WAVE OPTICS

1. WAVEFRONTS

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the phase speed. The energy of the wave travels in a direction perpendicular to the wavefront.

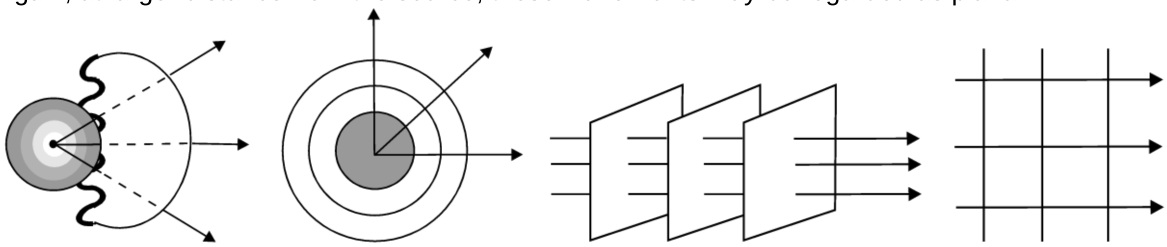
Figure (a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e.. radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b)

A linear source such as a slit Illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wave fronts may be regarded as planar.



(a) (b)

Figure : Wavefronts and the corresponding rays in. two cases: (a) diverging spherical wave. (b) plane wave. The figure on the left shows a wave (e.g.. light) in three dimensions. The figure on the right shows a wave in two dimensions (a water surface).

2. PRINCIPLE OF SUPERPOSITION :

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of E.M.W. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called *interference*.

2.1 SUPERPOSITION OF TWO SINUSOIDAL WAVES :

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \varphi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

$$x(t) = x_1(t) + x_2(t) = a_1 \sin \omega t + a_2 \sin (\omega t + \varphi) = A \sin (\omega t + \varphi_0)$$

where $A^2 = a_1^2 + a_2^2 + 2a_1 . a_2 \cos \varphi$ .... (1)

and $\tan \varphi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ....(2)

Solved Examples

Example 1. If $i_1 = 3\sin \omega t$ and $i_2 = 4 \cos \omega t$, find i_3.

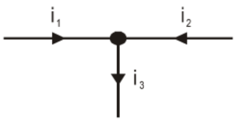


Figure 1.1

Solution : from kirchoff's current law,

$i_3 = i_1 + i_2$.

$= 3 \sin \omega t + 4 \sin (\omega t + \frac{\pi}{2})$

$= 5 \sin (\omega t + \tan^{-1})$

Example 2. S_1 and S_2 are two source of light which produce individually disturbance at point P given by $E_1 = 3\sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E_1}$ & $\vec{E_2}$ to be along the same line, find the result of their superposition.

Solution :

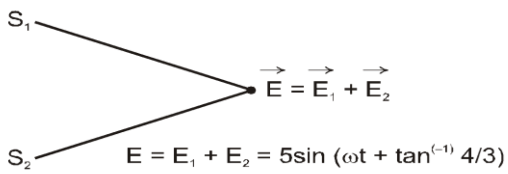


Figure 1.2

3. SUPERPOSITION OF PROGRESSIVE WAVES; PATH DIFFERENCE :

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

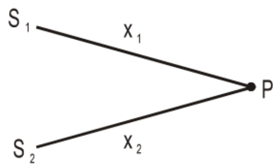


Figure: 1.3

At point P,

$y_1 = a_1 \sin (\omega t - kx_1 + \theta_1)$

$y_2 = a_2 \sin (\omega t - kx_2 + \theta_2)$

$y = y_1+y_2 = A \sin(\omega t + \Delta\varphi)$

Here, the phase difference,

$\Delta\varphi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$

$= k(x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p + \Delta\theta$

Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \frac{2\pi}{\lambda}$

$\Rightarrow \quad \Delta\varphi = k\Delta p = \frac{2\pi}{\lambda} \Delta x$ ..... (1)

For Constructive Interference :

$\Delta\varphi = 2n\pi$, $n = 0, 1, 2$ .... or, $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2} \quad \Rightarrow \quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ... (2)

For Destructive interference :

$\Delta\varphi = (2n + 1)\pi$, $n = 0, 1, 2$ ....

or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{min}} = \sqrt{I_1} - \sqrt{I_2} \quad \Rightarrow \quad I_{max} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ... (3)

Solved Example

Example 3. Light from two sources, each of same frequency and travelling in same direction, but with intensity in the ratio 4 : 1 interfere. Find ratio of maximum to minimum intensity.

Solution : $\dfrac{I_{max}}{I_{min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{\sqrt{\dfrac{I_1}{I_2}}+1}{\sqrt{\dfrac{I_1}{I_2}}-1}\right)^2 = \left(\dfrac{2+1}{2-1}\right)^2$ = 9 : 1.

4. COHERENCE :

Two sources which vibrate with a fixed phase difference between them are said to be coherent. The phase differences between light coming form such sources does not depend on time.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 ns. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1ns, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$$I = I_1 + I_2 \qquad \text{...... (1)}$$

5. YOUNG'S DOUBLE SLIT EXPERIMENT (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two; these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

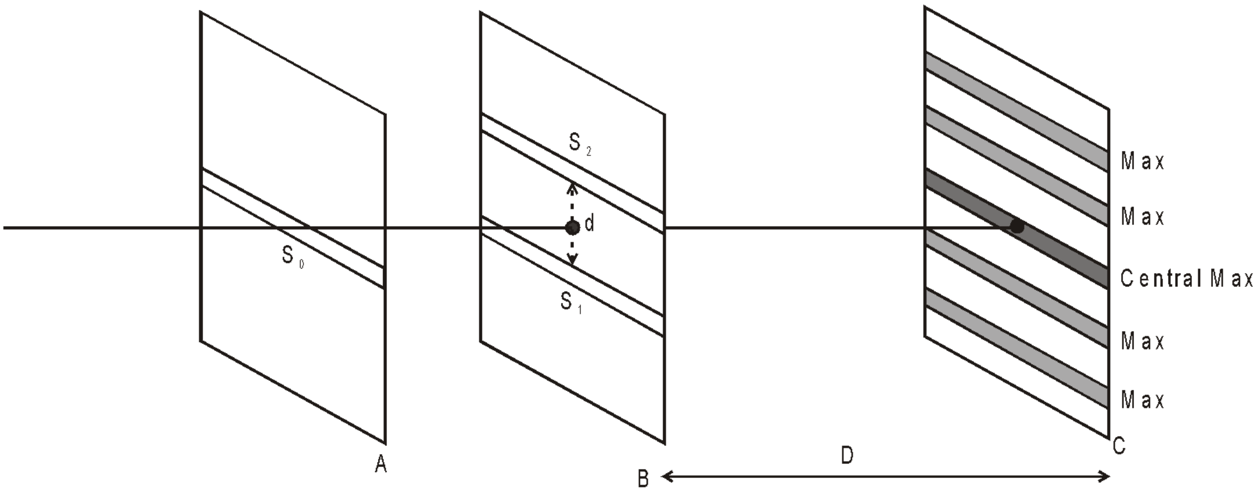


Figure : *Young's Arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2*

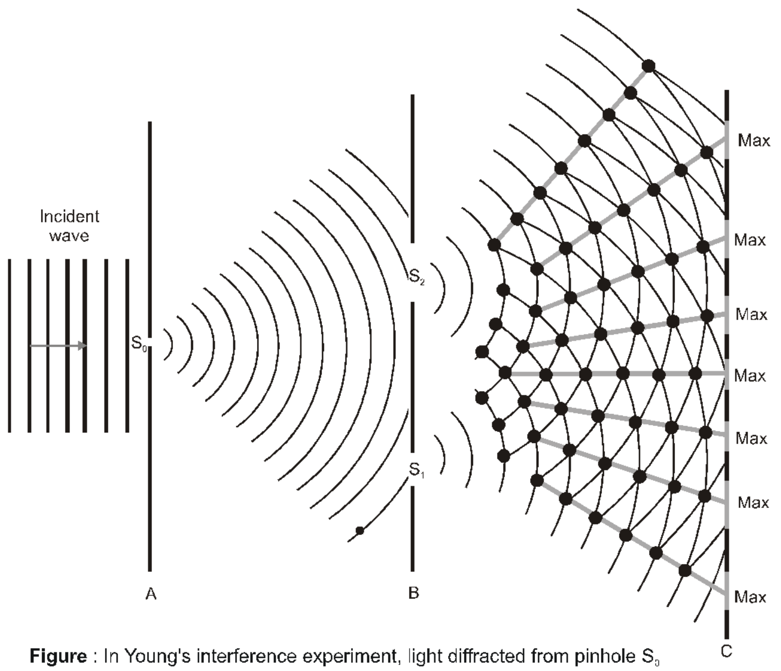


Figure : In Young's interference experiment, light diffracted from pinhole S_0 excounters pinholes S1 and S2 in screen B. Light diffracted from these two pinholes overlaps in the region between screen B and viewing screen C, producing an interference pattern on screen C.

5.1 ANALYSIS OF INTERFERENCE PATTERN

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However the wave reaching P from S_2 may not be in phase with the wave reaching P from S_1, because the latter must travel a longer path to reach P than the former. We have already discussed the phase-difference arising due to path difference. If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point P.

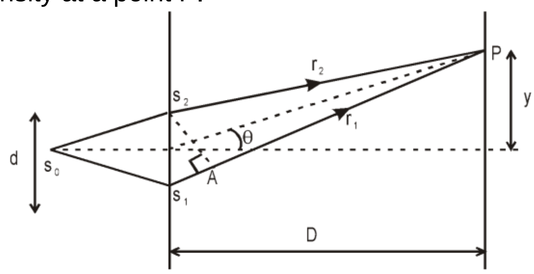


Figure : 4.3

Path difference $\Delta p = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$ ...(1)

Approximation I :

For D >> d, we can approximate rays $\vec{r}_1$ and $\vec{r}_2$ as being approximately parallel, at angle θ to the principle axis.

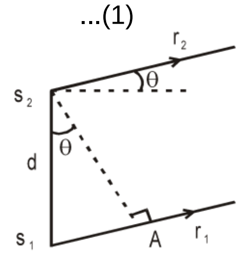


Figure : 4.4

Now, $S_1P - S_2P = S_1A = = S_1S_2 \sin\theta$

⇒ path difference = $d \sin\theta$ ...(2)

Approximation II :

further if θ is small, i.e.y << D, $\sin\theta = \tan\theta = \frac{y}{D}$

and hence, path difference = $\frac{dy}{D}$ ...(3)

for maxima (constructive interference),

$$\Delta p = \frac{d.y}{D} = n\lambda$$

$\Rightarrow$ $y = \frac{n\lambda D}{d}$, n = 0, ± 1, ± 2, ± 3 .....(4)

Here n = 0 corresponds to the central maxima
n = ±1 correspond to the 1st maxima
n = ±2 correspond to the 2nd maxima and so on.

for minima (destructive interference).

$$\Delta p = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2} \pm\frac{5\lambda}{2} \Rightarrow \Delta p = \begin{cases} (2n-1)\frac{\lambda}{2} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda}{2} & n = -1, -2, -3........ \end{cases}$$

$$\text{consequently,} \quad y = \begin{cases} (2n-1)\frac{\lambda D}{2d} & n = 1, 2, 3............ \\ (2n+1)\frac{\lambda D}{2d} & n = -1, -2, -3....... \end{cases} \quad .... (5)$$

Here n = ± 1 corresponds to first minima,
n = ± 2 corresponds to second minima and so on.

5.2 FRINGE WIDTH :

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

fringe width $\beta = \frac{\lambda D}{d}$ ... (1)

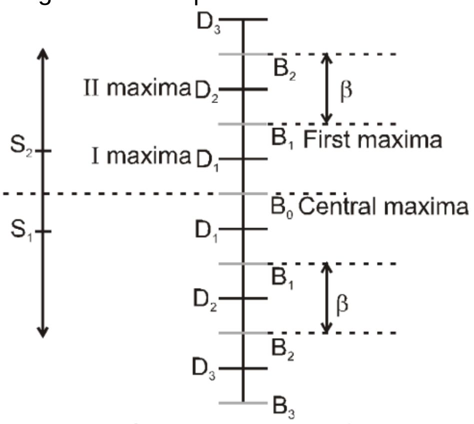


Figure : fringe pattern in YDSE

· Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

5.3 INTENSITY :

Suppose the electric field components of the light waves arriving at point P (in the Figure) from the two slits S_1 and S_2 vary with time as

$$E_1 = E_0 \sin \omega t \quad \text{and} \quad E_2 = E_0 \sin (\omega t + \varphi)$$

Here $\varphi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0); hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin (\omega t + \varphi) = E_0' \sin (\omega t + \varphi')$$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 . E_0 \cos \varphi = 4 E_0^2 \cos^2 \varphi/2$

Hence the resultant intensity at point P,

$I = 4I_0 \cos^2 \frac{\phi}{2}$ ....(2)

$I_{max} = 4I_0$ when $\frac{\phi}{2} = n\pi$, $n = 0, \pm1, \pm2,.......,$

$I_{min} = 0$ when $\frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi$ $n = 0, \pm1, \pm2$ ....

Here $\varphi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$

If $D >> d$, $\varphi = d\frac{2\pi}{\lambda}\sin\theta$

If $D >> d$ & $y << D$, $\varphi = \frac{2\pi}{\lambda} d \frac{y}{D}$

However if the two slits were of different intensities I_1 and I_2,

say $E_1 = E_{01}, \sin \omega t$

and $E_2 = E_{02}, \sin(\omega t + \varphi)$

then resultant field at point P,

$E = E_1 + E_2 = E_0 \sin(\omega t + \varphi)$

where $E_0^2 = E_{01}^2 + E_{O2}^2 + 2E_{01}E_{02}\cos\varphi$

Hence resultant intensity at point P,

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$ .... (3)

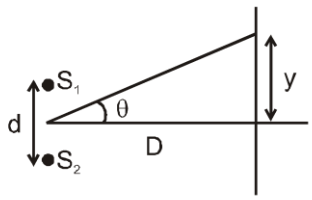
Solved Examples

Example 4. In a YDSE, D = 1m, d = 1mm and λ = 1/2 mm

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Solution : (i) $D >> d$



Hence $\Delta P = d \sin\theta$

$\frac{d}{\lambda} = 2,$

clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\Delta p = \frac{dy}{D}$ cannot be used for Ist maxima,

$\Delta p = d\sin\theta = \lambda$

$\Rightarrow$ $\sin\theta = \frac{\lambda}{d} = \frac{1}{2}$

$\Rightarrow$ $\theta = 30^o$

Hence, $y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(ii) Maximum path difference

$\Delta P_{max} = d = 1$ mm

$\Rightarrow$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$ and highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total no. of maxima = $2n_{max} + 1^* = 3$ *(central maxima).

Total no. of minima = $2n_{min} = 4$

Example 5. Monochromatic light of wavelength 5000 A° is used in Y.D.S.E., with slit-width, d = 1mm, distance between screen and slits, D = 1m. If intensity at the two slits are, $I_1 = 4I_0$, $I_2 = I_0$, find

(i) fringe width β

(ii) distance of 5th minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ mm

(iv) Distance of the 1000th maxima

(v) Distance of the 5000th maxima

Solution : (i) $\beta = \frac{\lambda D}{d} = \frac{5000 \times 10^{-10} \times 1}{1 \times 10^{-3}} = 0.5$ mm

(ii) $y = (2n-1)\frac{\lambda D}{2d}$, n = 5 $\Rightarrow$ y = 2.25 mm

(iii) At $y = \frac{1}{3}$ mm, y << D. Hence $\Delta p = \frac{d.y}{D}$

$\Delta\varphi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$

Now resultant intensity

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\varphi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5 \times 10^{-6}} = 2000$ n = 1000 is not << 2000

Hence now $\Delta p = d\sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda$

$\Rightarrow \sin\theta = 1000\frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30°$ $y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima $n_{max} = \left[\frac{d}{\lambda}\right] = 2000$

Hence, n = 5000 is not possible.

5.4 SHAPE OF INTERFERENCE FRINGES IN YDSE :

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$S_2P - S_1P = \Delta$ = constant ....(1)

If $\Delta = \pm \frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta = \pm \frac{3\lambda}{2}$ it represents 2nd minima

If $\Delta = 0$ it represents central maxima,

If $\Delta = \pm \lambda$, it represents 1st maxima etc.

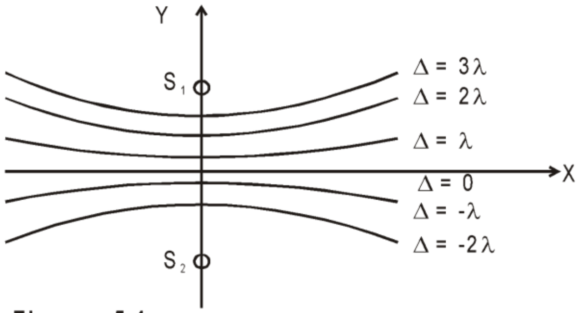


Figure : 5.1

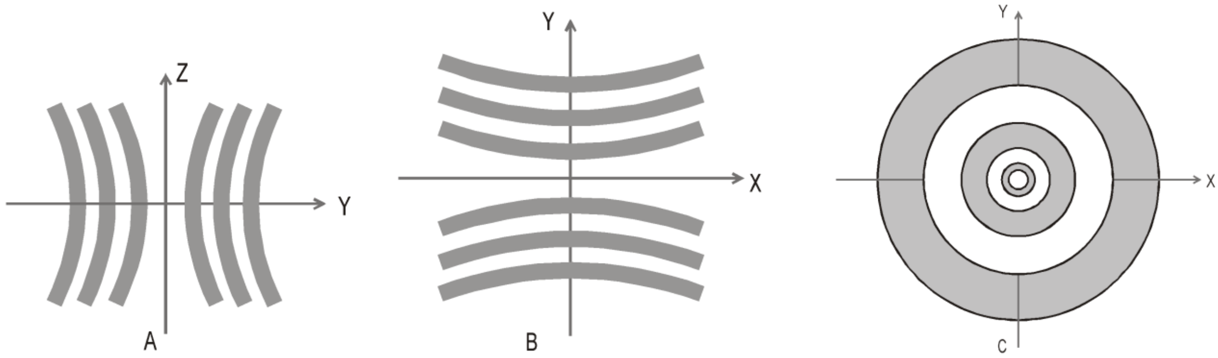
Equation (1) represents a hyperbola with its two foci at S_1 and S_2

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is ⊥er to the X axis, i.e. in the YZ plane, as is generally the case, firinges are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

C. If screen is ⊥er to Y axis (along S_1S_2), ie in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\frac{\lambda}{2}$.



5.5 YDSE WITH WHITE LIGHT :

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. for example if P is a point on the screen such that

$$S_2P - S_1P = \frac{\lambda_{violet}}{2} = 190 \text{ nm},$$

completely destructive interference will occur for violet light. Hence we will have a line devoid of violet colour that will appear reddish. And if

$$S_2P - S_1P = \frac{\lambda_{red}}{2} = 350 \text{ nm},$$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe; for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. for example if

$$S_2P - S_1P = 3000 \text{ nm},$$

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}$ nm. In the visible region these wavelength are 750 nm (red), 600 nm (yellow), 500 nm (greenish–yellow), 430 nm (violet). Clearly such a light will appear white to the unaided eye.

Thus with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Solved Examples

Example 6. A beam of light consisting of wavelengths 6000Å and 4500Å is used in a YDSE with D = 1m and d = 1 mm. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Solution : $\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6$ mm

$\beta_2 = \frac{\lambda_2 D}{d} = 0.45$ mm

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1P_1 = n_2P_2$ = LCM of β_1 and β_2

⇒ y = LCM of 0.6 cm and 0.45 mm

y = 1.8 mm **Ans.**

At this point 3rd maxima for 6000 Å & 4th maxima for 4500 Å coincide

Example 7. White light is used in a YDSE with D = 1m and d = 0.9 mm. Light reaching the screen at position y = 1 mm is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum. (A visible region from 350 nm to 750 nm)

Solution : $\Delta p = \frac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}$ m = 900 nm

for minima $\Delta p = (2n - 1)\lambda/2$

⇒ $\lambda = \frac{2\Delta P}{(2n-1)} = \frac{1800}{(2n-1)}$

$= \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{3}, \frac{1800}{7}$ ....

of these 600 nm and 360 nm lie in the visible range. Hence these will be missing lines in the visible spectrum.

6. GEOMETRICAL PATH & OPTICAL PATH :

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$$E = E_0 \sin(\omega t - kx + \varphi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v}\Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase

$$\Delta\varphi = k\Delta x = \frac{\omega}{c}(\mu\Delta x)$$

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance μΔx in vacuum. i.e. a path length of Δx in medium of refractive index μ is equivalent to a path length of μΔx in vacuum.

The quantity μΔx is called the optical path length of light, Δx_{opt} . And in terms of optical path length, phase difference would be given by,

$$\Delta\varphi = \frac{\omega}{c}\Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \qquad .... (1)$$

where λ_0 = wavelength of light in vacuum.

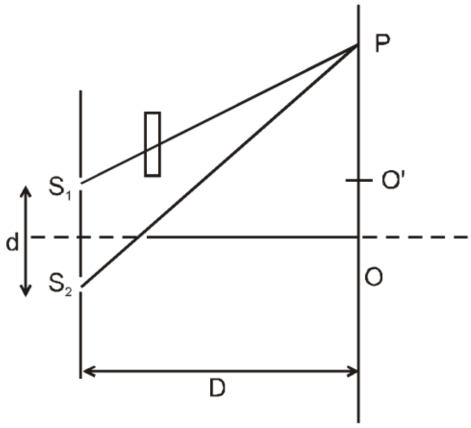
However in terms of the geometrical path length Δx,

$$\Delta\varphi = \frac{\omega}{c}(\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \qquad .....(2)$$

where λ = wavelength of light in the medium (λ =).

6.1 DISPLACEMENT OF FRINGE :

On introduction of a glass slab in the path of the light coming out of the slits–



On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by t(μ – 1). Now the path difference between waves coming form S_1 and S_2 at any point P is

$\Delta p = S_2P - (S_1P + t(\mu - 1))$

$= (S_2P - S_1P) - t(\mu - 1)$

⇒ $\Delta p = d\sin\theta - t(\mu - 1)$ if $d << D$

and $\Delta p = \frac{yd}{D} - t(\mu - 1)$ If $y << D$ as well.

for central bright fringe,

$\Delta p = 0 \quad \Rightarrow \quad \frac{yd}{D} = t(\mu - 1).$

⇒ $y = OO' = (\mu - 1)t\frac{D}{d} = (\mu - 1)\,t\,.\frac{\beta}{\lambda}$

The whole fringe pattern gets shifted by the same distance

$\Delta = (\mu - 1).\frac{D}{d} = (\mu - 1)t\,.\frac{B}{\lambda}$

***** Notice that this shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Solved Example

Example 8. In a YDSE with d = 1mm and D = 1m, slabs of (t = 1μm, μ = 3) and (t = 0.5 μm, μ = 2) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution : Optical path for light coming from upper slit S_1 is

$S_1P + 1\mu m\,(2 - 1) = S_2P + 0.5\,\mu m$

Similarly optical path for light coming from S_2 is

$S_2P + 0.5\,\mu m\,(2 - 1) = S_2P + 0.5\,\mu m$

Path difference : $\Delta p = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m) = (S_2P - S_1P) - 1.5\ \mu m$.

$$= \frac{yd}{D} - 1.5\ \mu m$$

for central bright fringe $\Delta p = 0$

$$\Rightarrow \quad y = \frac{1.5\ \mu m}{1\ mm} \times 1m = 1.5\ mm.$$

The whole pattern is shifted by 1.5 mm upwards. **Ans.**

7. YDSE WITH OBLIQUE INCIDENCE :

In YDSE, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up

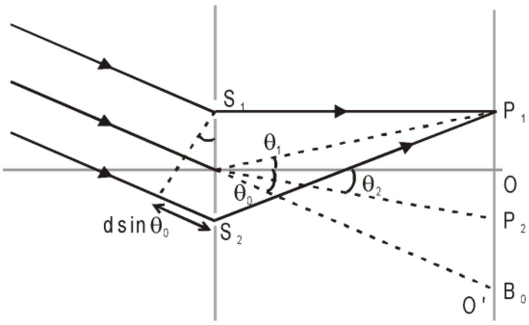


Figure : 8.1

for points above the central point on the screen, (say for P_1)

$\Delta p = d \sin\theta_0 + (S_2P_1 - S_1P_1)$

$\Rightarrow \quad \Delta p = d \sin\theta_0 + d\sin\theta_1$ (If $d << D$)

and for points below O on the screen, (say for P_2)

$\Delta p = |(d\sin\theta_0 + S_2P_2) - S_1P_2|$

$= |d \sin\theta_0 - (S_1P_2 - S_2P_2)|$

$\Rightarrow \quad \Delta p = |d \sin\theta_0 - d \sin\theta_2|$ (if $d << D$)

We obtain central maxima at a point where, $\Delta p = 0$.

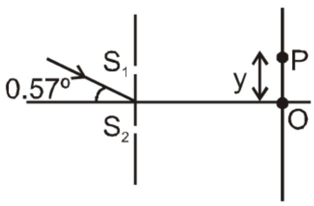
$(d \sin\theta_0 - d \sin\theta_2) = 0$

or $\theta_2 = \theta_0$.

This corresponds to the point O' in the diagram. Hence we have finally for path difference.

$$\Delta p = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O\&O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases} \quad \text{... (1)}$$

Solved Example



Example 9. In YDSE with D = 1m, d = 1mm, light of wavelength 500 nm is incident at an angle of 0.57º w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.

(i) find the position of central maxima

(ii) Intensity at point O in terms of intensity of central maxima I_0.

(iii) Number of maxima lying between O and the central maxima.

Solution :. (i) $\theta = \theta_0 = 0.57º$

$\Rightarrow \quad y = -D \tan\theta \simeq -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57} \text{rad}\right)$

$\Rightarrow \quad y = -1\text{cm}$.

(ii) for point 0, $\theta = 0$

Hence, $\Delta p = d \sin\theta_0$; $d\theta_0 = 1\ mm \times (10^{-2}\ rad)$

$= 10,000\ nm = 20 \times (500\ nm)$

$\Rightarrow \quad \Delta p = 20\ \lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ intensity at O = I_0

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

8. THIN-FILM INTERFERENCE :

In YDSE we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.

Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

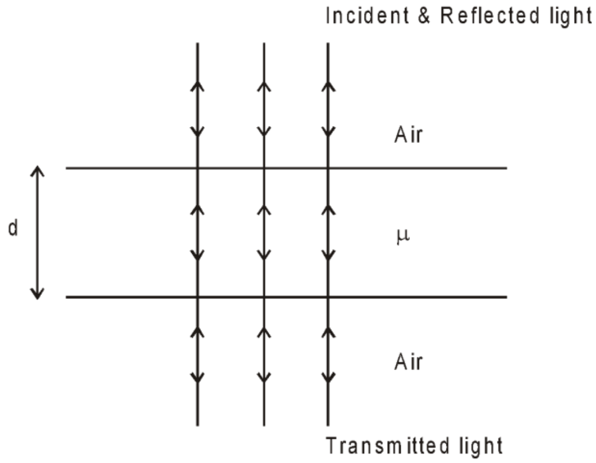


Figure : 9.1

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently condition for constructive and destructive interference in the reflected light is given by,

$2\mu d = n\lambda$ for destructive interference

and $2\mu d = (n + \frac{1}{2})\lambda$ for constructive interference ....(1)

where $n = 0, 1, 2$ ....,

and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (9.1)

i.e. $$2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ (n + \frac{1}{2}\lambda) & \text{for destructive interference} \end{cases}$$ ....(2)

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (1) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.

If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation (1).

However if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation 2 and not equation (1).

Solved Examples

Example 10. White light, with a uniform intensity across the visible wavelength range 430–690 nm, is perpendicularly incident on a water film, of index of refraction $\mu = 1.33$ and thickness $d = 320$ nm, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Solution : This situation is like that of Figure, for which euqation gives the interference maxima. Solving for λ and inserting the given data, we obtain

$$\lambda = \frac{2\mu d}{m+1/2} = \frac{(2)(1.33)(320\ \ nm)}{m+1/2} = \frac{851\ \ nm}{m+1/2}$$

for m = 0, this give us λ = 1700 nm, which is in the infrared region. For m = 1, we find l = 567 nm, which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 nm, which is in the ultraviolet region. So the wavelength at which the light seen by the observer is brightest is

λ = 567 nm. **Ans.**

Example 11. A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38; that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible specturm (λ = 550 nm)? Assume the light is approximately perpendicular to the lens surface.

Solution : The situation here differs from figure in that $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same (see figure 9.2). Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference 2L within the film must be equal to an odd number of half wavelengths:

$2L = (m + 1/2)\lambda_{n2}$,

or, with $\lambda_{n2} = \lambda/n_2$,

$2n_2 L = (m + 1/2)\lambda$.

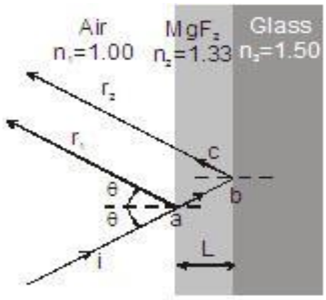


Figure : 9.2

We want the least thickness for the coating, that is, the smallest L. Thus we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\ \ nm}{(4)\ \ (1.38)} = 96.6\ nm$$ **Ans.**

9. FRESNEL'S BIRPISM EXPERIMENT

(1) It is optical device to obtain two coherent sources by refraction of lights.

(2) The angle of biprism is 179º & refracting angle is α = 1/2º.

(3) Distance between source & screen D = a + b.

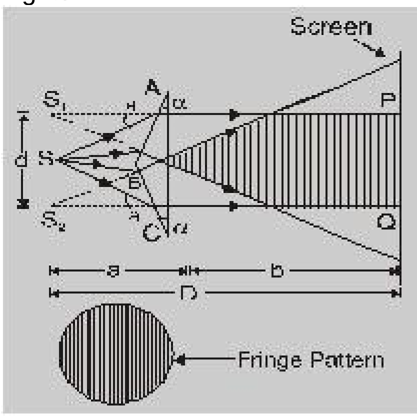
Distance between two coherent source = d = 2a (μ –1)α

Where a = distance between source & Biprism

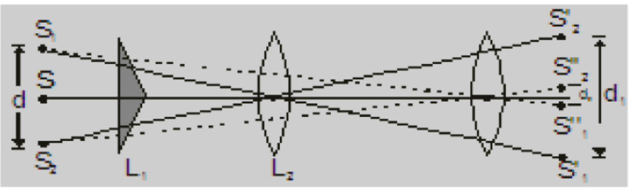
b = distance between screen & Biprism

μ = refractive index of the material of prism.

$$\lambda = \frac{d\beta}{D} = \frac{2a(\mu-1)\alpha\beta}{(a+b)} = \frac{\sqrt{d_1 d_2}\ .\beta}{(a+b)}$$



Note- α is in radian $\alpha^o = \alpha \times \frac{3.14}{180}$. Suppose refracting angle & refractive index is not known then d can be calculate by convex lens.



One convex lens whose focal length (f) and 4f < D.

First convex lens is kept near biprism & d_1 is calculated then it is kept near eyepiece & d_2 is calculated.

$$d = \sqrt{d_1 \quad d_2}$$

Application :
With the help of this experiment the wavelength of monochromatic light, thickness of thin films and their refractive index & distance between apparent coherent sources can be determined.
When **Fresnel's arrangement** is immersed in water

(1) Effect on d

$d_{water} < d_{air}$. Thus when the Fresnel's biprism experiment is immersed in water, then the separation between the two virtual sources decreases but in young's double slit experiment it does not change.

(2) In young's double slit experiment β decrease and in fresnel's biprism experiment β increases.

Solved Example

Example 12 In Fresnel's biprism experiment the width of 10 fringes is 2cm which are formed at a distance of two 2 meter from the slit. If the wavelength of light is 5100 Å then the distance between two coherent sources.

Solution :

$$d = \frac{D\lambda}{\beta} \qquad \text{........(1)}$$

According to question $\lambda = 5100 \times 10^{-10}$ m

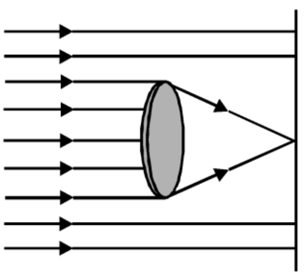
$$\beta = \frac{2}{10} \times 10^{-2} \text{m} \qquad \text{........(2)}$$

D = 2m d = ?

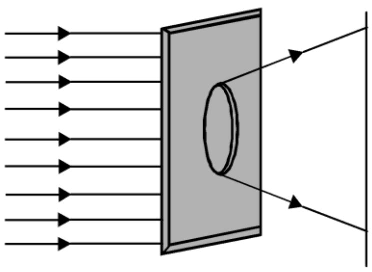
From eqs. (1) and (2) $d = \frac{2 \times 51 \times 10^{-8}}{2 \times 10^{-3}}$ $= 5.1 \times 10^{-4}$ m

DIFFRACTION OF LIGHT

Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the eometricle shaddow region is defined as diffraction of light or deviation of light from its rectilinear propogation tendency is defined as diffraction of light.



diffraction from obstacle



diffraction from aperture

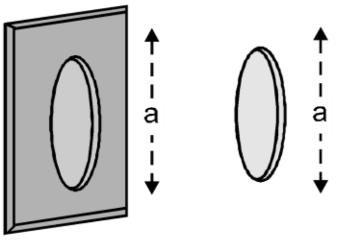
Diffraction was discovered by Grimaldi

Theoritically explained by Fresnel

Diffraction is possible in all type of waves means in mechanical or electromagnetic waves shows diffraction.

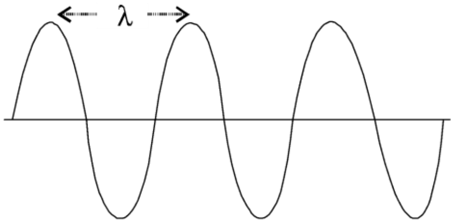
Diffraction depends on two factors :

(i) Size of obstacles or aperture



aperture obstacle

(ii) Wave length of the wave



Condition of diffraction. Size of obstacle or aperture should be nearly equal to the wave length of light

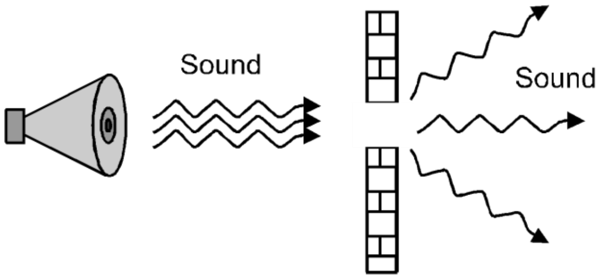
$\lambda \simeq a$ $\qquad \frac{a}{\lambda} \simeq 1$

If size of obstacle is much greater then wave length of light, the rectilinear motion of light is observed.

- It is practically observed when size of aperture or obstacle is greater than 50 λ then obstacle or aperture does not shows diffraction.

- Wave length of light is in the order 10^{-7}m. In general obstacle of this wave length is not present so light rays does not show diffraction and it appears to travel in straight line Sound wave shows more diffraction as compare to light rays because wavelength of sound is high (16 mm to 16m). So it is generally diffracted by the objects in our daily life.

- Diffraction of ultrasonic wave is also not observed as easily as sound wave because their wavelength is of the order of about 1 cm. Diffraction of radio waves is very conveniently observed because of its very large wavelength (2.5 m to 250 m). X-ray can be diffracted easily by crystel. It was discovered by Lave.



diffraction of sound from a window

TYPES OF DIFFRACTION

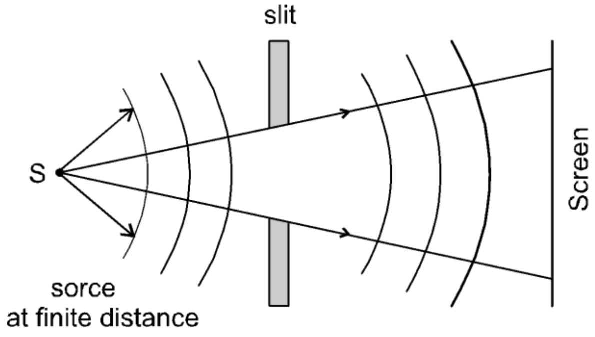
(i) There are two type of diffraction of light :

(a) Fresnel's diffraction (b) Fraunhofer's's diffraction

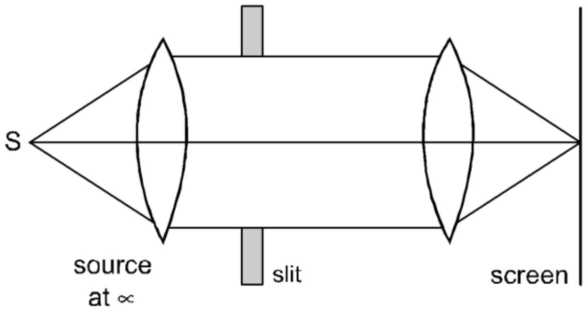
(a) Fresnel diffraction

If either source or screen or both are at finite from the diffracting device (obstacle or aperture), the diffraction is called fresnel diffraction and the pattern is the shadow of the diffracting device modified by diffraction effect.

Example :- Diffraction at a straight edge, small opaque disc, narrow wire are examples of fresnel diffraction.



Fresnel's diffraction



Fraunhofer's diffraction

(b) Fraunhofer diffraction

Fraunhofer diffraction is a particular limiting case of fresnel diffraction.

In this case, both source and screen are effectively at infinite distance from the diffracting device and pattern is the image of source modified by diffraction effects.

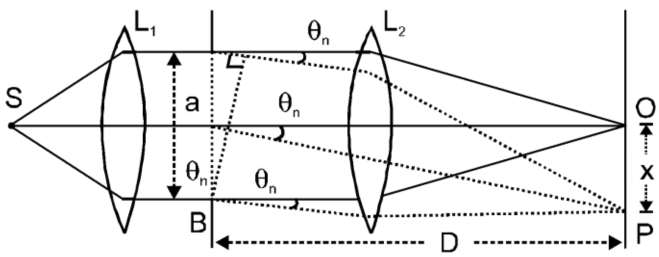
Example :- Diffraction at single slit, double slit and diffraction grating are the examples of fraunhofer diffraction.

Comparison between fresnel and fraunhofer diffraction

	Fresnel Diffraction	Fraunhofer Diffraction
(a)	Source and screen both are at finite distance from the diffractor	Source and screen both are at infinite distance from the diffractor
(b)	Incident and diffracted wave fronts are spherical or cylinderical	Incident and diffracted wavefronts are plane due to infinite distance from source
(c)	Mirror or lennses are not used for obtaining the diffraction pattern	Lens are used in this diffraction pattern
(d)	Centre of diffraction pattern is sometime bright and sometime dark depending on size of diffractor and distance of observation point.	Centre of diffraction is always bright
(e)	Amplitude of wave coming from different half period zones are different due to difference of obliquity	Amplitude of waves coming from different half period zones are same due to same obliquity

FRAUNHOFER DIFFRACTION DUE TO SINGLE SLIT

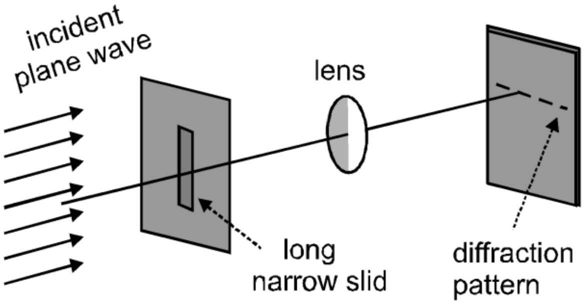
Ab is single slit of width a, Plane wavefront is incident on a slit AB. Secondary wavelets coming from every part of AB reach the axial point P in same phase forming the central maxima. The intensity of central maxima is maximum in this diffrection. Where θ_n represents direction of n^{th} minima Path difference BB' =a sin $\theta_{n\backslash}$



for n^{th} minima $a \sin \theta_n = n\lambda$ $\therefore$ $\sin \theta_n \approx \theta_n = \frac{n\lambda}{a}$ (if θ_n is small)

When path difference between the secondary wavelets coming from A and B is $n\lambda$ or $2n\left[\frac{\lambda}{2}\right]$

or even multiple of $\frac{\lambda}{2}$ then minima occurs



For minima $a \sin \theta_n = 2n\left[\frac{\lambda}{2}\right]$

- When path difference between the secondary wavelets coming from A and B is $(2n + 1)\frac{\lambda}{2}$

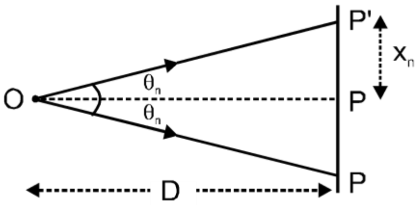
or odd multiple of $\frac{\lambda}{2}$ then maxima occurs

For maxima $a \sin \theta_n = (2n + 1)$ where n = 1, 2, 3 ......

n = 1 → first maxima and n = 2 → second maxima.

- In alternate order minima and maxima occurs on both sides of central maxima.

For nth minima



If distance of nth minima from central maxima = x_n

distance of slit from screen = D, width of silt = a

Path difference $\delta = a \sin\theta_n = \frac{2n\lambda}{2} \Rightarrow \sin\theta_n = \frac{n\lambda}{a}$

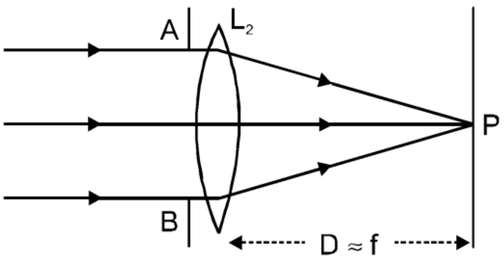
In Δ POP' $\tan \theta_n = \frac{x_n}{D}$ If θ_n is small $\Rightarrow \sin \theta_n \approx \tan\theta_n \approx \theta_n$

$X_n = \frac{n\lambda D}{a} \Rightarrow \theta_n = \frac{x_n}{D} = \frac{n\lambda}{a}$ First minima occurs both sides on central maxima.

For first minima $x = \frac{D\lambda}{a}$ and $\theta = \frac{x}{D} = \frac{\lambda}{a}$

- Linear width of central maxima $w_x = 2x \Rightarrow w_x = \frac{2d\lambda}{a}$

- Angular width of central maxima $w_\theta = 2\theta = \frac{2\lambda}{a}$



SPECIAL CASE

Lens L_2 is shifted very near to slit AB.

In this case distance between slit and screen will be nearly equal to the focal lenght of lense L_2 (i.e. D f)

$\theta_n = \frac{x_n}{f} = \frac{n\lambda}{a} \Rightarrow \quad x_n = \frac{n\lambda f}{a}$

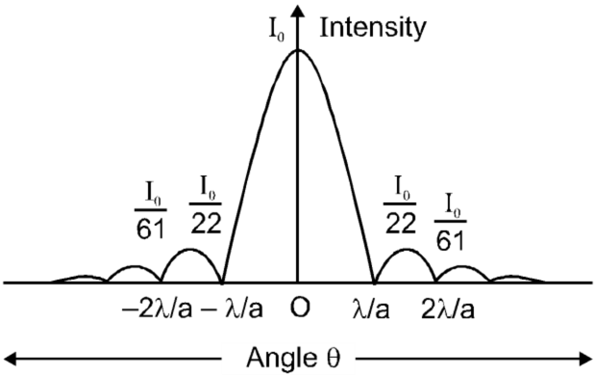
$w_x = \frac{2\lambda f}{a}$ and angular width of cnetral maxima $w_B = \frac{2x}{f} = \frac{2x}{a}$

Fringe width

Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width.

$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a}$

Intensity curve of Fraunhofer's diffraction



Intensity of maxima in Fraunhofer's diffrection is determined by $I = \left[\frac{2}{(2n+1)\pi}\right]^2 I_0$

I_0 = intensity of central maxima

n = order of maxima

intensity of first maxima $I_1 = d\theta \frac{4}{9\pi^2} I_0 \approx \frac{I_0}{22}$

intensity of second maxima $I_2 = \frac{4}{25\pi^2} I_0 \approx \frac{I_0}{61}$

Diffraction occurs in slit is always fraunhofer diffraction as diffraction pattern obtained from the cracks between the fingers, when viewed a distant tubelight and in YDSE experiment are fraunhofer diffraction

GOLDEN KEY POINTS

- The width of central maxima $\propto \lambda$, that is, more for red colour and less for blue.
  i.e., $w_x \propto \lambda$
  as $\lambda_{blue} < \lambda_{red} \Rightarrow w_{blue} < w_{red}$
- For obtaining the fraunhofer diffraction, focal length of second lens (L_2) is used.
  $w_x \propto \lambda \propto f \propto 1/a$
  width will be more for narrow slit
- By decreasing linear width of slit, the width of central maxima increase.

RESOLVING POWER (R.P.)

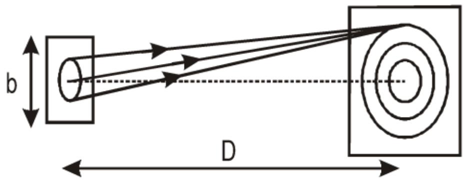
A large number of images are formed as consequence of light diffraction of from a source. If two sources are separated such that their central maxima do not overlap, their images can be distinguished and are said to be resolved R.P. of an optical instrument is its ability to distinguish two neighbouring points.

Linear R.P. $d / \lambda D$ here D = Observed distance

Angular R.P. d / λ d = Distance between two points,

Fraunhofer Differaction by a circular aperture :

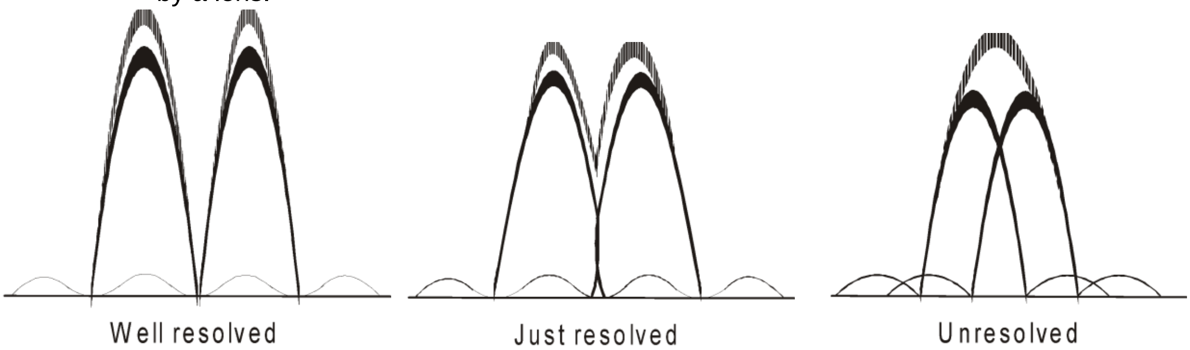
The mathematical analysis shows that the first dark ring is formed by the light diffeacted from at an angle θ with the axis $\sin\theta = \frac{1.22\lambda}{b}$



The radius of the diffraction disc is given by $R = \frac{1.22\lambda D}{b}$

Limits of Resolution

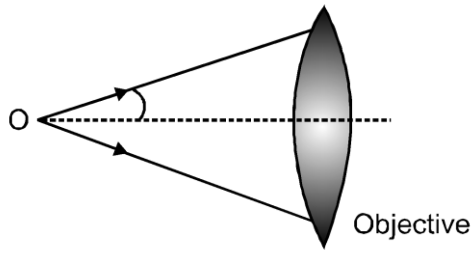
The fact that a lens forms a disc image of a point source, puts a limit on resolving two next points imaged by a lens.



For two objects to be barely resolved, the angular separation between then should be at $\theta_R = \sin^{-1}\left(\frac{1.22\lambda}{b}\right)$.

(1) **Microscope :** In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RPO)

R.L. = $\frac{\lambda}{2\mu \sin\theta}$ and R.P. = $\frac{2\mu \sin\theta}{\lambda}$ $\Rightarrow$ R.P. $\propto \frac{1}{\lambda}$



λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) **Telescope :** Smallest angular separations (dθ) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$ and resolving power (RP) $= \frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow$ R.P. $\propto \frac{1}{\lambda}$ where a = aperture of objective.

Solved Example

Example 13. Light of wavelength 6000Å is incident normally on a slit of width 24×10^{-5} cm. Find out the angular position of second minimum from central maximum ?

Solution : $a \sin\theta = 2\lambda$

given $\lambda = 6 \times 10^{-7}$ m, $a = 24 \times 10^{-5} \times 10^{-2}$ m

$\sin\theta = \frac{2\lambda}{a} = \frac{2 \times 6 \times 10^{-7}}{24 \times 10^{-7}}$ ∴ $\theta = 30^o$

Example 14. Light of wavelength 6328Å is incident normally on a slit of width 0.2 mm. Calculate the angular width of central maximum on a screen distance 9 m ?

Solution : given $\lambda = 6.328 \times 10^{-7}$ m, $a = 0.2 \times 10^{-3}$ m

$w_\theta = \frac{2\lambda}{a} = \frac{2 \times 6.328 \times 10^{-7}}{2 \times 10^{-4}}$ radian $= \frac{6.328 \times 10^{-3} \times 180}{3.14}$ = 0.36≡

Example 15. Light of wavelength 5000Å is incident on a slit of width 0.1 mm. Find out the width of the central bright line on a screen distance 2m from the slit ?

Solution : $w_x = \frac{2\lambda}{a} = \frac{2 \times 2 \times 5 \times 10^{-7}}{10^{-4}}$ = 20 mm

Example 16. The fraunhofer diffraction pattern of single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the thrid minimum is formed at a distance of 5 mm from the central maximum then calculate the wavelength of light.

Solution : $x_n = \frac{nf\lambda}{a} \Rightarrow \lambda = \frac{ax_n}{fn} = \frac{3 \times 10^{-4} \times 5 \times 10^{-3}}{3 \times 1}$ = 5000Å [∵ n = 3]

Example 17. Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

Solution : ∵ $\sin\theta = \frac{\lambda}{a}$ θ = half angular width of the central maximum.

$a = 12 \times 10^{-5}$ cm, $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm ∴ $\sin\theta = \frac{\lambda}{a} = \frac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50 \Rightarrow \theta = 30^o$

Example 18. Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.

Solution : The first fringe is on either side of the central bright fringe.

here $n = \pm 1$, D = 2m, λ = 6000 Å = 6 j 10^{-7} m

∴ $\sin\theta = \frac{x}{D}$ ⇒ $a = 0.30$ mm $= 3 \times 10^{-4}$ m $\Rightarrow a \sin\theta = n\lambda \Rightarrow \frac{ax}{D} = n\lambda$

(a) $x = \frac{n\lambda D}{a}$ ⇒ $x = \left[\frac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right] \pm = \pm 4 \times 10^{-3}$ m

The positive and negative signs corresponds to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe $y = 2x = 2 \times 4 \times 10^{-3} = 8 \times 10^{-3}$ m = 8 mm

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION :

	Interference		Diffraction
(1)	It is the phenomenon of superposition of two waves coming from two different coherent sources	(1)	It is the phenomenon of superposition of two waves coming from two different parts of the same wave front.
(2)	In intrference pattern, all bright lines are equally bright and equally spaced	(2)	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
(3)	All dark lines are totaally dark	(3)	Dark lines are not perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction.
(4)	In interference bands are large in number	(4)	In diffraction bands are a few in number

Solved Example

Example 19. A Slit of width a is illuminated by monochromatic light of wavelength 650 nm at normal incidence. Calculate the value of a when :

(a) the first minimum falls at an angle of diffraction of 30º

(b) the first maximum falls at an angles of diffraction of 30º

Solution : (a) for first minimum $\sin\theta_1 = \frac{\lambda}{a}$,

$$\therefore \quad a = \frac{\lambda}{\sin\theta_1} = \frac{650 \times 10^{-9}}{\sin\ 30^o} = \frac{650 \times 10^{-9}}{0.5} = 1.3 \times 10^{-6} \text{ m}$$

(b) for first maximum $\sin\theta_1 = \frac{3\lambda}{2a}$, $\therefore$ $a = \frac{3\lambda}{2\sin\theta} = \frac{3 \times 650 \times 10^{-9}}{2 \times 0.5} = 1.95 \times 10^{-6}$ m

Example 20. Red light of wavelength 6500 Å from a distance source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m. from the slit.

Solution : Given λ = 6500Å = 65×10^{-8} m, a = 0.5 mm = 0.5×10^{-3} m., D = 1.8 mm

Example 21. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

Solution : For minima in diffraction pattern $\quad d \sin\theta = n\lambda$

For first minima $\quad d \sin\theta_1 = (1)\lambda_1 \Rightarrow \sin\theta_1 = \frac{\lambda_1}{d}$

for first maxima $\quad d \sin\theta_2 = \frac{3}{2}\lambda_2 \Rightarrow \sin\theta_2 = \frac{3\lambda_2}{2d}$

Two will coincide if, $\theta_1 = \theta_2$ or $\sin\theta_1 = \sin\theta_2$

$$\therefore \quad \frac{\lambda_1}{d} = \frac{3\lambda_2}{2d} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660 \text{ nm} = 440 \text{ nm}.$$

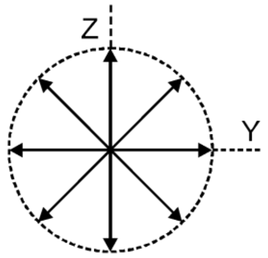
POLARISATION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e. whether the light waves are longitudinal or transverese.

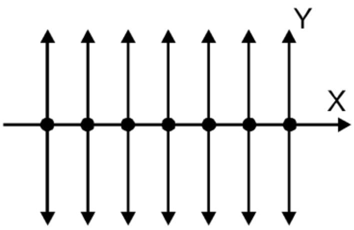
The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

UNPOLARISED LIGHT

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of $\vec{E}$ are equally probable.



unpolarised light propagating along. X-axis



unpolarised light

The resultant electromagnetic wave is a super position of waves produced by the individual atomic sources and it is called unpolarised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

POLARISATION

The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light.

In polarised light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions.)

After polarisation the vibrations become asymmetrical about the direction of propagation of light.

POLARISER

Tourmaline crystal

When light is passed through a tomaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism

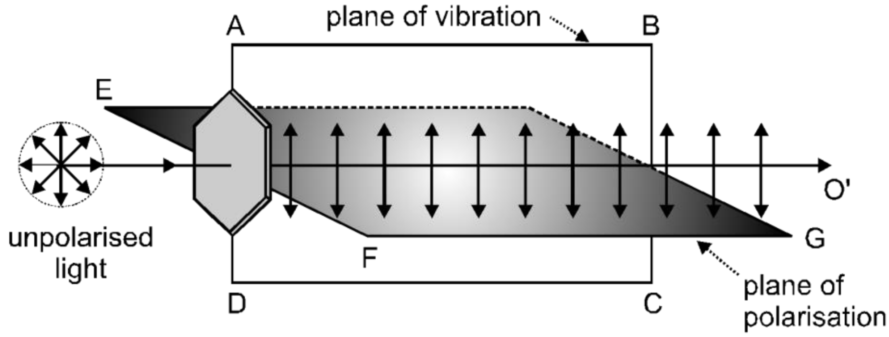
A nicol prism is an optical device which can be used for the production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid

A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

PLANE OF POLARISATION AND PLANE OF VIBRATION :

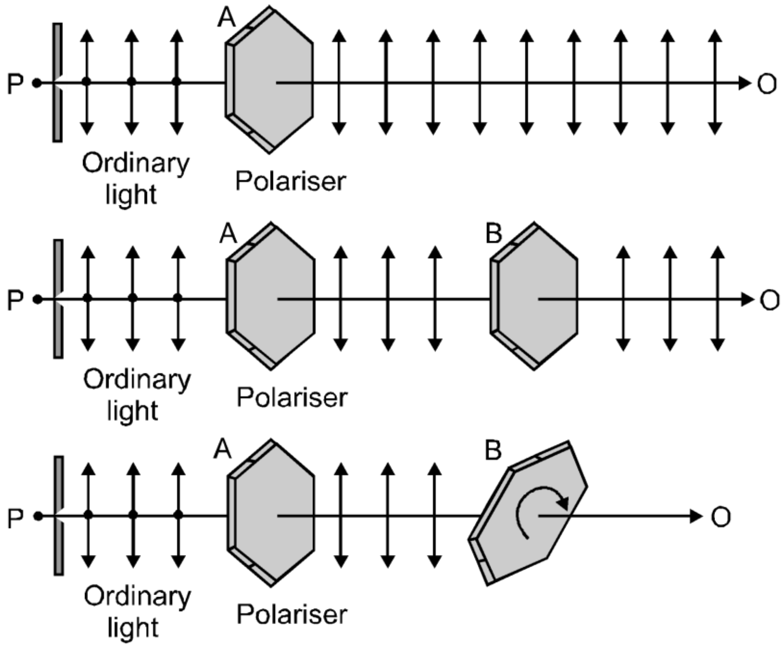
The plane in which vibrations of light vector and the direction of propogation lie is known as plane of vibration. A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarisation.



EXPERIMENTAL DEMONSTRATION OF POLARISATION OF LIGHT

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis)

First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured.



Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light. Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and outcoming light appears coloured.

Now, rotate the crystal B about the axis PO. It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.

If the crystal B is further rotated light reappears and intensity becomes maximum again when their axes are parallel. This occurs after a further rotation of B through 90º

This experiment confirms that the light waves are transverse in nature.

The vibrations in light waves are perpendicular to the direction of propogation of the wave.

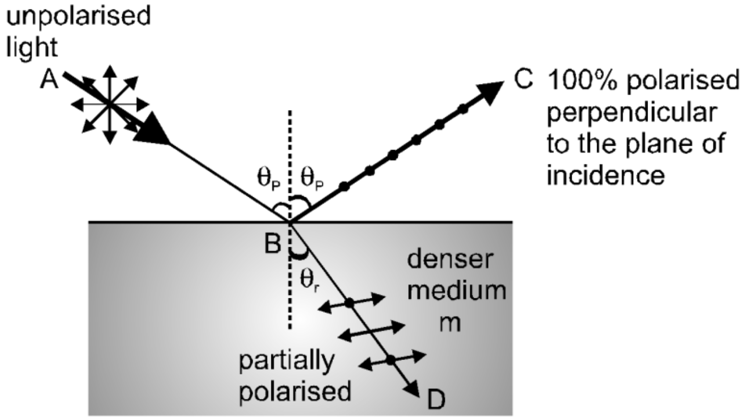
First crystal A polarises the light so it is called polariser.

Second crystal B, analyses the light whether it is polarised or not, so it is called analyser.

METHODS OF OBTAINING PLANE POLARISED LIGHT

Polarisation of reflection

The simplest method to produce plane polarised light is by reflection. This method was discovered by Malus in 1808. When a beam of ordinary light is reflected from a surface, the reflected light is partially polarised. The degree of polarisation of the polarised light in the reflected beam is greatest when it is incident at an angle called polarising angle or Brewster's angle.



Polarising angle

Polarising angle is that angle of incidence at which the reflected light is completely plane polarisation.

Brewster's Law

When unpolarised light strikes at polarising angle θ_P on a interface separating a rare medium from a denser medium of refractive index μ, such that $\mu = \tan\theta_P$ then the reflected light (light in rare medium) is completely polarised. Also reflected and refracted and refracted rays are normal to each other.

This relation is known as Brewster's Law.

The law state that the tangent of the polarising angle of incidence of a transparent medium is equal to tis refractive index $\mu = \tan\theta_P$

In case of polarisation by reflection :

(i) For $i = \theta_P$ refracted light is partially polarised.

(ii) For $i = \theta_P$ reflected and refracted rays are perpendicular to each other.

(iii) For $i < \theta_P$ or $i > \theta_P$ both reflected and refracted light become partially polarised.

According to snell's law $\mu = \dfrac{\sin\theta_P}{\sin\theta_r}$ ....(i)

But according to Brewster's law $\mu = \tan\theta_O = \dfrac{\sin\theta_P}{\cos\theta_P}$ ....(ii)

From equation (i) and (ii) $\dfrac{\sin\theta_P}{\sin\theta_r} = \dfrac{\sin\theta_P}{\cos\theta_P} \Rightarrow \sin\theta_r = \cos\theta_P$

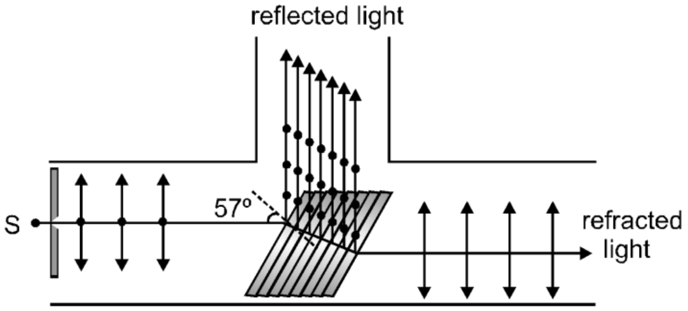
$\therefore \quad \sin\theta_r = \sin(90^o - \theta_P) \Rightarrow \theta_r = 90^o - \theta_P \quad$ or $\quad \theta_P + \theta_r = 90^o$

Thus reflected and refracted rays are mutually perpendicular

By Refraction

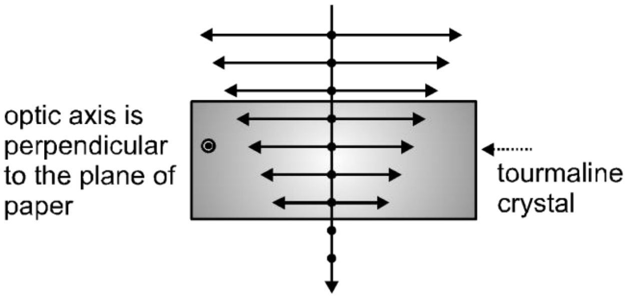
In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident a polarising angle 57º. According Brewster law, the reflected light will be plane polarised with vibrations perpendicualr to the plane of incidence and the transmitted light will be partially polarised.

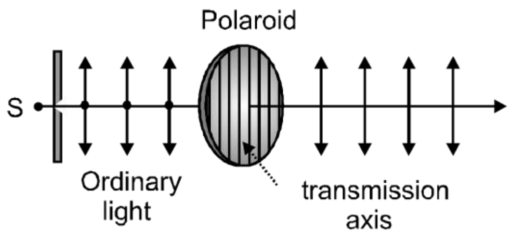
Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.



By Dichroism

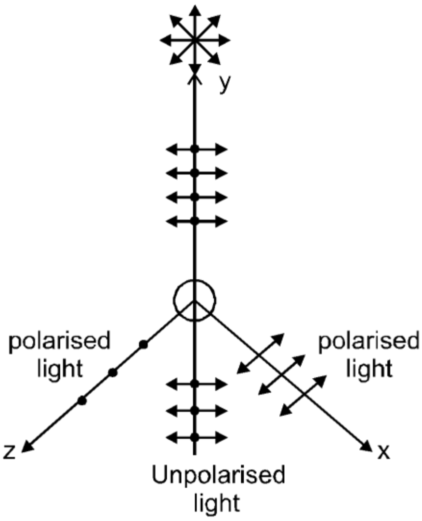
Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmision axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these, the transmitted light will plane polarised withvibrations parallel to transmission axis. Polaroids work onthis principle.





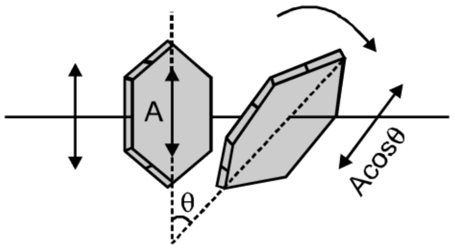
By scattering :

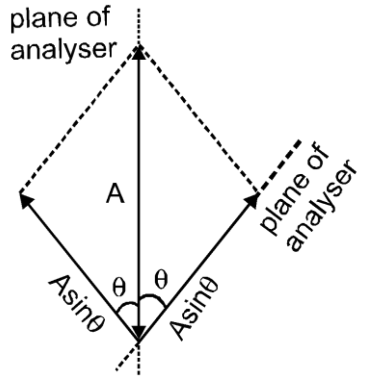
When light is incident on small particles of dust, air molecule etc. (having smaller size as compared to the wavelength of light,) it si absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angles to the incident light is always plane-polarised.



Law of Malus

When a completely plane polarised light beam light beam is incident analyser, then intensity intensity of emergent light varies as the square of cosine of the angle between the planes of transmision of the analyser and the polarizer. $I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$





(i) If $\theta = 0^o$ then $I = I_0$ maximum value (Parallel arrangement)

(ii) If $\theta = 90^o$ then, $I = 0$ minimum value (Crossed arrangement)

If plane polarised light of intensity I_0 (= KA^2) is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be A cosθ while perpendicular to it will be be A sinθ .

Polaroid will pass only those vibrations which are parallel to transmission axis i.e. Acos θ,

$\because$ $I_0 \propto A^2$

So the intensity of emergent light $I = K\,(A\cos\theta)^2 = KA^2\cos^2\theta$

If an unpolarised light is converted into plane polarised light its intensity becomes half.

If light of light emerging from the second polaroid is :

$I_2 = I_1 \cos^2\theta$ θ = angle between the transmission axis of the two polaroids.

APPLICATIONS AND USES OF POLARISATION

- By determining the polarising and and using Brewster's law $\mu = \tan\theta_P$ refractive index of dark transparent substance can be determined.
- In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display (L.C.D)
- In CD player polarised laser beam acts as needle for producing sound from compact disc.
- It has also been used in recording and reproducing three dimensional pictures.
- Polarised light is used in optical stress analysis known as photoelasticity.
- Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of optical activity.

Solved Example

Example 22. Two polaroids are crossed to each other. When one of them is rotated through 60^o, then what percentage of the incident unpolarised light will be transmitted by the polaroids ?

Solution : Initially the polaroids are crossed to each other, that is the angle between their polarising directions is 90^o. When one is rotated through 60^o, then the angle between their polarising directions directions will become 30^o.

Let the intensity of the incident unpolarised light = I_0

This light is plane polarised and passes through the second polaroid.

The intensity of light emerging form the second polaroid is $I_2 = I_1 \cos^2\theta$

θ = the angle between the polarising directions of the two polaroids.

$I_1 = \frac{1}{2}I_0$ and $\theta = 30^o$ so $O_2 = I_1 \cos^2 30^o \Rightarrow \frac{I_2}{I_0} = \frac{3}{8}$

$\therefore$ transmission percentage = $\frac{I_2}{I_0} \times 100 = \frac{3}{8} \times 100 = 37.5\%$

Example 23. At what angle of incidence will the light reflected from water ($\mu = 1.3$) be completely polarised ?

Solution : $\mu = 1.3$,

From Brewster's law $\tan \theta_P = \mu = 1.3 \Rightarrow \theta = \tan^{-1} 1.3 = 53^o$

Example 24. If light beam is incident at polarising angle (56.3º) on air-glass interface, then what is the angle of refraction in glass ?

Solution : $i_P + r_P = 90^o$ $\therefore r_P = 90^o - i_P = 90^o - 56.3^o = 33.7^o$

Example 25. A polariser and an analyser are oriented so that maximum light is transmitted, what will be the intensity of outcoming light when analyer is rotated through 60º

Solution : According to Malus Law $I = I = I_0 \cos^2\theta = I_0 \cos^2 60^o = I_0 \left[\frac{1}{2}\right]^2 = \frac{I_0}{4}$

Solved Miscellaneous Problems

Problem 1. Consider interference between waves from two sources of intensities I & 4I. Find intensities at points where the phase difference is π.

Solution : $I = R^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \delta = I + 4I + 4I \cos \pi$

$I = 5I - 4I = I$

Problem 2. The width of one of the two slits in a Young's double slits experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportion to slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

Solution : Suppose the amplitude of the light wave coming from the narrow slit is A and that coming from the wider slit is 2A. The maximum intensity occurs at a place where constructive interference takes place. Then the resultant amplitude is the sum of the individual amplitudes. Thus,

$A_{max} = 2A + A = 3A$

The minimum intensity occurs at a place where destructive interference takes place. The resultant amplitude is then difference of the individual amplitudes.

Thus, $A_{min} = 2A - A = A$. $\therefore$ $\frac{I_{max}}{I_{min}} = \frac{(A_{max})^2}{(A_{min})^2} = \frac{(3A)^2}{(A)^2} = 9$

Problem 3. In a Young's experiment, the separation between the slits is 0.10mm, the wavelength of light used is 600nm and the interference pattern is observed on a screen 1.0 m away. Find the separation between the successive bright frignes.

Solution : The separation between the successive bright fringes is

$$\beta = \frac{D\lambda}{d} = \frac{1 \times 600 \times 10^{-9}}{.1 \times 10^{-3}}$$

$\beta = 6.0$ mm

Problem 4. Two waves originating from source S_1 and S_2 having zero phase difference and common wavelength λ will show completely destructive interference at a point P if ($S_1 P - S_2 P$) is

Solution : For destructive interference :

Path difference = $S_1 P - S_2 P = (2n - 1)\lambda/2$

For $n = 1$, $S_1 P - S_2 P = (2 \times 1 - 1)\lambda/2 = \lambda/2$

$n = 2$, $S_1 P - S_2 P = (2 \times 2 - 1)\lambda/2 = 3\lambda/2$

$n = 3$, $S_1 P - S_2 P = (2 \times 3 - 1)\lambda/2 = 5\lambda/2$

$n = 4$, $S_1 P - S_2 P = (2 \times 4 - 1)\lambda/2 = 7\lambda/2$

$n = 5$, $S_1 P - S_2 P = (2 \times 5 - 1)\lambda/2 = 9\lambda/2$

$n = 6$, $S_1 P - S_2 P = (2 \times 6 - 1)\lambda/2 = 11\lambda/2$

So, destructive pattern is possible only for path difference = $11\lambda/2$.

Problem 5. In Young's experiment the wavelength of red light is 7.5×10^{-5} cm. and that of blue light 5.0×10^{-5} cm. The value of n for which $(n+1)^{th}$ the blue bright band coincides with n^{th} red band.

Solution : $n_1\lambda_1 = n_2\lambda_2$ for bright fringe

$$n(7.5\times10^{-5}) = (n+1)(5\times10^{-5})\ ;\quad n = \frac{5.0\times10^{-5}}{2.5\times10^{-5}} = 2.$$

Problem 6. In Young's slit experiment, carried out with lights of wavelength λ = 5000 Aº, the distance between the slit is 0.2 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0. The third maximum will be at x equal to.

Solution : $X_n = \frac{n\lambda D}{d}$ or $X_3 = \frac{3\lambda D}{d}$

$$x_3 = \frac{3x\ \ (5000\times10^{-8})\times200}{0.02} = 1.5 \text{ cm}$$

Problem 7. Two slits separated by a distance of 1mm are illuminated with red light of wavelength 6.5×10^{-7} m. The interference fringes are observed on a screen placed 1m from the slits. The distance between third dark fringe & the fifth bright fringe.

Solution : $\beta = \frac{\lambda D}{d} = \frac{6.5\times10^{-7}\times1}{10^{-3}}$

$\beta = .65\times10^{-3}$ m = .65 mm

The distance between the fifth bright fringe from third dark fringe

$= 5\beta - 2.5\beta \Rightarrow 2.5\beta = 2.5\times.65 = 1.63$ mm

Problem 8. In an experiment the two slits are 0.5 mm apart and the fringes are observed to 100 cm from the plane of the slits. The distance of the 11th bright fringe from the Ist bright fringe is 9.72 mm. Calculate the wavelength.

Solution : Given d = .5 mm = 5×10^{-2} cm,

D = 100 cm

$X_n = X_{11} - X_1 = 9.72$ mm

$\therefore\ X_n = \frac{n\lambda D}{d}$ n = 11 – 1 = 10 λ $\Rightarrow \lambda = \frac{X_n d}{nD} = \frac{.972\times5\times10^{-2}}{10\times100} = 4.86\times10^{-5}$ cm

Problem 9. In a Young's experiment, two coherent sources are placed 0.90 mm apart and the fringes are observed one meter away. If it produces the second dark fringe at a distance of 1mm from the central fringe, the wavelength of monochromatic light used.

Solution : D =1m, d = .90 mm = $.9\times10^{-3}$ m

The distance of the second dark ring from centre = 10^{-3} m

$$X_n = (2n-1)\frac{\lambda}{2}\ \frac{D}{d}$$

for n = 2,

$X_n = \frac{3\lambda}{2}\ \frac{D}{d} \Rightarrow \lambda = \frac{2X_n d}{3D} = \frac{2\times10^{-3}\times.9\times10^{-3}}{3}$

$\lambda = 6\times10^{-7}$ m $\Rightarrow$ $\lambda = 6\times10^{-5}$ cm

Problem 10. A beam of light consisting of two wavelength 6500Aº & 5200Aº is used to obtain interference fringes in a young's double slit experiment. The distance between the slits is 2.0 mm and the distance between the plane of the slits and the screen is 120 cm. What is the least distance from the central maximum where the bright fringes due to both the wave length coincide ?

Solution : Suppose the m^{th} bright fringe of 6500Aº coincides with the n^{th} bright fringe of 5200Aº.

$$X_n = \frac{m\lambda_1 D}{d} = \frac{n\lambda_2 D}{d} \Rightarrow \frac{m\times6500\times D}{d} = \frac{n\times5200\times D}{d} \Rightarrow \frac{m}{n} = \frac{5200}{6500} = \frac{4}{5}$$

distance y is $y = \frac{m\lambda_1 D}{d}$ $\Rightarrow$ y = 0.156 cm .

Problem 11. Interference fringes were produced in young's double slit experiment using light of wave length 5000 A°. When a film of material 2.5×10^{-3} cm thick was placed over one of the slits, the fringe pattern shifted by a distance equal to 20 fringe width. The refractive index of the material of the film is -

Solution : $n = \frac{(\mu - 1)tD}{d}$

but $\beta = \frac{\lambda D}{d} \Rightarrow \frac{D}{d} = \frac{\beta}{\lambda}$

$n = (\mu - 1)t\beta / \lambda$

$20\beta = (\mu - 1)\, 2.5 \times 10^{-3}\, (\beta/5000 \times 10^{-8})$

$\mu - 1 = \frac{20 \times 5000 \times x10^{-8}}{2.5 \times 10^{-3}}$ $\Rightarrow$ $\mu = 1.4$

Problem 12. The path difference between two interfering waves at a point on screen is 171.5 times the wavelength. If the path difference is 0.01029 cm. Find the wavelength.

Solution : Path difference = 171.5 λ

$= \frac{343}{2}\lambda$ = odd multiple of half wavelength .

It means dark finge is observed

According to question .

$0.01029 = \frac{343}{2}\lambda$

$\Rightarrow$ $\lambda = \frac{0.01029 \times 2}{343} = 6 \times 10^{-5}$ cm

$\Rightarrow$ λ = 6000A°

Problem 13. Find the minimum thickness of a film which will strongly reflect the light of wavelength 589 nm. The refractive index of the material of the film is 1.25

Solution : For strong reflection, the least optical path difference introduced by the film should be $\lambda/2$. The optical path difference between the waves reflected from the surfaces of the film is $2\mu d$.

Thus, for strong reflection,

$2\mu d = \lambda/2$

$d = \frac{\lambda}{4\mu} = \frac{589}{4 \times 1.25} = 118$ nm.