data in the table?

Most Common Pets in the U.S.

Urban	Rural
1. Dog	1. Dog
2. Cat	2. Cat
3. Fish	3. Fish
4. Bird	4. Bird
5. Reptile	5. Horse

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A(n) _____ is a collection of numbers or pieces of information.
2. A(n) _____ is a display that uses connected points to show how data change over time.
3. To find a(n) _____, divide the sum by the number of addends.

Vocabulary

average
data set
bar graph
line graph
pictograph

CONCEPTS AND SKILLS

Write in order from least to greatest.

4. 280, 218, 28, 300, 82, 48
5. 57, 54, 59, 65, 56, 52, 60
6. 77, 770, 17, 7, 71, 771, 701
7. 103, 301, 311, 113, 111, 303

Divide. Round each quotient to the nearest tenth.

8. 3,210 ÷ 8
9. 8,714 ÷ 25
10. 56.74 ÷ 9
11. 639.5 ÷ 6

Match. Write the letter of the average for each set of numbers.

12. 555, 648, 517, 212, 423
13. 72, 108, 96, 216, 118
14. 506, 425, 671, 324, 494
15. 393, 102, 115, 283, 442

a. 122
b. 267
c. 471
d. 484

Find the product.

16. $\frac{2}{3}$ of 810
17. $\frac{3}{4}$ of 700
18. $\frac{4}{5}$ of 2,695
19. $\frac{3}{8}$ of 8,032



20. Suppose you wanted to make a graph that compares the average lengths of several species of snakes. Would you make a bar graph or a line plot? Why?

Facts Practice, See Page 657.

Lesson 1

Surveys and Sampling

Objective Select a representative sample and use data from samples to predict data for an entire population.

Vocabulary

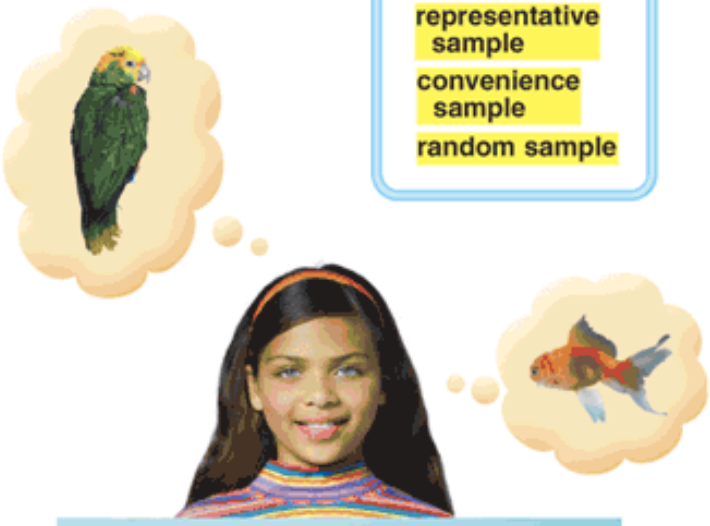
- population
- sample
- representative sample
- convenience sample
- random sample

Learn About It

Sometimes it isn't practical to survey all the members of a large group, or **population**. Researchers then select a small group of people called a **sample** to get information about the whole group.

Researchers want the answers given by the sample to be similar to those of the whole group. This kind of sample is called a **representative sample**.

Josh, Corey, and Lian each used a different method to poll students at their school. Identify each sample. Which sampling method provides a more representative sample? Explain.



Do birds or fish make better pets?

72 out of every 100 students, or 72% of students at Jackson School said birds make better pets.

Sampling Methods	Analysis of Methods
Josh asked the three other reporters working on the story with him if they thought fish or birds were better pets. 66% said *fish*; one had no opinion.	Josh's sample: 3 reporters Josh's sample is too small to represent all the students in the school.
Corey used a **convenience sample**. A convenience sample is easy to survey. Corey asked 20 of his friends if they thought birds or fish were better pets. 40% said *birds*.	Corey's sample: 20 of his close friends Corey asked only a small group of friends, whose interests may not be representative of the whole school.
Lian used a **random sample**. She randomly asked every tenth student, as they entered the school building, if birds or fish made better pets and recorded their answers: 65% said *birds*.	Lian's sample: every tenth student entering the school building In Lian's sample, each student polled has an equal chance of being chosen.

Solution: Of the three methods, Lian's random sample is more likely to be the most representative sample.

A representative sample often is used to make predictions.

The headline of an article in the school newspaper makes these predictions. Which predictions are reasonable?



Solution: This prediction is reasonable. Based on Lian's survey, a majority (65%) of the students preferred birds to fish.

Solution: This prediction is not reasonable. The sample does not give information about everyone at the school.

Guided Practice

Decide if you would use a sample to learn something about each group. Explain.

1. the students in your math class
2. the attendees at a major league baseball game
3. the employees of a large company

Ask Yourself

- Could I poll or survey each member of this population?
- Is the sample too small?
- What group of people is not included in the sample?

Explain why each sample may not be representative.

4. A written survey is mailed to 10 addresses on Main Street at random.
5. One hundred people are called at home between 3 P.M. and 4 P.M.

Explain Your Thinking Would a sample be needed for a population of 50 people? Why or why not?

Practice and Problem Solving

Decide if you would use a sample to learn about each group. Explain.

6. the account holders at a local bank
7. the players on your sports team
8. all the students in a high school
9. the teachers in your school

Go On

The school newspaper wants information about the Internet habits of sixth-graders. Identify whether each sample is a convenience sample or a random sample.

10. All the names of the students are put in a hat and 25 names are drawn.

11. The first 25 names are picked from an alphabetical list of students.

12. All the students in the Computer Club are given a survey.

13. One tenth of all the students in the cafeteria are given a survey.



The owners of a pet store conducted a random survey about the buying habits of cat owners at their store. Use this survey for Exercises 14 and 15.

14. Suppose the sample is representative. Identify the population and the sample.

15. Decide if each prediction can be reasonably made based on the survey.

a. More than half of all the cat owners who come to the store buy toys for their cat.

b. More than half of all cat owners buy toys for their cats.

Use the table on the right for Exercises 16 and 17.

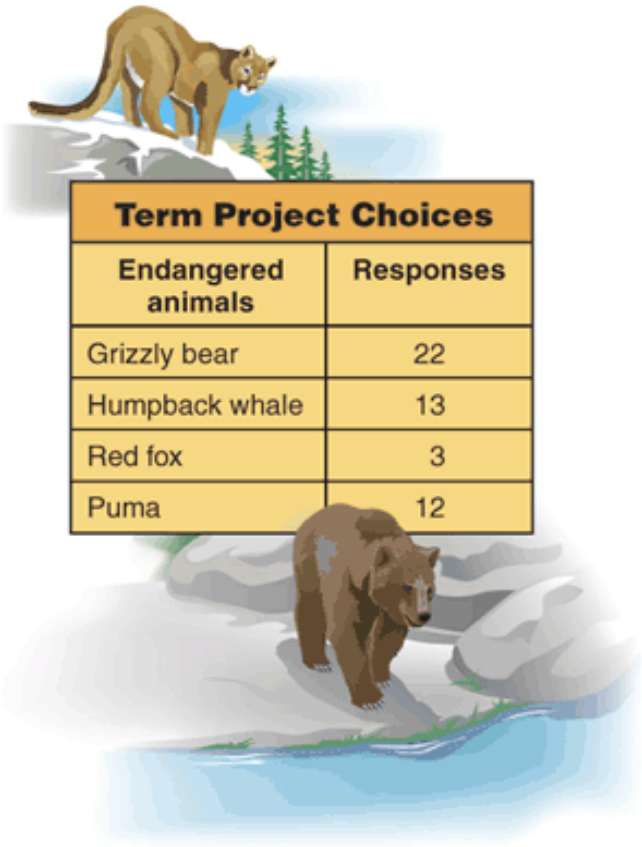
16. Out of 200 sixth-graders, 50 students were asked to choose one animal from a list of four endangered animals as a subject for a term project. Based on the responses, which of the following predictions are reasonable?

a. More students are interested in grizzly bears as a subject for their term project than any other animal.

b. Students don't like red foxes.

c. More than half the students surveyed will pick an animal other than the grizzly bear.

17. **Estimate** Based on the results of the survey, about how many out of 100 students might choose to do a term project on the humpback whale? Explain.



Term Project Choices

Endangered animals	Responses
Grizzly bear	22
Humpback whale	13
Red fox	3
Puma	12

Mixed Review and Test Prep

Open Response

Find the average of each set of numbers.
(Grade 4)

18. 29, 33, 26, 28

19. $82, $84, $78, $76

20. 148, 132, 167, 153

21. 17, 19, 21, 23, 25

22. $120, $196, $218, $175, $181

23. A company mails 600 copies of a written survey. They get 40 surveys back with *yes* circled and 10 with *no* circled. They report that 80% said *yes*. Explain whether the sample is representative or not.
(Ch. 9, Lesson 1)

Activity

Data Sense
Presidential Elections

In presidential elections, the voting public only votes for President and Vice President indirectly. Here are some facts about the 2000 presidential election.

- George W. Bush won the election with 271 electoral votes.
- Vice President Al Gore won the popular vote by 0.3% but lost the election with 266 electoral votes.
- People who voted: 126 million
- Registered voters: 130 million
- Eligible voters: 203 million
- U.S. Population: 281 million



In 2000, an estimated 51.2% of the nation's population voted in the presidential election.

1. **Calculator** The figure 51.2% is a misleading statistic since the eligible voters are all citizens over 18 years of age. What percent of the eligible voters registered to vote? What percent of the registered voters actually voted?
2. Based on the answers to Exercise 1, can you conclude that most people who register to vote actually do vote?
3. **Predict** In 1996, only 48.9% of the nation's population voted in the presidential election. In a random sample of 100 voters in one town, 10 said they had not voted in the election. Based on that sample, would you conclude that the town has a good voting record in presidential elections?
4. Besides the 2000 election, there were two other presidential elections where the electoral vote and the popular vote resulted in different results. Use data sources to find which two elections.

Extra Practice See page 231, Set A.

Lesson 2

Audio Tutor 1/23 Listen and Understand

Bias

Objective Identify bias in sampling.

Vocabulary

bias



Learn About It

The city council is going to vote on whether to keep its leash laws. The council is concerned that the samples chosen for the survey may lead to incorrect conclusions.

For a survey to lead to correct conclusions, it should avoid **bias**. Bias can occur in two ways. One way bias can occur is if the *sample* selected is not representative of the population to be surveyed. Another way bias can occur is if the *survey questions* are worded in ways meant to lead the people surveyed to give a certain answer.

Sampling the Population

Sample 1	Sample 2	Sample 3
Surveying dog owners	Surveying people who don't have any pets	Surveying some people who own dogs and some who don't
• Shows bias since many more dog owners than people with no pets would probably vote *no*	• Shows bias since more people who don't have any pets would probably vote *yes*	• Less likely to show bias since the chances of *yes* or *no* responses are equal

Wording the Survey Question

Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times, instead of running over people's property?	*Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times, instead of running freely to exercise?*
• Shows bias by making it sound like it's a bad idea to "let dogs run over people's property"	• Shows bias by making it sound like dogs would benefit by "running freely"

Solution: Either question would lead to biased results. A better question might be: *Should the mayor approve a bill requiring dogs that are outdoors to be on leashes at all times?*

Guided Practice

Identify each source of bias.

1. Shoppers in the cat food section at a supermarket are asked about favorite dog foods.

2. Students are asked, "Are you willing to waste weekend time volunteering at an animal shelter?"

Ask Yourself

- Is the sample representative?
- Did I look at the wording of the question?

Explain Your Thinking ▶ When is a sample or question biased?

Practice and Problem Solving

Identify each source of bias.

3. Parents are asked, "Should we develop hand-eye coordination in our children by teaching them how to play computer games?"

4. The president of a large pet supply company asks her employees, "Do you agree with my opinion each time I make a company decision?"

5. Reword the question in Exercise 3 to remove bias.

6. Reword the question in Exercise 4 to remove bias.

Solve.

7. A community is considering the creation of a small petting zoo for its children. To avoid bias, what kind of families should be included in a survey on this issue?

8. Shoppers are asked, "Does your cat prefer the exciting new taste of Tiger Tails moist cat food or ordinary kibble?" Find the bias. Then rewrite the question.

9. A local radio station invited people to call in to vote *for* or *against* a new community leash law. Two fifths of the callers voted against the leash law. The other 54 callers voted for a leash law. How many callers responded to the survey?

10. **Explain** In her survey, Sue asked her sixth-grade friends, "Should our school cafeteria replace their current menu, which everybody in the school dislikes?" Sue could not use her survey result to persuade the school administration to change the menu. Why?

Mixed Review and Test Prep

Open Response

Multiply. (Ch. 2, Lesson 2)

11. 26×34
12. 58×27
13. 75×65
14. 85×85
15. 47×72
16. 96×38

17. Is this a biased question?
Should children participate in the dangerous pastime of skateboarding?

Explain your answer. (Ch. 9, Lesson 2)

Extra Practice See page 231, Set B.

Lesson 3

Audio Tutor 1/24 Listen and Understand

Measures of Central Tendency

Objective Describe data, using mean, median, mode, and range.

Vocabulary
- outlier
- measures of central tendency



Learn About It

A data set is often a collection of numbers. **Outliers** are data points whose values are much greater or much less than the values of other data in the set.

Birds' eggs come in a variety of sizes. The largest egg from a non-extinct bird is the ostrich egg. The smallest is the hummingbird egg. This list shows the lengths in centimeters of eight ostrich eggs.

14.5, 15.2, 16.1, 16.5, 17.3, 18.6, 19.2, 21.0

How can you describe these data?

Different Ways to Describe a Data Set

Way 1

The **mean** of a data set is the quotient of the sum of the data divided by the number of data points in the set.

mean = 138.4 ÷ 8 = 17.3 cm

The mean length of the ostrich eggs is 17.3 cm.

The mean, or average, is a good way to describe a data set when there are few outliers.

Way 2

The **median** is the middle number in a data set when the data are arranged in numerical order. For an even number of data points, the median is the average of the two middle numbers.

median = (16.5 + 17.3) ÷ 2 = 16.9 cm

The median length of the ostrich eggs is 16.9 cm.

The median is a good way to describe a data set when there are outliers.

Way 3

The **mode** is the number or numbers that appear most often in the data set. Sometimes a data set has no mode or has more than one mode.

This data set has no mode.

The mode is a good way to describe a data set when many of the data points have the same value.

The mean, median, and mode are called **measures of central tendency** because they are meant to identify the center of the data. Another number used to describe a data set is the range.

Way 4 The **range** is the difference between the greatest and least values in a data set.

range = 21.0 − 14.5 = 6.5 cm.

The range describes the dispersion, or how spread out the data are.

Sometimes you want to use a single typical number to represent a data set. Which measure of central tendency best describes the typical length of an ostrich egg?

- The mean, 17.3 cm, shows what the length of each egg would be if the eggs were all the same length.
- The median, 16.9 cm, shows that the number of eggs longer than 16.9 cm equals the number of eggs shorter than 16.9 cm.
- There is no mode.



Solution: There is no mode. Since the data set is small, and there are no outliers, the mean best describes the typical length of an ostrich egg.

Guided Practice

Find the mean, median, mode, and range for each data set.

1. 7, 10, 3, 2, 5, 7, 1

2. 7, 11, 2, 3, 12, 1

The size of chicken eggs is usually given in ounces per dozen. There are six sizes: 15, 18, 21, 24, 27, and 30 ounces per dozen. Use these facts for Exercises 3 and 4.

3. Find the mean, median, mode, and range of the egg sizes.

4. Which statistic best describes the size of a chicken egg? Explain.

Ask Yourself

- How do I arrange the numbers?
- Which statistic best describes the center of the data?
- Which statistic is most useful for the given purpose?

Explain Your Thinking Can the mean, the median, and the mode of a set of data ever be the same number? Explain.

Go On

Practice and Problem Solving

Find the mean, median, mode, and range for each data set.

5. 1, 4, 19, 20, 9, 10, 14

6. 54, 72, 12, 90

7. 10, 54, 101, 6

8. 100, 90, 45, 88, 87

9. 94, 350, 20, 20, 106

10. 800, 1,000, 900, 860, 20, 40, 1,000

The data show the number of wild birds an ecologist banded. Use the data for Exercises 11 and 12.

80, 80, 87, 88, 90

11. Find the mean, median, mode, and range for the data set.

12. Which statistic best describes the data set?

13. Create a data set of 5 numbers with the median, range, and mean given at the right.

The median is 4.
The range is 8.
The mean is 5.

Algebra • Variables

Use the data set for Problems 14–16. In the data set, *n* represents an unknown length.

14, 46, 19, 10, 39, 22, 24, 77, *n*

14. What value of *n* will result in a mean of 31?

15. What value of *n* will result in a median of 23?

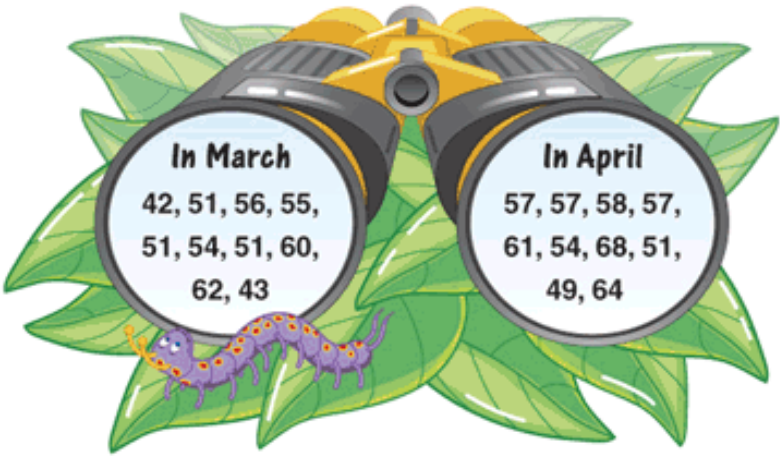
16. What value of *n* will result in a range of 84?

Data

Use the chart for Problems 17–19.

Bird watchers keep track of the numbers of wild birds they see. The table on the right shows the number of birds Ms. Horowitz sighted in two different months.

Number of Birds Sighted



According to the U.S. Fish and Wildlife Service, there are over 51 million Americans who watch birds.

17. Find the mean, median, mode, and range for each data set.

18. Which statistic best describes the data? Explain.

19. Analyze Were more birds sighted in March or in April? What can you conclude by comparing the measures of central tendency for the two data sets?

20. Create and Solve Create a data set and a story in which the mean is the most useful statistic. Have a classmate determine the most useful measure.

Extra Practice See page 231, Set C.

Game

Activity

Guess the Mode

2 players **What You'll Need** • Two number cubes labeled 1–6

How to Play

1. The object of the game is to make a prediction that is close to the mode of the sums after the two number cubes have been tossed 24 times. Before starting, each player predicts the mode of the sums and records the prediction.

2. Each player tosses the cubes and finds the sum of the two numbers tossed. Play continues until each player has tossed 24 sums.

3. Each player finds the mode. The player whose prediction is closest to the mode of his or her data set is the winner.



Quick Check

Check your understanding of Lessons 1–3.

Give a reason why these samples may not be representative. (Lesson 1)

1. Residents of one neighborhood are asked where the city landfill is to be built.

2. Students on a school's soccer team are asked if they like sports.

Identify the source of bias in Exercises 3 and 4. (Lesson 2)

3. A survey asked: "Should Green Street become a business zone so shopping will be more convenient for people on the east side of town?"

4. Parents are asked, "Should students be allowed to endanger other citizens by riding their bicycles on public walking paths?"

5. Find the mean, median, mode, and range for this data set. (Lesson 3)

49, 52, 57, 45, 49, 48, 51, 45, 45

Lesson 4

Clusters, Gaps, and Outliers

Objective Use a line plot to show how data are distributed.

Vocabulary
- line plot
- cluster
- gap

Learn About It

A **line plot** is a diagram that uses a number line to display data. The line plot below shows the masses in whole kilograms of some Coho salmons, caught by fishermen along the Columbia River in the Pacific Northwest.

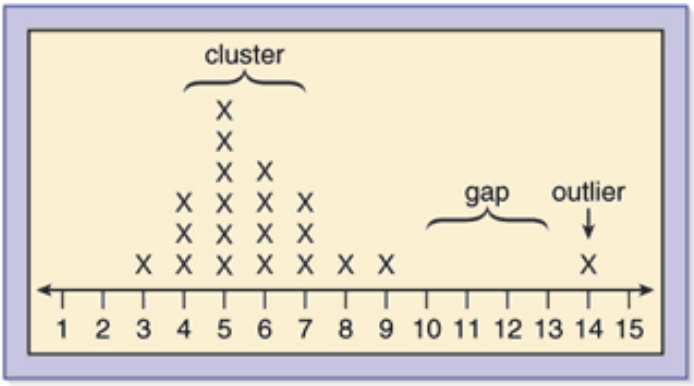
Use the line plot to describe the data set.

Using a Line Plot

A **cluster** is formed when several data points lie in a small interval.

A **gap** is an interval that contains no data.

Remember: An outlier is a data point that has a value much greater or much less than the other data in the set.



Solution: The mean for this data set is 121 ÷ 20, or about 6 kilograms. The median is 5.5 kilograms. The mode is 5 kilograms and the range is 11 kilograms.

How are the mean, the median, the mode, and the range affected if the outlier, 14 kilograms, is excluded from the data set? Which statistic changes the most?

Find the mean, median, mode, and range without the outlier.

- The mean changes to 107 ÷ 19 ≈ 5.6 kg.
- The median changes from 5.5 kg to 5 kg.
- The mode is unchanged.
- The range changes from 11 kg to 6 kg.

Solution: Without the outlier, the mean and median decrease, while the mode stays the same. The range decreases by 5 kilograms. The range changes the most.

Guided Practice

Use the data set 69, 70, 70, 71, 72, 74, and 92 for Exercises 1–3.

1. **Represent** Make a line plot of the data, and identify any clusters, gaps, and outliers.
2. Find the mean, median, mode, and range of the data.
3. Suppose the outlier is not in the data set. Which statistic represents the data best? Explain.

Ask Yourself

- How do I use the line plot to find the median, the mode, and the range?
- How do outliers affect the data?

Explain Your Thinking How will the mean be affected if 92 is replaced by 102 in the data set above?

Practice and Problem Solving

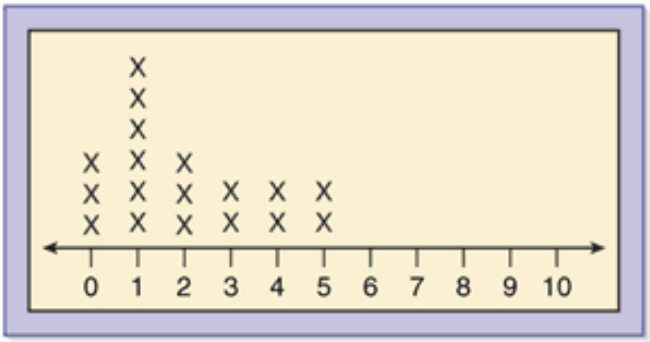
Last summer, Gil worked for 14 weeks at a pet store. Use the hours he worked, shown at the right, for Exercises 4 and 5.

5, 6, 21, 22, 23, 23, 24, 25, 25, 26, 26, 26, 27, 29

4. **Represent** Make a line plot of the data. Identify any clusters, gaps, and outliers. Find the mean, median, and mode.
5. Suppose the outliers were excluded from the data set. Compare the mean, median, and mode with and without the outliers.

Use the line plot for Problems 6–9.

Eighteen fishermen were asked how many fish they caught on their latest fishing trip.



6. Find the mean, median, mode, and range of the data.
7. Another 10 fishermen were asked how many fish they caught. The result was 0, 0, 0, 2, 3, 3, 3, 6, 8, and 9 fish. Find the mean, median, mode, and range.
8. Make a line plot that shows the results from all 28 fishermen. Find the mean, median, mode, and range of the combined data set.
9. **Explain** How did the mean, median, mode, and range change when the second data set was combined with the first?

Mixed Review and Test Prep

Open Response

Complete. (Ch. 8, Lesson 4)

10. 1,250 g = ■ kg

11. 1.8 L = ■ mL

12. 17 mm = ■ dm

13. 0.4 km = ■ m

Multiple Choice

14. What is the outlier in this set of data?

34, 37, 31, 19, 38, 35, 40, 42, 39

(Ch. 9, Lesson 4)

A 42 **B** 34 **C** 31 **D** 19

Extra Practice See page 231, Set D.

Lesson 5

Audio Tutor 1/25 Listen and Understand

Problem-Solving Strategy

Make an Organized List

Objective Solve a problem by making a list.

Sometimes you need to find all possible choices to solve a problem.

Problem Dinah has a set of mixed-up animal cards. The cards show the heads and bodies of an eagle, a panda, and an iguana separately. How many different animals can she build if she picks a head card and a body card?



UNDERSTAND

This is what you know:

- There are 3 different animals to pick from: an eagle, a panda, an iguana.
- Dinah arranges a head card and a body card to build an animal.

PLAN

You can make an organized list to organize the information to help you solve the problem.

SOLVE

Make a list.

Make a list with the headings *Head* and *Body*.

- There are three animals with eagle heads.
- There are three animals with panda heads.
- There are three animals with iguana heads.

Solution: There are nine possible animals that can be built.

Head	Body
eagle	eagle
eagle	panda
eagle	iguana
panda	panda
panda	eagle
panda	iguana
iguana	iguana
iguana	eagle
iguana	panda

LOOK BACK

Look back at the problem. Does the solution make sense?

Extra Help at **eduplace.com/map**

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. John has a set of head, body, and leg cards for a llama and a turkey. If he picks a head card, a body card, and a legs card without looking, how many different animals can he build?

2. A storeowner wanted to arrange the four animal magnets shown below in her store window. How many ways can she arrange the four magnets in a row so that the two birds are not next to each other?

Ask Yourself

UNDERSTAND
- What facts do I know?

PLAN
- Can I make a list?
- Did I use appropriate headings for my list?

SOLVE
- How did I start my list?
- How did I complete my list?

LOOK BACK
- Did I list all possibilities?



Independent Practice

Make a list to solve each problem.

3. The storeowner wants to arrange four stuffed animals in her window: a baby elephant, a mother elephant, a tiger cub, and a mother tiger. How many ways can she arrange the animals and keep each baby next to its mother?

4. The Cooper family paid $95 for a set of videos about animals in the wild. They used no $1 bills, no $2 bills, no $10 bills, and no bills greater than $50. How many different ways could the Cooper family have paid for the videos?

5. Larry buys a bottle of juice for $2. He pays with a $10 bill. In how many different ways can the clerk give Larry change, using only $1, $2, and $5 bills?

6. Chester has loose-leaf binders in four colors: red, blue, yellow, and green. How many different ways could he arrange the four binders in a row in a bookcase?

7. John paid 30 cents for his ticket to the petting zoo. How many different ways could he pay for the ticket if he had quarters, nickels, and dimes in his pocket?

8. CDs of animal sounds in the rain forest were on sale for $5 and $7 each. Russ bought 6 CDs and paid a total of $38. How many CDs did he buy at each price?

Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

9. A sixth-grade class raised $10 toward a class trip in the first week, $15 in the second week, $25 in the third week, and $40 in the fourth week. If the pattern continues, how much will the class raise in the sixth week?

10. Jon drove for $1\frac{1}{4}$ hours from his house to pick up his sister. Then they drove for $\frac{1}{2}$ hour to visit their cousin. They arrived at their cousin's house at 7:00 P.M. At what time did Jon leave his house to pick up his sister?

11. Connie has 4 more fish than Jack, who has 6 fewer fish than Nina. Mark has 3 times as many fish as Jack. If Nina has 15 fish, how many does Mark have?

You Choose

Strategy

- Draw a Diagram
- Find a Pattern
- Use Logical Reasoning
- Work Backward
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data Use the graph to solve Problems 12–16.



Service dogs assist physically disabled people with everyday tasks. The most popular breeds of service dogs are golden retrievers and labrador retrievers.

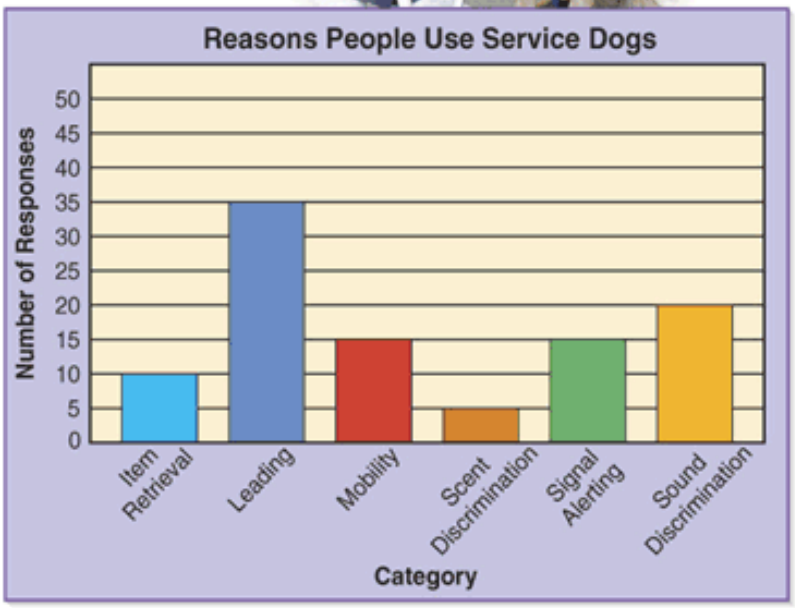
12. According to the survey, what is the most important reason people use service dogs?

13. What fraction of the people surveyed chose this reason?

14. According to the survey, which is the most common reason for using a service dog, mobility or sound discrimination?

15. Of the people surveyed, how many more listed signal alerting than item retrieval as a reason?

16. **Analyze** What fraction of the total number of people surveyed listed either scent discrimination or sound discrimination as a reason?



Problem Solving on Tests

Multiple Choice

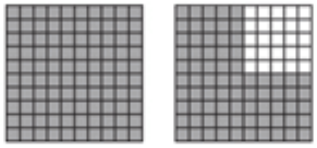
Choose the letter of the correct answer.

1. Tina has an 18-inch board that she wants to cut into pieces that are $2\frac{1}{4}$ inches long. How many pieces will she be able to cut?

A 7 **B** 8 **C** $8\frac{1}{2}$ **D** 9

(Chapter 6, Lesson 4)

2. Jamal has two square patches of land to plant. The shaded part of the diagram shows how much he has planted so far. What decimal represents the shaded parts?



F 0.25 **G** 0.75 **H** 1.75 **I** 1.95

(Chapter 4, Lesson 2)

Open Response

Solve each problem

3. On Monday 0.3 inches of rain was recorded. On Tuesday 1.4 inches of rain was recorded. What was the average rainfall for the two-day period?

Explain How did you find the average?

(Chapter 7, Lesson 4)

4. Rosa has half her garden planted. She put red flowers in $\frac{3}{4}$ of the sector. What fraction of the whole garden is represented by the portion that is already planted with red flowers?

Represent Draw a diagram to show your answer.

(Chapter 7, Lesson 1)

Extended Response

5. Worn out and torn currency, or paper money, is constantly being returned to the Federal Reserve and replaced with new bills. In 2001, \$566,074,787,441 was in circulation. There was also close to \$30,671,949,054 in coins in circulation.

a. Did you know that \$1 million in \$1 bills would weigh about 1 ton and if you put them in a pile, it would be about 361 feet high? About how many \$1 bills would stand 1 foot high? Write an equation to show your work.

b. If you had \$1 million in \$100 bills, it would weigh only 22 lb so you would be able to carry it. About how tall would they be if you stacked them? Tell how you know.

c. After making new coins, they are inspected then counted and bagged. The bags are then weighed before being sent to various banking centers. If you had \$1 million in one-cent coins, it would weigh about 46 tons. About how many pennies would you have in 1 ton?

(Chapter 9)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 6

Draw Conclusions and Make Predictions

Objective Draw and justify conclusions and make predictions based on data.



Learn About It

A veterinary clinic sent surveys to 500 pet owners asking:

Would you use our excellent new boarding facility to board your pet?

Surveys were returned by 161 pet owners. Based on the sample, can you predict that more than 300 of the 500 pet owners would use the facility?

Total population: 500 pet owners
Sample: 161 respondents
Results: For every 5 who said *yes*, 2 said *no*.

Making a Prediction

STEP 1 Find the fraction of respondents who said *yes*.

Given: There were 2 "no" responses for every 5 "yes" responses.

2 *no* + 5 *yes* = 7 responses

5 out of every 7 respondents said *yes*, so $\frac{5}{7}$ of them said *yes*.

STEP 2 Multiply the total population by the fraction.

$\frac{5}{7}$ of $500 = \frac{5}{7} \times 500 = \frac{2500}{7} = 357\frac{1}{7}$

STEP 3 Compare.

$357 > 300$

Solution: Since $357 > 300$, you can predict that more than 300 of the pet owners would use the facility.

Based on the results, the clinic concluded that the new boarding facility would be a success. To decide whether or not the conclusion based on the survey is valid, you need to analyze the data set and the way it was collected.

Analyzing a Conclusion

The following questions can help you decide if the conclusion is valid.

A. Is the sample large enough?

Estimate the sample size.

$\frac{161}{500} = \frac{332}{1{,}000}$ which is slightly less than $\frac{1}{3}$.

The sample represents nearly $\frac{1}{3}$ of the total population. If this is a random sample, it is probably large enough.

B. Is the sample representative or biased?

- If the clinic is interested only in current customers, then the sample may be representative.
- If they want to attract new business but surveyed only their customers, the sample could be biased.

C. Is the question biased?

The question shows bias because it refers to the new facility as "excellent." This wording might influence some of the responses.

Solution: The sample may be biased, and the survey question definitely is biased. The conclusion made by the clinic is questionable.

Guided Practice

Use these data for Exercises 1–2.

Total population: 280 students in the school

Random sample: 50 sixth-graders

Question: Should the students write to their state representative about funding for the municipal zoo?

Result: For every 7 *yes* responses, there were 3 *no* responses.

Conclusion: Students at our school think the school should write to their state representative about funding for the zoo.

Ask Yourself

- Is there bias in the sample or the question?
- How can I analyze the results?



1. Based on the results of the sixth-grade survey, can you conclude that more than 100 of the students in the school would write to the state representative?

2. Is the conclusion based on the survey results valid? Explain.

Explain Your Thinking ▶ What are some ways to analyze survey results?

Go On

Practice and Problem Solving

Use the data for Exercises 3 and 4.

Total population: 4,000 town residents

Random sample: 80 residents

Question: Should cats be licensed the same way as dogs?

Results: For every 13 *yes* responses, there were 7 *no* responses.

Conclusion: Most people in the town thought cats should be licensed.

3. Based on the results of the survey, can you conclude that more than 2,500 people think that cats should be licensed?

4. Is the conclusion based on the survey result reasonable? Explain.

Use the survey information on the right for Problems 5–8.

Students were asked to rate the statement on a scale of 1 to 5, where 1 = strongly disagree, 2 = disagree, 3 = neither agree nor disagree, 4 = agree, and 5 = strongly agree.

5. **Represent** Make a line plot of the data. Find the mean, median, and mode.

6. What can you conclude from the survey results and your analysis of the data?

7. **Predict** How many students in the total population would rate the statement a *3*?

8. **You Decide** Add 10 responses to the data that would change the conclusion. Explain how you made your choices.

Total Population: 2,000 students
Random Sample: 25 students

Statement:

Middle-school students do not know enough about pet nutrition.

Results:

3 students responded *1*

5 students responded *2*

4 students responded *3*

7 students responded *4*

6 students responded *5*

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Tell which method you chose.

9. **Explain** In a survey, students whose families own pets, were asked whether regular visits to the vet are important. There were 120 responses. For every 3 students who said *no,* four times as many said *yes.* How many said *no*? *yes*?

10. **Analyze** In his survey, Kyle asked if students think the after-school program should be expanded. He asked 5 students in each of grades 6, 7, and 8. Nine students said *yes.* The school has 328 students. Is Kyle's sample large enough?

11. Ms. Anderson spends $132 on boarding her dog and her cat for 6 days. The cost of boarding her cat is $10 a day. How much does it cost to board her dog per day?

12. A survey showed that out of 138 responses, twice as many people would board their dog as would board their cat. How many people would board their dog?

Extra Practice See page 231, Set E.

Mixed Review and Test Prep

Open Response

Subtract. Express each answer in simplest form. (Ch. 5, Lesson 4)

13. $\frac{7}{9} - \frac{1}{4}$

14. $2\frac{1}{4} - 1\frac{1}{6}$

15. $\frac{3}{4} - \frac{1}{3}$

16. $1\frac{5}{6} - \frac{2}{3}$

Multiple Choice

17. 2 quarts = ■ (Ch. 8, Lessons 1–3)

A 6 c

B 20 oz

C $\frac{1}{2}$ gal

D 2 pt

Activity

Real World Connection

A Survey

Suppose you are a reporter for the school newspaper. Your local youth center is planning a new addition. They want to add space that young people in your city or town would use the most.

List the types of activities the new space could be used for. Design a survey that will rate each activity.

- What population will you survey? How large should your sample be?
- How should you phrase the question in order to avoid bias?



Collect the data. Use tallies to record the responses. Discuss the results based on your data.

Continue to conduct interviews until you think you have enough data.

Draw a bar graph to present your data. Write a paragraph to answer the question "What activities could the new space be used for?"

Was there a common response from the people you interviewed?

WEEKLY WR READER eduplace.com/map

Chapter Review/Test

VOCABULARY

1. A(n) _____ has a value much greater or much less than the other data in the set.
2. The difference between the greatest and least numbers in a set of data is called the _____ of the data.
3. A(n) _____ is formed when several data points lie in a small interval.

Vocabulary

gap
mean
range
outlier
cluster

CONCEPTS AND SKILLS

Explain why the sample may not be representative. (Lesson 1, pp. 210–213)

4. Five people in a restaurant are asked to name their favorite dessert.

Identify the source of bias. (Lesson 2, pp. 214–215)

5. Students are asked, "Would you rather go to the latest hit movie or read your boring social studies book?"

Use this data set for Exercises 6 and 7. (Lessons 3–4, pp. 216–218 and pp. 220–221)

5, 26, 25, 21, 26, 22, 29, 6, 23, 25, 27, 23, 24, 26

6. Make a line plot of the data. Identify any clusters, gaps, and outliers.
7. Suppose there were no outliers in the data set. Compare the mean, median, and mode with and without the outliers.

Use the data at the right for Exercises 8 and 9. (Lesson 6, pp. 226–228)

Total population: 240 parents
Random sample: 24 parents
Question: Would you prefer a year-round school calendar?
Result: For every 6 *yes* responses, there were 2 *no* responses.
Conclusion: Parents would prefer their children to go to school year-round.

8. How many parents out of the total population would you predict are in favor of a year-round school calendar?
9. Based on the survey results, can you conclude that more than half the parents would prefer a year-round school calendar?

PROBLEM SOLVING

Solve. (Lesson 5, pp. 222–224)

10. Jody has 4 photos: a hawk, a manatee, a parakeet, and a sea otter. How many different ways can she arrange the photos in a row if the birds are not next to each other?

Write About It

Show You Understand

Make up a survey question that includes a source of bias. Then rewrite the question so that there is no source of bias.

Extra Practice

Set A (Lesson 1, pp. 210–212)

Decide if you would use a sample to learn about each group. Explain.

1. sixth-graders in the Midwest
2. members of your family

Explain why each sample may not be representative.

3. a poll conducted at a local senior citizens' home
4. a survey mailed to 20 people randomly selected by computer

Set B (Lesson 2, pp. 214–215)

Identify each source of bias.

1. Attendees at a local sports event are surveyed about whether a new sports arena should be built.
2. Middle- and high-school students are asked whether the minimum age for driving should be lowered.
3. Drivers are asked: "Should the speed limit on Main Street be lowered to reduce the number of auto accidents?"

Set C (Lesson 3, pp. 216–218)

Find the mean, median, mode, and range of each data set.

1. 48, 39, 59, 81, 62, 47, 51
2. 101, 99, 100, 98, 97, 89
3. 15, 16, 18, 16, 19, 20, 17

Use this data set for Exercises 4 and 5: 17, 10, 5, 9, 7.

4. Find the mean, median, mode, and range of the data set. Round to the nearest tenth.
5. Which statistic gives the most useful information about the typical number of species of birds in a family?

Set D (Lesson 4, pp. 220–221)

Use this data set for Exercises 1–3.

6, 10, 8, 12, 6, 8, 10, 4, 15, 7, 8, 9, 11, 7

1. Make a line plot of the data. Identify any clusters, gaps, and outliers.
2. Find the mean, median, mode, and range.
3. Suppose the outliers were not in the set. Which statistic changes the most?

Set E (Lesson 6, pp. 226–228)

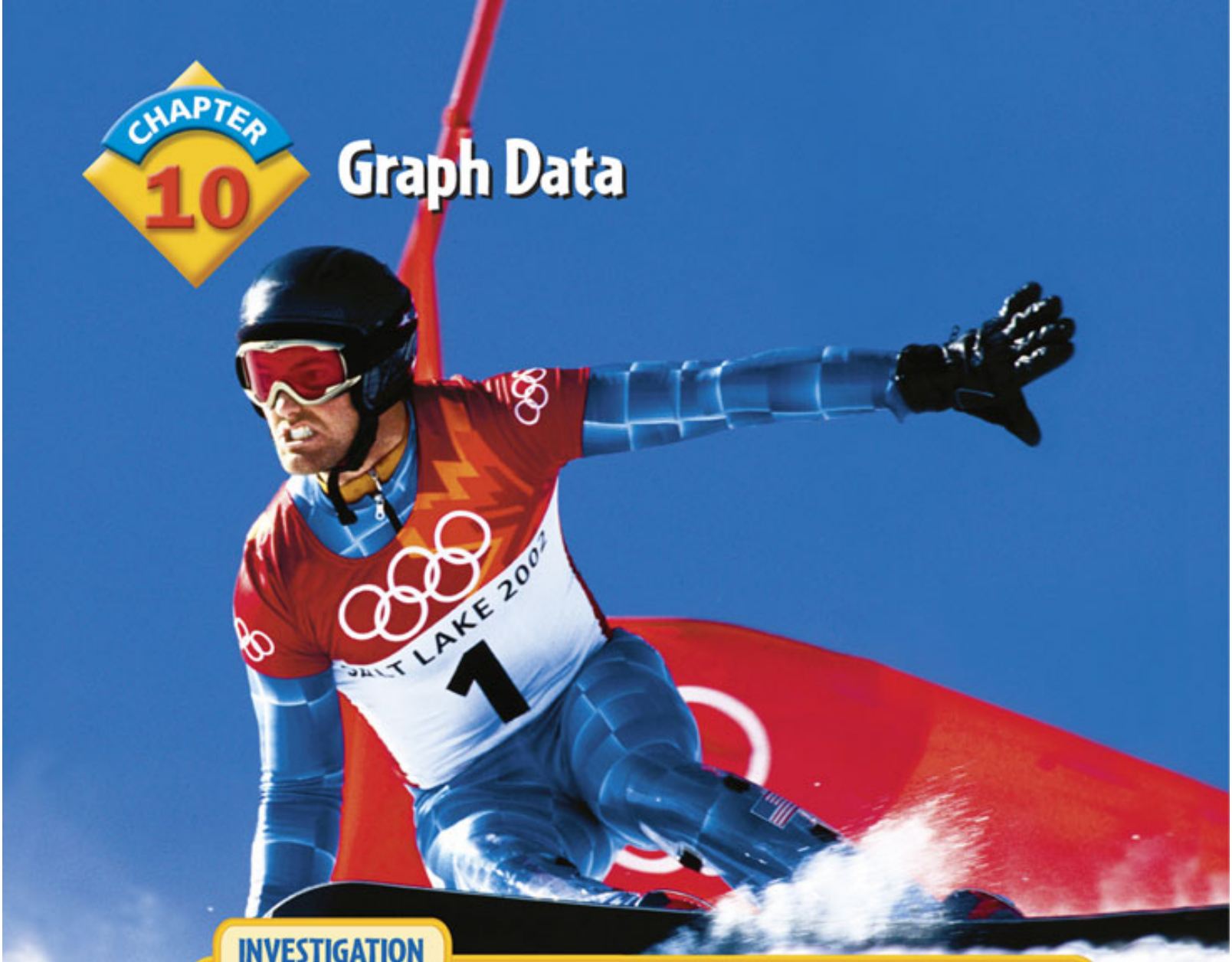
1. How many voters out of the total would you predict are in favor of increasing the library budget to buy books?
2. Based on the survey results, can you conclude that more than 750 voters will vote to increase the library budget? Explain.

Total population: 1,500 eligible voters
Random sample: 100 voters
Question: Should the library budget be increased to buy more books?
Result: For every 6 *yes* responses, there were 4 *no* responses.
Conclusion: Voters think the library budget should be increased to buy more books.

Extra Practice at **eduplace.com/map**

CHAPTER 10

Graph Data

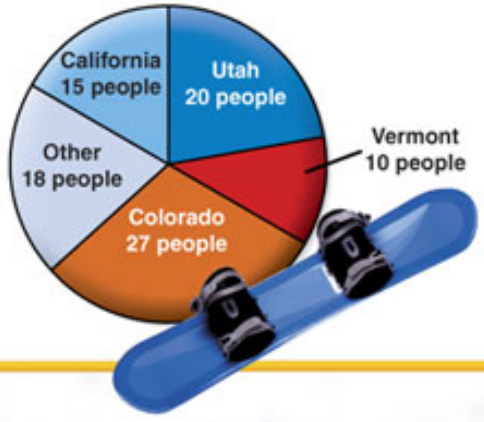


INVESTIGATION

Using Data

Ninety people were asked to name a state with great snowboarding. Based on the graph, can you determine the state with the best snowboarding? List some reasons why people would say a state has great snowboarding. Now make your own bar graph based upon these reasons.

Snowboarding Survey Results



State	People
California	15
Colorado	27
Utah	20
Vermont	10
Other	18

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

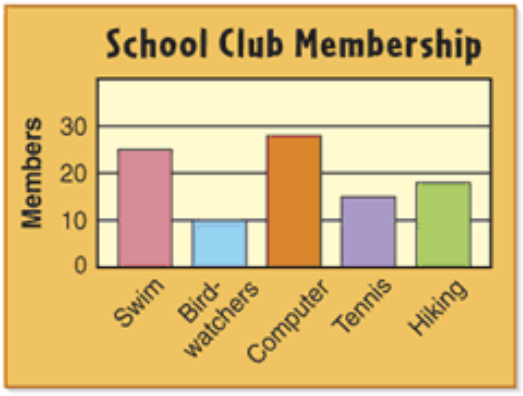
1. A _____ uses symbols to represent different quantities.
2. A display of data that uses connected points to show change over time is a _____.
3. A _____ compares the parts of a whole.

Vocabulary

bar graph
line graph
pictograph
circle graph

CONCEPTS AND SKILLS

Use the bar graph for Exercises 4–6.



4. Which club has the most members?
5. How many people belong to the swim club?
6. How many more people belong to the tennis club than the birdwatchers' club?

Use the line graph for Exercises 7–9.



7. About how many DVDs were sold in 2002?
8. How many more DVDs were sold in 2004 than 2002?
9. Between which two years was there the least change in the number of DVDs sold?

Write About It

10. Suppose you wanted to make a graph to show how many home runs each of the 5 top home-run leaders hit in their careers. What kind of graph would you make? Why?

Facts Practice, See page 657.

Lesson 1

Review Graphs

Objective Interpret and display data in graphs.

Vocabulary
- double bar graph
- scale
- interval
- pictograph

Learn About It

A graph is a visual way to display a data set. A **double bar graph** is useful when you want to compare two data sets on one graph.

Use the data in the table to make a double bar graph. Which event has the greatest difference in the number of male and female U.S. athletes?

2004/Participants (American)

Event	Men	Women
Basketball	12	12
Canoe/Kayak	8	6
Cycling	3	0
Equestrian	6	7
Sailing	11	7



Making a Double Bar Graph

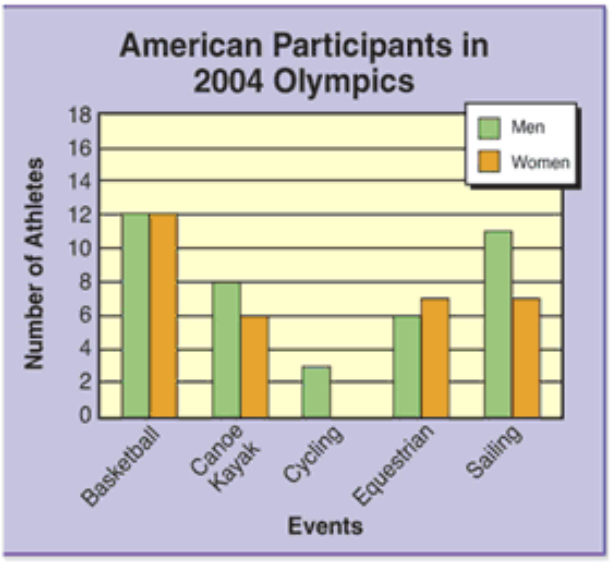
STEP 1 Choose a title.

STEP 2 Label the horizontal axis *Events* and the vertical axis *Number of Athletes*.

STEP 3 Choose a **scale** and mark equal **intervals**. The number of athletes should start at 0 and could extend by 2s to just above 18, the greatest number of athletes.

STEP 4 Make a key to show what the bars represent.

STEP 5 Draw the bars.



Solution: Look at the difference in the heights of the pairs of bars. Sailing has 4 more male U.S. athletes than female U.S. athletes.

Guided Practice

Ask Yourself
- How do I choose a scale?
- How do I use the key to help me interpret the data?

1. For the months September through December, the fifth grade raised $175, $400, $350, and $325 for the school Mini-Olympics. During the same four months, the sixth grade raised $225, $250, $300, and $200. Make a double bar graph to represent these data.

Use the double bar graph you drew to answer Questions 2–5.

2. How much money in all was raised by the fifth- and sixth-graders in December?
3. How much more money did the fifth grade raise than the sixth grade did in November?
4. In which month was there the greatest difference in the amount of money raised by the fifth and sixth grades?
5. In which month did the fifth grade raise twice the amount of money it raised in September?

Explain Your Thinking Why is a double bar graph a good way to compare two sets of data?

Practice and Problem Solving

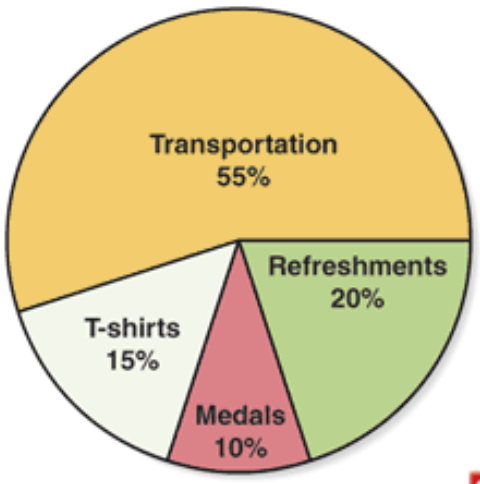
Use the table at right for Problems 6–8.

Skates Sales (Pairs)		
Month	**Ice Skates**	**In-line Skates**
January	9,000	1,000
February	5,000	4,000
March	2,000	10,000
April	500	12,000

6. Otis wants to display the data in a double bar graph. He plans to label the vertical axis *Number Sold*. How should he label the horizontal axis?
7. Which interval is the best choice for graphing the numbers of skates sold, 100, 1,000, or 10,000? Why?
8. **Represent** Make a double bar graph to display the data.

A circle graph is used to show parts of a whole. Use the circle graph for Problems 9–11.

Mini Olympics Budget



9. Which item accounted for about one half of the budget?
10. About what fraction of the budget was spent on medals and T-shirts?
11. Which single item represented about one fourth of the budget?

Go On

A **pictograph** uses pictures or symbols to represent data. There is always a key to indicate how many each symbol represents.

Use the table at right for Problems 12–14.

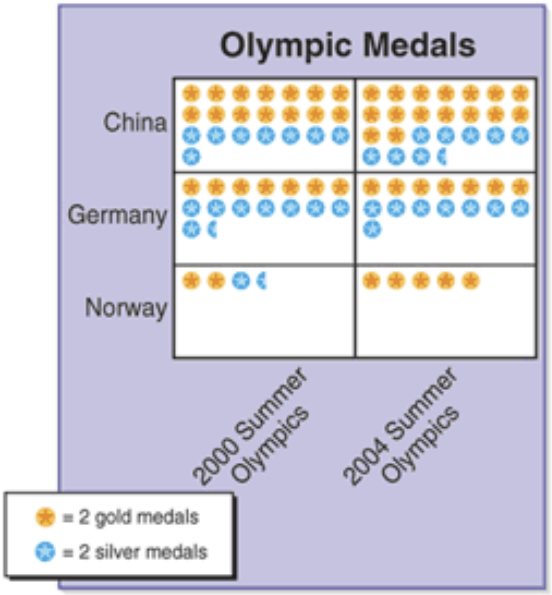
2004 Summer Olympics Gold Medals Won	
United States	35
Russia	27
China	32
Australia	17
South Korea	9

12. **Analyze** Jung Me wants to display the data in a pictograph. She wants to use a gold circle to represent a number of gold medals. Should each gold circle represent 20, 10, or 8 medals? Explain.

13. **Represent** Make a pictograph to display the data in the table.

14. **Explain** How could you use the information in your pictograph to find the mean, median, and mode?

Use the double pictograph at right for Problems 15–17.



15. How many gold and silver medals did Norway win in the 2000 Summer Olympics?

16. How many more silver medals did Germany win in the 2000 Summer Olympics than in the 2004 Summer Olympics?

17. **Create and Solve** Write a question that could be answered by reading the pictograph.

Use the table at the right for Problems 18 and 19.

Should there be longer gym periods?	
Answer	**Number of Votes**
Yes	30
No	15
Undecided	5

18. Dane wants to show how the number of votes in each category compares with the total vote. What kind of graph should he use?

19. What type of graph should Dane use if he wants to show which category got the most votes?

Mixed Review and Test Prep

Open Response

Find each missing number. (Ch. 8, Lesson 2)

20. 2 yd = ■ in.

21. 6 pt = ■ qt

22. 2.5 lb = ■ oz

23. ■ T = 3,000 lb

Multiple Choice

24. In the key to a pictograph, 1 symbol stands for 100 items. How many items does one fourth of a symbol stand for? (Ch. 10, Lesson 1)

A $\frac{1}{4}$ **B** 4 **C** 10 **D** 25

Extra Practice See page 259, Set A.

Data Sense

Different Graph Scales

Activity

People who make graphs sometimes use special techniques to make a graph easy to use. The table shows television viewing data for the Salt Lake City Winter Olympics in 2002 for some countries in the Americas.

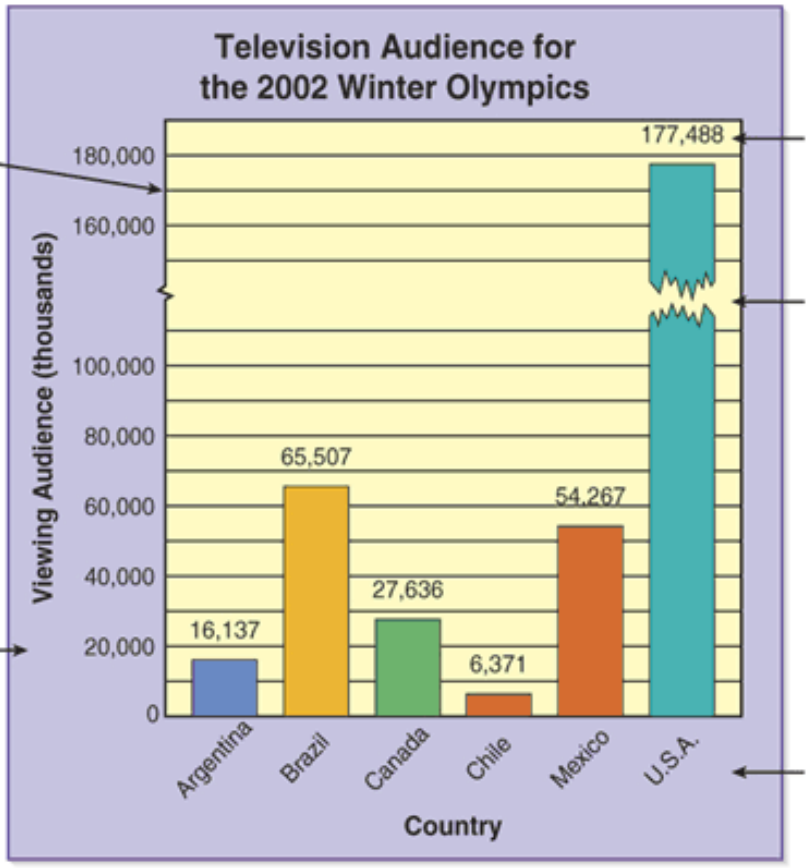
Television Viewing Data for the 2002 Winter Olympics

Country	Number of Viewers
Argentina	16,136,633
Brazil	65,507,012
Canada	27,635,883
Chile	6,371,047
Mexico	54,267,466
United States	177,488,385

Here's how you can graph these data.

The interval is 10,000. This allows the large numbers to be shown and keep the graph a reasonable size.

The scale is in thousands. The number 20,000 on the scale stands for 20,000 thousand, which is 20 million.



The numbers on the tops of the bars have been rounded to the nearest thousand.

The zigzag lines on the bar and the vertical axis show that some numbers have been left out.

These are the names of the countries.

Use the graph to answer the questions.

1. If the scale showed the viewing data in millions, how would you write the numbers on the scale?
2. How would you use the graph to write the viewing data for Canada in standard form?
3. What do you need to consider when you are choosing a scale?

Lesson 2

Audio Tutor 1/26 Listen and Understand

Frequency Tables and Histograms

Objective Make and interpret frequency tables and histograms.

Vocabulary
frequency table
histogram



Learn About It

You can organize data in a tally chart or **frequency table**. Another way to display the data is to use a **histogram**, a special bar graph that represents the frequency of events. There are no spaces between the bars of a histogram because there are no gaps between the intervals.

Olympic gymnasts spend many hours each day practicing. In a survey, Pang and Melba asked 25 members of the school Gymnastics Club how many hours they practiced last week. How many of the members practiced for at least 15 hours?

Practice Time Last Week
4, 5, 6, 8, 8, 9, 10, 10,
10, 11, 12, 12, 12, 12,
14, 15, 15, 16, 18, 18,
22, 24, 25, 26, 30

Different Ways to Display Frequency

Way 1 You can use a frequency table.

Melba chooses 5-hour intervals. She uses tally marks to record the number of responses in each interval.

Practice Time		
Intervals (hours)	Tally Marks	Frequency
0–4	|	1
5–9	𝍸	5
10–14	𝍸 ||||	9
15–19	𝍸	5
20–24	||	2
25 and more	|||	3

Way 2 You can use a histogram.

Pang divides the horizontal axis into 5-hour intervals. He labels the vertical axis *Frequency*. He draws bars to show the frequency in each interval.



Solution: Add the frequencies for the 15–19 hours, 20–24 hours, and 25 hours and more intervals. Ten members practiced for at least 15 hours.

Extra Help at eduplace.com/map

Guided Practice

Use the frequency table or the histogram on page 238 for Questions 1–3.

1. How many of the members surveyed practiced for less than 20 hours?

2. How many more members practiced 10–14 hours than 15–19 hours?

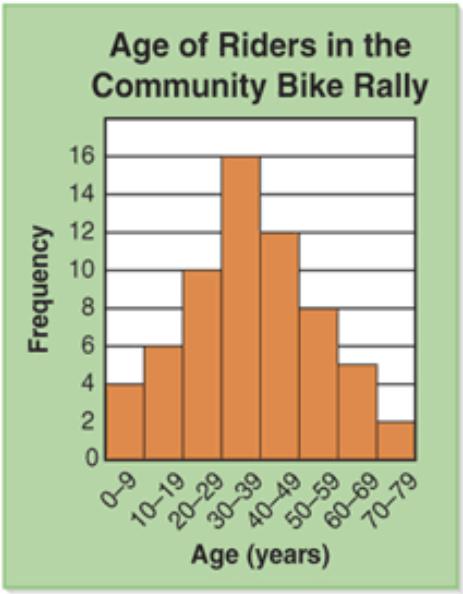
3. Which intervals have the same frequency?

Ask Yourself
- Which interval do I look at?
- Do I need to add or subtract?

Explain Your Thinking ▶ How are histograms and bar graphs alike? How are they different?

Practice and Problem Solving

Use the histogram at the right for Problems 4–9.



4. How many riders are from 20 to 49 years old?

5. How many of the riders are at least 40 years old?

6. **What's Wrong?** Lily says there are more than 75 riders in the community bike rally. Explain why she is wrong.

7. **Explain** Are most of the riders under 40 years of age?

8. **Analyze** Which bar of the histogram contains the median of the data? How do you know?

9. **Write About It** Redraw the histogram using four intervals instead of eight. Which histogram gives more information? Explain.

Mixed Review and Test Prep

Open Response

Find the mean, median, mode, and range for each data set. (Ch. 9, Lesson 3)

10. Test scores: 86, 69, 85, 91, 69

11. Ages (years): 10, 12, 15, 15, 10, 9

12. Prices: $22, $22, $11, $21, $20, $21, $23

13. Make a histogram to display these data. (Ch. 10, Lesson 2)

1, 2, 3, 4, 5, 6, 6, 9, 11, 14, 17, 19

Explain your choice of interval.

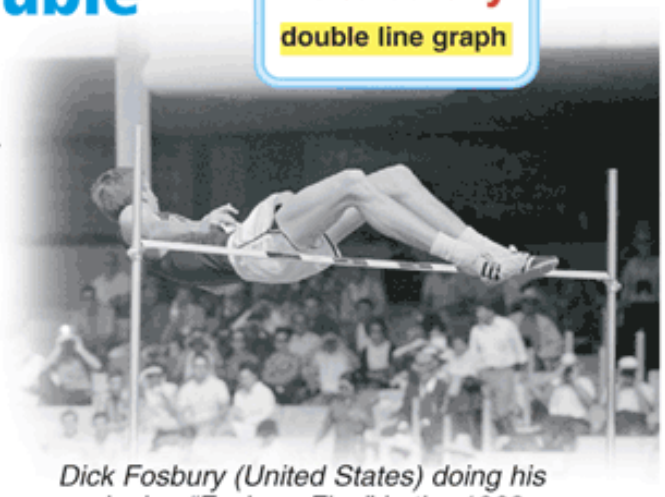
Extra Practice See page 259, Set B.

Lesson 3

Read and Make Double Line Graphs

Vocabulary
double line graph

Objective Make and interpret double line graphs.



Dick Fosbury (United States) doing his winning "Fosbury Flop" in the 1968 Mexico City summer games.

Learn About It

The winning heights in the Olympic high jump have changed over time for both men and women athletes. Use the table. In which year was the difference between men's and women's winning height the greatest?

To compare two sets of data that change over time, use a **double line graph**.

Olympic Winning High Jumps 1960–2004
(in centimeters)

Year	1960	1964	1968	1972	1976	1980	1984	1988	1992	1996	2000	2004
Men	216	218	224	223	225	236	235	238	234	239	235	236
Women	185	190	182	192	193	197	202	203	202	205	201	206

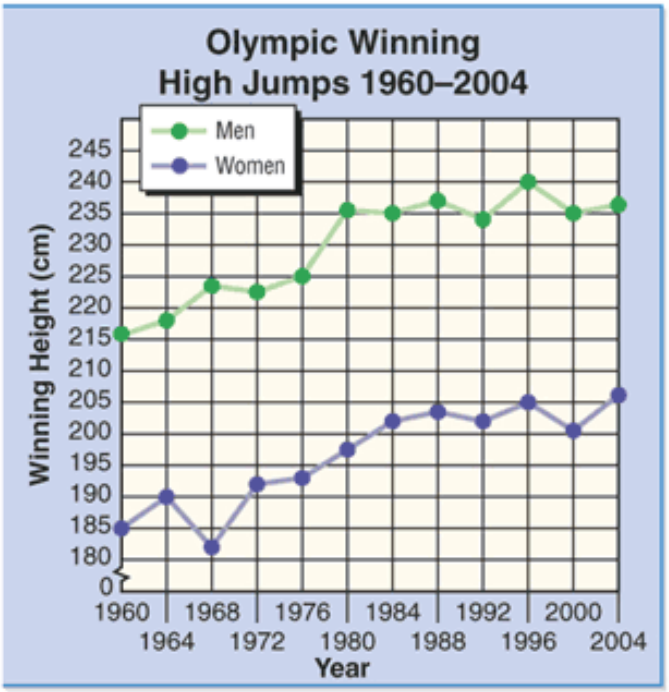
Making a Double Line Graph

STEP 1 Draw the axes. Label the horizontal axis *Year* and the vertical axis *Winning Height*. Choose an appropriate scale and mark equal intervals.

STEP 2 Plot the data for men. Use straight lines to connect the data points.

STEP 3 Repeat Step 2 for women.

STEP 4 Make a key to show what each line represents. Add a title to the graph.



Solution: Find the greatest distance between the two graphs.
The greatest difference in winning heights was in 1968.

Guided Practice

Use the graph on page 240 for Exercises 1 and 2.

1. What can you tell about the trends in winning jump height for men and women between 1960 and 2004?
2. Between which two Olympics did the men's winning height change the most?
3. Use the table or the graph on page 240 to find the difference in men's and women's winning heights for each year. Find the mean of the differences. What do you notice?
4. The table below gives the distances of Olympic winning javelin throws for men and women between 1960 and 2004, rounded to the nearest whole meter. Make a double line graph to compare the distances thrown.

Ask Yourself

- Is the information I need to know on the horizontal or vertical axis?
- Which interval should I choose?

Olympic Winning Javelin Throws 1960–2000
(in meters)

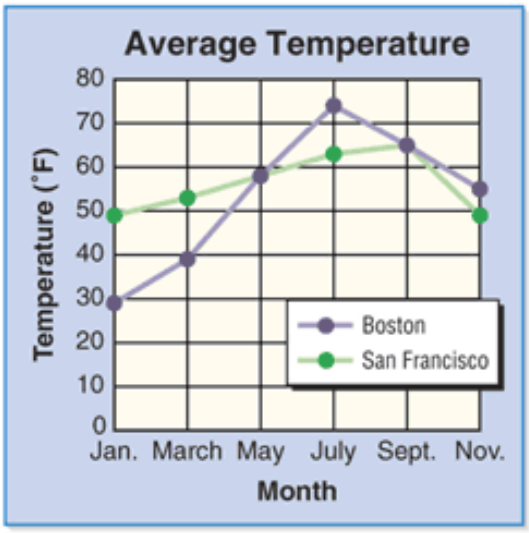
Year	1960	1964	1968	1972	1976	1980	1984	1988	1992	1996	2000	2004
Men	85	83	90	90	95	91	87	84	90	88	90	87
Women	56	61	60	64	66	68	70	75	68	68	69	72

5. **Write About It** Use your double line graph to describe the data.

Explain Your Thinking How can a double line graph help you compare data?

Practice and Problem Solving

Use the graph at the right for Exercises 6–9.



6. Runners training for marathons are affected by the temperature. In which city will runners train with the higher average temperature in July?
7. In which months will runners in Boston and San Francisco train with the same average temperature?
8. **Estimate** What is the greatest difference in temperature? In which month is it?
9. **Analyze** Which city has a greater range in average temperature? What is the range?

Go On

Use the table below for Problems 10–12.

Ticket Sales							
Day	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Janet	$40	$35	$25	$30	$30	$45	$55
Linda	$30	$40	$30	$25	$35	$55	$50

10. Janet and Linda sold tickets to a school athletic event. The table shows their ticket sales for each day during one week. Display the data in a double line graph.

11. **Analyze** How can you use the graph to tell whether Janet's and Linda's ticket sales were higher during midweek or during the weekend?

12. **Create and Solve** Write a problem that uses the double line graph to solve. Give your problem to a classmate to solve.

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Data **Use the table for Problems 13–16. Then tell which method you chose.**

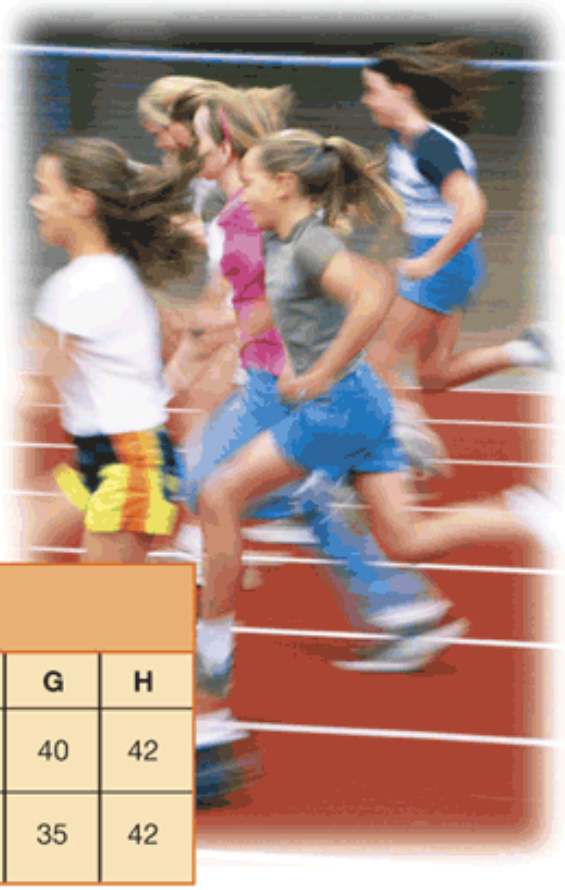
The track team held tryouts. Each of the eight candidates had two chances to complete a 200-meter run. The table below shows the time in seconds for each runner's tryouts.

13. Find the mean, median, mode, and range for each set of tryouts.

14. **Analyze** What would happen to the mean of the first set of tryouts if Runner *D* had not run?

15. **Explain** Look at all the data. Were most of the tryout times greater than or less than 35 seconds?

16. **You Decide** What kind of graph would you use to compare the two sets of tryouts? Explain your choice.



Results of Track Team Tryouts (in seconds)								
Runner	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
First Tryout	30	34	38	75	40	29	40	42
Second Tryout	28	30	40	35	36	32	35	42

Extra Practice See page 259, Set C.

Problem Solving

Logical Thinking

Race for the Gold!

Five students competed in a race—Alicia, Botan, Charles, Dwayne, and Ellen. Use the three clues below to find out in which order the runners crossed the finish line.

Clues

- Alicia did not finish immediately before or after Botan or Charles.
- Dwayne finished immediately before or immediately after Charles.
- Ellen finished first, and Alicia did not finish last.



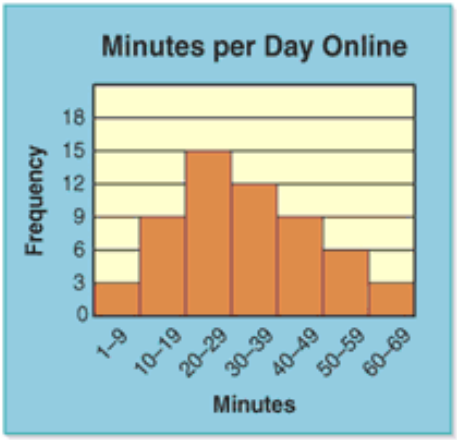
Quick Check

Check your understanding of Lessons 1–3.

Use this table for Problems 1 and 2. (Lesson 1)

Normal January Temperatures (°F)		
City	**High**	**Low**
Boston	35	25
Chicago	30	15
Houston	60	40
Miami	75	60
Seattle	45	38

Use this histogram for Problems 3 and 4. (Lesson 2)



1. Use this table to make a double bar graph.
2. Which city had the least difference between its normal high and low temperatures?
3. How many people spend at least 40 minutes per day online?
4. How many people spend more than 19 minutes but less than 50 minutes per day online?
5. Display the data below in a double line graph. (Lesson 3)

Total Sales for Six Months						
Month	Jan.	Feb.	Mar.	Apr.	May	June
Toy Town	$0	$10,000	$20,000	$25,000	$35,000	$45,000
Toy Mart	$0	$50,000	$10,000	$20,000	$45,000	$50,000

Lesson 4

Audio Tutor 1/27 Listen and Understand

Stem-and-Leaf Plots

Objective Make and interpret stem-and-leaf plots.

Vocabulary
- stem-and-leaf plot
- stem
- leaf



Learn About It

You can use a **stem-and-leaf plot** to organize data. A stem-and-leaf plot separates the data into groups and orders the data from least to greatest.

Alex and Mika found the following data set of U.S. athletes who competed in various sports at the 2004 Summer Olympic Games. How can Alex and Mika organize the data in a stem-and-leaf plot?

6, 25, 24, 12, 13, 27, 7, 15, 37, 15

Making a Stem-and-Leaf Plot

STEP 1 Write each number as tens and ones.

6 = 0 tens 6 ones
25 = 2 tens 5 ones
24 = 2 tens 4 ones
12 = 1 ten 2 ones
...
15 = 1 ten 5 ones

STEP 2 In the left column, list the tens digits from least to greatest. Each of these tens will be a **stem**.

Number of U.S. Athletes in 2004 Olympic Sports

Stem	Leaves
0	6 7
1	2 3 5 5
2	4 5 7
3	7

Key: 3|7 means 37 athletes

STEP 3 Next to each stem, write the ones digits for all numbers in the data set that begin with that stem. Order the digits from least to greatest. Each of these digits is called a **leaf**.

STEP 4 Create a key to show the meaning of each point in the stem-and-leaf plot.

Guided Practice

Use the stem-and-leaf plot for Questions 1–3.

1. The number of U.S. athletes on the 2004 Olympics Volleyball team was 24. Where would you put 24 in the stem-and-leaf plot?

2. What is the range?

3. What is the mode?

Ask Yourself
- What place does the stem represent?
- What place does the leaf represent?

Explain Your Thinking How would you use a stem-and-leaf plot to find the median?

Practice and Problem Solving

Use the data set at the right for Questions 4–7.

8, 15, 14, 49, 18, 8, 10, 111, 24

4. What place value should be used for the stem?

5. Make a stem-and-leaf plot. What place value does a leaf represent?

6. What is the least number in the data set? What is the greatest number in the set?

7. Are there any outliers? If so, predict how they affect the mean and median.

The stem-and-leaf plot shows the batting averages of some Olympic baseball players. Use the plot for Problems 8–11.

8. Find the median, mode, and range of the numbers in the data set.

9. **Calculator** Find the mean. Did the stem-and-leaf plot help you find the mean? Explain.

10. How can a stem-and-leaf plot help you draw a histogram?

11. **Analyze** Suppose you add one data point to the stem-and-leaf plot. The range of the data stays the same and no new mode is added. The new median is 0.250. Your new data point must be between what numbers?

12. **Write About It** Is it easier to use a stem-and-leaf plot or a line plot to identify clusters, gaps, and outliers?

Batting Averages

Stem	Leaves
.21	1 7
.22	6 9
.23	4
.24	8
.25	0 6 6 9
.26	
.27	
.28	6
.29	4 5 8

Key: .22|9 means .229 batting average

Mixed Review and Test Prep

Open Response

Identify each property. (Ch. 2, Lesson 6)

13. $(4 + 7) + 8 = 8 + (4 + 7)$

14. $(6 \times 3) \times 5 = 6 \times (3 \times 5)$

15. $9 \times 17 = 9 \times 10 + 9 \times 7$

16. $2 \times (7 \times 9) = (7 \times 9) \times 2$

17. This data set shows the average number of students in various schools. (Ch. 10, Lesson 4)

332, 345, 310, 357, 326, 345, 334, 321, 315, 348, 316, 348, 323, 359

Explain how to make a stem-and-leaf plot to show the data.

Extra Practice See page 259, Set D.

Hands On Lesson 5

Make a Box-and-Whisker Plot

Objective Make a box-and-whisker plot and understand its parts.

Vocabulary

box-and-whisker plot

Materials

large sheet of paper
markers
small adhesive notes

Work Together

You can use a **box-and-whisker plot** to show how data are spread out and where data cluster.

This data set shows the lengths (rounded to the nearest foot) of the winning distances in the women's Olympic shot-put event for the years 1960 to 2000.

57, 60, 64, 69, 69, 74, 67, 73, 69, 67, 67

STEP 1 Order the data from least to greatest and find the median. Next find the upper and lower *quartiles.* These are the medians, or middle values, of each half of the data.

57, 60, 64, 67, 67, 67, 69, 69, 69, 73, 74

lower quartile: 64 median: 67 upper quartile: 69

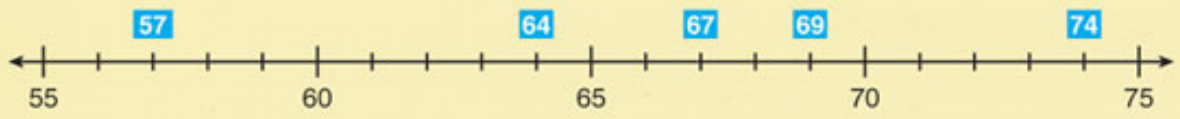
The lower quartile, median, and upper quartile separate the data set into four equal parts.

STEP 2 Find the *extremes,* or the greatest value and the least value in the data set.

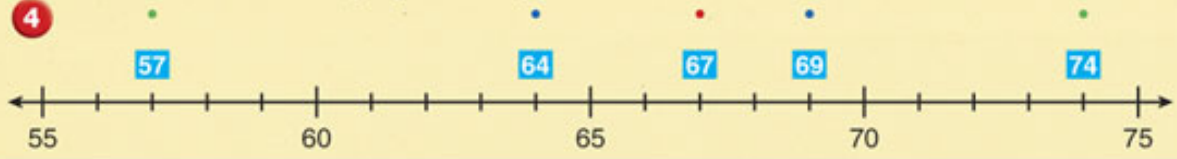
57, 60, 64, 67, 67, 67, 69, 69, 69, 73, 74

lower extreme: 57 upper extreme: 74

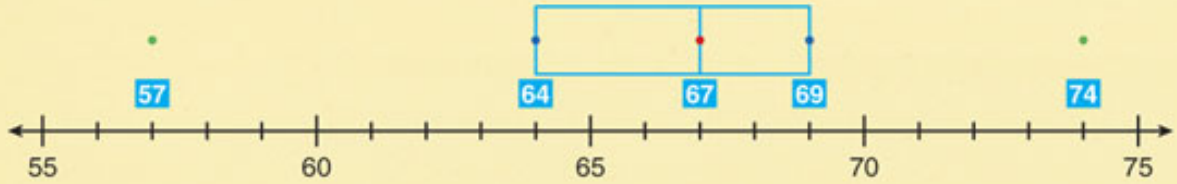
STEP 3 Draw a number line on a large sheet of paper. Begin your number line before 57 and end it after 74. Copy the values of the five points you found on adhesive notes. Place the notes above the number line.



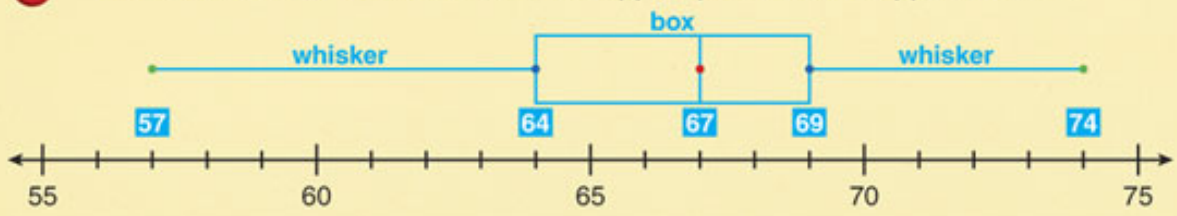
STEP 4 Use markers to display the five points.



STEP 5 Draw a box, using the lower and upper quartiles as sides. Then draw a vertical line segment through the median.



STEP 6 Draw a horizontal line from the lower extreme to the lower quartile. Draw another horizontal line from the upper quartile to the upper extreme.



On Your Own

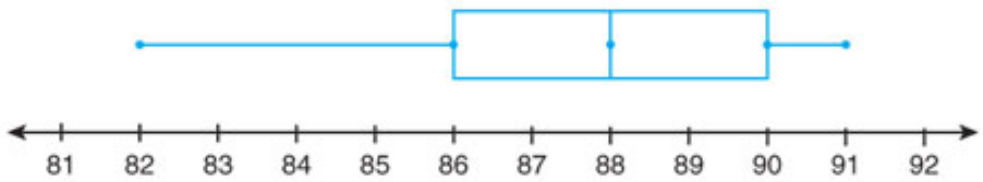
1. Look at the box-and-whisker plot you made. What fraction of the data are in the box? in the whiskers?
2. These data show the heights (in inches) of the U.S. Olympic women's basketball team members in 2000. Make a box-and-whisker plot to display the data.

68, 71, 75, 74, 77, 71, 73, 71, 66, 72, 74, 79

Talk About It • Write About It

You learned how to make a box-and-whisker plot to display data.

3. **Explain** Look at the box-and-whisker plot you made in Exercise 2. Is the median in the center of the box? How do you know?
4. **Create Your Own** Create a set of 12 numbers that this plot might represent. Explain how you chose your data.



Lesson 6

Audio Tutor 1/28 Listen and Understand

Box-and-Whisker Plots

Objective Interpret box-and-whisker plots.

Vocabulary

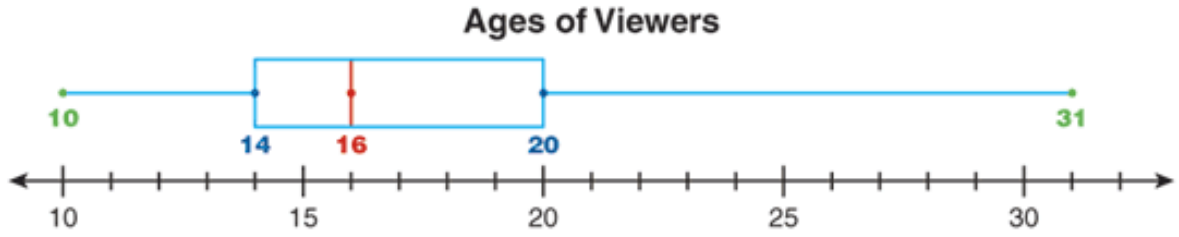
- lower extreme
- upper extreme
- lower quartile
- upper quartile

Learn About It

Bert surveyed 40 people about watching the Olympics on television. He collected data on the ages of the people who watched at least one Olympic event.

Bert displayed his data in this box-and-whisker plot. Tell what the data points represent. What does the plot tell you about how the data are distributed?

Ages of Viewers
10, 10, 11, 12, 12, 13, 13, 13,
14, 14, 14, 14, 14, 15, 15, 15,
16, 16, 16, 16, 16, 17, 17, 17,
18, 18, 18, 18, 19, 20, 21, 22,
26, 27, 27, 28, 29, 29, 30, 31



Interpreting a Box-and-Whisker Plot

- The **lower extreme** is 10 and the **upper extreme** is 31.

  The youngest person is 10 years old, and the oldest person is 31 years old. The range of ages is 31 − 10, or 21 years.

- The median age of the viewers is 16. The **lower quartile** is 14. The **upper quartile** is 20.

  Half the viewers are under 16 and half are over 16.

  Half of the viewers are between 14 and 20 years old.

- The part of the box between the lower quartile and the median is shorter than the other part of the box or either of the whiskers. The data cluster in the quarter that represents viewers aged 14–16.

- The right whisker is longer than the left whisker or any part of the box. The data are spread out in the quarter that represents viewers aged 20–31.

Guided Practice

Use the box-and-whisker plot below for Questions 1–3.

1. What are the least and greatest amounts of time spent exercising?

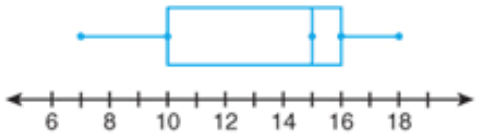
2. What is the median amount of time? the upper and lower quartiles?

3. Where are the data clustered? What does it tell you about minutes spent exercising?

Ask Yourself

- Where do I find the quartiles?
- What do the lengths of the whiskers and box parts mean?

Minutes Spent Exercising



Explain Your Thinking Can you determine mean and mode from a box-and-whisker plot?

Practice and Problem Solving

Use the test scores at the right for Problems 4 and 5.

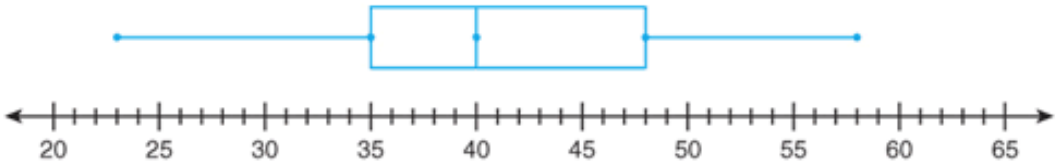
67, 76, 82, 80, 87, 83, 92, 88, 93, 89, 95, 92, 96, 92, 99

4. Make a box-and-whisker plot to represent the data.

5. **Write About It** Use the plot to describe the data set.

6. **Explain** What are the only true data points that can be identified from a box-and-whisker plot?

This box-and-whisker plot displays the number of sit-ups completed in one minute by each of 11 students. Use the plot for Problems 7–10.



7. What is the median number of sit-ups completed in one minute? What are the upper and lower quartiles?

8. What are the lower and the upper extremes of the data for sit-ups? What do the extremes mean?

9. **You Decide** Write a set of 11 numbers that this box-and-whisker plot could represent.

10. **Predict** What would happen to the box if data points representing 42 and 47 sit-ups were added?

Mixed Review and Test Prep

Open Response

Find each missing number. (Ch. 8, Lesson 2)

11. $\frac{1}{2}$ ft = ■ in.

12. $1\frac{1}{2}$ lb = ■ oz

13. 4 in. = ■ ft.

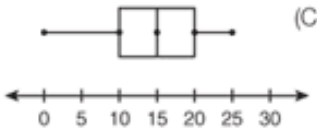
14. 40 oz = ■ lb

15. $1\frac{1}{4}$ T = ■ lb

16. 2 qt = ■ gal

Multiple Choice

17. What is the upper quartile of the data in this box-and-whisker plot? (Ch. 10, Lesson 6)



A 10 **B** 15 **C** 20 **D** 25

Extra Practice See page 259, Set D

Lesson 7

Misleading Data Displays

Objective Interpret data displays and draw accurate conclusions.



Learn About It

Misleading data displays can lead you to draw incorrect conclusions about the data. Therefore, you must be careful when you interpret or draw data displays.

Interpreting Data Displays

STEP 1 Read the title. Is the title a clear and fair description of the data?

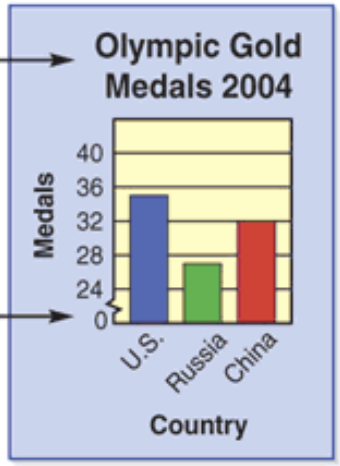
STEP 2 Look at the scale or the key carefully.

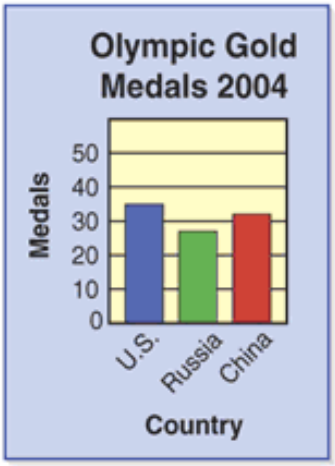
- Do the intervals fit the data?
- Does the scale begin at zero or is the scale broken?
- Is the key clearly labeled?

STEP 3 Decide whether there are irregularities in the graph that cause the graph to be misleading.

The two bar graphs below show the same information, the number of gold medals won by the United States, Russia, and China at the 2004 Summer Olympics. How is the first graph misleading?

- The title of both graphs is the same.
- The vertical scales are different. There is a break in the vertical scale of the first graph.
- The second graph shows the same data but in a fair way.





Solution: The first graph is misleading because the bars are drawn so that it appears that the United States won twice as many medals as Russia.

These pictographs compare the number of gold and silver medals won by the United States and Russia in the 2000 Summer Olympics. What makes these pictographs misleading?

Displaying Pictographs



$(8 \times 5) + (8 \times 3) = 40 + 24$, or 64.

The United States won 64 gold and silver medals combined.



$(4 \times 8) + (4 \times 7) = 32 + 28$, or 60.

Russia won 60 gold and silver medals combined.

Solution: Look at the keys. The graphs are misleading because the symbol stands for a different number of medals in the two graphs.

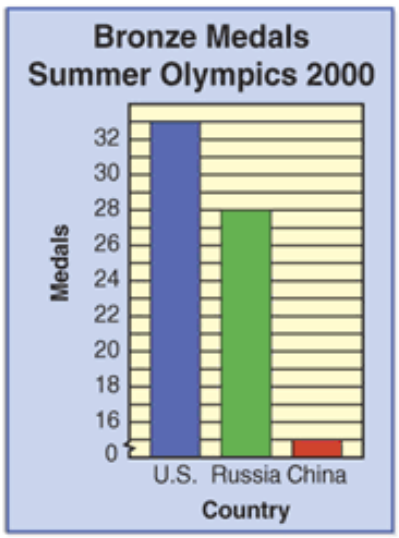
Guided Practice

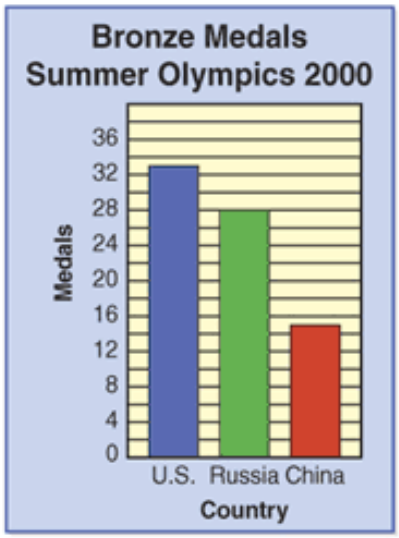
Use the bar graphs below for Problems 1 and 2.

Ask Yourself

- Are the scales the same?
- Do the symbols stand for the same number of objects?

1. What might you conclude from a quick glance at the graph on the left?
2. **Explain** Why is the graph on the left misleading?





3. **Represent** Look at the pictographs above, showing the number of medals won at the 2000 Olympics. Redraw the pictographs to show the data in a way that is not misleading.

Explain Your Thinking ▶ What other factors could cause a data display to be misleading?

Go On

Practice and Problem Solving

Use these pictographs for Problems 4–6.





4. What might you conclude from a quick glance at the two graphs?

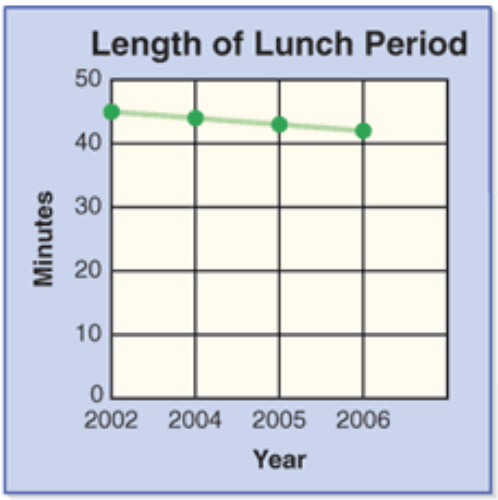
5. **Explain** Why are the two pictographs misleading?

6. **Represent** Draw pictographs that show the data in a way that is not misleading.

7. **Analyze** Over four years the average length of lunch periods in a school district decreased from 45 minutes to 42 minutes. Does the graph show the lunch period data in a fair way?

8. Redraw the graph in a way that makes it appear that the lengths of lunch periods has decreased greatly.

9. **Create and Solve** Draw a graph that may be misinterpreted. Then ask a classmate to explain why your graph is misleading.



Mixed Review and Test Prep

Open Response

Evaluate each expression and tell what properties you used. (Ch. 2, Lesson 7)

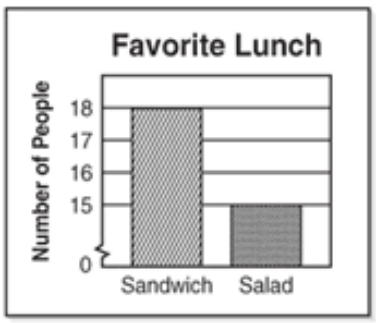
10. $4 + 79 + 16$

11. $16.8 \times 9 \times 0$

12. $19 \times 9 + 19 \times 1$

13. $(46.5 \times n) \times 2$, given $n = 5$

14. What might you conclude when you look at the bar graph? (Ch.10, Lesson 7)



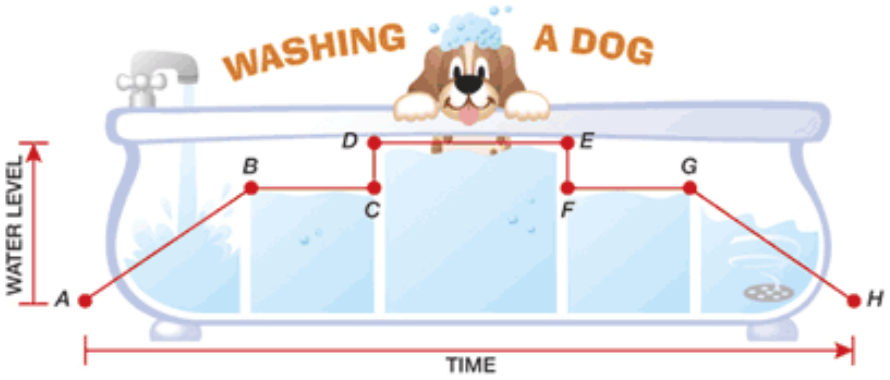
Explain why.

Extra Practice See page 259, Set E.

Visual Thinking

Graphs Without Numbers

Line graphs show changes over time. Between points *A* and *B* the tub was filling. Between points *B* and *C*, you went to get the dog.



1. The water level of the tub rises when you put the dog in the tub. At what point did you put the dog in the tub? At what point did the dog jump out of the tub?
2. What do you think happened between points *D* and *E*?
3. What do you think happened between points *G* and *H*?
4. **You Decide** Write a story and draw a graph to represent the story.

Math Reasoning

Guess the Weight



Mary, Joni, and Trey each guessed the weight of the pumpkin in pounds.

Mary	Joni	Trey
646 lb	523 lb	499 lb

One of the guesses is off by 124 pounds. One is off by 100 pounds. One is off by 23 pounds. What is the actual weight of the pumpkin?

Brain Teaser

In an ancient problem, a man wanted to carry wind, fire, and water across a stream. He could take only two of these in his boat at the same time.

- He could not leave fire and wind alone because wind blows out fire.
- He could not leave fire and water alone because water puts out fire.
- He could not leave wind and water alone because they would fight and cause a terrible storm.

How could the man carry all three across the stream?

Education Place

Check out **eduplace.com/map** for more brain teasers.

Lesson 8

Problem-Solving Application

Use a Graph

Objective Choose the most appropriate type of data display.



Problem This table shows the amounts that Grades 5 and 6 raised for the Special Olympics from September through May. What is the most appropriate graph for comparing the data? Use that type of graph to display the data.

The Special Olympics is an international organization that offers children and adults with special needs training and competition in Olympic-type sports.

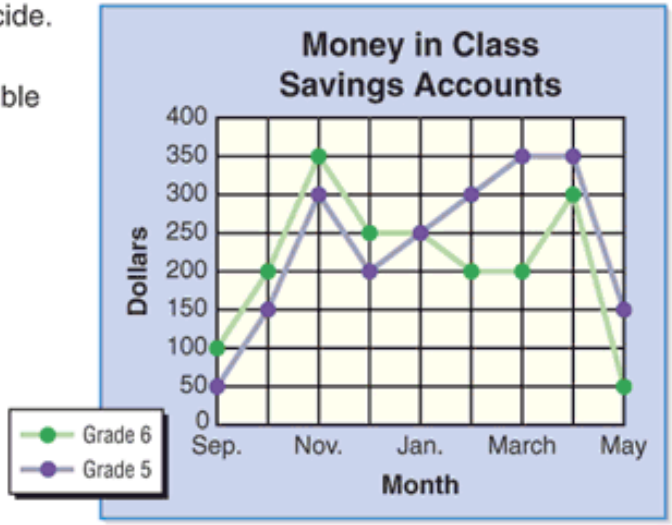
Money in Class Savings Accounts—September Through May

Month	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
Grade 5	$50	$150	$300	$200	$250	$300	$350	$350	$150
Grade 6	$100	$200	$350	$250	$250	$200	$200	$300	$50

- Look at the chart below to help you decide.
- The two sets of data change over time. So the most appropriate graph is a double line graph.
- Draw the graph.

Type of Graph

bar graph: data can be counted
line graph: data change over time
pictograph: data are in multiples
circle graph: data are part of a whole
histogram: data come in intervals



Look Back Is there another kind of graph you could have used?

Guided Practice

Use the data in the table on page 254 and the Ask Yourself questions to help you solve each problem.

Ask Yourself

- Am I comparing sets of data?
- What information is most important?

1. Which type of graph would you use to show how much money was in both accounts in the month of November? Explain.

2. Which type of graph would you use to show how much of the $350 raised by the sixth grade in November was due to a bake sale, a used book sale, or ticket sales for a class music performance? Explain.

Independent Practice

Use a graph to display each data set. Explain your choice of graph.

3. Twenty people sponsored Jamie in a charity walk. Eight people pledged from $1 to $5, 6 people pledged from $6 to $10, 2 people pledged from $11 to $15, and 4 people pledged between $16 and $40.

4. Last month a hiking club spent $100 on transportation, $50 on food, $25 on gear, and $25 on other expenses. Luz wants to prepare a display showing how each expense compares with the total amount.

5. Nick recorded the number of hits made by team members in a baseball game.

   0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 4, 4, 4, 5, 5, 5

   He wants to show clusters and gaps in the data.

6. Aki collected these data on contributions made to the Special Olympics.

   $1, $2, $2, $3, $4, $5, $5, $5, $6, $7, $7, $8, $10, $14, $15

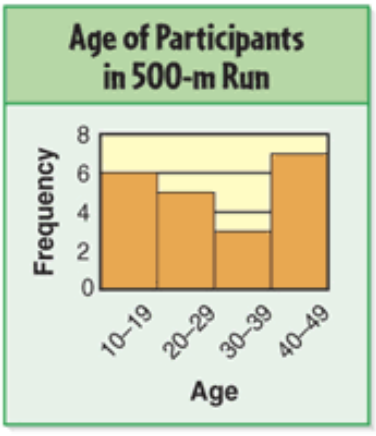
   She wants to show how the data are distributed.

The displays for Problems 7 and 8 represent the same data.

Age of Participants in 500-m Run

Stem	Leaves
1	2 3 4 8 9 9
2	0 4 5 6 8
3	2 3 4
4	1 3 7 7 7 8 9

Key: 1|2 means 12



7. When might you want to use the stem-and-leaf plot to analyze the data?

8. When might you want to use the histogram to analyze the data?

Go On

Solve. Show your work. Tell what strategy you used.

9. **Reasoning** Earl and his friends live in an apartment building. After visiting Gerry, Earl went down 3 floors to visit Tom, then up 7 floors to visit Dipak, and then down 9 floors to visit Jake, who lives on the second floor. On what floor does Gerry live?

10. How many different ways can you make change for 35 cents with quarters, dimes, and nickels?

11. At an amusement park, Joan spent half of her money on rides. She spent one third of her remaining money on refreshments. She left the amusement park with $10. How much money did Joan have to start with?

You Choose

Strategy

- Draw a Diagram
- Guess and Check
- Make an Organized List
- Use Logical Reasoning
- Work Backward

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator



Solve. Tell which method you chose.

12. Fruit juice is sold in 200-milliliter bottles. How many liters of juice are in 10,000 bottles?

13. Three packages weigh 2 pounds 7 ounces, 4 pounds 10 ounces, and 13 ounces. What is the total weight?

14. Tina can run one lap around a track in 1 minute 30 seconds. At that rate, how long will it take her to run three laps?

15. Gabe earned $78 for 8 hours of work and $30 for 4 hours of work. What was his average hourly wage?

16. The hourly wages of the 8 employees of a company are $10.50, $7.50, $13.80, $11.25, $15.75, $16.80, $16.00, and $22.00. What is the average hourly wage?

17. Jodi is saving for a CD player. The CD player costs $159.99. She can save about $40 per month. About how many months will it take Jodi to save enough money?

18. A recipe calls for $1\frac{2}{3}$ cups of vegetable broth. Jason wants to triple the recipe. Broth is sold in 1-pint cans. How many cans will Jason need to buy?

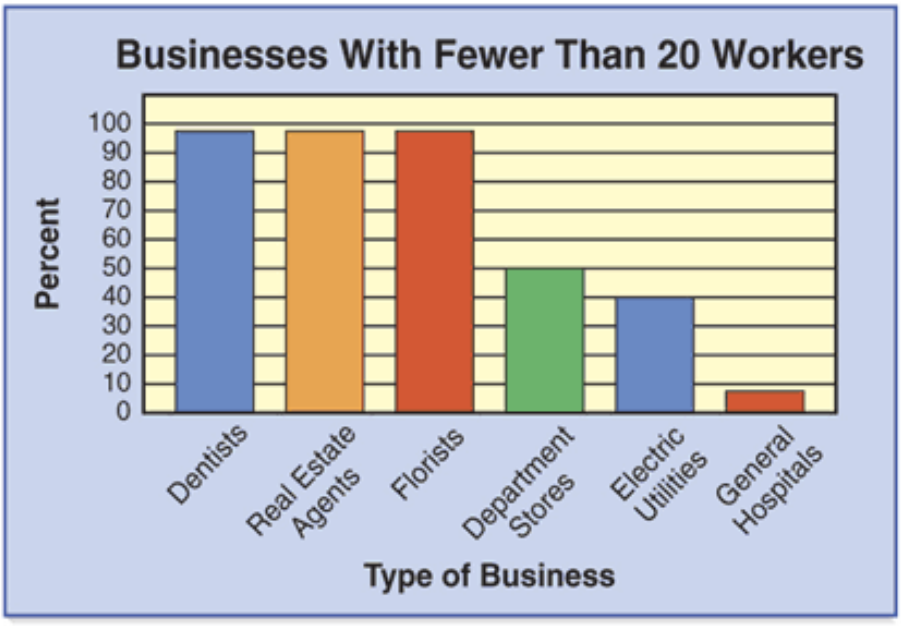
19. In a school with 576 students, $\frac{3}{4}$ of the students signed up to go on a field trip. How many students did not sign up for the field trip?

Activity

Real-World Connection

Taking Care of Business

A newspaper reported on the types of businesses most likely and least likely to be small (fewer than 20 workers). This graph shows the information for six types of businesses (rounded to the nearest percent).



1. **Explain** If you worked as a florist, would you be more likely to work at a small or a large company?

2. For which type of business would you be most likely to work at a large company?

3. Name some types of information that can be obtained by examining this bar graph.

4. Choose three businesses displayed in the bar graph, and survey 10 students in your class. Find which of the selected businesses the students surveyed have visited in the past 6 months. Create a bar graph to show your results.

5. Exchange your bar graph with another student. List three kinds of information that can be determined by the graph you received.

Chapter Review/Test

VOCABULARY

1. In a box-and-whisker plot, the greatest number is called the upper _____.
2. Each bar in a histogram represents a(n) _____.
3. Each ones digit represents a(n) _____ in a stem-and-leaf plot.

Vocabulary

leaf
stem
extreme
interval
quartile

CONCEPTS AND SKILLS

Use the data set at right. (Lesson 3, pp. 240–242)

4. Make a double line graph to represent the data.
5. What trends do you notice in the graph?
6. Use your graph. When was the difference in the number of push-ups the least? the greatest? Explain.

Push–ups per Minute

Week	1	2	3	4	5	6
Raj	10	20	20	30	40	42
Tim	5	10	20	15	25	30

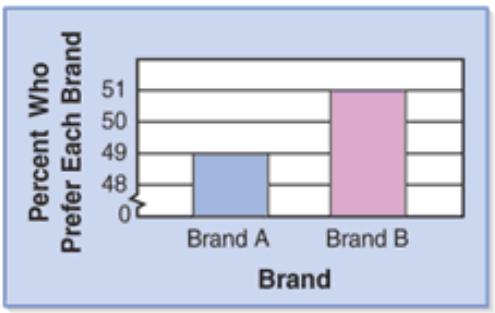
Use the data set at right for Exercises 7–10.
(Lesson 2, pp. 238–239, Lessons 4–6, pp. 244–249)

18, 16, 22, 14, 24, 16, 20, 21, 19, 25, 20

7. Make a stem-and-leaf plot of the data. Then find the median.
8. Choose an interval. Make a histogram. In which bar is the median? Explain.
9. Make a box-and-whisker plot. What are the lower and upper quartiles?
10. Use your box-and-whisker plot to describe the data.

Use the bar graph for Exercise 11.
(Lesson 1, pp. 234–236, Lesson 7, pp. 250–252)

11. What might you conclude when you look at the bar graph? What influenced your conclusion?



PROBLEM SOLVING

Solve. (Lesson 8, pp. 254–256)

12. Meg spent $15 of her birthday money on CDs, $5 on snacks, and saved the rest. What type of graph would you use to show these data? Explain your choice.

Write About It

Show You Understand

Why is it important to know how to use graphs to represent data?

Extra Practice

Set A (Lesson 1, pp. 234–236)

1. Make a double bar graph to display the data in the table at right.

2. Use your graph. On which day was the difference between fiction and nonfiction sales the greatest?

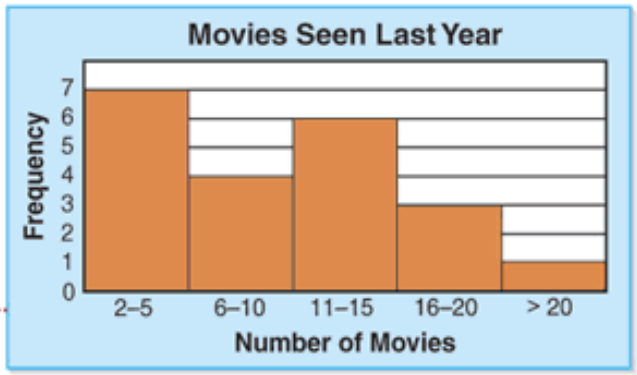
Books Sold by Clerk					
Type	**Mon.**	**Tue.**	**Wed.**	**Thu.**	**Fri.**
Fiction	16	18	22	19	24
Nonfiction	12	18	16	22	20

Set B (Lesson 2, pp. 238–239)

Use the histogram for Problems 1 and 2.

1. How many people saw fewer than 16 movies?

2. How many more people saw 5 or fewer movies than 16–20 movies?



Set C (Lesson 3, pp. 240–242)

Use the data at right for Problems 1 and 2.

1. Make a double line graph to display the data.

2. Would you expect the temperature in July to be higher in Fresno or in Los Angeles? Explain.

Average Monthly Temperature (°F)						
Month	**Jan.**	**Feb.**	**Mar.**	**Apr.**	**May**	**June**
Fresno	46	51	55	61	69	77
Los Angeles	58	60	61	63	66	70

Set D (Lessons 4 and 6, pp. 244–245 and 248–249)

Use the data set at right for Problems 1 and 2.

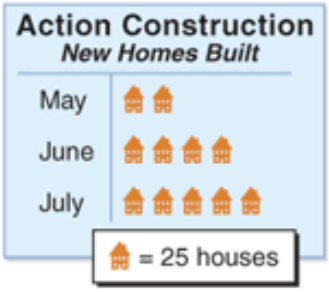
1. Make a stem-and-leaf plot. Then find the median number of points scored.

2. Make a box-and-whisker plot. What are the lower and upper quartiles and extremes?

Points Scored
21, 18, 27, 14, 35, 31, 27, 21, 30, 29

Set E (Lesson 7, pp. 250–252)

What makes these pictographs misleading? Why?





Extra Practice at **eduplace.com/map**

WEEKLY READER®
Social Studies Connection

Skiing through History

On the first Sunday in March each year, thousands of skiers from all over the world compete in a 90-kilometer cross-country ski race from Sälen to Mora in central Sweden. The race, called Vasaloppet, has become one of the most popular ski races in the world.

Vasaloppet commemorates the heroic journey of Gustav Eriksson Vasa in 1521. Legend has it that Vasa, the leader of a Swedish revolt against the Danish king, fled on skis from Mora towards Norway to avoid capture by the Danes. After Sweden's defeat of Denmark, Gustav Vasa became King of Sweden in 1523.





Problem Solving

1. **Solve** The data set at right shows the number of skiers who registered for each race 1992–2002. Find the mean, median, and mode.

2. The men's record time, set in 1998, is 3 h 38 min 57 s. The women's record time, also set in 1998, is 4 h 17 min 2 s. What is the difference between the record times?

3. The table at right shows stopping points along the Vasaloppet route. Use the data to graph the altitudes above sea level for each stop from start to finish.

4. **Estimate** In the 78 races between 1922 and 2002, 364,312 skiers have crossed the finish line in Mora. What is the average number of skiers per race that had crossed the finish line?

Vasaloppet 1992–2002
Number of Registered Skiers

12,277	13,014	15,826	13,551	15,729	15,058
13,807	15,883	16,402	15,000	16,486	

Vasaloppet Route

Food/Rest Stops	Altitude (m)	Distance (km)
Berga (Start)	350	0
Smågan	500	11
Mångsbodarna	450	24
Risberg	400	35
Evertsberg	425	48
Oxberg	225	62
Hökberg	275	71
Eldris	200	81
Mora (Finish)	150	90

Education Place
Check out **eduplace.com/map** for more brainteasers.

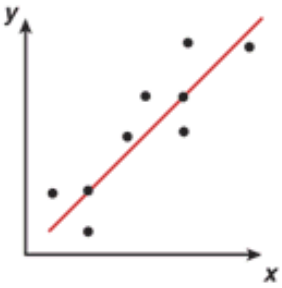
Enrichment: Data Sense

Scatter Plots

Materials
meter stick or tape measure
grid paper
ruler

A **scatter plot** shows the **correlation**, or relationship, between two sets of data. When you compare real-life data you will often get a scatter plot.

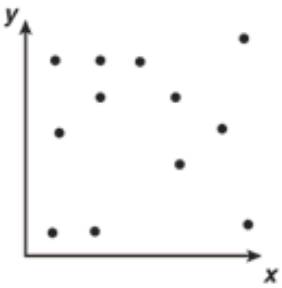
If the two data sets both increase, the scatter plot will show an upward trend, or *positive correlation*. If one data set increases as the other decreases, the scatter plot will show a downward trend, or *negative correlation*. Some scatter plots show no relationship, or *no correlation*, between the data sets.



positive correlation



negative correlation



no correlation

A **line of best fit** is a line that shows the trend. To find the line of best fit, draw a line that has an equal number of points above and below it.

Try These!

1. Work together to measure the arm span and height of each student in your class. Use a meter stick or tape measure to measure to the nearest centimeter. Record your measurements.

2. On grid paper, draw two axes. Label the vertical axis *Height (in centimeters)* and the horizontal axis *Arm Span (in centimeters)*. Choose a scale and intervals so as to make good use of the grid paper. Plot the data.

3. **Analyze** Look at your plot. Do you notice any correlation between arm span and height? Explain.

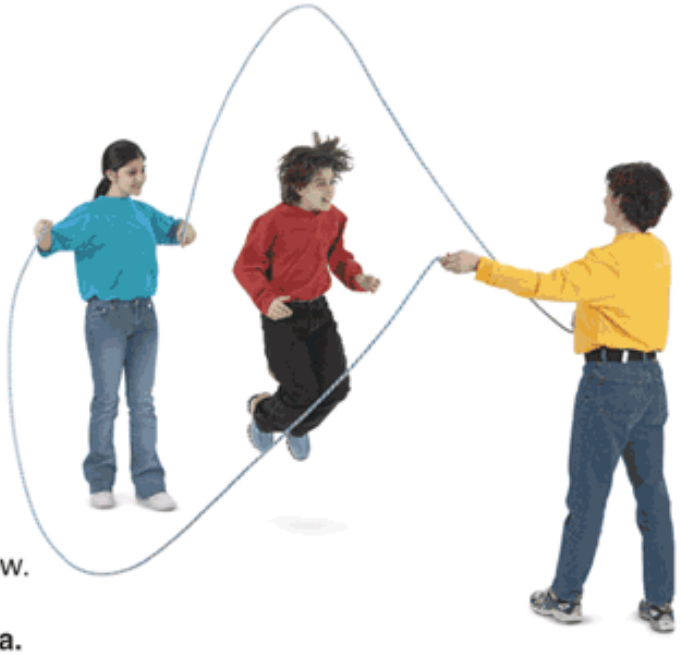
4. **Predict** Draw a line of best fit to your data. Then predict the height of a student whose arm span is 140 cm.



Explain Your Thinking

Why do the data points not all lie on a straight line?

Double Data



At a Double Dutch competition, one contest involves jumping rope as fast as possible for 2 minutes. The team names and the number of times the jumpers' feet hit the floor are shown in the table below.

You can use Easy Sheet to make a graph of the data.

- Type "Team Name" in cell A1. Then enter the team names in column A.
- Type "Number of Jumps" in cell B1. Then enter these data in column B.
- Click and drag from cell A1 to cell B10. Click  . Repeat twice, making a circle graph and a line graph.
- Double-click each graph. Choose **Labels** and then click **Label Data**.
- Type "Mean", "Median", and, "Mode" in cells A12–A14.
- To find the mean, click on cell B12. Click **Window**, then **Function List**. Double-click **Avg (range)**, then click and drag from cells B2 to B10 and press enter. Repeat in cells B13 and 14 to find the median and the mode.

Double Dutch Results

Team Name	Number of Jumps
Double Jeopardy	285
Double Trouble	293
Dutch Treat	296
Jive Jumpin'	300
Jump Masters	293
Jumpin' Beans	265
Jumpin' Jacks	289
Smoke Jumpers	283
Soul City Stars	306

Use the graphs you created to answer Problems 1–4.

1. What are the mean, median, mode, and range of the data?
2. Of the three graphs, which best suits these data? Explain your answer.
3. Which statistical measure would you use to find the number of jumps a team needed to make in order to be in the top half of the scores? Explain.
4. If you graphed these data as a histogram, a stem-and-leaf plot, and a line plot, how would each show a different aspect of the data?

Unit 3 Test

VOCABULARY Open Response

1. A(n) ____ is a number that is much greater or much less than other data in the set.
2. A ____ shows the extremes, the median, and the upper and lower quartiles of a data set.
3. Mean, median, and mode are ____.
4. The number of square units in a region is called the ____.

Vocabulary

area
outlier
perimeter
box-and-whisker plot
measures of central tendency
stem-and-leaf plot

CONCEPTS AND SKILLS Open Response

Find the sum or difference. (Chapter 8)

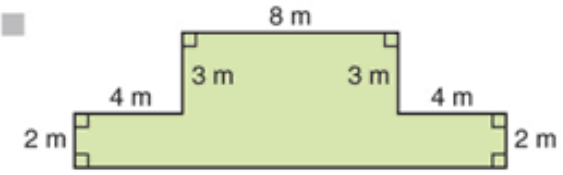
5. 4 mi 200 yd − 2 mi 300 yd
6. 9 lb 7 oz + 8 lb 10 oz
7. 12 min 37 s + 48 min 24 s
8. 9 qt 6 fl oz − 7 qt 25 fl oz

Complete. Use the conversion table on page 198. (Chapter 8)

9. 25 mi ≈ ■ km
10. 10 cm ≈ ■ in.
11. 8 kg ≈ ■ lb
12. 11 m ≈ ■ ft

Find the perimeter and area of the figure. (Chapter 8)

13. P = ■
14. A = ■

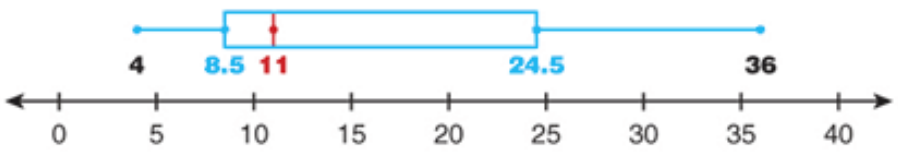


Find each measure of central tendency for the data set. (Chapter 9)

34, 24, 28, 34, 14, 36, 36, 34, 26, 26, 36, 24, 36, 26, 34, 36

15. range
16. mean
17. median
18. mode

Use the box-and-whisker plot for Problems 19–21. (Chapter 10)



19. median = ■
20. extremes = ■, ■
21. upper and lower quartiles = ■, ■

PROBLEM SOLVING Open Response

22. **Explain** A rectangle has a perimeter of 34 inches. One side of the rectangle has a measure of 4 inches. What are the measures of the other sides?

23. Five students want to make a graph to compare their batting averages. What type of graph would be their best choice? Explain.

24. **Write About It** Make a stem-and-leaf plot for the following data set.

2, 5, 18, 12, 21, 20, 33, 6, 17.

Identify any outliers. Use your plot to describe the data set.

25. A survey showed that 3 out of 7 sixth graders spend at least one hour a day playing sports. Thirty-five out of 100 sixth-graders at your school play sports. Are the data similar? Explain.

Performance Assessment

Extended Response



Task The student council wants to take a survey to find out in which Volunteer All-School Project students would like to take part.

Decide what volunteer projects to list on the survey. Explain your choices.

Use the data at right. Tell what steps the council might take to get a representative sample. Explain your thinking.

Information You Need

- Total student population: 1,100
- Sixth grade: 352 students
- Seventh grade: 358 students
- Eighth grade: 390 students

Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

Sometimes when you take a test, you can eliminate answer choices that are clearly wrong.

Look at the example below.

A field trip was supposed to last $2\frac{1}{2}$ hours, but instead lasted for $3\frac{3}{4}$ hours. By how many hours did the field trip run over?

A $\frac{3}{4}$ **C** $2\frac{3}{4}$

B $1\frac{1}{4}$ **D** $3\frac{1}{2}$

THINK

Use estimation: $3\frac{3}{4}$ is a little less than 4 and $2\frac{1}{2}$ is a bit more than 2. So the difference must be less than 2 hours and greater than 1 hour. Therefore, you can eliminate choices A, C and D.

Multiple Choice

1. Which is the least common multiple of 60 and 45?

A 45 **B** 90 **C** 180 **D** 240

(Chapter 3, Lesson 5)

2. The cafeteria had 64 apples for sale. They sold 48 apples. Which fraction, in simplest form, tells how many they've sold so far?

F $\frac{3}{4}$ **G** $\frac{6}{8}$ **H** $\frac{12}{16}$ **J** $\frac{7}{8}$

(Chapter 4, Lesson 1)

3. The students in Room 333 want to put a bookcase on a wall between the windows and a chalkboard. They measure from the windows to the chalkboard to find out how wide the bookshelf can be. How precise should their measurement be?

A to the nearest $\frac{1}{32}$ inch

B to the nearest inch

C to the nearest foot

D to the nearest yard

(Chapter 8, Lesson 2)

4. Pete wants to determine his classmates' favorite sport. He surveys his friends, especially members of his football team. Which best describes his sample?

F random **H** convenience

G biased **J** representative

(Chapter 9, Lesson 2)

For Test-Taking Tips, See page 648.

Open Response

5.

Year	2002	2003	2004	2005
Distance (in kilometers)	20.5	17.9	21.4	26.2

Each summer Winona and her father take a bicycle trip. The table shows how far they have traveled during each of the last four summers. How many kilometers have they biked in all?

(Chapter 1, Lesson 6)

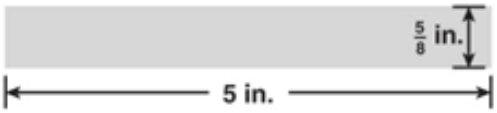
6. Every day, Bart buys 2 tuna sandwiches from the school cafeteria. If the tuna sandwiches cost $2.95 each, how much does he spend in five days?

(Chapter 7, Lesson 2)

7. On Tuesday, Antonio was halfway through a 500-page book. On Wednesday he read 125 pages. What fraction of the book had Antonio read by the end of Wednesday?

(Chapter 5, Lesson 2)

8.



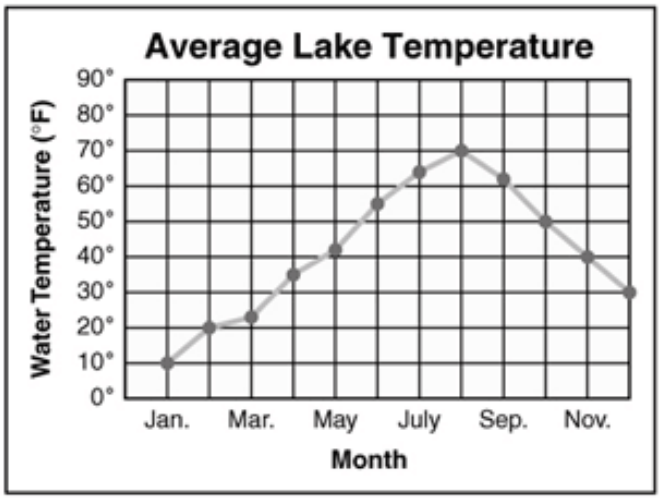
How many square inches is the area of the rectangle?

(Chapter 8, Lesson 7)

9. Fatima bought 6 CDs. Each CD cost $12.99. How much money did she spend?

(Chapter 7, Lesson 2)

Extended Response



10. The line graph shows how the water temperature in a Wisconsin lake varies from month to month.

A Identify the interval used for the vertical axis.

B Water freezes at 32°F. During which months is the lake most likely to be frozen over?

C Frida wears a wetsuit for water skiing only when the water temperature is less than 65°F. During which months is she most likely to wear a wetsuit?

D Do you think a line graph was the best choice for displaying these data? Explain.

(Chapter 10, Lesson 8)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 3

Look back at the big ideas and vocabulary in this unit.

Big Ideas

You can use different statistics to describe a data set, including mean, median, and mode.

You can use graphs to display data. A graph may be misleading if, for example, the intervals on the axes have been chosen to show bias.

Key Vocabulary

statistics
intervals
bias

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how understanding powers of 10 can help you convert metric units of measure.
2. Explain how the mean, median, and mode of a data set can help you draw conclusions and make predictions.
3. Explain how to tell whether a graph is misleading.
4. **Write About It** The metric system is used by people in most nations. Americans, however, still use customary measures most of the time. Describe the advantages and disadvantages of each system. Do you think Americans should use the metric system? Explain.



UNIT 4

Integers and Algebra

CHAPTER 11

Integers

page 272



CHAPTER 12

Algebra: Expressions and Equations

page 300



CHAPTER 13

Algebra: Patterns, Functions, and the Coordinate Plane

page 318

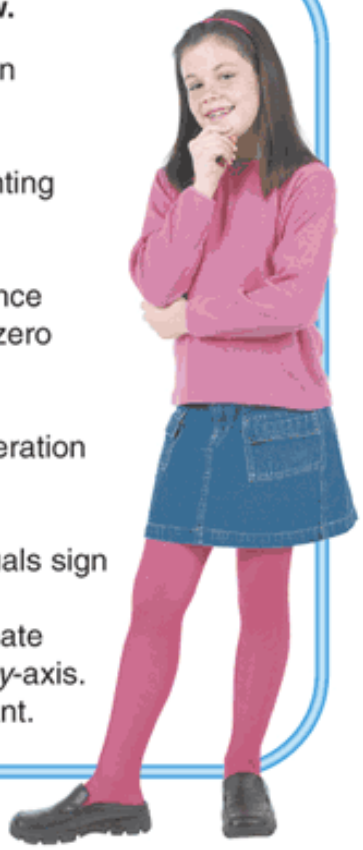


Reading Mathematics

Reviewing Vocabulary

Here are some math vocabulary words that you should know.

absolute value	the distance of a number from zero on a number line
integers	the set of numbers consisting of counting numbers, their opposites, and zero
opposite numbers	two numbers that are the same distance from zero, but on opposites sides of zero on a number line
expression	numbers and variables related by operation signs and grouping symbols
equation	a mathematical sentence with an equals sign
quadrant	one of four parts into which a coordinate plane is separated by the *x*-axis and *y*-axis. The axes are not parts of any quadrant.

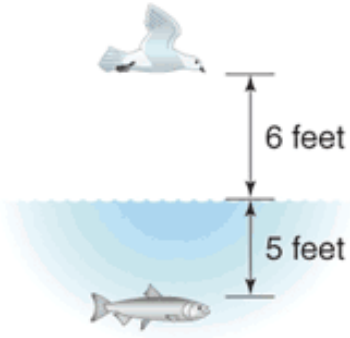


Reading Words and Symbols

You can use words, symbols, or a combination of words and symbols to express mathematical ideas such as integers.

All of these statements represent the same number.

- positive six
- $^{+}6$ or 6
- the opposite of $^{-}6$



All of these statements represent the same number.

- negative five
- $^{-}5$
- the opposite of 5

Write each number in two other ways.

1. negative nine **2.** $^{+}12$ **3.** 23 **4.** the opposite of $^{-}45$ **5.** $^{-}70$

Reading Test Questions

Choose the correct answer for each.

6. Which number sentence is related to $^{-}4 - {}^{+}7 = {}^{-}11$?

a. $^{+}4 + {}^{+}7 = {}^{+}11$

b. $^{-}4 + {}^{-}7 = {}^{-}11$

c. $^{-}4 - {}^{-}7 = {}^{+}3$

d. $^{+}4 - {}^{+}7 = {}^{-}3$

Related means "connected" or "linked."

Addition and subtraction are inverse operations. Every subtraction sentence is related to an addition sentence.

7. Which expression can you use to find the difference of 28 and 7?

a. $28 + 7$ **c.** $28 - 7$

b. $28 \div 7$ **d.** 28×7

A **difference** is the result of a subtraction.

8. What is the result when you evaluate the expression $2n - 3$ for $n = 3$?

a. 2 **c.** 4

b. 3 **d.** 5

Evaluate means "to find the value of."

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

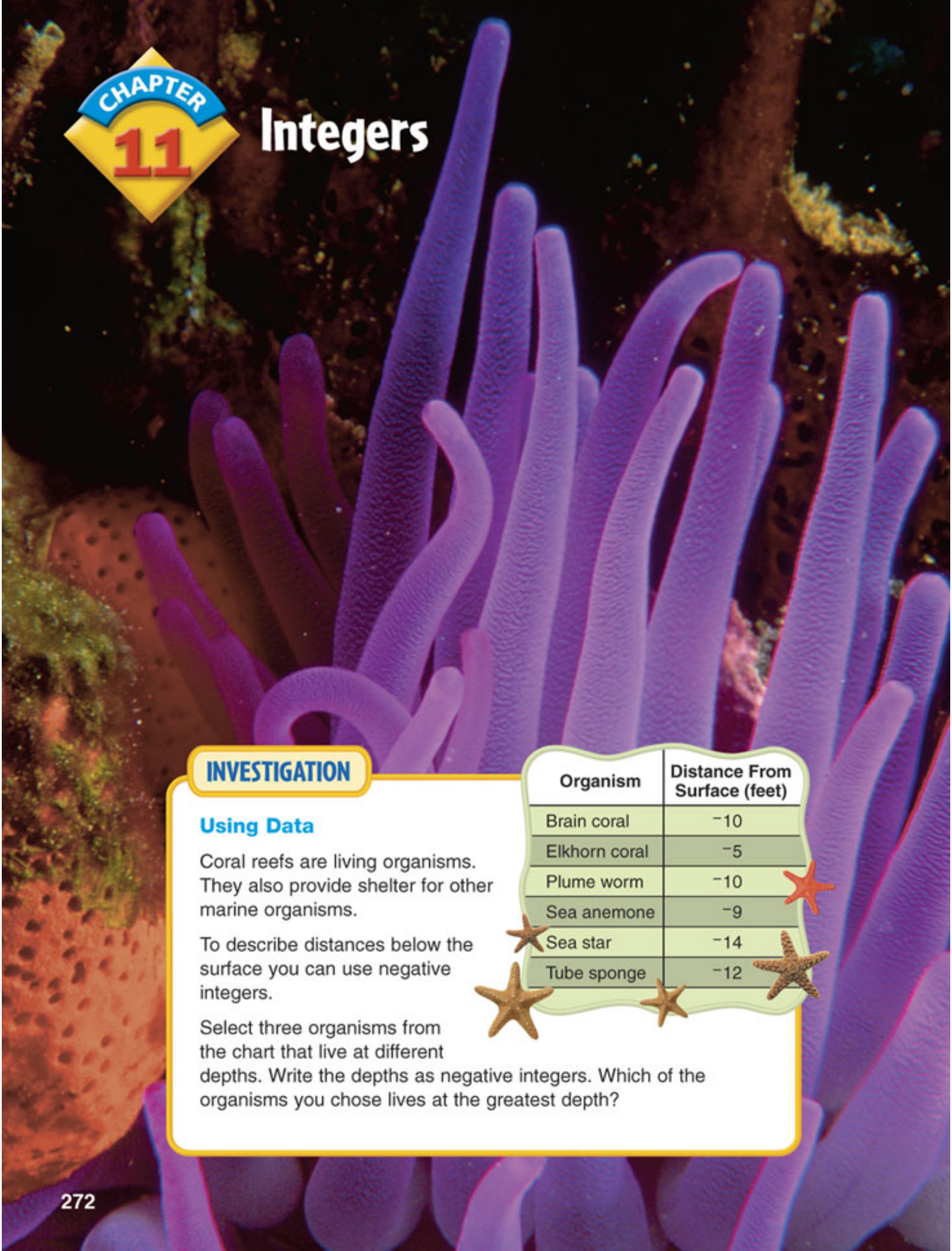
- **additive inverse**
- **algebraic expression**
- **sequence**
- **function**
- **coefficient**

Literature Connection

Read "The Great Figure" on page 639. Then work with a partner to answer the questions about the story.

Education Place

At **eduplace.com/map** see eGlossary and EGames—Math Lingo.



CHAPTER 11

Integers

INVESTIGATION

Using Data

Coral reefs are living organisms. They also provide shelter for other marine organisms.

To describe distances below the surface you can use negative integers.

Organism	Distance From Surface (feet)
Brain coral	−10
Elkhorn coral	−5
Plume worm	−10
Sea anemone	−9
Sea star	−14
Tube sponge	−12

Select three organisms from the chart that live at different depths. Write the depths as negative integers. Which of the organisms you chose lives at the greatest depth?

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

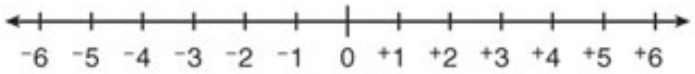
1. The set of positive whole numbers and their opposites are ____.
2. The distance of a number from zero on a number line is its ____.
3. The ____ include counting numbers and zero but not fractions, decimals, or negative numbers.

Vocabulary

integers
absolute value
whole numbers
opposite number

CONCEPTS AND SKILLS

Use the number line for Exercises 4–9.



For each group, write the numbers that are greater than 0.

4. $^{+}2, {}^{+}3, {}^{-}1$ **5.** $^{-}4, {}^{-}6, {}^{+}4$ **6.** $^{+}5, {}^{-}2, {}^{+}1$

For each group, write the numbers that are less than 0.

7. $^{-}2, {}^{-}4, {}^{+}6$ **8.** $^{+}5, {}^{+}3, {}^{-}1$ **9.** $^{+}4, {}^{-}2, {}^{-}5, 0$

Write the absolute value of each integer.

10. $^{+}4$ **11.** $^{+}6$ **12.** $^{-}5$ **13.** $^{-}11$ **14.** $^{+}15$

15. $^{-}5$ **16.** $^{+}7$ **17.** $^{+}9$ **18.** $^{-}14$ **19.** $^{-}8$

Write About It

20. Look at Jim's money record. What do the integers $^{+}10$, $^{-}5$, and $^{-}6$ show?

Jim's Money

Earned $10	→	$^{+}10$
Spent $5	→	$^{-}5$
Owes Tom $6	→	$^{-}6$

Facts Practice, See page 657.

Lesson 1

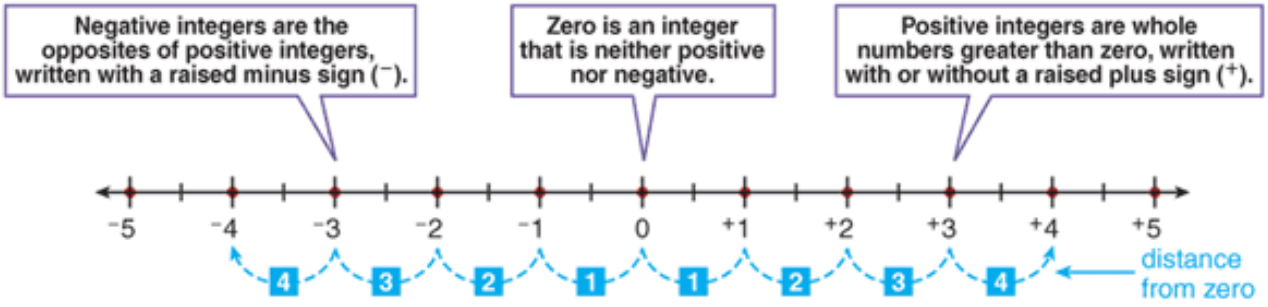
Integers

Objective Identify and use integers.

Vocabulary
- integers
- positive integers
- negative integers
- opposite numbers

Learn About It

You can use a number line to display numbers called **integers**. The set of integers include the counting numbers, their opposites, and zero.



⁻4 and ⁺4 are **opposite numbers**.
Opposite numbers are the same distance from zero.

For a science project, Diarra measured the height of the water above or below sea level at her town dock. At which times was the water height above sea level? At which times was the water height below sea level?

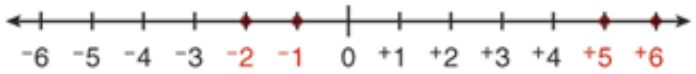
Water Height

Time	Height (feet)
2:30 A.M.	⁺6
8:30 A.M.	⁻2
2:30 P.M.	⁺5
8:30 P.M.	⁻1



Use a number line to analyze the height compared to sea level.

STEP 1 Plot the heights on a number line.



STEP 2 Identify the times when the heights were above sea level.

The heights at 2:30 A.M. and 2:30 P.M. are **positive integers**. They are placed to the *right* of zero since the water was above sea level.

STEP 3 Identify the times when the heights were below sea level.

The heights at 8:30 A.M. and 8:30 P.M. are **negative integers**. They are placed to the *left* of zero since the water was below sea level.

Solution: At 2:30 A.M. and 2:30 P.M. the water heights were above sea level. At 8:30 A.M. and 8:30 P.M. the water heights were below sea level.

Guided Practice

If the number is an integer, write its opposite. If it is not, write *no*.

1. $^{+}48$ **2.** $^{+}2.6$ **3.** $^{-}325$ **4.** $\frac{7}{10}$

Ask Yourself

- When is a number an integer?
- Is the integer positive or negative?

Explain Your Thinking ▶ What number is its own opposite?

Practice and Problem Solving

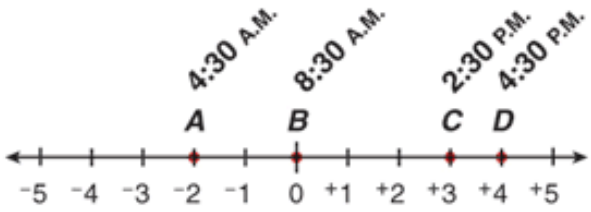
Write each number in word form. If the number is an integer, write its opposite. If it is not, write *no*.

5. $^{-}0.4$ **6.** $^{+}71$ **7.** $^{-}1{,}320$ **8.** $^{-}6\frac{1}{8}$ **9.** $^{+}2$

10. $^{-}29$ **11.** $^{-}4.65$ **12.** $^{-}625$ **13.** $^{+}11$ **14.** 0

15. $\frac{2}{3}$ **16.** $^{+}870$ **17.** $^{-}78$ **18.** $^{+}2\frac{1}{6}$ **19.** $^{+}31$

Use the number line for Problems 20 and 21.



Points *A*, *B*, *C*, and *D* represent changes in water height at sea level at four different times of day.

20. At which time had the water height not changed?

21. **Explain** At which time had the water height changed more, point *A* or *C*?

22. **Represent** What integer could you use to represent the phrase "5 feet below sea level"?

23. **Reasoning** In Problem 22, how did you determine whether the integer should be positive or negative?

Mixed Review and Test Prep

Open Response

Find the reciprocal of each. (Ch. 6, Lesson 4)

24. $\frac{1}{6}$ **25.** $9\frac{7}{8}$

26. $\frac{3}{11}$ **27.** 4

Multiple Choice

28. On a number line, which integer is the greatest distance from zero? (Ch. 11, Lesson 1)

A $^{+}1$ **C** $^{+}3$

B $^{-}2$ **D** $^{-}4$

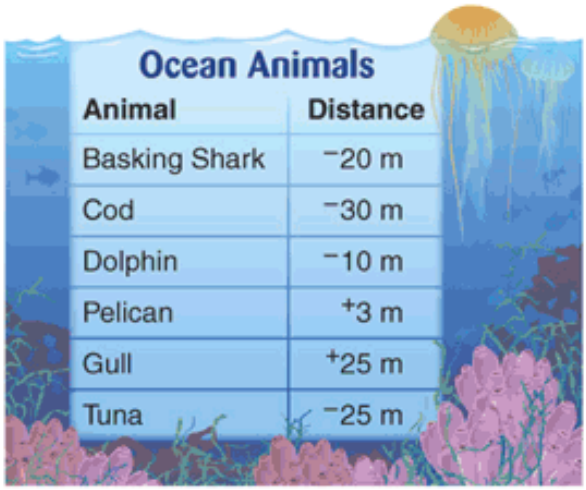
Extra Practice See page 299, Set A.

Lesson 2

Audio Tutor 1/29 Listen and Understand

Compare and Order Integers

Objective Compare and order integers.



Ocean Animals

Animal	Distance
Basking Shark	$^-20$ m
Cod	$^-30$ m
Dolphin	$^-10$ m
Pelican	$^+3$ m
Gull	$^+25$ m
Tuna	$^-25$ m

Learn About It

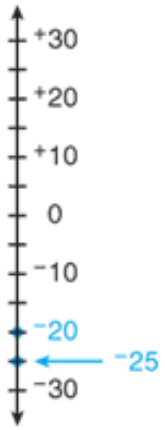
The table shows the distances relative to sea level (0 meters) of some living things found in or above the oceans. Which animal is found at a higher level, the tuna or the basking shark?

Use a number line to compare the two heights.

STEP 1 Locate the heights of the animals on a vertical number line.

STEP 2 Compare. On a number line, integers decrease in value from top to bottom.

$^-20 > {}^-25$



Solution: The basking shark is found at a level higher than the tuna.

Another Example

Order Integers

Order the integers $^-20$, $^+5$, $^-30$, $^+15$, $^-10$, and $^+25$ from least to greatest.



$^-30$, $^-20$, $^-10$, $^+5$, $^+15$, $^+25$

Guided Practice

Compare. Write > or < for each ◯.

1. $^+20$ ◯ $^-20$
2. $^+10$ ◯ $^-15$
3. $^-10$ ◯ 0

Order from least to greatest.

4. $^+8$, $^-3$, $^+11$, $^-7$
5. $^-3$, $^-14$, $^+14$
6. $^+9$, 0, $^+17$, $^-6$

Ask Yourself

- Did I look at the sign of the integers?

Explain Your Thinking How can you use a thermometer to decide if $^-15$°C is lower than $^-10$°C?

Extra Help at eduplace.com/map

Practice and Problem Solving

Compare. Write > or < for each ●.

7. $^{+}9$ ● $^{+}16$ **8.** $^{-}4$ ● $^{+}5$ **9.** $^{-}18$ ● $^{-}19$ **10.** $^{-}38$ ● $^{-}16$

11. $^{+}40$ ● $^{-}40$ **12.** $^{+}10$ ● $^{-}5$ **13.** $^{-}23$ ● 0 **14.** $^{-}10$ ● $^{+}14$

Order from least to greatest.

15. $^{-}6, ^{+}5, ^{-}1, ^{-}4$ **16.** $0, ^{-}4, ^{-}3, ^{+}14, ^{-}7, ^{+}7$ **17.** $^{+}15, 0, ^{-}21, ^{-}18$

18. $^{+}16, ^{+}13, ^{-}5, ^{-}6$ **19.** $^{+}10, ^{-}10, ^{-}20, ^{+}30$ **20.** $^{-}30, ^{-}1, ^{+}40, ^{-}8, ^{+}12$

21. $^{-}9, 0, ^{+}11, ^{+}23$ **22.** $^{+}5, ^{-}4, ^{+}1, ^{-}5, ^{-}2$ **23.** $^{-}11, ^{+}6, ^{-}9, ^{-}10$

Algebra • Inequalities

Compare, given $a = 0$, $b = 1$, and $c = 2$. Write <, >, or = for each.

24. 1 ● c **25.** a ● $^{-}3$ **26.** 3 ● c **27.** b ● $^{-}1$

Data

Use the table on page 276 for Problems 28 and 29.

28. Order the distances in the table from lowest to highest. Which animal was the highest? Which was lowest?

29. **You Decide** A sunfish is swimming at a water level between that of the shark and of the tuna. Use integers to name some possible depths for the sunfish.

30. Fishing rods rent at $18 a day. Bait costs $2.50 a pound. A bag of ice costs $1.98. What is the total cost of renting a rod and buying 2 bags of ice and 4 pounds of bait?



Mixed Review and Test Prep

Open Response

Divide. Write each quotient in simplest form. (Ch. 6, Lessons 4 and 5)

31. $\frac{14}{3} \div \frac{4}{21}$ **32.** $\frac{9}{10} \div \frac{1}{5}$

33. $5 \div \frac{1}{7}$ **34.** $3\frac{2}{3} \div 2\frac{1}{2}$

35. Explain how to use a number line to compare and order the integers $^{-}5$, $^{+}3$, and $^{-}14$. (Ch. 11, Lesson 2)

Extra Practice See page 299, Set B.

Lesson 3

Audio Tutor 1/30 Listen and Understand

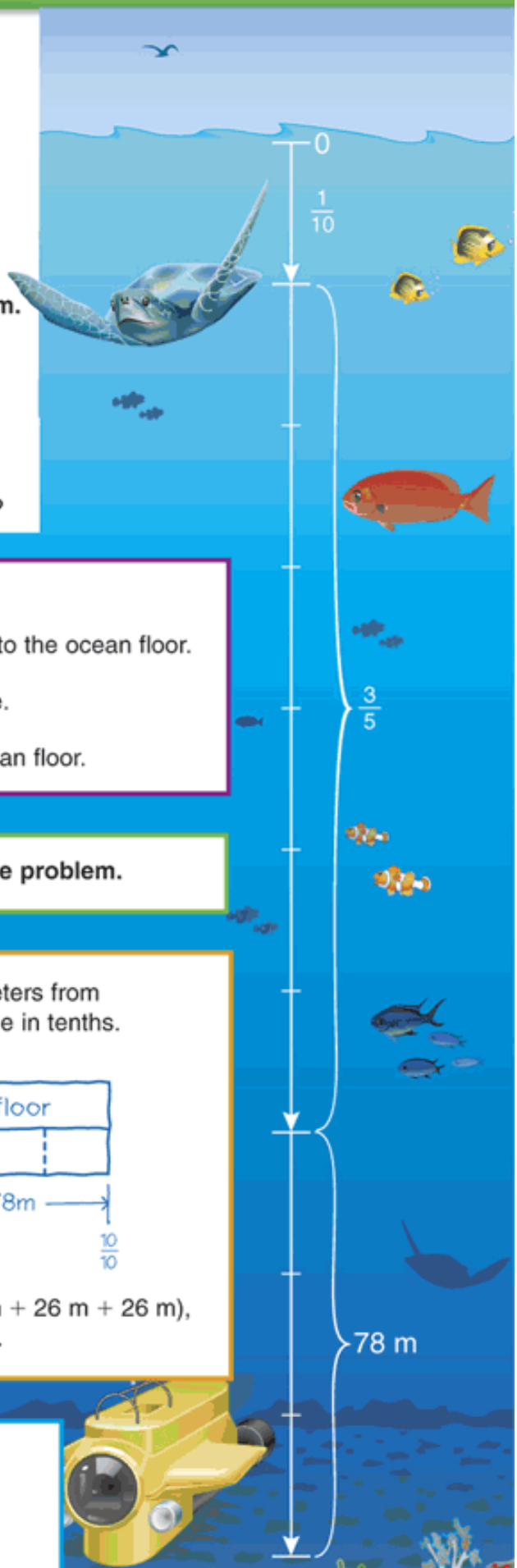
Problem-Solving Strategy

Draw a Diagram

Objective Solve a problem by drawing a diagram.

Sometimes drawing a diagram can help you solve a problem.

Problem A submersible stopped at different depths before settling on the ocean floor. First, it stopped at $\frac{1}{10}$ of the distance down. Next, it stopped after another $\frac{3}{5}$ of the distance. After descending the final 78 meters, it reached the ocean floor. How far below the surface is the ocean floor?



UNDERSTAND

This is what you know:

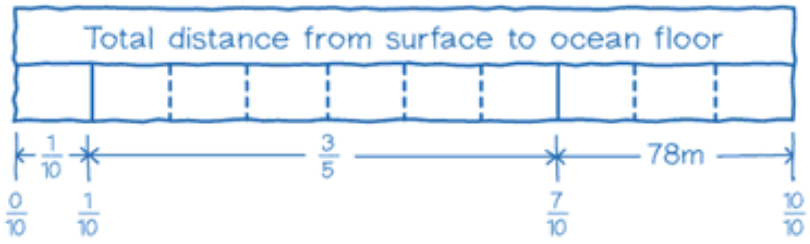
- The submersible stopped $\frac{1}{10}$ of the way down to the ocean floor.
- Next, it stopped after another $\frac{3}{5}$ of the distance.
- Then it descended 78 meters to reach the ocean floor.

PLAN

You can draw a diagram to help you solve the problem.

SOLVE

Draw a rectangle to represent the number of meters from the surface to the ocean floor. Mark the rectangle in tenths. Enter the given information.



Since three tenths of the distance is 78 m (26 m + 26 m + 26 m), the ocean floor is 260 meters below the surface.

LOOK BACK

Look back at the problem.

How far from the surface was the submersible after each stop?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Kyle dove 12 feet below the surface of the ocean. His dive was $\frac{1}{4}$ as deep as his deepest dive ever. How far below the surface was his deepest dive?

2. A submersible is 20 meters below sea level. Then it rises 5 meters. What integer expresses the position of the submersible compared to sea level?

   Hint Use 0 to represent sea level.

Ask Yourself

UNDERSTAND What facts do I know?

PLAN How can I use a diagram to organize the data?

SOLVE
- Into how many equal parts should the diagram be divided?
- Did I label the parts of my diagram correctly?

LOOK BACK Does my solution make sense?

Independent Practice

Draw a diagram to solve each problem.

3. Shark *A* was swimming 3 feet below the surface of the ocean. Shark *A*'s depth was $\frac{1}{7}$ the depth of Shark *B*. How far below the surface of the ocean was Shark *B*?

4. A pod of dolphins swam 182 meters above the ocean floor. This was $\frac{7}{10}$ of the distance to the surface. How far from the surface were the dolphins?

5. A *tsunami* is a very large ocean wave caused by underwater earthquakes or volcanic eruptions. One of the largest recorded tsunamis measured 210 feet above sea level. A tsunami occurred that was $\frac{2}{3}$ its height. How high did it measure?

6. A research vessel is lowered 15 meters below sea level. It is then lowered another 20 meters. What integer expresses the position of the research vessel compared to sea level?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Ty began scuba diving 7 years ago when he was 16 years old. Pat began 13 years ago. When she began, Pat was 4 years younger than Ty was when he began. How much older than Ty is Pat?

8. On Sunday it was 87°F. On Saturday, the temperature was 4° higher. On Friday, the temperature was 7° lower than on Saturday. Express each temperature.

9. **Analyze** The high temperature for a winter day was $^{+}3$°C. What was the low temperature that day if the range of temperatures for the day was 7°C?

You Choose

Strategy

- Draw a Diagram
- Use Logical Reasoning
- Make an Organized List
- Use Models
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the graph for Problems 10–15. Then tell which method you chose.**

10. How many more adults went whale watching on Thursday than on Friday?

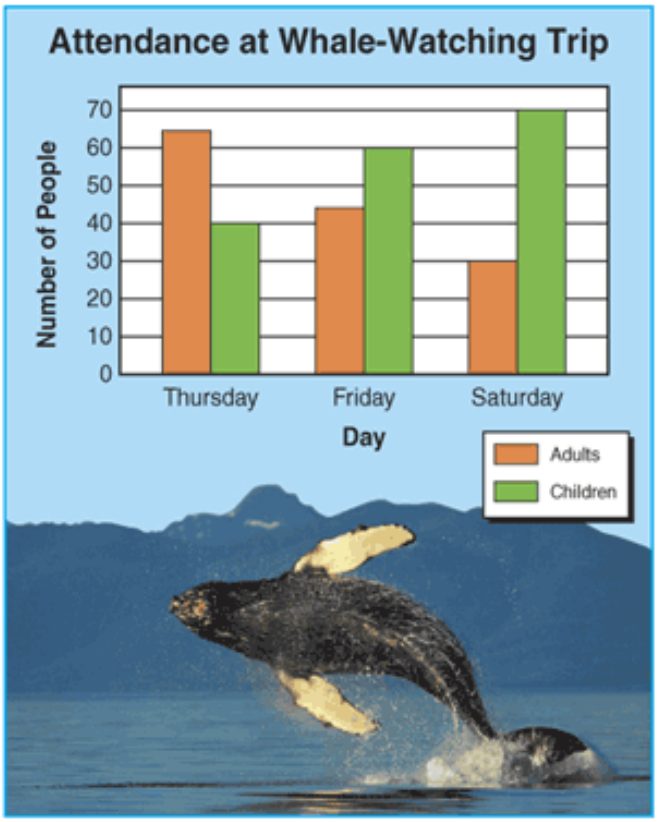
11. How many adults and children went whale watching on Saturday?

12. On which day did the most children go whale watching?

13. On which day was the number of adults who went whale watching $\frac{3}{4}$ of the number of children?

14. On which day did more adults than children go whale watching?

15. **Write About It** On which days did the same number of people go whale watching? Explain your reasoning.



Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. Which number makes the sentence true?

If 1 yd = 36 in., then $\frac{1}{4}$ yd = ■ in.

A 3 **B** 4 **C** 6 **D** 9

(Chapter 8, Lesson 2)

2. Jake's family bought 8.5 pounds of tomatoes at a farmers' market. If tomatoes are \$1.59 a pound, how much did they pay for the tomatoes?

F \$19 **G** \$13.58 **H** \$13.52 **J** \$13.42

(Chapter 7, Lesson 2)

Open Response

Solve each problem.

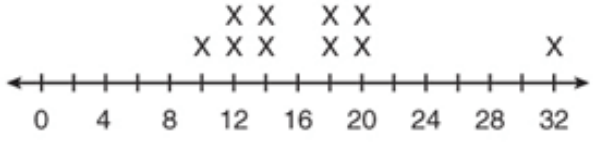
3. The stem-and-leaf plot shows the high temperatures for one January in Niagara Falls, New York. What are the highest and lowest high temperatures listed?

Stem	Leaves
0	5 6 8 8 9
1	0 1 1 1 3 4 5 6 7 7 8 9 9
2	0 1 3 3 4 5 5 6
3	1 1 2 4 6

Key: 3|1 means 31°F

(Chapter 10, Lesson 4)

4. Identify the outlier in the given data set.



Explain How did you make your choice?

(Chapter 9, Lesson 4)

Constructed Response

5. In a magic square the sum of the numbers in any row, column, or diagonal is the same.

Square *A*			
+5	−8	−9	+2
−6	−1	0	−3
−2	−5	−4	+1
−7	+4	+3	−10

Square *B*			
?	?	?	+4
?	?	?	−1
0	−3	−2	+3
−5	+6	+5	−8

a. Look at Square *A*. Determine whether or not it is a magic square.

b. Copy Square *B*. Find the missing integers to make it a magic square.

c. Make your own magic square using integers.

(Chapter 11, Lesson 3)



Education Place

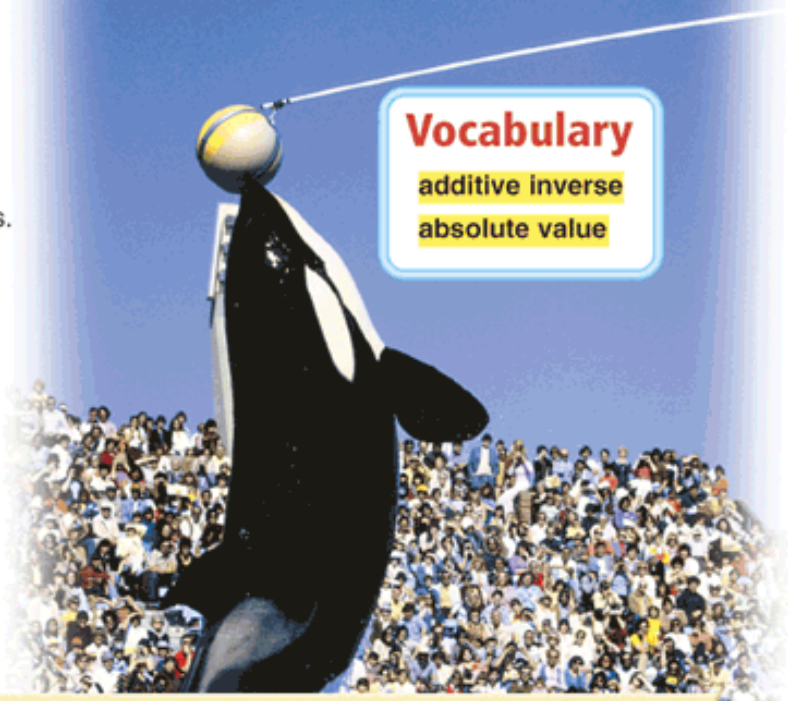
See **eduplace.com/map** for more Test-Taking Tips.

Hands On Lesson 4

Audio Tutor 1/31 Listen and Understand

Add Integers

Objective Find the sum of two integers.



Vocabulary

additive inverse

absolute value

Learn About It

Lynn measures an orca's performance on a point system. The orca scores 5 points for catching a ball then loses 3 points for missing her cue. What was her score during this routine?

You can use counters or a number line to add integers.

Add. $^{+}5 + {}^{-}3 = n$

Different Ways to Add $^{+}5 + {}^{-}3$

Way 1 Use counters.

STEP 1 Show $^{+}5$ and $^{-}3$ with counters.



STEP 2 Pair a yellow counter with a red counter until you have only one color left.



Each yellow–red pair represents $^{+}1 + {}^{-}1$, or 0.

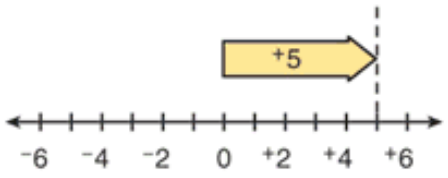
STEP 3 Remove the zero pairs.



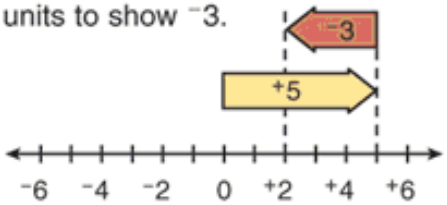
There are 2 yellow counters left. So $^{+}5 + {}^{-}3 = {}^{+}2$.

Way 2 Use a number line.

STEP 1 Begin at 0. Move right 5 units to show $^{+}5$.



STEP 2 Then, starting at $^{+}5$, move left 3 units to show $^{-}3$.

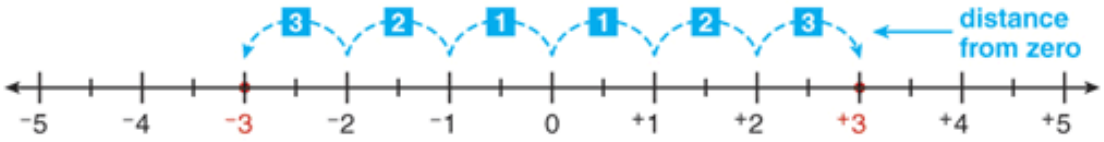


The point at which you stop is the sum $^{+}5 + {}^{-}3$.

$^{+}5 + {}^{-}3 = {}^{+}2$

Solution: The orca scored $^{+}2$ points.

The **additive inverse** of an integer is its opposite. The sum of an integer and its additive inverse is always 0. Additive inverses have the same **absolute value**, or distance from zero.



The absolute value of both $^{-}3$ and $^{+}3$ is 3 because both are 3 units from zero.

If you are adding two integers that have different signs, you can subtract the absolute values and use a rule to decide on the sign of the sum.

Rule:

The sum of a positive and a negative integer will have the same sign as the integer with the greater absolute value.

Find $^{+}4 + {}^{-}7$.

STEP 1 Subtract the lesser absolute value from the greater.

$^{+}4 + {}^{-}7 = n$

The absolute value of $^{+}4$ is 4.

The absolute value of $^{-}7$ is 7.

$7 - 4 = 3$

STEP 2 Use the rule to decide if the sum will be positive or negative.

Since $^{-}7$ has a greater absolute value than $^{+}4$, the sum will be negative.

$^{+}4 + {}^{-}7 = {}^{-}3$

Guided Practice

Ask Yourself

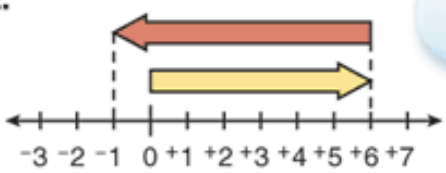
- Did I use the correct sign for the sum?

Write the addition problems shown by the models.

1.



2.



Add. Use counters or a number line to help.

3. $^{-}8 + {}^{+}7$ **4.** $^{-}11 + {}^{-}3$ **5.** $^{+}5 + {}^{+}8$ **6.** $^{-}9 + {}^{+}9$

Write the missing addend.

7. $^{+}9 + \square = {}^{-}4$ **8.** $^{-}2 + \square = {}^{+}6$ **9.** $\square + {}^{-}3 = {}^{+}7$ **10.** $\square + {}^{+}6 = 0$

Explain Your Thinking When is the sum of a positive integer and a negative integer positive?

Go On

Practice and Problem Solving

Add. Use counters or a number line to help.

11. $^{-}1 + {}^{-}2$ **12.** $^{+}8 + {}^{-}5$ **13.** $^{-}3 + {}^{-}2$ **14.** $^{-}7 + {}^{+}11$

15. $^{-}5 + {}^{+}9$ **16.** $^{-}16 + 0$ **17.** $^{+}4 + {}^{-}3$ **18.** $^{-}13 + {}^{-}1$

Add. Use the rule.

19. $^{+}10 + {}^{-}11$ **20.** $^{+}11 + {}^{+}10$ **21.** $^{-}17 + {}^{+}4$ **22.** $^{-}21 + {}^{+}22$

23. $^{+}19 + {}^{-}1$ **24.** $^{+}17 + {}^{-}17$ **25.** $^{-}11 + 0$ **26.** $^{-}3 + {}^{-}5$

Mental Math **Write the missing addend.**

27. $^{-}10 + \blacksquare = {}^{+}2$ **28.** $\blacksquare + {}^{+}7 = {}^{-}11$ **29.** $^{-}2 + \blacksquare = {}^{-}2$ **30.** $\blacksquare + {}^{-}8 = {}^{-}10$

31. $^{+}13 + \blacksquare = {}^{+}6$ **32.** $\blacksquare + {}^{-}1 = {}^{+}8$ **33.** $\blacksquare + {}^{+}5 = {}^{-}5$ **34.** $0 + \blacksquare = {}^{-}3$

35. $\blacksquare + {}^{-}6 = {}^{+}21$ **36.** $^{+}7 + \blacksquare = {}^{+}14$ **37.** $^{-}3 + \blacksquare = {}^{+}4$ **38.** $\blacksquare + {}^{-}5 = {}^{-}18$

Algebra • Functions Complete each table by following the rule.

39. Rule: Add $^{+}8$

Input	Output
$^{-}6$	■
$^{+}9$	■
$^{-}13$	■

40. Rule: Add $^{-}7$

Input	Output
$^{+}15$	■
$^{-}6$	■
$^{+}3$	■

41. Rule: Add $^{-}4$

Input	Output
$^{+}10$	■
$^{+}2$	■
$^{-}1$	■

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Tell which method you chose.

42. Two dolphins jumped through a ring 10 feet above the water. Then they turned and dove 26 feet below that height. How many feet below the surface did the dives take them?

43. Analyze A seal dives to a depth of 35 feet and then resurfaces. On its next dive, the seal dives 10 feet deeper and resurfaces. How many feet will the seal have traveled during the two dives?

44. A sea lion dove from a rock 12 feet above sea level to a reef 45 feet below that height. It then dove another 14 feet to a boulder 9 feet off the bottom. What is the elevation at the bottom?



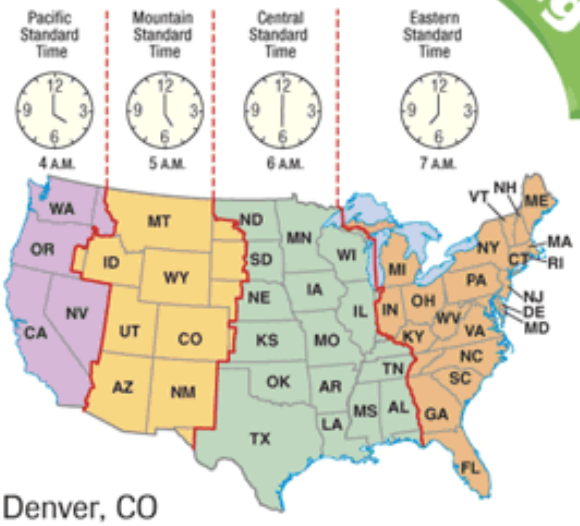
Extra Practice See page 299, Set C.

Problem Solving

Real World Connection

Time Zones

The system of time zones, which is used over the world, is called Standard Time. As you travel *west* through the time zones in the Continental U.S., the time changes to one hour earlier ($^-1$) than the previous time. As you travel *east* through each time zone, the time changes to one hour later ($^+1$).



Use the map of time zones to answer each question.

1. If it is 3:30 P.M. in New Orleans, LA, what time is it in each of these cities?

   a. Indianapolis, IN **b.** Eugene, OR **c.** Denver, CO

2. A plane from St. Louis, MO, arrived in Los Angeles, CA, at 10:00 A.M. What time did it leave St. Louis if it takes $3\frac{1}{2}$ hours to fly to Los Angeles?

WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

If the number is an integer, write its opposite. If it is not, write *no.* (Lesson 1)

1. $^-0.2$ **2.** $^+37$ **3.** $^-2{,}130$

Compare. Write > or < for each ●. (Lesson 2)

4. $^+8$ ● $^+14$ **5.** $^-3$ ● $^+7$ **6.** $^-12$ ● $^-16$

Draw a diagram to solve. (Lesson 3)

7. On his scuba dive, Einar spent $\frac{2}{5}$ of the time exploring a wreck, 15 minutes looking at coral, and 30 minutes descending and ascending. How long did Einar's dive last?

Add. (Lesson 4)

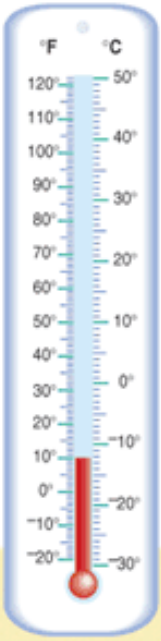
8. $^-6 + {}^+5$ **9.** $^+7 + {}^-9$ **10.** $^+3 + {}^-3$

Lesson 5

Audio Tutor 1/32 Listen and Understand

Subtract Integers

Objective Find the difference of two integers.



Learn About It

Addition and subtraction are inverse operations. Just as you can use a number line or a rule to add integers, you can use a number line or a rule to subtract integers.

Find $^-7 - {}^-10$.

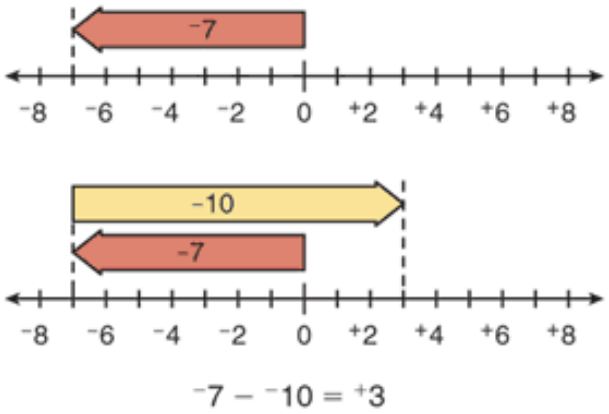
Different Ways to Find $^-7 - {}^-10$

Way 1 Use a number line.

STEP 1 Begin at 0. Move left 7 units to show $^-7$.

STEP 2 To add $^-10$, you would move left 10 units. To subtract $^-10$, you move right 10 units.

The point at which you stop is the difference $^-7 - {}^-10$.



$^-7 - {}^-10 = {}^+3$

Way 2 Use a rule.

$^-7 - {}^-10$

$^-7 + {}^+10 = {}^+3$

Rule:

Subtracting an integer is the same as adding its opposite.

Solution: $^-7 - {}^-10 = {}^-7 + {}^+10 = {}^+3$.

Guided Practice

Subtract. Use a number line to help.

1. $^-10 - {}^+7$ **2.** $^+8 - {}^-3$ **3.** $^-3 - {}^-2$ **4.** $^+5 - {}^+10$

5. $^-4 - 0$ **6.** $0 - {}^+7$ **7.** $^-7 - {}^-2$ **8.** $^-4 - {}^+8$

Ask Yourself

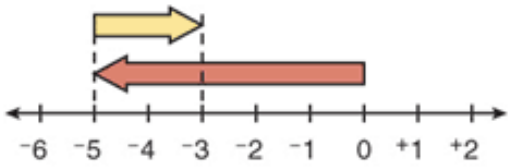
- Did I change the sign of the second integer and add?

Explain Your Thinking How are adding and subtracting integers related?

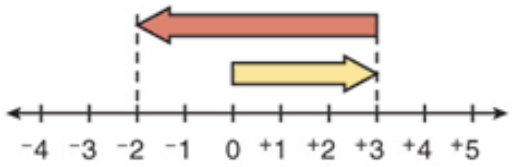
Practice and Problem Solving

Write a subtraction sentence for each.

9.



10.



Complete each related equation.

11. $^{+}9 - {}^{+}2 = \blacksquare$
$^{+}9 + {}^{-}2 = \blacksquare$

12. $^{-}9 - {}^{+}10 = \blacksquare$
$^{-}9 + {}^{-}10 = \blacksquare$

13. $^{-}5 - {}^{+}4 = \blacksquare$
$^{-}5 + {}^{-}4 = \blacksquare$

Subtract. Use a number line or the rule.

14. $^{+}11 - {}^{-}4$

15. $^{+}3 - {}^{+}10$

16. $0 - {}^{+}2$

17. $0 - {}^{-}13$

18. $^{-}4 - {}^{+}8$

19. $^{-}5 - {}^{-}5$

20. $^{+}4 - {}^{+}1$

21. $^{-}4 - {}^{+}1$

22. $^{-}11 - {}^{+}14$

23. $^{-}17 - {}^{-}9$

24. $^{-}9 - {}^{+}5$

25. $^{-}7 - {}^{-}8$

Algebra • Equations

Use mental math to solve for *n*.

26. $^{+}4 - n = {}^{-}4$

27. $^{+}12 - n = {}^{+}5$

28. $^{-}7 - n = {}^{-}8$

Solve.

29. Phil read his thermometer as 10°F. The radio meteorologist said wind chill made it feel like $^{-}$11°F. What is the difference in temperature?

30. Geoff owed Carola $14. He recorded this in his notebook as $^{-}$$14. Then he paid Carola $9 of his debt. How can Geoff record how much he owes Carola now?

31. Reasoning Two integers have a sum of $^{-}2$. The difference of the lesser integer and the greater integer is $^{-}2$. The sum of the absolute values of the integers is 2. What are the integers?

32. Create and Solve Choose an integer greater than 100. Choose an integer less than $^{-}10$. Write a situation that can be represented by the positive integer subtracted from the negative integer.

Mixed Review and Test Prep

Open Response

Multiply. Express each product in simplest form. (Ch. 6 Lessons 2 and 3)

33. $\frac{2}{5} \times \frac{1}{6}$

34. $\frac{3}{11} \times \frac{6}{7}$

35. $2\frac{1}{2} \times 3$

36. $\frac{1}{2} \times 3\frac{4}{5}$

37. At 5 P.M., the air temperature was $^{-}$5°C. At 10 P.M. it had decreased by 9°. What was the air temperature at 10 P.M.? Explain how you solved the problem. (Ch. 11, Lesson 5)

Extra Practice See page 299, Set D.

Hands On Lesson 6

Model Multiplication and Division

Objective Use models to multiply and divide integers.

Vocabulary
integers

Materials
red and yellow counters
work mat

Work Together

You can use counters to help you multiply and divide **integers**.

To model multiplication, recall that multiplication is repeated addition.

Find $^{+}2 \times {}^{-}3$.

Multiplying by a Positive Integer

$^{+}2 \times {}^{-}3$ means to add 2 groups of $^{-}3$ to zero. Model $^{+}2 \times {}^{-}3$ by adding 2 groups of 3 red counters to your workmat.



- How many red counters did you use?
- What is $^{+}2 \times {}^{-}3$?

Remember A yellow counter represents $^{+}1$ and a red counter represents $^{-}1$. A pair of 1 red and 1 yellow counter represents zero.

Find $^{-}2 \times {}^{+}3$.

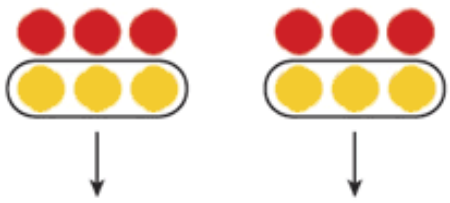
Multiplying by a Negative Integer

$^{-}2 \times {}^{+}3$ means to remove 2 groups of $^{+}3$ from zero. Start with 0. Use pairs of yellow and red counters to show 0.



- How many pairs of yellow and red counters do you need to be able to remove 2 groups of 3 yellow counters?

Now remove 2 groups of 3 yellow counters.



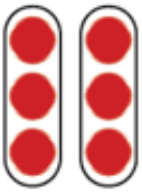
- What integer do the remaining counters represent?
- What is $^{-}2 \times {}^{+}3$?

$^{-}2 \times {}^{-}3$ means to remove 2 groups of $^{-}3$ from zero.

- How would you revise the activity to find $^{-}2 \times {}^{-}3$?
- What is $^{-}2 \times {}^{-}3$?

To model division, recall that division and multiplication are inverse operations. Use related division and multiplication facts.

$^{+}2 \times {}^{-}3 = {}^{-}6$ means that ${}^{-}6 \div {}^{+}2 = {}^{-}3$



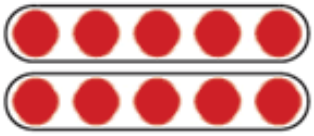
Find ${}^{-}10 \div {}^{-}5$.

Dividing by an Integer

${}^{-}10 \div {}^{-}5$ means how many sets of ${}^{-}5$ will you add to zero to make ${}^{-}10$.

Find a missing factor to help you divide.

- What factor would you multiply ${}^{-}5$ by to get ${}^{-}10$?
- What is ${}^{-}10 \div {}^{-}5$?



On Your Own

Multiply. Use counters to help you.

1. $^{+}3 \times {}^{-}5$ **2.** $^{-}5 \times {}^{-}4$ **3.** $^{-}4 \times {}^{+}3$ **4.** $^{-}2 \times {}^{-}6$

5. What multiplication is being modeled at right?



Divide by finding a missing factor.

6. ${}^{+}10 \div {}^{-}2$ **7.** ${}^{-}8 \div {}^{-}4$ **8.** ${}^{-}16 \div {}^{+}4$ **9.** ${}^{+}9 \div {}^{-}3$

10. Write a division sentence for the array shown at right.



Talk About It • Write About It

You learned how to model multiplication and division of integers.

11. Compare the quotients of ${}^{-}6 \div {}^{+}2$ and ${}^{+}6 \div {}^{-}2$. What do you notice?

12. ${}^{-}3 \times {}^{-}5$ means to remove 3 groups of ${}^{-}5$ from zero. Explain how you can multiply ${}^{-}3 \times {}^{-}5$. Use words or a picture with labels.

Lesson 7

Audio Tutor 1/33 Listen and Understand

Multiply Integers

Objective Find the product of two integers.



The Bay of Fundy in Canada has some of the highest tides in the world.

Learn About It

The difference in sea level between high and low tide can be as much as 50 feet. Suppose that the sea level at high tide is 48 feet and, as the tide goes out, the sea level changes by $^{-}2$ feet each hour. What will the change in sea level be after 6 hours?

Multiply. $^{+}6 \times {}^{-}2 =$ ■

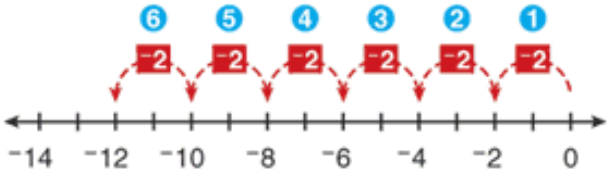
Different Ways to Multiply $^{+}6 \times {}^{-}2$

Way 1 Use repeated addition.

$^{+}6 \times {}^{-}2 = {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2$
$= {}^{-}12$

Way 2 Use a number line.

Begin at 0. Add $^{-}2$ six times. The result is $^{-}12$.



Way 3 You can use rules.

- First find the product of the absolute values of the integers.

$$6 \times 2 = 12$$

- Then use the rules to decide if the product will be positive or negative.

$$^{+}6 \times {}^{-}2 = {}^{-}12$$

Rules:

- The product of two integers with different signs is always negative.
- The product of two integers with the same sign is always positive.
- If one factor is zero, the product is zero.

Solution: The change in sea level will be $^{-}12$ feet after 6 hours.

Guided Practice

Write each expression as a multiplication problem. Then evaluate.

1. $^{+}8 + {}^{+}8 + {}^{+}8 + {}^{+}8$

2. $^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2 + {}^{-}2$

Ask Yourself

- What sign should I use?
- What is the product of the absolute values?

Find each product.

3. $^{+}7 \times {}^{-}4$ **4.** $^{+}3 \times {}^{+}12$ **5.** $^{-}2 \times {}^{-}6$

Explain Your Thinking How is multiplying integers different from multiplying whole numbers?

Practice and Problem Solving

Write each expression as a multiplication problem. Then evaluate.

6. $^{-}7 + {}^{-}7 + {}^{-}7$ **7.** $^{+}9 + {}^{+}9 + {}^{+}9 + {}^{+}9 + {}^{+}9$ **8.** $^{-}5 + {}^{-}5 + {}^{-}5 + {}^{-}5$

Find each product.

9. $^{-}4 \times {}^{-}7$ **10.** $^{-}8 \times {}^{+}9$ **11.** $^{+}5 \times {}^{+}6$ **12.** $^{-}6 \times {}^{+}6$

13. $^{+}7 \times {}^{-}8$ **14.** $^{-}1 \times {}^{+}4$ **15.** $^{+}11 \times {}^{-}4$ **16.** $^{+}9 \times 0$

17. $^{-}9 \times {}^{+}9$ **18.** $^{-}13 \times {}^{-}3$ **19.** $^{+}9 \times {}^{-}7$ **20.** $^{-}6 \times {}^{+}8$

Solve.

21. Jason stopped at 8 feet below sea level while exploring an underwater shipwreck. He decided to dive deeper. He descended by 3 feet each minute for 5 minutes. What was Jason's location compared to sea level after 5 minutes?

22. Ondrea found \$3.00 in coins while snorkeling in a lagoon. Four of the coins were quarters. The rest were dimes and nickels. She found 5 more dimes than nickels. How many of each type of coin did Ondrea find?

23. Create and Solve Write a problem with positive and negative integers that you or a classmate could solve by using repeated addition or a number line. Include a picture or diagram in your problem.

24. Mario dives 33 meters to the ocean floor to investigate a sunken ship. The top of the ship's deck is 7 meters above the ocean floor. How far below the surface is the deck of the ship?



Mixed Review and Test Prep

Open Response

Complete. (Ch. 8, Lesson 2)

25. 1 yd = ■ ft **26.** 18 in. = ■ yd

27. $\frac{1}{2}$ T = ■ lb **28.** 13 fl oz = ■ c

29. 7 pt = ■ qt **30.** $1\frac{1}{2}$ gal = ■ qt

Multiple Choice

31. The water level at high tide was $^{+}24$ feet. If the water level decreases by 2 feet each hour, what integer will describe the water level after 6 hours? (Ch. 11, Lesson 7)

A $^{-}12$ ft **C** $^{+}12$ ft

B $^{+}14$ ft **D** $^{-}14$ ft

Extra Practice See page 299, Set E.

Lesson 8

Audio Tutor 1/34 Listen and Understand

Divide Integers

Objective Find the quotient of two integers.



Learn About It

Just as repeated addition, number lines, and rules can help you multiply integers, repeated subtraction, number lines, and rules can help you divide integers.

Divide. $^{-}6 \div {}^{-}2 = \blacksquare$

Different Ways to Divide $^{-}6 \div {}^{-}2$

Way 1 Use repeated subtraction.

Begin at $^{-}6$. Subtract $^{-}2$ repeatedly until you reach 0.

$$^{-}6 \div {}^{-}2 = {}^{-}6 - {}^{-}2 - {}^{-}2 - {}^{-}2$$
$$= {}^{-}6 + {}^{+}2 + {}^{+}2 + {}^{+}2 = 0$$

The quotient is the number of times you subtracted $^{-}2$.

So, $^{-}6 \div {}^{-}2 = {}^{+}3$.

Way 2 Use a number line.

Begin at $^{-}6$. Subtract $^{-}2$ repeatedly until you reach 0.



The quotient is the number of times you subtracted $^{-}2$.

So, $^{-}6 \div {}^{-}2 = {}^{+}3$.

Way 3 You can use rules.

- First find the quotient of the absolute values of the integers. $6 \div 2 = 3$
- Then use the rules to decide if the quotient will be positive or negative.
- The two integers have the same sign so the quotient is positive. $^{-}6 \div {}^{-}2 = {}^{+}3$

Rules:

- The quotient of two integers with the same sign is always positive.
- The quotient of two integers with different signs is always negative.

Solution: $^{-}6 \div {}^{-}2 = {}^{+}3$.

Multiply to check.

$^{+}3 \times {}^{-}2 = {}^{-}6$

Another Example

Integers With Different Signs

$^{-}18 \div {}^{+}3 = \blacksquare$

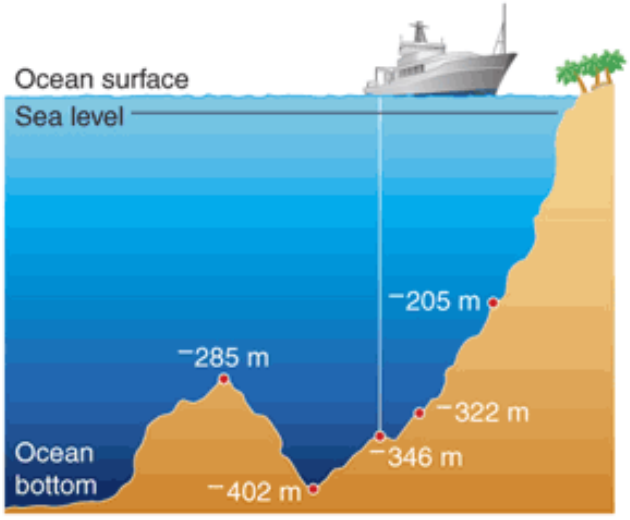
$^{-}18 \div {}^{+}3 = {}^{-}6$

Think

The absolute value of $^{-}18$ is 18 and of $^{+}3$ is 3. The signs are different. The quotient is $^{-}6$.

You can use division of integers to solve problems about averages.

A research ship is probing an area of the ocean floor. The researchers take five readings of the location of the bottom compared to sea level. The graph at right shows their results. What is their average reading for the ocean bottom?



Finding the Average

STEP 1 Add the readings.

$^{-}205 + {}^{-}322 + {}^{-}346 + {}^{-}402 + {}^{-}285$

$= {}^{-}1{,}560$

STEP 2 Divide by the number of locations.

$^{-}1{,}560 \div 5 = {}^{-}312$

Solution: The average reading is $^{-}312$ meters. On average, the bottom is 312 meters below sea level.

Guided Practice

Ask Yourself

- Are the signs the same or different?
- Which sign do I use for the quotient?

Find each quotient. Check your work.

1. $^{+}9 \div {}^{+}3$
2. $^{-}9 \div {}^{-}3$
3. $^{+}14 \div {}^{+}7$
4. $^{-}4 \div {}^{+}2$
5. $^{-}28 \div {}^{-}4$
6. $^{+}36 \div {}^{-}3$
7. $^{+}48 \div {}^{-}8$
8. $^{-}15 \div {}^{+}3$

Write each expression as a division problem. Then evaluate.

9. $^{-}12 - {}^{-}3 - {}^{-}3 - {}^{-}3 - {}^{-}3$
10. $^{+}24 - {}^{+}8 - {}^{+}8 - {}^{+}8$

Explain Your Thinking How can you tell if the quotient of two integers will be negative? positive?

Practice and Problem Solving

Find each quotient. Check your work.

11. $^{+}21 \div {}^{+}3$
12. $^{+}20 \div {}^{-}2$
13. $^{-}16 \div {}^{+}4$
14. $^{-}15 \div {}^{-}3$
15. $^{-}30 \div {}^{-}10$
16. $^{+}28 \div {}^{-}7$
17. $^{+}25 \div {}^{-}5$
18. $^{-}25 \div {}^{+}5$
19. $^{+}63 \div {}^{-}9$
20. $^{-}72 \div {}^{+}6$
21. $^{-}60 \div {}^{-}4$
22. $^{-}70 \div {}^{-}5$

Go On

Algebra • Equations Use mental math to find each divisor.

23. $63 \div n = {}^{-}9$

24. $9 \div n = 3$

25. ${}^{-}3 \div n = 1$

26. $18 \div n = 3$

27. $21 \div n = {}^{-}7$

28. ${}^{-}16 \div n = 4$

Evaluate each expression.

29. $({}^{-}7 + {}^{-}5) \div {}^{+}4$

30. ${}^{+}8 - ({}^{+}15 \div {}^{-}3)$

31. $({}^{-}2 \times {}^{+}8) \div {}^{+}4$

32. ${}^{-}8 \times {}^{+}5 \div {}^{-}2$

33. ${}^{-}18 \div {}^{-}6 \times {}^{-}5$

34. ${}^{-}7 + {}^{+}12 \div {}^{+}3$

35. Compare. Which has the greatest quotient? Explain.

${}^{-}14 \div 7$ 14 $\div$ ${}^{-}7$ ${}^{-}14 \div {}^{-}7$

Solve.

36. What's Wrong? Simon thinks that ${}^{-}12 \div {}^{-}2 \div {}^{-}3$ should have a positive answer because all the signs are the same. Explain, then correct Simon's mistake.

37. Estimate One week, Mr. Chen bought scuba diving equipment for $419.00. The next week, he rented a boat for $279.99 and bought fuel for $37.00. Estimate the amount of money Mr. Chen spent.

38. One day, Brianna went scuba diving three times. She dove to ${}^{-}24$ feet, ${}^{-}33$ feet, and ${}^{-}18$ feet, compared to the surface. Use an integer to describe the average depth of Brianna's dives.

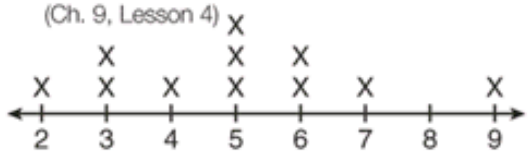
39. During its first year of business, Betty's Bait and Tackle Shop lost $2,064 for the year. What was the average amount of money Betty's lost per month?



Mixed Review and Test Prep

Open Response

40. Find the mean, median, and mode. (Ch. 9, Lesson 4)



41. A 150-gallon fish tank has a leak. The amount of water in the fish tank changes by ${}^{-}5$ gallons each hour. In how many hours will the tank be empty? (Ch. 11, Lesson 8)

Explain your answer.

Extra Practice See page 299, Set F.

Problem Solving

Calculator Connection
Calculators and Integers

You can use a calculator to add, subtract, multiply, and divide integers.

Working With Integers on a Calculator

To change the sign of a number, press the change sign key, then enter the number.

Find $^-4 + 6$.



Use your calculator to find each answer.

1. $^-14 + 25$

2. $98 - {}^-54$

3. $^-78 \times 421$

4. $6{,}765 \div {}^-5$

5. $^-809 \times {}^-36$

6. $^-120 + {}^-1{,}870$

7. $^-897 - {}^-68$

8. $^-9{,}802 \div {}^-2$

9. You work for a small shop that makes and sells special bicycles. Your job is to keep track of the money made from sales (credits) and the money spent on bills (debits). Some paint has spilled on your ledger.

Copy the ledger below. Use the change sign key on your calculator to fill in the missing entries.

Description	Credit(+)	Debit(–)	Balance
Handicap bicycle	467.58		+467.58
Box of tire tubes from WHEELS 'R US			+134.60
Mountain bicycle	375.72		
Monthly rent for shop			-389.68
Refund for a unicycle		251.92	
Welding materials		263.67	
4 boxes of 21" bike chains			-1,005.27
Tandem bicycle	848.48		
Circus cycle			+341.57
Sheet metal for frames 15 sheets @ 14.75		221.25	

Lesson 9

Problem-Solving Decision

Choose the Operation

Objective Choose operations to solve problems.

When you solve a multistep problem, you need to decide which operation to use at each step.

Terry is an oceanographer. One day he picks up a bottle floating near his ship. Inside the bottle is a note, which describes the location of a buried treasure. Here are the operations Terry chose to find the treasure.







Ask Yourself

What operation do I use for Step 1?

The position of the big oak tree is zero.

Use a positive integer to represent steps to the north.

Addition: $0 + {}^{+}15 = {}^{+}15$

What operation do I use for Step 2?

Use a negative integer to represent steps to the south.

Addition: ${}^{+}15 + {}^{-}30 = {}^{-}15$

What operations do I use for Step 3?

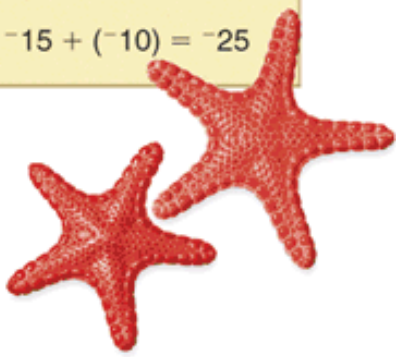
The number of steps to the south is one third of 30.

- Division: $30 \div 3 = 10$

Since these 10 steps are south, use ${}^{-}10$.

- Addition: ${}^{-}15 + ({}^{-}10) = {}^{-}25$

Solution: The treasure is located at ${}^{-}25$, or 25 steps south of the big oak tree. How did you decide what operation to choose?



Try These

Solve. Explain how you chose which operations to use.

1. Terry cannot find the treasure. He notices a smudge on the "3" in the directions for Step 2. He thinks it should read, "Walk 90 steps south." Where is the treasure if he is right?

2. To find a treasure, Yoko walks 50 steps to the east, 45 steps to the west, and then 20 steps to the east. Where is the treasure located compared to the place where Yoko started?

3. On Day 1 of a sailing trip, Ted sailed 10 miles north. On Day 2 he sailed 12 miles south. On Day 3 he sailed south $\frac{1}{4}$ as many miles as he sailed on Day 2. How far north or south is Ted from where he started?

4. Mario dives 33 meters to the bottom of the ocean to find a sunken treasure. The treasure chest sits on a rock 7 meters above the ocean floor. How far below the surface of the water is the treasure chest?

Problem Solving

Math Reasoning

IOU!

Four brothers often borrow money from one another. To keep a record, they write IOU notes whenever they borrow money. When money is paid back, the IOU is thrown away. Here are all the IOU notes the brothers now have.



1. Which brother borrowed more money than he is owed?

2. Which brother borrowed the least amount of money?

3. Which brother loaned out the most amount of money?

4. Which brother owes the least?

5. Find the total amount that each brother will owe or receive if the IOUs are paid. For example, Willie gets \$4 in the first note, owes \$5 in the third note, and gets \$3 in the fifth note. Since $4 + (^-5) + 3 = 2$, you know that Willie will receive \$2.

6. How can the brothers settle all their debts with only 3 payments?



Chapter Review/Test

VOCABULARY

1. ____ are the same distance from zero.
2. Zero is a(n) ____ that is neither positive nor negative.
3. Integers that are greater than zero are ____.
4. The integers $^-8$ and $^+8$ have a(n) ____ of 8.

Vocabulary
integer
positive
negative
absolute value
opposite numbers

CONCEPTS AND SKILLS

If the number is an integer, write its opposite. If it is not, write *no*. (Lesson 1, pp. 274–275)

5. $^-49$
6. $^+733$
7. $^-5\frac{2}{3}$
8. $^-18$
9. $^+2.88$

Order from least to greatest. (Lesson 2, pp. 276–277)

10. $^+6, ^-6, ^-11, ^+12$
11. $^+7, ^-4, ^+6, ^-8, ^-1$
12. $^+8, ^-4, ^-9, ^+7$
13. $^-2, ^-6, ^-5, ^+15, ^-9, ^+7$
14. $^-5, ^+6, ^-7, ^-4$
15. $^+30, ^-30, ^-40, ^+60$

Add or subtract. Use counters, a number line, or the rule. (Lessons 4–5, pp. 282–284, 286–287)

16. $^-8 + ^+7$
17. $^+18 + ^-14$
18. $^-12 + ^-3$
19. $^+9 + ^+8$
20. $^-15 - ^+11$
21. $^+7 - ^+3$
22. $^+14 - ^-4$
23. $^-11 - ^-2$

Find each product or quotient. Check your work. (Lessons 6–8, pp. 288–294)

24. $^-10 \times ^+11$
25. $^-6 \times ^-5$
26. $^+2 \times ^-16$
27. $^+8 \times ^+12$
28. $^-22 \div ^+2$
29. $^-8 \div ^-4$
30. $^+42 \div ^-7$
31. $^+18 \div ^+3$

PROBLEM SOLVING

Solve. (Lesson 3, pp. 278–280; Lesson 9, p. 296)

32. Jon and Anne had an equal amount of money. Jon spent $10. Anne spent $30. Now Jon has twice as much money as Anne. How much did they each have at first? Draw a diagram.
33. Anita rode her bike 5 blocks east from her house. Then she rode west twice as many blocks. She then rode 8 blocks east. How many blocks east or west is Anita from home?

Write About It

Show You Understand

Why is the product of any positive or negative integer and 0 always 0?

Extra Practice

Set A (Lesson 1, pp. 274–275)

If the number is an integer, write its opposite. If it is not, write *no*.

1. $^{+}9$ **2.** $^{-}4\frac{1}{2}$ **3.** $^{+}525$ **4.** $^{-}37$ **5.** $^{+}0.88$

6. 0 **7.** $^{+}7.33$ **8.** $^{-}2{,}422$ **9.** $^{-}75$ **10.** $^{-}2\frac{4}{5}$

Set B (Lesson 2, pp. 276–277)

Compare. Write > or < for each ●.

1. $^{+}10$ ● $^{+}20$ **2.** $^{-}7$ ● $^{+}9$ **3.** $^{-}11$ ● $^{-}22$ **4.** $^{+}29$ ● $^{-}29$

Order from least to greatest.

5. $^{-}7, ^{+}3, ^{-}2, ^{-}5$ **6.** $^{+}9, ^{-}6, ^{+}3, ^{-}9, ^{-}1$ **7.** $^{-}1, ^{-}5, ^{-}3, ^{+}11, ^{-}6, ^{+}8$

Set C (Lesson 4, pp. 282–284)

Add or subtract. Use a number line or the rule.

1. $^{-}6 + ^{-}5$ **2.** $^{+}14 + ^{-}7$ **3.** $^{+}9 + ^{-}2$ **4.** $^{+}16 + ^{-}16$ **5.** $^{-}5 + ^{+}11$

6. $^{-}5 + ^{+}9$ **7.** $^{-}11 + ^{-}4$ **8.** $^{+}8 + ^{-}9$ **9.** $^{-}4 + ^{-}4$ **10.** $^{+}5 + ^{+}3$

Set D (Lesson 5, pp. 286–287)

Subtract.

1. $^{-}4 - ^{+}3$ **2.** $^{-}18 - 0$ **3.** $^{+}10 - ^{-}6$ **4.** $^{+}5 - ^{+}7$ **5.** $^{-}9 - ^{-}8$

6. $^{+}5 - ^{+}3$ **7.** $^{-}6 - ^{+}2$ **8.** $^{+}8 - ^{+}10$ **9.** $^{-}18 + ^{+}17$ **10.** $^{-}10 - ^{+}12$

Set E (Lesson 7, pp. 290–291)

Find each product.

1. $^{+}4 \times ^{+}3$ **2.** $^{-}6 \times ^{+}5$ **3.** $^{-}4 \times ^{+}8$ **4.** $^{-}9 \times ^{-}2$ **5.** $0 \times ^{-}15$

6. $^{+}12 \times ^{-}3$ **7.** $^{-}5 \times ^{-}14$ **8.** $^{-}10 \times ^{+}7$ **9.** $^{+}3 \times ^{-}15$ **10.** $^{+}11 \times ^{-}11$

Set F (Lesson 8, pp. 292–294)

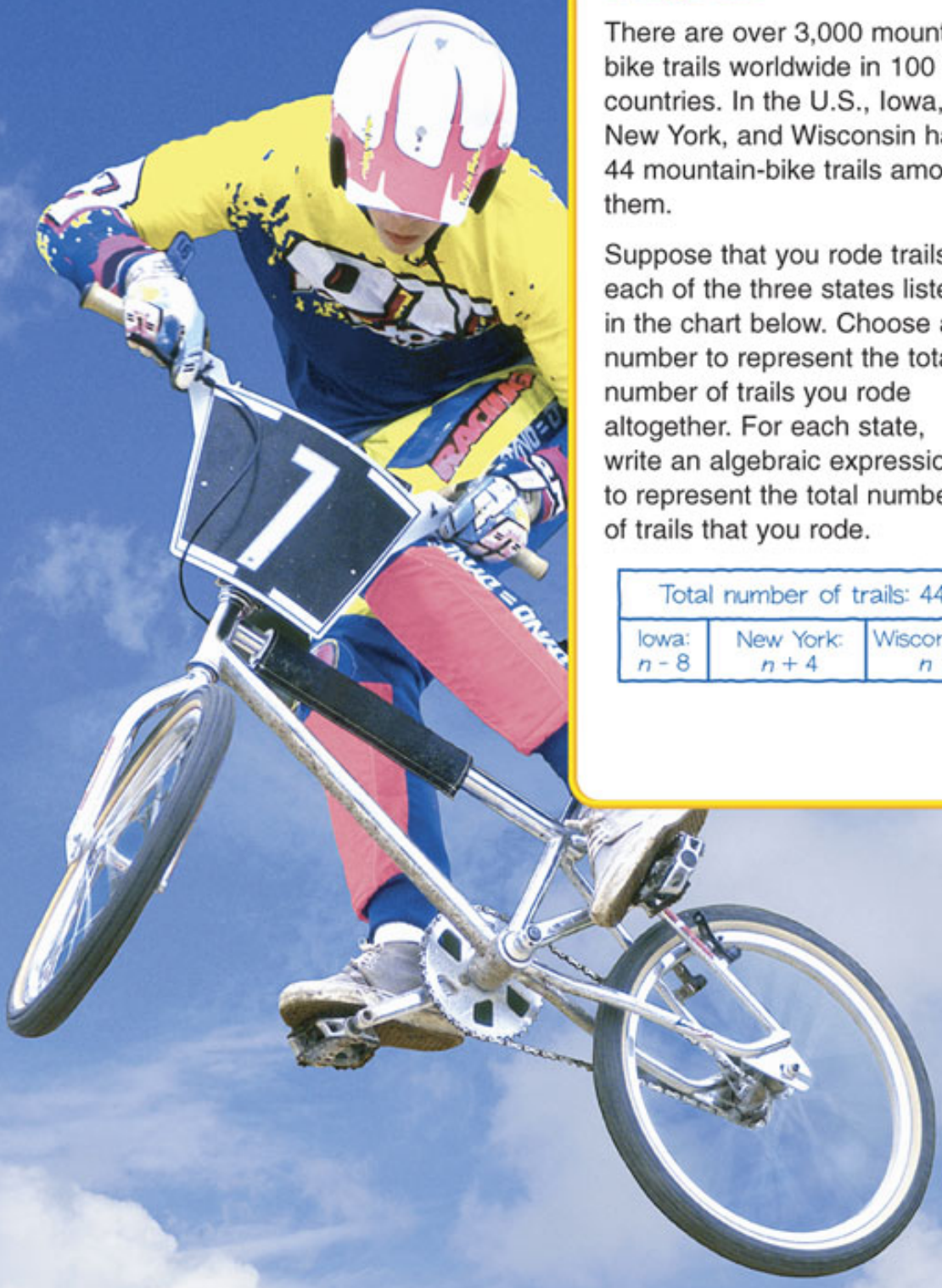
Find each quotient. Check your work.

1. $^{+}8 \div ^{+}4$ **2.** $^{-}7 \div ^{-}1$ **3.** $^{+}6 \div ^{-}2$ **4.** $^{-}4 \div ^{+}2$ **5.** $^{-}14 \div ^{+}2$

6. $^{-}12 \div ^{+}2$ **7.** $^{+}32 \div ^{-}8$ **8.** $^{+}24 \div ^{-}6$ **9.** $^{-}42 \div ^{+}3$ **10.** $^{-}35 \div ^{-}5$

CHAPTER 12

Algebra: Expressions and Equations



INVESTIGATION

Using Data

There are over 3,000 mountain-bike trails worldwide in 100 countries. In the U.S., Iowa, New York, and Wisconsin have 44 mountain-bike trails among them.

Suppose that you rode trails in each of the three states listed in the chart below. Choose a number to represent the total number of trails you rode altogether. For each state, write an algebraic expression to represent the total number of trails that you rode.

Total number of trails: 44		
Iowa: $n - 8$	New York: $n + 4$	Wisconsin: n

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A(n) ____ is a letter or symbol that represents a number in an algebraic equation.
2. A(n) ____ uses an equal sign to show that two expressions are equal.
3. A(n) ____ is a number that gives a true statement when substituted for a variable in an equation.

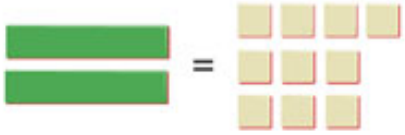
Vocabulary

- variable
- equation
- operation
- solution of an equation

CONCEPTS AND SKILLS

Match each model with the correct equation on the right.

4. 

5. 

a. $x + 3 = 5$

b. $4 + x = 7$

c. $2x = 10$

d. $3x = 9$

Match each equation with the correct solution for *x*.

6. $x + 2 = 5$
7. $2x = 12$
8. $3x = 12$
9. $x \div 4 = 7$
10. $x - 4 = 6$
11. $x + 1 = 10$
12. $4x = 20$
13. $x \div 2 = 10$
14. $8 + x = 15$
15. $x - 5 = 6$

a. 4
b. 11
c. 3
d. 28
e. 5
f. 10
g. 6
h. 20
i. 9
j. 7

Write *true* or *false* for each statement.

16. Another way of writing "40 plus a number" is $40 + n$.
17. Another way of writing "7 less than a number" is $7 - n$.
18. Another way of writing "3 times a number" is $n + 3$.
19. Another way of writing "a number divided by 2" is $2 \div n$.

Write About It

20. What everyday situations could you describe with the expression $3n$?

Facts Practice, See page 657.

Lesson 1

Write an Expression

Objective Write algebraic expressions with whole numbers, fractions, decimals, and integers.

Vocabulary
algebraic expression



Learn About It

Each student enrolled in a skateboard safety class pays a registration fee. An instructor is paid $100 from the total registration fees collected for each class. Write an **algebraic expression** to show how much money was left after paying the instructor.

STEP 1 Use words to represent the situation.

registration fees collected minus $100 paid to the instructor

STEP 2 Identify the variable. Write the expression.

The amount of the registration fees collected is unknown, so the variable should represent this amount.

Using f to represent the registration fees, the expression $f - 100$ represents the situation.

Solution: The expression $f - 100$ shows how much money was left after paying the instructor.

Other Examples

A. Division

The average speed in miles per hour of a car that traveled 520 miles in h hours

$520 \div h$

B. Two Operations

86 times the quantity $34 - n$

$86 \times (34 - n)$

Guided Practice

Write an algebraic expression for each phrase. Then evaluate, given $n = 5$.

1. 12.5 more than a number n
2. $3\frac{1}{2}$ added to n multiplied by $\frac{4}{5}$
3. a number n divided by 2
4. 22.76 less than a number n added to 36.78

Ask Yourself

- Did I substitute the correct mathematical symbols for words and phrases?
- Did I evaluate the expression correctly?

Explain Your Thinking How did you use the order of operations to evaluate the expressions?

Practice and Problem Solving

Write an algebraic expression for each. Then evaluate, given $n = 8$.

5. 1.5 less than n

6. 16 more than n minus $^-3$

7. $12\frac{1}{2}$ more than n

8. the sum of 6 and 1.28 divided by n

9. n less than n multiplied by 3

10. the sum of a number n and 2 times n

Data **Use the picture for Problems 11 and 12.**

11. When Jasmine went to the skateboard store, she bought 4 wheels, 2 helmets, and n decals. Write an expression that shows how much Jasmine spent. Evaluate the expression for $n = 2$.

12. Suppose you have x dollars. Write an expression to show how much money you would have left after buying 1 board, 2 trucks, 4 wheels, and 1 kneepad. Then evaluate the expression for $x =$ \$150.00.

13. The Club charged a registration fee of \$50 per student for one class. Write an expression that shows the money the Club had left after paying \$75 in costs. Let n represent the number of students who registered.

14. **You Decide** Describe in words a mathematical situation that involves multiplication and division. Then write and evaluate an expression that represents that situation.



Mixed Review and Test Prep

Open Response

Find the product or quotient.
(Ch. 7, Lessons 2, 5, and 6)

15. 12.6×1.2

16. $23.2 \div 0.3$

17. 3.6×2.4

18. $100 \div 0.2$

Multiple Choice

19. Evaluate the expression, "*the number* t *divided by the number* r," given $r = 1\frac{1}{2}$ and $t = 3$. (Ch. 12, Lesson 1)

A $4\frac{1}{2}$

C 1.5

B 2

D 3

Extra Practice See page 317, Set A.

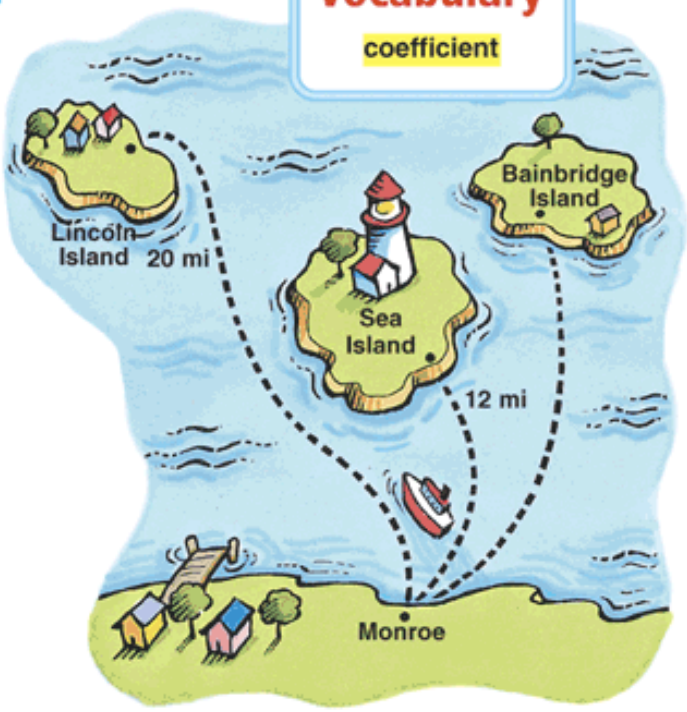
Lesson 2

Write an Equation

Objective Write equations by translating words into numbers, variables, and operations.

Vocabulary

coefficient



Learn About It

Safe Shuttle Ferry Service carries passengers to towns located on small islands along the coast. The ferry map is posted by the dock.

Sea Island is 5 miles closer to Monroe than Bainbridge Island is to Monroe. Write an equation that can be used to find the distance to Bainbridge Island from Monroe.

Write an Equation

STEP 1 Choose a variable to represent the distance to Bainbridge Island.

Since the problem is about distances, you can use the variable d.

STEP 2 Use the variable to write an equation.

Distance to Bainbridge Island minus 5 miles equals 12 miles.

$$d - 5 = 12$$

Solution: The equation $d - 5 = 12$ can be used to find the distance from Monroe to Bainbridge Island.

Other Examples

The sum of six and a number equals 238. → $6 + x = 238$

Three more than the product of 8 and a number equals 91. → $8r + 3 = 91$

▶ In the equation $8r + 3 = 91$, the number 8 is called a **coefficient** of the variable r. Coefficients are values by which the variables are multiplied.

Guided Practice

Write as an equation. Use x to represent the variable.

1. Eight more than a number equals 982.
2. Nine less than a number equals 27.
3. 0.5 times a number equals 300.

Ask Yourself

- Did I use the correct operations?
- Did I include coefficients where needed?

Explain Your Thinking ▶ How do you know when to use a coefficient in an equation?

Practice and Problem Solving

Write as an equation.

4. 1.25 times a number equals 103.75.

5. $\frac{3}{4}$ more than a number equals $16\frac{3}{4}$.

6. Seven less than a number equals 1.

7. A number divided by 13 equals 22.

Write each equation in words.

8. $\frac{1}{4}v = 88$

9. $3x = 15$

10. $83 - x = 899$

11. $5 + v = 90$

12. $4 + y = 82$

13. $x - 8 = 11$

14. $q \div 3.5 = 5$

15. $45 \div q = 2$

Solve.

16. Last week, Dave spent $48\frac{1}{2}$ hours riding the ferry. This week, he spent $3\frac{3}{4}$ more hours on the ferry. Write an equation in words to show how many hours Dave rode the ferry last week. Then write the algebraic equation.



17. Use the map and the example on page 304. Find the distance to Bainbridge Island. Then find the total distance a ferry would travel if it made one roundtrip to each island every day.

18. **Analyze** The ferry's operating costs are $31.00 per mile. Use your answer to Problem 17. How much does it cost the ferry company to make one roundtrip to each island every day?

19. A one-way ferry ticket for the Monroe–Lincoln route costs $8.50. A roundtrip ticket costs $15.00. How much more would it cost to make two one-way trips than one roundtrip?

20. **What's Wrong?** Harrison wrote the equation $25n - 2 = 98$ in words as "25 times a number minus 2 equals 98." Tell how his word equation can mean more than one thing.

Mixed Review and Test Prep

Open Response

Solve. (Ch. 8, Lesson 7)

21. Jacob's rectangular pool measures 16 feet by 20 feet. What is the perimeter of Jacob's pool?

22. Gary spent $90 on two new docklines and two new lamps for his ferry. Each lamp cost $28. Write an equation to show how much Gary spent on each dockline. Show all your work. (Ch. 12, Lesson 2)

Extra Practice See page 317, Set B.

Hands On Lesson 3

Audio Tutor 1/35 Listen and Understand

Solve Addition and Subtraction Equations

Vocabulary
inverse operations

Objective Solve addition and subtraction equations.

Learn About It

Inverse operations are operations that undo each other. Addition and subtraction are examples of inverse operations. To solve some equations, you can use inverse operations to isolate the variable on one side of the equals sign.

Try this activity to solve an addition equation by using inverse operations.

Materials: **algebra tiles** (Learning Tool 38), **workmat**

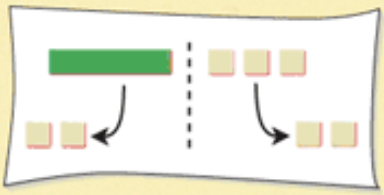
Solve the equation $x + 2 = 5$ for x.

STEP 1 Model the equation with algebra tiles.



$x + 2 = 5$

STEP 2 Subtract two 1-tiles from each side.



$x + 2 - 2 = 5 - 2$

STEP 3 The solution to the equation is 3.



$x = 3$

When you add or subtract the same number to each side of an equation, you are using a property of equality.

The Addition and Subtraction Properties of Equality

Adding or subtracting the same number from both sides of an equation results in a new equation, having the same solutions as the original.

Use properties to solve the equation $t - 5 + 2 = 11$.

STEP 1 Simplify each side of the equation.

$t - 5 + 2 = 11$
$t - 3 = 11$

STEP 2 Use the Addition Property of Equality. Add 3 to both sides.

$t - 3 + 3 = 11 + 3$

STEP 3 Isolate the variable. Solve for t.

$t = 11 + 3$
$t = 14$

Solution: 14 is a solution to the equation.

Extra Help at eduplace.com/map

Guided Practice

Solve. State the inverse operation you used.

1. $4.56 + n = 184.72$

2. $p + 5 - 4 = 23$

3. $s - 8 = 18$

4. $y + 6 - 12 = 38$

Ask Yourself

- Did I use inverse operations to isolate the variable?
- Did I perform the same operation on both sides of the equals sign?

Explain Your Thinking ▶ How can you check the solution to an equation?

Practice and Problem Solving

Solve. State the inverse operation you used. Check your work.

5. $y + 5 = 14$

6. $a - 8 = 12$

7. $s + 98 - 27 = 138$

8. $c + 45.6 - 38.25 = 112.76$

9. $12 + d = 45$

10. $3 + f - 17 = 43$

11. $t + 23 - 36 + 23 = 59$

12. $j + 38 = 4 + 5 \times 3$

13. $35 - 56 = h - 47$

Solve.

14. In his first flight, Orville Wright flew 120 feet. His brother Wilbur Wright went on to fly a distance that was 732 feet farther. How far did Wilbur fly?

15. Wilbur's flight lasted 59 seconds. Orville's flight took 47 seconds less time. How long was Orville's flight?

16. **Reasoning** Ann has decided to make a 55-mile bike trip to Kitty Hawk in 2 days. If Ann can ride 18 miles per day now, how many more miles will she need to be able to ride in one day in order to cover half the trip each day?



The Wright brothers tried several models before their first flight took place at Kitty Hawk, North Carolina, on Thursday, December 17, 1903.

17. **Analyze** Over six trial flights, a model airplane stayed airborne for an average of 26 seconds. For the first five trials, the flight times were 28 seconds, 10 seconds, 52 seconds, 20 seconds, and 26 seconds. What was the sixth time?

Mixed Review and Test Prep

Open Response

If the number is an integer, write its opposite. If it is not, write *no*. (Ch. 11, Lesson 1)

18. $43\frac{1}{2}$

19. $^{-}34$

20. $^{-}90.7$

21. 146

Multiple Choice

22. Solve $s + 18 - 11 = 123$ for s. (Ch. 12, Lesson 3)

A 94

C 130

B 152

D 116

Extra Practice See page 317, set C.

Hands On Lesson 4

Audio Tutor 1/36 Listen and Understand

Solve Multiplication and Division Equations

Objective Solve multiplication and division equations.



Learn About It

Winona rides a bus 12 miles a month to and from her guitar lessons. If she has 4 lessons each month, how can Winona write and solve an equation to find the number of miles she rides to and from each lesson?

Winona wrote the equation $4 \times x = 12$ to express *4 trips times a number of miles is 12 miles.*

You can solve a multiplication equation by using division, the inverse of multiplication.

Use algebra tiles to solve the equation $4 \times x = 12$ for x.

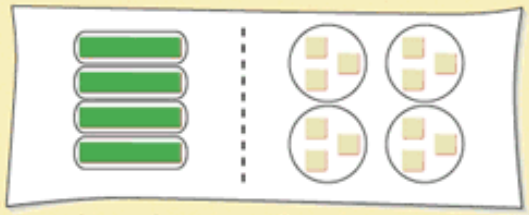
Materials • algebra tiles (Learning Tool 38)**, workmat**

STEP 1 Model the equation with algebra tiles.



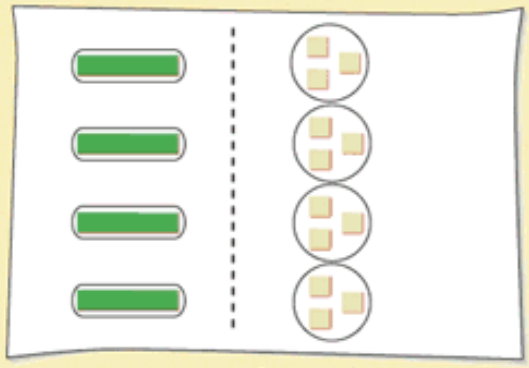
$4 \times x = 12$

STEP 2 Isolate the x-tile. Use the *inverse* of multiplication. Divide the tiles into 4 equal groups.



$(4 \times x) \div 4 = 12 \div 4$

STEP 3 Match each x-tile to a group of 1-tiles. Each x-tile is equivalent to 3 1-tiles.



$x = 3$

▶ When you multiply or divide each side of an equation by the same nonzero number, you are using a property of equality.

The Multiplication and Division Properties of Equality

Multiplying or dividing both sides of an equation by the same nonzero number results in a new equation, having the same solutions as the original.

Use properties to solve the equation $4 \times x = 12$.

Since the variable x is multiplied by a number, use the Division Property of Equality to isolate the variable.

$4 \times x = 12$

$4 \times x \div 4 = 12 \div 4$ ← Divide both sides by 4.

$x = 12 \div 4$ ← Isolate the variable.

$x = 3$ ← Solve for x.

Check: Check your work by substituting the solution into the original equation.

$4 \times x = 12$

$4 \times 3 = 12$

$12 = 12$

So the solution, 3, is correct.

Solution: Winona rides 3 miles to and from each lesson.

Another Example

Use Multiplication to Solve

$\frac{a}{1.5} = 22$

$\frac{a}{1.5} \times 1.5 = 22 \times 1.5$ ← Multiply both sides by 1.5. (Multiplication Property of Equality)

$a = 22 \times 1.5$ ← Isolate the variable.

$a = 33$

Check:

$\frac{a}{1.5} = 22$

$\frac{33}{1.5} = 22$

$22 = 22$ It checks.

Guided Practice

Solve. State the inverse operation you used.

1. $4.5 \times c = 18$

2. $a \times 4 = 24$

3. $n \div 7 = 8$

4. $m \div 7 = 30$

Ask Yourself

- Did I perform the same operation on both sides of the equals sign?
- Did I check my work?

Explain Your Thinking Why is it helpful to isolate the variable when solving an equation?

Practice and Problem Solving

Solve. State the inverse operation you used.

5. $d \div 9 = 2$

6. $c \div 3 = 9$

7. $2 \times e = 14$

8. $f \div 5 = 3$

9. $y \times 8 = 64$

10. $j \div 7 = 49$

11. $k \times 4 = 36$

12. $m \div 45 = 3$

13. $120 \times u = 240$

14. $m \div 12 = 36$

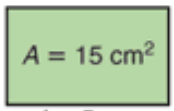
15. $p \times 18 = 18$

16. $t \div 15 = 10$

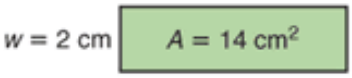
Go On

Remember that the area, A, of a rectangle is $l \times w$. Write and solve an equation to find each missing length or width.

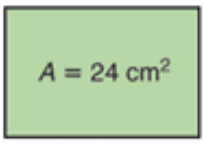
17.



18.



19.



Algebra • Functions

Write a function rule to show how the input (m) is changed to the output (n).

20.

Input (m)	Output (n)
12	24
1	2
51	102
12.6	25.2
m	?

21.

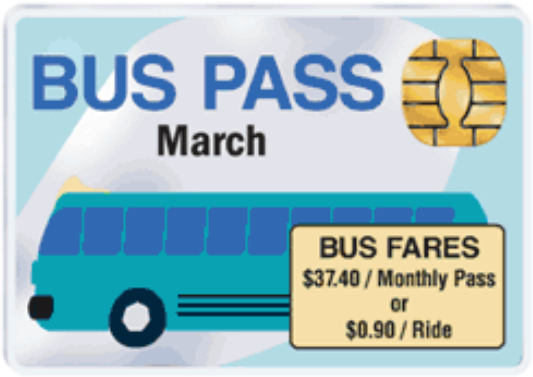
Input (m)	Output (n)
120	30
100	25
60	15
16	4
m	?

22.

Input (m)	Output (n)
60	30
55	25
50	20
40	10
m	?

Data

Use the advertisement for Problems 23–26.



23. Winona's brother Jon takes the bus to and from college each day. He rides the bus about 22 days each month. How much does it cost him to ride to and from college each day?

24. How much money would Jon save each 22-day month by buying the bus pass?

25. You Decide Kenji knows that because of vacations, he will only be going to college 15 days in December. Should he buy a bus pass for December? Explain.

26. Analyze How many days each month would a person need to ride to and from work to make buying a bus pass worthwhile?

Mixed Review and Test Prep

Open Response

Compute. (Chs. 6 and 7)

27. $33 \times \frac{1}{2}$

28. $4\frac{2}{3} \div 3$

29. $222 \div 0.2$

30. $81.2 \times 38\frac{1}{2}$

31. $7{,}200 \div 10^4$

32. 8.1×10^{-2}

33. Last month, Jenna rode a bus 7 times from her house to and from baseball practice. If she rode a total of $45\frac{1}{5}$ miles, how far is it from her house to practice? Explain how you found your answer. (Ch. 12, Lesson 4)

Extra Practice See page 317, Set D.

Problem Solving

Algebraic Thinking
Exploring Inequalities

Inequalities are similar to equations. The two expressions are separated by $<$ or $>$, instead of $=$. Solving inequalities is similar to solving equations in that you use inverse operations. To solve the inequality $5x > 25$, divide both ides by $5x > 5$. Notice that there are an infinite number of values for x that are greater than 5. An equation has only one solution, but an inequality has an infinite number of solutions, which can be shown on a number line.

Look at the number line below. The open circle indicates that the particular value is not included as a solution of the inequality.



1. What inequality is expressed by the number line?

2. Write and plot an inequality that expresses all the solutions of $x - 2 < 5$.

3. Using a number line, plot and solve the inequalities that are expressed by each statement.
   - seven minus a number x less than 4
   - a number x multiplied by 8 is less than 24
   - four plus a number x is less than negative ten

Quick Check

Check your understanding of Lessons 1–4.

Write an algebraic expression or equation for each phrase. Then evaluate, given $n = 4$. (Lessons 1 and 2)

1. 22.73 more than a number equals n
2. a number n is added to the product of $3\frac{1}{2}$ and n

Solve. State the inverse operation you used. (Lessons 3 and 4)

3. $y + 6 = 8$
4. $7a = 21$
5. $4 \times 9 \div 3 = r \times 6$

Lesson 5

Problem-Solving Application

Use Equations

Objective Use an equation to solve a problem.

Sometimes you can write and solve an equation to help you solve a problem.

Problem A-1 Bike Rentals has two bike rental plans. For each plan you pay a service charge and an hourly rate, depending on the day of the week and the type of bike.

On Saturday, Bill wants to rent a mountain bike. He has $85. What is the greatest number of hours that Bill can rent a mountain bike?



Bike Rental Plans

	Weekdays	Weekends
Service Charge	$5.00	$8.50
Mountain	$10.00/h	$12.00/h
Road	$7.50/h	$10.00/h

UNDERSTAND

What is the question?

What is the greatest number of hours that Bill can rent a mountain bike?

What do you know?

- Service charge: $8.50
- Per-hour charge: $12.00
- Maximum amount to spend: $85.00

PLAN

You can write and solve an equation to solve the problem.

SOLVE

- Write an equation.

  Let h represent the number of hours. Use the information you know to write an equation.

$$8.50 + 12 \times h = 85.00$$

- Solve the equation.

$$\begin{aligned} 8.50 - 8.50 + 12 \times h &= 85.00 - 8.50 \\ 12 \times h &= 76.50 \\ (12 \times h) \div 12 &= 76.50 \div 12 \\ h &= 6.375 \end{aligned}$$

Solution: Bill can rent a mountain bike on a Saturday for at most 6 hours.

LOOK BACK

Look back at the question.

Why didn't you round your answer to 7 hours?

Guided Practice

Use the table on page 312 to solve each problem.

1. Mandy wants to rent a road bike on Wednesday. She has $30. What is the longest time that she can rent the bike?

2. When Benjamin rented a mountain bike on Tuesday, he was charged $45. For how long did Benjamin rent the bike?

Hint Do you want to know how much he paid or how long he rented the bike?

Ask Yourself

UNDERSTAND What does the question ask me to find?

PLAN Did I write the correct equation to answer the question?

SOLVE
- Did I use the correct operation(s)?
- Did I correctly compute the answer?

LOOK BACK Does my answer make sense?

Independent Practice

Data **The *We Ride Bike Shoppe* rents bikes. The table shows the store's rental plans. Use the table to solve Problems 3–5.**



	Weekdays	Weekends
Service Charge	$10.00	$15.50
Mountain	$12.00/h	$15.00/h
Road	$9.50/h	$12.00/h

3. Cheyna wants to rent a mountain bike on Sunday for 4 hours. How much will she need to pay?

4. Darlene wants to rent a road bike for 4 hours on Sunday. Who will pay more for her rental, Cheyna or Darlene? How much more?

5. Suppose *We Ride Bike Shoppe* offers $\frac{1}{10}$ off their rates to senior citizens. If Jake's grandfather rented a road bike on Thursday for 3 hours, how much was his rental?

6. **Create and Solve** Make up a rental plan for another type of business or service, such as a car rental company or a skating rink that rents skates. Then write and solve a problem about the plan you created.

Go On

Mixed Problem Solving

Solve. Show your work. Tell which strategy you used.

7. Richard rented a mountain bike from *We Ride Bike Shoppe* for 5 hours on Saturday and a road bike on Tuesday for 2 hours. How much did he pay for both rentals? Use the table on page 313.

8. While training for a competition, Mel walked for 10 minutes, ran for 24 minutes, and swam for 18 minutes. Then he rested. If he trained for one hour, how long did Mel rest?

9. Three objects each weigh less than 12 pounds. Each weighs a whole number of pounds. If the objects are placed on a scale two at a time, the pairs weigh 13 pounds, 16 pounds, and 15 pounds. How much does each object weigh?

You Choose

Strategy

- Make a Table
- Use Logical Reasoning
- Use Models
- Work Backward
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the table for Problems 10–13. Then tell which method you chose.**

10. How many more calories are burned by a 180-pound cyclist than a 130-pound cyclist if both pedal 14 miles in an hour?

11. Suppose the same number of cyclists will continue to ride to work at least once a week each year. About how many cyclists will ride their bicycles to work at least once a week over the next 1.5 years? Round your answer to the nearest million.

12. **Analyze** How many bikes can fit in a parking lot with 52 spaces for automobiles?

13. About how many automobiles can fit in a parking lot with space for about 200 bicycles?

Bicycle Facts and Statistics

- Estimated number of cyclists in U.S. who bicycled to work at least once a week last year: **3.2 million**
- Calories burned by a 130-pound cyclist pedaling 14 mi/h: **402**
- Calories burned by a 180-pound cyclist pedaling 14 mi/h: **540**
- Bicycles that can be parked in the space required for one automobile: **about 12**



Problem Solving

Number Sense

Solving Two-Step Equations

Some equations involve more than one operation, such as $2c - 3 = 4$. To isolate the variable, you may have to use more than one inverse operation.

Inverse Operations

- Addition and Subtraction
- Multiplication and Division

Solve the equation $2c - 3 = 4$ for c.

$2c - 3 = 4$	Addition is the inverse of subtraction.
$2c - 3 + 3 = 4 + 3$	Add 3 to both sides of the equation. (Addition Property of Equality)
$2c = 7$	Simplify.
$2 \times c = 7$	Recall that $2c$ means $2 \times c$.
	Division is the inverse of multiplication.
$(2 \times c) \div 2 = 7 \div 2$	Divide both sides by 2. (Division Property of Equality)
$c = 3\frac{1}{2}$	Simplify.

Tell what is done in each step to solve the equation.

1.
$3n + 2 = 11$
$3n + 2 - 2 = 11 - 2$
$3n = 9$
$3 \times n = 9$
$(3 \times n) \div 3 = 9 \div 3$
$n = 3$

2.
$7c + 2 = 2$
$7c + 2 - 2 = 2 - 2$
$7c = 0$
$7 \times c = 0$
$(7 \times c) \div 7 = 0 \div 7$
$c = 0$

3.
$9y - 3 = 6$
$9y - 3 + 3 = 6 + 3$
$9y = 9$
$9 \times y = 9$
$(9 \times y) \div 9 = 9 \div 9$
$y = 1$

Solve each equation.

4. $2t + 3 = 5$

5. $8r - 14 = 10$

6. $3d = 8 + 1$

7. $5f - 5 = 10$

8. $4m + 4 = 4$

9. $0.5s + 3 = 5$

Chapter Review/Test

VOCABULARY

1. The value by which a variable is multiplied is called a ____.
2. A(n) ____ is an expression that consists of one or more variables.
3. Operations that undo each other are ____.
4. A(n) ____ is a mathematical sentence that uses an equals sign to show that two quantities are equal.

Vocabulary

- equation
- solution
- variable
- coefficient
- algebraic expression
- inverse operations

CONCEPTS AND SKILLS

Write an algebraic expression for each. Then evaluate, given $n = 9$. (Lesson 1, pp. 302–303)

5. 10 more than n minus 4
6. 3 less than 6 times n

Write as an equation. (Lesson 2, pp. 304–305)

7. A number times 3.3 equals 82.5.
8. A number divided by 28 equals 31.
9. Seven less than a number equals 15.
10. Eight times a number equals 144.

Write each equation in words. (Lesson 2, pp. 304–305)

11. $z \div 5.1 = 33$
12. $77 + x = 130$
13. $6w = 54$

Solve. State the inverse operation you used. (Lessons 3–4, pp. 306–310)

14. $56 + b = 34 + 27$
15. $y - 16 = 8$
16. $c - 8 = 78$
17. $g + 8 = 22$
18. $3 \times 4 \div 6 = d - 15$
19. $h + 35 + 14 = 119$
20. $j + 46.9 - 5.4 = 147.8$
21. $10 + x = 7 \times 3 - 8 \div 2$
22. $12 + a - 27 = 150$
23. $k \times 9 = 126$
24. $x \times 15 = 75$
25. $y \div 4 = 9$
26. $6 \times b = 42$
27. $d \times 205 = 820$
28. $a \div 5 = 12$
29. $f \div 7 = 8$
30. $w \times 3 = 90$
31. $c \times 8 = 88$

PROBLEM SOLVING

Use an equation to solve each problem. (Lesson 5, pp. 312–314)

32. A one-day ski pass from the Blue Mountain Ski Club costs $50. In January, Alma spent $250 on ski passes. How many did she buy?
33. Last season the Blue Mountain Ski Club had 110 members. The club collected a total of $4,950 in membership fees. How much did the club charge each member?

Write About It

Show You Understand

Explain why it is important to follow the rules for the order of operations when solving an equation. Use an example from this test in your explanation.

Extra Practice

Set A (Lesson 1, pp. 302–303)

Write an algebraic expression for each. Then evaluate, given $n = 6$.

1. $3\frac{1}{3}$ less than n
2. 18 less than 5 times n
3. 8 more than n plus $^{-}3$
4. $4\frac{1}{3}$ added to n times $\frac{2}{3}$
5. 34 less than n plus 42
6. 10.2 more than a number n
7. the sum of $\frac{1}{2}n$ and $5\frac{1}{2}$
8. n less than n multiplied by 98

Set B (Lesson 2, pp. 304–305)

Write as an equation.

1. Eight more than a number equals 12.
2. $\frac{2}{3}$ less than a number equals $7\frac{2}{3}$.
3. Nine less than a number equals 72.
4. Four times a number equals 72.
5. A number times 6.5 equals 42.25.
6. A number divided by 41 equals 15.

Write each equation in words.

7. $x - 5 = 8$
8. $4y = 48$
9. $58 + x = 420$
10. $v \div 6.2 = 5$

Set C (Lesson 3, pp. 306–307)

Solve. State the inverse operation you used.

1. $b + 9 = 24$
2. $x - 17 = 17$
3. $e - 13 = 33$
4. $c + 3.3 - 8.22 = 18.6$
5. $16 + f - 37 = 143$
6. $41 + x = 28 + 39$
7. $48 - 27 = z - 15$
8. $8 \times 4 \div 2 = 9 + g$
9. $7 + d = 8 \times 2 - 6 \div 2$

Set D (Lesson 4, pp. 308–310)

Solve. State the inverse operation you used.

1. $d \times 4 = 88$
2. $x \div 5 = 7$
3. $b \div 8 = 6$
4. $g \times 7 = 84$
5. $a \div 6 = 8$
6. $y \div 9 = 5$
7. $f \times 16 = 80$
8. $h \times 110 = 330$
9. $c \div 75 = 2$

CHAPTER 13

Algebra: Patterns, Functions, and the Coordinate Plane

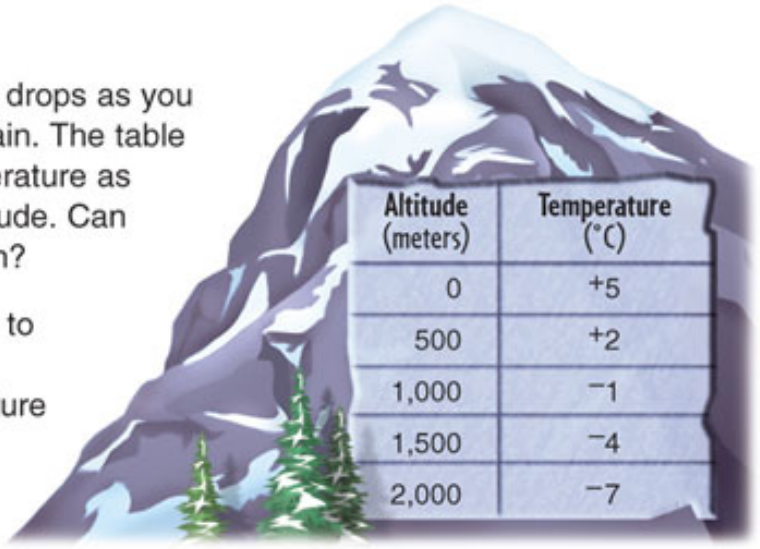


INVESTIGATION

Using Data

The temperature drops as you ascend a mountain. The table shows the temperature as a function of altitude. Can you find a pattern?

Use your pattern to predict what the Celsius temperature would be at an altitude of 3,000 meters.



Altitude (meters)	Temperature (°C)
0	+5
500	+2
1,000	−1
1,500	−4
2,000	−7

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

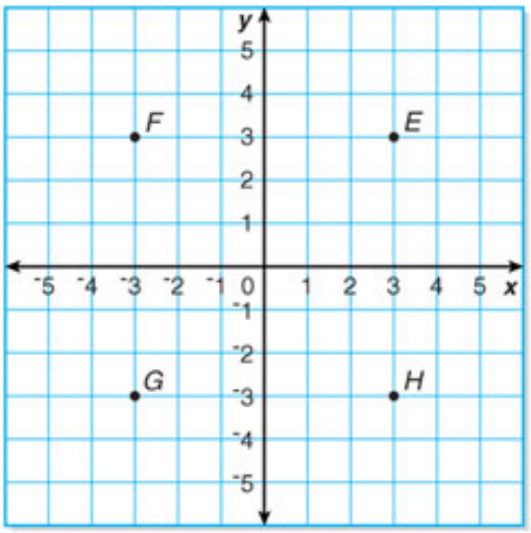
1. A(n) _____ contains a horizontal number line and a vertical number line that intersect at (0, 0).
2. The _____ is the vertical number line.
3. The _____ is the horizontal number line.

Vocabulary

- ***x*-axis**
- ***y*-axis**
- **origin**
- **quadrant**
- **coordinate plane**

CONCEPTS AND SKILLS

Use the graph at right for Exercises 4–7. Write the coordinates for each point.



4. *E*
5. *G*
6. *F*
7. *H*

Graph each ordered pair on a coordinate plane.

8. $(^{-}4, 2)$
9. $(4, 2)$
10. $(4, ^{-}2)$
11. $(^{-}4, ^{-}2)$

Write *true* or *false* for each statement. Plot the points on a graph if you need help.

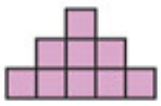
12. The point (2, 3) is to the right of the point (3, 3).
13. The coordinates (4, 1) are an ordered pair.
14. The point $(^{-}2, 3)$ is below point $(^{-}1, 4)$.
15. The point (0, 5) is above (0, 4).

Write the next likely term in each sequence.

16. 2, 4, 6, 8, ■
17. 5, 3, 1, $^{-}1$, ■
18. 1, 10, 100, 1,000, ■
19. 9, $^{-}18$, 27, $^{-}36$, ■

Write About It

20. What number pattern does this tile pattern show? Explain how you would continue the number pattern.



Facts Practice, See page 657.

Lesson 1

Graph Points in the Coordinate Plane

Objective Identify and graph points in a coordinate plane.

Vocabulary

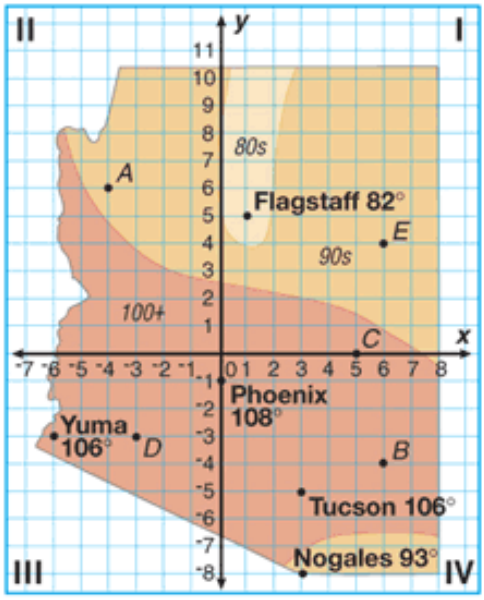
- coordinate plane
- origin
- ordered pair
- *x*-coordinate
- *y*-coordinate

Learn About It

A **coordinate plane** contains a horizontal number line called the *x*-axis and a vertical number line called the *y*-axis. The point where the two axes intersect, (0, 0), is called the **origin**.

The axes divide the coordinate plane into four quadrants, I, II, III, and IV.

The map shows daily high temperatures for one day in June in Arizona. Where is Flagstaff located on the coordinate plane?



Find the coordinates for Flagstaff.

STEP 1 Locate Flagstaff on the coordinate plane above. Flagstaff is in the first quadrant.

STEP 2 Find the distance left or right of the origin. Find the distance above or below the *x*-axis.

Flagstaff is located 1 unit to the right of the origin and 5 units above the *x*-axis.

Solution: The location of Flagstaff is given by the **ordered pair** (1, 5). The first number, 1, is the ***x*-coordinate** and the second number, 5, is the ***y*-coordinate** in the ordered pair.

Another Example

Distance Between Points

- Point *B* is at $(^{+}6, ^{-}4)$ and point *E* is at $(^{+}6, ^{+}4)$.
- For two points, if the *x*-coordinates or the *y*-coordinates are the same, you can count to find the distance between those two points.
- The distance between *B* and *E* is 8 units.

Name the city with coordinates (3, $^{-}$5) on the map.

STEP 1 Begin at the origin. Since 3 is a positive integer, move right three units.

STEP 2 Since $^{-}$5 is a negative integer, move down five units.

Solution: Tucson is the city with coordinates at (3, $^{-}$5).

Guided Practice

Use the map on page 320 for Exercises 1–7. Write the ordered pair for each city listed.

1. Phoenix **2.** Nogales **3.** Yuma

Name the letter at each ordered pair. Tell in which quadrant or on which axis the point lies.

4. (6, 4) **5.** ($^{-}$4, 6) **6.** (5, 0) **7.** ($^{-}$3, $^{-}$3)

Ask Yourself

- Did I count along the *x*-axis to find the first coordinate?
- Did I count along the *y*-axis to find the second coordinate?

Explain Your Thinking In which quadrant does the point ($^{-}$1, 3) lie? How do you know?

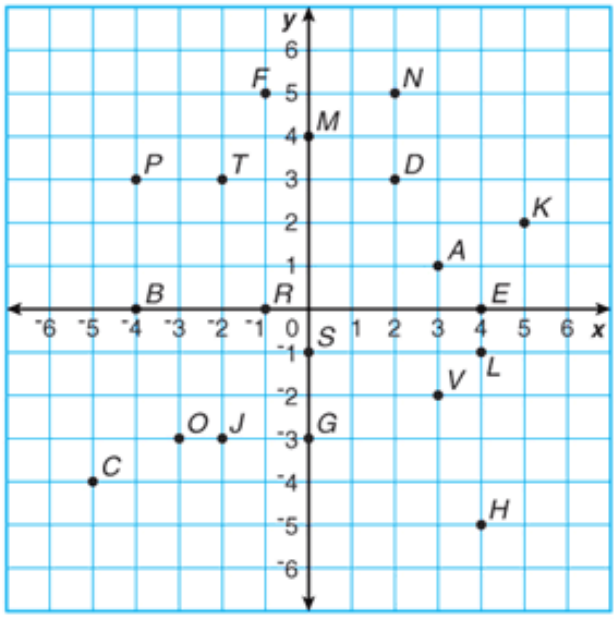
Practice and Problem Solving

Use the graph for Problems 8–23. Write the coordinates for each point.

8. *A* **9.** *B*

10. *C* **11.** *D*

12. *E* **13.** *F*



Find the distance between each pair of points.

14. *L* and *H* **15.** *P* and *D*

Name the letter of the point at each ordered pair.

16. (2, 5) **17.** ($^{-}$2, $^{-}$3)

18. ($^{-}$4, 3) **19.** (0, 4)

20. (4, $^{-}$1) **21.** ($^{-}$2, 3)

22. (5, 2) **23.** ($^{-}$1, 0)

Go On

Graph each ordered pair. Label the point with the given letter. Tell in which quadrant or on which axis the point lies.

24. Point A (1, 5)

25. Point B (3, $^{-}6$)

26. Point C (0, $^{-}2$)

27. Point D ($^{-}4$, $^{-}6$)

28. Point E ($^{-}5$, 4)

29. Point F ($^{-}2$, $^{-}5$)

30. Point G (5, 2)

31. Point H (0, 3)

32. Point J ($^{-}6$, 0)

Answer each question.

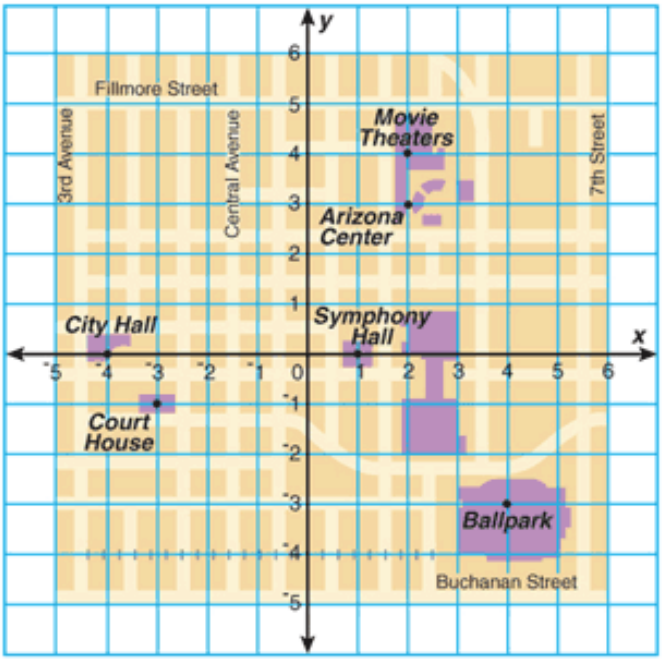
33. A point lies in Quadrant IV. What is the sign of the first coordinate? What is the sign of the second coordinate?

34. A point lies in Quadrant III. What is the sign of the first coordinate? What is the sign of the second coordinate?

35. The first coordinate of a point is negative, and the second coordinate is positive. In which quadrant does the point lie?

36. Both coordinates of a point are positive. In what quadrant does the point lie?

Data **Use the map of downtown Phoenix for Problems 37–41.**



37. Name the coordinates for the Arizona Center.

38. Which two buildings are located on the x-axis? Explain how you know.

39. **What's Wrong?** Farha says the Ballpark is located at ($^{-}3$, 4). Describe and correct her error.

40. **Analyze** What building is located 4 units to the left and 1 unit down from Symphony Hall?

41. **Create and Solve** Write a problem that uses the map and coordinate plane. Solve the problem.

Mixed Review and Test Prep

Open Response

Choose the appropriate expression to match each phrase. (Ch. 12, Lesson 1)

42. $d + 8$ **a.** eight times a number d

43. $8 \div d$ **b.** the sum of a number d and 8

44. $d - 8$ **c.** eight less than a number d

45. $8d$ **d.** eight divided by a number d

46. A point is located 3 units to the left of the origin and 3 units up. What ordered pair describes the location of the point? (Ch. 13, Lesson 1)

Explain how you found your answer.

Extra Practice See page 339, Set A.

Find The Direction

Game Activity

2 Players

What You'll Need • Learning Tool 39 or coordinate grid for each player

How to Play



1. The object of the game is to find the other player's lost barometer and weather map.

2. Each player marks the location of a barometer (1 point) and a weather map (3 points in a row) on his or her grid without letting the other person see the points.

3. First, take turns naming the coordinates of a point to try to locate the other player's barometer.

   After each try, the other player will tell whether it was found or whether it is above, below, to the right of, or to the left of the named point.

4. After finding the barometer, each player tries to locate all 3 points of the other player's weather map. The first player to find the lost barometer and weather map is the winner.

Lesson 2

Audio Tutor 1/37 Listen and Understand

Sequences and Patterns

Objective Find, describe, and extend patterns in number sequences.

Vocabulary
- sequence
- term

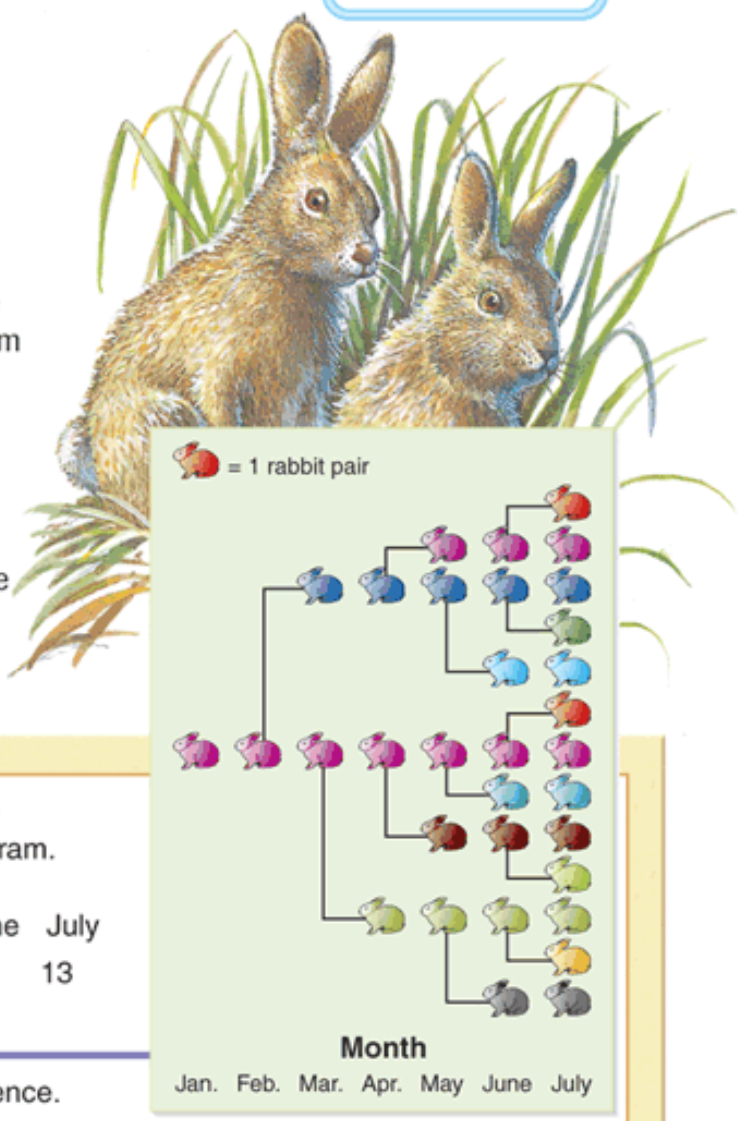
Learn About It

A **sequence** is an ordered set. The **terms** in a sequence often are numbers.

In the year 1202, a mathematician called Fibonacci made up a word problem about pairs of rabbits. The number sequence that was used to solve the problem became known as the *Fibonacci sequence:*

1, 1, 2, 3, 5, 8, 13, . . .

Suppose you have a pair of newly born rabbits on January 1. After the rabbits are two months old, the pair will start to produce a new pair of rabbits on the first of each month. Each new pair of rabbits follows the same pattern. If all the pairs of rabbits live, how many pairs of rabbits will there be on August 1?



STEP 1 List the number of pairs each month as the terms of a number sequence. Use the diagram.

Month:	Jan.	Feb.	Mar.	Apr.	May	June	July
Pairs:	1	1	2	3	5	8	13

STEP 2 Look for a pattern in the terms of the sequence.

$1 = 1 + 0$ $2 = 1 + 1$ $3 = 1 + 2$ $5 = 2 + 3$ . . .

The number of pairs each month is equal to the sum of the number of pairs in the two previous months. So on August 1 there will be 8 + 13, or 21 pairs.

Solution: There will be 21 rabbit pairs on August 1.

Other Examples

A. Find the Next Three Terms

2, 4, 8, 16, 32, ?, ?, ?

×2 ×2 ×2 ×2 ×2 ×2 ×2

The rule is "multiply the previous term by 2." The next three terms are 64, 128, and 256.

B. Find the Missing Terms

27, 9, 3, ?, ?, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$

÷3 ÷3 ÷3 ÷3 ÷3 ÷3 ÷3

The rule is "divide the previous term by 3." The missing terms are 1 and $\frac{1}{3}$.

Extra Help at eduplace.com/map

Guided Practice

Ask Yourself
- How are the terms related?
- What pattern can I find?

Write the next three likely terms in each sequence.

1. 5, 20, 35, 50, ... **2.** 5, 1, $^{-}3$, $^{-}7$, ... **3.** 1, 7, 1, 7, 1, ...

Write the missing terms.

4. 10, 8, 11, 9, ■, ■, ■, 11, 14, 12

Explain Your Thinking Could more than one rule be used to describe the same pattern? Give an example.

Practice and Problem Solving

Write the next three likely terms in each sequence.

5. 19, 17, 15, 13, ... **6.** 7, $^{-}14$, 28, $^{-}56$, ... **7.** \$0.50, \$1.50, \$4.50, \$13.50, ...

8. 4, 8, 3, 6, 1, 2, ... **9.** 10^9, 10^6, 10^3, ... **10.** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

Write the missing terms.

11. 63, 56, 49, ■, ■, 28, 21 **12.** 2.4, 4.2, 6.0, ■, ■, 11.4, 13.2

Algebra • **Expressions** **Write a rule for each sequence as an algebraic expression. Use *n* as the previous term. Use the rule to write the next three terms in the sequence.**

13. $\frac{1}{2}, \frac{1}{8}, \frac{1}{32}, \frac{1}{128}, \ldots$ **14.** 20, 14, 8, 2, ... **15.** 1,000, 200, 40, 8, ...

Solve.

16. Look back at the rabbit problem on page 324. If all pairs live, how many rabbit pairs will there be on October 1?

17. The third term in a sequence is 35. The rule is "add 8 to each term." What are the first and sixth terms in the sequence?

18. **Explain** You work one hour a day for 14 days. You have a choice of being paid \$5 for each hour worked or 1¢ for the first hour, 2¢ for the second, 4¢ for the third, 8¢ for the fourth, and so on. How would you rather be paid?

19. **You Decide** You are asked to add all the counting numbers from 1 to 100.

$1 + 2 + 3 + \ldots + 98 + 99 + 100$

Can you think of a way to use a pattern to solve this problem quickly?

Mixed Review and Test Prep

Open Response

Order the integers from least to greatest. (Ch. 11, Lesson 2)

20. $^{-}11$, $^{-}2$, $^{-}10$ **21.** 5, $^{-}2$, $^{-}5$

22. $^{-}16$, $^{-}7$, 4 **23.** $^{-}6$, $^{-}8$, 0, $^{-}2$

Multiple Choice

24. Find the next likely number. (Ch. 13, Lesson 2)

640, 565, 490, 415, ...

A 400 **B** 365 **C** 340 **D** 75

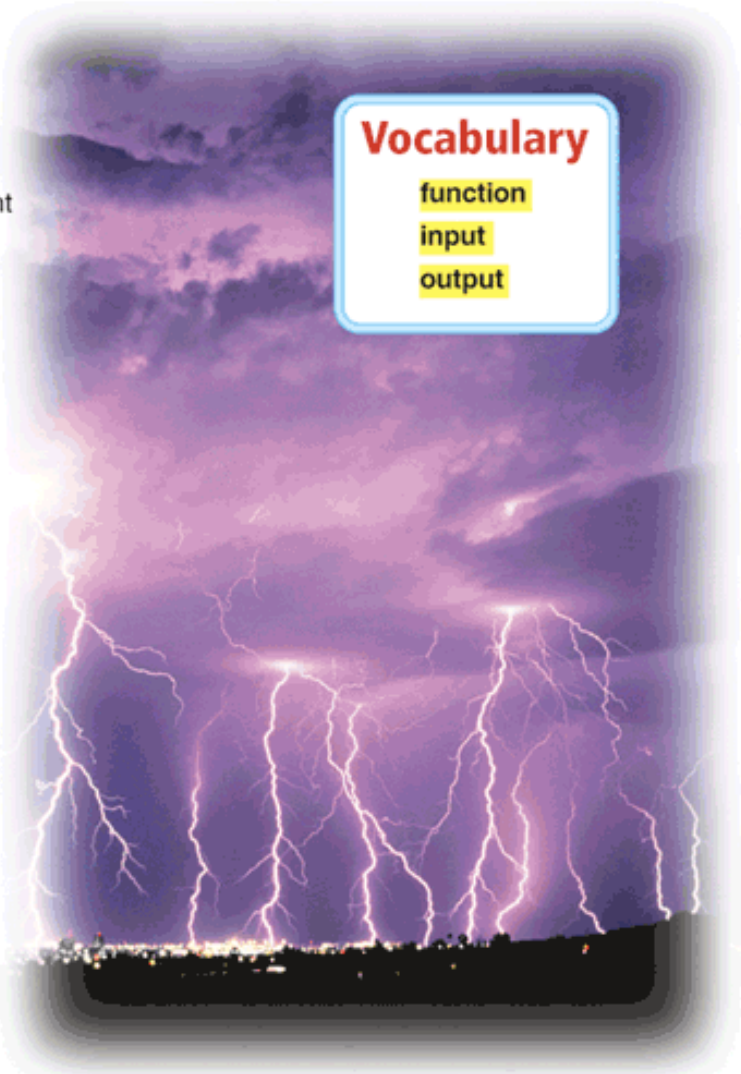
Extra Practice See page 339, Set B.

Lesson 3

Audio Tutor 1/38 Listen and Understand

Functions

Objective Investigate functions and different ways to represent functions.



Vocabulary

- function
- input
- output

Learn About It

A **function** is a rule that pairs each **input** with one and only one **output**.

Sound travels 0.2 mile each second. You can estimate how many miles you are from a thunderstorm by counting the seconds between seeing the lightning and hearing the thunder.

The distance to a thunderstorm is a *function* of the number of seconds counted. What is the distance to the storm if you count 5 seconds? 10 seconds? 15 seconds?

Lightning and thunder occur at the same time. Since light travels faster than sound, you see the flash before you hear the thunder.

Write and evaluate a function.

Different Ways to Represent a Function

Way 1 As a rule, using words

Distance to storm = 0.2 × number of seconds

Multiply the number of seconds by 0.2.

Number of seconds:	5	10	15
Distance to storm (miles):	1	2	3

Way 2 As an equation

Let d represent the distance and n the number of seconds. Then $d = 0.2 \times n$. Substitute.

For $n = 5$, $d = 0.2 \times 5$, or 1
For $n = 10$, $d = 0.2 \times 10$, or 2
For $n = 15$, $d = 0.2 \times 15$, or 3

Way 3 As a function table

Find the output values of the function $d = 0.2 \times n$ for the input values $n = 5$, 10, and 15.

Seconds counted (n)	5	10	15
Distance (d)	1	2	3

Way 4 As a set of ordered pairs

Use the input (n) as the first number and the output (d) as the second number in the ordered pair.

(5, 1), (10, 2), (15, 3)

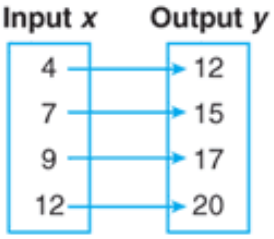
Solution: If you count 5 seconds, the storm is about 1 mile away, 2 miles away at 10 seconds, and 3 miles away at 15 seconds.

You can also show a function as a mapping.

Write a rule that shows how *y* is related to *x*.

The mapping at right shows an input *x* and an output *y*. This pairing of input and output is a function because for each input there is one and only one output.

- Look for a pattern in the *x* and *y* values to find the rule.
- Each *y* value is 8 more than the *x* value.



$4 + 8 \rightarrow 12$
$7 + 8 \rightarrow 15$
$9 + 8 \rightarrow 17$
$12 + 8 \rightarrow 20$

Solution: The function rule is $y = x + 8$.

Guided Practice

Tell whether each represents a function. Explain.

1. $a = 10b + 5$

2. Multiply by 4 and add 2.

3. $m = s \div 60$

4.

x	0	1	2	3	0
y	0	3	4	6	8

Ask Yourself
- Is there one and only one output for each input?
- How do I find the missing values?

5. Find the function rule. Then use the rule to find *i* when $f = 8$.

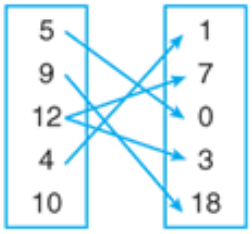
Feet (*f*)	1	2	3	4	5
Inches (*i*)	12	24	36	48	60

Explain Your Thinking How does finding a pattern help you find a function rule?

Practice and Problem Solving

Tell whether each represents a function. Explain.

6.



7. speed = distance ÷ time

8.

p	1	2	3	4
q	6	7	8	7

Go On

Show another way to represent each function.

9. (0,1), (1, 2), (2, 3), (3, 4)

10. $d = c \div 2$

11. cost = number of people × $8

Copy. Use the rule to complete each function table.

12. $y = x + 7$

x	y
3	
5	
7	
9	

13. $g = {}^{-}50f$

f	g
0	
1	
2	
3	

14. $b = \frac{2}{3}a$

a	b
0	
3	
6	
9	

Find the rule for each function table.

15.

x	⁻1	0	1	2
y	4	5	6	7

16.

Number of vans (v)	1	2	3	4
Number of students (n)	6	12	18	24

Solve.

17. Suppose ten inches of snow are equivalent to one inch of rain. Write an equation to show the amount of snow as a function of the amount of rain.

18. During four days a town got the following daily rainfall: 0.5 inches, 1.2 inches, 1.9 inches, and 0.8 inches. What was the average daily rainfall?

19. **Represent** Write a function for the pattern shown below. Use two different representations.

Figure 1



Figure 2



Figure 3



Figure 4



20. **What's Wrong?** Claire wrote the rule for the function table below as $y = 2x$. Explain why she is wrong.

x	y
8	4
22	11
34	17
48	24

Mixed Review and Test Prep

Open Response

Find each product or quotient.
(Ch. 11, Lessons 7–8)

21. $10 \times {}^{-}8$

22. $72 \div {}^{-}9$

23. ${}^{-}15 \div {}^{-}3$

24. ${}^{-}54 \div {}^{-}6$

25. Write an equation to represent the function shown in the table.
(Ch. 13, Lesson 3)

Explain how you found the equation.

x	y
7	2
12	7
15	10

Extra Practice See page 339, Set C.

Activity

Social Studies Connection

Where Are You?

Lines of *latitude* cross the globe from east to west between the equator and the poles. Lines of *longitude* run between the poles east and west of the prime meridian.

By using latitude as the first coordinate and longitude as the second coordinate, you can locate cities.



The prime meridian runs through western Europe and Africa at 0º longitude.



The equator circles the globe at 0º latitude.

Study the map of the United States. Can you identify these capital cities?

1. 41.4°N, 93.4°W **2.** 44.2°N, 69.4°W **3.** 39.4°N, 104.6°W **4.** 30.3°N, 84.2°W

5. Estimate the coordinates of your city or town. Use an atlas to check.

Math Challenge

Order, Order

Insert parentheses to make each statement true.

1. $1 + {}^{-}2 \times 3 - {}^{-}4 = {}^{-}1$

2. $1 + {}^{-}2 \times 3 - {}^{-}4 = 1$

3. $1 + {}^{-}2 \times 3 - {}^{-}4 = {}^{-}7$

4. $1 + {}^{-}2 \times 3 - {}^{-}4 = {}^{-}13$

Make up your own set of problems. Challenge a classmate to solve them.

Brain Teaser

Each different letter in this puzzle represents a different digit. What digits do each of the letters represent?

$$\begin{array}{r} \text{SEVEN} \\ -\ \text{NINE} \\ \hline \text{EIGHT} \end{array}$$

Hint: S + 1 = N

Education Place

Check out **eduplace.com/map** for more brain teasers.

Lesson 4

Graph Functions

Objective Represent functions with graphs.

Vocabulary
linear function
nonlinear function

Learn About It

In Lesson 3, you learned that functions can be represented by words, tables, equations, and ordered pairs. Functions also can be represented by graphs.

When you are given a rule for a function, you can graph the function by making a table of input-output values.

Draw a graph to represent the function $y = x + 5$.

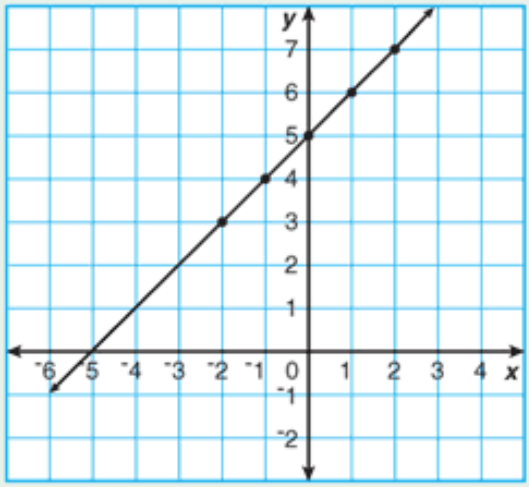
STEP 1 Make a table of values. Choose x-values that make the values of y easy to find, such as $^-2$, $^-1$, 0, 1, and 2. Write them in numerical order.

Substitute each value of x to find the corresponding value of y.

Input x	Output $y = x + 5$	Ordered Pairs (x, y)
$^-2$	$y = (^-2) + 5 = 3$	$(^-2, 3)$
$^-1$	$y = (^-1) + 5 = 4$	$(^-1, 4)$
0	$y = (0) + 5 = 5$	$(0, 5)$
1	$y = (1) + 5 = 6$	$(1, 6)$
2	$y = (2) + 5 = 7$	$(2, 7)$

STEP 2 Label the horizontal axis with the input values x. Label the vertical axis with the output values y.

STEP 3 Plot the ordered pairs. Use a ruler to see if the points lie on a line. If so, draw the line.



The graph of a function is the graph of all points whose coordinates are solutions to the equation representing the function. Note that the points of this graph lie on a straight line. Functions whose points lie along a straight line are called **linear functions**. Functions whose points do not lie along a straight line are called **nonlinear functions**.

Sometimes when you use a function to represent a real-life situation not all points are meaningful solutions to the equation of the function.

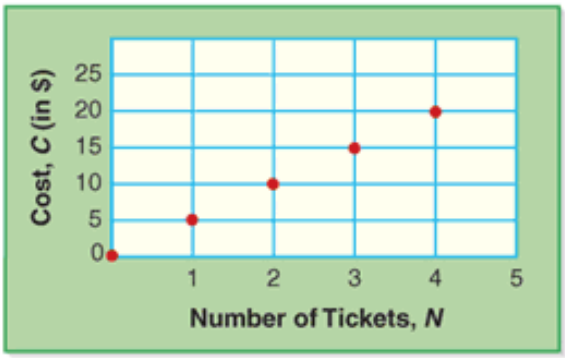
On a visit to a science museum, Morgan and her three friends decide to watch a show on storm chasers. Tickets to the show cost \$5 each. You can use the function $C = 5N$ to model the total cost C for N tickets. What is the cost of 4 tickets?



Make a table of values and graph the function.

Only non-negative values of N make sense in this example. Plot the points in the first quadrant.

N	C	(N, C)
0	0	(0, 0)
1	5	(1, 5)
2	10	(2, 10)
3	15	(3, 15)
4	20	(4, 20)



Note that the points are not connected by a line. Only points for whole-number values of N make sense, since you can buy only whole tickets.

Solution: Four tickets cost \$20.

Guided Practice

For each function, copy and complete a table of values like the one shown below. Use grid paper to graph the function.

1. $y = x + 3$

2. $y = x$

3. $y = {}^{-}2x$

4. $y = 4$

5. Copy the graph in the example above. Then extend the graph to find the cost of 8 tickets.

x	y
$^{-}2$	
$^{-}1$	
0	
1	
2	

Ask Yourself

- Did I use the horizontal axis for the x values and the vertical axis for the y values?
- Do the points lie in a straight line?

Explain Your Thinking What do you notice about the graph of the function in Exercise 4?

Go On

Practice and Problem Solving

For each function, make a table of values. Then use grid paper to graph the function.

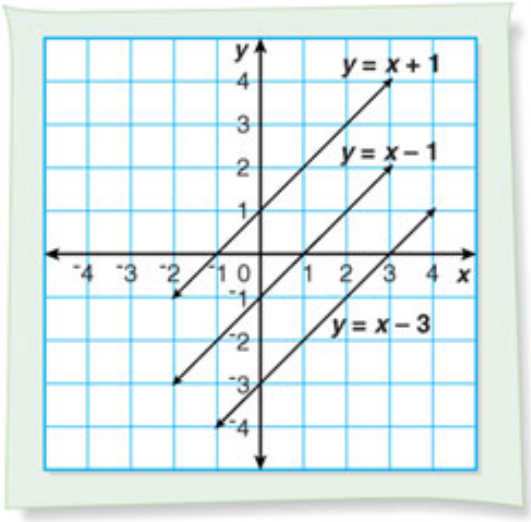
6. $y = x - 2$ **7.** $y = {}^{-}5x$ **8.** $y = x + 2$ **9.** $y = 3x$

10. $y = 2x - 1$ **11.** $y = 3 - x$ **12.** $y = 0$ **13.** $y = {}^{-}1$

Graph the function $y = {}^{-}2x + 1$. Tell whether or not each ordered pair lies on the line. Explain.

14. $(0, 1)$ **15.** $({}^{-}5, 3)$ **16.** $(1, {}^{-}1)$ **17.** $({}^{-}2, 5)$

18. **Reasoning** Look at the three graphs at right. Each graph is identified by its equation. What patterns do you see in the graphs and in the equations? Without graphing, describe what the graph of $y = x + 3$ would look like and where it would be located.



19. On grid paper, draw the graph of $y = 4x$ and the graph of $y = {}^{-}4x$ on the same set of axes. How are the graphs alike? How are they different?

20. During a blizzard, the snow depth increased steadily by 2 inches each hour for 5 hours. Write an equation to represent the snow depth as a function of time. Then graph the function.

Solve.

21. **Predict** Suppose the blizzard in Problem 20 continued and the snow depth increased in the same pattern. Extend your graph to predict the snow depth after 9 hours of snowing.

22. **Represent** A tile pattern starts with 6 tiles. Each new figure is formed by adding 3 tiles to the previous figure. Write a function rule for the pattern.

23. **Analyze** Suppose you graph the function in Problem 22. Would you connect the points or not? Explain your reasoning.



Problem Solving

Science Connection

Crickets and Temperature

Male crickets make chirping songs. Scientists have noticed this relationship between the number of chirps per minute (x) and the temperature in degrees Fahrenheit (F).

$$F = \frac{1}{4}x + 40$$



1. At about what temperature F will a cricket chirp 25 times a minute?
2. About how many times a minute will a cricket chirp when the temperature is 75°F?
3. What is the temperature when a cricket chirps 28 times a minute?

WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

Graph each ordered pair on a coordinate plane. Label the point. Tell in which quadrant or on which axis the point lies. (Lesson 1)

1. $A\ (5, 3)$
2. $B\ (^{-}4, ^{-}4)$
3. $C\ (^{-}2, 0)$

Write the next three likely terms in each sequence. (Lesson 2)

4. 31, 27, 23, 19, ...
5. $1, 1\frac{2}{3}, 2\frac{1}{3}, 3, \ldots$
6. 15, 19, 23, 27, ...
7. 31,250, 6,250, 1,250, 250, ...

Use the rule to make a function table. Then graph the function. (Lessons 3–4)

8. **Rule:** $y = x + 3$
9. **Rule:** $p = {}^{-}q$
10. **Rule:** $y = 2x - 1$

Lesson 5

Problem-Solving Application

Use Functions and Graphs

Objective Use graphs and functions to solve problems.



You can solve some problems by interpreting a graph of a function.

Problem For her science project on weather, Selena must cool water. She has a thermometer that measures temperature in degrees Celsius. If the solution has a temperature of 15°C, what would the temperature be in degrees Fahrenheit?

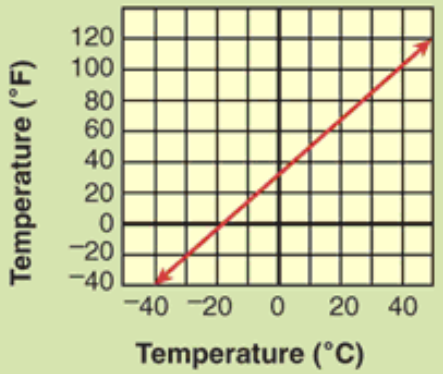
Use a graph.

The graph shows Fahrenheit temperature as a function of Celsius temperature.

- Find 15°C on the horizontal axis.
- Draw a vertical line from this point to the graph.
- From this point, draw a horizontal line across to the vertical axis.
- The line crosses the vertical axis at about 60°F.

Solution: 15°C is about 60°F.

Relationship of Fahrenheit and Celsius Temperatures



Use a conversion formula to convert 15°C to degrees Fahrenheit.

An equation that is used many times in similar problems is called a *formula*. The relationship between Fahrenheit (F) and Celsius (C) temperature is given by this formula.

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(15) + 32 \leftarrow \text{Substitute 15 for } C.$$

$$= 9 \times 3 + 32$$

$$= 27 + 32$$

$$= 59$$

Solution: 15°C is 59°F.

Look Back Which method should Selena use if she needs to know an exact temperature? Explain.

Guided Practice

Use the graph or the formula to convert each temperature.

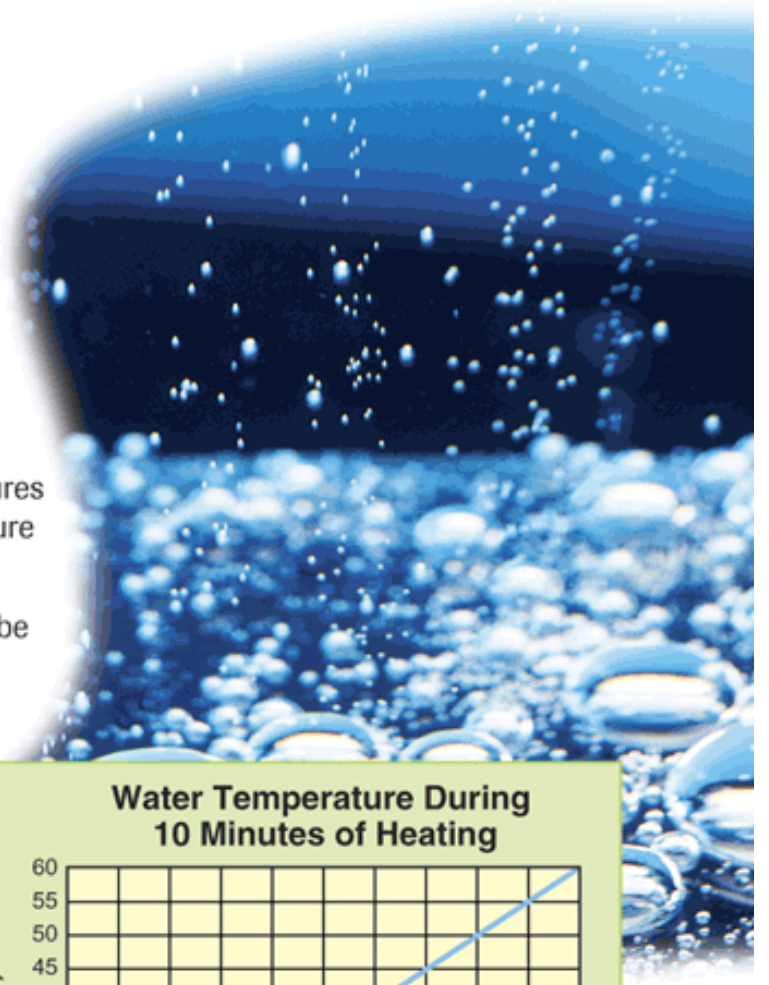
1. What is the temperature in degrees Fahrenheit when it is 0°C?

Hint Where is 0°C located on the horizontal axis?

2. In an experiment, Pierre needed to know the exact temperature in degrees Fahrenheit when it is 30°C. What is that temperature?

Ask Yourself

- Should I use the graph or the formula?
- Did I read the information in the graph correctly?
- Is my answer reasonable?



Independent Practice

Solve. Use the graph on page 334 for Problems 3–5.

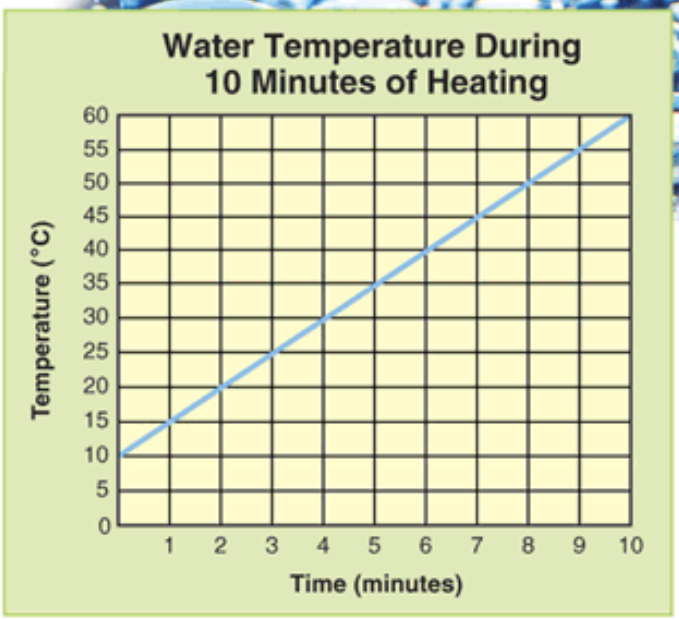
3. Debbie says that $^{-}20$°C is about 30°F. How could you use the graph to tell whether she is right or wrong?

4. **Estimate** Suppose you extend the horizontal axis of the graph to include Celsius temperatures greater than 50°C. Approximate the temperature in degrees Fahrenheit for 70°C.

5. **Create and Solve** Write a problem that can be solved by using the temperature graph or the conversion formula. Give the problem to a classmate to solve.

6. For the first 10 minutes, the increase in the temperature (T) of water that is heated can be described by the function $T = 5t$, where t is the time in minutes. If the water temperature was 25°C to start with, what was it after 7 minutes?

7. **What's Wrong?** The graph shows the increase in temperature of water that is heated for 10 minutes. Marcy looks at point (10, 60). She predicts that the water temperature will be 90°C after 15 minutes. What did she do wrong?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

8. Percy has 10 loose identical red socks and 10 loose identical blue socks in his drawer. Without looking, what is the least number of socks he would have to pick to be sure to have a pair of the same color?

9. There are two blinking lights on one block. One light blinks every 10 seconds, while the other light blinks every 6 seconds. How many times per minute do they blink at the same time?

10. On Sunday, Paul walked three times farther than Saturday. On Saturday, he walked half as far as he did on Friday. If Paul walked 2.5 miles on Friday, about how many miles did he walk in all?

11. Sue gave half of the trinkets she bought to Al. Then she gave half of what she had left to Dale. She gave 3 trinkets to Sal. Now she had two trinkets left. How many trinkets did she have to start with?

You Choose

Strategy

- Draw a Diagram
- Guess and Check
- Make an Organized List
- Use Logical Reasoning
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Solve. Then tell which method you chose.

12. At a park, 50 tulips are planted in equal rows. If the total number of tulips is 5 times the number of tulips in one row, how many tulips are in one row?

13. Hector and Nancy were paid $45.00 for painting a neighbor's fence. Hector worked 6 hours and Nancy worked 9 hours. If they earned the same hourly rate, how much does Nancy get?

14. Mu Tan and Shing collected $29 for tickets to the science museum exhibit on temperature. Adults' tickets cost $7 and children's tickets cost $5. How many adults and how many children plan to visit the museum?

15. Walter earns $25.50 a day working part-time in a shoe store. He gets $0.50 extra for each package of sports socks he sells. After 5 days of work, Walter had earned $157.50. How many packages of sports socks did he sell?

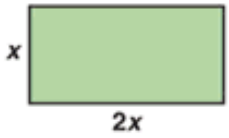
16. Hansa has $8 in her savings account and adds $1 each week. Natu has $12 in his savings account and adds $3 each week. After how many weeks will Natu's account have twice as much money as Hansa's account?

17. **Explain** Pedro plays checkers with a computer. If Pedro wins, he gets 5 points. If the computer wins, it gets 2 points. They play 49 games and end with a tie score. How many games did the computer win?

Problem Solving

Algebraic Thinking
Nonlinear Functions

Look at the rectangle on the right. The perimeter (P) and area (A) of the rectangle are functions of the value of x.



1. Write an equation to express P as a function of x.

2. Write an equation to express A as a function of x.

3. Copy and complete the table below.

x	P	A	(x, P)	(x, A)
0	0	0	(0, 0)	(0, 0)
1	6	2	(1, 6)	(1, 2)
2				
3				
4				

4. Graph the two sets of ordered pairs on the coordinate plane. Connect the points for the perimeter with a straight line. Connect the points for the area with a smooth curve.

5. **Analyze** Compare the two graphs. Describe how they are different.

The graph of the equation for the area represents a **nonlinear function**. For nonlinear functions, the rate of change from one point to the next is not constant.



6. **Calculator** The path of the water from a fountain can be modeled by the nonlinear equation $y = {}^{-}0.60x^2 + 3.73x$, where x is the horizontal distance in feet and y is the height of the water arc in feet. Graph the equation. Then estimate the maximum height of the water arc.



Extra Practice, See page 339, Set D.

Chapter Review/Test

VOCABULARY

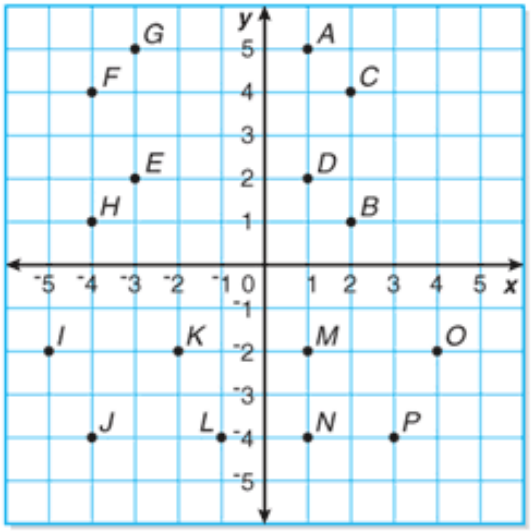
1. The _____ is the first number in an ordered pair.
2. The second number in an ordered pair is the _____ .
3. A _____ is one number in a sequence.

Vocabulary

term
ordered pair
x-coordinate
y-coordinate

CONCEPTS AND SKILLS

Use the graph on the right. Name the letter of the point at each ordered pair. (Lesson 1, pp. 320–322)



4. $(^-3, 5)$
5. $(^-3, 2)$
6. $(^-5, ^-2)$
7. $(^-2, ^-2)$
8. $(2, 1)$
9. $(1, 5)$
10. $(1, 2)$
11. $(^-4, ^-4)$

Write the next three likely terms in each sequence. (Lesson 2, pp. 324–325)

12. $1\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}, \ldots$
13. $8, 0, ^-8, ^-16, \ldots$
14. $40, 4, 0.4, 0.04, \ldots$

Find the rule for each function table. Then use the rule to complete each table. (Lesson 3, pp. 326–328)

15. Rule: $y =$ ■

x	$^-2$	$^-1$	0	1	2	3
y	$^-7$	$^-6$	$^-5$	$^-4$		

16. Rule: $b =$ ■

a	$^-2$	0	2	4	6	8
b	$^-1$	0	1	2		

17–18. Graph each function in Exercises 15 and 16. (Lesson 4, pp. 330–332)

PROBLEM SOLVING

Solve. (Lesson 5, pp. 334–336)

19. Al used a graph and put his house at (0, 0). Frank's house is at (2, 2), Ed's house is at $(^-1, ^-2)$, and Jim's is at $(^-2, 3)$. Whose house is closest to Al's?
20. Al's school is at $(^-3, 4)$ on the graph. Who lives closest to the school—Al, Frank, Ed, or Jim?

Write About It

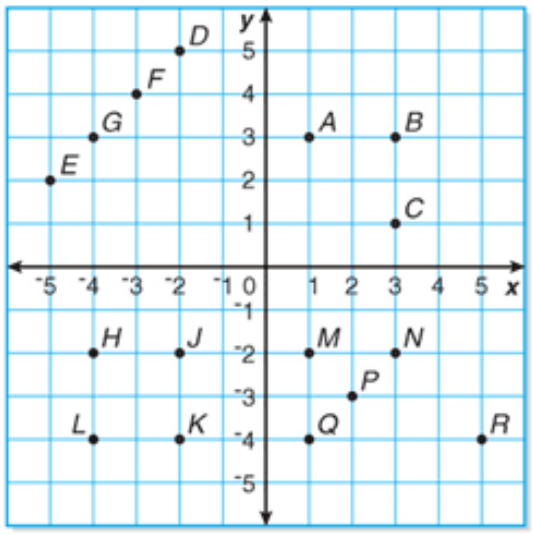
Show You Understand

Could Al make a function table to show the information on his graph in a different way? Explain.

Extra Practice

Set A (Lesson 1, pp. 320–322)

Use the graph at the right. Write the coordinates for each point.



1. B
2. K
3. F
4. P

Use the graph on the right. Name the letter of the point at each ordered pair.

5. $(3, 1)$
6. $(^{-}2, ^{-}2)$
7. $(1, ^{-}4)$
8. $(^{-}4, ^{-}2)$
9. $(3, ^{-}2)$
10. $(^{-}2, 5)$

Set B (Lesson 2, pp. 324–325)

Write the next three likely terms in each sequence.

1. 10, 40, 70, 100, . . .
2. 5, 1, $^{-}3$, $^{-}7$, . . .
3. 20, 22, 30, 33, . . .
4. 160, 80, 40, 20, . . .
5. 3, $^{-}9$, 27, $^{-}81$, . . .
6. 5, 7, 11, 19, . . .

Write the missing terms.

7. 10, 50, ■, ■, 30, 150
8. $6\frac{1}{2}$, 13, ■, ■, 104, 208
9. 8.1, 7.0, 5.9, ■, ■, 2.6, 1.5
10. 72, 36, 18, ■, ■, 2.25, 1.125

Set C (Lesson 3, pp. 326–328)

Write a rule for each function table.
Then use the rule to complete each table.

1. Rule: $y =$ ■

x	y
$^{-}1$	2
0	3
1	4
2	5
3	
4	

2. Rule: $y =$ ■

x	y
0	$^{-}2$
1	$^{-}1$
2	0
3	1
4	
5	

3. Rule: $b =$ ■

a	b
$^{-}1$	5
0	0
1	$^{-}5$
2	$^{-}10$
3	
4	

4. Rule: $d =$ ■

c	d
0	$^{-}1$
1	2
2	5
3	8
4	
5	

Set D (Lesson 4, pp. 330–332)

For each function, make a table of values.
Then graph the function.

1. $y = x - 3$
2. $y = 4 + x$
3. $y = 3x - 3$
4. $y = 2$

WEEKLY READER®
Reading Connection

The Chessboard Problem

There is an old Arabian tale about a king who wanted to reward a man for inventing the game of chess.

Chess is played on a board with 64 squares. As his reward, the inventor asked for 1 grain of wheat on the first square of the board, 2 grains of wheat on the second square, 4 grains on the third, and so on.

For each new square, the number of grains of wheat would be twice the number on the previous square until all 64 squares would be covered. The king, thinking this was a small request, granted the reward. What do you think happened?



Problem Solving

Calculator **Use the chessboard and the story on page 340 for Problems 1–6.**

Square	Number of Grains	Total Number of Grains
1	$1 = 2^0$	1
2	$2 = 2^1$	3
3	$4 = 2^2$	7
4	$8 = 2^3$	15
5	? ?	?

1. Make a table like the one at right. Find the number of grains of wheat on each of squares 1 through 10. Then find the total number of grains.

2. Recall that $2^0 = 1$, $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, and so on. Use exponents to rewrite the number of grains on each of squares 1–10.

3. **Analyze** Look for a pattern in the exponents. What relationship do you notice between square number and exponent?

4. **Calculator** Use the pattern and exponents to predict the number of grains on square 32. How big a number is that? Use the power key to find out!

5. **Calculator** You can use the algebraic expression $2^n - 1$ to find the total number of grains through square n. Use the expression to check the totals in your table.

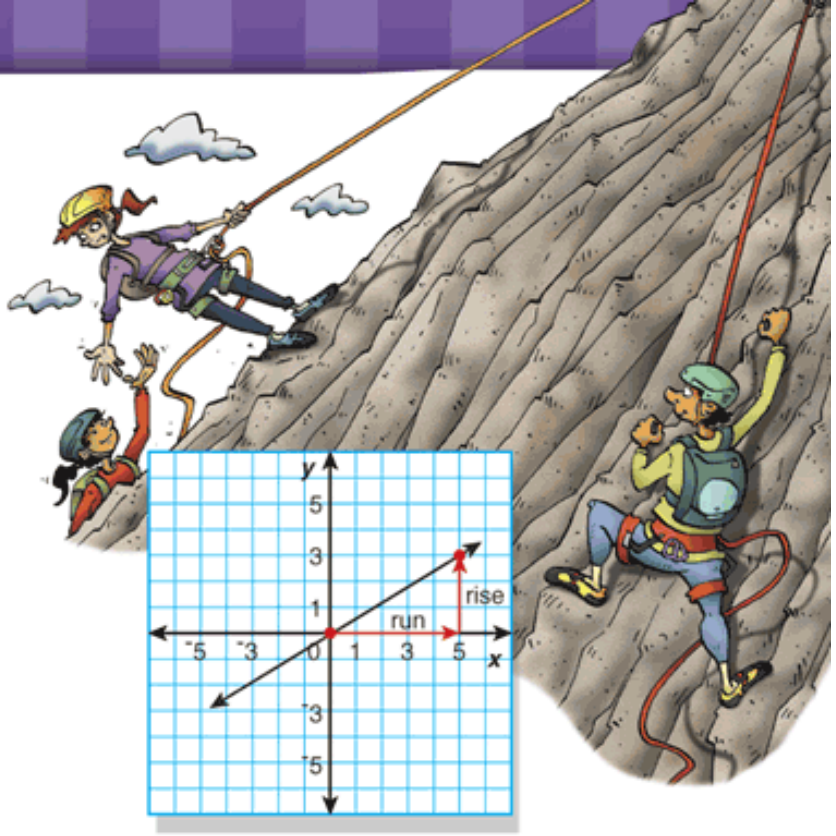
6. **Calculator** Find the number of grains on square 64 and the total number of grains on the chessboard. That's enough wheat to cover the state of California one foot deep!

Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic

Enrichment: Graph Sense

SLOPE OF A LINE

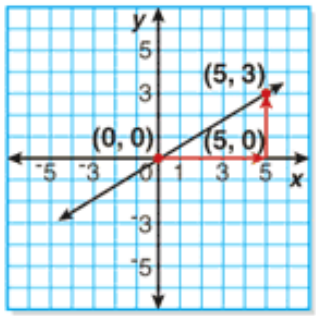


The **slope** of a line is a measure of the steepness, or slant, of the line. The slope is given as $\frac{\text{rise}}{\text{run}}$.

The **rise** is the vertical difference between any two points on the line, and the **run** is the horizontal difference between those same two points. If a line passes through the origin, it is easy to find its slope.

Find the slope of the line $y = \frac{3}{5}x$.

STEP 1 Graph the equation and choose two points on the graph. Label the coordinates of the points.



STEP 2 Find the slope. Subtract the *y*-coordinates to find the rise. Subtract the *x*-coordinates to find the run. Write $\frac{\text{rise}}{\text{run}}$.

rise: $3 - 0 = 3$ run: $5 - 0 = 5$

slope: $\frac{3}{5}$

Solution: The slope of the line $y = \frac{3}{5}x$ is $\frac{3}{5}$.

Try These!

Graph each line. Then find the slope of each line.

1. $y = x$
2. $y = 2x$
3. $y = 4x$
4. $y = {}^-2x$
5. $y = {}^-4x$

Explain Your Thinking

- What happens if you choose two different points?
- What connection do you see between the equations and the slopes of the corresponding lines?



Planting Profits



Shonda's family owns a business that sells garden tools all year round, but makes them only during the winter. Shonda's father uses a spreadsheet to record their company's finances.

You can create your own spreadsheet using Easy Sheet.

- Starting in cell A1, enter the information shown in the spreadsheet at right.
- Click on cell D2. Click on **Window**. Choose **Function List**.
- Scroll down on the function list and double-click **Sum (range)**.
- Click and drag from cell B2 to cell C2. Press enter to find the profit for January.
- Click on cell D2. Click **Edit**, then **Copy**. Now click on cell D3. Click **Edit**, then **Paste**. Repeat to paste into cells D4 through D13.

	A	B	C	D
1	Month	Income	Expenses	Profit
2	January	2250	-9220	
3	Februrary	3370	-10220	
4	March	2080	-3310	
5	April	15880	-1330	
6	May	16390	-2550	
7	June	10030	-3580	
8	July	16670	-3650	
9	August	9880	-4490	
10	September	5320	-1060	
11	October	5440	-3280	
12	November	2090	-4460	
13	December	4880	-15490	
14	Total			
15				

Solve.

1. What calculation is performed when you use the **Sum (range)** feature? Include an example from the spreadsheet.
2. June's income was entered incorrectly. Enter the correct amount, $18,030, and explain how the spreadsheet changes.
3. Which month has the highest income? Which month has the highest profit? How is it possible that they are not the same month?
4. Use **Sum (range)** to calculate the total income, expenses, and profits for the year. What two cells can be added to check the profit total in cell D14?

Write About It Why is it easier to make changes using a computerized spreadsheet than using a paper and pencil spreadsheet?

Unit 4 Test

VOCABULARY Open Response

1. The ____ of a number is the opposite of the number.
2. A(n) ____ is an ordered set of numbers.
3. A(n) ____ is a rule that pairs each input with one and only one output.
4. A(n) ____ is a value by which a variable is multiplied.

Vocabulary

- function
- sequence
- coefficient
- algebraic expression
- additive inverse
- inverse operation

CONCEPTS AND SKILLS Open Response

Order from least to greatest. (Chapter 11)

5. $^{-}7, {}^{+}4, {}^{-}2, {}^{-}5$
6. $^{-}20, {}^{-}1, {}^{+}30, {}^{-}7, {}^{+}8$
7. $0, {}^{-}3, {}^{-}2, {}^{+}15, {}^{-}6, {}^{+}6$

Add, subtract, multiply, or divide. (Chapter 11)

8. $^{-}16 + {}^{+}5$
9. $^{+}14 + {}^{-}5$
10. $^{-}5 - {}^{+}7$
11. $^{-}6 - {}^{-}8$
12. $^{-}9 \times {}^{-}7$
13. $^{-}52 \times 0$
14. $^{-}48 \div {}^{+}3$
15. $^{+}72 \div {}^{-}12$

Write an algebraic expression for each. Then evaluate, given $n = 6$. (Chapter 12)

16. 3.6 more than n
17. 3 times the sum of a number n and 3

Write each as an equation. (Chapter 12)

18. 1.4 less than a number equals 5.
19. $\frac{2}{3}$ more than a number equals $4\frac{1}{3}$.

Solve. State the inverse operation you used. (Chapter 12)

20. $y + 17 = 35$
21. $59 = n - 59$
22. $k \div 12 = 6$
23. $125 \times a = 500$

Write the next three likely terms in each sequence. (Chapter 13)

24. 33, 29, 25, 21, …
25. 6, $^{-}12$, 24, $^{-}48$, …

Graph each ordered pair. Label the point with the given letter. Tell in which quadrant or on which axis the point lies. (Chapter 13)

26. Point A $(2, {}^{-}5)$

27. Point B $({}^{-}4, 0)$

28. Find the rule for the function table. (Chapter 13)

x	$^{-}2$	$^{-}1$	0	1
y	1	2	3	4

29. Make a table of values for the function $y = 2 - x$. Then use grid paper to graph the function. (Chapter 13)

PROBLEM SOLVING Open Response

Solve.

30. A pigeon is flying 108 feet above a field. This is $\frac{4}{5}$ as high as a sparrow flying over the same field. How many feet higher was the sparrow flying?

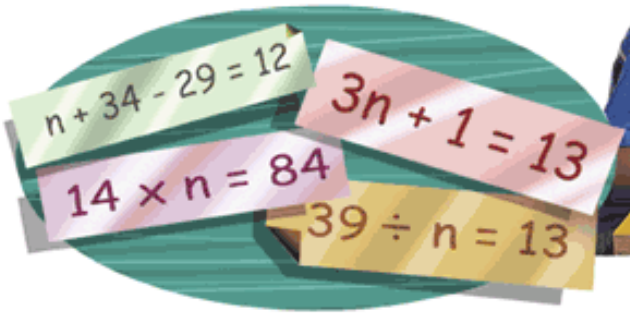
31. Canoes rent for a basic charge of \$12 plus \$8 per hour. Between them, Betty and Joyce have \$40. What is the greatest amount of time they can rent a canoe?

32. On a walking tour, Tod first walked 1 mile east, then 0.8 mile west. Then he walked east $\frac{1}{4}$ the distance he walked in the second part. How far east or west was Tod from his starting point?

33. The number of gallons of water (G) being filled into a pool is $G = 5t$, where t is the time in minutes. If there were 20 gallons in the pool to start with, how many gallons will there be after 8 minutes?

Performance Assessment

Extended Response





Task Ari, Beth, Cindy, and Don were chosen to represent their class in a math bee.

Use the equations above and the information at right. In one round of the math bee, each student solved one of the equations. Who solved which equation?

Explain your thinking.

Information You Need

- Ari's solution was twice Beth's solution.
- Beth's solution was 4 less than Cindy's solution.
- Cindy's solution was 1 more than Ari's solution.
- Don's solution was the difference between Cindy's and Beth's solutions.

Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

When a test question involves solving an equation, you can check your answer by substituting your value for the variable into the given equation.

Mr. Lee charges $22 per hour for carpentry work. In the equation

$$T = 22 \times h$$

T is the total amount he charges, and h is the number of hours he works. On one job, he charged $132. How many hours did he work?

A 110 **C** 6

B 11 **D** 4

THINK

Substitute each value into the equation. Which multiplication expression will have 132 as the product?

A: 22×110
B: 22×11
C: 22×6
D: 22×4

Since $22 \times 6 = 132$, answer choice C is the correct one.

Multiple Choice

1. In a school election, Flora got 139 votes and Lana got 98 votes. In the equation $139 = 98 + v$, the variable v represents how many more votes Flora got. How many more votes did she get?

A 31 **B** 41 **C** 141 **D** 237

(Chapter 12, Lesson 3)

2. Jerry has 848 baseball cards and Ping has 784 cards. Which equation can be used to represent the difference, n, in the number of cards the boys have?

F $848 - n = 784$

G $848 + 784 = n$

H $n - 784 = 848$

J $n - 848 = 784$

(Chapter 12, Lesson 2)

3. On a winter day, the temperature dropped by 12°F between 8 A.M. and 2 P.M. After rising 3°F, the temperature was $^{-}5$°F at 4 P.M. What was the temperature at 8 A.M.?

A $^{-}12$°F **C** $^{-}4$°F

B $^{-}5$°F **D** $^{+}4$°F

(Chapter 11, Lesson 5)

4. Judy bought 2.3 pounds of apples at $1.39 a pound and 3.1 pounds of bananas at $0.69 a pound. To the nearest cent, how much money did she spend?

F $2.14 **H** $3.20

G $5.34 **J** $6.85

(Chapter 7, Lesson 2)

For Test-Taking Tips, see page 648.

Open Response

5. This table shows the amounts raised by four students during a fundraising drive.

Student	Amount Raised
Erin	$102
Frank	$95
Gerry	$112
Harold	$87

What was the average amount raised?

(Chapter 2, Lesson 3)

6. Jill has a 15-inch piece of wire. She wants to cut pieces that are $1\frac{7}{8}$ inches long. How many pieces can she cut?

(Chapter 6, Lesson 5)

7. The bookcase in Steven's classroom is 2.4 meters high. How many centimeters high is the bookcase?

(Chapter 8, Lesson 4)

8. Tim is making a square dog run that has an area of 25 square feet. How many feet of fencing will he need to enclose it?

(Chapter 8, Lesson 7)

9. Penny made this stem-and-leaf plot of heights of students in her class.

Stem	Leaves
5	8 8
6	0 0 0 0 1 1 1 2 2 2 3 4 4 5

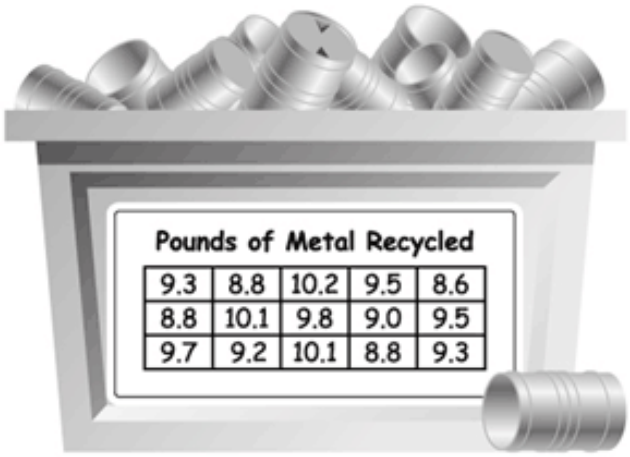
Key: 6|5 means 65 in.

What is the median of her data?

(Chapter 10, Lesson 4)

Extended Response

10. The data below show the number of pounds of metal recycled by several classes at the Murray Hill Middle School.



A Organize the data into a line plot.

B Find the mean, median, mode, and range of the data.

C Next year each class hopes to double the number of pounds of metal recycled. If they succeed, what will the line plot of the new data look like? Explain your answer.

D Describe how the mean, median, mode, and range will change if the numbers of pounds of recycled metal are doubled.

(Chapter 9, Lesson 4)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 4

Look back at the big ideas and vocabulary in this unit.

Big Ideas

You can use rules for signs to add, subtract, multiply, and divide integers.

You can solve equations by using inverse operations.

There are many ways to represent a function, such as an input/output rule, an equation, a function table, ordered pairs, or a graph.

Key Vocabulary

integer
equation
function

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how to graph this function.

$$y = 2x + 1$$

2. Explain how an equation can represent

Six more than a number is 73.

3. Explain what operation you can use to solve this equation.

$$n - 15 = 47$$

4. Explain how to find the next likely term in this sequence.

3, $^{-}6$, 12, $^{-}24$, 48, ...

5. **Write About It** Look for articles with facts that can be represented by equations. Some examples are television schedules, weather reports, and stock market data. Choose one or more articles and write the equations.



UNIT 5

Geometry

CHAPTER 14

Plane Figures

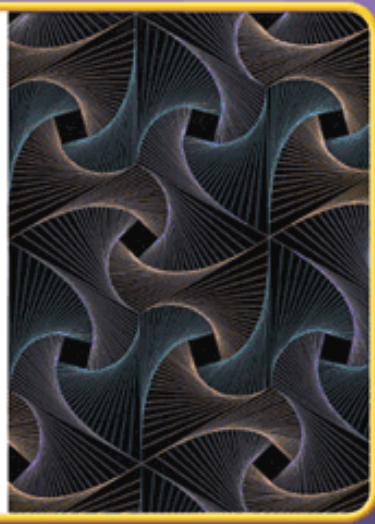
page 352



CHAPTER 15

Congruence and Construction

page 380



Reading Mathematics

Reviewing Vocabulary

Here are some math vocabulary words that you should know.

plane	a flat surface made up of a continuous and unending collection of points
angle	a figure formed by two rays with a common endpoint
vertex	a point common to two sides of an angle or polygon
polygon	a simple closed plane figure made up of three or more line segments
diameter	a line segment that connects two points on a circle and passes through the circle's center



Reading Words and Symbols

When you read mathematics, sometimes you read only words, sometimes you read words and symbols, and sometimes you read only symbols.

Each of these statements has the same meaning:

- $\angle c \cong \angle f$
- Angle c is congruent to angle f.
- Angle c has the same degree measure as angle f.

Write each as a word statement.

1. $\overline{PQ} \cong \overline{TU}$ **2.** $\overrightarrow{EF} \parallel \overrightarrow{WX}$ **3.** $\overleftrightarrow{AB} \perp \overleftrightarrow{JK}$

Write each statement using symbols.

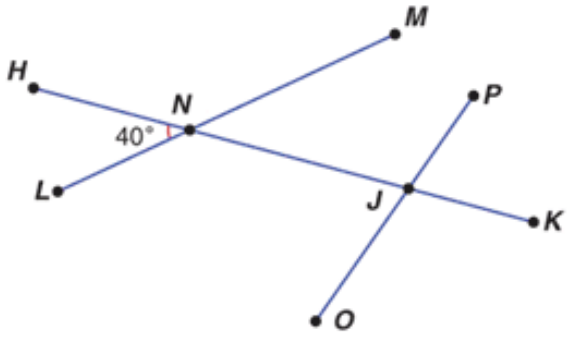
4. ray CD is perpendicular to segment GH

5. lines YZ and AB are parallel

Reading Test Questions

Choose the correct answer for each.

6. Which two line segments intersect in the figure at right?



a. *HK* and *LM*

b. *LN* and *JP*

c. *HN* and *OP*

d. *OJ* and *NM*

To **intersect** means "to meet or cross at a common point."

7. Determine at which point segments *HK* and *OP* intersect.

a. Point *H*

b. Point *N*

c. Point *J*

d. Point *K*

To **determine** means "to find out" or "to decide."

8. What is the degree measure of $\angle HNL$?

a. 40°

b. 140°

c. 80°

d. 160°

A **degree** is a unit used to measure angles.

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

transversal

adjacent angles

vertical angles

alternate exterior angles

alternate interior angles

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "A Gift of Liberty" on page 640. Then work with a partner to answer the questions about the story.

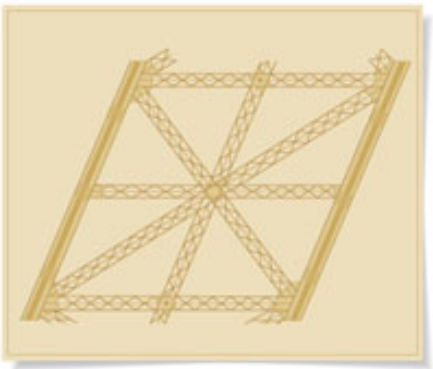
CHAPTER 14

Plane Figures

INVESTIGATION

Using Data

The Eiffel Tower was completed in 1889. The nearly 1,000-foot-tall tower is made from 18,038 iron pieces, held together by 2,500,000 rivets.



The geometric design of its structure gives the tower strength and stability. Look at the tower and the diagram at right. What geometric figures do you see?

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A line segment has two ____.
2. Two rays with a common endpoint form a(n) ____.
3. An angle that is less than 90° is ____.
4. Any chord that passes through the center of a circle is called a(n) ____.

Vocabulary

rays
angle
acute
radius
diameter
endpoints

CONCEPTS AND SKILLS

Match each figure with its correct name or symbol.

5. 

6. 

7. 

a. $\overleftrightarrow{AB}$ b. ray AB c. $\overline{AB}$

Draw each geometric figure. Use a ruler as needed.

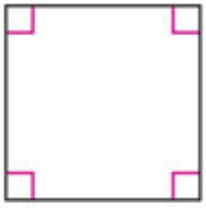
8. a scalene triangle
9. an acute angle
10. a trapezoid

Write *true* or *false*.

11. The angle sum in a triangle is 180°.
12. The angle sum in a quadrilateral is 270°.
13. All quadrilaterals have four sides.
14. Every angle has a vertex.



15. Describe the geometric figure at right in as many ways as you can.



Facts Practice, See Page 654.

Lesson 1

Basic Concepts

Objective Review space, point, plane, line, ray, and angle.

Vocabulary
angle
interior of an angle



Learn About It

This structure is a modern addition to the Louvre Museum in Paris, France. It was designed by the Chinese American architect I.M. Pei. Use the terms in the table below to help you describe the geometric concepts in the building.

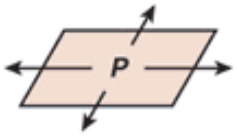
Geometric Concepts

Space is the collection of all points.
A **point** is a location in space.



Read: point *A*
Write: • *A*

A **plane** is a collection of points that forms a flat, continuous, and unending surface.



Read: plane *P*

A **vertex** is the common endpoint of two rays or two line segments.



Read: vertex *B*

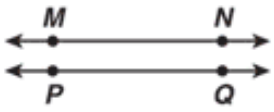
Two rays with a common endpoint form an **angle**. The part of the plane between the two rays is called the **interior of the angle**.



Read: angle *m*
Write: ∠*m*

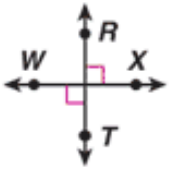
Point *Z* is in the interior of ∠*m*.

Parallel lines lie in the same plane but never intersect.



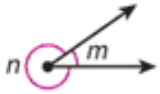
Read: $\overleftrightarrow{MN}$ is parallel to $\overleftrightarrow{PQ}$
Write: $\overleftrightarrow{MN} \parallel \overleftrightarrow{PQ}$

Perpendicular lines intersect to form right angles.



Read: $\overleftrightarrow{RT}$ is perpendicular to $\overleftrightarrow{WX}$
Write: $\overleftrightarrow{RT} \perp \overleftrightarrow{WX}$

Two rays with a common endpoint determine two different angles.



∠*m* ∠*n*

An angle can be named using three points, the vertex point, or a letter.



∠*RST*
∠*S*
∠*s*

Guided Practice

Draw and label each geometric figure.

1. line CD
2. vertex R
3. segment PQ
4. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
5. $\angle TOB$
6. $\overleftrightarrow{FG} \perp \overleftrightarrow{JK}$
7. ray MP
8. $\overrightarrow{XY}$ and $\overrightarrow{XZ}$
9. $\angle n$ with interior point B

Ask Yourself

- Did I remember about the endpoints of segments and rays?
- How are parallel and perpendicular lines different?

Explain Your Thinking When naming angles, why should the letter for the vertex be in the middle?

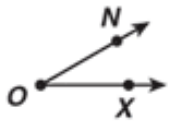
Practice and Problem Solving

Draw and label each geometric figure.

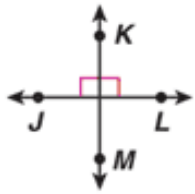
10. $\angle d$
11. $\angle XYZ$
12. $\overrightarrow{KB}$ and $\overrightarrow{KD}$
13. $\overleftrightarrow{RP} \perp \overleftrightarrow{ST}$
14. $\overleftrightarrow{CD} \parallel \overleftrightarrow{FG}$
15. $\overrightarrow{AJ}$ and $\overrightarrow{AK}$
16. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$
17. segment XY

Write the name and symbol for each figure.

18. 

19. 

20. 

21. 

Solve.

22. Suppose two rays intersect. What geometric concept could you use to describe the point where the rays meet?
23. **Represent** Draw two lines in the same plane that do not intersect. What can you say about the lines?
24. **Create and Solve** Choose a geometric concept. Then draw a building that shows the application of that concept. Have a classmate identify the geometric concept in your drawing.
25. **Analyze** Draw a point. How many different lines can you draw that pass through that point? Now draw two points. How many different lines can you draw that pass through both of the points?

Mixed Review and Test Prep

Open Response

Find the prime factorization for each. (Ch. 3, Lesson 3)

26. 12 **27.** 40 **28.** 60

29. Lines HT and RT intersect to form four square corners. Explain how the lines are related. (Ch. 14, Lesson 1)

Extra Practice See page 379, Set A.

Lesson 2

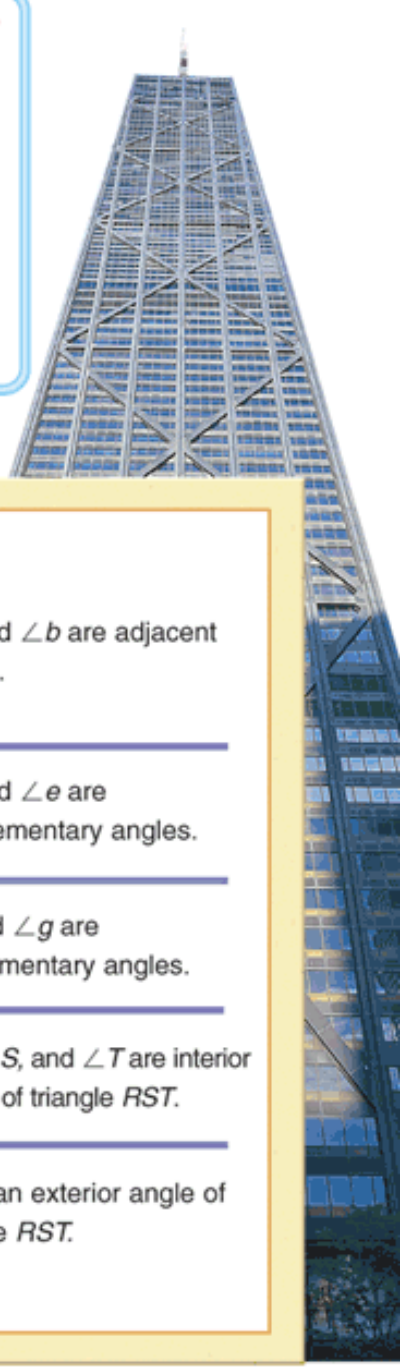
Audio Tutor 2/1 Listen and Understand

Angle Pairs

Objective Identify and explore special angle pairs.

Vocabulary

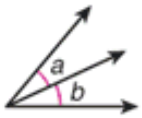
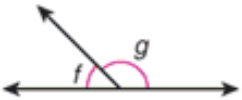
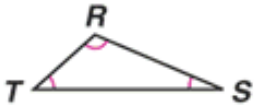
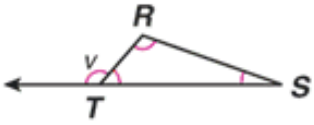
- angle pairs
- adjacent angles
- complementary angles
- supplementary angles
- interior angle
- exterior angle



Learn About It

Exterior beams add decoration to and provide support for some buildings. Angles are formed where the beams cross. Use the terms in the table below to help you identify some of the **angle pairs** in the skyscraper pictured at the right.

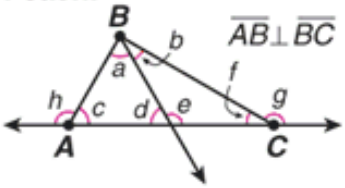
Pairs of Angles

Adjacent angles share a common side and a common vertex and their interiors do not overlap.		$\angle a$ and $\angle b$ are adjacent angles.
The sum of the measures of two **complementary angles** is 90°.		$\angle d$ and $\angle e$ are complementary angles.
The sum of the measures of two **supplementary angles** is 180°.		$\angle f$ and $\angle g$ are supplementary angles.
Each angle of a polygon is an **interior angle** of the polygon.		$\angle R$, $\angle S$, and $\angle T$ are interior angles of triangle *RST*.
An **exterior angle** of a polygon is formed when a side of a polygon is extended. An exterior angle is adjacent to an interior angle of the polygon.		$\angle v$ is an exterior angle of triangle *RST*.

Guided Practice

Use the diagram to give an example of each.

1. a pair of supplementary angles
2. a pair of complementary angles
3. an exterior angle of the triangle



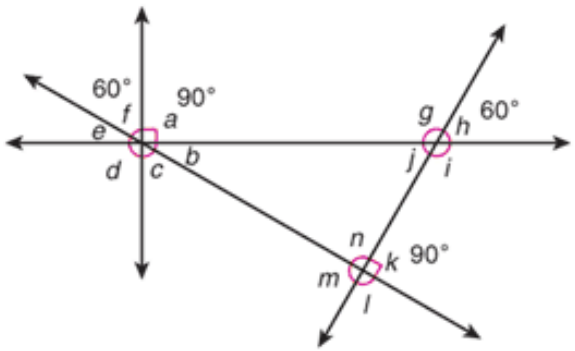
Ask Yourself

- Did I find the measure of one angle by subtracting its measure from 180°?

Explain Your Thinking Why are the adjacent angles formed by two intersecting lines always supplementary?

Practice and Problem Solving

Use the diagram to give an example of each.



4. a pair of complementary angles

5. a pair of supplementary angles

6. an interior angle of the triangle

Find the degree measure of each angle.

7. $\angle n$ **8.** $\angle l$ **9.** $\angle g$ **10.** $\angle e$

Algebra • Equations For Problems 11–14, write and solve an equation to find each angle measure.

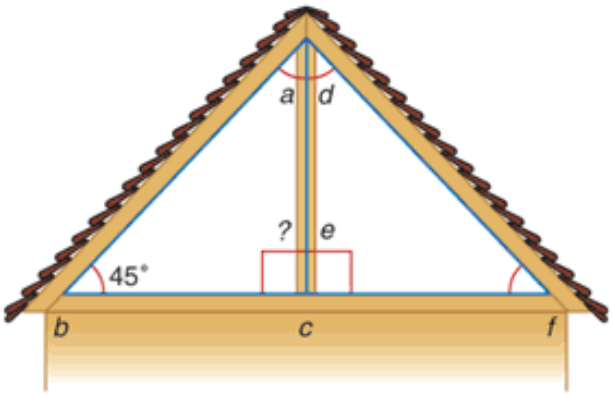
11. $\angle P$ and $\angle O$ are supplementary angles, and $m\angle P = 112°$. Find $m\angle O$.

12. $\angle R$ and $\angle V$ are complementary angles, and $m\angle V = 72°$. Find $m\angle R$.

13. $\angle M$ and $\angle N$ are complementary angles and have the same measure. Find $m\angle M$ and $m\angle N$.

14. An interior of a triangle has a measure of 63°. Find the measure of the adjacent exterior angle.

Solve. Use the drawing for Problems 15 and 16.



15. The angle formed by the horizontal beam and the roof is 45°. This angle and the angle formed by the vertical post and the roof are complementary angles. What is the measure in degrees of the angle formed by the vertical post and the roof?

16. What is the measure in degrees of the angle formed by the vertical post and the horizontal beam? Explain how you know.

17. What's Wrong? Andy drew a design that had three adjacent 30° angles. He claims that, since the sum of the angles is 90°, the angles are complementary. Do you agree with Andy's thinking? Explain.

18. Analyze One of two complementary angles has a degree measure that is four times the degree measure of the other angle. Find the measure in degrees of each angle. Explain your reasoning.

Mixed Review and Test Prep

Open Response

Add or subtract. (Ch. 1, Lesson 6)

19. 2.75 + 3.23 **20.** 2.8 + 6.787

21. 72.39 − 64.18 **22.** 3.004 − 1.337

Multiple Choice

23. $\angle X$ has half the degree measure of its supplementary angle. What is the measure of $\angle X$? (Ch.14, Lesson 2)

A 120° **C** 60°

B 90° **D** 30°

Extra Practice See page 379, Set B.

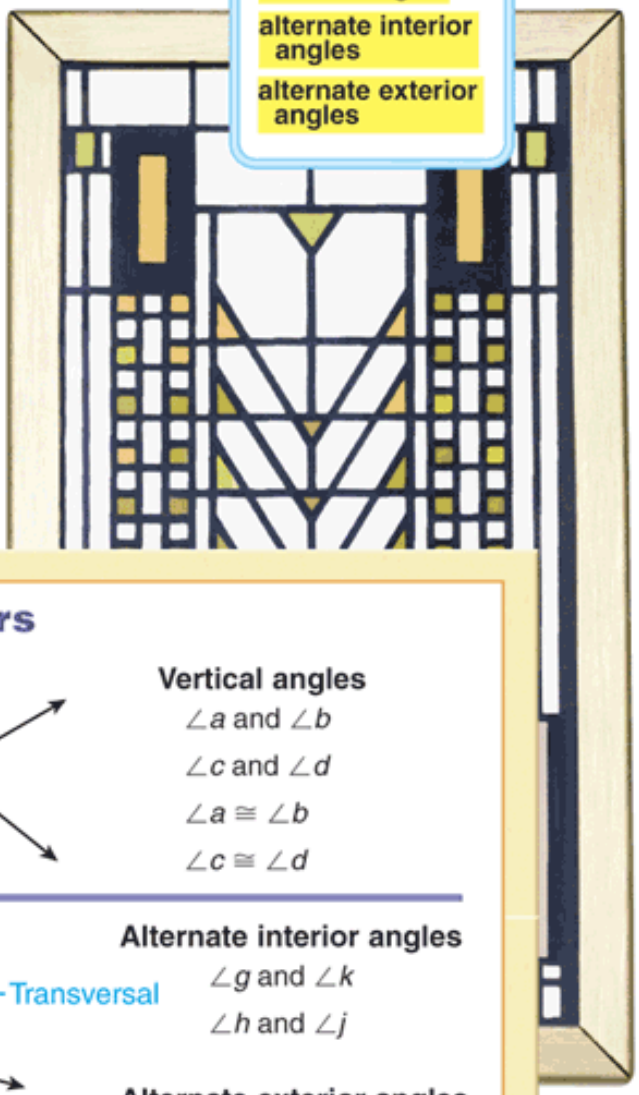
Lesson 3

Audio Tutor 2/12 Listen and Understand

Transversals and Angles

Objective Explore the properties of vertical, alternate interior, and alternate exterior angles.

Vocabulary
- transversal
- vertical angles
- alternate interior angles
- alternate exterior angles



Learn About It

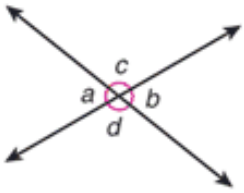
When two lines intersect, pairs of angles are formed. When two lines each are intersected by a third line, called a **transversal**, several angle pairs are formed.

Look at the picture of the stained-glass window. Find an example of two intersecting lines. Then look for an example of two lines that are intersected by a transversal.

Stained-glass window, Tree of Life, *designed by American architect Frank Lloyd Wright.*

More Angle Pairs

▶ When two lines intersect, two pairs of **vertical angles** are formed. Vertical angles are congruent. The symbol ≅ means *similar* **and** *equal to* **or** *congruent.*



Vertical angles

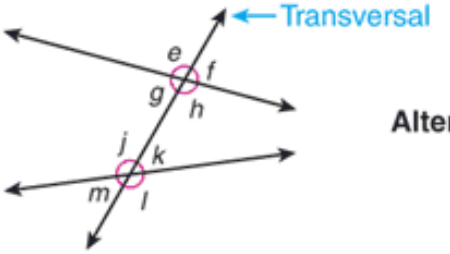
$\angle a$ and $\angle b$

$\angle c$ and $\angle d$

$\angle a \cong \angle b$

$\angle c \cong \angle d$

▶ When two lines are intersected by a transversal, two pairs of **alternate interior angles** and two pairs of **alternate exterior angles** are formed.



Alternate interior angles

$\angle g$ and $\angle k$

$\angle h$ and $\angle j$

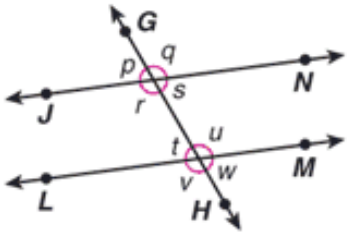
Alternate exterior angles

$\angle e$ and $\angle l$

$\angle f$ and $\angle m$

▶ When two parallel lines are intersected by a transversal, several pairs of congruent angles are formed.

In the diagram on the right, parallel lines $\overleftrightarrow{JN}$ and $\overleftrightarrow{LM}$ are intersected by the transversal $\overleftrightarrow{GH}$:



$\angle r \cong \angle u$ and $\angle s \cong \angle t$ and

$\angle p \cong \angle w$ and $\angle q \cong \angle v$

Extra Help at **eduplace.com/map**

Guided Practice

Use the diagram to give two examples of each.

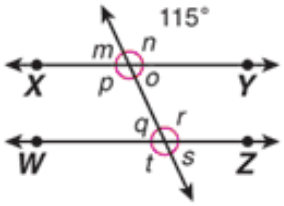
1. a pair of vertical angles

2. a pair of alternate exterior angles

3. a pair of alternate interior angles

Find the degree measure of each angle.

4. $\angle s$ **5.** $\angle p$ **6.** $\angle m$



$\overleftrightarrow{XY} \parallel \overleftrightarrow{WZ}$

Ask Yourself

- How can I use the one angle measure that is given to find the other three angle measures?

Explain Your Thinking ▶ Why are vertical angles congruent?

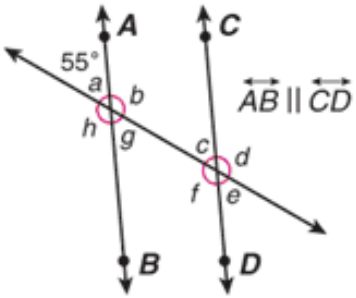
Practice and Problem Solving

Use the diagram to give two examples of each.

7. a pair of vertical angles

8. a pair of alternate interior angles

9. a pair of alternate exterior angles



$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Find the degree measure of each angle.

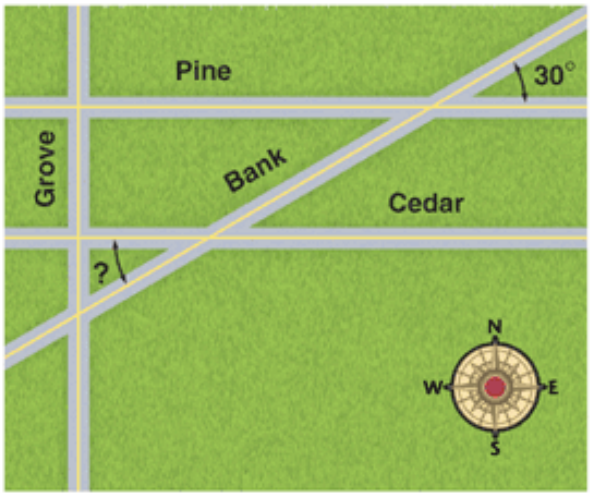
10. $\angle g$ **11.** $\angle c$ **12.** $\angle d$

Solve. Use the street map for Problems 13–15.

13. **Reasoning** The lot on the northeast corner of Pine and Bank has sides that form a 30° angle. What is the measure of the angle formed by the lot on the southwest corner of Cedar and Bank?

14. What is the angle measure of the corner of the lot on the northwest corner of Pine and Bank?

15. Name another lot on the street map with a measure of 30°.



Mixed Review and Test Prep

Open Response

Evaluate each expression. (Ch. 2, Lesson 7)

16. $4 + 3 \times (11 - 5)$ **17.** $7 + 5 \times (12 \div 4)$

18. $5 + (3 + 6 \div 3)$ **19.** $(8 \times 9) + (8 \times 1)$

20. If $\angle x$ and $\angle y$ are vertical angles, and $m\angle y$ is 47°, what is $m\angle x$? (Ch.14, Lesson 3)

Draw a diagram to explain.

Extra Practice See page 379, Set B.

Hands On Lesson 4

Measure, Draw, and Classify Angles

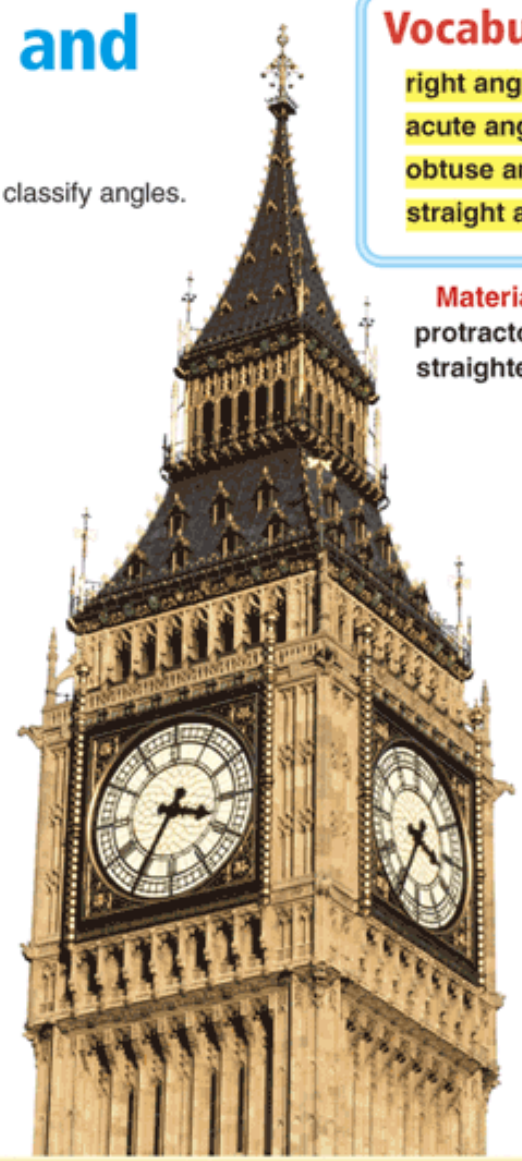
Objective Estimate, measure, draw and classify angles.

Vocabulary
- right angle
- acute angle
- obtuse angle
- straight angle

Materials
protractor
straightedge

Work Together

"Big Ben" is actually the name of the largest bell inside the clock tower at London's Houses of Parliament. The clock's hands are like two rays that form an angle at the center of a clock's face.



As the minute hand moves, or rotates, about its endpoint, the measure of the angle changes. In a fifteen-minute period, the minute hand rotates 90°, or one-quarter of a complete 360° rotation.

You can use a protractor to measure angles or to draw an angle of a given measure.

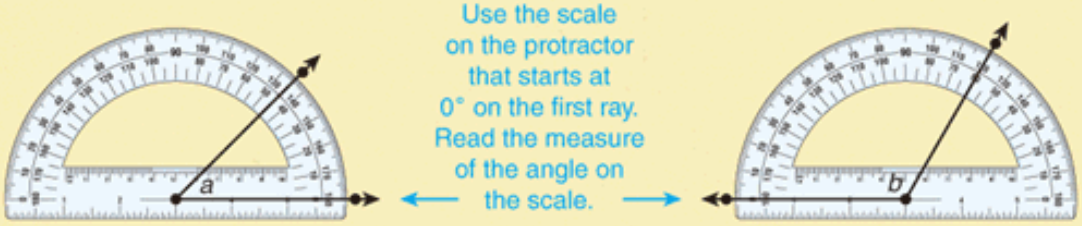
Work with a partner to find the degree measures of $\angle a$ and $\angle b$.



STEP 1 Place the center of the protractor at the vertex of the angle.

STEP 2 Align the 0° mark on the protractor with one ray of the angle to be measured.

STEP 3 Find where the other ray passes through the same scale.



Use the scale on the protractor that starts at 0° on the first ray. Read the measure of the angle on the scale.

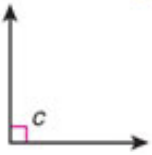
The measure of $\angle a$, $m\angle a = 45°$.

The measure of $\angle b$, $m\angle b = 120°$.

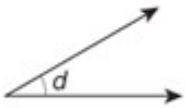
Once you know the measure of an angle, you can classify the angle.

Classifying Angles

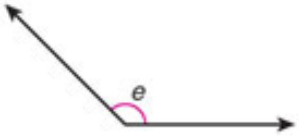
The measure of a **right angle** is 90°.



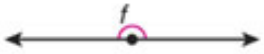
The measure of an **acute angle** is less than 90°.



The measure of an **obtuse angle** is greater than 90° and less than 180°.



The measure of a **straight angle** is 180°.



You can use a protractor and a straightedge to draw angles.

Draw an angle with a measure of 60°.

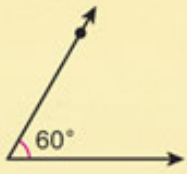
STEP 1 Use a straightedge to draw a ray. Place the center mark of the protractor on the endpoint of the ray. Align the ray with the 0° mark of one scale of the protractor.



STEP 2 Using the same scale, find the 60° mark. Place a point on your paper at the mark.



STEP 3 Use a straightedge to draw a ray from the endpoint of the first ray through the point you drew. Label the angle 60°.



Check your work. Use the protractor to check the measure of the angle you have just drawn.

Go On

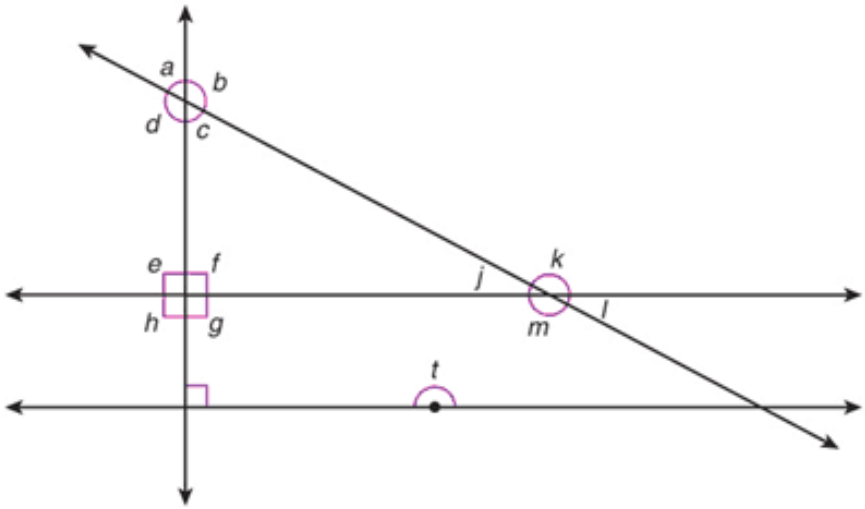
On Your Own

Measure each angle. Then classify each angle as *acute, right, obtuse,* or *straight.*

1. $\angle c$ **2.** $\angle h$

3. $\angle d$ **4.** $\angle g$

5. $\angle t$ **6.** $\angle l$



Use a protractor and a straightedge to draw each angle.

7. 65° **8.** 135° **9.** 180° **10.** 8° **11.** 20° **12.** 142°

Use a protractor and a straightedge to draw an example of each angle. Then measure the angle.

13. a right angle **14.** an acute angle **15.** an obtuse angle

16. Through how many degrees does a clock hand rotate as it moves from the 12 o'clock position to the 9 o'clock position?

17. Estimate each angle formed between the hands of a clock at these times, and classify it as acute, right, obtuse, or straight. Then use a protractor to find the exact measure of each angle.

a. 3:50 **b.** 1:12 **c.** 5:05

Talk About It • Write About It

You learned to use a protractor to measure angles and how to draw an angle with a given measure.

18. When you measure an angle, what happens if you align the first ray with the 0° mark, but use the scale that starts at 180° to measure?

19. How could you use a protractor to draw an angle greater than 180°?

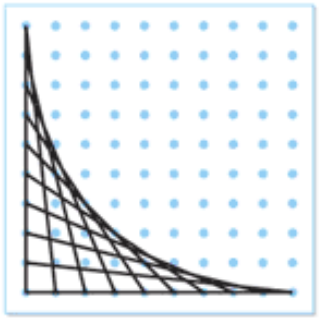
Art Connection

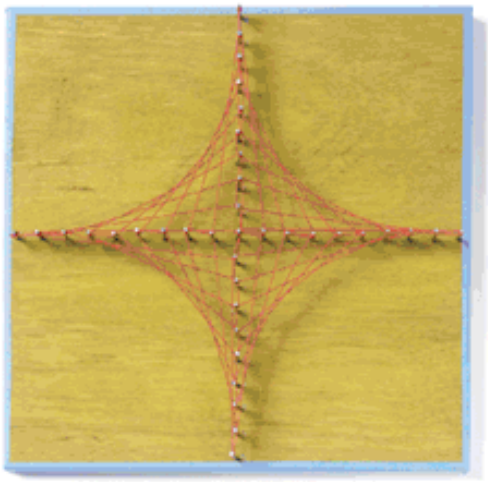
Activity

Design by Line

You can make arcs by using line segments.

Use dot paper. Copy the design in the picture and use it as the start of a larger design.





1. Name some of the figures in your design.
2. What types of angles can you find?
3. Color your design and compare it to your classmates' designs. How are the designs alike? How are they different?

WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

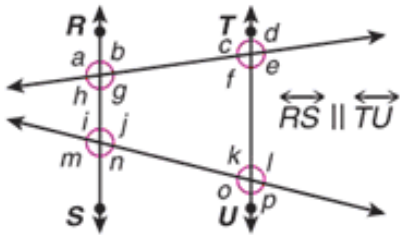
Draw and label each geometric figure. (Lesson 1)

1. $\overleftrightarrow{RS} \parallel \overleftrightarrow{TU}$
2. ray CD
3. $\overleftrightarrow{AB} \perp \overleftrightarrow{OP}$
4. $\angle HJK$

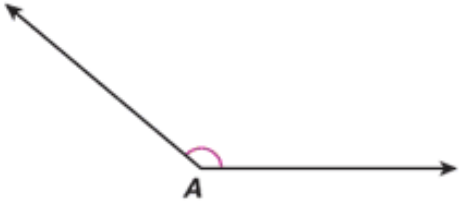
Find each angle measure. (Lesson 2)

5. $\angle n$ and $\angle p$ are supplementary angles, and $m\angle n = 56°$. Find $m\angle p$.
6. $\angle d$ and $\angle e$ are complementary angles, and $m\angle e = 14°$. Find $m\angle d$.

Use the diagram for Exercises 7 and 8. (Lesson 3)



7. Give an example of a pair of vertical angles.
8. Give two examples of a pair of alternate exterior angles.
9. Draw an angle that measures 70°. (Lesson 4)
10. Find the measure of angle A on the right. Classify the angle as *acute, obtuse, right,* or *straight.* (Lesson 4)



Hands On Lesson 5

Audio Tutor 2/3 Listen and Understand

Triangles and Angle Sums

Objective Find the measure of an angle of a triangle when the measures of the other two angles are known.

Vocabulary
- isosceles triangle
- acute triangle
- scalene triangle
- obtuse triangle

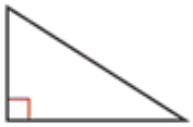
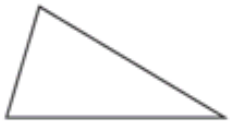
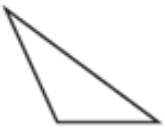
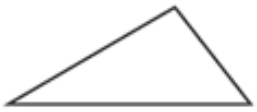
Learn About It

Architects may use triangles to give buildings visual interest. The East Building of the National Gallery of Art in Washington, D.C., has a glass ceiling of triangular shapes. Use the table below to help you find the kinds of triangles in the ceiling.



The East Building of the National Gallery of Art in Washington, D.C., was designed by architect I.M. Pei.

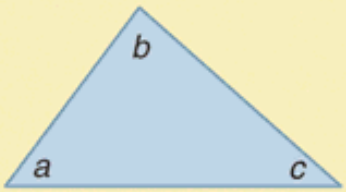
Classifying Triangles

By Side Measures		By Angle Measures	
Equilateral All sides congruent		**Right** One 90° angle	
Isosceles At least two sides congruent		**Acute** Three acute angles	
Scalene No sides congruent		**Obtuse** One obtuse angle	

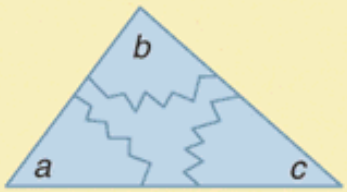
As you can see, triangles come in many different shapes and sizes. But all triangles have one thing in common—the sum of the angle measures is always 180°.

You can see this for yourself by following the steps shown here.

STEP 1 Draw and cut out a triangle. Label the three angles.



STEP 2 Tear off the three corners of the triangle.

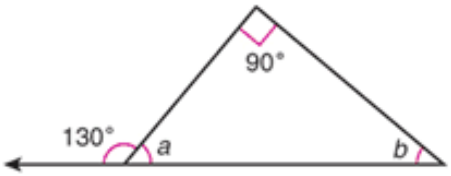


STEP 3 Arrange the pieces so that the three angles form a straight angle.



You can use what you know about angles and the sum of the angle measures to find missing angle measures.

What are the measures of $\angle a$ and $\angle b$ in this triangle?



Find m$\angle a$ and m$\angle b$.

STEP 1 Write an equation to find m$\angle a$.

A straight angle measures 180°.

$$m\angle a + 130° = 180°$$
$$m\angle a = 180° - 130°$$
$$m\angle a = 50°$$

STEP 2 Use the measure of $\angle a$ to find m$\angle b$.

The sum of the angle measures in a triangle is 180°.

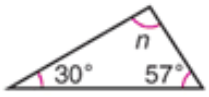
$$m\angle b + 50° + 90° = 180°$$
$$m\angle b = 180° - 140°$$
$$m\angle b = 40°$$

Solution: m$\angle a = 50°$ and m$\angle b = 40°$.

Guided Practice

Classify each triangle by its sides and by its angles. Then find the degree measure of $\angle n$ in each triangle.

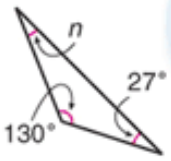
1.



2.



3.



Ask Yourself

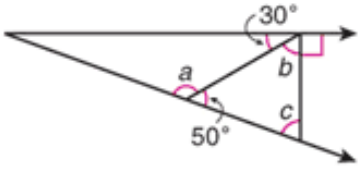
- Did I find the sum of the two known angles before subtracting?
- Did I remember how obtuse angles and acute angles differ?

Use the diagram to find the measure of each angle.

4. $\angle a$

5. $\angle b$

6. $\angle c$



Explain Your Thinking Why can't a triangle have two angles that each has a measure of 90°?

Practice and Problem Solving

Classify each triangle by its sides and by its angles. Then find the degree measure of *n* in each triangle.

7.



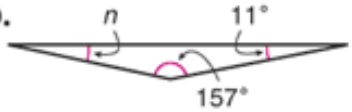
8.



9.



10.



Go On

Use the diagram to find each angle measure.

11. $\angle r$

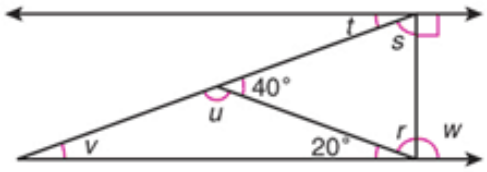
12. $\angle u$

13. $\angle s$

14. $\angle v$

15. $\angle t$

16. $\angle w$



Draw an example of each.

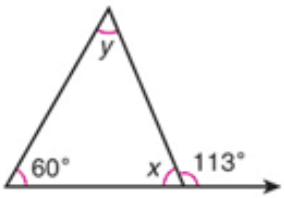
17. a triangle that is both scalene and acute

18. a triangle that is both scalene and obtuse

Algebra • **Equations** **Use the diagram. Write and solve an equation to find each angle measure.**

19. $m\angle x$

20. $m\angle y$



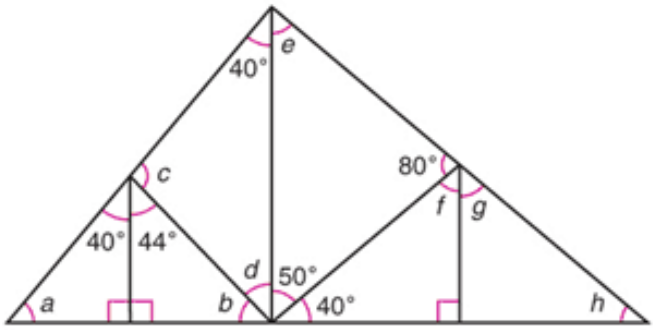
Solve.

21. **Explain** When is a right triangle also an isosceles triangle? Draw an example.

22. An architect is considering the design at right for the roof of a new skyscraper. Find each missing angle measure.

23. A skylight is made from triangular shapes. Some of the triangles are acute isosceles triangles with one angle measuring 30°. What are the measures of the other two angles?

24. **Create and Solve** Draw your own design, using triangles. Use your protractor to measure some of the angles. Then challenge a classmate to find the missing angle measures.



Mixed Review and Test Prep

Open Response

Check each number for divisibility by 2, 3, 4, 5, 9, and 10. (Ch. 3, Lesson 1)

25. 118

26. 295

27. 7,100

28. 1,278

Multiple Choice

29. Suppose you connect two opposite corners of a square. What is the measure of each acute angle in the isosceles triangles formed? (Ch. 14, Lesson 5)

A 30° **B** 45° **C** 60° **D** 90°

Extra Practice See page 379, Set C.

Activity

Social Studies Connection

Paths

In Königsberg, two islands were connected to one another and to the rest of the city by seven bridges. Is it possible to walk across all seven bridges, crossing each bridge only once before returning to the starting point? The answer is no.



This is a famous historical problem. A figure is traceable if it can be drawn without lifting the pencil off the paper or retracing any of the paths.

1. Copy each figure below. Which ones can you trace without lifting your pencil or retracing any line?

a. 

b. 

c. 

2. Look back at the problem about the Königsberg bridges. Can you add an eighth bridge so that it is possible to cross all the bridges once and finish at your starting point? Draw a picture to explain.

Visual Thinking

What's Your Angle?

Builders in ancient Egypt used cords with 12 equally spaced knots to make square corners.

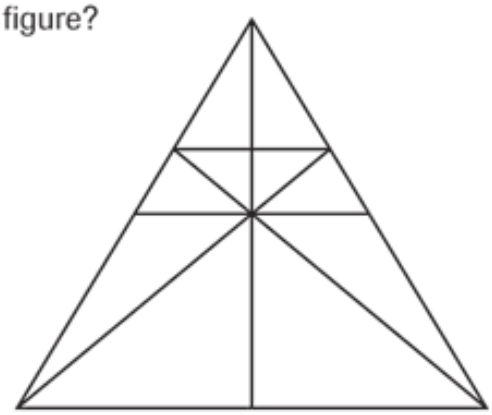


Use a length of rope or string to make your own Egyptian right-angle tool. Tie 12 equally spaced knots in the string.

- How many different right triangles can you make?
- Use your tool to check whether angles in your classroom are right angles or not.

Brain Teaser

How many triangles can you find in this figure?



Hint Have you counted the triangles that overlap?



Education Place

Check out **eduplace.com/map** for more brain teasers.

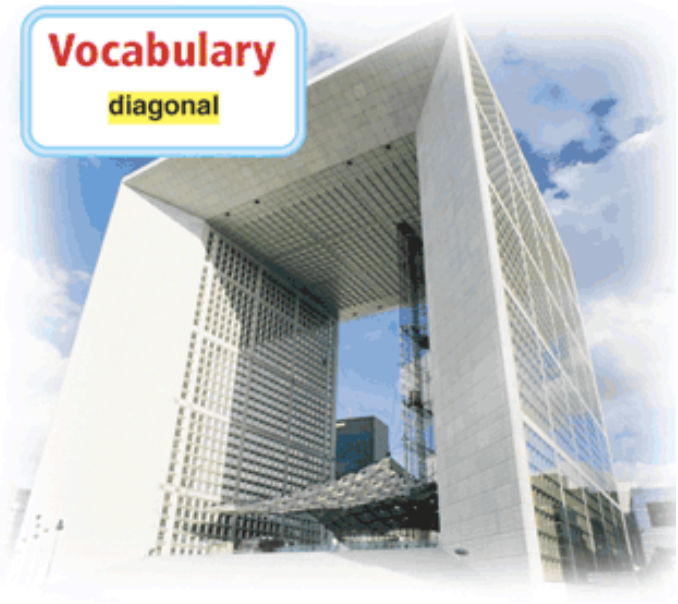
Lesson 6

Audio Tutor 2/4 Listen and Understand

Quadrilaterals and Angle Sums

Objective Use the properties of quadrilaterals to find angle measures.

Vocabulary
diagonal



Learn About It

This building contains many quadrilaterals. A quadrilateral is a closed plane figure with four line segments as its four sides and four angles. Quadrilaterals are given names based on the properties of their angles and sides.

La Grande Arche, Paris, France

Properties of Quadrilaterals

Parallelogram



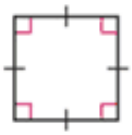
A quadrilateral with opposite sides parallel. The opposite sides and opposite angles are also congruent.

Rectangle



A quadrilateral with four right angles. A rectangle is also a parallelogram.

Square



A rectangle with all sides congruent

Rhombus



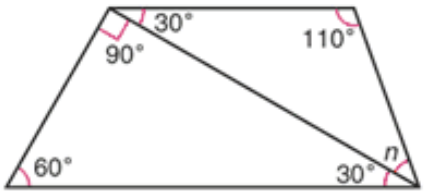
A parallelogram with all sides congruent (plural: *rhombi*)

Trapezoid



A quadrilateral with exactly one pair of parallel sides

A **diagonal** is a line segment that connects opposite vertices of a closed figure. A diagonal partitions any quadrilateral into two triangles. Each triangle has an angle sum of 180°. So the angle sum of any quadrilateral is $2 \times 180°$, or 360°.

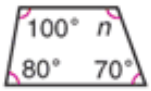


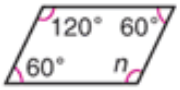
You can use this fact to help find the measures of the angles of a quadrilateral. In the pictured quadrilateral,

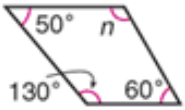
$$m\angle n = 180° - (110° + 30°) = 40°.$$

Guided Practice

Find each missing angle measure. Then classify each figure as a *quadrilateral, parallelogram, rectangle, rhombus, square,* or *trapezoid.* Include all classifications that apply.

1. 

2. 

3. 

Ask Yourself

- Are opposite angles of a parallelogram always congruent?

Explain Your Thinking Why is a rectangle also a parallelogram?

Practice and Problem Solving

Draw a quadrilateral that satisfies each set of conditions.

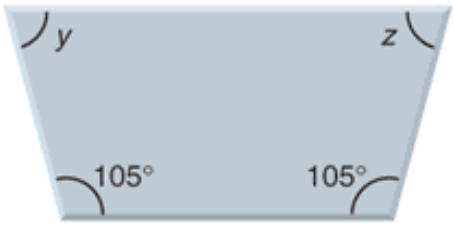
4. four congruent sides but no right angles

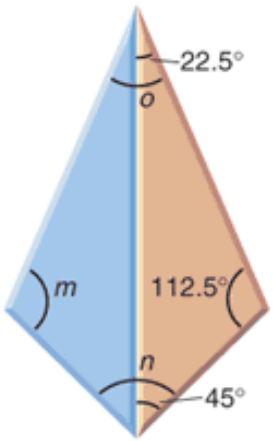
5. exactly three congruent sides

6. four congruent angles but not four congruent sides

7. Its diagonal forms two isosceles right triangles.

Data **Use the diagrams for Problems 8–10.**





8. The first diagram shows a tile to be used on a new office building. If angles *y* and *z* are equal, what is the angle measure of each?

9. The second diagram shows a tile made by joining two identical triangular tiles. Find the angle measures of the quadrilateral formed by the two tiles.

10. Classify each of the tiles as *quadrilateral, parallelogram, rectangle, square, rhombus,* or *trapezoid.* Include all classifications that apply.

11. Analyze Why is any square both a rhombus and a rectangle? Explain.

Mixed Review and Test Prep

Open Response

Subtract. (Ch. 1, Lesson 6)

12. 3.4 − 1.75

13. 4.56 − 1.583

14. 9 − 8.321

15. 3 − 2.008

16. A quadrilateral has exactly two right angles. Classify the quadrilateral. (Ch. 14, Lesson 6)

Use a drawing to explain your answer.

Extra Practice See page 379, Set D.

Lesson 7

Audio Tutor 2/5 Listen and Understand

Problem-Solving Strategy

Use Logical Reasoning

Objective Use logical reasoning to solve a problem.



You can use logical reasoning and generalizations to solve problems. A generalization is a statement that applies to all things of a certain type.

Problem A mosaic wall has tiles shaped like rectangles, rhombi, squares, and trapezoids. Richard says these tiles are all parallelograms. Is Richard's reasoning correct? Explain.

UNDERSTAND

This is what you know:

- The tiles are shaped like rectangles, rhombi, squares, and trapezoids.
- the properties of these quadrilaterals

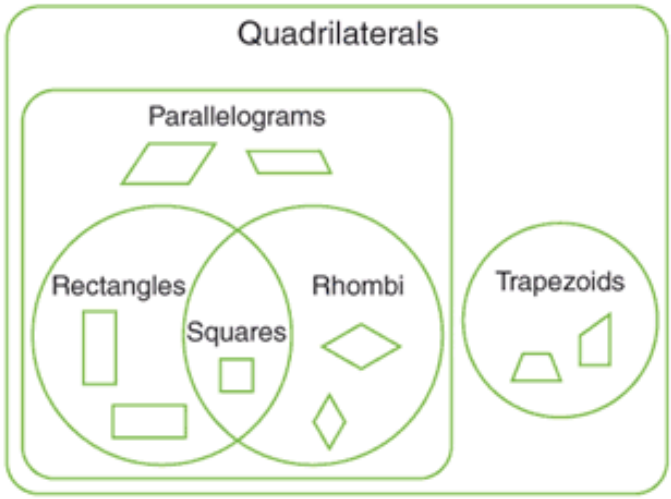
PLAN

Draw a Venn diagram and include the information you know. Then use logical reasoning to make generalizations.

SOLVE

Draw and use the Venn diagram. Use the words *all, some,* and *none* to state the generalizations.

- *All* parallelograms are quadrilaterals.
- *Some* rectangles and some rhombi are squares.
- *All* rectangles, rhombi, and squares are parallelograms.
- *None* of the trapezoids are parallelograms.



Solution: Richard's reasoning is not correct. Some of the quadrilaterals are parallelograms, but none of the trapezoids are parallelograms.

LOOK BACK

Look back at the problem. Does your answer seem reasonable?

Guided Practice

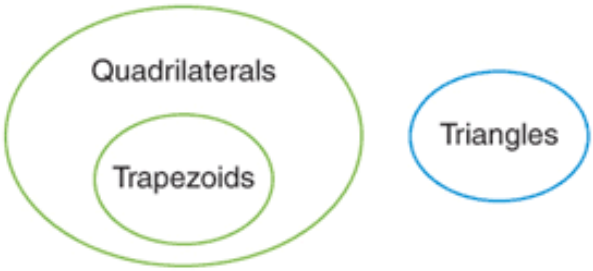
Use the Venn diagrams and the Ask Yourself questions to help you solve each problem.

Ask Yourself

- **UNDERSTAND** — What facts do I know?
- **PLAN** — How can I organize the facts?
- **SOLVE** — What do the different parts of the diagram represent?
- **LOOK BACK** — Did I answer the question that was asked?

1. Use the Venn diagram on page 370 to tell if each generalization is true or false and why.
   - **a.** Some rectangles are squares.
   - **b.** All rectangles are rhombi.
   - **c.** All squares are rectangles.
   - **d.** Some squares are not rhombi.
   - **e.** No squares are triangles.

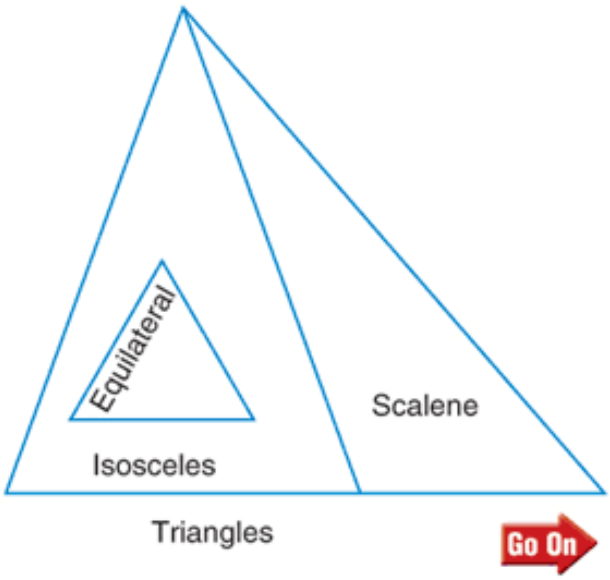
2. Use the Venn diagrams at the right. Copy and complete each generalization, using *All, Some,* or *No.* Explain your reasoning.
   - **a.** ____?____ trapezoids are quadrilaterals.
   - **b.** ____?____ trapezoids are triangles.
   - **c.** ____?____ quadrilaterals are trapezoids.
   - **d.** ____?____ triangles are quadrilaterals.



Independent Practice

Use the Venn diagram and logical reasoning to solve each problem.

3. Sajal says that all equilateral triangles are isosceles triangles, but that no equilateral triangles are scalene triangles. Explain.
4. **What's Wrong?** Jorge says that if all equilateral triangles are also isosceles triangles, then all isosceles triangles are also equilateral. Is he correct? Explain.
5. Does the Venn diagram include all types of triangles? Explain.
6. **Analyze** If you wanted to add a region representing right triangles to the Venn diagram, how would you draw it? Why?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Jarell has four sports trophies. How many different ways can he arrange the trophies in one row on a bookshelf?

8. Clarise, Deven, and Edmundo each participate in a sport. One student plays tennis, one plays baseball, and one swims. Edmundo's sport does not involve a ball. Clarise's brother is the tennis player. Who participates in each sport?

9. At Vernon's school, each class period is 45 minutes long. There is a 5-minute break between class periods. Period 8 ends at 3:00 P.M. At what time does Period 6 begin?

You Choose

Strategy

- Draw a Diagram
- Make a Table
- Make an Organized List
- Solve a Simpler Problem
- Use Logical Reasoning

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the table for Problems 10–13.**

10. How much longer is the main span of the Golden Gate Bridge than the main span of the Brooklyn Bridge?

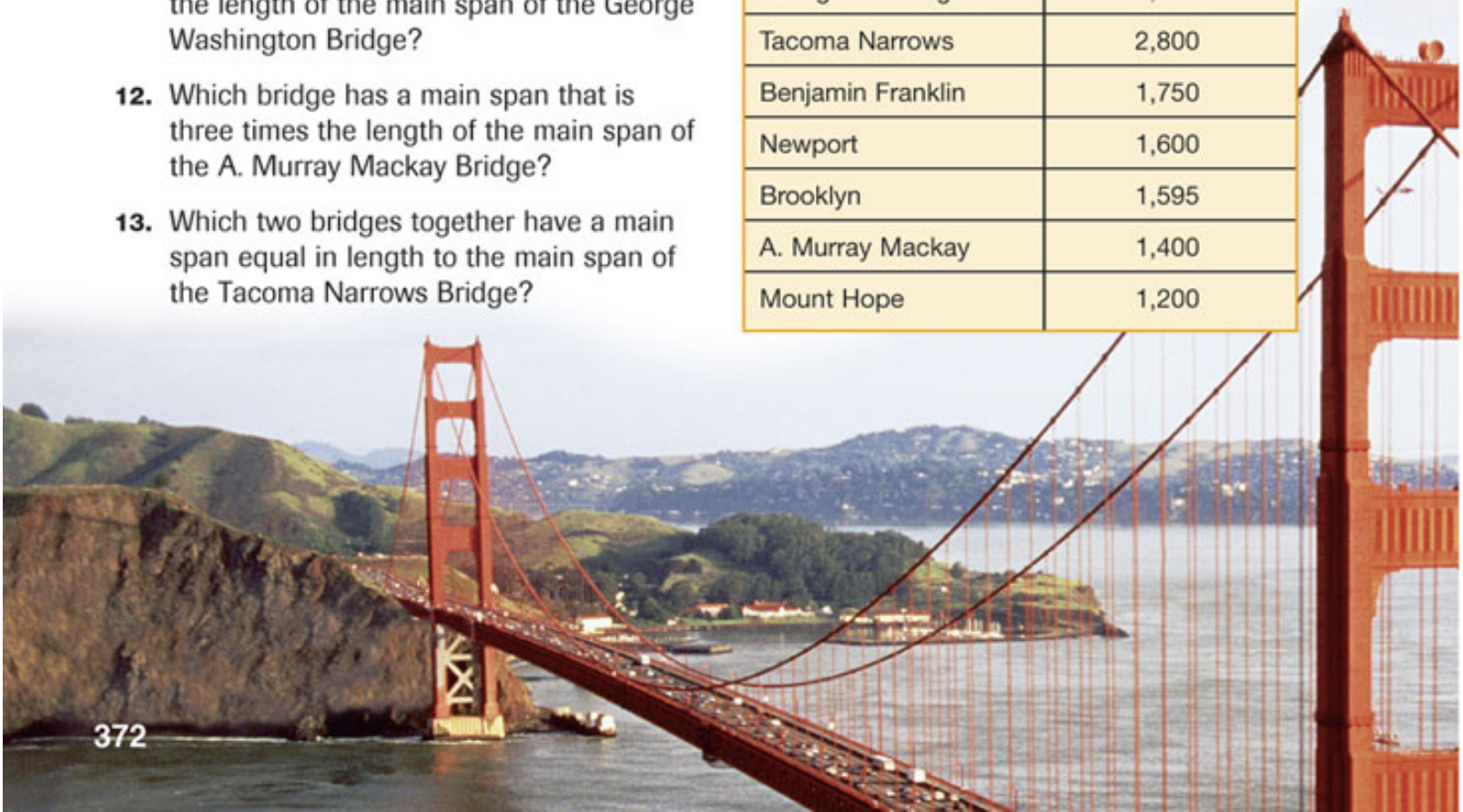
11. Which bridge has a main span that is half the length of the main span of the George Washington Bridge?

12. Which bridge has a main span that is three times the length of the main span of the A. Murray Mackay Bridge?

13. Which two bridges together have a main span equal in length to the main span of the Tacoma Narrows Bridge?

Some Notable Suspension Bridges in North America

Bridge	Main Span (feet)
Golden Gate	4,200
George Washington	3,500
Tacoma Narrows	2,800
Benjamin Franklin	1,750
Newport	1,600
Brooklyn	1,595
A. Murray Mackay	1,400
Mount Hope	1,200



Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. At 3:00 P.M., the temperature was 4°C. By 8:00 P.M. it had dropped by 7°. What was the temperature at 8:00 P.M.?

A 11°C **C** 3°C

B $^{-}$11°C **D** $^{-}$3°C

(Chapter 11, Lesson 5)

2. Lisa made a number scale on a 6-inch line segment. She labeled one endpoint 0 and made marks at $\frac{3}{4}$-inch intervals. If she used counting numbers to label the marks, how did she label the other endpoint?

F 6 **H** 10

G 8 **J** 12

(Chapter 6, Lesson 4)

Open Response

Solve each problem.

3. A survey about lengthening the school day showed 49% in favor and 51% against doing so.

Explain How could you draw a graph about the survey in a way that is misleading?

(Chapter 10, Lesson 7)

4. Camilla planted $\frac{1}{3}$ of her garden with flowers. She planted $\frac{3}{4}$ of the area with roses. What part of her whole garden is planted with roses?

Represent Support your answer with a diagram.

(Chapter 6, Lesson 2)

Extended Response

5. You are deciding how to cut a breakfast pie into slices to serve 7 people.

Information You need

- Two people want large slices.
- Three people want small slices.
- Two people don't care what size slice they get.
- There can be only two sizes.
- You must use the whole pie.

a. Draw a circle to represent the pie.

b. Decide how many people will get each size slice.

c. Cut the pie so that each slice represents a central angle. Find the degree measure for each size slice.

d. If the diameter of the pie is 13 inches, what is the length of each size of a slice? Explain.

(Chapter 14)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Hands On Lesson 8

Audio Tutor 2/6 Listen and Understand

Circles and Central Angles

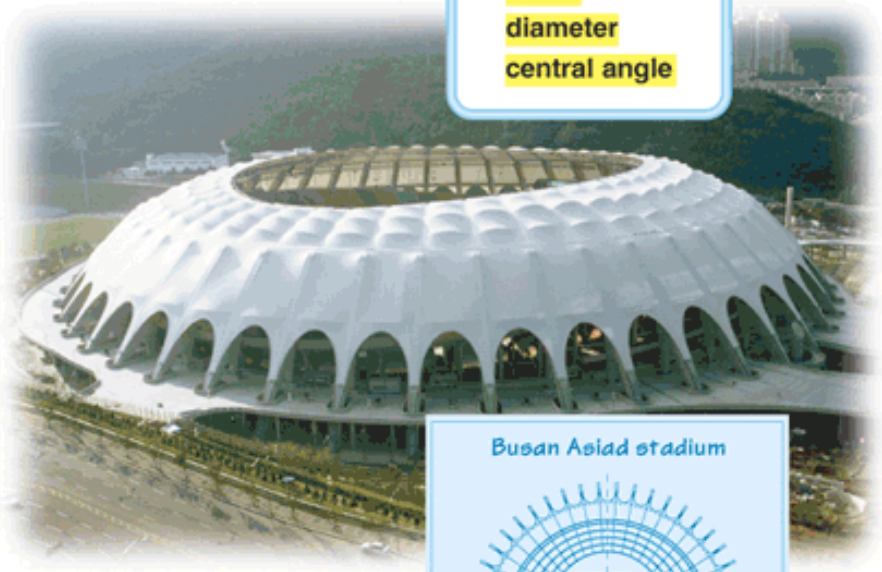
Objective Determine the measures of angles whose vertices are at the center of a circle.

Vocabulary
- center
- radius
- chord
- diameter
- central angle

Learn About It

The Busan Asiad stadium in South Korea, has a base in the shape of a circle. A circle is a geometric figure consisting of all points in a plane that are the same distance from a given point, called the **center**. A circle is named by its center.

A **radius** (plural: *radii*) is any segment that has one endpoint at the center of the circle and the other endpoint on the circle. The same word also describes the length of that segment.



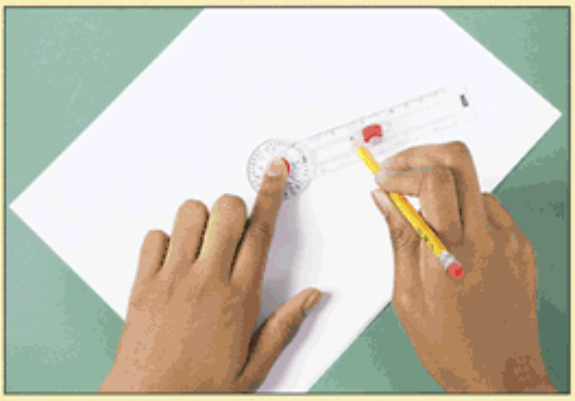
The Busan Asiad stadium was built for the Soccer World Cup in 2002.



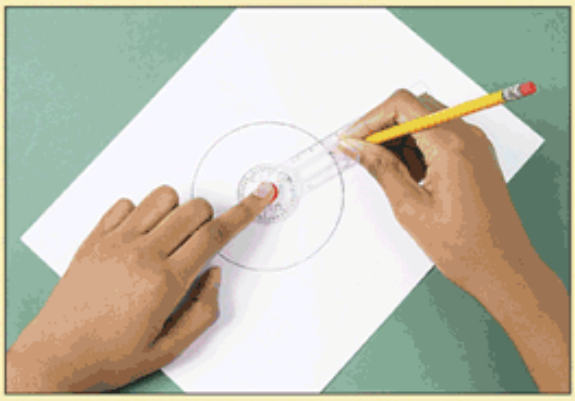
Try this activity to construct a circle with a radius of 5 centimeters.

Materials: safe drawing compass, paper and pencil

STEP 1 Draw a point and place the center of the compass over it. Set the compass slider to 5 cm. Lock the slider.

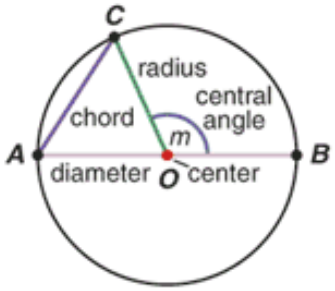


STEP 2 Move the compass in a circle around the center point. Then measure the radius to check.

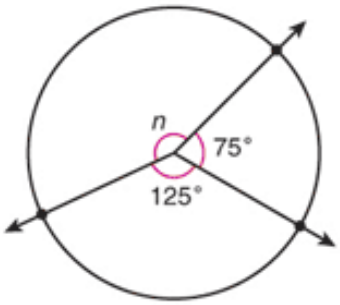


Here are the terms for some important parts of a circle. Look at circle *O*.

- A **chord** is any segment with both endpoints on the circle. *Example:* $\overline{AC}$
- A **diameter** is any chord that passes through the center of the circle. *Example:* $\overline{AB}$



A **central angle** is any angle formed by two radii. A central angle has its vertex at the center of the circle. *Example:* $\angle BOC$ or $\angle m$ in circle O on page 374.



If the radius of a circle is rotated one full turn about the center of a circle, the rotation measures 360°. You can use this fact to find a missing measure of one central angle. In the diagram at the right there are three central angles. The measures of two angles are known. Find the measure of the third angle.

Find m$\angle n$.

STEP 1 Write an equation. Simplify.

$$m\angle n + 125° + 75° = 360°$$
$$m\angle n + 200° = 360°$$

STEP 2 Subtract 200° from each side of the equation to find m$\angle n$.

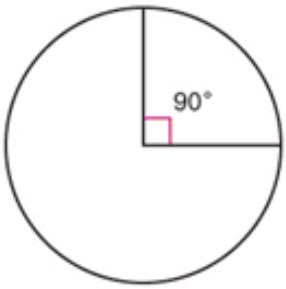
$$m\angle n + 200° - 200° = 360° - 200°$$
$$m\angle n = 160°$$

Solution: $m\angle n = 160°$.

You can use the measures of central angles to show parts of a circle as fractions.

- Since there are 360° in a circle, $\frac{90}{360}$ is the fraction that shows this part of the circle.
- When you simplify $\frac{90}{360}$, the result is $\frac{1}{4}$.

So 90° is $\frac{1}{4}$ of a complete rotation about the center of the circle.



Guided Practice

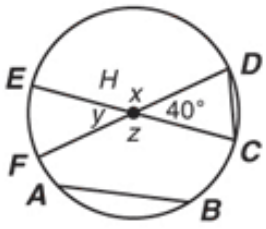
Use circle *H* for Exercises 1–6.

1. Find m$\angle x$. **2.** Find m$\angle y$.

3. Find m$\angle z$. **4.** Find m$\angle x$ + m$\angle y$.

5. Identify two chords that are not diameters.

6. Write m$\angle y$ as a fraction of the circle.



Ask Yourself

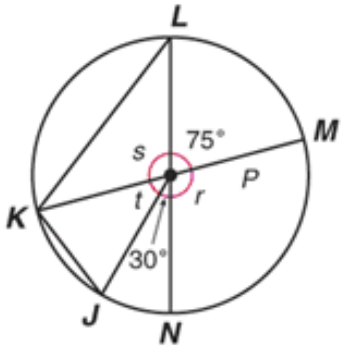
- Is the sum of the central angles 360°?
- Did I remember that the diameter forms a 180° central angle?

Explain Your Thinking Why must the sum of the degree measures of the central angles of a circle be 360°?

Go On

Practice and Problem Solving

Use circle *P* for Exercises 7–13.



7. Find m∠*s*.

8. Find m∠*r*.

9. Find m∠*t*.

10. Find m∠*s* + m∠*t*.

11. Identify two chords that are not diameters.

12. Write ∠*t* as a fraction of the circle.

13. Write 75° as a fraction of the circle.

Construct circles with the following radii.

14. 2 cm **15.** 3 cm **16.** 4 cm **17.** 3.5 cm

Solve.

18. The photo shows part of the parliament building in Bangladesh. Two staircases cross the circular opening in the wall. They form segments that connect points on the circle. What geometric term can you use to describe these segments?

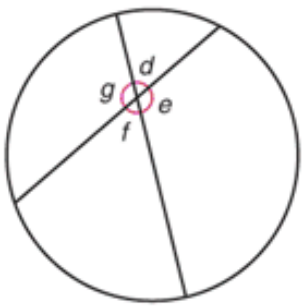


The National Assembly Building in Dacca (Dhaka), Bangladesh, designed by Estonian American architect Louis I. Kahn

19. **Measurement** Draw a circle. Draw an acute isosceles triangle and an obtuse isosceles triangle, using the center of the circle as one vertex and two points on the circle for the other vertices. Measure and label the angles to check your work.

20. **Analyze** Lizzy colors $\frac{1}{2}$ of a circle red. She then marks off an adjacent sector with a central angle of 40° and colors it blue. What is the measure of the central angle of the sector that is not red or blue?

21. **What's Wrong?** Fern drew the two intersecting chords in a circle and measured the angles formed. Fern claimed that the angles are central angles. Do you agree with Fern's thinking? Explain.



Mixed Review and Test Prep

Open Response

Determine if each number is a prime number. (Ch. 3, Lesson 2)

21. 29 **22.** 47 **23.** 53

24. 39 **25.** 41 **26.** 77

Multiple Choice

27. What is $\frac{8}{5} \div 6$? (Ch. 6, Lesson 5)

A $9\frac{2}{3}$ **C** $\frac{5}{48}$

B $\frac{4}{15}$ **D** $3\frac{3}{4}$

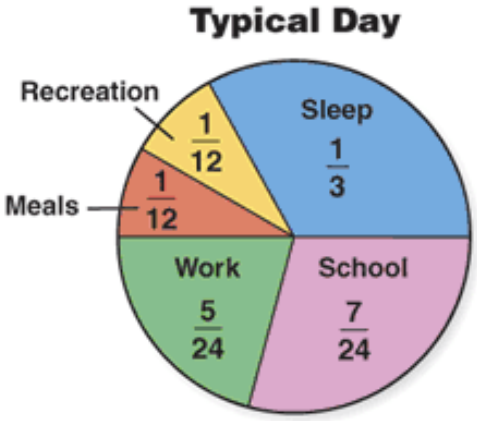
Problem Solving

Art Connection

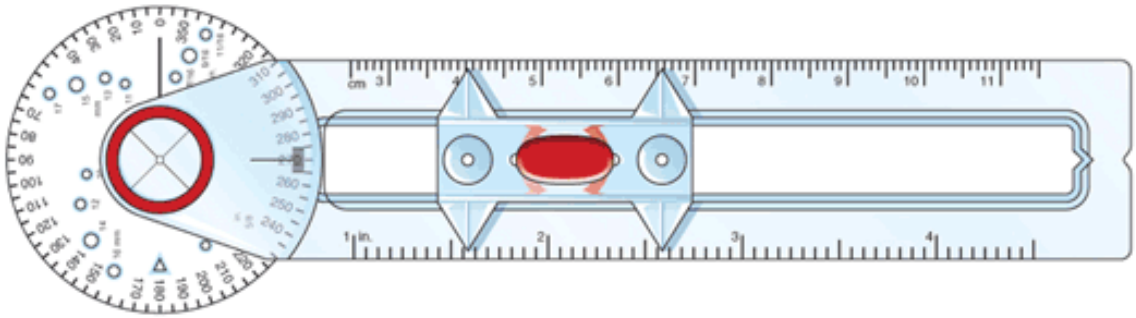
Design a Circle Graph

Take a survey of how your fellow students spend a typical day. Ask them how many hours they spend sleeping, at school, at work, in recreation, and for meals.

Gather the results of your survey. Present the results in the form of a circle graph. Use a compass to draw a circle. Mark its center.



Use a compass, a protractor, and a straightedge to mark the segments of each category on the graph.



WEEKLY WR READER eduplace.com/map

Chapter Review/Test

VOCABULARY

1. A triangle with no congruent sides is ____.
2. A line segment that connects opposite vertices of a closed figure is a(n) ____.
3. The part of the plane between two rays that form an angle is the ____.

Vocabulary

diagonal
diameter
interior of the angle
isosceles
scalene

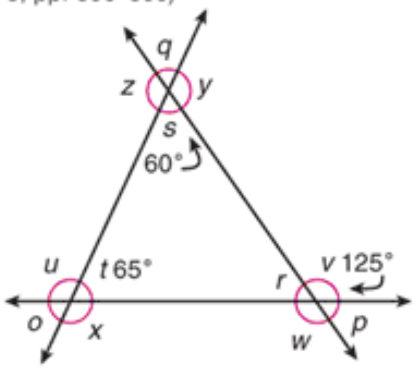
CONCEPTS AND SKILLS

Draw and label each geometric figure. (Lesson 1, pp. 354–355)

4. $\overleftrightarrow{RS} \parallel \overleftrightarrow{TU}$
5. $\overrightarrow{GH}$ and $\overrightarrow{GI}$
6. vertex J

Use the diagram at the right for Exercises 7–12. (Lessons 2–3, pp. 356–359)
Give an example of each.

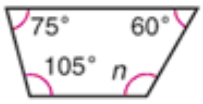
7. a pair of supplementary angles
8. an interior angle of a triangle
9. a pair of vertical angles



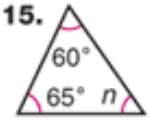
Find the degree measure of each angle.

10. $\angle r$
11. $\angle u$
12. $\angle w$

Find each missing angle measure. Classify each triangle and quadrilateral. Include all classifications that apply. (Lessons 5–6, pp. 364–366, 368–369)

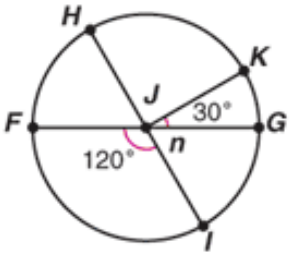
13. 

14. 

15. 

Use the diagram for Exercises 16–18. (Lesson 8, pp. 374–376)

16. Name a diameter.
17. Name a radius.
18. Find m$\angle n$.



PROBLEM SOLVING

Draw a Venn diagram and use logical reasoning to solve. (Lesson 7, pp. 370–372)

19. If all rectangles are parallelograms, are all squares parallelograms? Explain.
20. If all rhombi are parallelograms, are all rhombi rectangles? Explain.

Write About It

Show You Understand

Can you determine the missing angles in a quadrilateral when only two angles are known? Explain.

Extra Practice

Set A (Lesson 1, pp. 354–355)

Draw and label each geometric figure.

1. line segment GH **2.** vertex D **3.** $\overleftrightarrow{LM} \perp \overleftrightarrow{NO}$ **4.** $\overrightarrow{PQ}$ and $\overrightarrow{PR}$

Set B (Lessons 2–3, pp. 356–359)

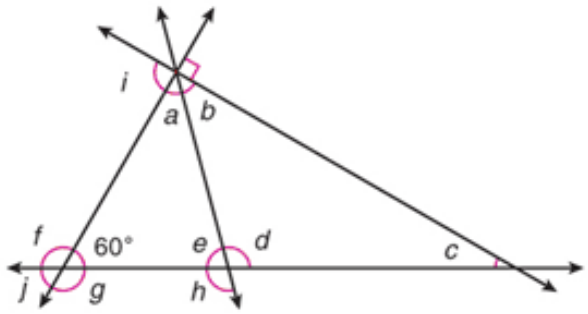
Use the diagram to give an example of each.

1. a pair of complementary angles

2. an exterior angle of the smaller triangle

3. a pair of vertical angles

4. a pair of alternate interior angles



Find the degree measure of each angle.

5. $\angle f$ **6.** $\angle i$ **7.** $\angle j$

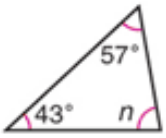
Set C (Lesson 5, pp. 364–366)

Classify each triangle by its sides and by its angles. Then find the degree measure of $\angle n$ in each triangle.

1.



2.



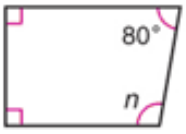
3.



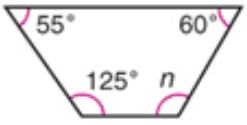
Set D (Lesson 6, pp. 368–369)

Find each missing angle measure. Classify each quadrilateral as a *parallelogram, rectangle, rhombus, square,* or *trapezoid.* Include all classifications that apply.

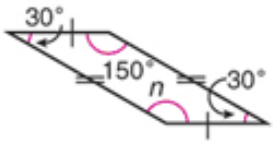
1.



2.



3.



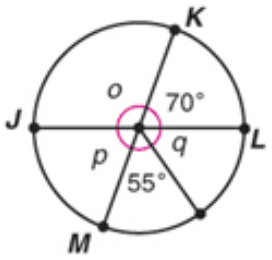
Set E (Lesson 8, pp. 374–376)

Use the diagram to find each angle measure.

1. m$\angle o$

2. m$\angle q$

3. m$\angle p$



Extra Practice at **eduplace.com/map**

CHAPTER 15

Congruence and Construction

INVESTIGATION

Using Data

Rutherford Boyd (1882–1951), an American artist, used mathematical curves and geometric shapes in many of his works, such as *Whirling Diamonds* shown.

Trace one of the curved shapes in the picture. Use your traced shape to decide what transformations Boyd might have used to create his picture.

Transformations
translation or *slide*
reflection or *flip*
rotation or *turn*

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

Vocabulary

similar
polygon
symmetry
congruent
line of symmetry

1. When a figure is folded to form two mirror images, the fold line is a _____.
2. Plane figures that have the same size and shape are _____.
3. When folded, a figure has _____ if its two parts match exactly.
4. A simple closed plane figure made up of three or more line segments is a _____.

CONCEPTS AND SKILLS

Match each figure with its correct name.

5. 
6. 
7. 
8. 
9. 
10. 

a. hexagon **b.** pentagon **c.** rectangle **d.** octagon **e.** triangle **f.** heptagon

Write each of the following in words.

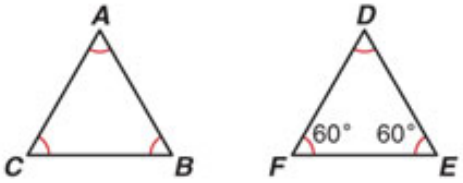
11. $\triangle ABC \cong \triangle DEF$ **12.** $\angle a \cong \angle b$ **13.** $\overline{JK} \cong \overline{LM}$

14. $\angle c \cong \angle d$ **15.** $\overline{AB} \cong \overline{CD}$ **16.** $\triangle STU \cong \triangle VWX$

17. $LMNO \cong PQRS$ **18.** $\triangle GHI \cong \triangle JKL$ **19.** $\overline{EF} \cong \overline{GH}$

Write About It

20. In $\triangle ABC$, $\angle A$ is congruent to $\angle B$. Angle B is congruent to $\angle E$ in $\triangle DEF$. How can you find the measure of $\angle A$?



Facts Practice, See Page 654.

Hands On Lesson 1

Polygons

Objective Identify, classify, and draw polygons.

Vocabulary
regular polygon
diagonal

Materials
regular polygons (Learning Tool 40), ruler or straightedge

Learn About It

A polygon is a simple closed plane figure formed by three or more line segments meeting only at their endpoints. A **regular polygon** is a polygon with all sides congruent and all angles congruent.

The painting on the right includes a variety of polygons. How many different polygons can you identify?

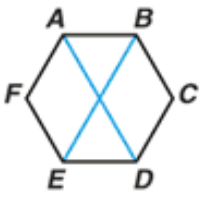


This painting is by Piet Mondrian (1872–1944).

Polygons

Name	Examples	Name	Examples
Triangle 3 sides		Octagon (8-gon) 8 sides	
Quadrilateral 4 sides		Nonagon (9-gon) 9 sides	
Pentagon (5-gon) 5 sides		Decagon (10-gon) 10 sides	
Hexagon (6-gon) 6 sides		Undecagon (11-gon) 11 sides	
Heptagon (7-gon) 7 sides		Dodecagon (12-gon) 12 sides	

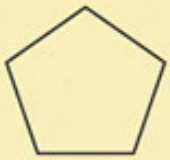
▶ A **diagonal** is a segment that joins two vertices of a polygon but is not a side. Segments *AD* and *BE* are two diagonals of the hexagon.



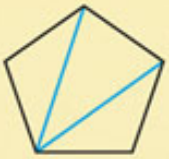
By drawing diagonals, you can calculate the sum of the angle measures in a polygon.

Try this activity to find the sum of the angle measures in a regular pentagon.

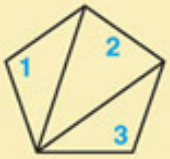
STEP 1 Use the regular pentagon from Learning Tool 40.



STEP 2 Choose one vertex and draw all of the possible diagonals from this vertex.



STEP 3 Count the triangles formed. Each angle of the pentagon is either an angle of one triangle, or is formed by 2 or more adjacent angles of the triangles.



STEP 4 Multiply the number of triangles in the pentagon by 180° to calculate the sum of all the angle measures.

$3 \times 180° = 540°$

The sum of the angle measures in any triangle is 180°.

Another Example

Measure of Each Angle in a Regular Pentagon

The sum of all angle measures in a pentagon is 540°. Since all angles in a regular polygon are congruent, divide by the number of angles in a pentagon.

$540° \div 5 = 108°$

Each angle in a regular pentagon is 108°.

Guided Practice

Find the sum of the angle measures in each regular polygon.

1. quadrilateral **2.** hexagon **3.** nonagon **4.** octagon

Find the measure of one angle in each polygon.

5. a regular octagon **6.** a regular heptagon

Ask Yourself

- Did I draw all of the diagonals?
- Did I remember that the angle sum in a triangle is 180°?

Explain Your Thinking Why do you multiply the number of triangles in a polygon by 180° to find the sum of the angle measures in the polygon?

Go On

Practice and Problem Solving

Find the sum of the angle measures in each regular polygon.

7. heptagon

8. decagon

9. undecagon

Find the measure of one angle in each polygon.

10. a regular hexagon

11. a regular nonagon

Give another name for each polygon.

12. 

13. 

14. a 9-sided figure

15. a 12-sided figure

Calculate the sum of the angle measures by dividing each figure into triangles.

16. Use the figure in Exercise 12.

17. Use the figure in Exercise 13.

Algebra • **Expressions** **If n is the number of sides, the sum of the angle measures in a polygon can be written as $(n - 2) \times 180°$. Use the expression to find the angle sum of each polygon.**

18. triangle

19. hexagon

20. pentagon

21. square

Solve.

22. **Write About It** Write a paragraph to explain how to find the sum of the angle measures of any polygon by drawing diagonals from one vertex. Include one example.

23. **Reasoning** Joni drew a circle and drew five points on the circle. Then she drew a line segment from each point to every other point. How many different line segments are in her design? Explain.

24. Look back at the picture of the Mondrian painting on page 382. How many triangles, quadrilaterals, and pentagons can you find in the painting?

25. **Analyze** Corey and Berta are drawing all the diagonals within a hexagon, using a different color for each diagonal. How many different colors will they need?

Extra Practice See page 407, Set A.

Tangram Tango

Game Activity

2 Players

What You'll Need • Two sets of tangram pieces (Learning Tool 42) and 6 sheets of manila paper

A tangram is an ancient Chinese mathematical puzzle. It is made up of seven geometric shapes.



How to Play

1. Use a set of seven tangram pieces to make three artistic designs. Each design should be on a separate sheet of manila paper. Trace each puzzle piece. Make sure that no pieces overlap. Place the completed design, facedown, in front of you.
2. Trade designs with the other player. Do not turn any designs over. Choose one design and, on the count of three, turn over your chosen tangram design.
3. The first player to reproduce the design correctly, using his or her tangram pieces, wins one point.
4. Repeat Steps 2 and 3 with the remaining two designs. The player with the most points wins.



Lesson 2

Congruent and Similar Figures

Objective Identify congruent and similar figures.

Vocabulary
- congruent
- similar
- corresponding parts



Learn About It

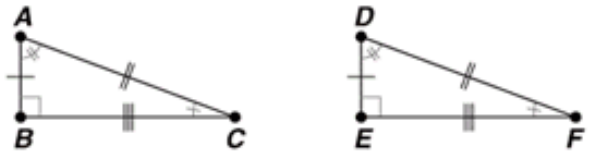
Figures that are the same size and shape are **congruent**. The symbol ≅ means "is congruent to." Figures that are the same shape but not the same size are **similar**. The symbol ~ means "is similar to."

Examine the Navaho rug at right. Look for examples of congruent figures and similar figures in the pattern.

When figures are congruent, their **corresponding parts** are also congruent.

Corresponding Parts of Congruent Figures

$\triangle ABC$ and $\triangle DEF$ are congruent triangles. Corresponding vertices are written in the same order.

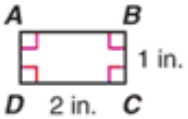


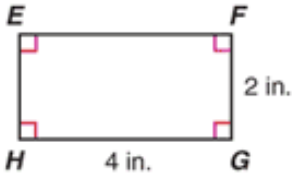
$\triangle ABC \cong \triangle DEF$

Corresponding Angles	Corresponding Sides
$\angle ABC \cong \angle DEF$	$\overline{AB} \cong \overline{DE}$
$\angle BCA \cong \angle EFD$	$\overline{AC} \cong \overline{DF}$
$\angle CAB \cong \angle FDE$	$\overline{BC} \cong \overline{EF}$

Determine whether *ABCD* ~ *EFGH*.

To decide whether the rectangles are similar, compare corresponding angles and corresponding sides. For similar figures, corresponding angles are congruent. The lengths of corresponding sides form equivalent fractions.





Corresponding Angles

$\angle A \cong \angle E$ $\quad \angle B \cong \angle F$
$\angle C \cong \angle G$ $\quad \angle D \cong \angle H$

All angles are congruent right angles.

Corresponding Sides

$\overline{AB}$ and $\overline{EF}$ $\quad \overline{BC}$ and $\overline{FG}$
$\overline{CD}$ and $\overline{GH}$ $\quad \overline{DA}$ and $\overline{HE}$

The lengths of the corresponding sides form equivalent fractions.

$$\frac{1}{2} = \frac{2}{4}$$

Rectangle *ABCD* is similar to rectangle *EFGH*.

Guided Practice

Decide whether the figures in each pair are congruent, similar, or neither. You may use tracing paper, a ruler, or a protractor.

1.



2.



Ask Yourself

- Are the corresponding sides and angles congruent?
- Do the lengths of the corresponding sides form equivalent fractions?

Explain Your Thinking Are all congruent figures also similar? Are all similar figures also congruent?

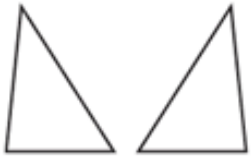
Practice and Problem Solving

Decide whether the figures in each pair are congruent, similar, or neither.

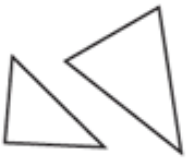
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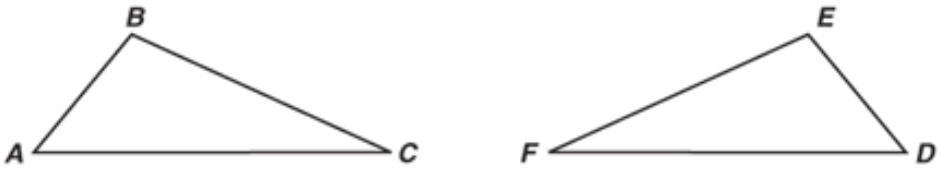
4.



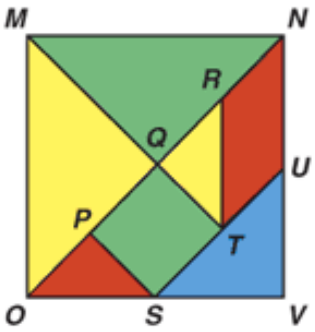
5.



6. Name the corresponding sides and angles of this pair of congruent triangles.



Seven tangram pieces are used to form square *MNVO*. Use the figure for Problems 7–10.



7. Given $\triangle OPS$ is congruent to $\triangle TQR$, name the corresponding sides and angles of the two triangles.

8. Use a ruler and a protractor to draw a square *WXYZ* with sides that are twice as long as in square *MNVO*. Are the two squares similar, congruent, or neither? Explain.

9. Reasoning Figures *MNVO* and *PQTS* are squares. Are the two figures similar or congruent? Are all squares similar? Explain.

10. Is $\triangle MOQ$ congruent to $\triangle MNQ$, $\triangle NMO$, or both?

Mixed Review and Test Prep

Open Response

Classify each triangle as *isosceles*, *equilateral*, or *scalene*. (Ch. 14, Lesson 5)

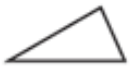
11.



12.



13.



14. Marisol thinks that two square picture frames are congruent. Explain how she could check for congruence. (Ch. 15, Lesson 2)

Extra Practice See page 407, Set B.

Lesson 3

Audio Tutor 2/7 Listen and Understand

Symmetry

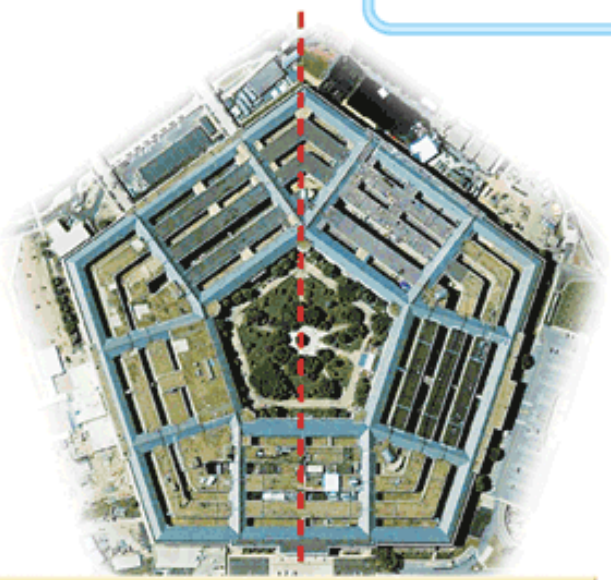
Objective Identify line and rotational symmetry in figures.

Vocabulary
- symmetry
- line of symmetry
- rotational symmetry

Learn About It

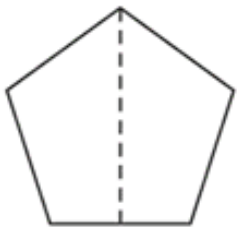
A figure has **symmetry** if, when folded, one half matches up exactly with the other half. The fold line is a **line of symmetry**.

The photograph shows the top view of the Pentagon. The two halves formed by the dashed line are mirror images. How many lines of symmetry does the Pentagon have?



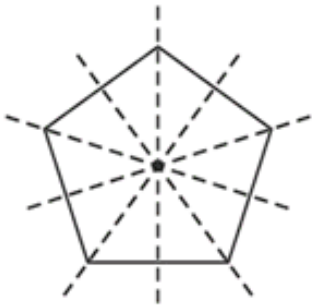
Test a Figure for Line Symmetry

STEP 1 Trace the outline of the Pentagon on paper. Draw a line of symmetry.



Fold your paper along this line. The halves should match exactly.

STEP 2 Look for other lines of symmetry in the Pentagon. Add each line of symmetry to your drawing.

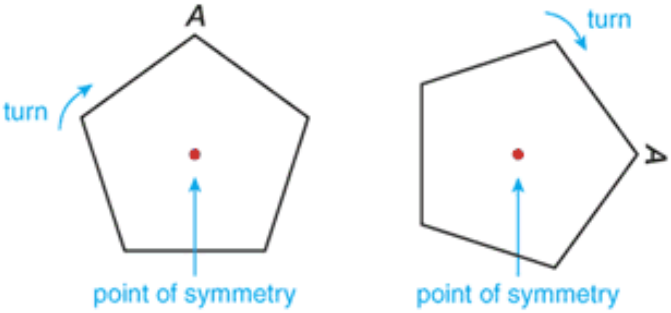


Solution: The Pentagon has 5 lines of symmetry.

Another Example

Rotational Symmetry

Find the point at which the lines of symmetry intersect in your drawing. When you rotate, or turn, the drawing about this point, the image should match the figure in the book before you have made a complete turn. So, the Pentagon has **rotational symmetry**.



Extra Help at **eduplace.com/map**

Guided Practice

Trace each figure. Draw and identify all the lines of symmetry.

1. 

2. 

3. 

Ask Yourself

- Does the fold line divide the figure into two mirror images?
- If I turn the figure, will the new image match up with the original image?

Explain Your Thinking If a figure has symmetry, are the matching parts congruent?

Practice and Problem Solving

Trace each figure. Draw and identify all the lines of symmetry.

4. 

5. 

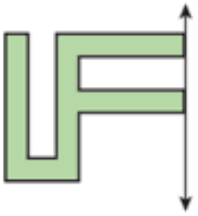
6. 

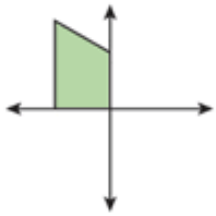
7. 

Trace each figure. Complete the design to make a symmetrical figure.

8. 

9. 

10. 

11. 

Solve.

12. The top view of many buildings is rectangular. How many lines of symmetry does a rectangle have?

13. **Create and Solve** Draw the top view of a building that has three lines of symmetry. Draw each line of symmetry.

14. **Explain** Write all the capital letters that have rotational symmetry. Explain why the other capital letters do not.

15. Which capital letters from N to Z have one or more lines of symmetry? Draw each letter and its line(s) of symmetry.

Mixed Review and Test Prep

Open Response

Classify each angle as *acute, obtuse,* or *right.* (Ch. 14, Lesson 4)

16. 45° **17.** 90° **18.** 101°

19. 5° **20.** 178° **21.** 91°

Multiple Choice

22. The top view of the dome on the Capitol Building is shaped like a circle. How many lines of symmetry does a circle have? (Ch. 15, Lesson 3)

A 2 **C** 6

B 4 **D** infinite number

Extra Practice See page 407, Set C.

Hands On Lesson 4

Audio Tutor 2/8 Listen and Understand

Transformations

Objective Explore transformations, such as translations, reflections, and rotations.

Vocabulary
- transformation
- translation
- reflection
- rotation

Materials
square dot paper
index cards
ruler
scissors

Work Together

A geometric **transformation** changes the position of a figure. In this lesson, you will model three transformations: a translation, a reflection, and a rotation.

A **translation** moves each point of a figure the same distance and direction.

Model a translation of a geometric shape.

STEP 1 Fold an index card in half. Then cut out a geometric shape along the fold line. Cut an extra piece from one side of the fold.

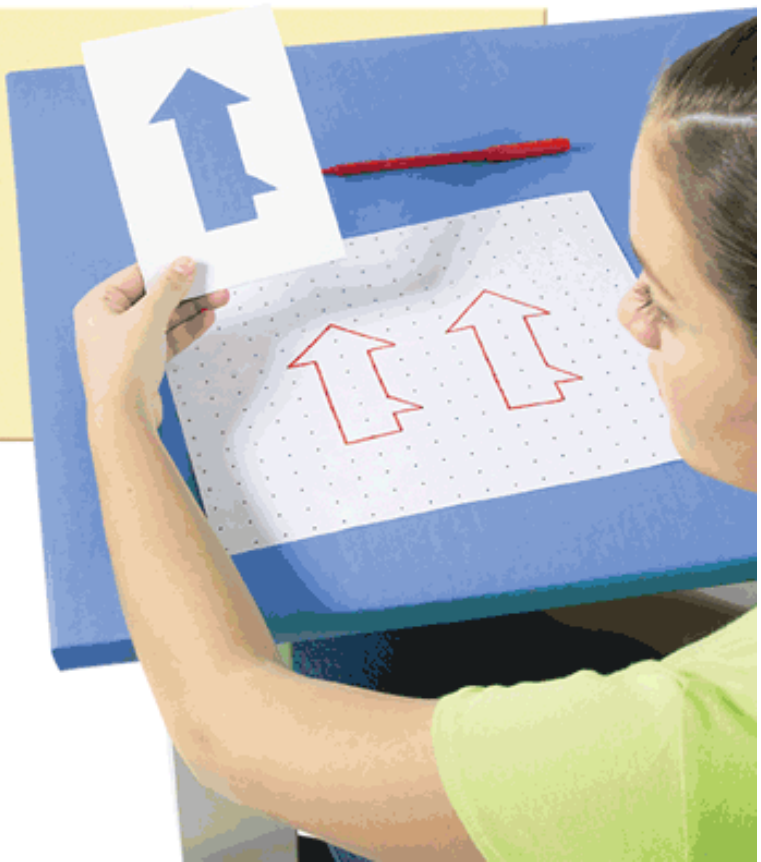


STEP 2 Unfold the index card. Using the card as a stencil, draw the outline of the shape on a sheet of dot paper.



STEP 3 Without turning or lifting it, slide the stencil a distance to the right. Draw the outline of the shape in its new position.

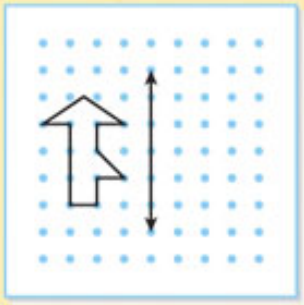
- Is the translated shape congruent to the original shape? Why or why not?
- Why, do you think, is a translation also called a *slide*? Explain.



A **reflection** moves every point of a figure to its mirror image across a line of reflection.

Use your stencil shape to model a reflection.

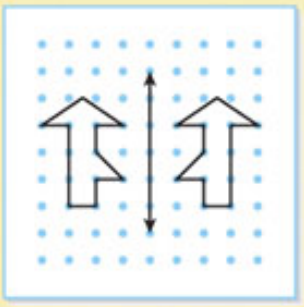
STEP 1 Draw the shape again on another area of your dot paper. Draw a line of reflection to the right of the shape you drew.



STEP 2 Flip the stencil over so that the front is facing down. Make sure that the stencil is to the right of the line of reflection. Draw the outline of the reflected shape.

Compare the reflected shape to the original.

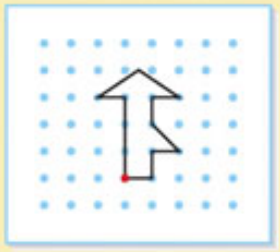
- Is the reflected shape congruent to the original shape? Why or why not?
- Why, do you think, is a reflection also called a *flip*? Explain.



A **rotation** turns a figure about a given point.

Use your stencil shape to model a rotation.

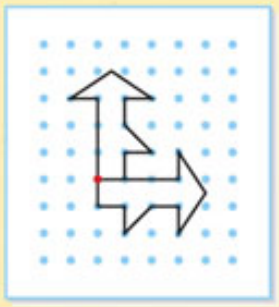
STEP 1 Draw the shape again. Draw a dot at the lower left-hand corner of the traced shape.



STEP 2 Place the stencil over the traced shape. Carefully turn the stencil clockwise 90°, using the dot as the center of rotation. Draw the outline of the shape in its new position.

Compare the rotated shape to the original.

- Is the rotated shape congruent to the original shape? Why or why not?
- Why, do you think, is a rotation also called a *turn*? Explain.

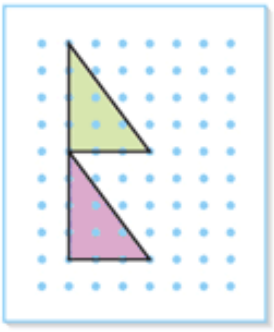


Go On

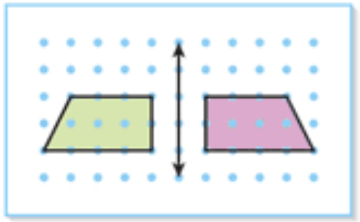
On Your Own

Write whether each pair of figures shows a translation, a reflection, or a rotation.

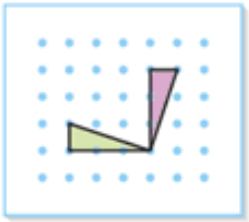
1.



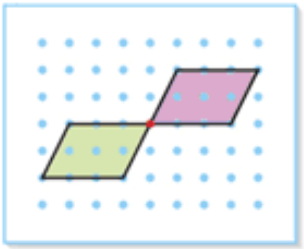
2.



3.



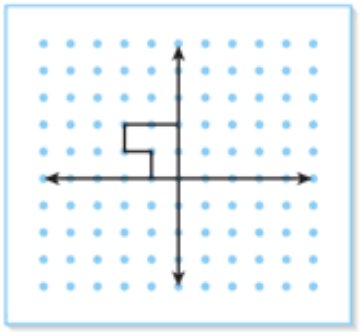
4.



5. Which image represents the first figure reflected across a horizontal line?



6. Copy the figure on the right onto dot paper. Reflect the figure four times, alternately using the vertical line and the horizontal line as lines of reflection. What image do you get after the fourth reflection?



7. **Create and Solve** Draw a geometric shape on dot paper. Draw a reflection, a rotation, and a translation of the geometric shape on the same sheet of paper. Label each transformation.

Talk About It • Write About It

You learned how to use models to show translations, reflections, and rotations.

8. How is a rotation different from a translation?

9. How is a reflection different from both a translation and a rotation?

10. Why is it important to identify the point about which a figure is being rotated?

Problem Solving

Visual Thinking

Round and Round

Visualizing an object in different positions sometimes helps to solve problems in geometry.

1. The Ferris wheel at right has 12 carriages that rotate about a single point. How many times will the four-color pattern repeat itself during a 360° rotation of the Ferris wheel?

2. How many 90° rotations of the design below are needed before the design appears as it does now?





WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

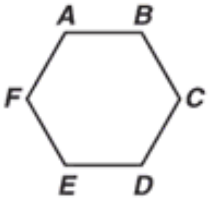
Use the polygons at right for Exercises 1–4.

1. Name polygon *M*. (Lesson 1)

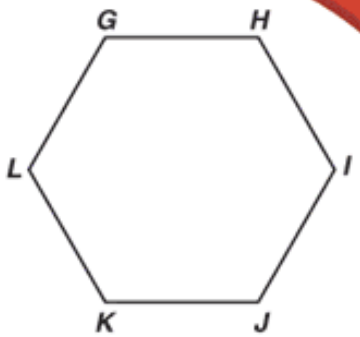
2. Find the measure of one angle in polygon *M*. (Lesson 1)

3. Decide whether polygons *M* and *N* are congruent or similar. (Lesson 2)

4. Trace polygon *N*. Draw and identify all the lines of symmetry. (Lesson 3)



Polygon *M*



Polygon *N*

Solve. (Lesson 4)

5. Draw a right scalene triangle on dot paper. Then draw and label a translation, a reflection, and a rotation of the triangle.

Lesson 5

Problem-Solving Strategy

Find a Pattern

Objective Find and use a pattern to solve a problem.



The honeycomb on the right is an example of a *tessellation*. A tessellation is formed when congruent geometric shapes fill a plane without overlapping or leaving gaps. In order for a set of polygons to create a tessellation, the sum of all the angles that come together at a common point must be 360°.

Problem Which regular polygons can be used alone to make a tessellation?

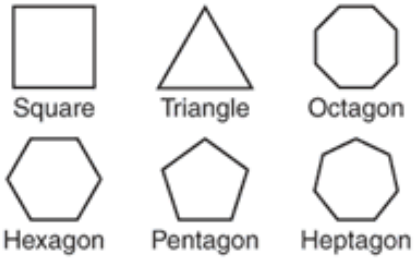
UNDERSTAND

This is what you know:

- There can be no gaps or overlaps.
- The sum of the angles at a common point must be 360°.
- In a regular polygon, all the sides and all the interior angles are congruent.

PLAN

Find the sum of the angle measures of the regular polygons. Use a pattern. Then find the measure of each interior angle.



SOLVE

Number of sides (n)	3	4	5	6	7	8
Sum of the angle measures	180°	360°	540°	720°	900°	1080°

+ 180° + 180° + 180° + 180° + 180°

Number of sides (n)	3	4	5	6	7	8
Interior angle measure	60°	90°	108°	120°	$128\frac{4}{7}°$	135°

Solution: 60°, 90°, and 120° are all factors of 360°. The only regular polygons that can be used alone to make a tessellation are the triangle, the square, and the hexagon.

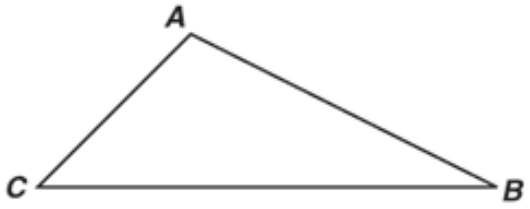
LOOK BACK

Did your geometric pattern form a tessellation or not?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Trace scalene triangle *ABC* below. Can you use the triangle to form a tessellation? Check your answer by translating, reflecting, or rotating the figure to form a pattern.



2. Trace parallelogram *PQRS*. Use it to make a geometric pattern. Does your geometric pattern form a tessellation? Explain.



Ask Yourself

UNDERSTAND — What facts do I know?

PLAN — What transformations can I use to make a tessellation?

SOLVE — What do I know about tessellations to help me solve the problem?

LOOK BACK — Are there any gaps or overlaps in my pattern?

Independent Practice

Use a pattern to solve each problem.

3. The picture on the right shows a set of pattern blocks. Which pattern-block shapes can be used to form tessellations?



Learning Tool 41

4. Lakisha says that any pattern made up of both regular hexagons and regular triangles must be a tessellation. Tim says it is impossible to do this using these two polygons. Who is correct? Explain.

5. **What's Wrong?** Jared tried to make a tessellation, using only a right triangle and a square. He decided it was not possible. Is Jared correct?

6. **You Decide** Design your own tessellation. Decide which shape or combinations of shapes you will use.

7. **Represent** Use a circle to make a geometric pattern. Does your pattern qualify as a tessellation? Explain.

Go On

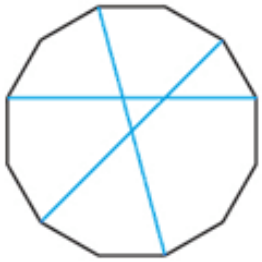
Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

8. Ty began collecting art prints 8 years ago when he was 33. Patricia began 10 years ago. When she began collecting, Patricia was 5 years younger than Ty was when he began collecting. How much older than Patricia is Ty?

9. Ellis borrowed $78 from his sister to buy a framed Escher print. If Ellis decreases his debt by $5 each week, how many weeks will it take him to pay off his debt?

10. Serena created a pattern using regular dodecagons. She decided to decorate it by drawing all possible diagonals in each dodecagon. How many diagonals did Serena draw in each dodecagon?



You Choose

Strategy

- Draw a Diagram
- Find a Pattern
- Make a Table
- Use Logical Reasoning
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data Use the graph to solve Problems 11–15.

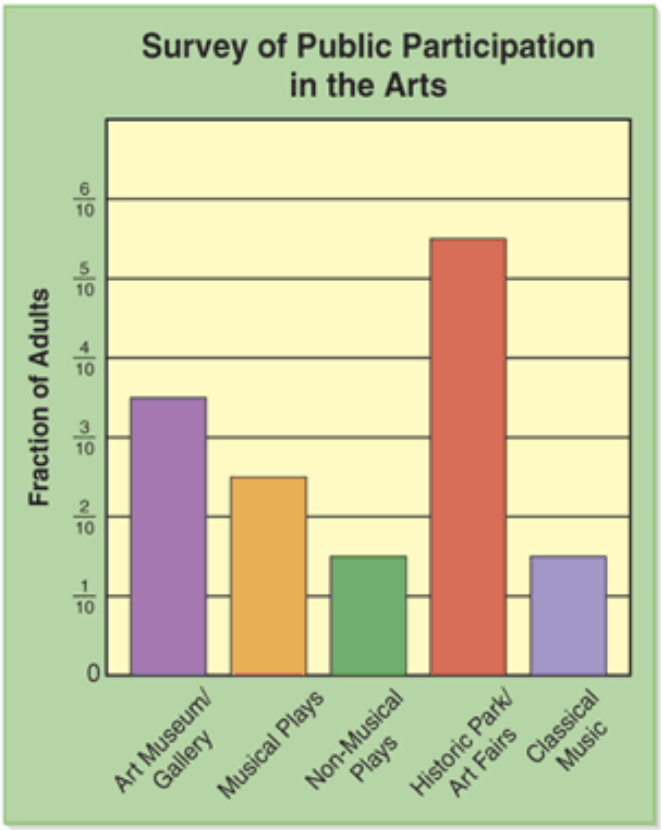
11. What fraction of the adults visited a historic park or an arts fair?

12. What is the difference in the fraction of adults who attended a musical play and those who listened to classical music?

13. **Predict** According to the survey results, would an adult be more likely to visit an art museum or attend a musical play?

14. Did more adults visit musical events or art events? Explain how you know.

15. **Analyze** Find the sum of the fractions in the bar graph. Explain why the sum is greater than 1.



Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. At a track meet, Michael jumped $3\frac{1}{4}$ feet, $4\frac{3}{4}$ feet, $5\frac{1}{3}$ feet, and 5 feet in the high jump. What is a reasonable average for Michael's jumps?

A 4 feet
C 5 feet
B $4\frac{1}{2}$ feet
D $5\frac{1}{2}$ feet

(Chapter 6, Lesson 5)

2. Phil bought five posters of his favorite football players to arrange on his bedroom wall. How many different ways can Phil arrange the posters?

F 8
H 16
G 12
J 120

(Chapter 9, Lesson 5)

Open Response

Solve each problem.

3. Cathy planned to bike 120 miles on her vacation. On the first day she biked $\frac{1}{4}$ of the distance. On the next day, she biked $\frac{1}{2}$ of the remaining distance. How far did Cathy have left to go?

Represent Draw a diagram to solve the problem.

(Chapter 11, Lesson 3)

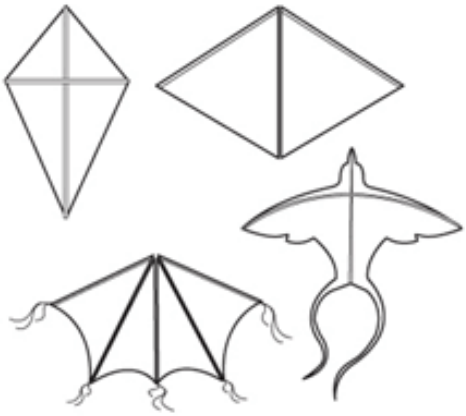
4. Travis, Sonya, April, and Don are going biking, boating, surfing, and hiking. Don does not know how to surf. Don and April talked with the hiker and the boater. Travis doesn't know how to swim and does not like boats. Tell who will do each activity.

Explain how to use logical reasoning to solve this problem.

(Chapter 14, Lesson 7)

Constructed Response

5. You want to make a pattern for a kite design.



a. Choose a design. Make a sketch of it and find a line of symmetry.

b. Explain how you could use transformations to draw the pattern.

c. On a coordinate grid, use the *y*-axis as your line of symmetry. Draw and label your kite pattern.

(Chapter 15)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Hands On Lesson 6

Construct Line Segments and Angles

Objective Use a compass and a straightedge to construct congruent line segments and congruent angles.

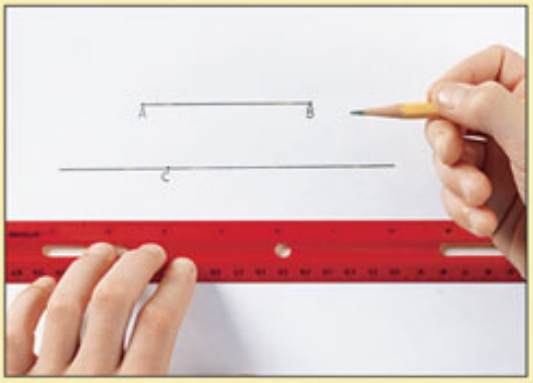
Materials
paper
pencil
straightedge
safe drawing compass
protractor

Work Together

Recall from Lesson 2 that two figures are congruent if they are the same size and shape. Corresponding parts of congruent figures are also congruent. You can construct congruent line segments and angles by using a straightedge and a safe drawing compass.

Construct a line segment congruent to $\overline{AB}$.

STEP 1 Trace $\overline{AB}$. Draw a line just below $\overline{AB}$. Label one point on the line Point *C*.



STEP 2 Place the compass on Point *A*. Move the slider to measure the length of segment $\overline{AB}$.



STEP 3 Using this measure, place the compass on Point *C*.



STEP 4 Draw an arc from Point *C*. Label the intersection Point *D*.

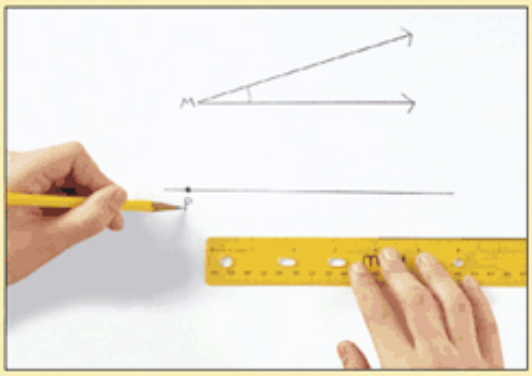


Check.
Measure $\overline{CD}$ and $\overline{AB}$.
If their lengths are the same, they are congruent.

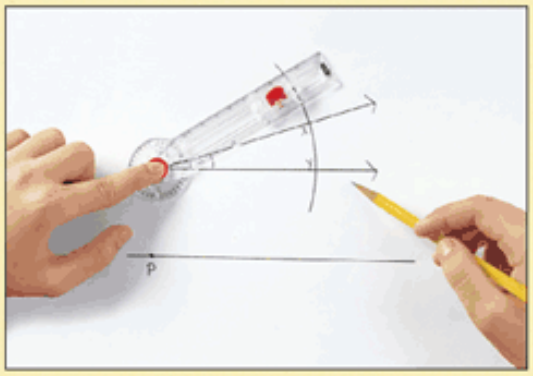
$\overline{CD}$ is congruent to $\overline{AB}$.

Construct an angle congruent to $\angle M$, using Point P as the vertex.

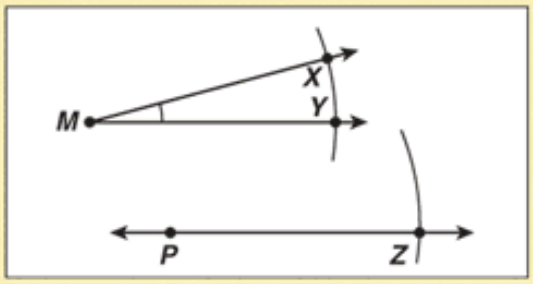
STEP 1 Trace $\angle M$. Draw a line below $\angle M$. Label one point on the line Point P.



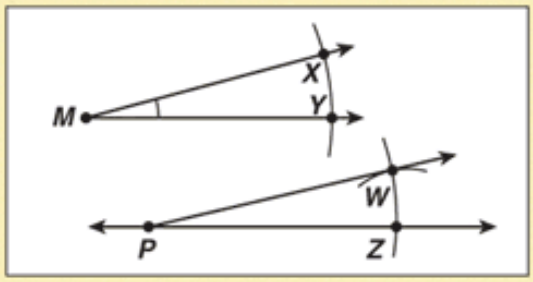
STEP 2 Place the compass on Point M and draw an arc that intersects both rays. Label the intersections Points X and Y.



STEP 3 Without changing the compass measure, place the compass on Point P. Draw a similar arc, intersecting the line at Point Z.



STEP 4 Use the compass to measure the distance from Point X to Point Y. Place the compass at Point Z. Draw an arc that intersects the larger arc at Point W. Draw ray PW.



$\angle P$ is congruent to $\angle M$.

On Your Own

Draw and label each figure. Then use a compass and a straightedge to draw a congruent figure for each.

1. line segment DE
2. $\angle F$, measuring 60°
3. $\angle GHI$, measuring 110°

Talk About It • Write About It

You learned how to construct congruent line segments and angles.

4. Look back at Step 4 for constructing a congruent angle. What can you tell about the line segments in the two figures?

5. Suppose a classmate is having difficulty with Step 4 of constructing an angle congruent to $\angle M$. Show how to complete the step. Describe the process in your own words.

Extra Practice See page 407, Set D.

Hands On Lesson 7

Construct Bisectors

Objective Use a compass and a straightedge to bisect line segments and angles.

Vocabulary
midpoint
bisect

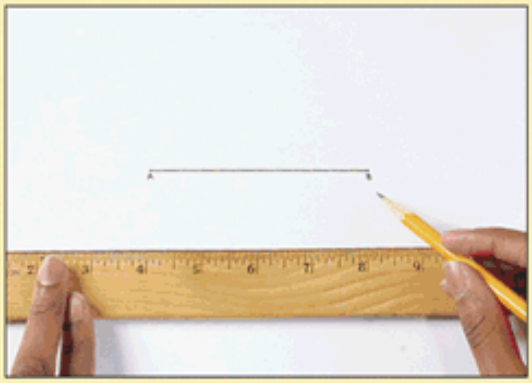
Materials
paper
pencil
straightedge
safe drawing compass
protractor

Work Together

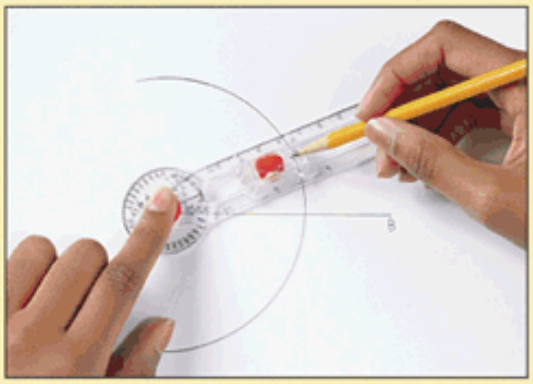
The **midpoint** of a line segment is the point that divides the segment into two congruent parts. You can find the midpoint of a line segment by constructing a perpendicular bisector. A perpendicular bisector **bisects**, or divides, the line segment into two congruent segments and forms four 90° angles.

Find the midpoint of $\overline{AB}$, using a straightedge and a compass.

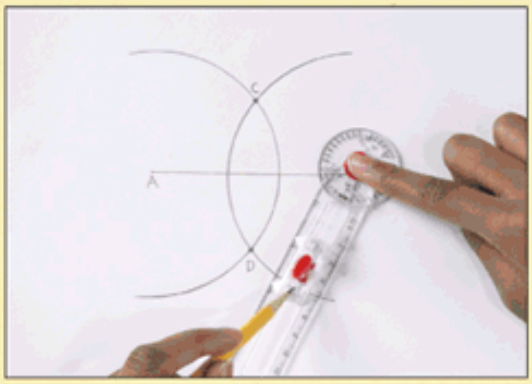
STEP 1 Draw a line segment. Label the endpoints *A* and *B*.



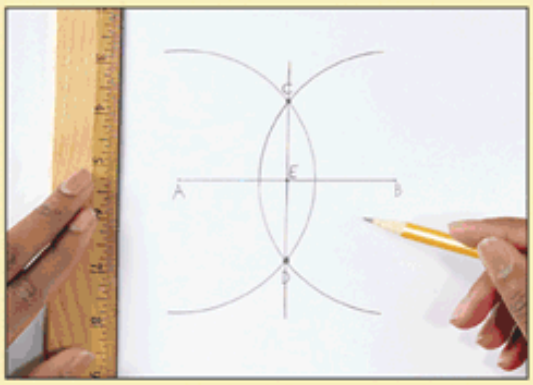
STEP 2 Place your compass on Point *A*. Move the slider to more than one half the length of $\overline{AB}$, then lock the compass. Draw an arc.



STEP 3 Place your compass on Point *B*. Draw another arc. Label the points where the arcs intersect Points *C* and *D*.



STEP 4 Use the straightedge to draw a line through Points *C* and *D*. Label the point where $\overleftrightarrow{CD}$ intersects $\overline{AB}$ Point *E*. Point *E* is the midpoint.

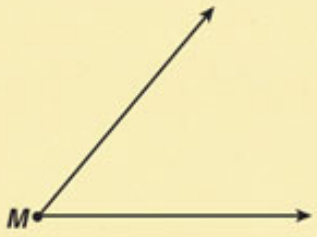


$\overline{CD}$ is the perpendicular bisector of $\overline{AB}$.

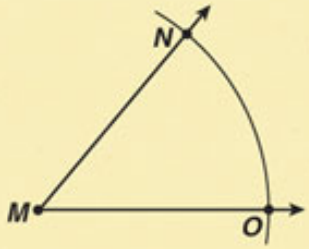
▶ You can divide an angle into two congruent angles by constructing the bisector of the angle.

Construct the angle bisector of $\angle M$.

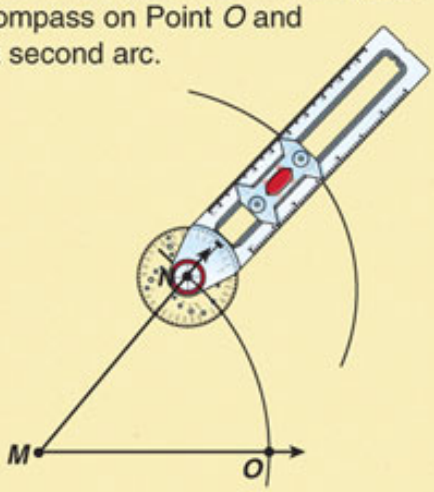
STEP 1 Trace $\angle M$ and label the vertex Point *M*. Place your compass at Point *M*, choose a setting, and lock the compass.



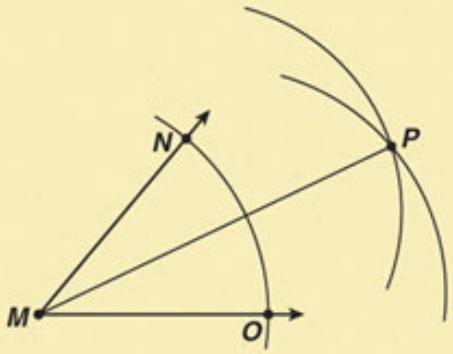
STEP 2 Draw an arc that intersects both rays. Label the intersections, Points *N* and *O*.



STEP 3 Place your locked compass on Point *N* and draw an arc as shown. Then place your compass on Point *O* and draw a second arc.



STEP 4 Label the point where the arcs intersect Point *P*. Draw a line segment from Point *M* to Point *P*.



Line segment *MP* is the bisector of $\angle M$.

On Your Own

Draw each figure. Then construct a bisector for each.

1. horizontal line segment *ST*

2. $\angle V$, measuring 125°

Talk About It • Write About It

You learned how to construct bisectors of line segments and angles.

3. Why is the bisector of a line segment called a perpendicular bisector?

4. If line segments intersect at their midpoints, must the segments be perpendicular or not? Use drawings to support your conclusion.

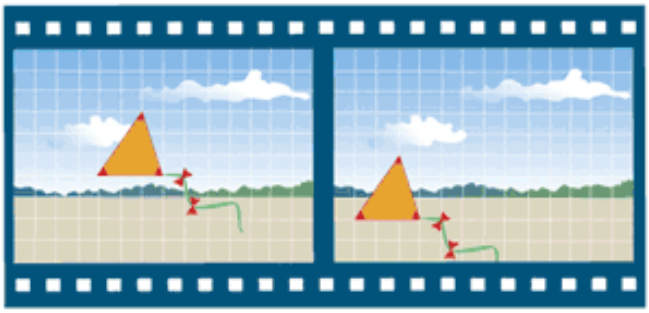
Lesson 8

Audio Tutor 2/9 Listen and Understand

Transformations in the Coordinate Plane

Objective Use coordinates to describe transformations of geometric figures.

Learn About It



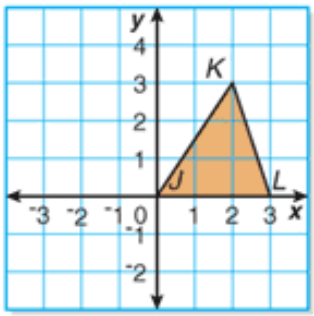
For the next frame in a computer-animated film, the animator wants to translate $\triangle JKL$ 3 units to the left and 2 units down, while keeping the background the same. How can she plot the translated $\triangle XYZ$?

Find and plot the vertices of $\triangle XYZ$.

STEP 1 Find the coordinates of the vertices of $\triangle JKL$.

J: (0, 0), K: ($^+2$, $^+3$)

L: ($^+3$, 0)



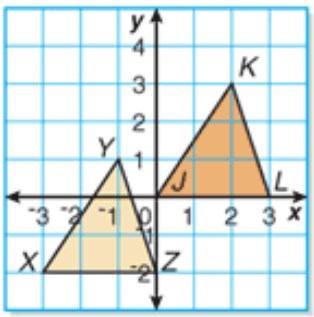
STEP 2 Translate each vertex 3 units left and 2 down.

J: (0, 0) → X: ($^-3$, $^-2$)

K: ($^+2$, $^+3$) → Y: ($^-1$, $^+1$)

L: ($^+3$, $^+0$) → Z: (0, $^-2$)

STEP 3 Plot and label the coordinates of the vertices.



Solution: The vertices of $\triangle XYZ$ are X ($^-3$, $^-2$), Y ($^-1$, $^+1$), and Z (0, $^-2$).

Other Examples

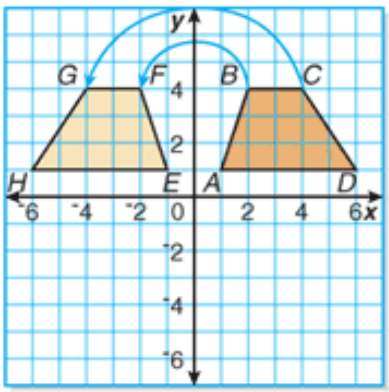
A. Reflection Across the *y*-axis

- Identify the coordinates of the vertices:

  A ($^+1$, $^+1$), B ($^+2$, $^+4$), C ($^+4$, $^+4$), and D ($^+6$, $^+1$)

- To reflect each vertex across the *y*-axis, plot the *opposite* *x*-coordinates and the *same* *y*-coordinates:

  E ($^-1$, $^+1$), F ($^-2$, $^+4$), G ($^-4$, $^+4$), and H ($^-6$, $^+1$)



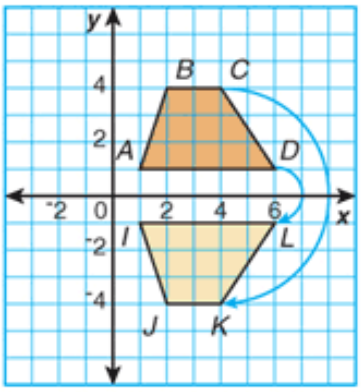
B. Reflection Across the *x*-axis

- Identify the coordinates of the vertices:

  $A\,(^{+}1, {}^{+}1)$, $B\,(^{+}2, {}^{+}4)$, $C\,(^{+}4, {}^{+}4)$, and $D\,(^{+}6, {}^{+}1)$

- To reflect each vertex across the *x*-axis, plot the *same* *x*-coordinates and the *opposite* *y*-coordinates:

  $I\,(^{+}1, {}^{-}1)$, $J\,(^{+}2, {}^{-}4)$, $K\,(^{+}4, {}^{-}4)$, and $L\,(^{+}6, {}^{-}1)$



Think
A rotation of 360° means the image has the same coordinates as the original figure.

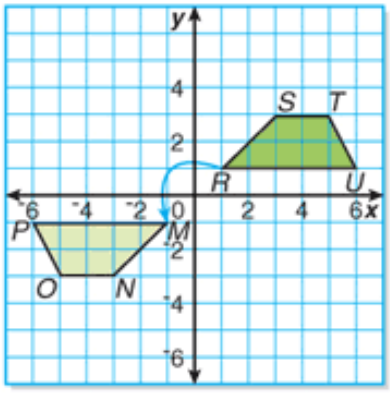
C. Rotation About the Origin

- Identify the coordinates of the vertices:

  $R\,(^{+}1, {}^{+}1)$, $S\,(^{+}3, {}^{+}3)$, $T\,(^{+}5, {}^{+}3)$, and $U\,(^{+}6, {}^{+}1)$

- To rotate 180° about the origin, plot the *opposite* *x*-coordinates and the *opposite* *y*-coordinates:

  $M\,(^{-}1, {}^{-}1)$, $N\,(^{-}3, {}^{-}3)$, $O\,(^{-}5, {}^{-}3)$, and $P\,(^{-}6, {}^{-}1)$

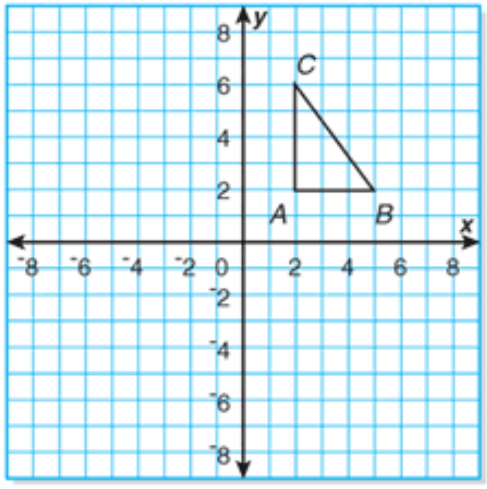


Guided Practice

Copy $\triangle ABC$ on graph paper. Identify the coordinates of $\triangle ABC$ after each transformation. Use $\triangle FGH$ for the transformed triangle.

Ask Yourself
- Did I draw each transformation correctly?
- Did I identify the coordinates of each vertex correctly?

1. Translate 3 units to the right.
2. Translate 4 units down.
3. Reflect across the *y*-axis.
4. Reflect across the *x*-axis.
5. Rotate 90° clockwise about the origin.
6. Rotate 180° about the origin.

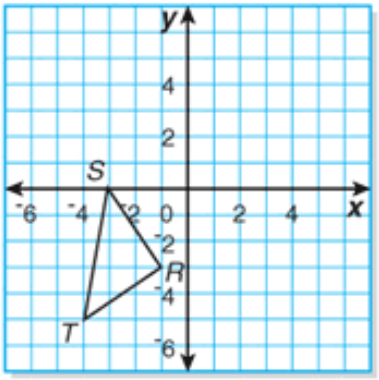


Explain Your Thinking How is a reflection across the *y*-axis different from a counterclockwise rotation of 90°?

Go On

Practice and Problem Solving

Copy $\triangle RST$ on graph paper. Identify the coordinates of $\triangle RST$ after each transformation. Use $\triangle ABC$ for the transformed triangle.



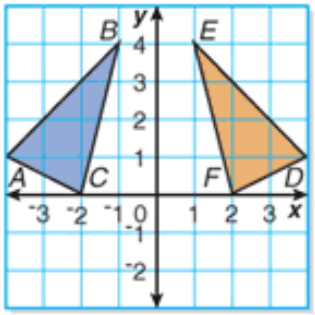
7. Translate right 5, then down 1.

8. Reflect across the *x*-axis.

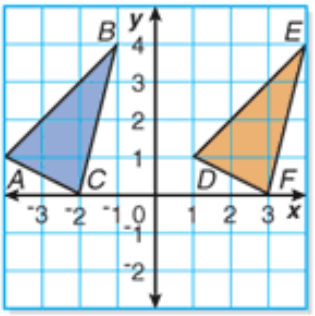
9. Reflect across the *y*-axis.

In Exercises 10–12, identify the transformation that changes the blue figure into the orange figure.

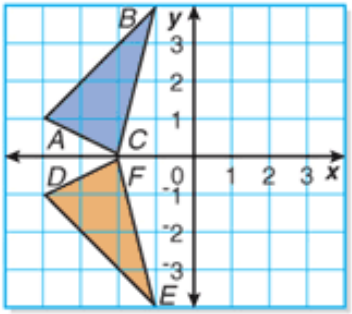
10.



11.



12.

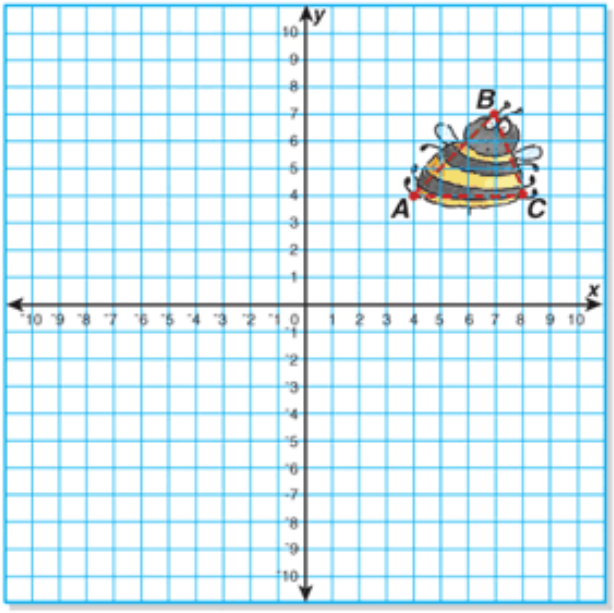


Draw $\triangle C$ (0, 2), D (3, 3), E (4, 1). Then complete each transformation, using the same coordinate grid.

13. Translate $\triangle CDE$ right 2 units and down 1 unit to form $\triangle RST$.

14. Reflect the translated image across the *y*-axis to form $\triangle XYZ$.

Solve. Use the diagram for Problems 15 and 16.



15. Suppose the animation artist wanted to move the bumblebee left 3 units and down 3 units. What would be the new coordinates of the bumblebee?

16. **Write About It** Describe the transformations the animation artist might use to move the bumblebee to Quadrant III.

17. **What's Wrong?** Dana is translating Point A (3, 2) 5 units right and 6 units up to Point B. She says the coordinates of Point B are (9, 7). Explain the error Dana made.

Extra Practice See page 407, Set E.

Mixed Review and Test Prep

Free Response

Identify each number as prime or composite.
(Ch. 3, Lesson 2)

18. 4 **19.** 11 **20.** 50

21. 139 **22.** 117 **23.** 96

24. 77 **25.** 83 **26.** 132

Multiple Choice

27. The shape of a mosaic tile from ancient Rome is a regular octagon. What is the measure of each angle?
(Ch. 15, Lesson 1)

A 108° **C** 135°

B 120° **D** 140°

Activity

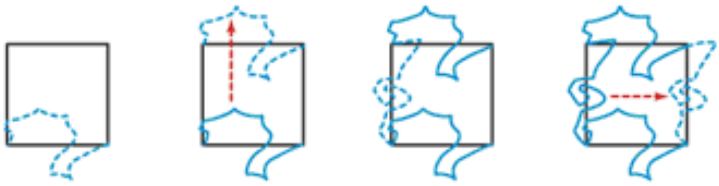
Art Connection
M.C. Escher

M.C. Escher (1898–1972) was a Dutch artist. Many of his designs are tessellations. The design at right is an example of a translation tessellation.



You can create your own Escher-type translation tessellation pattern by starting with a square and translating opposite sides.

Create a simple drawing of one of your favorite animals. Then experiment with altering your drawing so that it will tessellate. What do you need to consider when changing your drawing?



Step 1 Step 2 Step 3 Step 4

Exchange the drawing you created with the one created by a classmate. Will your classmate's drawing tessellate? Explain.

If not, revise your classmate's drawing so that it does tessellate. Explain what you did to revise your classmate's drawing to make it tessellate.

WEEKLY WR READER eduplace.com/map

Chapter Review/Test

VOCABULARY

1. A transformation that involves a turning movement of a figure about a point is a _____.
2. A polygon with all sides congruent and all angles congruent is a _____.
3. Figures that are the same shape but not the same size are _____.

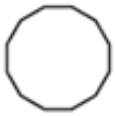
Vocabulary

similar
rotation
reflection
congruent
regular polygon

CONCEPTS AND SKILLS

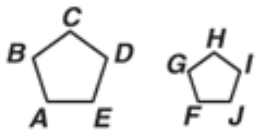
Name each regular polygon. Find the measure of one angle in each. (Lesson 1, pp. 382–384)

4. 

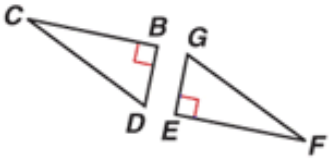
5. 

6. 

Decide whether the figures in each pair are congruent, similar, or neither. Then name the corresponding sides and angles. (Lesson 2, pp. 386–387)

7. 

8. 

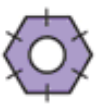
9. 

Trace each figure. Draw all the lines of symmetry. Does each figure have rotational symmetry? Write *yes* or *no*. (Lesson 3, pp. 388–389)

10. 

11. 

12. 

13. 

Draw $\triangle EFG$ with vertices E (1, 3), F (5, 4), and G (2, 1). Identify the coordinates of the triangle after each transformation. (Lesson 8, pp. 402–404)

14. Translate 2 to the right, then down 4. Name this triangle *MNO*.
15. Reflect across the *x*-axis. Name this triangle *PQR*.

PROBLEM SOLVING

Solve. (Lesson 5, pp. 394–396)

16. Suzanne wants to make a tessellation with regular pentagons and triangles. Explain how she can do this.

Write About It

Show You Understand

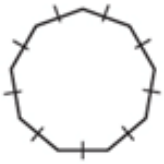
Suppose $\triangle ABC$ is similiar to $\triangle DEF$. If you know the lengths of the sides of $\triangle ABC$, could you find the lengths of the sides of $\triangle DEF$? Explain.

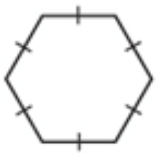
Extra Practice

Set A (Lesson 1, pp. 382–384)

Name each polygon. Then find the measure of one angle in each polygon.

1. 

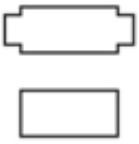
2. 

3. 

Set B (Lesson 2, pp. 386–387)

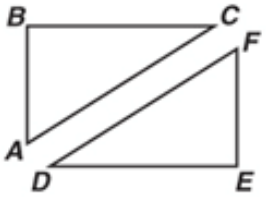
Decide whether the figures in each pair are congruent, similar, or neither.

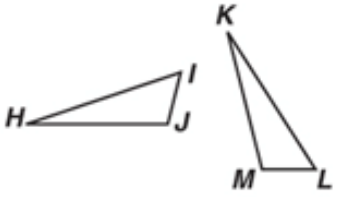
1. 

2. 

3. 

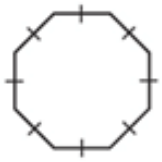
Name the corresponding sides and angles of each pair of congruent triangles.

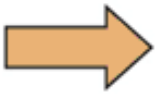
4. 

5. 

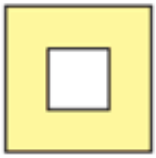
Set C (Lesson 3, pp. 388–389)

Trace each figure. Draw all the lines of symmetry. Does each figure have rotational symmetry? Write *yes* or *no*.

1. 

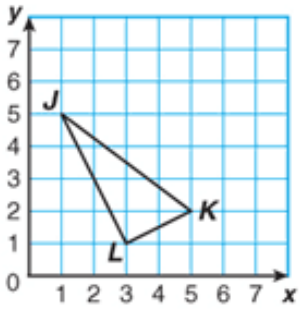
2. 

3. 

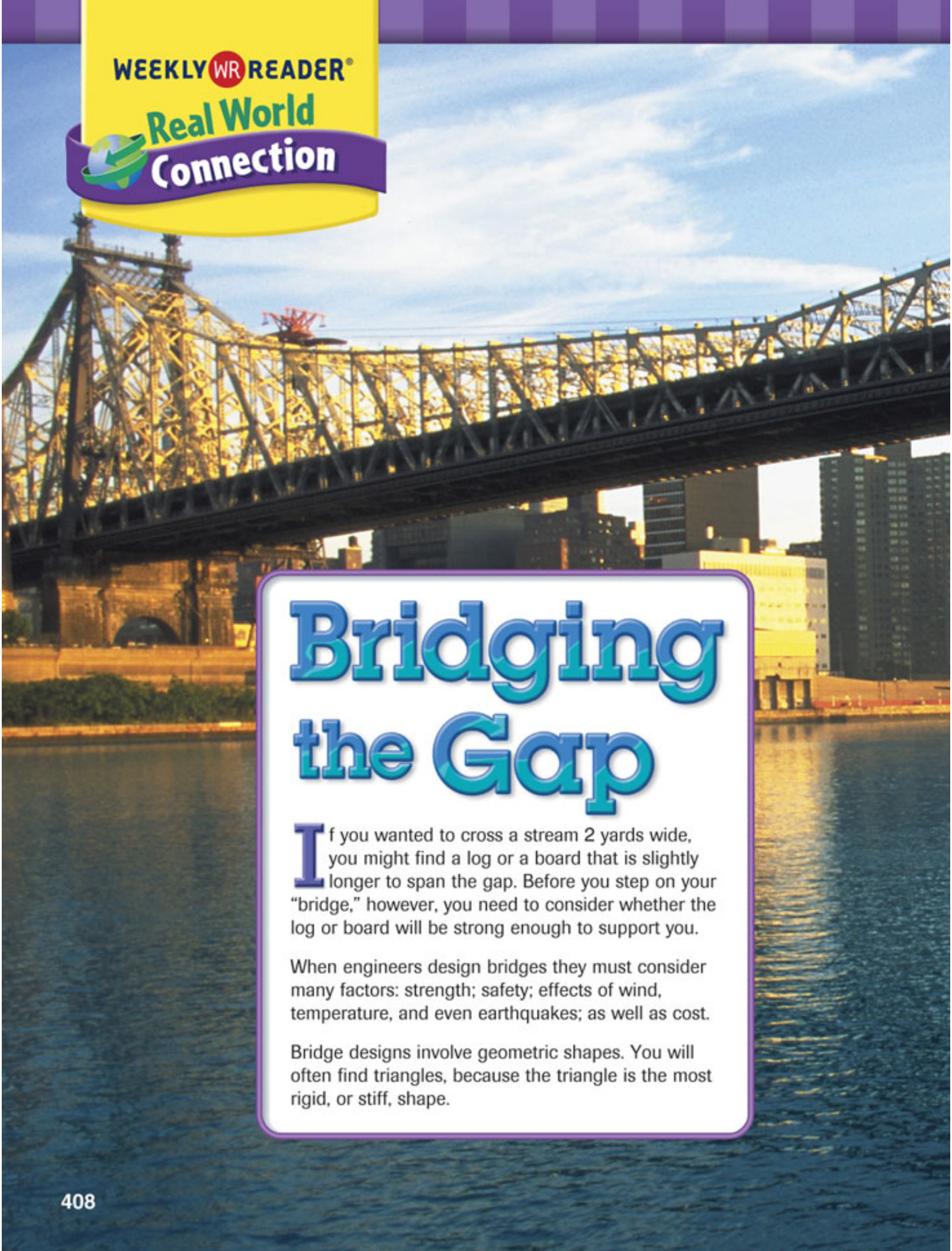
4. 

Set D (Lesson 8, pp. 402–404)

Copy $\triangle JKL$ on graph paper. Identify the coordinates of $\triangle JKL$ after each transformation.



1. Translate 3 to the left, then down 6. Name this triangle *RST*.
2. Translate 1 to the right, then up 2. Name this triangle *MNO*.
3. Reflect across the *y*-axis. Name this triangle *ABC*.
4. Reflect across the *x*-axis. Name this triangle *XYZ*.



WEEKLY WR READER®
Real World Connection

Bridging the Gap

If you wanted to cross a stream 2 yards wide, you might find a log or a board that is slightly longer to span the gap. Before you step on your "bridge," however, you need to consider whether the log or board will be strong enough to support you.

When engineers design bridges they must consider many factors: strength; safety; effects of wind, temperature, and even earthquakes; as well as cost.

Bridge designs involve geometric shapes. You will often find triangles, because the triangle is the most rigid, or stiff, shape.

Problem Solving

Materials

4–6 books, file cards, pennies

Make a file-card bridge and see how many pennies it can hold.

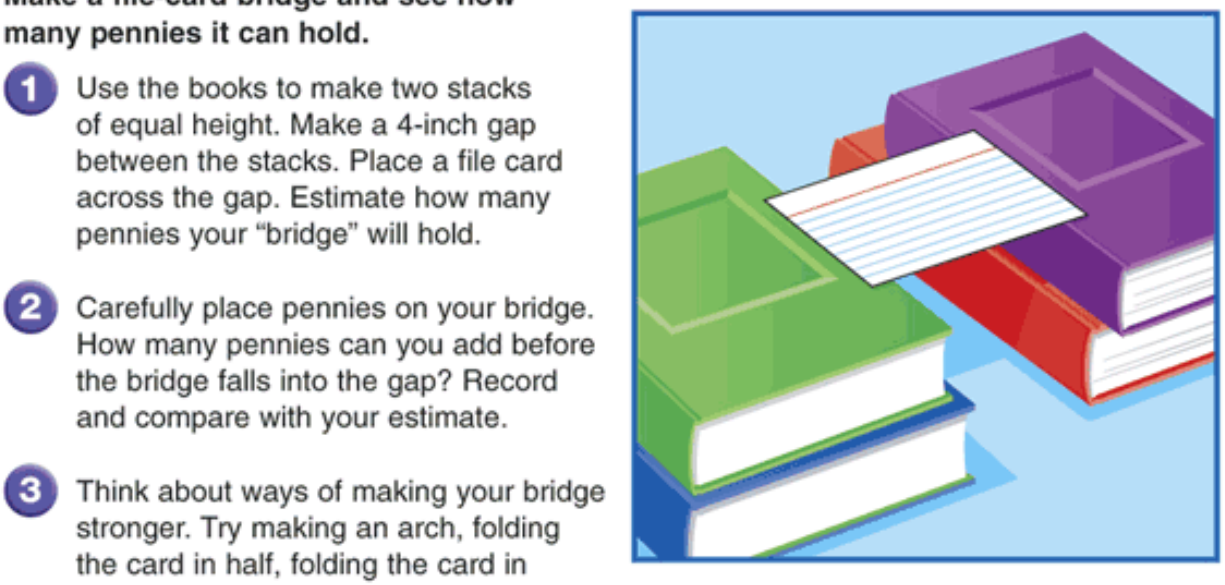
1. Use the books to make two stacks of equal height. Make a 4-inch gap between the stacks. Place a file card across the gap. Estimate how many pennies your "bridge" will hold.

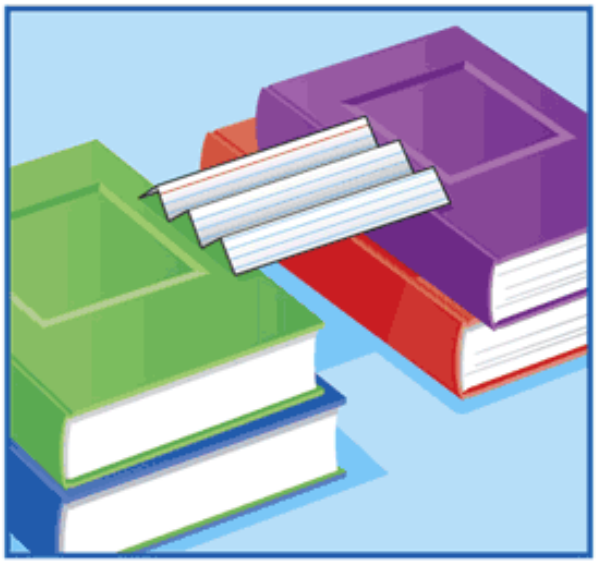
2. Carefully place pennies on your bridge. How many pennies can you add before the bridge falls into the gap? Record and compare with your estimate.

3. Think about ways of making your bridge stronger. Try making an arch, folding the card in half, folding the card in pleats, or any other design. List your ideas.

4. Make the bridges to test your ideas from Step 3. Record how many pennies each bridge will hold before falling into the gap.

5. **Analyze** Which of your file-card bridges could hold the most pennies? Explain why you think some designs make stronger bridges.



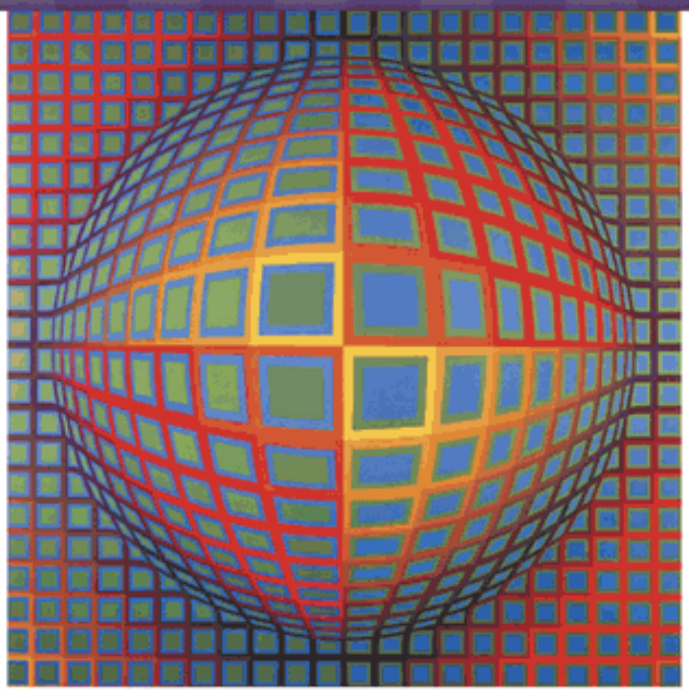


Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic.

Enrichment

Illusion as Art



Vega-Nor *by Victor Vasarely (1969)*

Does the image at right look flat to you? By arranging shapes and color in particular ways, painter Victor Vasarely has made it seem as if the round shape is actually bulging toward the viewer. It isn't. It's just an illusion.

Vasarely was the leader of an artistic movement called Op-Art, which makes use of geometric and optical techniques to create visual illusions.

Try This!

Use the painting for Problems 1–3.

1. The round shape in the painting resembles a half-sphere, and the "grid" pattern on its surface resembles the lines of longitude and latitude on a globe. If it were a complete sphere, about how many lines of longitude would it have? of latitude?

2. Locate the point "closest" to you at which two lines intersect. Use a protractor to find the measures of the angles formed. Measure the angles at other intersections and compare. Are they the same? Why or why not?

3. When we view circles from an angle, the shape isn't perfectly round. What is seen is a distorted circle called an ellipse. Explain how Vasarely's Vega-Nor incorporates an ellipse.

4. Use grid paper, a straightedge, a safe drawing compass, and colored pencils or markers to draw a half-sphere like Vasarely's. How can you draw "grid" lines and use color to make your half-sphere look three-dimensional?

Fractal Fun



A pattern with similar shapes that repeats on a smaller and smaller scale is called a fractal. Each new complete repetition of a fractal is known as an iteration. Natural fractal patterns can be found in snowflakes, ferns, and coastlines.

Use 2-D World in Shape-Up! to make a fractal called Sierpinski's Triangle.

- Click , then click the center of the screen.
- Click . Click the triangle 8 times.

To make the first iteration:

- Click . Place it next to the first one.
- Click . Click the triangle 4 times.
- Click . Click the triangle 2 times.
- Click . Click and drag the triangle into the first one.

Use the , , and buttons to complete the next two iterations of the pattern.

To make the second iteration:

- Click . Place it away from the others.
- Click . Click the triangle 2 times.
- Click . Click the triangle 2 times.
- Click . Click and drag each triangle into the center of one of the corner triangles.



Solve.

1. How many times could this pattern be repeated? Would it ever go outside the original triangle?
2. Look for examples of natural and human-made fractals on the Internet. Print three examples of each.
3. **Challenge** Use Shape-Up! to create your own fractal pattern.

Unit 5 Test

VOCABULARY Open Response

1. When a(n) ____ intersects two parallel lines, pairs of congruent corresponding and alternate angles are formed.
2. Angles that share a common side but no common interior points are called ____.
3. Congruent angles formed by two lines that intersect are called ____.
4. The part of a plane between two rays that share a common endpoint is called the ____.

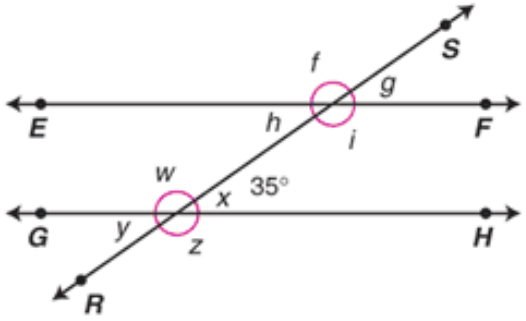
Vocabulary

- transversal
- vertical angles
- supplementary angles
- complementary angles
- adjacent angles
- interior of an angle

CONCEPTS AND SKILLS Open Response

Use the diagram at right to give two examples of each. (Chapter 14)

5. a pair of vertical angles
6. a pair of alternate interior angles
7. a pair of alternate exterior angles

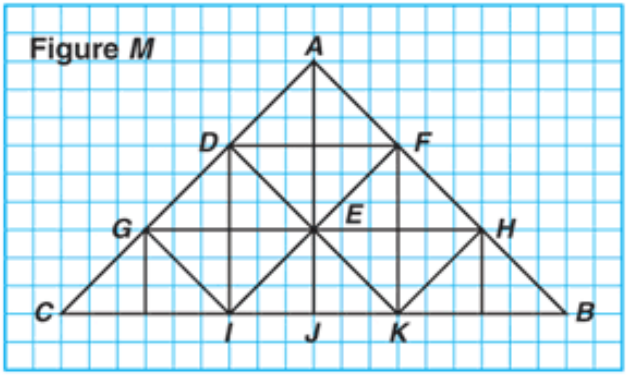


Use the diagram above to classify and find the degree measure of each angle. (Chapter 14)

8. $\angle h$
9. $\angle f$
10. $\angle y$

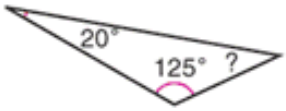
Classify each polygon in Figure *M* by its sides and by its angles in as many ways as you can. (Chapter 14)

11. *GDE*
12. *ADEF*
13. *CGHB*
14. *DEJIG*
15. *DFHE*

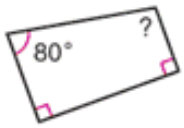


Find each missing angle measure. (Chapter 14)

16.



17.



Use Figure *M* on page 412. Tell whether the polygons are similar or congruent. (Chapter 15)

18. *DEF* and *IEK*

19. *EFHK* and *DFKI*

Draw $\triangle PQR$ with vertices *P* (+5, +7), *Q* (+6, +5), and *R* (+3, +4). Then complete each transformation using the same coordinate grid. (Chapter 15)

20. Translate $\triangle PQR$ left 2 units and down 3 units to form $\triangle STU$.

21. Reflect the translated image across the *x*-axis to form $\triangle VWX$.

PROBLEM SOLVING Open Response

22. How many triangles are there in Figure *M* on page 412? How many triangles are congruent to $\triangle AJC$? to $\triangle ABC$? to $\triangle AEH$?

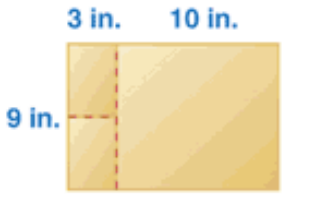
23. **Represent** Vu drew a design using isosceles and right triangles. Draw a Venn diagram to show that some isosceles triangles are right triangles.

24. Karen drew a trapezoid with two right angles, one obtuse angle, and one acute angle. The acute angle has a measure of 35°. Write an equation to find the measure of the obtuse angle.

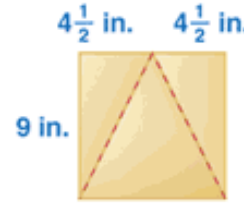
25. Sam ordered one large pizza cut into 12 equal slices, and one small pizza cut into 8 equal slices. Which pizza had central angles with a greater degree measure? Explain.

Performance Assessment

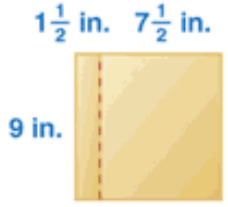
Extended Response













Task You want to make and sell specialty cakes. Use the diagrams above.

Decide which cake to make. Identify and classify the shapes in your cake.

Make a life-size drawing of your cake, using the measurements given in the diagram. Use what you know about constructing perpendicular lines to explain.

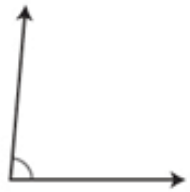
Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

When you take a test, sometimes you can use estimation to help you decide whether an answer choice is reasonable.

Look at the example below.



What is the measure of the angle?

A 40° **C** 170°

B 85° **D** 35°

THINK

The angle is not close to a straight line, so answer choice C, which is 10° less than a straight angle, cannot be correct.

The angle appears close to a right angle, which is 90°, so neither A nor D are correct.

The correct choice is B.

Multiple Choice

1. Which symbol makes a true statement?

978.2 ● 9,978.02

A < **B** = **C** > **D** +

(Chapter 1, Lesson 3)

2. Only 62 of 93 players showed up for practice. What fraction represents the number of players that showed up for practice?

F $\frac{2}{3}$ **G** $\frac{3}{5}$ **H** $\frac{3}{4}$ **J** $\frac{5}{6}$

(Chapter 4, Lesson 1)

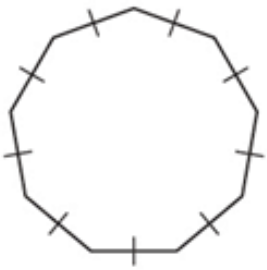
3. The temperature on Monday was 0°F. What was the temperature in °C? Use the formula, $F = \frac{9}{5}C + 32$.

A $^{-}11$°C **C** $^{-}29$°C

B $^{-}18$°C **D** $^{-}42$°C

(Chapter 13, Lesson 5)

4. Which is the angle measure of one angle in a regular nonagon?



F 60° **G** 90° **H** 120° **J** 140°

(Chapter 15, Lesson 1)

For Test-Taking Tips, See page 648.

Open Response

5. James spent $13.98 on three baseball posters. If each poster was the same price, how much did he spend on each poster?

(Chapter 7, Lesson 4)

6.

1 mi ≈ 1.6 km
1 km ≈ 0.62 mi

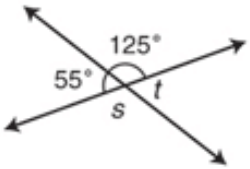
David walks one mile each day to make the round trip to and from school. How many kilometers does he walk in 5 days?

(Chapter 8, Lesson 5)

7. The temperature on Wednesday was $^{-}15°F$. On Thursday, it was $^{-}4°F$. How many degrees cooler was it on Wednesday?

(Chapter 11, Lesson 5)

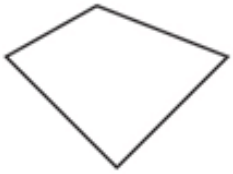
8.

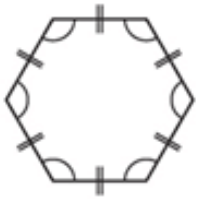


What is the measure in degrees of angle *s*?

(Chapter 14, Lesson 2)

Extended Response





9. Use the polygons above to answer the questions.

- **A** Identify the polygons. Are they regular or irregular? How do you know?
- **B** Copy the polygons. Draw diagonals from a single vertex to divide each polygon into triangles. How many triangles are there in each polygon?
- **C** Find the sum of the measures of the interior angles in each polygon. Explain how knowing the angle sum in a triangle helps you.

(Chapter 15, Lesson 1)



10. Use the angle above, a compass, and a straightedge to construct the following figures. Explain your methods.

- **A** Construct an angle congruent to $\angle D$.
- **B** Bisect $\angle D$.
- **C** Draw a line segment *EF*. Construct its perpendicular bisector.

(Chapter 15, Lessons 6–7)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 5

Look back at the big ideas and vocabulary in this unit.

Big Ideas

If you know the measures of two of the angles in a triangle, you can find the measure of the third angle.

Polygons are named according to their number of sides.

When two figures are congruent, the corresponding parts (sides and angles) are congruent.

Vocabulary

- angle
- polygon
- congruent
- corresponding parts

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain the difference between supplementary and complementary angles.
2. Explain how you would draw a circle with a diameter of 4 inches.
3. Explain the differences between similar and congruent figures.
4. Explain how you would draw congruent figures on a coordinate plane.
5. **Write About It** Geometric shapes can be found in many common household items, such as dinnerware and floor tiles. Find shapes in your home or classroom. Describe each shape and tell why you think each particular design is useful or pleasing.



UNIT 6

Ratio, Proportion, Percent, and Probability

CHAPTER 16

Ratio and Proportion

page 420



CHAPTER 17

Understand Percent

page 442



CHAPTER 18

Use Percent

page 460



CHAPTER 19

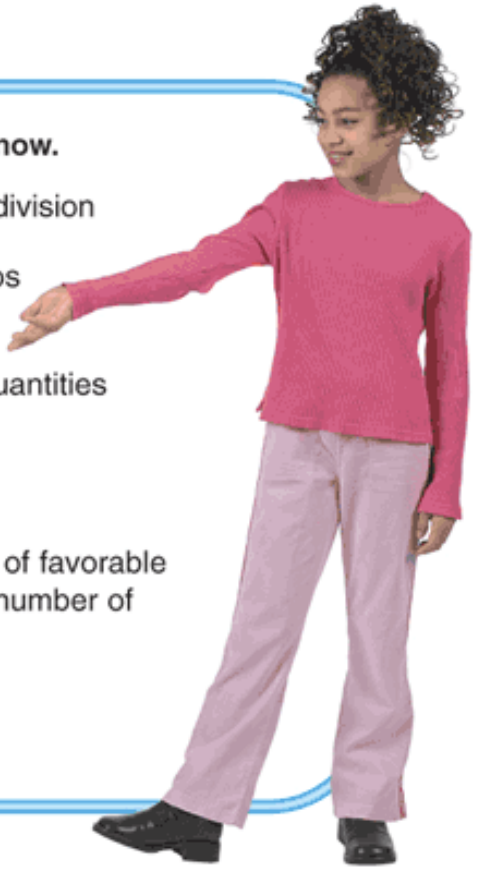
Probability

page 484



Reading Mathematics

Reviewing Vocabulary



Here are some math vocabulary words that you should know.

ratio	a comparison of two numbers by division
proportion	an equation showing that two ratios are equal
rate	a comparison by division of two quantities measured with different units
percent	the ratio of a number to 100
theoretical probability	the calculated ratio of the number of favorable outcomes in an event to the total number of outcomes
experimental probability	the actual results of a probability experiment

Reading Words and Symbols

Percent is a ratio. Look at the model at the right. These symbols and words all describe the diagram.

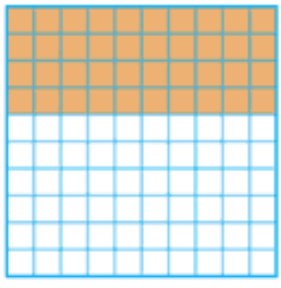
40% is shaded

$\frac{40}{100}$ is shaded

0.4 is shaded

40 out of 100 squares are shaded

The ratio of shaded squares to the total number of squares is 40 : 100.



Write each as a percent.

1. $\frac{2}{100}$ **2.** 0.75 **3.** 85 out of 100

4. 0.19 **5.** $\frac{100}{100}$ **6.** 27 out of 100

Reading Test Questions

Choose the correct answer for each.

7. If you spin the spinner at right, which numbers are equally likely to come up?



a. 1 and 2 **c.** 1 and 3

b. 2 and 3 **d.** 3 and 4

Equally likely means "having the same chance of happening."

8. Which is a good approximation for 10% of $87.95?

a. $90 **c.** $1

b. $9 **d.** $0.90

Approximation means "estimate."

9. What number should you substitute for n to make the equation true?

$$5\% = \frac{n}{100}$$

a. 500 **c.** 5

b. 50 **d.** 0.5

To **substitute** means "to replace," or "use in place of."

Learning Vocabulary

Watch for these words in this unit.
Write their definitions in your journal.

- base
- part of base
- principal
- simple interest
- discount
- independent events
- dependent events
- combination
- permutation

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "J is for Jupiter" on pages 641–642. Then work with a partner to answer the questions about the story.

CHAPTER 16

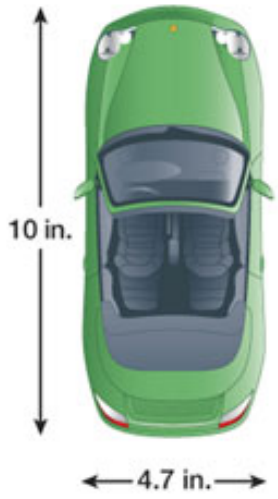
Ratio and Proportion

INVESTIGATION

Using Data

Model cars often come in 1:18 scale. This means that all measures on the model car are 18 times smaller than on the actual car.

How big is the actual car this model was based on? Can you find your answers in more than one way?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A ratio that compares different units is a ____.
2. In a ratio, the two numbers being compared are called ____.
3. Corresponding parts of a scale drawing and a real object have the same ratio. This ratio is called the ____.

Vocabulary

- rate
- scale
- terms
- proportion
- scale drawing

CONCEPTS AND SKILLS

Match each scale drawing with the correct description.

4. a rectangle 3 feet long × 2 feet high ■
5. a rectangle 1 foot long × 3 feet high ■
6. a rectangle 4 feet long × 1 foot high ■

a.



b.



c.



Write *true* or *false* for each statement.

7. A fraction is an example of a ratio.
8. You can find the rate of speed if you know the distance and time spent traveled.
9. You can find the distance somebody travels if you know the rate of speed.

Complete each set of equivalent ratios.

10. $\frac{1}{3} = \frac{\blacksquare}{6} = \frac{3}{9} = \frac{\blacksquare}{12}$
11. $\frac{1}{4} = \frac{\blacksquare}{8} = \frac{3}{12} = \frac{\blacksquare}{16}$
12. $\frac{1}{5} = \frac{2}{\blacksquare} = \frac{3}{15} = \frac{4}{\blacksquare}$
13. $\frac{1}{6} = \frac{2}{\blacksquare} = \frac{3}{18} = \frac{4}{\blacksquare}$
14. $\frac{1}{8} = \frac{2}{\blacksquare} = \frac{3}{24} = \frac{4}{\blacksquare}$
15. $\frac{1}{10} = \frac{2}{\blacksquare} = \frac{3}{30} = \frac{4}{\blacksquare}$
16. $\frac{2}{3} = \frac{\blacksquare}{6} = \frac{6}{9} = \frac{\blacksquare}{12}$
17. $\frac{3}{4} = \frac{\blacksquare}{8} = \frac{9}{12} = \frac{\blacksquare}{16}$
18. $\frac{3}{5} = \frac{6}{\blacksquare} = \frac{12}{20} = \frac{18}{\blacksquare}$
19. $\frac{4}{5} = \frac{\blacksquare}{10} = \frac{16}{20} = \frac{\blacksquare}{30}$

Write About It

20. Does this drawing show that the ratio 1:4 is the same as 2:8?



Facts Practice, See Page 660.

Lesson 1

Audio Tutor 2/10 Listen and Understand

Ratios and Equivalent Ratios

Objective Use ratios to compare numbers and equivalent ratios to find a missing term.

Vocabulary
- ratio
- terms of a ratio
- equivalent ratio



Learn About It

A **ratio** is a comparison of two numbers or quantities by division. The two numbers being compared are called the **terms of a ratio**.

A model train hobbyist set up her locomotives so that for every 10 cars that carry passengers, there are 12 cars that carry freight. You can write the ratio of passenger cars to freight cars in these three ways. The order of the terms is important.

10 passenger cars to 12 freight cars

$$10 \text{ to } 12 \rightarrow 10:12 \rightarrow \frac{10}{12}$$

You read each of these ratios as *ten to twelve*.

When you multiply or divide both terms of a ratio by a number other than 0, the resulting terms form an **equivalent ratio**. You find equivalent ratios the same way you find equivalent fractions.

$$\frac{10}{12} \overset{\times 4}{=} \frac{40}{48} \qquad \frac{10}{12} \overset{\div 2}{=} \frac{5}{6}$$

Other Examples

A. Writing and Simplifying

Simplify the ratio 2 feet to 4 yards.

Write the ratio using the same unit of measure.

2 feet to 4 yards → 2 feet to 12 feet

Simplify by dividing common factors of both terms.

$$\frac{2}{12} = \frac{\overset{1}{\cancel{2}} \times 1}{\underset{2}{\cancel{4}} \times 3} = \frac{1}{6}$$

B. Find a Missing Term

$$\frac{4}{15} = \frac{24}{\blacksquare}$$

Find the number that when multiplied by 4 will give the product 24. Use this factor to find the missing term.

$$\frac{4}{15} \overset{\times 6}{=} \frac{24}{90}$$

Guided Practice

Ask Yourself
- Do the terms have the same unit of measure?
- Have I multiplied or divided both terms by the same number?

Simplify each ratio if possible. Then express it in two other ways.

1. 5 kg to 30 kg **2.** 6:25 **3.** 9 in. to 1 ft **4.** $\frac{30}{2}$

Use equivalent ratios to find each missing term.

5. $\frac{3}{4} = \frac{n}{12}$ **6.** $\frac{72}{144} = \frac{n}{16}$ **7.** $\frac{21}{n} = \frac{147}{196}$ **8.** $\frac{5}{16} = \frac{25}{n}$

Explain Your Thinking Why are ratios like $\frac{3}{4}$ and $\frac{9}{12}$ said to be equivalent rather than equal?

Practice and Problem Solving

Simplify each ratio if possible. Then express it in two other ways.

9. 5 miles to 2 miles **10.** $\frac{20}{45}$ **11.** 75 : 25 **12.** 50:13

13. 3 hours to 2 days **14.** 2 to 1 **15.** $\frac{17}{2}$ **16.** 5 to 8

Use equivalent ratios to find each missing term.

17. $\frac{5}{8} = \frac{10}{k}$ **18.** $\frac{4}{6} = \frac{y}{30}$ **19.** $\frac{12}{3} = \frac{m}{1}$ **20.** $\frac{15}{9} = \frac{5}{p}$ **21.** $\frac{7}{14} = \frac{t}{2}$

22. $\frac{2}{5} = \frac{p}{25}$ **23.** $\frac{18}{6} = \frac{6}{d}$ **24.** $\frac{12}{3} = \frac{4}{q}$ **25.** $\frac{56}{8} = \frac{y}{2}$ **26.** $\frac{14}{2} = \frac{7}{n}$

Solve.

27. Ellis saves $60 in 3 weeks towards a new train set. If he saves the same amount each week, how much money will he save in 10 weeks?

28. Anne Marie's trains have 4 passenger cars to 1 caboose. How many passenger cars are on the track when there are 4 cabooses?

29. What's Wrong? Alexander's train set has 3 passenger cars and 2 freight cars. He wrote the ratio of passenger to freight cars this way: 2:3. Explain what Alexander did wrong.

30. Reasoning An O-gauge track is $\frac{1}{2}$ inch narrower than a #1-gauge track and $\frac{3}{8}$ inch wider than HO track. If a #1-gauge track is $1\frac{3}{4}$ inches wide, how wide are O and HO tracks?

Mixed Review and Test Prep

Open Response

Identify the equivalent fractions in each set. (Ch. 4, Lesson 1)

31. $\frac{6}{8}; \frac{12}{18}; \frac{20}{30}$ **32.** $\frac{1}{3}; \frac{8}{32}; \frac{30}{120}$

Multiple Choice

33. Find the missing term. (Ch. 16, Lesson 1)

$\frac{4}{160} = \frac{12}{\blacksquare}$

A 24 **B** 240 **C** 48 **D** 480

Extra Practice See page 441, Set A.

Lesson 2

Audio Tutor 2/11 Listen and Understand

Rates

Objective Use a rate to compare quantities with different units.

Vocabulary

rate

Learn About It

A **rate** is a comparison by division of two quantities measured with different units. A unit rate is a rate in which the second term is 1. For example, \$3 per hour is a unit rate.

You can buy 4 gallons of fuel for a radio-controlled speedboat for \$60. What is the cost per gallon of the fuel?



Many model speedboats are about $\frac{1}{27}$ actual size.

Find a Unit Rate

STEP 1 Identify the two ratios to compare. Use a ■ to represent the missing term.

$$\frac{\$60}{4 \text{ gal}} = \frac{\$ \blacksquare}{1 \text{ gal}}$$

STEP 2 Divide by 4 to make the second term 1.

$$\frac{\$60}{4 \text{ gal}} \overset{\div 4}{=} \frac{\$15}{1 \text{ gal}}$$

Solution: The cost of the fuel is \$15 per gallon.

Other Examples

A. Second Term Less Than One

Express "30 miles in $\frac{1}{2}$ hour" as a unit rate.

$$\frac{30}{\frac{1}{2} \text{ hour}} \overset{\times 2}{=} \frac{60}{1} = 60 \text{ miles per hour}$$

Multiply by 2 to make the second term 1.

B. Unit Rate

$$\frac{28 \text{ dollars}}{4 \text{ weeks}} = \frac{\blacksquare \text{ dollar}}{1 \text{ week}}$$

$$\frac{28}{4} \overset{\div 4}{=} \frac{7}{1}$$

7 dollars per week

Guided Practice

Write each as a unit rate.

1. 40 miles in 2 hours
2. 50 miles in $\frac{1}{2}$ hour
3. 1 inch in 2 months
4. 4 miles in 2 hours
5. 450 miles on 15 gallons
6. \$1.56 for 12

Ask Yourself

- Have I written the right second term?
- What should I do if the second term is less than 1?

Explain Your Thinking What do you know about the second term in the rate "7 dollars *per* hour"?

Practice and Problem Solving

Write each as a unit rate.

7. 36 for $12

8. $6 for 3 lb of grapes

9. 24 puppies in 3 days

10. 30 mi on 1.5 gal

11. 7 commercials in $\frac{1}{2}$ h

12. 25 yd in 30 s

13. $192 for 24 h

14. 350 mi in 5 h

15. 20 pillows for $5

16. $16 for 32 photos

17. 300 mi on 20 gal

18. $12 for 12 containers

19. 200 mi in 4 h

20. 8 tests in 2 weeks

21. $84 grocery bill for 7 days

Algebra • Equations Find each unknown.

22. $\frac{32 \text{ racers}}{8 \text{ teams}} = \frac{n \text{ racers}}{1 \text{ team}}$

23. $\frac{75 \text{ minutes}}{15 \text{ calls}} = \frac{n \text{ minutes}}{1 \text{ call}}$

24. $\frac{x \text{ books}}{5 \text{ shelves}} = \frac{9 \text{ books}}{1 \text{ shelf}}$

25. $\frac{x \text{ cookies}}{5 \text{ bags}} = \frac{12 \text{ cookies}}{1 \text{ bag}}$

26. $\frac{540 \text{ beads}}{r \text{ bracelets}} = \frac{36 \text{ beads}}{1 \text{ bracelet}}$

27. $\frac{24 \text{ boats}}{b \text{ heats}} = \frac{8 \text{ boats}}{1 \text{ heat}}$

Data The table shows data for three radio-controlled racing boats. Use the data for Problems 28–30.

RC Racing Boats		
Boat	**Length (in mm)**	**Mass (in g)**
Atlantio 600	635	2,000
Wavemaster	705	1,590
EP Viper R	650	1,360

28. **Estimate** Estimate the unit rate of the length in millimeters per gram for the EP Viper R.

29. **Calculator** How many inches longer is the Wavemaster than the Atlantio 600? Remember: 1 in. ≈ 25.4 mm.

30. You can buy 1 quart of fuel for a radio-controlled boat for $6.49 or 4 gallons for $51.95. Use unit prices to explain which is the better buy.

31. To run any of these boats, you need 2 servos, or motors. A hobby store sells servos for $14.99 each or 3 for $42.97. What is the unit price if you buy 5 servos?

Mixed Review and Test Prep

Open Response

Compare. Write >, <, or = for each ●.
(Ch. 4, Lesson 3)

32. $\frac{2}{3}$ ● $\frac{5}{6}$

33. 0.3 ● $\frac{3}{10}$

34. $\frac{3}{16}$ ● 0.125

35. $1\frac{5}{8}$ ● 1.6

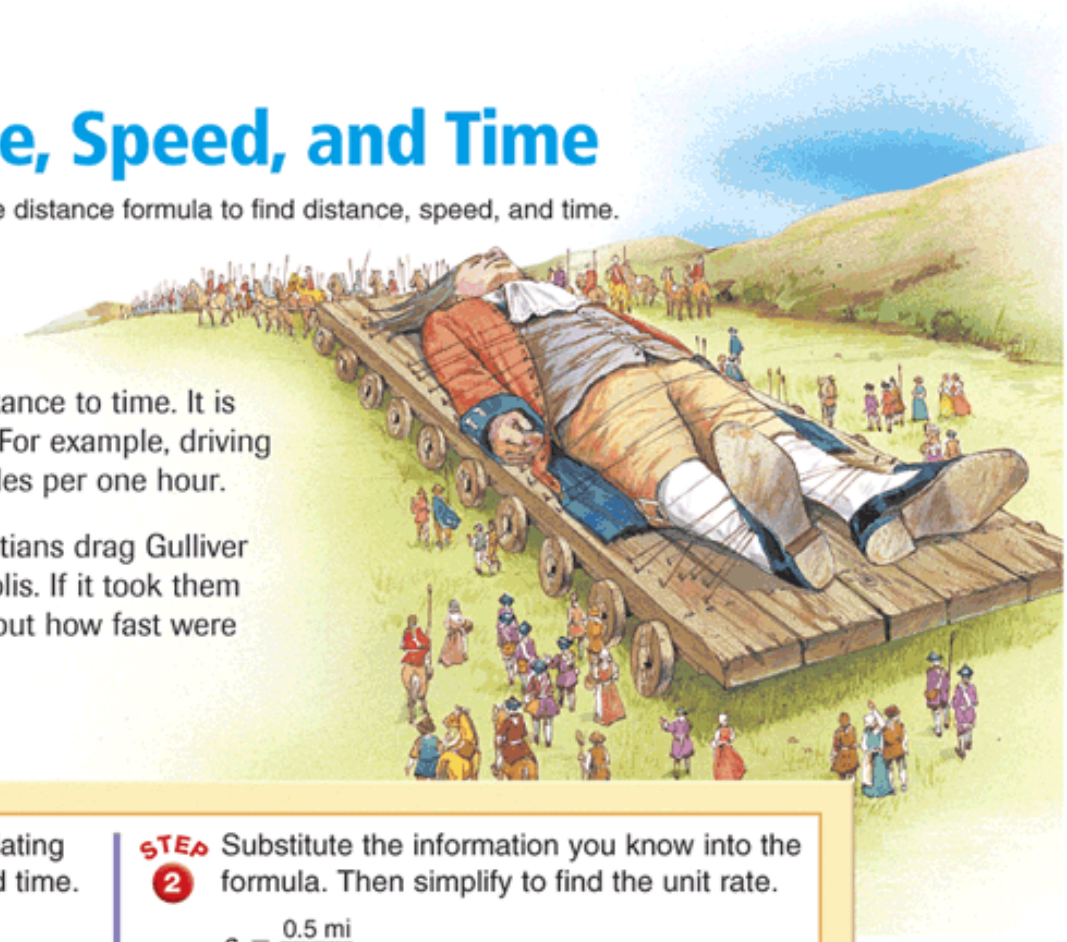
36. After driving 250 miles, Mr. Garcia bought 10 gallons of gas to refill the tank. How many miles did the car travel on 1 gallon of gas?
(Ch. 16, Lesson 2)

Extra Practice See page 441, Set B.

Lesson 3

Distance, Speed, and Time

Objective Use the distance formula to find distance, speed, and time.



Learn About It

Speed is a rate—a ratio of distance to time. It is usually written as a unit rate. For example, driving speed limits are written as miles per one hour.

In *Gulliver's Travels,* the Lilliputians drag Gulliver about 0.5 mile to the Metropolis. If it took them about 14 hours to do this, about how fast were they going in miles per hour?

Find the average speed.

STEP 1 Write the formula relating speed, distance, and time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$s = \frac{d}{t}$$

STEP 2 Substitute the information you know into the formula. Then simplify to find the unit rate.

$$s = \frac{0.5 \text{ mi}}{14 \text{ h}}$$

$$= \frac{0.036 \text{ mi}}{1 \text{ h}}$$ ← Divide each term by 14 to find the unit rate.

s = 0.036 miles per hour, or 0.036 mi/h

A slash, /, is often used in place of the word *per.*

Solution: The Lilliputians dragged Gulliver at a speed of about $\frac{1}{28}$ mile in an hour, or 0.036 mi/h.

Other Examples

A. Find Time

Find the time it takes to travel 150 miles at 30 mi/h.

$$s = \frac{d}{t}$$

$$30 = \frac{150}{t}$$

$$30 \times t = \frac{150}{t} \times t$$

$$30t = 150$$

$$30 \times t \div 30 = 150 \div 30$$

$$t = 5$$

It takes 5 hours.

B. Find Distance

Find how far you can walk in 20 minutes if your average speed is $\frac{1}{2}$ block per minute.

$$s = \frac{d}{t}$$

$$\frac{\frac{1}{2}}{1} = \frac{d}{20}$$

$$\frac{\frac{1}{2}}{1} \times 20 = \frac{d}{20} \times 20$$

$$\frac{1}{2} \times 20 = d$$

$$10 = d$$

You can walk 10 blocks.

Guided Practice

Ask Yourself
- Do I need to change the unit of measure?
- How do I label my answer?

Find the speed (s), distance (d), or time (t).

1. $s =$ ■, $d = 225$ mi, $t = 5$ h
2. $s = 300$ mi/h, $d =$ ■, $t = 6$ h
3. $s = 20$ mi/h, $d = 2$ mi, $t =$ ■

Explain Your Thinking How are the equations $d = s \times t$ and $s = \frac{d}{t}$ related?

Practice and Problem Solving

Find the speed (s), distance (d), or time (t).

4. $s =$ ■, $d = 25$ ft, $t = 1$ h
5. $s =$ ■, $d = 60$ mi, $t = 2.5$ h
6. $s = 250$ mi/h, $d = 500$ mi, $t =$ ■
7. $s =$ ■, $d = 42$ in., $t = 7$ min
8. $s = 45$ mi/h, $d =$ ■, $t = \frac{1}{3}$ h
9. $s = 40$ ft/s, $d = 100$ ft, $t =$ ■
10. $s = \frac{1}{4}$ mi/h, $d =$ ■, $t = \frac{3}{4}$ h
11. $s =$ ■, $d = 100$ yd, $t = 8$ s
12. $s = 2\frac{1}{2}$ ft/s, $d = 1$ mi, $t =$ ■
13. $s = 12$ mi/h, $d =$ ■, $t = 1$ day

Solve.

14. Four hours after the Lilliputians began dragging Gulliver, there was an accident. If they were traveling at a speed of 0.1 miles per hour, how far had they gone before the accident?
15. **Estimate** If the average pace a Lilliputian can walk is at a rate of 0.2 mile per hour, about how long would it take a Lilliputian to walk the 300 yards from the forest to the sea?
16. **Analyze** It took Gulliver 12 hours to sail 4 leagues form Blefuscu to the next island. If 1 league equals 3 miles, what was Gulliver's speed in miles per hour?
17. After Gulliver left the island, he sailed 24 leagues before meeting an English ship. If he sailed at the same rate as in Problem 16, how many hours did the trip take?
18. **Write About It** A Lilliputian mathematician calculated that 1 foot of Gulliver's height was equivalent to 1 inch for a Lilliputian. Write three equivalent to ratios and explain how to compare the heights of Gulliver and the Lilliputians.

Mixed Review and Test Prep

Open Response

Divide. (Ch. 7, Lessons 4–5)

19. $0.4 \div 8$
20. $0.9 \div 0.3$
21. $6 \div 2.4$
22. $5 \div 0.02$
23. $0.24 \div 0.003$
24. $3.6 \div 2.4$

Multiple Choice

25. A truck travels at 50 mi/h. A car travels at 60 mi/h. How many hours less will it take the car than the truck to go 300 miles? (Ch. 16, Lesson 3)

A 1 **B** 5 **C** 6 **D** 10

Extra Practice See page 441, Set C.

Lesson 4

Audio Tutor 2/12 Listen and Understand

Write and Solve Proportions

Vocabulary
proportion

Objective Use proportions to find relationships and solve problems.



An actual Brachiosaurus skeleton is 40 feet tall and 75 feet long.

Learn About It

A **proportion** is a statement that says two ratios are equivalent. You can use proportions to describe relationships and solve problems.

Animators often build clay models that they pose and photograph. An animator built a clay dinosaur that is about 8 inches tall. About how many inches long should he make the model?

To find out, write a proportion by using two ratios.

actual height (feet) → $\frac{40}{75} = \frac{8}{x}$ ← height of model (inches)
actual length (feet) → ← length of model (inches)

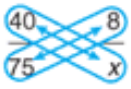
You can use cross multiplication to find the missing term in a proportion.

Solve for *x*. $\frac{40}{75} = \frac{8}{x}$

STEP 1 Write the proportion.

$$\frac{40}{75} = \frac{8}{x}$$

STEP 2 Identify the cross products—the terms to be multiplied.



75×8

$40 \times x$

STEP 3 Use the cross products to write an equation. Solve for *x*.

$$75 \times 8 = 40 \times x$$
$$600 = 40x$$
$$600 \div 40 = 40x \div 40$$
$$15 = x$$

Solution: The clay dinosaur should be about 15 inches long.

Another Example

Find a missing term.

$\frac{3}{4} = \frac{9}{s}$

$\frac{3}{4} \times \frac{9}{s}$

Cross multiply. $3 \times s = 36$

Solve the equation. $3 \times s \div 3 = 36 \div 3$

$s = 12$

Remember that the numbers 3 and $\frac{1}{3}$ are multiplicative inverses because their product is 1. You can use multiplicative inverses to understand why the cross products of a proportion are equal.

$\frac{a}{b} = \frac{c}{d}$	a, b, c, and d, form a proportion, where $b \neq 0$ and $d \neq 0$.
$\frac{a}{b} \times \frac{b}{a} = \frac{c}{d} \times \frac{b}{a}$	If you multiply both sides of the equation by the multiplicative inverse of $\frac{a}{b}$, which is $\frac{b}{a}$, the products are equal.
$1 = \frac{c \times b}{d \times a}$	Because $\frac{c \times b}{d \times a}$ is equal to 1, the terms of the ratio must be equal.
$c \times b = d \times a$ $a \times d = b \times c$	The cross products are equal.

Do $\frac{14}{35}$ and $\frac{2}{5}$ form a proportion?

STEP 1 Write the two ratios.

$\frac{14}{35}$ $\frac{2}{5}$

STEP 2 Find the cross products.

$\frac{14}{35}$ $\frac{2}{5}$

$14 \times 5 = 35 \times 2$

STEP 3 If the cross products are equal, the ratios are equivalent.

$70 = 70$

The cross products are equal.

Solution: Since the ratios are equivalent, they form a proportion.

Guided Practice

Use cross products to find each missing term.

1. $\frac{t}{12} = \frac{3}{4}$ **2.** $\frac{18}{f} = \frac{3}{1}$ **3.** $\frac{2}{5} = \frac{p}{25}$ **4.** $\frac{8}{16} = \frac{5}{h}$

Find the cross products for each pair of ratios. Do the two ratios form a proportion? Write *yes* or *no*.

5. $\frac{5}{15}$ $\frac{10}{30}$ **6.** $\frac{2}{3}$ $\frac{4}{9}$ **7.** $\frac{12}{3}$ $\frac{6}{2}$ **8.** $\frac{3}{8}$ $\frac{12}{32}$

Ask Yourself

- Did I write the cross products correctly?
- Are the cross products equal?

Explain Your Thinking Why can you multiply or divide both sides of an equation by the same number and still have the equation be true?

Go On

Practice and Problem Solving

Use cross products to find each missing term.

9. $\frac{5}{2} = \frac{10}{a}$ **10.** $\frac{3}{h} = \frac{9}{3}$ **11.** $\frac{12}{c} = \frac{3}{1}$ **12.** $\frac{7}{m} = \frac{2}{4}$ **13.** $\frac{e}{15} = \frac{6}{5}$

14. $\frac{f}{15} = \frac{8}{5}$ **15.** $\frac{5}{5} = \frac{7}{p}$ **16.** $\frac{12}{5} = \frac{24}{d}$ **17.** $\frac{27}{18} = \frac{6}{p}$ **18.** $\frac{30}{25} = \frac{u}{10}$

Find the cross products for each pair of ratios. Do the two ratios form a proportion? Write *yes* or *no*.

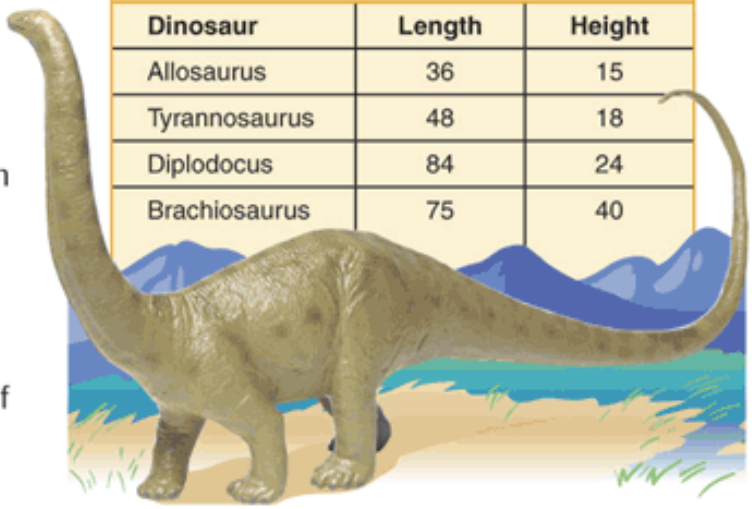
19. $\frac{4}{5}$ $\frac{8}{15}$ **20.** $\frac{25}{15}$ $\frac{5}{3}$ **21.** $\frac{32}{24}$ $\frac{8}{6}$ **22.** $\frac{19}{11}$ $\frac{57}{33}$ **23.** $\frac{24}{36}$ $\frac{2}{3}$

Data

The table shows some actual measurements for dinosaurs. Use the table for Problems 24–27.

Dinosaur Measurements (in feet)

Dinosaur	Length	Height
Allosaurus	36	15
Tyrannosaurus	48	18
Diplodocus	84	24
Brachiosaurus	75	40



24. To advertise a movie, a *Tyrannosaurus* balloon proportional to the actual dinosaur's measurements is made. If the balloon is 72 feet long, what is its height?

25. You Decide An animator wants to design a virtual *Diplodocus* that will fit on her 15-inch-square monitor. Choose a length for the model. Then use cross products to find its proportional height.

26. Analyze A toy company makes models of the dinosaurs in a movie. The *Diplodocus* model is 3 feet long and 1 foot tall. Is it proportional to the actual dinosaur's measurements? Explain how you know.

27. Predict Suppose a *Brachiosaurus* could have a length of 130 feet. Predict the height of a *Brachiosaurus* that was 130 feet long. Explain your prediction.

Mixed Review and Test Prep

Open Response

Use cross products to find the missing term in each proportion. (Ch. 16, Lesson 4)

28. $\frac{78}{c} = \frac{26}{3}$ **29.** $\frac{3}{7} = \frac{21}{d}$

30. $\frac{30}{25} = \frac{u}{30}$ **31.** $\frac{16}{b} = \frac{128}{40}$

Multiple Choice

32. Ali bought a new book and read $\frac{1}{5}$ of the book that day. That evening, he read $\frac{1}{6}$ of the book. During the next day, he read $\frac{1}{3}$ of it. How much of the book does Ali have left to read? (Ch. 16, Lesson 3)

A $\frac{1}{3}$ **B** $\frac{21}{30}$ **C** $\frac{3}{10}$ **D** $\frac{2}{5}$

Extra Practice See page 441, Set D.

Game

Activity

Bargain Hunting

2 Players

What You'll Need • Learning Tool 44 or shopping list



How to Play

1. Each player estimates the least expensive way to buy the items on the grocery list.
2. One player acts as the cashier who determines the cost of the other's groceries.
3. Players then trade roles.
4. The player who spends the least amount of money during his/her shopping trip wins the game.



Quick Check

Check your understanding of Lessons 1–4.

Write each ratio in two other ways. Then simplify the ratio. (Lesson 1)

1. 24 eggs to 2 boxes

2. 15:18

Write each rate as a unit rate. (Lesson 2)

3. \$10 for 4 tickets

4. 224 kilometers in 4 hours

Find the missing distance (d), speed (s), or time (t). (Lesson 3)

5. $s =$ ■, $d = 120$ mi, $t = 3$ h

6. $s = 15$ ft/s, $d =$ ■, $t = 1$ minute

7. $s = 40$ mi/h, $d = 220$ mi, $t =$ ■

Use cross products to find each missing term. (Lesson 4)

8. $\frac{7}{8} = \frac{35}{n}$

9. $\frac{16}{5} = \frac{n}{1.25}$

10. $\frac{n}{18} = \frac{3}{2}$

Lesson 5

Problem-Solving Strategy

Write an Equation

Objective Solve a problem by writing an equation.

Sometimes writing an equation is the easiest way to solve a problem. Before you write an equation, you need to understand the problem.

Problem Mariela loves car races, so she buys a model of the winning car from a race she watched. On the box she sees that the length of the model is about 8.5 inches. She knows that the actual car is 208 inches long and 72.5 inches wide. About how wide should the model be?



UNDERSTAND

This is what you know:

- The model is 8.5 inches long.
- The actual car is 208 inches long.
- The actual car is 72.5 inches wide.

PLAN

You can write an equation to find the width of the model car.

SOLVE

Write an equation to represent the problem.

$$\frac{\text{length of model}}{\text{length of real car}} \rightarrow \frac{8.5}{208} = \frac{n}{72.5} \leftarrow \frac{\text{width of model}}{\text{width of real car}}$$

$8.5 \times 72.5 = 208 \times n$ Use a calculator.

$616.25 = 208 \times n$ Use the Division Property of Equality

$n \approx 3$

Solution: The model should be about 3 inches wide.

LOOK BACK

Look back at the problem.
How can I check my answer?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. One model of an actual race car is 12 cm long. A smaller model is 4 cm long. The hood on the larger model is 3.5 cm long. To the nearest tenth, how long will the hood on the smaller model be?

   Hint What are you comparing?

2. Kaya has 255 car models on display in her shop. She puts the same number of cars in each of 3 display cases. Only 2 of her cases are lighted. How many cars are in those cases?

Ask Yourself

UNDERSTAND — What facts do I know?

PLAN — What equation can I write to help me solve the problem?

SOLVE
- Does the order of the terms on each side of the equation match?
- Did I cross multiply correctly?

LOOK BACK — Did I check that my proportion works?

Independent Practice

Write an equation to solve each problem.

3. Serena and Chad each collect model racing cars. For every 3 model cars that Chad has, Serena has 5. If Chad has 12 model cars, how many does Serena have?

4. One popular car model size is $\frac{1}{24}$ actual size. A larger model is $\frac{1}{18}$ actual size. To the nearest millimeter, how long will the same 1-centimeter part from a 1:18 model be on the 1:24 model?

5. The Davis family is driving across the country. Their truck's gas tank holds 28 gallons. If they traveled 504 miles on one tank of gas, what was their gas mileage?

6. Look back at Problem 5. If the Davis family had used only $\frac{3}{4}$ tank of gas for the same number of miles, what would their gas mileage have been?



Chad's

Serena's



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Cheryl spends $35.92 on 2 different kinds of paint for her model cars. She buys 4 cans of spray paint and 4 jars of airbrush paint. The spray paint costs $1 more than the airbrush paint. How much does each kind of paint cost?

8. Becky multiplies a length in centimeters by 30, adds 800 to the product, and divides the sum by 50. If the final quotient is 1 meter, what length did Becky start with?

9. Your friend says that if fraction *A* is less than fraction *B*, then the reciprocal of fraction *A* is less than the reciprocal of fraction *B*. Is your friend correct? Justify your answer.

You Choose

Strategy
- Draw a Diagram
- Make a Table
- Work Backward
- Use Logical Reasoning
- Write an Equation

Computation Method
- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the table to solve Problems 10–12.**

The table shows the different ticket prices to Sea Life shows.

10. Suppose you bring $20 to a Sea Life show and buy an adult ticket and a snack. If you have $5.50 left, how much did you spend on the snack?

11. **Reasoning** Three different groups of people each buy one kind of ticket. Each group spends $84. How many tickets does each group buy?

12. **Write About It** At one ticket booth, the ratio of adult tickets sold to senior tickets sold is 6:1. If senior-ticket sales are $1,120, how many adult tickets were sold at the booth? Explain your answer.



Sea Life Ticket Prices

Ticket	Price
Adults	$10
Children under 10	Free with paying adult
Seniors	$8
College students	$6
Reserved seats	$12
VIP seats	$14

Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. A function can be represented by the following set of ordered pairs: $(^{-}2, ^{-}1)$, $(^{-}1, 1)$, $(0, 3)$, $(1, 5)$, $(2, 7)$.

Which equation can represent the function?

A $y = 2x - 1$ **C** $y = 3x + 2$
B $y = 2x + 3$ **D** $y = 3x + 1$

(Chapter 13, Lesson 3)

2. A square and a rectangle each have a perimeter of 12 centimeters. The rectangle is 1 centimeter longer than the square. What is the rectangle's area?

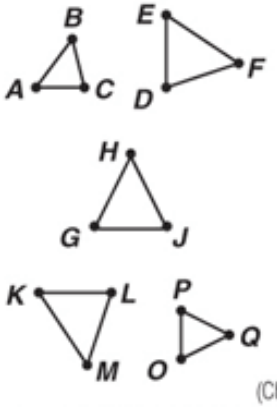
F 4 cm^2 **G** 8 cm^2 **H** 9 cm^2 **J** 12 cm^2

(Chapter 8, Lesson 7)

Open Response

Solve each problem.

3. Identify the congruent triangles. You may use a protractor and ruler.



(Chapter 15, Lesson 3)

4. Tamara subtracted $^{-}8$ from $^{+}2$ and said that the difference is $^{+}10$. Is Tamara correct? Explain.

(Chapter 11, Lesson 5)

Extended Response

5. To study beach erosion, you are using an instrument to measure the height of sand dunes above and below sea level. You made note of the different heights.

Heights of Dunes
$^{-}2$ ft
11 ft
$\frac{3}{4}$ ft
13 ft
$^{-}4$ ft
$\frac{7}{8}$ ft
8 ft
0 ft
$\frac{1}{2}$ ft

a. Of all the data you collected, which are integers? Order the integers from greatest to least. What is the product of the integers?

b. What is the difference between the highest and the lowest elevations?

c. How many measurements did you make altogether? Is it possible to find the mean of all the measurements? Explain. If it is possible, find the mean. If it is not possible, tell why not.

(Chapter 11, Lesson 5)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 6

Audio Tutor 2/13 Listen and Understand

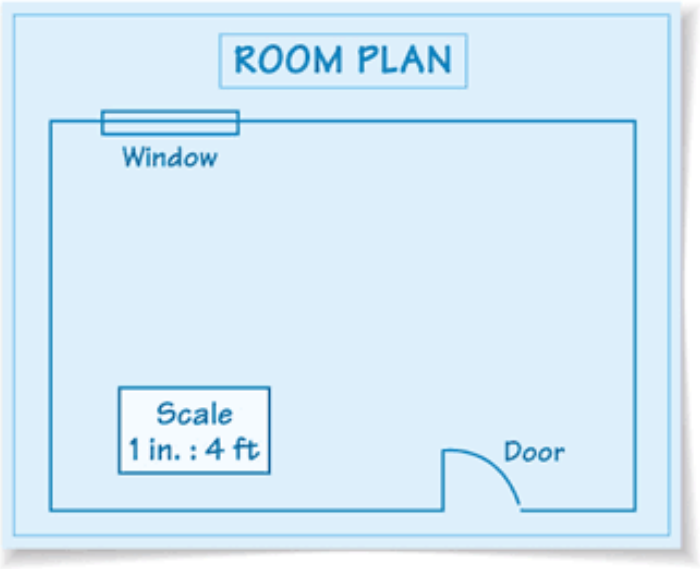
Scale Drawings

Objective Use scale drawings to represent larger or smaller objects.

Vocabulary
- scale
- scale drawing

Learn About It

The floor plan on the right is a model of an actual room. The model and the actual room are similar figures. The lengths of the corresponding sides of similar polygons are proportional.



Corresponding parts of models and similar real objects have the same ratio. This ratio is called a **scale**. You can use the scale on the drawing at the right to find the length and width of the actual room.

Find the actual length and width.

STEP 1 Write a ratio.

Find the length, l, of the actual object.

length in drawing → $\frac{3 \text{ in.}}{l \text{ ft}}$ ← actual length

STEP 2 Solve a proportion to find the length l.

length in drawing → $\frac{3 \text{ in.}}{l \text{ ft}} = \frac{1 \text{ in.}}{4 \text{ ft}}$ ← length in drawing

actual length → actual length

$\frac{3}{l} \times \frac{1}{4}$

$l = 12$ ft

STEP 3 Repeat to find the width w.

$\frac{2 \text{ in.}}{w \text{ ft}} = \frac{1 \text{ in.}}{4 \text{ ft}}$

$\frac{2}{w} \times \frac{1}{4}$

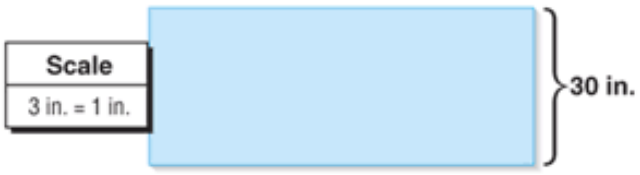
$w = 8$ ft

Solution: The actual room is 8 feet wide and 12 feet long.

When you create a drawing using a scale, it is called a **scale drawing**. You can choose a scale to obtain a scale drawing that is smaller or larger than the actual object.

Another Example

Drawing is Larger Than Object



Find the width, w, of the actual object.

width in drawing → $\frac{30}{w} = \frac{3}{1}$ ← scale 3 in.: 1in.

actual width →

$30 \times 1 = w \times 3$ ← Cross multiply.

$30 = w \times 3$

$10 = w$

The actual width is 10 in.

A map is a special kind of scale drawing. This map shows the Washington, D.C. Mall, a 2-mile-long park that extends from the Capitol Building to the Lincoln Memorial.

No scale is shown, but you can use a ruler to measure distances on the map to estimate actual distances.



Estimate the actual distance, *d*, to the White House.

$$\frac{\text{Map distance to Lincoln Memorial}}{\text{Actual distance to Lincoln Memorial}} = \frac{\text{Map distance to White House}}{\text{Actual distance to White House}}$$

$$\frac{5 \text{ in.}}{2 \text{ mi}} = \frac{3\frac{1}{2} \text{ in.}}{d}$$

$$5 \times d = 2 \times 3\frac{1}{2} \quad \leftarrow \text{Cross multiply.}$$

$$5 \times d = 7$$

$$d = 1\frac{2}{5}$$

Solution: The actual distance from the Capitol to the White House is $1\frac{2}{5}$ miles or 1.4 miles when written in decimal form.

Guided Practice

Use the scales to find the actual measures of each object.

1. 1 in. : 5 ft for a house drawing that is $6\frac{1}{2}$ in. × 5 in.

2. $\frac{1}{2}$ in. : 1 ft for a dollhouse bed that is 3 in. × $2\frac{1}{2}$ in.

Use the scale 1 in. : 3 ft to find each missing measure.

3. 3 in. to ■ ft

4. ■ in. to 150 ft

Ask Yourself

- Did I use the correct units (inches or feet) in my answers?

Explain Your Thinking If the scale 1 in. : 1 in. is used to make a scale drawing, what do you know about the object(s) in the drawing?

Go On

Practice and Problem Solving

Use the scales in the boxes below to find the actual measure(s) of each object.

1 in.:2 ft
$\frac{1}{2}$ in.:1 ft

5. room is $7\frac{1}{2}$ in. × 8 in.

6. chair is 1 in. wide

7. rug is 5 in. × 6 in.

8. model dresser is 3 in. × 1.5 in.

9. diameter of model table is 4 in.

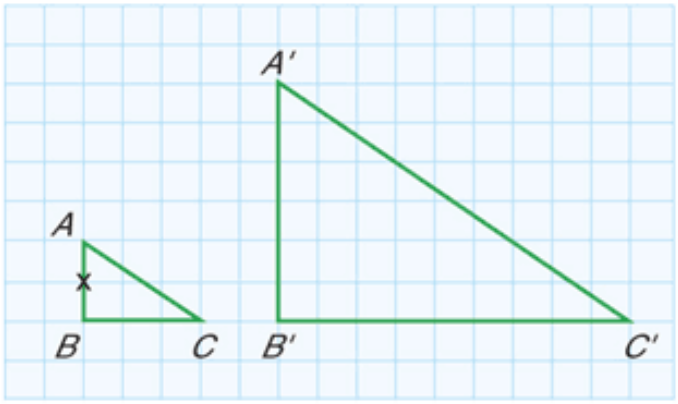
Data **The figures show two similar triangles.**

10. Compare the two similar triangles to the right. What is the ratio between $\overline{AB}$ and $\overline{A'B'}$?

11. If $\overline{AB}$ is 8 in. long, how many inches is it from point B to the location marked by "x"?

12. If $\overline{BC}$ is about 12 in. long, estimate the length of $\overline{B'C'}$.

13. **Create Your Own** Use the figure to the right to create your own enlargement. Exchange enlargements with a partner and determine the scale used.



Solve.

14. The wingspan of an airplane measures 90 feet. A model of the airplane has a wingspan of $4\frac{1}{2}$ inches. What scale was used to build the model?

15. One drawing of a park uses the scale 1 in.:12 ft. Another uses 1 in.:1 ft. If both are drawn on $8\frac{1}{2}$ in. by 11 in. paper, which drawing shows more of the park?

16. In 1912, President William H. Taft approved regulations for the shape of the United States flag. An official U.S. flag must have a width-to-length ratio of 1:1.9. What is the width of an official U.S. flag that is 9.5 meters long?

17. **Analyze** Anna and Ralph are making scale drawings. Anna uses a scale of 2 in.:6 ft. Ralph uses a scale of 3 in.:6 ft. What is the ratio of the measures on Anna's drawings to the corresponding measures on Ralph's?

18. **Represent** Use a tape measure or yardstick to measure the length and width of your classroom. Make sure you note key features, such as doors and windows. Then choose a scale and make a scale drawing.

Extra Practice See page 441, Set E.

Mixed Review and Test Prep

Open Response

Solve. (Ch. 2, Lesson 7)

19. $3 + (2 + 3)$

20. $42 \div (12 \div 2)$

21. $8 \div 2 + 6 \times 2$

22. $4 \times 5 - (4 + 5)$

Multiple Choice

23. Jerry weighs 126 pounds on Earth. If his weight on the moon is $\frac{1}{6}$ of that, how much does he weigh on the moon? (Ch. 16, Lesson 4)

A 19 lb **B** 21 lb **C** 22 lb **D** 23 lb

Problem Solving

Real World Connection
Direct Proportion

Sometimes when one quantity increases, another quantity increases by the same percentage. These two quantities are then said to be in direct proportion.





Since the cost of both items is in direct proportion, you can determine the cost of buying model cars by examining the relation between the price per model car and the number of model cars purchased.

This may be written as an equation, $y = kx$, where k is the price per model car.

1. If you were to purchase 25 model cars, what would the cost of the cars be?

2. What would happen if the price of model cars increased by 5%? Explain.

3. Suppose you paid $72.00 for the cars. How many cars did you purchase?

WEEKLY WR READER eduplace.com/map

Chapter Review/Test

VOCABULARY

1. A drawing that uses a scale is called a(n) ____.
2. When you multiply or divide both terms of a ratio by a number other than 0, the resulting terms form a(n) ____.
3. A statement that says two ratios are equivalent is a(n) ____.
4. A(n) ____ is a comparison of two numbers or quantities by division.

Vocabulary

equivalent ratio
proportion
rate
ratio
scale
scale drawing

CONCEPTS AND SKILLS

Simplify each ratio if possible. Then express it in two other ways. (Lesson 1, pp. 422–423)

5. $\frac{27}{63}$
6. 5 min to 1 h
7. 7 to 21
8. $\frac{110}{10}$

Use equivalent ratios to find each missing term. (Lesson 1, pp. 422–423)

9. $\frac{5}{6} = \frac{a}{12}$
10. $\frac{48}{8} = \frac{12}{c}$
11. $\frac{15}{25} = \frac{b}{5}$
12. $\frac{28}{36} = \frac{d}{9}$
13. $\frac{15}{55} = \frac{3}{f}$

Write each as a unit rate. (Lesson 2, pp. 424–425)

14. \$2.95 for 5 lb apples
15. 770 mi in $3\frac{1}{2}$ h
16. 80 jets in 5 h

Find the speed *(s)*, distance *(d)*, or time *(t)*. (Lesson 3, pp. 426–427)

17. $s = 20$ mi/h, $d = 90$ mi, $t =$ ■
18. $s =$ ■, $d = 2{,}100$ ft, $t = 2$ h 20 min
19. $s = 35$ mi/h, $d =$ ■, $t = 3$ h
20. $s = 5$ yd/s, $d =$ ■, $t = 1$ min
21. $s =$ ■, $d = 275$ m, $t = 2.5$ h

Use cross products to find each missing term. (Lesson 4, pp. 428–430)

22. $\frac{17}{2} = \frac{a}{6}$
23. $\frac{5}{6} = \frac{10}{c}$
24. $\frac{15}{3} = \frac{b}{1}$
25. $\frac{240}{d} = \frac{12}{1}$
26. $\frac{16}{4} = \frac{8}{f}$

Use the scale 1 in. : $1\frac{1}{2}$ ft to find the actual length. (Lesson 6, pp. 436–438)

27. Model of couch: 6 in. long.
28. Model of table: $1\frac{1}{2}$ in. long.
29. Model of piano: 3 in. long.

PROBLEM SOLVING

Write an equation to solve. (Lesson 5, 432–434)

30. Jaime is making a scale drawing of his bedroom. His drawing is 25 inches long and 20 inches wide. His real room is 10 feet long. How wide is it?

Write About It

Show You Understand

A recreation room is 36 feet long and 24 feet wide. Suppose you wanted to prepare a scale drawing of this room. Tell what scale you would use, and give the dimensions of your drawing.

Extra Practice

Set A (Lesson 1, pp. 422–423)

Simplify each ratio if possible. Then express it in two other ways.

1. 4 days to 2 weeks **2.** 9 to 18 **3.** $\frac{35}{63}$ **4.** $\frac{16}{4}$ **5.** 7:9

Use equivalent ratios to find each missing term.

6. $\frac{2}{3} = \frac{4}{b}$ **7.** $\frac{3}{5} = \frac{c}{10}$ **8.** $\frac{16}{2} = \frac{d}{1}$ **9.** $\frac{21}{24} = \frac{7}{e}$ **10.** $\frac{39}{9} = \frac{f}{3}$

Set B (Lesson 2, pp. 424–425)

Write each as a unit rate.

1. 40 mi in 8 h **2.** \$7.50 for 5 gal **3.** 16 trains in 4 hours

4. \$7.20 for 12 bottles **5.** \$62.00 phone bill for 31 days

Set C (Lesson 3, pp. 426–427)

Find the speed *(s)*, distance *(d)*, or time *(t)*.

1. $s =$ ■, $d = 30$ mi, $t = 2$ h **2.** $s = 10$ ft/s, $d = 200$ ft, $t =$ ■ **3.** $s =$ ■, $d = 50$ yd, $t = 2.5$ min **4.** $s = 10$ mi/h, $d =$ ■, $t = 3\frac{1}{2}$ h **5.** $s = 48$ in./min, $d = 240$ in., $t =$ ■

6. $s =$ ■, $d = 490$ mi, $t = 3.5$ h **7.** $s = 36$ mi/h, $d = 120$ mi, $t =$ ■ **8.** $s = 7$ ft/min, $d =$ ■, $t = 1.2$ h **9.** $s = 1.5$ m/min, $d =$ ■, $t = 5$ h **10.** $s = 8$ yd/s, $d = 2{,}400$ yd, $t =$ ■

Set D (Lesson 4, pp. 428–430)

Use cross products to find each missing term.

1. $\frac{9}{2} = \frac{18}{z}$ **2.** $\frac{28}{4} = \frac{y}{1}$ **3.** $\frac{45}{25} = \frac{9}{x}$ **4.** $\frac{16}{3} = \frac{32}{w}$ **5.** $\frac{150}{50} = \frac{v}{2}$

Find the cross products for each pair of ratios. Do the two ratios form a proportion? Write *yes* or *no*.

6. $\frac{3}{4}$ $\frac{9}{12}$ **7.** $\frac{14}{4}$ $\frac{7}{2}$ **8.** $\frac{34}{4}$ $\frac{8}{1}$ **9.** $\frac{2}{3}$ $\frac{50}{75}$ **10.** $\frac{36}{81}$ $\frac{4}{9}$

Set E (Lesson 6, pp. 436–438)

Use the scales in the boxes below to find the actual measure(s) of each object.

1 in. : 3 ft

1. Model bus: 10 in. long. **2.** Table: $3\frac{1}{2}$ in. long **3.** Counter: 4 in. × $\frac{2}{3}$ in.

$\frac{1}{2}$ in. : 1 ft

4. Model tree: $27\frac{1}{2}$ in. tall. **5.** Model truck: 6 in. × 11 in. × 8 in.

CHAPTER 17

Understand Percent



INVESTIGATION

Using Data

When you go to the mall to shop for CDs, what type of music do you buy?

The table shows the percents different types of music made up of total sales in 2000, rounded to the nearest whole percent.

What type(s) of music accounted for less than 10% of total sales? What type accounted for $\frac{1}{4}$ sales?

Music Sales in 2000

Type of Music	Percent of Total
Classical	3
Country	11
Pop	11
Rap/Hip Hop	13
Rock	25

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best term to complete each sentence.

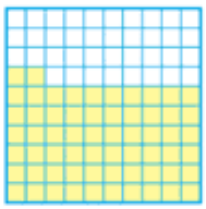
1. A number that names a part of a whole is a(n) ____.
2. The values of the digits to the right of a ____ are less than 1.
3. A ratio that compares a number to 100 is a(n) ____.

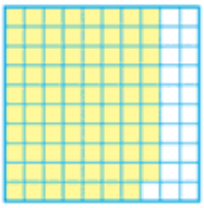
Vocabulary

- percent
- fraction
- equivalent
- decimal point

CONCEPTS AND SKILLS

Write the percent shown in each model.

4. 

5. 

Match each percent with an equivalent fraction.

6. 10%
7. 95%
8. 30%
9. 40%
10. 50%
11. 75%

a. $\frac{3}{10}$
b. $\frac{1}{2}$
c. $\frac{1}{10}$
d. $\frac{3}{4}$
e. $\frac{2}{5}$
f. $\frac{95}{100}$

Identify the larger ratio in each pair.

12. $\frac{3}{5}$ 75%
13. 85% $\frac{9}{10}$
14. 35% $\frac{1}{4}$
15. 10% $\frac{1}{5}$
16. 90% $\frac{95}{100}$
17. 30% $\frac{2}{5}$

Write About It

18. Does this picture show that the ratio of yellow buttons to the entire set of buttons is 1 to 7? Explain.



Facts Practice, See Page 660.

Lesson 1

Audio Tutor 2/14 Listen and Understand

Percent as a Ratio

Objective Relate ratios and percents.

Vocabulary

percent



Learn About It

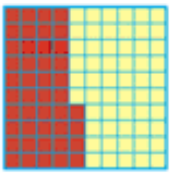
A **percent (%)** is a ratio that compares a number to 100.

In a survey, 44 percent of the teenagers interviewed said they enjoy spending time at the local mall on weekends.

What does 44 percent mean?

Look at the picture on the right. Write a ratio to compare the number of red squares to the total number of squares in the grid.

$$\frac{\text{number of red squares}}{\text{number of squares in the grid}} = \frac{44}{100} \rightarrow \text{44 percent, or 44\%}$$



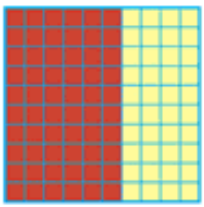
Solution: 44% means that 44 out of every 100 teenagers surveyed enjoy spending time at the mall.

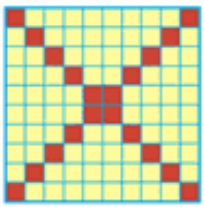
Guided Practice

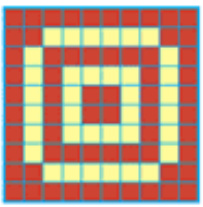
Ask Yourself

- What should the second term of the ratio be?
- Did I use the part as the first term of the ratio?

For each grid, write the percent of squares that are shaded red and the percent that are shaded yellow.

1. 

2. 

3. 

Write each ratio as a percent.

4. 56 out of 100
5. 29 out of 100
6. 100 out of 100
7. $\frac{2}{100}$
8. $\frac{33}{100}$

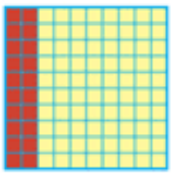
Write each percent as a ratio in word form and in fraction form.

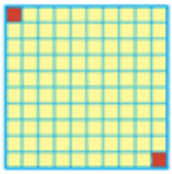
9. 38 percent
10. 70 percent
11. 6%
12. 19%
13. 94%

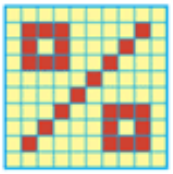
Explain Your Thinking How would you write x% as a ratio?

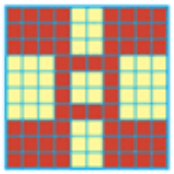
Practice and Problem Solving

For each grid, write the percent of squares that are shaded red and the percent that are shaded yellow.

14. 

15. 

16. 

17. 

Write each ratio as a percent.

18. 35 out of 100 **19.** 18 out of 100 **20.** 24 out of 100 **21.** $\frac{7}{100}$ **22.** $\frac{60}{100}$

23. $\frac{90}{100}$ **24.** $\frac{87}{100}$ **25.** $\frac{13}{100}$ **26.** $\frac{9}{100}$ **27.** $\frac{78}{100}$

Write each percent as a ratio in word form and in fraction form.

28. 45 percent **29.** 20 percent **30.** 60% **31.** 72% **32.** 96%

33. 55% **34.** 17% **35.** 32% **36.** 8% **37.** 85%

Write a ratio and a percent of the squares for each color in the grid at right.

38. red

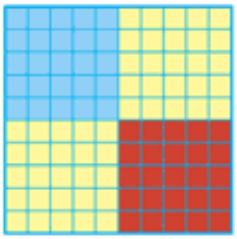
39. blue

40. yellow

41. red and blue

42. green

43. red, blue, and yellow



Solve.

44. Only 17 out of every 100 teens said they enjoy going to the mall after school. What percent is that?

45. **Analyze** If 17% of teens go to the mall after school, what percent does not go to the mall?

46. **Predict** Suppose 1,000 teens were surveyed in Problem 44. How many of the teens said they like to spend time at the mall after school? Explain.

47. **Represent** Girls made up 1% less than half of the 44% of teens who said they like to spend weekend time at the mall. Use a 10 × 10 grid to show this percent.

Mixed Review and Test Prep

Open Response

Evaluate each expression, given $a = \frac{2}{3}$, $b = \frac{1}{8}$, and $c = 2\frac{1}{4}$. (Ch. 6, Lesson 7)

48. $a + b$

49. $2c - b$

50. $6c \div 3$

51. $b \times c$

Multiple Choice

52. In a survey of 100 sixth-graders, 4 said they never have been to a mall. What percent of the students never have been to a mall? (Ch. 17, Lesson 1)

A 0.4% **B** 4% **C** 40% **D** 400%

Extra Practice See page 459, Set A.

Lesson 2

Relate Fractions, Decimals, and Percents

Objective Write equivalent fractions, decimals, and percents.



Learn About It

For a project about consumer spending, sixth-grade students collected data about the kinds of stores at a local mall. How can you find the percent of each kind of store at the mall?



Any fraction or decimal can be written as an equivalent percent.

8 out of 100 $= \frac{8}{100}$ → 8 *per hundred*, or 8%

$0.12 = \frac{12}{100} = 12\%$

Different Ways to Write the Fraction $\frac{3}{4}$ as a Percent

Way 1 Use equivalent fractions.

$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100}$

$\frac{75}{100} = 75\%$

Way 2 Use division.

Divide the numerator by the denominator. Write the decimal quotient as a percent.

$$\begin{array}{r} 0.75 \\ 4\overline{)3.00} \\ -28 \\ \hline 20 \\ -20 \\ \hline 0 \end{array} \rightarrow 75\%$$

Solution: Food stores represent 8%, movie and music stores represent 12%, and clothing stores represent 75% of the stores at the local mall.

Other Examples

Percent as a Decimal

Remove the percent sign. Move the decimal point two places to the left. You may need to write one or more zeros.

58% → .58. = 0.58

8% → .08. = 0.08

Guided Practice

Write as a percent.

1. 0.56 **2.** 0.37 **3.** 1 out of 4 **4.** $\frac{2}{5}$

Write each percent as a ratio in fraction and decimal form.

5. 48% **6.** 80% **7.** 2% **8.** 1.5%

Ask Yourself

- Did I move the decimal point correctly?
- Did I divide the first term by the second term?

Explain Your Thinking In 58%, why does moving the decimal point two places to the left change the percent into its decimal form?

Practice and Problem Solving

Write as a percent.

9. 0.05 **10.** 0.75 **11.** 0.4 **12.** 0.02 **13.** $\frac{9}{10}$

14. 3 out of 10 **15.** $\frac{3}{50}$ **16.** $\frac{1}{2}$ **17.** $\frac{50}{200}$ **18.** 17 out of 20

Write each percent as a ratio in fraction and decimal form.

19. 20% **20.** 50% **21.** 3% **22.** 33% **23.** 99%

24. 60% **25.** 32% **26.** 10% **27.** 14% **28.** 5%

Data **The circle graph shows the percent of the space at a mall leased by clothing stores. Use the graph for Problems 29–31.**

29. What kind of store leased 0.2 of the space?

30. Analyze Which three stores together lease $\frac{1}{4}$ of the space occupied by clothing stores?

31. Write About It Do the department stores and the casual clothing stores together have $\frac{7}{10}$ or $\frac{3}{4}$ of the space? Explain.



Mixed Review and Test Prep

Open Response

Solve for *n*. (Ch. 16, Lesson 4)

32. $\frac{25}{100} = \frac{n}{20}$ **33.** $\frac{3}{8} = \frac{9}{n}$

34. $\frac{4}{n} = \frac{2}{3}$ **35.** $\frac{n}{6} = \frac{20}{24}$

36. Jane got 2 hits in 8 times at bat. In what percent of her times at bat did Jane get a hit? (Ch. 17, Lesson 2)

Explain how you found your answer.

Extra Practice See page 459, Set B.

Lesson 3

Audio Tutor 2/15 Listen and Understand

Compare and Order Fractions, Decimals, and Percents

Objective Compare and order fractions, decimals, and percents.



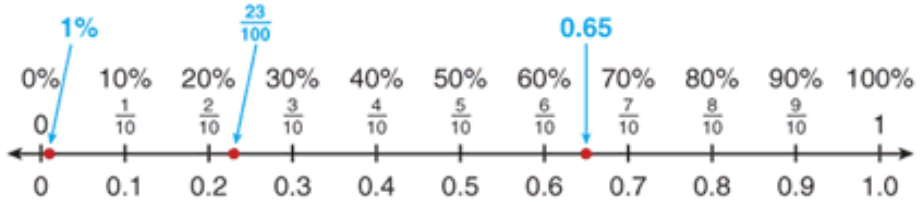
Of more than 45,000 malls in the United States, 3,000 are in Texas alone.

Learn About It

In Texas, 1% of malls are mega-malls with more than a million square feet, 0.65 are small malls of 100,000 square feet or less, and $\frac{23}{100}$ are between 100,001 and 200,000 square feet. Of which size mall does Texas have the greatest number? the least?

Different Ways to Compare Fractions, Decimals, and Percents

Way 1 You can use a number line to show parts of a unit.



Way 2 You can use place value.

- Write all the numbers as decimals in hundredths.
- Compare.

	ones		tenths	hundredths
1%	0	.	0	1
.65	0	.	6	5
$\frac{23}{100}$	0	.	2	3

Solution: $0.65 > \frac{23}{100} > 1\%$, so most of the malls in Texas are small.
The least number of malls are mega-malls.

Another Example

Compare With Mixed Numbers

To compare 20%, 1.10, 0.6%, and $1\frac{4}{5}$, write each number in the same form. In this case, it is easiest to use the decimal form.

$20\% = 0.20$
$1.10 = 1.10$
$0.6\% = 0.006$
$1\frac{4}{5} = \frac{9}{5} = 1.8$

$1\frac{4}{5} > 1.10 > 20\% > 0.6\%$

Extra Help at eduplace.com/map

Sometimes you need to find ratios that are greater than 100% or less than 1%.

Percents greater than 100% represent ratios greater than $\frac{100}{100}$, or 1.

$120\% = \frac{120}{100}$

$120\% = 1.20. = 1.2$

Remove the percent sign. Move the decimal point two places to the left.

Percents less than 1% represent ratios less than $\frac{1}{100}$, or 0.01.

$0.9\% = \frac{0.9}{100} = \frac{9}{1000}$

$0.9\% = .00.9 = 0.009$

Guided Practice

Ask Yourself

- Can I use a number line to compare the numbers?
- Can I use place value to compare the numbers?

Write > or < for each ●.

1. 49% ● 0.51 **2.** 0.1 ● 1%

Write each in two different forms.

3. 0.3% **4.** 200% **5.** $1\frac{3}{4}$ **6.** 0.6

Order each set from the least to the greatest part of a unit.

7. 0.3% 0.65 $\frac{3}{5}$ 3.0 **8.** $\frac{5}{6}$ $\frac{80}{100}$ 1.2 1.9% **9.** 0.12 $\frac{1}{10}$ 11% 0.20

Explain Your Thinking In Exercise 9, in what form did you choose to write the numbers before comparing? Explain your choice.

Practice and Problem Solving

Write > or < for each ●.

10. 25% ● 2.5% **11.** 0.001 ● 1% **12.** $\frac{67}{100}$ ● 6.7%

Write each in two different forms.

13. 165% **14.** 275% **15.** 0.7% **16.** 0.2

Go On

Order each set from the least to the greatest part of a unit.

17. 1.1 $\frac{7}{10}$ 99%

18. 4% 0.05 $\frac{2}{20}$

19. $\frac{80}{200}$ 51% $\frac{1}{2}$

20. $1\frac{4}{5}$ 3.0 250%

21. 0.13 0.31 0.31%

22. $\frac{1}{10}$ 0.15 13% 1.0

Algebra • Inequalities

Compare, given $a = 116\%$, $b = 0.6\%$, $c = \frac{3}{5}$, and $d = 1.6$. Write >, <, or = for each ●.

23. 1.4 ● a

24. c ● 90%

25. d ● 150%

26. $\frac{1}{3}$ ● b

Solve.

27. Texas has $\frac{7}{100}$ of the country's malls. California has 0.13 and Florida has 8% of the malls. Which state has the most malls? the least?

28. An old mall is torn down and a new one is built that is $2\frac{1}{2}$ times as large as the old one. What percent of the size of the old mall is the new mall?

29. Analyze In 2001, 0.01 of the new malls built were mega-malls, 58% were 100,000 ft^2 or less, and $\frac{1}{5}$ were 100,001 to 200,000 ft^2. Order the groups of malls from greatest to least.

30. Represent In a contest, 32% of those who entered won gift certificates, $\frac{2}{5}$ won coupons for free food, and 0.28 won T-shirts. Which prize was awarded most often? Show how you know.

31. What's Wrong? When the new mall opened, Tony counted its stores. Tony said that 150% of the stores were open. Explain why Tony's statement is incorrect.

32. Create and Solve Write a problem that involves comparing fractions, decimals, and percents. Challenge a partner to solve your problem.

Mixed Review and Test Prep

Open Response

Use equivalent ratios to find each missing term. (Ch. 16, Lesson 1)

33. $\frac{5}{20} = \frac{a}{100}$

34. $\frac{4}{3} = \frac{b}{12}$

35. $\frac{2}{5} = \frac{m}{15}$

36. $\frac{18}{60} = \frac{3}{x}$

Multiple Choice

37. Emily uses 15% of her tickets for roller coaster rides, 0.05 for the merry-go-round, $\frac{3}{5}$ for train rides, and the rest for wall climbing. On which ride did she use the greatest part of her roll of tickets? (Ch. 17, Lesson 3)

A roller coaster **C** train

B merry-go-round **D** wall climbing

Extra Practice See page 459, Set C.

Problem Solving

Math Challenge
The Golden Ratio

Suppose a rectangle has length L and width S.
The ratio $\frac{L+S}{L}$, is called the **Golden Ratio.**
This ratio is about 1.618 to 1.

The ratio is often used in art and architecture because of its pleasing proportions.

Does a rectangular building with a length of 55 meters and a width of 34 meters follow the Golden Ratio?



The ratio is about 1.6:1, so the rectangular building follows the Golden Ratio.



Parthenon, Greece, built in about 440 B.C., follows the Golden Ratio.

Use a calculator. Write each ratio as a decimal. Write *yes* or *no* to show if the ratio is a Golden Ratio.

1. 13:8 **2.** 14:6 **3.** $\frac{21}{13}$

4. $\frac{24}{18}$ **5.** $\frac{89}{55}$ **6.** $\frac{233}{144}$

Logical Thinking
Check That Spiral

The Golden Ratio also occurs in poetry and music and is found many places in nature.



The numbers in the spiral pattern are part of the Fibonacci Sequence.

1, 1, 2, 3, 5, 8, 13, 21, 34,
55, 89, 144, 233, 377, 610, 987...

Calculator Starting with 144, show how the ratios of pairs of adjacent numbers are related in the sequence.

Brain Teaser

A painter wants to mix a special shade of green that is 30% yellow, 50% blue, and 20% white. She has 2 quarts of yellow paint. How much of the other colors should she use?



Education Place

Check out **eduplace.com/map** for more brain teasers.

Lesson 4

Estimation With Percents

Vocabulary
tip

Objective Use mental math to estimate with percents.



Learn About It

Camila and her brother Emilio just finished lunch at a café in the mall. Their bill comes to $18.96. They want to leave a **tip** of no less than 15% for their waitress. How can they use estimation and mental math to estimate 15% of $18.96?

The term tip *stands for "to insure promptness."*

Estimate 15% of $18.96.

STEP 1 Use compatible numbers.

$18.96 is about $20.

STEP 2 Find 15% of $20.

10% of $20 = $0.1 \times \$20 = \$2$

5% of $20 = $\frac{1}{2} \times \$2 = \$1$

15% of $20 = $\$2 + \$1 = \$3$

5% is $\frac{1}{2}$ of 10%

Solution: Fifteen percent of $20.00 is $3, so $3 is slightly more than 15% of $18.96.

Other Examples

A. Percent Only

Estimate 48% of $90.

48% is about 50%.

50% of $90 = $\frac{1}{2} \times \$90 = \$45$

B. Percent and Number

Estimate 11% of 147.

11% is about 10%.

147 is about 150.

10% of 150 = $0.1 \times 150 = 15$

C. Fraction Form

Estimate 78% of 40.

78% ≈ 75%.

75% of 40 = $\frac{3}{4} \times 40 = 30$

Guided Practice

Estimate. Tell whether the exact percent would be greater than, less than, or about the same as your estimate.

1. 120% of 400 **2.** 25% of 41 **3.** 9% of 106

4. 33% of 91 **5.** 23% of 99 **6.** 54% of 78

Ask Yourself
- Did I use compatible numbers?
- Is my answer reasonable?

Explain Your Thinking ▶ How do you decide which estimation method to use?

Practice and Problem Solving

Estimate. Tell whether the exact percent would be greater than, less than, or about the same as your estimate.

7. 50% of 120 **8.** 110% of 86 **9.** 120% of 32 **10.** 20% of 77

Estimate a 15% tip and a 20% tip for each amount.

11. $59.43 **12.** $16.40 **13.** $32.14 **14.** $74.13

Data **Use the restaurant bill for Problems 15–17.**

15. **Estimate** Lauren, Jason, and their Aunt Anne are having lunch in the mall. Lauren wants to leave a 15% tip. Use mental math to estimate a 15% tip for their meal, excluding tax.

16. **Analyze** To the nearest whole percent, what tax rate was applied to the bill?

17. **Write About It** Suppose Aunt Anne decides to leave a 20% tip. Explain how she can use mental math to estimate 20% of the whole bill.

The Lunch Line
Guest Receipt

Persons 3

1 chef's salad	$6.50
1 tuna platter	7.95
1 vegetarian sandwich	5.95
2 iced teas	3.60
1 apple juice	1.95
subtotal	$25.95
tax	1.30
Total	$27.25

THANK YOU

Quick Check

Check your understanding of Lessons 1–4.

Write each as a percent. (Lessons 1 and 2)

1. 1 out of 5 **2.** 0.6 **3.** 1.4

Write as a ratio in decimal and fraction form. (Lesson 2)

4. 30% **5.** 75% **6.** 9%

Order each set of ratios from greatest to least. (Lesson 3)

7. 20% 1.2 0.15 **8.** 0.45 0.8% $\frac{1}{4}$ 40%

Solve. (Lesson 4)

9. Use mental math to estimate a 15% and a 20% tip on a bill of $16.29.

Extra Practice See page 459, Set D.

Lesson 5

Audio Tutor 2/16 Listen and Understand

Problem-Solving Application

Use Proportions

Objective Solve problems by using proportions.



When you use a proportion to solve a problem, you need to know what information to use.

The zoning laws in a town set a limit of 35 feet for the height of all signs. To find the height of a road sign, an inspector and a helper measure the length of the sign's shadow (28 feet), the height of the helper (5 feet), and the length of the helper's shadow (4 feet).

Does the sign meet the zoning laws?

UNDERSTAND

What do you know?

- Length of the sign's shadow is 28 feet.
- Helper's height is 5 feet. Length of the helper's shadow is 4 feet.

PLAN

Represent the problem using two similar triangles.

SOLVE

- Draw and label two similar right triangles.
- Write and solve a proportion.

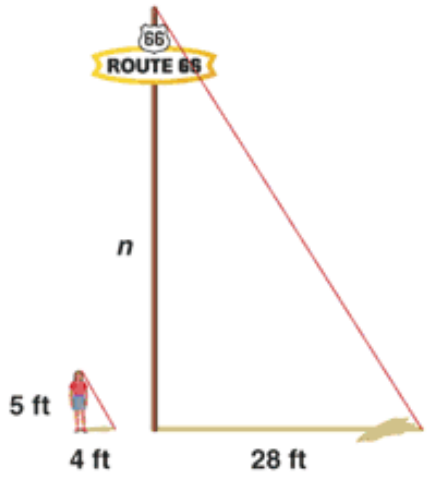
$$\frac{\text{helper's height}}{\text{sign's height}} \rightarrow \frac{5}{n} = \frac{4}{28} \leftarrow \frac{\text{helper's shadow}}{\text{sign's shadow}}$$

$$\frac{5}{n} \times \frac{4}{28}$$

$$5 \times 28 = 4 \times n$$

$$35 = n$$

Remember
Ratios of corresponding sides are equivalent.



Solution: Yes, the sign is 35 feet tall.

LOOK BACK

Look back at the problem. How can you check your solution?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. The post office at the mall has a flagpole in front. It casts a shadow that is 30 feet long. Your friend is 4 feet tall. When he stands by the flagpole, his shadow is 6 feet long. How tall is the flagpole?

Hint Which ratio do you know?

2. Lampposts in a mall parking lot cast shadows that are 8 feet long. A trash can by the lamppost is 3 feet tall and casts a shadow that is $1\frac{1}{3}$ feet long. How tall are the lampposts?

Ask Yourself

- **UNDERSTAND** — What do I need to find or do?
- **PLAN** — Did I use the correct data?
- **SOLVE** — Did I set up and solve the proportions correctly?
- **LOOK BACK** — Did I substitute my solution into the proportion to check my answer?

Independent Practice

Data **Use the data in the table for Problems 3–6.**

Trees for Sale

Tree	Shadow Length (in feet)
Ash	32
Crab apple	16
Dogwood	12
Hawthorn	20

3. **Analyze** Marty is 6 feet tall. He measured the shadow lengths for some trees on sale at the nursery at the mall. At the time he measured the trees, Marty's own shadow was 8 feet long. Which tree is 12 feet tall?

4. Which tree is twice as tall as one of the other trees? What is the height of each?

5. Suppose Marty's own shadow had been 10 feet long when he measured the trees' shadows. How tall would the hawthorn tree be then?

6. **Reasoning** A cherry tree is twice as tall as the dogwood. How tall is the cherry tree?

7. Have a partner stand by a flagpole, sign, lamppost, or other tall, hard-to-measure object. Measure your partner's height, the length of your partner's shadow, and the length of the shadow of the object. Find the object's height.



Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

8. Several mall walkers walk 2 laps around the mall on Mondays, 4 laps on Wednesdays, and 6 laps on Fridays. If the laps on these three days total 6 miles in all, how much is 1 lap around the mall?

9. There is a roller coaster ride inside a mega-mall. A shadow from the peak of its highest hill is 10 feet long. Standing nearby, a 6-foot-tall man casts a shadow 1 foot long. How high is the peak of the highest hill?

10. On Sunday, the workers at the mall's aquarium feed the animals every 1.5 hours. If there are 5 feedings and the last one begins at 5:30 P.M., at what time is the first feeding?

You Choose

Strategy

- Draw a Diagram
- Make an Organized List
- Use Models
- Work Backward
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the table of facts for Problems 11–15. Then tell which method you chose.**

11. On average, about how many visitors come to the mall each day?

12. If you spent 10 minutes in each store, how long would it take to visit each store at the mall?

13. It took about twice as much steel to build the mall as it did to build the Eiffel Tower. About what percent of the amount of steel used for the Eiffel Tower was needed to build the mega-mall?

14. In addition to the hanging plants there are about 400 trees in the mall. If a tree casts a 9-foot shadow at the same time a 5-foot-tall girl casts a 3-foot-long shadow, how tall is the tree?

Mega-Mall Facts

Visitors per year: 42,000,000
Specialty stores: 525
Tons of steel: 13,300
Hanging plants: 30,000

15. **You Decide** Decide how many people are needed to water the hanging plants and how many hours each person must work. Make a plan for watering each hanging plant twice a week.

Problem Solving

Number Sense

The Best Representation

You can express quantities, relationships, and comparisons, using whole numbers, fractions, decimals, percents, and ratios. In some situations, one representation is better than another.

- The population is generally **50%** female.
- The probability of getting a head when you flip a coin 10 times is 5 to 10, or $\frac{5}{10}$ or 50%.
- You might run a **0.5**-kilometer race.
- 50 cents is written as $\frac{50}{100}$ dollars and as **.50** on a bank check.



Choose $\frac{25}{100}$, $\frac{1}{4}$, 0.25, or 25% as the best representation for each.

1. in a recipe: ■ cup of sugar
2. on a bank check: five and ■ dollars
3. on a sale poster: ■ off the regular price
4. a metric length: ■ meter
5. on a sales receipt: ■ tax
6. a model racing car: ■ m long
7. a class survey: ■ students who prefer math
8. a ■ score at the gymnastics championship

9. **Write About It** Find an example of a percent in a newspaper or magazine. Explain why the percent is or is not the best representation for the situation.
10. **Analyze** Can you think of any situations where more than one representation of a quantity could be equally effective.

Chapter Review/Test

VOCABULARY

1. At restaurants, people include a percentage of their total bill as a(n) ____.

2. A(n) ____ is a ratio that compares a number to 100.

Vocabulary

tip
rate
ratio
percent

CONCEPTS AND SKILLS

Write each ratio as a percent. (Lesson 1, pp. 444–445)

3. $\frac{3}{100}$ **4.** $\frac{60}{100}$ **5.** 45 out of 100 **6.** $\frac{57}{100}$

Write each percent as a ratio in word form and in fraction form. (Lesson 1, pp. 444–445)

7. 73% **8.** 90% **9.** 88% **10.** 16% **11.** 12%

Write as a percent. (Lesson 2, pp. 446–447)

12. 0.45 **13.** $\frac{70}{200}$ **14.** 0.01 **15.** $\frac{29}{50}$ **16.** $\frac{4}{25}$

Write each percent as a ratio in fraction and decimal form. (Lesson 2, pp. 446–447)

17. 87.5% **18.** 30% **19.** 12% **20.** 73%

Order each set from the least to the greatest part of a unit. (Lesson 3, pp. 448–450)

21. 0.37 $\frac{2}{5}$ 38% **22.** $\frac{3}{5}$ $\frac{1}{2}$ 65% 0.59 **23.** 3.233 3.7 350% $\frac{10}{3}$

24. $\frac{7}{8}$ 0.90 $\frac{7}{9}$ 70% **25.** 0.45 0.52 5.4% **26.** $\frac{150}{200}$ 70% $\frac{2}{3}$

Estimate a 15% and a 20% tip for each amount. (Lesson 4, pp. 452–453)

27. $59.03 **28.** $31.89 **29.** $48.75 **30.** $9.15 **31.** $14.98

PROBLEM SOLVING

Solve (Lesson 5, pp. 454–456)

32. Marisa is standing next to a stop light. It casts a shadow 40 feet long. She is 5 feet tall, and her shadow is 8 feet long. How tall is the stop light?

33. Michael is 6 feet tall, and his shadow is 9 feet long. A pole next to Michael casts a 30-foot shadow. How tall is the pole?

Write About It

Show You Understand

Describe some real-life situations where it is helpful to know how to estimate percents.

Extra Practice

Set A (Lesson 1, pp. 444–445)

Write each ratio as a percent.

1. 15 out of 100 **2.** $\frac{70}{100}$ **3.** $\frac{93}{100}$ **4.** $\frac{7}{100}$ **5.** $\frac{42}{100}$

Write each percent as a ratio in word form and in fraction form.

6. 25 percent **7.** 70% **8.** 92% **9.** 66% **10.** 23%

Set B (Lesson 2, pp. 446–447)

Write as a percent.

1. 5 out of 10 **2.** 0.95 **3.** 0.08 **4.** $\frac{60}{200}$ **5.** $\frac{7}{50}$

Write each percent as a ratio in fraction and decimal form.

6. 70% **7.** 19% **8.** 22% **9.** 8% **10.** 66%

Set C (Lesson 3, pp. 448–450)

Write each in two different forms.

1. 125% **2.** 0.6 **3.** 425% **4.** $\frac{1}{5}$ **5.** 0.9%

Order each set from least to greatest.

6. 0.24 0.42 2.4% **7.** 0.18 $\frac{1}{5}$ 19% **8.** $\frac{90}{200}$ 55% $\frac{1}{2}$

9. $\frac{4}{5}$ $\frac{2}{3}$ 75% 0.60 **10.** $\frac{7}{3}$ 2.133 2.31 340%

Set D (Lesson 4, pp. 452–453)

Estimate. Tell whether the exact percent would be greater than, less than, or about the same as your estimate.

1. 40% of 200 **2.** 33% of 150 **3.** 150% of 48 **4.** 10% of 72 **5.** 60% of 300

Use mental math to estimate a 15% tip and a 20% tip for each amount.

6. \$9.88 **7.** \$4.25 **8.** \$29.21 **9.** \$11.91 **10.** \$46.20

11. \$39.10 **12.** \$58.12 **13.** \$46.50 **14.** \$31.75 **15.** \$62.18



CHAPTER 18

Use Percent

INVESTIGATION

Using Data

Most antique dolls are between 50 and 250 years old. What percent of the dolls listed are more than 100 years old?

Suppose a collector bought the doll from 1912 and paid 5% sales tax. What would be her total cost, including tax? Tell two different ways to find the answer.

Antique Dolls	
Year Made	Price
1800	$700
1880	$1,500
1895	$140
1912	$200
1920	$600

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A(n) ____ is a comparison of two numbers by division.
2. A ratio of a number to 100 is a(n) ____.
3. A comparison by ratio of two quantities using different kinds of units is a(n) ____.

Vocabulary

- rate
- ratio
- interest
- percent
- proportion

CONCEPTS AND SKILLS

Use the dimes to help you find the percent of each amount.



4. 10% of $1.00
5. 20% of $1.00
6. 30% of $1.00
7. 40% of $1.00
8. 50% of $1.00
9. 60% of $1.00
10. 70% of $1.00
11. 80% of $1.00
12. 90% of $1.00
13. 100% of $1.00

Match each question with its correct answer.

a. 180 **b.** 200 **c.** 240 **d.** 25% **e.** 300 **f.** 100%

14. 10% of what number is 20?
15. What percent of 400 is 100?
16. What is 30% of 600?
17. 50% of what number is 150?
18. What is 60% of 400?
19. What percent of 200 is 200?

Write About It

20. How is 1% of a whole like 1¢ out of a dollar?

Facts Practice, See Page 660.

Lesson 1

Audio Tutor 2/17 Listen and Understand

Find a Percent of a Number

Vocabulary
proportion

Objective Find a percent of a number.



Learn About It

Collecting and studying postcards is the fourth most popular hobby in the United States. Many collectors buy postcards through Internet auction sites.

If Jameson's e-bid on a postcard collection was 20% of the final selling price of $47.50, how much was his bid?

Find 20% of $47.50.

Different Ways to Find a Percent of a Number

Way 1 You can write and solve a proportion.

STEP 1 Write a **proportion**, using part-to-whole ratios.

part → $\frac{20}{100} = \frac{n}{47.50}$ ← part (Jameson's bid)

whole → ← whole (final selling price)

STEP 2 Solve for n.

$\frac{20}{100} \times \frac{n}{47.50}$

$20 \times 47.50 = 100 \times n$ ← Cross multiply.

$950 = 100n$

$9.50 = n$

Way 2 You can write and solve an equation.

STEP 1 Write an equation, using the decimal form of the percent.

$n = 0.20 \times 47.50$

$20\% = \frac{20}{100} = 0.20$

STEP 2 Solve for n.

$n = 9.50$

Check your work.

- 10% of $47.50 is $4.75, or about $5.
- So, 20% of $47.50 is about $2 \times \$5 = \$10$.
- The answer is reasonable.

Solution: Jameson's bid was $9.50.

Guided Practice

Find the percent of each number.

1. 70% of 100
2. 35% of 317
3. $6\frac{1}{2}\%$ of 88
4. 45% of 1
5. 250% of 18.5
6. 1.8% of 50

Ask Yourself

- How do I write a percent as a ratio?
- How do I write a percent as a decimal?

Explain Your Thinking Why can a ratio represent a percent?

Extra Help at eduplace.com/map

Practice and Problem Solving

Use a proportion to find the percent of each number.

7. 40% of 220 **8.** 20% of 66 **9.** 100% of 15 **10.** 99% of 1

11. 1% of 19 **12.** 105% of 6 **13.** 382% of 1,000 **14.** 23.5% of 48

15. 3.6% of 998 **16.** 4.4% of 112 **17.** 87% of 78 **18.** 33% of 33

Use an equation to find the percent of each number.

19. 60% of 150 **20.** 10% of 212 **21.** 90% of 210 **22.** 10% of 189

23. 45% of 92 **24.** 18% of 18 **25.** 1% of 300 **26.** 6% of 124

27. 101% of 5 **28.** 115% of 200 **29.** 11.5% of 76 **30.** 3.5% of 160

Find the percent of each number. Use either method.

31. 12.1% of 100 **32.** 15.6% of 47 **33.** 97.3% of 17 **34.** 56% of 143

Solve.

35. Felicia has 325 postcards in her collection. Sixty percent of the cards are from Japan. How many postcards are from Japan? How many are from places other than Japan?

36. Bill has 40 postcards from Mexico. He bought 20 percent of the cards at collectors' shows and 65 percent at online auctions. The rest were gifts from friends. How many postcards were gifts?

37. **Analyze** The table shows the prices of postcards at Hector's Hobby Shop. Given this pattern of pricing, how much will 7 cards cost? Explain your answer.

Number of Postcards	1	2	3	4	5
Price	45¢	85¢	$1.20	$1.50	$1.75

38. **Explain** When you find a percent of a number, how can you tell if your answer will be less than or greater than the given number?

Mixed Review and Test Prep

Open Response

Complete. (Ch. 8, Lesson 2)

39. 1 gal = ■ qt **40.** 3 pt = ■ c

41. ■ pt = 2 gal **42.** 12 c = ■ qt

43. 32 oz = ■ lb **44.** ■ T = 10,000 lb

45. 36 in. = ■ yd **46.** ■ ft = 36 in.

Multiple Choice

47. A mural covers 96 square feet of a wall. Twenty-five percent of the mural is black. How many square feet are not black? (Ch. 18, Lesson 1)

A 24 ft^2 **C** 93.6 ft^2

B 72 ft^2 **D** 2,496 ft^2

Extra Practice See page 483, Set A.

Lesson 2

Audio Tutor 2/18 Listen and Understand

Find a Percent

Objective Find a percent.



Learn About It

Robots are becoming "smarter." Designers of artificial intelligence software have even created robots that can play soccer without any remote control.

Elisa spends $40 on a remote control cat and a remote-free cyber dog. If the cat cost $15, what percent of the total cost is the cost of the cat?

$15 = *n*% of $40. Find *n*.

Different Ways to Find a Percent

Way 1 You can write and solve a proportion.

STEP 1 Write a proportion, using part-to-whole ratios.

$$\frac{n}{100} = \frac{15}{40}$$

$\frac{n}{100}$ ← part (cost of remote control cat)

$\frac{15}{40}$ ← whole (total cost)

STEP 2 Solve for n.

$\frac{n}{100} \times \frac{15}{40}$

$n \times 40 = 100 \times 15$ ← Cross multiply.

$40n = 1{,}500$

$n = 37.5$

Way 2 You can write and solve an equation.

STEP 1 Write an equation.

$$15 = \frac{n}{100} \times 40$$

STEP 2 Solve for n.

$\frac{15}{40} \times 100 = n$

$37.5 = n$

Check your work.
Fifteen is a little less than half of 40, so the answer is reasonable.

Solution: The cost of the cat is 37.5% of the total cost.

Guided Practice

Use a proportion or an equation to find each percent.

1. What percent of 40 is 30?

2. What percent of 300 is 24?

3. 1.6 is what percent of 12.8?

4. $\frac{5}{6}$ is what percent of $3\frac{1}{3}$?

Ask Yourself

- Which number is the part and which number is the whole?
- How do I express a decimal as a percent?

Explain Your Thinking What information do you need to find a percent?

Practice and Problem Solving

Use a proportion or an equation to find each percent.

5. What percent of 65 is 13?
6. 96 is what percent of 120?
7. 44 is what percent of 44?
8. 15 is what percent of 125?
9. What percent of 90 is 63?
10. 2 is what percent of 1?
11. What percent of 80 is 88?
12. What percent of 200 is 45?
13. 4.6 is what percent of 0.8?
14. What percent of 80 is 24.4?
15. What percent of $3\frac{1}{3}$ is 3?
16. $1\frac{1}{2}$ is what percent of $3\frac{3}{4}$?

Algebra • Equations

17. Write an equation and then a proportion to find b in the statement $a = b\%$ of c.

Data Use the picture to solve Problems 18–20.



18. Rosa wants to buy a cyber puppy. So far, she has saved $34. What percent of the price of the cyber puppy do Rosa's savings make up?
19. Franklin saved $9 by using a coupon to buy a cyber cat. What percent of the cost did Franklin save?
20. Bob bought 5 batteries. The clerk gave him $2.50 in change. What three bills did Bob use to pay for the batteries?
21. **Write About It** Explain how you could use mental math to find what percent of 2 is 4.

Mixed Review and Test Prep

Open Response

Write each as a unit rate. (Ch. 16, Lesson 3)

22. $30 for 5 books
23. 400 miles in 8 hours
24. 20 pounds in 4 months
25. $48 in 8 hours
26. Seventy-five robots were entered in a science fair. Forty-five received an honorable mention, 15 got a ribbon, and the rest won a prize. What percent of the robots won a prize? (Ch. 18, Lesson 2)

Explain how you got your answer.

Extra Practice See page 483, Set B.

Lesson 3

Audio Tutor 2/19 Listen and Understand

Find a Number When a Percent is Known

Objective Find a number when you know a percent.



Learn About It

Students are using squares that are the same size to design a quilt for an Indiana history project. They have finished designing 35% of the quilt, using 42 squares. How can you find the number of squares there will be in the quilt when it is completed?

35% of n = 42. Find n.

Different Ways to Find a Number When a Percent is Known

Way 1 You can write and solve a proportion.

STEP 1 Write a proportion.

part → $\frac{42}{n} = \frac{35}{100}$ ← part

whole → whole

STEP 2 Solve for n.

$\frac{42}{n} \times \frac{35}{100}$

$42 \times 100 = n \times 35$ ← Cross multiply.

$4{,}200 = 35n$

$120 = n$

Way 2 You can write and solve an equation.

STEP 1 Write an equation.

$0.35 \times n = 42$

STEP 2 Solve for n.

$\frac{0.35 \times n}{0.35} = \frac{42}{0.35}$

$n = \frac{42}{0.35}$

$n = 120$

Check your work.

Estimate: 42 rounds to 40

0.35 rounds to 0.4

$40 \div 0.4 = 100$

120 is close to 100, so the answer is reasonable.

Solution: There will be 120 squares in the quilt when it is completed.

Guided Practice

Use a proportion to find each number.

1. 75% of what number is 75? **2.** 93 is $15\frac{1}{2}$% of what number?

Ask Yourself

- Did I write the proportion correctly?

Explain Your Thinking How can you use estimation to check your answer to Exercise 2?

Practice and Problem Solving

Use a proportion to find each number.

3. 9 is 15% of what number?

4. 220 is 110% of what number?

5. 25 is 125% of what number?

6. 64% of what number is 24?

7. 2.5% of what number is 2.5?

8. 80% of what number is 7.2?

9. 20.3 is 35% of what number?

10. $\frac{1}{2}$% of what number is 6?

Use an equation to find each number.

11. 57 is 19% of what number?

12. 14% of what number is 56?

13. 22 is 55% of what number?

14. 1% of what number is 6?

15. 112% of what number is 84?

16. 3.1 is 15.5% of what number?

17. 92% of what number is 142.6?

18. 90% of what number is 72.9?

Solve.

19. Five squares in the quilt will show flowers that grow in the state, 4 will show birds, and 7 will show landmarks. These make up 80 percent of the total quilt squares that teachers will make in all. How many quilt squares will teachers make?

20. **Represent** The border of the quilt will be made up of alternating 6-inch and 4-inch squares. The quilt will measure 60 inches by 80 inches including the border. How many of each size square are needed for the border? Show your work.

21. Three students, or 5 percent of the students making the quilt, are embroidering pictures of the state's agricultural products. What is the total number of students making the quilt?

22. **Explain** Do you prefer to use a proportion or an equation to find a number when you know the percent? Explain by giving examples to support your choice.

Mixed Review and Test Prep

Open Response

Order each set from the least to the greatest part of a unit. (Ch.17, Lesson 3)

23. $\frac{1}{2}$ 0.1 2%

24. 0.4 $\frac{4}{5}$ 50%

25. 0.33 3% $\frac{3}{10}$

26. 0.7 $\frac{1}{20}$ 7%

Multiple Choice

27. There are 27 more students enrolled at Jefferson School than there were last year. This represents an increase of 6%. How many students were enrolled last year? (Ch.18, Lesson 3)

A 45 students **C** 450 students

B 423 students **D** 477 students

Extra Practice See page 483, Set C.

Lesson 4

Problem-Solving Decision

Choose a Method

Objective Review how to decide which computation method to use to solve a problem.



Before you solve a percent problem, you need to decide which method to use.

Problem Cora knows what the clues for 16 words in this crossword puzzle mean. What percent of the clues does Cora know?

This first ever crossword puzzle, by Arthur Wynne, appeared in a New York newspaper on December 21, 1913.

Ask Yourself

Should I use mental math?

16 is what percent of 32?
16 is $\frac{1}{2}$ of 32.
So 16 is 50% of 32.

Do I need to use paper and pencil?

$$16 = \frac{n}{100} \times 32$$
$$\frac{16}{32} \times 100 = n$$
$$50 = n$$

So 16 is 50% of 32.

Does it make sense to use a calculator?

Enter:





Display: 50

Solution: Cora knows 50% of the clues. Which method would you have chosen? Explain.

Try These

Solve. Explain which method you used.

1. Bo finished 110 puzzles in her book, *200 Crossword Puzzles*. What percent of the book's puzzles did Bo complete?
2. Domingo filled in 102 words, or 85% of the words, in a crossword puzzle. How many words are in the crossword puzzle?
3. **Explain** Suppose the number of words in the crossword puzzle in Problem 2 is doubled and Domingo fills in twice as many words in the new puzzle. Will the percent of words in the new puzzle that Domingo fills in also double?
4. **Reasoning** In a student crossword puzzle competition, 20% of 400 students qualified for the final round. Twenty-five percent of the finalists received a medal. What percent of all the students received a medal?

Problem Solving

Calculator Connection

Percents and Sports

It is often helpful to use a calculator to find percents in real-world situations, such as sports. Here's how you can use the % key on your calculator.

Note: On some calculators, the percent key [%] is used alone *instead of* the [=] .

Solving Percent Problems on the Calculator

To solve:	Enter	Display
0.9% of 5 = n	5 × 0 . 9 % or 5 × 0 · 9 % Enter	0.045
32 = n% of 15	3 2 ÷ 1 5 % or 3 2 ÷ 1 5 % Enter	213.33333
15 is 25% of n	1 5 ÷ 2 5 % or 1 5 ÷ 2 5 % Enter	60

1. The basketball team made 35.4% of the 82 shots they attempted. How many baskets did they make?
2. There were 22,512 people at the game. The stadium was only 72% filled. About what is the seating capacity of the stadium?



Quick Check

Check your understanding of Lessons 1–4.

Use a proportion to find the percent of each number. (Lesson 1)

1. 60% of 420 **2.** 4% of 333 **3.** 3.1% of 1 **4.** $5\frac{1}{2}$% of 72

Use a proportion or an equation to find each percent. (Lesson 2)

5. What percent of 60 is 12? **6.** 2.4 is what percent of 3?

Use a proportion or an equation to find each number. (Lesson 3)

7. 20 is 40% of what number? **8.** 120% of what number is 18?

Solve. (Lesson 4)

9. Thirty-five percent of the 1,120 coins in Jason's coin collection were minted before 1940. How many coins in Jason's collection were minted *after* 1940?

Lesson 5

Percent Relationships

Objective Use an equation to solve percent problems.

Vocabulary
- percent
- base
- part of base



Learn About It

You have learned how to find each of the three parts of a percent problem: the **percent**; the original number, or **base**; and the percent of the number, or **part of base**. The relationship of part of base to the percent and base can be written as an equation.

$$\text{Part of Base} = \text{Percent} \times \text{Base}$$
$$A = P \times B$$

Corey's father collects hand-made African masks. He wants to buy a mask for $280, which is 80 percent of the original catalog price. What was the original price of the mask?

Find the original price of the mask.

STEP 1 Substitute the given values in the percent equation.

$$A = P \times B$$
$$\$280 = 80\% \times B$$

Sale price — Percent of original price — Original price

STEP 2 Solve the equation to find the original price, or base.

$$280 = 0.80 \times B$$
$$280 \div 0.80 = B$$
$$350 = B$$

Check your work.
80 percent of $350
$= 0.8 \times \$350 = \$280$
The answer checks.

Solution: The original price of the mask was $350.

Other Examples

A. Find the Percent

$1,200 is what percent of $2,000?

$$A = P \times B$$
$$1{,}200 = P \times 2{,}000$$
$$\frac{1{,}200}{2{,}000} = P$$
$$0.60 = P = 60\%$$

$1,200 is 60% of $2,000.

B. Find the Part of Base

What is 85% of $3,000?

$$A = P \times B$$
$$A = 0.85 \times 3{,}000$$
$$A = 2{,}550$$

85% of $3,000 is $2,550.

Guided Practice

Use the percent equation to solve.

1. 12% of what number is \$450?

2. What is 38% of \$2,000?

3. \$1,400 is what percent of \$1,600?

4. \$4,500 is 60% of what number?

5. What is 18% of \$5,000?

6. \$770 is what percent of \$2,200?

Ask Yourself

- Did I substitute the values in the percent equation correctly?
- Is my answer reasonable?

Explain Your Thinking ▶ Is there more than one way to solve each equation?

Practice and Problem Solving

Use the percent equation to solve.

7. What is 15% of 2,300?

8. 48 is what percent of 320?

9. 10% of what number is \$750?

10. 120 is what percent of 1,000?

11. What percent of \$700 is \$245?

12. What is 7% of 4,900?

13. What percent of \$793 is \$1,586?

14. What is 180% of \$1,650?

15. \$2,748 is 120% of what number?

16. 35% of what number is \$664.65?

Solve.

17. Maria has a collection of 16 African musical instruments. Four of the instruments are drums. What percent of Maria's collection are drums?

18. An African beaded necklace is marked down from \$60 to \$48. A sand painting is marked down from \$35 to \$26.25. Which price was reduced by a greater percent?

19. How many different arrays can Jim make with his collection of 24 African baskets?

20. **Analyze** Is it possible for an item to be "marked down" for sale at 105% of its original price? Explain.

Mixed Review and Test Prep

Open Response

Use cross products to find each missing term. (Ch.16, Lesson 4)

21. $\frac{5}{h} = \frac{15}{24}$

22. $\frac{6}{8} = \frac{9}{b}$

23. $\frac{s}{64} = \frac{5}{8}$

24. $\frac{4}{5} = \frac{t}{15}$

25. $\frac{2}{3} = \frac{n}{21}$

26. $\frac{9}{d} = \frac{3}{1}$

27. Bruce paid \$44 for a pair of jeans on sale. The price he paid was 80% of the original price. What was the original price?

Explain how you got your answer.

(Ch.18, Lesson 5)

Extra Practice See page 483, Set D.

Lesson 6

Simple Interest

Objective Calculate simple interest.

Vocabulary
- principal
- interest
- interest rate

Learn About It

Dave sold his comic book collection for $150. If he deposits the money at this bank, how much interest will he earn in one year? What will his total account balance be if he does not withdraw any money?

Let your money work for you!

We pay 2.1% interest per year.

Principal:	Amount of money deposited or borrowed.
Interest:	An amount paid for the use of money. It is calculated only on the original principal.
Interest Rate:	The rate of interest paid over a given period of time, usually one year.

Find the interest and the total of principal and interest.

STEP 1 Calculate the simple interest.

$$\begin{aligned} \text{Interest } (I) &= \text{principal } (p) \times \text{rate } (r) \times \text{time } (t) \\ &= \$150 \quad \times 2.1\% \quad \times 1 \text{ yr} \\ &= 150 \quad \times 0.021 \quad \times 1 \\ &= \$3.15 \end{aligned}$$

STEP 2 Find the total.

$$\begin{aligned} \text{Total} &= \text{principal } (p) + \text{interest } (I) \\ &= \$150 \quad + \$3.15 \\ &= \$153.15 \end{aligned}$$

Check your work.
2% of 100 is 2 and 2% of 200 is 4. So 2% of 150 is 3.

$150 + $3 is close to $153.15, so the solution is reasonable.

Solution: Dave will earn $3.15 in interest in one year. His account balance will then be $153.15.

Guided Practice

Find the simple interest and the sum of the principal and the interest for each.

Ask Yourself
- How do I change the percent to a decimal?
- How do I represent the months as part of a year?

	Principal	Interest Rate	Time
1.	$3,000	3.5%	1 yr
2.	$80	2.9%	6 mo
3.	$500	2.0%	3 mo

6 mo = 0.5 yr

Explain Your Thinking In Exercise 3, what steps did you follow to represent the time in decimal form?

Practice and Problem Solving

Find the simple interest and the sum of the principal and the interest for each.

	Principal	Interest Rate	Time
4.	$1,070	3.0%	1 yr
5.	$230	2.4%	1 yr
6.	$72	1.5%	6 mo
7.	$2,000	3.25%	1 yr
8.	$10,060	3.3%	6 mo
9.	$950	1.2%	9 mo



Data **The table shows the simple interest rate for three different savings accounts. Use the table for Problems 10–13.**

10. Melba deposited $200 in a Super Saver account. What will her account balance be after 6 months if she does not make any more deposits or any withdrawals?

11. Lydia opened an account with a $300 deposit. After 6 months her balance was $303.75. She made no more deposits and no withdrawals. Which account plan did she choose?

12. Manuel deposited $175 in a Simple Save account and $150 in a Savings Plus account. If he does not deposit or withdraw any money, which account will earn more interest in one year? How much more?

SAVINGS BANK

Savings Accounts	
Account	Simple Interest Rate (per year)
Simple Save	2.0%
Savings Plus	2.5%
Super Saver	2.75%

13. **Analyze** Dave deposited $400 in a Simple Save account. After 6 months, he withdrew the entire balance and deposited it in a Savings Plus account. After another 6 months, how much interest had Dave earned in all? Explain.

Mixed Review and Test Prep

Open Response

Write each as a unit rate. (Ch.16, Lesson 2)

14. $12 for 4 comic books

15. 49 flowers in 7 vases

16. 120 students in 6 buses

17. $320 in 4 months

18. 84 players on 7 teams

19. 48 beads on 6 necklaces

Multiple Choice

20. Cathy put $450 in a bank at a simple interest rate of 3.2% per year. She withdrew $225 after 6 months. If Cathy doesn't make any more deposits or withdrawals, what will her balance be after 1 year? (Ch.18, Lesson 6)

A $228.60 **C** $460.80

B $235.80 **D** $464.40

Extra Practice See page 483, Set E.

Lesson 7

Problem-Solving Strategy

Make a Table

Objective Make a table to solve a problem.



Most bank savings accounts pay you "interest on your interest." This is called compounding.

Problem The Stamp Club has saved $700 from the proceeds of events it hosts. It deposits the money in a savings account that pays an interest rate of 2.25% per year. If the Club makes no withdrawals or deposits, how much money will be in its account after 4 years?

UNDERSTAND

This is what you know:

- The amount to be deposited (the principal) is $700.
- The interest rate is 2.25% compounded, paid yearly on the entire balance.
- The money will be invested for 4 years (4 time periods).

PLAN

You can make a spreadsheet-like table to solve the problem.

SOLVE

- Find the interest and new balance for the first time period.
- Use the new balance as the beginning balance for the second time period.
- Repeat the steps to find the new balance at the end of the fourth time period.

Interest Paid = Beginning Balance × 0.0225

New Balance = Beginning Balance + Interest Paid

Easy Sheet

	A	B	C	D
1	Time Period	Beginning Balance	Interest Paid	New Balance
2	Year 1	$ 700.00	$ 15.75	$ 715.75
3	Year 2	$ 715.75	$ 16.10	$ 731.85
4	Year 3	$ 731.85	$ 16.47	$ 748.32
5	Year 4	$ 748.32	$ 16.84	$ 765.16

Solution: The Club will have $765.16 at the end of 4 years.

LOOK BACK

Did I record data in the correct rows and columns?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.



1. **Calculator** Paco deposited $100 in a savings account that pays 1.75% interest, compounded yearly. If he makes no withdrawals or deposits, how much interest will Paco have earned after 3 years?

   Hint Do you need to find the interest or the new balance after 3 years?

2. A store reduces the price of a jacket by 10% of its original price each week it is not sold. If the original price of the jacket is $79, what will its price be during Week 5?

Ask Yourself

UNDERSTAND — What do I know about Paco's investment?

PLAN — How will I set up a table so that the data are organized?

SOLVE — What calculations should I perform in each row of the table?

LOOK BACK — Did I answer the question I was asked?

Independent Practice

Make a table to solve each problem.

3. Each player in a game of darts throws three darts at the board shown at the right. What are all the possible scores that a player can get if all three darts hit the board?

4. Percy buys 7 sheets of posterboard, 12 packs of markers, and 8 bottles of glitter. Markers cost $3.29 a pack, glitter is $2.99 a bottle, and a sheet of posterboard costs $1.69. Percy uses a 10% off coupon. To the nearest cent, what is the total cost?



5. **Calculator** Ella lent her brother $300. She charged him 1.5 percent interest on the balance each month. If Ella's brother waits and pays back the entire loan after 6 months, how much will he owe Ella?

6. **Analyze** Min Soo has $0.18 in his pocket. How many different combinations of coins could be in Min Soo's pocket? Explain how you organized data in a table to solve the problem.



Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Patrick walked 4 blocks west, 5 blocks north, 3 blocks east, 2 blocks south, and 1 block east. How many blocks does Patrick need to walk to get back to his starting point? In what direction should he walk?

8. The houses on Elm Street are numbered from 1 through 100. How many times does the digit 5 appear in the Elm Street house numbers?

9. **Analyze** There are 30 red or blue T-shirts in a closet. At least one T-shirt is blue. If you pick any two T-shirts, at least one will be red. How many blue T-shirts are in the closet? How many red T-shirts are in the closet?

You Choose

Strategy
- Draw a Diagram
- Use Logical Reasoning
- Use Models
- Solve a Simpler Problem

Computation Method
- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data Use the graph to solve Problems 10–13.

This line graph shows the sales for January through April for two types of sneakers.



10. Between which two months did the sales of Superwalks decrease and the sales of Roadracers increase?

11. Which brand of shoes was the first to reach $10,000 in sales?

12. In what month were the sales of Roadracers less than half the sales of Superwalks?

13. **Calculator** What percent of Superwalks' total sales for January through April was made in March?

14. **Create and Solve** Write your own word problem using data from the line graph.

Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer. If a correct answer is not here, choose NH.

1. George makes $5.75 an hour doing yard work. How much will George earn if he works 3 hours on Monday, 1.5 hours on Wednesday, and 2.5 hours on Saturday?

A $17.25 **C** $39.95

B $23.00 **D** $40.25

(Chapter 7, Lesson 2)

2. Daniel is covering a floor with square tiles that have 6-inch sides. The floor is 12 feet by 14 feet. How many tiles does Daniel need to cover the floor?

F 14 **H** 672

G 504 **J** NH

(Chapter 8, Lesson 7)

Open Response

Solve each problem.

3. If the range in temperature is 11 degrees and the high temperature is 7°F, what is the low temperature?

Represent Draw a picture to solve the problem.

(Chapter 11, Lesson 5)

4. What is the least number of 90° counterclockwise rotations that are needed to change the letter from the left position to the right position?

Explain How did you find your answer?



(Chapter 15, Lesson 4)

Constructed Response

5. You are an architect working for a firm that makes pre-fabricated houses. Use an inch ruler to measure the drawing of the house below.



Scale: 1 in. = 16 ft

a. What is the actual height of the house from the ground to the rooftop?

b. You need to include a large window on the ground floor of the house. The actual dimensions of the window are 8 feet long and 6 feet high. What will be the dimensions in the drawing?

c. Your firm is designing a new house with dimensions that will be $6\frac{1}{4}\%$ greater than the house above. What will be the actual length of the side of the new house? What will be its height from ground to rooftop? Show your work.

d. What will be the side length of the new house on a drawing that uses the same scale? Explain.

(Chapter 18)



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Lesson 8

Audio Tutor 2/20 Listen and Understand

Sales Tax and Discounts

Objective Use percents to calculate total cost with sales tax and sale prices.

Vocabulary
discount

Learn About It

A **discount** decreases the price of an item. Discount is often expressed as a rate, using percent. The sale price is the price of the item after the discount.



What is the sale price of a model pick-up truck if the discount is 25% of the original price of $14.89?

Find the sale price of the model pick-up truck.

STEP 1 Determine the discount. Round to the nearest cent.

Discount = Discount Rate × Original Price
= 25% of $14.89
= 0.25 × $14.89
= $3.72

STEP 2 Subtract the discount from the original price to find the sale price.

Sale price = Original Price − Discount
= $14.89 − $3.72
= $11.17

Check your work.
25% of $14.89 → $\frac{1}{4}$ of $16 = $4
$16 − $4 = $12; $11.17 is close to $12, so the answer is reasonable.

Solution: The sale price of the model pick-up truck is $11.17.

Sales tax is a fee that a state government adds to the cost of many items to help pay for government programs. A percent of your total purchases is added as sales tax.

Suppose you have $6.50 and want to buy a model race car that costs $5.94. Your state has a 6% sales tax. Do you have enough money?

Find the sales tax and total cost.

STEP 1 Find the sales tax.

Sales Tax = Tax Rate × Price
Sales Tax = 0.06 × $5.94
= $0.36

STEP 2 Find the total cost.

Total Cost = Original Price + Sales Tax
Total Cost = $5.94 + $0.36
= $6.30

Solution: Since $6.30 is less than $6.50, you have enough money.

Another Example

Subtract Percent First

Find the sale price of a $38.00 model airplane if the discount is 20%.

$$100\% - 20\% = 80\%$$

100% represents the original price.

A 20% discount means that the sale price is 80% of the regular price.

Regular Price: 100% = $38.00	
20% off	Sale Price: 80% of $38.00

$$\begin{aligned}\text{Sale Price} &= 80\% \text{ of } \$38.00 \\ &= 0.8 \times \$38.00 \\ &= \$30.40\end{aligned}$$

Guided Practice

Find each sale price or total cost with sales tax. Round your answer to the nearest cent.

1. 20% off $15.95

2. $29.49 and 5% sales tax

3. 35% off $13.35

4. $9.99 and 6.25% tax

Ask Yourself

- How do I use the discount to find the sale price?
- How do I find the total cost, including sales tax?

Explain Your Thinking What information do you need to find the sale price?

Practice and Problem Solving

Find each sale price or total cost with sales tax. Round your answer to the nearest cent.

5. 50% off $21.50

6. 5% off $8.99

7. 30% off $32.48

8. 75% off $54.63

9. 20% off $17.79

10. 20% off $61.56

11. $5.00 and 7% tax

12. $31.04 and 6% tax

13. $13.48 and 5% tax

14. $15.75 and 4.5% tax

15. $7.99 and 5.5% tax

16. $6.45 and 5.5% tax

Estimate each sale price or total cost with sales tax.

17. 33% off $42.75

18. 13% off $5.99

19. 9% off $89.99

20. 22% off $101.66

21. 11% off $512.35

22. 15% off $6.50

23. $6.32 and 8% tax

24. $24.75 and 1.2% tax

25. $366.00 and 4.6% tax

26. $5.15 and 6.8% tax

27. $441.04 and 3.25% tax

28. $113.48 and 5.19% tax

Go On

Which computation method would you use to find each total cost? Write *mental math, paper and pencil*, or *calculator*. Then find the total cost.

29. 20% off \$40.00 and 5% tax

30. 25% off \$100.00 and 10% tax

31. 15% off \$53.75 and 8% tax

32. 10% off \$430.00 and 6.8% tax

Algebra • Equations

Solve each equation for *n*.

33. $\$36.00 = n \times \$80.00$

34. $\$3.75 = 0.25 \times n$

35. $n = 0.30 \times \$15.40$

36. $\$16.34 = n \times \$32.68$

Data

Use the sales-tax table to solve Problems 37–39. Round your answer to the nearest cent. Assume that the sales tax is computed on the original price.

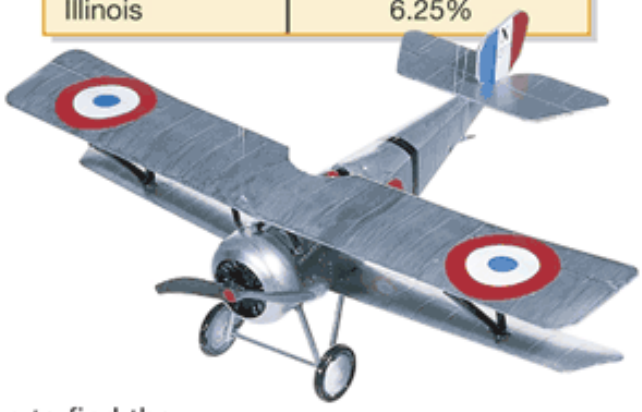
37. Tim has a coupon for \$5.00 off a model airplane. How much will he pay for a \$46.00 model airplane plus tax in Illinois?

38. A model railroad set costs \$150 in both Florida and Illinois. How much would you save in sales tax by purchasing the model railroad set in Florida?

39. A model kit sells for \$35.00 in North Carolina. In New Jersey, the same kit is \$34.00. In which state is the price of the model kit plus sales tax less?

State Sales Tax Rates

State	Tax Rate
Florida	6%
North Carolina	4.5%
New Jersey	6%
Illinois	6.25%



40. Write About It Write an equation that relates the sale price (s) to the original price (p) and discount (d%). Explain how you found the equation and why you think it would help to find the sale price.

41. You Decide Mark and Adela used different ways to find the sale price of a \$72.95 item marked down 20%. Explain the way each student solved the problem. Explain which way you prefer.

Mark

100% − 20% = 80%
80% of \$72.95
0.80 × \$72.95 = \$58.36

Adela

20% of \$72.95
0.20 × \$72.95 = \$14.59
\$72.95 − \$14.59 = \$58.36

Extra Practice See page 483, Set F.

Mixed Review and Test Prep

Open Response

Solve. (Ch. 12, Lesson 3)

42. $12 + 4 + j = 28$

43. $w = 11 + 50$

44. $h + 32 = 54 - 8$

45. $p - 9 + 16 = 24$

Multiple Choice

Choose the letter of the correct answer.

46. What is 80% of 445? (Ch. 18, Lesson 3)

A 55.6

B 3.56

C 356

D 5.56

Problem Solving

Real World Connection

Credit Card Debt

When you use a credit card to pay for a purchase, the bank pays the store for you. You must repay the debt, plus interest, on the money you owe.

1. Suppose you have a credit card with 2% monthly interest. If you make a $50 purchase and make $10 monthly payments, how many months will it take you to pay off your debt?

Number of Month	Beginning Balance	Monthly Interest at 2%	Balance with Interest	Payment	New Balance
1	$50.00	$1.00	$51.00	$10.00	$41.00
2	$41.00	$0.82	$41.82	$10.00	

2. Why does the money you pay in interest decrease each month?

3. **Analyze** Do you think you would pay more or less total interest if you reduced your credit card payments from $10 to $5 per month?

4. If you have a credit card with a beginning balance of $1,700 that charges 2% monthly interest and you have made 3 payments of $20, how much do you owe?

5. In Exercise 4, is what you owe after 3 payments more or less than your original balance? Explain.

6. **Create Your Own** Develop a chart similar to the one above. Determine a beginning balance, monthly interest, monthly payment, and calculate the balance with interest for a period of one year. What do you notice about the chart that you created? Explain.

WEEKLY WR READER eduplace.com/map

Chapter Review/Test

VOCABULARY

1. The rate of interest paid for a given period of time is the ____.
2. An amount of money invested or borrowed is the ____.
3. A(n) ____ is a decrease applied to the price of an item.

Vocabulary

discount
sales tax
principal
proportion
interest rate
simple interest

CONCEPTS AND SKILLS

Find the percent of each number. (Lesson 1, pp. 462–463)

4. 70% of 120
5. 110% of 150
6. 27% of 210
7. 8% of 98
8. 12.5% of 88
9. 4.4% of 220

Find each percent or number. (Lessons 2 and 3, pp. 464–467)

10. What percent of 115 is 23?
11. 10.5 is what percent of 70?
12. What percent of 240 is 210?
13. 18 is 45% of what number?
14. 72% of what number is 43.2?
15. 112 is 56% of what number?

Use the percent equation to solve. (Lesson 5, pp. 470–471)

16. What is 30% of 3,650?
17. 105 is what percent of 700?
18. What percent of 900 is 198?
19. 65% of what number is 552.5?

Find the simple interest and the sum of the principal and interest for each. (Lesson 6, pp. 472–473)

	Principal	*Interest Rate*	*Time*
20.	$2,150	5.0%	1 year
21.	$10,400	2.5%	6 months

Find each sale price or total cost with sales tax. Round your answer to the nearest cent. (Lesson 8, pp. 478–480)

22. 18% off $62.70
23. 7% off $8.50
24. $520.20 and 3.75% tax

PROBLEM SOLVING

Solve. (Lesson 4, p. 468; Lesson 7, pp. 474–476)

25. Benjamin has completed 70 percent of 60 math problems. How many problems does he still have to do?
26. How many ways could you pick 2 cards out of 6 if each card is a different color? Make a table.

Write About It

Show You Understand

Is it useful to have different ways to solve percent problems? Explain.

Extra Practice

Set A (Lesson 1, pp. 462–463)

Find the percent of each number.

1. 30% of 330
2. 350% of 100
3. 66% of 97
4. 4.5% of 480
5. 28.5% of 92
6. 75% of 250
7. 35% of 89
8. 28% of 261
9. 5% of 172
10. 2.5% of 88

Set B (Lesson 2, pp. 464–465)

Find each percent.

1. What percent of 60 is 105?
2. 35 is what percent of 140?
3. What percent of 87 is 26.1?
4. 30.8 is what percent of 77?

Set C (Lesson 3, pp. 466–467)

Find each number.

1. 18 is 20% of what number?
2. 70% of what number is 10.5?
3. 396 is 120% of what number?
4. $\frac{1}{4}$% of what number is 2?

Set D (Lesson 5, pp. 470–471)

Use the percent equation to solve.

1. What is 35% of 3,100?
2. 115 is what percent of 1,000?
3. What percent of 995 is 2,985?
4. 45% of what number is $562.50?

Set E (Lesson 6, pp. 472–473)

Find the simple interest and the sum of the principal and interest for each.

	Principal	*Interest Rate*	*Time*
1.	$250	5.0 %	1 yr
2.	$2,200	4.0 %	1 yr
3.	$970	1.5 %	1 yr
4.	$4,500	3.5 %	6 mo

Set F (Lesson 8, pp. 478–480)

Find each sale price or total cost with sales tax.
Round your answer to the nearest cent.

1. 25% off $30.80
2. $19.65 and 5.5% tax
3. 20% off $55.76
4. $9.95 and 6% tax
5. 30% off $8.90
6. $30.00 and 6.25% tax

CHAPTER 19

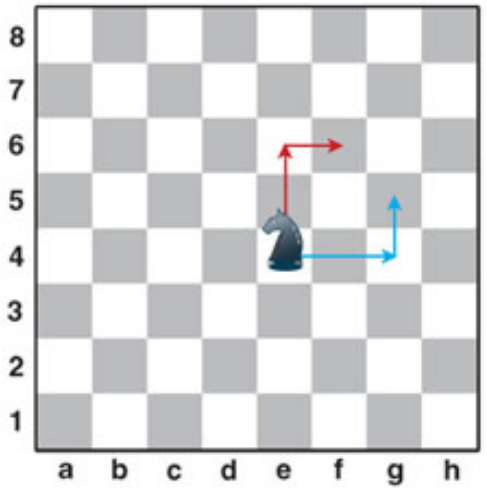
Probability

INVESTIGATION

Using Data

The knight is the only chess piece that may jump over other pieces. The knight moves like the letter "L." The "L" can be sideways, upside-down, or backwards.

The diagram shows two possible moves. Find all other possible moves. How many are there altogether?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. A(n) _____ is an event with a likelihood of 0.
2. The _____ gives the likelihood that an event will occur.
3. A diagram that shows all possible outcomes of an event is a(n) _____.

Vocabulary

- probability
- likely event
- tree diagram
- certain event
- impossible event
- dependent events

CONCEPTS AND SKILLS

Refer to the spinner for Exercises 4–10. Write *true* or *false* for each statement.



4. The probability of landing on red is 1 out of 4.
5. P(green) = 25%
6. P(blue) = 0.33
7. P(yellow) = $\frac{1}{4}$
8. The probability of landing on green or blue is 1 out of 4.
9. The probability of landing on green or yellow is 2 out of 4, or $\frac{1}{2}$.
10. The probability of landing on orange is 0.

Use the spinner for Exercises 11–19. Tell whether each event is *likely*, *unlikely*, *certain*, or *impossible*.



11. P(purple)
12. P(white)
13. P(not red)
14. P(blue)
15. P(yellow)
16. P(green)
17. P(not blue)
18. P(red)
19. P(purple, white, or yellow)

Write About It

20. The cards shown will be placed inside the bag. How likely is it that Ann will pick a red card from the bag?



Facts Practice, See Page 657.

Lesson 1

Possible Outcomes

Objective Find all possible outcomes of a multistep experiment.

Vocabulary

- sample space
- fundamental counting principle

Learn About It

Jerome and Tanya make up a game in which they first toss a number cube, labeled with the numbers 1, 1, 3, 3, 5, and 5, and then flip a coin. What are the possible outcomes of tossing a number cube and then flipping a coin?



A listing of all possible outcomes is called a **sample space**. There are different ways to find the sample space for this game.



Different Ways to Find Possible Outcomes

Way 1 You can make a list.

For each number tossed, list heads and tails.

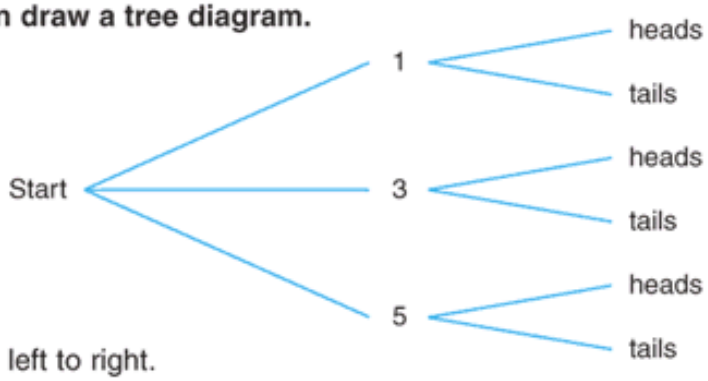
Number tossed,	Coin flipped
1,	heads
1,	tails
3,	heads
3,	tails
5,	heads
5,	tails

Way 2 You can use a grid.

Write each different pair of outcomes.

	heads	tails
1	1, heads	1, tails
3	3, heads	3, tails
5	5, heads	5, tails

Way 3 You can draw a tree diagram.



Trace each path from left to right.

Solution: There are six different possible outcomes: 1 and heads, 1 and tails, 3 and heads, 3 and tails, 5 and heads, and 5 and tails.

Sometimes you do not need to know the specific outcomes as just listed. If you only need to know the *number* of possible outcomes, you can use the **fundamental counting principle**.

> If an experiment or problem has two steps, with m different ways to do the first step and n different ways to do the second step, the total number of possible outcomes for the two-step sequence is $m \times n$.

In the previous example, there are 3 different outcomes (1, 3, or 5) when tossing the number cube ($m = 3$) and 2 different outcomes (heads, tails) when flipping the coin ($n = 2$).

$$\begin{aligned}\text{Total number of possible outcomes} &= m \times n \\ &= 3 \times 2 \\ &= 6\end{aligned}$$

Guided Practice

Ask Yourself
- Did I include all possibilities?
- Did I duplicate anything?

Make a list, a grid, and a tree diagram to show all possible outcomes.

1. Toss a number cube with numbers 1–6, then spin this spinner.





2. Flip a coin, then pick a card.



Use the fundamental counting principle to find the number of possible choices when you choose one item from each category.

3. 6 shirts, 3 pairs of shorts, 4 hats

4. 3 sizes, 4 colors

5. 3 soups, 4 main dishes, 4 desserts

6. 5 flowerpots, 8 kinds of plants

Explain Your Thinking What advantages are there for each of the three methods used for finding all possible outcomes?

Practice and Problem Solving

Make a list, a grid, and a tree diagram to show all possible outcomes.

7. Pick a card and toss a 1–6 number cube.



8. Spin the spinner and then flip a penny.





Go On

Use the fundamental counting principle to find the number of possible choices when you choose one item from each category.

9. 3 salads, 7 dressings

10. 4 pastas, 5 sauces

11. 3 cars, 5 colors, 4 options

12. 6 shirts, 5 ties, 3 jackets

13. 10 ice cream flavors, 5 toppings, 4 syrups, 3 portion sizes

Solve.

14. In Nigeria, people played the game Igba-Ita (which means to pitch and toss). In the game, players tossed 4 shells. Shells could land opening up or opening down. How many possible outcomes of tossing 4 shells are there?

15. In Igba-Ita, players won shells if all 4 shells landed up or landed down, or if 2 landed up and 2 landed down. How many winning outcomes are there?

16. **Explain** Elisa and Hector each toss two 1–6 number cubes and find the product of the two numbers. If the product is even, the player scores 1 point. If the product is odd, the player scores 3 points. Find all possible outcomes in this game. How many are even numbers? odd numbers?

17. **Write About It** Write a paragraph that explains why using the fundamental counting principle works. Include one example, using both a number cube and a spinner.



In some parts of Africa, people once used small shells called cowries as money. They also used them to play games.

Mixed Review and Test Prep

Open Response

Multiply. (Ch. 6 Lessons 2–3)

18. $\frac{5}{8} \times 1\frac{1}{2}$

19. $\frac{7}{10} \times 2\frac{4}{7}$

20. $9\frac{9}{10} \times 1\frac{2}{3}$

21. $\frac{8}{11} \times 10\frac{3}{4}$

22. $3\frac{5}{9} \times 1\frac{1}{8}$

23. $1\frac{7}{20} \times 3\frac{2}{3}$

Multiple Choice

24. Deli customers have a choice of 6 fillings, 5 types of bread, and 4 toppings. How many different sandwiches, consisting of 1 filling, 1 bread type, and 1 topping are there? (Ch. 19, Lesson 1)

A 15 **B** 30 **C** 24 **D** 120

Extra Practice See page 511, Set A.

Data Sense
Fair Games

Activity

A game is fair if all players have the same chance of winning.

Don and Melissa take turns tossing two 1–6 number cubes. If the sum of the two numbers tossed is 6, 7, or 8, Don scores 1 point. If any other sum comes up, Melissa scores 1 point. The winner is the player with the higher score after 15 rolls.

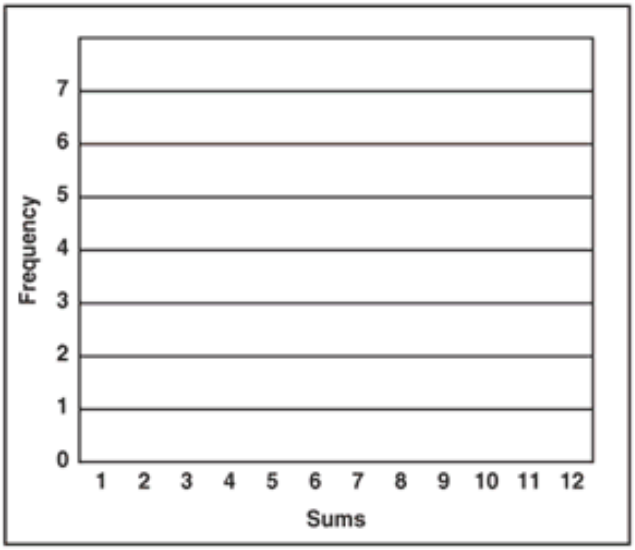
1 Copy and complete the table at right to find all possible outcomes. Then copy and complete the graph below.

2 How many of the possible outcomes are sums of 6, 7, or 8?

Second Cube

First Cube +	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

(Learning Tool 45)



(Learning Tool 45)

3 How many of the possible outcomes are sums other than 6, 7, or 8?

4 **Analyze** When Don and Melissa toss the number cubes, do they have the same chance of winning the game? Explain.

5 Do you think this game is fair? Explain why or why not. If you think the game is not fair, how could you make it fair?

6 **Create and Solve** Create two spinner games. One game should be fair, the other unfair. Ask another student to determine which is the fair game. For the unfair game, ask your partner to suggest a way to make it fair.



Lesson 2

Audio Tutor 2/21 Listen and Understand

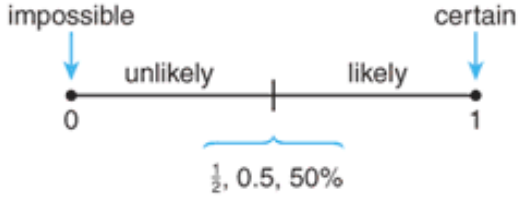
Theoretical Probability

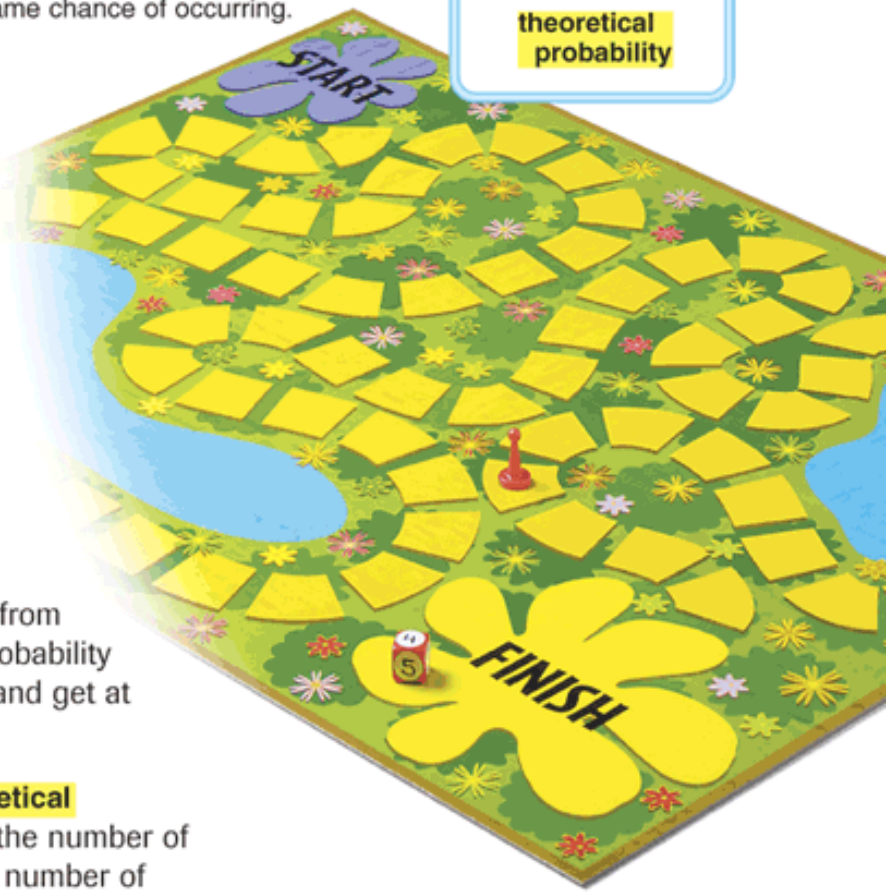
Objective Find probabilities if each outcome of an experiment has the same chance of occurring.

Vocabulary
- event
- probability
- theoretical probability

Learn About It

An **event** is one or more outcomes of an experiment. The **probability** of an event is a measure of the likelihood that the event will occur. A probability can be expressed as a ratio between 0 and 1. It can also be expressed as a decimal or as a percent.





Suppose a game piece is only four spaces from the last space on the board. What is the probability that a player will toss a 1–6 number cube and get at least a 4 to win the game?

If each outcome is equally likely, the **theoretical probability** P of an event A is the ratio of the number of possible outcomes in the event to the total number of possible outcomes.

$$P(A) = \frac{\text{number of outcomes in the event}}{\text{total number of possible outcomes}}$$

To read P(A), say "The probability of event A."

Find P(4 or higher).

STEP 1 Find the sample space.



There are 6 equally likely possible outcomes: 1, 2, 3, 4, 5, and 6.

STEP 2 Count the number of outcomes in the event *4 or higher*. Write the probability as a ratio.

There are three outcomes of 4 or higher: 4, 5, or 6

$$P(\text{4 or higher}) = \frac{\text{outcomes in event}}{\text{possible outcomes}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

Solution: For the event *4 or higher* on a number cube, the probability is 3 out of 6. $P(\text{4 or higher}) = \frac{3}{6} = \frac{1}{2} = 0.5 = 50\%$.

Extra Help at eduplace.com/map

Sometimes you need to find the probability that an event will *not* occur.

Suppose that there are 4 marbles of identical size—1 red, 1 blue, 1 green, and 1 yellow. Bill picks one marble without looking. What is the probability that he does not pick a blue marble?

If P(A) is the probability of an event A occurring, then the probability that the event A will *not* occur is 1 − P(A).

$$\text{P(not blue)} = 1 - \text{P(blue)}$$
$$= 1 - \frac{1}{4} = \frac{3}{4}$$

The probability that Bill will not pick a blue marble is $\frac{3}{4} = 0.75 = 75\%$.



Other Examples

A. Probability of 0

In the example above, what is the probability that Bill will pick a purple marble?

$$\text{P(purple)} = \frac{0 \leftarrow \text{outcomes in event}}{4 \leftarrow \text{possible outcomes}}$$

$$\text{P(purple)} = 0$$

B. Probability of 1

In the example above, what is the probability that Bill will pick a marble that is either red, blue, green, or yellow?

$$\text{P(red, blue, green, or yellow)} = \frac{4 \leftarrow \text{outcomes in event}}{4 \leftarrow \text{possible outcomes}}$$

$$\text{P(red, blue, green, or yellow)} = 1$$

Guided Practice

Use the following data for Problems 1–6.
Find each probability.

There are 20 pennies in a jar. Two were minted in 2002, 4 in 2003, 5 in 2004, 8 in 2005, and 1 in 2006. A penny is drawn at random and its date checked.

Ask Yourself
- How many outcomes are there in the event?
- How many possible outcomes are there?

1. P(2002) **2.** P(1995) **3.** P(not 2004)

4. P(2006) **5.** P(2003) **6.** P(not 2002)

Explain Your Thinking What is the mathematical relationship between the probability that an event will happen and the probability that the event will not happen?

Go On

Practice and Problem Solving



Find all possible outcomes for flipping three identical coins at once. Find each probability.

7. P(3 heads)

8. P(1 head, 2 tails)

9. P(no heads)

A jar contains 20 marbles that are the same size. There are 6 blue, 5 yellow, and 9 red marbles. Find each probability.

10. P(blue)

11. P(red)

12. P(yellow)

13. P(green)

14. P(not red)

15. P(not blue)

16. P(not yellow)

17. P(not white)

18. P(white)

Find the sample space for flipping four identical coins at once. Find each probability.

19. P(4 heads)

20. P(1 head, 3 tails)

21. P(2 tails, 2 heads)

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Tell which method you chose.

22. In the game *Products*, each player tosses two 1–6 number cubes and finds the product of the numbers tossed. The player who tosses the greater product scores 1 point. Find the probability of tossing a product greater than 18.

23. **Explain** If you draw a marble without looking from a jar filled with black and white marbles, you are 3 times more likely to draw a black marble than a white one. If there are 60 marbles in the jar, how many are black?

24. **Reasoning** At the beginning of a basketball game, each of the 11 players on the home team shook hands with each of the 11 players on the visiting team. How many handshakes took place?

25. The spinner for the board game *Did the Butler Do It?* is divided into 10 equal sectors, numbered from 1 through 10. Find the probability of the spinner *not* landing on an even number.

Mixed Review and Test Prep

Open Response

Use a proportion or an equation to find each percent. (Ch. 18, Lesson 1)

26. 25% of 200

27. 50% of 160

28. 33% of 33

29. 12% of 86

30. 23% of 70

31. 10% of 420

32. 1.8% of 500

33. 205% of $10

34. A spinner is divided into 3 equal sectors that are colored red, blue, and yellow. What is the probability that the arrow will land on red on the next spin? (Ch. 19, Lesson 2)

Explain how you found your answer.

Extra Practice See page 511, Set B.

Problem Solving

Data Sense

Disjoint Events

The mathematical study of probability grew out of the study of outcomes of games of chance. Probability theory didn't begin until after the 1700s and is continuing to the present day.



Hilda Geiringer von Mises (1893–1973) was an Austrian-born mathematician. She made important contributions to probability theory.

If A and B are events with no outcome in common, then A and B are said to be **disjoint events**. The probability of *either* event occurring is the sum of the individual probabilities:

$$P(A \text{ or } B) = P(A) + P(B).$$

The table at right shows all the possible outcomes for tossing two 1–6 number cubes. What is the probability of tossing a sum less than 7 or a sum greater than 7?

Think of a sum < 7 as event A.
Think of a sum > 7 as event B.

The two events have no outcome in common, because it is impossible for a sum to be less than 7 and greater than 7 at the same time.

Results of Tossing 2 Number Cubes

Second Number Cube \ First Number Cube	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

$$P(A \text{ or } B) = P(A) + P(B) = P(< 7) + P(> 7)$$
$$= \frac{15}{36} + \frac{15}{36} = \frac{5}{12} + \frac{5}{12}$$
$$= \frac{10}{12} = \frac{5}{6}$$

Use the table above to find the probability of the following disjoint events.

1. sum is odd or sum is even
2. sum is 3 or sum is 10
3. sum is 2 or sum is > 9
4. **Analyze** Use the table. Are events C (sum is not 7) and D (sum is odd) disjoint or not? Explain.
5. Describe how finding the probability of disjoint events could be related to the formula for finding probability.

Hands On Lesson 3

Experimental Probability

Objective Identify the difference between experimental and theoretical probability.

Vocabulary

experiment

experimental probability

Work Together

Materials

2 identical coins
2 number cubes
1 blank spinner
red, blue, and green markers
Learning Tool 46

In Lesson 2 you learned about theoretical probability. You can also do an **experiment** and collect data to find **experimental probability**.

Theoretical probability tells you what the outcome of an experiment should be when the number of trials is large. In an actual experiment, you may not get the results predicted by the theoretical probability because the number of trials is not large enough.



Work with a partner to find the experimental probability, P(red), of spinning red.

STEP 1 Make a spinner with 8 equal sectors. Color 4 sectors red, 3 sectors blue, and 1 sector green.

STEP 2 Calculate the theoretical probability of spinning red. Then use this result to calculate the result you would expect if you spin the spinner 24 times.

STEP 3 Spin the spinner 24 times. Use tally marks to record the number of times you spin red. Note: Record only the number of favorable outcomes.

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
P(red) = $\frac{1}{2}$	24	$\frac{1}{2} = \frac{12}{24}$		

STEP 4 Find the experimental probability of spinning red. Write the probability as a ratio in decimal or percent form.

STEP 5 Compare the experimental probability with the theoretical probability of spinning red. What do you notice?

- How might the experimental probability change if you were to complete twice as many trials? four times as many? Explain.



On Your Own

Copy and complete the table for each probability experiment. Record each probability.

1. Toss two coins and get heads both times, P(heads, heads).

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
$\frac{1}{4}$	60			

2. Toss two number cubes and get a sum of 7, P(sum of 7).

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
$\frac{6}{36} = \frac{1}{6}$	60			

3. Toss two number cubes and get an even sum, P(even sum).

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
$\frac{18}{36} = \frac{1}{2}$	20			

4. Repeat Exercise 3, but complete 100 trials.

Theoretical Probability	Number of Trials	Expected Results	Actual Results	Experimental Probability
$\frac{18}{36} = \frac{1}{2}$	100			

Talk About It • Write About It

You learned how to find experimental probability.

5. In your own words, explain the difference between experimental probability and theoretical probability.

6. Use your results from Exercises 3 and 4. Does the experimental probability get closer to or farther from the theoretical probability as the number of trials increases? Add your results to those of a classmate and compare the experimental and theoretical probabilities.

Lesson 4

Audio Tutor 2/22 Listen and Understand

Compound Events

Objective Find the probability of two independent and two dependent events.

Vocabulary
- compound event
- independent
- dependent event

Learn About It

A **compound event** consists of the outcomes of two or more events. Sometimes the events that make up a compound event are independent events. Two events are **independent** if the outcome of one does not change the outcome of the other.



To find the probability of two independent events occurring together, you multiply the individual probabilities.

$$P(A \text{ and } B) = P(A) \times P(B)$$

Vince and Linda are playing a game of backgammon. Vince needs to roll doubles two times in a row to win. What is the probability that Vince will roll a double on his next turn (call it N) *and* again on the following turn (call it F)?

If Vince rolls a double on his next turn, it does not affect the likelihood that he will roll a double on the following turn. The events are *independent*.

Find P(N and F).

STEP 1 Determine the theoretical probability of each event. Simplify, if possible.

$$P(N) = \frac{6}{6} \times \frac{1}{6} = \frac{6}{36} = \frac{1}{6}$$

$$P(F) = \frac{6}{6} \times \frac{1}{6} = \frac{6}{36} = \frac{1}{6}$$

STEP 2 The events are independent, so find the product of the individual probabilities.

$$P(N \text{ and } F) = P(N) \times P(F)$$

$$= \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Solution: The probability that Vince will roll doubles on his next turn and on the following turn is $\frac{1}{36}$.

Sometimes the events that make up a compound event are dependent. **Dependent events** are events in which the outcome of the first event *does* affect the outcomes that may occur in the second event.

Helen and Tim are playing a game called *Guess My Identity*. From 26 cards labeled *A* through *Z*, Tim draws a card with the letter *G*. Without replacing the first card, he draws a second card with the letter *W*. What is the probability of Tim drawing cards with the letters *G* and *W*?

Since the first card drawn is not replaced, the outcome of the first event affects the outcome of the second event. The events are *dependent*.

Find P(G and W).

STEP 1 Determine the theoretical probability of each event. Simplify, if possible.

$$P(G) = \frac{\text{number of cards with } G}{\text{total number of cards}} = \frac{1}{26}$$

Tim draws the letter *G* and keeps it, so

$$P(W) = \frac{\text{number of cards with letter } W}{\text{total number of cards left in deck}} = \frac{1}{25}$$

STEP 2 Find the product of the individual probabilities.

$$P(G \text{ and } W) = P(G) \times P(W)$$

$$P(G \text{ and } W) = \frac{1}{26} \times \frac{1}{25} = \frac{1}{650}$$

Solution: The probability of Tim drawing cards with the letter *G* first and the letter *W* second is $\frac{1}{650}$.

Other Examples

A. Probabilities With *Not*

Flip a coin and toss a 1–6 number cube. Find the probability of the event *heads and a number that is not 4*.

$$P(H \text{ and not } 4) = P(H) \times P(\text{not } 4)$$

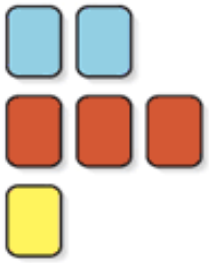
$$= \frac{1}{2} \times (1 - \frac{1}{6})$$

$$= \frac{1}{2} \times \frac{5}{6}$$

$$= \frac{5}{12}$$

B. Picking Two Cards in Order

Find P(blue, red) if the first card is not returned.



$$P(\text{blue}) = \frac{2}{6} = \frac{1}{3}$$

$$P(\text{red}) = \frac{3}{5} \leftarrow \text{one fewer card}$$

$$P(\text{blue, red}) = \frac{1}{3} \times \frac{3}{5} = \frac{3}{15}$$

$$= \frac{1}{5}$$

Guided Practice

For tossing a 1–6 number cube and spinning the spinner, find each probability.

1. P(1 and blue)
2. P(4 and not blue)
3. P(yellow and an odd number)



Ask Yourself

- Does the outcome of the first event affect the outcome of the second event?
- Did I subtract from 1 to find probabilities with *not*?

Go On

Use the cards to find the probability of each compound event. The first card is not returned before the second card is picked.

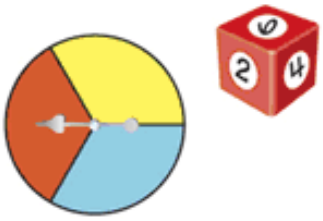


4. P(blue, yellow)
5. P(red, blue)
6. P(yellow, yellow)

Explain Your Thinking How can the events that make up a compound event be either dependent or independent?

Practice and Problem Solving

For tossing a number cube and spinning the spinner, find each probability.



7. P(6 and red)
8. P(1 and yellow)
9. P(a multiple of 3 and blue)
10. P(1 and orange)
11. P(an even number and not red)
12. P(blue and 15)

Use the cards to find the probability of picking two cards in the order and colors listed. The first card is not returned.

13. P(blue, yellow)
14. P(red, blue)
15. P(blue, not yellow)



Data **Use the table for Problems 16 and 17.**

16. Carla and Hannah are the first two game winners to choose prizes from the treasure bag. What is the probability that they both will pick spiders?
17. **Analyze** After five spiders and eight stickers have been chosen, Brian picks two prizes. What is the probability that both are key rings?
18. **Write About It** Describe a real-life situation that involves a compound event with two independent events. Then describe a situation that involves a compound event with two dependent events.

Type of Boxed Prize	Number in Bag
Press-on sticker	21
Glow-in-the-dark spider	25
Key ring	3
$20 bill	1
Total Number of Prizes	**50**

Extra Practice See page 511, Set C.

Data Sense

What Are the Odds?

Activity

Odds compare favorable outcomes to unfavorable outcomes.

On the spinner at the right, the odds in favor of spinning an even number are 4 to 5, or $\frac{4}{5}$. The odds of spinning an odd number are 5 to 4, or $\frac{5}{4}$. Finding odds is related to finding probability but is not the same: P (even) $= \frac{4}{9}$ and P (odd) $= \frac{5}{9}$.



1. In the example above, what are the *odds against* spinning an even number?
2. In the example above, what are the *odds in favor* of spinning a 5? a 0?

Calculate the odds in favor of and against each event.

3. Spin a fair spinner with letters *A* through *Z*. The spinner lands on the first letter of your last name.
4. Toss a 1–6 number cube. The number tossed is a multiple of 2.

Quick Check

Check your understanding of Lessons 1–4.

Solve.

1. List the possible outcomes of tossing two coins. (Lesson 1)
2. Find P(all heads or all tails) if two identical coins are tossed. (Lessons 2 and 3)



3. Use the cards to find the probability of picking a green card first and then an orange card. The first card is not returned before the second card is picked. (Lesson 4)



For tossing the 1–6 number cube and spinning the spinner, find each probability. (Lesson 4)

4. P(5 and blue)
5. P(not an odd number and yellow)

Lesson 5

Problem-Solving Strategy

Use Models

Objective Use a model to simulate the solution to a problem.



Problem Fred's baseball coach estimates that the team has a 50% chance of winning on Friday. Fred's batting average (the ratio of hits per times at bat) is .200. What is the probability that Fred will get a hit *and* his team will win on Friday?

UNDERSTAND

This is what you know:

- The chance of winning is 50%.
- The chance that Fred will get a hit is 20%.

PLAN

Use two spinners, one to represent each event. Spinning each spinner once simulates what may happen in any given game.

SOLVE

- Model the team's chance of winning the game. 50% is $\frac{1}{2}$.
- Model Fred's chance of getting a hit. 20% $= \frac{1}{5}$.





Trial	1	2	3	4	5	6	7	8	9	10
Outcome	L, N	***W, H***	L, H	L, N	W, N	L, N	L, N	L, N	***W, H***	W, N
Trial	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
Outcome	L, N	L, N	***W, H***	W, N	L, N	W, N	W, N	L, N	W, N	L, N

- Perform 20 trials. This table shows the results of an experiment.

$$\text{Experimental Probability} = \frac{\text{Both } \textit{Win} \textbf{ and } \textit{Hit}}{\text{Total Number of Trials}} = \frac{3}{20}$$

Solution: Based on 20 trials, the experimental probability that the team will win and Fred will get a hit is $\frac{3}{20}$ or 15%.

LOOK BACK

Look back. Is the simulation a good representation of the problem?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Your dad estimates that there is a 40% chance that he will be able to attend your baseball game on Thursday. The forecast calls for a 50% chance of rain. What is the probability that it won't rain and your dad will attend the game?

2. There is a 60% chance that your team will win the last game of the season and a 75% chance of winning the championship. What is the probability that your team will win both the last game and the championship?

Ask Yourself

UNDERSTAND — What facts do I know?

PLAN — How can I use models to simulate the problem?

SOLVE — Did I accurately record the results of my simulation?

LOOK BACK — Are the models a good representation of the problem?

Independent Practice

Use models to solve each problem. Express each probability as a ratio, a decimal, and a percent.

3. Alan is playing a board game with his friends. To move, he needs to roll two 1–6 number cubes. What is the probability that Alan will roll doubles on each of his next two turns?

4. To win, Alan needs to roll precisely the sum 8 on his next turn. What is the probability that Alan will win the game?

5. Brittany is playing a game in which a spinner is divided into ten equal regions numbered 1 through 10. To win, she must spin a 2 and a 7 on her next two turns. What is the probability that she will win?

6. Vernon estimates that if he gets a math question in a trivia game, with an equal number of questions in 6 categories, he has an 80% chance of getting the answer correct. What is the probability that Vernon will get a math question and answer it correctly on his next turn?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

7. Kate wants to collect all 6 different prizes that come with a new brand of cereal. There is an equally likely chance of any one prize being in any one box. What is the probability that the first 3 boxes Kate buys will contain the same prize?

8. **Analyze** Suppose you schedule the games for your volleyball league's tournament. If a team loses a game, it is eliminated. There are 32 teams. How many games do you need to schedule to determine the league champion?

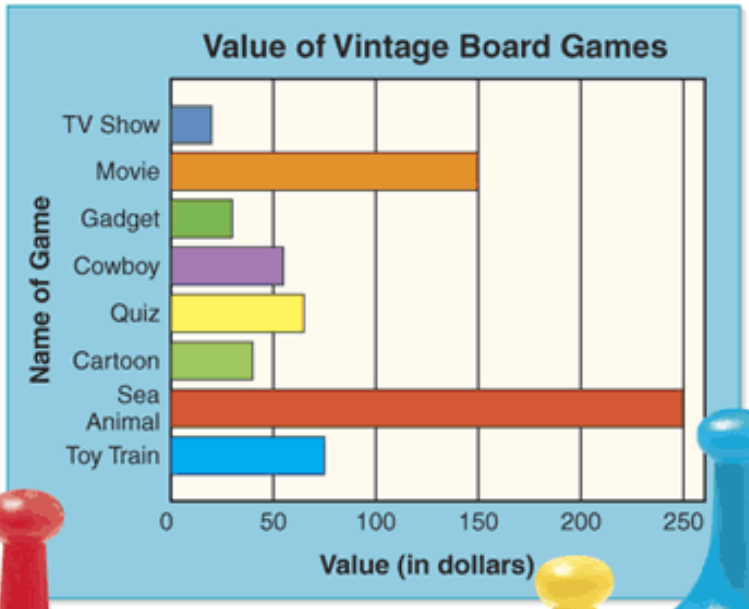
9. Jack, Jill, Jean, and Joe won first, second, third, and fourth place in a chess tournament. Jack did not do as well as Jill. Jean did better than Jack but did not win the first place. Joe came in fourth. What is the correct order of the winners?

You Choose

Strategy
- Draw a Diagram
- Use Models
- Solve a Simpler Problem
- Make an Organized List
- Use Logical Reasoning

Computation Method
- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the bar graph to solve Problems 10–14.**



10. Which are the most expensive and the least expensive board game for sale?

11. To the nearest dollar, what is the difference in price between the most expensive and least expensive games?

12. Suppose that the movie game originally sold for $10. By what percent has its value increased?

13. **Explain** Find the mean and the median price of the games. Which statistic best describes the price data, the mean or the median?

14. **Create and Solve** Write a problem that can be solved by using the graph. Give the problem to a classmate to solve.



Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer. If a correct answer is not here, choose NH.

1. There are 32 students in the band and 44 students in the chorus. Eight students are in both the band and the chorus. How many students are enrolled in these two programs?

A 24 **B** 36 **C** 68 **D** 6

(Chapter 11, Lesson 9)

2. Otis runs 4 kilometers in 18 minutes. He plans to run in a 15-kilometer race. If he runs at the same pace, how long will it take him to complete the race?

F 67.5 minutes **H** 72 minutes

G 65 minutes **J** NH

(Chapter 17, Lesson 5)

Open Response

Solve each problem.

3. At a garage sale, paperback books were priced at \$0.60 each and hardcover books were priced at \$1.00 each. Ella spent \$4.40. How many of each kind of book did she buy?

Explain how you found the answer.

(Chapter 9, Lesson 5)

4. Suppose you have 24 feet of fencing to make a rectangular kennel for your puppy. Each side will be a whole number of feet. List all possible dimensions for the kennel. Which will give your puppy the greatest area?

(Chapter 11, Lesson 3)

Extended Response

5. You have kept a record of the daily temperature for the past week.

Daily Temperatures Fahrenheit

Day	High	Low
Sunday	14°	⁻9°
Monday	17°	⁻4°
Tuesday	30°	11°
Wednesday	33°	15°
Thursday	26°	7°
Friday	20°	⁻2°
Saturday	21°	⁻3°

a. Make a double line graph showing the highest and lowest temperatures for last week. Label each axis and make a key.

b. What day had the greatest difference between the highest and lowest temperature? What was the difference?

c. It snowed last night and you have to shovel the snow from your patio. The patio measures 8 ft by 10 ft. The snow is about 18 inches deep. What do you estimate the volume of snow is that you need to shovel off the patio? Explain how you know.

(Chapter 19)



Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 6

Audio Tutor 2/23 Listen and Understand

Combinations and Permutations

Vocabulary
combination
permutation

Objective Find combinations and permutations.



Learn About It

Combinations are ways of selecting items from a set when the order is not important.

On Game Day, five students want to be on the chess team. Only 4 students will be chosen. How many different chess teams of 4 students can be formed from a group of 5 students?

Find the number of combinations of 4 students chosen from a group of 5.

List the choices of 4 students from among students A, B, C, D, E:

A-B-C-D **A-B-C-E** **A-C-D-E** **A-B-D-E** **B-C-D-E**

The combination D-C-B-A is not listed because it has the same letters as A-B-C-D.

Solution: Five different 4-student chess teams can be formed from a group of 5 students.

Permutations are ways of arranging items chosen from a set when the order *is* important.

Find all the permutations of the 4 students A, B, C, D.

List the arrangements for students A, B, C, D:

A-B-C-D	**B-C-D-A**	**C-A-B-D**	**D-A-B-C**
A-B-D-C	**B-C-A-D**	**C-A-D-B**	**D-A-C-B**
A-C-D-B	**B-A-C-D**	**C-B-A-D**	**D-B-A-C**
A-C-B-D	**B-A-D-C**	**C-B-D-A**	**D-B-C-A**
A-D-B-C	**B-D-A-C**	**C-D-A-B**	**D-C-A-B**
A-D-C-B	**B-D-C-A**	**C-D-B-A**	**D-C-B-A**

You can use the fundamental counting principle to find the number of permutations.

$4 \times 3 \times 2 \times 1 = 24$ permutations.

(1st place) (2nd place) (3rd place) (4th place)



Solution: There are 24 different ways in which the 4 students can be arranged.

Guided Practice

Find the number of combinations.

1. Choose 2 people from 3.

2. Choose 3 people from 4.

Ask Yourself

- Does the order matter?
- Did I make an organized list to find all possible combinations?

Find the number of permutations for each group of letters. You may use a calculator.

3. B, A, T

4. S, A, C, K

5. B, E, A, U, T, Y

Explain Your Thinking What is the difference between finding all possible combinations and finding all possible permutations?

Practice and Problem Solving

Find the number of combinations.

6. Choose 2 people from 5.

7. Choose 3 people from 5.

8. Choose 4 people from 5.

9. Choose 5 people from 5.

10. Choose 3 people from 6.

11. Choose 1 person from 3.

Find the number of permutations for each group of letters. You may use a calculator.

12. M, A, T, H

13. I, N, C, L, U, D, E

14. F, U, T, U, R, E

15. A, S, T, R, O, N, O, M, Y

16. L, A, N, G, U, A, G, E

17. W, O, R, L, D

Solve.

18. Four students want to be in the water-balloon toss. Only two students can participate. How many different pairs of students can the teacher select?

19. There are three different track events to be held on Games Day. In how many different orders can the three track events take place?

20. **Reasoning** In the 100-yard dash, Tyrone came in after Judith. Marissa was ahead of Tyrone. Ron came in last. What was the correct order in which the four crossed the finish line?

21. **Create and Solve** Write a realistic problem that involves combinations and another that involves permutations. Exchange papers with a classmate and solve each other's problems.

Mixed Review and Test Prep

Open Response

Find each total cost with sales tax. Round your answer to the nearest cent. (Ch. 18 Lesson 8)

22. $8.99 and 5% tax

23. $42.88 and 7% tax

24. $15 and 4.5% tax

Multiple Choice

25. How many different arrangements of 3 winners are possible out of 8 runners in a race? (Ch. 19, Lesson 6)

A 336 **B** 224 **C** 56 **D** 12

Extra Practice See page 511, Set D.

Lesson 7

Problem-Solving Application

Use Data to Make Predictions

Objective Use data to estimate probability.

You can use what you know about probability to make predictions.

Sally wants to schedule the three-day softball championship playoffs in June. In the past 3 years, there were 72 sunny days out of 90 in June. What is the probability of having 3 sunny days in a row in June?



UNDERSTAND

This is what you know:

- The championship playoffs last 3 days.
- In the past 3 years, 72 out of 90 June days were sunny.

PLAN

First, estimate the probability that any given day in June will be sunny. Then, use the formula for the probability of independent events.

SOLVE

Find the probability of events A, B, and C, where each event represents a sunny day.

Event	Description	Probability
A	First day is sunny.	$\frac{72}{90} = \frac{4}{5}$
B	Second day is sunny.	$\frac{72}{90} = \frac{4}{5}$
C	Third day is sunny.	$\frac{72}{90} = \frac{4}{5}$

Probability of one sunny day in June $= \frac{72}{90} = \frac{4}{5}$

$$P(\text{A and B and C}) = P(A) \times P(B) \times P(C)$$

$$= \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$$

$$= \frac{64}{125} \text{ or } 0.512 \approx 50\%$$

Solution: The probability of 3 sunny days in a row is about 50%.

LOOK BACK

Look back at the problem. Is your answer reasonable?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. In her past 9 games, Pat had 9 hits in 30 times at bat. What is a reasonable prediction of the probability that Pat will have no hits during her next 3 times at bat?

   Hint What is Pat's batting average?

2. What is a reasonable prediction of the probability that Pat will have exactly 1 hit in her next 3 times at bat? exactly 2 hits?

Ask Yourself

- UNDERSTAND — What events make up the compound events?
- PLAN — Are the events independent or dependent?
- SOLVE — Are my computations correct?
- LOOK BACK — Is my answer reasonable?

Independent Practice

Solve.

3. Bob's batting average is .200. Predict the probability that Bob will have no hits in his next 3 times at bat.

4. Use the batting average data in Problem 3 to estimate the probability that Bob will have 5 hits in a row.

5. During the past 4 years, it has snowed 40 out of 120 days during November. This year, the football coach thinks his team has a 60% chance of winning. Find the probability that it will not snow and the team will win.

6. Paula has a batting average of .325. Predict how many hits Paula will get if she bats 40 times. What is a reasonable prediction of the probability that she will get 2 hits in her next 3 times at bat?

7. Rodney had 2 hits in 8 at bats in the first game. He got 1 hit in 6 at bats in the second game, and 3 hits in 10 at bats in the third game. Predict the probability that he will have exactly 1 hit in 3 times at bat in his fourth game.

8. The Kennedy High School basketball team has won 8 games and lost 4 games so far this season. What is the probability that the team will win its next two games based on its record so far this season?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

9. Ahmed can run 5 kilometers in a half hour. In the same amount of time, Drew can run 3 kilometers. How much of a head start does Drew need in order for both runners to finish an 8-kilometer race at the same time?

10. The town of Thornville is 15 miles south of Chesterton. Thompson is 12 miles north of Thornville. Limburgh is 4.5 miles north of Thompson. Order the towns from south to north.

11. **Analyze** Amanda is creating a mosaic design in which there are 49 red tiles in the first row, 42 red tiles in the second row, and 35 red tiles in the third row. If her pattern continues, how many red tiles will there likely be in the seventh row?

12. The cost of renting a canoe can be represented by the expression $\$12 + \$10h$, where \$12 is the base charge and h is the number of hours the canoe is rented. What is the cost of renting a canoe for 7 hours?

You Choose

Strategy

- Draw a Diagram
- Find a Pattern
- Use Logical Reasoning
- Use Models
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Solve. Tell which method you chose.

13. Of all the performers in a community theater musical, 20% are children. If there are 8 children in the musical, how many performers are in the musical altogether?

14. Yolanda has saved \$50.38 from her baby-sitting jobs. How many CDs can she buy with \$50.38 if one CD costs \$12.95 plus 6.25% sales tax?

15. Of the 42 teachers at Sandy Lane School, 50% have been at the school for 10 years or more. How many teachers have been at the school for less than 10 years?



16. Seth plans to write a book report on the book he is currently reading. He has read 42 out of the 178 pages. What percent of the book does Seth have left to read?

Mixed Review and Test Prep

Open Response

Find the total with simple interest for each situation. (Ch. 18, Lesson 7)

17. $180 with 7% interest for 3 years

18. $556 with 2.5% interest for 2 years

Multiple Choice

19. A train travels 488 miles in 8 hours. What is its rate of travel? (Ch. 16, Lesson 2)

A 80 mi/h **C** 60 mi/h

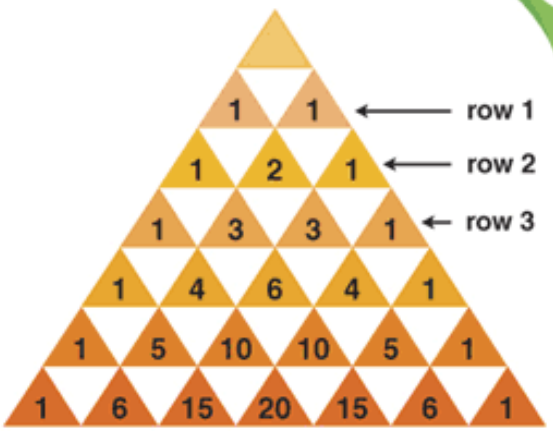
B 61 mi/h **D** 10 mi/h5

Problem Solving

Math Challenge

Pascal's Triangle

The triangular pattern at right is called Pascal's Triangle. It was named after Blaise Pascal, a French mathematician who lived 1623–1662. He was one of the first mathematicians to write on probability.



There are many patterns in the rows and diagonals in this triangle. For instance, the first and last numbers in each row are 1. Every other number is the sum of the two numbers that lie directly above it.

1. **Represent** Write rows 8 and 10.
2. Find the sum of the numbers in each row. What do you notice? Predict the sum for row 7.
3. At right is a diagram of 6 city blocks in a 2×3 array. How many different ways can you find to walk from *A* to *B* without ever moving down or to the left? Two different routes are shown.



4. **Analyze** How is the answer to Problem 3 related to Pascal's triangle?

Create Your Own Extend Pascal's Triangle containing at least 8 more rows. What are the numbers in each row?

Can you design a Pascal's Triangle in another way than the one that is shown? Explain.

Chapter Review/Test

VOCABULARY

1. One or more outcomes of an experiment is a(n) _____.
2. A(n) _____ consists of the outcomes of two or more events.
3. Two or more _____ have no effect on each other.

Vocabulary

- event
- probability
- sample space
- compound event
- dependent event
- independent events

CONCEPTS AND SKILLS

Find the number of possible choices when you choose one item from each category. (Lesson 1, pp. 486–488)

4. 3 hats, 2 scarves, 3 pairs of gloves
5. 8 pairs of socks, 4 pairs of shoes
6. 2 coats, 2 pairs of boots, 2 vests
7. 5 T-shirts, 4 pairs of shorts

A jar contains 30 marbles that are the same size. There are 7 purple, 8 red, and 15 pink marbles. Find each probability. (Lesson 2, pp. 490–492)

8. P(purple)
9. P(red)
10. P(yellow)
11. P(not red)
12. P(not pink)
13. P(not green)

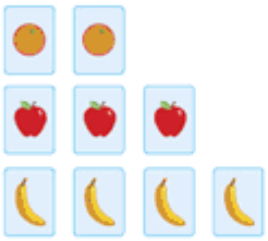
For tossing a coin and spinning the spinner, find each probability. (Lesson 4, pp. 496–498)





14. P(tails, green)
15. P(not heads, white)
16. P(heads, not white)
17. P(not tails, not blue)

Use the cards to find the probability of picking two cards in the order listed. The first card is not returned before the second card is picked. (Lesson 4, pp. 496–498)



18. P(apple, orange)
19. P(apple, not banana)
20. P(banana, not apple)

Find the number of combinations. (Lesson 6, pp. 504–505)

21. Choose 2 people from 7.
22. Choose 1 person from 6.

Find the number of permutations. (Lesson 6, pp. 504–505)

23. G, A, L, A, X, I, E, S
24. R, O, C, K, E, T

PROBLEM SOLVING

Solve. (Lesson 5, pp. 500–502; Lesson 7, 506–508)

25. Jessica says that there's a 50% chance that she'll go to a movie this weekend and a 30% chance that she'll go to the park. Determine the probability that she will do both.

Write About It

Show You Understand

Tell about some real-life situations, besides sports and entertainment, in which you might use probability to determine the likelihood of events.

Extra Practice

Set A (Lesson 1, pp. 486–488)

Use the fundamental counting principle to find the number of possible choices when you choose one item from each category.

1. 4 soups, 3 salads
2. 5 vegetables, 6 dips
3. 4 pies, 6 ice creams
4. 3 cracker types, 8 cheeses, 6 fruits
5. 15 main courses, 4 sides, 3 appetizers

Set B (Lesson 2, pp. 490–492)

A jar contains 25 beads that are the same size. There are 8 green, 10 white, and 7 orange beads. Find each probability.

1. P(green)
2. P(white)
3. P(orange)
4. P(yellow)
5. P(not green)
6. P(not white)
7. P(not orange)
8. P(not yellow)
9. P(red)
10. P(not red)

Set C (Lesson 4, pp. 496–498)

For tossing a 1–6 number cube and spinning the spinner, find each probability.

1. P(1 and blue)
2. P(4 and white)
3. P(5 and red)
4. P(a multiple of 2 and blue)
5. P(3 and not white)
6. P(6 and not blue)



Use the cards to find the probability of picking two cards in the order listed. The first card is not returned before the second card is picked.

7. P(star, diamond)
8. P(diamond, happy face)
9. P(diamond, not diamond)
10. P(happy face, happy face)



Set D (Lesson 6, pp. 504–505)

Find the number of combinations.

1. Choose 2 people from 6.
2. Choose 1 person from 4.
3. Choose 4 people from 6.
4. Choose 3 people from 4.
5. Choose 5 people from 6.
6. Choose 2 people from 4.

Find the number of permutations for each group of letters. You may use a calculator.

7. S, T, U, D, Y
8. M, A, R, S
9. U, N, I, V, E, R, S, E
10. S, C, I, E, N, T, I, S, T

WEEKLY READER®
Social Studies Connection

Our Crowded Planet

If you've ever been to a big city, you know how crowded certain places can get. In 2000, the world's population exceeded 6 billion people. The world's population is expected to keep growing by about 1 billion people every 15 years.

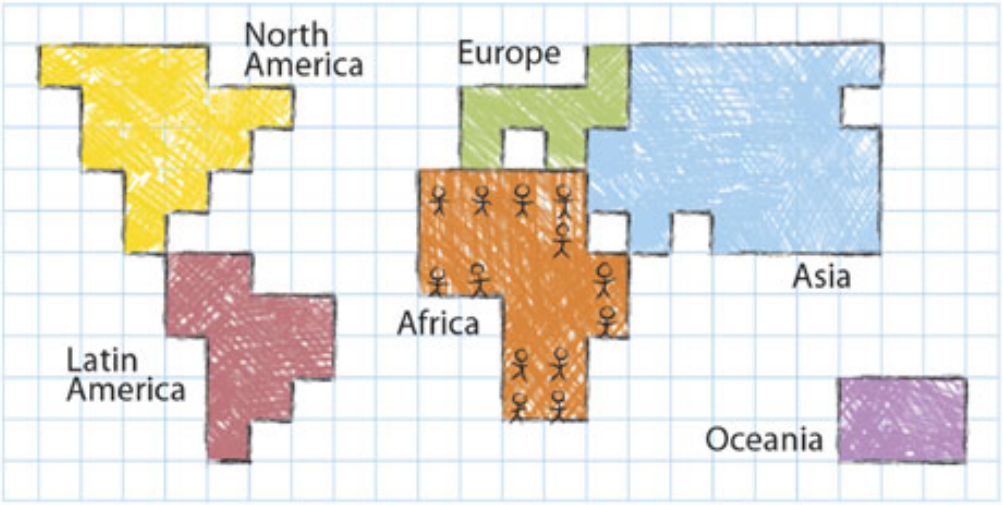
What regions are the most crowded? the least crowded?



Region	% of Land Area	Population (2000)	% of Population
World	100%	6,056,715,000	100%
Africa	20%	793,627,000	13%
Asia	30%	3,672,342,000	?
Europe	7%	727,304,000	?
Latin America	12%	518,809,000	?
North America	16%	314,113,000	?
Oceania	6%	30,521,000	?

Problem Solving

The map below shows the percent of the world's land in six regions. Each square represents 1% of the world's land area.



Use the table on page 512 and the map above for Problems 1–4.

1. **Calculator** Find the percent of the world's population in each region to complete the table. Round to the nearest whole percent.

2. Copy the map above onto grid paper. Draw stick figures to represent the percent of the world's population in each region. Let one stick figure represent 1% of the world's population.

3. Which regions are the most crowded? the least crowded?

4. **Estimate** By 2050, the world's population may reach 9 billion people. Suppose the percents stay the same. Predict what each region's population will be in 2050.



Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic

Enrichment: Number Sense

percent of change



When an amount changes, the percent of increase or decrease of that change is called the *percent of change*. The percent of change describes what percent of the original amount the change represents.

Suppose you want to enlarge a 2-inch × 3-inch photo to a 20-inch × 30-inch poster. What is the percent of change in size?

STEP 1 Find the change in each measure.

width: $20 - 2 = 18$

length: $30 - 3 = 27$

STEP 2 Write a ratio.

width: change in size → $\frac{18}{2} = \frac{9}{1}$
original size →

length: change in size → $\frac{27}{3} = \frac{9}{1}$
original size →

STEP 3 Write a proportion to find p, the percent of change.

$$\frac{9}{1} = \frac{p}{100}$$

STEP 4 Solve the proportion for p.

$$9 \times 100 = p \times 1$$

$$900 = p$$

Solution The percent of change in size is 900%.

An increase of 900% is an example of a percent of increase. A percent of decrease is a change that causes an amount to decrease. Reducing a photo's size or the price of a sale item are examples of percents of decrease.

Try These!

Find the percent of change for the following situations. Classify each as a percent of increase or decrease.

1. A \$10 poster is on sale for \$7.50.
2. The number of photos a photo lab processes per hour goes from 550 to 715.
3. A 12-inch × 18-inch poster is reduced to 3 inches × $4\frac{1}{2}$ inches.

Explain Your Thinking ▶ In what situations is a percent of change 100%? What is the relationship of the original amount to the amount of change?



Pay Day



Between her allowance and baby-sitting, Kathryn earns $50 a month. She wants to budget her money so she can put 20% in the bank and divide the rest between each of the following categories: clothes, entertainment, other shopping, and charity.

Use a spreadsheet program to make Kathryn's monthly budget.

- Type "Categories" in cell A1. Enter the categories in column A.
- Type "$ Amount" in cell B1. To find how much Kathryn puts in the bank each month enter "= .20*50" in cell B2.
- Enter the rest of the information in the table as shown on the right. Calculate and enter the amount left for charity.
- To find the percent spent on each category, enter "= B2/50" in cell C2. Copy and paste this into cells C3–C6.
- Click and drag from cell A1 to cell B6. Then click  .
- Double-click on the graph. Choose **Labels**. Give your graph a title. Then click the box next to **Label Data**.

	A	B	C
1	Categories	$ Amount	% Amount
2	Bank		
3	Clothes	18	
4	Entertainment	12	
5	Other Shopping	8	
6	Charity		

Use the graph you made to answer Problems 1–4.

1. What does the graph tell you?
2. What is the sum of the percents?
3. Look at the amounts shown in column C. How do they relate to the numbers shown in the graph key?
4. Kathryn wants to buy new sneakers that cost $39.95. How many months must she save her clothing money for the sneakers?
5. **Challenge** Kathryn wants to buy a $100 stereo. If she puts 50% of the money she now puts into the bank into "Other Shopping" what percent of her money will be in "Other Shopping"? How many months will she have to save?

Unit 6 Test

VOCABULARY Open Response

1. A ratio that compares favorable outcomes to unfavorable outcomes is called ____.
2. An amount paid for the use of money that is calculated only on the original principal is ____.
3. A way of arranging items in a set when the order is important is a(n) ____.
4. A(n) ____ uses the same ratio to represent the corresponding parts of real objects.

Vocabulary

- odds
- principal
- discount
- combination
- permutation
- scale drawing
- simple interest

CONCEPTS AND SKILLS Open Response

Simplify each ratio if possible. Then express it in two other ways. (Chapter 16)

5. 4 hours to 1 day **6.** $\frac{19}{3}$ **7.** 25 : 5

Write each as a unit rate. (Chapter 16)

8. 270 mi in 3 h **9.** $6 for 12 boxes

Use cross products to find each missing term. (Chapter 16)

10. $\frac{140}{20} = \frac{k}{1}$ **11.** $\frac{w}{24} = \frac{4}{16}$

Write each ratio as a percent. (Chapter 17)

12. $\frac{25}{500}$ **13.** 13 out of 20 **14.** 0.44

Write each percent as a ratio in fraction and decimal form. (Chapter 17)

15. 62% **16.** 0.9%

Order each set from the least to the greatest part of a unit. (Chapter 17)

17. 82% 0.815 $\frac{4}{5}$ 8.2 **18.** 3.4% 4.3 $\frac{43}{100}$ $\frac{21}{50}$

Use an equation to find each. (Chapter 18)

19. 50% of 94 **20.** 30 is what percent of 125? **21.** 126 is 30% of what number?

Find the sale price or total cost with sales tax. (Chapter 18)

22. 25% off $52.80

23. 18.99 and 8.25% sales tax

A jar contains 16 same-size marbles. There are 4 blue, 3 yellow, and 9 red marbles. Find each probability. (Chapter 19)

24. P(blue)

25. P(not yellow)

In Exercises 26–27, find each probability. (Chapter 19)

26. tossing a 6 on a 1–6 number cube and heads on a coin

27. picking 8 from a deck of cards numbered 1–12, then picking an odd number from the remaining cards

28. How many ways can you choose 4 people from 6? (Chapter 19)

29. Find the number of permutations for the letters **C, A, T, C, H**. (Chapter 19)

PROBLEM SOLVING Open Response

30. An airplane is 30 meters long and has a wingspan of 24 meters. A model of the airplane is 2.5 meters long and has a wingspan of 1.5 meters. Are the airplane and its model proportional? Explain.

31. José collected 84 different stamps from around the world. Of his stamps, 25% are from Asia and 23 are from Europe. Does José have more stamps from Asia or from Europe in his collection?

32. Twenty-seven species of spider, or 36% of those displayed in Tricia's local natural history museum, can be found in her home state. How many species of spiders are displayed in the museum?

33. Mr. Hamilton's sixth-grade class has chosen four different topics to be presented during the school's History Day. In how many different orders can the topics be presented?

Performance Assessment

Constructed Response



Task You have taken a poll to find out about the most popular vegetables among sixth-graders.

Use the information at the right. Arrange the vegetables in order from last to first choice. Explain how you found your answer.

Information You Need

- 16% chose green beans.
- $\frac{1}{4}$ chose carrots.
- 0.1 chose spinach.
- The probability that broccoli was chosen is $\frac{1}{8}$.
- The remaining students chose corn.

Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

Sometimes one of the answer choices may be "none of the above," or a choice such as "only A and C." Look at all the choices carefully before deciding.

Look at the example below.

Yusef discounted his T-shirts by reducing the price of each from \$12 to \$9.60. Which is an expression of the discount?

A 25%

B $\frac{3}{10}$

C 0.15

D none of the above

THINK

Use a proportion or the percent equation to find the discount. The discount is 20%. Compare 20% to the answer choices. Since 0.15 is 15% and $\frac{3}{10}$ is 30%, none of the answer choices is equivalent to 20%.

Answer choice D is correct.

Multiple Choice

1. Dorothea used the Distributive Property to multiply $50 \times 2{,}300$. Which of the following is a partial product she used to compute?

A 10,000

B 15,000

C 100,000

D both B and C

(Chapter 2, Lesson 6)

2. What are the next 3 likely numbers in this sequence: 0, $^{-}2$, 3, 1, 6, 4…?

F $^{-}1$, 2, 5

G 9, 7, 12

H 2, 7, 5

J none of the above

(Chapter 13, Lesson 2)

3. Which of the following are *both* outliers in this set of data?

2, 1, 7, 2, 6, $^{-}8$, 2, 8, 29, 12, 4

A $^{-}8$ and 8

B 4 and 29

C 29 and 12

D none of the above

(Chapter 9, Lesson 4)

4. Quadrilateral *ABCD* has two obtuse angles. How could you classify quadrilateral *ABCD*?

F as a trapezoid

G as a parallelogram

H as a rhombus

J as all of the above

(Chapter 14, Lesson 6)

For Test-Taking Tips, see page 648.

Open Response

5. A charity has $108\frac{3}{4}$ pounds of rice to distribute among six soup kitchens. If the charity gives each kitchen the same amount of rice, how much rice will each kitchen receive?

(Chapter 6, Lesson 5)

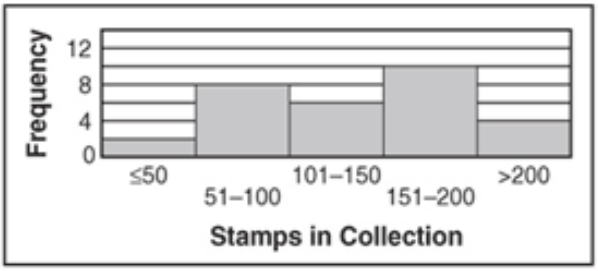
6. Cathy constructed bisector $\overrightarrow{BD}$ for $\angle ABC$. Then she constructed bisector $\overrightarrow{BR}$ of $\angle DBC$. If the measure of $\angle ABC$ is 164°, what is the measure of $\angle RBC$?

(Chapter 15, Lesson 7)

7. Ruth Ann shoveled the sidewalk in front of her grandfather's house at a rate of 120 feet per hour. How many minutes did it take her to shovel all 28 feet of the sidewalk?

(Chapter 16, Lesson 3)

8. How many Stamp Club members have between 51 and 100 stamps? In which bar in the histogram is the median of the data? Explain.

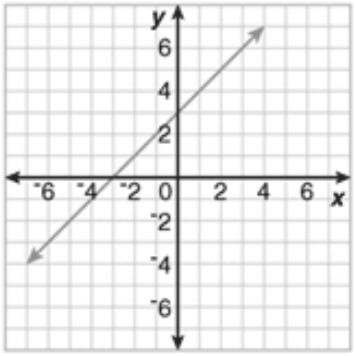


(Chapter 10, Lesson 2)

9. Gerardo wants to add a rectangular border around a drawing. If the drawing is 1 foot 10 inches wide and the border is $2\frac{3}{4}$ inches thick, how wide will the drawing be with the border?

(Chapter 8, Lesson 3)

Extended Response



10. The line shown on the grid is a graph of the function $y = x + 3$.

A Make a function table with values for at least 5 coordinates on the graph.

B Make a table of values for the function $y = x - 3$. Compare these coordinates with the ones in the function table you made in part A. How are they different?

C Use the coordinates from your table for $y = x - 3$ to graph the function.

D What relationship does this graph have with the graph of $y = x + 3$?

(Chapter 13, Lesson 4)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for test prep practice.

Vocabulary Wrap-Up for Unit 6

Look back at the big ideas and vocabulary in this unit.

Big Ideas

You can use cross products to find the missing term in a proportion.

You can use the percent equation to find the percent, the base, or part of the base.

To find the probability of two or more independent events, multiply the individual probabilities.

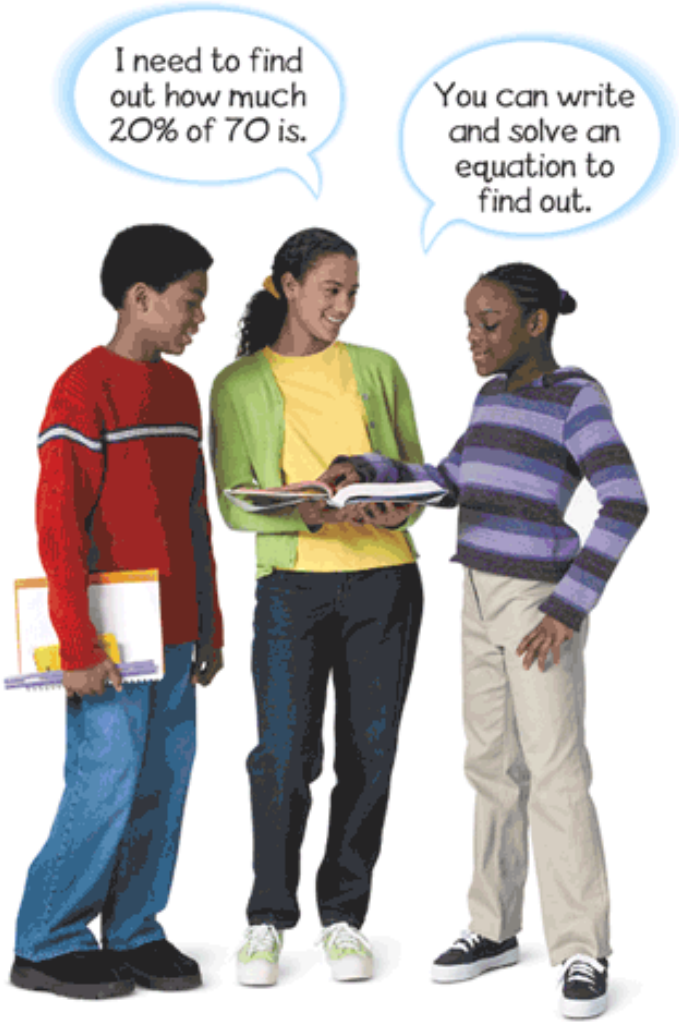
Key Vocabulary

- proportion
- percent
- event
- probability

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how to write a unit rate to express the mileage for a car that travels 312 miles on 13 gallons of gasoline.
2. Explain how to use a proportion to find the percent of discount if you paid $53.55 for an item whose original price was $63.00.
3. Explain how to find the sample space for tossing a coin three times in a row.
4. Explain how to find the probability of two dependent events. Use an example.
5. **Write About It** Look for advertisements that use percents, discounts, and interest rates. What information is each providing? How can you use what you know about percents to help you decide when an offer is a good deal?



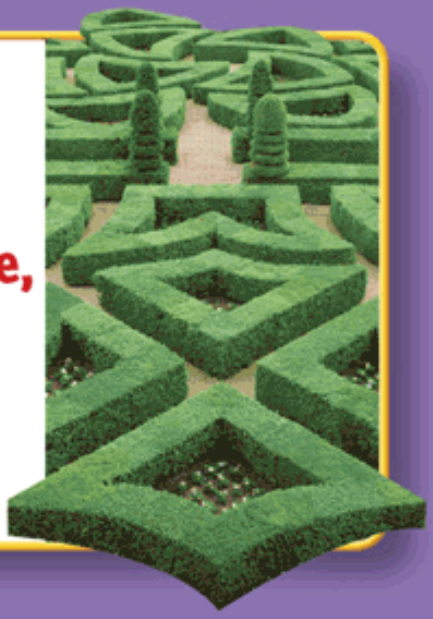
UNIT 7

Geometry and Measurement

CHAPTER 20

Perimeter, Circumference, and Area

page 524



CHAPTER 21

Volume and Surface Area

page 550



Reading Mathematics

Reviewing Vocabulary

Here are some math vocabulary words that you should know.

circle	a closed figure in which every point is the same distance from a given point called the center of the circle
parallelogram	a quadrilateral in which both pairs of opposite sides are parallel
perimeter	the distance around a figure
area	the number of square units that cover a figure without overlapping
volume	the number of cubic units that make up a solid figure



Reading Words and Symbols

In mathematics, many words have symbols that represent them. The word and the symbol mean the same thing.

Word	Symbol	Word	Symbol
• perimeter	P	• base	b
• area	A	• height	h
• volume	V	• radius	r
• length	l	• diameter	d
• width	w	• circumference	C

Write each statement, using symbols from the table or any others you know.

1. The area is the length times the width.
2. The volume of a cube is 27 cm^3.
3. The diameter of the circle is six feet.
4. The circumference of the circle is approximately 6.28 inches.

Reading Test Questions

Choose the correct answer for each.

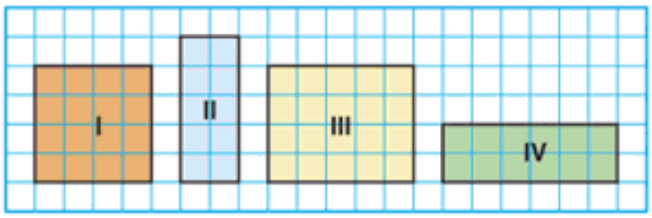
5. Which of the figures has a perimeter of at least 16 units?

a. II only

b. I and II

c. III and IV

d. I, III, and IV



At least means "equal to or greater than."

6. A rectangle is 6 feet long and 3 feet wide. What would be its area if its dimensions were doubled?

a. 18 ft^2

b. 36 ft^2

c. 54 ft^2

d. 72 ft^2

Dimensions are measures such as length, width, and height.

7. Which is the formula for the perimeter, P, of a rectangle with length l and width w?

a. $P = l + w$

b. $P = l \times w$

c. $P = 2l + 2w$

d. $P = 2l \times 2w$

A **formula** is an equation that is used often.

Learning Vocabulary

Watch for these words in this unit.
Write their definitions in your journal.

pi (π)	**cone**
prism	**pyramid**
cylinder	**polyhedron**
square root	**perfect square**

Literature Connection

Read "Freeze! Icebergs Ahead" on page 643. Then work with a partner to answer the questions about the story.

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

CHAPTER 20

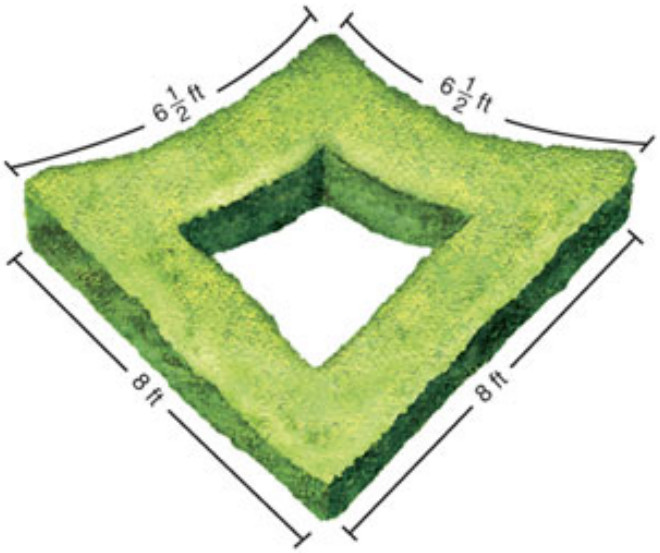
Perimeter, Circumference, and Area

INVESTIGATION

Using Data

Renaissance gardens were popular in Europe in the 16th and 17th centuries. These gardens used a geometrical design of shrubs, herbs, and gravel.

What is the outer perimeter of the shrub shown in the diagram? If the width of the shrub is 2 ft, what is the inner perimeter?



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

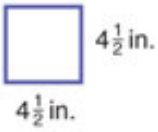
Vocabulary

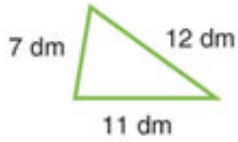
base
chord
radius
diameter
exponent

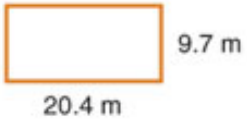
1. A line segment with one endpoint at the center of a circle and the other endpoint on the circle is a(n) _____ of the circle.
2. In the expression 10^2, the 2 is called the _____.
3. A(n) _____ is a line segment with both endpoints on a circle that may not pass through the center of the circle.

CONCEPTS AND SKILLS

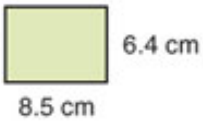
Find the perimeter of each figure.

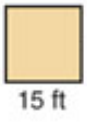
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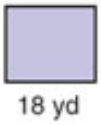
5. 

6. 

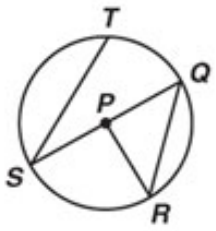
Find the area of each figure.

7. 

8. 

9. 

Use the circle for Exercises 10–13.



10. Name a diameter.

11. Name two chords.

12. Name three radii.

13. Name a central angle.

Find the percent of each number.

14. 25% of 300

15. 67% of 200

16. 15% of 360

17. 45% of 100

18. 60% of 360

19. 35% of 360



20. For a given perimeter of a rectangle, is there only one area? Explain. Give two or three examples.

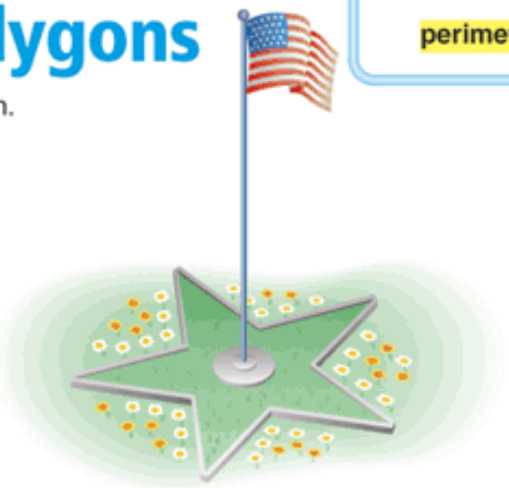
Facts Practice, See page 654.

Lesson 1

Perimeter of Polygons

Objective Find the perimeter of a polygon.

Vocabulary
perimeter



Learn About It

A school is creating a star-shaped garden around its flagpole. Each of the star's 10 sides is 3 feet long. The distance around a polygon is its **perimeter**. How many 1-foot-long blocks are needed to go around the star's perimeter?

Different Ways to Find the Perimeter

Way 1 Use addition.

Add the lengths of the sides.

$P = 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3$
$= 30$ feet

You can use addition to find the perimeter of any polygon.

Way 2 Use multiplication.

Multiply the length of a side, s, by the number of sides, n.

$P = n \times s$
$P = 10 \times 3 = 30$ feet

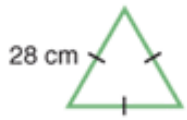
You can use multiplication to find the perimeter of any *regular* polygon.

Solution: They will need 30 blocks.

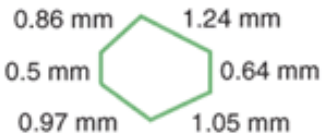
Guided Practice

Find the perimeter of each polygon.

1.



2.



Ask Yourself

- Is the figure a regular polygon?
- Do I add or multiply to find the perimeter?

Explain Your Thinking Why can you use the formula $P = n \times s$ to find the perimeter of a regular polygon but not of an irregular polygon?

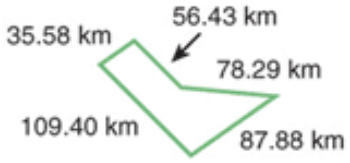
Practice and Problem Solving

Find the perimeter of each polygon.

3.



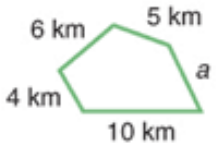
4.



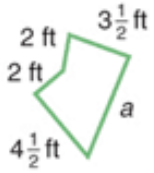
5. a regular hexagon given $s = 9.8$ meters

Use mental math to find the length of each missing side.

6. side a, given $P = 32$ kilometers



7. side a, given $P = 17$ feet



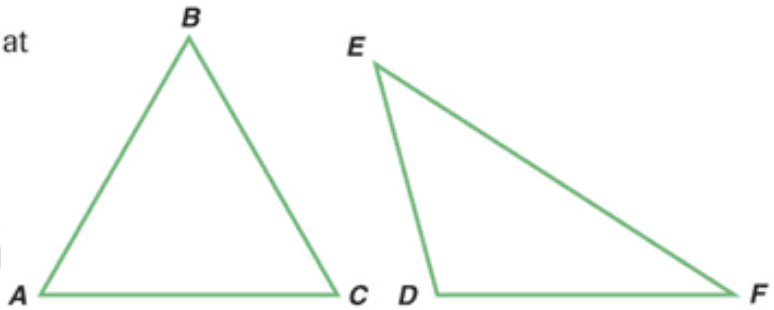
8. any side of a regular octagon, given $P = 9.6$ miles

Measurement

Solve. Use a metric ruler and tracing paper for Problems 9–11.

9. Measure the sides of $\triangle ABC$ and $\triangle DEF$ at right to the nearest millimeter. Find the perimeter of each triangle. Record the measurements and perimeters.

10. Trace both triangles at right. For $\triangle ABC$, find the midpoint of each side and label the midpoints X, Y, and Z. Draw $\triangle XYZ$. Find the length of each side and the perimeter of $\triangle XYZ$. Repeat for $\triangle DEF$.



11. Analyze Compare the perimeter of each $\triangle XYZ$ to the original triangles. How are the two perimeters related?

12. Explain An equilateral triangle and a regular hexagon have the same length sides. How do their perimeters compare?

Mixed Review and Test Prep

Open Response

Classify each polygon. (Ch. 15, Lesson 1)

13.



14.



Multiple Choice

15. A regular polygon has a perimeter of 24 feet. It has a side of 4 feet. What kind of polygon is it? (Ch. 20, Lesson 1)

A a triangle

B a square

C a hexagon

D an octagon

Extra Practice See page 549, Set A.

Hands On Lesson 2

Audio Tutor 2/24 Listen and Understand

Circumference

Objective Find the circumference of a circle.

Vocabulary
- radius
- diameter
- circumference
- pi (π)

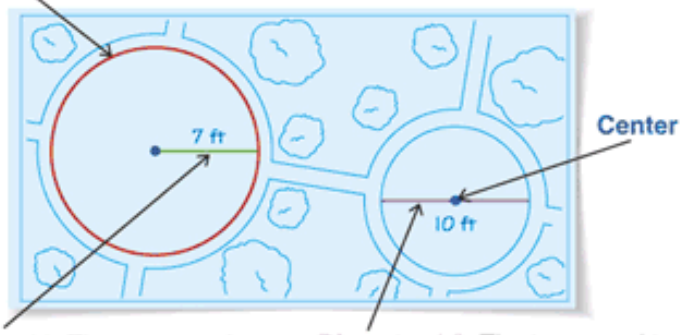
Materials
various circular objects
measuring tape

Learn About It

A landscape architect is estimating the cost of digging two fishponds. One pond has a **radius** of 7 feet. The other has a **diameter** of 10 feet. The architect needs to find the **circumference** of each pond.

The ratio of the circumference, C, to the diameter, d, of any circle is always the same. The Greek letter **pi (π)** is used to name the ratio $\frac{C}{d}$.

Circumference (C): The distance around a circle.



Radius (r): The term used to describe both a line segment from the center of a circle to any point on the circle and the length of that line segment.

Diameter (d): The term used to describe both a line segment through the center of a circle with endpoints on the circle and the length of that line segment.

Try this activity to estimate π.

- Make a 3-column table with the headings *Circumference*, *Diameter*, and $\frac{\textit{Circumference}}{\textit{Diameter}}$.
- Measure the circumference and diameter of three circular objects. Record your measurements.
- For each circle, compute the ratio $\frac{\text{circumference}}{\text{diameter}}$ to estimate π.
- Describe any pattern you see.



The value you found for π should be slightly greater than 3. You can approximate the value of the constant π using a fraction or decimal. Use $\pi \approx \frac{22}{7}$ or $\pi \approx 3.14$. (The symbol ≈ means "is approximately equal to.")

▶ Since $\pi = \frac{C}{d}$, you can use the formula $C = \pi d$ to find circumference. Since the diameter of a circle is twice its radius, you can also write the formula as $C = 2\pi r$.

Extra Help at eduplace.com/map

Different Ways to Find Circumference

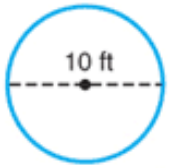
Way 1 Find the circumference, using $C = 2\pi r$.



$$\begin{aligned} C &= 2\pi r \\ &= 2 \times \pi \times 7 \\ &= 2 \times 7 \times \pi \\ &\approx 14 \times \frac{22}{7} \\ &= 44 \end{aligned}$$

It is easy to multiply the radius, 7, by the fraction $\frac{22}{7}$.

Way 2 Find the circumference, using $C = \pi d$.



$$\begin{aligned} C &= \pi d \\ &= 10 \times \pi \\ &\approx 10 \times 3.14 \\ &= 31.4 \end{aligned}$$

It is easy to multiply the diameter, 10, by the decimal 3.14.

Solution: The fishpond with a radius of 7 feet has a circumference of about 44 feet. The pond with the diameter of 10 feet has a circumference of about 31.4 feet.

Other Examples

A. Find the Radius of a Circle

Given $C = 18.8$ in.

$$\begin{aligned} C &= 2\pi r \\ \frac{C}{2\pi} &= r \\ \frac{18.8}{2 \times 3.14} &\approx r \\ 3 \text{ in.} &= r \end{aligned}$$

The radius is about 3 inches.

B. Use Significant Digits

Use a calculator to find the diameter of a circle with a circumference of 12 meters.

$$\begin{aligned} C &= \pi d \\ \frac{C}{\pi} &= d \\ \frac{12}{3.14} &\approx d \\ 3.821656051 &\approx d \\ d &\approx 3.82 \end{aligned}$$

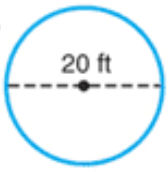
Hundredths is the significant place, so round to the nearest hundredth.

The diameter is about 3.82 meters.

Guided Practice

Find the circumference or missing measure for each circle. Use 3.14 or $\frac{22}{7}$ as approximations for π.

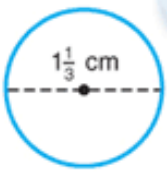
1.



2.



3.



Ask Yourself
- Which formula should I use?
- Which value for π is better to use?

4. $r = 70$ m
$C \approx$ ■

5. $d = 0.35$ ft
$C =$ ■

6. $C = 176$ cm
$r =$ ■

7. $C = d^2$
$d =$ ■

Explain Your Thinking ▶ When is using $\frac{22}{7}$ a better choice than using 3.14 as an approximation for π?

Go On

Practice and Problem Solving

Find the circumference or missing measure for each circle.
Use 3.14 or $\frac{22}{7}$ as approximations for π.

8. 

9. 

10. 

11. 

12. $d = 3$ m
$C =$ ■

13. $C = 90.12$ yd
$d =$ ■

14. $r = 3\frac{3}{4}$ m
$C =$ ■

15. $C = 110$ cm
$d =$ ■

16. $C = 220$ ft
$r =$ ■

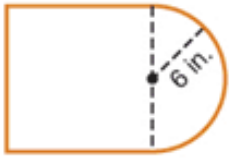
17. $C = 4$ yd
$r =$ ■

18. $d = 67$ in.
$C =$ ■

19. $r = \frac{1}{2}$ m
$C =$ ■

Estimate the perimeter of the figure. Use 3.14 or $\frac{22}{7}$ as approximations for π. Explain your choice.

20. 

21. 

22. 

Solve.

23. **Estimate** Tara is building a fence for a circular fishpond with a diameter of 40 feet. If fencing costs $3.50 per foot, estimate to the nearest dollar the cost of enclosing the pond.

24. **Reasoning** When Dave pushes a lawn roller 100 yards, the drum makes 50 complete revolutions. What is the drum's circumference? What is its diameter? Round to the nearest inch.

25. **Write About It** Flora buys a 6-hour gardening videotape. As it plays, one reel decreases in diameter as the other increases. After 3 hours, how do the diameters of the two reels compare?

26. **Explain** The diameter of a circle and the side of a square have the same length. How can you show that the circumference of the circle is less than the perimeter of the square without calculating?

Mixed Review and Test Prep

Open Response

Write the missing number. (Ch. 8, Lesson 4)

27. 42 cm = ■ mm

28. 6.3 km = ■ m

29. 735 mm = ■ cm

30. 75 m = ■ km

31. 2.2 m = ■ cm

32. 526 cm = ■ m

33. What is the circumference of a circle with a radius that is twice as large as this one? Explain. (Ch. 20, Lesson 2)



Extra Practice See page 549, Set B.

Competing Circles

Game Activity

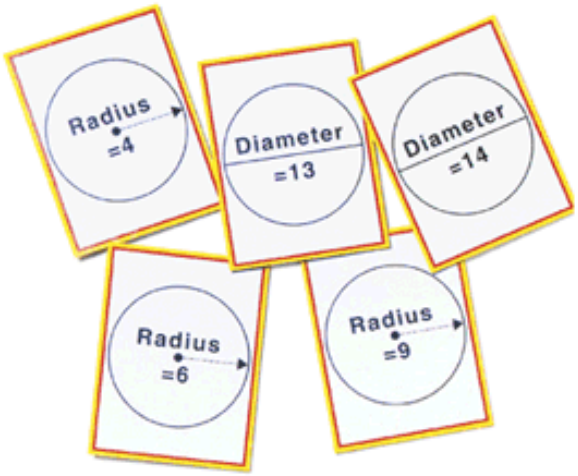
2 Players

What You'll Need • 5 "diameter" cards (Learning Tool 56)
• 5 "radius" cards (Learning Tool 57)

How to Play

1. Shuffle the cards and deal 5 cards to each player.
2. Each player places one card faceup.
3. Each player calculates the circumference of a circle with the radius or diameter listed on his or her card. Use 3.14 or $\frac{22}{7}$ as an approximation for π. The player with the circle having the greater circumference takes both cards.

Repeat Steps 2 and 3. The player with more cards after five turns wins.





Lesson 3

Problem-Solving Application

Make a Circle Graph

Objective Solve problems by making and using a circle graph.

When you have data that represent part of a whole, the data can be displayed in a circle graph.

Problem Maria has a yard service business. She wants to make a circle graph to show her parents how she has managed the money she has earned.

To make a circle graph, you need to know what size to make each sector of the circle to represent the data accurately.

Maria's Yard Care

Use of Earnings	% of Earnings
Savings	47%
Pocket Money	28%
Advertising	8%
Equipment	17%

Find the measures of the central angles that represent each category of use.

Write a proportion to find the number of degrees for each central angle. Round your answers to the nearest degree.

Remember There are 360° in a circle.

Savings 47%

$$\frac{47}{100} = \frac{\blacksquare}{360°}$$
$$47 \times 360° = 100 \times \blacksquare$$
$$\blacksquare = 169.2° \approx 169°$$

Advertising 8%

$$\frac{8}{100} = \frac{\blacksquare}{360°}$$
$$8 \times 360° = 100 \times \blacksquare$$
$$\blacksquare = 28.8° \approx 29°$$

Pocket Money 28%

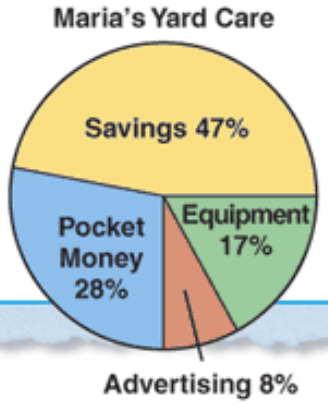
$$\frac{28}{100} = \frac{\blacksquare}{360°}$$
$$28 \times 360° = 100 \times \blacksquare$$
$$\blacksquare = 100.8° \approx 101°$$

Equipment 17%

$$\frac{17}{100} = \frac{\blacksquare}{360°}$$
$$17 \times 360° = 100 \times \blacksquare$$
$$\blacksquare = 61.2° \approx 61°$$

Draw the circle graph.

- Draw a circle and one radius.
- Use that radius as a ray of one central angle.
- Measure and draw the other angles.
- Label the graph with the categories and the percents.



Look Back Look back at the circle graph. Do the sizes of the sectors make sense?

Guided Practice

Ask Yourself

- Did I use the correct data?
- Did I set up and solve the proportions correctly?
- Do the sizes of the circle sectors make sense?

1. Of Maria's jobs, 78% are mowing lawns, 21% are yard cleanup, and 1% is planting. In a circle graph of these data, what should the measure of the central angle be for each job category?

   Hint How can $\frac{\blacksquare}{100} = \frac{\blacksquare}{360}$ help you set up your proportions?

2. Use the measures of central angles you calculated in Problem 1 to make a circle graph of the data.

Independent Practice

Data **Use the table to solve Problems 3–6.**

Maria asked new customers how they heard about her yard service. She wrote the results in the table at the right.

How New Customers Heard About Maria's Yard Care

Way	Percent
Supermarket flyers	24%
Give-away newspapers	18%
Local newspaper	2%
Satisfied customers	56%



3. If Maria makes a circle graph of the data, what should the measure of each central angle be for each response category?

4. **Represent** Draw a circle graph of the data in the table.

5. Maria surveyed more new customers. She found that the percent who heard of her service from satisfied customers jumped 8%. Find the new measure of the central angle for this sector.

6. **Analyze** In her new survey, Maria finds that one way that customers found out about her drops 3%. Its new central angle measure is 54°. Which way is it?

7. **Explain** Once you have found all the central angle measures for your data, how can you check that they are correct before drawing your circle graph?

Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

8. Of Maria's savings, she saved half for college, a third for a new set of in-line skates, and the rest for a family trip. Draw a circle graph to represent these data.

9. To help her check whether her graphs are correct, Maria calculates the central angle measure for each 10 percent from 10%, 20%, 30%, and so on to 100%. What are these measures?

10. Some of Maria's 35 customers pay by check. Others pay by cash. Maria makes a graph of these data. The sector representing people who pay by check has a central angle measure of 72°. How many customers pay by check?

You Choose

Strategy

- Draw a Diagram
- Use Logical Reasoning
- Use Models
- Work Backward
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Data **Use the table for Problems 11–16. Then tell which method you chose.**

11. Find the total lawn mower expenses.

12. **Analyze** What percent of the total lawn mower expenses does each expense category make up?

13. Make a circle graph of the data.

14. **Write About It** In your circle graph, which expense is represented by a sector with a central angle measure of 180°? Explain how you know.

15. **What's Wrong?** When Maria first drew her graph of the data, she made the central angle representing blade-sharpening 324°. What did she do wrong?

16. **You Decide** Maria says that if she gets a new lawn mower, the percents for these expenses will change. Decide which categories will change and by how much.



Lawn Mower Expenses

Expense	Cost
Gas/Oil	$52
Tune-ups	$30
Blade-sharpening	$18
Repairs	$100

17. A circle graph is not labeled with percents. The central angles measure 90°, 45°, 180°, and 45°. What percents do these central angles represent?

Calculator Connection
Calculating Circumference

Activity

Pi is an approximation. It can be shown in three different ways on a calculator.

Find the circumference of a circle with a 9-inch radius using each of the following 3 methods. Copy the complete answer for each one.

1. Press: 2 × 9 × π Enter F↔D

2. Press: 2 × 9 × 2 2nd 7 d Enter F↔D

3. Press: 2 × 9 × 3 . 1 4 Enter

4. At what decimal place do the answers start to differ?

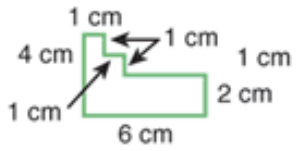
5. Yu's driveway is 100 ft long. She rides four different vehicles. How many complete revolutions does each type of wheel make if she rides the length of her driveway?

Vehicle	Wheel diameter
Bicycle	20 in.
Rollerblade	2.5 in.
Scooter	4 in.
Skateboard	2 in.

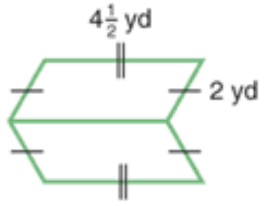
Quick Check

Check your understanding of Lessons 1–3.

Find the perimeter or circumference of each figure. Use 3.14 or $\frac{22}{7}$ as approximations for π. (Lessons 1 and 2)

1. 

$P =$ ■

2. 

$P =$ ■

3. $r = 2.4$ m
$C =$ ■

4. $r = 3\frac{1}{2}$ mi
$C =$ ■

Solve. (Lesson 3)

5. Dewi earns $50 babysitting. She keeps $5 in her pocket, spends $5 on a craft kit, and puts $30 in the bank. She spends the rest on a gift for her mom. Draw a circle graph to show how Dewi used her money.

Lesson 4

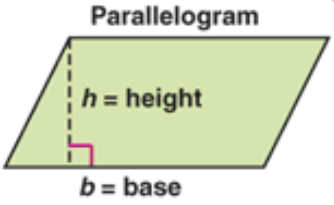
Area of a Parallelogram

Vocabulary
parallelogram

Objective Find the area of a parallelogram.

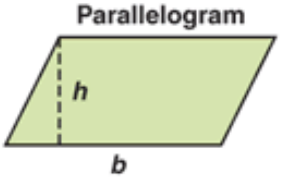
Learn About It

A **parallelogram** is a quadrilateral with opposite sides that are parallel and congruent. You can use what you know about the area of a rectangle, $A = lw$, to find the area of a parallelogram.

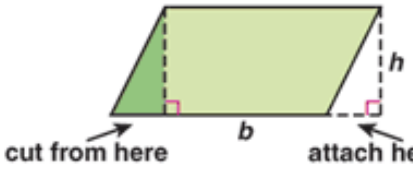


The height is the length of a line segment that is perpendicular to both bases.

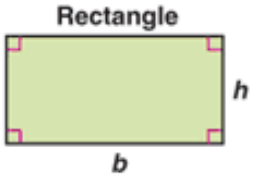
- Draw the height from a vertex to the base.



- Cut off the right triangle and attach it to the other side.



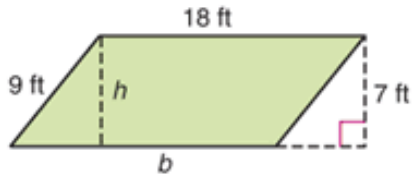
- The rectangle and parallelogram have the same area.



▶ The formula for the area of a parallelogram is $A = bh$.

Use the formula to find the area of the parallelogram.

STEP 1 Identify the length of the base and the perpendicular height.



$b = 18$ ft
$h = 7$ ft

STEP 2 Use the formula to find the area.

$A = bh$
$= 18 \times 7 = 126$

Solution: The area of the parallelogram is 126 square feet, or 126 ft^2.

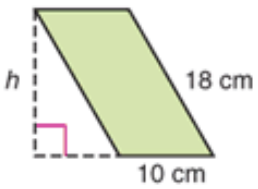
Guided Practice

Find the missing measure for each parallelogram, given base *b*, height *h*, or area *A*.

1. $b = 25$ m
$h = 30$ m
$A =$ ■

2. $b = 8.2$ yd
$h = 1.25$ yd
$A =$ ■

3. $A = 150$ cm^2



Ask Yourself
- Did I identify the height?
- Did I use the correct numbers in the formula?

Explain Your Thinking ▶ Does it matter where or how you draw a line segment to measure the height?

Practice and Problem Solving

Find the missing measure for each parallelogram or combination of parallelograms.

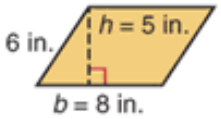
4. $b = 1{,}000$ ft
$h = 2.5$ ft
$A =$ ■

5. $A = 30$ ft^2
$h = 5$ ft
$b =$ ■

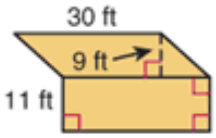
6. $b = 5\frac{1}{4}$ in.
$h = 7\frac{3}{7}$ in.
$A =$ ■

7. $b = 6.32$ m
$h = 2.85$ m
$A =$ ■

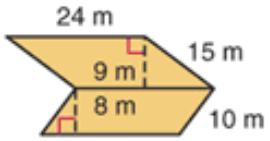
8. $A =$ ■



9. $A =$ ■

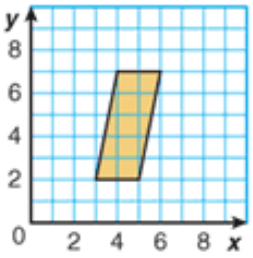


10. $A =$ ■

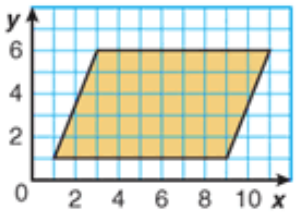


Find the area of each parallelogram.

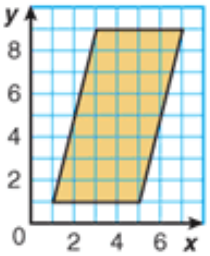
11.



12.



13.

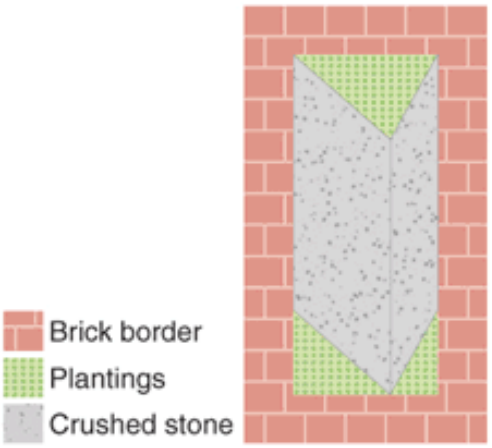


Measurement Use the landscape design for Problems 14–16.

14. Measure the base and height of the parallelograms that form the crushed stone patio design to the nearest quarter inch. Find the total area of the two parallelograms.

15. Suppose the scale in the drawing were $\frac{1}{4}$ inch = 1 yard. What would the actual area of the two parallelograms be?

16. Measure the dimensions of the brick border to the nearest quarter inch. Find the area of the border.



Mixed Review and Test Prep

Open Response

Find the perimeter and area of each figure. (Ch. 8, Lesson 7)

17. rectangle *RSTU*: $l = 24$ cm; $w = 16$ cm

18. square *JKLM*: $s = 3.4$ m

19. A tulip garden is in the shape of a parallelogram. The area of the garden is 14 square meters. What could be the garden's dimensions? Explain. (Ch. 20, Lesson 4)

Extra Practice See page 549, Set C.

Lesson 5

Area of a Triangle

Objective Find the area of a triangle.



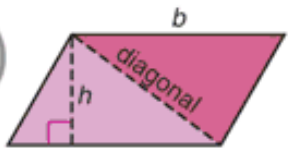
Learn About It

On their schoolyard, a sixth-grade class marks off a parallelogram-shaped garden with a base of 20 feet and a height of 6 feet. The garden is divided into two congruent triangles. One triangular part is a butterfly garden. What is its area?

Find the area of a triangle.

STEP 1 Use a diagonal to divide the parallelogram into two congruent triangles.

The area of each triangle is one half the area of the parallelogram.



STEP 2 Find a formula for the area of a triangle.

Let b = base

Let h = perpendicular height

A of the parallelogram $= bh$

A of the triangle $= \frac{1}{2}bh$

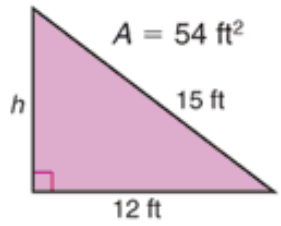
STEP 3 Find the area of the triangular garden.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(20 \times 6)$$
$$= \frac{1}{2}(120)$$
$$= 60 \text{ ft}^2$$

Solution: The area of the triangular butterfly garden is 60 square feet.

Another Example

Find the Height of a Triangle



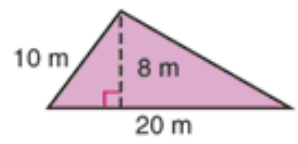
$$A = \frac{1}{2}bh$$
$$54 = \frac{1}{2}(12h)$$
$$54 = 6h$$
$$9 = h$$

The height is 9 feet.

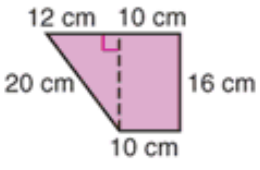
Guided Practice

Find the area of each figure.

1.



2.



Ask Yourself

- Did I use the correct formula?
- Did I use the correct values for b, h, and A?

Find the missing measure for each triangle.

3. $A = \blacksquare$
$b = 20$ ft
$h = 12$ ft

4. $A = 28$ m^2
$b = 8$ m
$h = \blacksquare$

5. $A = 60.16$ in.^{2}
$b = \blacksquare$
$h = 9.4$ in.

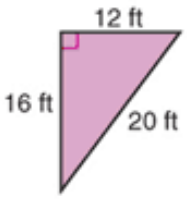
6. $A = 7\frac{1}{2}$ in.^{2}
$b = 5$ in.
$h = \blacksquare$

Explain Your Thinking Draw a parallelogram and one of its diagonals. Explain how you know that two congruent triangles are formed.

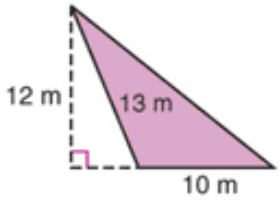
Practice and Problem Solving

Find the area of each figure or missing measure of each triangle.

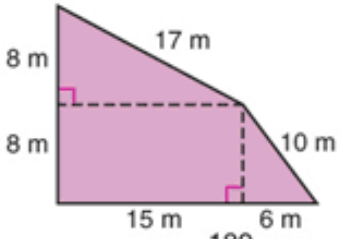
7.



8.



9.



10. $A = \blacksquare$
$b = 4$ yd
$h = 2$ yd

11. $A = \blacksquare$
$b = 15$ cm
$h = 18$ cm

12. $A = 156.16$ ft^2
$b = 12.2$ ft
$h = \blacksquare$

13. $A = \frac{180}{40}$ in.^{2}
$b = \blacksquare$
$h = 2\frac{1}{2}$ in.

Solve.

14. Next to each plant in the garden is a triangular label stating the plant's name. What is the height of each label if its base is 8 inches and its area is 24 in.^{2}?

15. The height of triangle *DEF* is 4 times the height of triangle *ABC*. If the bases have equal lengths, how do the areas of triangles *DEF* and *ABC* compare?

16. Reasoning The banner over the garden entrance is a triangle with a height that is 3 times the length of its base. If the area of the banner is 54 square meters, what is its height?

17. Analyze Cindy and Eric each drew a triangle with an area of 120 square feet. The base of Cindy's triangle is half as long as the base of Eric's triangle. Compare the heights of the triangles.

Mixed Review and Test Prep

Open Response

Draw and name a polygon with the given attributes. (Ch. 15, Lesson 1)

18. 2 pairs of congruent parallel sides; 2 acute and 2 obtuse angles

19. 5 congruent sides and angles

20. 3 sides, 2 of which are congruent; 1 obtuse angle

21. A diagonal divides a square patch of garden that is 4 feet on a side. What is the area on either side of the diagonal? (Ch. 20, Lesson 5)

Explain how you found your answer.

Extra Practice See page 549, Set D.

Hands On Lesson 6

Audio Tutor 2/25 Listen and Understand

Squares and Square Roots

Objective Find the square of a number and the square root of a number.

Vocabulary
perfect square
square root

Materials
grid paper
colored pencils

Learn About It

A **perfect square** is the product of a number and itself. For example, 25 is a perfect square because 5×5, or 5^2, equals 25.

Try this activity to explore perfect squares.

STEP 1 On your grid paper, color a square area 5 units long and 5 units wide.

- How many square units did you color?



STEP 2 Color a square area 4 units long and 4 units wide.

- How many square units did you color?
- What is 4^2?

Color square areas to show that the numbers 1, 4, and 9 are all square numbers. Then write each perfect square, using the power 2.



You know that inverse operations "undo" each other. Squaring a number and taking the square root of a number are inverse operations.

You can use a square root to find the length of the side of a square.

Rule:

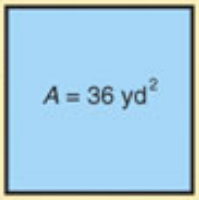
The **square root** of a number n is a number that when multiplied by itself equals n. The square root of a number is denoted by the symbol $\sqrt{\ }$.

Find $\sqrt{36}$.

Use the area to find the length of one side.

Think
What number multiplied by itself equals 36?

$6 \times 6 = 36$, so $\sqrt{36} = 6$



Solution: The length of the side of the square is 6 yards.

Sometimes you need to find the square root of a number that is not a perfect square.

Different Ways to Find $\sqrt{18}$

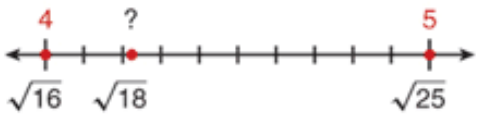
Way 1 Use a number line and estimation

You know that $\sqrt{16} = 4$ and $\sqrt{25} = 5$.
So $\sqrt{18}$ is between 4 and 5.
$\sqrt{18}$ is a little more than 4. Try some values.

$4.1^2 = 16.81$ Too low.

$4.2^2 = 17.64$ Close.

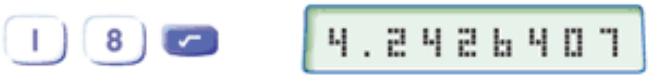
$4.3^2 = 18.49$ Too high.



Since 18 is closer to 16 than to 25, $\sqrt{18}$ must be closer to $\sqrt{16}$ than to $\sqrt{25}$.

Way 2 Use a calculator.

Use the $\sqrt{\ }$ key on your calculator. Round to the nearest tenth.



$4.2426407 \approx 4.2$

Solution: $\sqrt{18} \approx 4.2$

Guided Practice

Write each perfect square, using the power 2.

1. 49 **2.** 81 **3.** 144 **4.** 625

Find each square root.

5. $\sqrt{36}$ **6.** $\sqrt{100}$ **7.** $\sqrt{64}$ **8.** $\sqrt{400}$

Estimate each square root to the nearest tenth. Use a number line.

9. $\sqrt{2}$ **10.** $\sqrt{22}$ **11.** $\sqrt{33}$ **12.** $\sqrt{50}$

Ask Yourself

- What number can I multiply by itself to find the perfect square?
- Do I know what two numbers a square root is between?

Explain Your Thinking In Exercises 9–12, how can you check your estimates?

Go On

Practice and Problem Solving

Write each perfect square, using the power 2.

13. 9 **14.** 64 **15.** 121 **16.** 900 **17.** 10,000

Find each square root.

18. $\sqrt{1}$ **19.** $\sqrt{49}$ **20.** $\sqrt{169}$ **21.** $\sqrt{1{,}600}$ **22.** $\sqrt{2{,}500}$

Estimate the side length of each square given its area. Use a calculator to check your estimate.

23. Area is 12 square units.

24. Area is 21 square units.

25. Area is 30 square units.

26. Area is 40 square units.

Calculator **Evaluate each square root. Round to the nearest tenth.**

27. $\sqrt{3}$ **28.** $\sqrt{5}$ **29.** $\sqrt{10}$ **30.** $\sqrt{90}$ **31.** $\sqrt{725}$

Solve.

32. **Estimate** Jessica wants to turn part of her yard into a square vegetable garden. If the area of the square is 150 square feet, about how long is each side?



Area = 150 ft^2

33. **Analyze** One of Jessica's square gardens has an area of 225 square feet. How many feet of fencing will she need to enclose this garden?

34. Find the number each symbol represents.

▲ × ▲ = 441

$\sqrt{★} = 19$

$●^2 = 1{,}225$

35. **Create and Solve** Create a problem that requires finding a perfect square or a square root. Give your problem to a partner to solve.

Mixed Review and Test Prep

Open Response

Write each missing number. (Ch. 8, Lesson 4)

36. 765 mL = ■ L

37. 8.2 L = ■ dL

38. 47.8 g = ■ kg

39. 0.065 g = ■ mg

Multiple Choice

40. A square garden has an area of 45 square meters. Which is the best estimate for the length of one side of the garden? (Ch. 20, Lesson 6)

A about 6.0 m **C** about 7.1 m

B about 6.7 m **D** about 7.5 m

Extra Practice See page 549, Set E.

Problem Solving

Math Reasoning

The Pythagorean Theorem

Pythagorean Theorem

For any right triangle, the square of the hypotenuse (c^2) equals the sum of the squares of the two legs ($a^2 + b^2$), or

$$c^2 = a^2 + b^2$$



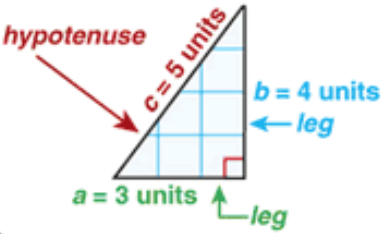
Pythagoras

Materials
grid paper
ruler

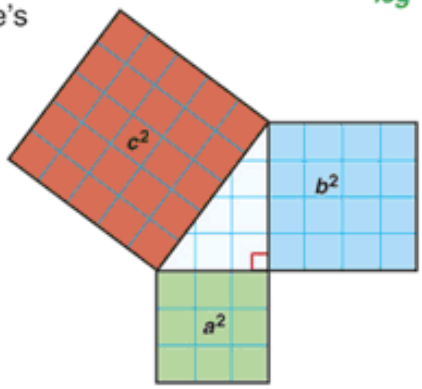
▶ The **Pythagorean Theorem** is a relationship between the perpendicular legs of a right triangle and its *hypotenuse*, the longest side of the triangle.

You can explore this relationship.

Step 1 Copy the scalene right triangle at right onto grid paper. Label the perpendicular legs *a* and *b*, and the hypotenuse *c*.



Step 2 Draw a square with side *a* on the triangle's leg *a*. Repeat for leg *b* and side *c*. Label the areas a^2, b^2, and c^2 as shown.



Step 3 Find the area of each square by counting the number of square units. Record the areas. Compare the areas. What do you notice?

If you know the lengths of two legs of a right triangle, you can use the Pythagorean Theorem to find the length of the hypotenuse.

If $a = 6$ and $b = 8$, what is c?

$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = 10$$

Think $\sqrt{100} = 10$

Calculator **To the nearest tenth, find the length of the hypotenuse (*c*) for each right triangle, given the lengths of legs *a* and *b*.**

1. $a = 7, b = 7$ **2.** $a = 18, b = 24$ **3.** $a = 9, b = 12$

4. Analyze A triangle has the side lengths $a = 5$ cm, $b = 6$ cm, and $c = 8$ cm. Is the triangle a right triangle? Explain.

Hands On Lesson 7

Audio Tutor 2/26 Listen and Understand

Area of a Circle

Objective Calculate the area of a circle.

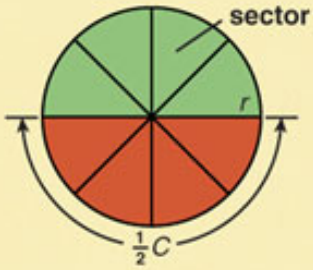
Materials
paper circle (Learning Tool 20)
colored pencils
scissors

Learn About It

The formulas for circumference of a circle and area of a parallelogram can help you find the formula for the area of a circle.

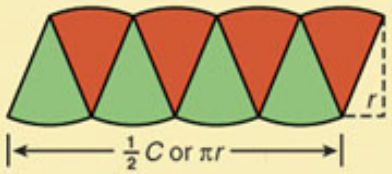
Try this activity to find the area of a circle.

STEP 1 Divide a circle into 8 equal sectors. Shade each half-circle.



If $C = 2\pi r$, then $\frac{1}{2}C = \pi r$.

STEP 2 Rearrange the sectors as shown. The figure resembles a parallelogram.



- How is the area of the parallelogram related to the area of the circle?

The area of a parallelogram $= b \times h$, so the area of a circle $= (\pi r) \times r$, or πr^2.

The formula for the area of a circle is $A = \pi r^2$.

Find the area of a circle with a diameter of 16 inches. Use 3.14 for π.

STEP 1 Find the radius of the circle.



$r = \frac{1}{2}d$

$= \frac{1}{2} \times 16$

$r = 8$ in.

STEP 2 Use the formula to find the area.

$A = \pi r^2$

$= \pi \times 8^2$

$\approx 3.14 \times 64$ ← Use 3.14 for π.

$A = 200.96$ in.^{2}

Solution: The area of the circle is about 201 square inches.

Guided Practice

Find the area of each circle. Use 3.14 or $\frac{22}{7}$ for π.

1. $r = 5$ in. **2.** $r = \frac{3}{4}$ cm **3.** $d = 24$ yd **4.** $d = 4\frac{1}{2}$ m

Ask Yourself

- Did I use the correct formula?
- Which value of π do I use?

Explain Your Thinking ▶ What happens to the area of a circle when its radius doubles?

Practice and Problem Solving

Calculator **Find the area of each circle. Use 3.14 or $\frac{22}{7}$ for π.**

5. $r = 28$ cm

6. $d = 30$ ft

7. $r = 5\frac{2}{3}$ in.

8. $d = 64.2$ m

9. $r = 84$ ft

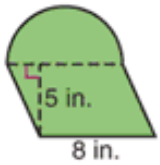
10. $r = 7$ m

11. $d = 25$ yd

12. $d = 3{,}500$ cm

Find the area of each figure. Use 3.14 for π. Round to the nearest tenth.

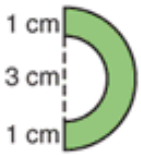
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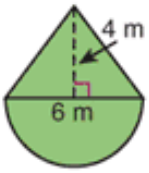
14.



15.



16.



Data **Use the price chart from The Gardener's Shed to solve Problems 17–19.**

17. How many square inches greater is the area of a 16-inch planter than the area of a 12-inch planter?

18. Which planter costs more per square inch?

19. **Write About It** Daphne spends about $50 on 4 planters with saucers. Their combined area is about 669 square inches. What did she buy? How did you decide?

20. The Gardener's Shed has free delivery of flowers within a 10-mile radius of the shop. How large is the free-delivery area in square miles?

21. **Analyze** It takes 1 shovelful of gravel to cover a circular area in a flowerbed that is 8 inches in diameter. How many shovelfuls will it take to fill a circular area that is 16 inches in diameter?



The Gardener's Shed

Circular Flower Pot Prices

Diameter	Planter	Saucer
12 in.	$7.99	$1.00
14 in.	$9.99	$1.25
16 in.	$11.99	$1.50

Mixed Review and Test Prep

Open Response

Find the GCF for each pair of numbers. (Ch. 3, Lesson 4)

22. 24 and 72

23. 26 and 39

24. 18 and 35

25. 12 and 42

26. 56 and 96

27. 14 and 36

28. A circular bush has a diameter of 4 feet. There is an open space of 2 feet all around the bush. Find the total area for the bush and the open space. Draw a diagram and show your work. (Ch. 20, Lesson 7)

Extra Practice See page 549, Set F.

Lesson 8

Problem-Solving Decision

Reasonable Answers

Objective Use units of measure to decide whether or not an answer is reasonable.



Once you have solved a problem, you can use units of measure to help you decide whether or not your answer is reasonable.

Problem Lucine walks at a rate of 240 feet per minute. She says that she is walking at a rate of 40 feet per second. Is her answer reasonable?

To check whether the answer is reasonable, solve the problem by including the units in your work.

Ask Yourself

What units do I need to compare?

$$\frac{240 \text{ feet}}{1 \text{ minute}} \times \frac{1 \text{ minute}}{60 \text{ seconds}}$$

Did I divide units appropriately?

$$\frac{240 \text{ feet}}{1 \cancel{\text{minute}}} \times \frac{1 \cancel{\text{minute}}}{60 \text{ seconds}}$$

Do the units in my answer match those in the question?

$$\frac{240 \text{ feet}}{60 \text{ seconds}} = \frac{4 \text{ feet}}{1 \text{ second}}$$

Solution: Lucine's answer is not reasonable. She is walking at a rate of 4 feet per second. The unit (feet/second) matches the unit in the question.

Try These

Solve. Use the units in the problem to decide whether your answer is reasonable. Show your work.

1. Sally cycled a distance of 5 miles in 30 minutes. Her cousin Hal in Canada cycled 8 kilometers in 25 minutes. Sally claims she cycled faster than Hal. Is her claim reasonable? (Hint: 1 mi ≈ 1.6 km)
2. A sixth-grade class will take a 400-mile bus trip as part of a field trip. If the bus travels at a rate of 50 miles per hour, is 8 hours a reasonable estimate for the length of the bus trip?
3. Can you earn $500 in the next 3 weeks if you make $5.25 per hour and work 6 hours per day, 5 days per week? Explain.
4. Jay says that 60 miles per hour is the same as 88 feet per second. Is he right? (Hint: 1 mile = 5,280 feet)

Math Challenge
Area Hysteria

To find the area of a circle on your calculator, enter the radius and then press:



The F↔D key converts pi to its decimal value.

Use a calculator to solve. Round each answer to the nearest tenth.

1. Which has the greater area, a square with 150-cm sides, or a circle with a 150-cm diameter?

2. A square backyard is 576 ft^2. Find the diameter and area of the largest circular pool that can fit in the yard.

3. Find the area of the shaded regions.

a.



radius = 5 in.

b.



length of triangle sides: 12 cm, 5 cm, 13 cm

4. To the nearest whole number, what length diameter would a circle need in order to have as great an area as a square with 200-cm sides?

5. Suppose the patio of a home is shaped like a right triangle with two sides measuring 6 feet each. A circular table with a radius of 2 feet sits upside down on the patio. Explain how you would find the area of the patio that is not covered by the table.

Chapter Review/Test

VOCABULARY

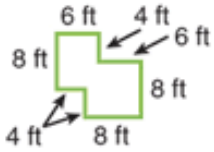
1. When a factor is multiplied by itself, the product is a(n) _____.
2. _____ is the number of square units needed to cover a region.
3. The Greek letter _____ is used to name the ratio $\frac{C}{d}$.

Vocabulary

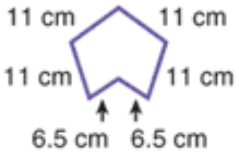
pi
area
perimeter
perfect square

CONCEPTS AND SKILLS

Find the perimeter. (Lesson 1, pp. 526–527)

4. 

5. 

6. 

Find the circumference of each circle. Then find the area. Use 3.14 for π. (Lesson 2, pp. 528–530; Lesson 7, pp. 544–545)

7. $r = 12$ in.
8. $r = 9$ cm
9. $d = 8$ yd
10. $d = 22$ m

Find the missing measure for each parallelogram. (Lesson 4, pp. 536–537)

11. $h = 11\frac{1}{2}$ in.
$A = 138$ in.^{2}
$b =$ ■

12. $b = 10.7$ m
$h = 9$ m
$A =$ ■

13. $A = 405$ ft^2
$b = 27$ ft
$h =$ ■

14. $b = 17$ mm
$A = 680$ mm^2
$h =$ ■

Find the area of each triangle. (Lesson 5, pp. 538–539)

15. $b = 18$ cm
$h = 15$ cm

16. $b = 24$ mi
$h = 24$ mi

17. $b = 150$ mm
$h = 75$ mm

18. $b = 230$ in.
$h = 27$ in.

Estimate each square root to the nearest tenth. (Lesson 6, pp. 540–542)

19. $\sqrt{22}$
20. $\sqrt{40}$
21. $\sqrt{54}$
22. $\sqrt{7}$
23. $\sqrt{11}$

PROBLEM SOLVING

Solve. (Lesson 3, pp. 532–534; Lesson 8, p. 546)

24. Of the people who voted for mayor, 45% voted for Shinn, 30% voted for Ames, and 25% voted for Gomez. Use the data to make a circle graph.

25. The Patel family will drive 300 miles at an average rate of 45 miles per hour. Is 5 hours a reasonable estimate for the length of the trip? Explain.

Write About It

Show You Understand

A goat is tethered with a 20-foot chain to the corner of a barn as shown below. Explain how to find the area in which the goat can graze. Then find the area.



Extra Practice

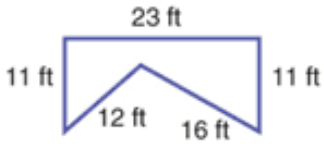
Set A (Lesson 1, pp. 526–527)

Find the perimeter.

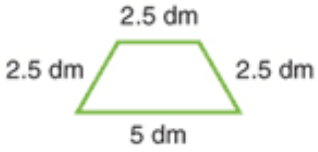
1.



2.



3.



Set B (Lesson 2, pp. 528–530)

Find the circumference. Use 3.14 for π.

1.



2.



3. $d = 0.45$ m

4. $r = 25$ mm

Set C (Lesson 4, pp. 536–537)

Find the missing measure for each parallelogram.

1. $b = 20$ in.
$h = 16$ in.
$A =$ ■

2. $h = 13$ cm
$b = 11$ cm
$A =$ ■

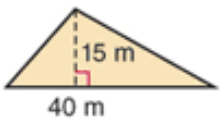
3. $b = 1\frac{1}{2}$ ft
$A = 4\frac{1}{2}$ ft^2
$h =$ ■

4. $h = 18$ m
$A = 145.8$ m^2
$b =$ ■

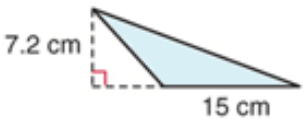
Set D (Lesson 5, pp. 538–539)

Find the area of each triangle.

1.



2.



3. $b = 12$ ft
$h = 9$ ft

4. $b = 32$ in.
$h = 36$ in.

Set E (Lesson 6, pp. 540–542)

Write each square number, using the power 2.

1. 36 **2.** 16 **3.** 121 **4.** 1,600 **5.** 2,500

Find each square root.

6. $\sqrt{81}$ **7.** $\sqrt{100}$ **8.** $\sqrt{10{,}000}$ **9.** $\sqrt{400}$ **10.** $\sqrt{900}$

Set F (Lesson 7, pp. 544–545)

Find the area of each circle. Use 3.14 for π.

1. $r = 30$ dm **2.** $r = 6$ in. **3.** $d = 20$ ft **4.** $d = 14$ in.



CHAPTER 21

Volume and Surface Area

INVESTIGATION

Using Data

"Machinable" parcels are those that fit through sorting machines. If a parcel is 14 in. high, 8 in. long, 18 in. thick, and weighs 8 oz, is it machinable? Explain.

How would you find the volume of the parcel?

Requirements for Machinable Parcels

Dimension	Minimum	Maximum
Height	3 in.	17 in.
Length	6 in.	34 in.
Thickness	$\frac{1}{4}$ in.	17 in.
Weight	6 or 8 oz	35 lb

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

1. The ____ of a circle is equal to πr^2.
2. One of two equal factors of a number is the ____ of the number.
3. $P = (2 \times l) + (2 \times w)$ is the ____ for the perimeter of a rectangle.

Vocabulary

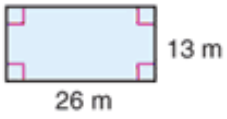
- area
- square root
- formula
- circumference

CONCEPTS AND SKILLS

Name each polygon.

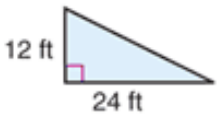
4. 
5. 
6. 
7. 

Find the area of each figure.

8. 

9. 

10. 

11. 

Find the area of each circle to the nearest tenth. Use 3.14 for π.

12. $r = 15$ yd
13. $d = 14$ dm
14. $r = 20$ in.
15. $d = 36$ ft

Name each solid figure.

16. 
17. 
18. 
19. 

Write About It

20. Write a definition of a polygon. Use it to explain why a circle is not a polygon.

Facts Practice, See Page 660.

Lesson 1

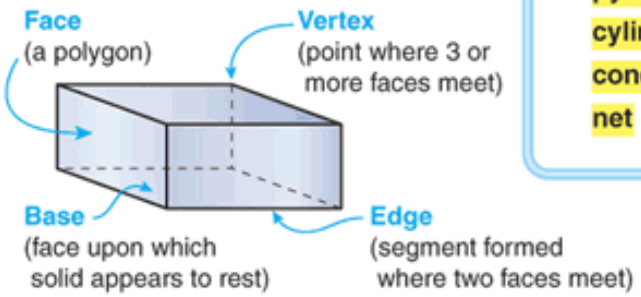
Solid Figures

Objective Identify, describe, and classify solid figures.

Vocabulary
- solid figure
- polyhedron
- prism
- pyramid
- cylinder
- cone
- net

Learn About It

Solid figures occupy space. A **polyhedron** is a solid figure enclosed by polygons.




Rectangular prism


Triangular prism


Square pyramid


Triangular pyramid

A **prism** is a polyhedron with two congruent and parallel bases joined by rectangular faces.

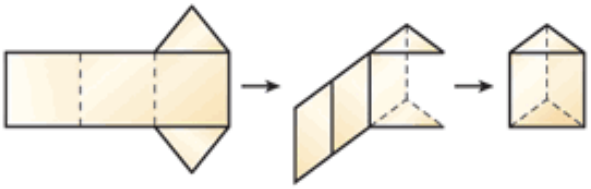
A **pyramid** is a polyhedron whose base is a polygon and whose other faces are triangles that share a common vertex.

Some solid figures have curved surfaces. They are not polyhedra (plural of *polyhedron*).
A **cylinder** has two congruent circular bases.
A **cone** has only one circular base.


Cylinder


Cone

▶ A **net** is a plane pattern that can be folded to make a solid figure. When folded, the net at the right forms a triangular prism.

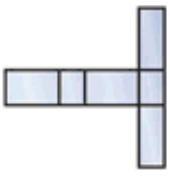


Guided Practice

Draw a net for each solid figure or identify the solid figure each net represents.

1. 

2. 

3. 

Ask Yourself
- Did I include all of the faces and bases in my net?

Explain Your Thinking ▶ The two bases of a solid figure are congruent pentagons. Which type of solid figure is this?

Practice and Problem Solving

Draw a net for each solid figure or identify the solid figure each net represents.

4. 

5. 

6. 

7. 

Refer to the solid figures on page 552 to help you complete the table. Use the table for Problem 9.

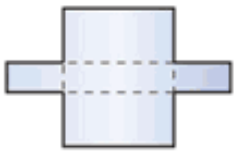
8.

Solid Figure	Rectangular Prism	Triangular Prism	Square Pyramid	Triangular Pyramid
Number of Faces				
Number of Vertices				
Number of Edges				

9. **Algebra** Compare the number of faces, vertices, and edges of a rectangular prism. Repeat this step for the other solid figures in the table. Write a formula to represent the relationship.

Solve.

10. **Analyze** Tara is having trouble folding this flattened-out box into a rectangular prism. What is wrong with it?



11. **Reasoning** What solid shape does a roll of postage stamps represent? Why do you think that postage stamps are not sold in packages that are the shape of pyramids?

12. **Represent** Kim is mailing a drum. Its height is $5\frac{1}{2}$ inches and its diameter is 14 inches. Draw and label a net for a rectangular box to hold the drum.

13. **Write About It** How are prisms and pyramids alike? How are they different?

Mixed Review and Test Prep

Open Response

Write each fraction as a decimal. (Ch. 7, Lesson 6)

14. $\frac{9}{10}$

15. $\frac{6}{100}$

16. $\frac{3}{5}$

17. $\frac{1}{8}$

18. $\frac{2}{9}$

19. $\frac{5}{12}$

20. Draw a net for a box in the shape of a cube, where each edge must be 18 inches long. The piece of cardboard you are using is 90 inches long and 36 inches wide. (Ch. 21, Lesson 1)

Extra Practice See page 573, Set A.

Hands On Lesson 2

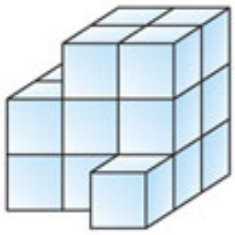
Two-Dimensional Views of Solid Figures

Objective Draw and identify different views of a solid figure.

Materials
centimeter cubes
centimeter grid paper (Learning Tool 15)
triangle dot paper (Learning Tool 19)

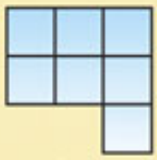
Work Together

When you look at a solid figure, parts of the figure are hidden from view. Looking at different views of a figure can help you visualize the hidden parts.



Work with a partner to draw the top, side, and front views of this figure.

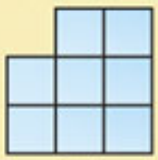
STEP 1 Look at this figure from above. This top view shows the number and position of cubes in the bottom layer of the figure. Draw this view on grid paper.


top

STEP 2 Look at this figure from the right side.This side view shows three layers in the two back columns and one layer in the front column. Draw this view.


right side

STEP 3 Look at this figure from the front. This front view shows that there are no cubes on the left side of the top layer. Draw this view.


front

Now use triangle dot paper to show how each layer of the figure is formed.

STEP 1 Draw the bottom layer of the figure.



STEP 2 The middle layer consists of three cubes in the two back columns. Draw the middle layer.



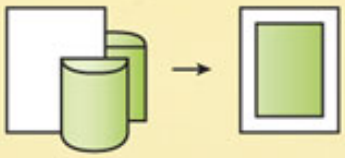
STEP 3 The top layer has no cubes along the left side. Draw the top layer.



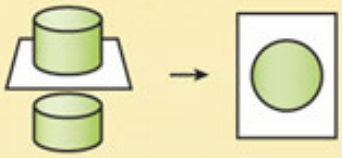
A cross section is a cut through a solid object.
You can think of it as one flat slice through a solid.

Sketch three cross sections of a cylinder.

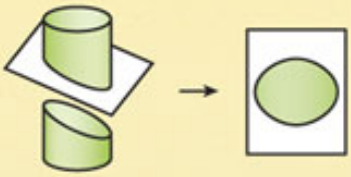
STEP 1 Think of cutting a vertical slice through a cylinder. Sketch the shape of the surface formed by the slice.



STEP 2 Think of cutting a horizontal slice through a cylinder. Sketch the shape of the surface formed by the slice.



STEP 3 Think of slicing through the cylinder in a way that is not vertical or horizontal. Sketch the shape of the surface formed by the slice.



On Your Own

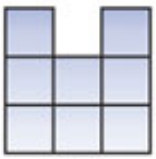
Draw the top, front, and side views of each solid figure.

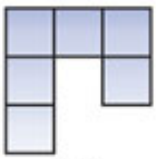
1. 

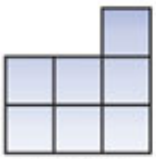
2. 

3. 

4. Use centimeter cubes to build the figure shown in the three different views. Then draw the figure on triangle dot paper.


front


top


right side

Go On

Which cross sections are possible for each solid?

5.



a.



b.



c.



6.



a.



b.



c.



7. This picture shows the base of a figure made from centimeter cubes. The numbers indicate how many cubes are stacked on each square. Build the figure. Then draw its front and side views.

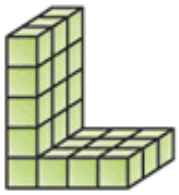
3	3	3
2	2	2
1	1	1

How many small cubes are used? Explain how you know.

8.

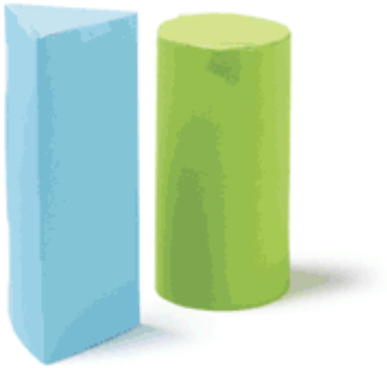


9.



10. **Create and Solve** Use centimeter cubes to build a solid figure. Draw the base of the figure. Label your drawing to show how many cubes are stacked on each cube of the base. Then give your drawing to a classmate to use as a blueprint to build your solid figure.

11. Create paper models of a prism and a cylinder. Sketch the top and side views of each. What are the shapes of the views?



Talk About It • Write About It

You learned how to draw and identify different views of solid figures.

12. How many cross-section shapes does a cylinder have? Explain.

13. Do any different solid figures have the same front, side, and top views?

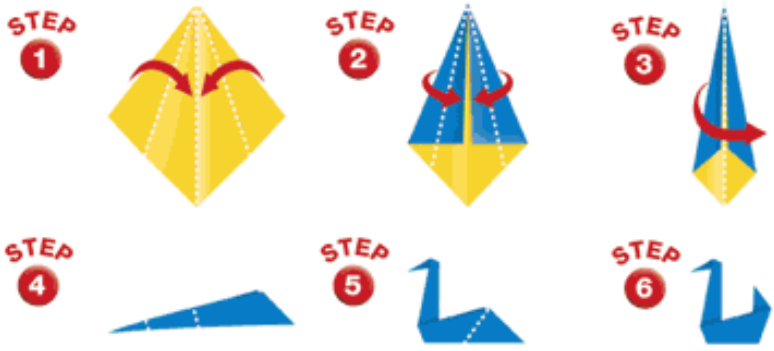
Activity

Visual Thinking

The Art of Paper Folding

Origami is the Japanese art of folding paper to make animals and other figures. Historians believe that origami, like paper, originated in China.

To make a paper swan, start with a square sheet of paper and follow the steps below.





Now unfold your swan. What types of triangles are formed by the creases in the paper?

Logical Thinking

A Good Day for Pi

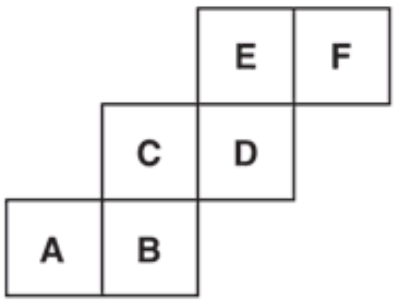
Besides being Albert Einstein's birthday, March 14 is Pi Day. Pi Day isn't an official holiday anywhere–yet. It is just a day for celebrating the number π (3.14).

The class at the Taft School ordered pizza to celebrate Pi Day. Jo said that a medium pizza (12 inches in diameter) is more than twice the area of a small pizza (8 inches in diameter). Is she right?



Brain Teaser

Imagine this net is folded to make a cube.



Which face is opposite

a. face A? **b.** face B? **c.** face C?

Education Place

Check out **eduplace.com/map** for more brain teasers.

Hands On Lesson 3

Estimate Volume

Objective Use centimeter cubes to estimate the volume of a solid figure.

Vocabulary
volume
unit cube

Materials
centimeter cubes
boxes, cans, and other empty containers

Work Together

The **volume** of a solid figure is a measure of the space enclosed by the figure. Volume is measured in cubic units. A **unit cube** is a cube with edges that are 1 unit in length.

Work with a partner to estimate the volume of different containers.

STEP 1 Count the number of unit cubes you use as you fill a box with cubes.
- What is the approximate volume?
- Record your estimate.



STEP 2 Count the number of unit cubes you use as you fill a can with cubes.
- What is the approximate volume?
- Record your estimate.
- Which container—the box or the can—had fewer gaps between the cubes? Explain.

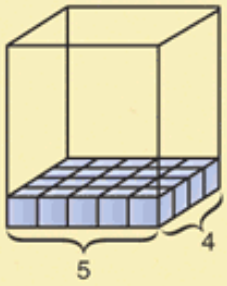


STEP 3 Choose some other containers. Estimate the volumes of these containers. Record your estimates. Then count the cubes as you fill each container.
- How did your estimates compare with the number of unit cubes it took to fill each container?



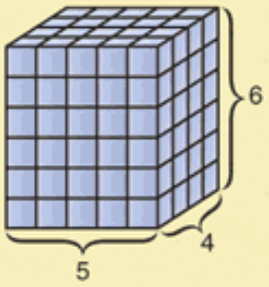
Find a better estimate of the volume of a box.

STEP 1 Place a layer of unit cubes to cover the bottom of a small box.



- How many unit cubes did you need to cover the bottom of the box?
- What is the length of the layer?
- What is the width of the layer?
- What is the area of the layer?

STEP 2 Continue to add layers of unit cubes to the box until it is full.



This box has 6 layers.

- How many layers of unit cubes did you place in the box?
- What is the height of the box?
- How can you use the height and the area of the base to find the volume?
- What is the volume of the box?

On Your Own

1. Empty the cubes from the box in the activity above. Use all of the cubes to build another solid figure. What is the volume of the new figure? Explain.
2. Build at least two rectangular prisms that each have a volume of 24 unit cubes and bases with different dimensions. What are the dimensions of each prism?



3. **Reasoning** If you had 24 unit cubes, would you be able to build a cube using all 24? Why or why not?
4. **Explain** If you fill a cylindrical can with unit cubes, will you get an underestimate or an overestimate of its volume?

Talk About It • Write About It

You learned how to estimate and find the volume of some solid figures.

5. Describe in your own words how to find the volume of a solid figure that has a length of 6 units, a width of 3 units, and a height of 7 units.
6. If two solid figures have the same volume, must they also have the same shape?

Lesson 4

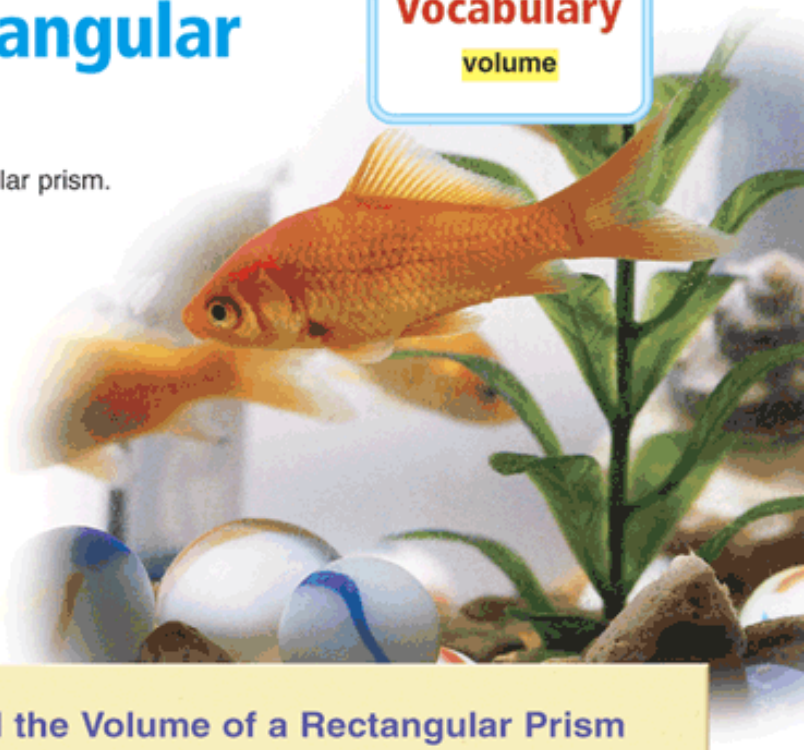
Audio Tutor 2/27 Listen and Understand

Volume of Rectangular Prisms

Vocabulary

volume

Objective Find the volume of a retangular prism.



Learn About It

You can have an aquarium custom built in almost any size. To determine how much water is needed to fill an aquarium, you need to know the volume of the aquarium.

Find the volume of a rectangular aquarium with a length $l = 4$ feet, a width $w = 2$ feet, and a height $h = 3$ feet.

Different Ways to Find the Volume of a Rectangular Prism

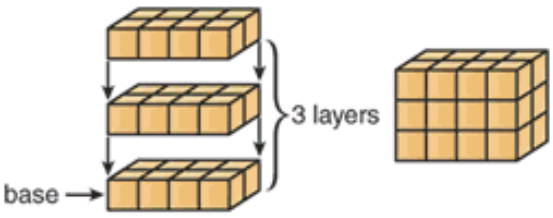
Way 1 Use unit cubes.

bottom layer: $4 \times 2 = 8$ ft^3

middle layer: $4 \times 2 = 8$ ft^3

top layer: $4 \times 2 = 8$ ft^3

$3 \times 8 = 24$ ft^3



Way 2 Use a formula.

Volume (V) = Base area (B) × height (h)

$V = B \times h$

$= 8 \times 3$

$= 24$ ft^3

This is the same as:

$V = lwh$

$= 4 \times 2 \times 3$

$= 24$ ft^3

Solution: The volume of the aquarium is 24 ft^3.

Other Examples

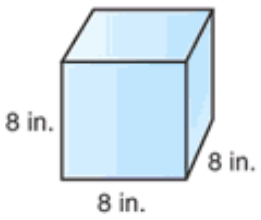
A. Volume of a Cube

$V = s \times s \times s = s^3$

$V = s^3$

$= 8 \times 8 \times 8$

$= 512$ in.^{3}



B. Finding an Unknown Dimension of a Rectangular Prism

$V = 60$ cm^3, $l = 6$ cm, and $w = 5$ cm. Find h.

$V = lwh$

$60 = 6 \times 5 \times h$ ← Substitute into the formula.

$60 = 30 \times h$

$\frac{60}{30} = \frac{30}{30} \times h$ ← Divide both sides by 30.

$2 = h$

$h = 2$ cm

Suppose each dimension of the aquarium were doubled. Would the volume of the aquarium also be doubled?

Find the volume if the length, width, and height are doubled.

STEP 1 Double the length, width, and height.

$l = 4 \times 2 = 8$ ft

$w = 2 \times 2 = 4$ ft

$h = 3 \times 2 = 6$ ft

STEP 2 Use the formula to find the volume.

$V = lwh$

$= 8 \times 4 \times 6$

$= 192$ ft^3

STEP 3 Find how many times greater 192 is than 24.

$192 = 24x$

$x = \frac{192}{24} = 8$

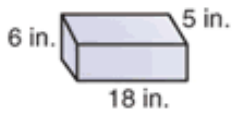
192 is 8 times 24.

Solution: No. If the dimensions of the aquarium were doubled, the volume would be 8 times the volume of the original aquarium.

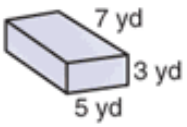
Guided Practice

Find the volume of each rectangular prism.

1.



2.



3. $l = 8$ ft
$w = 2$ ft
$h = 13$ ft

Ask Yourself

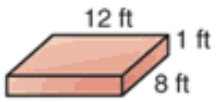
- Did I use the correct cubic units in my answer?

Explain Your Thinking Does it matter which dimension of a rectangular prism is used as the height? Explain why or why not.

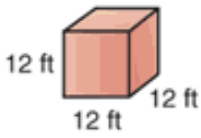
Practice and Problem Solving

Find the volume of each rectangular prism.

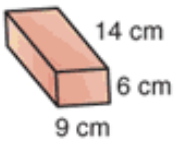
4.



5.



6.



7. $l = 16$ cm
$w = 10$ cm
$h = 8$ cm

Algebra • Equations

Find the missing measure of each rectangular prism.

8. $V = 81$ m^3
$w = 9$ m
$h = 3$ m
$l =$ ■

9. $l = 8$ in.
$V = 320$ in.^{3}
$h = 5$ in.
$w =$ ■

10. $l = 7$ yd
$w = 7$ yd
$V = 343$ yd^3
$h =$ ■

11. $l = 9$ cm
$V = 216$ cm^3
$h = 4$ cm
$w =$ ■

12. What can you tell about the rectangular prism in Exercise 10?

Go On

Find the volume of each rectangular prism. Then find the volume of the rectangular prisms with the dimensions doubled.

	Original Dimensions				Doubled Dimensions			
	l (ft)	w (ft)	h (ft)	V (ft^3)	l (ft)	w (ft)	h (ft)	V (ft^3)
13.	1	1	1	■	2	2	2	■
14.	3	8	3	■	■	■	■	■
15.	2.5	4	2.5	■	■	■	■	■
16.	6	7	8	■	■	■	■	■

17. **Algebra** Look back at Exercises 13–16. Suppose the original dimensions of a rectangular prism are l, w, and h. Write a formula for the volume of a rectangular prism when the dimensions are doubled.

18. **Predict** What would happen to the volume of a rectangular prism if all the dimensions were doubled again? Use an example to support your prediction.

Calculator **Find the volume or the missing dimension of each rectangular prism.**

19. $l = 4.7$ cm
$w = 2.5$ cm
$h = 3.2$ cm
$V =$ ■

20. $V = 253.5$ ft^3
$l = 7.8$ ft
$w = 6.5$ ft
$h =$ ■

21. $V = 2{,}910.6$ m^3
$l = 14$ m
$h = 12.6$ m
$w =$ ■

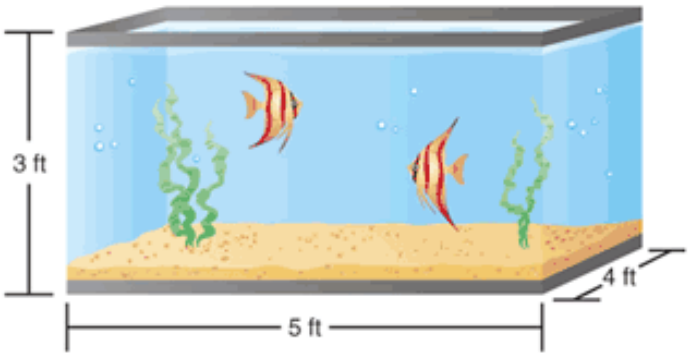
22. $V = 4{,}973.94$ cm^3
$w = 12.2$ cm
$h = 22.65$ cm
$l =$ ■

Choose a Computation Method

Mental Math • Estimation • Paper and Pencil • Calculator

Solve. Explain which method you chose.

23. A gallon of water uses 231 cubic inches of space. About how many gallons of water are needed to fill this aquarium?



24. At least 8 gallons of water should be allowed for each fish in an aquarium. Using this rule, what is the maximum number of fish that should be in the aquarium shown in Problem 23?

25. **Analyze** What is the greatest number of $3 \times 4 \times 2$-inch rectangular boxes of fish food that can fit in a carton with a volume of 3 cubic feet and a width of 1 foot?

26. The volume of a carton is 384 cubic inches. The height of the carton is 6 inches. The base is a square. What are the dimensions of its base?

Extra Practice See page 573, Set B.

Problem Solving

Social Studies Connection

Miniature Villages

Miniland in Carlsbad, California, includes famous landmarks from Washington, D.C., New Orleans, New York, California, and New England. The buildings and landscapes are built entirely to 1:20 scale.

1. The Washington Monument is about 555.4 feet high. What would be its height in feet (h) at Miniland?

Scale → $\frac{1}{20} = \frac{h}{555.4 \text{ ft}}$ ← Model



Find the dimensions in inches of the following prisms, given a 1:20 scale.

2. $l = 2$ ft, $w = 16$ in., $h = 1$ ft

3. $l = 3$ ft, $w = \frac{2}{3}$ ft, $h = 6$ in.

WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–4.

Draw a net for each solid figure. (Lesson 1)

1.



2.

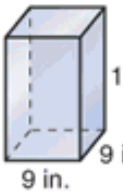


3. Draw the top, front, and side view of the solid figure at the right. (Lesson 2)



Find the volume of each rectangular prism. (Lessons 3–4)

4.



$V =$ ■

5. $l = 12$ m
$w = 9$ m
$h = 3$ m

$V =$ ■

Extra Practice at eduplace.com/map

Lesson 5

Audio Tutor 2/28 Listen and Understand

Surface Area

Objective Find the surface area of a rectangular prism, a triangular prism, and a square pyramid.

Vocabulary
- surface area
- net

Learn About It

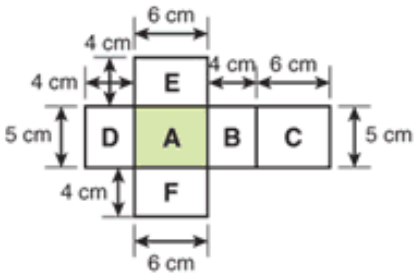
Surface area (SA) is the sum of the areas of all the faces of a solid figure.

Kim made this memory box in the shape of a rectangular prism. To find the least amount of hand-made paper she needs to cover the outside surfaces of the box, Kim has to find the surface area.



Find the surface area of the memory box.

STEP 1 Draw a **net** of the rectangular prism.



STEP 2 Find the area of each pair of opposite faces using the formula for the area of a rectangle.

Faces A and C	Faces B and D	Faces E and F
$A = 2lw$	$A = 2wh$	$A = 2lh$
$= 2(6 \times 5)$	$= 2(5 \times 4)$	$= 2(6 \times 4)$
$= 60 \text{ cm}^2$	$= 40 \text{ cm}^2$	$= 48 \text{ cm}^2$

STEP 3 Add the areas of all the faces to find the surface area.

$SA = 2lw + 2wh + 2lh$

$SA = 60 \text{ cm}^2 + 40 \text{ cm}^2 + 48 \text{ cm}^2$

$SA = 148 \text{ cm}^2$

Solution: The surface area is 148 cm^2, so Kim needs at least 148 square centimeters of paper.

Other Examples

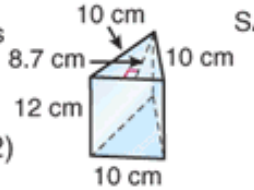
A. Surface Area of a Triangular Prism With Equilateral Triangular Base

SA = areas of bases + areas of faces

$= 2(\frac{1}{2}bh) + 3(lw)$

$= 2(\frac{1}{2} \times 10 \times 8.7) + 3(10 \times 12)$

$= 447 \text{ cm}^2$



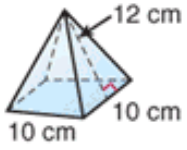
B. Surface Area of a Square Pyramid

SA = area of base + areas of faces

$= s^2 + 4(\frac{1}{2}bh)$

$= (10 \times 10) + 4(\frac{1}{2} \times 10 \times 12)$

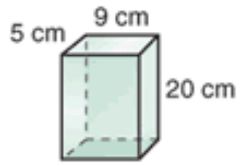
$= 340 \text{ cm}^2$



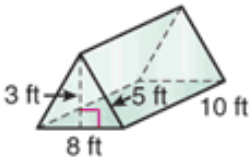
Guided Practice

Find the surface area of each figure.

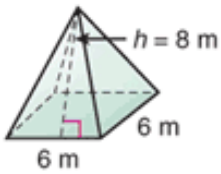
1.



2.



3.



Ask Yourself

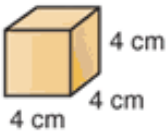
- Did I include the areas of all the faces?
- Could I have used a formula?

Explain Your Thinking How is finding the surface area of a rectangular prism like finding the surface area of a square pyramid?

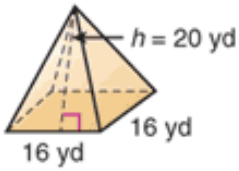
Practice and Problem Solving

Find the surface area of each figure.

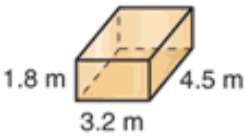
4.



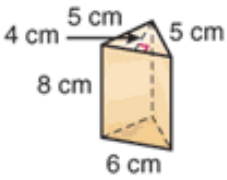
5.



6.



7.



Solve.

8. Reasoning To find the surface area of the figure in Exercise 4, Juanita used the formula $SA = 6s^2$ and Sam used the formula $SA = 2(lw) + 2(wh) + 2(lh)$. Do both students' formulas work? Explain.

9. Unfold an empty cereal box or cracker box to show a net. Measure and label the dimensions of each face of the net. Use the dimensions to find the surface area of the cereal box.

10. Double each dimension of the box you unfolded in Problem 9. How many times greater is the surface area? Compare your results with your classmates' results. How does doubling the dimensions of a rectangular prism change its surface area?

11. Estimate The edges of a square pyramid are each 7.89 cm. The perpendicular height of each triangular face is 6.83 cm. Estimate the surface area if the lengths of each edge and the height are tripled. Support your answer.

Mixed Review and Test Prep

Open Response

Find the circumference and area. Use 3.14 or $\frac{22}{7}$ for π. (Ch. 20, Lessons 2 and 7)

12. $d = 12$ m

13. $r = 2.5$ m

14. $r = 14$ m

15. $r = 9$ m

16. $d = 3.6$ m

17. $d = 14$ m

Multiple Choice

18. The surface area of a cube is 96 cm^2. What is the length of each edge? (Ch. 21, Lesson 5)

A 1 cm

C 3 cm

B 2 cm

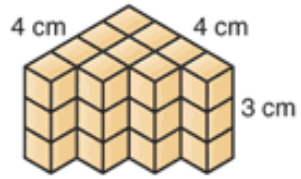
D 4 cm

Extra Practice See page 573, Set C.

Lesson 6

Volume of Other Solids

Objective Find the volume of a triangular prism, a cylinder, a pyramid, and a cone.



Right Prism

Learn About It

The solid at right has a base as shown and a height of 3 cm. Each unit square is 1 cm by 1 cm, so the area of the base is 10 cm^2. Each unit square in the base is also the base of a stack of 3 unit cubes. Therefore, the volume V of the solid is 30 cubic centimeters (30 cm^3).

$$V = 10 \times 3 = 30 \text{ cm}^3$$

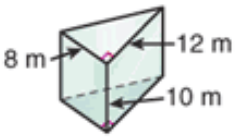


For solids like prisms and cylinders with congruent bases,

V = area of base (B) × height (h)

Volumes of Triangular Prisms and Cylinders

Triangular Prism



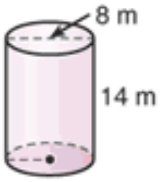
V = area of base (B) × height

$= \frac{1}{2}bh_1 \times h_2$

$= \frac{1}{2} \times 12 \times 8 \times 10$

$V = 480 \text{ m}^3$

Cylinder



V = area of base (B) × height

$= \pi r^2 \times h$

$\approx 3.14 \times 4^2 \times 14$

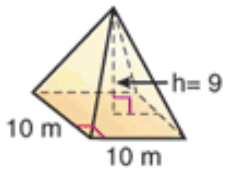
$V \approx 703.36 \text{ m}^3$

The formula for the volume of a pyramid or a cone is related to the formula for the volume of a prism or a cylinder.

$$V = \frac{1}{3}Bh$$

Volumes of Pyramids and Cones

Square Pyramid



$V = \frac{1}{3} \times$ area of base (B) × perpendicular height (h)

$= \frac{1}{3} \times s^2 \times h$

$= \frac{1}{3} \times 10 \times 10 \times 9$

$V = 300 \text{ m}^3$

Cone



$V = \frac{1}{3} \times$ area of base (B) × perpendicular height (h)

$= \frac{1}{3} \times \pi r^2 \times h$

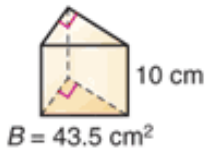
$\approx \frac{1}{3} \times 3.14 \times 6^2 \times 16$

$V \approx 602.88 \text{ m}^3$

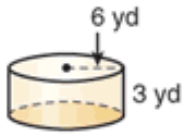
Guided Practice

Find the volume of each figure. Use 3.14 as an approximation of π.

1.



2.



3.



Ask Yourself

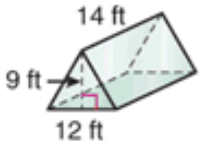
- Did I choose the correct formula?
- Did I find the area of the base first?

Explain Your Thinking How could the figure in Exercise 1 and a copy of itself be used to form a rectangular prism?

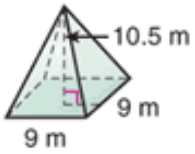
Practice and Problem Solving

Find the volume of each figure. Use 3.14 as an approximation of π.

4.



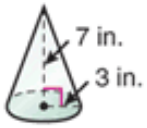
5.



6.



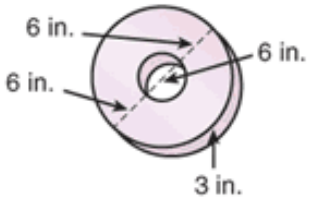
7.



Solve.

8. A mold for a toy is in the shape of a cylinder with a radius of 9 inches and a height of 3 inches. Another mold is in the shape of a triangular prism with a base of 324 square inches and a height of 2 inches. Which mold has the greater volume?

9. Analyze Ellis made this ring-shaped mold. What is the volume of the mold?



10. Write About It How are the formulas for the volume of a square pyramid and the volume of a cone similar? How do they compare to the volume of a rectangular prism?

11. Rose has a paper circle that is 10 inches in diameter. She uses it as the base for a cone with a height of 9 inches. What is the volume of Rose's cone?

Mixed Review and Test Prep

Open Response

Find the perimeter of each figure.
(Ch. 20, Lesson 1)

12. an equilateral triangle with 7-inch sides

13. a rhombus with 1.75-yard sides

14. a hexagon with two 9-inch sides and the remaining sides 4.65 inches each

Multiple Choice

15. A square pyramid has a volume of 1,200 cubic centimeters. If the base has sides 15 centimeters in length, what is the height of the pyramid?
(Ch. 21, Lesson 6)

A 5 cm **C** 16 cm

B 5.3 cm **D** 240 cm

Extra Practice See page 573, Set D.

Lesson 7

Audio Tutor 2/29 Listen and Understand

Problem-Solving Application

Use a Formula

Objective Use a formula to solve a problem.



You can often use formulas to solve problems.

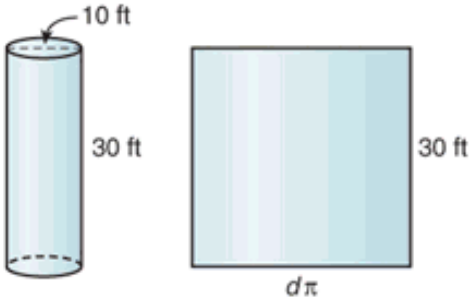
Problem Hector is helping his father paint the silo on their farm. The silo's top is a circular dome with a diameter of 10 feet. The silo is 30 feet tall. The silo will be painted red. The dome will not be painted. What is the area of the part of the silo that Hector and his father will paint red?

▶ It helps to draw a diagram first.

Draw a net to represent the silo.

The curved surface becomes a rectangle with a length equal to the silo's circumference (πd) and a width equal to the silo's height (30 ft).

Remember: Hector will not paint the top or bottom of the silo.



▶ Find the area of the silo's curved surface.

Recall that the formula for the area of a rectangle is $A = lw$. Substitute the silo's circumference (πd) for l and the silo's height $h = 30$ ft for w.

A = circumference of the base × height $= \pi d \times h$

$A \approx 3.14 \times 10 \times 30$

$= 942 \text{ ft}^2$

Solution: The area to be painted is about 942 square feet.

Look Back Look back at the problem. How could you use estimation to check your answer?

Extra Help at eduplace.com/map

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

Ask Yourself

- Should I use the formula for surface area or volume?
- What numbers do I substitute in the formula?
- Did I answer the question I was asked?

1. Hector's mother bought a storage box for the barn. The box is a rectangular prism with a volume of 45 cubic feet. It is 10 feet long and 3 feet wide. What is the height of the box?

2. Hector's sister painted the outside of the box described in Problem 1. She painted the sides and the bottom of the box. Find the area she painted.

   Hint Surface area is the total area of all the surfaces of a solid figure. Volume is the amount of space enclosed by the figure.



Independent Practice

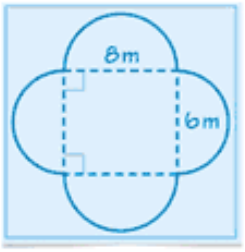
Solve. Use a formula when possible.

3. **Estimate** Hector's family needs to know the volume of their barn. The barn is a rectangular prism that is 120 feet long, 70 feet wide, and 30 feet tall. What is the volume of the barn?

4. Hector's family wants to paint all four walls of the barn in Problem 3 once. A gallon of paint covers 315 square feet. The paint comes in 5-gallon buckets. How many buckets of paint will they need?

5. Hector's father is drilling holes in the ground for the foundation of a windmill. One hole is to be 3 meters deep and have a radius of 0.5 meters. Find the volume of the cement needed to fill the hole.

6. At right is a plan for a garden. Find the area of the garden.



7. A cardboard box holds one slice of a pie. The box is a triangular prism that is $1\frac{1}{2}$ inches high. The base of the prism is an isosceles triangle with a base of 3 inches and a height of 4 inches. Find the volume of the box.

8. The formula for the area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$. The height of the trapezoid is h, and the lengths of the bases are b_1 and b_2. Find the area of a trapezoid with a height of 6 meters and bases of 145 meters and 16 meters.

Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

9. **Explain** A set of containers comes in these sizes: 12 × 8 × 2 inches, 10 × 6 × 4 inches, and 8 × 4 × 6 inches. If the fourth container in the set follows the same pattern, will it be large enough to hold a 100 cubic-inch serving of potato salad?

10. A container is in the shape of a cube. George designs and decorates the cardboard patterns that when folded become the cubes. How many different cardboard patterns are possible?

11. Suppose you have a rectangular sheet of paper that measures 2 feet × $2\frac{1}{2}$ feet. What is the greatest number of 3-inch × 5-inch labels that can be cut from the sheet?

You Choose

Strategy

- Draw a Diagram
- Make a Table
- Make an Organized List
- Fina a Pattern
- Use Models

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Solve. Tell which method you chose.

12. Four people exited the elevator on the second floor. On the third floor, 2 people entered and 5 exited. After 2 people entered on the fourth floor, there were 6 people on the elevator. How many people got on at the first floor?

13. Cary is planning a party. His recipe box contains recipes for 3 vegetable dishes, 4 salads, 2 casseroles, 3 pasta dishes, and 2 desserts. How many different possible menu choices does Cary have if he chooses one of each kind of recipe?

14. A 1-pound 10-ounce box of salt has 491 servings. Each serving contains 25% of the daily requirement of sodium, or 590 milligrams. How many milligrams of sodium are in half of a box of salt?

15. A basket has twice as many apples as oranges and two more bananas than apples. Six pieces of fruit are either bananas or pears. There are two pears. How many oranges are there?



Mixed Review and Test Prep

Open Response

Order these integers from least to greatest.
(Ch. 11, Lesson 2)

16. $^{-}7$ 0 $^{+}4$ $^{+}6$

17. $^{-}9$ $^{+}3$ $^{-}3$ $^{+}4$

18. $^{-}8$ $^{-}1$ 0 $^{+}2$

Multiple Choice

19. What is the distance between two towns if they are 4.5 inches apart on a map with a scale of 0.5 in. = 75 mi?
(Ch. 16, Lesson 6)

A 16.67 mi **C** 168.75 mi

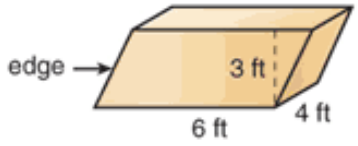
B 337.5 mi **D** 675 mi

Problem Solving

Measurement Sense

Volume of Any Prism

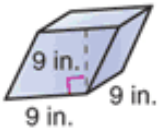
The height of any prism is given by the perpendicular distance between the bases. So the volume of any prism is the same as the volume of a right prism with the same base area and height.



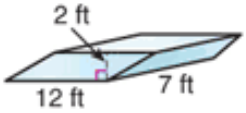
$V = 6 \times 4 \times 3 = 72 \text{ ft}^3$

Find the volume of each prism.

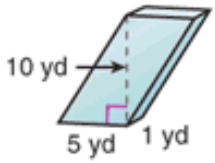
1.



2.



3.



4. Reasoning A package of paper that contains 500 sheets is called a ream of paper. What happens to the volume of the paper if it is tilted to one side as shown below?



5. Explain how you would find the volume of a triangular pyramid. How does the volume of the pyramid compare to the volume of a square prism with the same base area and the same height?

Chapter Review/Test

VOCABULARY

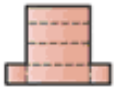
1. A _____ is a solid figure enclosed by polygons.
2. A _____ is formed by unfolding a solid figure and laying it flat.
3. A solid figure with 1 curved surface and 2 circular bases is called a _____.
4. The _____ is a measure of the number of cubic units contained in a solid figure.

Vocabulary

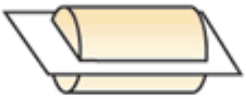
net
cone
volume
cylinder
polyhedron

CONCEPTS AND SKILLS

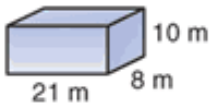
Identify the solid figure each net represents. (Lesson 1, pp. 552–553)

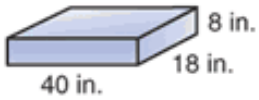
5. 
6. 
7. 

Draw the cross section formed by each given cut. (Lesson 2, pp. 554–556)

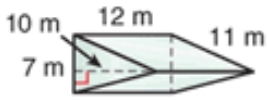
8. 
9. 
10. 

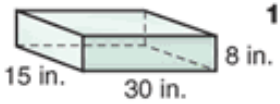
Estimate the volume for Exercise 11. Find the exact volume in Exercises 12–13. (Lessons 3–4, pp. 558–562)

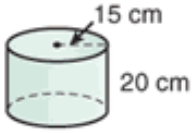
11. 
12. 

13. 

Find the surface area of each figure. (Lesson 5, pp. 564–565)

14. 

15. 

16. 

PROBLEM SOLVING

Solve. (Lesson 7, pp. 568–570)

17. A box shaped like a rectangular prism is 12 inches wide and 6 inches long. It has a volume of 1,296 cubic inches. Can it fit in a $12\frac{1}{2}$-inch-wide space on the bottom shelf of a storage bin that has 16 inches of space between shelves?

Write About It

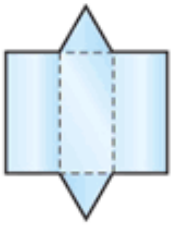
Show You Understand

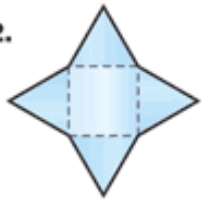
How can you find the length of the edge of a cube when the surface area is known?

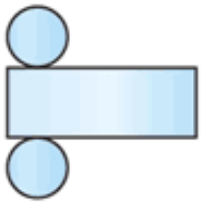
Extra Practice

Set A (Lesson 1, pp. 552–553)

Identify the solid figure each net represents.

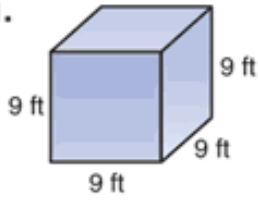
1. 

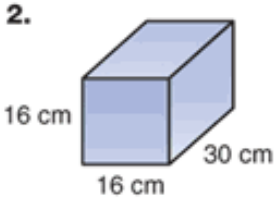
2. 

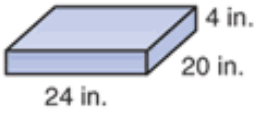
3. 

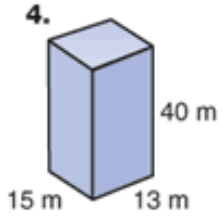
Set B (Lesson 4, pp. 560–562)

Find the volume of each rectangular prism.

1. 

2. 

3. 

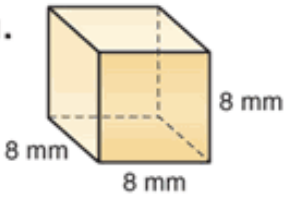
4. 

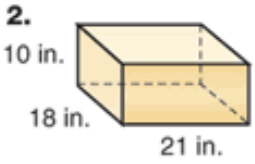
5. a cube with edges of 10 km

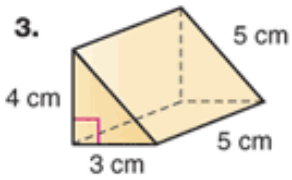
6. a cube with edges of 0.1 cm

Set C (Lesson 5, pp. 564–565)

Find the surface area of each figure.

1. 

2. 

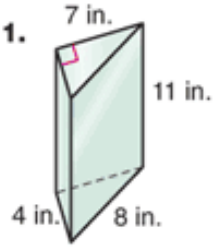
3. 

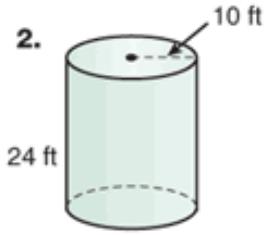
4. 

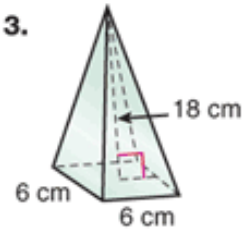
5. a rectangular prism with a length of 20 in., a width of 40 in., and a height of 10 in.

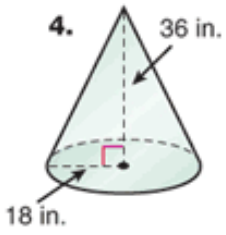
Set D (Lesson 6, pp. 566–567)

Find the volume of each figure.

1. 

2. 

3. 

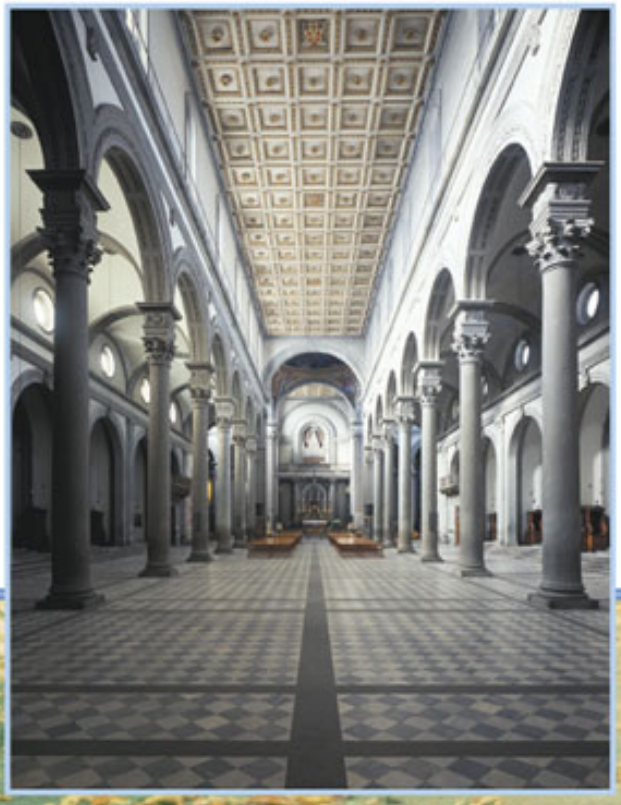
4. 

WEEKLY WR READER®
Art Connection

Put It in Perspective

It wasn't until the 1500s that artists began to use geometry to create something called *perspective*. They used a *horizon line* and a *vanishing point* so that solid objects would appear the way they look in real life when drawn on a flat surface.

Imagine looking down a straight street or road. The horizon is the line where land and sky seem to meet. The vanishing point is where the two parallel sides of the street seem to come together on the horizon.



Problem Solving

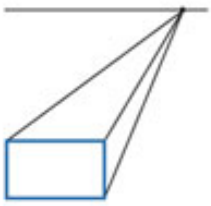
Materials
grid paper
pencil
straightedge or ruler

Complete Steps 1–3 to create a perspective drawing of a rectangular prism.

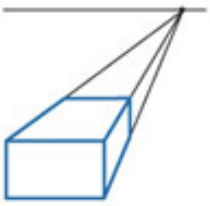
Step 1
Draw a rectangle. Then draw a horizon line. Pick and mark a vanishing point.



Step 2
Lightly draw lines to connect the vertices of the rectangle to the vanishing point.



Step 3
Draw line segments to complete the rectangular prism. Erase the guidelines.



Solve. Follow Steps 1–3 to draw each object.

1. On a new sheet of paper, draw two rectangular prisms of different sizes and in different positions, using the same vanishing point.

2. **Analyze** How would you draw a "transparent" prism?

3. Copy the block letter at right onto grid paper. Use perspective to make the letter look solid.



4. **Create and Solve** Use what you have learned to create a perspective drawing of a street or a house. Include trees and people in your drawing.

Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic.

Enrichment

Möbius Strip



A Möbius strip has only one side and one edge.

Can an object have only one surface? Look at a sheet of paper. It has two surfaces, a front and a back. In the 1800s, German mathematician August Ferdinand Möbius (1790–1868) discovered the properties of one-sided surfaces. One shape he wrote about bears his name: the Möbius strip.

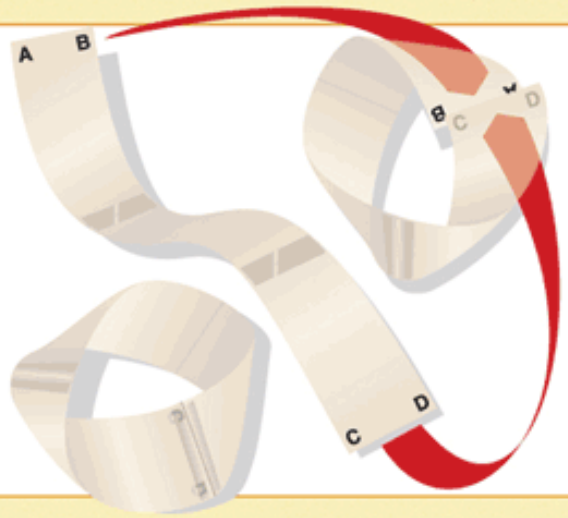
Follow these steps to make your own Möbius strip.

Materials: paper, scissors, tape, pencil or marker

Step 1 Cut a sheet of paper into thirds lengthwise. Tape the ends together to make one long strip. Then label the corners *A*, *B*, *C*, and *D* as shown.

Step 2 Bring the ends of the strip together. Twist one end a half turn so that the letters *A* and *D* touch and the letters *B* and *C* touch. Tape the ends together securely.

Now you have a Möbius strip!



Try These!

Solve. Use your Möbius strip.

1. Draw a line the long way down the center of the strip without taking your pencil off the paper. Stop when you get back where you started. Are there any sides without a line? Explain.

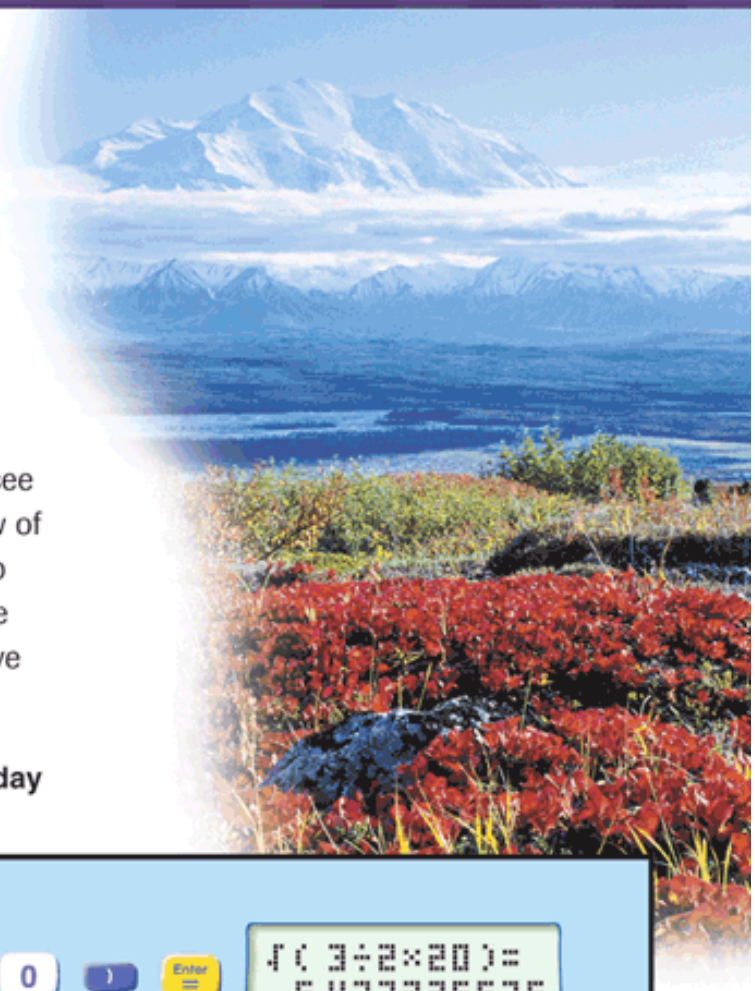
2. Predict what will happen if you cut the strip along the line you drew in Problem 1. Try it. Describe what happens.

3. Predict what will happen if you take the strip in Problem 2 and cut it along its centerline. Try it. Describe your results.

4. Make a new Möbius strip. This time, predict what will happen if you cut $\frac{1}{3}$ of the way in from an edge. Try it and describe what happens.

Explain Your Thinking Describe an experiment that would show that a Möbius strip has only one edge.

What a View



On a clear day it sometimes seems like you can see for miles! Assuming nothing is blocking your view of the horizon, you can use the formula $d^2 = (\frac{3}{2})h$ to estimate the distance in miles, d, that you can see on a clear day, from any given height in feet above sea level, h.

About how many miles can you see on a clear day at a height of 20 feet?

Use a calculator.

Press: (3 ÷ 2 × 2 0) Enter



Solution: You can see about 5.5 miles.

Use a calculator to find the distance you could see on a clear day if you were at the height of the objects in the table below. Round to the nearest tenth.

	Object	Height	Distance
1.	Sand Sculpture, DuQuoin, Illinois	24 feet	
2.	Roller Coaster, Sandusky, Ohio	300 feet	
3.	Tallest Living Tree, Ukiah, California	368.5 feet	
4.	Hoover Dam, Arizona–Nevada border	725 feet	
5.	Petronas Towers, Kuala Lumpur, Malaysia	1,483 feet	
6.	CN Tower, Toronto, Canada	1,815 feet	
7.	Mauna Loa volcano, Hilo, Hawaii	13,677 feet	
8.	Mt. Everest, Nepal, Tibet	29,028 feet	

Unit 7 Test

VOCABULARY Open Response

1. A number defined by the ratio of the circumference of any circle to its diameter is ____.
2. The ____ of a number *n* is the number that multiplied by itself equals *n*.
3. A polyhedron with two congruent parallel bases joined by rectangular faces is a ____.
4. A ____ is any solid figure enclosed by polygons.

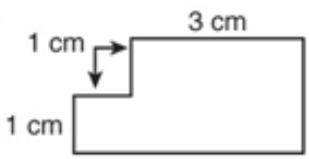
Vocabulary

- pi (π)
- prism
- pyramid
- polyhedron
- square root
- perfect square

CONCEPTS AND SKILLS Open Response

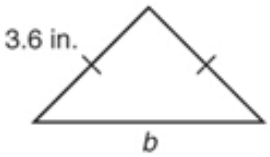
Find the perimeter or missing measure of each polygon. (Chapter 20)

5.



$P =$ ■

6.



$P = 12.3$ inches
$b =$ ■

7.



$P = 13.5$ feet
$s =$ ■

Find the circumference or missing measure for each circle. Use 3.14 or $\frac{22}{7}$ as approximations for π. (Chapter 20)

8. $r = 1.8$ cm; $C \approx$ ■

9. $d = 21$ in.; $C \approx$ ■

10. $C = 44$ ft; $d \approx$ ■

Find the area or missing measure for each figure. (Chapter 20)

11. Parallelogram *RSTU*
$h = 14$ ft
$b = 7$ ft
$A =$ ■

12. Square *ABCD*
$A = 0.49$ cm^2
$s =$ ■

13. Triangle *EFG*
$b = 3.5$ yd
$h = 6$ yd
$A =$ ■

14. Triangle *XYZ*
$A = 31.5$ m^2
$h = 7$ m
$b =$ ■

Write each perfect square, using the power 2. (Chapter 20)

15. 144

16. 400

17. 90,000

Find each square root. If necessary, round to the nearest tenth. (Chapter 20)

18. $\sqrt{81}$

19. $\sqrt{112}$

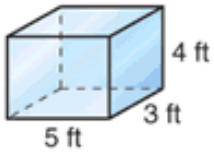
20. $\sqrt{4{,}900}$

Find the area of each circle. Use 3.14 or $\frac{22}{7}$ for π. (Chapter 20)

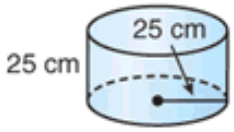
21. $d = 14$ m **22.** $r = 15$ ft **23.** $d = 168$ cm

Find the volume of each figure. Use 3.14 as an approximation for π. (Chapter 21)

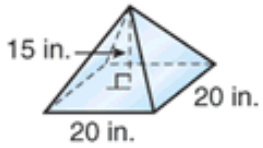
24.



25.

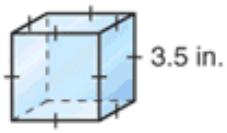


26.

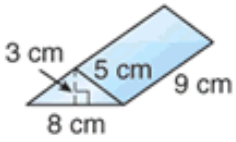


Find the surface area of each figure. (Chapter 21)

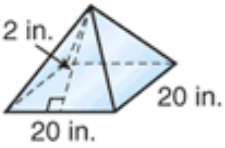
27.



28.



29.



PROBLEM SOLVING Open Response

30. Ted is making a circle graph to compare the hours each grade spent tutoring. Hours for the sixth grade represent 36% of the total. What measure central angle should Ted use for the sixth grade in his graph?

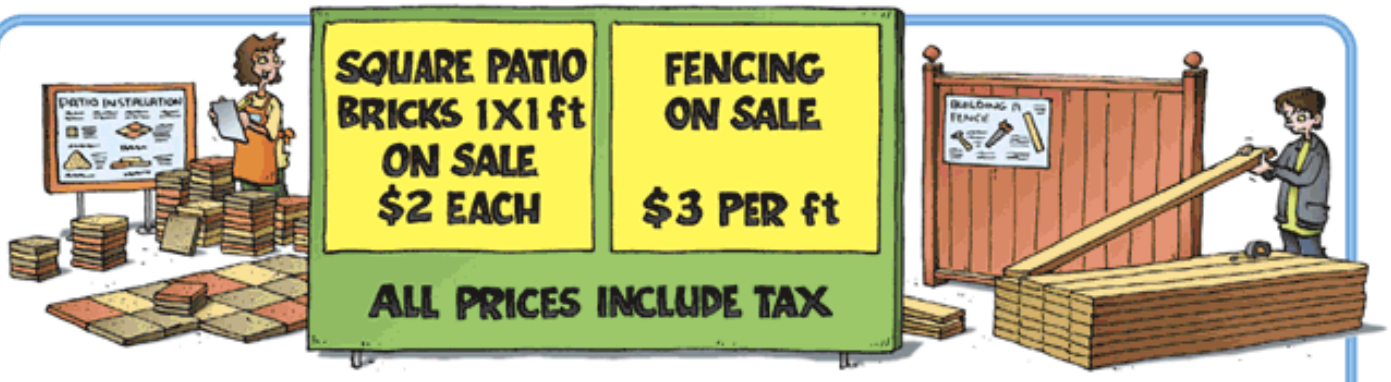
31. Last week Wendy earned $60. She put $30 in the bank, spent $15 on school supplies, kept $5 in her pocket, and used the rest to buy a birthday gift. Draw a circle graph to show how Wendy used her money.

32. A hatbox has the shape of a cylinder. The diameter of the base is 14 inches, and the height is 8 inches. What is the volume of the hatbox?

33. Karen's goal is to jog 100 kilometers in the next 6 weeks. Will she be able to reach her goal if she jogs 3.5 kilometers a day, 5 days a week? Explain.

Performance Assessment

Extended Response



Task You are making a patio with paving stones and surrounding the patio with fencing.

Use the prices above and the information at right to decide on the possible dimensions for the patio. Explain your thinking.

Information You Need

- Your budget for paving stones is $128 and you want to spend all of it.
- You want a rectangular-shaped patio.
- Your budget for fencing is no more than $125.

Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

Sometimes when you take a test, it is better to work out the problem before looking at the answer choices. Then compare your answer with the answer choices.

Look at the example below.

Cal bought some birdseed. It came in a package shaped like a square pyramid. The base was 4 inches on a side and the box was 6 inches tall. What was the volume of the package? Use the formula to help you solve the problem.

$$V = \frac{1}{3}Bh$$

A 96 in.^{3} **C** 32 in.^{3}

B 48 in.^{3} **D** 24 in.^{3}

THINK

Use the dimensions given and the formula to find the volume. Compare. Eliminate choices that are very different from your answer.

If your answer matches one of the choices, your answer is correct. If not, go back to the formula and try again.

Multiple Choice

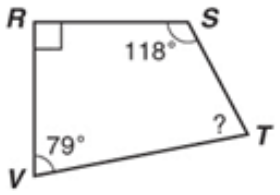
1. A triangle has a height of $3\frac{1}{2}$ ft and an area of $10\frac{1}{2}$ ft^2. What is the length of its base?

A 6 ft **C** $\frac{2}{3}$ ft

B 3 ft **D** $\frac{1}{3}$ ft

(Chapter 20, Lesson 5)

2. What is the measure of angle T in this quadrilateral?



F 63° **G** 67° **H** 73° **J** 83°

(Chapter 14, Lesson 6)

3. A drawing of a house has the scale $\frac{1}{4}$ in. : 1 foot. The actual length of the house is 43 feet. What is the length of the house in the drawing?

A $9\frac{3}{4}$ in. **C** $10\frac{3}{4}$ in.

B $10\frac{1}{4}$ in. **D** $11\frac{1}{4}$ in.

(Chapter 16, Lesson 6)

4. Ruth bought a sweater on sale for $24.50. The sales tax was 6%. What was the total cost of the sweater?

F $30.50 **H** $26.46

G $26.95 **J** $25.97

(Chapter 18, Lesson 8)

For Test-Taking Tips, See page 648.

Open Response

5. The Nelson family still owes $2,374.80 on their car. They have 24 equal monthly payments remaining. How much is each payment?

(Chapter 7, Lesson 4)

6. The tree outside Diane's house is 5.8 meters tall. How many centimeters tall is the tree?

(Chapter 8, Lesson 4)

7. These were the grades on a math quiz given in Evan's class.

83, 80, 76, 80, 80, 74, 75, 98, 82, 73, 73, 79, 80, 78, 72, 79, 81, 80

What is an outlier in the data?

(Chapter 9, Lesson 4)

8.

Ticket Prices ($)

Stem	Leaves
1	7 9
2	2 5 5
3	4 8 9
4	3 6 6

Key: 2 \| 5 means $25

What is the median ticket price?

(Chapter 10, Lesson 4)

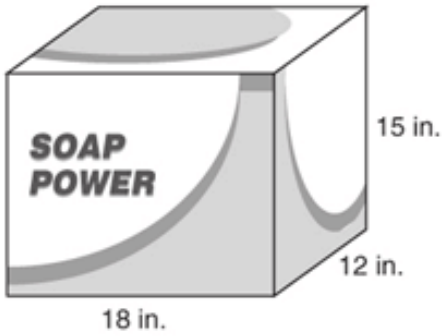
9. Brenda researched bus fares in her town.

Year	1980	1990	2000
Bus Fare	$0.65	$0.95	$1.25

If the pattern continues, what will the bus fare likely be in the year 2020?

(Chapter 13, Lesson 2)

Extended Response



10. A manufacturer wants to ship boxes of powdered soap in cartons like the one shown above.

A The cargo hold in one kind of truck is 21 feet long, 9 feet wide, and 8 feet high. What is the greatest number of cartons that will fit into this truck? (Cartons must rest on their 18 in. × 12 in. bases.)

B How much empty space will there be in the cargo hold after the truck has been loaded?

C The cargo hold of the truck has a weight capacity of 15,000 pounds. Each carton weighs 20 pounds. How does the total weight of the shipment compare with the truck's weight capacity?

D How many boxes will have to be removed so that the total weight is equal to the truck's capacity?

(Chapter 21, Lesson 4)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 7

Look back at the big ideas and vocabulary in this unit.

Big Ideas

In a circle, the ratio of the circumference to the diameter is always the same value, called π (pi).

Squaring a number and taking the square root of a number are inverse operations.

You can use formulas for finding the volume of a prism, a square pyramid, a cylinder, and a cone.

Key Vocabulary

- pi (π)
- circumference
- diameter
- square root
- volume

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how to find the area and circumference of a circular table top with a diameter of 30 inches.
2. Explain how to find the surface area of a rectangular prism with the base 9 feet by 8 feet and the height 5 feet.
3. Explain how the volume of a prism changes when each of its dimensions is doubled.
4. Explain how you could find the square root of 70 to the nearest tenth without using a calculator.
5. **Write About It** Look for photos showing examples of the solid figures you have studied. Classify the solid figures by name. Explain how you can estimate their volumes and surface areas.



UNIT 8

Rational Numbers and Algebra

CHAPTER 22

Rational Numbers

page 586



CHAPTER 23

Algebra: Expressions, Equations, and Inequalities

page 604



Reading Mathematics

Reviewing Vocabulary



Here are some math vocabulary words that you should know.

order of operations	the order in which operations in an expression to be evaluated are carried out
evaluate	to find the numerical value of an expression
equation	a mathematical sentence with an equals sign
addition and multiplication properties	The commutative, associative, and identity properties of addition and multiplication, the Zero Property of Multiplication, and the Distributive Property
integers	the set of positive whole numbers and their opposites (negative numbers) and zero

Reading Words and Symbols

In mathematics, the relationships among numbers can be expressed using words, words and symbols, and symbols alone.

Words	Symbols
• The sum of five and six is greater than one third of twenty-eight.	$(5 + 6) > \frac{28}{3}$
• Eight less than a number is equal to negative seventeen.	$n - 8 = {}^{-}17$

Write each statement, using symbols.

1. Fifty subtracted from thirty-one is less than negative six.
2. Two times a number is greater than ten.

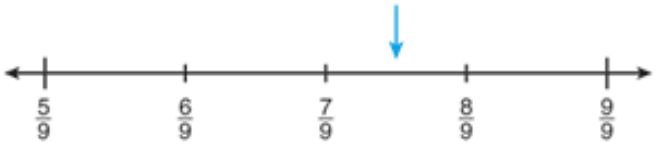
Write each statement, using words.

3. $70 - 1.2n = 46$ **4.** $({}^{-}4 + {}^{-}3) > {}^{-}8$ **5.** $4{,}900 = 70^2$

Reading Test Questions

Choose the correct answer for each.

6. Which fraction is located between $\frac{7}{9}$ and $\frac{8}{9}$ on the number line?



a. $\frac{13}{18}$ **c.** $\frac{24}{27}$

b. $\frac{21}{27}$ **d.** $\frac{30}{36}$

Located means "can be found," "is situated," or "is placed."

7. Which number could you use to express a loss of 4 yards in a football game?

a. $\frac{1}{4}$ **c.** 0.25

b. $^{-}4$ **d.** 0.4

Express means to "show" or "represent."

8. Solve for y.

$$43.5 - (3y + 1) = 24.5$$

a. 5 **c.** 6

b. 5.5 **d.** 6.5

To **solve for** a variable means to "find the value for the variable that will make the equation true."

Learning Vocabulary

Watch for these words in this unit. Write their definitions in your journal.

- **rational number**
- **additive inverse**
- **multiplicative inverse**
- **inequality**

Education Place

At **eduplace.com/map** see eGlossary and eGames—Math Lingo.

Literature Connection

Read "Wildlife Count-a-thon" on pages 644–645. Then work with a partner to answer the questions about the story.

CHAPTER 22

Rational Numbers



INVESTIGATION

Using Data

Jean Driscoll is the only athlete to win the Boston Marathon wheelchair race 8 times.

Round each finish time in the table to the nearest whole minute. Then use positive and negative numbers to express the change in Jean's finish times from one year to the next.

Jean Driscoll's Finish Times

Year	Time (h: min: s)
1990	1:43:17
1991	1:42:42
1992	1:36:52
1993	1:34:50
1994	1:34:22
1995	1:40:41
1996	1:52:56
2000	2:00:52

Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

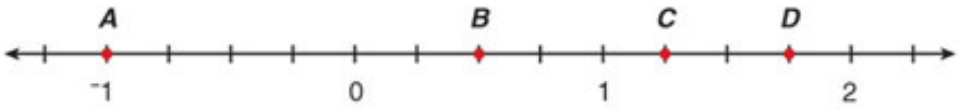
1. The ____ states that when you add 0 to a number the result is the same number or variable.
2. The set of positive whole numbers, their opposites (negative numbers), and zero are ____.
3. The ____ states that when you multiply a number or variable by 1, the result is the same number or variable.

Vocabulary

- integers
- prime numbers
- Zero Property
- Identity Property of Addition
- Identity Property of Multiplication

CONCEPTS AND SKILLS

Match each number with a letter on the number line below.



4. $1\frac{1}{4}$
5. 1.75
6. $^{-}1$
7. 0.5

Identify the property that each equation illustrates.

8. $55 \times 0 = 0$
9. $7 \times (4 \times 3) = (7 \times 4) \times 3$
10. $17 + 12 = 12 + 17$
11. $(5 + 3) + 2 = 5 + (3 + 2)$
12. $4 \times 16 = 16 \times 4$
13. $0 \times 18 = 0$

Identify the first operation you would use to evaluate each expression.

14. $(7 + 2) \div 5$
15. $5 - 4 \div 2$
16. $3 + 2 \times 8$
17. $(8 - 4) \times 2$
18. $30 \div 5 \times 2$
19. $5 \times 9 \div 3$

Write About It

20. Does this picture illustrate the Associative Property of Addition? Explain.



Facts Practice, See page 660.

Lesson 1

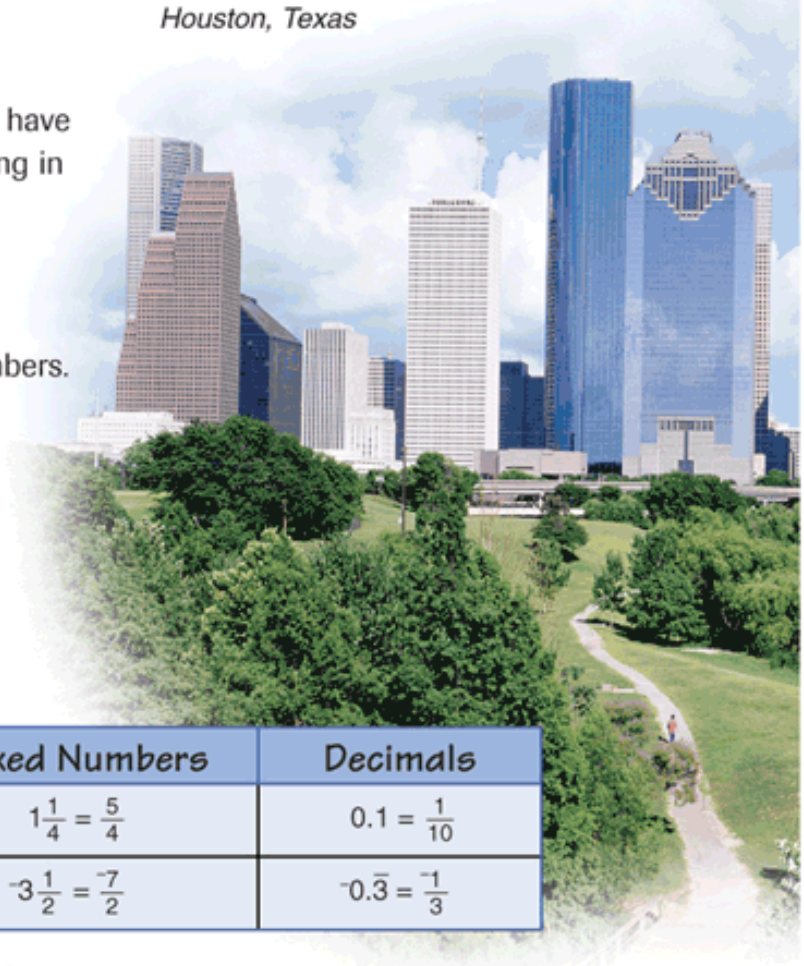
Audio Tutor 2/3 Listen and Understand

Rational Numbers

Objective Identify and graph rational numbers on a number line.

Vocabulary
rational number

Learn About It



Houston, Texas

According to the 2000 census report, Americans have left older cities, such as Philadelphia, and are living in newer cities, such as Houston. The population of Philadelphia changed by $^{-}0.6$ million while the population of Houston changed by $^{+}1.4$ million.

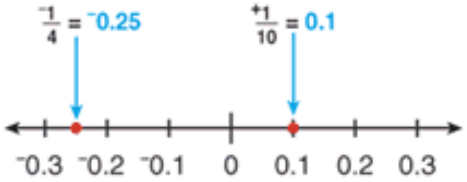
Numbers such as $^{+}1.4$ and $^{-}0.6$ are rational numbers. A **rational number** is a number that can be expressed in the form $\frac{a}{b}$, where a and b are any integers and b is not equal to zero.

$1.4 = 1\frac{4}{10} = \frac{14}{10}$ $\quad -0.6 = \frac{^{-}6}{10} = \frac{^{-}3}{5}$

Integers, fractions, mixed numbers, and many decimals are examples of rational numbers.

Integers	Fractions	Mixed Numbers	Decimals
$46 = \frac{46}{1}$	$\frac{4}{5}$	$1\frac{1}{4} = \frac{5}{4}$	$0.1 = \frac{1}{10}$
$^{-}215 = \frac{^{-}215}{1}$	$\frac{^{-}9}{2}$	$^{-}3\frac{1}{2} = \frac{^{-}7}{2}$	$^{-}0.\overline{3} = \frac{^{-}1}{3}$

You can use a number line to graph rational numbers.



Guided Practice

Rewrite each rational number in the form $\frac{a}{b}$.

1. $^{-}3$ **2.** $2\frac{1}{4}$ **3.** $^{-}1.5$ **4.** the opposite of $\frac{3}{10}$

5. Graph each number in Exercises 1–4 on a number line.

Ask Yourself

- Did I express each number in the form $\frac{a}{b}$?
- Is the number I'm graphing an integer or does it lie between two integers?

Explain Your Thinking When you write an integer or a whole number using the form $\frac{a}{b}$, what can you use as the denominator?

Extra Help at eduplace.com/map

Practice and Problem Solving

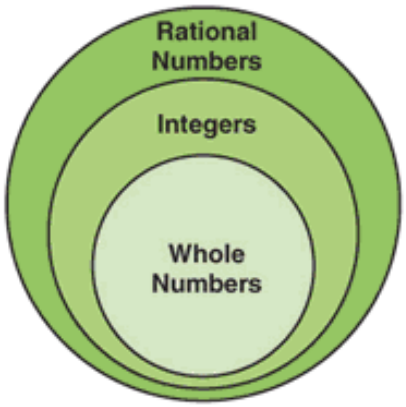
Rewrite each rational number in the form $\frac{a}{b}$.

6. 2 **7.** $3.\overline{6}$ **8.** $^{-}0.8$ **9.** $^{-}4$

10. $3\frac{3}{5}$ **11.** $\frac{^{-}23}{5}$ **12.** the opposite of 0.2 **13.** the opposite of $^{-}1\frac{7}{10}$

14. Draw a number line from $^{-}5$ through $^{+}5$. Graph each number in Exercises 6–13 on your number line.

Reasoning Use the Venn diagram at the right for Exercises 15–18. Write *true* or *false*.



15. All rational numbers are integers.

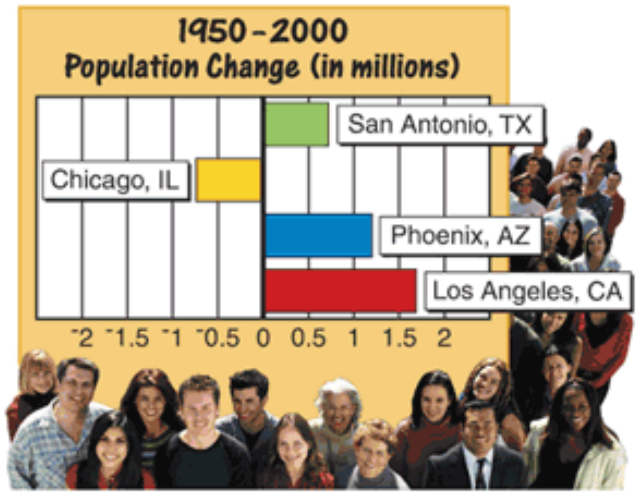
16. No integers are rational numbers.

17. Some rational numbers are whole numbers.

18. No whole numbers are integers.

Data Use the graph to solve Problems 19–21.

The bar graph shows the change in population between 1950 and 2000 in some of the largest cities in the U.S.



19. Explain How do the bars in the graph show an increase in population? a decrease in population?

20. Write a rational number to describe the population increase or decrease in millions in each city.

21. In which two cities were the population changes opposite?

Mixed Review and Test Prep

Open Response

Compare. Write >, <, or = for each ●.
(Ch. 4, Lesson 3)

22. $1\frac{4}{5}$ ● 1.7 **23.** 0.65 ● 0.85

24. 1.18 ● $\frac{9}{4}$ **25.** $\frac{24}{1{,}000}$ ● 0.24

Multiple Choice

26. A scuba diver is 23 feet below the surface. What rational number could you use to describe her location?
(Ch. 22, Lesson 1)

A $^{-}23$ **C** 0.23

B $\frac{^{-}1}{23}$ **D** 23

Extra Practice See page 603, Set A.

Lesson 2

Compare and Order Rational Numbers

Objective Compare and order rational numbers.



Learn About It

In the game *Spin for the Sign*, students pick number cards and use them in order to build rational numbers. Then they spin the spinner for the sign.

Melvin used the cards to build the number $^{-}1\frac{2}{5}$. Angela made the number $^{-}1.25$. Who built the greater number?

Compare $^{-}1.25$ and $^{-}1\frac{2}{5}$.

STEP 1 Express each rational number to be compared in the same form.

Write $^{-}1\frac{2}{5}$ as a decimal.

$$^{-}1\frac{2}{5} = {}^{-}1.40$$

STEP 2 Compare.

$^{-}1.40 < {}^{-}1.25$, so $^{-}1\frac{2}{5} < {}^{-}1.25$

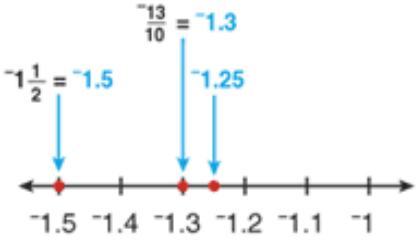
Solution: Since $^{-}1\frac{2}{5} < {}^{-}1.25$, Angela built the greater number.

Another Example

Order $^{-}1\frac{1}{2}$, $^{-}1.25$, and $^{-}\frac{13}{10}$ from least to greatest.

Locate each rational number. Approximate the number when necessary.

$$^{-}1\frac{1}{2} < {}^{-}\frac{13}{10} < {}^{-}1.25$$



Guided Practice

Compare. Write >, <, or = for each ●. Use a number line to help, if necessary.

1. $\frac{1}{3}$ ● 0.3
2. $\frac{1}{8}$ ● $^{-}\frac{8}{5}$
3. $^{-}\frac{7}{4}$ ● $^{-}1\frac{2}{3}$
4. $^{-}1\frac{2}{5}$ ● 1.4

Ask Yourself

- Is the pointed end of the greater than or less than symbol pointing toward the lesser number?
- Did I locate the numbers correctly on the number line?

Explain Your Thinking How can you tell when a rational number is less than, greater than, or equal to another rational number?

Practice and Problem Solving

Compare. Write >, <, or = ●. Use a number line if necessary.

5. $0.2 \bullet \frac{1}{5}$ **6.** $\frac{-3}{3} \bullet \frac{-4}{3}$ **7.** $\frac{5}{2} \bullet 2\frac{1}{2}$ **8.** $-1.6 \bullet \frac{-9}{5}$ **9.** $-0.5 \bullet \frac{-3}{4}$

Order the numbers from least to greatest. Use a number line if necessary.

10. $\frac{27}{21}, 1\frac{1}{3}, 140\%$ **11.** $-1\frac{7}{12}, -1.85, \frac{-9}{5}$ **12.** $\frac{7}{8}, 0.6, \frac{13}{25}$ **13.** $-2.45, \frac{-32}{15}, -5\frac{7}{19}$

Reasoning **Use the digits 4, 5, 6, 7 to complete each inequality.**

14. $0 < \frac{\blacksquare}{\blacksquare} < 1$ **15.** $-2 < \frac{-\blacksquare}{\blacksquare} < -1$ **16.** $0 > \frac{-\blacksquare}{\blacksquare} > -1$ **17.** $1 < \frac{\blacksquare}{\blacksquare} < 2$

Algebra • Inequalities

Find a value of *n* so that each set of numbers are in order from greatest to least.

18. $\frac{1}{2}, \frac{n}{16}, \frac{3}{8}$ **19.** $-1\frac{3}{8}, -1\frac{5}{8}, -1\frac{n}{6}$ **20.** $\frac{-1}{2}, \frac{-n}{5}, \frac{-7}{10}$ **21.** $1\frac{1}{2}, 1\frac{n}{20}, 1\frac{2}{5}$

Data

Use the *Spin for the Sign* game scorecard at the right for Problems 22 and 23.

22. Write About It The player with the greatest number of points is the winner of the game. Angela says that she can determine the winner by comparing only two scores. Explain how she can do that. Who is the winner?

23. Which player's score is closest to the median score?

Spin for the Sign Scorecard

Player	Score
Nicky	9
Angela	-2
Melvin	$-3\frac{1}{2}$
Linda	11
Flora	$-5\frac{1}{4}$

Mixed Review and Test Prep

Open Response

Write each probability as a percent or as a ratio. (Ch. 19, Lesson 2)

24. $P = \frac{0}{6}$ **25.** $P = \frac{3}{8}$ **26.** $P = \frac{2}{5}$

27. $P = 50\%$ **28.** $P = 45\%$ **29.** $P = 100\%$

30. A meteorologist said the temperature decreased from 38.2°F to $38\frac{1}{2}$°F. Explain whether the statement is correct. (Ch. 22, Lesson 2)

Extra Practice See page 603, Set B.

Lesson 3

Audio Tutor 2/31 Listen and Understand

Rational Numbers and Their Properties

Objective Use the properties of rational numbers.

Vocabulary
additive inverse
multiplicative inverse

Learn About It

Some properties of whole numbers and integers that you have learned earlier also apply to rational numbers.

Properties of Rational Numbers

Property	Addition	Multiplication
Commutative	$\frac{3}{4} + \frac{5}{6} = \frac{5}{6} + \frac{3}{4}$	$^{-}2 \times 3 = 3 \times {}^{-}2$
Associative	$(\frac{1}{2} + \frac{3}{5}) + \frac{3}{4} = \frac{1}{2} + (\frac{3}{5} + \frac{3}{4})$	$(7.5 \times 4) \times \frac{1}{6} = 7.5 \times (4 \times \frac{1}{6})$
Identity	$\frac{2}{3} + 0 = \frac{2}{3}$	$2.16 \times 1 = 2.16$
Distributive	$5(\frac{1}{5} + \frac{2}{3}) = (5 \times \frac{1}{5}) + (5 \times \frac{2}{3})$	

The set of rational numbers has other properties.

Property	Addition	Multiplication
Inverse	The **additive inverse** of any number is the opposite of that number. $^{+}11 + (^{-}11) = 0$ $^{-}11 + {}^{+}11 = 0$ The sum of a number and its additive inverse is zero.	The **multiplicative inverse** of any nonzero number is its reciprocal. $\frac{3}{7} \times \frac{7}{3} = 1$ $\frac{7}{3} \times \frac{3}{7} = 1$ The product of a number and its multiplicative inverse is 1.

Guided Practice

Complete each equation. Identify the property you used.

1. $\frac{5}{6} \times {}^{-}\frac{3}{4} = \blacksquare \times \frac{5}{6}$

2. $(\frac{1}{2} \times \frac{2}{3}) \times \blacksquare = \blacksquare \times (\blacksquare \times \frac{6}{7})$

3. $\blacksquare + 16 = 0$

4. $\frac{7}{1} \times \frac{1}{7} = \blacksquare$

Ask Yourself

- Does each side of the equation have the same value?

Explain Your Thinking Why is the sum of any number and its additive inverse zero?

Practice and Problem Solving

Complete each equation. Identify the property you used.

5. $\frac{4}{5} + \blacksquare = 0$

6. $\frac{2}{3} + (\frac{1}{4} + \blacksquare) = (\blacksquare + \frac{1}{4}) + \frac{2}{7}$

7. $\blacksquare \times 3.4 = 3.4$

8. $^{-}2\,(\blacksquare + \frac{1}{3}) = (\blacksquare \times \frac{3}{5}) + (^{-}2 \times \blacksquare)$

9. $\frac{^{-}5}{8} \times {}^{-}\blacksquare = 1$

10. $(^{-}4 \times \frac{1}{3}) \times \blacksquare = \blacksquare \times (\blacksquare \times \frac{4}{5})$

11. $\frac{3}{7} + \frac{^{-}2}{3} = {}^{-}\blacksquare + \frac{3}{7}$

12. $\blacksquare + 8 = 8$

Data **Use the chart at the right for Problems 13 and 14.**

13. **Explain** A running back carried a football 12 times. Did his carries result in a total gain or total loss of yards?

14. How many yards would the running back need to gain on one additional carry to have a mean of $^{+}4$ yards per carry?

15. A jogger ran around a track $1\frac{1}{2}$ times to go a distance of 1 mile. What is the distance once around the track? What property can you use to solve the problem using mental math?

16. **Analyze** Pick a rational number between $\frac{1}{2}$ and $\frac{3}{4}$. Then pick another between $\frac{1}{2}$ and the number you picked. Repeat five more times. How many rational numbers do you think there are between any two rational numbers? Explain.



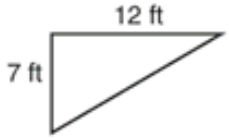
Yards Gained or Lost			
$^{+}2$	$^{-}1$	$^{+}5$	0
$^{-}4$	$^{+}9$	$^{-}3$	$^{-}3$
$^{+}6$	$^{-}12$	0	$^{+}1$

Mixed Review and Test Prep

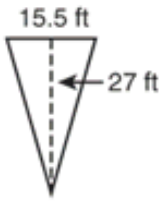
Open Response

Find the area of each triangle. (Ch. 20, Lesson 5)

17.



18.



19. Why do the Additive Inverse Property and the Multiplicative Inverse Property not apply to the set of whole numbers? (Ch. 22, Lesson 3)

Explain your reasoning.

Extra Practice See page 603, Set C.

Lesson 4

Audio Tutor 2/32 Listen and Understand

Order of Operations With Rational Numbers

Objective Use the order of operations to evaluate expressions with rational numbers.

Learn About It

You add and subtract rational numbers using the same sign rules you use to add and subtract integers.

Similarly, you multiply and divide rational numbers using the same sign rules you use to multiply and divide integers.

When an expression with rational numbers involves more than one operation, use the order of operations to evaluate the expression.

Evaluate $2(^-7 + {}^-0.5) \div 5 + (0.25 + {}^-3.25)^2 - 4$.

Examples:

$^-2 + 3 = 1$ $\quad \frac{3}{4} + \frac{^-1}{4} = \frac{2}{4} = \frac{1}{2}$

$^-2 - 3 = {}^-5$ $\quad \frac{3}{4} - \frac{^-1}{4} = \frac{4}{4} = 1$

Examples:

$^-12 \times 4 = {}^-48$ $\quad ^-1.2 \times 0.4 = {}^-0.48$

$^-12 \div 4 = {}^-3$ $\quad ^-1.2 \div 0.4 = {}^-3$

Complete operations within parentheses.	$2(^-7 + {}^-0.5) \div 5 + (0.25 + {}^-3.25)^2 - 4$ $2 \times {}^-7.5 \div 5 + (^-3)^2 - 4$
Evaluate powers.	$2 \times {}^-7.5 \div 5 + 9 - 4$
Multiply and divide in order from left to right.	$^-15 \div 5 + 9 - 4$ $^-3 + 9 - 4$
Add and subtract in order from left to right.	$^-3 + 9 - 4$ $6 - 4$ 2

Solution: $2(^-7 + {}^-0.5) \div 5 + (0.25 + {}^-3.25)^2 - 4 = 2$

Other Examples

A. Expression with Fractions

$$\left(\frac{1}{4} + \frac{3}{8}\right) \times \left(^-\frac{1}{2}\right)^2$$
$$= \frac{5}{8} \times \left(^-\frac{1}{2}\right)^2$$
$$= \frac{5}{8} \times \frac{1}{4}$$
$$= \frac{5}{32}$$

B. Expression with Fractions and Decimals

$$^-\frac{3}{4}(87.2 + 12.8) + (^-2.6 \div \frac{1}{2})$$
$$= {}^-\frac{3}{4} \times 100 + {}^-5.2$$
$$= {}^-75 + {}^-5.2$$
$$= {}^-80.2$$

Guided Practice

Ask Yourself

- Which operation should I complete first?
- Is the sign in my answer correct?

Evaluate each expression.

1. $4 + 18 \div 25$
2. $(17 + 3) - {}^{-}24.72$
3. $(0.6 + 0.8) \times 2^2$
4. $(1 - 3) \times \frac{3}{4}$
5. $0.9 \div (11 - 8) \times ({}^{-}0.3)^2$
6. $(\frac{2}{5} + \frac{1}{10}) \times 5 - (0.2 + 0.3)$

Explain Your Thinking Why do you need to follow the order of operations when evaluating expressions?

Practice and Problem Solving

Evaluate each expression.

7. $6(2 - 1) - 6$
8. $3({}^{-}6 - 3) - 8$
9. $4.26 \div 3 + 1.9$
10. $(5.75 - 6.9) \times 4.1$
11. $\frac{2}{32} + \frac{4}{5} \times \frac{1}{5}$
12. $\frac{{}^{-}3}{8} + \frac{1}{16} - (\frac{1}{8} + \frac{1}{8})$
13. $\frac{1}{3} + (\frac{{}^{-}4}{9}) - \frac{2}{3}$
14. $\frac{1}{4}(22.5 + 6.2) - 3^2$
15. $(6.6 \times 4) + (\frac{1}{2} \times 5)^2$

Solve.

16. **What's Wrong?** Emilio and Dorothy evaluated the same expression, but they both made mistakes by not following the order of operations. Explain what each student did incorrectly. Then evaluate the expression correctly.

Emilio

$3(4-8) + {}^{-}6(5 \times 2)^2$
$3 \times {}^{-}4 + {}^{-}6 \times 10^2$
$3 \times {}^{-}4 + {}^{-}6 \times 100$
$3 \times {}^{-}10 \times 100$
${}^{-}30 \times 100$
${}^{-}300$

$3(4-8) + {}^{-}6(5 \times 2)^2 = {}^{-}300$

Dorothy

$3(4-8) + {}^{-}6(5 \times 2)^2$
$3 \times {}^{-}4 + {}^{-}6 \times 10^2$
${}^{-}12 + {}^{-}6 \times 10^2$
${}^{-}18 \times 10^2$
${}^{-}18 \times 100$
${}^{-}1{,}800$

$3(4-8) + {}^{-}6(5 \times 2)^2 = {}^{-}1{,}800$

17. Neil says $(2 - 1)^2$ and $2^2 - 1^2$ have the same value. John says they do not. Who is correct? Why?

Go On

Calculator **Fill in each ■ with parentheses, the correct sign, or the correct operation to make each statement true.**

18. $3 \blacksquare 4 \blacksquare 2 = 5$

19. $^{-}9 \blacksquare 10 \blacksquare 2 = 3$

20. $8.1 \blacksquare 0.9 \blacksquare {}^{-}3 = 7.8$

21. $1\frac{1}{8} \blacksquare \blacksquare \frac{3}{4} \blacksquare \frac{1}{2} \blacksquare = {}^{-}\frac{1}{8}$

Write a rational number to represent each term shown.

In golf, each hole on a course is assigned a *par*. Par is the number of strokes an expert is expected to make to complete the hole. Some terms used to represent the number of strokes above or below par are *birdie, bogey, eagle,* and *double bogey.*

22. birdie: 1 stroke below par

23. bogey: 1 stroke above par

24. eagle: 2 strokes below par

25. double bogey: 2 strokes above par

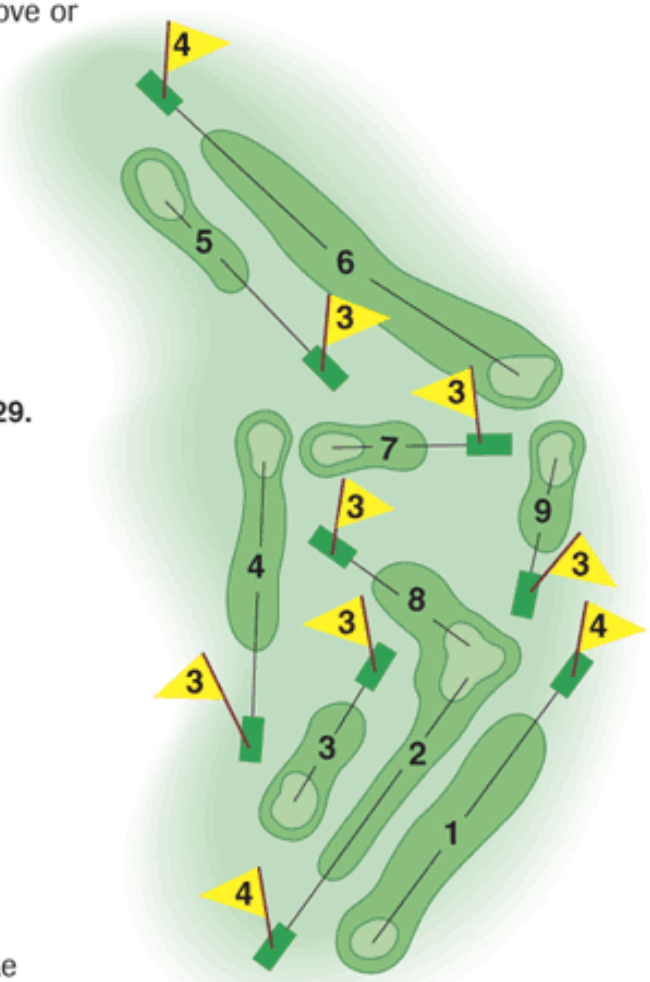
Data **Use the rational numbers in Problems 22–25 and the diagram at the right for Problems 26–29.**

26. **Reasoning** Francesca didn't get any birdies on the first four holes of the course but her total score after the fourth hole is 1 below par. How many shots did she make on each of the four holes?

27. Min Soo got birdies on the first two holes, bogeys on the next three holes, and a double bogey on Hole 6. What does he need to score on each remaining hole to end up with a total score of 30?

28. Albert got a birdie on holes 1, 2, and 6; a bogey on holes 3, 5, and 8; and a double bogey on holes 4, 7, and 9. Find Albert's final score.

29. **Create and Solve** Use the golf terms above to make a golfer's scorecard for nine holes at Balgove Golf Course. Have a classmate find the golfer's final score.



During school summer holidays, adults can only play the Balgove course in Scotland when accompanied by a child.

Mixed Review and Test Prep

Open Response

Name each solid figure. (Ch. 21, Lesson 1)

30.



31.



Multiple Choice

32. Which of the following expressions has the same value as $({}^{-}1 + 4)^2$? (Ch. 22, Lesson 4)

A $({}^{-}4 + 1)^2$

B $({}^{-}1 - 4)^2$

C $2({}^{-}4 + 1)$

D $-4^2 + 1^2$

Extra Practice See page 603, Set D.

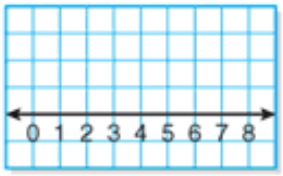
Activity

Math Challenge

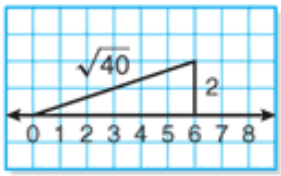
Graphing Square Roots

Materials: safe drawing compass; straightedge; grid paper

You can estimate square roots by graphing. Graph $\sqrt{40}$.



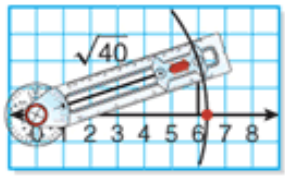
Use a straightedge to draw a number line on grid paper. Number the line from 0 to 8.



Write 40 as the sum of two squares.

$40 = 4 + 36 = 2^2 + 6^2$

Draw a right triangle whose sides have lengths 2 and 6.



Use a compass to copy the hypotenuse's length onto the number line. From the number line, you can estimate that $\sqrt{40}$ is about 6.3.

Graph to estimate the value of each square root.

1. $\sqrt{61}$ **2.** $\sqrt{20}$ **3.** $\sqrt{2}$ **4.** $\sqrt{32}$

5. $\sqrt{85}$ **6.** $\sqrt{17}$ **7.** $\sqrt{58}$ **8.** $\sqrt{37}$

9. $\sqrt{80}$ **10.** $\sqrt{29}$ **11.** $\sqrt{40}$ **12.** $\sqrt{41}$

13. Describe your strategy for writing a number as the sum of two squares. Can you explain why this method works?

Quick Check

Check your understanding of Lessons 1–4.

Rewrite each rational number in the form $\frac{a}{b}$. (Lesson 1)

1. $^-5$ **2.** $^-2\frac{4}{5}$ **3.** the opposite of $\frac{2}{3}$

Order the numbers from least to greatest. Use a number line if necessary. (Lesson 2)

4. 1.2, $^-0.7$, $^-1$ **5.** $\frac{^-3}{5}$, 0.4, $^-0.1$

Complete each equation. Identify the property you used. (Lesson 3)

6. $\frac{^-5}{6} \times {}^-\square = 1$ **7.** $\frac{1}{8} + \square = 0$

Evaluate each expression. (Lesson 4)

8. $\frac{2}{3} + \frac{4}{5} \times \frac{1}{5}$ **9.** $(6.6 \times 4) + (\frac{1}{2} \times 5^2)^2$ **10.** $5.15 \div 4 + 3.9$

Extra Practice at **eduplace.com/map**

Lesson 5

Problem-Solving Application

Use Rational Numbers

Objective Solve problems using rational numbers.



Sometimes you can solve problems by representing values as rational numbers.

Problem Michelle and Kelly are mountain climbing. When they begin their climb at the foot of the mountain, the temperature is 10°F. Suppose that for each increase in elevation of 1,000 feet, the temperature decreases by about 5°F. About what will the temperature be at 4,000 feet?

UNDERSTAND

What is the question?

About what will the temperature at 4,000 feet?

What do you know?

- The temperature at the foot of the mountain is 10°F.
- For each increase in elevation of 1,000 feet, the temperature decreases by about 5°F.

PLAN

Identify the sequence of steps needed to solve the problem.

- First, find the number of 1,000-foot increases to reach 4,000 feet.
- Next, find the temperature decrease for 4,000 feet.
- Finally, find the temperature at 4,000 feet.

SOLVE

- $4{,}000 \div 1{,}000 = 4$ ← There are 4 increases of 1,000 feet in the climb.
- $4 \times {}^{-}5° = {}^{-}20°$ ← The temperature has dropped 20° by 4,000 feet.
- $10° + {}^{-}20° = {}^{-}10°$

The temperature is about ${}^{-}10$°F at 4,000 feet.

LOOK BACK

Look back at the problem. Would you expect the temperature to be higher or lower as elevation increases?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. During $6\frac{1}{2}$ hours, the temperature at the foot of the mountain fell from 10°F to $^{-}16$°F. If the rate of decrease was constant, by how many degrees did the temperature decrease each half hour?

2. **Estimate** At 3,000 feet, the temperature is $^{-}7.5$°F. What would you estimate the temperature to be at 0 feet, at the bottom of the mountain slope?

Hint Will the temperature increase or decrease as you go down the mountain?

Ask Yourself

- **UNDERSTAND** — What do I know about the temperature?
- **PLAN** — What steps do I need to follow to solve the problem?
- **SOLVE** — In what form should I write the numbers to solve the problem?
- **LOOK BACK** — Does my answer make sense?

Independent Practice

Data **Use the table for Problems 3–7. Round to the nearest tenth when necessary.**

3. Do the temperature changes in the table represent increases or decreases? Explain how you know.

4. How much did the temperature in each city change?

5. In which city was the average hourly temperature change about half of the total temperature change?

6. **Estimate** If the temperature in Spearfish had changed by the same amount each minute, about what would the temperature have been 15 minutes after the temperature started to decrease?

7. **Analyze** Kevin says that the temperature change in Browning, Montana, was greater than that of Fairfield, Montana. Peggy says that Fairfield had the greater temperature change over time. Justify both students' statements.

Exreme Temperature Changes

Place	Time Span (h)	Temp. Change
Browning, MT	24	44°F to $^{-}56$°F
Fairfield, MT	12	63°F to $^{-}21$°F
Rapid City, SD	2	49°F to $^{-}13$°F
Spearfish, SD	about $\frac{1}{2}$	54°F to 4°F

Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

8. A football team has four tries, called downs, to move the ball forward 10 more yards. On the first down, the home team lost 4 yards. They gained 8 yards on the second down and 5 more yards on the third down. How close is the home team to making the 10-yard goal?

9. The length of a football field, including the end zones, is 360 feet. That is 2.25 times greater than the width of the field. What is the perimeter of the football field?

10. In the final standings for the season, Jim's team won 8 more games than they lost. They played a total of 16 games. How many games did they win? How many games did they lose?

You Choose

Strategy

- Draw a Diagram
- Use Logical Reasoning
- Use Models
- Work Backward
- Write an Equation

Computation Method

- Mental Math
- Estimation
- Paper and Pencil
- Calculator

Solve. Tell which method you chose.

Before 2001, stock prices were quoted as mixed numbers. Now decimals are used.



11. Which stock transaction shows a greater profit: 100 shares purchased at \$31.22 per share and sold at \$33.05 per share or 200 shares purchased at \$19.80 per share and sold at \$21.99 per share? How much greater is the profit?

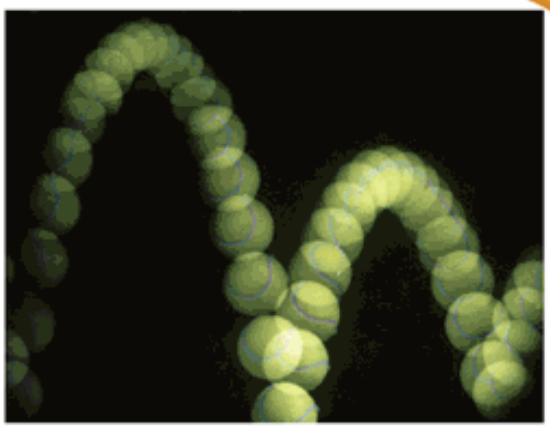
12. Over 4 days, the price of a share of a certain transportation stock changed as follows: $^{+}$\$0.07, $^{-}$\$0.02, $^{-}$\$0.13, $^{+}$\$0.06. How much does the price need to change on Day 5 for the week's gains and losses to cancel out?

13. Suppose you own 20 shares of stock in a publishing company. The company pays a dividend (part of the profits) of \$0.78 per share each quarter. How much would you receive in dividends in a year?

14. The 52-week high for one share of a technology stock was \$21.28, or 40% higher than the 52-week low. What is the difference between the 52-week high and 52-week low prices?

Activity

Visual Thinking
Up and Down



When a ball is dropped from a height, it bounces—but not up to its original height. On each bounce, a certain ball reached at most $\frac{4}{5}$ of the height of its previous bounce.

1. How high will the ball bounce on its third bounce?
2. How many bounces will it take the ball to bounce to a height that is less than $\frac{1}{2}$ of the height of the first bounce?
3. Suppose the ball was dropped from a height of 2 meters. When the ball hits the floor after the third bounce, what is the total vertical distance it has traveled?
4. Will the ball described ever stop bouncing? Explain.

Logical Thinking
Rational Reality

Can you think of some real-life situations that use rational numbers?

Look around your school, your home, and your shopping mall.

Do your examples have anything in common with each other?



Brain Teaser

Max's robot can cross a yard by walking on stepping stones.

- It can only move forward 3 stones or back 2 stones at a time.
- It cannot move forward more than 3 times without moving backward at least once.
- It starts on the first stone and cannot step on the ninth stone.

What is the least number of moves the robot can make to land on the fifteenth stone?

Education Place

Check out **eduplace.com/map** for more brain teasers.

Chapter Review/Test

VOCABULARY

1. The ____ of any number is the opposite of that number.
2. The ____ of any nonzero number is its reciprocal.
3. A(n) ____ is a number that can be expressed in the form $\frac{a}{b}$, where a and b are any integers and b is not equal to zero.

Vocabulary

integer
rational number
additive inverse
multiplicative inverse

CONCEPTS AND SKILLS

Rewrite each rational number in form $\frac{a}{b}$. (Lesson 1, pp. 588–589)

4. $\frac{14}{4}$
5. $^{-}6$
6. $^{-}3\frac{3}{8}$
7. opposite of 1.4

Compare. Write <, >, or =. Use a number line if necessary. (Lesson 2, pp. 590–591)

8. $\frac{^{-}4}{7} \bigcirc \frac{^{-}3}{8}$
9. $^{-}3.2 \bigcirc \frac{^{-}11}{15}$
10. $\frac{2}{5} \bigcirc 0.4$
11. $\frac{2}{9} \bigcirc \frac{2}{11}$

Order the numbers from least to greatest. Use a number line if necessary. (Lesson 2, pp. 590–591)

12. $^{-}1.45, ^{-}1\frac{1}{2}, \frac{^{-}4}{3}$
13. $\frac{2}{3}, 0.6, \frac{4}{5}$
14. $^{-}5.3, \frac{^{-}7}{2}, ^{-}5\frac{1}{8}$

Complete each equation. Identify the property you used. (Lesson 3, pp. 592–593)

15. $\frac{3}{5} + \blacksquare = 0$
16. $\frac{^{-}2}{9} \times \blacksquare = 1$
17. $\frac{5}{8} \times \blacksquare = \frac{5}{8}$
18. $3(\blacksquare - \frac{1}{4}) = (3 \times 2) + (\blacksquare \times \frac{^{-}1}{4})$

Evaluate each expression. (Lesson 4, pp. 594–596)

19. $7(5 - 2) + 4$
20. $4(4 + 1)^2$
21. $\frac{1}{5} + \frac{2}{5} \times (6.7 + 4.3)$
22. $(3.3 - 2.1) \times (^{-}4 + 6)$
23. $(2.56 + 2.44) \times 5.8$

PROBLEM SOLVING

Solve. (Lesson 5, pp. 598–600)

24. Lisa's track team competed in 250 races during the season. They won 96 more races than they lost. How many did they lose?
25. Lisa won 8 races and lost 2 during the first third of the season. During the second third, she won $\frac{1}{4}$ more races and lost $\frac{1}{2}$ fewer races. Her total for the season was 27 wins and 5 losses. What was her record the last third of the season?



Show You Understand

The number $\frac{0}{4}$ is a rational number, but $\frac{4}{0}$ is not. Why can't a rational number in the form $\frac{a}{b}$ have 0 in the denominator?

Extra Practice

Set A (Lesson 1, pp. 588–589)

Rewrite each rational number in form $\frac{a}{b}$.

1. 4
2. $^-5$
3. $^-0.5$
4. 0.3
5. 1.2
6. $^-3\frac{1}{3}$
7. $\frac{15}{6}$
8. the opposite of 0.25
9. the opposite of $\frac{^-21}{5}$
10. Draw a number line from $^-6$ to 6. Place each number in exercises 1–9 on your number line.

Set B (Lesson 2, pp. 590–591)

Compare. Write <, >, or =. Use a number line if necessary.

1. $\frac{3}{10}$ ● 0.3
2. $2\frac{1}{3}$ ● $\frac{^-7}{3}$
3. $\frac{^-5}{4}$ ● $\frac{^-6}{4}$
4. $\frac{8}{11}$ ● 0.8
5. $^-2.4$ ● $\frac{^-5}{2}$

Order the numbers from least to greatest. Use a number line if necessary.

6. $\frac{3}{5}$, 0.7, $\frac{11}{20}$
7. $2\frac{1}{2}$, $\frac{19}{16}$, $\frac{16}{10}$
8. $^-2.68$, $^-2\frac{2}{3}$, $^-2\frac{4}{5}$
9. $^-1.28$ $-1\frac{1}{4}$, $\frac{^-9}{8}$
10. $\frac{^-27}{10}$, $^-2.66$, $^-3\frac{1}{9}$
11. $\frac{^-1}{11}$, $\frac{2}{12}$, $^-0.1$

Set C (Lesson 3, pp. 592–593)

Complete each equation. Identify the property you used.

1. $\frac{^-4}{7} + ■ = \frac{^-4}{7}$
2. $(\frac{1}{3} \times ■) \times \frac{2}{5} = \frac{1}{3} \times (\frac{4}{7} \times ■)$
3. $\frac{1}{2} + ■ = 0$
4. $4(■ + \frac{1}{2}) = (4 \times 3) + (■ \times \frac{1}{2})$
5. $\frac{^-7}{9} \times ■ = \frac{^-7}{9}$
6. $(^-5 \times \frac{^-3}{10}) \times ■ = ■ \times (■ \times 2)$
7. $\frac{^-7}{5} \times ■ = 1$
8. $^-3(■ + \frac{1}{3}) = (■ \times \frac{4}{3}) + (^-3 \times ■)$
9. $\frac{5}{9} + \frac{5}{8} = \frac{5}{8} + ■$
10. $\frac{2}{3} \times \frac{3}{4} = ■ \times \frac{2}{3}$

Set D (Lesson 4, pp. 594–596)

Evaluate each expression.

1. $8(3 + 5) - ^-4$
2. $7(^-2 - 5) + 4$
3. $2(33.8 + 9.6) - 2.1^2 \div 3$
4. $6.625 \div 5 + 2.25$
5. $(4.83 + 1.67) \times 2.2$
6. $9.84 \div 3 + 11$
7. $(^-2.9 + 5.9) \times (3 - ^-2)^2$
8. $(\frac{1}{4} + 2\frac{1}{8}) - (\frac{5}{6} + \frac{5}{6})$
9. $3(3 + 1)^2$
10. $\frac{3}{4} + \frac{1}{6} \times (5.7 - 0.3)$

CHAPTER 23

Expressions, Equations, and Inequalities

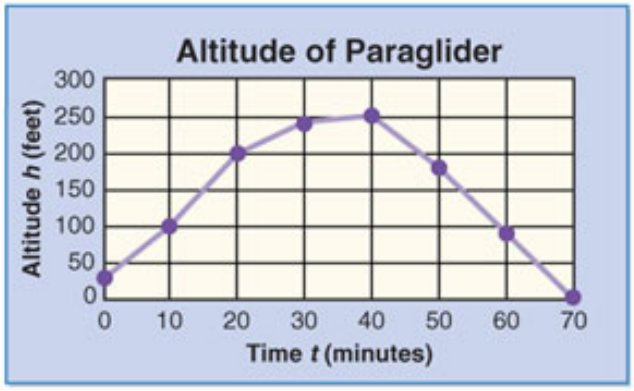
INVESTIGATION

Using Data

Using their canopies, paragliders can remain airborne for an hour or more. The graph shows a paraglider's altitude every 10 minutes during a 70-minute ride. Use the graph to find her altitude at each 10-minute interval. The equation

$$h = {}^{-}9t + 630$$

models one paraglider's altitude in feet between 50 and 70 minutes. Use the equation to find her altitude at 55 minutes.



Use What You Know

Use this page to review and remember what you need to know for this chapter.

VOCABULARY

Choose the best word to complete each sentence.

Vocabulary

variable
equation
expression
coefficient

1. A(n) ____ is a number, a variable, or any combination of numbers, variables, operation signs, and grouping symbols.
2. A(n) ____ is a letter or symbol that represents a number in an algebraic expression.
3. A ____ is a value by which a variable is multiplied.

CONCEPTS AND SKILLS

Match each phrase with an expression.

4. three plus a number
5. five times a number
6. a number divided by eight
7. the sum of a number and eight
8. the difference between a number and three

a. $x - 3$ **d.** $3 + x$
b. $5x$ **e.** $\frac{x}{8}$
c. $x + 8$

Tell whether each of the following equations is accurate. Write *true* or *false*.

9. $(4 + n) + 7 = 4 + (n + 7)$
10. $n \times 4 + 7 = 4n + 7$
11. $n \div 3 + 6 = \frac{1}{3}n + 6$
12. $3n + 2n + 4 = n(3 + 2) + 4$
13. $(9 \times 5) + n = 9 \times (5 + n)$

Match each equation with its solution.

14. $x - 3 = 11$
15. $\frac{x}{4} = 5$
16. $7 + x = 19$
17. $6x = 30$
18. $x + 9 = 12$
19. $x \div 5 = 8$

a. 12 **d.** 40
b. 3 **e.** 20
c. 14 **f.** 5

Write About It

20. Shawn wrote an equation to tell about his weekend baby-sitting jobs. What information can you tell from the equation below?

$$3x + 4x = 28$$

Facts Practice, See page 661.

Lesson 1

Expressions With Rational Numbers

Vocabulary
evaluate
expression

Objective Use properties and order of operations to evaluate algebraic expressions.

Learn About It

To **evaluate** an algebraic **expression** you must substitute given values for variables. You use the order of operations and addition and multiplication properties to make the calculations easier.

Evaluate $6(^{-}x) + 17y + 98$, given $x = 17$ and $y = 6$.

STEP 1 Substitute the given values for x and y.	$6(^{-}17) + 17(6) + 98$
STEP 2 Use the Commutative Property of Multiplication, $ab = ba$.	$6(^{-}17) + 17(6) + 98$ $6(^{-}17) + 6(17) + 98$
STEP 3 Use the Distributive Property, $ab + ac = a(b + c)$.	$6(^{-}17) + 6(17) + 98$ $6(^{-}17 + 17) + 98$ $6(0) + 98$ 98

Solution: The value of the expression is 98.

Other Examples

A. Same Variable Used More Than Once

Evaluate $6n + 5 + 4n$, given $n = \frac{^{-}3}{4}$.

$6n + 5 + 4n$
$6n + 4n + 5$ ← Commutative Property
$(6 + 4)n + 5$ ← Distributive Property
$(10)n + 5$
$(10 \times \frac{^{-}3}{4}) + 5$ ← Substitution
$\frac{^{-}30}{4} + 5$
$^{-}2\frac{1}{2}$

B. Expressions Using Multiplication

Evaluate $2n \times 3 + 4n$, given $n = 0.5$.

$2n \times 3 + 4n$
$6n + 4n$ ← Multiply first.
$(6 + 4)n$ ← Distributive Property
$10n$
$10(0.5)$ ← Substitution
5

Guided Practice

Evaluate each expression for the given values.

1. $^-x - 7$, given $x = 6$

2. $x \div 2$, given $x = \frac{^-2}{3}$

3. $8(x - y)$, given $x = 2.4$ and $y = {}^-0.5$

4. $x^2 + y$, given $x = \frac{1}{2}$ and $y = \frac{^-1}{2}$

Ask Yourself

- Did I use the correct signs?
- Did I consider that the product is zero when one of the factors is zero?

Explain Your Thinking What happens if you evaluate $6(^-17) + 17(6) + 98$ without using the Distributive Property? Which way do you prefer?

Practice and Problem Solving

Evaluate each expression for the given values.

5. $7 + x - y$, given $x = {}^-7$ and $y = 10$

6. $10x + 3(^-y)$, given $x = \frac{^-1}{5}$ and $y = \frac{1}{5}$

7. $x^2 + 10$, given $x = {}^-0.3$

8. $x + y - 2x$, given $x = \frac{^-5}{8}$ and $y = 1\frac{3}{8}$

9. $^-5x + (3y - 2)$, given $x = {}^-2$ and $y = 1$

10. $9.02 - (^-x) + y$, given $x = 6.2$ and $y = 5$

11. $\frac{^-4}{5}x + (\frac{3}{10} - y)$, given $x = 4$ and $y = \frac{1}{2}$

12. $x^2 + 0.4 - y$, given $x = {}^-3$ and $y = 2.9$

Solve.

13. What's Wrong? Jeff evaluated the expression at the right for $x = \frac{1}{2}$ and $y = \frac{^-1}{2}$. Explain what he did wrong. Then find the correct answer.

$$8(^-x) + 5y + 2$$
$$8(\tfrac{^-1}{2}) + 5(\tfrac{1}{2}) + 2$$
$$^-4 + 2\tfrac{1}{2} + 2$$
$$\tfrac{1}{2}$$

14. Carla earns \$8 per hour in take-home pay. Last week she worked 19 hours. She saved 10% of her take-home pay. How much did she save last week?

15. Analyze Melita says the expression $(x - y)^2$ evaluates to 16 when $x = 2$ and $y = {}^-2$. Max says it evaluates to 0. Who is correct? Show your work to explain.

Mixed Review and Test Prep

Open Response

Express each decimal as a fraction and as a percent. (Ch. 17, Lesson 2)

16. 0.19 **17.** 0.04 **18.** 1.7

Express each percent as a fraction and as a decimal.

19. 45% **20.** 3% **21.** 300%

Multiple Choice

22. One share of Investo stock was bought for \$22.25. It rose by \$0.35. Then its price dropped by \$0.65. What was the final price? (Ch. 1, Lesson 6)

A \$21.95 **C** \$22.60

B \$22.25 **D** \$22.95

Extra Practice See page 623, Set A.

Lesson 2

Write Expressions With Rational Numbers

Vocabulary
rational number
additive inverse

Objective Write algebraic expressions with rational numbers.

Learn About It

You can translate phrases into algebraic expressions involving **rational numbers**.

Word Description	Algebraic Expression
• two and three tenths times some number r	$2.3r$
• m marbles shared equally among six people	$m \div 6$
• the cost of three shirts of equal value	$s + s + s$ or $3s$
• five times the quantity x minus y	$5(x - y)$
• seven times the **additive inverse** of some number n	$7(^{-}n)$

Guided Practice

Write an expression for each phrase or situation. Tell what each variable represents.

Ask Yourself
- Did I use the correct operation?
- Did I use variables for numbers that change?

1. the sum of the additive inverses of two numbers divided into three equal parts
2. four times the difference of two numbers
3. negative one fifth increased by three times a number
4. The price of a stock is $35.50 per share. The price decreases by a certain amount and then increases by half that amount. What is the price of a share of stock now?
5. The price of another stock is $65. The price climbs by two dollars and twenty cents each day. What is the price of the stock after n days?



Explain Your Thinking How is the expresson "five times the quantity x minus y" different from the expression "five times x minus y"?

Practice and Problem Solving

Write an expression for each phrase. Tell what each variable represents.

6. a number times the difference between that number and seven

7. divide the sum of four fifths and a number into equal parts

8. four and three tenths times the sum of a gain of one amount and the loss of a different amount

9. negative nine increased by a number then decreased by another number

10. the additive inverse of a number increased by half that number

11. one third times the sum of two different numbers

Solve.

12. Marisa gets an allowance of $20.50 each month. She spends all of it. Write an expression to show how much money Marisa will have saved after x months.

13. Kim has $150 in her savings account. She withdraws $5 each month and she saves nothing. Write an expression for the amount of money she will have in her savings account after x months.

14. Floyd bought the same number of shares of stock each day for 4 days. Then he sold a different number of shares on the fifth day. Write an expression to represent the number of shares of stock he has now.

15. George earns $15 per day. He spends $120 for a new CD player and $4 every time he visits the mystery bookstore. Write an expression to represent the money he earns and spends. Tell what each variable represents.

16. **Analyze** Pamela saved $50. Each time she goes to a movie she spends $6.25. Each time she goes ice-skating she spends $6.25. Does the expression $50 - 6.25(x + y)$ describe her saving and spending habits? Explain.

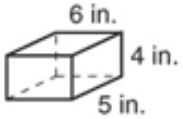


Mixed Review and Test Prep

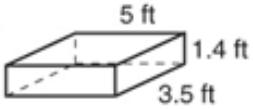
Open Response

Find the volume of each rectangular prism. (Ch. 21, Lesson 4)

17.



18.



19. Bob bought 4 T-shirts for $12.99 each and 3 hats for $7.99 each. Write an expression for the total amount of money Bob spent. (Ch. 23, Lesson 2)

Extra Practice See page 623, Set B.

Lesson 3

Audio Tutor 2/33 Listen and Understand

Equations With Rational Numbers

Objective Solve equations with rational numbers.



Learn About It

You can use what you know about solving equations with whole numbers and integers to solve equations with rational numbers.

From his kitchen window in the city, Luis watches a peregrine falcon swoop 475 feet from its nest on a building ledge to a prey that is 175 feet below his window. How far above Luis's kitchen window is the falcon's nest?

Write and solve an equation.

STEP 1 Let h stand for the height of the falcon's nest above the kitchen window.

h = height in feet of nest above kitchen window.

STEP 2 Write an addition equation to represent the distance between the falcon's nest and the falcon's prey.

$h + 175 = 475$

STEP 3 Solve. Use the Additive Inverse Property. Add $^-175$ to both sides of the equation.

Simplify.

$$h + 175 = 475$$
$$h + 175 + {}^-175 = 475 + {}^-175$$
$$h = 300$$

Check:

$n + 175 = 475$

$300 + 175 = 475$

$475 = 475$

It checks.

Solution: The falcon's nest is 300 feet above the kitchen window.

Other Examples

A. Use Division

Solve. $^-3b = {}^-540$

$$\frac{^-3b}{^-3} = \frac{^-540}{^-3}$$
$$b = 180$$

B. Use Multiplication

Solve. $\frac{y}{0.6} = {}^-1.2$

$$\frac{y}{0.6} \times 0.6 = {}^-1.2 \times 0.6$$
$$y = {}^-0.72$$

C. Use Subtraction

$$n + \frac{3}{4} = 8$$
$$n + \frac{3}{4} - \frac{3}{4} = 8 - \frac{3}{4}$$
$$n = 7\frac{1}{4}$$

Extra Help at eduplace.com/map

Guided Practice

Solve each equation.

1. $2 + x = 4$ **2.** $^-10 + x = ^-20$ **3.** $\frac{2}{3}x = 6$

4. $^-3x = 9$ **5.** $^-4.1x = ^-8.2$ **6.** $^-x = \frac{1}{10}$

Ask Yourself

- Which signs do I use?
- Which inverse operations do I use?

Explain Your Thinking ▶ How can you check your solutions in Exercises 1–6?

Practice and Problem Solving

Solve each equation.

7. $x + 4 = ^-3$ **8.** $x - ^-1 = 4$ **9.** $^-4 + s = ^-3$ **10.** $h - 4 = 0.2$

11. $t + \$0.80 = \$8.00$ **12.** $x - ^-4.1 = 4.1$ **13.** $\frac{3}{4}x = 12$ **14.** $\frac{1}{2}n = ^-4$

15. $y \div 2 = \$2.00$ **16.** $\frac{3}{5}p = ^-6$ **17.** $x \div 16 = ^-2$ **18.** $81t = ^-8.1$

19. $x \div 3 = 7$ **20.** $0.55 + y = ^-1.3$ **21.** $n \div ^-8 = ^-16$ **22.** $^-5q = 30$

What operation was performed on both sides of the first equation to produce the second equation?

23. $7x = 49, x = 7$ **24.** $x \div 9 = 9, x = 81$ **25.** $n + ^-4.3 = ^-6.8, n = ^-2.5$

26. $\frac{2}{5}s = 1\frac{1}{5}, s = 3$ **27.** $x - ^-\frac{1}{4} = ^-1\frac{1}{4}, x = ^-1\frac{1}{2}$ **28.** $5.07n = ^-11.661, n = ^-2.3$

29. $t - 3\frac{1}{2} = ^-5, t = ^-1\frac{1}{2}$ **30.** $\frac{5}{7}x = 1, x = \frac{7}{5}$ **31.** $b \div 12 = ^-5.1, b = ^-61.2$

Choose the equation that correctly models each situation. Explain what the variable represents. Then solve.

32. A vessel was discovered 125 meters below the water surface. This was 3 times deeper than expected. What was the expected depth of the vessel?

a. $3 + x = ^-125$

b. $3x = ^-125$

c. $^-125x = 3$

d. $x - 3 = ^-125$

33. Doing yard work, Jack earned enough money to pay off an \$85.75 loan and still have \$15.50 left. How much money did Jack earn?

a. $x + 85.75 = 15.50$

b. $15.50x = 85.75$

c. $x - 85.75 = 15.50$

d. $\frac{85.75}{x} = 15.50$

Go On

Practice and Problem Solving

Calculator **A calculator may be helpful when you solve equations with decimals. Solve each equation.**

34. $5.9x = 31.86$

35. $69.6 \div d = {}^{-}1.2$

36. $6.01s = 0.1202$

37. $0.04 + s = 1.1$

38. $33.9 - s = {}^{-}4.6$

39. $\$4.55 \div x = \$1.82$

40. $x \div 0.3 = 7$

41. ${}^{-}0.1x = 100$

42. $0.42x = 0.126$

Solve.

43. **Write About It** Explain how you would solve for b in the equation $ab = c$.

44. **What's Wrong?** Describe the error each student made. Then solve the equation correctly.

Mary

$6.3x = 32.13$
$6.3x \div 6.3 = 32.13 \times 6.3$
$x = 202.419$

Raj

$6.3x = 32.13$
$\frac{x}{6.3} = 32.13 - 6.3$
$x = 25.83$

45. **Analyze** A pelican is hovering above the water surface. It dives 180 inches to catch a fish swimming 30 inches below the surface of the water. How many inches above the water is the pelican?

46. During a flood, a river first rose to $9\frac{1}{2}$ feet above normal. The next day, the water level fell by $\frac{3}{4}$ foot. The following day the river fell by three times that amount. How much above normal was the river then?

47. A shark dives from the surface of the water to a depth of 27 meters. The depth of the shark is four times the depth of a stingray. How many meters below the surface is the stingray swimming?

48. An elevator takes 10 seconds to rise from a subway 20 feet below ground to a shopping mall. The elevator rises 6 feet per second. How many feet above ground is the shopping mall?

Mixed Review and Test Prep

Open Response

Order the numbers from least to greatest.
(Ch. 22, Lesson 2)

49. $3.22, {}^{-}3, 0$

50. $\frac{5}{8}, \frac{1}{4}, {}^{-}\frac{3}{4}$

51. ${}^{-}7.1, {}^{-}6.4, 6\frac{1}{5}$

52. $\frac{7}{5}, 1\frac{1}{2}, 0.2$

Multiple Choice

53. The temperature in the shade is 72°F. This is 8° less than the temperature in the sun. What is the temperature in the sun?
(Ch. 23, Lesson 3)

A ${}^{-}80°F$ **C** 64°F

B 9°F **D** 80°F

Extra Practice See page 623, Set C.

Problem Solving

Math Challenge

Weighty Matters

Use the scales to find the weight of each object.

1.  = ■ oz

2.  = ■ oz

3.  = ■ oz

4.  = ■ oz



WEEKLY WR READER eduplace.com/map

Quick Check

Check your understanding of Lessons 1–3.

Evaluate each expression for the given values. (Lesson 1)

1. $5 - x + y$, given $x = {}^{-}2$ and $y = 4$

2. $2x - 4y$, given $x = {}^{-}0.4$ and $y = 0.1$

Write an expression for each phrase. Tell what each variable represents. (Lesson 2)

3. a number subtracted from the sum of twelve and another number

4. the product of a number and the difference between seven and another number

Solve each equation. Explain how you found your solution. (Lesson 3)

5. $x + 5 = {}^{-}2$

6. $x - {}^{-}8 = {}^{-}1$

7. ${}^{-}4x = {}^{-}1.2$

Solve. (Lesson 3)

8. Two integers have a sum of 1 and a difference of 7. What are the integers?

Lesson 4

Audio Tutor 2/34 Listen and Understand

Problem-Solving Strategy

Guess and Check

Objective Use the Guess-and-Check strategy to solve problems.

Sometimes you can solve a problem by making and checking reasonable guesses.

Problem Arrange the numbers $^{-}4$, $^{-}5$, $^{-}6$, $^{-}7$, $^{-}8$, and $^{-}9$ in the circles so that the sum of the numbers along each side is $^{-}17$. Use each number once and only once.



UNDERSTAND

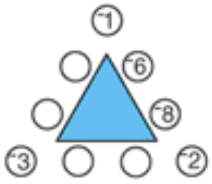
This is what you know:

- There are two missing numbers along each side of the triangle.
- The sum along each side of the triangle should be $^{-}17$.
- $^{-}4$, $^{-}5$, $^{-}6$, $^{-}7$, $^{-}8$, and $^{-}9$ can be used only once.

PLAN

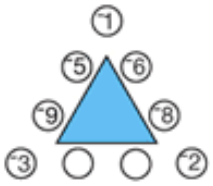
Start on one side of the triangle. Make a reasonable guess as to which two numbers belong in the empty circles. Check to see if the sum along each side is $^{-}17$. Continue with the other sides.

SOLVE



- Try $^{-}6$ and $^{-}8$ on one side of the triangle.

Check:
$^{-}1 + {}^{-}6 + {}^{-}8 + {}^{-}2 = {}^{-}17$



- Try $^{-}5$ and $^{-}9$ on another side.

Check:
$^{-}1 + {}^{-}5 + {}^{-}9 + {}^{-}3 = {}^{-}18$

Since $^{-}17 > {}^{-}18$, change one of the numbers you guessed to a number that is 1 greater.



- Try $^{-}4$ in place of $^{-}5$.

Check:
$^{-}1 + {}^{-}4 + {}^{-}9 + {}^{-}3 = {}^{-}17$

Use the Guess-and-Check strategy to fill in the circles on the last side of the triangle.

Solution: The last triangle above is a solution.

LOOK BACK

Look back at the problem.

Did you use each number only once?

Guided Practice

Use the Ask Yourself questions to help you solve each problem.

1. Two rational numbers have a difference of 0.5 and a sum of $^{-}0.9$. What are the rational numbers?

2. Find the two integers made up of the digits 1, 2, 3, 4, and 5 that have the greatest possible product. Use each digit once and only once.

   Hint Use estimation to help you find the integers that have the greatest possible product.

Ask Yourself

UNDERSTAND
- What do I know about the sum and difference of the numbers?

PLAN
- How will I make my first guess reasonable?

SOLVE
- If the first two numbers I guessed don't work, how can I make a more reasonable guess?

LOOK BACK
- Does my answer make sense?

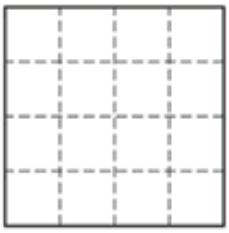
Independent Practice

Use the Guess-and-Check strategy to solve each problem.

3. Raymond has as many brothers as sisters, but each of his sisters has only half as many sisters as brothers. How many brothers and sisters are in Raymond's family?

4. Linda sold two used printers for $600 each. She made a 20% loss on the more expensive printer, but made a 20% profit on the other printer. How much was Linda's total profit or loss?

5. Find three different ways to divide the square below into four congruent parts by following the dashed lines.



6. Two customers purchased all but one of the keychains shown below. The first customer paid twice as much as the second customer. Which keychains did each customer buy? Which keychain did neither customer buy?



Go On

Mixed Problem Solving

Solve. Show your work. Tell what strategy you used.

You Choose

Strategy
- Guess and Check
- Make an Organized List
- Use Logical Reasoning
- Work Backward
- Write an Equation

Computation Method
- Mental Math
- Estimation
- Paper and Pencil
- Calculator

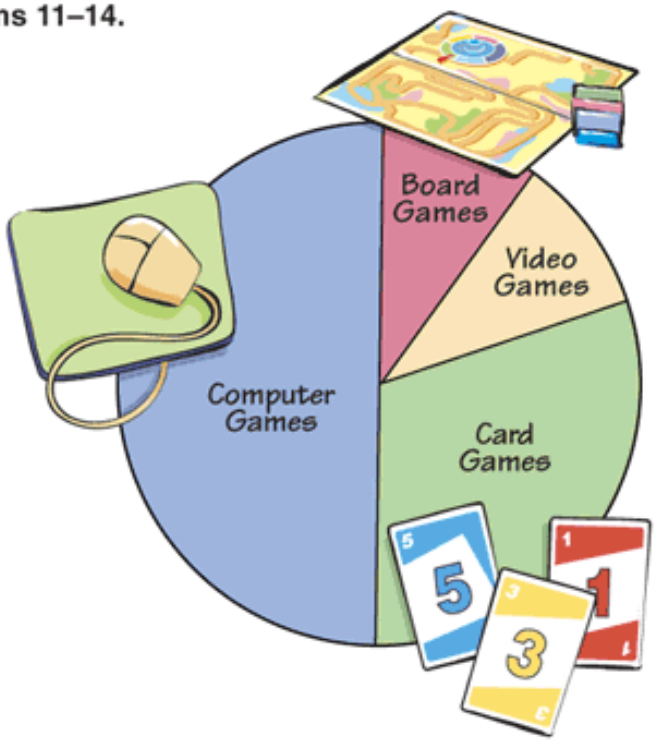
7. Use the digit 5 five times, operation signs, and parentheses to make an expression that has a value of 100.

8. There are two ways to combine addition signs and the digits 1, 2, 3, 4, 5, 6, and 7 in order to make a sum of 100. Here is one way: $1 + 2 + 34 + 56 + 7 = 100$. Find the other way.

9. Ten students in a class play soccer and 12 students play basketball. Seven students play both soccer and basketball. Half of the students in the class don't play either sport. How many students are in the class?

10. **Reasoning** A concert begins at 8:00 P.M. The concert is a 45-minute drive from Sean's house, but he wants to get there 15 minutes early. He needs a half-hour to get dressed before he leaves. What time should Sean start to get dressed?

Data Use the circle graph to solve Problems 11–14.



11. **Explain** How can you tell which type of game received the most votes?

12. Which two types of games each received less than half as many votes as card games?

13. **Estimate** If the number of students in the survey was 120, about how many students voted for each type of game?

14. **Create and Solve** Write your own word problem, using the circle graph.

Problem Solving on Tests

Multiple Choice

Choose the letter of the correct answer.

1. Pedro left school at noon and walked 3 miles per hour to the recreation center. Becky left at 1:30 P.M. to go there and rode her bike 6 miles per hour. At what time did Becky catch up with Pedro?

A 3:00 A.M. **C** 4:00 P.M.

B 3:00 P.M. **D** 5:00 P.M.

(Chapter 16, Lesson 2)

2. The mean of four numbers is 83. One of the numbers is 92. If the number 92 is removed, by how much will the mean of the remaining three numbers drop?

F by 2 **H** by 3

G by 4 **J** by 6

(Chapter 9, Lesson 3)

Open Response

Solve each problem.

3. Flora has 25¢ in her purse, but no quarters. How many different combinations of coins could be in her purse?

Explain How can making an organized list help you solve this problem?

(Chapter 9, Lesson 5)

4. What percent was added to this bill for tax?



(Chapter 18, Lesson 8)

Constructed Response

5. You are studying the growth of four annual plants in your garden. The table shows their heights above ground at four different times.



Height of Annual Plants				
	Week 1	Week 3	Week 6	Week 10
Amaranthus	2.0 cm	9.9 cm	34.5 cm	62.3 cm
Cleome	1.5 cm	16.7 cm	47.5 cm	89.3 cm
Cosmos	3.5 cm	27.4 cm	76.1 cm	152.6 cm
Salvia	1.8 cm	12.2 cm	31.1 cm	37.8 cm

a Which plant grew the slowest? The fastest?

b Which two plants grew about the same rate between Week 1 and Week 6? Explain.

c Convert the growth shown at Week 10 from metric to customary units. Round the decimal to the nearest tenth. Remember that 1 cm ≈ 0.4 in.

(Chapter 23)

Education Place

See **eduplace.com/map** for more Test-Taking Tips.

Lesson 5

Audio Tutor 2/35 Listen and Understand

Inequalities

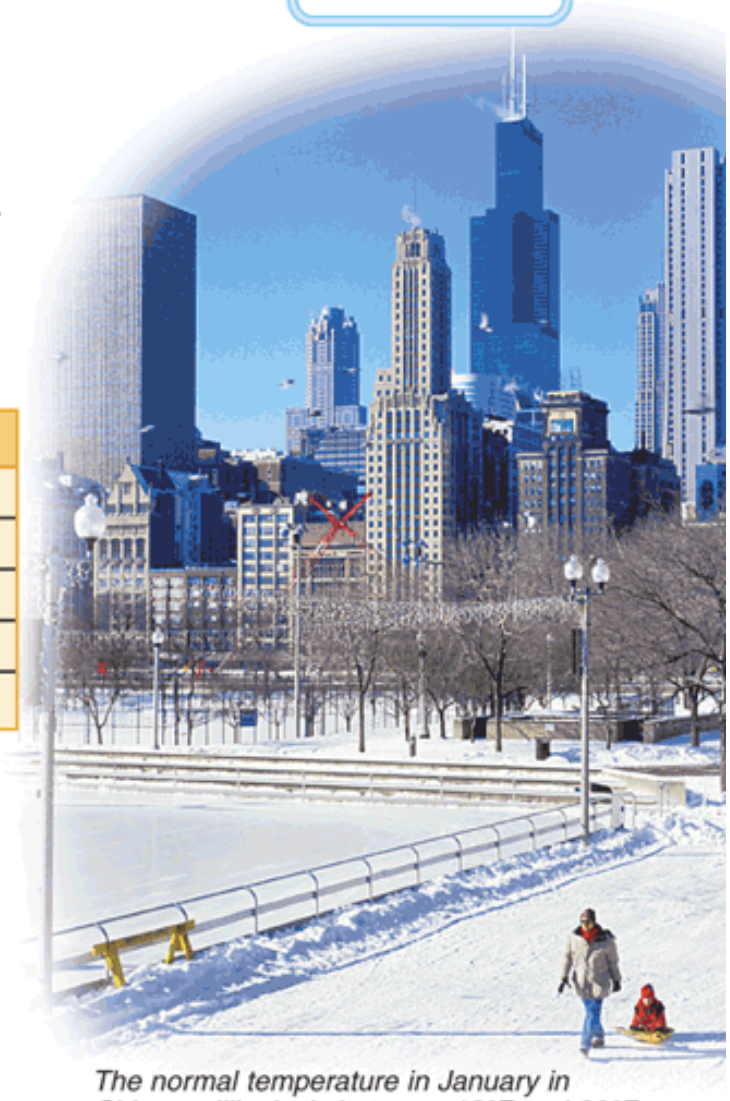
Objective Use inequality symbols and solve inequalities.

Vocabulary
- inequality
- solution to an inequality

Learn About It

An **inequality** is a relation that is expressed by placing an inequality symbol between two expressions. A **solution to an inequality** is any value of the variable that makes the inequality true. Inequalities may have many solutions.

Inequality	Read as:
$a \geq 4$	*a is greater than or equal to* 4
$d \leq 5$	*d is less than or equal to* 5
$7x \neq 21$	*7x is not equal to* 21
$12.5 \geq x$	12.5 *is greater than or equal to x*
$5 < c < 8$	*c* is *greater than* 5 and *less than* 8



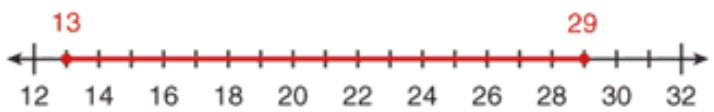
The normal temperature in January in Chicago, Illinois, is between 13°F and 29°F.

If T represents the temperature, then the inequality $13 \leq T \leq 29$ represents normal January temperatures in Chicago. What temperatures are solutions to this inequality?

▶ You can graph the solutions to inequalities on a number line.

Graph $13 \leq T \leq 29$.

The closed circles at 13 and 29 mean that $T = 13$ and $T = 29$ are solutions to the inequality.

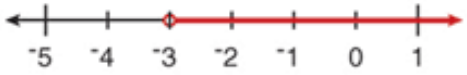


Solution: The solutions to $13 \leq T \leq 29$ are any temperatures that are greater than or equal to 13°F and less than or equal to 29°F.

Other Examples

A. Write an Inequality.

The graph shows all possible values for the variable x.



The open circle at -3 means that $x = -3$ is not part of the graph. Possible values of x are any number greater than -3.

$$x > -3$$

B. Check a Solution.

Is 3 a solution to $s + 3 \leq 9$?

$$s + 3 \leq 9$$

Substitute for s. $3 + 3 = 6$

Compare. $6 \leq 9$

Since $6 \leq 9$, 3 is a solution to $s + 3 \leq 9$.

Guided Practice

Graph each inequality. Write a phrase for each.

1. $s < -7$ **2.** $n \geq -4$ **3.** $x \leq 4$ **4.** $y > 3$

Ask Yourself

- Should I use a closed circle or an open circle for the endpoint?

Explain Your Thinking Why is graphing the solutions to an inequality easier than listing the solutions?

Practice and Problem Solving

Graph each inequality. Write a phrase to represent the solutions.

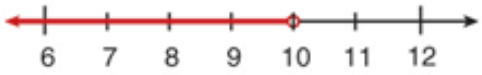
5. $p \leq 9$ **6.** $r > -4$ **7.** $x < 6$

8. $y \geq -3$ **9.** $a < 5$ **10.** $n \leq 2$

11. $x < 0.2$ **12.** $s > -2$ **13.** $-1\frac{1}{2} < y \leq -\frac{1}{2}$

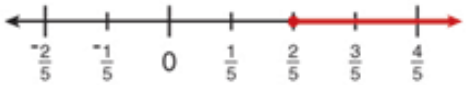
Choose the inequality that matches each graph.

14.



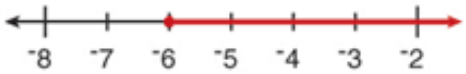
a. $x > 10$ **b.** $x < 10$ **c.** $x \geq 10$ **d.** $x \leq 10$

15.



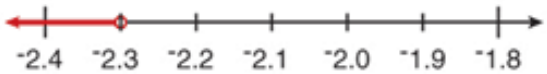
a. $x < \frac{2}{5}$ **b.** $x > \frac{2}{5}$ **c.** $x \geq \frac{2}{5}$ **d.** $x \geq -\frac{2}{5}$

16.



a. $x \leq -6$ **b.** $x > -6$ **c.** $x < -6$ **d.** $x \geq -6$

17.



a. $x \geq -2.3$ **b.** $x \leq -2.3$ **c.** $x > -2.3$ **d.** $x < -2.3$

Go On

Practice and Problem Solving

Use mental math to decide which values are solutions to each inequality.

18. $^-3, 2, 3, 4$
$5x > 15$

19. $^-7, ^-12, ^-18, ^-22$
$p + 9 \geq ^-2$

20. $6, 5, 0, ^-1$
$^-2 < y - 1 < 4.5$

Solve.

21. Explain Are there any other solutions to the inequality in Exercise 18? How do you know?

22. Analyze Write an inequality to represent any number that is *not* a solution to the inequality in Exercise 19.

Complete. Write $>$, $<$, $\geq$, $\leq$, or $\neq$ to make each inequality true for all values of *a*, *b*, and *c*.

23. If $a < b$, then b ● a.

24. If $a > b$, and $b > c$, then a ● c.

25. If $a \neq b$, then b ● a.

26. If $a \leq b$, then b ● a.

Write and graph an inequality. Identify three solutions for each.

27. At least 6 students entered a math competition. Let s represent the number of students who entered the competition.

28. No more than 10 books can fit on a shelf. Let b represent the number of books that can fit on the shelf.

29. After a snowstorm, the height of a snowdrift in a storm was more than 42 inches. Let h represent the height of the snowdrift in inches.

30. Maria is shorter than Joe but taller than Tina. Joe is 5 feet 2 inches tall and Tina is 4 feet 11 inches tall. Let m represent Maria's height in inches.

Find a real-life situation that could be represented by the graph.

31.



32.



Data **Use the step graph at right for Problems 33–35.**

33. How much does the band charge to play for $1\frac{1}{4}$ hours?

34. How much does the band charge to play for $2\frac{1}{2}$ hours?

35. Explain The band earned \$250. For how long could the band have played to earn that amount?



Extra Practice See page 623, Set D.

Think Rationally!

Game Activity

Players: 2

What You'll Need • 30 index cards, each with a different rational number.

How to Play

1. Divide the cards into two equal stacks and place them facedown on the table.
2. Each player turns over the first card from his or her stack and places it faceup on the table. The first player to call out the greater number keeps the cards.
3. Play continues until all cards have been used.
4. The player with the greater number of cards wins.



Now Play Again

Each student turns over three cards and arranges the cards in order from least to greatest. The first student to order the numbers correctly scores one point.

Players return the cards to their stacks and shuffle the cards.

The player with the most points after 5 rounds wins.



Chapter Review/Test

VOCABULARY

1. A number sentence that contains the symbols $>$ or $<$ is a(n) ____.
2. To ____ an algebraic expression, substitute the values given for the variables and perform the operations to find the value of the expression.
3. The opposite of a number is its ____.

Vocabulary

evaluate
inequality
expression
rational number
additive inverse

CONCEPTS AND SKILLS

Evaluate each expression for the given values. (Lesson 1, pp. 606–607)

4. $x - y + 4$, given $x = {}^{-}5$ and $y = 4$
5. $8.3 + ({}^{-}x) - y$, given $x = 2.2$ and $y = {}^{-}2$
6. $3x - 2 + ({}^{-}y)$, given $x = \frac{1}{3}$ and $y = \frac{1}{2}$
7. $x^2 - 7$, given $x = {}^{-}1.1$

Write an expression for each phrase. Tell what each variable represents. (Lesson 2, pp. 608–609)

8. negative three decreased by a number, then increased by another number
9. the multiplicative inverse of a number plus the number

Solve each equation. (Lesson 3, pp. 610–612)

10. $k - \$1.90 = \$12.25$
11. $q \div {}^{-}21 = {}^{-}8$
12. $1 - x = 15$
13. $77p = {}^{-}7.7$
14. $\frac{{}^{-}1}{5}n = 35$
15. $\frac{5}{6}m = 85$

Graph each inequality. Write a phrase for each. (Lesson 5, pp. 618–620)

16. $b > {}^{-}6$
17. $d < 4$
18. ${}^{-}3 \leq f \leq \frac{{}^{-}1}{2}$

PROBLEM SOLVING

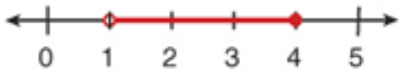
Use the Guess-and-Check strategy to solve each problem. (Lesson 4, pp. 614–616)

19. The product of two rational numbers is 1.5. Their sum is 3.5. What are the numbers?
20. The quotient of two rational numbers is 4. Their sum is ${}^{-}10$. What are the numbers?

Write About It

Show You Understand

Jim made this graph about his weekly swimming practice. What could you tell about the number of hours he practices each week?



Extra Practice

Set A (Lesson 1, pp. 606–607)

Evaluate each expression for the given values.

1. $5 - x + y$, given $x = 4$ and $y = {}^{-}8$
2. $x^2 + 18$, given $x = {}^{-}4$
3. ${}^{-}6x + 4y$, given $x = \frac{{}^{-}1}{2}$ and $y = \frac{1}{2}$
4. $x^2 + y + 8$, given $x = 0.2$ and $y = 3$
5. $x - 3y - 5$, given $x = \frac{{}^{-}3}{4}$ and $y = \frac{{}^{-}2}{3}$
6. $x - y + 3x$, given $x = \frac{5}{6}$ and $y = \frac{{}^{-}1}{3}$
7. ${}^{-}2x + 3y - x$, given $x = 3$ and $y = {}^{-}2$
8. $\frac{1}{2}x + (2y - 2)$, given $x = 2$ and $y = {}^{-}1$
9. $\frac{3}{4}x - (\frac{5}{8} - y)$, given $x = {}^{-}4$ and $y = {}^{-}5$
10. $x^2 + 2.5 + y$, given $x = {}^{-}4$ and $y = {}^{-}2.25$

Set B (Lesson 2, pp. 608–609)

Write an expression for each phrase. Tell what each variable represents.

1. a number increased by the three times the sum of that number plus eight
2. a number multiplied by the difference between another number and four
3. the difference between three and a half times a number and one-half another number plus negative eight
4. divide negative four by a negative number, then add another number
5. the additive inverse of a number decreased by one fourth the number

Set C (Lesson 3, pp. 610–612)

Solve each equation.

1. $x + 8 = {}^{-}2$
2. $x - 5 = {}^{-}3$
3. $8 - y = 12$
4. $x - {}^{-}3 = {}^{-}5$
5. $a - 3.3 = 2.1$
6. $b + \$1.75 = \$7.90$
7. $\frac{1}{3}c = 11$
8. $d \div 4 = 120$
9. $\frac{7}{8}e = 42$
10. $a - 3.3 = {}^{-}2$

Set D (Lesson 5, pp. 618–620)

Graph each inequality. Write a phrase for each.

1. $g \leq 4$
2. $h > {}^{-}3$
3. $j \geq 5$
4. $k < 2$
5. $x \leq {}^{-}1$
6. $y \geq {}^{-}1$
7. $m < 0.5$
8. $n \geq 3$
9. $\frac{{}^{-}1}{2} \leq x \leq \frac{1}{2}$
10. ${}^{-}5 \leq y \leq {}^{-}1$

WEEKLY WR READER®
Science Connection



Temperature, Absolutely!

The temperature of an object is a measure of the motion of the atoms or molecules that make up that object. The greater the motion, the higher the temperature.

The Fahrenheit and Celsius temperature scales are relative scales. This means that the temperatures are measured relative to some fixed points, such as the freezing and boiling points of water.

Scientists often use the absolute temperature, or Kelvin (K) scale. Zero temperature on this scale is the point where all molecular motion stops. 0 K = $^{-}$273.15°C.

Problem Solving

Use the formula K = °C + 273.15 for Problems 1–5.

1. Find the missing temperatures in the table at the right.

2. What is the difference in degrees Celsius of the temperatures on the Moon at noon and at night?

3. The average nighttime temperature on Mercury is 63.15 K. Is that lower or higher than the lowest temperature on Earth? Explain how you know.

4. The average temperature on Jupiter is ⁻153°C. The average temperature on Venus is ⁺482°C. What is the difference in their temperatures in degrees Celsius? in Kelvin?

5. Rewrite the formula K = °C + 273.15 to show how to find a temperature in degrees Celsius when you know the temperature in Kelvin.

	Kelvin (K)	Celsius (°C)
Noon on Moon	374.15	
Water boils		100
Human body		37
Water freezes		0
Coldest on Earth		⁻89.2
Oxygen boils	90.18	
Night on Moon	89.15	
Nitrogen boils		⁻195.79
Absolute zero	0	



Education Place

Visit Weekly Reader Connections at **eduplace.com/map** for more on this topic.

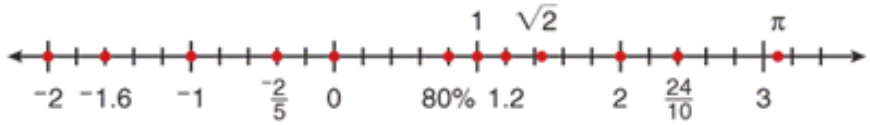
Enrichment: Number Sense

The Set of Real Numbers

Vocabulary
real numbers
irrational numbers

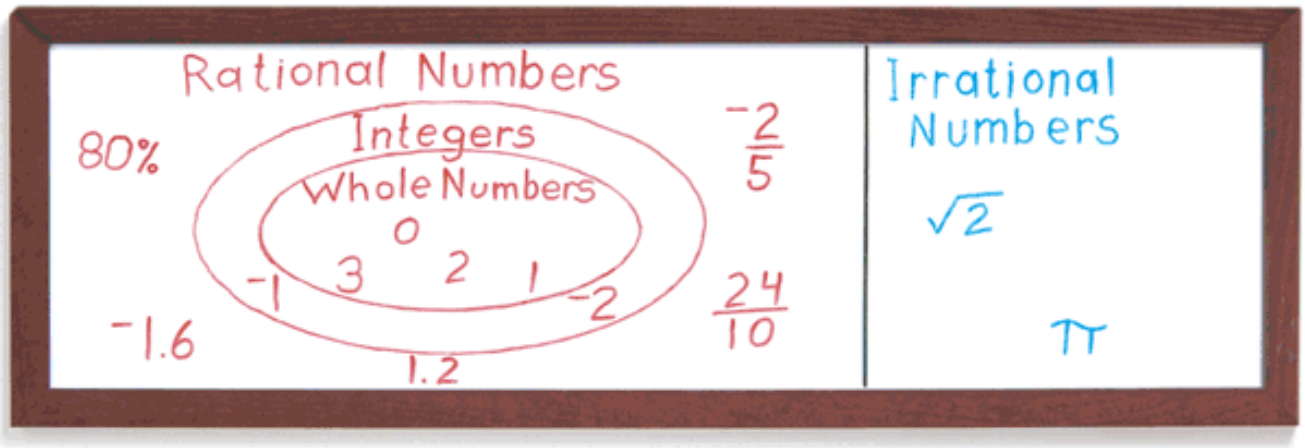
The set of rational numbers is part of an even larger set of numbers, called the **real numbers**. The set of real numbers consists of all the rational numbers and all the irrational numbers. **Irrational numbers** cannot be expressed as the ratio of two integers. They are *nonterminating, nonrepeating* decimals such as π.

The number line below shows the locations of several real numbers.



A Venn diagram can help you classify a real number as rational or irrational. Notice the way in which each number from the number line above is classified in the diagram.

Real Numbers



Try These!

Use the Venn diagram to help you classify each number as *rational* or *irrational*.

1. $\sqrt{4}$
2. $6.\overline{3}$
3. $\sqrt{5}$
4. $-2\frac{3}{4}$
5. $\sqrt{0.25}$
6. $-\frac{7}{9}$

Explain Your Thinking ▶ Explain how you would locate the numbers in Exercises 2, 3, and 6 on a number line.



Let's Be Rational



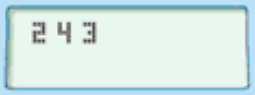
You can use your calculator to solve problems with rational numbers.

Remember this when working with rational numbers on a calculator.

Entering Exponents

Find 3^5.

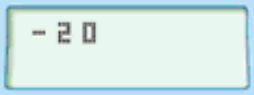
Press: 3 ^ 5 Enter



Entering a Negative Number

Find $4 \times {}^{-}5$.

Press: 4 × (−) 5 Enter



Use your calculator to solve each problem.

1. $7 - 1 + 3^2 \div 4 + (8.4 + {}^{-}5)$

2. $(6 + 4) \times 8 - 5^3 \times {}^{-}\frac{1}{2} \div 4$

Add parentheses to each equation to get each of the answers shown.

3. Equation: $1 + 4 \times 5 \div 8$
Answers: 3.125, 3.5, 2.625

4. Equation: $6 + 21 \div 3^2 - 6$
Answers: ${}^{-}3$, 9, 13

5. Equation: $9 - 7 \times 5 \div 2$
Answers: 5, ${}^{-}8.5$, ${}^{-}13$

6. Equation: $15 - 6^2 + 4 - 3$
Answers: ${}^{-}22$, ${}^{-}20$, 82

A magic square is an arrangement of numbers in which the sum of each row, column, or diagonal is the same. This sum is called the magic number. A number can only appear once in any magic square.

Copy the squares below. Then use your calculator to complete.

7. Magic number: 15

	6^0	$5\frac{4}{5}$
$4\frac{1}{5}$		

8. Magic number: ${}^{-}15$

${}^{-}5.8$	${}^{-}7.4$	
		-3^2

Unit 8 Test

VOCABULARY Open Response

1. A(n) ____ belongs to the set of whole numbers and their opposites.
2. A(n) ____ is a number than can be expressed in the form $\frac{a}{b}$, where a and b are any integers and b is not equal to zero.
3. A(n) ____ is a mathematical relation in which two expressions are separated by $>$, $<$, $\geq$, or $\leq$.
4. The ____ of a number is the opposite of that number.

Vocabulary

integer
equation
expression
inequality
rational number
additive inverse

CONCEPTS AND SKILLS Open Response

Rewrite each rational number in the form $\frac{a}{b}$. (Chapter 22)

5. 5 **6.** $^-7.5$ **7.** $^-3\frac{5}{6}$ **8.** $0.\overline{6}$

Compare. Write >, <, or =. Use a number line if necessary. (Chapter 22)

9. $^-0.75 \bigcirc \frac{-33}{44}$ **10.** $8.875 \bigcirc 8\frac{7}{9}$ **11.** $^-4\frac{2}{3} \bigcirc ^-4.6$

Order the numbers from least to greatest. Use a number line if necessary. (Chapter 22)

12. $\frac{-11}{7}$, $^-1.5$, $^-1\frac{29}{50}$ **13.** $\frac{-5}{6}$, $^-0.8$, $\frac{-19}{25}$ **14.** 125%, 1.275, $1\frac{5}{8}$

Complete each equation. Identify the property you used. (Chapter 22)

15. $\frac{-7}{9} \times \square = 1$ **16.** $\square + \frac{-2}{3} = \frac{-2}{3}$ **17.** $\frac{2}{5} + ^-3 = \square + \frac{2}{5}$

Simplify each expression. (Chapter 22)

18. $4(^-7 - 4) + 3$ **19.** $\frac{1}{16} + \frac{3}{4} \times 4$ **20.** $(2.4 + 0.8) \div (1 + ^-3)^2$

Evaluate each expression for the given values. (Chapter 23)

21. $^-x + ^-19$, given $x = 6$ **22.** $5(x + y^2)$, given $x = 0.5$ and $y = ^-1$

Write an expression for each phrase. Tell what each variable represents. (Chapter 23)

23. the sum of five eighths plus a number divided into three equal parts

24. the multiplicative inverse of a number decreased by one quarter of the number

Solve each equation. (Chapter 23)

25. $24.6 - s = {}^{-}3$ **26.** ${}^{-}0.2y = 9$ **27.** $d \div 1.3 = 67.6$

Graph each inequality. Write a phrase to represent the solutions. (Chapter 23)

28. $y \leq 2$ **29.** $^{-}\frac{1}{2} < x < 3$

PROBLEM SOLVING Open Response

30. The air temperature at ground level is 4°C. For every 5,000-foot increase in altitude, the air temperature drops by 6.5°. What will the temperature be at an altitude of 30,000 feet?

31. Each weekend Sal works half as many hours as Debbie does. Write an expression to represent the total number of hours the two work every weekend. Tell what each variable represents.

32. The floor of a cave is 0.76 miles below the ground. This is $2\frac{1}{2}$ times deeper than the second deepest cave. Write an equation to find the depth of the second deepest cave.

33. **Analyze** Write an inequality to represent any number that is *not* a solution to this inequality:

$$^{-}7 + b \leq 4$$

Performance Assessment

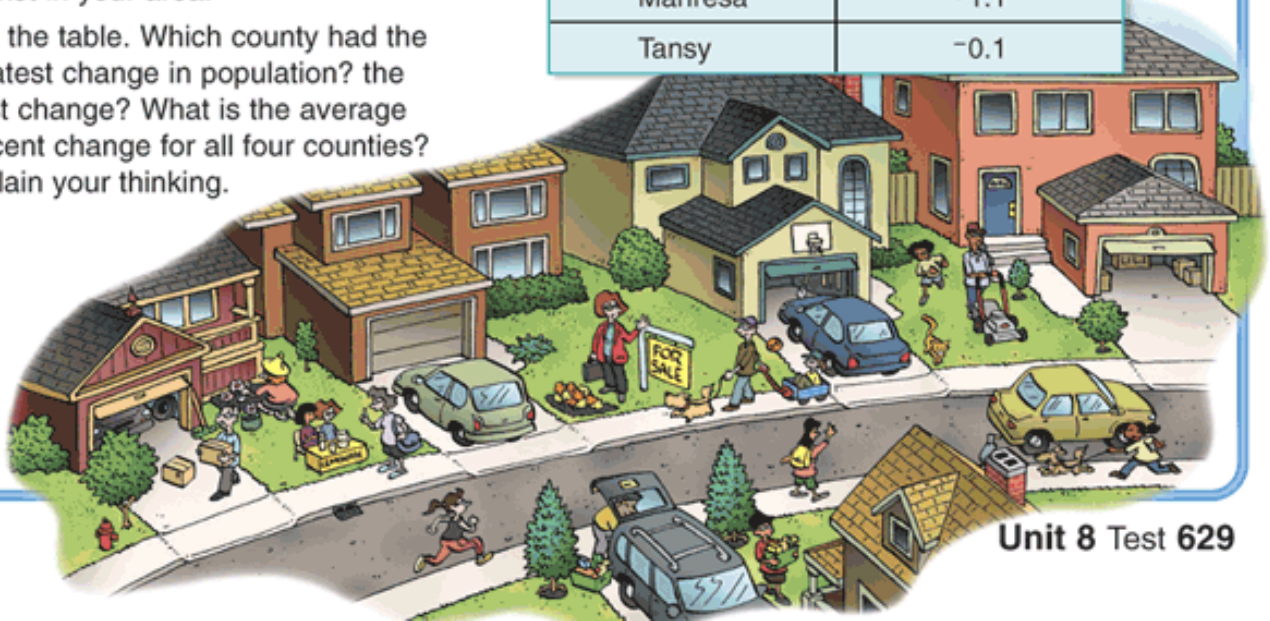
Extended Response

Task As a real estate broker, you want to use population trends to predict what might happen to the housing market in your area.

Use the table. Which county had the greatest change in population? the least change? What is the average percent change for all four counties? Explain your thinking.

Percent Change in Population

County	1985–2004
Samson	⁻1.2
Vanderwal	⁺3.2
Manresa	⁺1.1
Tansy	⁻0.1



Cumulative Test Prep

Solve Problems 1–10.

Test-Taking Tip

Look for certain qualifying words, such as *always*, *not*, *all*, *none*, *except*, *least*, and *best*. These words may be important in deciding which answer choice is correct.

Look at the example below.

Which of the following rational numbers is the least accurate expression of π?

A 3.14 **C** $\frac{22}{7}$

B $\sqrt{9}$ **D** 1.8^2

THINK

Since the question asks for the *least* accurate choice, find the value farthest away from π. You know that 3.14 and $\frac{22}{7}$ are close approximations of π. Convert choices B and D to decimal form and compare.

$\sqrt{9} = 3$; $1.8^2 = 3.24$

3 is farther away from 3.14 than 3.24, so answer choice B is the least accurate expression of π.

Multiple Choice

1. Stacey had $240 in savings. She spent $92 to buy art supplies. Which is the best expression for the part of her savings Stacey spent?

A about 40% **C** about 0.38

B about $\frac{1}{2}$ **D** about $\frac{1}{3}$

(Chapter 4, Lessons 2 and 4)

2. Which is not a measure of central tendency for this data set?

5, 15, 17, 18, 18, 20, 22, 22, 22

F 5 **G** $17\frac{2}{3}$ **H** 18 **J** 22

(Chapter 9, Lesson 3)

3. Which expression does not evaluate to a positive product?

A $^{-}3 \times 1 \times {}^{-}3 \times 1$

B 77×0

C $^{-}9 \times {}^{-}30$

D 8×1

(Chapter 11, Lesson 7)

4. Four rectangles have different perimeters but the same area. What is the area of each rectangle if their perimeters are 74 ft, 40 ft, 30 ft, and 26 ft?

F 24 ft^2 **H** 32 ft^2

G 30 ft^2 **J** 36 ft^2

(Chapter 8, Lesson 7)

For Test-Taking Tips, see page 648.

Open Response

5. Jared, Martha, and Sue walked in a charity walk. Together they raised $87. Jared's portion was 32% and Martha's was $\frac{2}{5}$. How much more than Sue did Jared and Martha raise?

(Chapter 17, Lesson 5)

6. Graph the equation $y = 2x - 3$ in the coordinate plane.

(Chapter 13, Lesson 4)

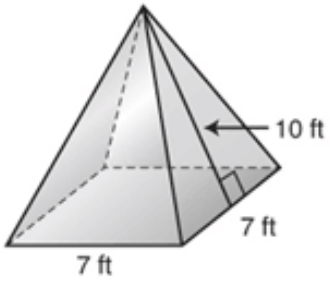
7. The low temperature in Coyote Ridge was $^{-}7°C$ at 8 A.M. Over the next hour, it rose 2 degrees. During each of the next two hours, it rose by twice as many degrees. What was the low temperature at 11 A.M.?

(Chapter 11, Lesson 4)

8. Derek picks a card from a stack of 9 cards, numbered 1–9, without looking. He then picks a second card without returning the first card to the stack. Find the probability P (3, odd number).

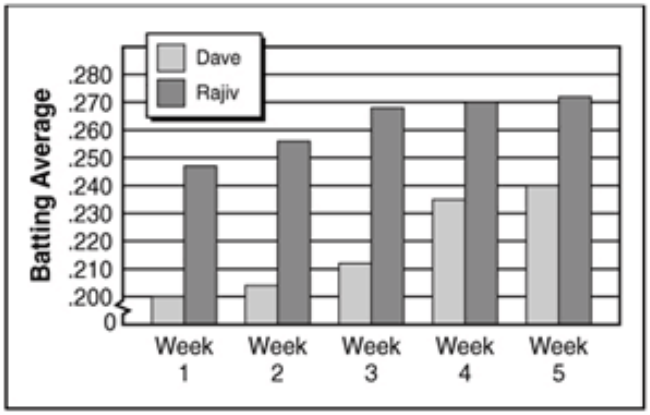
(Chapter 19, Lesson 4)

9. What is the surface area of this pyramid?



(Chapter 21, Lesson 5)

Extended Response



10. The graph shows batting averages for Dave and Rajiv for part of the baseball season.

A Who had the higher batting average during the third week? How much higher?

B Who had the higher batting average overall during the weeks shown on the graph? Whose batting average improved the most? Explain how you found your answers.

C Do you think the graph displays the data in a fair or misleading way? Explain your thinking.

D Draw your own version of the graph. Which version is more fair? Why?

(Chapter 10, Lesson 7)



Education Place

Look for Cumulative Test Prep at **eduplace.com/map** for more practice.

Vocabulary Wrap-Up for Unit 8

Look back at the big ideas and vocabulary in this unit.

Big Ideas

A number is rational if it can be expressed as a ratio of integers $\frac{a}{b}$, where b is not 0.

You can use inverse properties to help you compute with rational numbers.

You can graph the solution to an inequality on a number line.

Key Vocabulary

inverse properties
rational numbers
inequality

Math Conversations

Use your new vocabulary to discuss these big ideas.

1. Explain how to order these rational numbers: 4.12, 22.14, $\frac{21}{4}$, 41%.
2. Explain how to use the Distributive Property to multiply $5(^{-}\frac{1}{3} + \frac{2}{5})$.
3. Explain how you would solve the equation $^{-}0.2x = 350$.
4. Explain the difference between the inequalities $^{-}3.25 < y < 1.25$ and $^{-}3.25 \leq y \leq 1.25$.
5. **Write About It** Look for examples of rational numbers in newspapers and magazine articles. For example, you may find negative decimals or fractions in financial tables. List the different ways rational numbers are used.



Student Resources

Literature 634

Student Handbook 646

Facts Practice 654

Table of Measures 662

Glossary 663



READING MATH

THE FESTIVAL OF MURALS

Source of information: Chemainus Festival of Murals Society

Early in the 20th century, Chemainus was typical of many towns along British Columbia's coast. Its economy centered on one thing: harvesting timber. It was home to the largest "covered-in" sawmill in North America. Most of the townspeople worked in the mill. The lumber company even owned many of the stores where people bought their food and clothing.

But in 1980 the timber industry began to decline. The sawmill closed. People moved to Victoria and Vancouver to find work. The village was in danger of becoming a ghost town!

Chemainus needed to change to survive. The townspeople decided to transform their quiet mill town to an artists' center and tourist destination. They created the Chemainus Festival of Murals and each summer invited artists to paint larger-than-life artwork on the sides of the town's buildings.

The first five murals, commissioned in 1982, depicted local historic people, places and events. Each year following, the festival tried to add at least one new painting using ideas from old photographs and historic images.

Today, on the side of the Fire Hall Tower, a 33-foot-tall mural depicts two men, called fallers, logging a large tree. In a mural down the street, a young Native American woman watches a European ship sailing into Chemainus' Horseshoe Bay.

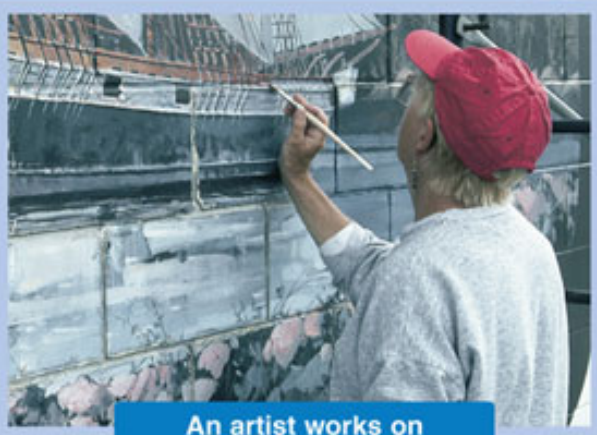
During the festival, visitors watch the newest mural in progress. An artist begins with a sketch known as a cartoon. Gridded squares from the cartoon are transferred to gridded squares on the wall. Using this method, a small drawing can be expanded to a large surface. The 25-foot-tall face of Chief Ce-who-latza or the bustling yard of the Chemainus sawmill is then brought to life with paintbrushes or airbrushes.

Each summer, about 400,000 visitors come to view the 33 large paintings. "Paint-in" events encourage artists to set up easels throughout the town, so visitors can watch art being created. Chemainus is prospering as an "open-air gallery" where thousands of visitors travel each year to enjoy the art of the little town that transformed itself.





This mural, painted in 1983, is titled "Native Heritage."



An artist works on one of the murals.

1. Five murals were commissioned in 1982. The festival tries to add one new painting each year. About how many paintings might be in the town by now?
2. About 400,000 visitors come to the festival each year. How many years would it take to have about 5 million visitors attend?
3. What was the problem the people of Chemainus had to face? How did they solve that problem?
4. What do you predict will happen to the town of Chemainus in the next ten years?

READING MATH

Tusks, Teeth, and Fractions

BY ANTHONY WILSON

Have you ever wondered what happens to a tiger with a toothache? Veterinary dentists care for giraffes, lions, bears—even elephants.

But how do these dentists make their patients hold still? Though animals are sometimes taken to specially designed facilities, in some instances procedures must be performed at the zoo. There, the animals are anesthetized. After the procedure, of course, the dentist leaves the holding cage before the animal wakes up. Imagine the surprise of veterinary dentists at the Oklahoma City Zoo the day a gorilla sat up sooner than expected after a root canal!

The teeth of animals come in all shapes and sizes. Some are adapted for grinding and chewing plants, some for tearing flesh. Bengal tigers have 32 teeth, the same as human adults. Sharks rely on their many rows of teeth not just to attack prey, but also to defend themselves against other sea creatures. When a shark loses a tooth, a new one soon grows in its place.

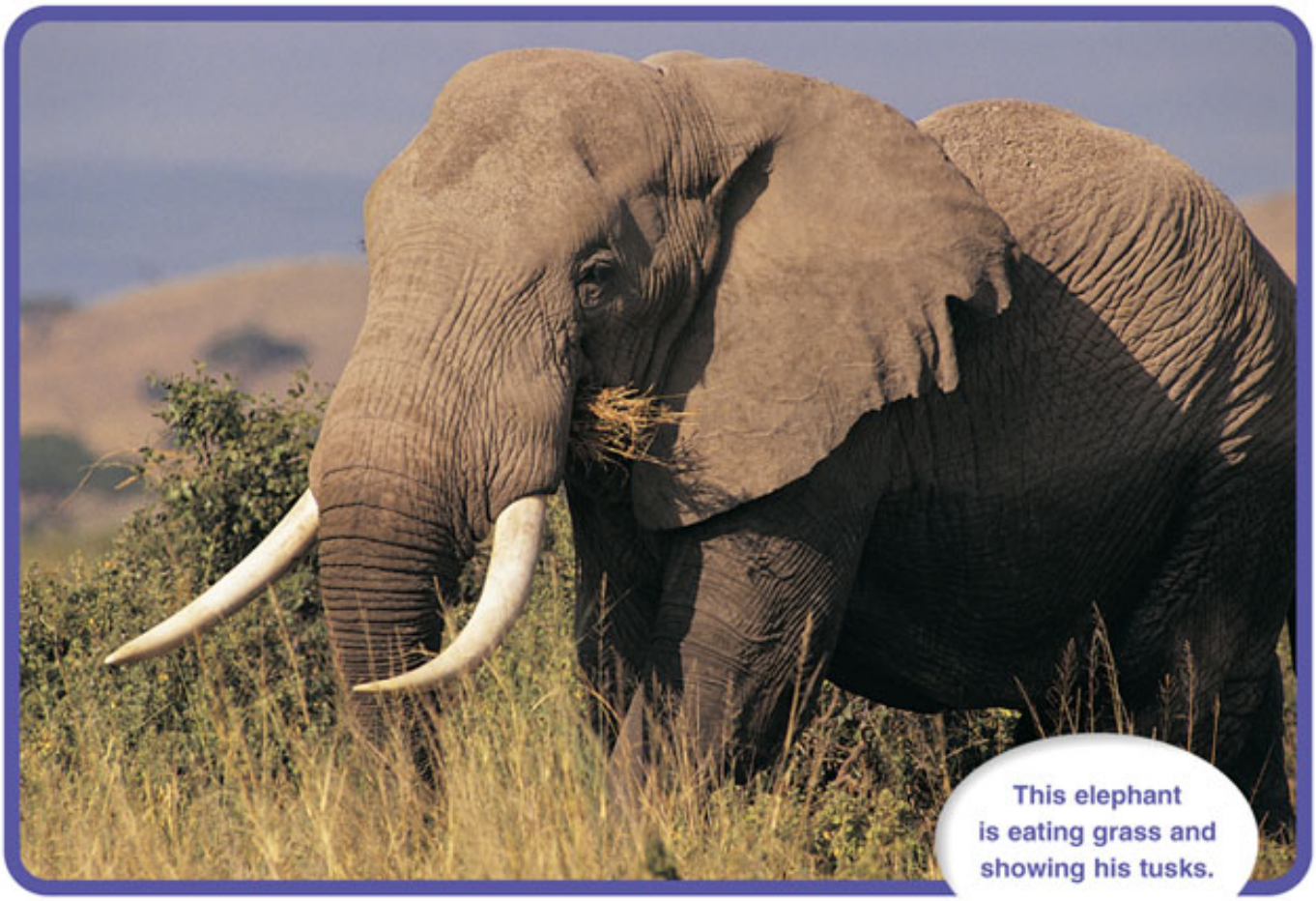


Dr. Kertesz inspects the teeth of a European brown bear named Mitsos. He holds up the bear's diseased tooth after extracting it. It's about 7 centimeters long.

The diseased tooth of a European brown bear was nearly 7 centimeters long. One Asian elephant's tooth, extracted at the Shanghai Zoo in the People's Republic of China, was 6.5 centimeters long.

Elephants' tusks appear at birth as milk teeth, which fall out after about a year, when the tusks are about 5 centimeters long. When the elephant is two to three years old, permanent tusks start to protrude beyond its lips. The tusks continue to grow from 15 to 18 centimeters per year throughout the elephant's life.

1. The tusks an elephant is born with grow more slowly than its permanent tusks. Express approximately how much more slowly, as both a fraction and a decimal.
2. An adult human's canine teeth are about a quarter of an inch long. A half-grown Bengal tiger's canine teeth are about 2.5 inches long. Estimate the difference in length between the canine teeth of a half-grown Bengal tiger and an adult human.



This elephant is eating grass and showing his tusks.

READING MATH

Skateboard and Scooter Alert!

If you are like many kids, you do sports on wheels. Activities such as biking, in-line skating, and skateboarding can be lots of fun. But not all kids are "wheel safe."

With any activity on wheels, a fall or collision can happen. Each year, thousands of kids are rushed to emergency rooms because of wheel-related injuries.

Wearing a helmet correctly can save your life. To work properly, the chin strap should be tight. Also, it is important that the helmet covers as much of your head as possible. If the helmet is too loose, it can slip to one side.

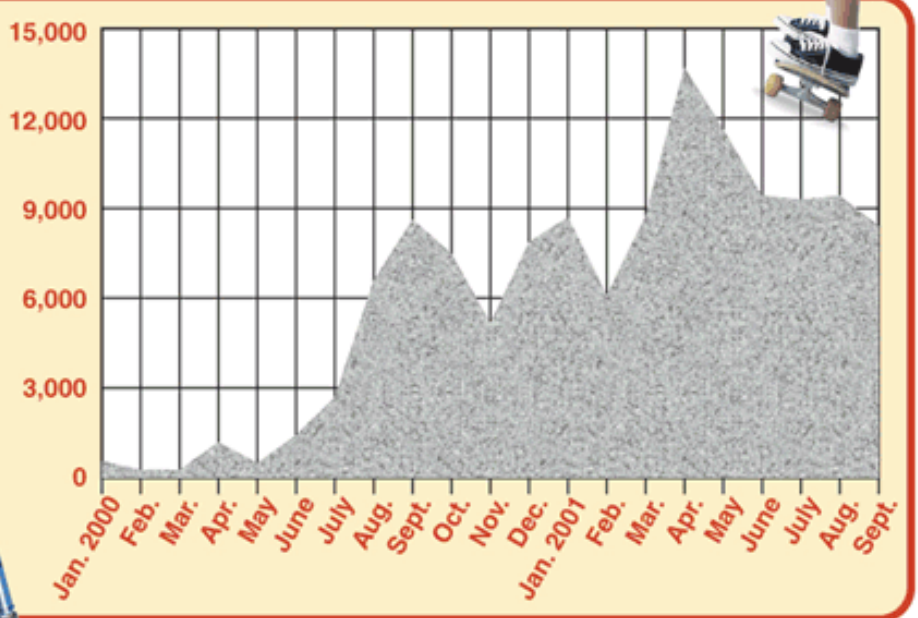
Because a helmet only covers your head, you still need to wear other safety gear. It's also important to wear kneepads and elbow pads to protect your body if you crash.

Being careful is the best way to avoid getting hurt. But even great riders can fall. Wearing a helmet and safety gear will keep you in one piece!



Scooter Injuries





1. What happened to the number of scooter injuries from January 2000 to January 2001?
2. Why do you think this change occurred?
3. If later years were added to the graph, do you think the numbers would increase or decrease? Explain.

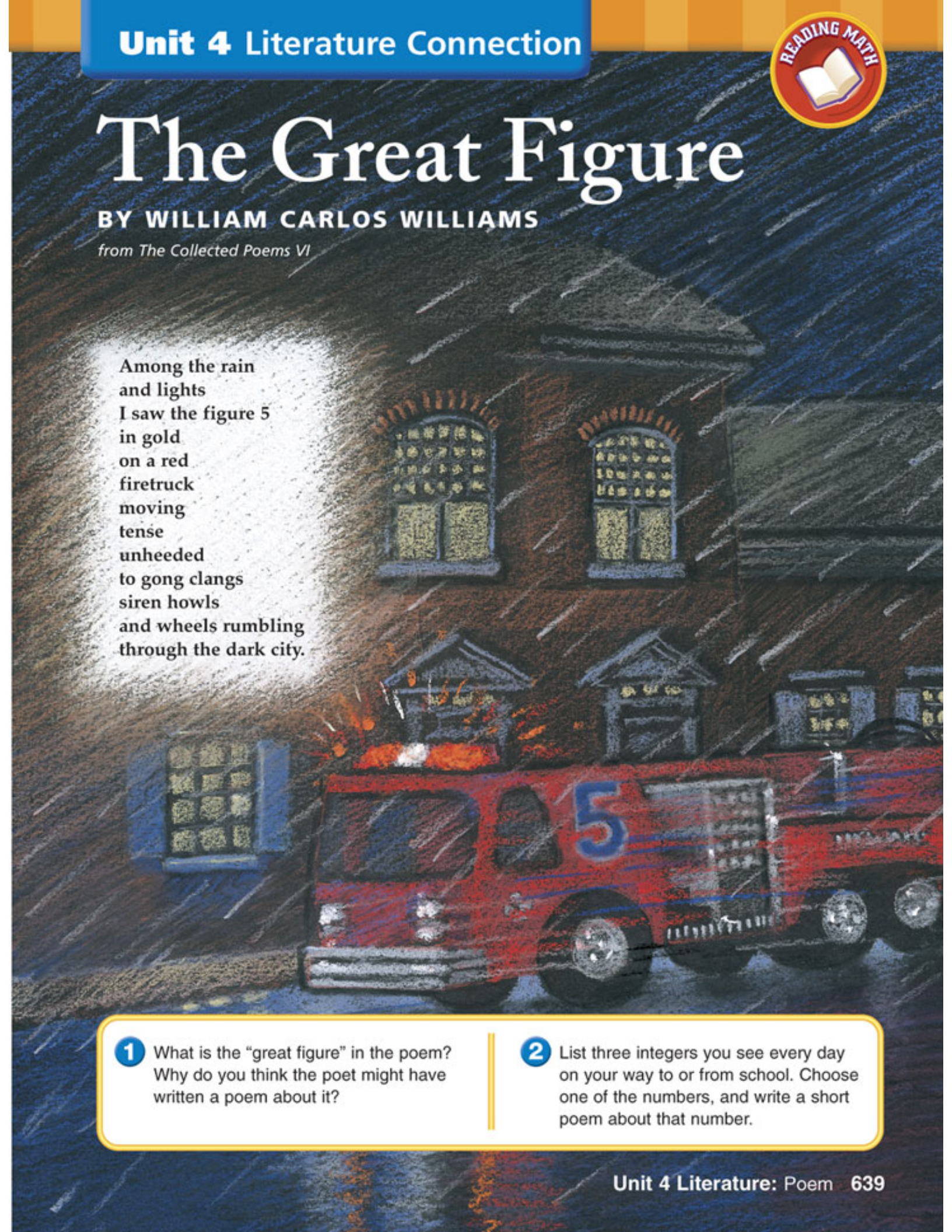
READING MATH

The Great Figure

BY WILLIAM CARLOS WILLIAMS

from The Collected Poems VI

Among the rain
and lights
I saw the figure 5
in gold
on a red
firetruck
moving
tense
unheeded
to gong clangs
siren howls
and wheels rumbling
through the dark city.



1. What is the "great figure" in the poem? Why do you think the poet might have written a poem about it?

2. List three integers you see every day on your way to or from school. Choose one of the numbers, and write a short poem about that number.

READING MATH

A Gift of Liberty

BY KIT CASEY

Source of information: The National Park Service

The Statue of Liberty stands holding her torch about 305 feet over New York harbor. It is considered by many to be a symbol of freedom and democracy. But perhaps most don't remember that the statue, completed in 1886, was a gift from France to the American people.

The building of the statue was a joint effort. It was agreed that the Americans would build the pedestal, and the French would design and assemble the statue itself. Sculptor Frèderic Auguste Bartholdi was hired to design the statue. Huge fundraising efforts took place in both countries to pay for construction costs.

After years of hard work, the Statue of Liberty was finally completed in 1884. In June of 1885, the heavy iron frame and copper "skin" was broken down into 350 pieces and put on a French ship bound for New York.

It took four months to reassemble the statue on its new pedestal. At last, the gift was completed. On October 28, 1886, President Grover Cleveland accepted it, saying, "We will not forget that Liberty has here made her home; nor shall her chosen altar be neglected."



1. List or draw three angles that you see in the shape of the Statue of Liberty. Describe each angle as obtuse, acute, straight, or right.

2. Why do you think the people of France wanted to give this gift to the American people?

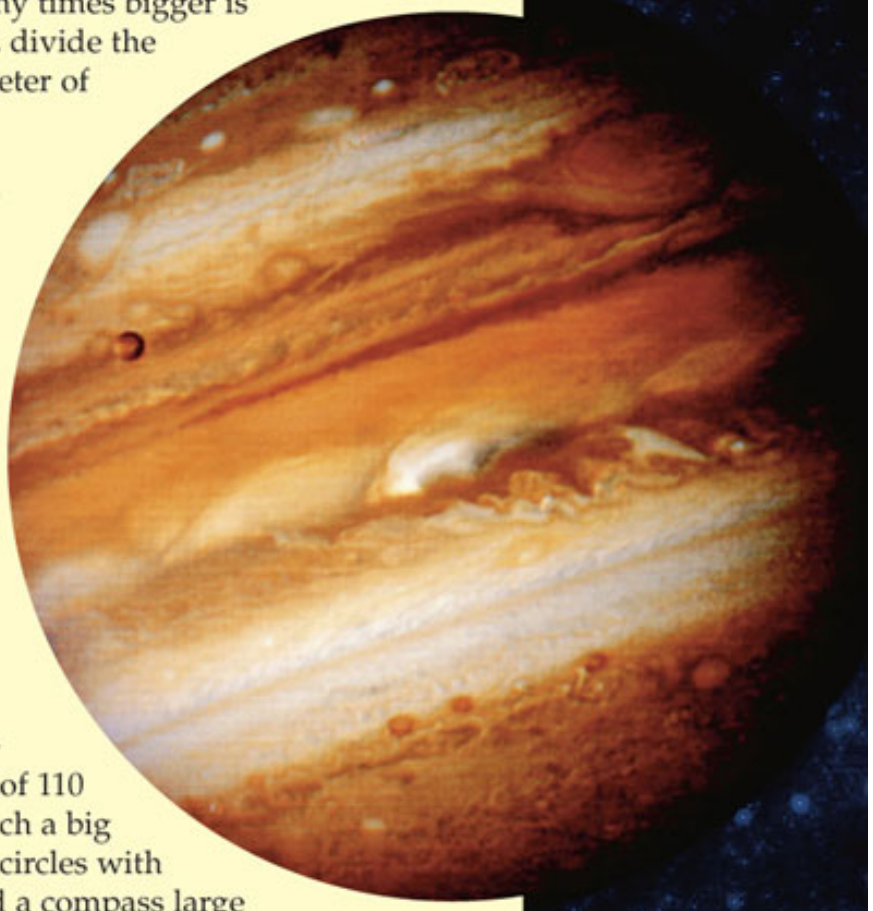
READING MATH

J is for Jupiter

BY DAVID M. SCHWARTZ *from G is for Googol*

Earth is a big place, but Jupiter is much bigger. How much bigger? You probably know Jupiter is the largest planet in our solar system, but that doesn't tell you how big it is compared to Earth. Math can help you see the difference.

Let's draw a circle around a button to represent Earth. The diameter of Earth is about 8,000 miles. The diameter of Jupiter is about 88,000 miles. How many times bigger is Jupiter than Earth? To find out, divide the diameter of Jupiter by the diameter of Earth. Jupiter is about 11 times bigger. Now draw Jupiter by making a circle with a diameter that's equal to 11 buttons. Compare the two circles. Do you still think Earth is such a big place?



So does that mean Jupiter is a giant? Let's compare it to the sun.

The sun's diameter is about 880,000 miles. How much bigger is the sun than Jupiter? Divide to find out. The sun is about 10 times bigger than Jupiter.

Draw a circle to represent the sun. It should have a diameter of 110 buttons. How can you draw such a big circle? You've probably drawn circles with a compass, but you'll never find a compass large enough to draw this circle! Instead, you could tie a string to a pencil or chalk. Think about how long the string should be. (Answer: 55 buttons.) We can't draw the entire circle representing the sun in this book. It's too big.

Hmmm. Jupiter isn't such a giant, after all. But the sun sure is!

continued on next page

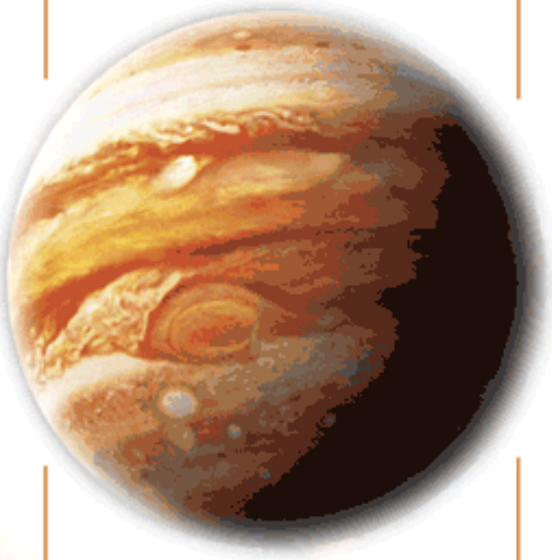
Big, Bigger, Biggest

Sun

Diameter = 880,000 miles
The Sun is a yellow dwarf star. If these pictures of the sun, Jupiter, and Earth were shrunk to fit the whole sun on this page, Earth would be almost too small to see!

Jupiter

Diameter = 88,000 miles
Jupiter is the fifth planet from the sun. It revolves around the sun once every 11.82 years. Jupiter is the biggest planet in the solar system.



Earth

Diameter = 8000 miles
Earth is the third planet from the sun. It revolves around the sun once every 365 days. Earth may seem big to us, but it is one of the smallest planets in the solar system.



1. Why do you think the author would tell you to use something small like a button to measure something as large as Jupiter?
2. What is the ratio of the sun's diameter to Jupiter's diameter?
3. What is the ratio of the sun's diameter to the diameter of Earth?

READING MATH

Freeze! Icebergs Ahead



A thunderous roar recently rumbled across part of Antarctica. A sheet of ice the size of Rhode Island had broken away and split into thousands of icebergs floating in the Weddell Sea.

The ice had been part of the Larsen Ice Shelf, on the east side of the Antarctic Peninsula. Antarctica's ice shelves are huge, thick areas of ice along the coasts. The section of the ice shelf that collapsed was named Larsen B. It weighed about 720 billion tons.

Shedding Ice

Larsen B wasn't the first giant chunk of ice to break off from Antarctica. In 1995, Larsen A, a similar-sized ice chunk, also broke away. Some scientists worry that a third section, called Larsen C, will break away next.

Many scientists say the collapse of the ice shelves in Antarctica in 1995 and this year is part of a natural cycle. They say new ice builds up in the middle of the continent. The new ice puts pressure on older ice. The pressure pushes the older ice down and out and causes ice to break away from Antarctica.

1. According to the article, how were some icebergs formed?
2. What do you think the word *peninsula* means?
3. The sheet of ice that broke off was about 2,000 square miles. Draw two different shapes that could represent 2,000 square miles. Be sure to label the measurements of each of the sides.

READING MATH

WILDLIFE Count-a-thon

BY SHARON L. BARRY *from Ranger Rick*

Seventy-four scientists are gathered at a city park in Hartford, Connecticut. But this is no picnic. It's a mission: Find and count as many kinds of living things in the park as possible. And the group has just 24 hours to do it!

The big event is called a BioBlitz. (*Bio* means "living," and a *blitz* is a "fast and furious attack.") The information collected helps local scientists figure out what lives in the area — and what needs to be protected.

At 3 P.M. on a summery Friday, the Connecticut scientists are off and running. The plants aren't going anywhere, of course. But wild animals sure don't line up to be counted.

So the scientists set traps and use binoculars and telescopes. They crawl through fields. They wade through streams. They sift through soil and peer under stones and logs. Sometimes they're lucky. One scientist finds a tick attached to him. Hey, parasites count too! Another scientist catches a garter snake that coughs up a frog. That's two species in one!

Night falls, but the count continues. Scientists hang nets to nab bats and bed sheets to attract insects. When the sheets are brightly lit, beetles, moths, spiders, and other creepy-crawlies go for the glow.

All morning and into the afternoon, the scientists rush to beat the clock. By 3 P.M. on Saturday, it's all over. The final count: 1,369 different species!

The bald eagle and three other endangered species made it onto the list. There were some more surprises too. For example, scientists didn't expect to find coyotes or fragile orchids called pink lady's slippers in this city park. But there they were!

The insect count was totally amazing — almost 800 species. That included four kinds of moths never before found in Connecticut, plus 27 different kinds of ants! Not bad for one day's work, huh?

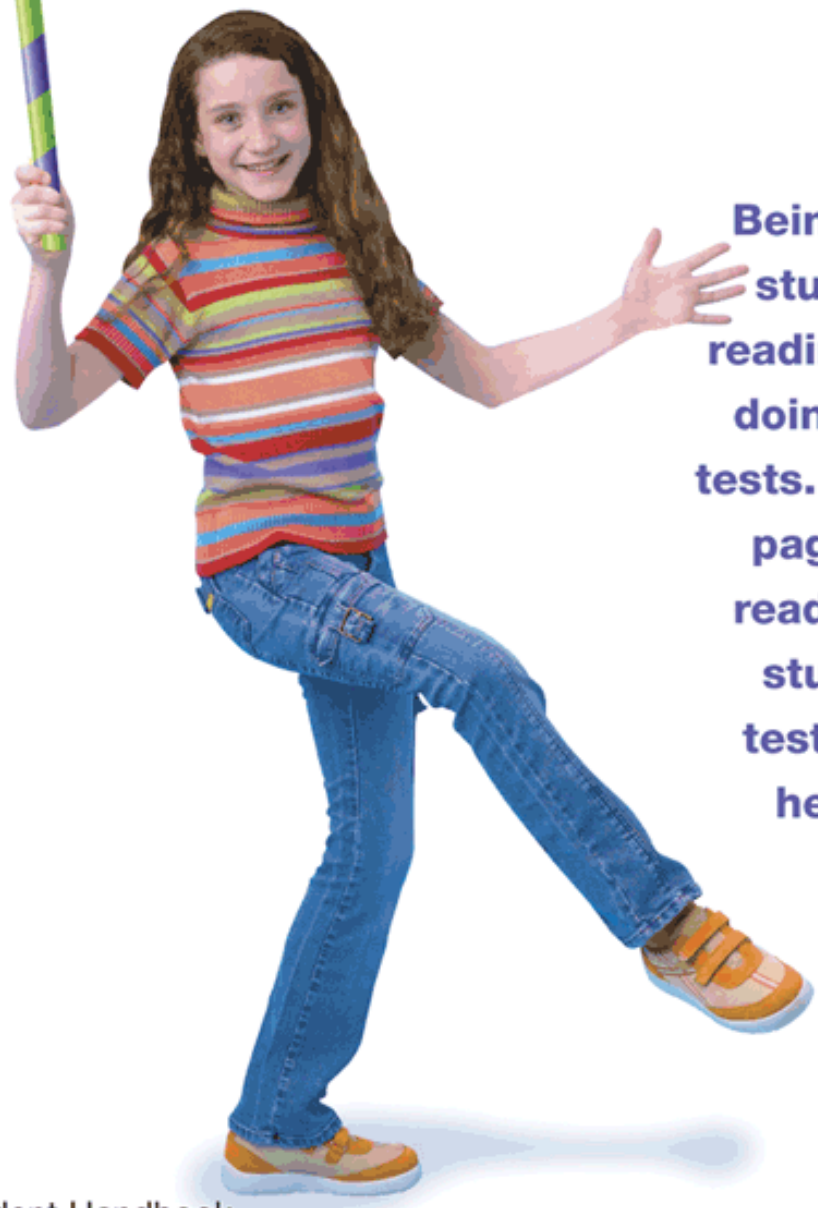


Final Count: 1,369 total species

285 plants
36 fungi
307 butterflies and moths
86 other invertebrates
75 birds
6 amphibians
13 fish
20 algae
25 one-celled animals
492 other insects
20 mammals
4 reptiles

1. What fraction of the total species counted were amphibians?
2. If 15 of the total birds counted were types of sparrow, what fraction of the birds were sparrows? How would you express this fraction in decimal form?
3. What details in the story tell the reader that the scientists were in a hurry to count the living things in the park?

Student Handbook



Being a good math student includes reading carefully and doing your best on tests. On the next few pages, you'll find reading strategies, study hints, and test-taking tips to help you learn.

Use Reading Strategies to Think About Math

What you learn during reading class can help you understand how to solve word problems.



Understand What the Question Is

Read the problem once to be sure it makes sense to you. Ask yourself the question in your own words. Picture the situation and make a drawing if it helps.

Think About the Words

As you read, pay attention to the mathematical terms. If you don't understand a word, try to decide what it means by looking at the words around it.

Be Sure You Have Enough Information

Identify the information you need. Look at tables or graphs as well as the words. Think about what you already know that may help.

Plan What You Will Do

Think about the problem-solving plan and strategies. Decide what computation method is needed. Then make a plan and follow it.

Evaluate Your Work

Look back at what the question asked, and check that your answer really answers that question. Be sure you have labeled your answer.

Strategies for Taking Tests

You need to think differently about how to answer various kinds of questions.

All Questions

If you can't answer a question, go on to the next question. You can return to it if there is time.

Always check your computation.

Multiple-Choice Questions

Estimate the answer. This can help eliminate any unreasonable choices.

On bubble sheets, be sure you mark the bubble for the right question and for the right letter.

Short-Answer Questions

Follow the directions carefully. You may need to show your work, write an explanation, or make a drawing.

If you can't give a complete answer, show what you do know. You may get credit for part of an answer.

Long-Answer Questions

Take time to think about these questions because you often need to explain your answer.

When you finish, reread the question and answer to be sure you have answered the question correctly.

Student Scoring Rubric

Your teacher may use a scoring rubric to evaluate your work. An example is on the next page. Not all rubrics are the same, so your teacher may use a different one.

Scoring Rubric

Rating	My work on this problem
Exemplary (full credit)	• has no errors, has the correct answer, and shows that I checked my answer. • is explained carefully and completely. • shows all needed diagrams, tables, or graphs.
Proficient (some credit)	• has small errors, has a close answer, and shows that I checked only the math. • is explained but may have missing parts. • shows most needed diagrams, tables, or graphs.
Acceptable (little credit)	• has some errors, has an answer, and shows that I did not check my answer. • is not explained carefully and completely. • shows few needed diagrams, tables, or graphs.
Limited (very little credit)	• has many errors and may not have an answer. • is not explained at all. • shows no needed diagrams, tables, or graphs.

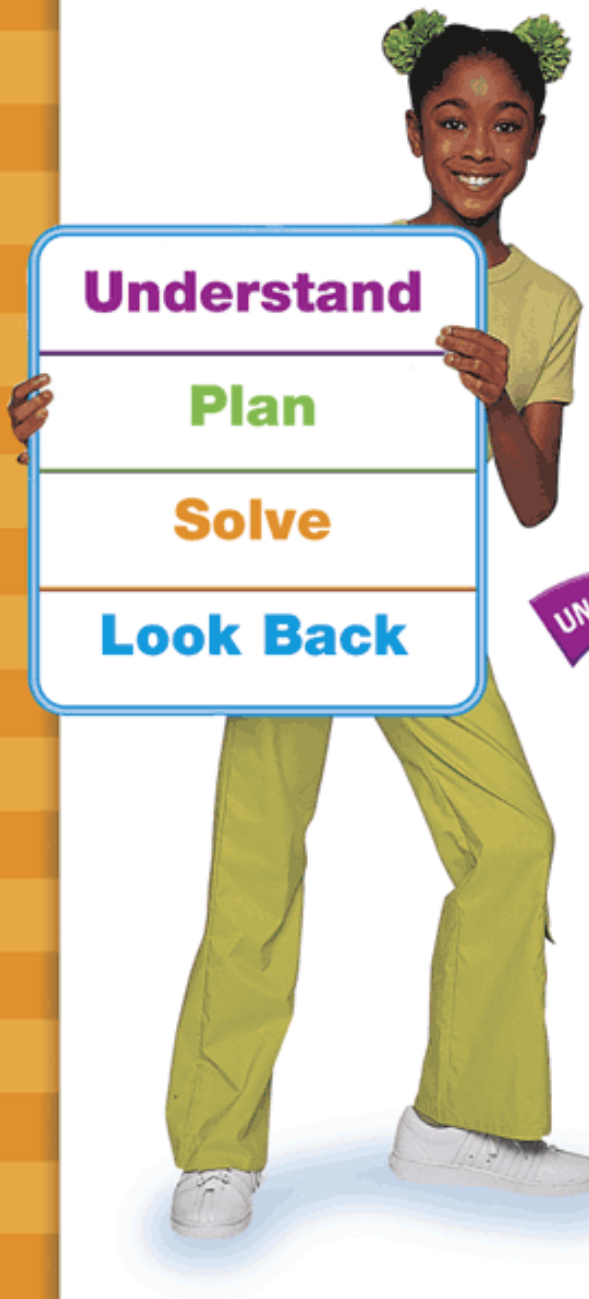
Two Important Things You Can Do Before a Test

- Get plenty of sleep the night before.
- Eat a good breakfast in the morning.



Your Plan For Problem Solving!

Follow this four-part plan and you'll become a problem-solving superstar!



Remember!

Always START at the "Understand" step and move on. But if you can't get an answer, don't give up. Just go back and start again.

UNDERSTAND

Always be sure you know what the question means! Here are some hints to help you:

- Read the problem and imagine the situation. Draw a picture if it helps.
- Replace any hard names you can'tread with easier ones.
- Identify what the question is asking and say it in your own words.
- Look for words that help you decide whether to add, subtract, multiply, or divide.

PLAN

Start by making a plan! Ask yourself:

- What strategy should I use?
- Do I have too much or too little information?
- Should I do more than one step?
- Which operation should I use?
- Should I use paper and pencil, mental math, or a calculator?

SOLVE

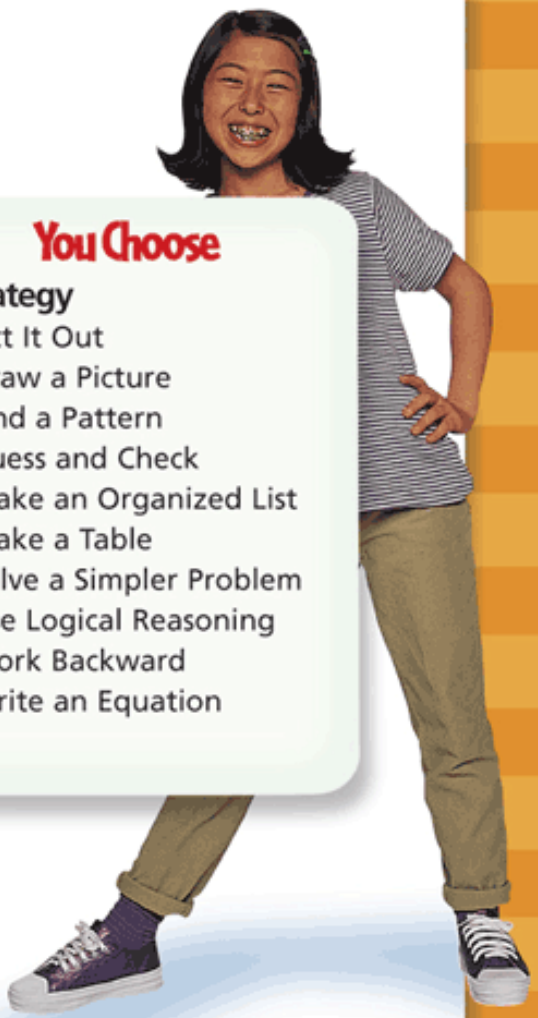
Finally! Now you're ready to solve the problem!

- Carry out your plan.
- Adjust your plan if needed.
- Check your calculations.

LOOK BACK

Congratulations! You've solved the problem. But is it correct? Once you have an answer, ask:

- Is my answer reasonable?
- Is my answer labeled correctly?
- Did I answer the question that was asked?
- Do I need to explain how I found the answer?



You Choose

Strategy

- Act It Out
- Draw a Picture
- Find a Pattern
- Guess and Check
- Make an Organized List
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Work Backward
- Write an Equation

Study Skills

Knowing how to study math will help you do well in math class.



To be a good math student, you need to learn

- ★ how to listen when your teacher is teaching.
- ★ how to work alone and with others.
- ★ how to plan your time.

Listening Skills

Listen carefully when your teacher is showing the class how to do something new. Try to understand what is being taught as well as how to do each step.

If you don't understand what your teacher is showing the class, ask a question. Try to let your teacher know what you don't understand.

Listening carefully will also help you be ready to answer any questions your teacher may ask. You may be able to help another student by explaining how you understand what your teacher is saying.

Working Alone and with Others

When you work alone, try to connect the math you are learning to math you already know. Knowing how parts of mathematics fit together helps you remember and understand.

When you work with others, help as much as you can. *Cooperating* is another word for working together. When people cooperate, they often learn more because they share ideas.



Planning Your Time

Doing your homework on time is part of being a good math student. Make sure that you take the assignment home with you.

Have a place at home to do your homework—it could be in your room or at the kitchen table or anywhere that works for your family.

Get extra help if you are having trouble. Write questions about what you don't understand. This will help your teacher give you the extra help you need.

Mixed Addition and Subtraction

- To practice adding, do columns A, C, and E of rows 1–5.
- To practice subtracting, do columns B, D, and F of rows 1–5.
- For mixed practice, choose rows to do.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	3 + 3	12 − 8	4 + 7	13 − 6	7 + 6	10 − 7
Row 2.	4 + 4	13 − 4	8 + 7	14 − 5	8 + 6	13 − 5
Row 3.	0 + 2	14 − 6	9 + 8	15 − 7	9 + 5	11 − 5
Row 4.	4 + 5	11 − 8	5 + 5	12 − 4	8 + 2	14 − 7
Row 5.	3 + 9	16 − 9	6 + 9	13 − 8	9 + 9	15 − 9
Row 6.	12 − 9	4 + 6	6 + 7	12 − 5	15 − 8	6 + 6
Row 7.	0 + 0	17 − 8	12 − 7	2 + 9	2 + 7	17 − 9
Row 8.	14 − 8	3 + 7	7 + 8	14 − 9	6 + 8	9 + 4
Row 9.	1 + 9	12 − 3	13 − 9	6 + 5	4 + 8	11 − 7
Row 10.	13 − 7	10 − 6	8 + 8	11 − 9	7 + 7	10 − 8
Row 11.	12 − 6	8 + 5	3 + 8	5 + 7	15 − 6	9 + 7
Row 12.	16 − 7	15 − 6	8 + 9	18 − 9	9 + 6	16 − 8

More Practice

Work with a partner. Make flash cards for the facts that give you trouble. Practice your facts by quizzing each other with the flash cards.

Multiplication Facts

- For mixed practice, choose rows or columns to do.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	3×2	8×5	6×3	4×8	9×7	6×8
Row 2.	4×2	2×9	8×1	6×2	2×2	0×9
Row 3.	3×3	5×2	6×0	4×3	7×1	9×2
Row 4.	4×4	2×6	7×7	1×1	6×5	4×5
Row 5.	8×6	7×4	3×8	9×5	2×0	7×9
Row 6.	7×5	3×9	6×4	7×3	8×8	5×1
Row 7.	1×9	0×8	7×2	2×8	3×7	5×3
Row 8.	0×0	8×9	5×6	9×3	3×4	7×6
Row 9.	9×6	5×9	9×9	7×8	5×4	9×8
Row 10.	8×4	6×7	3×5	4×9	5×7	8×3
Row 11.	5×5	4×3	6×9	3×6	8×2	4×6
Row 12.	8×7	9×4	0×3	5×8	6×6	4×7

More Practice

Work with a partner. Make flash cards for the facts that give you trouble. Practice your facts by quizzing each other with the flash cards.

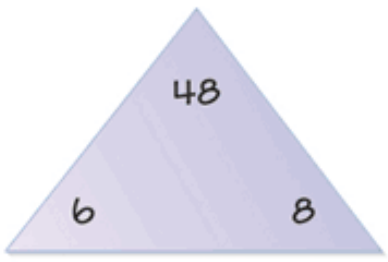
Division Facts

- For mixed practice, choose rows or columns to do.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	18 ÷ 9	16 ÷ 4	30 ÷ 5	42 ÷ 6	72 ÷ 8	14 ÷ 7
Row 2.	20 ÷ 5	48 ÷ 6	24 ÷ 8	63 ÷ 9	21 ÷ 3	27 ÷ 9
Row 3.	18 ÷ 3	35 ÷ 7	36 ÷ 9	25 ÷ 5	18 ÷ 6	40 ÷ 8
Row 4.	56 ÷ 8	0 ÷ 5	30 ÷ 6	28 ÷ 7	49 ÷ 7	24 ÷ 4
Row 5.	0 ÷ 3	12 ÷ 6	15 ÷ 5	7 ÷ 7	16 ÷ 8	1 ÷ 1
Row 6.	21 ÷ 7	5 ÷ 5	14 ÷ 2	0 ÷ 6	45 ÷ 9	20 ÷ 4
Row 7.	4 ÷ 4	54 ÷ 9	64 ÷ 8	15 ÷ 3	40 ÷ 5	0 ÷ 8
Row 8.	54 ÷ 6	16 ÷ 2	27 ÷ 3	63 ÷ 7	32 ÷ 4	42 ÷ 7
Row 9.	28 ÷ 4	48 ÷ 8	24 ÷ 6	18 ÷ 2	21 ÷ 7	72 ÷ 9
Row 10.	9 ÷ 9	35 ÷ 5	8 ÷ 1	56 ÷ 7	20 ÷ 4	36 ÷ 6
Row 11.	49 ÷ 7	36 ÷ 4	32 ÷ 8	24 ÷ 3	81 ÷ 9	45 ÷ 5

More Practice

Make triangular flash cards for multiplication and division fact families. Place all cards face down. Without looking at the numbers, pick a card up by a corner so that one number is covered up. Use the numbers you can see to decide what the unknown number is.



Multiplication and Division

- For mixed practice, choose rows or columns to do.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	8 ÷ 2	4 × 8	9 × 3	40 ÷ 5	9 × 6	49 ÷ 7
Row 2.	5 × 5	7 × 9	24 ÷ 6	36 ÷ 4	12 ÷ 3	5 × 6
Row 3.	45 ÷ 9	20 ÷ 4	4 × 4	6 × 6	48 ÷ 6	8 × 9
Row 4.	4 × 7	64 ÷ 8	9 × 9	10 ÷ 5	0 × 3	24 ÷ 3
Row 5.	35 ÷ 5	6 × 5	18 ÷ 3	2 × 9	56 ÷ 8	4 × 9
Row 6.	27 ÷ 3	16 ÷ 4	2 × 1	3 × 6	1 × 1	42 ÷ 6
Row 7.	0 × 0	5 × 8	54 ÷ 6	32 ÷ 4	21 ÷ 3	7 × 5
Row 8.	7 × 8	4 ÷ 4	3 × 3	25 ÷ 5	7 × 7	81 ÷ 9
Row 9.	8 × 3	72 ÷ 9	30 ÷ 5	8 × 8	9 × 5	28 ÷ 7
Row 10.	36 ÷ 6	6 × 8	18 ÷ 9	4 × 6	63 ÷ 7	4 × 5
Row 11.	9 × 5	63 ÷ 9	0 ÷ 6	54 ÷ 9	40 ÷ 8	6 × 7
Row 12.	36 ÷ 9	9 × 4	8 × 5	48 ÷ 8	56 ÷ 7	9 × 8

More Practice

Work with a partner. Take turns making up multiplication and division problems and solving them.

Adding Fractions Practice

- To practice adding fractions with like denominators, do columns A and B.
- To practice adding fractions with unlike denominators, do columns C and D.
- To practice adding mixed numbers, do columns E and F.
- For mixed practice, choose rows to do.

Write each answer in simplest form.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	$\frac{1}{4} + \frac{3}{4}$	$\frac{2}{6} + \frac{5}{6}$	$\frac{8}{9} + \frac{2}{5}$	$\frac{4}{5} + \frac{6}{8}$	$1\frac{3}{8} + \frac{6}{9}$	$3\frac{1}{6} + \frac{5}{9}$
Row 2.	$\frac{3}{7} + \frac{1}{7}$	$\frac{7}{9} + \frac{4}{9}$	$\frac{5}{9} + \frac{3}{5}$	$\frac{1}{9} + \frac{3}{7}$	$1\frac{3}{5} + 1\frac{1}{4}$	$4\frac{3}{10} + \frac{1}{6}$
Row 3.	$\frac{4}{12} + \frac{1}{12}$	$\frac{4}{10} + \frac{1}{10}$	$\frac{1}{3} + \frac{5}{8}$	$\frac{2}{3} + \frac{4}{5}$	$3\frac{2}{5} + \frac{2}{5}$	$1\frac{5}{8} + \frac{1}{4}$
Row 4.	$\frac{6}{8} + \frac{2}{8}$	$\frac{1}{4} + \frac{3}{4}$	$\frac{7}{9} + \frac{1}{8}$	$\frac{4}{16} + \frac{1}{5}$	$4\frac{4}{7} + \frac{3}{6}$	$2\frac{5}{9} + \frac{3}{5}$
Row 5.	$\frac{10}{12} + \frac{11}{12}$	$\frac{2}{5} + \frac{1}{5}$	$\frac{3}{5} + \frac{1}{4}$	$\frac{5}{6} + \frac{6}{7}$	$2\frac{5}{9} + \frac{5}{7}$	$3\frac{3}{8} + \frac{8}{9}$
Row 6.	$\frac{3}{7} + \frac{2}{7}$	$\frac{4}{9} + \frac{6}{9}$	$\frac{3}{8} + \frac{1}{2}$	$\frac{2}{3} + \frac{5}{6}$	$2\frac{2}{5} + \frac{5}{7}$	$3\frac{2}{3} + 1\frac{1}{4}$
Row 7.	$\frac{3}{10} + \frac{7}{10}$	$\frac{2}{6} + \frac{1}{6}$	$\frac{9}{10} + \frac{3}{4}$	$\frac{2}{3} + \frac{3}{8}$	$3\frac{3}{7} + \frac{4}{7}$	$\frac{5}{8} + 1\frac{2}{5}$
Row 8.	$\frac{2}{8} + \frac{5}{8}$	$\frac{4}{7} + \frac{2}{7}$	$\frac{5}{12} + \frac{1}{11}$	$\frac{1}{4} + \frac{5}{7}$	$4\frac{2}{6} + \frac{5}{8}$	$2\frac{3}{4} + 1\frac{7}{8}$

More Practice

Work with a partner. Make flash cards for the problems.
Practice by quizzing each other with the flash cards.

Subtracting Fractions Practice

- To practice subtracting fractions with like denominators, do columns A and B.
- To practice subtracting fractions with unlike denominators, do columns C and D.
- To practice subtracting mixed numbers, do columns E and F.
- For mixed practice, choose rows to do.

Write each answer in simplest form.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	$\frac{4}{8} - \frac{1}{8}$	$\frac{7}{9} - \frac{3}{9}$	$\frac{3}{6} - \frac{1}{12}$	$\frac{3}{4} - \frac{1}{3}$	$3\frac{5}{8} - 2\frac{1}{8}$	$1\frac{1}{5} - \frac{3}{4}$
Row 2.	$\frac{3}{6} - \frac{2}{6}$	$\frac{4}{7} - \frac{2}{7}$	$\frac{3}{5} - \frac{1}{10}$	$\frac{2}{3} - \frac{1}{5}$	$7\frac{3}{10} - \frac{1}{2}$	$2\frac{1}{6} - \frac{2}{6}$
Row 3.	$\frac{9}{12} - \frac{1}{12}$	$\frac{5}{9} - \frac{3}{9}$	$\frac{8}{14} - \frac{5}{9}$	$\frac{3}{8} - \frac{1}{4}$	$3\frac{3}{12} - \frac{3}{4}$	$4\frac{5}{6} - \frac{7}{8}$
Row 4.	$\frac{2}{3} - \frac{1}{3}$	$\frac{7}{8} - \frac{1}{8}$	$\frac{8}{10} - \frac{1}{9}$	$\frac{1}{2} - \frac{1}{3}$	$6\frac{2}{5} - \frac{1}{8}$	$5\frac{1}{8} - \frac{3}{5}$
Row 5.	$\frac{8}{12} - \frac{2}{12}$	$\frac{5}{8} - \frac{2}{8}$	$\frac{5}{6} - \frac{3}{4}$	$\frac{4}{9} - \frac{3}{7}$	$1\frac{5}{9} - \frac{5}{9}$	$1\frac{5}{8} - \frac{1}{4}$
Row 6.	$\frac{4}{5} - \frac{2}{5}$	$\frac{3}{4} - \frac{2}{4}$	$\frac{7}{12} - \frac{1}{4}$	$\frac{1}{3} - \frac{1}{4}$	$5\frac{2}{5} - \frac{6}{7}$	$2\frac{1}{3} - \frac{4}{7}$
Row 7.	$\frac{4}{6} - \frac{1}{6}$	$\frac{5}{7} - \frac{1}{7}$	$\frac{11}{12} - \frac{5}{6}$	$\frac{3}{4} - \frac{5}{7}$	$2\frac{3}{8} - \frac{3}{4}$	$2\frac{3}{4} - \frac{4}{5}$
Row 8.	$\frac{3}{4} - \frac{1}{4}$	$\frac{5}{8} - \frac{3}{8}$	$\frac{2}{3} - \frac{1}{9}$	$\frac{1}{3} - \frac{1}{6}$	$4\frac{3}{4} - \frac{3}{4}$	$1\frac{1}{4} - \frac{1}{8}$

More Practice

Work with a partner. Take turns making up subtraction problems and solving them.

Multiplying Fractions

- To practice multiplying fractions, do columns A and B.
- To practice multiplying whole numbers and fractions or mixed numbers, do columns C and D.
- To practice multiplying with mixed numbers, do columns E and F.
- For mixed practice, choose rows to do.

Write each answer in simplest form.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	$\frac{1}{3} \times \frac{2}{3}$	$\frac{1}{2} \times \frac{1}{2}$	$4 \times \frac{2}{7}$	$9\frac{1}{5} \times 3$	$6\frac{7}{8} \times \frac{1}{4}$	$3\frac{5}{6} \times \frac{1}{7}$
Row 2.	$\frac{3}{10} \times \frac{1}{2}$	$\frac{1}{3} \times \frac{3}{5}$	$4 \times 1\frac{2}{5}$	$6 \times \frac{4}{7}$	$4\frac{3}{5} \times \frac{1}{4}$	$1\frac{2}{8} \times \frac{3}{4}$
Row 3.	$\frac{3}{4} \times \frac{2}{4}$	$\frac{2}{9} \times \frac{1}{9}$	$5 \times \frac{3}{4}$	$\frac{6}{7} \times 2$	$7\frac{1}{8} \times \frac{4}{9}$	$5\frac{5}{6} \times \frac{9}{10}$
Row 4.	$\frac{2}{3} \times \frac{1}{12}$	$\frac{1}{3} \times \frac{5}{6}$	$1\frac{4}{5} \times 5$	$6 \times \frac{7}{15}$	$2\frac{1}{6} \times 1\frac{3}{4}$	$1\frac{5}{8} \times 3\frac{1}{4}$
Row 5.	$\frac{7}{9} \times \frac{4}{9}$	$\frac{2}{3} \times \frac{2}{3}$	$1 \times 6\frac{3}{5}$	$4\frac{2}{5} \times 5$	$8\frac{2}{5} \times 5\frac{1}{4}$	$4\frac{3}{4} \times \frac{5}{6}$
Row 6.	$\frac{6}{7} \times \frac{7}{8}$	$\frac{5}{8} \times \frac{1}{4}$	$3 \times \frac{2}{3}$	$4 \times 5\frac{5}{16}$	$3\frac{1}{5} \times 1\frac{7}{10}$	$1\frac{4}{5} \times \frac{3}{7}$
Row 7.	$\frac{3}{5} \times \frac{1}{5}$	$\frac{8}{9} \times \frac{2}{3}$	$4 \times \frac{6}{18}$	$1 \times \frac{3}{4}$	$3\frac{3}{8} \times 1\frac{1}{8}$	$1\frac{4}{5} \times 3\frac{3}{4}$
Row 8.	$\frac{1}{12} \times \frac{3}{7}$	$\frac{4}{5} \times \frac{2}{5}$	$5 \times 2\frac{4}{15}$	$1\frac{8}{9} \times 8$	$6\frac{3}{5} \times 5\frac{2}{9}$	$9\frac{3}{4} \times \frac{5}{6}$

More Practice

Work with a partner. Make flash cards for the problems.
Practice by quizzing each other with the flash cards.

Dividing Fractions

- To practice dividing fractions, do columns A and B.
- To practice dividing whole numbers and fractions or mixed numbers, do columns C and D.
- To practice dividing with mixed numbers, do columns E and F.
- For mixed practice, choose rows to do.

Write each answer in simplest form.

	Column A	Column B	Column C	Column D	Column E	Column F
Row 1.	$\frac{1}{3} \div \frac{1}{3}$	$\frac{2}{10} \div \frac{2}{5}$	$4 \div \frac{1}{2}$	$\frac{1}{4} \div 4$	$1\frac{5}{8} \div \frac{1}{4}$	$5\frac{1}{4} \div 1\frac{1}{6}$
Row 2.	$\frac{1}{2} \div \frac{3}{4}$	$\frac{1}{2} \div \frac{1}{3}$	$4 \div 2\frac{1}{5}$	$3 \div \frac{8}{9}$	$\frac{3}{4} \div 1\frac{3}{4}$	$4\frac{3}{8} \div \frac{3}{7}$
Row 3.	$\frac{2}{5} \div \frac{1}{5}$	$\frac{1}{2} \div \frac{7}{8}$	$4\frac{1}{4} \div 3$	$3 \div \frac{1}{2}$	$1\frac{3}{8} \div \frac{1}{4}$	$1\frac{1}{2} \div 6\frac{1}{2}$
Row 4.	$\frac{7}{12} \div \frac{2}{3}$	$\frac{1}{3} \div \frac{1}{2}$	$5 \div 5\frac{3}{5}$	$1\frac{1}{2} \div 5$	$2\frac{1}{2} \div 4\frac{2}{7}$	$2\frac{1}{2} \div \frac{1}{12}$
Row 5.	$\frac{1}{10} \div \frac{9}{10}$	$\frac{1}{4} \div \frac{5}{8}$	$5 \div \frac{8}{10}$	$\frac{2}{5} \div 4$	$1\frac{1}{2} \div \frac{7}{8}$	$10\frac{3}{5} \div \frac{2}{8}$
Row 6.	$\frac{2}{9} \div \frac{1}{3}$	$\frac{1}{2} \div \frac{1}{2}$	$3\frac{2}{3} \div 9$	$4 \div \frac{2}{9}$	$6\frac{2}{3} \div 5\frac{3}{4}$	$2\frac{2}{3} \div \frac{5}{6}$
Row 7.	$\frac{3}{5} \div \frac{4}{5}$	$\frac{1}{2} \div \frac{5}{6}$	$1\frac{3}{9} \div 7$	$1\frac{2}{6} \div 10$	$4\frac{5}{7} \div \frac{3}{4}$	$3\frac{1}{3} \div \frac{1}{10}$
Row 8.	$\frac{1}{3} \div \frac{2}{3}$	$\frac{2}{3} \div \frac{7}{9}$	$8\frac{2}{6} \div 4$	$5 \div 2\frac{2}{3}$	$\frac{1}{2} \div 1\frac{1}{6}$	$5\frac{2}{3} \div \frac{3}{17}$

More Practice

Work with a partner. Make flash cards for the problems.
Practice by quizzing each other with the flash cards.

Table Of Measures

Customary Units of Measure

Length

1 foot (ft)	=	12 inches (in.)
1 yard (yd)	=	36 inches
1 yard	=	3 feet
1 mile (mi)	=	5,280 feet
1 mile	=	1,760 yards

Area

144 square inches ($in.^2$)	=	1 square foot (ft^2)
9 square feet	=	1 square yard (yd^2)

Volume

1,728 cubic inches ($in.^3$)	=	1 cubic foot (ft^3)
27 cubic feet	=	1 cubic yard (yd^3)

Capacity

1 tablespoon (tbsp)	=	3 teaspoons (tsp)
1 fluid ounce (fl oz)	=	2 tablespoons
1 cup (c)	=	8 fluid ounces
1 pint (pt)	=	2 cups
1 quart (qt)	=	2 pints
1 quart	=	4 cups
1 gallon (gal)	=	4 quarts
1 gallon	=	8 pints

Weight/Mass

1 pound (lb)	=	16 ounces (oz)
1 ton (T)	=	2,000 pounds (lb)

Metric Units of Measure

Length

1 centimeter (cm)	=	10 millimeters (mm)
1 decimeter (dm)	=	10 centimeters
1 meter (m)	=	1,000 millimeters
1 meter	=	100 centimeters
1 meter	=	10 decimeters
1 kilometer (km)	=	1,000 meters

Area

1 square centimeter (cm^2)	=	100 square millimeters (mm^2)
1 square decimeter (dm^2)	=	100 square centimeters
1 square meter (m^2)	=	100 square decimeters

Volume

1 cubic centimeter (cm^3)	=	1,000 cubic millimeters (mm^3)
1 cubic decimeter (dm^3)	=	1,000 cubic centimeters
1 cubic meter (m^3)	=	1,000 cubic decimeters

Capacity

1 liter (L)	=	1,000 milliliters (mL)
1 liter	=	10 deciliters (dL)
1 liter	=	1 cubic decimeter (dm^3)
1,000 liters	=	1 cubic meter (m^3)

Weight/Mass

1 gram (g)	=	1,000 milligrams (mg)
1 kilogram (kg)	=	1,000 grams
1 metric ton (t)	=	1,000 kilograms

Units of Time

1 minute (min)	=	60 seconds (s)
1 hour (h)	=	60 minutes
1 day	=	24 hours
1 week (wk)	=	7 days
1 year (yr)	=	12 months (mo)
1 year	=	365 days
1 leap year	=	366 days
1 decade	=	10 years
1 century	=	100 years
1 millennium	=	1,000 years

Glossary

A

absolute value The distance of a number from zero on a number line.

accuracy The degree of closeness of a measurement to its true measure.

acute angle An angle with a measure less than that of a right angle.

acute triangle A triangle in which each of the three angles is acute.

addend A number to be added in an addition expression. In $7 + 4 + 8$, the numbers 7, 4, and 8 are addends.

Addition Property of Equality Adding the same number to both sides of an equation results in a new equation, having the same solution(s) as the original.

additive inverse The opposite of a number. The sum of a number and its additive inverse is always zero.

adjacent angles Angles that share a common side and a common vertex. Their interiors do not overlap.

algebraic expression An expression that consists of one or more variables. It could contain some constants and some operations.

Example: $2x + 3y + 6$.

alternate exterior angles Exterior angles formed when two lines are intersected by a transversal.

alternate interior angles Interior angles formed when two lines are intersected by a transversal.

angle An angle is formed by two rays with a common endpoint.



angle pairs Pairs of related angles.

area The number of square units that cover a figure completely without overlapping.

array An arrangement of objects, pictures, or numbers in columns and rows.

Associative Property of Addition Changing the grouping of addends does not change their sum. It is also called the *Grouping Property of Addition.*

Example: For all numbers a, b and c, $a + (b + c) = (a + b) + c$.

Associative Property of Multiplication Changing the grouping of factors does not change their product. It is also called the *Grouping Property of Multiplication.*

Example: For all numbers a, b and c, $a \times (b \times c) = (a \times b) \times c$.

average The number found by dividing the sum of a group of numbers by the number of addends.

B

bar graph A graph in which information is shown by means of rectangular bars.

base of a geometric figure A bottom side or face of a geometric figure.

base of a percent The original number in a percent problem.

base of a power The number that is used as a factor when evaluating powers.

Example: 10^3. 10 is the base of the power.

bias The difference between what the responses of a sample are and what the actual responses from the entire population would be.

binary system A system using only the digits 0 and 1 in which each place has a value 2 times greater than the place to its right. It is also known as *base two.*

bisect(s) To partition a line segment or angle into two congruent parts.

box-and-whisker plot A data display that shows the extremes, the median, and the lower and upper quartiles in a simple and clear diagram.

C

capacity The amount a container can hold.

Celsius A temperature scale with the freezing point of water set to 0 degrees, and the boiling point set to 100 degrees.

center of a circle A point lying in the middle of a circle that is the same distance from all points on the circle.

central angle Any angle with a vertex at the center of a circle.

chord A line segment that connects two points on a circle.

circle A closed figure in which every point is the same distance from a given point called the center of the circle.

circle graph A graph used to display data that are parts of a whole.

circumference The distance around a circle.

cluster In a data display, a group of data points that are close to each other.

coefficient A value by which a variable is multiplied.

combination A way of selecting members of a set when the order is not important.

common denominator Any common multiple of the denominators of two or more factors.

common factor A number that is a factor of two or more numbers.

common multiple(s) A number that is shared as a multiple of two or more numbers.

Commutative Property of Addition Changing the order of addends does not change their sum. It is also called the *Order Property of Addition.*

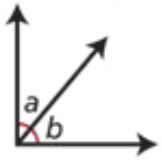
Example: For all numbers a and b, $a + b = b + a$.

Commutative Property of Multiplication Changing the order of factors does not change their product. It is also called the *Order Property of Multiplication.*

Example: For all numbers a and b, $a \times b = b \times a$.

compatible numbers Numbers that are close to the original numbers and are easy to divide.

complementary angles Two angles for which the sum of the angle measures is 90°.



composite number A whole number that has factors other than 1 and itself.

compound event In probability, an event that is made up of two or more events.

cone A solid with a curved surface that has one circular base and one vertex.



congruent figures Figures that have the same size and the same shape.

convenience sample In statistics, a sample that is easy to survey. A convenience sample allows results to be obtained quickly.

coordinate plane A system of coordinates formed by the perpendicular intersection of horizontal and vertical number lines.

coordinates An ordered pair of numbers that locates a point in the coordinate plane with respect to the x- and y-axis.

corresponding parts In geometry, matching sides and angles of polygons.

cross product A product obtained by multiplying the second term of one ratio by the first term of another.

cube A solid figure that has six congruent square faces.



cubic unit A unit for measuring volume. It is the volume of a cube with each edge 1 unit long.

customary system The measurement system that uses foot, quart, pound, and degrees Fahrenheit.

cylinder A solid figure with two circular faces that are congruent and a cylindrical surface connecting the two faces.



D

data A set of numbers or pieces of information.

data set A collection of numbers or pieces of information.

decagon A polygon with 10 sides.



decimal A number with one or more digits to the right of a decimal point.

Example: 0.3

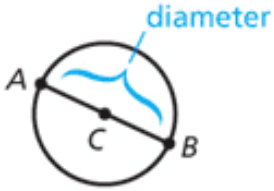
decimal point

denominator The number below the bar in a fraction.

dependent event An event that is affected by the outcome of another event.

diagonal A line segment that connects two vertices of a polygon and is not a side.

diameter Any chord that passes through the center of a circle.



difference The result of subtraction.

Example: $5 - 2 = 3$

difference

discount A decrease in the price of an item.

disjoint events Two or more events with no outcomes in common.

Distributive Property The product of a factor and a sum (or difference) equals the sum (or difference) of the products.

Example: For all numbers a, b and c, $a \times (b + c) = (a \times b) + (a \times c)$

dividend The number that is divided in division problem.

divisible One number is divisible by another if the quotient is a whole number and the remainder is 0.

Example: 10 is divisible by 2, since $10 \div 2 = 5$ R0.

Division Property of Equality Dividing both sides of an equation by the same nonzero number results in a new equation, having the same solution(s) as the original.

divisor The number by which a number is being divided.

Example: $40 \div 5$

dividend divisor

dodecagon A polygon with 12 sides.



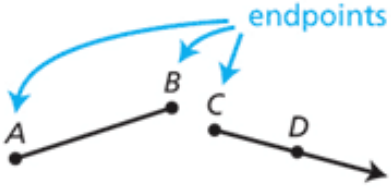
double bar graph A graph in which two data sets are compared by means of pairs of rectangular bars drawn next to each other.

double line graph A graph that is used to compare two data sets by means of two broken lines.

E

edge The segment formed where two faces of a solid figure meet.

endpoint The point at either end of a line segment. The beginning point of a ray.



equally likely Having the same chance of occurring.

equation A mathematical sentence that uses an equals sign to show that two expressions are equal.

Examples: $3 + 1 = 4$ and $2x + 5 = 9$

equilateral triangle A triangle that has three congruent sides and three congruent angles.



equivalent fractions Different fractions that have the same value. Equivalent fractions represent the same number.

Example: $\frac{1}{2}$ and $\frac{4}{8}$ are equivalent fractions.

equivalent ratios Ratios that are formed when both terms are multiplied or divided by a number other than zero.

estimate A number close to an exact amount. An estimate tells *about* how much or *about* how many.

evaluate To calculate the numerical value of.

evaluating an expression For a numerical expression, performing the operations to find the value of the expression. For an algebraic expression, substituting number(s) for the variable(s) and then performing the operations to find the value of the expressions.

even number A whole number that is a multiple of 2. The ones digit in an even number is 0, 2, 4, 6, or 8. The numbers 56 and 48 are examples of even numbers.

event One or more outcomes of an experiment.

experiment A series of repeated trials performed to determine whether a theory is valid.

experimental probability The probability of an event that is based upon the results of an experiment.

exponent The number in a power that tells how many times the base is used as a factor.

Example: In 10^3, 3 is the exponent.

expression A number, a variable, or any combination of numbers, variables, operation signs, and grouping symbols.

exterior angle of a polygon An angle formed when a side of a polygon is extended.

F

face A flat surface of a solid figure.

fact family Facts that are related, using the same numbers.

Examples:

$1 + 4 = 5$ $\quad$ $4 + 1 = 5$

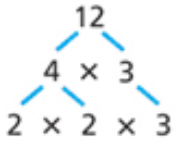
$5 - 1 = 4$ $\quad$ $5 - 4 = 1$

$3 \times 5 = 15$ $\quad$ $5 \times 3 = 15$

$15 \div 3 = 5$ $\quad$ $15 \div 5 = 3$

factor One of two or more numbers that are multiplied to give a product.

factor tree A diagram that is used to show the prime factorization of a number.



factorization A number written as a product of its factors.

factor of a number A number that divides evenly into a given number.

Fahrenheit A temperature scale with the freezing point of water set to 32 degrees and the boiling point set to 212 degrees.

fraction A number that can be expressed in the form $\frac{a}{b}$, where a and b are whole numbers, and b is not equal to 0. A fraction can name a part of a whole, a part of a collection, or a part of a region.

Examples: $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{3}$

frequency In surveys, the number of times a response is chosen.

frequency table A way of summarizing data that shows how often data occur in each interval.

front-end estimation Estimation by looking at the digits in the greatest place of each number.

function A rule that pairs each input with one and only one output.

function table A table that uses functions to represent and describe problems.

Fundamental counting principle A general rule that states that if a choice can be made in m different ways and for each of these choices a second choice can be made in n different ways, then the choices can be made in $m \times n$ different ways. Also known as the *Basic counting principle.*

G

gap In a data display, a region where there are no data points.

greatest common divisor (GCD) The greatest whole number that is a common factor of two or more numbers. It is also called the *greatest common factor.*

H

heptagon A polygon with seven sides.



regular irregular

hexagon A polygon with six sides.



regular irregular

histogram A graph in which bars are used to display how frequently data occur within equal intervals.

horizontal axis The x-axis in a coordinate system. It is a number line that is used to locate points to the left or to the right of the origin.

hundred thousandth One of 100,000 equal parts.

I

Identity Property of Addition The sum of any number and 0 is that number.

Example: For any number f, $f + 0 = f$.

Identity Property of Multiplication The product of any number and 1 is that number.

Example: For any number f, $f \times 1 = f$.

improper fraction A fraction in which the numerator is greater than or equal to the denominator.

independent event In probability, when the outcome of one event is not dependent upon the outcome of another event.

inequality A relation that is expressed by placing an inequality symbol between two expressions.

Examples: $8 \neq 2$, $5 + 7 > 6 + 4$, $n \leq 7$

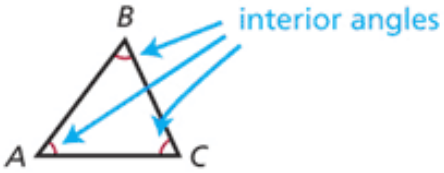
input In a function, a value that is entered into the function to yield an output.

integers The set of positive whole numbers, their opposites (negative numbers), and 0.

Examples: ..., $^{-}3$, $^{-}2$, $^{-}1$, 0, $^{+}1$, $^{+}2$, $^{+}3$, ...

interest rate The rate of interest paid for a given period of time, usually one year.

interior angle of a polygon An angle formed by two sides of a polygon.



interior of an angle The part of the plane between the two rays forming the angle.

intersecting lines Lines that meet or cross at a common point.



interval A measure of space between two or more numbers.

inverse operations Two operations that undo each other, such as addition and subtraction or multiplication and division.

invert To interchange the numerator and the denominator.

irrational numbers Non-terminating, non-repeating decimals that cannot be expressed as the ratio of two integers.

irregular polygon A polygon with one or more sides or angles that are not congruent to the others.

isolate the variable In algebra, to perform operations on an equation until the variable stands alone on one side of the equation.

isosceles triangle A triangle that has at least two congruent sides.



L

leaf The group of digits after the first digit (or stem) in a stem-and-leaf plot.

least common denominator (LCD) For two or more fractions, the least common multiple of the denominators.

least common multiple (LCM) The least number that is a multiple of two or more numbers.

Example: 12 is the least common multiple of 3 and 4.

like denominators In two or more fractions, denominators that are the same.

line A straight, continuous, and unending set of points on a plane.

line graph A graph that uses a broken line to show changes in data. A line graph is often used to display data that vary with time.

line of best fit A line used to show the trend in a scatter plot.

line of symmetry The line along which a figure can be folded so that the two halves match exactly.



line plot A diagram that organizes data using a number line.

line segment A part of a line that has two endpoints.

line symmetry A figure has line symmetry if it can be folded in half and the two halves are congruent.

linear equation An equation that represents the graph of a straight line.

Example: $y = x + 3$

linear function A function whose graph is a straight line.

lower extreme The least value in a box-and-whisker plot.

lower quartile The median of the lower half of a data set in a box-and-whisker plot.

M

mass The amount of matter in an object.

maximum The greatest value in a set of numbers.

mean Arithmetic mean, also called *average*. The number found by dividing the sum of a group of numbers by the number of addends.

measures of central tendency Single numbers that represent the center of a set of numerical data. These include the mean, median, and mode.

median The middle number when a set of numbers is arranged in order from least to greatest.

Examples: The median of 2, 5, 7, 9, and 10 is 7. For an even number of numbers, the median is the average of the two middle numbers. The median of 2, 5, 7, and 12 is $\frac{5+7}{2}$, or 6.

metric system A system of measurement in which the basic units of length, mass, and capacity are the meter, gram, and liter.

midpoint The point that divides the segment into two congruent parts.



millionth One of 1,000,000 equal parts.

minimum value The least value in a set of numbers.

mixed number A number that is the sum of a whole number and a fraction.

mode The number or numbers that occur most often in a set of data.

multiple of a number A number that is the product of a given number and any whole number.

Multiplication Property of Equality Multiplying both sides of an equation by the same nonzero number results in a new equation, having the same solution(s) as the original.

multiplicative inverse Any two numbers whose product is 1. For any nonzero number x, the reciprocal is $\frac{1}{x}$. The multiplicative inverse of a fraction $\frac{a}{b}$ is the fraction $\frac{b}{a}$. Also called the *reciprocal*.

N

negative correlation In a scatter plot, if the y-coordinates decrease as the x-coordinates increase.

negative integer(s) Any integer less than 0.

Examples: $^{-}4$, $^{-}7$, $^{-}100$

negative number Any number that is less than 0.

Examples: $\frac{^{-}7}{8}$, $^{-}5$, and $^{-}2.8$ are negative numbers.

net A flat pattern that can be folded to make a solid.



nonagon A polygon with nine sides.



regular irregular

nonlinear function A function whose graph is not a straight line.

number line A line on which real numbers are assigned to points.

numerator The number above the bar in a fraction.

numerical expression An expression that contains at least one number. It can contain operations and grouping symbols.

O

obtuse angle An angle with a measure greater than that of a right angle and less than 180°.

obtuse triangle A triangle that has one obtuse angle.

octagon A polygon with eight sides.



regular irregular

odd number A whole number that is not a multiple of 2. The ones digit in an odd number is 1, 3, 5, 7, or 9. The numbers 67 and 493 are examples of odd numbers.

odds The comparison between favorable and unfavorable outcomes.

opposite numbers Two numbers that are the same distance from zero but are on opposite sides of zero on a number line.

opposite of a number The same number but with the opposite sign. The opposite of a number is also called its *additive inverse*.

Examples: $^{+}2$ and $^{-}2$, $^{-}7$ and $^{+}7$, and $^{-}12$ and $^{+}12$.

order of operations The order in which operations in an expression to be evaluated are carried out. Begin with the operation in parentheses; next evaluate exponents; then multiply and divide from left to right; and finally add and subtract from left to right.

ordered pair A pair of numbers in which the first number is the x-coordinate and the second number is the y-coordinate.

origin A point assigned to zero on the number line or the point where the x- and y-axis intersect in a coordinate system.

outcome A result of a probability experiment.

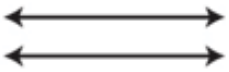
output The amount produced from a specific input.

outlier A number or numbers whose values are much less or much greater than the other numbers in the data set.

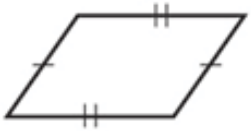
overestimate An estimate that is greater than the exact amount.

P

parallel lines Lines that lie in the same plane and do not intersect. The distance between the lines is everywhere the same.

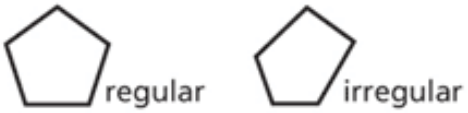


parallelogram A quadrilateral in which both pairs of opposite sides are parallel and opposite angles are congruent.



part of base In a percent problem the part of the problem represented by the percent of the number.

pentagon A polygon with five sides.



percent Per hundred. Ratio of a number to 100.

Example: 7% means 7 out of 100 or $\frac{7}{100}$.

percent of change The percent of increase or decrease of an amount.

perfect square The product of a number and itself.

perimeter The distance around a figure.

period In a multidigit number, a group of three digits set off by commas.

perpendicular Two lines or line segments that intersect or meet to form right angles.

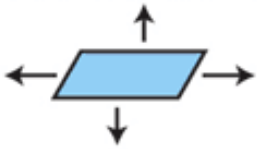


permutation A way of arranging items in a set when the order is important.

pi (π) A number defined by the ratio of the circumference of any circle to its diameter. Two common approximations used for pi are $\frac{22}{7}$ and 3.14.

pictograph A graph that uses pictures or symbols to represent data.

plane A flat surface made up of a continuous and unending collection of points.



point An exact location in space, represented by a dot.

polygon A simple closed plane figure made up of three or more line segments.

polyhedron (polyhedra) A solid figure enclosed by polygons.

population In statistics, all the individuals that fit a given description.

positive correlation In a scatter plot, if the *y*-coordinates increase as the *x*-coordinates increase.

positive integer An integer greater than 0.

positive number A number greater than 0.

power of ten A number that can be written as a product of tens.

precision A term used to refer to the accuracy of a measurement. A smaller unit produces a more precise measurement that a larger unit.

prime factorization Writing a number as a product, using only prime factors.

Example: the prime factorization of 30 is $2 \times 3 \times 5$.

prime number A whole number, greater than 1, that has exactly two factors, 1 and itself.

principal An amount of money invested or borrowed.

prism A solid figure that has two parallel congruent bases and rectangles and parallelograms for faces.



probability The likelihood of an event occurring. A probability can be any number from 0 through 1.

product The answer in a multiplication problem.

proper fraction A fraction in which the numerator is less than the denominator.

Example: $\frac{4}{7}$

proportion An equation showing that two ratios are equal.

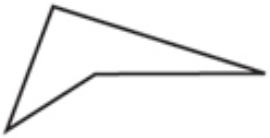
pyramid A solid figure whose base can be any polygon and whose faces are triangles.



Q

quadrant Each of the four regions into which the coordinate plane is separated by the *x*-axis and the *y*-axis. The axes are not part of any quadrant.

quadrilateral A polygon with four sides.



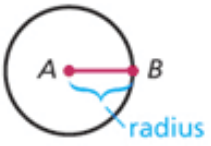
quotient The answer in a division problem.

Example: $45 \div 5 = 9$

quotient

R

radius (radii) A segment that connects the center of a circle to any point on the circle.



random sample In statistics, a sample in which every individual in the population has the same chance of being included.

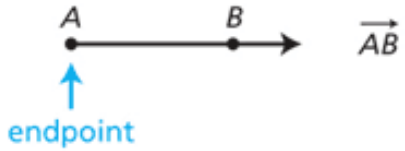
range The difference between the greatest and the least numbers in a set of data.

rate A comparison by division of two quantities measured with different units.

ratio A comparison of two numbers by division.

rational number A number that can be expressed in the form $\frac{a}{b}$, where a and b are any integers and b is not equal to zero.

ray Part of a line that starts at an endpoint and goes on without end in one direction.



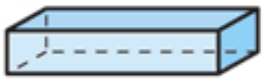
real numbers The set of all rational and irrational numbers.

reciprocal The product of a number and its reciprocal is 1. The reciprocal of a number is also called its *multiplicative inverse*.

Example: $\frac{2}{3} \times \frac{3}{2} = 1$, so $\frac{2}{3}$ and $\frac{3}{2}$ are reciprocals of each other.

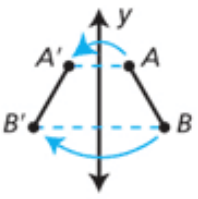
rectangle A polygon with opposite sides parallel and four right angles.

rectangular prism A solid figure with six faces that are rectangles.



rectangular pyramid A solid figure whose base is a rectangle and whose faces are triangles.

reflection A transformation that involves a flipping movement of a figure over a line.



regular polygon A polygon with all sides having the same length and all angles having the same measure.

remainder The number that is left after one whole number is divided by another.

Example: $4\overline{)37}$ R1 (9)

remainder

repeating decimal A decimal quotient whose last digit or block of digits repeat without end.

Examples: 2.4545..., $3.\overline{6}$

representative sample A sample for which the results from the sample can be considered the same as the results from the entire population.

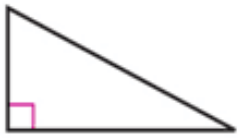
rhombus (rhombi) A parallelogram with all four sides the same length.



right angle An angle that measures 90°.

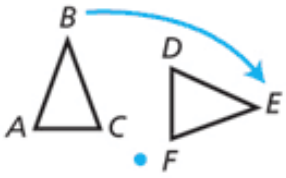


right triangle A triangle that has one right angle.



rise The vertical difference between any two points on a line.

rotation A transformation that involves a turning movement of a figure about a point.



rotational symmetry A figure has rotational symmetry if, after the figure is rotated about a point of less than a full turn, the figure is the same as when in its original position.

rounding To find about how many or how much by expressing a number to the nearest ten, hundred, thousand, and so on.

run The horizontal difference between any two points on a line.

S

sale price The price of an item after the discount is subtracted.

sample In statistics, a smaller group used to gather data about a population.

sample space A list of all possible outcomes.

scale The ratio of the size in a drawing or model to the actual size of an object.

scale drawing When a scale is used to obtain a drawing of an object that is smaller or larger than the actual object.

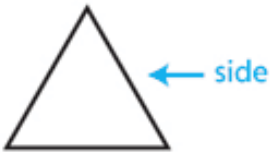
scalene triangle A triangle whose sides are all different lengths.



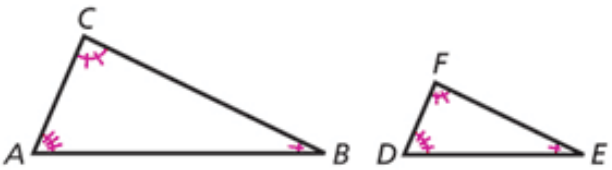
scatter plot A collection of data that shows the correlation, or relationship between two sets of data.

sequence An ordered set of numbers.

side of a polygon One of the line segments that make up a polygon.



similar figures Figures that are the same shape but not necessarily the same size.



simple interest The amount of money a bank pays to use money or the amount paid to borrow money, often calculated once a year.

simplest form (of a fraction) A fraction whose numerator and denominator have the number 1 as the only common factor.

simplest form (of an algebraic expression) An algebraic expression is in simplest form if no terms can be combined.

slope A measure of the slant or steepness of a line. Slope is the ratio of rise to run.

solid figure A three-dimensional figure in space.

solution (to an equation) A number or numbers that, when substituted for the variable or variables in an equation, give a true statement.

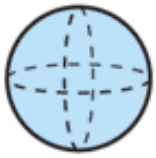
Example: 5 is a solution to the equation $x + 3 = 8$ because $5 + 3 = 8$.

solution to an inequality Any value of a variable that makes an inequality true.

space figure A figure with three dimensions: length, width, and height.

speed A ratio of distance to time.

sphere A solid figure that is shaped like a round ball.



square A polygon with four right angles and four congruent sides.

square root The square root of a number n is a number that when multiplied by itself equals the number n.

square unit A unit used to measure area.

stem The first digit in a stem-and-leaf plot.

stem-and-leaf plot A frequency distribution that arranges data in order of place value. The last digits of each number make up the leaves. The digits to the left of the leaves are the stems.

straight angle An angle whose measure is 180°.

substitute To put in place of.

Subtraction Property of Equality Subtracting the same number from both sides of an equation results in a new equation, having the same solution(s) as the original.

supplementary angles Two angles for which the sum of the angle measures is 180°.



surface area The total area of the surface of a solid.

survey In statistics, a method used to gather information.

symmetric figure A figure that has line or rotational symmetry.

T

ten thousandth One of 10,000 equal parts.

term(s) A number in a sequence.

terminating decimal A decimal quotient that has a limited number of nonzero digits.

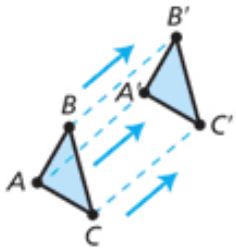
terms of a ratio The numerator and denominator of a ratio expressed in fraction form. The numerator is the first term and the denominator is the second term.

theoretical probability The ratio of the number of outcomes in an event to the total number of outcomes.

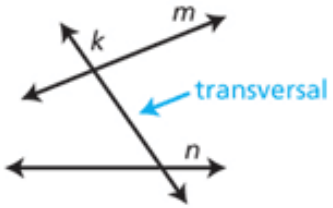
tip A percentage portion of a total bill, customarily left after service.

transformation A geometric change in the position of a figure.

translation A transformation that involves a sliding movement of a figure in any direction.



transversal A line intersecting two other lines.



trapezoid A quadrilateral with exactly one pair of parallel sides.

tree diagram A diagram used to display all possible choices or all possible outcomes of an event.



triangle A polygon with three sides.

triangular prism A prism whose bases are triangles.

triangular pyramid A pyramid whose base is a triangle.

two-variable equation An equation that has two different variables.

U

undecagon A polygon with 11 sides.



regular irregular

underestimate An estimate that is less than the exact amount.

unit cost The cost of a single item.

unit cube A cube with an edge length of 1.

unit fraction A fraction in which the numerator is 1.

Example: $\frac{1}{5}$

unit rate A rate with the denominator 1.

upper extreme The greatest value in a whisker in a box-and-whisker plot.

upper quartile The median of the upper half of a data set in a box-and-whisker plot.

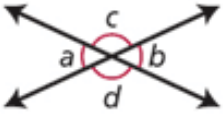
V

variable In an algebraic expression, a letter or a symbol that represents a number.

vertex A point common to two sides of an angle, polygon, or solid figure.



vertical angles Angles formed by intersecting lines. Vertical angles are always congruent.



$\angle a$ and $\angle b$
$\angle c$ and $\angle d$

vertical axis The *y*-axis in the coordinate system. It is a number line used to locate points above or below the origin.

volume The number of cubic units that make up a solid figure.

W

weight The measure of how heavy an object is.

X

***x*-axis** The horizontal number line in a coordinate system.

***x*-coordinate** The first number of an ordered pair of numbers that names a point in a coordinate system.

***x*-intercept** The point on a graph where the line crosses the *x*-axis.

Y

***y*-axis** The vertical number line in a coordinate system.

***y*-coordinate** The second number of an ordered pair of numbers that names a point in a coordinate system.

***y*-intercept** The point on a graph where the line crosses the *y*-axis.

Z

Zero Property of Multiplication The product of any number and 0 is 0.

Example: For any number g, $g \times 0 = 0$.

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