ELECTROSTATICS

1. INTRODUCTION

The branch of physics which deals with electric effect of static charge is called electrostatics.

2. ELECTRIC CHARGE

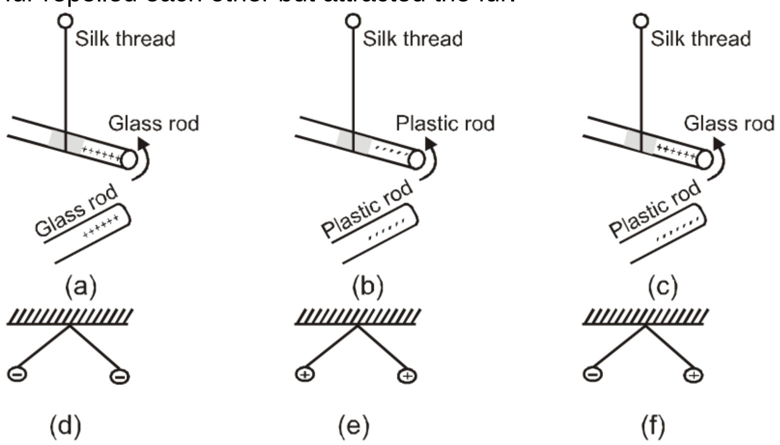
Charge of a material body or particle is the property (acquired or natural) due to which it produces and experiences electrical and magnetic effects. Some of naturally charged particles are electron, proton, α-particle etc.

Charge is a derived physical quantity. Charge is measured in coulomb in S.I. unit. In practice we use mC (10^{-3}C), μC (10^{-6}C), nC(10^{-9}C) etc.
C.G.S. unit of charge = electrostatic unit = esu.
1 coulomb = 3×10^{9} esu of charge
Dimensional formula of charge = $[M^0L^0T^1I^1]$

2.1 Electric Charges

a. It was observed that if two glass rods rubbed with wool or silk cloth are brought close to each other, they repel each other.

b. The two strands of wool or two pieces of silk cloth, with which the rods were rubbed, also repel each other. However, the glass rod and wool attracted each other. Similarly, two plastic rods rubbed with cat's fur repelled each other but attracted the fur.



c. It was concluded, after many careful studies by different scientists, that there were only two kinds of an entity which is called the electric charge.

d. We say that the bodies like glass or plastic rods, silk, fur and pith balls are electrified. They acquire an electric charge on rubbing.

e. The experiments on pith balls suggested that there are two kinds of electrification and we find that (i) like charges repel and (ii) unlike charges attract each other

f. The charges were named as positive and negative by the American scientist Benjamin Franklin.
By convention,

g. The charge on glass rod or cat's fur is called positive and that on plastic rod or silk is termed negative. If an object possesses an electric charge, it is said to be electrified or charged. When it has no charge it is said to be neutral

2.2 Properties of Charge

(i) Charge is a scalar quantity : It adds algebraically and represents excess, or deficiency of electrons.

(ii) Charge is of two types : (i) Positive charge and (ii) Negative charge Charging a body implies transfer of charge (electrons) from one body to another. Positively charged body means loss of electrons, i.e., deficiency of electrons. Negatively charged body means excess of electrons. This also shows that **mass of a negatively charged body > mass of a positively charged identical body.**

(iii) Charge is conserved : In an isolated system, total charge (sum of positive and negative) remains constant whatever change takes place in that system.

(iv) Charge is quantized : Charge on any body always exists in integral multiples of a fundamental unit of electric charge. This unit is equal to the magnitude of charge on electron ($1e = 1.6 \times 10^{-19}$ coulomb). So charge on anybody $Q = \pm ne$, where n is an integer and e is the charge of the electron. **Millikan's oil drop** experiment proved the quantization of charge or atomicity of charge

Note : Recently, the existence of particles of charge $\pm\frac{1}{3}e$ and $\pm\frac{2}{3}e$ has been postulated. These particles are called quarks but still this is not considered as the quantum of charge because these are unstable (They have very short span of life).

(v) Like point charges repel each other while unlike point charges attract each other.

(vi) Charge is always associated with mass, i.e., charge can not exist without mass though mass can exist without charge. The particle such as photon or neutrino which have no (rest) mass can never have a charge.

(vii) Charge is relativistically invariant: This means that charge is independent of frame of reference, i.e., charge on a body does not change whatever be its speed. This property is worth mentioning as in contrast to charge, the mass of a body depends on its speed and increases with increase in speed.

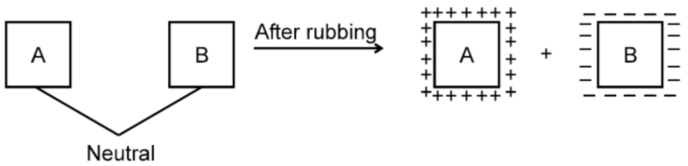
(viii) A charge at rest produces only electric field around itself; a charge having uniform motion produces electric as well as magnetic field around itself while a charge having accelerated motion emits electromagnetic radiation.

2.3 Charging of a body

A body can be charged by means of (a) friction, (b) conduction, (c) induction, (d) thermionic ionization or thermionic emission (e) photoelectric effect and (f) field emission.

(a) Charging by Friction :

When a neutral body is rubbed against other neutral body then some electrons are transferred from one body to other. The body which can hold electrons tightly, draws some electrons and the body which can not hold electrons tightly, looses some electrons. The body which draws electrons becomes negatively charged and the body which looses electrons becomes positively charged.



For example : Suppose a glass rod is rubbed with a silk cloth. As the silk can hold electrons more tightly and a glass rod can hold electrons less tightly (due to their chemical properties), some electrons will leave the glass rod and gets transferred to the silk. So in the glass rod their will be deficiency of electrons, therefore it will become positively charged. And in the silk there will be some extra electrons, so it will become negatively charged

(b) **Charging by conduction (flow):** There are three types of material in nature

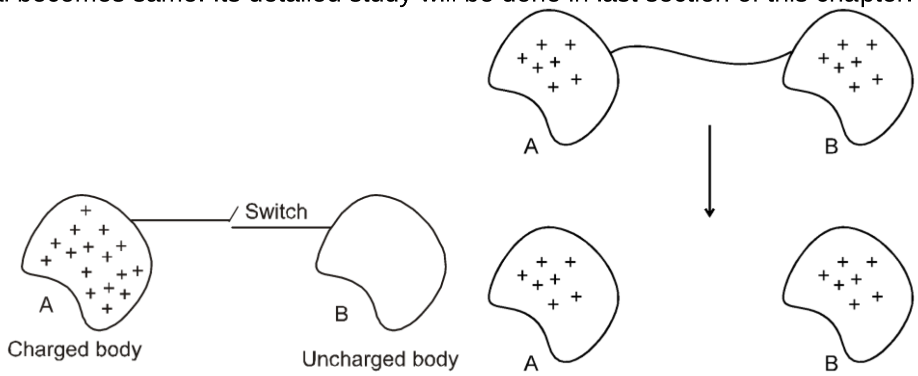
(i) Conductor : Conductors are the material in which the outer most electrons are very loosely bounded, so they are free to move (flow). So in a conductors, there are large number of free electrons.

Ex. Metals like Cu, Ag, Fe, Al.............

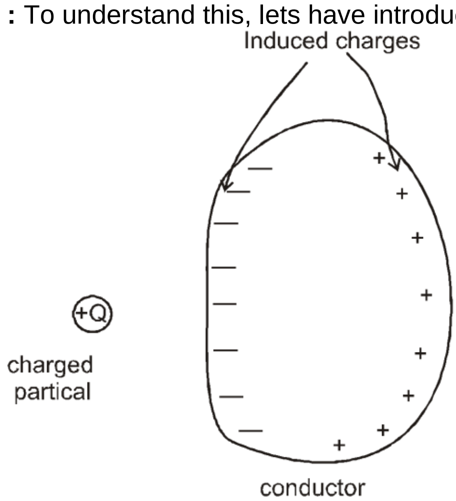
(ii) Insulator or Dielectric or Nonconductor : Non-conductors are the materials in which outer most electrons are very tightly bounded, so they cannot move (flow). Hence in a non-conductor there is no free electrons. Ex. plastic, rubber, wood etc.

(iii) Semi conductor : Semiconductor are the materials which have free electrons but very less in number.

Now lets see how the charging is done by conduction. In this method we take a charged conductor 'A' and an uncharged conductor 'B'. When both are connected some charge will flow from the charged body to the uncharged body. If both the conductors are identical & kept at large distance, if connected to each other, then charge will be divided equally in both the conductors otherwise they will flow till their electric potential becomes same. Its detailed study will be done in last section of this chapter.



(c) **Charging by Induction :** To understand this, lets have introduction to induction.

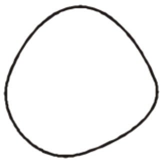


We have studied that there are lot of free electrons in the conductors. When a charge particle +Q is brought near a neutral conductor. Due to attraction of +Q charge, many electrons (–ve charges) come closer and accumulate on the closer surface. On the other hand a positive charge (deficiency of electrons) appears on the other surface. The flow of charge continues till there is resultant force on free electrons of the conductor becomes zero. This phenomena is called induction, and charges produced are called induced charges.

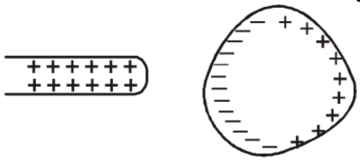
A body can be charged by induction in the following two ways :

Method I :

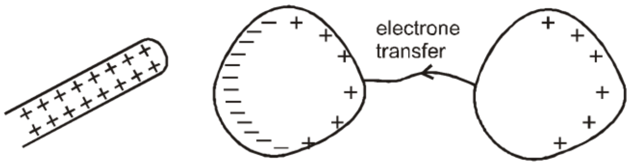
Step 1. Take an isolated neutral conductor..



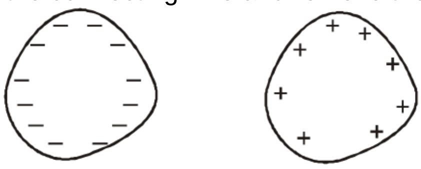
Step 2. Bring a charged rod near to it. Due to the charged rod, charges will induce on the conductor.



Step 3. Connect another neutral conductor with it. Due to attraction of the rod, some free electrons will move from the right conductor to the left conductor and due to deficiency of electrons positive charges will appear on right conductor and on the left conductor there will be excess of electrons due to transfer from right conductor..



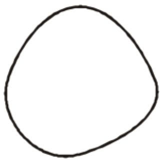
Step 4. Now disconnect the connecting wire and remove the rod.



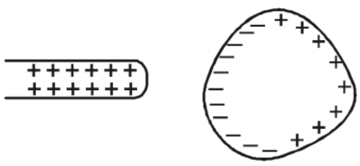
The first conductor will be negatively charged and the second conductor will be positively charged.

Method II

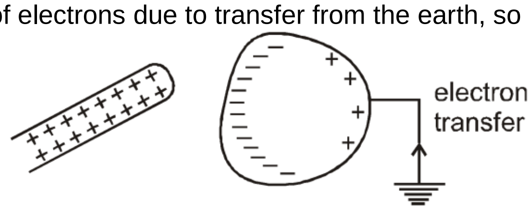
Step 1. Take an isolated neutral conductor..



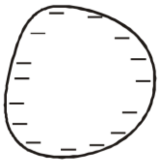
Step 2. Bring a charged rod near to it. Due to the charged rod, charges will induce on the conductor.



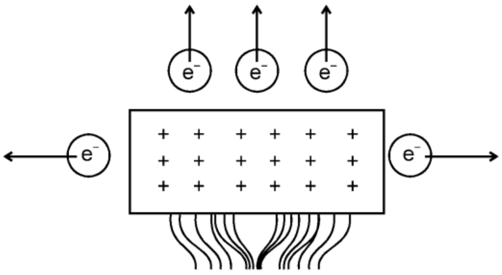
Step 3. Connect the conductor to the earth (this process is called grounding or earthling). Due to attraction of the rod, some free electrons will move from earth to the conductor, so in the conductor there will be excess of electrons due to transfer from the earth, so net charge on conductor will be negative.



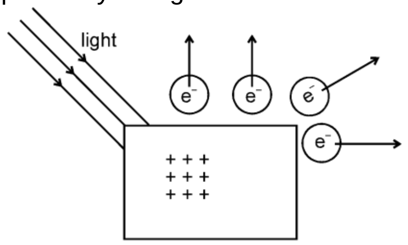
Step 4. Now disconnect the connecting wire. Conductor becomes negatively charge.



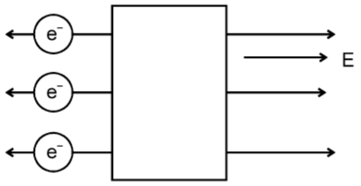
(d) Thermionic emission : When the metal is heated at a high temperature then some electrons of metals are ejected and the metal becomes positively charged.



(e) Photoelectric effect : When light of sufficiently high frequency is incident on metal surface then some electrons gains energy from light and come out of the metal surface and remaining metal becomes positively charged.



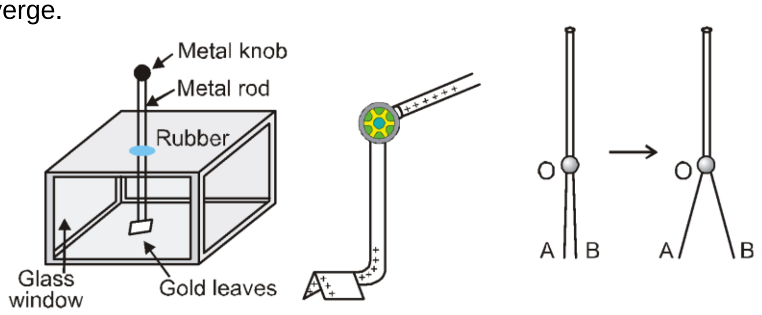
(f) Field emission : When electric field of large magnitude is applied near the metal surface then some electrons come out from the metal surface and hence the metal gets positively charged.



2.4 Gold Leaf Electroscope (GLE)

a. A simple apparatus to detect charge on a body is the gold-leaf electroscope

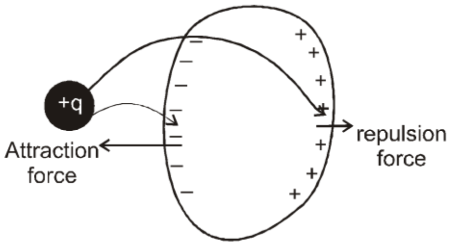
b. It consists of a vertical metal rod housed in a box, with two thin gold leaves attached to its bottom end. When a charged object touches the metal knob at the top of the rod, charge flows on to the leaves and they diverge.



Solved Examples

Example 1. If a charged body is placed near a neutral conductor, will it attract the conductor or repel it?

Solution :



If a charged body (+ve) is placed leftside near a neutral conductor, (–ve) charge will induce at left surface and (+ve) charge will induce at right surface. Due to positively charged body –ve induced charge will feel attraction and the +ve induced charge will feel repulsion. But as the –ve induced charge is nearer, so the attractive force will be greater than the repulsive force. So the net force on the conductor due to positively charged body will be attractive. Similarly we can prove for negatively charged body also.

From the above example we can conclude that. "A charged body can attract a neutral body."

If there is attraction between two bodies then one of them may be neutral. But if there is repulsion between two bodies, both must be charged (similarly charged).

So **"repulsion is the sure test of electrification".**

Example 2. A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:

(1) positive (2) negative (3) zero (4) can't say

Ans. (2, 3)

Example 3. Five styrofoam balls A, B, C, D and E are used in an experiment. Several experiments are performed on the balls and the following observations are made :

(i) Ball A repels C and attracts B.

(ii) Ball D attracts B and has no effect on E.

(iii) A negatively charged rod attracts both A and E.

For your information, an electrically neutral Styrofoam ball is very sensitive to charge induction, and gets attracted considerably, if placed nearby a charged body. What are the charges, if any, on each ball ?

	A	B	C	D	E
(1)	+	–	+	0	+
(2)	+	–	+	+	0
(3)	+	–	+	0	0
(4)	–	+	–	0	0

Ans. 3

Solution: From (i), As A repels C, so both A and C must be charged similarly. Either both are +ve or both are

–ve. As A also attract B, so charge on B should be opposite of A or B may be uncharged conductor.

From (ii) As D has no effect on E, so both D and E should be uncharged, and as B attracts uncharged D, so B must be charged and D must be on uncharged conductor.

From (iii) a –ve charged rod attract the charged ball A, so A must be +ve, and from exp. (i) C must also be +ve and B must be –ve.

Example 4. Charge conservation is always valid. Is it also true for mass?

Solution: No, mass conservation is not always. In some nuclear reactions, some mass is lost and it is converted into energy.

Example 5. What are the differences between charging by induction and charging by conduction ?

Solution: Major differences between two methods of charging are as follows :

(i) In induction, two bodies are close to each other but do not touch each other while in conduction they touch each other. (or they are connected by a metallic wire)

(ii) In induction, total charge of a body remains unchanged while in conduction it changes.

(iii) In induction, induced charge is always opposite in nature to that of source charge while in conduction charge on two bodies finally is of same nature.

Example 6. If a glass rod is rubbed with silk it acquires a positive charge because :

(1) protons are added to it (2) protons are removed from it

(3) electrons are added to it (4) electrons are removed from it.

Ans. 4

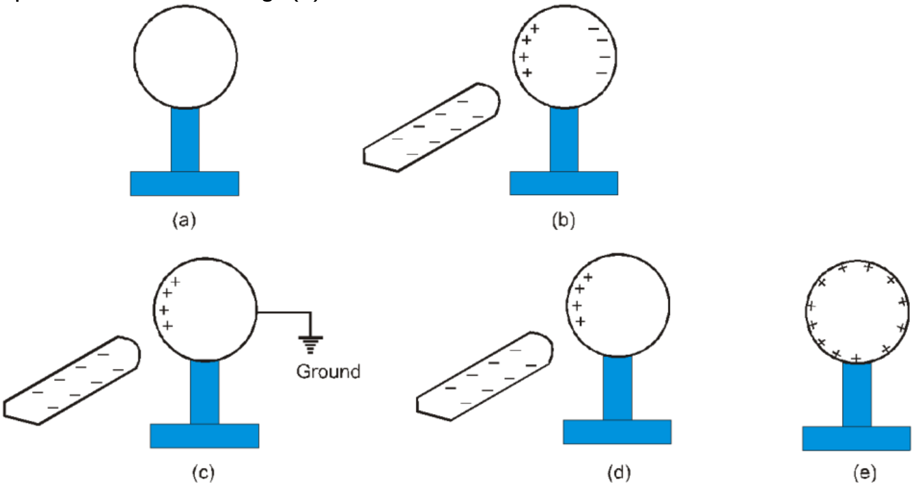
Example 7. How can you charge a metal sphere positively without touching it?

Solution : Figure (a) shows an uncharged metallic sphere on an insulating stand.

Bring a negatively charged rod close to the metallic sphere, as shown in Fig. (b). As the rod is brought close to the sphere, the free electrons in the sphere move away due to repulsion and start piling up at the farther end. The near end becomes positively charged due to deficit of electrons. This process of charge distribution stops when the net force on the free electrons inside the metal is zero.

Connect the sphere to the ground by a conducting wire. The electrons will flow to the ground while the positive charges at the near end will remain held there due to the attractive force of the negative charges on the rod, as shown in Fig. (c).

Disconnect the sphere from the ground. The positive charge continues to be held at the near end Fig.(d) Remove the electrified rod. The positive charge will spread uniformly over the sphere as shown in Fig. (e).



Example 8. If 10^{9} electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body ?

Solution In one second 10^{9} electrons move out of the body. Therefore the charge given out in one second is $1.6 \times 10^{-19} \times 10^{9}$ C = 1.6×10^{-10} C.

The time required to accumulate a charge of 1 C can then be estimated to be

$$\frac{1C}{1.6 \times 10^{-10} C/s} = 6.25 \times 10^{9} \text{ s} = \frac{6.25 \times 10^{9}}{365 \times 24 \times 3600} \text{ years} = 198 \text{ years.}$$

Thus to collect a charge of one coulomb, from a body from which 10^{9} electrons move out every second, we will need approximately 200 years. One coulomb is, therefore, a very large unit for many practical purposes.

Example 9. How much positive and negative charge is there in a cup of water?

Solution : Let us assume that the mass of one cup of water is 250 g.

The molecular mass of water is 18g.

One mole(= 6.02×10^{23} molecules) of water is 18 g. Therefore the number of molecules in one cup of water is $\frac{250 \times 10^{9}}{18} \times 6.02 \times 10^{23}$

Each molecule of water contains two hydrogen atoms and one oxygen atom, i.e., 10 electrons and 10 protons. Hence the total positive and total negative charge has the same magnitude. It is equal to $\frac{250 \times 10^{9}}{18} \times 6.02 \times 10^{23} \times 10 \times 1.6 \times 10^{-19}$ C = 1.34×10^{7} C.

Example 10. Which is bigger, a coulomb or charge on an electron? How many electronic charges form one coulomb of charge ?

Solutions : A coulomb of charge is bigger than the charge on an electron.

Magnitude of charge on one electron, e = 1.6×10^{-19} coulomb

Number of electronic charge in one coulomb, $n = \frac{q}{e} = \frac{1}{1.6\times10^{-19}}$ = 0.625×10^19^

Example 11. Assume that each atom in a copper wire contributes one free electron. Estimate the number of free electrons in a copper wire having a mass of 6.4 g (take the atomic weight of copper to be 64g/mol).

Solutions : Number of atoms in 64 g of copper = 6.023×10^{23}

Number of atoms in 6.4 g of copper = $\frac{6.023\times10^{23}}{64}\times6.4$ = 6.023×10^{22}

As each atom contributes one free electron, therefore, number of free electrons in copper wire = 6.023×10^{22}.

Self Practice Problems

1. A body can be negatively charged by

(1) Giving excess of electrons to it (2) Removing some electrons from it
(3) Giving some protons from it (4) Removing some neturons from it

2. The minimum charge on an object is

(1) 1 coulomb (2) 1 stat coulomb (3) 1.6×10^{-19} coulomb (4) 3.2×10^{-19} coulomb

Ans. **1. (1) 2. (3)**

3. COULOMB'S LAW (INVERSE SQUARE LAW)

On the basis of experiments Coulomb established the following law known as Coulomb's law.
The magnitude of electrostatic force between two point charges is directly proportional to the product of charges and inversely proportional to the square of the distance between them.

i.e. $F \propto q_1q_2$ and $F \propto \frac{1}{r^2}$ $\Rightarrow$ $F \propto \frac{q_1q_2}{r^2}$ $\Rightarrow$ $F = \frac{Kq_1q_2}{r^2}$

Important points regarding Coulomb's law :

(i) It is applicable only for point charges.

(ii) The constant of proportionality K in SI units in vacuum is expressed as $\frac{1}{4\pi\varepsilon_0}$ and in any other medium expressed as $\frac{1}{4\pi\varepsilon}$. If charges are dipped in a medium then electrostatic force on one charge is $\frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1q_2}{r^2}$. ε_0 and ε are called permittivity of vacuum and absolute permittivity of the medium respectively. The ratio $\varepsilon/\varepsilon_0 = \varepsilon_r$ is called relative permittivity of the medium, which is a dimensionless quantity.

(iii) The value of relative permittivity ε_r is constant for medium and can have values between 1 to ∞. For vacuum, by definition it is equal to 1. For air it is nearly equal to 1 and may be taken to be equal to 1 for calculations. For metals the value of ε_r is ∞ and for water is 81. The material in which more charge can induce ε_r will be higher.

(iv) The value of $\frac{1}{4\pi\varepsilon_0}$ = 9×10^9 $Nm^2 C^{-2}$ $\Rightarrow$ $\varepsilon_0 = 8.855 \times 10^{-12}$ C^2/Nm^2.
Dimensional formula of ε is $M^{-1} L^{-3} T^4 A^2$

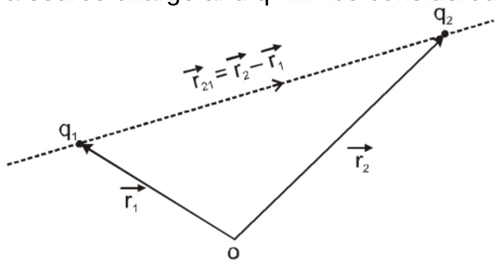
(v) The force acting on one point charge due to the other point charge is always along the line joining these two charges. It is equal in magnitude and opposite in direction on two charges, irrespective of the medium, in which they lie.

(vi) The force is conservative in nature i.e., work done by electrostatic force in moving a point charge along a close loop of any shape is zero.

(vii) Since the force is a central force, in the absence of any other external force, angular momentum of one particle w.r.t. the other particle (in two particle system) is conserved,

(viii) **Coulomb low in vector form**

Let us consider q_1 and q_2 are placed at positions $\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ and $\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ respectively. If we want to calculate coulomb force on q_2 due to q_1 then q_1 will be considered as a source charge and q_2 will be considered as test charge.



(a)

$\vec{F} = \dfrac{kq_1q_2}{|\vec{r}|^3}\vec{r}$ where $\vec{r}$ = position vector of test charge – position vector source charge.

($\vec{r}$ = position vector of test charge w.r.t. source charge)

(i) When we will use this formula in vector form then we have to put value of charges with their sign.

(ii) If the force $\vec{F}_{12}$ on charge q_1 due to charge q_2, and $\vec{F}_{21}$ is force on charge q_2 due to charge q_1 then $\vec{F}_{12} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_{21}^2}\hat{r}_{12} = -\vec{F}_{21}$

Solved Examples

Example 12. Find out the electrostatics force between two point charges placed in air (each of +1 C) if they are separated by 1m .

Solution : $F_e = \dfrac{kq_1q_2}{r^2} = \dfrac{9\times10^9\times1\times1}{1^2} = 9\times10^9$ N

☞ From the above result we can say that 1 C charge is too large to realize. In nature, charge is usually of the order of μC

Example 13. Two particles having charges q_1 and q_2 when kept at a certain distance, exert a force F on each other. If the distance between the two particles is reduced to half and the charge on each particle is doubled then what will be the force between the particles :

Ans. 16 F

Solution : $\because \quad F = \dfrac{kq_1q_2}{r^2}$

If $q'_1 = 2q_1$, $q'_2 = 2q_2$ $r' = \dfrac{r}{2}$,

then $F' = \dfrac{kq'_1\,q'_2}{r'^2} = \dfrac{k(2q_1)(2q_2)}{\left(\dfrac{r}{2}\right)^2}$ $F' = \dfrac{16kq_1q_2}{r^2}$ $F' = 16F$

Example 14. A particle of mass m carrying charge q_1 is revolving around a fixed charge $-q_2$ in a circular path of radius r. Calculate the period of revolution and its speed also.

Solution : $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2} = mr\omega^2 = \dfrac{4\pi^2mr}{T^2}$, $T^2 = \dfrac{(4\pi\varepsilon_0)r^2(4\pi^2mr)}{q_1q_2}$ or $T = 4\pi r\sqrt{\dfrac{\pi\varepsilon_0 mr}{q_1q_2}}$

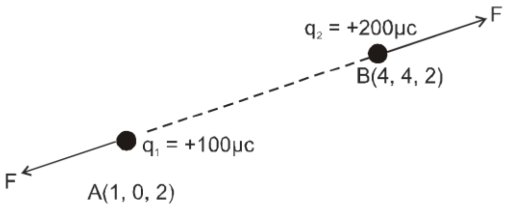
and also we can say that $\dfrac{q_1q_2}{4\pi\varepsilon_0 r^2} = \dfrac{mv^2}{r}$ $\Rightarrow$ $V = \sqrt{\dfrac{q_1q_2}{4\pi\varepsilon_0 mr}}$

Example 15. A point charge q_A = + 100 μc is placed at point A (1, 0, 2) m and an another point charge q_B = +200μc is placed at point B (4, 4, 2) m. Find :

(i) Magnitude of Electrostatic interaction force acting between them

(ii) Find $\vec{F}_A$ (force on A due to B) and $\vec{F}_B$ (force on B due to A) in vector form

Solution :

(i) 

Value of F : $|F| = \frac{kq_Aq_B}{r^2} = \frac{(9\times10^9)(100\times10^{-6})(200\times10^{-6})}{\sqrt{(4-1)^2+(4-0)^2+(2-2)^2}} = 7.2$ N

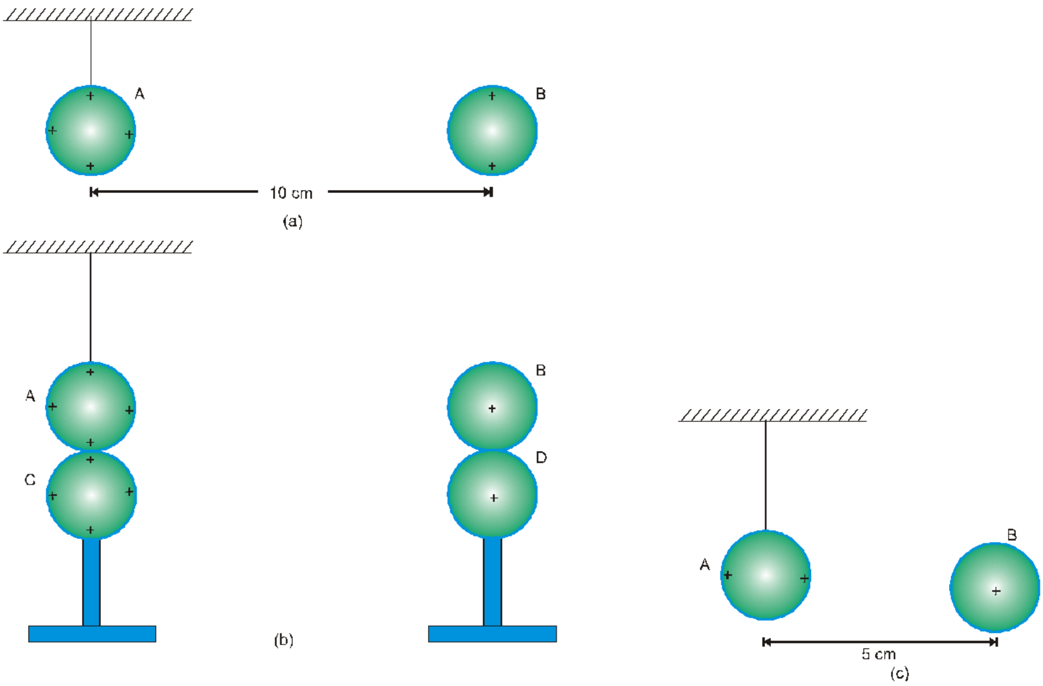
(ii) Force on B $\vec{F}_B = \frac{kq_Aq_B}{|\vec{r}|^3}\vec{r} = \frac{(9\times10^9)(100\times10^{-6})(200\times10^{-6})}{\sqrt{(4-1)^2+(4-0)^2+(2-2)^2}}\left[(4-1)\hat{i}+(4-0)\hat{j}+(2-2)\hat{k}\right]$

$= 7.2\left(\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}\right)$ N

Similarly $\vec{F}_A = 7.2\left(-\frac{3}{5}\hat{i}-\frac{4}{5}\hat{j}\right)$ N

Action($\vec{F}_A$) and Reaction ($\vec{F}_B$) are equal but in opposite direction.

Example 16. A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating handle is brought close to A such that the distance between their centres is 10 cm, as shown in Fig. (a). The resulting repulsion of A is noted (for example, by shining a beam of light and measuring the deflection of its shadow on a screen). Spheres A and B are touched by uncharged spheres C and D respectively, as shown in Fig. (b). C and D are then removed and B is brought closer to A to a distance of 5.0 cm between their centres, as shown in Fig. (c). What is the expected repulsion of A on the basis of Coulomb's law? Spheres A and C and spheres B and D have identical sizes. Ignore the sizes of A and B in comparison to the separation between their centres.



Solution : Let the original charge on sphere A be q and that on B be q'. At a distance r between their centres, the magnitude of the electrostatic force on each is given by

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq'}{r^2}$$

neglecting the sizes of spheres A and B in comparison to r. When an identical but uncharged sphere C touches A, the charges redistribute on A and C and, by symmetry, each sphere carries a charge q/2. Similarly, after D touches B, the redistributed charge on each is q'/2. Now, if the separation between A and B is halved, the magnitude of the electrostatic force on each is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(q'/2)}{(r/2)^2} = \frac{1}{4\pi\varepsilon_0}\frac{(qq')}{r^2} = F$$

Thus the electrostatic force on A, due to B, remains unaltered.

Self Practice Problems

3. A total charge Q is broken in two parts Q_1 and Q_2 and they are placed at a distance R from each other. the maximum force of repulsion between them will occur, when

(1) $Q_2 = \frac{Q}{R}$, $Q_1 = Q - \frac{Q}{R}$ (2) $Q_2 = \frac{Q}{4}$, $Q = Q - \frac{2Q}{3}$

(3) $Q_2 = \frac{Q}{4}$, $Q_1 = \frac{3Q}{4}$ (4) $Q_1 = \frac{Q}{2}$, $Q_2 = \frac{Q}{2}$

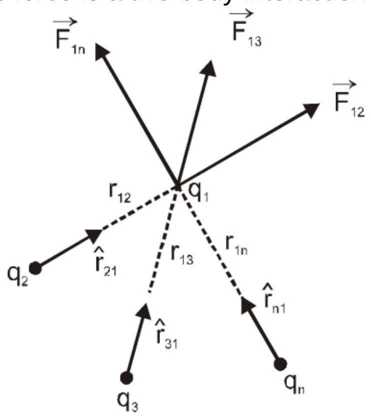
4. +2C and +6C two charge are repelling each other with a force of 12N. if each charge is given –2C of charge,the the value of the force will be

(1) 4N(Attractive) (2) 4N (Repulsive) (3) 8N (Repulsive) (4) Zero

Ans. : 3. (4) 4. (4)

4. PRINCIPLE OF SUPERPOSITION

The electrostatic force is a two body interaction i.e. electrical force



between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid i.e. force on charged particle due to number of point charges is the resultant of forces due to individual point charges.

Consider that n point charges $q_1, q_2, q_3, ... q_n$ are distributed in space in a discrete manner. The charges are interacting with each other. Let us find the total force on the charge, say q_1 due to all other remaingin charge. If the charge $q_2, q_3, ... q_n$ exert forces $\vec{F}_{12}, \vec{F}_{13}, ... \vec{F}_{1n}$ on the charge q_1, then according to principle of super-position, the total force on charge q_1 is given by

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...\vec{F}_{1n}$$

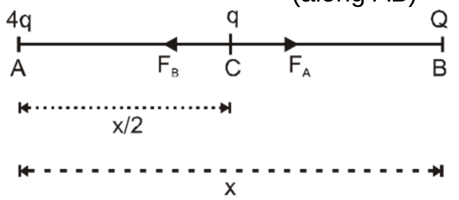
Solved Examples

Example 17. Two point charge of charge value Q and q are placed at a distance of x and x/2 respectively from a third charge of charge value 4q, all charges being in the same straight line. Calculate the magnitude and nature of charge Q, such that the net force experienced by the charge q is zero.

Solution : Suppose that the charge 4q is located at point A. The charges Q and q are placed at the points B and C, such that AB = x and AC = x/2. Also, all the charges lie on the same straight line. We assume that the charges of 4q and q are of same nature, a say positive.

Then, force on the charge q due to 4q,

$F_A = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q \cdot q}{(x/2)^2}$ (along AB)



The net force experienced by charge q will be zero only if the charge Q exerts force on the charge q equal and opposite to that exerted by the charge 4q. Thus, the charge Q should exert force F_B on charge q equal to F_A (in magnitude) and along CA. For this, charge Q has to be positive (i.e. of the nature same as that of 4q or q).

Now, force on the charge q due to charge Q,

$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(BC)^2}$

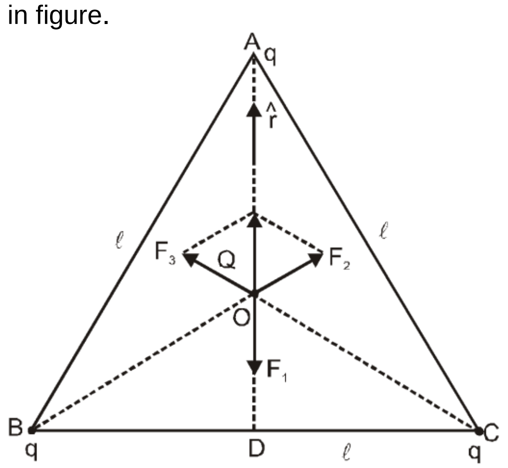
or

$F_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(x/2)^2}$ (along CA)

For net force on the charge q to be zero, $F_B = F_A$

$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q.q}{(x/2)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4q.q}{(x/2)^2}$ = Q = 4q

Example 18. Consider three point charges each having charge q at the vertices of an equilateral triangle of side ℓ. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, as shown in figure.



Solution : In the given equilateral triangle ABC of sides of length l, if we draw a perpendicular AD to the side BC, AD = AC cos 30º = $(\sqrt{3}/2)$ l and the distance AO of the centroid O from A is (2/3) AD = $(1/\sqrt{3})$ l. By symmetry AO = BO = CO.

Thus,

Force $\vec{F_1}$ on Q due to charge q at A = $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$ along AO

Force $\vec{F_2}$ on Q due to charge q at B = $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$ along BO

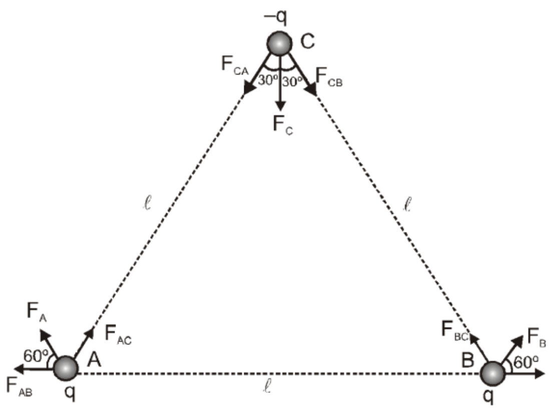
Force $\vec{F_3}$ on Q due to charge q at C = $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$ along CO

The resultant of forces $\vec{F_2}$ and $\vec{F_3}$ is $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}$ along OA, by the parallelogram law. Therefore, the total force on Q = $\frac{3}{4\pi\varepsilon_0}\frac{Qq}{\ell^2}(\vec{r}-\vec{r})$ = 0, where $\hat{r}$ is the unit vector along OA.

It is clear also by symmetry that the three forces will sum to zero. Suppose that the resultant force was non-zero but in some direction. Consider what would happen if the system was rotated through 60º about O.

Example 19. Consider the charges q, q, and –q placed at the vertices of an equilateral triangle of side ℓ. Calculate force on each charge?

Solution :



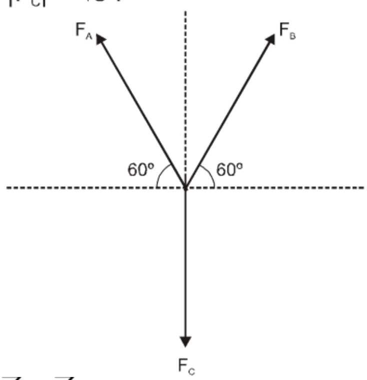
The forces acting on charge q at A due to charges q at B and –q at C are $\vec{F_{AB}}$ along BA and $\vec{F_{AC}}$ along AC respectively as shown in Fig.

The force of attraction or repulsion for each pair of charges has the same magnitude

$$F = \frac{q^2}{4\pi\varepsilon_0\ell^2}$$

$|F_A| = |F_B| = F$

$|F_C| = \sqrt{3}\,F$

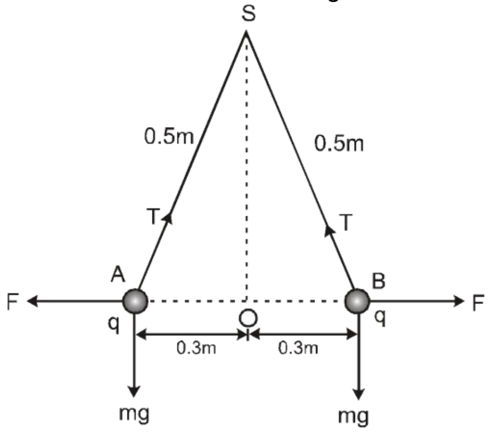


as shown in figure $\vec{F_A} + \vec{F_B} + \vec{F_C} = 0$.

It is interesting to see that the sum of the forces on three charges is zero.

Example 20. Two pith-balls each of mass weighing 10^{-4} kg are suspended from the same point by means of silk threads 0.5 m long. On charging the pith-balls equally, they are found to repel each other to a distance of 0.6 m. Calculate the charge on each ball. ($g = 10 m/s^2$)

Solution : Consider two pith balls A and B each having charge q and mass 10^{-13} kg. When the pith balls are suspended from point S by two threads each 0.5 m long, they repel each other to the distance AB = 0.2 m as shown in Fig.



Each of the two pith-balls is in equilibrium under the action of the following three forces :

(i) The electrostatic repulsive force F.
(ii) The weight mg acting vertically downwards.
(iii) The tension T in the string directed towards point S. The three forces mg, F and T can be represented by the therefore, according to triangle law of forces,

$$\frac{F}{OA} = \frac{mg}{SO} = \frac{T}{AS} \qquad \text{.............(i)}$$

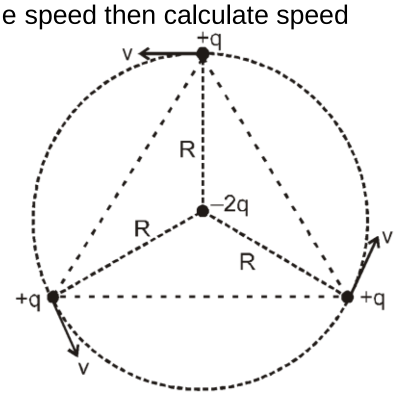
Here, $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{(AB)^2} = 9 \times 10^9 \times \frac{q^2}{(0.6)^2} N$

$mg = 10^{-4} \times 10 = 10^{-3}$ N

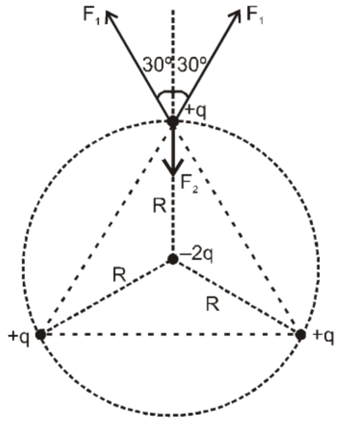
From the equation (i), we have

$F = mg \times \frac{OA}{SO}$ or $9 \times 10^9 \times \frac{q^2}{(0.6)^2} = 10^{-3} \times \frac{0.3}{\sqrt{(0.5)^2 - (0.3)^2}}$ or $q = \sqrt{3} \times 10^{-6}$ C

Example 21 Three equal point charges of charge +q are moving along a circle of radius R and a point charge –2q is also placed at the centre of circle as (shown in figure), if charges are revolving with constant and same speed then calculate speed



Solution :

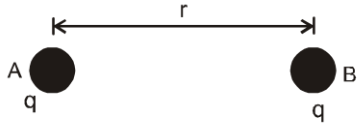


$F_2 - 2F_1 \cos 30 = \frac{mv^2}{R} \Rightarrow \frac{K(q)(2q)}{R^2} - \frac{2(Kq^2)}{(\sqrt{3}R)^2} \cos 30 = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{kq^2}{Rm}\left[2 - \frac{1}{\sqrt{3}}\right]}$

Example 22 Two equally charged identical small metallic spheres A and B repel each other with a force 2×10^{-5}N when placed in air (neglect gravitation attraction). Another identical uncharged sphere C is touched to B and then placed at the mid point of line joining A and B. What is the net electrostatic force on C?

Solution : Let initially the charge on each sphere be q and separation between their centres be r; then according to given problem.

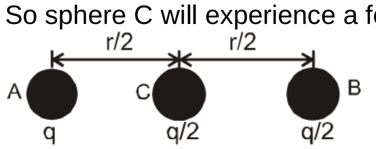
$F = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{r^2} = 2 \times 10^{-5}$ N



When sphere C touches B, the charge of B, q will distribute equally on B and C as sphere are identical conductors, i.e., now charges on spheres;

$q_B = q_C = (q/2)$

So sphere C will experience a force



$F_{CA} = \frac{1}{4\pi\varepsilon_0} \frac{q(q/2)}{(r/2)^2} = 2F$ along $\overrightarrow{AB}$ due to charge on A

and, $F_{CB} = \frac{1}{4\pi\varepsilon_0} \frac{(q/2)(q/2)}{(r/2)^2} = F$ along $\overrightarrow{BA}$ due to charge on B

So the net force F_C on C due to charges on A and B,

$F_C = F_{CA} - F_{CB} = 2F - F = 2 \times 10^{-5}$ N along $\overrightarrow{AB}$.

Example 23 Five point charges, each of value q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value –q coulomb placed at the centre of the hexagon?

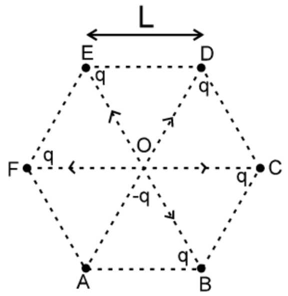
Solution : **Method : I**

If there had been a sixth charge +q at the remaining vertex of hexagon force due to all the six charges on –q at O would be zero (as the forces due to individual charges will balance each other), i.e.,

$$\vec{F}_R = 0$$

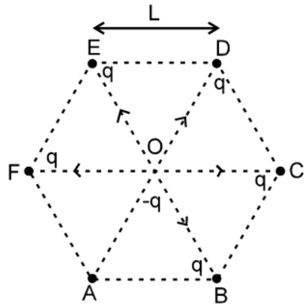
Now if $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

$\vec{F} + \vec{f} = 0$ i.e. $\vec{F} = -\vec{f}$ or, $|F| = |f| = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{L^2} = \frac{1}{4\pi\varepsilon_0} \frac{q^2}{L^2}$



$\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$ along CO

Method : II



In the diagram we can see that force due to charge A and D are opposite to each other

$\vec{F}_{DO} + \vec{F}_{AO} = 0$ ....(i)

Similarly $\vec{F}_{BO} + \vec{F}_{EO} = 0$ ....(ii) So $\vec{F}_{AO} + \vec{F}_{BO} + \vec{F}_{CO} + \vec{F}_{DO} + \vec{F}_{EO} = \vec{F}_{Net}$

Using (i) and (ii) $\vec{F}_{Net} = \vec{F}_{CO} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2}$ along CO.

Note : (1) **The total charge of A rod cannot be considered to be placed at the centre of the rod as we do in mechanics for mass in many problems.**

Note : (2) If a >> l then $F = \frac{KQq}{a^2}$ behaviour of the rod is just like a point charge.

5. ELECTROSTATIC EQUILIBRIUM

The point where the resultant force on a charged particle becomes zero is called equilibrium position.

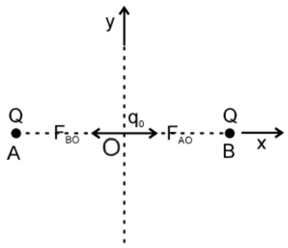
5.1 Stable Equilibrium : A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of stable equilibrium.

5.2 Unstable Equilibrium : If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position.

5.3 Neutral Equilibrium : If charge is displaced by a small distance and it is still in equilibrium condition then it is called neutral equilibrium.

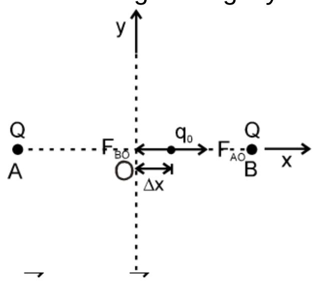
Solved Examples

Example 24 Two equal positive point charges 'Q' are fixed at points B(a, 0) and A(–a, 0). Another test charge q_0 is also placed at O(0, 0). Show that the equilibrium at 'O' is
(i) stable for displacement along X-axis.
(ii) unstable for displacement along Y-axis.



Solution : (i) Initially $\vec{F}_{AO} + \vec{F}_{BO} = 0 \Rightarrow |\vec{F}_{AO}| = |\vec{F}_{BO}| = \frac{KQq_0}{a^2}$

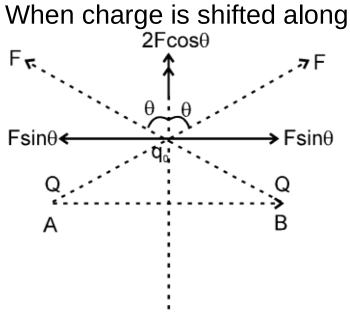
When charge is slightly shifted towards + x axis by a small distance Δx, then.



$|\vec{F}_{AO}| < |\vec{F}_{BO}|$

Therefore the particle will move towards origin (its original position) hence the equilibrium is stable.

(ii) When charge is shifted along y axis



After resolving components net force will be along y axis so the particle will not return to its original position so it is unstable equilibrium. Finally the charge will move to infinity.

Example 25. Two point charges of charge q_1 and q_2 (both of same sign) and each of mass m are placed such that gravitation attraction between them balances the electrostatic repulsion. Are they in stable equilibrium? If not then what is the nature of equilibrium?

Solution : In given example :

$$\frac{K q_1 q_2}{r^2} = \frac{Gm^2}{r^2}$$

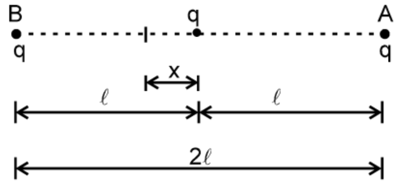
We can see that irrespective of distance between them charges will remain in equilibrium. If now distance is increased or decreased then there is no effect in their equilibrium. Therefore it is a neutral equilibrium.

Example 26. A particle of mass m and charge q is located midway between two fixed charged particles each having a charge q and a distance 2ℓ apart. Prove that the motion of the particle will be SHM if it is displaced slightly along the line connecting them and released. Also find its time period.

Solution : Let the charge q at the mid-point the displaced slightly to the left.

The force on the displaced charge q due to charge q at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell + x)^2}$$



The force on the displaced charge q due to charge at B,

$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell - x)^2}$

Net restoring force on the displaced charge q.

$F = F_2 - F_1$ or $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell - x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(\ell + x)^2}$

or $F = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(\ell - x)^2} - \frac{1}{(\ell + x)^2}\right] = \frac{q^2}{4\pi\varepsilon_0}\frac{4\ell x}{(\ell^2 - x^2)^2}$

Since $\ell >> x$, $\therefore F = \frac{q^2 \ell x}{\pi\varepsilon_0 \ell^4}$ or $F = \frac{q^2 x}{\pi\varepsilon_0 \ell^3}$

We see that $F \propto x$ and it is opposite to the direction of displacement. Therefore, the motion is SHM. $T = 2\pi\sqrt{\frac{m}{k}}$, here $k = \frac{q^2}{\pi \in_0 \ell^3}$ $= 2\pi\sqrt{\frac{m\pi \in_0 \ell^3}{q^2}}$

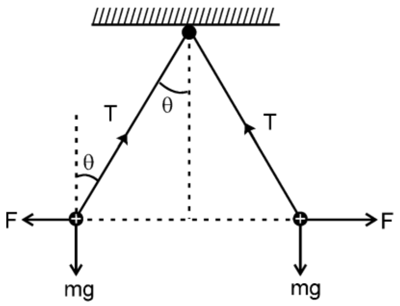
Example 27. Two identical charged spheres are suspended by strings of equal length. Each string makes an angle θ with the vertical. When suspended in a liquid of density σ = 0.8 gm/cc, the angle remains the same. What is the dielectric constant of the liquid? (Density of the material of sphere is ρ = 1.6 gm/cc.)

Solution : Initially as the forces acting on each ball are tension T, weight mg and electric force F, for its equilibrium along vertical,

$T\cos\theta = mg$ ...(1)

and along horizontal

$T\sin\theta = F$ ...(2)



Dividing Eqn. (2) by (1), we have

$\tan\theta = \frac{F}{mg}$ ... (3)

When the balls are suspended in a liquid of density σ and dielectric constant K, the electric force will become (1/K) times, i.e., F' = (F/K) while weight

$mg' = mg - F_B = mg - V\sigma g$ [as $F_B = V\sigma g$, where σ is density of material of sphere]

i.e., $mg' = mg\left[1 - \frac{\sigma}{\rho}\right]$ $\left[\text{as } V = \frac{m}{\rho}\right]$ So for equilibrium of ball,

$\tan\theta' = \frac{F'}{mg'} = \frac{F}{Kmg[1 - (\sigma/\rho)]}$ ... (4)

According to given information θ' = θ; so from equations (4) and (3), we have

$K = \frac{\rho}{(\rho - \sigma)} = \frac{1.6}{(1.6 - 0.8)} = 2$ **Ans.**

6. ELECTRIC FIELD

Electric field is the region around charged particle or charged body in which if another charge is placed, it experiences electrostatic force.

6.1 Electric field intensity $\vec{E}$:

Electric field intensity at a point is equal to the electrostatic force experienced by a unit positive point charge both in magnitude and direction.

If a test charge q_0 is placed at a point in an electric field and experiences a force $\vec{F}$ due to some charges (called source charges), the electric field intensity at that point due to source charges is given by

$$\vec{E} = \frac{\vec{F}}{q_0};$$

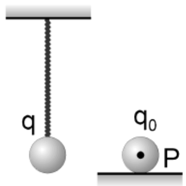
If the $\vec{E}$ is to be determined practically then the test charge q_0 should be small otherwise it will affect the charge distribution on the source which is producing the electric field and hence modify the quantity which is measured.

Solved Examples

Example 28. A positively charged ball hangs from a long silk thread. We wish to measure E at a point P in the same horizontal plane as that of the hanging charge. To do so, we put a positive test charge q_0 at the point and measure F/q_0. Will F/q_0 be less than, equal to, or greater than E at the point in question?

Solution : When we try to measure the electric field at point P then after placing the test charge at P it repels the source charge (suspended charge) and the measured value of electric field

$E_{measured} = \frac{F}{q_0}$ will be less than the actual value E_{act} that we wanted to measure.



6.2 Properties of electric field intensity $\vec{E}$:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Direction of electric field due to positive charge is always away from it while due to negative charge always towards it.

(iii) Its S.I. unit is Newton/Coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

(v) Electric force on a charge q placed in a region of electric field at a point where the electric field intensity is $\vec{E}$ is given by $\vec{F} = q\vec{E}$.
Electric force on point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

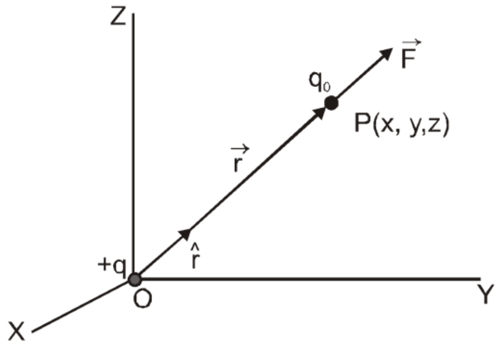
(vi) It obeys the superposition principle, that is, the field intensity at a point due to a system of charges is vector sum of the field intensities due to individual point charges.

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$$

(vii) It is produced by source charges. The electric field will be a fixed value at a point unless we change the distribution of source charges.

6.3 Electric field due to a point charge :

Consider that a point charge +q is placed at the origin O of the co-ordinate frame. Let P be the point, where electric field due to the point charge +q is to be determined. Let $\overline{OP} = \vec{r}$ be the position vector of the point P.



To find electric field at point P, place a vanishingly small positive test charge q_0 at point P. According to Coulomb's law, force on the test charge q_0 due to charge q is given by :

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\hat{r}$$

where $\hat{r}$ is unit vector along OP. If $\vec{E}$ is the electric field at point P, then

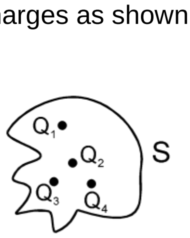
$$\vec{E} = \frac{\vec{F}}{q_0} = \left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{qq_0}{r^2}\hat{r}\right) \quad \text{or} \quad \vec{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^3}\vec{r}$$

The magnitude of the electric field at point P is given by

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

Solved Examples

Example 29. Electrostatic force experienced by –3μC charge placed at point 'P' due to a system 'S' of fixed point charges as shown in figure is $\vec{F} = (21\hat{i} + 9\hat{j})$ μN.



(i) Find out electric field intensity at point P due to S.
(ii) If now 2μC charge is placed and –3 μC is removed at point P then force experienced by it will be.

Solution : **(i)** $\vec{F} = q\vec{E}$ $\Rightarrow$ $(21\hat{i} + 9\hat{j})$μN = -3μC $(\vec{E})$

$\Rightarrow$ $\vec{E} = -7\hat{i} - 3\hat{j}\ \frac{\mu N}{C}$

(ii) Since the source charges are not disturbed the electric field intensity at 'P' will remain same.

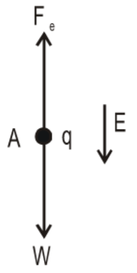
$\vec{F}_{2\mu C} = +2(\vec{E}) = 2(-7\hat{i} - 3\hat{j}) = -14\hat{i} - 6\hat{j}$ μN

Example 30. Calculate the electric field intensity which would be just sufficient to balance the weight of a particle of charge –10 μc and mass 10 mg. (take g = 10 ms^2)

Solution : As force on a charge q in an electric field $\vec{E}$ is

$\vec{F}_q = q\vec{E}$

So according to given problem



$|\vec{F_q}| = |\vec{W}|$ i.e., $|q|E = mg$

i.e., $E = \frac{mg}{|q|}$ = 10 N/C., in downward direction.

Self Practice Problems

5. The maguitude of elective field intensity E is such that an electron placed in it would experience an electrical force equal to its weight is given by

(1) mge (2) $\frac{mg}{e}$ (3) $\frac{e}{mg}$ (4) $\frac{e^2}{m^2}g$

6. The distance between the two charges 25μC and 36μC is 11cm At what point on the line joining the two, the intensity will be zero

(1) At a distance of 5cm from 25μC (2) At a distance of 5 cm from 36μC

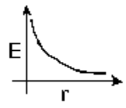
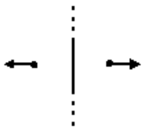
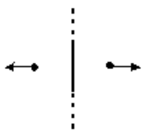
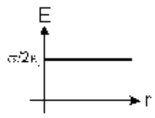
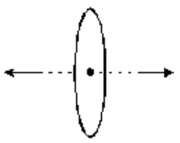
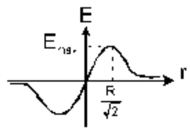
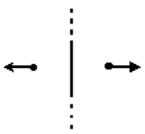
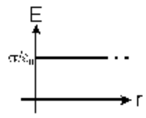
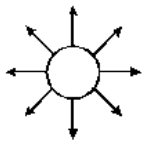
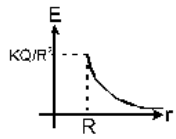
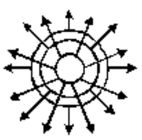
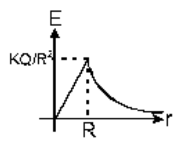
(3) At a distance of 10cm from 25μC (4) At a distance of 11cm from 36μC

7. A charge produce an electric field of 1 N\C at a point distant 0.1 m from it. The megnitude of charge is

(1) 1.11×10^{-12}C (2) 9.11×10^{-12}C (3) 7.11×10^{-6}C (4) None of these

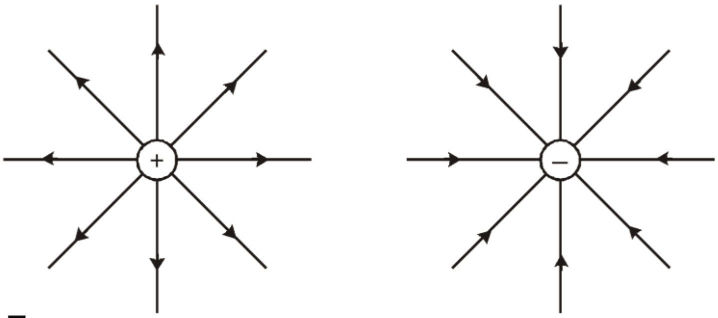
Ans. **5. (2) 6. (1) 7. (1)**

List of formula for Electric Field Intensity due to various types of charge distribution :

Name / Type	Formula	Note	Graph
Point charge 	$\vec{E}=\frac{Kq}{\lvert\vec{r}\rvert^2}\cdot\hat{r}$	* q is source charge. * $\vec{r}$ is vector drawn from source charge to the test point. outwards due to +charges & inwards due to –charges.	
Infinitely long line charge 	$\frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}=\frac{2K\lambda\hat{r}}{r}$	* η is linear charge density (assumed uniform) * r is perpendicular distance of point from line charge * is radial unit vector drawn from the charge to test point.	
Infinite non-conducting thin sheet 	$\frac{\sigma}{2\varepsilon_0}\hat{n}$	* σ is surface charge density. (assumed uniform) * is unit normal vector. * x – distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Uniformly charged ring 	$E=\frac{KQx}{\left(R^2+x^2\right)^{3/2}}$ $E_{centre}=0$	* Q is total charge of the ring * x – distance of point on the axis from centre of the ring. * electric field is always along the axis.	
Infinitely large charged conducting sheet 	$\frac{\sigma}{\varepsilon_0}\hat{n}$	* σ is the surface charge density (assumed uniform) * $\hat{n}$ is the unit vector perpendicular is the surface.	
Uniformly charged hollow conducting/ nonconducting /solid conducting sphere 	(i) for r ≥ R $\vec{E}=\frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r < R E = 0	* R is radius of the sphere. * $\vec{r}$ is vector drawn from centre of sphere to the point. * Sphere acts like a point charge placed at centre for points outside the sphere * $\vec{E}$ is always along radial direction. * Q is total charge (= σ4πR²). (σ = surface charge density)	
Uniformly charged solid nonconducting sphere (insulating material) 	(i) for r ≥ R $\vec{E}=\frac{kQ}{\lvert\vec{r}\rvert^2}\hat{r}$ (ii) for r < R $\vec{E}=\frac{kQ}{R^3}\vec{r}$	* $\vec{r}$ is vector drawn from centre of sphere to the point * Sphere acts like a point charge placed at the centre for points outside the sphere $\vec{E}$ is always along radial dirⁿ * Q is total charge (ρ $\frac{4}{3}\pi R^3$). (ρ = volume charge density) * Inside the sphere E ∝ r. * Outside the sphere E ∝ 1/r².	

Electric field due to point charge

$E = \dfrac{kq}{r^2} \Rightarrow \vec{E} = \dfrac{kq}{r^2}\hat{r} \Rightarrow \vec{E} = \dfrac{kq}{r^3}\vec{r}$



$\vec{r}$ = position vector of test point with respect to source charge

$\vec{r} = \vec{r}_{\text{test point}} - \vec{r}_{\text{source charge}}$

Solved Examples

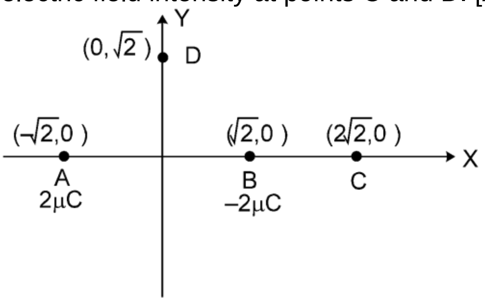
Example 31. Find out electric field intensity at point A (0, 1m, 2m) due to a point charge –20μC situated at point B($\sqrt{2}$ m, 0, 1m).

Solution : $\vec{E} = \dfrac{KQ}{|\vec{r}|^3}\vec{r} = \dfrac{KQ}{|\vec{r}|^2}\hat{r} \Rightarrow$ $\vec{r}$ = P.V. of A – P.V. of B (P.V. = Position vector)

$= (-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k})$ $|\vec{r}|$ = = 2

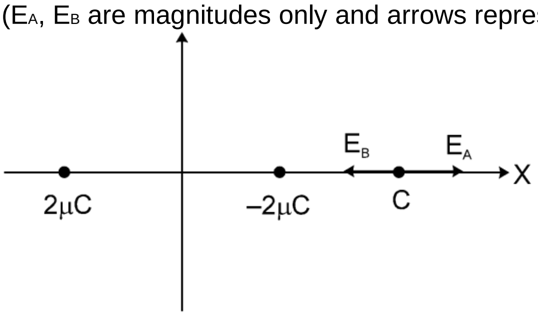
$\vec{E} = \dfrac{9\times10^9\times(-20\times10^{-6})}{8}(-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k}) = -22.5\times10^3(-\sqrt{2}\,\hat{i} + \hat{j} + \hat{k})$ N/C.

Example 32. Two point charges 2μc and – 2μc are placed at point A and B as shown in figure. Find out electric field intensity at points C and D. [All the distances are measured in meter].



Solution : Electric field at point C

(E_A, E_B are magnitudes only and arrows represent directions)

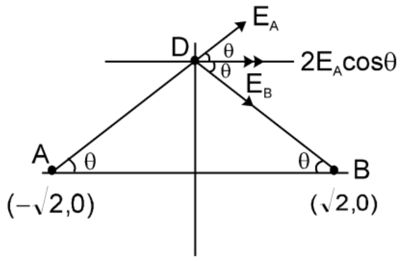


Electric field due to positive charge is away from it while due to negative charge it is towards the charge. It is clear that $E_B > E_A$.

∴ $E_{Net} = (E_B - E_A)$ towards negative X-axis

$= \dfrac{K(2\mu c)}{(\sqrt{2})^2} - \dfrac{K(2\mu c)}{(3\sqrt{2})^2}$ towards negative X-axis = $8000\,(-\hat{i})$ N/C

Electric field at point D :



Since magnitude of charges are same and also AD = BD

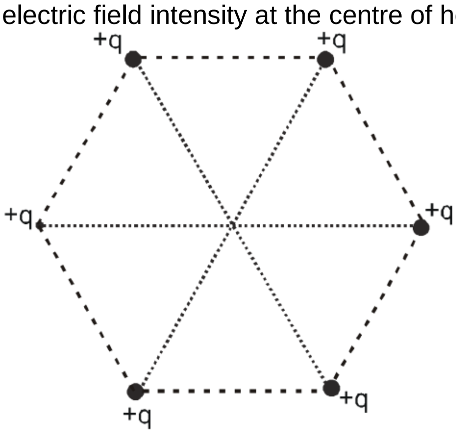
So $E_A = E_B$

Vertical components of $\vec{E}_A$ and $\vec{E}_B$ cancel each other while horizontal components are in the same direction.

So, $E_{net} = 2E_A\cos\theta = \frac{2.K(2\mu c)}{2^2}\cos45^0$

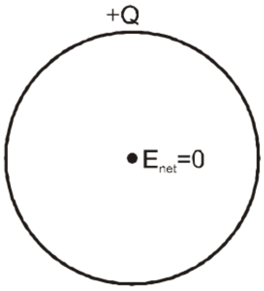
$= \frac{K\times10^{-6}}{\sqrt{2}} = \frac{9000}{\sqrt{2}}\hat{i}$ N/C.

Example 33. Six equal point charges are placed at the corners of a regular hexagon of side 'a'. Calculate electric field intensity at the centre of hexagon?



Ans Zero

Similarly electric field due to a uniformly charged ring at the centre of ring :



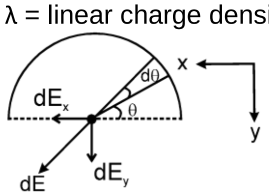
Note : **(i) Net charge on a conductor remains only on the outer surface of a conductor. This property will be discussed in the article of the conductor. (article no.17)**
(ii) On the surface of isolated spherical conductor charge is uniformly distributed.

6.4 Electric field due to a uniformly charged ring and arc.

Solved Examples

Example 34. Find out electric field intensity at the centre of uniformly charged semicircular ring of radius R and linear charge density λ.

Solution : λ = linear charge density.



The arc is the collection of large no. of point charges. Consider a part of ring as an element of length Rdθ which subtends an angle dθ at centre of ring and it lies between θ and θ + dθ

$\vec{dE} = dE_x\,\hat{i} + dE_y\,\hat{j}$ $\qquad E_x = \int dE_x = 0$ (due to symmetry)

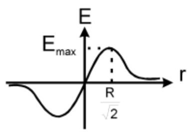
$E_y = \int dE_y = \int_0^{\pi} dE\sin\theta = \frac{K\lambda}{R}\int_0^{\pi}\sin\theta.d\theta = \frac{2K\lambda}{R}$

Example 35. Find out electric field intensity at the centre of uniformly charged quarter ring of radius R and linear charge density λ.

Solution : Refer to the previous quation $\vec{dE} = dE_x\,\hat{i} + dE_y\,\hat{j}$ on solving $\quad E_{net} = \frac{K\lambda}{R} = (\hat{i} + \hat{j})$,

Electric field due to ring on its axis :

$E_{net} = \frac{KQx}{[R^2 + x^2]^{3/2}}$



E will be max when $\frac{dE}{dx} = 0$, that is at $x = \frac{R}{\sqrt{2}}$ and $E_{max} = \frac{2KQ}{3\sqrt{3}\,R^2}$

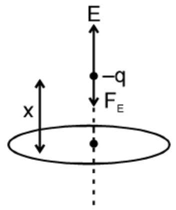
Case (i) : if x>>R, $E = \frac{KQ}{x^2}$ Hence the ring will act like a point charge

Case (ii) : if x<<R, $E = \frac{KQ\,x}{R^3}$

Solved Examples

Example 36. Positive charge Q is distributed uniformly over a circular ring of radius R. A point particle having a mass m and a negative charge –q, is placed on its axis at a distance x from the centre. Find the force on the particle. Assuming x << R, find the time period of oscillation of the particle if it is released from there. (Neglect gravity)

Solution : When the negative charge is shifted at a distance x from the centre of the ring along its axis then force acting on the point charge due to the ring:



$F_E = qE$ (towards centre) $= q\left[\frac{KQx}{(R^2+x^2)^{3/2}}\right]$

if R >>x then

$R^2 + x^2 \simeq R^2$

$F_E = \frac{1}{4\pi\varepsilon_0}\frac{Qqx}{R^3}$ (Towards centre)

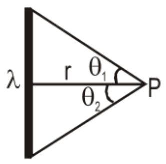
Since restoring force $F_E \propto x$, therefore motion of charge the particle will be S.H.M.

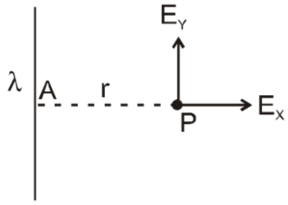
Time period of SHM.

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{\left(\frac{Qq}{4\pi\varepsilon_0 R^3}\right)}} = \left[\frac{16\pi^3\varepsilon_0 mR^3}{Qq}\right]^{1/2}$

6.5 Electric field due to uniformly charged wire

(i) Line charge of finite length : Derivation of expression for intensity of electric field at a point due to line charge of finite size of uniform linear charge density λ. The perpendicular distance of the point from the line charge is r and lines joining ends of line charge distribution make angle θ_1 and θ_2 with the perpendicular line.





$E_x = \frac{K}{r}[\sin\theta_1 + \sin\theta_2]$ ....(1)

$E_y = \frac{K}{r}[\cos\theta_2 - \cos\theta_1]$

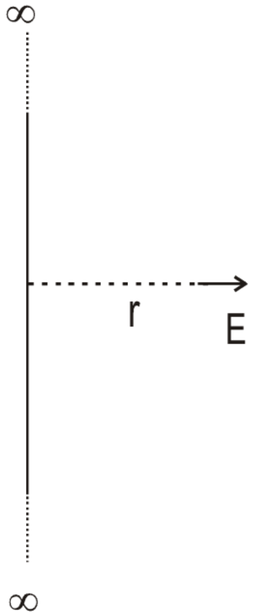
Net electric field at the point

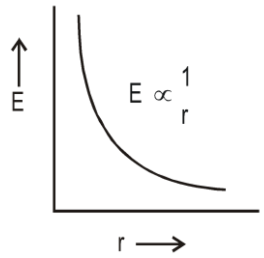
$E_{net} = \sqrt{E_x^2 + E_y^2}$

(ii) We can derive a result for infinitely long line charge

In above eq. (1) & (2) if we put $\theta_1 = \theta_2 = 90^o$ we can get required result.

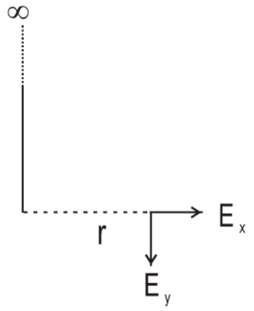
$E_{net} = E_x = \frac{2K\lambda}{r}$





(iii) **For Semi- infinite wire**

$\theta_1 = 90^o$ and $\theta_2 = 0^o$ so $E_x = \frac{K\lambda}{r}$, $E_y = \frac{K\lambda}{r}$

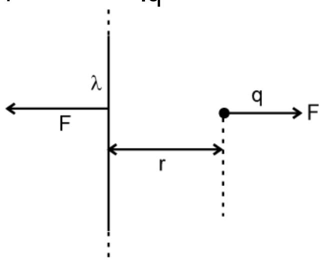


Solved Examples

Example 37. A point charge q is placed at a distance r from a very long charge thread of uniform linear charge density λ. Find out total electric force experienced by the line charge due to the point charge. (Neglect gravity).

Solution : Force on charge q due to the thread,

$F = \left(\frac{2K\lambda}{r}\right).q$



By Newton's III law, every action has equal and opposite reaction so force on the thread = $\frac{2K\lambda}{r}.q$ (away from point charge)

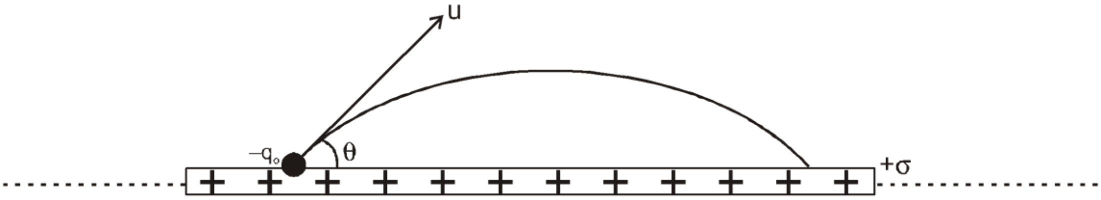
6.6 Electric field due to uniformly charged infinite sheet

$E_{net} = \frac{\sigma}{2\varepsilon_0}$ toward normal direction

Note: **(1) The direction of electric field is always perpendicular to the sheet.**
(2) The magnitude of electric field is independent of distance from sheet.

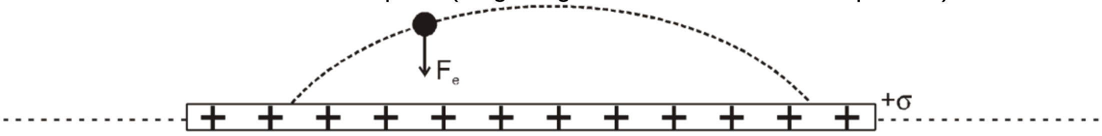
Solved Examples

Example 38. An infinitely large plate of surface charge density $+\sigma$ is lying in horizontal xy plane. A particle having charge $-q_0$ and mass m is projected from the plate with velocity u making an angle θ with sheet. Find :



(i) the time taken by the particle to return on the plate..
(ii) maximum height achieved by the particle.
(iii) At what distance will it strike the plate (Neglect gravitational force on the particle)

Solution :



Electric force acting on the particle $F_e = q_0E$: $F_e = (q_0)\left(\frac{\sigma}{2\varepsilon_0}\right)$ downward

So acceleration of the particle : $a = \frac{F_e}{m} = \frac{q_0\sigma}{2\varepsilon_0 m}$ = uniform

this acceleration will act like 'g' (acceleration due to gravity)

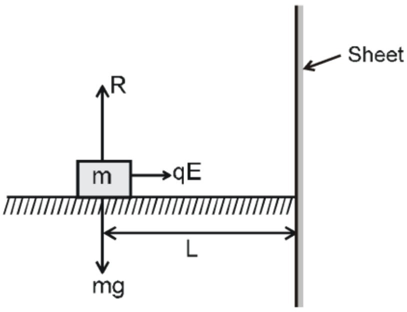
So the particle will perform projectile motion.

(i) $T = \frac{2u\sin\theta}{g} = \frac{2u\sin\theta}{\left(\frac{q_0\sigma}{2\varepsilon_0 m}\right)}$ (ii) $H = \frac{u^2\sin^2\theta}{2g} = \frac{2u^2\sin^2\theta}{2\left(\frac{q_0\sigma}{2\varepsilon_0 m}\right)}$ (iii) $R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin 2\theta}{\left(\frac{q_0\sigma}{2\varepsilon_0 m}\right)}$

Example 39. A block having mass m and charge –q is resting on a frictionless plane at a distance L from fixed large non-conducting infinite sheet of uniform charge density σ as shown in Figure. Discuss the motion of the block assuming that collision of the block with the sheet is perfectly elastic. Is it SHM?

Solution : The situation is shown in Figure. Electric force produced by sheet will accelerate the block towards the sheet producing an acceleration. Acceleration will be uniform because electric field E due to the sheet is uniform.

$a = \frac{F}{m} = \frac{qE}{m}$, where $E = \sigma/2\varepsilon_0$



As initially the block is at rest and acceleration is constant, from second equation of motion, time taken by the block to reach the wall

$L = \frac{1}{2}at^2$ i.e., $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{aE}} = \sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$

As collision with the wall is perfectly elastic, the block will rebound with same speed and as now its motion is opposite to the acceleration, it will come to rest after travelling same distance L in same time t. After stopping it will be again accelerated towards the wall and so the block will execute oscillatory motion with 'span' L and time period.

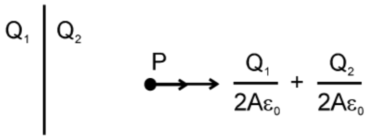
$$T = 2t = 2\sqrt{\frac{2mL}{aE}} = 2\sqrt{\frac{4mL\varepsilon_0}{a\sigma}}$$

However, as the restoring force F = qE is constant and not proportional to displacement x, the motion is not simple harmonic.

Example 40. If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Solution : Electric field at point P :

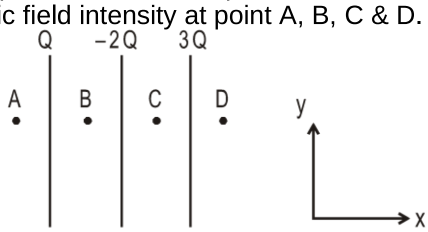
$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$



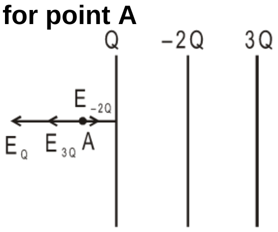
$$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example 41. Three large conducting parallel sheets are placed at a finite distance from each other as shown in figure. Find out electric field intensity at point A, B, C & D.

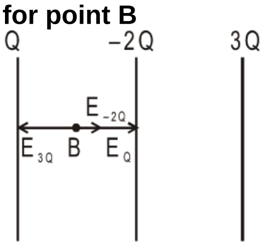


Solution : **for point A**



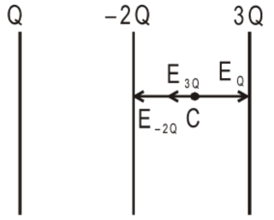
$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = -\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{Q}{A\varepsilon_0}\hat{i}$$

for point B



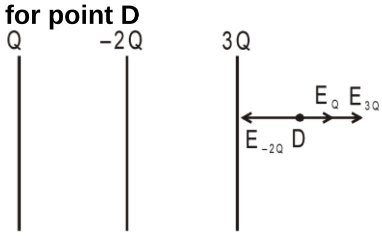
$$\vec{E}_{net} = \vec{E}_{3Q} + \vec{E}_{-2Q} + \vec{E}_Q = -\frac{3Q}{2A\varepsilon_0}\hat{i} + \frac{2Q}{2A\varepsilon_0}\hat{i} + \frac{Q}{2A\varepsilon_0}\hat{i} = 0$$

for point C



$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} - \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = -\frac{2Q}{A\varepsilon_0}\hat{i}$$

for point D

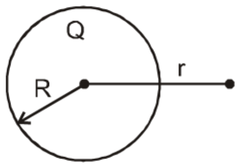


$$\vec{E}_{net} = \vec{E}_Q + \vec{E}_{3Q} + \vec{E}_{-2Q} = +\frac{Q}{2A\varepsilon_0}\hat{i} + \frac{3Q}{2A\varepsilon_0}\hat{i} - \frac{2Q}{2A\varepsilon_0}\hat{i} = \frac{Q}{A\varepsilon_0}\hat{i}$$

6.7 Electric field due to uniformly charged spherical shell

$E = \frac{KQ}{r^2}$ $\quad r \geq R \quad \Rightarrow$ **For the out side points & point on the surface the uniformly charged spherical shell behaves as a point charge placed at the centre**

$E = 0$ $\quad r < R$

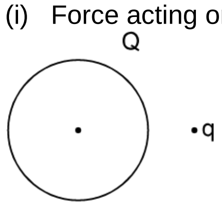


Electric field due to spherical shell out side it is always along the radial direction.

Solved Examples

Example 42. Figure shows a uniformly charged sphere of radius R and total charge Q. A point charge q is situated outside the sphere at a distance r from centre of sphere. Find out the following :

(i) Force acting on the point charge q due to the sphere.



(ii) Force acting on the sphere due to the point charge.

Solution : (i) Electric field at the position of point charge

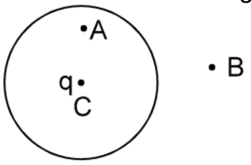
$\vec{E} = \frac{KQ}{r^2}\hat{r}$ so, $\vec{F} = \frac{KqQ}{r^2}\hat{r}$ $\quad |\vec{F}| = \frac{KqQ}{r^2}$

(ii) Since we know that every action has equal and opposite reaction so

$$\vec{F}_{sphere} = -\frac{KqQ}{r^2}\hat{r}$$

$$|\vec{F}_{sphere}| = \frac{KqQ}{r^2}$$

Example 43. Figure shows a uniformly charged thin sphere of total charge Q and radius R. A point charge q is also situated at the centre of the sphere.
Find out the following :



(i) Force on charge q (ii) Electric field intensity at A. (iii) Electric field intensity at B.

Solution : (i) Electric field at the centre of the uniformly charged hollow sphere = 0
So force on charge q = 0
(ii) Electric field at A

$\vec{E}_A = \vec{E}_{Sphere} + \vec{E}_q = 0 + \frac{Kq}{r^2}$; r = CA

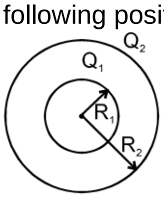
E due to sphere = 0 , because point lies inside the charged hollow sphere.

(iii) Electric field $\vec{E}_B$ at point B = $\vec{E}_{Sphere} + \vec{E}_q$

$= \frac{KQ}{r^2}.\hat{r} + \frac{Kq}{r^2}.\hat{r} = \frac{K(Q+q)}{r^2}.\hat{r}$; r = CB

Note : Here we can also assume that the total charge of sphere is concentrated at the centre, for calculation of electric field at B.

Example 44. Two concentric uniformly charged spherical shells of radius R_1 and R_2 ($R_2 > R_1$) have total charges Q_1 and Q_2 respectively. Derive an expression of electric field as a function of r for following positions.



(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$

Solution : (i) for $r < R_1$,
therefore point lies inside both the spheres

$E_{net} = E_{Inner} + E_{outer} = 0 + 0$

(ii) for $R_1 \le r < R_2$,
therefore point lies outside inner sphere but inside outer sphere:

$E_{net} = E_{inner} + E_{outer}$

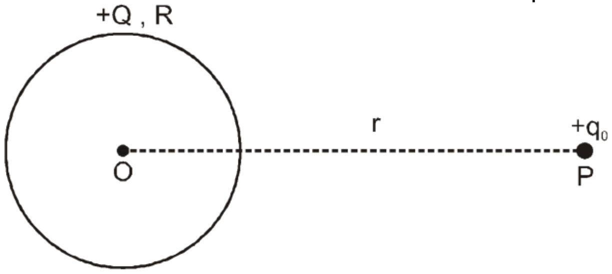
$= \frac{KQ_1}{r^2} + 0 = \frac{KQ_1}{r^2}\hat{r}$

(iii) for $r \ge R_2$
point lies outside inner as well as outer sphere therefore.

$E_{Net} = E_{inner} + E_{outer}$

$= \frac{KQ_1}{r^2}\hat{r} + \frac{KQ_2}{r^2}\hat{r} = \frac{K(Q_1+Q_2)}{r^2}\hat{r}$

Example 45. A spherical shell having charge +Q (uniformly distributed) and a point charge + q_0 are placed as shown. Find the force between shell and the point charge(r>>R).

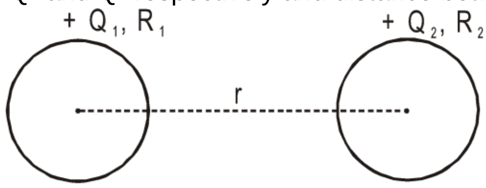


(i) Force on the point charge + q_0 due to the shell

$= q_0\vec{E}_{shell} = (q_0)\left(\frac{KQ}{r^2}\right)\hat{r} = \frac{KQq_0}{r^2}\hat{r}$ where $\hat{r}$ is unit vector along OP.
From action - reaction principle, force on the shell due to the point charge will also be $F_{shell} = \frac{KQq_0}{r^2}(-\hat{r})$

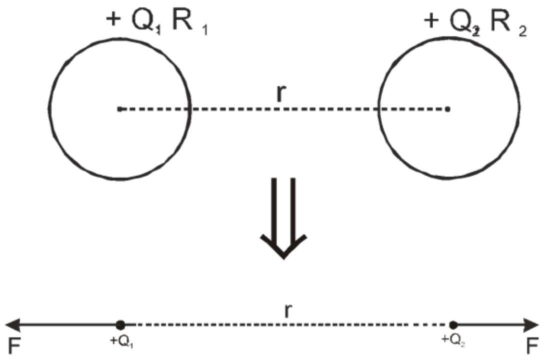
Conclusion - To find the force on a hollow sphere due to outside charges , we can replace the sphere by a point charge kept at centre.

Example 46. Find force acting between two shells of radius R_1 and R_2 which have uniformly distributed charges Q_1 and Q_2 respectively and distance between their centre is r.



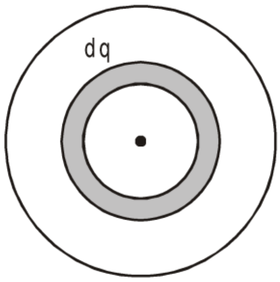
Solution : The shells can be replaced by point charges kept at centre so force between them

$F = \frac{KQ_1Q_2}{r^2}$



6.8 Electric field due to uniformly charged solid sphere

Derive an expression for electric field due to solid sphere of radius R and total charge Q which is uniformly distributed in the volume,
at a point which is at a distance r from centre for given two cases.



(i) $r \geq R$ (ii) $r \leq R$

Assume an elementry concentric shell of charge dq. Due to this shell the electric field at the point (r > R) will be

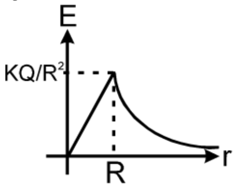
$dE = \frac{Kdq}{r^2}$ [from above result of hollow sphere]

$E_{net} = \int dE = \frac{KQ}{r^2}$

For r < R, there will be no electric field due to shell of radius greater than r, so electric field at the point will be present only due to shells having radius less than r.

$$E'_{net} = \frac{KQ'}{r^2}$$

here $Q' = \dfrac{\dfrac{Q}{\frac{4}{3}\pi R^3} \times \dfrac{4}{3}\pi r^3}{} = \dfrac{Qr^3}{R^3}$



$E'_{net} = \dfrac{KQ'}{r^2} = \dfrac{KQr}{R^3}$ away from the centre.

Note : The electric field inside and outside the sphere is always in radial direction.

7. ELECTRIC POTENTIAL

In electrostatic field the electric potential (due to some source charges) at a point P is defined as the work done by external agent in taking a point unit positive charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy..

7.1 Mathematical representation :

If $(W_{\infty \to P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{W_{\infty p})_{ext}}{q}\right]_{\Delta K=0} = \frac{-W_{elc})_{\infty \to p}}{q}$$

Note : (i) $(W_{\infty \to P})_{ext}$ can also be called as the work done by external agent against the electric force on a unit positive charge due to the source charge.

(ii) Write both W and q with proper sign.

7.2 Properties :

(i) Potential is a scalar quantity, its value may be positive, negative or zero.

(ii) S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulmb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

(iii) Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

(iv) Electric potential due to a positive charge is always positive and due to negative charge it is always negative except at infinite. (taking $V_\infty = 0$).

(v) Potential decreases in the direction of electric field.

(vi) $V = V_1 + V_2 + V_3 + ...$

7.3 Use of potential :

If we know the potential at some point (in terms of numerical value or in terms of formula) then we can find out the work done by electric force when charge moves from point 'P' to ∞ by the formula

$W_{el})_{p\infty} = qV_p$

Solved Examples

Example 47. A charge 2μC is taken from infinity to a point in an electric field, without changing its velocity. If work done against electrostatic forces is –40μJ then find the potential at that point.

Solution : $V = \frac{W_{ext}}{q} = \frac{-40\mu J}{2\mu C} = -20\text{ V}$

Example 48. When charge 10 μC is shifted from infinity to a point in an electric field, it is found that work done by electrostatic forces is –10 μJ. If the charge is doubled and taken again from infinity to the same point without accelerating it, then find the amount of work done by electric field and against electric field.

Solution : $W_{ext})_{\infty p} = -W_{el})_{\infty p} = W_{el})_{p\infty} = 10\ \mu J$

because ΔKE = 0

$V_p = \frac{W_{ext})_{\infty p}}{q} = \frac{10\mu J}{10\mu C} = 1V$

So if now the charge is doubled and taken from infinity then

$1 = \frac{w_{ext})_{\infty p}}{20\mu C}$ ⇒ $W_{ext})_{\infty P} = 20\ \mu J$ ⇒ $W_{el})_{\infty P} = -20\ \mu J$

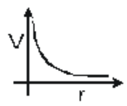
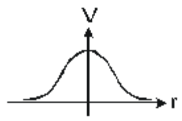
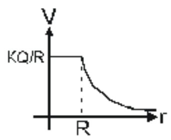
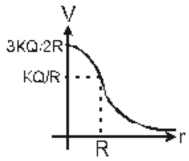
Example 49. A charge 3μC is released at rest from a point P where electric potential is 20 V then its kinetic energy when it reaches to infinite is :

Solution : $W_{el} = \Delta K = K_f - 0$

$W_{el})_{P\to\infty} = qV_P = 60\ \mu J$

so, $K_f = 60\ \mu J$

Electric Potential due to various charge distributions are given in table.

Name / Type	Formula	Note	Graph
Point charge	$\frac{Kq}{r}$	* q is source charge. * r is the distance of the point from the point charge.	
Ring (uniform/nonuniform charge distribution)	at centre $\frac{KQ}{R}$ at the axis $\frac{KQ}{\sqrt{R^2+x^2}}$	* Q is source chage. * x is the distance of the point on the axis	
Uniformly charged hollow conducting/nonconducting /solid conducting sphere	for $r \geq R$ $V = \frac{kQ}{r}$ for $r \leq R$ $V = \frac{kQ}{R}$	* R is radius of sphere * r is the distance from centre of sphere to the point * Q is total charge $= \sigma 4\pi R^2$.	
Uniformly charged solid nonconducting	for $r \geq R$ $V = \frac{kQ}{r}$ for $r \leq R$ $\frac{KQ(3R^2 - r^2)}{2R^3}$ $= \frac{\rho}{6\varepsilon_0}(3R^2 - r^2)$	* R is radius of sphere * r is distance from centre to the point * $V_{centre} = \frac{3}{2} V_{surface}$. * Q is total charge $= \rho \frac{4}{3}\pi R^3$. * Inside sphere potentail varies parabolically * outside potential varies hyperbolically.	
Infinite line charge	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -2K\lambda \ln(r_B/r_A)$	
Infinite nonconducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{2\varepsilon_0}(r_B - r_A)$	
Infinite charged conducting thin sheet	Not defined	* Absolute potential is not defined. * Potential difference between two points is given by formula $V_B - V_A = -\frac{\sigma}{\varepsilon_0}(r_B - r_A)$	

7.4 Potential due to a point charge :

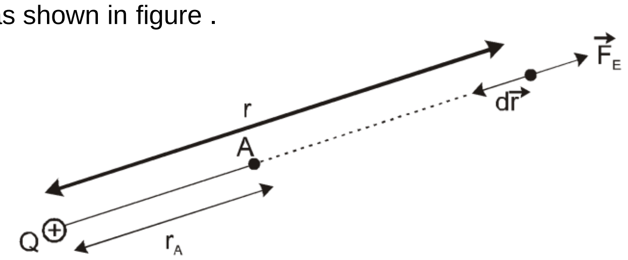
The electrostatic potential at a point in an electric field due to the point charge may be defined as the amount of work done per unit positive test charge in moving it from infinity to that point (without acceleration) against the electrostatic force due to the electric field of point charge.
It is a scalar quantity.
Consider a point charge +q placed at point O. Suppose that V_A is electric potential at point A, whose distance from the source charge +q is r_A
If $W_{\infty A}$ is work done in moving a vanishingly small positive test charge q_0 from infinity to point A, then

$$V_A = \frac{W_{\infty A}}{q_0}$$

Derivation : (i) Consider a positive point charge Q at the origin. We wish to determine the potential at any point A with position vector r from the origin.
(ii) Work done in bringing a unit positive test charge from infinity to the point A. For Q > 0. The work done against the repulsive force on the test charge is positive.
(iii) Since work done is independent of the path, we choose a convenient path - along the radial direction from infinity to the point A.
(iv) At some intermediate point A' on the path, the electrostatic force on a unit positive charge is

$$\vec{F}_E = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$$

where $\hat{r}$ is the unit vector along OP'. Work done by electric field on test charge for small displacement $(d\vec{r})$ as shown in figure .



$$\vec{F}_{ext} = -\vec{F}_E$$

$$dw = \vec{F}_{ext} \cdot d\vec{r} \Rightarrow (-\vec{F}_E) \cdot (d\vec{r})$$

$$dw = F_E(-dr)$$ (Here $\vec{r}$ is decreasing so we will take dr negative)

$$w_{\infty A} = -\int_{\infty}^{r_A} dw = -\int_{\infty}^{r_A} \frac{Q}{4\pi\varepsilon_0 r^2} dr$$

$$= V_A - V_\infty = \frac{Q}{4\pi\varepsilon_0 r_A}$$ ($V_\infty = 0$) (reference point is taken at infinity)

$$= V_A = \frac{Q}{4\pi\varepsilon_0 r_A}$$

In case, the distance of point A is from the charge +Q is denoted by r

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

Electric potential due to a system of charges

Let us now find electrostatic potential at a point P due to a group of point charges q_1, q_2, q_3 ... q_n lying at distances r_1, r_2, r_3.... r_n from point P (fig.). The electrostatic potential at point P due to these charges is found by calculating electrostatic potential P due to each individual charge, considering the other charges to be absent and then adding up these electrostatic potentials algebraically.
The electrostatic potential at point P due to charge q_1, when other charges are considered absent,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1}$$

Similarly, electrostatic potentials at point P due to the individual charges $q_2, q_3, ...q_n$ (when other charges are absent) are given by

$$V_2 = \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{r_2} \;;\; V_3 = \frac{1}{4\pi\varepsilon_0}.\frac{q_3}{r_3} \;;\; ....;\; V_n = \frac{1}{4\pi\varepsilon_0}.\frac{q_n}{r_n}$$

Hence, electrostatic potential at point P due to the group of n point charges,

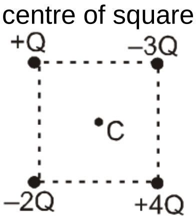
$$V = V_1 + V_2 + V_3 + ... + V_n$$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}.\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}.\frac{q_3}{r_3} + ... + \frac{1}{4\pi\varepsilon_0}.\frac{q_n}{r_n}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ... + \frac{q_n}{r_n}\right) \Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

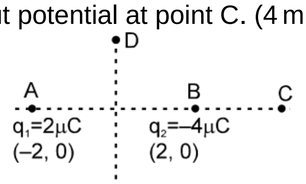
Solved Examples

Example 50. Four point charges are placed at the corners of a square of side ℓ calculate potential at the centre of square.



Solution : $V = 0$ at 'C'.

Example 51. Two point charges 2μC and – 4μC are situated at points (–2m, 0m) and (2 m, 0 m) respectively. Find out potential at point C. (4 m, 0 m) and. D (0 m, $\sqrt{5}$ m).



Solution : Potential at point C

$$V_C = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{6} + \frac{K(-4\mu C)}{2} = \frac{9\times10^9\times2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2} = -15000\text{ V.}$$

Similarly, $V_D = V_{q_1} + V_{q_2} = \frac{K(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{K(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{K(2\mu C)}{3} + \frac{K(-4\mu C)}{3} = -6000\text{ V.}$

Finding potential due to continuous charges

If formula of E is tought, then we take a small element and integrate

$$V = \int dv$$

If formula of E is easy then we use

$$V = -\int_{r\to\infty}^{r=r} \vec{E}\cdot\vec{dr}$$

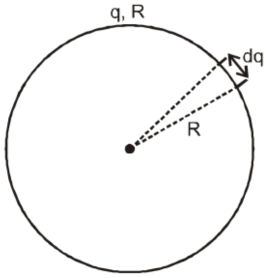
(i.e. for sphere, plate infinite wire etc.)

7.5 Potential due to a ring :

(i) Potential at the centre of uniformly charged ring :

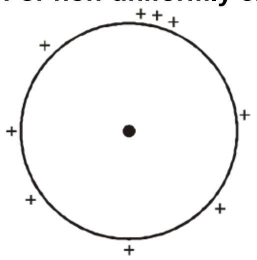
Potential due to the small element dq

$dV = \frac{Kdq}{R}$



Net potential $V = \int \frac{K\,dq}{R}$ $\Rightarrow$ $V = \frac{K}{R}\int dq = \frac{Kq}{R}$

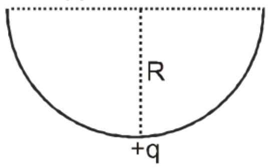
(ii) For non-uniformly charged ring potential at the center is



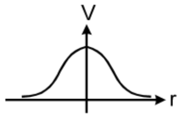
$V = \frac{Kq_{total}}{R}$

(iii) Potential due to half ring at center is :

$V = \frac{Kq}{R}$



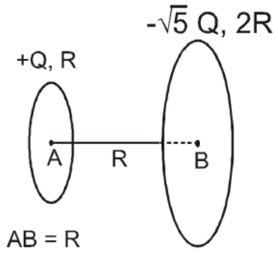
(iv) Potential at the axis of a ring:



$V = \frac{KQ}{\sqrt{R^2 + x^2}}$

Solved Examples

Example 52. Figure shows two rings having charges Q and – $\sqrt{5}$ Q. Find Potential difference between A and B (V_A - V_B).



Solution : $V_A = \frac{KQ}{R} + \frac{K(-\sqrt{5}\,Q)}{\sqrt{(2R)^2+(R)^2}}$ $\quad V_B = \frac{K(-\sqrt{5}\,Q)}{2R} + \frac{K(Q)}{\sqrt{(R)^2+(R)^2}}$

From above we can easily find $V_A - V_B$.

Example 53. A point charge q_0 is placed at the centre of uniformly charged ring of total charge Q and radius R. If the point charge is slightly displaced with negligible force along axis of the ring then find out its speed when it reaches to a large distance.

Solution : Only electric force is acting on q_0

$\therefore \quad W_{el} = \Delta K = \frac{1}{2} mv^2 - 0 \Rightarrow \quad$ Now $W_{el})_{c\to\infty} = q_0 V_c = q_0 \cdot \frac{KQ}{R}$

$\therefore \quad \frac{Kq_0Q}{R} = \frac{1}{2} mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2Kq_0Q}{mR}}$

7.6 Potential due to uniformly charged disc :

$\mathbf{V} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+x^2} - x\right)$, where σ is the charged density and x is the distance of the point on the axis from the center of the disc, R is the radius of disc.

7.7 Potential Due To Uniformly Charged Spherical shell :

Derivation of expression for potential due to uniformly charged hollow sphere of radius R and total charge Q, at a point which is at a distance r from centre for the following situation
(i) r > R (ii) r < R

As the formula of E is easy , we use $V = -\int_{r\to\infty}^{r=r} \vec{E}\cdot\vec{dr}$

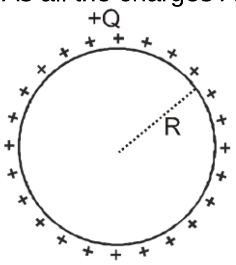
(i) At outside point (r ≥ R):

$V_{out} = -\int_{r\to\infty}^{r=r} \left(\frac{KQ}{r^2}\right) dr \quad \Rightarrow \quad V_{out} = \frac{KQ}{r} = \frac{KQ}{(\text{Distance from centre})}$

For outside point, the hollow sphere act like a point charge.

(ii) Potential at the centre of the sphere (r=0) :

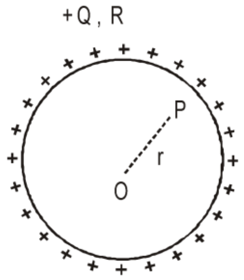
As all the charges Are at a distance R from the centre ,



So $V_{centre} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphare})}$

(iii) Potential at inside point (r<R) :

Suppose we want to find potential at point P, inside the sphere.



Potential difference between Point P and O :

$$V_P - V_O = -\int_O^P \vec{E}_{in} \cdot \vec{dr} \qquad \text{Where } E_{in} = 0$$

So $\quad V_P - V_O = 0$

$\Rightarrow \quad V_P = V_O = \frac{KQ}{R}$

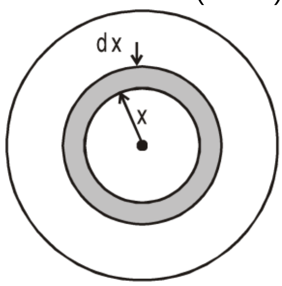
$\Rightarrow \quad V_{IN} = \frac{KQ}{R} = \frac{KQ}{(\text{Radius of the sphere})}$

7.8 Potential Due To Uniformly Charged Solid Sphere :

(i) for $r \geq R$ (outside)

$V = \frac{KQ}{r}$

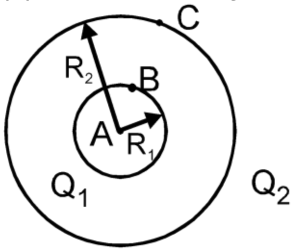
(ii) for $r \leq R$ (inside)



$V = \frac{KQ}{2R^3}(3R^2 - r^2) \qquad \Rightarrow \text{Here } \rho = \frac{Q}{\frac{4}{3}\pi R^3}$

Solved Examples

Example 54. Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out potential
(i) at point A
(ii) at surface of smaller shell (i.e. at point B)
(iii) at surface of larger shell (i.e. at point C)



(iv) at $r \leq R_1$ (v) at $R_1 \leq r \leq R_2$ (vi) at $r \geq R_2$

Solution : Using the results of hollow sphere as given in the table 7.4.

(i) $V_A = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$ (ii) $V_B = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

(iii) $V_C = \frac{KQ_1}{R_2} + \frac{KQ_2}{R_2}$ (iv) for $r \le R_1$ $V = \frac{KQ_1}{R_1} + \frac{KQ_2}{R_2}$

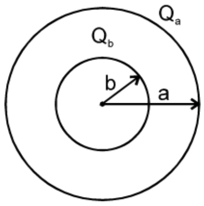
(v) for $R_1 \le r \le R_2$ $V = \frac{KQ_1}{r} + \frac{KQ_2}{R_2}$

(vi) for $r \ge R_2$ $V = \frac{KQ_1}{r} + \frac{KQ_2}{r}$

Example 55. Two hollow concentric nonconducting spheres of radius a and b (a > b) contains charges Q_a and Q_b respectively. Prove that potential difference between two spheres is independent of charge on outer sphere. If outer sphere is given an extra charge, is there any change in potential difference?

Solution : $V_{\text{inner sphere}} = \frac{KQ_b}{b} + \frac{KQ_a}{a}$

$V_{\text{outer sphere}} = \frac{KQ_b}{a} + \frac{KQ_a}{a}$



$V_{\text{inner sphere}} - V_{\text{outer sphere}} = \frac{KQ_b}{b} - \frac{KQ_b}{a}$

$\Delta V = KQ_b \left[\frac{1}{b} - \frac{1}{a}\right]$

Which is independent of charge on outer sphere.
If outer sphere in given any extra charge then there will be no change in potential difference.

8. POTENTIAL DIFFERENCE

The potential difference between two points A and B is work done by external agent against electric field in taking a unit positive charge from A to B without acceleration (or keeping Kinetic Energy constant or $K_i = K_f$))

(a) Mathematical representation :

If $(W_{A \to B})_{ext}$ = work done by external agent against electric field in taking the unit charge from A to B

$$V_B - V_A = \left.\frac{(W_{A\to B})_{ext}}{q}\right)_{\Delta K=0} = \frac{-(W_{A\to B})_{electric}}{q} = \frac{U_B - U_A}{q} = \frac{-\int_A^B \vec{F}_e \cdot d\vec{r}}{q} = -\int_A^B \vec{E} \cdot d\vec{r}$$

Note : Take W and q both with sign

(b) Properties :

(i) The difference of potential between two points is called potential difference. It is also called voltage.

(ii) Potential difference is a scalar quantity. Its S.I. unit is also volt.

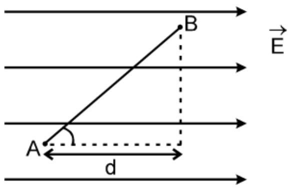
(iii) If V_A and V_B be the potential of two points A and B, then work done by an external agent in taking the charge q from A to B is
$(W_{ext})_{AB} = q\,(V_B - V_A)$ or $(W_{el})_{AB} = q\,(V_A - V_B)$.

(iv) Potential difference between two points is independent of reference point.

8.1 Potential difference in a uniform electric field :

$\mathbf{V_B - V_A = -}\ \vec{E} \cdot \overrightarrow{AB}$

⇒ $V_B - V_A = -|E|\,|AB| \cos\theta$
$= -|E|\, d$
$= -Ed$



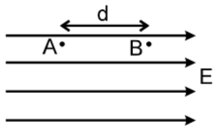
d = effective distance between A and B along electric field.

or we can also say that $E = \frac{\Delta V}{\Delta d}$

Special Cases :

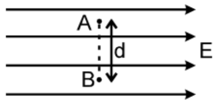
Case 1. Line AB is parallel to electric field.

$\therefore \quad V_A - V_B = Ed$



Case 2. Line AB is perpendicular to electric field.

$\therefore \quad V_A - V_B = 0 \quad \Rightarrow \quad V_A = V_D$



Note : In the direction of electric field potential always decreases.

Solved Examples

Example 56. 1μC charge is shifted from A to B and it is found that work done by an external force is 40μJ in doing so against electrostatic forces then, find potential difference $V_A - V_B$

Solution : $(W_{AB})_{ext} = q(V_B - V_A) \quad \Rightarrow \quad 40\ \mu J = 1\mu C\ (V_B - V_A) \quad \Rightarrow \quad V_A - V_B = -40$

Example 57. A uniform electric field is present in the positive x-direction. If the intensity of the field is 5N/C then find the potential difference ($V_B - V_A$) between two points A (0m, 2 m) and B (5 m ,3 m)

Solution : $V_B - V_A = -\vec{E}.\vec{AB} \quad = -(5\hat{i})\ .\ (5\hat{i} + \hat{j}) \quad = -25V.$

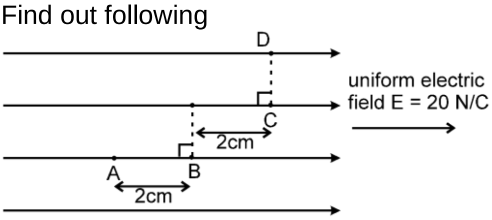
The electric field intensity in uniform electric field, $E = \frac{\Delta V}{\Delta d}$

Where ΔV = potential difference between two points.

Δd = effective distance between the two points.

(projection of the displacement along the direction of electric field.)

Example 58. Find out following



(i) $V_A - V_B$ (ii) $V_B - V_C$

(iii) $V_C - V_A$ (iv) $V_D - V_C$

(v) $V_A - V_D$ (vi) Arrange the order of potential for points A, B, C and D.

Solution : (i) $|\Delta V_{AB}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$

so, $V_A - V_B = 0.4$ V

because **In the direction of electric field potential always decreases.**

(ii) $|\Delta V_{BC}| = Ed = 20 \times 2 \times 10^{-2} = 0.4$ so, $V_B - V_C = 0.4$ V

(iii) $|\Delta V_{CA}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_C - V_A = -0.8$ V

because **In the direction of electric field potential always decreases.**

(iv) $|\Delta V_{DC}| = Ed = 20 \times 0 = 0$ so, $V_D - V_C = 0$

because the effective distance between D and C is zero.

(v) $|\Delta V_{AD}| = Ed = 20 \times 4 \times 10^{-2} = 0.8$ so, $V_A - V_D = 0.8$ V

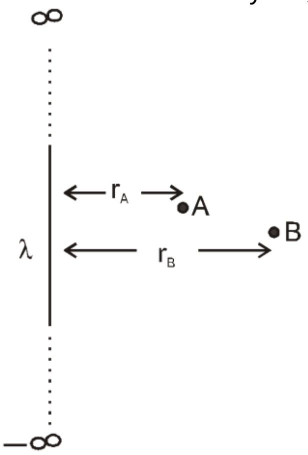
because **In the direction of electric field potential always decreases.**

(vi) The order of potential

$V_A > V_B > V_C = V_D$.

8.2 Potential difference due to infinitely long wire :

Derivation of expression for potential difference between two points, which have perpendicular distance r_A and r_B from infinitely long line charge of uniform linear charge density λ..



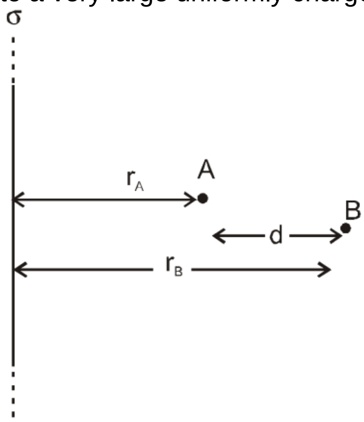
From definition of potential difference

$$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{2K\lambda}{r}\hat{r}\cdot d\vec{r}$$

$$V_{AB} = -2K\lambda\, \ell n\left(\frac{r_B}{r_A}\right)$$

8.3 Potential difference due to infinitely long thin sheet:

Derivation of expression for potential difference between two points, having separation d in the direction perpendicularly to a very large uniformly charged thin sheet of uniform surface charge density σ.



Let the points A and B have perpendicular distance r_A and r_B respectively then from definition of potential difference.

$$V_{AB} = V_B - V_A = -\int_{r_A}^{r_B} \vec{E}\cdot d\vec{r} = -\int_{r_A}^{r_B} \frac{\sigma}{2\varepsilon_0}\hat{r}\cdot d\vec{r} \quad \Rightarrow \quad V_{AB} = -\frac{\sigma}{2\varepsilon_o}(r_B - r_A) \quad = -\frac{\sigma d}{2\varepsilon_o}$$

9. EQUIPOTENTIAL SURFACE :

9.1 Definition : If potential of a surface (imaginary or physically existing) is same throughout then such surface is known as a equipotential surface.

9.2 Properties of Equipotential Surfaces :

The following properties are associated with the equipotential surfaces :

(i) No work done in moving a test charge over an equipotential surface.

Let A and B be two points on an equipotential surface (Fig.). If a positive test charge q_0 is moved from point A to B, then work done in moving the test charge is related to electrostatic potential difference between the two points as

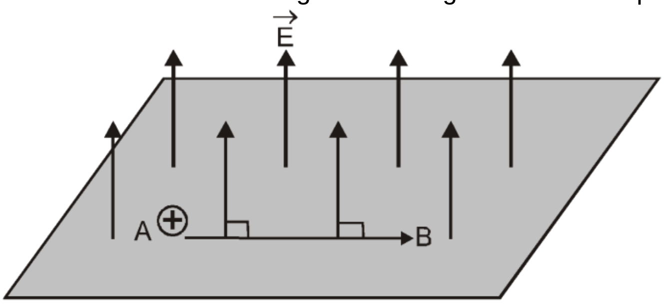
$$V_B - V_A = \frac{W_{AB}}{q_0}$$

Since the two points A and B are on the same equipotential surface ,

$$V_B - V_A = 0$$

$$\therefore \quad \frac{W_{AB}}{q_0} = 0 \quad \text{or} \quad W_{AB} = 0$$

Hence , no work is done in moving a test charge between two points on an equipotential surface.



(ii) The electric field is always at right angles to the equipotential surface

Since work done in moving a test charge between two pints on an equipotential surface is zero, the displacement of the test charge and the force applied on it must be perpendicular to each other. Since displacement is along the equipotential surface, and force on test charge is $q_0\vec{E}$, then the electric field ($\vec{E}$) must be at right angles to the equipotential surface.

(iii) **The equipotential surfaces help to distinguish regions to strong field from those of weak field**.

We know that

$$E = -\frac{dV}{dr} \quad \text{or} \quad dr = -\frac{dV}{E}$$

For same change in value of dV i.e. dV = constant, we have

$$dr \propto \frac{1}{E}$$

i.e. the spacing between the equipotential surfaces will be denser in the regions, where the electric field is stronger and vice-versa. Therefore, the equipotential surfaces are closer together, where the electric field is stronger and farther apart, where the field is weaker.

(iv) The equipotential surfaces tell the direction of the electric field.

Again $E = -\frac{dV}{dr}$

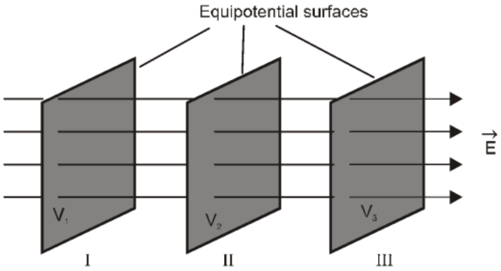
The negative sign tells that electric field is directed in the direction of electric potential with distance. Therefore, direction of electric field is from the equipotential surfaces which are close to each other to those which are more and more away from each other, provided such surfaces having been drawn for same change in value of dV.

(v) **No two equipotential surfaces can intersect each other .**

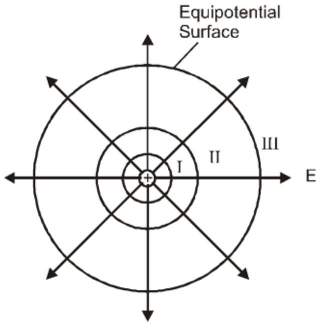
In case, two equipotential surfaces intersect each other, then at their point of intersection, there will be two values of electric potential. As it is not possible, the two equipotential surfaces can not intersect each other.

9.3 Examples of equipotential surfaces :

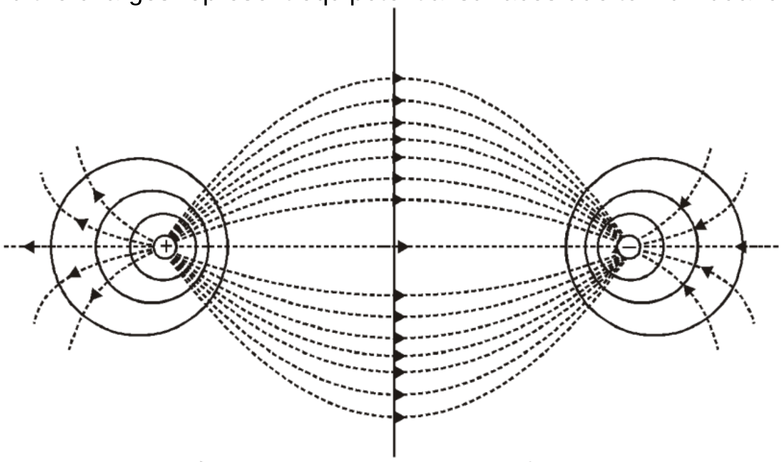
(i) For a uniform electric field : In a uniform electric field, the strength and direction of the field is same at every point inside it.
In a uniform electric field, equipotential surfaces differing by same amount of potential difference will be equidistant from each other.

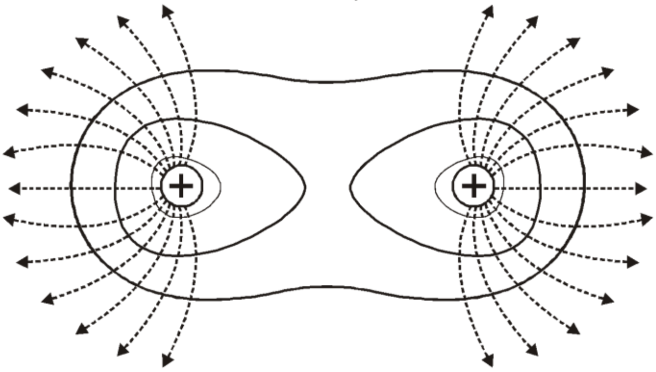


(ii) **For an isolated point charge :** The electric field due to an isolated point charge is radial in nature and varies inversely as the square of the distance from the charge. The potential at all the points equidistant from the charge is same. All such points lie on the surface of a spherical shell, such that the charge lies at its centre. Therefore, for a point charge, equipotential surfaces will be a series of concentric spherical shells.

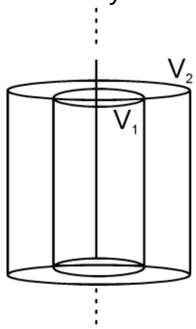


(iii) **For a system of two point charges :** the dotted lines represent electric lines of force. The thick circles around the charges represent equipotential surfaces due to individual charges.



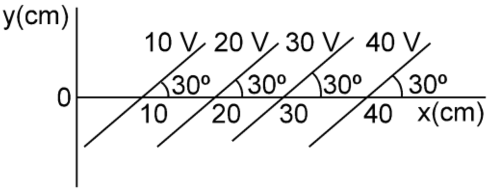


(iv) Line charge :
Equipotential surfaces have curved surfaces as that of coaxial cylinders of different radii.



Solved Examples

Example 59. Some equipotential surfaces are shown in figure. What can you say about the magnitude and the direction of the electric field ?



Solution : Here we can say that the electric will be perpendicular to equipotential surfaces.

Also $|\vec{E}| = \frac{\Delta V}{\Delta d}$

where ΔV = potential difference between two equipotential surfaces.

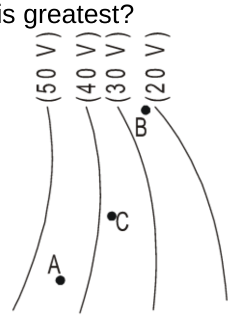
Δd = perpendicular distance between two equipotential surfaces.

So $|\vec{E}| = \frac{10}{(10 \sin 30^\circ) \times 10^{-2}}$ = 200 V/m

Now there are two perpendicular directions either direction 1 or direction 2 as shown in figure, but since we know that in the direction of electric field electric potential decreases so the correct direction is direction 2.

Hence E = 200 V/m, making an angle 120° with the x-axis

Example 60. Figure shows some equipotential surface produce by some charges. At which point the value of electric field is greatest?



Solution : E is larger where equipotential surfaces are closer. ELOF are ⊥ to equipotential surfaces. In the figure we can see that for point B they are closer so E at point B is maximum

Self Practice Problems

8. Angle between equipotenital surface and lines of force is
(1) Zero (2) 180_0 (3) 90_0 (4) 45_0

9. A charge of 5C expriences a force of 5000N when it is kept in unifrom electric filed. What is the potential difference between two points separated by a distance of 1cm
(1) 10V (2) 250 V (3) 1000 V (4) 2500 V

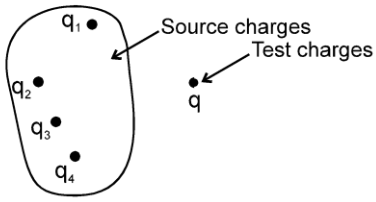
Ans. **8. (3)** **9. (1)**

11. ELECTROSTATIC POTENTIAL ENERGY

11.1 Electrostatic potential energy of a point charge due to many charges :

The electrostatic potential energy of a point charge at a point in electric field is the work done in taking the charge from reference point (generally at infinity) to that point without acceleration (or keeping KE const. or $K_i = K_f$)).

Its Mathematical formula is



$U = W_{\infty P)ext}]_{acc=0} = qV = -W_{P\infty)el}$

Here q is the charge whose potential energy is being calculated and V is the potential at its position due to the source charges.

Note : **Always put q and V with sign.**

11.2 Properties :

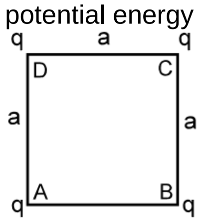
(i) Electric potential energy is a scalar quantity but may be positive, negative or zero.

(ii) Its unit is same as unit of work or energy that is joule (in S.I. system).
Some times energy is also given in electron-volts.
$1eV = 1.6 \times 10^{-19}$ J

(iii) Electric potential energy depends on reference point. (Generally Potential Energy at r= ∞ is taken zero)

Solved Examples

Example 61 The four identical charges q each are placed at the corners of a square of side a. Find the potential energy of one of the charges due to the remaining charges.



Solution : The electric potential of point A due to the charges placed at B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

∴ Potential energy of the charge at A is = $qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Example 62 A particle of mass 40 mg and carrying a charge 5×10^{-9} C is moving directly towards a fixed positive point charge of magnitude 10^{-8} C. When it is at a distance of 10 cm from the fixed point charge it has speed of 50 cm/s. At what distance from the fixed point charge will the particle come momentarily to rest? Is the acceleration constant during the motion?

Solution : If the particle comes to rest momentarily at a distance r form the fixed charge, then from conservation of energy' we have

$$\frac{1}{2}mu^2 + \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

Substituting the given data, we get

$$\frac{1}{2} \times 40 \times 10^{-6} \times \frac{1}{2} \times \frac{1}{2} = 9 \times 10^{9} \times 5 \times 10^{-8} \times 10^{-9}\left[\frac{1}{r}-10\right]$$

or, $\frac{1}{r} - 10 = \frac{5\times10^{-6}}{9\times5\times10^{-8}} = \frac{100}{9}$ $\Rightarrow \frac{1}{r} = \frac{190}{9}$ $\Rightarrow r = \frac{9}{190}$ m

or, i.e., $r = 4.7 \times 10^{-2}$ m

As here, $F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ so acc. $= \frac{F}{m} \propto \frac{1}{r^2}$

i.e., acceleration is not constant during the motion.

Example 63 A proton moves from a large distance with a speed u m/s directly towards a free proton originally at rest. Find the distance of closet approach for the two protons in terms of mass of proton m and its charge e.

Solution : As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, then by 'conservation of momentum' of the two protons system.

$mu = mv + mv$ i.e., $v = \frac{1}{2}u$

And by conservation of energy'

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\frac{1}{2}mu^2 - m\left(\frac{u}{2}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad [\text{as } v = \frac{u}{2}] \quad \Rightarrow \quad \frac{1}{4}mu^2 = \frac{e^2}{4\pi\varepsilon_0 r} \Rightarrow \quad r = \frac{e^2}{\pi m\varepsilon_0 u^2}$$

12. ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

(This concept is usefull when more than one charges move.)

It is the work done by an external agent against the internal electric field required to make a system of charges in a particular configuration from infinite separation without accelerating it.

12.1 Types of system of charge

(i) Point charge system

(ii) Continuous charge system.

12.2 Derivation for a system of point charges:

(i) Keep all the charges at infinity. Now bring the charges one by one to its corresponding position and find work required. PE of the system is algebric sum of all the works.

Let W_1 = work done in bringing first charge

W_2 = work done in bringing second charge against force due to 1st charge.

W_3 = work done in bringing third charge against force due to 1st and 2nd charge.

PE = $W_1 + W_2 + W_3 + ...$. (This will contain $\frac{n(n-1)}{2} = {}_nC_2$ terms)

(ii) Method of calculation (to be used in problems)

U = sum of the interaction energies of the charges.

$= (U_{12} + U_{13} + ... + U_{1n}) + (U_{23} + U_{24} + ... + U_{2n}) + (U_{34} + U_{35} + ... + U_{3n})$

(iii) Method of calculation useful for symmetrical point charge systems.

Find PE of each charge due to rest of the charges.

If U_1 = PE of first charge due to all other charges.

$= (U_{12} + U_{13} + ... + U_{1n})$

U_2 = PE of second charge due to all other charges.

$= (U_{21} + U_{23} + ... + U_{2n})$ then U = PE of the system

$= \frac{U_1 + U_2 + ...U_n}{2}$

Solved Examples

Example 64 Find out potential energy of the two point charge system having q_1 and q_2 charges separated by distance r.

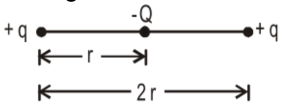
Solution : Let both the charges be placed at a very large separation initially.

Let W_1 = work done in bringing charge q_1 in absence of $q_2 = q(V_f - V_i) = 0$

W_2 = work done in bringing charge q_2 in presence of $q_1 = q(V_f - V_i) = q_1(Kq_2/r - 0)$

$PE = W_1 + W_2 = 0 + Kq_1q_2 / r = Kq_1q_2 / r$

Example 65 Figure shows an arrangement of three point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$.

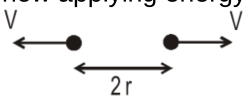


Solution : $U_{sys} = \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r} + \frac{(+q)(+q)}{2r} + \frac{Q(-q)}{r}\right] = 0$

$-Q + \frac{q}{2} - Q = 0$ or $2Q = \frac{q}{2}$ or $\frac{q}{Q} = \frac{4}{1}$.

Example 66 Two point charges each of mass m and charge q are released when they are at a distance r from each other. What is the speed of each charge particle when they are at a distance 2r?

Solution : According to momentum conservation both the charge particles will move with same speed now applying energy conservation.



$0 + 0 + \frac{Kq^2}{r} = 2\,\frac{1}{2}mv^2 + \frac{Kq^2}{2r} \Rightarrow v = \sqrt{\frac{Kq^2}{2rm}}$

Example 67 Two charged particles each having equal charges 2×10^{-5} C are brought from infinity to within a separation of 10 cm. Calculate the increase in potential energy during the process and the work required for this purpose.

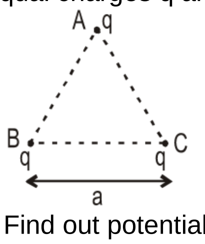
Solution : $\Delta U = U_f - U_i = U_f - 0 = U_f$

We have to simply calculate the electrostatic potential energy of the given system of charges

$\Delta U = U_f = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} = \frac{9\times10^9\times2\times10^{-5}\times2\times10^{-5}\times100}{10}$ J = 36 J

work required = 36 J.

Example 68 Three equal charges q are placed at the corners of an equilateral triangle of side a.



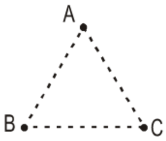
(i) Find out potential energy of charge system.

(ii) Calculate work required to decrease the side of triangle to a/2.

(iii) If the charges are released from the shown position and each of them has same mass m then find the speed of each particle when they lie on triangle of side 2a.

Solution : **(i) Method I** (Derivation)

Assume all the charges are at infinity initially.



work done in putting charge q at corner A

$W_1 = q\,(v_f - v_i) = q\,(0 - 0)$

Since potential at A is zero in absence of charges, work done in putting q at corner B in presence of charge at A :

$W_2 = \left(\frac{Kq}{a} - 0\right) = \frac{Kq^2}{a}$

Similarly work done in putting charge q at corner C in presence of charge at A and B.

$W_3 = q(v_f - v_i) \quad = q\left[\left(\frac{Kq}{a} + \frac{Kq}{a}\right) - 0\right]$

So net potential energy PE = $W_1 + W_2 + W_3 \quad = 0 + \frac{Kq^2}{a} + \frac{2Kq^2}{a} = \frac{3Kq^2}{a}$

Method II (using direct formula)

$U = U_{12} + U_{13} + U_{23} \quad = \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a} = \frac{3Kq^2}{a}$

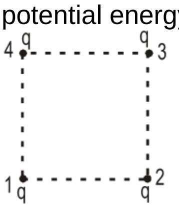
(ii) Work required to decrease the sides

$W = U_f - U_i \quad = \frac{3Kq^2}{a/2} - \frac{3Kq^2}{a} = \frac{3Kq^2}{a}$

(iii) Work done by electrostatic forces = change is kinetic energy of particles.

$U_i - U_f = K_f - K_i \qquad \Rightarrow \qquad \frac{3Kq^2}{a} - \frac{3Kq^2}{2a} = 3\left(\frac{1}{2}mv^2\right) - 0 \quad \Rightarrow \qquad v = \sqrt{\frac{Kq^2}{am}}$

Example 69 Four identical point charges q are placed at four corners of a square of side a. Find out potential energy of the charge system



Solution : **Method 1 (using direct formula) :**

$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$= \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{a\sqrt{2}} + \frac{Kq^2}{a} = \left[\frac{4Kq^2}{a} + \frac{2Kq^2}{a\sqrt{2}}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$

Method 2 [using $U = \frac{1}{2}(U_1 + U_2 + ...)$] **:**

U_1 = total P.E. of charge at corner 1 due to all other charges

U_2 = total P.E. of charge at corner 2 due to all other charges

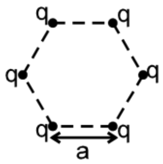
U_3 = total P.E. of charge at corner 3 due to all other charges

U_4 = total P.E. of charge at corner 4 due to all other charges

Since due to symmetry $U_1 = U_2 = U_3 = U_4$

$U_{Net} = \frac{U_1 + U_2 + U_3 + U_4}{2} = 2U_1 = 2\left[\frac{Kq^2}{a} + \frac{Kq^2}{a} + \frac{Kq^2}{\sqrt{2}a}\right] = \frac{2Kq^2}{a}\left[2 + \frac{1}{\sqrt{2}}\right]$

Example 70 Six equal point charges q are placed at six corners of a hexagon of side a. Find out potential energy of charge system



Solution : $U_{Net} = \dfrac{U_1 + U_2 + U_3 + U_4 + U_5 + U_6}{2}$

Due to symmetry $U_1 = U_2 = U_3 = U_4 = U_5 = U_6$ so $U_{net} = 3U_1 = \dfrac{3Kq^2}{a}\left[2 + \dfrac{2}{\sqrt{3}} + \dfrac{1}{2}\right]$

Self Practice Problems

10. Two equal charges q are placed at a distance of 2a and a third charge –2q is placed at the midpoint, The potential energy of the system is

(1) $\dfrac{q^2}{8\pi\varepsilon_0 a}$ (2) $\dfrac{6q^2}{8\pi\varepsilon_0 a}$ (3) $-\dfrac{7q^2}{8\pi\varepsilon_0 a}$ (4) $\dfrac{9q^2}{8\pi\varepsilon_0 a}$

11. A particle of mass 'm' and charge 'q' is accelerated throught a potenital difference of V uoit, its engery will be

(1) qV (2) mqV (3) $\left(\dfrac{q}{m}\right)V$ (4) $\dfrac{q}{mV}$

12. When one electron is taken towards the other electron, then the electric potenital energy of the system

(1) Decreases (2) Increase

(3) Remains unchanged (4) Becomes zero

Ans. **10. (3) 11. (1) 12. (2)**

12.3 Electric potential energy for continues charge system :

This energy is also known as self energy.

(i) P.E. (Self Energy) of a uniformly Charged spherical shell :-

$U_{self} = \dfrac{KQ^2}{2R}$

(ii) Self energy of uniformly charged solid sphere :

for a solid sphere P.E. is $U_{self} = \dfrac{3}{5}\dfrac{KQ^2}{R}$

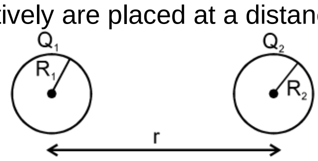
Solved Examples

Example 71 A spherical shell of radius R with uniform charge q is expanded to a radius 2R. Find the work performed by the electric forces and external agent against electric forces in this process.

Solution : $W_{ext} = U_f - U_i = \dfrac{q^2}{16\pi\varepsilon_0 R} - \dfrac{q^2}{8\pi\varepsilon_0 R} = -\dfrac{q^2}{16\pi\varepsilon_0 R}$

$W_{ext} = U_i - U_f = \dfrac{q^2}{8\pi\varepsilon_0 R} - \dfrac{q^2}{16\pi\varepsilon_0 R} = \dfrac{q^2}{16\pi\varepsilon_0 R}$

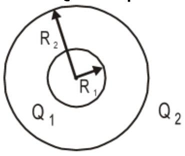
Example 72 Two nonconducting hollow uniformly charged spheres of radii R_1 and R_2 with charge Q_1 and Q_2 respectively are placed at a distance r. Find out total energy of the system.



Solution : $U_{total} = U_{self} + U_{Interaction} = \dfrac{Q_1^2}{8\pi\varepsilon_0 R_1} + \dfrac{Q_2^2}{8\pi\varepsilon_0 R_2} + \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r}$

Example 73

Two concentric spherical shells of radius R_1 and R_2 ($R_2 > R_1$) are having uniformly distributed charges Q_1 and Q_2 respectively. Find out total energy of the system.



Solution : $U_{total} = U_{self\ 1} + U_{self\ 2} + U_{Interaction} = \frac{Q_1^2}{8\pi\varepsilon_0 R_1} + \frac{Q_2^2}{8\pi\varepsilon_0 R_2} + \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_2}$

12.4 Energy density :

Def: Energy density is defined as energy stored in unit volume in any electric field. Its mathematical formula is given as following

Energy density = $\frac{1}{2}\varepsilon E^2$ where E = electric field intensity at that point

$\varepsilon = \varepsilon_0\varepsilon_r$ electric permittivity of medium

Example 74 Find out energy stored in an imaginary cubical volume of side a in front of a infinitely large nonconducting sheet of uniform charge density σ.

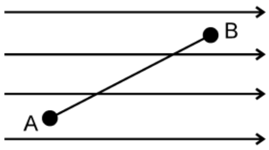
Solution : Energy stored

$U = \int \frac{1}{2}\varepsilon_0 E^2\, dV$ where dV is small volume $= \frac{1}{2}\varepsilon_0 E^2 \int dV$

∵ E is constant $= \frac{1}{2}\varepsilon_0 \frac{\sigma^2}{4\varepsilon_0^2} . a^3 . = \frac{\sigma^2 a^3}{8\varepsilon_0}$

13. RELATION BETWEEN ELECTRIC FIELD INTENSITY AND ELECTRIC POTENTIAL

13.1 For uniform electric field :



(i) Potential difference between two points A and B

$V_B - V_A = -\vec{E} . \vec{AB}$

13.2 Non uniform electric field

(i) $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, $E_z = -\frac{\partial V}{\partial z}$ ⇒ $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

$= -\left[\hat{i}\frac{\partial}{\partial x}V + \hat{j}\frac{\partial}{\partial y}V + \hat{k}\frac{\partial}{\partial z}V\right] = -\left[\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right]V = -\nabla V = -\text{grad } V$

Where $\frac{\partial V}{\partial x}$ = derivative of V with respect to x (keeping y and z constant)

$\frac{\partial V}{\partial y}$ = derivative of V with respect to y (keeping z and x constant)

$\frac{\partial V}{\partial z}$ = derivative of V with respect to z (keeping x and y constant)

13.3 If electric potential and electric field depends only on one coordinate, say r :

(i) $\vec{E} = -\frac{\partial V}{\partial r}\hat{r}$

where $\hat{r}$ is a unit vector along increasing r.

(ii) $\int dV = -\int \vec{E}\cdot\vec{dr}$ ⇒ $V_B - V_A = -\int_{r_A}^{r_B} \vec{E}.\vec{dr}$

$\vec{dr}$ is along the increasing direction of r.

(iii) The potential of a point $V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$

Example 75 A uniform electric field is along x – axis. The potential difference $V_A - V_B$ = 10 V between two points A (2m, 3m) and B (4m, 8m). Find the electric field intensity.

Solution : $E = \frac{\Delta V}{\Delta d} = \frac{10}{2}$ = 5 V / m. It is along + ve x-axis.

Example 76 $V = x^2 + y$, Find $\vec{E}$.

Solution : $\frac{\partial V}{\partial x} = 2x$, $\frac{\partial V}{\partial y} = 1$ and $\frac{\partial V}{\partial z} = 0$

$\vec{E} = -\left(\hat{i}\frac{\partial V}{\partial x} + \hat{j}\frac{\partial V}{\partial y} + \hat{k}\frac{\partial V}{\partial z}\right) = -(2x\hat{i} + \hat{j})$ Electric field is nonuniform.

Example 77 For given $\vec{E} = 2x\hat{i} + 3y\hat{j}$ find the potential at (x, y) if V at origin is 5 volts.

Solution : $\int_5^V dV = -\int \vec{E}\cdot\vec{dr} = -\int_0^x E_x dx - \int_0^y E_y dy$

$V - 5 = -\frac{2x^2}{2} - \frac{3y^2}{2}$ ⇒ $V = -\frac{2x^2}{2} - \frac{3y^2}{2} + 5$.

14. ELECTRIC DIPOLE

14.1 Electric Dipole

If two point charges equal in magnitude q and opposite in sign separated by a distance a such that the distance of field point r>>a, the system is called a dipole. The electric dipole moment is defined as a vector quantity having magnitude p = (q × a) and direction from negative charge to positive charge.

Note: [In chemistry, the direction of dipole moment is assumed to be from positive to negative charge.] The C.G.S unit of electric dipole moment is **debye** which is defined as the dipole moment of two equal and opposite point charges each having charge 10^{-10} frankline and separation of 1 Å, i.e.,

1 debye (D) = $10^{-10} \times 10^{-8} = 10^{-18}$ Fr × cm

1 D = $10^{-18} \times \frac{C}{3\times10^9} \times 10^{-2}$ m = 3.3×10^{-30} C × m.

S.I. Unit is coulomb × metre = C . m

Solved Examples

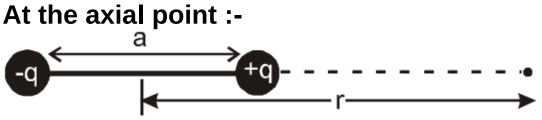
Example 78 A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A : (0, 0, – 0.15 m) and B ; (0, 0, + 0.15 m) respectively. What is the net charge and electric dipole moment of the system ?

Solution : Net charge = $2.5 \times 10^{-7} - 2.5 \times 10^{-7} = 0$

Electric dipole moment,
P = (Magnitude of charge) × (Separation between charges)
= 2.5×10^{-7} [0.15 + 0.15] C m = 7.5×10^{-8} C m
The direction of dipole moment is from B to A.

14.2 Electric Field Intensity Due to Dipole :

(i) At the axial point :-



$$\vec{E} = \frac{Kq}{\left(r - \frac{a}{2}\right)} - \frac{Kq}{\left(r + \frac{a}{2}\right)^2} \text{ along the } \hat{P} = \frac{Kq(2ra)}{\left(r^2 - \frac{a^2}{4}\right)^2}\hat{P}$$

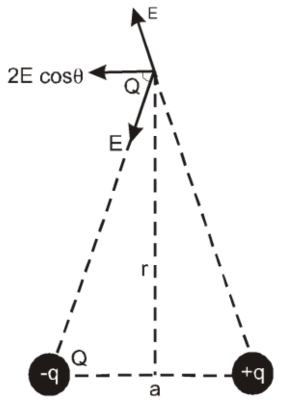
If r >> a then

$$\vec{E} = \frac{Kq\,2ra}{r^4}\hat{P} = \frac{2K\vec{P}}{r^3},$$

As the direction of electric field at axial position is along the dipole moment ($\vec{P}$)

$$\vec{E}_{axial} = \frac{2K\vec{P}}{r^3}$$

(ii) Electric field at perpendicular Bisector (Equitorial Position)



E_{net} = 2 E cos θ (along – $\hat{P}$)

$$\vec{E}_{net} = 2\left(\frac{Kq}{\left(\sqrt{r^2 + \left(\frac{a}{2}\right)^2}\right)^2}\right)\frac{\frac{a}{2}}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}}(-\hat{P}) = 2\frac{Kqa}{\left(r^2 + \left(\frac{a}{2}\right)^2\right)^{3/2}}(-\hat{P})$$

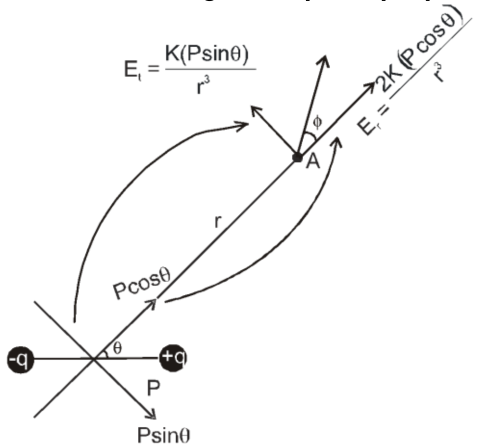
If r >> a then

$$\vec{E}_{net} = \frac{KP}{r^3}(-\hat{P})$$

As the direction of $\vec{E}$ at equitorial position is opposite of $\vec{P}$ so we can write in vector form:

$$\vec{E}_{eqt} = -\frac{K\vec{P}}{r^3}$$

(iii) Electric field at general point (r, θ) :



$E_{net} = \frac{KP}{r^3}\sqrt{1+3\cos^2\theta}$; $\tan\phi = \frac{\tan\theta}{2}$

Solved Examples

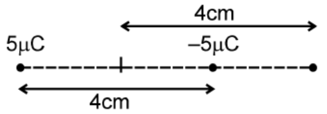
Example 79 The electric field due to a short dipole at a distance r, on the axial line, from its mid point is the same as that of electric field at a distance r', on the equatorial line, from its mid-point. Determine the ratio $\frac{r}{r'}$.

Solution : $\frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r'^3}$ or $\frac{2}{r^3} = \frac{1}{r'^3}$ or $\frac{r^3}{r'^3} = 2$ or $\frac{r}{r'} = 2^{1/3}$

Example 80 Two charges, each of 5 μC but opposite in sign, are placed 4 cm apart. Calculate the electric field intensity of a point that is at a distance 4 cm from the mid point on the axial line of the dipole.

Solution : We can not use formula of short dipole here because distance of the point is comparable to the distance between the two point charges.

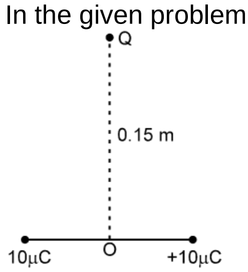
$q = 5 \times 10^{-6}$ C, $a = 4 \times 10^{-2}$ m, $r = 4 \times 10^{-2}$ m



$E_{res} = E_+ + E_- = \frac{K(5\mu C)}{(2cm)^2} - \frac{K(5\mu C)}{(6cm)^2} = \frac{144}{144\times10^{-8}}$ $NC^{-1} = 10^8$ N C^{-1}

Example 81 Two charges ± 10 μC are placed 5×10^{-3} m apart. Determine the electric field at a point Q which is 0.15 m away from O, on the equatorial line.

Solution : In the given problem, r >> a



∴ $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3} = \frac{1}{4\pi\varepsilon_0}\frac{q(a)}{r^3}$

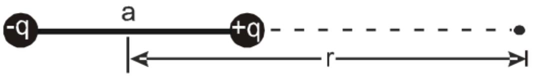
or $E = 9 \times 10^9 \frac{10\times10^{-6}\times5\times10^{-3}}{0.15\times0.15\times0.15}$ NC^{-1}

$= 1.33 \times 10^5$ NC^{-1}

14.3 Electric Potential due to a small dipole :

(i) Potential at axial position :

$$V = \frac{Kq}{\left(r - \frac{a}{2}\right)} + \frac{K(-q)}{\left(r + \frac{a}{2}\right)}$$



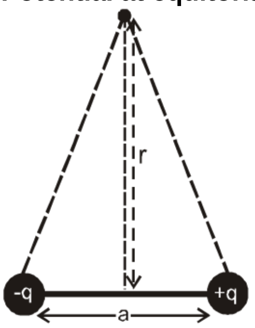
$$V = \frac{Kqa}{\left(r^2 - \left(\frac{a}{2}\right)^2\right)}$$

If r >> a than

$$V = \frac{Kqa}{r^2} \quad \text{where } qa = p$$

$$V_{axial} = \frac{KP}{r^2}$$

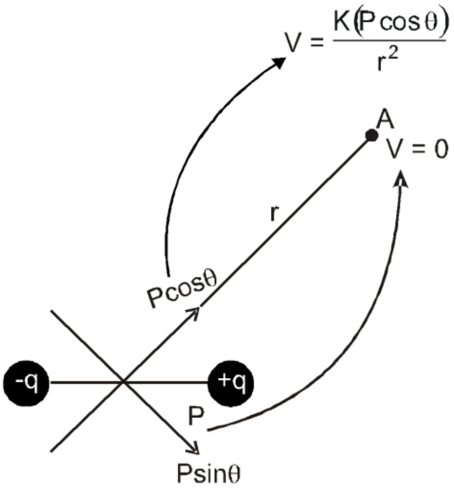
(ii) Potential at equitorial position :



$$V = \frac{Kq}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} + \frac{K(-q)}{\sqrt{r^2 + \left(\frac{a}{2}\right)^2}} = 0$$

$V_{eqt} = 0$

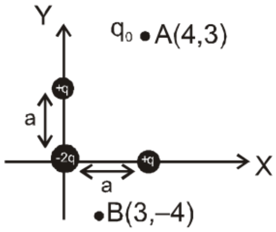
(iii) Potential at general point (r,θ) :



$$V = \frac{K\left(\vec{P}.\vec{r}\right)}{r^3}$$

Solved Examples

Example 82 (i) Find potential at point A and B due to the small charge - system fixed near origin.(distance between the charges is negligible).
(ii) Find work done to bring a test charge q_0 from point A to point B, slowly. All parameters are in S.I. units.



Solution : (i) Dipole moment of the system is

$\vec{P} = (qa)\,\hat{i} + (qa)\,\hat{j}$

Potential at point A due to the dipole

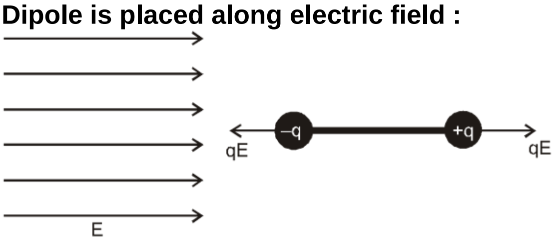
$$V_A = K\frac{(\vec{P}\cdot\vec{r})}{r^B} = \frac{K[(qa)\hat{i}+(qa)\hat{j}]\cdot(4\hat{i}+3\hat{j})}{5^3} = \frac{k(qa)}{125}(7)$$

$$\Rightarrow \quad V_B = \frac{K\left[(qa)\hat{i}+(qa)\hat{j}\right]\cdot\left(3\hat{i}-4\hat{j}\right)}{(5)^3} = \frac{K(qa)}{125}$$

(ii) $$W_{A\to B} = U_B - U_A = q_0(V_B - V_A) = \left[-\frac{K(qa)}{125} - \left(\frac{K(qa)(7)}{125}\right)\right] \Rightarrow W_{A\to B} = \frac{K\,qq_0a}{125}(8)$$

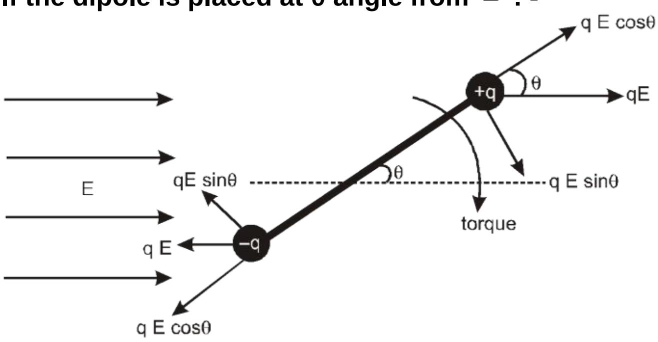
14.4 Dipole in uniform electric field

(i) Dipole is placed along electric field :



In this case $F_{net} = 0$, $\tau_{net} = 0$ so it is an equilibrium state. And it is a stable equilibrium position.

(ii) If the dipole is placed at θ angle from $\vec{E}$: -



In this case $F_{net} = 0$ but

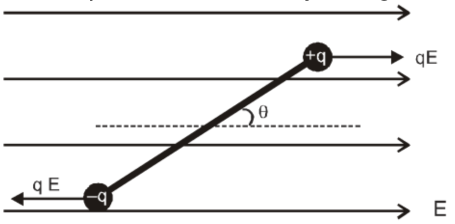
Net torque $\tau = (qE\sin\theta)\,(a)$

Here qa = P ⇒ τ = PE sinθ in vector form $\vec{\tau} = \vec{P} \times \vec{E}$

Solved Examples

Example 83 A dipole is formed by two point charge –q and +q, each of mass m, and both the point charges are connected by a rod of length ℓ and mass m_1. This dipole is placed in uniform electric field $\vec{E}$. If the dipole is disturbed by a small angle θ from stable equilibrium position, prove that its motion will be almost SHM. Also find its time period.

Solution : If the dipole is disturbed by θ angle,



τ_{net} = –PE sinθ (here – ve sign indicates that direction of torque is opposite of θ). If θ is very small, sinθ = θ

τ_{net} = –(PE)θ

$\tau_{net} \propto (-\theta)$ so motion will be almost SHM.

$T = 2\pi\sqrt{\frac{I}{K}}$

(iii) Potential energy of a dipole placed in uniform electric field :

$U_B - U_A = -\int_A^B \vec{F}\cdot\vec{dr}$ Here $U_B - U_A = -\int_A^B \vec{\tau}\cdot\vec{d\theta}$

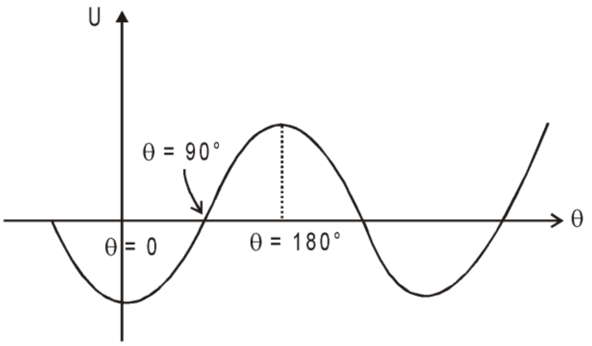
In the case of dipole, at θ = 90° , P.E. is assumed to be zero.

$U_\theta - U_{90^\circ} = -\int_{\theta=90^\circ}^{\theta=\theta} (-PE\sin\theta)(d\theta)$

(As the direction of torque is opposite of θ)

$U_\theta - 0 = -PE\cos\theta$

θ = 90° is chosen as reference, so that the lower limit comes out to be zero.



$U_\theta = -\vec{P}\cdot\vec{E}$

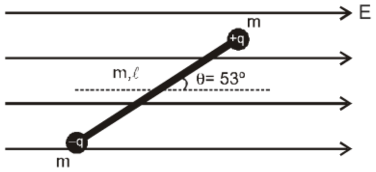
From the potential energy curve, we can conclude :

(i) at θ = 0, there is minimum of P.E. so it is a stable equilibrium position.

(ii) at θ = 180° , there is maxima of P.E. so it is a position of unstable equilibrium.

Solved Examples

Example 84 Two point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in figure from θ = 53°



(i) Find its angular acceleration of the rod just after releasing
(ii) What will be its angular velocity of the rod when it passes through stable equilibrium.
(iii) Find work required to rotate the system it by 180°.

Solution : (i) τ_{net} = PE sin53° = I α

$$\alpha = \frac{(q\ell)E\left(\frac{4}{5}\right)}{\frac{m\ell^2}{12}+m\left(\frac{\ell}{2}\right)^2+m\left(\frac{\ell}{2}\right)^2} = \frac{48qE}{35\,m\ell}$$

(ii) from energy conservation :

$K_i + U_i = K_f + U_f$

$$0 + (-PE\cos 53°) = \frac{1}{2}I\omega^2 + (-PE\cos 0°)$$

where $I = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2$ ⇒ $\omega = \sqrt{\frac{48qE}{35\,m\ell}}$

(iii) $W_{ext} = U_f - U_i$

W_{ext} = (–PE cos(180° + 53°)) – (–PEcos 53°)

$W_{ext} = (q\ell)E\left(\frac{4}{5}\right) + (q\ell)E\left(\frac{4}{5}\right)$ ⇒ $W_{ext} = \left(\frac{8}{5}\right)q\ell E$

Self Practice Problems

13. The electric potenital at a point on the axis of an electric dipole depends on the distance r of the point from the dipole as

(1) $\propto \frac{1}{r}$ (2) $\propto \frac{1}{r^2}$ (3) ∝r (4) $\propto \frac{1}{r^3}$

14. An electric dipole when placed in a uniform electric field E will have minimum potenital energy if the positive direction of dipole moment makes the following angle with E

(1) π (2) π/2 (3) Zero (4) 3π/2

15. An electric dipole of moment P is placed in the position of stable equilibrium in uniform electric field of intensity E. It is rotated through an angle θ from the initial position. the potenital energy of electric dipole in the final position is :

(1) PE cos θ (2) PE sin θ (3) PE (1-cos θ) (4) – PE cos θ

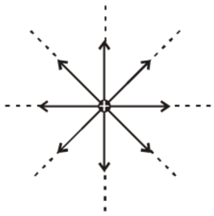
Ans. **13. (2) 14. (3) 15. (4)**

15. ELECTRIC LINES OF FORCE (ELOF)

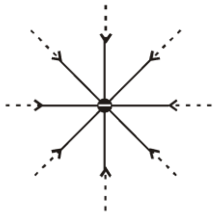
The line of force in an electric field is an imaginary line, the tangent to which at any point on it represents the direction of electric field at the given point.

15.1 Properties :

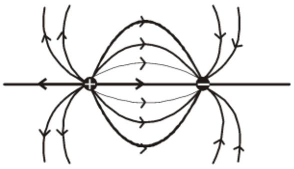
(i) Line of force originates out from a positive charge and terminates on a negative charge. If there is only one positive charge then lines start from positive charge and terminate at ∞. If there is only one negative charge then lines start from ∞ and terminates at negative charge.



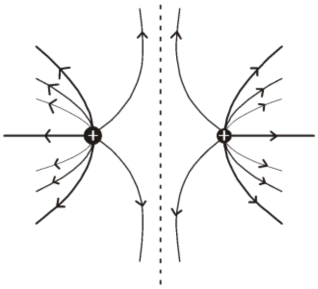
ELOF of Isolated positive charge



ELOF of Isolated negative charge

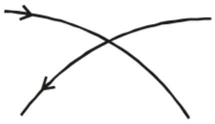


ELOF due to positive and negative charge

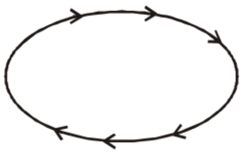


ELOF due to two positive charges

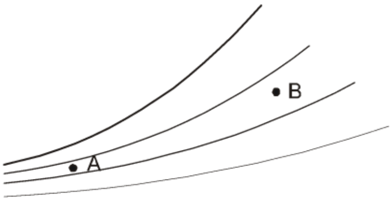
(ii) Two lines of force never intersect each other because there cannot be two directions of $\vec{E}$ at a single Point



(iii) Electric lines of force produced by static charges do not form close loop.
If lines of force make a closed loop, than work done to move a +q charge along the loop will be non-zero. So it will not be conservative field. So these type of lines of force are not possible in electrostatics.



(iv) The Number of lines per unit area (line density) represents the magnitude of electric field.
If lines are dense, ⇒ E will be more
If Lines are rare, ⇒ E will be less
and if E = O, no line of force will be found there



$E_A > E_B$

(v) Number of lines originating (terminating) is proportional to the charge.

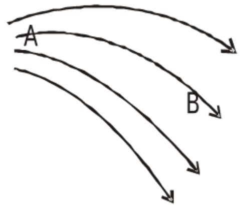
Example 85 If number of electric lines of force from charge q are 10 then find out number of electric lines of force from 2q charge.

Solution : No. of ELOF ∝ charge
$10 \propto q \quad \Rightarrow \quad 20 \propto 2q$
So number of ELOF will be 20.

(vi) Electric lines of force end or start perpendicularly on the surface of a conductor.
(vii) Electric lines of force never enter into conductors.

Solved Examples

Example 86 Some electric lines of force are shown in figure, for point A and B



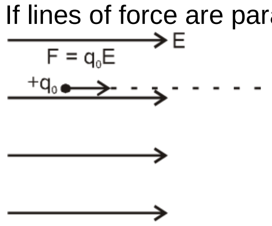
(1) $E_A > E_B$ (2) $E_B > E_A$ (3) $V_A > V_B$ (4) $V_B > B_A$

Solution : lines are more dense at B so $E_A > E_B$ In the direction of Electric field, potential decreases so $V_A > V_B$

Example 87 If a charge is released in electric field, will it follow lines of force?

Solution : **Case I :**

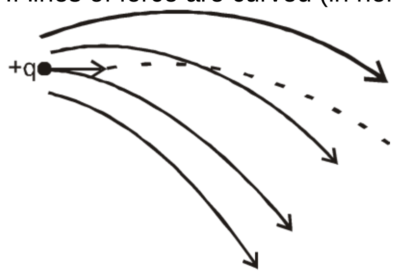
If lines of force are parallel (in uniform electric field) :-



In this type of field, if a charge is released, force on it will be q_0E and its direction will be along $\vec{E}$. So the charge will move in a straight line, along the lines of force.

Case II : -

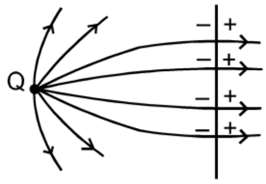
If lines of force are curved (in non-uniform electric field) :-



The charge will not follow lines of force

Example 88 A charge + Q is fixed at a distance of d in front of an infinite metal plate. Draw the lines of force indicating the directions clearly.

Solution : There will be induced charge on two surfaces of conducting plate, so ELOF will start from +Q charge and terminate at conductor and then will again start from other surface of conductor.

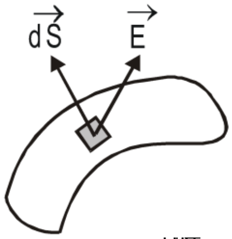


15.2 SOLID ANGLE :

Solid angle is a measure of a cone. Consider the intersection of the given cone with a sphere of radius R. The solid angle $\Delta\Omega$ of the cone is defined to be equal to $\Delta S/R^2$, where ΔS is the area on thesphere cut out by the cone.

16. ELECTRIC FLUX

Consider some surface in an electric field $\vec{E}$. Let us select a small area element $\vec{dS}$ on this surface. The electric flux of the field over the area element is given by $d\phi_E = \vec{E}.\vec{dS}$



Direction of $\vec{dS}$ is normal to the surface. It is along $\hat{n}$

or $d\phi_E = EdS\cos\theta$ or $d\phi_E = (E\cos\theta)\, dS$ or $d\phi_E = E_n\, dS$

where E_n is the component of electric field in the direction of $\vec{dS}$.

The electric flux over the whole area is given by $\phi_E = \int_S \vec{E}.\vec{dS} = \int_S E_n dS$

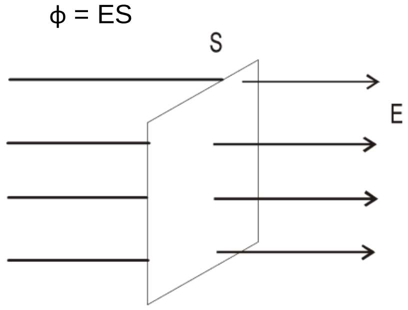
If the electric field is uniform over that area then $\phi_E = \vec{E}\cdot\vec{S}$

Special Cases :

Case I : If the electric field in normal to the surface, then angle of electric field $\vec{E}$ with normal will be zero

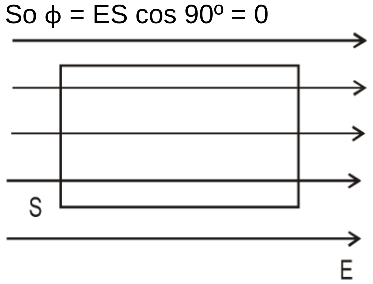
So $\phi = ES\cos 0$

$\phi = ES$



Case II : If electric field is parallel of the surface (glazing), then angle made by $\vec{E}$ with normal = 90º

So $\phi = ES\cos 90º = 0$



16.1 Physical Meaning :

The electric flux through a surface inside an electric field represents the total number of electric lines of force crossing the surface. It is a property of electric field

16.2 Unit

(i) The SI unit of electric flux is $Nm^2\, C^{-1}$ (gauss) or $J\, m\, C^{-1}$.

(ii) Electric flux is a scalar quantity. (It can be positive, negative or zero)

Solved Examples

Example 89. If the electric field is given by ($6\hat{i}+3\hat{j}+4\hat{k}$)N/C, calculate the electric flux through a surface of area 20 m^2 lying in YZ plane.

Solution : Here, $\vec{E}=6\hat{i}+3\hat{j}+4\hat{k}$

The are vector representing the surface of area 20 units in YZ-plane is given by

$\vec{S}=20\hat{i}$

Therefore, electric flux through the surface,

$\phi=\vec{F}.\vec{S}=(6\hat{i}+3\hat{j}+4\hat{k}).20\hat{i}=120$ N-m^2/C

Example 90. A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric filed of 25 Vm^{-1}, such that normal to the surface makes an angle of 60º with the direction of electric field. Find the flux of the electric field through the rectangular surface.

Solution : The flux through the rectangular surface given by

$$\phi=\vec{E}.\Delta\vec{S}=E\Delta\ S\cos\theta$$

Here, $E = 25\ V\ m^{-1}$;

$\Delta S = 10 \times 15 = 150\ cm^2 = 150 \times 10^{-4}\ m^2$

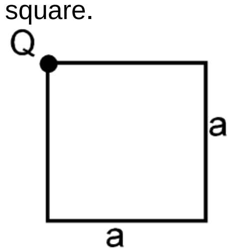
and $\theta = 60^o$

$\therefore\quad \phi = 25 \times 150 \times 10^{-4} \cos 60^o = \frac{3\sqrt{3}}{16}\ Nm^2\ C^{-1}$

Example 91 The electric field in a region is given by $\vec{E}=\frac{3}{5}E_0\vec{i}+\frac{4}{5}E_0\vec{j}$ with $E_0 = 2.0 \times 10^3$ N/C. Find the flux of this field through a rectangular surface of area $0.2m^2$ parallel to the Y–Z plane.

Solution : $\phi_E = \vec{E}\cdot\vec{S} = \left(\frac{3}{5}E_0\vec{i}+\frac{4}{5}E_0\vec{j}\right).\left(0.2\hat{i}\right) = 240\frac{N-m^2}{C}$

Example 92 A point charge Q is placed at the corner of a square of side a, then find the flux through the square.

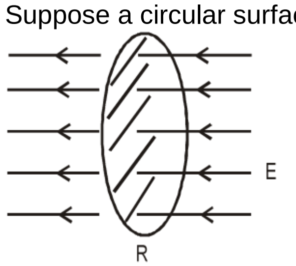


Solution : The electric field due to Q at any point of the square will be along the plane of square and the electric field line are perpendicular to square ; so $\phi = 0$.

In other words we can say that no line is crossing the square so flux = 0.

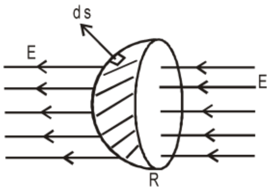
Case-III : Curved surface in uniform electric field

Suppose a circular surface of radius R is placed in a uniform electric field as shown.



Flux passing through the surface $\phi = E\ (\pi R^2)$

(ii) Now suppose, a hemispherical surface is placed in the electric field flux through hemispherical surface



$\phi = \int E ds \cos\theta$ $\qquad \phi = E \int ds \cos\theta$

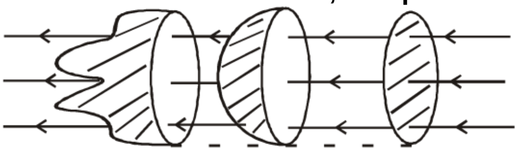
where $\int ds \cos\theta$ is projection of the spherical surface Area on base.

$\int ds \cos\theta = \pi R^2$

so $\phi = E(\pi R^2)$ = same Ans. as in previous case

so we can conclude that

If the number of electric field lines passing through two surfaces are same, then flux passing through these surfaces will also be same, irrespective of the shape of surface

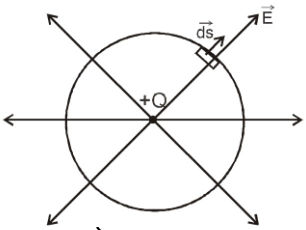


$\phi_1 = \phi_2 = \phi_3 = E(\pi R^2)$

Case IV:

Flux through a closed surface :

Suppose there is a spherical surface and a charge 'q' is placed at centre. flux through the spherical surface



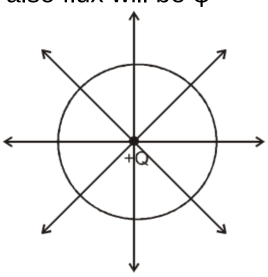
$\phi = \int \vec{E} \cdot \vec{ds} = \int E\, ds$ $\qquad$ as $\vec{E}$ is along $\vec{ds}$ (normal)

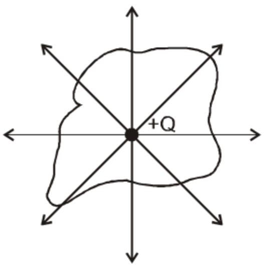
$\phi = \frac{1}{4\pi\varepsilon_o} \frac{Q}{R^2} \int ds$ $\qquad$ where $\int ds = 4\pi R^2$

$\phi = \left(\frac{1}{4\pi R^2} \frac{Q}{R^2}\right)\left(4\pi R^2\right) \Rightarrow \phi = \frac{Q}{\varepsilon_o}$

Now if the charge Q is enclosed by any other closed surface, still same lines of force will pass through the surface.

So here also flux will be $\phi = \frac{Q}{\varepsilon_o}$, that's what Gauss Theorem is.

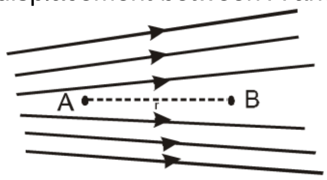




$\phi = \frac{Q}{\varepsilon_o}$ $\qquad\qquad$ $\phi = \frac{Q}{\varepsilon_o}$

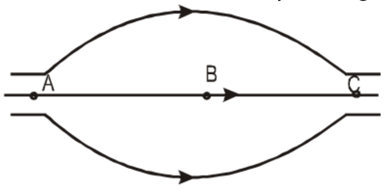
Self Practice Problems

16. Figure shows the electric lines of force emerging from a charged body. If the electric field at A and B are E_A and E_B respectively and if the displacement between A and B is r then



(1) $E_A > E_B$ (2) $E_A < E_B$ (3) $E_A = \frac{E_B}{r}$ (4) $E_A = \frac{E_B}{r^2}$

17. The figure shows some of the elective field lines corresponding to an elective filed. The figure suggests



(1) $E_A > E_B > E_C$ (2) $E_A = E_B = E_C$ (3) $E_A = E_C > E_B$ (4) $E_A = E_C < E_B$

Ans. 16. (1) 17. (3)

17. GAUSS'S LAW IN ELECTROSTATICS OR GAUSS'S THEOREM

This law was stated by a mathematician Karl F Gauss. This law gives the relation between the electric field at a point on a closed surface and the net charge enclosed by that surface. This surface is called Gaussian surface. It is a closed hypothetical surface. Its validity is shown by experiments. It is used to determine the electric field due to some symmetric charge distributions.

17.1 Statement and Details :

Gauss's law is stated as given below.

The surface integral of the electric field intensity over any closed hypothetical surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the total charge enclosed within the surface. Here, ε_0 is the permittivity of free space.

If S is the Gaussian surface and $\sum_{i=1}^{n} q_i$ is the total charge enclosed by the Gaussian surface, then according to Gauss's law,

$$\phi_E = \oint \vec{E} \cdot d\vec{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i .$$

The circle on the sign of integration indicates that the integration is to be carried out over the closed surface.

Note : (i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on total charge present inside gaussian surface.

(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.

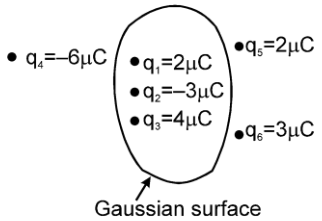
(iv) Electric field intensity at the gaussian surface is due to all the charges present inside as well as outside the gaussian surface.

(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive, because $\hat{n}$ is taken positive in outward direction.

(vi) In a gaussian surface $\phi = 0$ does not imply E = 0 at every point of the surface but E = 0 at every point implies $\phi = 0$.

Solved Examples

Example 93 Find out flux through the given gaussian surface.



Solution : $\phi = \frac{Q_{in}}{\varepsilon_0} = \frac{2\mu C - 3\mu C + 4\mu C}{\varepsilon_0} = \frac{3 \times 10^{-6}}{\varepsilon_0}$ Nm^2/C

Example 94 If a point charge q is placed at the centre of a cube then find out flux through any one surface of cube.

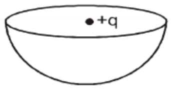
Solution : Flux through 6 surfaces = $\frac{q}{\varepsilon_0}$. Since all the surfaces are symmetrical

so, flux through one surfaces = $\frac{1}{6}\frac{q}{\varepsilon_0}$

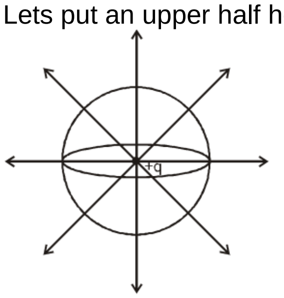
17.2 Flux through open surfaces using Gauss's Theorem :

Solved Examples

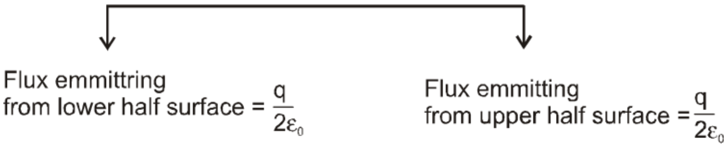
Example 95 A point charge +q is placed at the centre of curvature of a hemisphere. Find flux through the hemispherical surface.



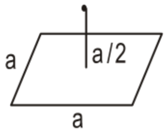
Solution : Lets put an upper half hemisphere. Now flux passing through the entire sphere = $\frac{q}{\varepsilon_0}$



As the charge q is symmetrical to the upper half and lower half hemispheres, so half-half flux will emit from both the surfaces.

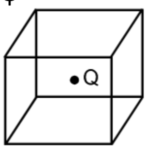


Example 96 A charge Q is placed at a distance a/2 above the centre of a horizontal, square surface of edge a as shown in figure. Find the flux of the electric field through the square surface.

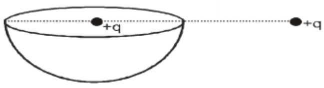


Solution : We can consider imaginary faces of cube such that the charge lies at the centre of the cube. Due to symmetry we can say that flux through the given area (which is one face of cube)

$\phi = \frac{Q}{6\varepsilon_0}$

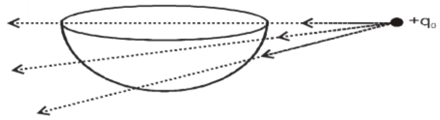


Example 97 Find flux through the hemispherical surface



Solution : (i) Flux through the hemispherical surface due to +q = $\frac{q}{2\varepsilon_0}$ (we have seen in previous examples)

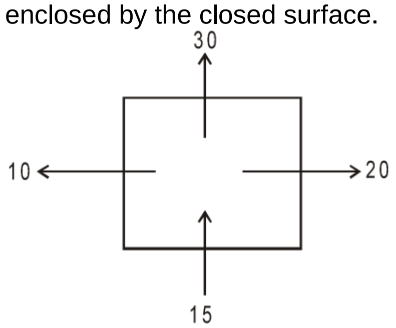
(ii) Flux through the hemispherical surface due to $+q_0$ charge = 0, because due to $+q_0$ charge field lines entering the surface = field lines coming out of the surface.



17.3 Finding q_{in} from flux :

Solved Examples

Example 98 Flux (in S.I.units) coming out and entering a closed surface is shown in the figure . Find charge enclosed by the closed surface.



Solution : Net flux through the closed surface = + 20 + 30 + 10 -15 = 45 N.m^2/c from Gauss`s theorem

$\phi_{net} = \frac{q_{in}}{\varepsilon_0}$ ⇒ $45 = \frac{q_{in}}{\varepsilon_0}$ ⇒ $q_{in} = (45)\varepsilon_0$

17.4 Finding electric field from Gauss`s Theorem :

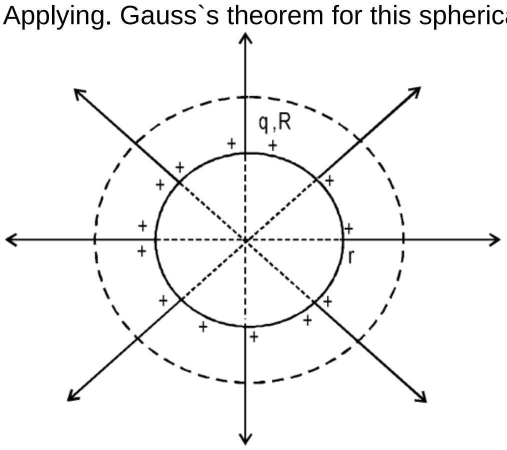
From gauss`s theorem, we can say

$$\int \vec{E}.\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0}$$

17.4.1 Finding E due to a spherical shell :-

Electric field outside the Sphere :

Since, electric field due to a shell will be radially outwards. So lets choose a spherical Gaussian surface Applying. Gauss`s theorem for this spherical Gauss`s surface,



$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{q}{\varepsilon_o}$$

↓

$\int |\vec{E}||\vec{ds}|\cos 0$ (because the $\vec{E}$ is normal to the surface)

↓

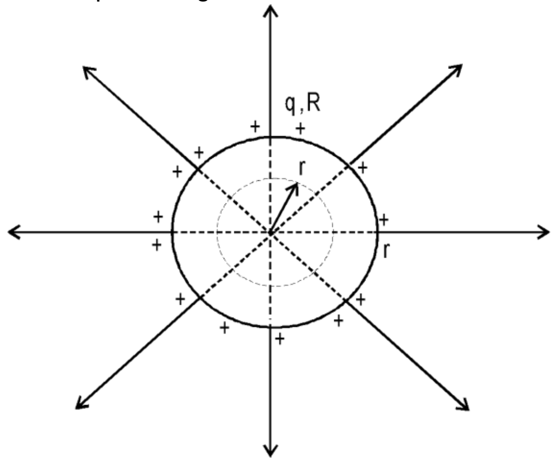
$E\int ds$ (because value of E is constant at the surface)

$E(4\pi r^2)$ ($\int ds$ total area of the spherical surface = $4\pi r^2$)

$\Rightarrow \quad E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0} \qquad \Rightarrow \quad E_{out} = \frac{q}{4\pi\varepsilon_o r^2}$

Electric field inside a spherical shell :

Lets choose a spherical gaussian surface inside the shell. Applying Gauss`s theorem for this surface



$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_0} = 0$

$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$

$\downarrow$

$E\int ds$

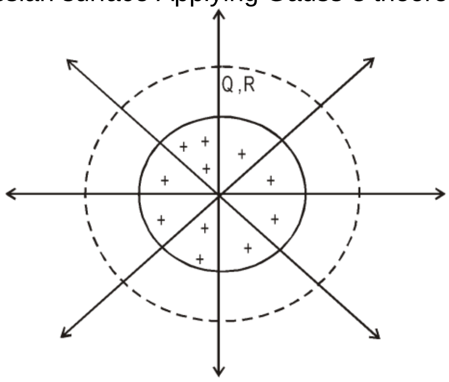
$\downarrow$

$E\,(4\pi r^2) \qquad \Rightarrow \qquad E(4\pi r^2) = 0 \qquad \Rightarrow \qquad E_{in} = 0$

17.4.2 Electric field due to solid sphere (having uniformly distributed charge Q and radius R) :

Electric field outside the sphere :

Direction of electric field is radially outwards, so we will choose
a spherical gaussian surface Applying Gauss`s theorem



$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{Q}{\varepsilon_o}$

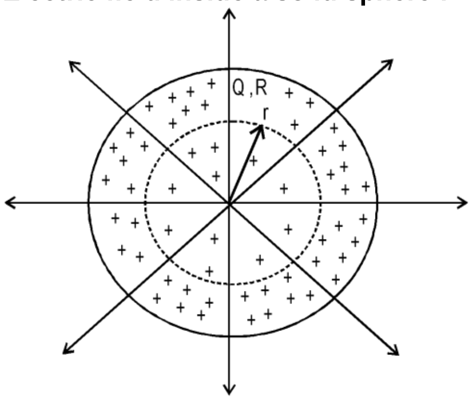
$\downarrow$

$\int |\vec{E}||\vec{ds}|\cos 0$

$$\downarrow$$
$$E\int ds$$
$$\downarrow$$
$$E\,(4\pi r^2)$$

$\Rightarrow \quad E\,(4\pi r^2) = \dfrac{Q}{\varepsilon_o} \quad \Rightarrow \quad E_{out} = \dfrac{Q}{4\pi\varepsilon_o r^2}$

Electric field inside a solid sphere :



For this choose a spherical gaussian surface inside the solid sphere Applying gauss`s theorem for this surface

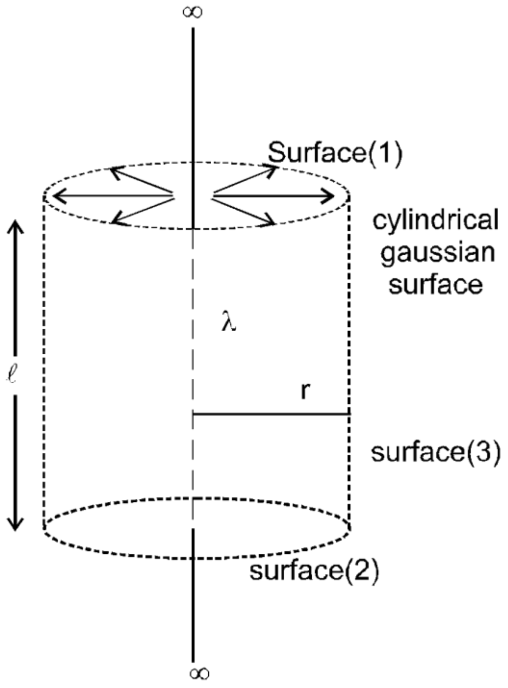
$$\int \vec{E}\,\vec{ds} = \phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\dfrac{q}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3}{\varepsilon_o} = \frac{qr^3}{\varepsilon_o R^3}$$

$$\downarrow$$
$$\int E\,ds$$
$$\downarrow$$

$E\,(4\pi r^2) \quad \Rightarrow \quad E(4\pi r^2) = \dfrac{qr^3}{\varepsilon_o R^3}$

$E = \dfrac{q\ r}{4\pi\varepsilon_o R^3} \quad \Rightarrow \quad E_{in} = \dfrac{kQ}{R^3}\ r$

17.4.3 Electric field due to infinite line charge (having uniformly distributed charged of charge density λ) :


∞
Surface(1)
cylindrical
gaussian
surface
λ
ℓ
r
surface(3)
surface(2)
∞

Electric field due to infinite wire is radial so we will choose cylindrical Gaussian surface as shown is figure.

$$= \frac{q_{in}}{\varepsilon_o} = \frac{\lambda \ell}{\varepsilon_o}$$

ϕ_{net}

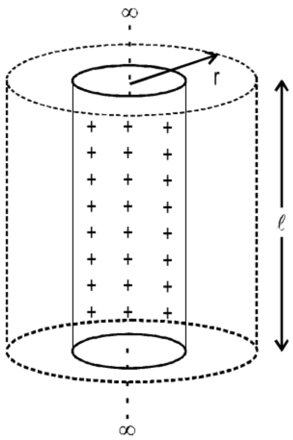
$\phi_1 = 0$ $\phi_2 = 0$ $\phi_3 \neq 0$

$$\phi_3 = \int \vec{E}.\vec{ds} = \int E\,ds = E\int ds = E\,(2\pi r \ell)$$

$$E\,(2\pi r\ell) = \frac{\lambda \ell}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r} = \frac{2k\lambda}{r}$$

17.4.4 Electric field due to infinity long charged tube (having uniform surface charge density σ and radius R)):



(i) **E out side the tube :-** lets choose a cylindrical gaussian surface

$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

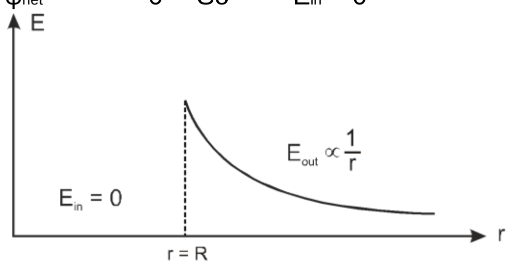
$$E_{out} \times 2\pi r\ell = \frac{\sigma 2\pi R\ell}{\varepsilon_o}$$

$$E = \frac{\sigma R}{r\,\varepsilon_0}$$

(ii) **E inside the tube :**

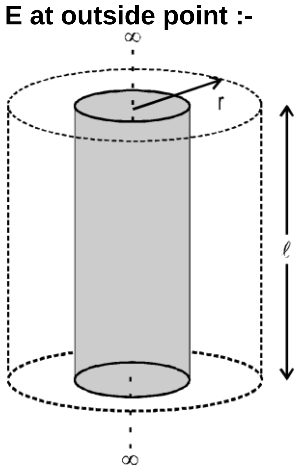
lets choose a cylindrical gaussian surface in side the tube.

$$\phi_{net} = \frac{q_{in}}{\varepsilon_o} = 0 \quad \text{So} \quad E_{in} = 0$$



17.4.5 E due to infinitely long solid cylinder of radius R (having uniformly distributed charge in volume (charge density ρ)) :

(i) E at outside point :-

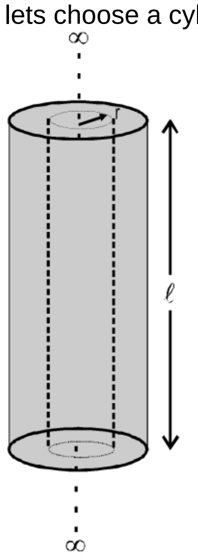


Lets choose a cylindrical gaussian surface. Applying gauss`s theorem

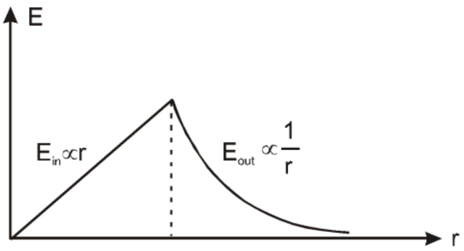
$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi R^2 \ell}{\varepsilon_o} \Rightarrow E_{out} = \frac{\rho R^2}{2r\,\varepsilon_0}$$

(ii) E at inside point :

lets choose a cylindrical gaussian surface inside the solid cylinder. Applying gauss`s theorem



$$E \times 2\pi r\ell = \frac{q_{in}}{\varepsilon_o} = \frac{\rho \times \pi r^2 \ell}{\varepsilon_o} \Rightarrow E_{in} = \frac{\rho r}{2\varepsilon_0}$$



Self Practice Problems

18. A cylinder of radius R and length L is placed in a uniform electric field E parallel to the cylinder axis.The total flux for the surface of the cylinder is given by

(1) $2\pi R_2 E$ (2) $\pi R_2/E$ (3) $(\pi R_2 - \pi R)E$ (4) zero

19. Electric field at a point varies as r_0 for
(1) An electric dipole (2) A point charge
(3) A plane infinite sheet of charge (4) A line charge of infinite length

20. An electric charge q is placed at the centre of a cube of side α. The electric fiux on one of its faces will be
(1) $\frac{q}{6\varepsilon_0}$ (2) $\frac{q}{\varepsilon_0 a^2}$ (3) $\frac{q}{4\pi\varepsilon_0 a^2}$ (4) $\frac{q}{\varepsilon_0}$

21. Total electric fiux coming out of a unit positive charge put in air is
(1) ε_0 (2) ε_0^{-1} (3) $(4\pi\varepsilon_0)_{-1}$ (4) $4\pi\varepsilon_0$

22. According to Gauss Theorem electric field of an infinitely long staight wire is proportional to
(1) r (2) $\frac{1}{r^2}$ (3) $\frac{1}{r^3}$ (4) $\frac{1}{r}$

23. The S.I unit of electric flux is
(1) Weber (2) Newton per coulomb (3) Volt ×meter (4) Joule per coulomb

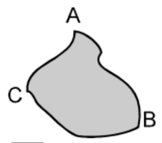
Ans. **18. (4) 19. (3) 20. (1) 21. (2) 22. (4) 23. (3)**

18. CONDUCTOR

18.1 Conductor and it's properties [For electrostatic condition]

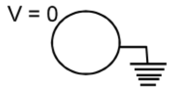
(i) Conductors are materials which contains large number of free electrons which can move freely inside the conductor.
(ii) In electrostatics conductors are always equipotential surfaces.
(iii) Charge always resides on outer surface of conductor.
(iv) If there is a cavity inside the conductor having no charge then charge will always reside only on outer surface of conductor.
(v) Electric field is always perpendicular to conducting surface.
(vi) Electric lines of force never enter into conductors.
(vii) Electric field intensity near the conducting surface is given by formula

$$\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$



$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n}$; $\vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n}$ and $\vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$

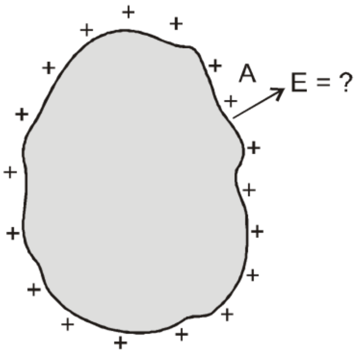
(viii) When a conductor is grounded its potential becomes zero.



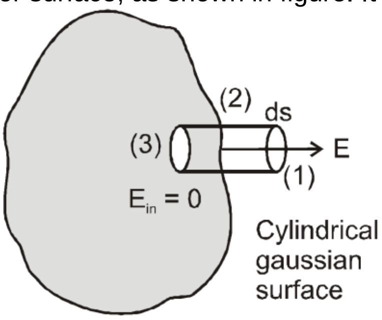
(ix) When an isolated conductor is grounded then its charge becomes zero.
(x) When two conductors are connected there will be charge flow till their potential becomes equal
(xi) Electric pressure : Electric pressure at the surface of a conductor is givey by formula P = $\frac{\sigma^2}{2\varepsilon_0}$ where σ is the local surface charge density.

18.2 Finding field due to a conductor

Suppose we have a conductor, and at any 'A', local surface charge density = σ. We have to find electric field just outside the conductor surface.



For this lets consider a small cylindrical gaussian surface, which is partly inside and partly outside the conductor surface, as shown in figure. It has a small cross section area ds and negligeable height.



Applying gauss's theorem for this surface

$$\phi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma\, ds}{\varepsilon_0}$$

- flux through surface (1): $\phi_2 = Eds$ (because $\vec{E}$ is normal to the surface of conductor)
- flux through surface (2): $\phi_2 = 0$ ($\vec{E}$ is normal to curved Gaussian surface)
- flux through surface (3): $\phi_3 = 0$ (as E inside the conductor = 0)

So, $Eds = \dfrac{\sigma ds}{\varepsilon_0}$ $\quad E = \dfrac{\sigma}{\varepsilon_0}$

Electric field just outside the surface of conductor $E = \dfrac{\sigma}{\varepsilon_0}$ direction will be normal to the surface

in vector form $\vec{E} = \dfrac{\sigma}{\varepsilon_0}\hat{n}$ (here $\hat{n}$ = unit vector normal to the conductor surface)

18.3 Electrostatic pressure at the surface of the conductor

Electrostatic pressure at the surface of the conductor $P = \dfrac{\sigma^2}{2\varepsilon_o}$

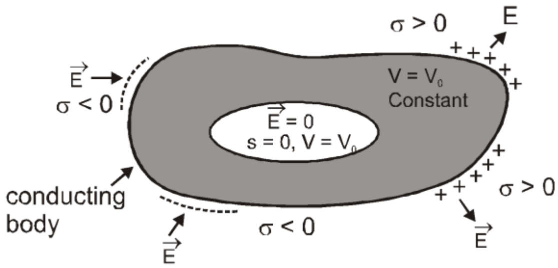
where σ = local surface charge density.

Electrostatic shielding

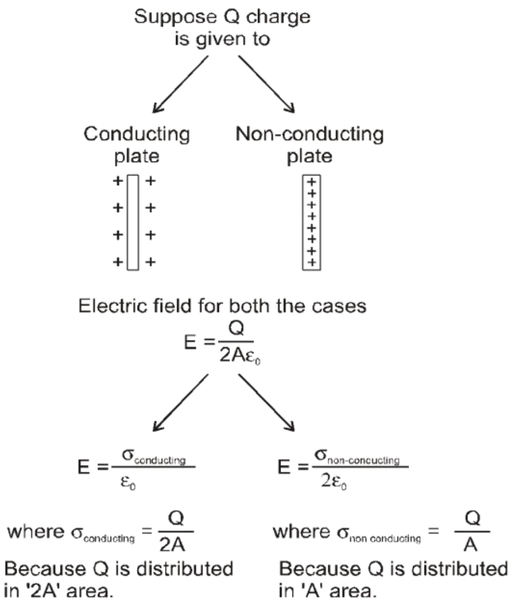
Consider a conductor with a cavity of any shape and size, with no charges inside the cavity. The electric field inside the cavity is zero, whatever be the charge on the conductor and the external fields in which it might be placed.

Any cavity in a conductor remains shielded from outside electric influence: the field inside the cavity is always zero (If cavity having no charge). This is known as electrostatic shielding.

This effect can be made use of in protecting sensitive instruments from outside electrical influence.



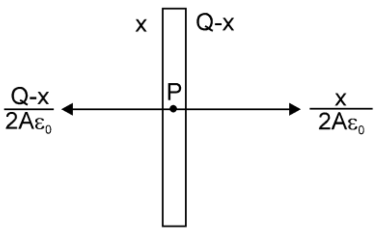
18.4 Electric field due to a conducting and nonconducting uniformaly charge infinite sheets



Solved Examples

Example 99 Prove that if an isolated (isolated means no charges are near the sheet) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Solution :



Let there is x charge on left side of sheet and Q–x charge on right side of sheet. Since point P lies inside the conductor so

$E_P = O$

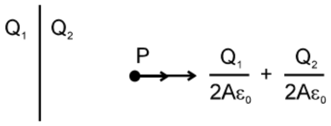
$\frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} = 0 \quad \Rightarrow \frac{2x}{2A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \quad \Rightarrow x = \frac{Q}{2} \qquad Q - x = \frac{Q}{2}$

So charge in equally distributed on both sides

Example 100 If an isolated infinite sheet contains charge Q_1 on its one surface and charge Q_2 on its other surface then prove that electric field intensity at a point in front of sheet will be $\frac{Q}{2A\varepsilon_0}$, where $Q = Q_1 + Q_2$

Solution : Electric field at point P :

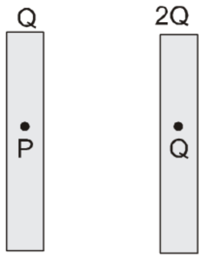
$\vec{E} = \vec{E}_{Q_1} + \vec{E}_{Q_2}$



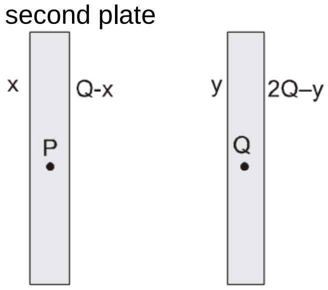
$= \frac{Q_1}{2A\varepsilon_0}\hat{n} + \frac{Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q_1 + Q_2}{2A\varepsilon_0}\hat{n} = \frac{Q}{2A\varepsilon_0}\hat{n}$

[This shows that the resultant field due to a sheet depends only on the total charge of the sheet and not on the distribution of charge on individual surfaces].

Example 101 Two large parallel conducting sheets (placed at finite distance) are given charges Q and 2Q respectively. Find out charges appearing on all the surfaces.



Solution : Let there is x amount of charge on left side of first plate, so on its right side charge will be Q–x, similarly for second plate there is y charge on left side and 2Q – y charge is on right side of second plate



$E_p = 0$ (By property of conductor)

$\Rightarrow \frac{x}{2A\varepsilon_0} - \left\{\frac{Q-x}{2A\varepsilon_0} + \frac{y}{2A\varepsilon_0} + \frac{2Q-y}{2A\varepsilon_0}\right\} = 0$

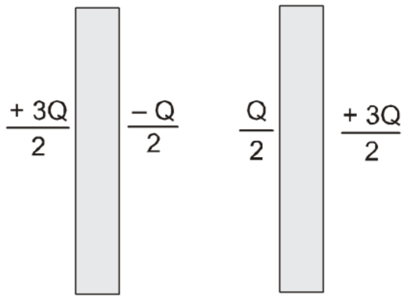
we can also say that charge on left side of P = charge on right side of P

$x = Q - x + y + 2Q - y \qquad \Rightarrow x = \frac{3Q}{2}, \; Q - x = \frac{-Q}{2}$

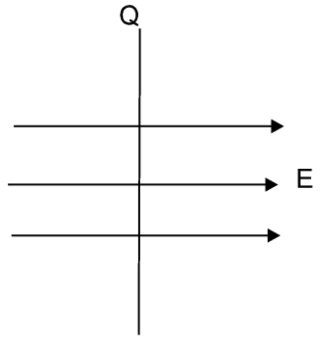
Similarly for point Q:

$x + Q - x + y = 2Q - y \qquad \Rightarrow y = Q/2, \; 2Q - y = 3Q/2$

So final charge distribution of plates is : -

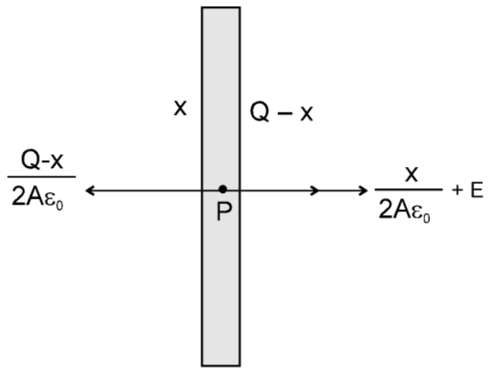


Example 102 An isolated conducting sheet of area A and carrying a charge Q is placed in a uniform electric field E, such that electric field is perpendicular to sheet and covers all the sheet. Find out charges appearing on its two surfaces.



Solution : Let there is x charge on left side of plate and Q – x charge on right side of plate

$E_P = 0$



$$\frac{x}{2A\varepsilon_0} + E = \frac{Q - x}{2A\varepsilon_0}$$

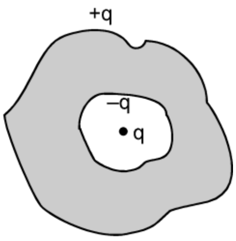
$$\Rightarrow \quad \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E \quad \Rightarrow \quad x = \frac{Q}{2} - EA\varepsilon_0 \text{ and } Q - x = \frac{Q}{2} + EA\varepsilon_0$$

So charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and other side $\frac{Q}{2} + EA\varepsilon_0$

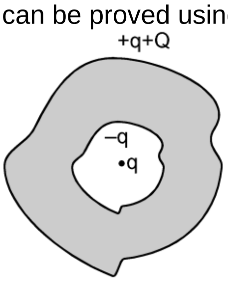
Note : Solve this question for Q = 0 without using the above answer and match that answers with the answers that you will get by putting Q = 0 in the above answer.

18.5 Some other important results for a closed conductor.

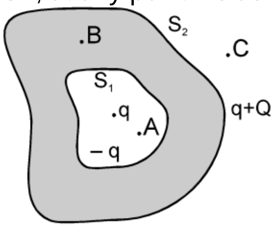
(i) If a charge q is kept in the cavity then –q will be induced on the inner surface and +q will be induced on the outer surface of the conductor (it can be proved using gauss theorem)



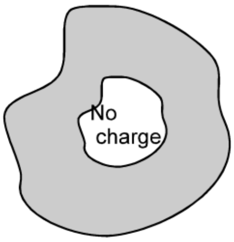
(ii) If a charge q is kept inside the cavity of a conductor and conductor is given a charge Q then –q charge will be induced on inner surface and total charge on the outer surface will be q + Q. (it can be proved using gauss theorem)



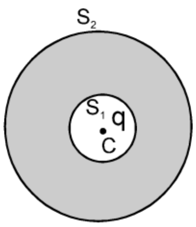
(iii) Resultant field, due to q (which is inside the cavity) and induced charge on S_1, at any point outside S_1 (like B,C) is zero. Resultant field due to **q + Q on S_2 and any other charge outside S_2** , at any point inside of surface S_2 (like A, B) is zero



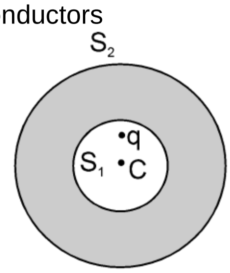
(iv) Resultant field in a charge free cavity in a closed conductor is zero. There can be charges outside the conductor and on the surface also. Then also this result is true. No charge will be induced on the inner most surface of the conductor.



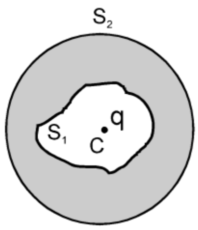
(v) Charge distribution for different types of cavities in conductors



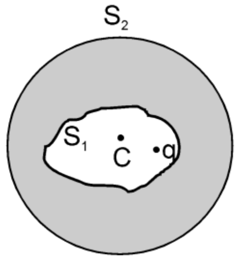
(A) charge is at the common centre (S_1, S_2 → spherical)



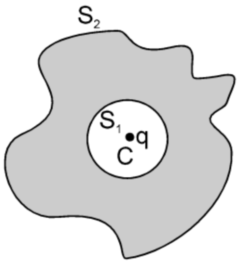
(B) charge is not at the common centre (S_1, S_2 → spherical)

(C) 

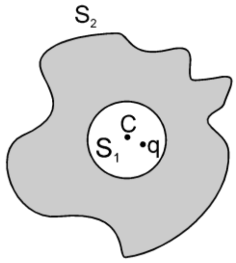
charge is at the centre of S_2
($S_2 \rightarrow$ spherical)

(D) 

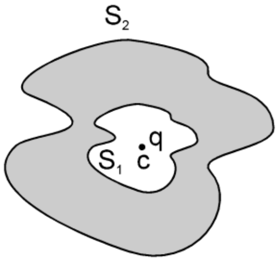
charge is not at the centre of S_2
($S_2 \rightarrow$ spherical)

(E) 

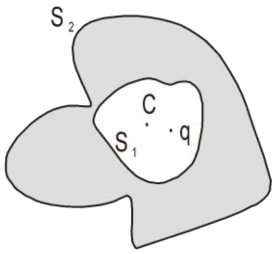
charge is at the centre of S_1 (Spherical)

(F) 

charge not at the centre of S_1(Spherical)

(G) 

charge is at the geometrical centre

(H) 

charges is not at the geometrical centre

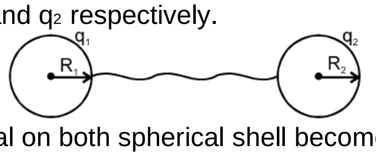
Using the result that $\vec{E}_{res}$ in the conducting material should be zero and using result (iii) We can show that

Case	A	B	C	D	E	F	G	H
S_1	Uniform	Nonuniform	Nonuniform	Nonuniform	Uniform	Nonuniform	Nonuniform	Nonuniform
S_2	Uniform	Uniform	Uniform	Uniform	Nonuniform	Nonuniform	Nonuniform	NonUniform

Note : In all cases charge on inner surface $S_1 = -q$ and on outer surface $S_2 = q$. The distribution of charge on 'S_1' will not change even if some charges are kept outside the conductor (i.e. outside the surface S_2). But the charge distribution on 'S_2' may change if some charges(s) is/are kept outside the conductor.

(vi) Sharing of charges :

Two conducting hollow spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and seperated by large distance, are joined by a conducting wire Let final charges on spheres are q_1 and q_2 respectively.



Potential on both spherical shell become equal after joining, therefore

$$\frac{Kq_1}{R_1} = \frac{Kq_2}{R_2} \Rightarrow \frac{q_1}{q_2} = \frac{R_1}{R_2} \qquad ......(i)$$

and, $q_1 + q_2 = Q_1 + Q_2$ ......(ii)

from (i) and (ii) $q_1 = \dfrac{(Q_1+Q_2)R_1}{R_1+R_2}$ $q_2 = \dfrac{(Q_1+Q_2)R_2}{R_1+R_2}$

ratio of charges $\dfrac{q_1}{q_2} = \dfrac{R_1}{R_2} \Rightarrow \dfrac{\sigma_1 4\pi R_1^2}{\sigma_2 4\pi R_2^2} = \dfrac{R_1}{R_2}$

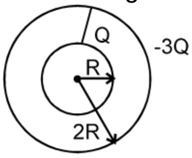
ratio of surface charge densities $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

Ratio of final charges $\frac{q_1}{q_2} = \frac{R_1}{R_2}$

Ratio of final surface charge densities. $\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$

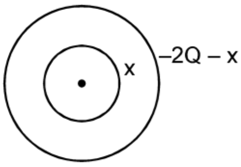
Solved Examples

Example 103 The two conducting spherical shells are joined by a conducting wire and cut after some time when charge stops flowing.Find out the charge on each sphere after that.



Solution : After cutting the wire, the potential of both the shells is equal

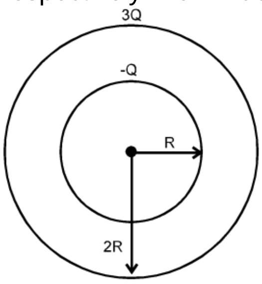
Thus, potential of inner shell $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{k(x-2Q)}{2R}$ and potential of outer shell $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$



As $V_{out} = V_{in}$ $\Rightarrow$ $\frac{-KR}{R} = \frac{K(x-2Q)}{2R}$ $\Rightarrow -2Q = x - 2Q \Rightarrow$ $x = 0$

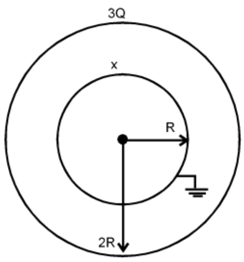
So charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

Example 104 Two conducting hollow spherical shells of radii R and 2R carry charges – Q and 3Q respectively. How much charge will flow into the earth if inner shell is grounded ?



Solution : When inner shell is grounded to the Earth then the potential of inner shell will become zero because potential of the Earth is taken to be zero.

$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$ $\Rightarrow$ $x = \frac{-3Q}{2}$, the charge that has increased



$= \frac{-3Q}{2} - (-Q) = \frac{-Q}{2}$ hence charge flows into the Earth = $\frac{Q}{2}$

Example 105 An isolated conducting sphere of charge Q and radius R is connected to a similar uncharged sphere (kept at a large distance) by using a high resistance wire. After a long time what is the amount of heat loss ?

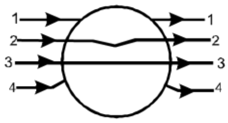
Solution : When two conducting spheres of equal radius are connected charge is equally distributed on them (Result VI). So we can say that heat loss of system

$\Delta H = U_i - U_f$

$$= \left(\frac{Q^2}{8\pi\varepsilon_0 R} - 0\right) - \left(\frac{Q^2/4}{8\pi\varepsilon_0 R} + \frac{Q^2/4}{8\pi\varepsilon_0 R}\right) = \frac{Q^2}{16\pi\varepsilon_0 R}$$

Self Practice Problems

24. A metallic soild sphere is placed in a unifrom elective field. The lines of force follow the path (s) shown in figure as



(1) 1 (2) 2 (3) 3 (4) 4

25. Inside a hollow charged spherical conductor, the potenital
(1) Is constant
(2) Varies directly as the distance from the centre
(3) Varies inversely as the distance from the centre
(4) Varies inversely as the spuare of the distance from the centre

26. If q is the charge per unit area on the surface of a conductor,then the electric field intensity at a point on the surface is
(1) $\left(\frac{q}{\varepsilon_0}\right)$ normal to surface (2) $\left(\frac{q}{2\varepsilon_0}\right)$ normal to surface
(3) $\left(\frac{q}{\varepsilon_0}\right)$ tangenital to surface (4) $\left(\frac{q}{2\varepsilon_0}\right)$ tangenital to surface

27. A hollow conducting sphere of radius R has a charge (+Q) on its surface.What is the electric potential within the sphere at a distance r = $\frac{R}{3}$ From its centre
(1) Zero (2) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ (3) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ (4) $\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$

Ans. 24. (4) 25. (1) 26. (1) 27. (3)

10. VAN DE GRAFF GENERATOR

This is a machine that can build up high voltages of the order of a few million volts. The resulting large electric fields are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter.

(i) Designed by R.J. Van de Graaff in 1931.
(ii) It is an electrostatic generator capable of generating very high potential of the order of 5×10^6 V.
(iii) This high potential is used in accelerating the charged particles.

Principle : It is based on the following two electrostatic phenomena :
(1). The electric discharge takes place in air or gases readily at pointed conductors.
(2) (i) If a hollow conductor is in contact with an other conductor, which lies inside the hollow conductor. Then as charge is supplied to inner conductor. The charge immediately shifts to outer surface of the hollow conductor.
Consider a large spherical conducting shell A having radius R and charge +Q, potential inside the shell is constant and it is equal to that at its surface.
Therefore, potential inside the charged conducting shell A,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R}$$

Suppose that a small conducting sphere B having radius r and charge +q is placed at the centre of the shell A.
Then, potential due to the sphere B at the surface of shell A,

$$V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

and potential due to the sphere B at its surface,

$$V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Thus, total potential at the surface of shell A due to the charges Q and q,

$$V_A = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R} \quad \text{or} \quad V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

and the total potential at the surface of sphere B due to the charges Q and q,

$$V_B = V_1 + V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \quad \text{or} \quad V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

It follows that $V_B > V_A$. Hence, potential difference between the sphere and the shell,

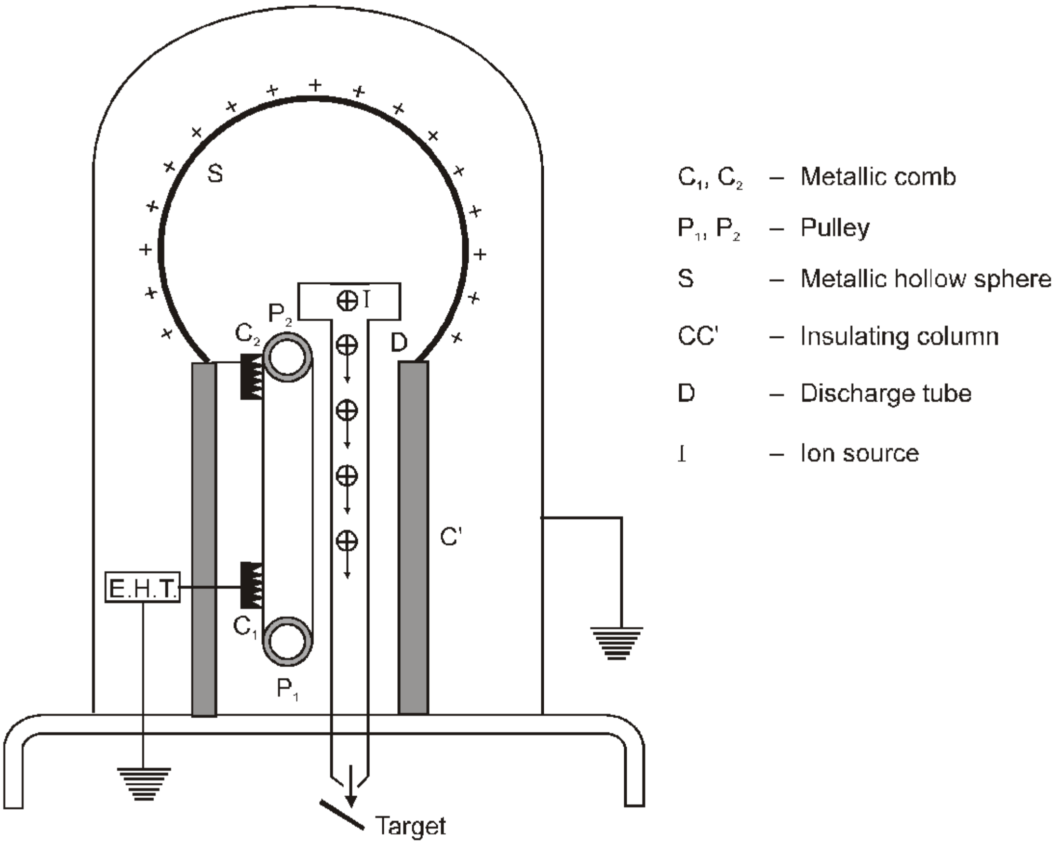
$$V = V_B - V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right) \Rightarrow V = \frac{1}{4\pi\varepsilon_0} . q\left(\frac{1}{r} - \frac{1}{R}\right)$$

It follows that potential difference between the sphere and the shell is independent of the charge Q on the shell. Therefore, if the sphere is connected to the shell by a wire, the charge supplied to the sphere will immediately flow to the shell.

It is because, the potential of the sphere is higher than that of the shell and the charge always flows from higher to lower potential.

It forms the basic principle of Van de Graaff generator.

Construction :



Working :

(i) An endless belt of an insulating material is made to run on two pulleys P_1 and P_2 with the help of an electric motor.

(ii) The metal comb C_1, called spray comb is held potential with the help of E.H.T. source ($\approx 10^4$ V), it produces ions in its vicinity. The positive ions get sprayed on the belt due to the repulsive action of comb C_1.

(iii) These positive ions are carried upward by the moving belt. A comb C_2, called collecting comb is positioned near the upper end of the belt, such that the pointed ends touch the belt and the other end is in contact with the inner surface of the metallic sphere S. The comb C_2 collects the positive ions and transfers them to the metallic sphere.

(iv) The charge transferred by the comb C_2 immediately moves on to the outer surface of the hollow sphere. As the belt goes on moving, the accumulation of positive charge on the sphere also keeps on taking place continuously and its potential rises considerably.

(v) With the increase of charge on the sphere, its leakage due to ionisation of surrounding air also becomes faster.

(vi) The maximum potential to which the sphere can be raised is reached, when the rate of loss of charge due to leakage becomes equal to the rate at which the charge is transferred to the sphere.

(vii) To prevent the leakage of charge from the sphere, the generator is completely enclosed inside an earth-connected steel tank, which is filled with air under pressure.

(viii) If the charged particles, such as protons, deutrons, etc. are now generated in the discharge tube D with lower end earthed and upper end inside the hollow sphere, they get accelerated in downward direction along the length of the tube. At the other end, they come to hit the target with large kinetic energy.

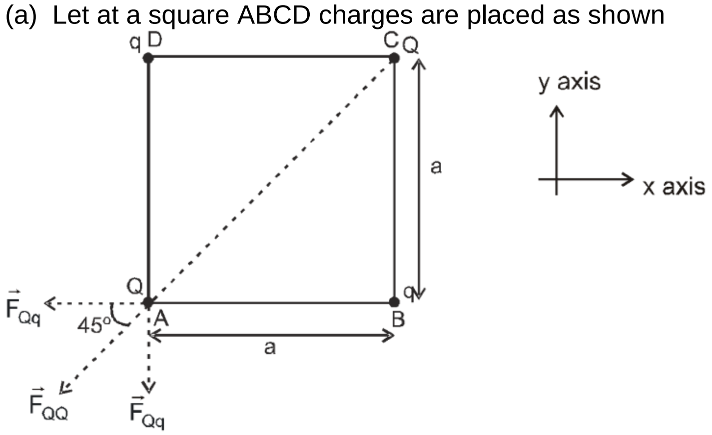
(ix) Van de Graaff generator of this type was installed at the Carnegie institute in Washington in 1937. One such generator was installed at Indian Institute of Technology, kanpur in 1970 and it accelerates particles to 2 MeV energy.

Problem 1. Two charges of Q each are placed at two opposite corners of a square. A charge q is placed at each of the other two corners.

(a) If the resultant force on Q is zero, how are Q and q related ?

(b) Could q be chosen to make the resultant force on each charge zero ?

Solution : (a) Let at a square ABCD charges are placed as shown



Now forces on charge Q (at point A) due to other charge are $\vec{F}_{QQ}$, $\vec{F}_{Qq}$ and F_{Qq} respectively shown in figure.

F_{net} on Q = $\vec{F}_{Q.Q} + \vec{F}_{Qq} + \vec{F}_{Qq}$ (at point A)

But $F_{net} = 0$ So, $\Sigma F_x = 0$

$\Sigma F_x = -F_{QQ}\cos 45° - F_{Qq}$

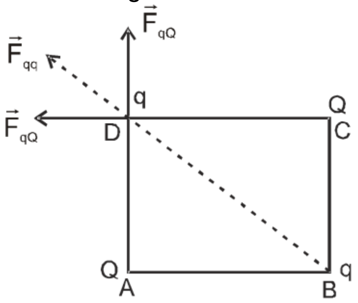
$\Rightarrow \dfrac{KQ^2}{(\sqrt{2}a)^2}\cdot\dfrac{1}{\sqrt{2}} + \dfrac{KQq}{a^2} = 0 \quad \Rightarrow \quad q = -\dfrac{Q}{2\sqrt{2}}$ **Ans.**

(b) For resultant force on each charge to be zero :

From previous data, force on charge Q is zero when $q = -\dfrac{Q}{2\sqrt{2}}$ if for this value of charge q, force on q is zero then and only then the value of q exists for which the resultant force on each charge is zero.

Force on q :-

Forces on charge q (at point D) due to other three charges are $\vec{F}_{qQ}$, $\vec{F}_{qq}$ and $\vec{F}_{qQ}$ respectively shown in figure.



Net force on charge q :-

$\vec{F}_{net} = \vec{F}_{qq} + \vec{F}_{qQ} + \vec{F}_{qQ}$ But $\vec{F}_{net} = 0$

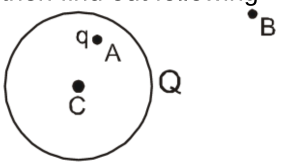
So, $\Sigma F_x = 0$

$$\Sigma F_x = -\frac{Kq^2}{(\sqrt{2}a)^2}\cdot\frac{1}{\sqrt{2}} - \frac{KQq}{(a)^2} \Rightarrow \quad q = -2\sqrt{2}\,Q$$

But from previous condition, $q = -\dfrac{Q}{2\sqrt{2}}$

So, no value of q makes the resultant force on each charge zero.

Problem 2. Figure shows a uniformly charged thin non-conducting sphere of total charge Q and radius R. If point charge q is situated at point 'A' which is at a distance r < R from the centre of the sphere then find out following



(i) Force acting on charge q.
(ii) Electric field at centre of sphere.
(iii) Electric field at point B.

Solution : (i) Electric field inside a hollow sphere = 0

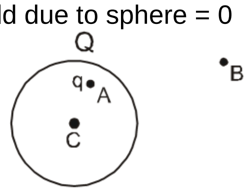
∴ Force on charge q.

F = qE = q × 0 = 0

(ii) Net electric field at centre of sphere

$E_{net} = \vec{E}_1 + \vec{E}_2$

E_1 = field due to sphere = 0



E_2 = field due to this charge = $\dfrac{Kq}{r^2}$

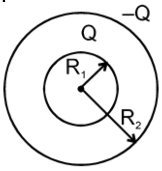
$E_{net} = \dfrac{Kq}{r^2}$

(iii) Electric field at B due to charge on sphere , $\vec{E}_1 = \dfrac{KQ}{r_1^2}\hat{r}_1$

and due to charge q at A , $\vec{E_2} = \frac{Kq}{r_2^2}\hat{r}_2$ So, $\vec{E}_{net} = \vec{E_1} + \vec{E_2} = \frac{KQ}{r_1^2}\hat{r}_1 + \frac{Kq}{r_2^2}\hat{r}_2$

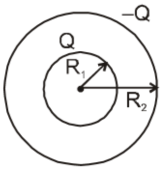
where r_1 = CB and r_2 = AB

Problem 3. Figure shows two concentric sphere of radius R_1 and R_2 ($R_2 > R_1$) which contains uniformly distributed charges Q and –Q respectively. Find out electric field intensities at the following positions :



(i) $r < R_1$ (ii) $R_1 \le r < R_2$ (iii) $r \ge R_2$

Solution : Net electric field = $E_1 + E_2$



E_1 = field due to sphere of radius R_1
E_2 = field due of sphere of radius R_2

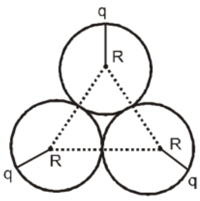
(i) $E_1 = 0, E_2 = 0$
$E_{net} = 0$

(ii) $E_1 = \frac{KQ}{r^2}$, $E_2 = 0$ $\Rightarrow$ $\vec{E} = \frac{Kq}{r^2}\hat{r}$

(iii) $\vec{E}_1 = \frac{Kq}{r^2}\hat{r}$ $\vec{E}_2 = \frac{Kq}{r^2}(-\hat{r})$ $\Rightarrow$ $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 = 0$

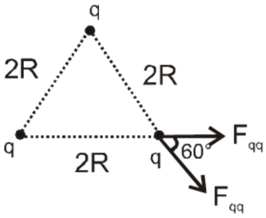
Problem 4. Three identical spheres each having a charge q (uniformly distributed) and radius R, are kept in such a way that each touches the other two. Find the magnitude of the electric force on any sphere due to other two.

Solution : Given three identical spheres each having a charge q and radius R are kept as shown :-



For any external point ; sphere behaves like a point charge. So it becomes a triangle having point charges on its corner.

$|\vec{F}_{qq}| = \frac{kq^2}{4R^2}$



So net force (F) = $2.\frac{kq^2}{4R^2}.\cos\frac{60}{2} = 2.\frac{kq^2}{4R^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}\frac{kq^2}{R^2}$. **Ans.**

Problem 5. A uniform electric field of 10 N/C exists in the vertically downward direction. Find the increase in the electric potential as one goes up through a height of 50cm.

Solution : $E = -\frac{dv}{dr} \Rightarrow dv = -\vec{E}\,.\,\vec{dr}$

for $\vec{E}$ = constant $\Rightarrow$ $\Delta v = -\vec{E}\,.\,\vec{\Delta r}$

$\Delta v = -10(-\hat{j}).(50 \times 10^{-2})\,\hat{j} = 5$ volts.

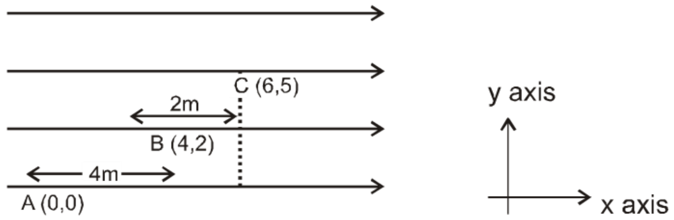
Problem 6. An electric field of 20 N/C exists along the x-axis in space. Calculate the potential difference $V_B - V_A$ where the point A and B are given by –

(a) A = (0,0) ; B = (4m , 2m)

(b) A = (4m,2m) ; B = (6m , 5m)

(c) A = (0,0) ; B = (6m , 5m)

Solution : Electric field in x - axis mean $\vec{E} = 20\hat{i}$

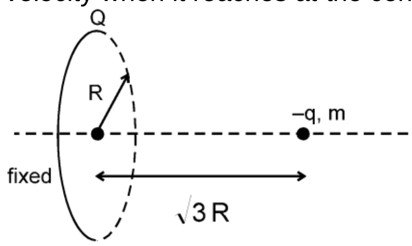


(a) $|\Delta V_{AB}| = \vec{E}.\vec{d} = 20\hat{i}\ .\ 4\hat{i} = 80\text{ V} \Rightarrow$ $V_B - V_A = -80V$

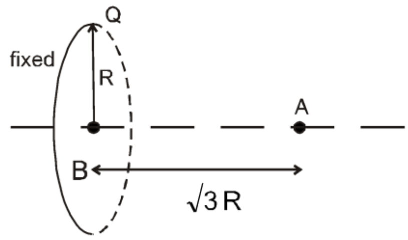
(b) $|\Delta V_{BC}| = \vec{E}.\vec{d} = 20\hat{i}\ .\ 2\hat{i} = 40\text{ volt}$ $\Rightarrow$ $V_C - V_B = -40V$

(c) $|\Delta V_{AC}| = \vec{E}.\vec{d} = 20\hat{i}\ .\ 6\hat{i} = 120\text{ volt}$ $\Rightarrow$ $V_C - V_A = -120\text{ V}$

Problem 7. A point charge of charge –q and mass m is released with negligible speed from a distance $\sqrt{3}R$ on the axis of fixed uniformly charged ring of charge Q and radius R. Find out its velocity when it reaches at the centre of the ring.



Solution :



As potential due to uniform charged ring at its axis (at x distance)

$V = \frac{kQ}{\sqrt{R^2 + x^2}}$;

So potential at point A due to ring

$V_1 = \frac{kQ}{\sqrt{R^2 + 3R^2}} = \frac{kQ}{2R}$

So potential energy of charge –q at point A

$P.E._1 = \frac{-kQq}{2R}$

and potential at point B $V_2 = \frac{kQ}{R}$

So potential energy of charge –q at point B

$P.E._2 = \frac{-kQq}{R}$

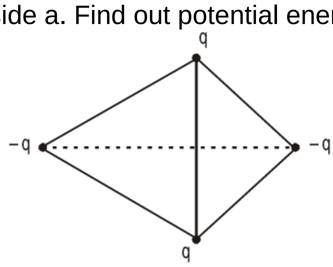
Now by energy conservation

$P.E._1 + K.E._1 = P.E._2 + K.E._2$

$$\frac{-kQq}{2R} + 0 = \frac{-kQq}{R} + \frac{1}{2}mv^2 \quad \Rightarrow \quad v^2 = \frac{kQq}{mR}$$

So velocity of charge – q at point B $\quad v = \sqrt{\frac{kQq}{mR}}$ **Ans.**

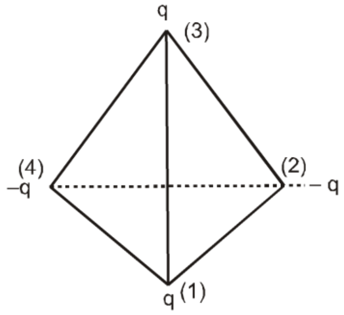
Problem 8. Four small point charges each of equal magnitude q are placed at four corners of a regular tetrahedron of side a. Find out potential energy of charge system



Solution : Potential energy of system :

$U = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

$$U = \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a} + \frac{-kq^2}{a} + \frac{kq^2}{a} + \frac{-kq^2}{a}$$



Total potential energy of this charge system $U = \frac{-2kq^2}{a}$

Problem 9. If $V = x^2y + y^2z$ then find $\vec{E}$ at (x, y, z)

Solution : Given $V = x^2y + y^2z$ and $\vec{E} = -\frac{\partial V}{\partial r}$

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

$$\vec{E} = -[2xy\,\hat{i} + (x^2+2yz)\,\hat{j} + y^2\,\hat{k}]$$

Problem 10. If $E = 2r^2$ then find V(r)

Solution : Given $E = 2r^2$

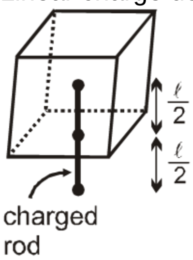
we know that $\int dv = -\int \vec{E}.\vec{dr} = -\int 2r^2\, dr$

$V(r) = \frac{-2r^3}{3} + c$ **Ans.**

Problem 11. A charge Q is uniformly distributed over a rod of length ℓ. Consider a hypothetical cube of edge ℓ with the centre of the cube at one end of the rod. Find the minimum possible flux of the electric field through the entire surface of the cube.

Solution : According to Gauss law : flux depend upon charge inside the closed hypothetical surface so for minimum possible flux through the entire surface of the cube = charge inside should be minimum.

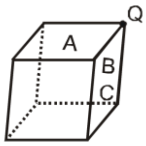
Linear charge density of rod = $\frac{Q}{\ell}$ and minimum length of rod inside the cube = $\frac{\ell}{2}$



So charge inside the cube = $\frac{\ell}{2} . \frac{Q}{\ell} = \frac{Q}{2}$

so flux through the entire surface of the cube = $\frac{\Sigma q}{\epsilon_0} = \frac{Q}{2\epsilon_0}$

Problem 12. A charge Q is placed at a corner of a cube. Find the flux of the electric field through the six surfaces of the cube.



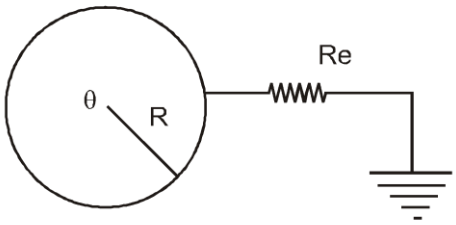
Solution :

By Gauss law, $\phi = \frac{q_{in}}{\varepsilon_0}$

Here, since Q is kept at the corner so only $\frac{q}{8}$ charge is inside the cube. (since complete charge can be enclosed by 8 such cubes) $\therefore q_{in} = \frac{Q}{8}$

So, $\phi = \frac{q_{in}}{\varepsilon_0} = \frac{Q}{8\varepsilon_0}$ **Ans.**

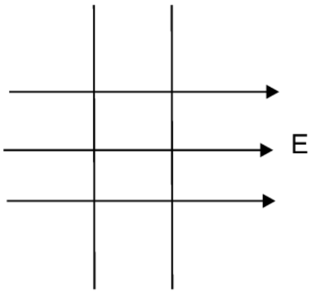
Problem 13. An isolated conducting sphere of charge Q and radius R is grounded by using a high resistance wire. What is the amount of heat loss ?



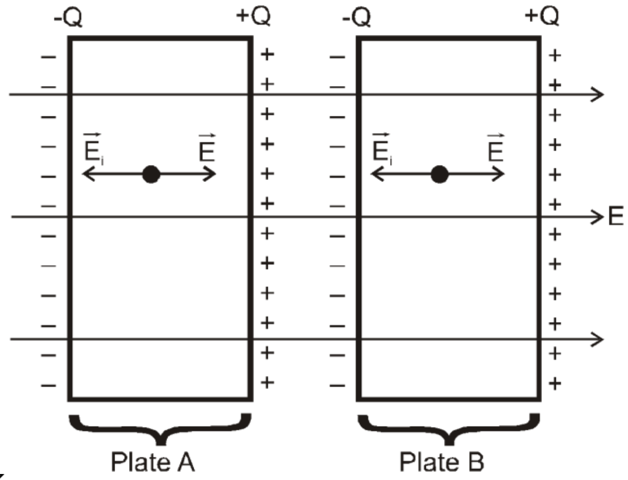
Solution :

When sphere is grounded it's potential become zero which means all charge goes to earth (due to sphere is conducting and isolated) so all energy in sphere is converted into heat so, total heat loss = $\frac{kQ^2}{2R}$

Problem 14. Two uncharged and parallel conducting sheets each of area A are placed in a uniform electric field E at a finite distance from each other. Such that electric field is perpendicular to sheets and covers all the sheets. Find out charges appearing on its two surfaces.



Solution : Plates are conducting so net electric field inside these plates should be zero. So, electric field due to induced charges (on the surface of the plate) balance the outside electric field.



Here $\vec{E}_i$ = induced electrid field

for both plates $\vec{E}_i + \vec{E} = 0$ $\Rightarrow$ $\vec{E}_i = -\vec{E}$ .... (1)

Let charge induced on surfaces are +Q and – Q then

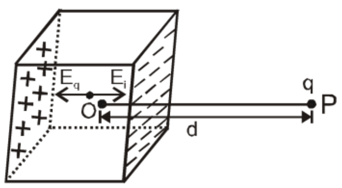
$|\vec{E}_i| = \frac{Q}{A\varepsilon_0}$

by equation (1)

$\frac{Q}{A\varepsilon_0} = E$ $\Rightarrow$ $Q = AE\varepsilon_0$ **Ans.**

Problem 15. A positive charge q is placed in front of a conducting solid cube at a distance d from its centre. Find the electric field at the centre of the cube due to the charges appearing on its surface.

Solution :



Here E_i = electric field due to induced charges

and E_q = electric field due to charge q

We know that net electric field in a conducting cavity is equal to zero.

Means $\vec{E} = 0$ at the centre of the cube

$\vec{E}_i + \vec{E}_q = 0$

$\vec{E}_i = -\vec{E}_i$

$\vec{E}_i = -\frac{kq}{d^2}\hat{PO}$ **Ans.**