



Table of Contents

GRADE K • MODULE 1

Numbers to 10

Module Overview	2
Topic A: Attributes of Two Related Objects	16
Topic B: Classify to Make Categories and Count	43
Topic C: Numbers to 5 in Different Configurations, Math Drawings, and Expressions	69
Topic D: The Concept of Zero and Working with Numbers 0–5	120
Mid-Module Assessment and Rubric	163
Topic E: Working with Numbers 6–8 in Different Configurations	175
Topic F: Working with Numbers 9–10 in Different Configurations	230
Topic G: <i>One More</i> with Numbers 0–10	284
Topic H: <i>One Less</i> with Numbers 0–10	324
End-of-Module Assessments and Rubric	366
Answer Key	375

Grade K • Module 1

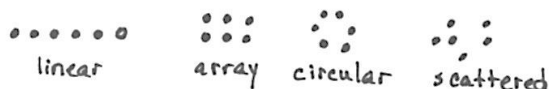
Numbers to 10

OVERVIEW

The first day of Kindergarten is long anticipated by parents and young students. Students expect school to be a dynamic and safe place to learn, an objective that is realized immediately by their involvement in purposeful and meaningful action.

In Topics A and B, classification activities allow students to analyze and observe their world and articulate their observations. Reasoning and dialogue begin immediately. “These balloons are exactly the same.” “These are the same but a different size.” As Topic B closes, students recognize cardinalities as yet one more lens for classification (**K.MD.3**). “I put a pencil, a book, and an eraser, three things, in the backpack for school.” “I put five toys in the closet to keep at home.” From the moment students enter school, they practice the counting sequence so that when counting a set of objects, their attention can be on matching one count to one object, rather than on retrieving the number words (**K.CC.4a**).

In Topics C, D, E, and F, students order, count (**K.CC.1**), and write (**K.CC.3**) up to ten objects to answer *how many* questions from linear, to array, to circular, and finally to scattered configurations wherein they must devise a path through the objects as they count. Students use their understanding of numbers and matching numbers with objects to answer *how many* questions about a variety of objects, pictures, and drawings (**K.CC.5**).



They learn that the last number name said tells the number of objects counted (**K.CC.4b**). Daily, they engage in mathematical dialogue. They might compare their seven objects to a friend’s. For example, “My cotton balls are bigger than your cubes, but when we count them, we both have seven!”

Very basic expressions and equations are introduced early in order to ensure students’ familiarity with numbers throughout the entire year so that they exit fluent in sums and differences to 5 (**K.OA.5**). Decomposition is modeled with small numbers with materials and drawings and as addition equations. Students see that both the expression $2 + 1$ (Topic C) and the equation $3 = 2 + 1$ (Topic D) describe a stick of three cubes decomposed into two parts (**K.OA.3**). Emphasis is not placed on the expressions and equations or using them in isolation from the concrete and pictorial—they are simply included to show another representation of decompositions alongside counters and drawings.

In Topics G and H, students use their understanding of relationships between numbers to recognize that each successive number name refers to a quantity that is one greater and that the number before is one less (**K.CC.4c**). This important insight leads students to use the Level 2 strategy of counting on rather than counting all, later in the year and on into Grade 1.



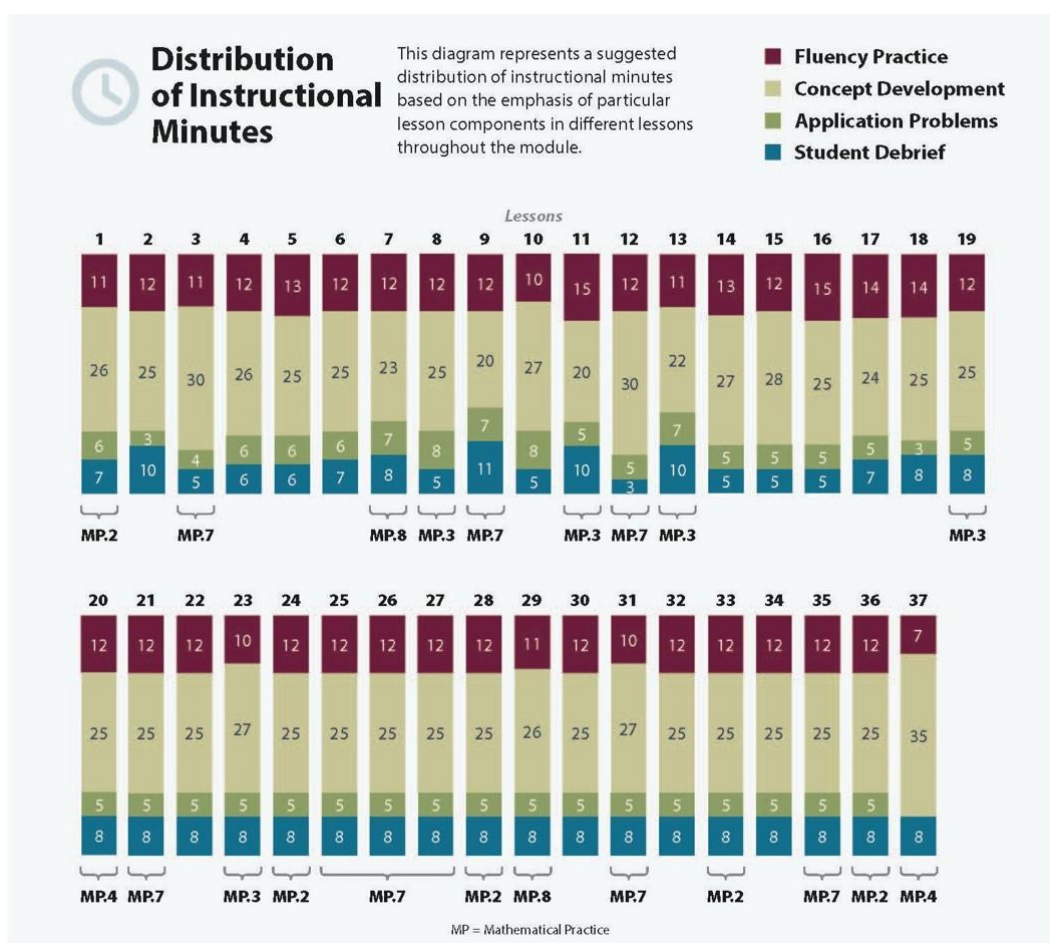
In this module, daily fluency activities with concentration and emphasis on counting (**K.CC.4ab**, **K.CC.5**) are integrated throughout the concept development: “I counted six beans in a row. I counted six beans in a circle and then squished them together and counted again. There were still six!” “I can make my six beans into rows, and there are no extras.” Students complete units of five using the fingers of their left hand and 5-groups. The numbers 6, 7, 8, and 9 are introduced relative to the number 5: “Five fingers and ____ more.” Students also explore numbers 5 to 9 in relation to 10, or two complete fives: “Nine is missing one to be ten or two fives.” (**K.OA.4**)

As students begin to master writing numbers to 10, they practice with paper and pencil. This is a critical daily fluency that may work well to close lessons, since management of young students is generally harder toward the end of math time. The paper and pencil work is calming, though energized.

Notes on Pacing for Differentiation

If pacing is a challenge, consider the following modifications. Consider consolidating Lessons 1 and 2 if students are competent in recognizing and discussing subtle differences in the attributes of objects.

Lessons 12, 13, 15, 18, 20, 22, 24, and 26 include numeral formation along with counting and cardinality concepts. In prioritizing a focus within each lesson (e.g., if reducing the instructional time for numeral writing), take care not to inadvertently omit the teaching of math concepts within the same lesson (e.g., cardinality, conservation, and counting in varied configurations).



Focus Grade Level Standards¹

Know number names and the count sequence.²

- K.CC.3** Write numbers from 0 to 20. Represent a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).

Count to tell the number of objects.³

- K.CC.4** Understand the relationship between numbers and quantities; connect counting to cardinality.
- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
 - Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
 - Understand that each successive number name refers to a quantity that is one larger.
- K.CC.5** Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.⁴

- K.OA.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., $5 = 2 + 3$ and $5 = 4 + 1$).

Classify objects and count the number of objects in each category.

- K.MD.3** Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. (Limit category counts to be less than or equal to 10.)

¹In this module work is limited to within 10.

²The balance of this cluster is addressed in Module 5.

³K.CC.4d is addressed in Module 6.

⁴The balance of this cluster is addressed in Module 4.

Foundational Standards

- PK.CC.1** Count to 20.
- PK.CC.2** Represent a number of objects with a written numeral 0–5 (with 0 representing a count of no objects).
- PK.CC.3** Understand the relationship between numbers and quantities to 10; connect counting to cardinality.
- When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
 - Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
 - Understand that each successive number name refers to a quantity that is one larger.
- PK.CC.4** Count to answer “how many?” questions about as many as 10 things arranged in a line, a rectangular array, or a circle, or as many as 5 things in a scattered configuration; given a number from 1–10, count out that many objects.
- PK.CC.6** Identify “first” and “last” related to order or position.

Focus Standards for Mathematical Practice

- MP.2 Reason abstractly and quantitatively.** Students represent quantities with numerals.
- MP.3 Construct viable arguments and critique the reasoning of others.** Students reason about each other’s ways of counting fingers or a scattered set of objects. They reason about counting fingers by comparing the fingers counted and about scattered objects by comparing counting paths through a set of up to 10 scattered objects.
- MP.4 Model with mathematics.** Students model decompositions of three objects as math drawings and addition equations.
- MP.7 Look for and make use of structure.** Students use the 5-group to reason about numbers within 10.
- MP.8 Look for and express regularity in repeated reasoning.** Students build a number stair to reason about 1 more and 1 less than each number within 10.

Overview of Module Topics and Lesson Objectives

Standards	Topics and Objectives		Days
K.MD.3	A	Attributes of Two Related Objects Lesson 1: Analyze to find two objects that are <i>exactly the same</i> or <i>not exactly the same</i> . Lesson 2: Analyze to find two similar objects— <i>these are the same but....</i> Lesson 3: Classify to find two objects that share a visual pattern, color, and use.	3
K.CC.4a K.CC.4b K.MD.3	B	Classify to Make Categories and Count Lesson 4: Classify items into two pre-determined categories. Lesson 5: Classify items into three categories, determine the count in each, and reason about how the last number named determines the total. Lesson 6: Sort categories by count. Identify categories with 2, 3, and 4 within a given scenario.	3
K.CC.4a K.CC.4b K.CC.5 K.OA.3 K.MD.3	C	Numbers to 5 in Different Configurations, Math Drawings, and Expressions Lesson 7: Sort by count in vertical columns and horizontal rows (linear configurations to 5). Match to numerals on cards. Lesson 8: Answer <i>how many</i> questions to 5 in linear configurations (5-group), with 4 in an array configuration. Compare ways to count five fingers. Lesson 9: Within linear and array dot configurations of numbers 3, 4, and 5, find <i>hidden partners</i> . Lesson 10: Within circular and scattered dot configurations of numbers 3, 4, and 5, find <i>hidden partners</i> . Lesson 11: Model decompositions of 3 with materials, drawings, and expressions. Represent the decomposition as $1 + 2$ and $2 + 1$.	5



Standards	Topics and Objectives	Days
K.CC.3 K.CC.4a K.CC.4b K.CC.5	D The Concept of Zero and Working with Numbers 0–5 Lesson 12: Understand the meaning of zero. Write the numeral 0. Lesson 13: Order and write numerals 0–3 to answer <i>how many</i> questions. Lesson 14: Write numerals 1–3. Represent decompositions with materials, drawings, and equations, $3 = 2 + 1$ and $3 = 1 + 2$. Lesson 15: Order and write numerals 4 and 5 to answer <i>how many</i> questions in categories; sort by count. Lesson 16: Write numerals 1–5 in order. Answer and make drawings of decompositions with totals of 4 and 5 without equations.	5
	Mid-Module Assessment: Topics A–D (Interview style assessment: 3 days)	3
K.CC.3 K.CC.4a K.CC.4b K.CC.5 K.MD.3	E Working with Numbers 6–8 in Different Configurations Lesson 17: Count 4–6 objects in vertical and horizontal linear configurations and array configurations. Match 6 objects to the numeral 6. Lesson 18: Count 4–6 objects in circular and scattered configurations. Count 6 items out of a larger set. Write numerals 1–6 in order. Lesson 19: Count 5–7 linking cubes in linear configurations. Match with numeral 7. Count on fingers from 1 to 7, and connect to 5-group images. Lesson 20: Reason about sets of 7 varied objects in circular and scattered configurations. Find a path through the scattered configuration. Write numeral 7. Ask, “How is your seven different than mine?” Lesson 21: Compare counts of 8. Match with numeral 8. Lesson 22: Arrange and strategize to count 8 beans in circular (around a cup) and scattered configurations. Write numeral 8. Find a path through the scattered set, and compare paths with a partner.	6
K.CC.3 K.CC.4a K.CC.4b K.CC.5	F Working with Numbers 9–10 in Different Configurations Lesson 23: Organize and count 9 varied geometric objects in linear and array (3 threes) configurations. Place objects on 5-group mat. Match with numeral 9. Lesson 24: Strategize to count 9 objects in circular (around a paper plate) and scattered configurations printed on paper. Write numeral 9. Represent a path through the scatter count with a pencil. Number each object.	6

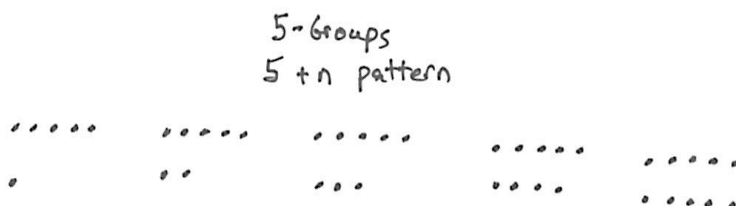


Standards	Topics and Objectives	Days
	<p>Lessons 25–26: Count 10 objects in linear and array configurations (2 fives). Match with numeral 10. Place on the 5-group mat. Dialogue about 9 and 10. Write numeral 10.</p> <p>Lesson 27: Count 10 objects, and move between all configurations.</p> <p>Lesson 28: Act out <i>result unknown</i> story problems without equations.</p>	
K.CC.4a K.CC.4b K.CC.4c K.CC.2 K.CC.5	G One More with Numbers 0–10 <p>Lesson 29: Order and match numeral and dot cards from 1 to 10. State 1 more than a given number.</p> <p>Lesson 30: Make math stairs from 1 to 10 in cooperative groups.</p> <p>Lesson 31: Arrange, analyze, and draw 1 more up to 10 in configurations other than towers.</p> <p>Lesson 32: Arrange, analyze, and draw sequences of quantities of 1 more, beginning with numbers other than 1.</p>	4
K.CC.4a K.CC.4b K.CC.4c K.CC.5	H One Less with Numbers 0–10 <p>Lesson 33: Order quantities from 10 to 1, and match numerals.</p> <p>Lesson 34: Count down from 10 to 1, and state 1 less than a given number.</p> <p>Lesson 35: Arrange number towers in order from 10 to 1, and describe the pattern.</p> <p>Lesson 36: Arrange, analyze, and draw sequences of quantities that are 1 less in configurations other than towers.</p> <p>Lesson 37: Culminating task</p> <p><i>Decide how to classify the objects in your bag into two groups. Count the number of objects in each group. Represent the greater number in various ways. Next, remove the card from your pack that shows the number of objects in the smaller group. Put your remaining cards in order from smallest to greatest. Your friends will have to figure out what card is missing when they visit your station!</i></p>	5
	End-of-Module Assessment: Topics E–H (Interview style assessment: 3 days)	3
Total Number of Instructional Days		43

Terminology

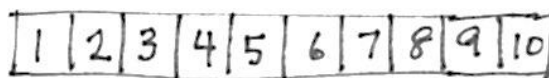
New or Recently Introduced Terms

- Exactly the same, not exactly the same, and the same, but... (ways to analyze objects to match or sort)
- Match (group items that are the same or that have the same given attribute)
- Sort (group objects according to a particular attribute)
- How many? (with reference to counting quantities or sets)
- Hidden partners (embedded numbers)
- Counting path (with reference to order of count)
- Number story (stories with *add to* or *take from* situations)
- Zero (understand the meaning of, write, and recognize)
- Number sentence ($3 = 2 + 1$)
- 5-group (pictured right)
- Rows and columns (linear configuration types)
- Number path
- 1 more (e.g., 4. 1 more is 5.)
- 1 less (e.g., 4. 1 less is 3.)



Suggested Tools and Representations

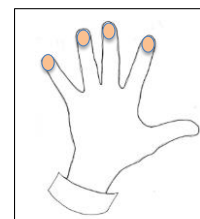
- Rulers for use as a straightedge
- Five dot mat
- Five-frame and ten-frame cards
- Number path
- Left hand mat
- Two hands mat
- 5-group cards
- Rekenrek (Slavonic abacus having beads with a color change at the five)
- Concrete materials in individual bags for counting and sorting (white beans painted red on one side, bags of twigs, dried leaves, dry pasta, pennies, plates, forks, spoons, cups, etc.)
- Commercial concrete materials (linking cubes in tens, non-linking cubes, square-inch tiles, etc.)



Number Path



20-Bead Rekenrek



Left Hand Mat



100-Bead Rekenrek

Suggested Methods of Instructional Delivery

Personal White Boards

Materials Needed for Personal White Boards

- 1 heavy duty, clear sheet protector
- 1 piece of stiff red tag board $11" \times 8 \frac{1}{4}"$
- 1 piece of stiff white tag board $11" \times 8 \frac{1}{4}"$
- 1 $3" \times 3"$ piece of dark synthetic cloth for an eraser (e.g., felt)
- 1 low odor blue dry erase marker, fine point

Directions for Creating Personal White Boards

Cut the white and red tag to specifications. Slide into the sheet protector. Store the eraser on the red side. Store markers in a separate container to avoid stretching the sheet protector.

Frequently Asked Questions About Personal White Boards

Why is one side red and one white?

- The white side of the board is the “paper.” Students generally write on it, and if working individually, turn the board over to signal to the teacher they have completed their work. The teacher then says, “Show me your boards,” when most of the class is ready.

What are some of the benefits of a personal white board?

- The teacher can respond quickly to gaps in student understandings and skills. “Let’s do some of these on our personal white boards until we have more mastery.”
- Students can erase quickly so that they do not have to suffer the evidence of their mistake.
- They are motivating. Students love both the drill and thrill capability and the chance to do story problems with an engaging medium.
- Checking work gives the teacher instant feedback about student understanding.

What is the benefit of this personal white board over a commercially purchased dry erase board?

- It is much less expensive.
- Templates such as place value charts, number bond mats, hundreds boards, and number lines can be stored between the two pieces of tag board for easy access and reuse.
- Worksheets, story problems, and other problem sets can be done without marking the paper so that students can work on the problems independently at another time.
- Strips with story problems, number lines, and arrays can be inserted and still have a full piece of paper on which to write.
- The red versus white side distinction clarifies expectations. When working collaboratively, there is no need to use the red. When working independently, the students know how to keep their work private.
- The tag board can be removed if necessary to project the work.

Homework

Homework at the K–1 level is not a convention in all schools. In this curriculum, homework is an opportunity for additional practice of the content from the day's lesson. The teacher is encouraged, with the support of parents, administrators, and colleagues, to discern the appropriate use of homework for his students. Fluency exercises can also be considered as an alternative homework assignment.

Scaffolds⁵

The scaffolds integrated into *A Story of Units* give alternatives for how students access information as well as express and demonstrate their learning. Strategically placed margin notes are provided within each lesson elaborating on the use of specific scaffolds at applicable times. They address many needs presented by English language learners, students with disabilities, students performing above grade level, and students performing below grade level. Many of the suggestions are organized by Universal Design for Learning (UDL) principles and are applicable to more than one population. To read more about the approach to differentiated instruction in *A Story of Units*, please refer to “How to Implement *A Story of Units*.”

⁵Students with disabilities may require Braille, large print, audio, or special digital files. Please visit the website www.p12.nysed.gov/specialed/aim for specific information on how to obtain student materials that satisfy the National Instructional Materials Accessibility Standard (NIMAS) format.

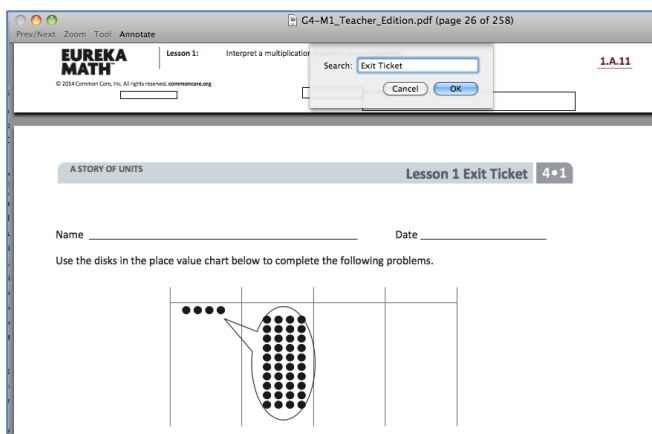
Preparing to Teach a Module

Preparation of lessons will be more effective and efficient if there has been an adequate analysis of the module first. Each module in *A Story of Units* can be compared to a chapter in a book. How is the module moving the plot, the mathematics, forward? What new learning is taking place? How are the topics and objectives building on one another? The following is a suggested process for preparing to teach a module.

Step 1: Get a preview of the plot.

- A: Read the Table of Contents. At a high level, what is the plot of the module? How does the story develop across the topics?
- B: Preview the module's Exit Tickets⁶ to see the trajectory of the module's mathematics and the nature of the work students are expected to be able to do.

Note: When studying a PDF file, enter "Exit Ticket" into the search feature to navigate from one Exit Ticket to the next.



Step 2: Dig into the details.

- A: Dig into a careful reading of the Module Overview. While reading the narrative, *liberally* reference the lessons and Topic Overviews to clarify the meaning of the text—the lessons demonstrate the strategies, show how to use the models, clarify vocabulary, and build understanding of concepts. Consider searching the video gallery on *Eureka Math's* website to watch demonstrations of the use of models and other teaching techniques.
- B: Having thoroughly investigated the Module Overview, read through the chart entitled Overview of Module Topics and Lesson Objectives to further discern the plot of the module. How do the topics flow and tell a coherent story? How do the objectives move from simple to complex?

Step 3: Summarize the story.

Complete the Mid- and End-of-Module Assessments. Use the strategies and models presented in the module to explain the thinking involved. Again, liberally reference the work done in the lessons to see how students who are learning with the curriculum might respond.

⁶A more in-depth preview can be done by searching the Problem Sets rather than the Exit Tickets. Furthermore, this same process can be used to preview the coherence or flow of any component of the curriculum, such as Fluency Practice or Application Problems.

Preparing to Teach a Lesson

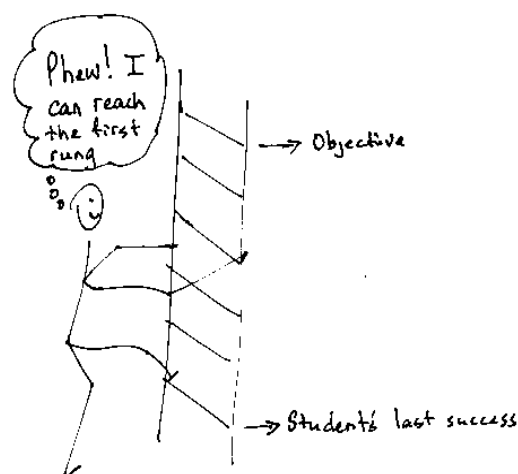
A three-step process is suggested to prepare a lesson. It is understood that at times teachers may need to make adjustments (customizations) to lessons to fit the time constraints and unique needs of their students. The recommended planning process is outlined below. Note: The ladder of Step 2 is a metaphor for the teaching sequence. The sequence can be seen not only at the macro level in the role that this lesson plays in the overall story, but also at the lesson level, where each rung in the ladder represents the next step in understanding or the next skill needed to reach the objective. To reach the objective, or the top of the ladder, all students must be able to access the first rung and each successive rung.

Step 1: Discern the plot.

- A: Briefly review the module's Table of Contents, recalling the overall story of the module and analyzing the role of this lesson in the module.
- B: Read the Topic Overview related to the lesson, and then review the Problem Set and Exit Ticket of each lesson in the topic.
- C: Review the assessment following the topic, keeping in mind that assessments can be found midway through the module and at the end of the module.

Step 2: Find the ladder.

- A: Complete the lesson's Problem Set.
- B: Analyze and write notes on the new complexities of each problem as well as the sequences and progressions throughout problems (e.g., pictorial to abstract, smaller to larger numbers, single- to multi-step problems). The new complexities are the rungs of the ladder.
- C: Anticipate where students might struggle, and write a note about the potential cause of the struggle.
- D: Answer the Student Debrief questions, always anticipating how students will respond.



Step 3: Hone the lesson.

At times, the lesson and Problem Set are appropriate for all students and the day's schedule. At others, they may need customizing. If the decision is to customize based on either the needs of students or scheduling constraints, a suggestion is to decide upon and designate "Must Do" and "Could Do" problems.

- A: Select "Must Do" problems from the Problem Set that meet the objective and provide a coherent experience for students; reference the ladder. The expectation is that the majority of the class will complete the "Must Do" problems within the allocated time. While choosing the "Must Do" problems, keep in mind the need for a balance of calculations, various word problem types⁷, and work at both the pictorial and abstract levels.

⁷See the Progression Documents "K, Counting and Cardinality" (pp. 9) and "K-5, Operations and Algebraic Thinking" (pp. 23).

- B: “Must Do” problems might also include remedial work as necessary for the whole class, a small group, or individual students. Depending on anticipated difficulties, those problems might take different forms as shown in the chart below.

Anticipated Difficulty	“Must Do” Remedial Problem Suggestion
The first problem of the Problem Set is too challenging.	Write a short sequence of problems on the board that provides a ladder to Problem 1. Direct the class or small group to complete those first problems to empower them to begin the Problem Set. Consider labeling these problems “Zero Problems” since they are done prior to Problem 1.
There is too big of a jump in complexity between two problems.	Provide a problem or set of problems that creates a bridge between the two problems. Label them with the number of the problem they follow. For example, if the challenging jump is between Problems 2 and 3, consider labeling the bridging problems “Extra 2s.”
Students lack fluency or foundational skills necessary for the lesson.	Before beginning the Problem Set, do a quick, engaging fluency exercise, such as a Rapid White Board Exchange, “Thrilling Drill,” or Sprint. Before beginning any fluency activity for the first time, assess that students are poised for success with the easiest problem in the set.
More work is needed at the concrete or pictorial level.	Provide manipulatives or the opportunity to draw solution strategies. Especially in Kindergarten, at times the Problem Set or pencil and paper aspect might be completely excluded, allowing students to simply work with materials.
More work is needed at the abstract level.	Hone the Problem Set to reduce the amount of drawing as appropriate for certain students or the whole class.

- C: “Could Do” problems are for students who work with greater fluency and understanding and can, therefore, complete more work within a given time frame. Adjust the Exit Ticket and Homework to reflect the “Must Do” problems or to address scheduling constraints.
- D: At times, a particularly tricky problem might be designated as a “Challenge!” problem. This can be motivating, especially for advanced students. Consider creating the opportunity for students to share their “Challenge!” solutions with the class at a weekly session or on video.
- E: Consider how to best use the vignettes of the Concept Development section of the lesson. Read through the vignettes, and highlight selected parts to be included in the delivery of instruction so that students can be independently successful on the assigned task.
- F: Pay close attention to the questions chosen for the Student Debrief. Regularly ask students, “What was the lesson’s learning goal today?” Help them articulate the goal.

Assessment Summary

Assessment Type	Administered	Format	Standards Addressed
Mid-Module Assessment Task	After Topic D	Interview with Rubric	(Numbers 1–5) K.CC.3 K.CC.4ab K.CC.5 K.OA.3 K.MD.3
End-of-Module Assessment Task	After Topic H	Interview with Rubric	(Numbers 0–10) K.CC.3 K.CC.4abc K.CC.5
Culminating Task	Lesson 37	<i>Decide how to classify the objects in your bag into two groups. Count the number of objects in each group. Represent the greater number in various ways. Next, remove the 5-group card from your pack that shows the number of objects in the smaller group. Put your remaining cards in order from smallest to greatest. Your friends will have to figure out what card is missing when they visit your station!</i>	K.CC.3 K.CC.4abc K.CC.5 K.MD.3