Sets, Relations & Function

SET

A set is a collection of well defined objects which are distinct from each other. Sets are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by small letters a, b, c .... etc.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A.

If a does not belong to A then we write a A,

e.g. the collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

METHODS TO WRITE A SET :

(i) **Roster Method or Tabular Method :** In this method a set is described by listing elements, separated by commas and enclose then by curly brackets. Note that while writing the set in roster form, an element is not generally repeated e.g. the set of letters of word SCHOOL may be written as {S, C, H, O, L}.

(ii) **Set builder form (Property Method) :** In this we write down a property or rule which gives us all the element of the set.
$A = \{x : P(x)\}$ where P(x) is the property by which $x \in A$ and colon (:) stands for 'such that'

Example # 1 : Express set $A = \{x : x \in N \text{ and } x = 2n \text{ for } n \in N\}$ in roster form
Solution : $A = \{2, 4, 8, 16, ...\}$

Example # 2 : Express set $B = \{x3 : x < 5, x \in W\}$ in roster form
Solution : $B = \{0, 1, 8, 27, 64\}$

Example # 3 : Express set $A = \{0, 7, 26, 63, 124\}$ in set builder form
Solution : $A = \{x : x = n3 – 1, n\in N, 1 \le n \le 5\}$

TYPES OF SETS

Null set or empty set : A set having no element in it is called an empty set or a null set or void set, it is denoted by φ or { }. A set consisting of at least one element is called a non-empty set or a non-void set.

Singleton set : A set consisting of a single element is called a singleton set.

Finite set : A set which has only finite number of elements is called a finite set.

Order of a finite set : The number of distinct elements in a finite set A is called the order of this set and denoted by O(A) or n(A). It is also called cardinal number of the set.
e.g. $A = \{a, b, c, d\}$ $\Rightarrow$ $n(A) = 4$

Infinite set : A set which has an infinite number of elements is called an infinite set.

Equal sets : Two sets A and B are said to be equal if every element of A is member of B, and every element of B is a member of A. If sets A and B are equal, we write A = B and if A and B are not equal then
$A \neq B$

Equivalent sets : Two finite sets A and B are equivalent if their cardinal number is same
i.e. $n(A) = n(B)$
e.g. $A = \{1, 3, 5, 7\}, B = \{a, b, c, d\}$ $\Rightarrow$ $n(A) = 4$ and $n(B) = 4$
$\Rightarrow$ A and B are equivalent sets

Note - Equal sets are always equivalent but equivalent sets may not be equal

Example # 4 : Identify the type of set :

(i) $A = \{x \in W : 3 \leq x < 10\}$ (ii) $A = \{\alpha, \beta, \gamma, \delta\}$
(iii) $A = \{1, 0, -1, -2, -3, ...\}$ (iv) $A = \{1, 8, -2, 6, 5\}$ and $B = \{1, 8, -2, 1, 6, 5\}$
(v) $A = \{x :$ x is number of students in a class room$\}$

Solution : (i) finite set (ii) finite set
(iii) infinite set (iv) equal sets
(v) singleton set

Self Practice Problem :

(1) Write the set of all integers 'x' such that $-2 < x - 4 < 5$.
(2) Write the set $\{1, 2, 5, 10\}$ in set builder form.
(3) If $A = \{x : x^2 < 9, x \in Z\}$ and $B = \{-2, -1, 1, 2\}$ then find whether sets A and B are equal or not.

Answers (1) $\{3, 4, 5, 6, 7, 8\}$
(2) $\{x :$ x is a natural number and a divisor of $10\}$
(3) Not equal sets

SUBSET AND SUPERSET :

Let A and B be two sets. If every element of A is an element of B then A is called a subset of B and B is called superset of A. We write it as $A \subseteq B$.

e.g. $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$ $\Rightarrow$ $A \subseteq B$

If A is not a subset of B then we write $A \not\subset B$

PROPER SUBSET :

If A is a subset of B but $A \neq B$ then A is a proper subset of B and we write $A \subset B$. Set A is not proper subset of A so this is improper subset of A

Note : (i) Every set is a subset of itself
(ii) Empty set φ is a subset of every set
(iii) $A \subseteq B$ and $B \subseteq A \Leftrightarrow A = B$
(iv) The total number of subsets of a finite set containing n elements is 2^n.
(v) Number of proper subsets of a set having n elements is $2^n - 1$.
(vi) Empty set φ is proper subset of every set except itself.

POWER SET :

Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

Example # 5 : Examine whether the following statements are true or false :

(i) $\{a\} \not\subset \{b, c, a\}$
(ii) $\{x, p\} \not\subset \{x :$ x is a consonant in the English alphabet$\}$
(iii) $\{\alpha, \beta, \gamma, \delta\} \subseteq \{\alpha, \beta, \varphi, \psi\}$
(iv) $\{a, b\} \in \{a, \{a\}, b, c\}$

Solution : (i) False as $\{a\}$ is subset of $\{b, c, a\}$
(ii) False as x, p are consonant
(iii) False as element γ, δ is not in the set $\{\alpha, \beta, \varphi, \psi\}$
(iv) False as $a, b \in \{a, \{a\}, b, c\}$ and $\{a, b\} \subseteq \{a, \{a\}, b, c\}$

Example # 6 : Find power set of set $A = \{1, 2, 3\}$

Solution : $P(A) = \{\varphi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

Example # 7 : If φ denotes null set then find

	(a)	$P(\varphi)$	(b)	$P(P(\varphi))$
	(c)	$n(P(P(P(\varphi))))$	(d)	$n(P(P(P(P(\varphi)))))$
Solution :	(a)	$P(\varphi) = \{\varphi\}$	(b)	$P(P(\varphi)) = \{\varphi, \{\varphi\}\}$
	(c)	$n(P(P(P(\varphi)))) = 2^2 = 4$	(d)	$n(P(P(P(P(\varphi))))) = 2^4 = 16$

Self Practice Problem :

(4) State true/false : $A = \{p, q, r, s\}$, $B = \{p, q, r, p, t\}$ then $A \subseteq B$.

(5) State true/false : $A = \{p, q, r, s\}$, $B = \{s, r, q, p\}$ then $A \subset B$.

(6) State true/false : $[4, 15) \subseteq [-15, 15]$

Answers (4) False (5) False (6) True

UNIVERSAL SET :

A set consisting of all possible elements which occur in the discussion is called a universal set and is denoted by U.

e.g. if $A = \{1, 2, 3\}$, $B = \{2, 4, 5, 6\}$, $C = \{1, 3, 5, 7\}$ then $U = \{1, 2, 3, 4, 5, 6, 7\}$ can be taken as the universal set.

SOME OPERATION ON SETS :

(i) **Union of two sets :** $A \cup B = \{x : x \in A \text{ or } x \in B\}$
e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cup B = \{1, 2, 3, 4\}$

(ii) **Intersection of two sets :** $A \cap B = \{x : x \in A \text{ and } x \in B\}$
e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ then $A \cap B = \{2, 3\}$

(iii) **Difference of two sets :** $A - B = \{x : x \in A \text{ and } x \notin B\}$. It is also written as $A \cap B'$.
Similarly $B - A = B \cap A'$ e.g. $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$; $A - B = \{1\}$

(iv) **Symmetric difference of sets :** It is denoted by $A \Delta B$ and $A \Delta B = (A - B) \cup (B - A)$

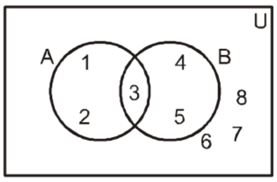
(v) **Complement of a set :** $A' = \{x : x \notin A \text{ but } x \in U\} = U - A$
e.g. $U = \{1, 2, ..., 10\}$, $A = \{1, 2, 3, 4, 5\}$ then $A' = \{6, 7, 8, 9, 10\}$

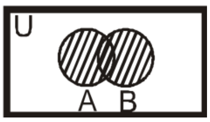
(vi) **Disjoint sets :** If $A \cap B = \varphi$, then A, B are disjoint sets.
e.g. If $A = \{1, 2, 3\}$, $B = \{7, 8, 9\}$ then $A \cap B = \varphi$

VENN DIAGRAM :

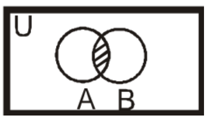
Most of the relationships between sets can be represented by means of diagrams which are known as venn diagrams. These diagrams consist of a rectangle for universal set and circles in the rectangle for subsets of universal set. The elements of the sets are written in respective circles.

For example If $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$, $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$ then their venn diagram is

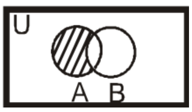




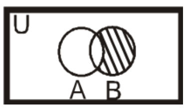
A ∪ B



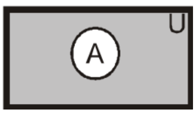
A ∩ B

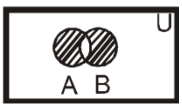


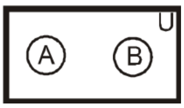
A – B



B – A







A' $(A \Delta B) = (A - B) \cup (B - A)$ Disjoint

LAWS OF ALGEBRA OF SETS (PROPERTIES OF SETS):

(i) **Commutative law :** $(A \cup B) = B \cup A$; $A \cap B = B \cap A$

(ii) **Associative law :** $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(iii) **Distributive law :** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(iv) **De-morgan law :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

(v) **Identity law :** $A \cap U = A$; $A \cup \varphi = A$

(vi) **Complement law :** $A \cup A' = U$, $A \cap A' = \varphi$, $(A')' = A$

(vii) **Idempotent law :** $A \cap A = A$, $A \cup A = A$

NOTE :

(i) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(ii) $A \cap \varphi = \varphi$, $A \cup U = U$

Example # 8 : Let A = {1, 2, 3, 4, 5, 6} and B = {4, 5, 6, 7, 8, 9} then find $A \cup B$

Solution : $A \cup B$ = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Example # 9 : Let A = {1, 2, 3, 4, 5, 6}, B = {4, 5, 6, 7, 8, 9}. Find A – B and B – A.

Solution : A – B = {x : x ∈ A and x B} = {1, 2, 3}

similarly B – A = {7, 8, 9}

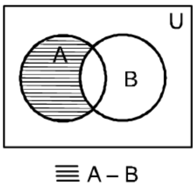
Example # 10 : State true or false :

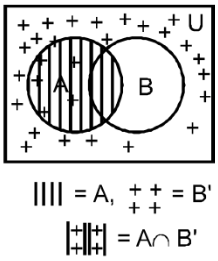
(i) $A \cup A' = A$ (ii) $U \cap A = A$

Solution : (i) false because $A \cup A' = U$

(ii) true as $U \cap A = A$

Example # 11 : Use Venn diagram to prove that $A - B = A \cap B'$.





Solution :

From venn diagram we can conclude that $A - B = A \cap B'$.

Self Practice Problem :

(7) Find $A \cup B$ if A = {x : x = 2n + 1, n ≤ 5, n ∈ N} and B = {x : x = 3n – 2, n ≤ 4, n ∈ N}.

(8) Find A – (A – B) if A = {5, 9, 13, 17, 21} and B = {3, 6, 9, 12, 15, 18, 21, 24}

Answers (7) {1, 3, 4, 5, 7, 9, 10, 11} (8) {9, 21}

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B, C are finite sets and U be the finite universal set then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A - B) = n(A) - n(A \cap B)$

(iii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(iv) Number of elements in exactly two of the sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(v) Number of elements in exactly one of the sets A, B, C

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

Example # 12: In a group of 60 students, 36 read English newspaper, 22 read Hindi newspaper and 12 read neither of the two. How many read both English & Hindi news papers ?

Solution : $n(U) = 60$, $n(E) = 36$, $n(H) = 22$

$n(E' \cap H') = 12 \Rightarrow n(E \cup H)' = 12$

$\Rightarrow \quad n(U) - n(E \cup H) = 12$

$\Rightarrow \quad n(E \cup H) = 48$

$\Rightarrow \quad n(E) + n(H) - n(E \cap H) = 48$

$\Rightarrow \quad n(E \cap H) = 58 - 48 = 10$

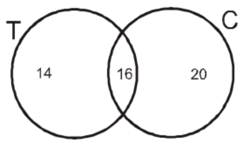
Example#13 : In a group of 50 persons, 14 drink tea but not coffee and 30 drink tea. Find

(i) How many drink tea and coffee both ? (ii) How many drink coffee but not tea ?

Solution : T : people drinking tea

C : people drinking coffee

(i) $n(T) = n(T - C) + n(T \cap C) \Rightarrow 30 = 14 + n(T \cap C) \Rightarrow n(T \cap C) = 16$



(ii) $n(C - T) = n(T \cup C) - n(T) = 50 - 30 = 20$

Self Practice Problem :

(9) Let A and B be two finite sets such that $n(A - B) = 15$, $n(A \cup B) = 90$, $n(A \cap B) = 30$. Find n(B)

(10) A market research group conducted a survey of 1000 consumers and reported that 720 consumers liked product A and 450 consumers liked product B. What is the least number that must have liked both products ?

Answers (9) 75 (10) 170

RELATIONS

ORDERED PAIR :

A pair of objects listed in a specific order is called an ordered pair. It is written by listing the two objects in specific order separating them by a comma and then enclosing the pair in parentheses.

In the ordered pair (a, b), a is called the first element and b is called the second element.

Two ordered pairs are set to be equal if their corresponding elements are equal.

i.e. (a, b) = (c, d) if a = c and b = d.

CARTESIAN PRODUCT :

The set of all possible ordered pairs (a, b), where $a \in A$ and $b \in B$ i.e. $\{(a, b) ; a \in A \text{ and } b \in B\}$ is called the Cartesian product of A to B and is denoted by $A \times B$. Usually $A \times B \neq B \times A$.

Similarly $A \times B \times C = \{(a, b, c) : a \in A, b \in B, c \in C\}$ is called ordered triplet.

RELATION:

Let A and B be two sets. Then a relation R from A to B is a subset of $A \times B$. Thus, R is a relation from A to B $\Rightarrow$ $R \subseteq A \times B$. The subsets is derived by describing a relationship between the first element and the second element of ordered pairs in $A \times B$ e.g. if A = {1, 2, 3, 4, 5, 6, 7, 8} and B = {1, 2, 3, 4, 5} and $R = \{(a, b) : a = b^2, a \in A, b \in B\}$ then R = {(1, 1), (4, 2), (9, 3)}. Here a R b $\Rightarrow$ 1 R 1, 4 R 2, 9 R 3.

NOTE :

(i) Let A and B be two non-empty finite sets consisting of m and n elements respectively. Then $A \times B$ consists of mn ordered pairs. So total number of subsets of $A \times B$ i.e. number of relations from A to B is 2^{mn}.

(ii) A relation R from A to A is called a relation on A.

DOMAIN AND RANGE OF A RELATION :

Let R be a relation from a set A to a set B. Then the set of all first components of coordinates of the ordered pairs belonging to R is called to domain of R, while the set of all second components of coordinates of the ordered pairs in R is called the range of R.

Thus, Dom (R) = $\{a : (a, b) \in R\}$ and Range (R) = $\{b : (a, b) \in R\}$

It is evident from the definition that the domain of a relation from A to B is a subset of A and its range is a subset of B.

Example#14 : If A = {1, 2} and B = {3, 4}, then find $A \times B$.

Solution : $A \times B$ = {(1, 3), (1, 4), (2, 3), (2, 4)}

Example#15 : Let A = {1, 3, 5, 7} and B = {2, 4, 6, 8} be two sets and let R be a relation from A to B defined by the phrase "$(x, y) \in R \Rightarrow x > y$". Find relation R and its domain and range.

Solution : Under relation R, we have 3R2, 5R2, 5R4, 7R2, 7R4 and 7R6

i.e. R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)}

$\therefore$ Dom (R) = {3, 5, 7} and range (R) = {2, 4, 6}

Example#16 : Let A = {2, 3, 4, 5, 6, 7, 8, 9}. Let R be the relation on A defined by
$\{(x, y) : x \in A, y \in A \ \& \ x^2 = y \text{ or } x = y^2\}$. Find domain and range of R.

Solution: The relation R is
R = {(2, 4), (3, 9), (4, 2), (9, 3)}
Domain of R = {2, 3, 4, 9}
Range of R = {2, 3, 4, 9}

Self Practice Problem :

(11) If $(2x + y, 7) = (5, y - 3)$ then find x and y.

(12) If $A \times B = \{(1, 2), (1, 3), (1, 6), (7, 2), (7, 3), (7, 6)\}$ then find sets A and B.

(13) If $A = \{x, y, z\}$ and $B = \{1, 2\}$ then find number of relations from A to B.

(14) Write $R = \{(4x + 3, 1 - x) : x \le 2, x \in N\}$

Answers (11) $x = -\frac{5}{2}$, $y = 10$ (12) $A = \{1, 7\}$, $B = \{2, 3, 6\}$
(13) 64 (14) $\{(7, 0), (11, -1)\}$

TYPES OF RELATIONS :

In this section we intend to define various types of relations on a given set A.

(i) **Void relation :** Let A be a set. Then $\varphi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on A.

(ii) **Universal relation :** Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

(iii) **Identity relation :** Let A be a set. Then the relation $I_A = \{(a, a) : a \in A\}$ on A is called the identity relation on A. In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

(iv) **Reflexive relation :** A relation R on a set A is said to be reflexive if every element of A is related to itself. Thus, R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

Note : Every identity relation is reflexive but every reflexive relation in not identity.

(v) **Symmetric relation :** A relation R on a set A is said to be a symmetric relation iff $(a, b) \in R \Rightarrow (b ,a) \in R$ for all $a, b \in A$. i.e. $a R b \Rightarrow b R a$ for all $a, b \in A$.

(vi) **Transitive relation :** Let A be any set. A relation R on A is said to be a transitive relation iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$
i.e. $a R b$ and $b R c \Rightarrow a R c$ for all a, b, $c \in A$

(vii) **Equivalence relation :** A relation R on a set A is said to be an equivalence relation on A iff
(i) it is reflexive i.e. $(a, a) \in R$ for all $a \in A$
(ii) it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$
(iii) it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b \in A$

Example#17 : Which of the following are identity relations on set $A = \{1, 2, 3\}$.
$R_1 = \{(1, 1), (2, 2)\}$, $R_2 = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$, $R_3 = \{(1, 1), (2, 2), (3, 3)\}$.

Solution: The relation R_3 is identity relation on set A.
R_1 is not identity relation on set A as $(3, 3) \notin R_1$.
R_2 is not identity relation on set A as $(1, 3) \in R_2$

Example#18 : Which of the following are reflexive relations on set $A = \{1, 2, 3\}$.
$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (2, 1)\}$, $R_2 = \{(1, 1), (3, 3), (2, 1), (3, 2)\}$..

Solution : R_1 is a reflexive relation on set A.
R_2 is not a reflexive relation on A because $2 \in A$ but $(2, 2) \notin R_2$.

Example#19 : Prove that on the set N of natural numbers, the relation R defined by $x R y \Rightarrow x$ is less than y is transitive.

Solution : Because for any $x, y, z \in N$ $x < y$ and $y < z \Rightarrow x < z \Rightarrow x R y$ and $y R z \Rightarrow x R z$. so R is transitive.

Example#20 : Let T be the set of all triangles in a plane with R a relation in T given by $R = \{(T_1 , T_2) : T_1$ is congruent to $T_2\}$. Show that R is an equivalence relation.

Solution : Since a relation R in T is said to be an equivalence relation if R is reflexive, symmetric and

transitive.
(i) Since every triangle is congruent to itself
$\therefore$ R is reflexive
(ii) $(T_1, T_2) \in R \Rightarrow T_1$ is congruent to T_2 $\Rightarrow$ T_2 is congruent to T_1 $\Rightarrow$ $(T_2, T_1) \in R$
Hence R is symmetric
(iii) Let $(T_1, T_2) \in R$ and $(T_2, T_3) \in R$ $\Rightarrow$ T_1 is congruent to T_2 and T_2 is congruent to T_3
$\Rightarrow T_1$ is congruent to T_3 $\Rightarrow$ $(T_1, T_3) \in R$
$\therefore$ R is transitive
Hence R is an equivalence relation.

Example#21 : Show that the relation R in R defined as $R = \{(a, b) : a \le b\}$ is transitive.
Solution : Let $(a, b) \in R$ and $(b, c) \in R$
$\therefore$ $(a \le b)$ and $b \le c \Rightarrow a \le c$ $\therefore$ $(a, c) \in R$ Hence R is transitive.

Example#22 : Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric.
Solution : Let $(a, b) \in R$ $[\because (1, 2) \in R]$
$\therefore$ $(b, a) \in R$ $[\because (2, 1) \in R]$
Hence R is symmetric.

Self Practice Problem :

(15) Let L be the set of all lines in a plane and let R be a relation defined on L by the rule $(x, y) \in R \Rightarrow$ x is perpendicular to y. Then prove that R is a symmetric relation on L.

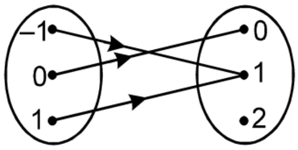
(16) Let R be a relation on the set of all lines in a plane defined by $(\ell_1, \ell_2) \in R \Rightarrow$ line ℓ_1 is parallel to line ℓ_2. Prove that R is an equivalence relation.

FUNCTION

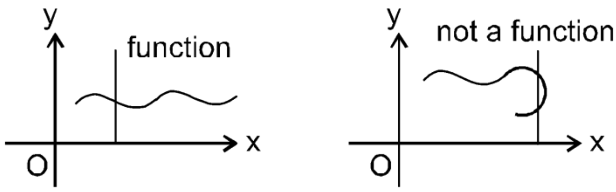
Definition :

Function is a rule (or correspondence), from a non empty set A to a non empty set B, that associates each member of A to a unique member of B. Symbolically, we write f: A → B. We read it as "f is a function from A to B".

For example, let A ≡ {–1, 0, 1} and B ≡ {0, 1, 2}.
Then A × B ≡ {(–1, 0), (–1, 1), (–1, 2), (0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2)}
Now, " f : A → B defined by $f(x) = x^2$ " is the function such that
f ≡ {(–1, 1), (0, 0), (1, 1)}
f can also be shown diagrammatically by following mapping.



Note : Every function say y = f(x) : A → B. Here x is independent variable which takes its values from A while 'y' takes its value from B. A relation will be a function if and only if
(i) x must be able to take each and every value of A and
and (ii) one value of x must be related to one and only one value of y in set B.



(a) (b)

Graphically : If any vertical line cuts the graph at more than one point, then the graph does not represent a function.

Example#23 : (i) Which of the following correspondences can be called a function ?

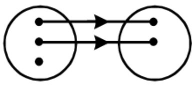
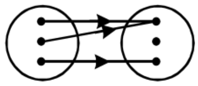
(A) $f(x) = x^3$; {–1, 0, 1} → {0, 1, 2, 3}

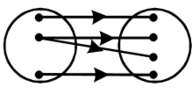
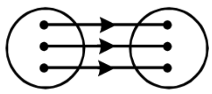
(B) $f(x) = \pm\sqrt{x}$; {0, 1, 4} → {–2, –1, 0, 1, 2}

(C) $f(x) = \sqrt{x}$; {0, 1, 4} → {–2, –1, 0, 1, 2}

(D) $f(x) = -\sqrt{x}$; {0, 1, 4} → {–2, –1, 0, 1, 2}

(ii) Which of the following pictorial diagrams represent the function

(A)  (B) 

(C)  (D) 

Solution :

(i) f(x) in (C) and (D) are functions as definition of function is satisfied. while in case of (A) the given relation is not a function, as f(–1) $\notin$ 2nd set. Hence definition of function is not satisfied. While in case of (B), the given relation is not a function, as f(1) = ± 1 and f(4) = ± 2 i.e. element 1 as well as 4 in 1st set is related with two elements of 2nd set. Hence definition of function is not satisfied.

(ii) B and D. In (A) one element of domain has no image, while in (C) one element of 1st set has two images in 2nd set

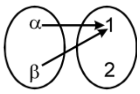
Self practice problem :

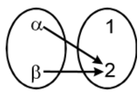
(17) Let g(x) be a function defined on [−1, 1]. If the area of the equilateral triangle with two of its vertices at (0,0) and **(**x,g(x)**)** is $\sqrt{3}/4$ sq. unit, then the function g(x) may be.

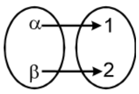
(A) g(x)= $\pm\sqrt{(1-x^2)}$ (B) g(x) = $\sqrt{(1-x^2)}$ (C) g(x) = $-\sqrt{(1-x^2)}$ (D) g(x) = $\sqrt{(1+x^2)}$

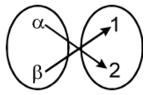
(18) Represent all possible functions defined from {α, β} to {1, 2}.

Answers : (17) A, B, C

(18) (i)


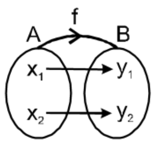
(ii)


(iii)


(iv)


Domain, Co-domain and Range of a Function :

Let y = f(x) : A → B, then the set A is known as the domain of f and the set B is known as co-domain of f.



If x_1 is mapped to y_1, then y_1 is called as image of x_1 under f. Further x_1 is a pre-image of y_1 under f.

If only expression of f (x) is given (domain and co-domain are not mentioned), then domain is **complete** set of those values of x for which f (x) is real, while co domain is considered to be (– ∞, ∞) (except in inverse trigonometric functions).

Range is the complete set of values that y takes. Clearly range is a subset of Co-domain.

A function whose domain and range are both subsets of real numbers is called **a real function**.

Example#24 : Find the domain of following functions :

(i) f(x) = $\sqrt{x^2-5}$ (ii) $\sin^{-1}(2x-1)$

Solution : (i) f(x) = $\sqrt{x^2-5}$ is real iff $x^2 - 5 \geq 0$

$\Rightarrow$ $|x| \geq \sqrt{5}$ $\Rightarrow$ $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$

$\therefore$ the domain of f is $(-\infty, -\sqrt{5}] \cup [\sqrt{5}, \infty)$

(ii) $\sin^{-1}(2x - 1)$ is real iff $-1 \le 2x - 1 \le +1$

$\therefore$ domain is $x \in [0, 1]$