# CURRENT ELECTRICITY

## 1. ELECTRIC CURRENT

Flow of charge is called **electric current** and the rate of flow of charge in a circuit represents the magnitude of the electric current in that circuit. It is generally represented by I.

If dq amount of charge flows in the circuit in time dt then the magnitude of electric current will be $I = \frac{dq}{dt}$

The direction of electric current is taken to be opposite to be direction of the flow of electrons or the direction of flow of positive charge in a conductor is taken as the direction of the electric current.

Electric current is a scalar quantity.

S.I. unit of electric current is coulomb/second or ampere.

Dimensions of electric current = $M^0L^0T^0A^1$

$$1A = \frac{1\text{ coulomb}}{1\text{ second}}$$

If n is the number of free electrons passing through a point in t seconds, then the total charge passing through that point in t seconds is

$$q = ne$$

and the current flowing through the conductor $I = \frac{q}{t} = \frac{ne}{t}$

$\therefore$ For one ampere current $n = \frac{1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$ electrons/sec

The number of electrons flowing through a conductor in t seconds.

$$n = \frac{It}{e}.$$

## Solved Examples

**Example.1** How many electrons flow per second through any cross-section of a wire, if it carries a current of one ampere?

**Solution :**

$$I = \frac{q}{t} = \frac{ne}{t}$$

$$n = \frac{It}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}} = 6.25 \times 10^{18}$$

**Example.2** How many electrons pass through a heater wire in one minute, if current flowing is 8 ampere?

**Solution :**

$$n = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$$

## 2. CONDUCTOR

In some materials, the outer electrons of each atom or molecule are only weakly bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons drift in a direction opposite to the field. Such materials are called conductors.

## 3. INSULATOR

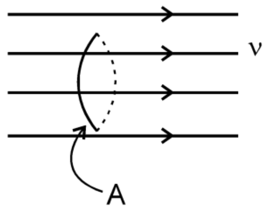
Another class of materials is called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they can’t leave their parent atoms or molecules and hence can’t move through long distances. Such materials are also called dielectrics.

# 4. SEMICONDUCTOR

In semiconductors, the behavior is like an insulator at low levels of temperature. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its behavior is in between a conductor and an insulator and hence, the name semiconductor. A free electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction.

## Current, velocity and current density

n → no. of free charge particles per unit volume
q → charge of each free particle
i → charge flow per unit time
i = nqvA




Current density, a vector, at a point have magnitude equal to current per unit normal area at that point and direction is along the direction of the current at that point

$\vec{J} = \frac{di}{ds}\,\hat{n}$ so $di = \vec{J}.\,\vec{ds}$

Current is flux of current density.

Due to principle of conservation of charge :

Charge entering at one end of a conductor = charge leaving at the other end, so current does not change with change in cross section and conductor remains uncharged when current flows through it.

## Solved Examples

**Example 3.** Find free electrons per unit volume in a metallic wire of density $10^4$ kg/m$^3$, atomic mass number 100 and number of free electron per atom is one.

**Solution :** Number of free charge particle per unit volume

$(n) = \frac{\text{total free charge particle}}{\text{total volume}}$

∴ Number of free electron per atom means total free electrons = total number of atoms.

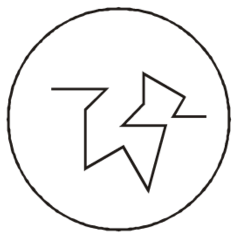
$= \frac{N_A}{M_W} \times M$

So $n = \frac{\frac{N_A}{M_W} \times M}{V} = \frac{N_A}{M_W} \times d = \frac{6.023 \times 10^{23} \times 10^4}{100 \times 10^{-3}}$

$n = 6.023 \times 10^{28}\ m^{-3}$

# 5. MOVEMENT OF ELECTRONS INSIDE CONDUCTOR

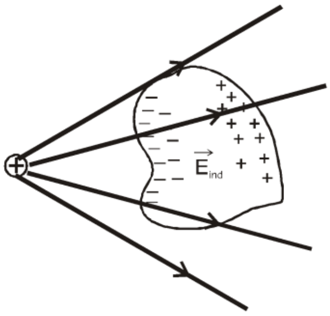
All the free electrons are in random motion due to the thermal energy and relationship in given by $\frac{3}{2}KT = \frac{1}{2}mv^2$



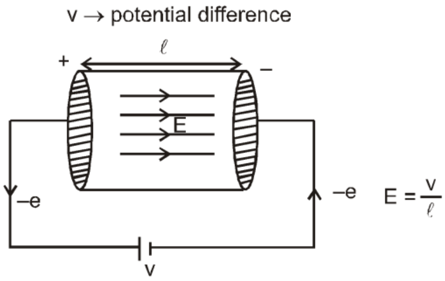
At room temperature its speed is around $10^6$ m/sec or $10^3$ km/sec

but the average velocity is zero so current in any direction is zero.

When a conductor is placed in an electric field. Then for a small duration electrons, do have an average velocity but its average velocity becomes zero within short interval of time.




When by some means a constant potential difference is applied across the conductor, then the electrons start moving with an acceleration and due to collision with other atoms & electrons, its average velocity becomes nearly constant and is called as drift velocity.




the electric field between the plate = $E = \frac{v}{\ell}$

$V_d$ = drift velocity = average velocity along the wire

hence $i = nAeV_d$ $\quad$ $V_d$ is of the order $10^{-3}$ m/s

## Solved Examples

**Example 4.** Find the approximate total distance travelled by an electron in the time-interval in which its displacement is one meter along the wire.

**Solution :** time = $\frac{\text{displacement}}{\text{drift velocity}} = \frac{S}{V_d}$

$\therefore$ $V_d = 1$ mm/s $= 10^{-3}$ m/s (normally the value of drift velocity is 1 mm/s)

S = 1 m

time = $\frac{1}{10^{-3}} = 10^3$ s

distance travelled = speed × time $\quad\therefore\quad$ speed $= 10^6$ m/s

So required distance $= 10^6 \times 10^3$ m $= 10^9$ m

## 6. RELATION BETWEEN I & V IN A CONDUCTOR

In absence of potential difference across a conductor no net current flows through a cross section. When a potential difference is applied across a conductor the charge carriers (electrons in case of metallic conductors) start drifting in a direction opposite to electric field with average drift velocity. If electrons are moving with velocity $v_d$, A is area of cross section and n is number of free electrons per unit volume then,

$I = nAev_d$. $\Rightarrow$ $v_d = \frac{\lambda}{\tau}$

λ → average displacement of electron along the wire between two successive collisions. It is also called **mean free path**.

τ → the time in which the particle does not collide with any other particle and is called as **relaxation time.**

$$\lambda = \frac{1}{2}\left(\frac{eE}{m}\right)\tau^2 = \frac{1}{2}\frac{e\tau^2}{m}.E = \frac{1}{2}\frac{e\tau^2}{m}\times\frac{V}{\ell}$$

$$i = nAe\,.\,\frac{1}{2}\frac{e\tau^2}{m}\times\frac{V}{\ell}\times\frac{1}{\tau} = \left(\frac{nAe^2\tau}{2m\ell}\right)V \quad\Rightarrow\quad i = \frac{nAe^2\tau}{2m\ell}V$$

As temperature (T) ↑ , τ ↓

## 7. ELECTRICAL RESISTANCE

The property of a substance by virtue of which it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have i = $\frac{nAe^2\tau}{2m\ell}V$

Here $i \propto V$

it is known as Ohm's law

$i = \frac{V}{R}$ ⇒ $R = \frac{2m\ell}{nAe^2\tau}$ ⇒ V = IR

hence $R = \frac{2m\ell}{nAe^2\tau}.\frac{\ell}{A}$ so Here $R = \frac{\rho\ell}{A}$ ⇒ $V = I \times \frac{\rho\ell}{A}$

⇒ $\frac{V}{\ell} = \frac{I}{A}\rho$ ⇒ E = J ρ ⇒ $J = \frac{I}{A}$ = current density

ρ is called resistivity (it is also called specific resistance), and $\rho = \frac{2m}{ne^2\tau} = \frac{1}{\sigma}$, σ is called conductivity. Therefore current in conductors is proportional to potential difference applied across its ends. This is **Ohm's Law**. Units: $R \to ohm(\Omega),\ \rho \to ohm-meter(\Omega-m)$ also called siemens, $\sigma \to \Omega^{-1}m^{-1}$.

## Solved Examples

**Example 5.** The resistance of a rectangular block of copper of dimensions 2 mm x 2 mm x 5 metre between two square faces is 0.02 ohm. What is the resistivity of copper?

**Solution :** $\rho = \frac{R.A}{\ell} = \frac{0.02 \times 4 \times 10^{-6}}{5}$

**Example 6.** What is the resistance between two rectangular faces of a block of dimensions 4 cm × 4 cm × 10 cm of manganin $\left(\rho = 48 \times 10^{-8}\,\Omega m\right)$ ?

**Solution :** $R = \frac{\rho.\ell}{A} = \frac{48 \times 10^{-8} \times 4 \times 10^{-2}}{4 \times 10 \times 10^{-4}} = 4.8\mu\Omega$

## 7.1 EFFECT OF STRETCHING OF A WIRE ON RESISTANCE

In stretching, the density of wire usually does not change. Therefore

Volume before stretching = Volume after

$\ell_1 A_1 = \ell_2 A_2$ and $\frac{R_2}{R_1} = \frac{\ell_2}{\ell_1} \times \frac{A_1}{A_2}$

If information on lengths before and after stretching is given, then use $\frac{A_1}{A_2} = \frac{\ell_2}{\ell_1}$

$\frac{R_2}{R_1} = \left(\frac{\ell_2}{\ell_1}\right)^2$

If information on radius $r_1$ and $r_2$ is given then use $\frac{\ell_2}{\ell_1} = \frac{A_1}{A_2}$ ; $\frac{R_2}{R_1} = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4$

**CONDUCTIVITY**

**(a)** Reciprocal of resistivity of a conductor is called its conductivity. It is generally represented by σ.

**(b)** $\sigma = \frac{1}{\rho}$

**(c)** Unit : $ohm^{-1}.\ \ metre^{-1}$

## Solved Examples

**Example 7.** Resistance of a wire is 20 ohm, it is stretched upto, three times of its length, then its new resistance will be

(1) 6.67 Ω (2) 60 Ω (3) 120 Ω (4) 180 Ω

**Solution :** Wire is stretched, therefore its volume remains unchanged

$A_1 l_1 = A_2 l_2$

$A_1 l = A_2 \times 3l$

$A_2 = \frac{A_1}{3}$

Ratio of resistance $\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{l}{3l} \times \frac{1}{3} = \frac{1}{9} = \frac{20}{R_2} = \frac{1}{9} \Rightarrow R_2 = 180\ \Omega$

## 7.2 EFFECT OF TEMPERATURE ON RESISTANCE AND RESISTIVITY :

The resistance of a conductor depends upon the temperature. As the temperature increases, the random motion of free electrons also increases. If the number density of charge carrier electrons remains constant as in the case of a conductor, then the increase of random motion increases the resistivity. The variation of resistance with temperature is given by the following relation

$$R_t = R_0\left(1 + \alpha t + \beta t^2\right)$$

where $R_t$ and $R_0$ are the resistance at $t^0C$ and $0^0C$ respectively and α and β are constants. The constant β is very small so its may be assumed negligible.

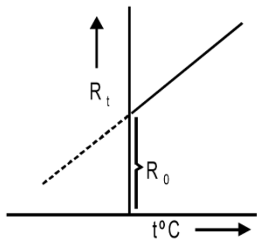
∴ $R_t = R_0(1 + \alpha t)$ or $\alpha = \frac{R_t - R_0}{R_0 \times t}$

This constant α is called as temperature coefficient of resistance of the substance. If $R_0$ = 1 ohm, t = $1^0C$, then $\alpha = (R_t - R_0)$

Thus, the temperature coefficient of resistance is equal to the increase in resistance of a conductor having a resistance of one ohm on raising its temperature by $1^0C$. The temperature coefficient of resistance may be positive or negative.

From calculations it is found that for most of the metals the value of α is nearly $\frac{1}{273}/{}^\circ C$. Hence substituting α in the above equation

$$R_t = R_0\left(1 + \frac{t}{273}\right) = R_0\left(\frac{273 + t}{273}\right) = R_0\frac{T}{273}$$




where T is the absolute temperature of the conductor.

∴ $R_t \propto T$

Thus, the resistance of a pure metallic wire is directly proportional to its absolute temperature.

The graph drawn between the resistance $R_t$ and temperature t is found to be a straight line

The resistivity or specific resistance varies with temperature. This variation is due to change in resistance of a conductor with temperature. The dependence of the resistivity with temperature is represented by the following equation.

$$\rho_t = \rho_0(1 + \alpha t)$$

With the rise of temperature the specific resistance or resistivity of pure metals increases and that of semi-conductors and insulators decrease. The resistivity of alloys increases with the rise of temperature but less than that of metals.

On applying pressure on pure metals, its resistivity decreases but on applying tension, the resistivity increases.

The resistance of alloys such as eureka, manganin etc., increases in smaller amount with the rise in temperature. Their temperature coefficient of resistance is negligible. On account of their high resistivity and negligible temperature coefficient of resistance these alloys are used to make wires for resistance boxes, potentiometer, metre bridge etc.,

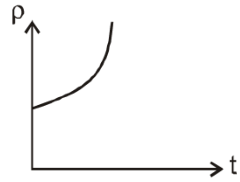
The resistance of semiconductors, insulators, electrolytes etc., decreases with the rise in temperature. Their temperature coefficients of resistance are negative.

On increasing the temperature of semi conductors a large number of electrons get free after breaking their bonds. These electrons reach the conduction band from valence band. Thus conductivity increases or resistivity decreases with the increase of free electron density.

## Solved Examples

**Example 8.** The specific resistance of a conductor increases with :
(1) Increase in temperature (2) Increases in cross-sectional area
(3) Decreases in length (4) Decrease in cross-sectional area

**Solution :** The specific resistance (resistivity) of a metallic conductor nearly increases with increasing temperature as shown in figure. This is because, with the increases in temperature the ions of the conductor vibrate with greater amplitude, and the collision between electrons and ions become more frequent, over all small temperature range (up to 100ºC). The resistivity of a metal can be represented approximately by the equation




$\rho_t = \rho_0 (1 + \alpha t)$

The factor α is called the temperature coefficient of resistivity.

## 7.3. COLOUR CODE FOR CARBON RESISTORS :

A colour code is used to indicate the resistance value of a carbon resistor and its percentage accuracy.

| Colour | Letter as an aid to memory | Number | Multiplier | Colour | Tolerance |
|---|---|---|---|---|---|
| Black | B | 0 | $10^0$ | Gold | 5% |
| Brown | B | 1 | $10^1$ | Silver | 10% |
| Red | R | 2 | $10^2$ | No fourth band | 20% |
| Orange | O | 3 | $10^3$ | | |
| Yellow | Y | 4 | $10^4$ | | |
| Green | G | 5 | $10^5$ | | |
| Blue | B | 6 | $10^6$ | | |
| Violet | V | 7 | $10^7$ | | |
| Grey | G | 8 | $10^8$ | | |
| White | W | 9 | $10^9$ | | |

A set of coloured co-axial rings or bands is printed on the resistor which reveals the following facts :

1. The first band indicates the first significant figure.
2. The second band indicates the second significant figure.
3. The third band indicates the power of ten with which the above two significant figures must be multiplied to get the resistance value in ohms.
4. The fourth band indicates the tolerance or possible variation in percent of the indicated value. If the fourth band is absent, it implies a tolerance of ± 20%

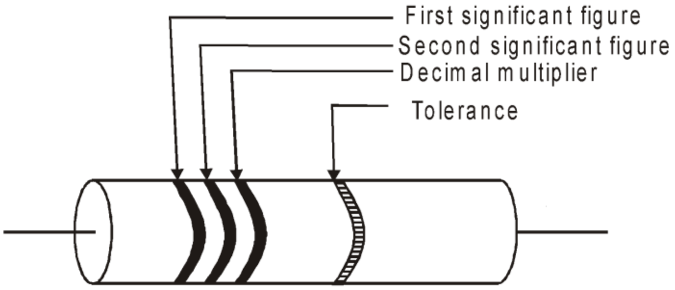



**Figure. Meanings of four bands.**

**Illustrations:**

1. In Figure. 3.13, the colours of the four bands are red, red, red and silver; the resistance value is :

| Red | Red | Red | Silver |
|---|---|---|---|
| ↓ | ↓ | ↓ | ↓ |
| 2 | 2 | 2 | ±10% |

$R = 22 \times 10^2\,\Omega \pm 10\%$

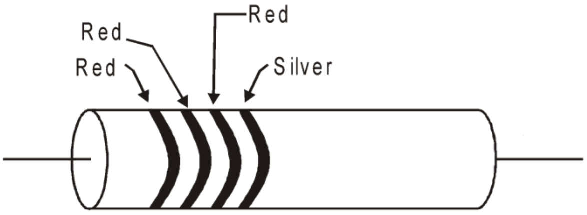
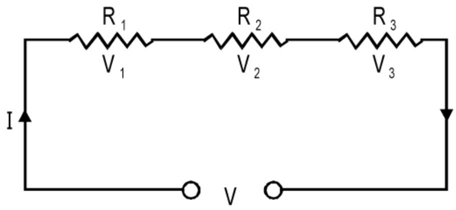



**Figure.**

## 8. COMBINATION OF RESISTANCES :




**(a) Series Combination :**

(i) Resistance are connected in series in the following way:

(ii) Same amount of current flows through each resistance.

(iii) The potential difference across each resistance depends upon the value of resistance.

(iv) The sum of potential differences across the resistances is equal to the voltage applied in the circuit, ie.,

$V = V_1 + V_2 + V_3$

(v) Total resistance of the circuit is $R = R_1 + R_2 + R_3$

Thus, the equivalent resistance of the resistances connected in series is equal to the sum of all resistances.

(vi) If identical resistances of resistance R' are connected in series, then total resistance will be

$R = nR'$

(vii) On joining the resistances in series total resistance in the circuit increases and current decreases in the circuit.

**(b) Parallel Combination :**

(i) Resistances are connected in parallel in the following way:

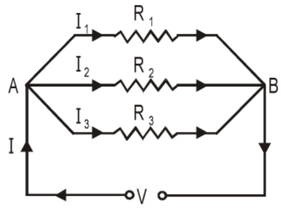
(ii) Potential difference across each resistance is same.

(iii) Current flowing through each resistance is inversely proportional to the resistance.

(iv) The sum of currents flowing through different resistances is equal to the total current flowing in the circuit, ie.,

$I = I_1 + I_2 + I_3$

(v) If R is the equivalent resistance of the circuit, then




$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Thus, the reciprocal of equivalent resistance of the resistances connected in parallel is equal to the sum of the reciprocals of those resistances.

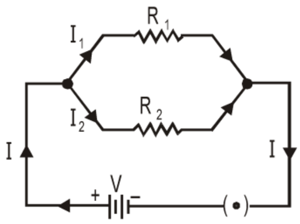
(vi) The equivalent resistance of the resistances connected in parallel is less than the smallest resistance among those resistances.

(vii) If n identical resistances of resistance R' are connected in parallel, then the equivalent resistance of the circuit will be

$R = \frac{R'}{n}$

(viii) On joining the resistances in parallel, the total resistance in the circuit decreases and the current taken from the cell increases.

(ix) When two resistances are connected in parallel, the current flowing through these resistances is inversely proportional to the resistance.




Thus, $I_1 = \frac{V}{R_1}$ and $I_2 = \frac{V}{R_2}$

and $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

∴ $\frac{I_1}{I_2} = \frac{R_2}{R_1}$

And equivalent resistance $R = \frac{R_1R_2}{R_1 + R_2}$

Thus, V = IR

∴ $I_1 = \frac{IR}{R_1} = \frac{I}{R_1}\left(\frac{R_1R_2}{R_1+R_2}\right) = \frac{IR_2}{(R_1+R_2)}$ and $I_2 = \frac{IR}{R_2} = \frac{I}{R_2}\left(\frac{R_1R_2}{R_1+R_2}\right) = \frac{IR_1}{(R_1+R_2)}$

## Solved Examples

**Example 9 :** The resistance of two conductors in series is 40 ohm and their resistance becomes 7.5 ohm when connected in parallel. What are the resistances?

**Solution :** Series $R_1 + R_2 = 40\Omega$ .............(1)

Parallel $\frac{R_1 + R_2}{R_1 + R_2} = 7.5\Omega$

$\therefore\ R_1R_2 = 7.5 \times 40 = 300\Omega$

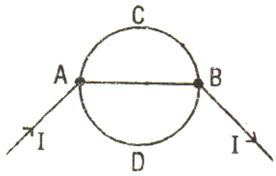
Since $(R_1 - R_2)^2 = (R_1 + R_2)^2 - 4R_1R_2$

$= 40^2 - 1200$ = 400

$R_1 - R_2 = 20$ ...............(2)

Solve (1) and (2), to get $R_1 = 30\Omega$ and $R_2$ = 10 Ω

**Example 10.** A wire of resistance $0.5\Omega m^{-1}$ is bent into a circle of radius 1 m. The same wire is connected across a diameter AB as shown in figure. The equivalent resistance is (in ohms)




(1) π (2) π + 1 (3) $\frac{\pi}{(\pi + 2)}$ (4) $\frac{\pi}{(\pi + 4)}$

**Solution :** There are three resistances ACB, AB and ADB, that are in parallel. Length ACB = πr = π, length ADB = π and AB = 2 (because r = 1m). The resistances are $R_{ACB}$ = 0.5 × π, $R_{ADB}$ = 0.5π, and $R_{AB}$ = 0.5 × 2 = 1.

The equivalent resistance is $\frac{1}{R} = \frac{1}{R_{ACB}} + \frac{1}{R_{ADB}} + \frac{1}{R_{AB}} = \frac{1}{0.5\pi} + \frac{1}{0.5\pi} + \frac{1}{1} = \frac{4+\pi}{\pi}$

or $R = \frac{\pi}{\pi + 4}$. The correct Answer is (**4**)

**Example 11.** Resistance R, (R + 1), (R + 2) ........(R + n) Ω are connected in series, their resultant resistance will be -

(1) $(n+1)\left[R + \frac{n}{2}\right]$ (2) $(n-1)\left[R - \frac{n}{2}\right]$ (3) n (R + n) (4) n (R – n)

**Solution :** Suppose the resultant resistance of the given resistance be R', then

R' = R + (R + 1) + (R + 2) + ........... (R + n)

$= \frac{(n+1)}{2}$ [2R + (n + 1) – 1] = $\frac{(n+1)}{2}$ [2R + n] = $(n+1)\left[R + \frac{n}{2}\right]$

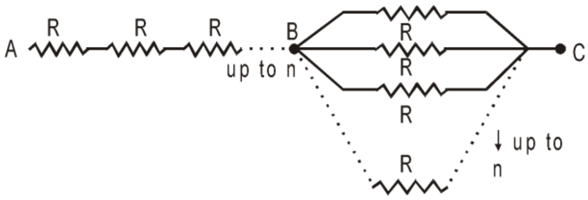
[∵ sum of n terms in arithmetic series is $S_n = \frac{n}{2}$ [2a + (n – 1) d]

Where a → is first term

d → is common difference

In the given question total terms are (n + 1)

**Example 12.** In the following figure the resultant Resistance between A and C will be -



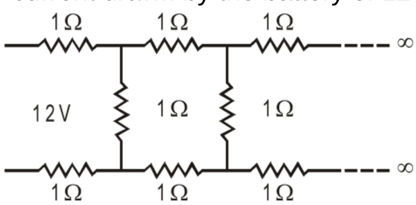

(1) $R\left(\frac{n^2+1}{n}\right)$ (2) $R\left(\frac{n+1}{n}\right)$ (3) $R\left(\frac{n^2-1}{n}\right)$ (4) $R\frac{n-1}{n}$

**Solution :** The resistance are connected in series between the points A and B and those between B and C are in parallel. Let $R_1$ and $R_2$ to be the resultant to these two combinations, then

$R_1 = nR$ and $R_2 = R/n$

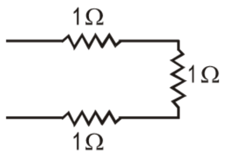
$R' = R_1 + R_2 = nR + \frac{R}{n} = R\left(\frac{n^2+1}{n}\right)$

**Example 13.** In the following fig, the current drawn by the battery of 12 V supply (in amp) will be -




(1) $6(\sqrt{3}-1)$ (2) $6(\sqrt{3}+1)$ (3) $12(\sqrt{3}-1)$ (4) $12(\sqrt{3}+1)$

**Solution :** Let x the resultant resistance. If in the following small combination (a) is added, the value of x will remain unaffected. Hence the resultant circuit will be as shown in fig. from fig (b) the resultant resistance



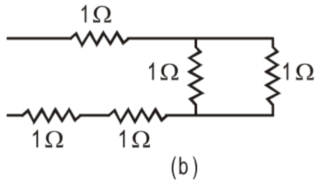

(a)

$x = 1 + \frac{1.x}{1+x} + 1 = 2 + \frac{x}{1+x}$

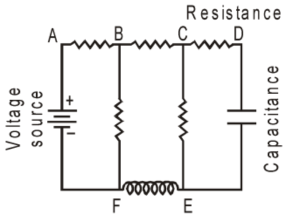
$x = 1 + \sqrt{3}$

Hence current

$$I = \frac{V}{R} = 12/(1+\sqrt{3}) = 6(\sqrt{3}-1)A$$




## 9. TERMS RELATED WITH ELECTRICAL CIRCUITS




**(a) Net work :** A group of circuits connected to each other and formed by connecting electric source and different components such as resistance, inductor, capacitor, diode, transistor etc. is called network. In the following figure a network has been shown

**(b) Branch :** A part of the network through which same current flows, is called branch of the network. In the figure, AB, BC, CD, BF and BE are branches of the network.

**(c) Node :** The junction at which two or more branches of the network meet, is called node. In the figure B, C, E etc., are nodes.

**(d) Loop :** The closed path of the current in the circuit consisting of some branches, is called loop. In the figure ABFA, BCEFB are loops.

### SOURCE OF EMF

It is a device which maintains a potential difference between two points in the circuit. Example. Primary cell, battery, solar cells, electric generators, Thermopile.

**(i) EMF (Electromotive force)**

In the interior of source of emf, positive charges move from a point of low potential (negative terminal) to a point of higher potential (positive terminal). The source of emf by doing work on the charges maintains a potential difference between its terminals. The amount of work done per unit charge is equal to the emf of the source.

$$E = \frac{W}{Q}$$

Unit of emf = volt = joule / coulomb

Dimensions $ML^2T^{-3}A^{-1}$

The emf of a cell is the potential difference between the two electrodes in an open circuit, i.e, when no current is drawn from the cell.

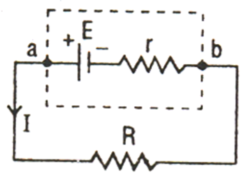
The source of energy required for moving charges from negative to positive terminal may be chemical (as in a cell or a battery), mechanical (in a generator), thermal (in a thermopile) or radiation (in a solar cell).

**(ii) Internal Resistance (r) :**

It is the resistance offered by the electrolyte of the cell to flow of charges (ions). The internal resistance can not be separated from the cell. It reduces the current that the emf can supply to the external circuit.

$$I = \frac{E}{R+r}$$

**(iii) Terminal Potential Difference (V) :**




The potential difference between the two electrodes of a cell in a closed circuit (when current is drawn from the cell) is called terminal potential difference (V). For figure, the terminal potential difference

$V = V_a - V_b$

$V = IR = E - Ir$

## Solved Examples

**Example 14.** A battery of emf 10 V and internal resistance 3 ohm is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of resistor? What is the terminal voltage of the battery when the circuit is closed?

**Solution :** (i) $I = \frac{E}{R+r}$ or $0.5 = \frac{10}{3+R}$

Solve to get $R = 17\Omega$

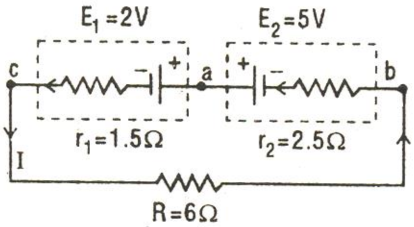
(ii) $V = E - Ir = 10 - 0.5 \times 3 = 8.5$ Volt

**Example 15.** See Fig.,

(i) What is the current in the circuit,

(ii) What is the potential difference between points a and b,

(iii) What is the potential difference between points a and c.




**Solution :** (i) $I = \frac{E_2 - E_1}{r_1 + r_2 + R}$

(ii) Imagine going from a to $E_2$ to $r_2$ to b.

or $V_a - E_2 + Ir_2 = V_b$

or $V_a - V_b = E_2 - Ir_2$

(When the cell is being discharged. The terminal potential difference is $V = E - Ir$)

$V_a - V_b = 5 - 0.3 \times 2.5 = 4.25$ Volt

[Alternatively, go from a to $E_1$ to $r_1$ to c to R to b, you will get the same answer,

$V_a - E_1 - Ir_1 - IR = V_b$

or $V_a - V_b = E_1 + I(r_1 + R) = 4.25$ Volt

(iii) Similarly, for $V_a - V_c$, go from a to $E_1$ to $r_1$ to c, then

$V_a - E_1 - Ir_1 = V_c$

(when the cell is being charged, then the terminal potential difference $V = E + Ir$)

$V_a - V_c = E_1 + Ir_1 = 2 + 0.45 = 2.45$ Volt

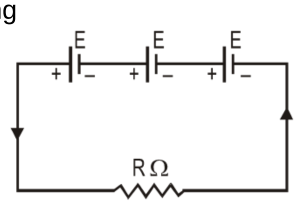
## 10. GROUPING OF CELLS :

**(a)** Each cell has definite emf E and internal resistance r.

**(b)** Cells can be grouped in the following three ways :

(i) Series grouping (ii) Parallel grouping (iii) Mixed grouping

**(c)** **Series grouping :** Total emf of all cells connected in series is equal to the sum of the emfs of individual cells and the total internal resistance is equal to the sum of the internal resistances of individual cells.



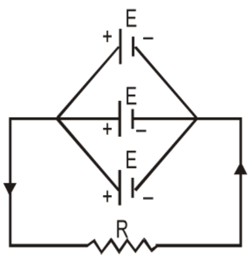

If n identical cells grouped in series are connected with an external resistance R, then the current

$$I = \frac{nE}{R+nr}$$

If r is negligible, then $I = \frac{nE}{R}$ = nx current obtained from one cell.

When the internal resistance is negligible, the current becomes n times the current due to one cell.

**(d)** **Parallel grouping :** In parallel combination for identical cells the total emf is equivalent to the emf E of one cell but equivalent internal resistance r' can be obtained as




$$\frac{1}{r'} = \frac{1}{r} + \frac{1}{r} + ..... + \frac{1}{r} = \frac{n}{r}$$

$$\therefore \quad r' = \frac{r}{n}$$

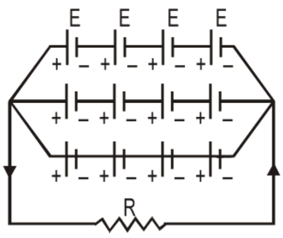
The current flowing from this parallel combination of cells through external resistance R

$$I = \frac{E}{R + \frac{r}{n}} = \frac{nE}{nR + r}$$

If $\frac{r}{n} > R$, the parallel combination is advantageous because the current becomes n times the current due to one cell.

**(e) Mixed grouping :** If m rows of N cells are formed, then in each row there will be $n = \frac{N}{m}$ cells connected in series.

Equivalent emf = nE




Equivalent internal resistance = $\frac{nr}{m}$

The current through external resistance R

$$I = \frac{nE}{R + \frac{nr}{m}} = \frac{mnE}{mR + nr} = \frac{mnE}{\left(\sqrt{nr} - \sqrt{mR}\right)^2 + 2\sqrt{nmrR}}$$

Current will be maximum when

$nr = mR$ or $R = \frac{nr}{m}$

Maximum current will be

$$I_{max} = \frac{e\sqrt{N}}{2\sqrt{rR}}$$

**Maximum Power Transfer from a D.C. Source :**

If an D.C. source is connected to a load resistance, then power is transferred from source to load. When load is varied, the current flowing through it changes and the power transferred to the load also changes. There is always internal resistance in each electric source. Due to variation of current the magnitude of power dissipation inside the source also changes. In order to provide more power to the load, there is more power dissipation in the source. Hence the power transfer to the load can be increased upto a certain limit.

Power transfer to the load from a D.C. source is maximum when the resistance of the load is equal to the internal resistance of the source. This is called maximum power transfer theorem.

If the internal resistance of a D.C. source of emf E is X and variable load resistance is R, then the current in the circuit

$$I = \frac{E}{X + R}$$

and the potential difference across the resistance R

$$V = IR = \frac{ER}{X + R}$$

∴ The power given to the load

$$P = I^2R = IV = \frac{V^2}{R} \quad \text{or} \quad P = \frac{E^2R}{(X + R)^2}$$

In case of maximum transfer of power

$$\frac{dP}{dR} = 0 \quad \text{or} \quad \frac{dP}{dR} = E^2\left[\frac{-2R}{(x + R)^3} + \frac{1}{(X + R)^2}\right] = 0 = E^2\frac{(X - R)}{(X + R)^3} = 0$$

∴ R = X

The process of obtaining maximum power by varying load resistance is called matching of load and the load in this condition is called matched load.

## Solved Examples

**Example.16** Twelve cells each having the same emf and negligible internal resistance are kept in a closed box. Some of the cells are connected in the reverse order. This battery is connected in series with an ammeter, an external resistance R and two cells of the same type as in the box. The current when they aid the battery is 3 ampere and when they oppose, it is 2 ampere. How many cells in the battery are connected in reverse order ?

**Solution :** Let n cells are connected in reverse order. Then emf of the battery is

$E' = (12-n)E - nE = (12-2n)E$

In case (i) ; $I = \frac{E' + 2E}{R} = 3$

or $E' + 2E = 3R$,

or $(14-2n)\ E = 3R$ ............. (1)

In case (ii) ; $I = \frac{E' - 2E}{R} = 2$ or $E' - 2E = 2R$,

or $(10-2n)\ E = 2R$ .............. (2)

Dividing (1) and (2)

$$\frac{14-2n}{10-2n} = \frac{3}{2}$$

or n = 1

One cell is connected in reverse order.

**Example.17** 8 cells are grouped to obtained the maximum current through a resistance of 2 ohm. If the emf of each cell is 2 volt and internal resistance is 1 ohm. Grouping of cells will have :

(1) all cells in series (2) all cells in parallel

(3) two rows of four cells (4) four rows of two cells

**Solution :** For maximum current, the number of rows

$$m = \sqrt{\frac{Nr}{R}} = \sqrt{\frac{8 \times 1}{2}} = 2$$

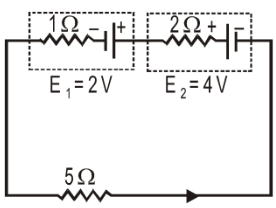
∴ They will be grouped in two rows of 4 cells.

Answer will be (3)

**Example.18** Current flowing in the following circuit will be

(1) 7.5 (2) 0.75 A (3) 2.5 A (4) 0.25 A

**Solution.** $i = \frac{E_2 - E_1}{1+2+5} = \frac{4-2}{8}$




$= \frac{2}{8} = 0.25A$

## 11. KIRCHHOFF'S LAWS :

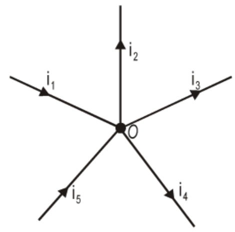
Kirchhoff proposed two laws by application of which the distribution of current among the conductors of complex electrical circuit or networks can be found out. These are:

(a) Current law, (b) Voltage law

**(a) Current law :**

(i) In an electric circuit, the algebraic sum of the currents meeting at any junction is zero. ie., $\Sigma i = 0$

(ii) While applying this law following sign convention is used : The current going towards the junction is taken as positive while that going away from the junction is taken as negative.




From Kirchhoff's law for the point O,

$i_1 - i_2 - i_3 - i_4 + i_5 = 0$

or $i_1 + i_5 = i_2 + i_3 + i_4$

(iii) In other words, the sum of the currents flowing towards the junction is equal to the sum of currents flowing away from the junction.

(iv) This law represents the law of conservation of charge, i.e., when a constant current flows in a circuit, the charge does not accummulate at a junction or at any point of the circuit.

**(b) Voltage law :**

(i) The algebraic sum of the products of the current and the resistance in a closed circuit is equal to the algebraic sum of the applied emf i.e.,

$\Sigma iR = \Sigma emf$

or $\Sigma iR - \Sigma emf = 0$

(ii) This law is a general form of Ohm's law.

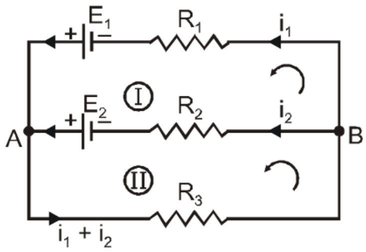
(iii) This law is applicable only in closed circuits.

(iv) Sign convention is as follows:

(A) When we traverse in the direction of current the product of the current and the resistance is taken as positive while that in the opposite direction of flow of current it is taken as negative.

(B) When we traverse from negative electrode to positive electrode of a cell the emf is taken as positive while that in the opposite direction emf is taken as negative.

(v) In the following circuit :



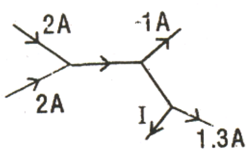

For (I) mesh : $i_1R_1 - i_2R_2 = E_1 - E_2$

For (II) mesh : $i_2R_2 + (i_1 + i_2)R_3 = E_2$

## Solved Examples

**Example.19** Find the value of I in the circuit below :

**Solution :**

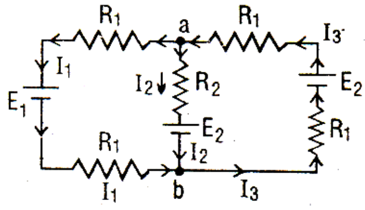



I = 4 – 1 – 1.3 = 1.7 A

**Example.20** Figure shows a circuit whose elements have the following vaues :

$E_1$ = 2V, $E_2$ = 6V, $R_1$ = 1.5 ohm and
$R_2$ = 3.5 ohm. Find the curents in the three branches of the circuit.

**Solutoin :** From junction rule at **a**




$I_3 = I_1 + I_2$ .............. (1)

For the left hand loop **aE₁ba,**

$-I_1R_1 - E_1 - I_1R_1 + E_2 + I_2R_2 = 0$

or $E_2 - E_1 = 2I_1R_1 - I_2R_2$

or $4 = 3I_1 - 3.5I_2$ ............... (2)

For the loop on right hand side starting from **a** (clockwise)

$I_3R_1 - E_2 + I_3R_1 + E_2 + I_2R_2 = 0$

$2I_3R_1 + I_2R_2 = 0$

Use $I_3 = I_1 + I_2$ Eq. (1)

$2I_1R_1 + I_2(2R_1 + R_2) = 0$

$3I_1 + 6.5I_2 = 0$ ................ (3)

From (2), use $3I_1 = 4 + 3.5I_2$, in (3), to get $4 + 3.5I_2 + 6.5I_2 = 0$

or $I_2 = -0.4\text{Amp}$

Substitute it in (2), to get

$I_1 = 0.87\text{Amp}$

Therefore, $I_3 = I_1 + I_2 = 0.87 + (-0.4) = 0.47\text{Amp}$

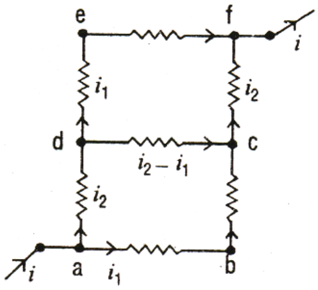
**Example 21.** What is the potential difference between points **a** and **b** in the circuit of above figure.

**Solution :** In going from a (potential $V_a$) to b (potential $V_b$), we have

$V_a - I_2R_2 - E_2 = V_b$

$V_a - V_b = E_2 + I_2R_2 = 6 + (-0.4) \times (3.5) = 6 - 1.4 = 4.6\text{volt}$

**Example 22.** The current in a, b for the circuit given here is (each resistance of R ohm).




(1) i/5 (2) 2i/5 (3) 3i/5 (4) i

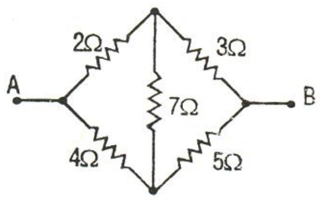
**Solution :** From symmetry the currents in various branches are as shown. Now for path abc, $V_a - V_c = 2Ri_1$, and for path adc, $V_a - V_c = 2Ri_2 - Ri_1$. Therefore,

$2Ri_1 = 2Ri_2 - Ri_1$ or $3i_1 = 2i_2$

Further $i_1 + i_2 = i$. Thus $i_1 + \left(\frac{3}{2}\right)i_1 = i$ or $i_1 = \left(\frac{2}{5}\right)i$.

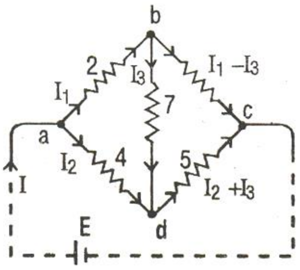
The answer is (2) 2/5 i irrespective of the value of R]

**Example.23** Five resistances are connected as shown in the figure. The equivalent resistance between the point A and B will be




(1) 3 ohm (2) 3.2 ohm (3) 2.5 ohm (4) 4 ohm

**Solution :** It is unbalanced Wheatstone bridge with current distributions shown in figure. Consider loops
(i) abda,
(ii) bcdb and (iii) adcEa



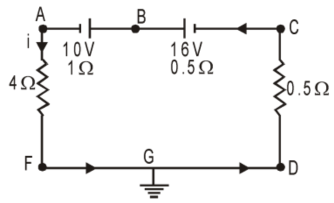

$2I_1 - 4I_2 + 7I_3 = 0$ .........(1)
$3I_1 - 5I_2 - 15I_3 = 0$ .........(2)
$E = 9I_2 + 5I_3$ .........(3)

from (1) $I_3 = \frac{-2}{7} I_1 + \frac{4}{7} I_2$ .........(4)
use it in (2) and solve
$51I_1 = 95 I_2$
use (4) and (5) in (3), solve to obtain
$E = (3283/357)I_2$
use $I = I_1 + I_2$ with $I_1 = (95I_2 / 5I)$
It is given $I = (146/51) I_2$ use it in (6)

$E = \frac{3283}{357} \times \frac{51}{146} I = 3.2\ I$

The equvialent resistance is E/I = 3.2 ohm. The correct answer is (2)

**Example 24.** From the fig. determine
(i) potential at A
(ii) potential at C, and
(iii) reading of the voltmeter connected across the 10V bettery



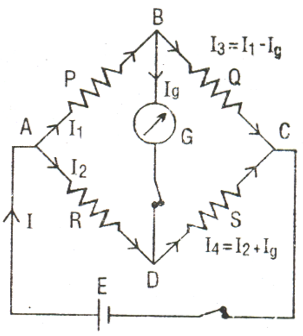

**Solution** The current in circuit is (consider loop (CBAFGDC)

$$I = \frac{E_2 - E_1}{r_1 + r_2 + R_1 + R_2} = \frac{16 - 10}{1 + 0.5 + 4 + 0.5} = \frac{6}{6} = 1A$$

(i) $V_A - V_F = IR = 4$ volt
Because $V_F = 0$ (grounded), therefore $V_A = 4$ volt
(ii) $V_D - V_C = 1 \times 0.5 = 0.5$ volt
∴ $V_D = 0$ (grounded), So $V_C = -0.5$ volt
(iii) The 10V battery is being charged therefore $V = E + Ir = 10 + 1 \times 1 = 11$ volt

## 12. WHEATSTONE'S BRIDGE

Four resistances P, Q, R S connected as in figure forms a Wheatstone bridge. Conventionally P, Q are called ratio arms, R is a variable resistance and S is some unknown resistance.




**Principle**

When no current flows in the galvanometer G($I_g$ = 0), then the potentials of points B and D are equal. Then, the bridge is said to be balanced, and

$$\frac{P}{Q} = \frac{R}{S}$$

Knowledge of any three, say, P, Q, R determines the fourth S.

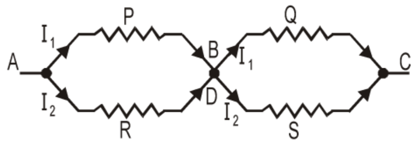
**Proof :** From Kirchhoff's second rule, for loop ABDA,

$-I_1P + 0 \times G + I_2R = 0$ , or $\quad I_1P = I_2R$

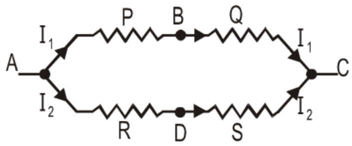
For loop BCDB, $(\because I_g = 0, I_3 = I_1,\ I_4 = I_2)$ , $= I_1Q + I_2S + 0 \times G = 0$ , or

$I_1Q = I_2S \quad$ Thus $\quad \dfrac{P}{Q} = \dfrac{R}{S}$

**Comment (View-I)** Since B and D are at the same potential for a balanced bridge, they can be thought to be the same point, so that Wheatstone bridge appears as in figure.



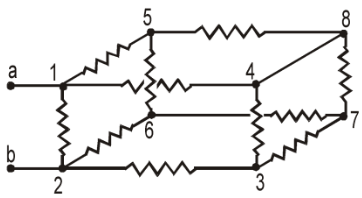

(View-2) Since no current flows through the galvanometer, we can ignore the G arm (as if it is not connected). Then, the balanced Wheatstone bridge, appears as in figure.



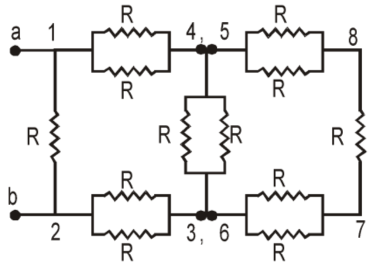

These views help a great deal in determining equivalent resistance.

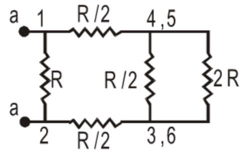
## Solved Examples

**Example.25** Figure shows a cube made of 12 resistances, each of resistance R. Find the equivalent resistance across a cube edge ab.



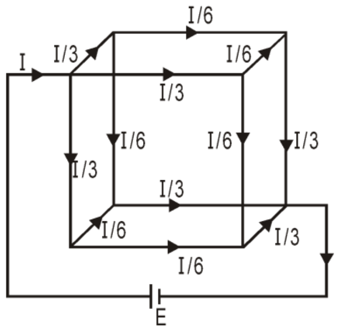

**Solution :** From the consideration of symmetry alone we notice that points 4 and 5 must be at the same potential. So must be points 3 and 6. This implies that the circuit can be redrawn with points 4 and 5 connected, and points 3 and 6 connected, as shown in figure. This figure further reduces to the combination shown in figure.




The equivalent resistance between a and b is (calculate) = $\left(\frac{7}{12}\right)R$

**Example.26** Twelve equal wires, each of resistance R ohm are connected so as to form a skeleton cube. An electric current enters this cube from one corner and leaves out the diagonally opposite corner. Calculate the total resistance of this assembly.




**Solution :** Let ABCDEFGH be skeleton cube formed of twelve equal wires each of resistance R. Let a battery of e.m.f. E be connected across A and G. Let the total current entering at the corner A and leaving the diagonally opposite corner G be I. By symmetry the distribution of currents in wires of cube, according to Kirchoff's I law is shown in fig. Applying Kirchoff's II law to mesh ADCGEA, we get

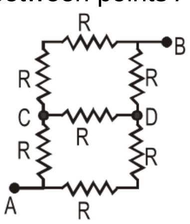
$$-\frac{1}{3}R-\frac{1}{6}R-\frac{1}{3}R+E=0$$

or $E=\frac{5}{6}IR$ .............(1)

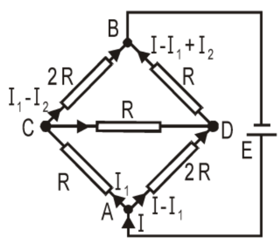
If $R_{AB}$ is equivalent resistance between corners A and B, then from ohm's law comparing (1) and (2), we get

$$IR_{AB}=\frac{5}{6}IR$$

**Example 27.** Find the equivalent resistance between points A and B in the following circuit.




**Solution :** This is Wheatstone bridge but is unbalanced. To find equivalent resistance, we imagine, that a cell of emf E is connected between points A and B. Then the combination looks like figure.




For the loop **ACDA**

$$\boxed{2I=3I_1+I_2}$$

For the loop **BDCB**

$\boxed{I = 3I_1 - 4I_2}$

Solving the two we get

$I_1$ (3/5) I, $I_2$ = (1/5)I

Consider loop **ADBEA,** then

$E = (I - I_1)\, 2R + (I - I_1 + I_2)R = (4R/5)I + (3R/5)I$

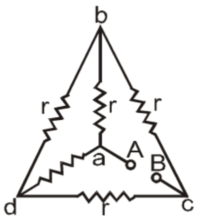
$\boxed{E = (7R/5)I}$

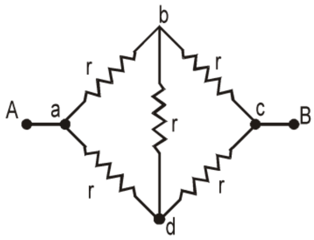
Therefore the effective resistance is

$R_{eq} = E/I$

$= 7R/5$

**Example 28.** In the adjoining network of resistors, each is of resistance r ohm, the equivalent resistance between points A and B is



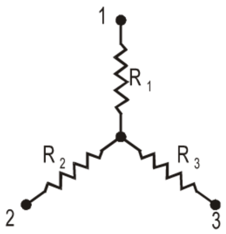

(1) 5r (2) 2r/3 (3)r (4) r/2

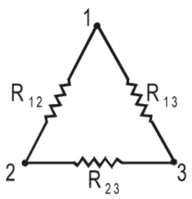
**Solution :** Imagine, Aa being pulled on the left side, then abcd becomes a balanced Wheatstone bridge (figure). The arm bd can be ignored. Then resistance between A, B becomes = r. The answer is (3)

**UNBALANCED WHEATSTONE BRIDGE**

**(i) STAR DELTA CONVERSIONS**

The combination of resistances shown in figure (a) is called a star connection and that shown in figure (b) is called a delta connection.

(a) 


(b) 


These two arrangements are electrically equivalent for the resistances measured between any pair of terminals. A star connection can be replaced by a delta, and a delta can be replaced by a star.

**STAR to DELTA**

If resistances $R_1$, $R_2$ and $R_3$ are known and connected in star configuration (as in figure (a)) then it can be replaced by a delta configuration with following resistances.

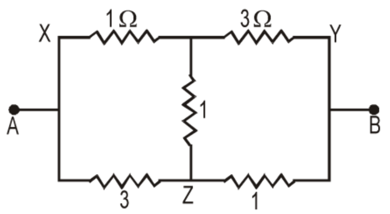
$$R_{12} = R_1 + R_2 + \frac{R_1R_2}{R_3} \qquad R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_{13} = R_1 + R_3 + \frac{R_1R_3}{R_2}$$

(Note $R_{12} = R_{21}$, $R_{13} = R_{31}$, $R_{23} = R_{32}$)

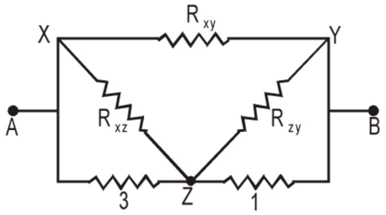
## Solved Examples

**Example.29** Consider the unbalanced wheatstone bridge shown in Fig. (a) Find its equivalent resistance between A and B (all values are in ohm)




(a)

**Solution :** Consider the star combination between XYZ. It can be reduced into a delta combination. Then the (see fig.a) looks like (see fig.b) with

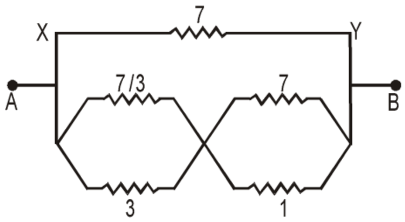



$R_{xy} = 1 + 3 + \frac{1 \times 3}{1} = 7$

$R_{xz} = 1 + 1 + \frac{1 \times 1}{3} = 7/3$

$R_{zy} = 1 + 3 + \frac{1 \times 3}{1} = 7$ (b)

Thus the equivalent diagram now looks like Fig. (c). The resistance between ends A and B is then easily determined. Its value is.

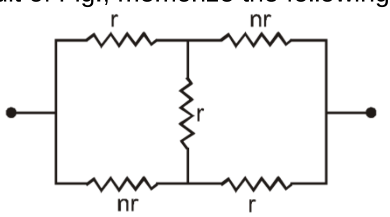



(c)

$$R_{AB} = \frac{5}{3}$$

Thus the answer is $R_{AB}$ = 5/3 ohm. (This problem can also be solved by delta to star conversion, see example 39).

**Comment.** For the circuit of Fig., memorize the following formula for equivalent




resistance. (If you are aspiring for engineering, prove it by using star-delta conversion).

$$R_{equivalent} = \left(\frac{3n+1}{n+3}\right)r$$

**(ii) DELTA to STAR Conversion**

Consider Fig. (a) and (b) again. If resistance $R_{12}$, $R_{23}$ and $R_{13}$ are known and connected in a delta configuration as in Fig. (b), then it can be replaced by a star connection with the following resistances.

$$R_1 = \frac{R_{12}R_{13}}{R_{12}+R_{13}+R_{23}}$$

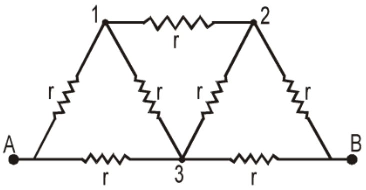
$$R_2 = \frac{R_{23}R_{21}}{R_{12}+R_{13}+R_{23}}$$

$$R_3 = \frac{R_{31}R_{32}}{R_{12}+R_{13}+R_{23}}$$

(Note $R_{13} = R_{31}$, $R_{23} = R_{32}$, $R_{12} = R_{21}$)

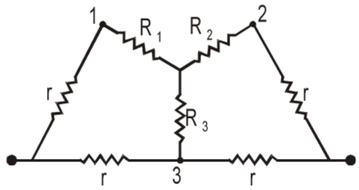
## Solved Examples

**Example.30** Each resistance in the network is of r ohm. Then calculate the equivalent resistance between the terminals A and B.

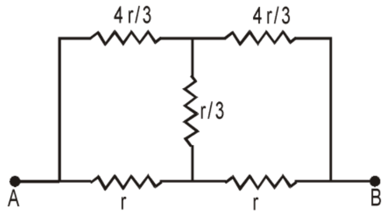



(a)

**Solution :** Consider the delta connection between points 123. Converting it into a star connection, the Fig. (a) now looks like Fig. (b), with resistances.




(b)



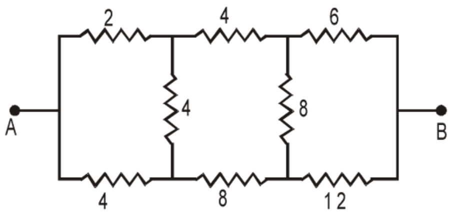

(c)

$R_1 = \frac{r \times r}{r+r+r} = \frac{r}{3}$, $R_2 = \frac{r}{3}$, $R_3 = \frac{r}{3}$

Thus Fig. (b), reduce to a balanced wheatstone bridge see fig. (c), whose equivalent resistance is $\boxed{R_{eq} = \frac{8}{7}r}$

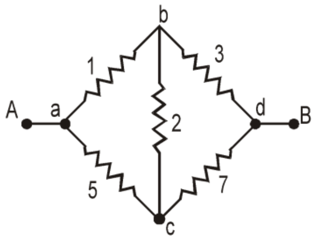
(Memorize the result)

(ii) The equivalent resistance for the circuit of Fig. is (all resistance are in ohm),




$R_{AB}$ = 8 ohm.

**Example 31.** Consider the unbalanced wheatstone bridge shown in Fig. (a). Find the equivalent resistance between the points A and B (All resistances are in ohms).




(a)

**Solution :** Consider one of the delta combination, say abc. Then converting it into equivalent star combination, we find fig. (b). A direct use of the conversion formula give.

$$R_1 = \frac{1\times 2}{1+2+5} = \frac{1}{4}$$

$$R_2 = \frac{5\times 1}{8} = \frac{5}{8}$$

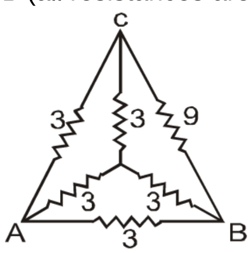
$$R_3 = \frac{2\times 5}{8} = \frac{5}{8}$$

The $R_{AB}$, now can be calculated from the simplied Fig. (b). It gives

$$R_{AB} = \frac{5}{8} + \frac{13}{4} \,||\, \frac{33}{4} = \frac{5}{8} + \frac{429}{184} = \frac{544}{184}$$ or **$R_{AB}$ = 2.96 Ω**

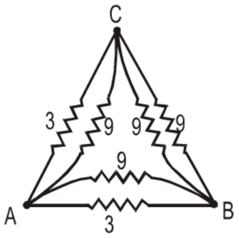
Solve example 37 by the above procedure

**Example.32** Consider the resistance combination shown in Fig.(a) Then determine the equivalent resistance between the terminals A and B (all resistances are in ohms).




(a)

**Solution :** In Fig. (a), there is a star connection at the centre. This can be easily converted into a delta connection, with




(b)

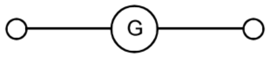
$$R_{12} = R_{13} = R_{23} = 3 + 3 + \frac{3\times 3}{3} = 9$$

the resulting diagram is then shown in Fig. (b). Therefore the equivalent resistance between terminal is (solve)

$$R_{AB} = \frac{27}{16}\text{ohm.}$$

## 13. GALVANOMETER

Galvanometer is represented as follow :




It consists of a pivoted coil placed in the magnetic field of a permanent magnet. Attached to the coil is a spring. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

When coil rotates the spring is twisted and it exerts an opposing torque on the coil.

There is a resistive torque also against motion to damp the motion. Finally in equilibrium

$\tau_{magnetic} = \tau_{spring} \Rightarrow$ BINA sin θ = Cϕ

But by making the magnetic field radial θ = 90º.

∴ BINA = C ϕ

$I \propto \phi$

here B = magnetic field | A = Area of the coil

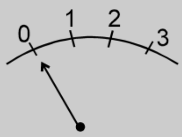
I = Current | C = torsional constant

N = Number of turns | ϕ = angle rotate by coil.

- **Current sensitivity**

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer CS = $\frac{\phi}{I} = \frac{BNA}{C}$

**Note:** Shunting a galvanometer decreases its current sensitivity. A linear scale is obtained. The marking on the galvanometer are proportionate.

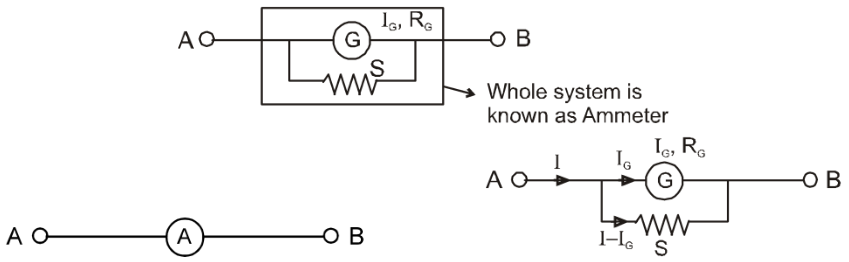



The galvanometer coil has some resistance represented by $R_g$ . It is of the order of few ohms. It also has a maximum capacity to carry a current known as $I_g$. $I_g$ is also the current required for full scale deflection. This galvanometer is called moving coil galvanometer.

## 14. AMMETER

A shunt (small resistance) is connected in parallel with galvanometer to convert it into ammeter; An ideal ammeter has zero resistance

Ammeter is represented as follow -



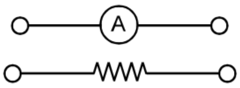

If maximum value of current to be measured by ammeter is I then

$I_G \cdot R_G = (I - I_G)S$

$S = \frac{I_G . R_G}{I - I_G}$

$S = \frac{I_G \times R_G}{I}$ when $I >> I_G$.

where I = Maximum current that can be measured using the given ammeter.




For measuring the current the ammeter is connected is series.

In calculation it is simply a resistance

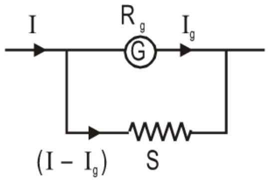
Resistance of ammeter $R_A = \frac{R_G.S}{R_G + S}$

for $S << R_G \Rightarrow R_A = S$

## Solved Examples

**Example 33.** What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm ?

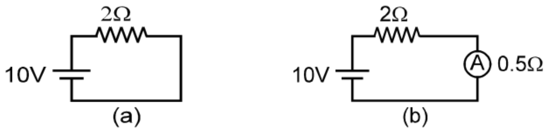
**Solution :** As in figure $R_g I_g = (I - I_g)S$




$\Rightarrow \quad 99 \times \frac{I}{10} = \left(I - \frac{I}{10}\right) \times S$

$\Rightarrow \quad S = 11\ \Omega$.

**Example 34.** Find the current in the circuit (a) & (b) and also determine percentage error in measuring the current through an ammeter.



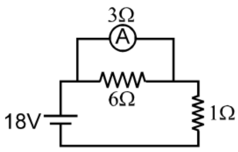

**Solution :** In A $\quad I = \frac{10}{2} = 5A$

In B $I = \frac{10}{2.5} = 4A$

Percentage error is $= \frac{i - i'}{i} \times 100 = 20\%$ **Ans.**

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

**Example 40.** Find the reading of ammeter ? Is this the current through 6 Ω ?




**Solution :** $R_{eq} = \frac{3 \times 6}{3 + 6} + 1 = 3\ \Omega$

Current through battery $I = \frac{18}{3} = 6$ A

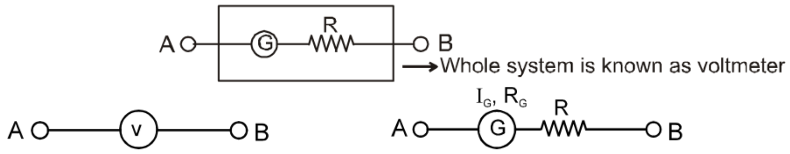
So, current through ammeter $= 6 \times \frac{6}{9} = 4$ A

No, it is not the current through the 6 Ω resistor.

**Note:** Ideal ammeter is equivalent to zero resistance wire for calculation potential difference across it is zero.

## 15. VOLTMETER

A high resistance is put in series with galvanometer. It is used to measure potential difference across a resistor in a circuit.

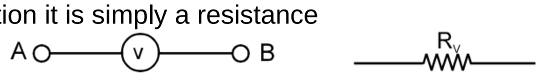



For maximum potential difference $V = I_G . R + I_G R_G$ $\qquad R = \frac{V}{I_G} - R_G$

If $R_G << R \Rightarrow R_S \approx \frac{V}{I_G}$

For measuring the potential difference a voltmeter is connected across that element. (parallel to that element it measures the potential difference that appears between terminals 'A' and 'B'.)

For calculation it is simply a resistance




Resistance of voltmeter $R_V = R_G + R \approx R$

$I_g = \frac{V_o}{R_g + R}$ . $R \to \infty \Rightarrow$ Ideal voltmeter.

A good voltmeter has high value of resistance.

Ideal voltmeter → which has high value of resistance.

**Note :**

- For calculation purposes the current through the ideal voltmeter is zero.
- Percentage error in measuring the potential difference by a voltmeter is = $\frac{V - V'}{V} \times 100$

## Solved Example

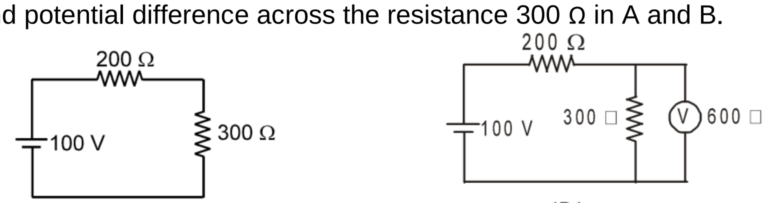
**Example 36.** A galvanometer has a resistance of G ohm and range of V volt. Calculate the resistance to be used in series with it to extend its range to nV volt.

**Solution :** Full scale current $i_g = \frac{V}{G}$

to change its range

$V_1 = (G + R_s)i_g \Rightarrow nV = (G + R_s)\frac{V}{G} \Rightarrow R_s = G(n - 1)$ **Ans.**

**Example 37.** Find potential difference across the resistance 300 Ω in A and B.




(A) (B)

**Solution :** In (A) : Potential difference = $\frac{100}{200 + 300} \times 300$ = 60 volt

In (B) : Potential difference = $\frac{100}{200 + \frac{300 \times 600}{300 + 600}} \times \frac{300 \times 600}{300 + 600}$ = 50 volt

We see that by connecting voltmeter the voltage which was to be measured has changed. Such voltmeters are not good. If its resistance had been very large than 300 Ω then it would not have affected the voltage by much amount.

### Current sensitivity

The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (C.S.) of the galvanometer CS = $\frac{\theta}{I}$

**Note :** Shunting a galvanometer decreases its current sensitivity.

## Solved Examples

**Example 38.** A galvanometer with a scale divided into 100 equal divisions, has a current sensitivity of 10 division per mA and voltage sensitivity of 2 division per mV. What adoptions are required to use it (a) to read 5A full scale and (b) 1 division per volt ?

**Solution :** Full scale deflection current $i_g = \frac{\theta}{cs} = \frac{100}{10}$ mA = 10 mA

Full scale deflection voltage $V_g = \frac{\theta}{vs}$
= mv = 50 mv

So galvanometer resistance $G = \frac{V_g}{i_g} = \frac{50mV}{10mA} = 5\ \Omega$

(a) to convert the galvanometer into an ammeter of range 5A, a resistance of value SΩ is connected in parallel with it such that

$(I - i_g)\ S = i_g G$

$(5 - 0.01)\ S = 0.01 \times 5$

$S = \frac{5}{499} \cong 0.01\ \Omega$ **Ans.**

(b) To convert the galvanometer into a voltmeter which reads 1 division per volt, i.e. of range 100 V,

$V = i_g\ (R + G)$

$100 = 10 \times 10^{-3}\ (R + 5)$

$R = 10000 - 5$

$R = 9995\ \Omega \cong 9.995\ k\Omega$ **Ans.**

## 16. POTENTIOMETER

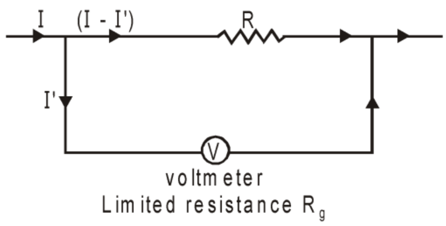
**(a)** Potentiometer is an electrical instrument which measures emf or potential difference between any two points accurately, that is, it is an ideal instrument for measuring emf or potential difference.

**(b)** The emf or potential difference measured by potentiometer is more accurate in comparison to that measured by a voltmeter.

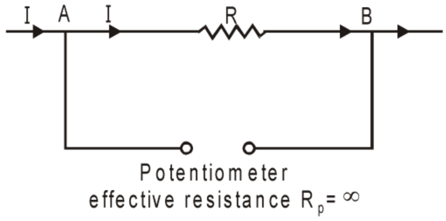
**(c)** Potentiometer is based on no deflection method. When the potentiometer gives zero deflection, it does not drawn any current form the cell or the circuit.

**(d)** Potentiometer is effectively an ideal instrument of infinite resistance for measuring potential difference. Therefore potentiometer is also said to be an ideal voltmeter.

**(e)** Voltmeter is made by connecting a proper high resistance in series with a galvanometer. If a voltmeter is used for measuring potential difference produced due to current I flowing through a resistance R, then a small part of current, say I', passes through the voltmeter. Therefore, the potential difference across the resistance R becomes (I – I')R instead of IR. In this way the potential difference decreases and voltmeter measures this decreased potential difference



**(f)** When a potentiometer is used for measuring the same potential difference, it compares unknown potential difference with known potential difference, when they become equal, the galvanometer connected in the circuit gives zero deflection. In this state the current I' is zero and as a result the current flowing through the resistance R does not decrease and the potential difference across the resistance R is equivalent to the real potential difference. In this way the potential difference measured by the potentiometer is more accurate in comparison to that measured by voltmeter

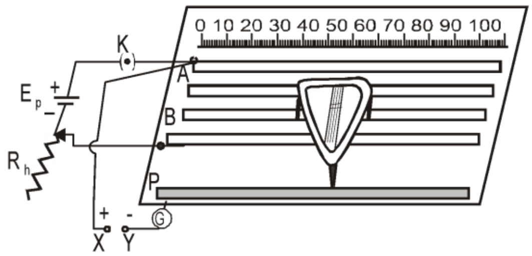



## 16.1 CONSTRUCTION OF POTENTIOMETER

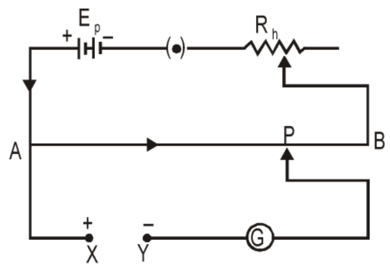
**(a)** Main part of a potentiometer is a wire made from material such as manganin, eureka or constantan. Their temperature coefficient of resistance is very small but the specific resistance is quite high.

**(b)** Potentiometer wire is of uniform cross section whose length is 10 m and resistance is about 5 ohm. This wire is bent in the form of 10 parallel wires each one metre long connected in series and stretched on a wooden board.

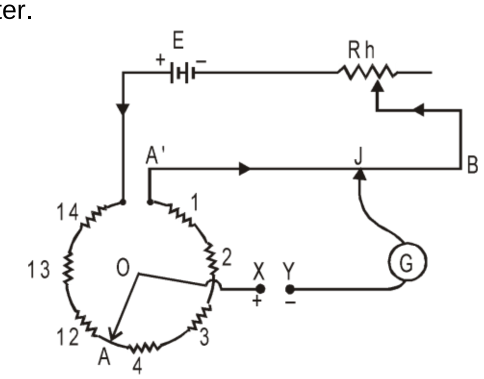
**(c)** A metre scale is also fixed parallel to the wires on a wooden board. A sliding jockey is placed on the wires. It is so placed that it can establish contact with the wire at any desired point. Connecting terminals are fitted at the ends A and B of the wire.




**(d)** Theoretically the potentiometer wire is usually represented by a straight line AB while drawing electrical circuit.




**(e)** In a modified form of potentiometer only one wire A'B and 14 resistance coils are used. The resistance of each coil is equivalent to the resistance of wire A'B. From this arrangement the effective length of potentiometer wire can be changed as desired hence the sensitivity of potentiometer can be increased or decreased according to need. Such type of potentiometer is called dial potentiometer or Crompton potentiometer.

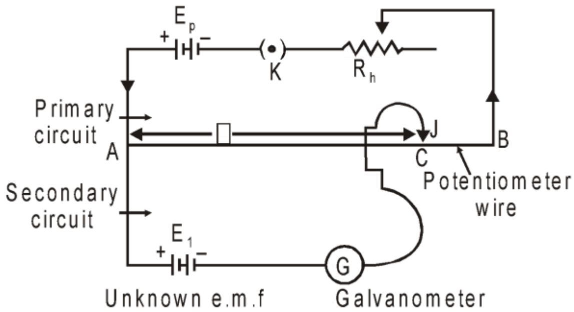



## 16.2 PRINCIPLE AND WORKING OF POTENTIOMETER

**(a)** The unknown emf or potential difference is determined with the help of a potentiometer by comparing it with emf or potential difference uniformly distributed along the wire.

**(b)** Potentiometer works on no deflection method, that is, when the potentiometer is in a state of balance, the galvanometer connected in its circuit does not show any deflection and does not draw any current from the circuit.

**(c)** Circuit diagram of potentiometer :




**(d) Potential gradient (x) :**

(i) The fall of potential per unit length of potentiometer wire is called potential gradient.

(ii) Its unit is V/m or V/cm

(iii) Potential gradient (x) = $\dfrac{\text{emf applied in the primary circuit}}{\text{total length of potentiometer wire}} = \dfrac{E}{L} \text{V/m}$

(iv) If R be the resistance of potentiometer wire, then

$$x = \left(\frac{E}{R}\right)\left(\frac{R}{L}\right)$$

(v) If i is the current flowing through the potentiometer wire or in the primary circuit and is the resistance of unit length of potentiometer wire, then

$i = \dfrac{E}{R}$ and $\rho = \dfrac{R}{L}$ $\therefore$ $x = i\rho$ V/m

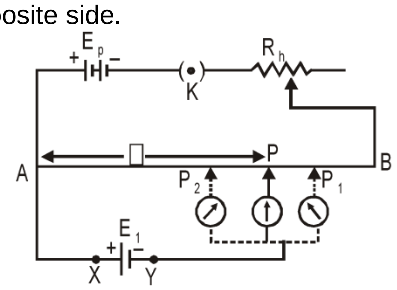
**(e)** If a point P is considered at a distance $\ell$ from a point A on the potentiometer wire, then the potential difference between A and P will be

$$V = x\ell$$

**(f)** If jockey is pressed at a distance $\ell$ on the potentiometer wire, then two potential differences are obtained at this length. One will be due to potential difference V due to a battery $E_\rho$ and second will be unknown potential difference $E_1$. The resultant current due to these potential differences will flow in the circuit whose direction is obtained as follows:

(i) If V > $E_1$, then the current will flow in the circuit along the path $AE_1GP_1A$ and galvanometer will show deflection in one side.

(ii) If V < $E_1$, then the current will flow in the circuit along the path $AP_2GE_1A$ and galvanometer will show deflection in opposite side.




(iii) In this way a point can be obtained on the potentiometer wire with the help of jockey where on pressing the jockey the galvanometer will not show any deflection. This point is called balancing point. The length of the wire from the point A to the balancing point is called balancing length. The potential difference V across this length due to battery $E_P$ is equal to the unknown potential difference or emf $E_1$, i.e., V = $E_1$.

**(g)** If the balancing length for unknown emf $E_1$ is $\ell_1$ then the potential difference across this length will be

$V_1 = x\ell_1$ but $V_1 = E_1$ $\therefore$ $E_1 = V_1 = x\ell_1$ or $E_1 = x\ell_1$

From this equation unknown potential difference can be determined. This is the principle of potentiometer.

**(h)** Potentiometer works on no deflection method because when unknown emf or potential difference is obtained from potentiometer, in this state no deflection is produced in the galvanometer, that is, the potentiometer is in the state of zero deflection. This state is called balancing state of potentiometer.

## 16.3 POTENTIAL GRADIENT

The fall of potential per unit length of potentiometer wire is called potential gradient. In some specific conditions its value is:

**(a)** If potentiometer wire having resistance R is connected in series with an external resistance $R_1$ and a battery of emf $E_\rho$ of internal resistance r in the primary circuit, then the current flowing in the potentiometer wire will be

$$\frac{E_\rho}{(R + r + R_1)}$$

From the definition of potential gradient

$$x = i\rho = i\frac{R}{L}$$

$$\therefore \quad x = \left(\frac{E_\rho}{R + r + R_1}\right)\left(\frac{R}{L}\right)$$

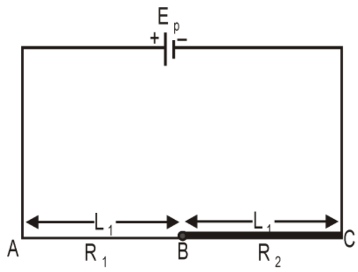
**(b)** If a rheostat of range $(0 – R_1)$ is connected in the primary circuit, then the potential gradient on the wire can be changed by the rheostat. When $R_1 = 0$ then current i will be maximum and also potential gradient is maximum.

$$x_{max} = \left(\frac{E_\rho}{R + r}\right)\left(\frac{R}{L}\right)$$

When the external resistance $R_1$ inserted by rheostat is maximum, the current will be minimum and the potential gradient is also minimum.

$$\therefore \quad x_{min} = \left(\frac{E_\rho}{R + r + R_1}\right)\left(\frac{R}{L}\right)$$

**(c)** In the following figure two wires AB and BC are connected at B and then they are connected to a battery of emf $E_\rho$ in series.




Potential gradient on part AB

$$x = i\rho$$

$$x_{AB} = \left(\frac{E_\rho}{R_1 + R_2}\right)\left(\frac{R_1}{L_1}\right)$$

and the potential gradient on part BC

$$x_{BC} = \left(\frac{E_\rho}{R_1 + R_2}\right)\left(\frac{R_2}{L_2}\right)$$

$$\therefore \quad \frac{x_{AB}}{x_{BC}} = \frac{R_1}{L_1} \times \frac{L_2}{R_2}$$

**(d)** Keeping the length of potentiometer wire constant if its thickness (diameter or radius) is varied, then the potential gradient does not change on the wire, that is, x remains constant because it does not depend on the thickness of the potentiometer wire. In practice the resistance of potentiometer wire changes with the change of its thickness as a result the current in the primary circuit will also change. Hence the potential gradient will change due to change in current.

**(e)** If the specific resistance of material of a potentiometer wire is K, its area of cross–section is A and the current flowing in the wire is i, then the resistance of potentiometer wire

$$R = K\frac{L}{A}$$

Hence $\rho = \frac{R}{L} = \frac{K}{A}$

Potential gradient $x = i\frac{K}{A}$

**(f)** If the length L of potentiometer wire and the current i flowing in the primary circuit are kept constant and temperature of the wire is increased, then the potential gradient does not change. But if current i changes due to change in resistance with temperature, then the potential gradient will not remain constant.

**(g)** Keeping the thickness of the potentiometer wire constant, if its length is changed from $L_1$ and $L_2$, then the ratio of the potential gradients will be $\frac{x_1}{x_2} = \frac{L_1}{L_2}$

**(h)** The potential gradient depends on the following factors:
(i) emf (E) and internal resistance (r) of the battery used in the primary circuit.
(ii) the length of wire (L)
(iii) the resistance of wire (R)
(iv) the current flowing through the wire (i)
(v) the resistance of unit length of wire (ρ)
(vi) the specific resistance of material of wire (K)
(vii) the resistance connected in series with the wire ($R_1$)

## Solved Examples

**Example 39.** Resistivity of potentiometer wire is $10^{-7}$ ohm-metre and its area of cross-section is $10^{-6}m^2$. When a current i = 0.1A flows through the wire, its potential gradient is :
(1) $10^{-2}$ V/m (2) $10^{-4}$ V/m (3) 0.1 V/m (4) 10V/m

**Solution:** Potential gradient of a wire equal to potential fall per unit length.
Potential gradient = Potential fall per unit length
= Current × Resistance per unit length

$= i \times \frac{R}{\ell}$

but $R = \frac{\rho\ell}{A}$

$\Rightarrow \frac{R}{\ell} = \frac{\rho}{A}$

∴ Potential gradient $= i \times \frac{\rho}{A}$

Here, $\rho = 10^{-7}$ Ω–m, i = 0.1 A × A = $10^{-6}$ m²

Hence, potential gradient $= 0.1 \times \frac{10^{-7}}{10^{-6}} = \frac{0.1}{10} = 0.01 = 10^{-2}$V/m

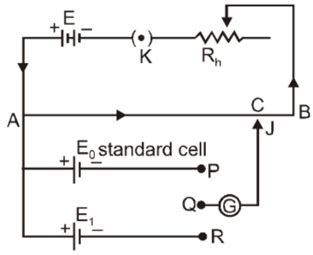
## 16.4 STANDARDISATION OF POTENTIOMETER

**(a)** For measurement of emf or potential difference by potentiometer first of all the potentiometer is standardised. Actually the process of determining potential gradient is called standardisation.

**(b)** The standardisation of potentiometer is performed with the help of a standard cell which is connected in the secondary circuit.

**(c)** Standard cell is that type of cell whose emf remains constant. Cadmium cell is such type of standard cell. Its emf is 1.018 volt at $20^0$C. In laboratories ordinary Daniel cell is used as a standard cell.

**(d)** For standardisation of potentiometer following circuit is used:




**(e)** Let the balancing length for the standard emf $E_0$ is $\ell_0$. If the potential gradient is x, then from the principle of potentiometer, $E_0 = x\ell_0$

Hence, $x = \frac{E_0}{\ell_0}$

Thus by knowing $E_0$ and $\ell_0$ the potential gradient x can be determined. This process is called standardisation.

**(f)** If the balancing length for the unknown emf $E_1$ is $\ell_1$, then

$$E_1 = x\ell_1 = \left(\frac{E_0}{\ell_0}\right)\ell_1$$

## 16.5 SENSITIVITY OF POTENTIOMETER

**(a)** If a potentiometer measures low value of emf or potential difference with high accuracy, then it is said to be more sensitive. Hence the sensitivity of a potentiometer depends on the potential gradient.

**(b)** Lower the potential gradient, higher will be its sensitivity. If the potential gradient is higher, then the potentiometer will be less sensitivie.

**(c)** On increasing the length of a potentiometer wire its potential gradient $\left(x = \frac{E}{L}\right)$ can be decreased and its sensitivity can be increased.

**(d)** By reducing the current flowing through the wire and decreasing the resistance per unit length of the wire, the potential gradient can be decresed or the sensitivity of potentiometer can be increased upto a certain limit.

## 16.6 USES OF POTENTIOMETER

**(a)** To determine unknown emf
**(b)** To determine unknown potential difference.
**(c)** To compare emf's of two cells.
**(d)** To determine current in the circuit.
**(e)** To compare two low resistances.
**(f)** To determine the internal resistance of a primary cell.
**(g)** To calibrate an ammeter.
**(h)** To calibrate a voltmeter.
**(i)** To determine temperature by measuring thermo emf of a thermocouple.

## 16.7 DETERMINATION OF EMF OR POTENTIAL DIFFERENCE AND CURRENT BY POTENTIOMETER

**(a)** To determine the unknown emf by potentiometer the circuit for standardisation of potentiometer is used.

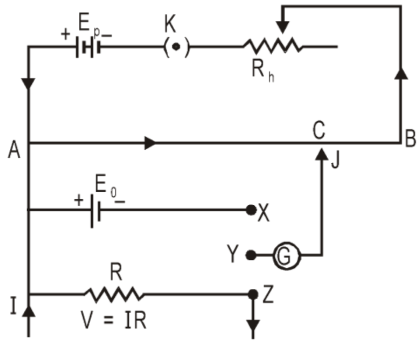
**(b)** If the balancing length for emf $E_0$ is $\ell_0$ then

$$E_0 = x\ell_0 \quad \text{or} \quad x = \frac{E_0}{\ell_0}$$

**(c)** If balancing length for unknown emf $E_1$ is $\ell_1$ then

$$E_1 = x\ell_1 \quad \text{or} \quad E_1 = \frac{E_0}{\ell_0}\ell_1$$

**(d)** For determining the unknown potential difference by potentiometer the following circuit is used. The unknown potential difference is the potential difference across the resistance R connected in the secondary circuit.




**(e)** Let the unknown potential difference be V and the balancing length for it be then

$$V = x\ell = \frac{E_0}{\ell_0}\ell$$

**(f)** Let the length of potentiometer wire be L and the balancing length for the unknown potential difference V be $\ell$. If the length of the potentiometer wire is increased to L' then the new balancing length for the same potential difference will be

$$\ell' = \frac{L'}{L}\ell$$

If L' > L then $\ell' > \ell$

If L' < L then $\ell' < \ell$

**(g)** The change in balancing length $\Delta\ell = (\ell' - \ell)$

**(h)** **Measurement of current :** If the unknown potential difference V is produced across a resistance R due to flow of current I through it then

$$V = IR = x\ell = \frac{E_0}{\ell_0}\ell \quad \text{or} \quad I = \frac{x\ell}{R}$$

In order to determine I, resistance R is taken as standard resistance whose magnitude should be known accurately. For simplicity one ohm standard resistance is used in measuring current.

## Solved Examples

**Example 40.** The length of a wire in a potentiometer is 100 cm, and the emf of its standard cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at $\ell$ = 30 cm from the positive end, the emf of the battery is

(1) $\frac{30E}{100.5}$

(2) $\frac{30E}{100 - 0.5}$

(3) $\frac{30(E - 0.5i)}{100}$, where i is the current in the potentiometer wire

(4) $\frac{30E}{100}$

**Solution :** $V \propto l \Rightarrow \frac{V}{E} = \frac{l}{L}$

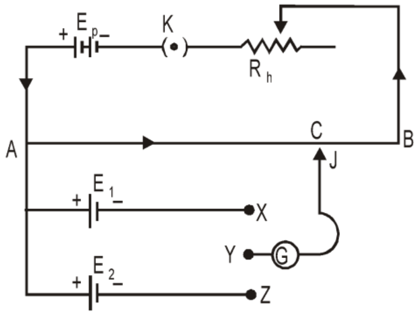
Where, L = balance point

L = length of potentiometer wire

or $V = \frac{l}{L}E$ or $V = \frac{30 \times E}{100} = \frac{30}{100}E$

## 16.8 COMPARISON OF EMF'S OF TWO CELLS WITH THE HELP OF POTENTIOMETER

**(a)** For comparing emf's of two cells with the help of potentiometer the following circuit is used:

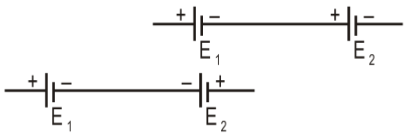



**(b)** If the balancing length for the emf $E_1$ is $\ell_1$ and for the emf $E_2$ is $\ell_2$ then

$$\frac{E_1}{E_2} = \frac{\ell_1}{\ell_2} \quad \text{or} \quad E_1 : E_2 :: \ell_1 : \ell_2$$

**(c)** The ratio of emf's of two cells is equal to the ratio of balancing lengths for them

**(d)** The balancing length for combination of two cells shown in the following figure is $\ell_1$ and on reversing the terminals of one cell ($E_1 > E_2$) the balancing length becomes $\ell_1$ then



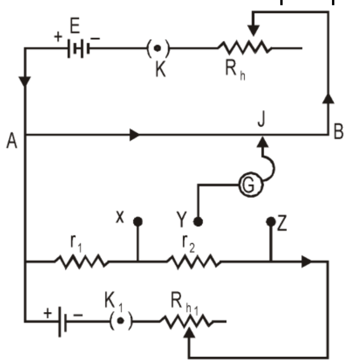

$$E_1 + E_2 = x\ell_1$$

$$E_1 - E_2 = x\ell_2$$

$$\therefore \quad \frac{\ell_1}{\ell_2} = \frac{E_1 + E_2}{E_1 - E_2} \qquad \therefore \quad \frac{E_1}{E_2} = \frac{\ell_1 + \ell_2}{\ell_1 - \ell_2}$$

## 16.9 COMPARISON OF TWO SMALL RESISTANCES

**(a)** For comparison of two small resistances with the help of potentiometer the following circuit is used:




**(b)** If the potential difference across the ends of resistance $r_1$ is $V_1$ and balancing length for it is $\ell_1$ then

$$V_1 = ir_1 = x\ell_1 \qquad \text{............. (1)}$$

**(c)** The potential difference across the resistances $r_1$ and $r_2$ connected in series is $V_2$ and balancing length for them is $\ell_2$ then

$$V_2 = i(r_1 + r_2) = x\ell_2 \qquad \text{............. (2)}$$

From equation (1) and (2)

$$\frac{i(r_1 + r_2)}{ir_1} = \frac{x\ell_2}{x\ell_1} \quad \text{or} \quad \frac{r_1 + r_2}{r_1} = \frac{\ell_2}{\ell_1}$$

$$\frac{r_2}{r_1} = \frac{\ell_2 - \ell_1}{\ell_1}$$

**(d)** If $r_1$ is known then $r_2$ will be

$$r_2 = \left(\frac{\ell_2 - \ell_1}{\ell_1}\right) r_1$$

**(e)** An unknown small resistance can also be determined with the help of potentiometer. The small resistance r is connected in series with a known resistance R and the balancing lengths for the potential differences across R and (R + r) are found out. If the balancing lengths are respectively $\ell_1$ and $\ell_2$ then

$iR = x\ell_1$ and $i(r + R) = x\ell_2$ hence $r = \frac{\ell_2 - \ell_1}{\ell_1} R$

**16.10 DETERMINATION OF INTERNAL RESISTANCE OF A CELL BY POTENTIOMETER**

**(a)** Internal resistance of a cell : When a cell is connected to an external resistance, the ions flow through the solution from cathode to anode inside the cell and hindrance is produced in their flow due to their collisions with molecules. This hindrance is called internal resistance of a cell. It is generally represented by r. Its unit is ohm.

**(b)** The internal resistance of a cell depends upon the following factors:

(i) The nature of solution (concentration and temperature) in the cell.

(ii) The distance between the electrodes.

(iii) The area of immersed part of electrodes in the solution.

(iv) The current drawn from the cell.

(v) Defects of cell such as polarisation etc.,

(vi) Temperature

**(c)** Internal resistance of a primary cell is higher and that of secondary cell is lower.

**(d)** Small current is obtained from the primary cell while high current is obtained from the secondary cell of same emf.

**(e)** EMF of a cell is always greater than its terminal voltage.

**(f)** The emf of a cell does not depend upon the current drawn from the cell while the terminal voltage depends upon the current drawn from the cell.

**(g)** **Formula for the internal resistance of a cell :** A cell can be represented by a source of emf along with an internal resistance r connected in series. When no current is drawn from the cell, its internal resistance remains ineffective and there is no potential drop. In this case the potential difference between the terminals of the cell is equivalent to its emf. When current, let I flows through the external circuit from the cell, there is potential drop $I_r$ across the internal resistance r. Thus, the potential difference bewteen the terminals of the cell will now be (E – Ir) = V. The emf is equivalent to the work done in flowing a unit positive charge through the complete circuit while the potential difference V is equivalent to the work done in flowing a unit positive charge through the external resistance R. If current I flows through the external resistance R from the cell, then

$E = IR + Ir$

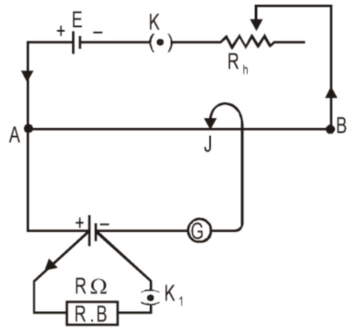
but $IR = V$

$E = V + Ir$ or $Ir = E - V$ or $r = \frac{E - V}{I}$

but $I = \frac{V}{R}$

$\therefore$ $r = \left(\frac{E - V}{V}\right) R$

**(h)** For determining the internal resistance of a cell by potentiometer the following circuit is used:




**(i)** When key $K_1$ is open, the balancing length for the emf E of the cell is found to be $\ell_1$ then $E = x\ell_1$. When key $K_1$ is closed, the balancing length for the potential difference V across the external resistance R is $\ell_2$ then

$V = x\ell_2$

From the formula of internal resistance

$$r = \left(\frac{E-V}{V}\right)R = \left(\frac{x\ell_1 - x\ell_2}{x\ell_2}\right)R \quad \text{or} \quad r = \left(\frac{\ell_1-\ell_2}{\ell_2}\right)R$$

## Solved Examples

**Example 41.** A cell can be balanced against 110 cm and 100 cm of potentiometer wire, respectively with and without being short circuited through a resistance of 10 Ω. Its internal resistance is

(1) 1.0 Ω (2) 0.5 Ω (3) 2.0 Ω (4) zero

**Solution :** $\frac{E}{V} = \frac{l_1}{l_2}$

where $l_1$ and $l_2$ are lengths of potentiometer wire with and without short circuited through a resistance.

Since, $\frac{E}{V} = \frac{R+r}{R}$ [∴ E = I (R + r) and V = IR]

∴ $\frac{R+r}{R} = \frac{l_1}{l_2}$ or $1 + \frac{r}{R} = \frac{110}{100}$ or $\frac{r}{R} = \frac{10}{100}$ or $r = \frac{1}{10} \times 10 = 1\Omega$

**Example 42.** A cell has an emf 1.5V. When connected across an external resistance of 2Ω, the terminal potential difference falls to 1.0V. The internal resistance of the cell is :

(1) 2Ω (2) 1.52Ω (3) 1.0Ω (4) 0.5Ω

**Answer.** **(3)**

**Solution** Internal resistance of the cell is given by $r = \left(\frac{E-V}{V}\right)R$

Given E = 1.5V, V = 1.0V, R = 2Ω

∴ $r = \left(\frac{1.5-1.0}{1.0}\right) \times 2 = \frac{0.5}{1.0} \times = 1.0\Omega$

## 16.13 Advantages and Disadvantages of Potentiometer

**Advantages :**

(i) The measurements by potentiometer are more accurate.

(ii) It works on the principle of no deflection method. Thus there is no possibility of error in measurements.

(iii) It does not drawn any current from that circuit which is used for measurement.
(iv) Its standardisation can be performed directly with the help of standard cell.
(v) It is very sensitive so the measurement of very low values is possible by it.

**Disadvantages:**

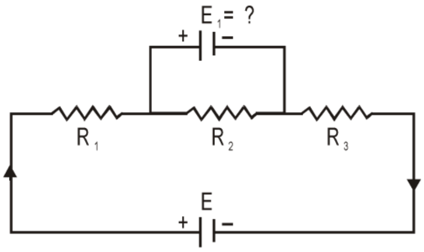
(i) It is cumbersome.
(ii) The area of cross–section of potentiometer wire should be uniform but it is not possible always.
(iii) The temperature of potentiometer wire should remain constant during the experiment but it does not remain constant due to flow of current through it.

**Some Other Important Facts Related to Potentiometer**

**(a)** Let three resistances $R_1$, $R_2$ and $R_3$ be connected in series and current be passed through them by connected a constant voltage battery. On connecting another battery across the ends of resistance $R_2$, if the current flowing in the circuit remains unchanged, then the voltage of battery can determined as,

The current flowing in the circuit $I = \dfrac{E}{R_1 + R_2 + R_3}$

The potential difference across the ends of the resistance $R_2$




$$V_2 = IR_2 = \frac{ER_2}{R_1 + R_2 + R_3}$$

If voltage of unknown battery is $E_1$, then on connecting it to the ends of resistance $R_2$, current will remain unchanged only when

$$V_2 = E_1 = \frac{ER_2}{R_1 + R_2 + R_3}$$

**(b)** In case of zero deflection in the galvanometer current flows in the primary circuit of the potentiometer, not in the galvanometer circuit.

**(c)** Internal resistance of a cell is a defect of the cell. As a result some part of the electrical energy is dissipated in the form of heat.

**(d)** In the calibration of an ammeter if a coil of R ohm resistance is used instead of one ohm coil and the balancing length for the potential difference V' across this is $\ell'$, then

$$V' = I'R = x\ell'$$

The current flowing through the coil will be $I' = \left(\dfrac{E_0}{R_0}\right)\left(\dfrac{\ell'}{R}\right)$

**(e)** The internal resistance of a primary cell increase due to polarisation as the current increases.

**(f)** Copper wire is not used as potentiometer wire because temperature coefficient of resistance of copper is more and its specific resistance is less.

## 17. METRE BRIDGE (USE TO MEASURE UNKNOWN RESISTANCE)

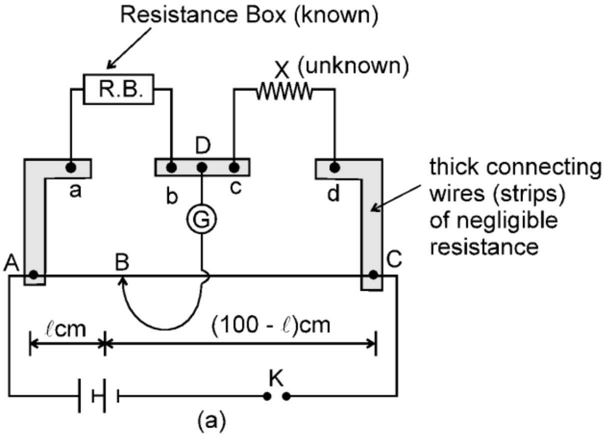
If AB = $\ell$ cm, then BC = $(100 - \ell)$ cm.

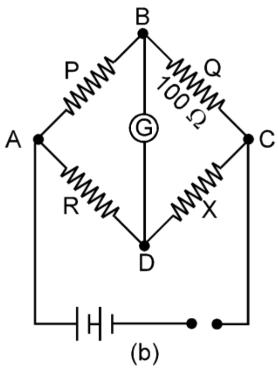
Resistance of the wire between A and B $R \propto \ell$

[∵ Specific resistance ρ and cross-sectional area A are the same for the whole of the wire]

or $R = \sigma\ell$ .....(1)

where σ is resistance per cm of wire.




Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - \ell$$

$\therefore \quad Q = \sigma(100 - \ell)$ ....(2)

Dividing (1) by (2), $\frac{P}{Q} = \frac{\ell}{100-\ell}$

Applying the condition for balanced Wheatstone bridge, we get

$$R\ Q = P\ X \ \therefore \ x = R\frac{Q}{P} \quad \text{or} \quad X = \frac{100-\ell}{\ell} R$$

Since R and $\ell$ are known, therefore, the value of X can be calculated.

**Note :** For batter accuracy, R is so adjusted that $\ell$ lies between 40 cm and 60 cm.

## Solved Examples

**Example 43.** In a meter bridge experiment, the value of unknown resistance is 2Ω. To get the balancing point at 40cm distance from the same end, the resistance in the resistance box will be :

(1) 0.5 Ω (2) 3 Ω (3) 20 Ω (4) 80 Ω

**Solution :** Apply condition for balance wheat stone bridge,

$$\frac{P}{Q} = \frac{\ell}{100-\ell} = \frac{P}{2} = \frac{100-40}{40}$$

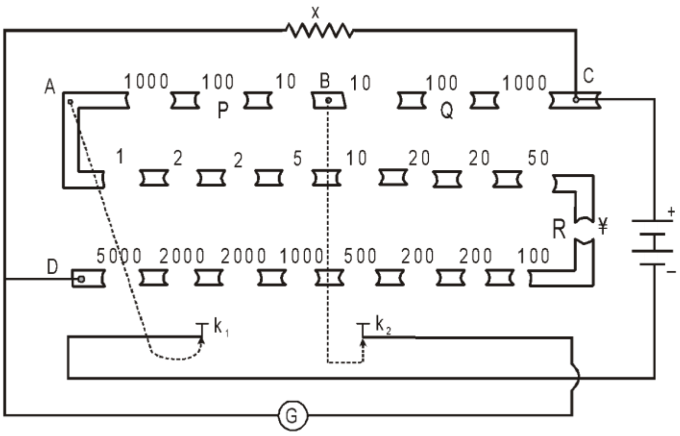
**Ans.** P = 3Ω.

## 18. POST-OFFICE BOX

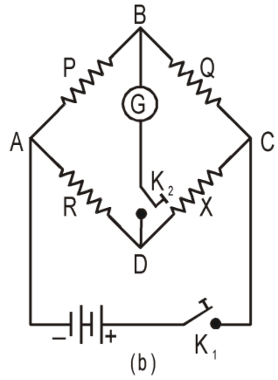
**Introduction :** It is so named because its shape is like a box and it was originally designed to determine the resistances of electric cables and telegraph wires. It was used in post offices to determine the resistance of transmission lines.

**Construction :** A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly. Two types of post office box are available - plug type and dial type. In the plug-type instrument shown in figure (a), each of the arms AB and BC contains three resistances of 10, 100 and 1000 ohm. These arms are called the ratio arms. While the resistance P can be introduced in the arm AB, the resistance Q can be introduced in the arm BC. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1 Ω to 5,000 Ω. In this arm, the resistance R is introduced by taking out plugs of suitable values. The unknown resistance X constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are infect the four arms of the Wheatstone bridge (figure (b)). Two tap keys $K_1$ and $K_2$ are also provided. While $K_1$ is connected internally to the terminal A, $K_2$ is connected internally to B. These internal connections are shown by dotted lines in figure (a).

A battery is connected between C and key $K_1$ (battery key). A galvanometer is connected between D and key $K_2$ (galvanometer key). Thus, the circuit is exactly the same as that shown in figure (b). It is always the battery key which is pressed first and then the galvanometer key. This is because a self-induced current is always set up in the circuit whenever the battery key is pressed or released. If we first press the galvanometer key, the balance point will be disturbed on account of induced current. If the battery key is pressed first, then the induced current becomes zero by the time the galvanometer key is pressed. So, the balance point is not affected. $\frac{100-\ell}{\ell}$




(a)




(b)

**Working :** The working of the post office box involves broadly the following four steps :

**I.** Keeping R zero, each of the resistances P and Q are made equal to 10 ohm by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.

**II.** Keeping both P and Q equal to 10Ω, the value of R is adjusted, beginning from 1Ω, till 1 Ω increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R.

$$\left[X = R\frac{Q}{P} = R\frac{10}{10} = R\right]$$

As an illustration, suppose with 3Ω resistance in the arm AD, the deflection is towards left and with 4Ω, it is towards right. The unknown resistance lies between 3Ω and 4Ω.

**III.** Making P 100 Ω and keeping Q 10 Ω, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[X = R\frac{Q}{P} = R\frac{10}{100} = \frac{R}{10}\right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between 30 Ω, and 40 Ω. So, in this step, we have to start adjusting R from 30 Ω onwards. If 32 Ω and 33 Ω are the two values of R which give opposite deflections, then the unknown resistance lies between 3.2 Ω and 3.3 Ω.

**IV.** Now, P is made 1000 Ω and Q is kept at 10 Ω. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[X = R\frac{10}{1000} = \frac{R}{100}\right]$$

In the illustration under consideration, the resistance in the arm AD will lie between 320 Ω and 330Ω. Suppose the deflection is to the right for 326 ohm, towards left for 324 ohm and zero deflection for 325Ω then, the unknown resistance is 3.25 Ω.

The post office box method is a less accurate method for the determination of unknown resistance as compared to a metre bridge. This is due to the fact that it is not always possible to arrange resistance in the four arms to be of the same order. When the arms ratio is large, large resistance are required to be introduced in the arm R.

## Solved Examples

**Example 44.** The post office box works on the principle of :
(A) Potentiometer (B) Wheatstone bridge (C) Matter waves (D) Ampere's law

**Answer :** (B)

**Example 45.** While using a post office box the keys should be switched on in the following order :
(A) first cell key the and then galvanometer key.
(B) first the galvanometer key and then cell key.
(C) both the keys simultaneously.
(D) any key first and then the other key.

**Answer :** (A)

**Example 46.** In a post office box if the position of the cell and the galvanometer are interchanged, then the :
(A) null point will not change (B) null point will change
(C) post office box will not work (D) Nothing can be said.

**Answer :** (A)

## 19. THERMAL AND CHEMICAL EFFECTS OF CURRENT

### 19.1 Chemical effect of current :

**(i)** Electrolyte consits of two equally but oppositely charged groups of atoms. When a potential difference is applied across a pair of electrodes introduced in the electrolyte, the two groups of ions get attracted towards their respective electrodes. Positive ions drift towards the cathode and are called cations whereas the negative ions move towards anode and are called anions.

**(ii) Faraday's first law :** For a given electrolyte, the mass of the substance deposited at or liberated from an electrode is directly proportional to the product of current I and time t for which it flows. Mathematically,

$m \propto It$ or $m = ZIt$ ......(1)

where Z is a constant, called the electrochemical equivalent of the element. Z is different for different elements. The unit of Z in MKS is kg/coulomb and in CGS system is g/ab-coulomb.

One (kg/coulomb) = $10^4$ (g/ab-coulomb)

For hydrogen, Z is given by : $Z_H = \frac{1}{96500000}$ kg/coulomb

**(iii) Faraday's second law :**

(a) The masses of different substances liberated or deposited by the same quantity of charge are proportional to their chemical equivalents. Mathematically,

$$\frac{m_1}{m_2} = \frac{W_1}{W_2}$$ ......(2)

where $W_1$ and $W_2$ are the chemical equivalents (= atomic weight/valency) of the two elements.

As m = Zq, so $\frac{m_1}{m_2} = \frac{Z_1 q}{Z_1 q} = \frac{Z_1}{Z_2}$

$\therefore$ $$\frac{m_1}{m_2} = \frac{Z_1}{Z_2} = \frac{W_1}{W_2}$$ ......(3)

i.e., electrochemical equivalent is directly proportional to the chemical equivalent of that element.

Hence $\frac{Z}{Z_h} = \frac{W}{W_h} = \frac{W}{1}$ So $Z = Z_h W = \frac{W}{96500000}$ kg/coul

(b) From eq. (3), we get : $\frac{W}{2}$ = constant, i.e., the ratio (W/Z) is same for all the elements and is called the Faraday constant F. The Faraday constant F represents the amount of charge required to deposit or liberate one kilogram equivalent of any element. F = 9.65 × $10^7$ coul.

(c) charge on n valent ion = ne.

Number of ions in one kilogram equivalent = $\frac{N}{n}$ (where N represents Avogadro's number).

$\therefore$ F = number of ions in one kg-equivalent × charge on one ion = $\left(\frac{N}{n}\right)$ × ne = Ne.

## Solved Examples

**Example 47.** When 1 gm hydrogen (ECE = $1.044 \times 10^{-8}$ kg/coulomb) forms water, 34 kcal heat is liberated. The minimum voltage required to decompose water is :
(1) 4.5 V (2) 3 V (3) 1.5 V (4) 0.75 V

**Solution :** Charge required to liberate 1 gm of $H_2$, Q = $\frac{1}{1.044 \times 10^{-8}} \times 10^{-3}$ C = $\frac{10^5}{1.044}$ C
Energy = 34000 × 4.2 J = QV

$34000 \times 4.2 = \frac{10^5}{1.044} \times V$ $\therefore$ $V = \frac{34000 \times 4.2 \times 1.044}{10^5}$ = 1.49 V = 1.5 V

**Example 48.** When a copper voltameter is connected with a battery of e.m.f. 12 volt, 2 gm of copper is deposited in 30 minutes. If the same voltameter is connected across a 6 V battery, then the mass of the copper deposited in 45 minutes would be: **[SCRA 1994]**
(1) 1 gm (2) 1.5 gm (3) 2 gm (4) 2.5 gm

**Solution :** We know that m = Zlt ∝ ZVt

$\therefore \frac{m_1}{m_2} = \frac{V_1 t_1}{V_2 t_2}$ or $m_2 = \frac{V_2 t_2}{V_1 t_1} \times m_1$

$\therefore$ $m_2 = \frac{6 \times 45}{12 \times 30} \times 2 = 1.5$ gm

**Example 49.** ECE of copper is $3.3 \times 10^{-7}$ kg/coulomb. If 100 kWh energy is consumed at 33 volt in a copper voltameter, then mass of copper liberated is
(1) 3.6 kg (2) 3.3 kg (3) 1 kg (4) 1 mg

**Solution :** We know that m = Zlt = $Z\left(\frac{P}{V}\right)t = (3.3 \times 10^{-7})\left(\frac{100 \times 36 \times 10^5}{33}\right)$ = 3.6 kg

**19.2 Thermoelectric effects :**

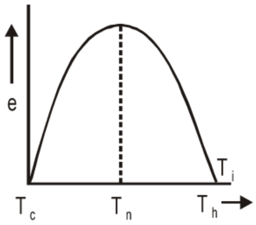
**(i) Seebeck effect :** The production of e.m.f. by maintaining a difference of temperature between two junctions of two different metals is known as Seebeck effect.

**(ii)** Seebeck arranged the metals in a series such that when any two of them are arranged to form a thermocouple, the current flows at the hot junction from the metal occurring earlier in the series to one occurring later. The series of some elements is shown below.
Bi, Ni, Co, Pt, Cu, Mn, Pb, Au, Ag, Zn, Cd, Fe, Sb

**(iii)** The farther the metals are in the series, the greater will be thermo e.m.f. produced in the circuit for a given difference of temperature between the junctions. So the e.m.f. developed in Bi-Sb thermocouple is greater than in Cu-Fe thermocouple.

**(iv)** Seebeck thermoelectric e.m.f. is given by : $e = at + \frac{1}{2}bt^2$
where t is the temperature of the hot junction when the temperature of the cold junction is maintained at 0ºC. a and b are constants which are the characteristics of the pair of metals used.

**(v)** The temperature of hot junction at which the thermo e.m.f. produced in the thermocouple becomes maximum is called neutral temperature ($T_N$). At neutral temperature $T_N$ :




$\left.\frac{de}{dt}\right|_{t=T_N} = 0$ or $\left.(a + 2bt)\right|_{t=T_N} = 0$ or $a + 2bT_N = 0$

$\therefore$ $T_N = -\left(\frac{a}{2b}\right)$

**(vi)** Further, $e = at + bt^2 = t(a + bt)$. Now reversal in the direction of the thermo e.m.f. takes place at inversion temperature $T_i$ At $t = T_i$, $e = 0$

$\therefore \quad a + bT_i = 0$ or $T_i = -\frac{a}{b}$

**(vii)** The neutral temperature $T_N$ is constant for a given thermocouple. The inversion temperature changes with the temperature of cold junction.

$T_i - T_N = T_N - T_C$

where $T_C$ is the temperature of cold junction.

**(viii)** The rate of change of thermo e.m.f. with temperatrue is called thermoelectric power, P, i.e., $P = (de/dt)$. At neutral temperature, thermoelectric power is zero.

**(ix)** Peltier effect :

(a) This effect is the reverse of Seebeck effect.

(b) When a current is passed through a thermocouple, heat is absorbed at one junction and evolved at the other junction. This is called Peltier effect.

(c) This effect is reversible, i.e., if the direction of current is reversed, the heat evolved and absorbed are interchanged.

(b) The amount of heat energy absorbed or evolved per second at a junction due to Peltier effect, when a unit current is passed through it, is known as Peltier coefficient ($\pi$).

$\pi = T\left(\frac{de}{dt}\right)$

**(x) Thomson effect :**

(a) The absorption or evolution of heat along the length of a conductor (part of same metal) when current is passed through it, whose one end is hot and the other is cold, is known as Thomson effect.

(b) Thomson effect for lead is zero and it is positive for metals below lead and negative for metals above lead in Seebeck series.

(c) The amount of heat energy absorbed or evolved per second between between two points of a con ductor having a unit temperature difference when a unit current is passed, is known as Thomson coefficient for the material of the conductor. This is represented by $\sigma$.

$\sigma = T\,(d^2 e/dt^2)$

## Solved Examples

**Example 50.** The thermo e.m.f. of a thermocouple is 25 $\mu V/^oC$ at room temperature. A galvanometer of 40 $\Omega$ capable of detecting current as low as $10^{-5}$ A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is :

(1) 20ºC (2) 16ºC (3) 12ºC (4) 8ºC

**Solution :** Given thermo e.m.f. of the thermocouple (V) = 25 $\mu V/^oC$ resistance of galvanometer (G) = 40 $\Omega$ and detecting current of galvanometer ($I_g$) = $10^{-5}$ A. We know that voltage of galvanometer ($V_g$) = $GI_g = 40 \times 10^{-5} = 4 \times 10^{-4}$ V. We also know that the smallest temperature difference that can be detected by this system,

$(T) = (V_g/V) = \frac{4\times10^{-4}}{25\times10^{-6}} = 16^oC$