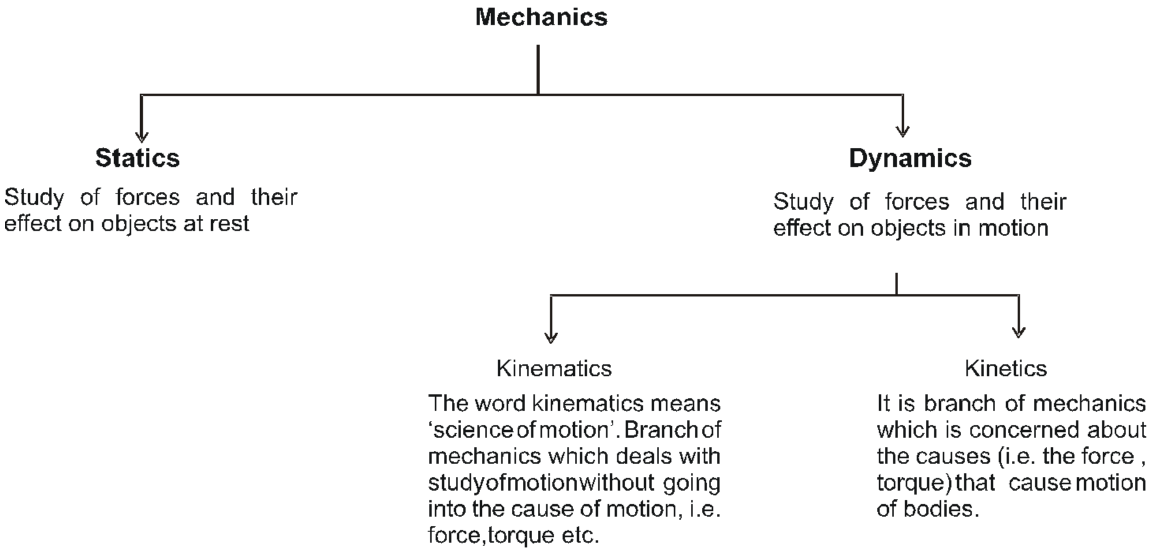
INFINITY NOTES (RECTILINEAR MOTION)

MECHANICS

Mechanics is the branch of physics which deals with the cause and effects of motion of a particle, rigid objects and deformable bodies etc. Mechanics is classified under two streams namely Statics and Dynamics. Dynamics is further divided into Kinematics and Kinetics.



1. MOTION AND REST

Motion is a combined property of the object and the observer. There is no meaning of rest or motion without the observer. Nothing is in absolute rest or in absolute motion.

An object is said to be in motion with respect to a observer, if its position changes with respect to that observer. It may happen by both ways either observer moves or object moves.

2. RECTILINEAR MOTION

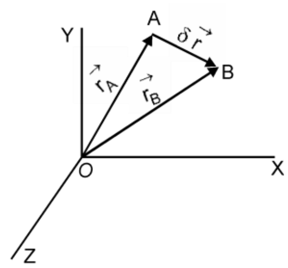
Rectilinear motion is motion, along a straight line or in one dimension. It deals with the kinematics of a particle in one dimension.

For example the motion of an ant on a wire is a rectilinear motion.

2.1 Position : The position of a particle refers to its location in the space at a certain moment of time. It is concerned with the question - *"where is the particle at a particular moment of time?"*

2.2 Displacement : The shortest distance from the initial position to the final position of the particle is called displacement. The displacement of a particle is measured as the change in the position of the particle in a particular direction over a given time interval. It depends only on final & initial positions.

Displacement of a particle is a position vector of its final position w.r.t. initial position.



Position vector of A w.r.t. $O = \overrightarrow{OA}$

$\Rightarrow \quad \vec{r}_A = x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}$

Position vector of B w.r.t. $O = \overrightarrow{OB}$

Displacement = $\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

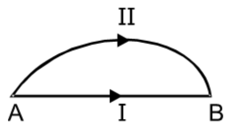
Characteristics of Displacement:

(i) It is a vector quantity.
(ii) The displacement of a particle between any two points is equal to the shortest distance between them.
(iii) The displacement of an object in a given time interval can be positive, negative or zero.
(iv) Dimension : $M^0L^1T^0$
(v) Unit : In C.G.S. centimeter (cm), In S.I. system meter (m).

2.3 **Distance :** The length of the actual path between initial & final positions of a particle in a given interval of time is called distance covered by the particle. Distance is the actual length of the path. It is the characteristic property of any path ie., path is always associated when we consider distance between two positions.

Distance beteen A & B while moving through path (1) may or may not be equal to the distance between A & B while moving through path (2)

Characteristics of Distance :



(i) It is a scalar quantity.
(ii) It depends on the path.
(iii) It never reduces with time.
(iv) Distance covered by a particle is always positive & can never be negative or zero.
(v) Dimension : $[M^0L_1T^0]$
(vi) Unit : In C.G.S.system centimeter (cm), In S.I. system metre (m).

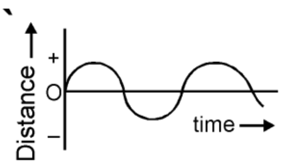
COMPARATIVE STUDY OF DISPLACEMENT AND DISTANCE

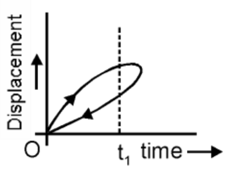
S.No.	Displacement	Distance
1.	It has single value bewteen two points	It may have more than one value between two points
2.	$0 \leq$ Displacement ≤ 0	Distance > 0
3.	Displacement can decrease with time	It can never decrease with time.
4.	It is a vector quantity	It is a scalar quantity

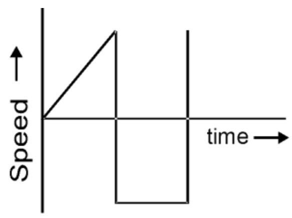
Special Note

1. The actual disatnce travelled by a particle in the given interval of time is always equal to or greater than the magnitude of the displacement and in no case, it is less than the magnitude of the displacement, i.e., Distance $\geq$ | Displacement |
2. Displacement may be + ve, – ve or zero.
3. Distance, speed and time can never be negative.
4. At the same time particle cannot have two positions.

SOME IMPOSSIBLE GRAPHS :

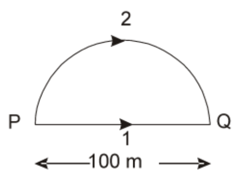






Solved Examples

Example-1. Ram takes path 1 (straight line) to go from P to Q and Shyam takes path 2 (semicircle).
(a) Find the distance travelled by Ram and Shyam?
(b) Find the displacement of Ram and Shyam?



Solution. (a) Distance travelled by Ram = 100 m
Distance travelled by Shyam = π(50 m) = 50π m
(b) Displacement of Ram = 100 m
Displacement of Shyam = 100 m

Example-2. An old person moves on a semi circular track of radius 40 m during a morning walk. If he starts at one end of the track and reaches at the other end. Find the displacement of the person.

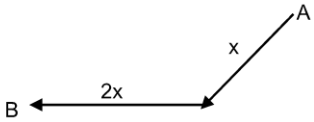
Solution. Displacement = 2R = 2 x 40 = 80 meter.

Example-3. An athelete is running on a circular track of radius 50 meter. Calculate the displacement of the athlete after completing 5 rounds of the track.

Solution. Since final and initial positions are same .

Hence displacement of athlete will be $\Delta r = r - r = 0$

Example-4. If a particle moves from point A to B then distance covered by particle will be.



Solution. D = x + 2x = 3x

Example-5. A monkey is moving on circular path of radius 80 m. Calcualte the distance covered by the monkey in a complete cycle.

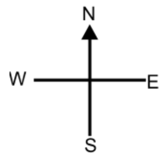
Solution. Distance = Circumference of the circle

$D = 2\pi R \quad \Rightarrow \quad D = 2\pi \times 80 \quad = 160 \times 3.14 \quad = 502.40\text{m}$

Example-6. A man has to go 50 m due north, 40 m due east and 20 m due south to reach a field.

(a) What distance he has to walk to reach the field?

(b) What is his displacement from his house to the field?

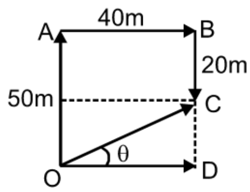


Solution. Let origin be O then

(a) Distance covered = OA + AB + BC

= 50 + 40 + 20

= 110 meter



(b) **First method :**

Displacement OC = $\sqrt{OD^2 + CO^2}$

= $\sqrt{40^2 + 30^2}$

= $10\sqrt{25} = 50$ meter

Second method :

Displacement $\vec{d} = 50\hat{j} + 40\hat{i} - 20\hat{j}$

= $30\,\hat{j} + 40\,\hat{i}$

$|\vec{d}| = \sqrt{40^2 + 30^2}$ = 50 meter

Example-7. A body covers $\frac{1}{4}$ th part of a circular path. Calulate the ratio of distance and displacement.

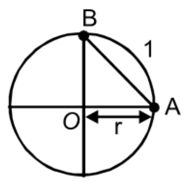
Solution. Distance = AB from path (1)

= $\frac{2\pi r}{4} = \frac{\pi r}{2}$

Displacement = AB

= $\sqrt{OA^2 + OB^2}$

= $\sqrt{r^2 + r^2} = r\sqrt{2}$



$\therefore \frac{\text{Distance}}{\text{Displacement}} = \frac{\pi r / 2}{r\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$

2.4 Speed :

Speed of an object is defined as the time rate of change of position of the object in any direction. It is measured by the distance travelled by the object in unit time in any direction. i.e.,

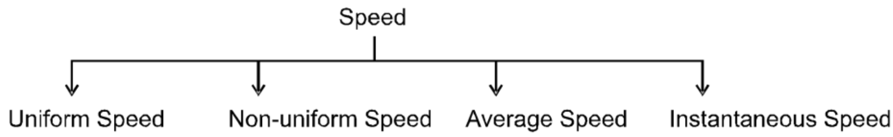
$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

(i) It is a scalar quantity.
(ii) It gives no idea about the direction of motion of the object.
(iii) It can be zero or positive but never negative.
(iv) Unit: C.G.S. cm/sec, S.I. m/sec,

$$1\text{km / h} = \frac{100}{60 \times 60} = \frac{5}{18}\text{m/s} \quad \Rightarrow \quad 1\text{ km/h} = \frac{5}{18}\text{m/s}$$

(v) Dimension : $M^0 L^1 T^{-1}$

Types of speed :



(a) **Uniform speed :** An object is said to be moving with a uniform speed, if it covers equal distances in equal intervals of time, howsoever small these intervals may be. The uniform speed is shown by straight line in distance time graph.

For example: suppose a train travels 1000 metre in 60 second. The train is said to be moving with uniform speed, if it travels 500 metre in 30 second, 250 metre in 15 second, 125 metre in 7.5 second and so on.

(b) **Non Uniform Speed :** An object is said to be moving with a variable speed if it covers equal distance in unequal intervals of time or unequal distances in equal intervals of time, howsoever small these intervals may be.

For example : suppose a train travels first 1000 metre in 60 second, next 1000 metre in 120 second and next 1000 metre in 50 second, then the train is moving with variable speed.

(c) **Average Speed :** When an object is moving with a variable speed, then the average speed of the object is that constant speed with which the object covers the same distance in a given time as it does while moving with variable speed during the given time. Average speed for the given motion is defined as the ratio of the total distance travelled by the object to the total time taken i.e.,

$$\text{Average speed } \bar{V} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Note : If any car covers distance x_1, x_2, ... in the time intervals t_1, t_2, ... then.

$$\bar{V} = \frac{x_1 + x_2 + x_3 + ... + x_n}{t_1 + t_2 + ... + t_n}$$

SOME IMPORTANT CASES RELATED TO AVERAGE SPEED :

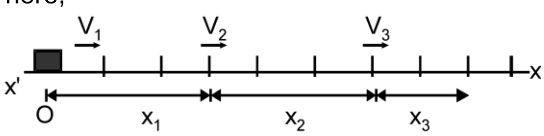
Case : 1

If body covers distances x_1, x_2, and x_3 with speeds v_1, v_2, and v_3 respectively in same direction then average speed of body.

$$\Rightarrow \quad \bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3}$$

here, $t_1 = \frac{x_1}{v_1}, t_2 = \frac{x_2}{v_2}, t_3 = \frac{x_3}{v_3}$

$$\bar{V} = \frac{x_1 + x_2 + x_3}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3}}$$



If body covers equal distances with different speeds then, $x_1 = x_2 = x_3 = x$

$$\bar{V} = \frac{3x}{\frac{x}{v_1} + \frac{x}{v_2} + \frac{x}{v_3}} = \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}} = \frac{3v_1v_2v_3}{v_1v_2 + v_2v_3 + v_3v_1}$$

Case : 2

If any body travels with speeds v_1, v_2, v_3 during time intervals t_1, t_2, t_3 respectively then the average speed of the body wil be

Average speed $\bar{V} = \frac{x_1 + x_2 + x_3}{t_1 + t_2 + t_3} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3}{t_1 + t_2 + t_3}$ If $t_1 = t_2 = t_3 = t$

$$= \frac{(v_1 + v_2 + v_3) \times t}{3 \times t} = \frac{(v_1 + v_2 + v_3)}{3}$$

(d) Instantaneous speed :

The speed of the body at any instant of time or at a particular position is called instantaneous speed.

Let a body travel a distance Δx in the time interval Δt, then its average speed $= \frac{\Delta x}{\Delta t}$.

When $\Delta t \to 0$, then average speed of the body becomes the instantaneous speed.

$\therefore$ Instantaneous speed $= \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

2.5 Velocity

It is defined as rate of change of displacement.

Characteristics of Velocity :

(i) It is a vector quantity.

(ii) Its direction is same as that of displacement.

(iii) Unit and dimension : Same as that of speed.

Types of Velcoity :

(a) Instantaneous Velocity (b) Average Velocity

(c) Uniform Velocity (d) Non-uniform Velocity

(a) Instantaneous Velocity : It is defined as the velocity at some particular instant.

Instantaneous velocity $= \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

(b) Average Velocity : Average Velocity = $\frac{\text{Total Displacement}}{\text{Total time}}$

(c) Uniform Velocity : A particle is said to have uniform velocity, if magnitudes as well as direction of its velocity remains same and this is possible only when the particles moves in same straight line reservering its direction.

(d) Non-uniform Velocity : A particle is said to have non-uniform velocity, if either of magnitude or direction of velocity changes (or both changes).

COMPARATIVE STUDY OF SPEED AND VELOCITY

S.No.	Speed	Velocity
1.	It is the time rate of change of distance of a body.	It is the time rate of change of displacement of a particle
2.	It tells nothing about the direction of motion of the particle	It tells the direction of motion of the particle
3.	It can be positive or zero	It can be positive or negative or zero
4.	It is a scalar quantity	It is a vector quantity

Note : The magnitude of instantaneous velocity and instantaneous speed are equal.

The determination of instantaneous velocity by using the definition usually involves calculation of derivative. We can find v = $\frac{dx}{dt}$ by using the standard results from differential calculus.

It is always tangential to the path.

Average speed is always positive in contrast to average velocity which being a vector, can be positive or negative.

If the motion of a particle is along a straight line and in same direction then,
average velocity = average speed.
Average speed is, in general, greater than the magnitude of average velocity.

Solved Examples

Example-8. In the example 1, if Ram takes 4 seconds and Shyam takes 5 seconds to go from P to Q, find
(a) Average speed of Ram and Shyam?
(b) Average velocity of Ram and Shyam?

Solution. (a) Average speed of Ram = $\frac{100}{4}$ m/s = 25 m/s

Average speed of Shyam = $\frac{50\pi}{5}$ m/s = 10π m/s

(b) Average velocity of Ram = $\frac{100}{4}$ m/s = 25 m/s (From P to Q)

Average velocity of Shyam = $\frac{100}{5}$ m/s = 20 m/s (From P to Q)

Example-9. A particle travels half of total distance with speed v_1 and next half with speed v_2 along a straight line. Find out the average speed of the particle?

Solution. Let total distance travelled by the particle be 2s.

Time taken to travel first half = $\frac{s}{v_1}$ Time taken to travel next half = $\frac{s}{v_2}$

Average speed = $\frac{\text{Total distance covered}}{\text{Total time taken}} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$ (harmonic progression)

2.6 Acceleration :

It is defined as the rate of change of velocity.
(i) It is a vector quantity.
(ii) Its direction is same as that of change in velocity and not of the velcoity (That is why, acceleration in uniform circular motion is towards the centre).
(iii) There are three ways possible in which change in velocity may occur.

When only direction	When only magnitude changes	When both the direction changes and magnitude change
To change the direction, net acceleration or net force should be perpendicular to direction of velocity.	In this case, net force or net acceleration should be parallel or antiparallel to the direction of velocity. (straight line motion)	In this case, net force or net acceleration has two components. One component is parallel or antiparallel to velocity and another one is perpendicular to velocity.
Ex: Uniform circular	**Ex:** When ball is thrown up	**Ex:** Projectile motion

Types of acceleration-

(a) Instantaneous acceleration : It is defined as the acceleration of a body at some particular instant.

Instantaneous acceleration = $\lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$

(b) Average acceleration = $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$

(c) **Uniform acceleration :** A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during particle motion.

Note : If a particle is moving with uniform acceleration, this does not necessarily imply that particle is moving in straight line.

Example : Parabolic motion

(d) **Non-uniform acceleration:** A body is said to have non-uniform acceleration, if magnitude or direction or both, change during motion.

Note :

(i) Acceleration is a vector with dimensions $[LT^{-2}]$ and SI units (m/s^2)

(ii) If acceleration is zero, velocity will be constant and motion will be uniform.

(iii) However if acceleration is constant acceleration is uniform but motion is non-uniform and if acceleration is not constt. both motion and acceleration are non-uniform.

Special note - I

If the motion of a particle is accelerated translatory (without change in direction) $\vec{v} = |\vec{v}|\ \hat{n}$

$\Rightarrow \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}[|\vec{v}|\ \hat{n}] = \hat{n}\ \dfrac{d|\vec{v}|}{dt}$ [as $\vec{n}$ is constant] $\Rightarrow \left|\dfrac{d\vec{v}}{dt}\right| = \dfrac{d\vec{v}}{dt} (\neq 0)$

Howeverm, if motion is uniform translatory, both these will still be equal but zero.

Special note - II

(i) $\dfrac{d|\vec{v}|}{dt} = 0$ while $\left|\dfrac{d\vec{v}}{dt}\right| \neq 0$ (it is possible.)

(ii) $\dfrac{d|\vec{v}|}{dt} \neq 0$ while $\left|\dfrac{d\vec{v}}{dt}\right| = 0$ (it is not possible.)

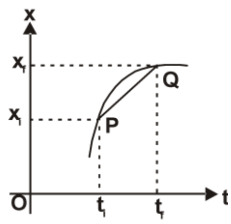
3. GRAPHICAL INTERPRETATION OF SOME QUANTITIES

3.1 Average Velocity

If a particle passes a point P (x_i) at time $t = t_i$ and reaches Q (x_f) at a later time instant

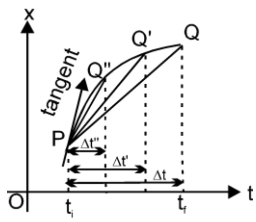
$t = t_f$, its average velocity in the interval PQ is $V_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t_f - t_i}$

This expression suggests that the average velocity is equal to the slope of the line (chord) joining the points corresponding to P and Q on the x-t graph.



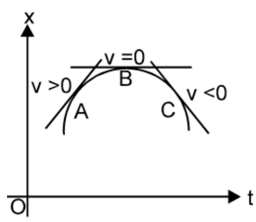
3.2 Instantaneous Velocity

Consider the motion of the particle between the two points P and Q on the x-t graph shown. As the point Q is brought closer and closer to the point P, the time interval between PQ (Δt, Δt′, Δt″,......) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line (PQ, PQ′, PQ″......)



As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the tangent to the curve at the point P. As $\Delta t \to 0$, $V_{av} (=\Delta x/\Delta t) \to V_{inst.}$

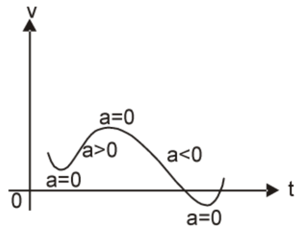
Geometrically, as $\Delta t \to 0$, chord PQ → tangent at P.



Hence the instantaneous velocity at P is the slope of the tangent at P in the x - t graph. When the slope of the x - t graph is positive, v is positive (as at the point A in figure). At C, v is negative because the tangent has negative slope. The instantaneous velocity at point B (turning point) is zero as the slope is zero.

3.3 Instantaneous Acceleration :

The derivative of velocity with respect to time is the slope of the tangent in velocity time (v-t) graph.



Solved Examples

Example-10. Position of a particle as a function of time is given as $x = 5t^2 + 4t + 3$. Find the velocity and acceleration of the particle at t = 2 s?

Solution. Velocity; $v = \frac{dx}{dt} = 10t + 4$

At $t = 2$ s

$v = 10(2) + 4$ $v = 24$ m/s

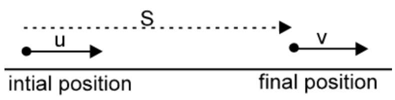
Acceleration; $a = \frac{d^2x}{dt^2} = 10$

Acceleration is constant, so at t = 2 s $a = 10$ m/s^2

4. UNIFORMLY ACCELERATED MOTION :

If a particle is accelerated with constant acceleration in an interval of time, then the motion is termed as uniformly accelerated motion in that interval of time.

For uniformly accelerated motion along a straight line (x-axis) during a time interval of t seconds, the following important results can be used.



(a) $a = \frac{v - u}{t}$

(b) $V_{av} = \frac{v + u}{2}$

(c) $S = (v_{av})t$

(d) $S = \left(\frac{v + u}{2}\right) t$

(e) $v = u + at$

(f) $s = ut + 1/2\ at^2$

$s = vt - 1/2\ at^2$

$x_f = x_i + ut + 1/2\ at^2$

(g) $v^2 = u^2 + 2as$

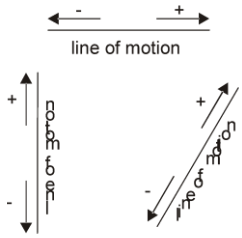
(h) $s_n = u + a/2\ (2n - 1)$

u = initial velocity (at the beginning of interval)
a = acceleration
v = final velocity (at the end of interval)

s = displacement (x_f - x_i)

x_f = final coordinate (position)
x_i = initial coordinate (position)
s_n = displacement during the n^{th} sec

5. DIRECTIONS OF VECTORS IN STRAIGHT LINE MOTION



In straight line motion, all the vectors (position, displacement, velocity & acceleration) will have only one component (along the line of motion) and there will be only two possible directions for each vector.

For example, if a particle is moving in a horizontal line (x-axis), the two directions are right and left. Any vector directed towards right can be represented by a positive number and towards left can be represented by a negative number.

For vertical or inclined motion, upward direction can be taken +ve and downward as -ve

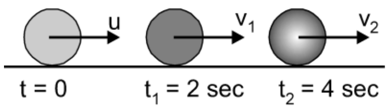
For objects moving vertically near the surface of the earth, the only force acting on the particle is its weight (mg) i.e. the gravitational pull of the earth. Hence acceleration for this type of motion will always be a = -g i.e. a = - 9.8 m/s^2 (-ve sign, because the force and acceleration are directed downwards, If we select upward direction as positive).

Note : If acceleration is in same direction as velocity, then speed of the particle increases.
If acceleration is in opposite direction to the velocity then speed decreases i.e. the particle slows down. This situation is known as retardation.

Solved Examples

Example-11. The velocity acquired by a body moving with uniform acceleration is 20 m/s in first 2 sec and 40 m/s in first 4 sec. Calculate initial velocity.

Solution.
$$a = \frac{v_2 - v_1}{t_2 - t_1}$$



$$a = \frac{40-20}{4-2} = \frac{20}{2} = 10\,m/s^2$$

Now, $v = u + at$

$v_1 = u + at_1$

$\Rightarrow \quad 20 = u + 10 \times 2$

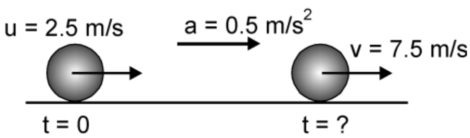
$\Rightarrow \quad 20 = u + 20 \quad \Rightarrow \quad u = 0\,m/s$ **Ans**

Example-12. A particle starts with initial velocity 2.5 m/s along the x direction and accelerates uniformly at the rate 50 cm/s^2. Find time taken to increase the velocity to 7.5 m/s.

Solution.
$v = u + at$

$7.5 = 2.5 + 0.5\,t$

$5.0 = 0.5\,t$



$$t = \frac{50}{5} = 10\,sec$$

Example-13. A particle starts with a constant acceleration. At a time t second speed is found to be 100 m/s and one second later speed becomes 150 m/s. Find acceleration of the particle.

Solution. From equation (1) of motion v = u + at

$\Rightarrow$ 100 = 0 + at $\Rightarrow$ 100 = at ... (1)

Now consider velocity one second later

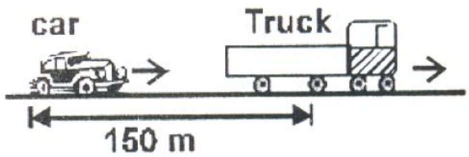
$v' = 0 + a(t+1)$ $\Rightarrow$ $150 = a(t+1)$ ... (2)

On subtracting equation (1) from equation (2)

$a = 50\,m/s^2$

Example-14. A truck starts from rest with an acceleration of 1.5 ms^{-2} while a car 150 metre behind starts from rest with an acceleration of 2 ms^{-2}. (a) How long will it take before both the truck and car are side by side and (b) How much distance is travelled by each.

Solution. **(a)** $s_T = \frac{1}{2}at^2$



$s_T = \frac{1}{2}(1.5)t^2$ .... (1)

Distance covered by car when car overtakes the truck

$s_c = \frac{1}{2}(2)t^2$ $\Rightarrow$ $(s_T + 150) = \frac{1}{2}(2)t^2$ .... (2)

Divide equation (2) by equation (1) $\frac{s_T + 150}{s_T} = \frac{2}{1.5}$ $\Rightarrow$ $1 + \frac{150}{s_T} = \frac{20}{15} = \frac{4}{3}$

$\Rightarrow$ $\frac{150}{s_T} = \frac{4}{3} - 1 = \frac{1}{3}$ or $s_T = 450$

Distance travelled by car = 450 + 150 = 600 metre

(b) Now by equation (1) $s_T = \frac{1}{2}at^2$ $\Rightarrow$ $450 = \frac{1}{2} \times 1.5 \times t^2$

$t^2 = \frac{450 \times 2}{1.5}$ $\Rightarrow$ $t = \sqrt{300 \times 2} = 24.5\,sec$

Therefore car will overtake the truck after 24.5 second.

Example-15. A body travels a distance of 2 m in 2 seconds and 2.2 m in next 4 seconds. What will be the velocity of the body at the end of 7th second from the start.

Solution. Here, case (i) s = 2m, t = 2s

case (ii) t = 2 + 4 = 6s

Let u and a be the initial velocity and uniform acceleration of the body.

we know that, $s = ut + \frac{1}{2}at^2$

Case (i) $2 = u \times 2 + \frac{1}{2}a \times 2^2$ or 1 = u + a .... (1)

Case (ii) $4.2 = u \times 6 + \frac{1}{2}a \times 6^2$ or $0.7 = u + 3a$ ... (2)

Subtracting (2) from (1), we get

$0.3 = 0 - 2a = -2a$ or $a = -0.3/2 = -0.15\,ms^{-2}$

From (i), $u = 1 - a = 1 + 0.15$

$u = 1.15\,ms^{-1}$

For the velocity of body at the end of 7th second, we have

$u = 1.15 ms^{-1}; a = -0.15 ms^{-2}$,

v = ?, t = 7s

As, v = u + at ∴ v = 1.15 + (–0.15) x 7

v = 0.1 m/s

Example-16. A body travels a distance of 20 m in the 7th second and 24 m in 9th second. How much distance shall it travel in the 15th second?

Solution. Here, $s_7 = 20$ m ; $s_9 = 24$ m, $s_{15} = ?$

Let u and a be the initial velocity and uniform acceleration of the body.

We know that, $s_n = u + \frac{a}{2}(2n-1)$ ∴ $s_7 = u + \frac{a}{2}(2 \times 7 - 1)$

or $20 = u + \frac{13a}{2}$ .... (i) and $s_9 = u + \frac{a}{2}(2 \times 9 - 1)$ or $24 = u + \frac{17}{2}a$ .... (ii)

Subtracting (ii) form (i), we get 4 = 2a or $a = 2 ms^{-2}$

Putting this value in (i), we get $20 = u + \frac{13}{2} \times 2$ or 20 = u + 13 or $u = 20 - 13 = 7 ms^{-1}$

Hence, $s_{15} = u + \frac{a}{2}(2 \times 15 - 1) = 7 + \frac{2}{2} \times 29$ ⇒ $s_{15} = 36$ m **Ans.**

Example-17. A person travelling at 43.2 km/h applies the brakes giving a deceleration of 6 m/s^2 to his scooter. How far will it travel before stopping?

Solution. Here, $u = 43.2 km/h = 43.2 \times \frac{5}{18} m/s$

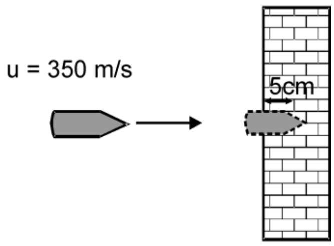
Deceleration; a = 6 m/s^2 v = 0 s = ?

Using $v^2 = u^2 - 2as$

$0 = (12)^2 - 2 \times 6 s$ ⇒ $144 = 2 \times 6s$ ⇒ $s = \frac{144}{12} = 12m$ **Ans.**

Example-18. A bullet going with speed 350 m/s enters in a concrete wall and penetrates a distance of 5 cm before coming to rest. Find deceleration.

Solution. Here, u = 350 m/s, s = 5 cm, v = 0 m/s and a = ?



By using $v^2 = u^2 - 2as$

$0 = u^2 - 2as$

$u^2 = 2as$ or $a = \frac{u^2}{2s}$

$a = \frac{350 \times 350}{2 \times 0.05} = 12.25 \times 10^5$ m/sec^2

Example-19. A driver takes 0.20 s to apply the brakes after he sees a need for it. This is called the reaction time of the driver. If he is driving a car at a speed of 54 km/h and the brakes cause a deceleration of 6.0 m/s^2, find the distance travelled by the car after he sees the need to put the brakes on.

Solution. Distance covered by the car during the application of brakes by driver -

$u = 54$ km/h $= 54 \times \frac{5}{18} m/s = 15$ m/s

$s_1 = ut$ or $s_1 = 15 \times 0.2 = 3.0$ meter

After applying the brakes;

v = 0 u = 15 m/s, a = 6 m/s^2 $s_2 = ?$

Using $v^2 = u^2 - 2as$

$0 = (15)^2 - 2 \times 6 \times s_2$

$$\Rightarrow s_2 = \frac{225}{12} = 18.75 \text{ metre}$$

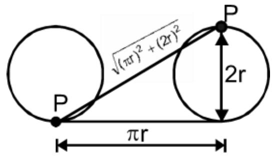
$12 s_2 = -225$

Distance travelled by the car after driver sees the need for it

$s = s_1 + s_2$

$s = 3 + 18.75 = 21.75$ metre. **Ans.**

Example-20. A point P consider at contact point of a wheel on ground which rolls on ground without sliping then value of displacement of point P when wheel completes half of rotation - [If radius of wheel is 1 m]



Solution. Displacement = $\sqrt{\pi^2 + 4}$ **Ans.**

Example-21. A person travelling on a straight line moves with a uniform velocity v_1 for some time and with uniform velocity v_2 for the next equal time. The average velocity v is given by

Solution. $v = \frac{v_1 + v_2}{2}$ (Arithmetic progression)

Example-22. A particle moving rectilinearly with constant acceleration is having initial velocity of 10 m/s. After some time, its velocity becomes 30 m/s. Find out velocity of the particle at the mid point of its path?

Solution. Let the total distance be 2x. ∴ distance upto midpoint = x

Let the velocity at the mid point be v and acceleration be a.

From equations of motion

$v^2 = 10^2 + 2ax$ ____ (1) $30^2 = v^2 + 2ax$ ____ (2)

(2) - (1) gives

$v^2 - 30^2 = 10^2 - v^2$ ⇒ $v^2 = 500$ ⇒ $v = 10\sqrt{5}$ m/s

Example-23. Mr. Sharma brakes his car with constant acceleration from a velocity of 25 m/s to 15 m/s over a distance of 200 m.

(a) How much time elapses during this interval?

(b) What is the acceleration?

(c) If he has to continue braking with the same constant acceleration, how much longer would it take for him to stop and how much additional distance would he cover?

Solution. (a) We select positive direction for our coordinate system to be the direction of the velocity and choose the origin so that $x_i = 0$ when the braking begins. Then the initial velocity is $u_x = +25$ m/s at t = 0, and the final velocity and position are $v_x = +15$ m/s and x = 200 m at time t.

Since the acceleration is constant, the average velocity in the interval can be found from the average of the initial and final velocities.

∴ $v_{av,x} = \frac{1}{2}(u_x + v_x) = \frac{1}{2}(15 + 25) = 20$ m/s.

The average velocity can also be expressed as

$v_{av,x} = \frac{\Delta x}{\Delta t}$. With $\Delta x = 200$ m and $\Delta t = t - 0$, we can solve for t:

$t = \frac{\Delta x}{v_{av,x}} = \frac{200}{20} = 10$ s.

(b) We can now find the acceleration using $v_x = u_x + a_x t$

$a_x = \frac{v_x - u_x}{t} = \frac{15 - 25}{10} = -1$ m/s^2.

The acceleration is negative, which means that the positive velocity is becoming smaller as brakes are applied (as expected).

(c) Now with known acceleration, we can find the total time for the car to go from velocity u_x = 25 m/s to v_x = 0. Solving for t, we find

$$t = \frac{v_x - u_x}{a_x} = \frac{0-25}{-1} = 25 \text{ s.}$$

The total distance covered is

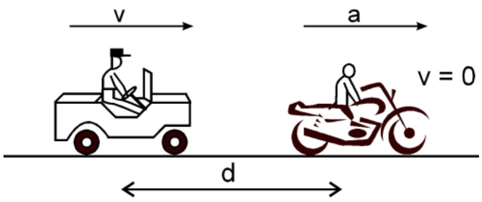
$$x = x_i + u_x t + \frac{1}{2} a_x t^2$$

$$= 0 + (25)(25) + \frac{1}{2}(-1)(25)^2 = 625 - 312.5 = 312.5 \text{ m.}$$

Additional distance covered
= 312.5 – 200 = 112.5 m.

Example-24. A police inspector in a jeep is chasing a pickpocket an a straight road. The jeep is going at its maximum speed v (assumed uniform). The pickpocket rides on the motorcycle of a waiting friend when the jeep is at a distance d away, and the motorcycle starts with a constant acceleration a. Show that the pick pocket will be caught if $v \geq \sqrt{2ad}$.

Solution. Suppose the pickpocket is caught at a time t after motorcycle starts. The distance travelled by the motorcycle during this interval is



$$s = \frac{1}{2}at^2 \quad \text{\_\_\_\_\_ (1)}$$

During this interval the jeep travels a distance

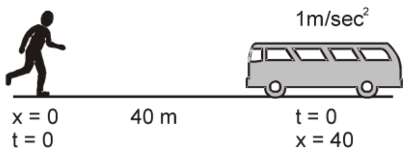
$$s + d = vt \quad \text{\_\_\_\_\_ (2)}$$

By (1) and (2), $\Rightarrow$ $\frac{1}{2}at^2 + d = vt$ or $t = \frac{v \pm \sqrt{v^2 - 2ad}}{a}$

The pickpocket will be caught if t is real and positive.

This will be possible if $v^2 \geq 2ad$ or $v \geq \sqrt{2ad}$

Example-25. A man is standing 40 m behind the bus. Bus starts with 1 m/sec² constant acceleration and also at the same instant the man starts moving with constant speed 9 m/s. Find the time taken by man to catch the bus.



Solution. Let after time 't' man will catch the bus

For bus $x = x_0 + ut + \frac{1}{2}at^2$, $x = 40 + 0(t) + \frac{1}{2}(1)t^2$

$x = 40 + \frac{t^2}{2}$ .... (i)

For man, $x = 9t$ .... (ii)

From (i) & (ii) $\Rightarrow$ $40 + \frac{t^2}{2} = 9t$ or t = 8 s or t = 10s.

Example-26. A particle is dropped from a tower. It is found that it travels 45 m in the last second of its journey. Find out the height of the tower?

Solution. Let the total time of journey be n seconds.

Using ; $s_n = u + \frac{a}{2}(2n-1)$ $\Rightarrow$ $45 = 0 + \frac{10}{2}(2n-1)$ $\Rightarrow$ n = 5 sec

Height of tower; $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m

Example-27. A ball is projected vertically up with an initial speed of 20 m/s on a planet where acceleration due to gravity is 10 m/s^2.
(a) How long does it takes to reach the highest point?
(b) How high does it rise above the point of projection?
(c) How long will it take for the ball to reach a point 10 m above the point of projection?

Solution. As here motion is vertically upwards,
$a = -g$ and $v = 0$
(a) From 1st equation of motion, i.e., $v = u + at$,
$0 = 20 - 10t$
i.e. $t = 2$ sec. **Ans.**
(b) Using $v^2 = u^2 + 2as$
$0 = (20)^2 - 2 \times 10 \times h$
i.e $h = 20$ m. **Ans.**
(c) Using $s = ut + \frac{1}{2}at^2$
$10 = 20t (-) \times 10 \times t^2$
i.e. $t^2 - 4t + 2 = 0$ or $t = 2 \pm \sqrt{2}$
i.e. $t = 0.59$ sec. or 3.41 sec.
i.e., there are two times, at which the ball passes through h = 10 m, once while going up and then coming down.

Example-28. A ball is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is 9.8 m/s^2 (a) What is the height of the bridge? (b) With which velocity does the ball strike the water?

Solution. Taking the point of projection as origin and downward direction as positive,
(a) Using $s = ut + \left(\frac{1}{2}\right)at^2$ we have $h = -4.9 \times 2 + \left(\frac{1}{2}\right)9.8 \times 2^2 = 9.8\,m$
(u is taken to be negative as it is upwards)
(b) Using $v = u + at$
$v = -4.9 + 9.8 \times 2 = 14.7$ m/s

Example-29. A rocket is fired vertically up from the ground with a resultant vertical acceleration of 10 m/s^2. The fuel is finished in 1minute and it continues to move up.
(a) What is the maximum height reached?
(b) After how much time from then will the maximum height be reached? (Take g = 10 m/s^2)

Solution. (a) The distance travelled by the rocket during burning interval (1 minute = 60 s) in which resultant acceleration is vertically upwards and 10 m/s^2 will be
$h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000\,m$ ...(1)
Velocity acquired by it is
$v = 0 + 10 \times 60 = 600$ m/s ...(2)
After one minute the rocket moves vertically up with initial velocity of 600 m/s and continues till height h_2 till its velocity becomes zero.
$0 = (600)^2 - 2gh_2$
or $h_2 = 18000\,m$ [as g = 10 m/s^2] ...(3)
From equations (1) and (3) the maximum height reached by the rocket from the ground is
$H = h_1 + h_2 = 18 + 18 = 36\,km$
(b) The time to reach maximum height after burning of fuel is
$0 = 600 - gt$
$t = 60$ s

After finishing fuel the rocket goes up for 60 s.

Example-30. A body is released from a height and falls freely towards the earth. Exactly 1 sec later another body is released. What is the distance between the two bodies after 2 sec the release of the second body, if g = 9.8 m/s^2.

Solution. The 2nd body falls for 2s, so

$$h_2 = \frac{1}{2}g(2)^2 \quad ...(1)$$

While 1st has fallen for 2 + 1 = 3 sec so

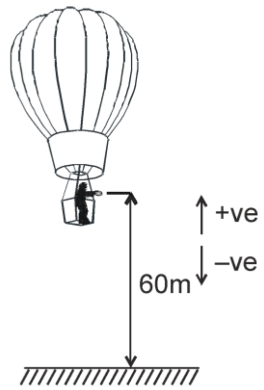
$$h_1 = \frac{1}{2}g(3)^2 \quad ...(2)$$

∴Separation between two bodies after 2 sec the release of 2nd body,

$$d = h_1 - h_2 = \frac{1}{2}g\left(3^2 - 2^2\right) = 4.9 \times 5 = 24.5\text{m}$$

Example-31. A stone is dropped from a balloon going up with a uniform velocity of 5 m/s. If the balloon was 60 m high when the stone was dropped, find its height when the stone hits the ground. Take g = 10 m/s^2.

Solution. $S = ut + \frac{1}{2}at^2$



$-60 = 5(t) + \frac{1}{2}(-10)t^2$

$-60 = 5t - 5t^2$

$5t^2 - 5t - 6f0 = 0$

$t^2 - t - 12 = 0$

$t^2 - 4t + 3t - 12 = 0$

$(t - 4)(t + 3) = 0$

$t = 4$

Height of balloon from ground at this instant

= 60 + 4 × 5 = 80 m

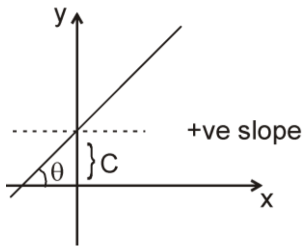
Note : As the particle is detached from the balloon it is having the same velocity as that of balloon, but its acceleration is only due to gravity and is equal to g.

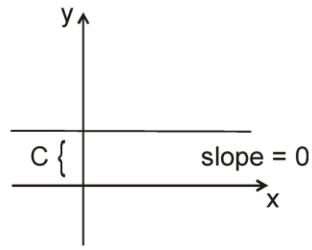
6. STRAIGHT LINE-EQUATION, GRAPH, SLOPE (+ve, –ve, zero slope).

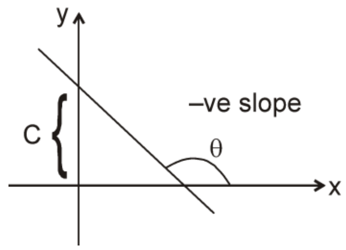
If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0° \le \theta < 180°$, $\theta \ne 90°$, then the slope of the line, denoted by m, is defined by m = tan θ. If θ is 90°, m does not exist, but the line is parallel to the y-axis. If θ = 0, then m = 0 & the line is parallel to the x-axis.

Slope - intercept form : y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

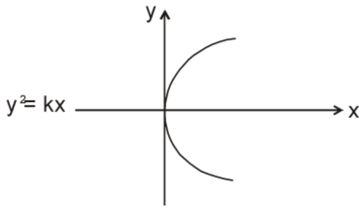
$m = \text{slope} = \tan\theta = \frac{dy}{dx}$

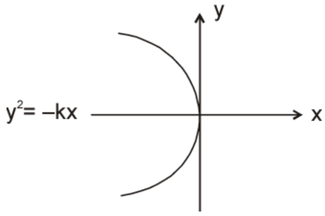


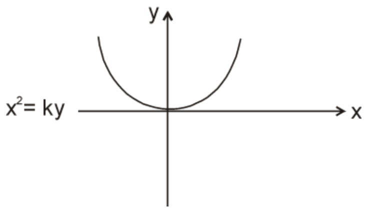


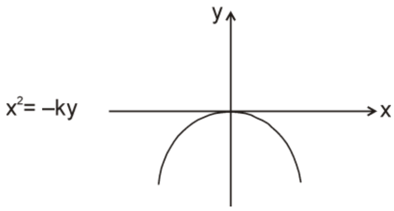


7. PARABOLIC CURVE-EQUATION, GRAPH (VARIOUS SITUATIONS UP, DOWN, LEFT, RIGHT WITH CONDITIONS)









Where k is a positive constant.

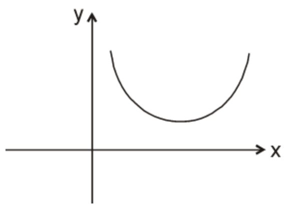
Equation of parabola :

Case (i) : $y = ax^2 + bx + c$

For $a > 0$

The nature of the parabola will be like that of the of nature $x^2 = ky$

Minimum value of y exists at the vertex of the parabola.

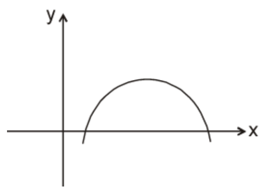


$y_{min} = \frac{-D}{4a}$ where $D = b^2 - 4ac$ $\therefore$ Coordinates of vertex = $\left(\frac{-b}{2a}, \frac{D}{4a}\right)$

Case (ii) : $a < 0$

The nature of the parabola will be like that of the nature of $x^2 = -ky$

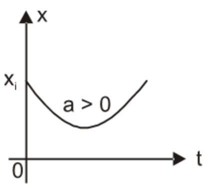
Maximum value of y exists at the vertex of parabola.

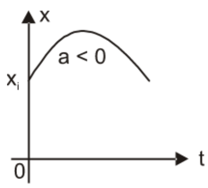


$y_{max} = \frac{D}{4a}$ where $D = b^2 - 4ac$

8. GRAPHS IN UNIFORMLY ACCELERATED MOTION ($a \neq 0$)

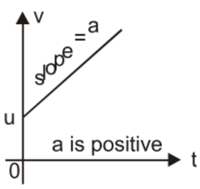
· x is a quadratic polynomial in terms of t. Hence x - t graph is a parabola.

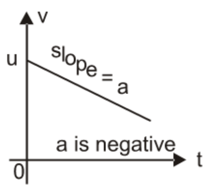




x-t graph

· v is a linear polynomial in terms of t. Hence v-t graph is a straight line of slope a.





v-t graph

· a-t graph is a horizontal line because a is constant.

a-t graph

9. INTERPRETATION OF SOME MORE GRAPHS

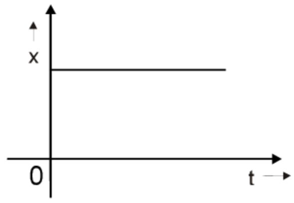
9.1 Position vs Time graph

(i) Zero Velocity

As position of particle is fixed at all the time,
so the body is at rest.

Slope $\frac{dx}{dt}$ = tanθ = tan 0º = 0

Velocity of particle is zero

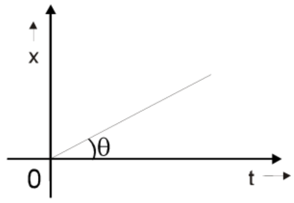


(ii) Uniform Velocity

Here tanθ is constant tanθ = $\frac{dx}{dt}$

∴ $\frac{dx}{dt}$ is constant.

∴ velocity of particle is constant.



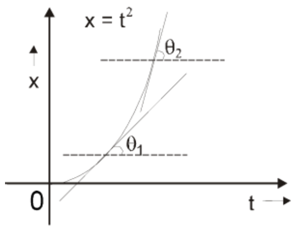
(iii) Non uniform velocity (increasing with time)

In this case;

As time is increasing, θ is also increasing.

∴ $\frac{dx}{dt}$ = tanθ is also increasing

Hence, velocity of particle is increasing.



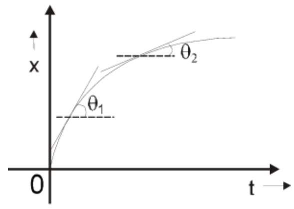
(iv) Non uniform velocity (decreasing with time)

In this case;

As time increases, θ decreases.

∴ $\frac{dx}{dt}$ = tanθ also decreases.

Hence, velocity of particle is decreasing.



9.2 Velocity vs time graph

(i) Zero acceleration

Velocity is constant.

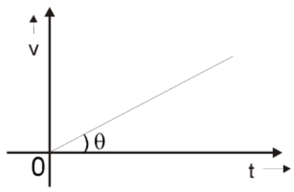
tanθ = 0

∴ $\frac{dv}{dt}$ = 0 Hence, acceleration is zero.

(ii) Uniform acceleration

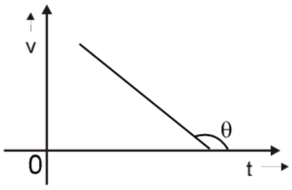
tanθ is constant.

∴ $\frac{dv}{dt}$ = constant



Hence, it shows constant acceleration.

(iii) Uniform retardation



Since $\theta > 90^o$

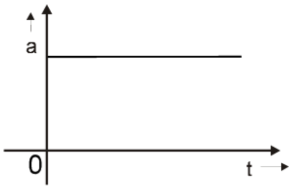
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \frac{dv}{dt}$ = negative constant

Hence, it shows constant retardation.

9.3 Acceleration vs time graph

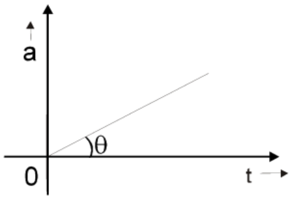
(i) Constant acceleration



$\tan\theta = 0$

$\therefore \frac{da}{dt} = 0$ Hence, acceleration is constant.

(ii) Uniformly increasing acceleration



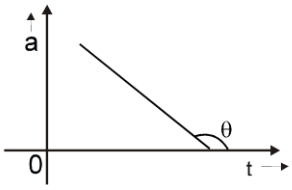
θ is constant.

$0^o < \theta < 90^o$ $\tan\theta > 0$

$\therefore \frac{da}{dt} = \tan\theta$ = constant > 0

Hence, acceleration is uniformly increasing with time.

(iii) Uniformly decreasing acceleration



Since $\theta > 90^o$

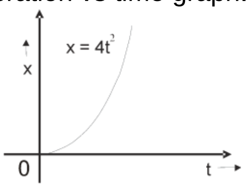
$\therefore$ $\tan\theta$ is constant and negative.

$\therefore \frac{da}{dt}$ = negative constant

Hence, acceleration is uniformly decreasing with time

Solved Examples

Example-32. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.



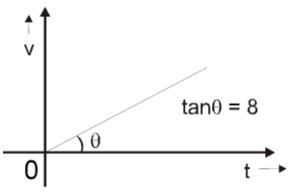
Solution. $x = 4t^2$ $\Rightarrow$ $v = \frac{dx}{dt} = 8t$

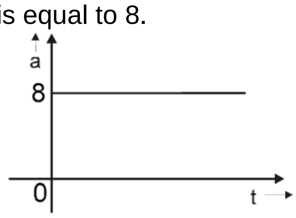
Hence, velocity-time graph is a straight line

having slope i.e. $\tan\theta = 8$

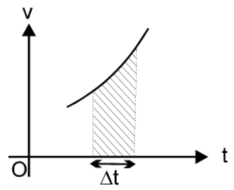
$a = \frac{dv}{dt} = 8$

Hence, acceleration is constant throughout and is equal to 8.





10. DISPLACEMENT FROM v - t GRAPH & CHANGE IN VELOCITY FROM a -t GRAPH

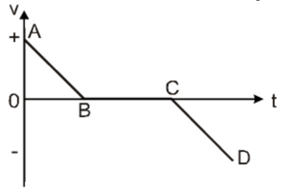


Displacement = Δx = area under v-t graph.

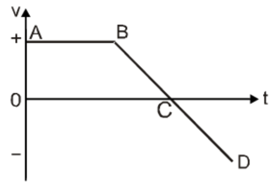
Since a negative velocity causes a negative displacement, areas below the time axis are taken negative. In similar way, can see that Δ v = a Δt leads to the conclusion that **area under a - t graph gives the change in velocity Δv during that interval.**

Solved Examples

Example-33. Describe the motion shown by the following velocity-time graphs.



(a)



(b)

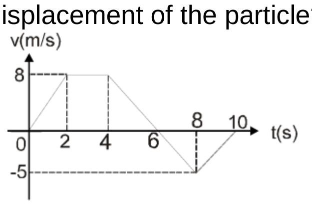
Solution. (a) **During interval AB:** velocity is +ve so the particle is moving in +ve direction, but it is slowing down as acceleration (slope of v-t curve) is negative. **During interval BC:** particle remains at rest as velocity is zero. Acceleration is also zero. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

(b) **During interval AB:** particle is moving in +ve direction with constant velocity and acceleration is zero. **During interval BC:** particle is moving in +ve direction as velocity is +ve, but it slows down until it comes to rest as acceleration is negative. **During interval CD:** velocity is -ve so the particle is moving in -ve direction and is speeding up as acceleration is also negative.

Important Points to Remember

- For uniformly accelerated motion ($a \neq 0$), x-t graph is a parabola (opening upwards if $a > 0$ and opening downwards if $a < 0$). The slope of tangent at any point of the parabola gives the velocity at that instant.
- For uniformly accelerated motion ($a \neq 0$), v-t graph is a straight line whose slope gives the acceleration of the particle.
- In general, the slope of tangent in x-t graph is velocity and the slope of tangent in v-t graph is the acceleration.
- The area under a-t graph gives the change in velocity.
- The area under the v-t graph gives the distance travelled by the particle, if we take all areas as positive.
- Area under v-t graph gives displacement, if areas below the t-axis are taken negative.

Example-34. For a particle moving along positive x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle?



Solution. Distance travelled = Area under v-t graph (taking all areas as +ve.)

Distance travelled = Area of trapezium + Area of triangle

$= \frac{1}{2}(2+6)\times 8 + \frac{1}{2}\times 4\times 5$

= 32 + 10

= 42 m

Displacement = Area under v-t graph (taking areas below time axis as –ve.)

Displacement = Area of trapezium - Area of triangle

$= \frac{1}{2}(2+6)\times 8 - \frac{1}{2}\times 4\times 5$ = 32 - 10 = 22 m

Hence, distance travelled = 42 m and displacement = 22 m.

11. MOTION WITH NON-UNIFORM ACCELERATION (USE OF DEFINITE INTEGRALS)

$\Delta x = \int_{t_i}^{t_f} v(t)dt$ (displacement in time interval t = t_i to t_f)

The expression on the right hand side is called the definite integral of v(t) between t = t_i and t = t_f. . Similarly change in velocity

$\Delta v = v_f - v_i = \int_{t_i}^{t_f} a(t)dt$

Solving Problems which Involves Nonuniform Acceleration

(i) Acceleration depending on velocity v or time t

By definition of acceleration, we have $a = \frac{dv}{dt}$. If a is in terms of t,

$\int_{v_0}^{v} dv = \int_0^t a(t)dt$. If a is in terms of v, $\int_{v_0}^{v} \frac{dv}{a(v)} = \int_0^t dt$.

On integrating, we get a relation between v and t, and then

using $\int_{x_0}^{x} dx = \int_0^t v(t)dt$, x and t can also be related.

(ii) Acceleration depending on velocity v or position x

$a = \frac{dv}{dt} \Rightarrow a = \frac{dv}{dx}\frac{dx}{dt}$

$\Rightarrow a = \frac{dx}{dt}\frac{dv}{dx}$

$\Rightarrow a = v\frac{dv}{dx}$

This is another important expression for acceleration.

If a is in terms of x,

$$\int_{v_0}^{v} v\,dv = \int_{x_0}^{x} a(x)\,dx$$

If a is in terms of v, $\int_{v_0}^{v} \frac{v\ dv}{a(v)} = \int_{x_0}^{x} dx$

On integrating, we get a relation between x and v.

Using $\int_{x_0}^{x} \frac{dx}{v\ (x)} = \int_{0}^{t} dt$, we can relate x and t.

Solved Examples

Example-35. An object starts from rest at t = 0 and accelerates at a rate given by a = 6t. What is (a) its velocity and (b) its displacement at any time t?

Solution. As acceleration is given as a function of time,

$\therefore \int_{v(t_0)}^{v(t)} dv = \int_{t_0}^{t} a(t)dt$

Here $t_0 = 0$ and $v(t_0) = 0$ ∴ $v(t) = \int_{0}^{t} 6t\,dt = 6\left(\frac{t^2}{2}\right)\Big|_0^t = 6\left(\frac{t^2}{2} - 0\right) = 3t^2$

So, $v(t) = 3t^2$

As $\Delta x = \int_{t_0}^{t} v(t)dt$ ∴ $\Delta x = \int_{0}^{t} 3t^2 dt = 3\left(\frac{t^3}{3}\right)\Big|_0^t = 3\left(\frac{t^3}{3} - 0\right) = t^3$

Hence, velocity $v(t) = 3t^2$ and displacement $\Delta x = t^3$.

Example-36. For a particle moving along x-axis, acceleration is given as a = x. Find the position as a function of time? Given that at t = 0 , x = 1 v = 1.

Solution. $a = x \Rightarrow \frac{vdv}{dx} = x \Rightarrow \frac{v^2}{2} = \frac{x^2}{2} + C$

t = 0, x = 1 and v = 1 ∴ C = 0 $\Rightarrow v^2 = x^2$

$v = x \Rightarrow \frac{dx}{dt} = x \Rightarrow \frac{dx}{x} = t$

$\ell nx = t + C \Rightarrow 0 = 0 + C \Rightarrow \ell nx = t \quad x = e^t$

Example-37. For a particle moving along x-axis, acceleration is given as a = v. Find the position as a function of time ?
Given that at t = 0 , x = 0 v = 1.

Solution. $a = v \Rightarrow \frac{dv}{dt} = v \Rightarrow \int \frac{dv}{v} = \int dt$

$\ell nv = t + c \Rightarrow 0 = 0 + c \Rightarrow c = 0$

$v = e^t \Rightarrow \frac{dx}{dt} = e^t \Rightarrow \int dx = \int e^t dt$

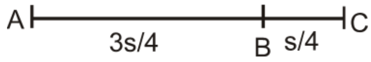
$\Rightarrow x = e^t + c \Rightarrow 0 = 1 + c \Rightarrow c = 0 \quad x = e^t - 1$

Solved Miscellaneous Problems

Problem 1. A particle covers $\frac{3}{4}$ of total distance with speed v_1 and next $\frac{1}{4}$ with v_2. Find the average speed of the particle?

Answer : $\frac{4v_1v_2}{v_1+3v_2}$

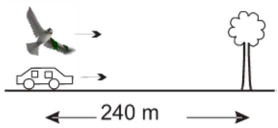
Solution : Let the total distance be s



average speed (< v >) = $\frac{\text{Total distance}}{\text{Total time taken}}$

$$< v > = \frac{s}{\frac{3s}{4v_1}+\frac{s}{4v_2}} = \frac{1}{\frac{3}{4v_1}+\frac{1}{4v_2}} = \frac{4v_1v_2}{v_1+3v_2}$$

Problem 2. A car is moving with speed 60 Km/h and a bird is moving with speed 90 km/h along the same direction as shown in figure. Find the distance travelled by the bird till the time car reaches the tree?



Answer : 360 m

Solution : Time taken by a car to reaches the tree (t) = $\frac{240\,m}{60\,km/hr} = \frac{0.24}{60}hr$

Now, the distance travelled by the bird during this time interval (s)

$= 90 \times \frac{0.24}{60}$ = 0.12 × 3 km = 360 m.

Problem 3. The position of a particle moving on X-axis is given by

$x = At^3 + Bt^2 + Ct + D$.

The numerical values of A, B, C, D are 1, 4, -2 and 5 respectively and SI units are used. Find (a) the dimensions of A, B, C and D, (b) the velocity of the particle at t = 4 s, (c) the acceleration of the particle at t = 4s, (d) the average velocity during the interval t =0 to t = 4s, (e) the average acceleration during the interval t = 0 to t = 4 s.

Answer : **(a)** $[A] = [LT^{-3}]$, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;
(b) 78 m/s ; **(c)** 32 m/s^2 ; **(d)** 30 m/s ; **(e)** 20 m/s^2

Solution : As $x = At^3 + Bt^2 + Ct + D$

(a) Dimensions of A, B, C and D ,

$[At^3] = [x]$ (by principle of homogeneity)

$[A] = [LT^{-3}]$

similarly, $[B] = [LT^{-2}]$, $[C] = [LT^{-1}]$ and $[D] = [L]$;

(b) As $v = \frac{dx}{dt} = 3At^2 + 2Bt + C$

velocity at t = 4 sec.

$v = 3(1)(4)^2 + 2(4)(4) - 2 = 78$ m/s.

(c) Acceleration (a) = $\frac{dv}{dt} = 6At + 2B$; $a = 32\ m/s^2$

(d) average velocity as $x = At^3 + Bt^2 + Ct = D$

position at t = 0, is x = D = 5m.

Position at t = 4 sec is (1)(64) + (4)(16) – (2) (4) + 5 = 125 m
Thus the displacement during 0 to 4 sec. is 125 – 5 = 120 m
$\therefore \langle v \rangle = 120 / 4 = 30$ m/s
(e) $v = 3At^2 + 20t + C$, velocity at t = 0 is c = – 2 m/s

velocity at t = 4 sec is 78 m/s $\therefore \langle a \rangle = \frac{v_2 - v_1}{t_2 - t_1} = 20$ m/s^2

Problem 4. For a particle moving along x-axis, velocity is given as a function of time as $v = 2t^2 + \sin t$. At t = 0, particle is at origin. Find the position as a function of time?

Solution : $v = 2t^2 + \sin t \Rightarrow \frac{dx}{dt} = 2t^2 + \sin t$

$$\int_0^x dx = \int_0^t (2t^2 + \sin t)dt = x = \frac{2}{3}t^3 - \cos t + 1$$ **Ans.**

Problem 5. A car deccelerates from a speed of 20 m/s to rest in a distance of 100 m. What was its acceleration, assumed constant?

Solution : v = 0 u = 20 m/s s = 100 m $\Rightarrow$ as $v^2 = u^2 + 2as$
0 = 400 + 2a × 100 $\Rightarrow$ a = – 2 m/s
$\therefore$ acceleration = 2 m/s^2 **Ans.**

Problem 6. A 150 m long train accelerates uniformly from rest. If the front of the train passes a railway worker 50 m away from the station at a speed of 25 m/s, what will be the speed of the back part of the train as it passes the worker?

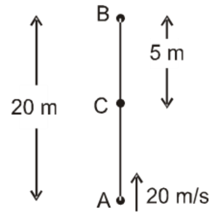
Solution : $v^2 = u^2 + 2as$
25 × 25 = 0 + 100 a
$a = \frac{25}{4}$ m/s^2
Now, for time taken by the back end of the train to pass the worker

we have $v'^2 = v^2 + 2al = (25)^2 + 2 \times \frac{25}{48} \times 150$
$v'^2 = 25 \times 25 \times 4 \Rightarrow v' =$ **50 m/s.** **Ans.**

Problem 7. A particle is thrown vertically with velocity 20 m/s . Find (a) the distance travelled by the particle in first 3 seconds, (b) displacement of the particle in 3 seconds.

Answer : 25m, 15m

Solution : Heighest point say B
$V_B = 0$
v = u + gt $\Rightarrow$ 0 = 20 – 10 t $\Rightarrow$ t = 2 sec.
$\therefore$ distance travel in first 2 seconds.
s = s(t - 0 to t = 2 sec) + 5 (2 sec. to 3 sec.)
$s = [ut + 1/2\, at^2]_{t = 0 \text{ to } t = 2\,s} + [ut + 1/2 at^2]_{t = 2 \text{ to } t = 3\,s}$
$s = 20 \times 2 - 1/2 \times 10 \times 4 + 1/2 \times 10 \times 1^2$
= (40 – 20) + 5 = 25 m.
and displacement = 20 – 5 = 15 m.



Problem 8. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time elapsed is t. Find the maximum velocity acquired by the car.

Solution : $t = t_1 + t_2$

slope of OA curve = $\tan\theta = \alpha = \frac{v_{max}}{t_1}$

slope of AB curve = $\beta = \dfrac{v_{max}}{t_2}$ $\Rightarrow$ $t = t_1 + t_2$

$\Rightarrow$ $t = \dfrac{v_{max}}{\alpha} + \dfrac{v_{max}}{\beta}$ $\Rightarrow$ $v_{max} = \left(\dfrac{\alpha\ \beta}{\alpha+\beta}\right) t$

Problem 9. In the above question find total distance travelled by the car in time ‘t’ .

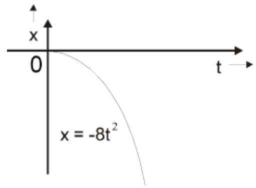
Solution : $v_{max} = \dfrac{\alpha\ \beta}{(\alpha+\beta)} t$ $\Rightarrow$ $t_1 = \dfrac{v_{max}}{\alpha} = \dfrac{\beta t}{(\alpha+\beta)}$ $\Rightarrow$ $t_2 = \dfrac{v_{max}}{\beta} = \dfrac{\alpha t}{(\alpha+\beta)}$

∴ Total distance travelled by the car in time ‘t’

$= \dfrac{1}{2}\alpha t_1^2 + v_{max} t_2 - \dfrac{1}{2}\beta t_2^2 = \dfrac{1}{2}\dfrac{\alpha\beta^2 t^2}{(\alpha+\beta)^2} + \dfrac{\alpha^2\beta t^2}{(\alpha+\beta)^2} - \dfrac{1}{2}\dfrac{\beta\alpha^2 t^2}{(\alpha\ +\ \beta)^2}$

$= \dfrac{1}{2}\dfrac{\alpha\beta t^2}{(\alpha+\beta)} = \dfrac{\alpha\beta t^2}{2(\alpha+\beta)}$ **Ans.**

Problem 10. The displacement vs time graph of a particle moving along a straight line is shown in the figure. Draw velocity vs time and acceleration vs time graph.
Upwards direction is taken as positive, downwards direction is taken as negative.

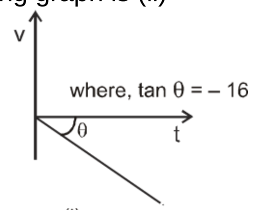


Solution : (a) The equation of motion is : $x = -8t^2$

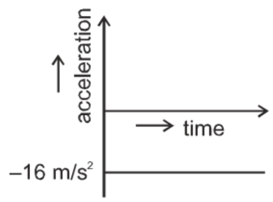
$\therefore v = \dfrac{dx}{dt} = -16\,t$; this shows that velocity is directly proportional to time and slope of velocity-time curve is negative i.e. – 16. Hence, resulting graph is (i)

(b) Acceleration of particle is : $a = \dfrac{dv}{dt} = -16$

This shows that acceleration is constant but negative.
Resulting graph is (ii)

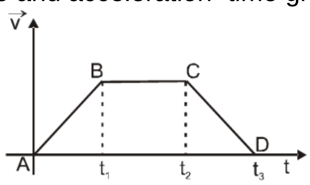


(i)

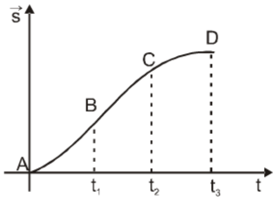


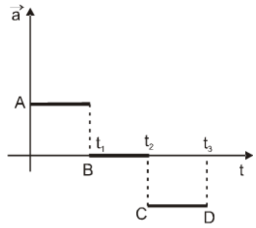
(ii)

Problem 11. Draw displacement–time and acceleration–time graph for the given velocity–time graph.

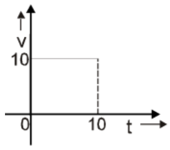


Solution : **Part AB :** v-t curve shows constant slope
i.e. constant acceleration or Velocity increases at constant rate with time.
Hence, s-t curve will show constant increase in slope
and a-t curve will be a straight line.
Part BC : v-t curve shows zero slope i.e. constant velocity. So, s-t curve will show constant slope and acceleration will be zero.
Part CD : v-t curve shows negative slope i.e. velocity is decreasing with time or acceleration is negative.
Hence , s-t curve will show decrease in slope becoming zero in the end.
and a-t curve will be a straight line with negative intercept on y-axis.
RESULTING GRAPHS ARE :

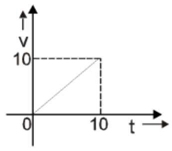




Problem 12. For a particle moving along x-axis, following graphs are given. Find the distance travelled by the particle in 10 s in each case?



(a)



(b)

Solution : **(a)** Distance area under the v - t curve

∴ distance = $10 \times 10 = 100$ m **Ans.**

(b) Area under v – t curve

∴ distance = $\frac{1}{2} \times 10 \times 10 = 50$ m **Ans.**

Problem 13. For a particle moving along x-axis, acceleration is given as $a = 2v^2$. If the speed of the particle is v_0 at x = 0, find speed as a function of x.

Solution : $a = 2v^2 \Rightarrow$ or $\frac{dv}{dt} = 2v^2$ or $\frac{dv}{dx} \times \frac{dx}{dt} = 2v^2$

$v\frac{dv}{dx} = 2v^2 \Rightarrow \frac{dv}{dx} = 2v$

$\int_{v_0}^{v} \frac{dv}{v} = \int_0^x 2dx \Rightarrow [\ell nv]_{v_0}^{v} = [2x]_0^x$

$\ell n\frac{v}{v_0} = 2x \Rightarrow v = v_0e^{2x}$ **Ans.**

Problem 14. The velocity of any particle is related with its displacement As; , Calculate acceleration at x = 5 m.

Solution $x = \sqrt{v+1}$ $x^2 = v+1$ $v = (x^2 - 1)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}(x^2-1) = 2x\frac{dx}{dt} = 2x \ v = 2x(x^2-1)$

at x = 5 m $a = 2 \times 5(25-1) = 240\,m/s^2$

Problem 15. If the displacement of a particle is $(2t^2 + t + 5)$ meter then, what will be acceleration at t = 5 sec.

Solution $v = \frac{dx}{dt} = \frac{d}{dt}(2t^2 + t + 5) = 4t + 1\ m/s$ and $a = \frac{dv}{dt} = \frac{d}{dt}(4t+1)$ $a = 4\ m/s$ $a = 4\ m/s^2$