ELECTROMAGNETIC INDUCTION

1. MAGNETIC FLUX

- The concept of magnetic lines of force was first proposed by Faraday. Faraday tried to provide the lines of force a real form assuming them as stretched rubber bands. In modern physics the concept of magnetic lines of force is used in visualization or explaination of principles only.
- The tangent drawn at any point on a line of force in a magnetic field shows the direction of magnetic field shows the direction of magnetic field at that point and the density of lines of force, i.e., the number of lines of force crossing normally a unit area indicates the intensity of magnetic field.
- The lines of force in a uniform magnetic field are parallel straight lines equidistant from each other. Where the lines of force are near each other, B is higher and where the lines of force are far apart, B is lesser.
- The number of lines of force crossing a given surface is called flux from that surface. If it generally represented by φ. Flux is a property of a vector field. If the vector field is a magnetic field, then the flux is called magnetic flux.
- The magnetic flux crossing a certain area is equal to the scalar product of the vector field $(\vec{B})$ and the vector area $(d\vec{A})$, that is

  Magnetic flux $d\phi = \vec{B}.d\vec{A}$ = BdA cosθ

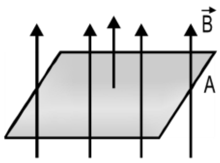
  where θ is the angle between the vector field $(\vec{B})$ and the vector area $d\vec{A}$.

  $\phi = \int \vec{B}.d\vec{A}$

  For a uniform magnetic field $\vec{B}$ and plane surface $\vec{A}$ $\quad \phi = \vec{B}.\vec{A} = BA\cos\theta$

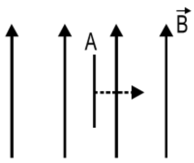
  (Note : In real sense area is a scalar quantity, but it can be treated as whose direction is in the direction of perpendicular pointing outward from the surface)
- Magnetic flux is a scalar quantity.
- If a plane surface of area A is imagined in a uniform magnetic field $\vec{B}$, then

  (a) when a surface is perpendicular to the magnetic filed, the lines of force crossing that area, i.e., the magnetic flux is

  φ = BA because θ = 0, cos θ = 1

  (b) if the surface is parallel to the field, then

  θ = 90°, cosθ = 0

  $\therefore \quad \varphi$ = BA cos 90° = 0

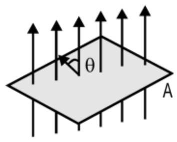
  (c) when the normal to the surface makes an angle θ with the magnetic field, the magnetic flux is φ = BA cos θ

- If the magnetic flied is not uniform and the surface in not plane, then the element $\vec{dA}$ of the surface may be assumed as plane and magnetic field $\vec{B}$ may also be assumed as uniform over his element.

  Thus the magnetic flux coming out from this element is $d\phi = \vec{B}.\vec{dA}$

  Hence magnetic flux coming out from the entire surface

  $$\phi = \int_S \vec{B}.\vec{dA}$$



- For a closed surface the vector area element pointing outward is positive and the vector area element pointing inward is negative.
- Magnetic lines of force are closed curves because free magnetic poles do not exist. Thus for a closed surface whatever is the number of the lines of force entering it, the same number of lines of force come out from it. As a result for a closed curve

  $$\phi = \oint_S \vec{B}.\vec{dA} = 0 \quad \text{or} \quad \nabla.\vec{B} = 0$$

  Thus the net magnetic flux coming out of a closed surface is equal to zero.

- For a normal plane surface in a magnetic field φ = BA. Hence B = $\frac{\phi}{A}$

  Thus the magnetic flux passing normally from a surface of unit area is equal to magnetic induction B.

  Therefore $\frac{\phi}{A}$ is also called flux density.

- **Unit of magnetic flux** - In M.K.S. system the unit of magnetic flux is weber (Wb) and in C.G.S. system unit of magnetic flux is maxwell.

  1 weber = 10^8 maxwell

  The M.K.S unit of flux density or magnetic induction is weber/m^2. It is also called tesla.

  1 tesla = 1 weber/m^2

  The C.G.S unit of magnetic flux density is gauss.

  1 gauss = 1 maxwell/cm^2

  1 tesla = 1 weber/m^2 = 10^4 gauss

- **Dimensions of magnetic flux** :

  φ = BA

  $$[\phi] = \frac{N}{A-m} \times m^2 = \frac{N-m}{A}$$

  $$\frac{(kg-m-s^2)\times m}{A} = kg\text{-}m^2\text{-}s^2\text{-}A^{-1} = M^1L^2T^{-2}A^{-1}$$

Solved Examples

Example 1. The plane of a coil of area 1m^2 and having 50 turns is perpendicular to a magnetic field of 3×10^{-5} weber/m^2. The magnetic flux linked with it will be -

(1) 1.5×10^{-3} weber (2) 3×10^{-5} weber (3) 15×10^{-5} weber (4) 150 weber

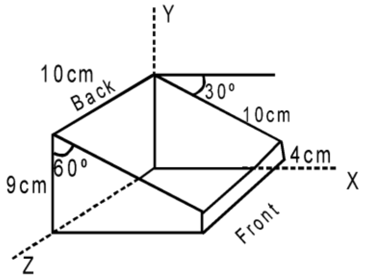
Solution : φ = NBA cosθ

but N = 50, B = 3×10^{-5} wb/m^2,

A = 1m^2, θ = 0 or φ = NBA

= $50 \times 3 \times 10^{-5} \times 1 = 150 \times 10^{-5}$ weber ∴ Answer will be (1)

Example 2. Consider the fig. A uniform magnetic field of 0.2 T is directed along the +x axis. Then what is the magnetic flux through top surface of the figure ?



(1) zero (2) 0.8m Wb (3) 1.0m Wb (4) –1.8m Wb

Solution : the magnetic flux is $\varphi = BA\cos\theta$

for the top surface, the angle between normal to the surface and the x-axis is

$\theta = 60^o$, and

$B = 0.2$ T, $A = 10 \times 10 \times 10^{-4}$ m^2

Thus $\varphi = 0.2 \times 10^{-2} \times \cos(60)$

$= 10^{-3}$ Wb.

The correct answer is thus (3)

1.1 FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

(i) When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf. If the circuit is closed then the current will be called induced current.

magnetic flux = $\int B.d\vec{s}$

(ii) The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire

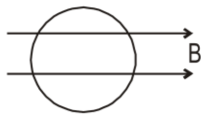
$E = -\dfrac{d\phi}{dt}$

(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.

SI unit of magnetic flux = Weber.

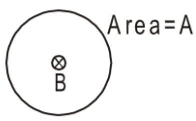
Solved Example

Example 3. A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



Solution : $\varphi = 0$ (always) since area is perpendicular to magnetic field. $\therefore$ emf = 0

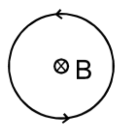
Example 4. Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



Solution : $\varphi = BA$ (always)

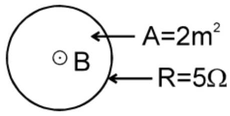
= const. $\therefore$ emf = 0

Example 5. Find the direction of induced current in the coil shown in figure.Magnetic field is perpendicular to the plane of coil and it is increasing with time.



Solution : Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

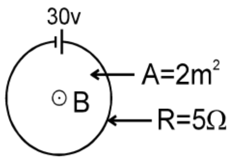
Example 6. Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil .The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction



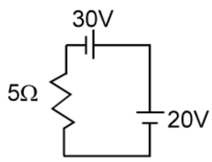
Solution : $\varphi = B.A \quad \Rightarrow \quad \text{emf} = A \cdot \frac{dB}{dt} = 2 \times 10 = 20$ v

$\therefore$ i = 20/ 5 = 4 amp. From Lenz's law direction of current will be anticlockwise.

Example 7. Figure shows a coil placed in a magnetic field decreasing at a rate of 10T/s. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.

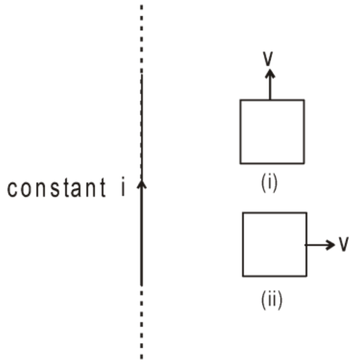


Solution :



Induce emf = 20V
equivalent i = 2A clockwise

Example 8. Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.

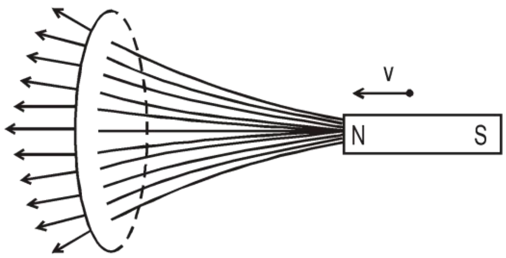


Solution : In loop (i) no emf will be induced because there is no flux change.
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magneticfield inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

2. LENZ'S LAW (CONSERVATION OF ENERGY PRINCIPLE)

According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.

Figure shows a magnet approaching a ring with its north pole towards the ring.



We know that magnetic field lines come out of the north pole and magnetic field intensity decreases as we move away from magnet. So the magnetic flux (here towards left) will increase with the approach of magnet. This is the cause of flux change. To oppose it, induced magnetic field will be towards right. For this the current must be anticlockwise as seen by the magnet.

If we consider the approach of North pole to be the cause of flux change, the Lenz's law suggests that the side of the coil towards the magnet will behave as North pole and will repel the magnet. We know that a current carrying coil will behave like North pole if it flows anticlockwise. Thus as seen by the magnet, the current will be anticlockwise.

If we consider the approach of magnet as the cause of the flux change, Lenz's law suggest that a force opposite to the motion of magnet will act on the magnet, whatever be the mechanism.

Lenz's law tells that if the coil is set free, it will move away from magnet, because in doing so it will oppose the 'approach' of magnet.

If the magnet is given some initial velocity towards the coil and is released, it will slow down .It can be explained as the following.

The current induced in the coil will produce heat. From the energy conservation, if heat is produced there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus the magnet must slow down. So we can justify that the **Lenz's law is conservation of energy principle**.

2.1 INDUCED EMF, CURRENT AND CHANGE IN A CIRCUIT

- If e.m.f induced in a circuit is E and rate of change of magnetic flux is $d\varphi/dt$, then from Faraday's and Lenz's law

  $E \propto -\left(\frac{d\phi}{dt}\right)$ or $E = -K\left(\frac{d\phi}{dt}\right)$ where K is constant, equal to one.

  Thus $E = -\left(\frac{d\phi}{dt}\right)$

- If there are N turns in the coil, then induced e.m.f will be $E = -N\left(\frac{d\phi}{dt}\right)$
- If the magnetic flux linked with the circuit changes from φ_1 to φ_2, in time t, then induced e.m.f will be

  $E = -N\left(\frac{d\phi}{dt}\right)$

  $= -N\left(\frac{\phi_2 - \phi_1}{t}\right)$

- If the resistance of the circuit is R, then the current induced in the circuit will be

  $I = \frac{E}{R} = -\frac{N(\phi_2 - \phi_1)}{tR}$ ampere $= -\frac{N}{R}\left(\frac{d\phi}{dt}\right)$ ampere

- Induced current depends upon $\frac{1}{R}$

  (a) the resistance of the circuit $I \propto$ $I \propto \left(\frac{d\phi}{dt}\right)$

  (b) the rate of change of magnetic flux

  (c) the number of turns (N) ; $I \propto N$

- If $R = \infty$, that is, the circuit is open, then the current will not flow and if the circuit if closed, then current will flow in the circuit.
- If change dq flows in the circuit in time dt, then the induced current will be

  $I = \left(\frac{dq}{dt}\right)$ or $dq = I\,dt$

but $I = \frac{1}{R}\left(\frac{d\phi}{dt}\right)$

$\therefore \quad dq = \frac{1}{R}\left(\frac{d\phi}{dt}\right)dt = \frac{1}{R}d\phi$ or $q = \int \frac{d\phi}{R} = \frac{\phi_2 - \phi_1}{R}$

If N is the number of turns, then $dq = \frac{Nd\phi}{R}$, $q = \frac{N(\phi_2 - \phi_1)}{R}$

- Charge flowing due to induction does not depend upon the time but depends upon the total change in the magnetic flux. It does not depend upon the rate or time interval of the change in magnetic flux. Whether the change in magnetic flux be rapid or slow, the charge induced in the circuit will remain same.

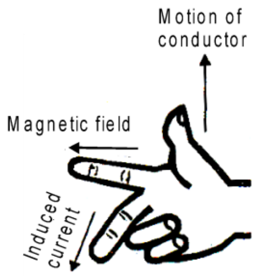
  Thus $q \propto d\varphi$ or $q \propto (\varphi_2 - \varphi_1)$
- Induced charge depends upon the resistance of the circuit, i.e., $q \propto 1/R$

  If $R = \infty$ or circuit is open, $q = 0$ that is charge will not flow in the circuit.

  If $R \neq \infty$ or circuit is closed, then $q \neq 0$, that is, induced charge will flow in the circuit
- The e.m.f induced in the circuit does not depend upon the resistance of the circuit.
- The e.m.f induced in the circuit depends upon the following factors -
  (a) Number of turns (N) in the coil,
  (b) Rate of change of magnetic flux,
  (c) Relative motion between the magnet and the coil,
  (d) Cross-sectional area of the coil,
  (e) Magnetic permeability of the magnetic substance or material placed inside the coil.

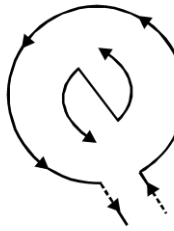
2.2 FLEMING'S RIGHT HAND RULE

This law is used for finding the direction of the induced e.m.f or current. According to this law, if we stretch the right hand thumb and two nearby fingers perpendicular to one another and first finger points in the direction of magnetic field and the thumb in the direction of motion of the conductor then the central finger will point in the direction of the induced current.

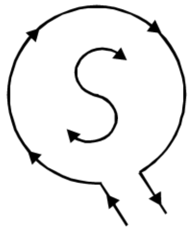


2.3 DIRECTION OF INDUCED EMF AND CURRENT (Applications of lenz's law)

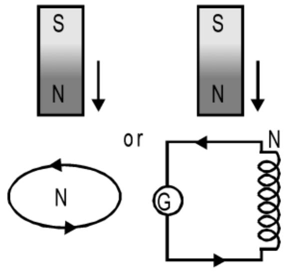
- If current flowing in a coil appears anti-clockwise, then that plane of coil will behave like a N-pole.



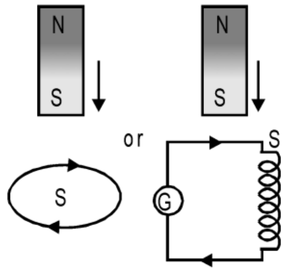
- If current flowing in the coil appears clock-wise, then that plane of coil will behave like a S-pole.



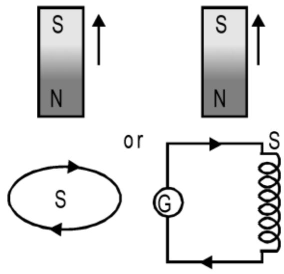
➢ If the north pole of magnet is moved rapidly towards the coil, then according to Lenz's law the induced current will flow in the coil in such a direction so as to oppose the motion of the magnetic. This will happen only when the face of the coil towards the magnet behaves as a north pole, that is, the induced current will appear flowing in anti-clockwise direction as seen from the side of magnet. Thus a force of repulsion will be produced between the magnet and the coil coming near each other which will oppose the motion of the magnet. Hence some mechanical work has to be done to move the magnet near the coil against this opposing force and this work (mechanical energy) is converted into current (electrical energy)



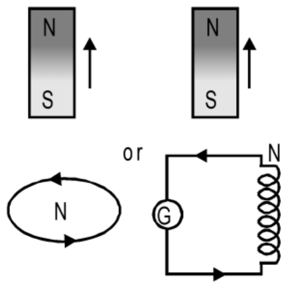
➢ On bringing a south pole towards a coil the current induced in the coil will appear flowing in clockwise direction as observed from the side of magnet and the face of the coil towards the magnet will behave as a south pole.



➢ On moving the north pole of a magnet away from the coil the current induced in the coil will appear flowing in the clockwise direction as seen from the side of magnet and the face of the coil towards the magnet will behave as a south pole.



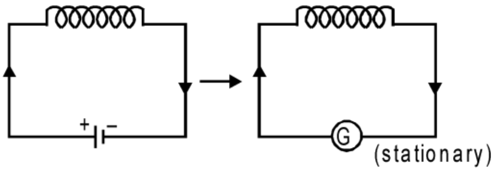
➢ On moving the south pole of a magnet away from the coil the current induced in the coil will appear flowing in the anticlockwise direction as seen from the side of magnet and the face of the coil towards the magnet will behave like a north pole.



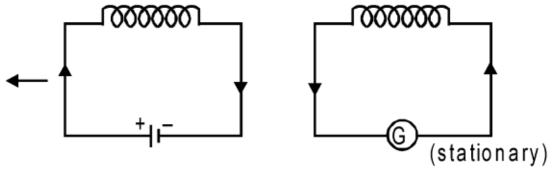
➢ If a magnet is allowed to drop freely through a copper coil, then an induced current will be produced in the coil. This current will oppose the motion of the magnet, as a result the acceleration of the falling magnet due to gravity will be less than 'g'. If coil is cut somewhere, then the emf will be induced in the coil only but current will not be induced. In absence of induced current the coil will not oppose the motion of magnet and the magnet will fall through the coil with the acceleration equal to g.

➢ If a magnet is dropped freely in a hollow long metal cylinder, then the acceleration of falling magnet will be less than gravitational acceleration. As the magnet keeps on falling inside a tube, its acceleration will continue to decrease and after traversing a certain distance the acceleration will become zero. Now the magnet will fall with constant velocity. This constant velocity is called terminal velocity.

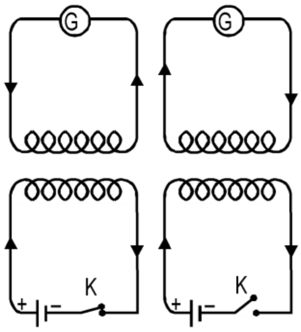
- If a current carrying coil is brought near another stationary coil, then the direction of induced current in the second coil will be in the direction of current in the moving coil.



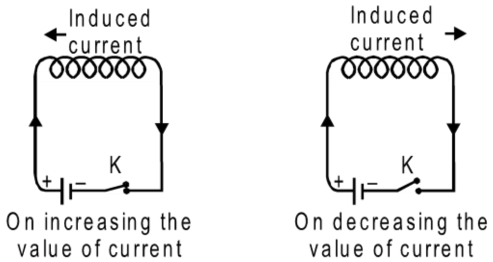
- If a current carrying coil is taken away from a stationary coil, then the direction of induced current in the second coil will be opposite to the direction of current in the moving coil.



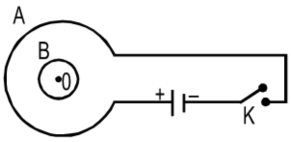
- In the coils arranged in the following way, when the key K connected in the circuit of primary coil, is pressed, an induced current is produced in the secondary coil. The direction of induced current in the secondary coil is opposite to the direction of the current in the primary coil. (From lenz's law)
  When the key is opened, then the current in the primary coil is reduced to zero but current is induced in the secondary coil. The direction of this induced current is same as the direction of current in the primary coil. (Form Lenz's law).



- When current is passed through a coil, the current flowing through the coil changes. As a result the magnetic flux linked with the coil changes. Due to this a current is induced in the coil. If the current induced in the coil flows in the opposite direction of the applied current. If the current flowing in the coil is decreased, then the current induced in the coil flows in the direction of the applied current so as to oppose the decrement of the applied current.

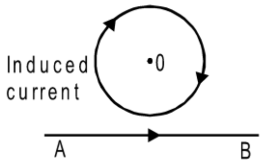


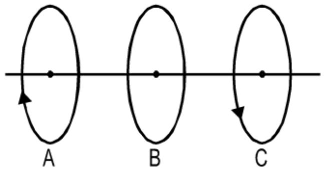
- Two coil A and B are arranged as shown in the figure. On pressing the key K current flows through the coil A in the clockwise direction and the current induced in the coil B will flow in the anticlockwise direction . (From Lenz's law)



  On opening the key K the current flowing through the coil A will go on decreasing. Thus the current induced in the coil B will flow in the clockwise direction.

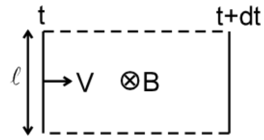
- If current flows in a straight conductor from A to B as shown in figure, then the direction of current induced in the loop placed near it will be clockwise. (From Lenz's law).





- Three indentical circular coils A, B and C are arranged coaxially as shown in figure. The coils A and C carry equal currents as shown. Coils B and C are fixed in position. If coil A is moved towards B, then the current induced in coil B will be in clockwise wise direction because the direction of current induced in the coil B will be to oppose the motion of coil A. (The face of A towards B is south pole, then the face of B towards A is south pole). There is no relative motion between B and C so current will not be induced in coil B due to coil C.

3. MOTIONAL EMF

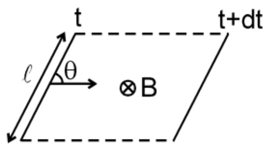


We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length ℓ moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to ℓv and hence it will cut B ℓ v lines per unit time.

Hence emf induced between the ends of the rod = $Bv\ell$

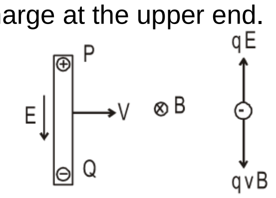
Also emf = $\frac{d\phi}{dt}$. Here φ denotes flux passing through the area,swept by the rod.The rod sweeps an area equal to ℓ vdt in time interval dt.Flux through this area = $B\ell$ vdt. Thus $\frac{d\phi}{dt} = \frac{B\ell vdt}{dt} = Bv\ell$

If the rod is moving as shown in the following figure, it will sweep area per unit time = $v\ell\sin\theta$ and hence it will cut B v ℓ $\sin\theta$ lines per unit time. Thus emf = $Bv\ell\sin\theta$.



3.1 EXPLANATION OF EMF INDUCED IN ROD ON THE BASIS OF MAGNETIC FORCE:

If a rod is moving with velocity v in a magnetic field B, as shown, the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end.



These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other.

Thus E = v B.

$V_P - V_Q = VB\ell$

The moving rod is equivalent to the following diagram, electrically.

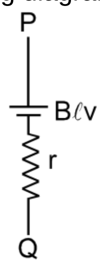
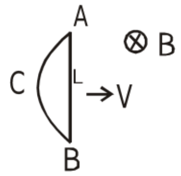
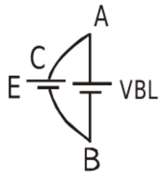


Figure shows a closed coil ABCA moving in a uniform magnetic field **B** with a velocity **v. The** flux passing through the coil is a constant and therefore the induced emf is zero.



Now consider rod AB, which is a part of the coil. Emf induced in the rod =B L v

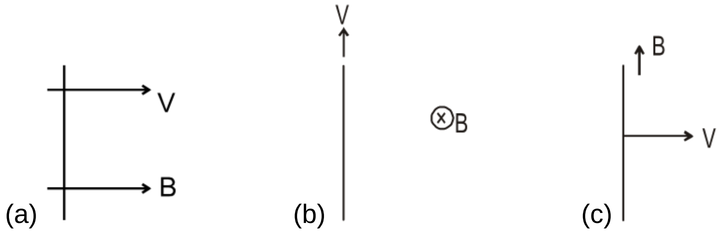


Suppose the emf induced in part ACB is E , as shown.
Since the emf in the coil is zero, Emf (in ACB) + Emf (in BA) = 0
or –E + vBL = 0 or E = vBL

Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is BLv (here the two emf's are in parallel)

Solved Examples

Example 9. Find the emf induced in the rod in the following cases. The figures are self explanatory.



Solution : (a) here $\vec{v} \parallel \vec{B}$ so $\vec{v} \times \vec{B} = 0$

emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

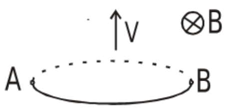
(b) here $\vec{v} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(c) here $\vec{B} \parallel \vec{\ell}$

so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

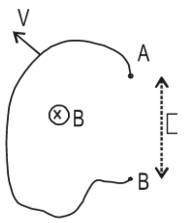
Example 10. A circular coil of radius R is moving in a magnetic field **B** with a velocity **v** as shown in the figure.



Find the emf across the diametrically opposite points A and B.

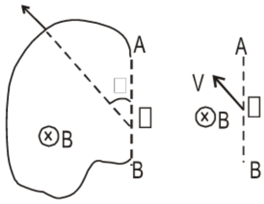
Solution : emf = $BVl_{effective}$ = 2 R v B

Example 11. Figure shows an irregular shaped wire AB moving with velocity v, as shown.

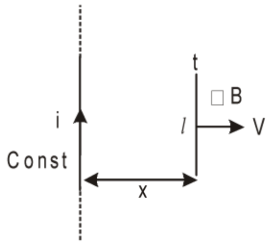


Find the emf induced in the wire.

Solution : The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin\theta$



Example 12. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.



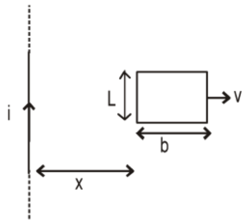
Solution : $E = B\,l\,V = \frac{\mu_0 i l v}{2\pi x}$

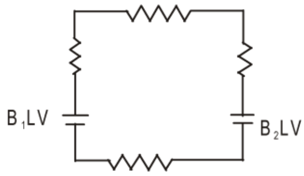
OR

Emf is equal to the rate with which magnetic field lines are cut. In dttime the area swept by the rod is l v dt. The magnetic field lines cut in dt time $= B\,l\,vdt = \frac{\mu_0 i l vdt}{2\pi x}$.

∴ The rate with which magnetic field lines are cut = $\frac{\mu_0 i l v}{2\pi x}$

Example 13. A rectangular loop ,as shown in the figure ,moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.

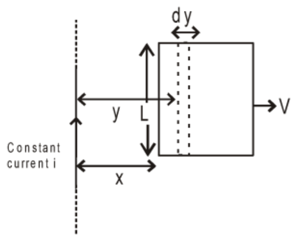




Solution : $E = B_1\,LV - B_2Lv = \frac{\mu_0 i}{2\pi x}Lv - \frac{\mu_0 i}{2\pi(x+b)}Lv = \frac{\mu_0 iLbv}{2\pi x(x+b)}$

Aliter : Consider a small segment of width dy at a distance y from the wire.

Let flux through the segment be dφ.



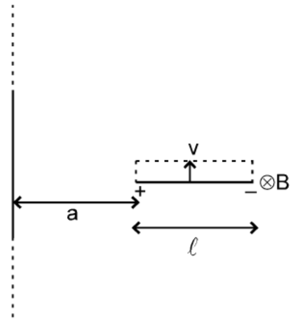
∴ $d\varphi = \frac{\mu_0 i}{2\pi y} L\,dy$

∴ $\phi = \frac{\mu_0 iL}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 iL}{2\pi}\left(\ell n(x+b) - \ell n x\right)$

Now $\frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]_v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$

∴ induced emf = $\frac{\mu_0 ibLv}{2\pi x(x+b)}$

Example 14. A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance ‘a’ from the wire.

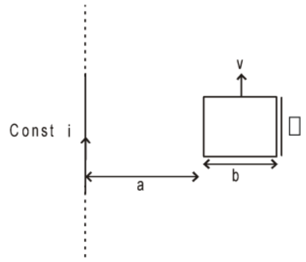


Solution : Consider a segment of rod of length dx, at a distance x from the wire. Emf induced in the segment

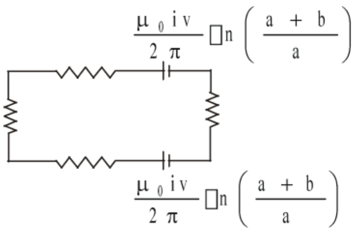
$$dE = \frac{\mu_0 i}{2\pi x}\,dx.v$$

$$\therefore\ E = \int_a^{a+\ell}\frac{\mu_0 i v dx}{2\pi x} = \frac{\mu_0 i v}{2\pi}\ell n\left(\frac{\ell + a}{a}\right)$$

Example 15. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop, if its nearest end is at a distance ‘a’ from the wire. Draw equivalent electrical diagram.



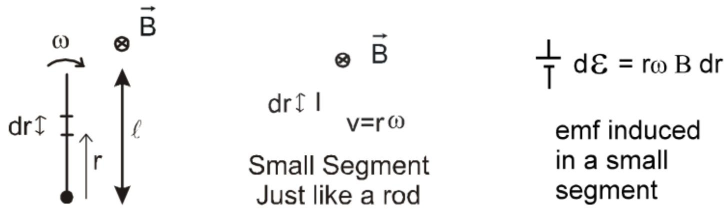
Solution : emf = 0 ;



4. INDUCED EMF DUE TO ROTATION

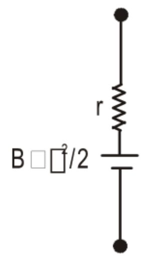
4.1 ROTATION OF THE ROD

Consider a conducting rod of length l rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod = v B dr = rω B dr

$\therefore$ emf induced in the rod = $\omega B\int_0^{\ell} r dr = \frac{1}{2}B\omega\ell^2$



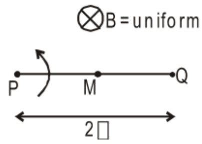
equivalent of this rod is as following or $\varepsilon = \frac{d\Phi}{dt}$

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}\ell^2\omega dt}{dt} = \frac{1}{2}B\omega\ell^2$$

Solved Example

Example 16. A rod PQ of length 2ℓ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .

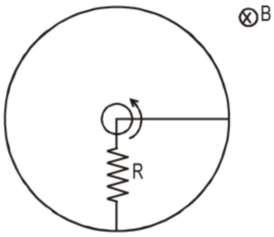


Solution : $E_{MQ} + E_{PM} = E_{PQ}$

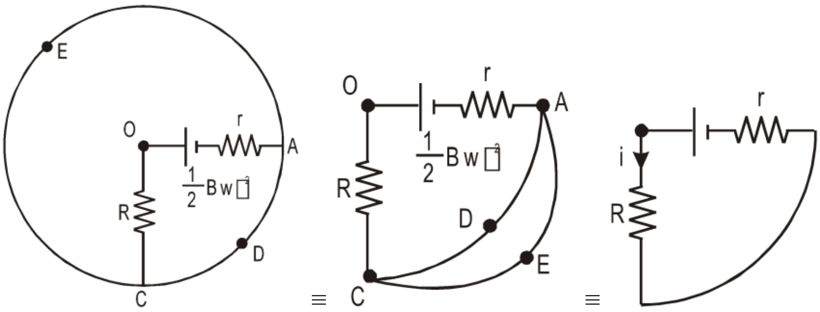
$$E_{PM} = \frac{Bw\ell^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2} \quad \Rightarrow \quad E_{MQ} = \frac{3}{8}B\omega\ell^2 = \frac{3}{4}\times 100\text{ V} = 75\text{ V}$$

Example 17. A rod of length ℓ and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

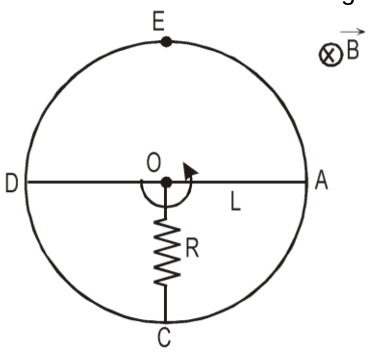


Solution :

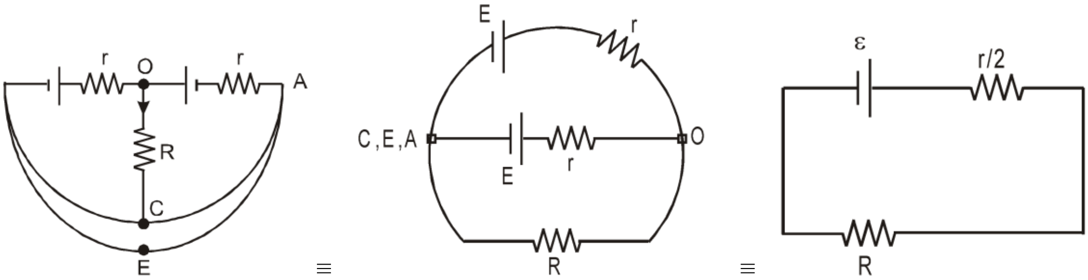


$$\text{current } i = \frac{\frac{1}{2}B\omega\ell^2}{R+r}$$

Example 18. Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring

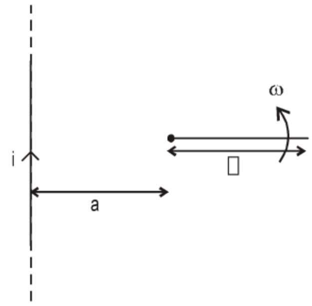


Solution :

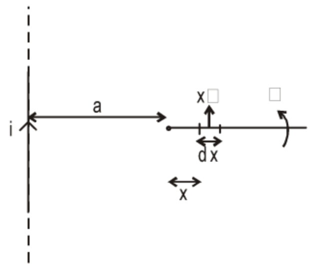


$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Example 19. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.



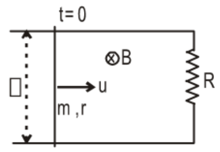
Solution : Consider a small segment of rod of length dx, at a distance x from one end of the rod. Emf induced in the segment



$$dE = \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx$$

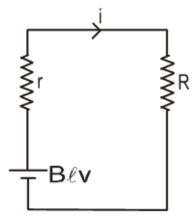
$$\therefore \quad E = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}(x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a.\ell n\left(\frac{\ell + a}{a}\right)\right].$$

Example 20. A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u .Find its velocity as a function of time.



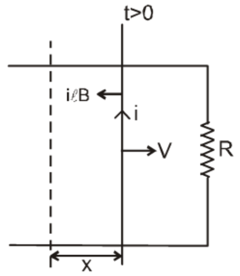
Solution : Let at an instant the velocity of the rod be v .The emf induced in the rod will be vBl.

The electrically equivalent circuit is shown in the following diagram.



$\therefore$ Current in the circuit $i = \dfrac{B\ell v}{R + r}$

At time t



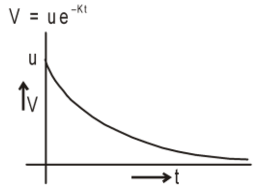
Magnetic force acting on the rod is F = $i\ell B$, opposite to the motion of the rod.

$i\ell B = -m\frac{dV}{dt}$ .....(1)

$i = \frac{B\ell v}{R+r}$ .....(2)

Now solving these two equation $\frac{B^2\ell^2V}{R+r} = -m.\frac{dV}{dt}$

$-\frac{B^2\ell^2}{(R+r)m}.dt = \frac{dV}{V}$



let $\frac{B^2\ell^2}{(R+r)m} = k$

$-K.dt = \frac{dV}{V}$

$\int_u^v \frac{dV}{V} = \int_0^t -K.dt$; $\ln\left(\frac{v}{u}\right) = -Kt$

$V = ue^{-Kt}$

Example 21. In the above question find the force required to move the rod with constant velocity v, and also find the power delivered by the external agent.

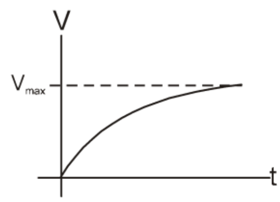
Solution : The force needed to keep the velocity constant $F_{ext} = i\ell B = \frac{B^2\ell^2v}{R+r}$

Power due to external force = $\frac{B^2\ell^2v^2}{R+r} = \frac{\varepsilon^2}{R+r} = i^2(R+r)$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

Example 22. In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Solution : $m\frac{dv}{dt} = F - i\ell B$ ....(1) $i = \frac{B\ell v}{R+r}$ ....(2)



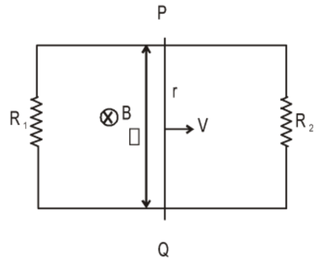
$m\frac{dv}{dt} = F - \frac{B^2\ell^2v}{R+r}$

let $K = \frac{B^2\ell^2}{R+r}$ $\Rightarrow$ $\int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$

$-\left[\ell n(F-KV)\right]_0^V = \frac{t}{m}$ $\Rightarrow$ $\ell n\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m}$

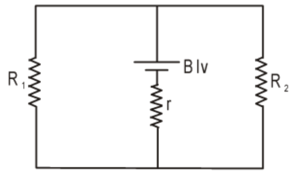
$F - KV = Fe^{-kt/m}$ $\Rightarrow$ $V = \frac{F}{K}(1 - e^{-kt/m})$

Example 23. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2) .Find the current in the rod at the instant its velocity is v.



Solution : $i = \dfrac{B\ell V}{r + \dfrac{R_1R_2}{R_1+R_2}}$

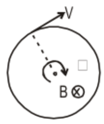
this circuit is equivalent to the following diagram.



4.2. EMF INDUCED DUE TO ROTATION OF A COIL

Solved Example

Example 24. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring



Solution : Flux passing through the ring φ = B.A is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

4.3 EMF INDUCED IN A ROTATING DISC :

Consider a disc of radius r rotating in a magnetic field B.

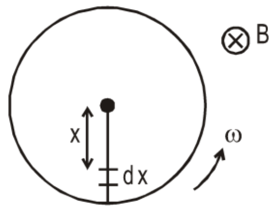
Consider an element dx at a distance x form the centre. This element is moving with speed v = ωx.

∴ Induced emf across dx

= B(dx) v = Bdxωx = Bωxdx

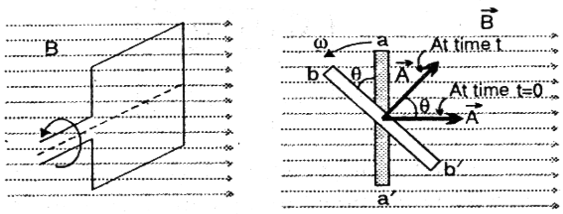
∴ emf between the centre and the edge of disc.

$= \int_0^r B\omega x dx = \dfrac{B\omega r^2}{2}$



4.4. ROTATION OF A RECTANGULAR COIL IN A UNIFORM MAGNETIC FIELD

➢ If the figure a conducting rectangular coil of area A and turns N is shown. It is rotated in a uniform magnetic field B about a horizontal axis perpendicular to the field with an angular velocity ω. The magnetic flux linked with the coil is continuously changing due to rotation.

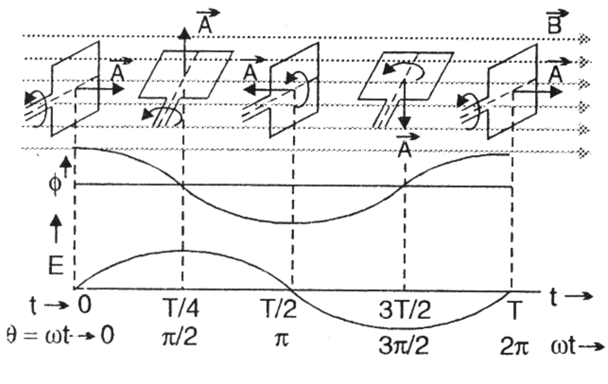


θ is the angle between the perpendicular to the plane of the coil and the direction of magnetic field.

- The magnetic flux passing through the rectangular coil depends upon the orientation of the plane of the coil about its axis.
- Magnetic flux passing through the coil $\phi = \vec{B}.\vec{A} = BA\cos\theta = BA\cos\omega t$
  If there are N turns in the coil, then the flux linked with the coil $\varphi = BAN \cos \omega t$
- Since φ depends upon the time t, the rate of change of magnetic flux
  $\frac{d\phi}{dt} = -BAN\omega \sin \omega t$
- According to Faraday's law, the emf induced in the coil
  $\epsilon = -\frac{d\phi}{dt}$ or $\epsilon = BAN\,\omega \sin \omega t$
  BAN ω is the maximum value of emf induced, Thus writing
  $BAN\omega = \epsilon_0$
  $\therefore \quad \epsilon = \epsilon_0 \sin \omega t$
  This equation represents the instantaneous value of emf induced at time t.
- If the total resistance of circuit along with the coil is R, then the induced current due to alternating voltage
  $I = \frac{\epsilon}{R} = \frac{\epsilon_0}{R} \sin \omega t$ or $I = I_0 \sin\omega t$
  where $I_0 = \frac{\epsilon_0}{R}$ is the maximum value of current.
- The magnetic flux linked with coil and the emf induced at different positions of the coil in one rotational cycle are shown in the following table :

Time	Position of coil	Magnetic flux	Induced emf
t = 0	Plane of the coil normal to $\vec{B}(\theta = 0)$	φ = NBA = maximum flux	= 0
t = T/4	Plane of the coil parallel to $\vec{B}(\theta = 90^\circ)$	φ = 0	=NBA ω = maximum
t = T/2	Plane of the coil normal to $\vec{B}$ again (θ = 180º)	φ = –NBA	= 0
t = 3T/4	Plane of the coil parallel to $\vec{B}$ again (θ = 270º)	φ = 0	= –NBA ω
t = T	Plane of the coil normal to $\vec{B}$ (θ = 360º)	φ = NBA	= 0

- The variations of magnetic flux linked with the coil and induced e.m.f at different times given in the above table are shown in the following figure.



- The phase difference between the instantaneous magnetic flux and induced emf is $\pi/2$.
- The ratio of ϵ_{max} and φ_{max} is equal to the angular velocity of the coil, Thus
  $\frac{\epsilon_{max}}{\phi_{max}} = \frac{NBA\omega}{NBA} = \omega$

- If $\theta = \frac{\pi}{4} = 45^o$, then

  $\varphi = \frac{NBA}{\sqrt{2}}$ and $\in = \frac{NBA\omega}{\sqrt{2}}$

  In this case the ratio of the induced emf and the magnetic flux is equal to the angular velocity of the coil. Thus

  $\frac{\in}{\phi} = \frac{NBA\omega}{\sqrt{2}} / \frac{NBA}{\sqrt{2}} = \omega$

- The direction of induced emf in the coil changes during one cycle so it is called alternating emf and current induced due to it is called alternating current. This is the principle of AC generator.

Solved Examples

Example 25. The phase difference between the emf induced in the coil rotating in a uniform magnetic field and the magnetic flux associated with it, is

(1) π (2) π/2 (3) π/3 (4) zero

Solution : $\varphi = NAB \cos \omega t$

and $\in = NAB\,\omega \sin \omega t$

Hence the phase difference between φ and ∈ will be π/2.

∴ Answer will be (2)

Example 26. A coil has 20 turns and area of each turn is 0.2 m^2. If the plane of the coil makes an angle of 60º with the direction of magnetic field of 0.1 tesla, then the magnetic flux associated with the coil will be -

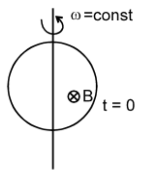
(1) 0.4 weber (2) 0.346 weber (3) 0.2 weber (4) 0.02 weber

Solution : $\varphi = n(B\, da \cos\theta)$

$= 20 \times 0.1 \times 0.2 \cos (90^o - 60^o) = 20 \times 0.1 \times 0.2 \times \frac{\sqrt{3}}{2} = 0.346$ weber

∴ Answer will be (2)

Example 27. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



Solution : At any time t , $\varphi = BA \cos \theta = BA \cos \omega t$

Now induced emf in the loop

$e = \frac{-d\phi}{dt} = BA\,\omega \sin \omega t$

If there are N turns

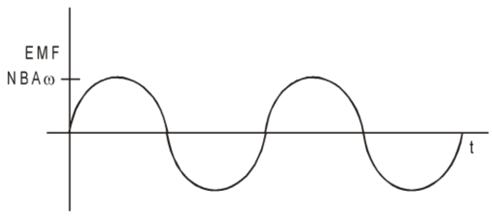
emf = $BA\omega N \sin \omega t$

BA ωN is the amplitude of the emf

$e = e_m \sin \omega t$

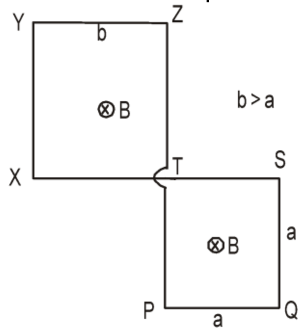
$i = \frac{e}{R} = \frac{e_m}{R} \sin wt = i_m \sin wt$

$i_m = \frac{e_m}{R}$



The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Example 28. Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B=\beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.



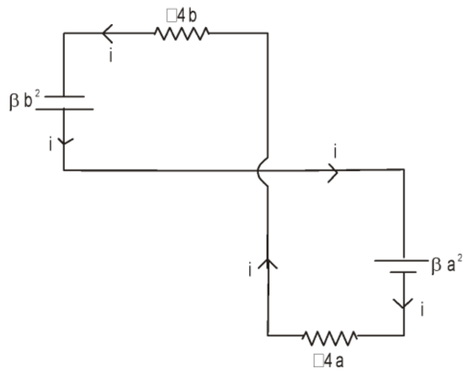
Solution : Induced emf in part PQST = βa^2 (in anticlockwise direction, from Lenz's Law)

Similarly Induced emf in part TXYZ = βb^2

(in anticlockwise direction, from Lenz's Law)

Total resistance of the part PQST $=\lambda 4a$.

Total resistance of the part TXYZ = $\lambda 4b$. The equivalent circuit is as shown in the following diagram.



writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda\, 4ai - \lambda\, 4bi = 0$$

$$i = \frac{\beta}{4\lambda}(b - a)$$

5. FIXED LOOP IN A VARYING MAGNETIC FIELD

Now consider a circular loop, at rest in a varying magnetic field. Suppose the magnetic field is directed inside the page and it is increasing in magnitude. The emf induced in the loop will be

$\varepsilon = -\frac{d\phi}{dt}$. Flux through the coil will be $\varphi = -\pi r^2 B$; $\frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}$; $\varepsilon = -\frac{d\phi}{dt}$ $\therefore \varepsilon = \pi r^2 \frac{dB}{dt}$

$\therefore$ $E\, 2\pi r = \pi r^2 \frac{dB}{dt}$ or $E = \frac{r}{2}\frac{dB}{dt}$

Thus changing magnetic field produces electric field which is non conservative in nature. The lines of force associated with this electric field are closed curves.

6. SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change.Total flux $N\varphi$ passing through a coil due to its own current is proportional to the current and is given as $N\varphi = Li$ where L is called coefficient of self induction or inductance**.**The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as $\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$

S.I Unit of inductance is wb/amp or Henry(H)

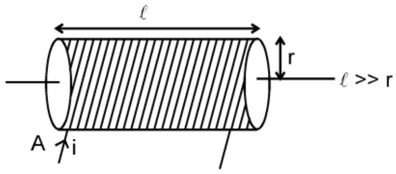
L - self inductance is +ve quantity .

L depends on : (1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current.

L is a scalar quantity.

6.1 SELF INDUCTANCE OF SOLENOID



Let the volume of the solenoid be V, the number of turns per unit length be n.

Let a current I be flowing in the solenoid.Magnetic field in the solenoid is given as $B=\mu_0 ni$.The magnetic flux through one turn of solenoid $\varphi = \mu_0 niA$.

The total magnetic flux through the solenoid

$= N\varphi = N\mu_0 ni A = \mu_0 n^2 i A \ell$

$\therefore \quad L = \mu_0 n^2 \ell A = \mu_0 n^2 V \quad \Rightarrow \quad \varphi = \mu_0 n i \pi r^2 (n\ell) \quad \Rightarrow \quad L = \frac{\phi}{i} = \mu_0 n^2 \pi r^2 \ell.$

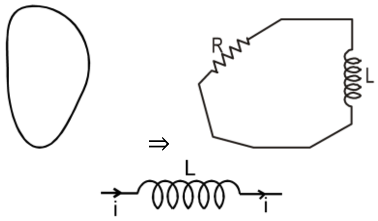
Inductance per unit volume = $\mu_0 n^2$.

Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant.Current can not change suddenly in the inductor.

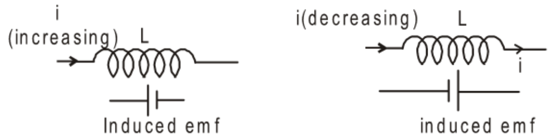
7. INDUCTOR :

It is represent by 

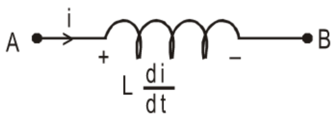
electrical equivalence of loop



If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current.If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.

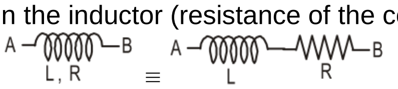


Over all result



$V_A - L\frac{di}{dt} = V_B$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :



Solved Example

Example 29. A B is a part of circuit. Find the potential difference $v_A - v_B$ if

(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution : $L\frac{di}{dt} = 1\frac{di}{dt}$

writing KVL from A to B

$V_A - 1\frac{di}{dt} - 5 - 2i = V_B$.

(i) Put i = 2, $\frac{di}{dt} = 0$

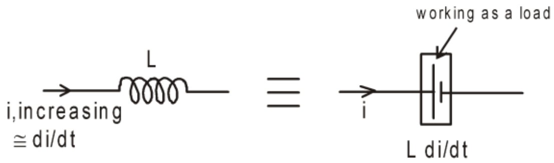
$V_A - 5 - 4 = V_B$ ∴ $V_A - V_B = 9$ volt

(ii) Put i = 2 , $\frac{di}{dt} = 1$; $V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\ V_0$

(iii) Put i = 2 , $\frac{di}{dt} = -1$; $V_A + 1 - 5 - 2 \times 2 = V_B$ or $V_A = 8$ volt.

7.1 ENERGY STORED IN AN INDUCTOR:

If current in an inductor at an instant is i and is increasing at the rate di/dt,the induced emf will oppose the current . Its behaviour is shown in the figure.



Power consumed by the inductor = $i\,L\frac{di}{dt}$

Energy consumed in dt time = $i\,L\frac{di}{dt}dt$

∴ total energy consumed as the current increases from 0 to I = $\int_0^I iL\,di = \frac{1}{2}LI^2 = \frac{1}{2}Li^2$

⇒ $U = \frac{1}{2}LI^2$

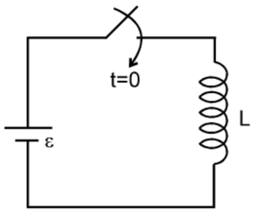
Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy $U = \int \frac{B^2}{2\mu_0\mu_r}\,dV$

Solved Example

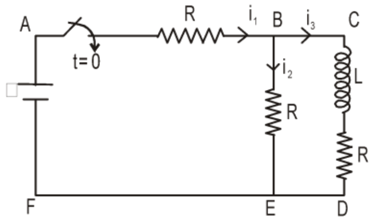
Example 30. A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open .It is closed at t=0.Find the current as a function of time.



Solution : $\varepsilon = L\frac{di}{dt}$ ⇒ $\int_0^i \varepsilon\,dt = \int_0^i L\,di$

$\varepsilon t = Li$ ⇒ $i = \frac{\varepsilon t}{L}$

Example 31. In the following circuit, the switch is closed at t = 0. Find the currents i_1, i_2, i_3 and $\frac{di_3}{dt}$ at t = 0 and at t = ∞. Initially all currents are zero.

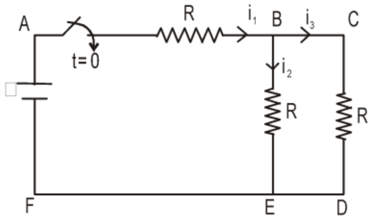


Solution : At t = 0

i_3 is zero ,since current cannot suddenly change due to the inductor.

$\therefore$ $i_1 = i_2$ (from KCL)

applying KVL in the part ABEF we get $i_1 = i_2 = \frac{\varepsilon}{2R}$.



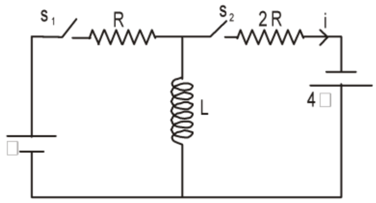
$i_3 = 0$, $\frac{di_3}{dt} = \frac{\epsilon}{2L}$

At t = ∞

i_3 will become constant and hence potential difference across the inductor will be zero.It is just like a simple wire and the circuit can be solved assuming it to be like shown in the following diagram.

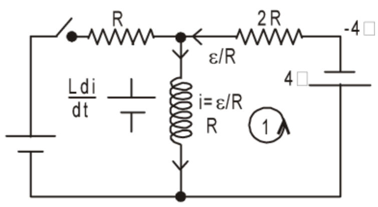
$i_2 = i_3 = \frac{\varepsilon}{3R}$, $i_1 = \frac{2\varepsilon}{3R}$, $\frac{di_3}{dt} = 0$.

Example 32. In the circuit shown in the figure, S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.



Solution : Before S_2 is closed and S_1 is opened current in the left part of the circuit = $\frac{\varepsilon}{R}$. Now when S_2 closed S_1 opened, current through the inductor can not change suddenly, current $\frac{\varepsilon}{R}$ will continue to move in the inductor.

Applying KVL in loop 1.



$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0$$

$$\frac{di}{dt} = -\frac{6\varepsilon}{L}$$

current inductor long after this moment $i = \frac{4\varepsilon}{2R} = \frac{2\varepsilon}{R}$.

7.2 GROWTH OF CURRENT IN SERIES R–L CIRCUIT :

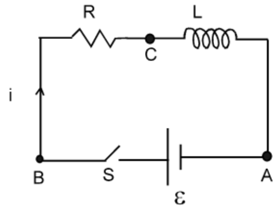
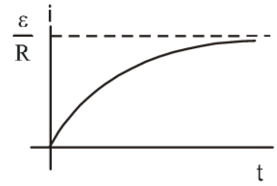


Figure shows a circuit consisting of a cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at an instant current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit, we have $\varepsilon - L\frac{di}{dt} - iR = 0$

On solving we get, $i = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.

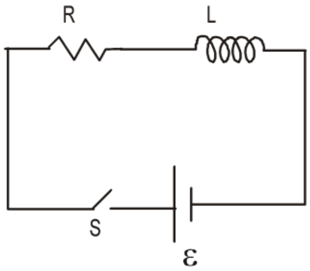
Note : 1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.

2. After one time constant, current in the circuit =63% of the final current (verify yourself)
3. More time constant in the circuit implies slower rate of change of current.
4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$L_1 i_1 = L_2 i_2$

Solved Examples

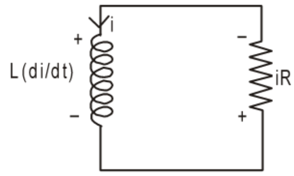
Example 33. At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser (L/η) then its initial value, find out instant current just after the operation.



Solution : Using above result (note 4)

$L_1 i_1 = L_2 i_2 \Rightarrow i_2 = \frac{\eta\varepsilon}{R}$

7.3 DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR :



Let the initial current in the circuit be I_0.At any time t,let the current be i and let its rate of change at this instant be di/dt.

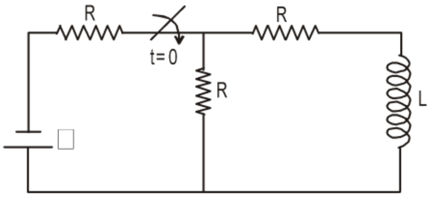
$L.\frac{di}{dt} + iR = 0, \qquad \frac{di}{dt} = -\frac{iR}{L}$

$\int_{I_0}^{i}\frac{di}{i} = -\int_0^t \frac{R}{L}.dt \Rightarrow \ln\left(\frac{i}{I_0}\right) = -\frac{Rt}{L}$ or $i = I_0 e^{\frac{-Rt}{L}}$

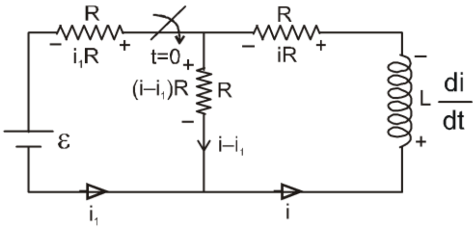
Current after one time constant :$i = I_0$=0.37% of initial current.

Solved Examples

Example 34. In the following circuit the switch is closed at t = 0.Intially there is no current in inductor. Find out current the inductor coil as a function of time.



Solution :



At any time t

$-\varepsilon + i_1R - (i - i_1)R = 0$

$-\varepsilon + 2i_1R - iR = 0$

$i_1 = \frac{iR+\varepsilon}{2R}$ Now, $-\varepsilon + i_1R + iR + L.\frac{di}{dt} = 0$

$-\varepsilon + \left(\frac{iR+\varepsilon}{2}\right) + iR + L.\frac{di}{dt} = 0$ $\Rightarrow$ $-\frac{\varepsilon}{2} + \frac{3iR}{2} = -L.\frac{di}{dt}$

$\left(\frac{-\varepsilon+3iR}{2}\right)dt = -L.di$ $\Rightarrow$ $-\frac{-dt}{2L} = \frac{di}{-\varepsilon+3iR}$

$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon+3iR}$ $\Rightarrow$ $-\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right)$

$-\ln\left(\frac{-\varepsilon+3iR}{-\varepsilon}\right) = \frac{3Rt}{2L}$ $\Rightarrow$ $i = +\frac{\varepsilon}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$

Aliter : By replacing inductance by it resistance (i.e. zero) resultant resistance of circuit is 3R/2

so I_{max} through battery is $I_{max} = \frac{2\varepsilon}{3R}$,

This current is equally distributed at junction so through L maximum current is $i_{max} = \frac{\varepsilon}{3R}$.

So, At any time t current through inductance is $i = \frac{\varepsilon}{3R}(1 - e^{-t/\tau})$ ....(i)

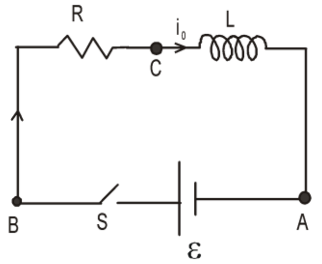
To determine τ we replace battery by it's internal resistance so resistance across inductor is 3R/2.

So $\tau = \frac{L}{3R/2} = \frac{2L}{3R}$

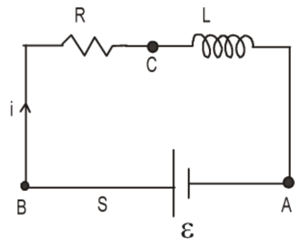
put in equation (1)

$i = \frac{\varepsilon}{3R}\left(1-e^{-\frac{3Rt}{2L}}\right)$

Example 35. Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Solution : Let an instant t current in the circuit is i which is increasing at the rate di/dt.

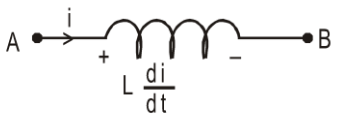


Writing KVL along the circuit , we have $\varepsilon - L\frac{di}{dt} - iR = 0$

$\Rightarrow \quad L\frac{di}{dt} = \varepsilon - iR \qquad \Rightarrow \quad \int_{i_0}^{i}\frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$

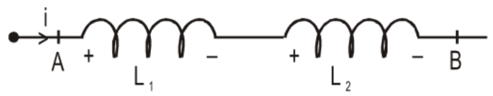
$\Rightarrow \quad \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L} \Rightarrow \varepsilon - iR = (\varepsilon - i_0R)\,e^{-Rt/L} \qquad \Rightarrow \qquad i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$

Equivalent self inductance :



$L = \frac{V_A - V_B}{di/dt}$ ..(1)

Series combination

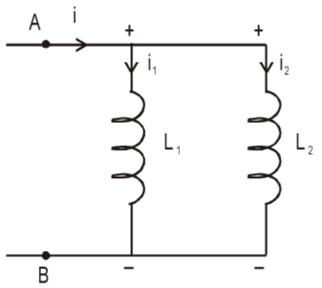


$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B$ ....(2)

from (1) and (2)

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination :



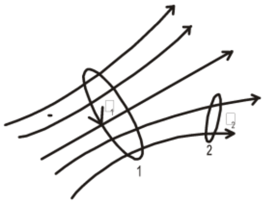
From figure $V_A - V_B = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt}$ ..... (3)

also $i = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ or $\frac{V_A - V_B}{L} = \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2}$

$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (Neglecting mutual inductance)

8. MUTUAL INDUCTANCE



Consider two arbitrary conducting loops 1 and 2. Suppose that I_1 is the instantaneous current flowing around loop 1. This current generates a magnetic field $\mathbf{B_1}$ which links the second circuit, giving rise to a magnetic flux φ_2 through that circuit. If the current I_1doubles, then the magnetic field $\mathbf{B_1}$ doubles in strength at all points in space, so the magnetic flux φ_2 through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux φ_2 through the second circuit is directly proportional to the current I_1 flowing around the first circuit. Hence, we can write $\varphi_2=M_{21}I_1$ where the constant of proportionality M_{21} is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux φ_2 through the first circuit due to the instantaneous current I_2 flowing around the second circuit is directly proportional to that current, so we can write $\varphi_1=M_{12}I_2$ where $M_{12}$is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that **$M_{21}= M_{12}$ (Reciprocity Theorem)**. Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The S.I. unit of mutual inductance is called Henry (H). One Henry is equivalent to a volt-second per ampere.

Suppose that the current flowing around circuit 1 changes by an amount ΔI_1 in a small time interval Δt. The flux linking circuit 2 changes by an amount $\Delta\varphi_2=M\Delta I_1$in the same time interval. According to Faraday's law, an emf $\varepsilon_2 = -\frac{\Delta\phi_2}{\Delta t}$ is generated around the second circuit due to the changing magnetic flux linking that circuit. Since, $\Delta\phi_2$ = $M\Delta I_1$, this emf can also be written $\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t}$.

Thus, the emf generated around the second circuit due to the current flowing around the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current I_2 flowing around the second circuit changes by an amount ΔI_1in a time interval Δt then the emf generated around the first circuit is $\varepsilon_1 = -M\frac{\Delta I_2}{\Delta t}$ Note that there is no direct physical connection(coupling) between the two circuits: the coupling is due entirely to the magnetic field generated by the currents flowing around the circuits.

Note : (1) $M \le \sqrt{L_1L_2}$

(2) For two coils in series if mutual inductance is considered then

$L_{eq} = L_1 + L_2 \pm 2M$

➢ **Unit of M** : In M.K.S. system unit of mutual inductance is henry

$$M = \frac{E_B}{-(dI_A/dT)} = \frac{\phi_B}{I_A}$$

$$\therefore \quad 1 \text{ henry} = \frac{1\text{volt}}{1\text{ampere/s}} = \frac{1\text{weber}}{\text{ampere}}$$

$$= \frac{(\text{joule/coulomb})\text{s}}{\text{ampere}} = \text{J/A}^2$$

➢ **Dimensions of M** :

$$M = \frac{J}{A^2} = \frac{\text{joule}}{\text{ampere}^2} = \frac{\text{newton} \times \text{metre}}{\text{ampere}^2}$$

$$= \frac{\text{kg} \times \text{metre} \times \text{sec}^{-2} \times \text{metre}}{\text{ampere}^2}$$

$$= ML^2T^{-2}A^{-2}$$

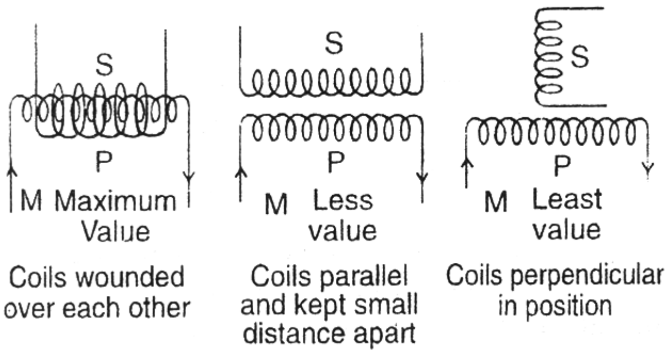
- Mutual inductance between the coils depends upon the number of turns in the coils, area and the permeability of the core placed inside the coils. Larger is the magnitude of M, more is the emf induced in the secondary coil.
- Out of the two coils coupled magnetically one coil can be taken as primary and the other coil as secondary. Thus mutual inductance

  $M_{AB} = M_{BA} = M$
- Mutual inductance between two coaxial solenoids of length ℓ and cross-sectional area A is

  $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

  where N_1 and N_2 are the number of turns in the two coils respectively.
- If two coils are wound one over the other, then mutual inductance will be maximum and it will be less in other arrangements.

  M and L have the following relation :



$M \propto \sqrt{L_1 L_2}$

$M = K\sqrt{L_1 L_2}$

where K is a coupling constant of coils and its value varies from 0 to 1.

(a) If K = 0, then there will be no coupling between the coils, that is magnetic flux produced by the primary coil is not linked with the secondary coil.

(b) If K = 1, then both coils are coupled together with maximum transfer to energy, that is, magnetic flux produced by the primary coil is totally linked with the secondary coil.

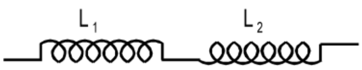
- If two coils of self inductances L_1 and L_2 are coupled in series such that their windings are in the same sense and mutual inductance between them is M, then the equivalent inductance will be

  $L = L_1 + L_2 + 2M$

(coils wound in same direction)

If two coils are coupled in series such that their windings are in opposite sense then equivalent inductance will be

$L = L_1 + L_2 - 2M$



(coils wound in oppsite direction)

Solved Example

Example 36. A coil of radius 1 cm and 100 turns is placed at the centre of a long solenoid of radius 5 cm and 8 turn/cm. The value of coefficient of mutual induction will be -

(1) 3.15×10^{-5} H (2) 6×10^{-5} H (3) 9×10^{-5}H (4) zero

Solution : $M = \mu_0 n_1 N_2 \pi r^2 = 4\pi \times 10^{-7} \times 800 \times 100\, \pi \times (0.01)^2 = 3.15 \times 10^{-5}$ H

Hence the correct answer will be (1)

Example 37. The coefficients of self induction of two coils are 0.01 H and 0.03 H respectively. If they oppose each other then the resultant self induction will be, if M = 0.01H

(1) 2H (2) 0.02H (3) 0.02H (4) zero

Solution : $L = L_1 + L_2 - 2M = 0.01 + 0.03 - 2 \times 0.01$

Hence the correct answer will be (3)

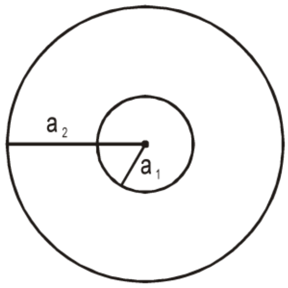
Example 38. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let I be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids,neglecting the end effects.

Solution : If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\ell}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is $B_1 A$. Thus, the flux linking all N_2 turns of the second wire is

$$\varphi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = M I_1 \qquad \therefore \quad M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

Example 39. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution : Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil = $\frac{\mu_0 i}{2a_2}\, \pi a_1^2$

or $Mi = \frac{\mu_0 i}{2a_2}\, \pi a_1^2$ or $M = \frac{\mu_0 \pi a_1^2}{2a_2}$

Example 40. Solve the above question, if the planes of coil are perpendicular.

Solution : Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0.

Example 41. Solve the above problem if the planes of coils make θ angle with each other.

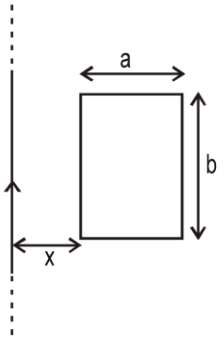
Solution : If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

Thus flux = $\vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2} . \pi a_1^2 . \cos\theta$

or $M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

Example 42. Find the mutual inductance of a straight long wire and a rectangular loop, as shown in the figure

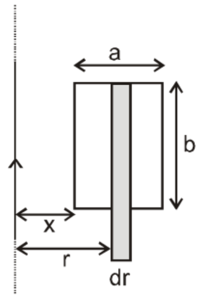


Solution : $d\varphi = \frac{\mu_0 i}{2\pi r} \times bdr$

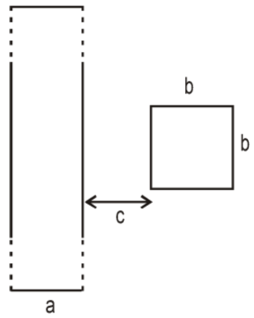
$\varphi = \int_x^{x+a} \frac{\mu_0 i}{2\pi r} \times bdr$

$M = \varphi/i$

$M = \frac{\mu_0 b}{2\pi} \ln\left(1+\frac{a}{x}\right)$



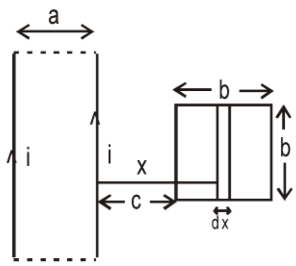
Example 43. Find the mutual inductance between two rectangular loops, shown in the figure



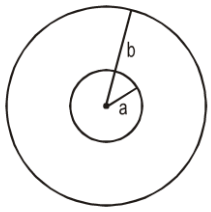
Solution : Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$d\varphi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b \ dx$

$\Rightarrow \quad \varphi = \int_c^{c+b} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] bdx = \frac{\mu_0 ib}{2\pi}\left[\ell n\frac{c+b}{c} - \ell n\frac{a+b+c}{a+c}\right]$.



Example 44. Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils
(b) Find the emf induced in the larger coil
(c) If the resistance of the larger loop is R find the current in it as a function of time

Solution : (a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 i}{2b}$.

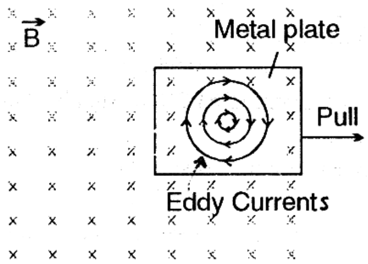
flux through the smaller coil = $\frac{\mu_0 i}{2b}\pi a^2$ $\therefore$ $M = \frac{\mu_0}{2b}\pi a^2$

(b) $\left|\text{emf induced in larger coil}\right| = M\left[\left(\frac{di}{dt}\right)\text{ in smaller coil}\right] = \frac{\mu_0}{2b}\pi a^2$ (2) $= \frac{\mu_0 \pi a^2}{b}$

(c) current in the larger coil $= \frac{\mu_0 \pi a^2}{b\ R}$.

9. EDDY CURRENT

- When a conductor is placed in a changing magnetic field, induced emf is produced in it. As a result local currents are produced in the conductor. These local currents are called eddy currents.
- If a conducting material is moved in a magnetic field, then eddy currents are also produced.
- Eddy currents flows in closed paths.
- There is loss of energy due to eddy currents and it appears in the form if heat.
- In order to minimize the energy loss in the form of heat due to eddy currents the core of dynamo, motor or transformer is not taken as a single piece of soft iron but in the form of a peck of thin sheets insulated from each other by a layer of insulating varnish, called laminated core. This device increases the resistance for the eddy currents. In this way eddy currents are considerably reduced and loss of energy becomes less.



- **Uses of eddy currents :**

(a) Moving coil galvanometer (b) Induction furnace

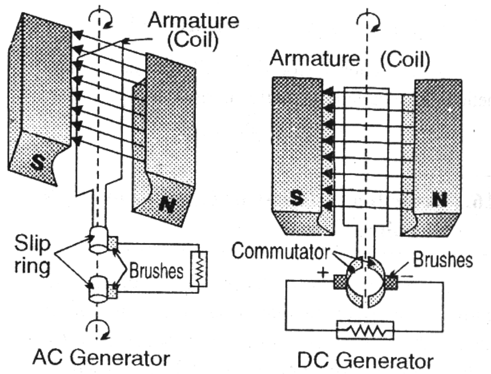
(c) Dead beat galvanometer (d) Speedometer (e) Electric brakes

10. GENERATOR OR DYNAMO

- Generator or dynamo is an electrical device which converts mechanical energy into electrical energy.
- Working of generators is based on the principle of electromagnetic induction.
- **Generators are of two types :**

(a) A.C. generator : If the current produced by the generator is alternating, then the generator is called A.C. generator.

(b) D.C. generator : If the current produced by the generator is direct current, then the generator is called D.C. generator.



- Generator consists of the following parts.

  (a) Armature (coil) (b) Magnet (c) Slip rings (d) Brushes

  In D.C. generator commutator is used in place of slip rings.
- In order to produce the magnetic field in big generators several magnetic poles are used. In these generators the armature coils are kept stationary and magnetic pole pieces are made to rotate around the armature. The frequency of alternating current produced by generator of multi poles is

  $$= \frac{\text{number of poles} \times \text{rotational frequency}}{2} = \frac{Nn}{2}$$
- **Energy loss in generators :** The loss of energy is due to the following reasons :

  (a) Flux leakage, (b) Copper losses, (c) Eddy current losses,

  (d) Hysteresis losses, (e) Mechanical losses
- **Efficiency of generator :** Practical efficiency of a generator

  $$= \frac{\text{Electrical power generated by the generator}}{\text{Mechanical energy given to the generator}}$$

  Practical efficiencies of big generators are about 92% to 95%.

11. MOTOR

- It converts electrical energy into mechanical energy.
- When a current carrying conductor (coil) is placed in a magnetic field, a couple acts on it which makes the coil to rotate.
- Electric motors are of two types :

  (a) Alternating current motor (AC motor) (b) Direct current motor (DC motor)
- D.C. motor consists of the following parts :

  (a) armature (b) magnet (c) commutator (d) brushes
- **Back E.M.F :** When current from an external electric source is passed through the armature of the electric motor, armature coil rotates in the magnetic field. In cuts the magnetic lines of force as a result emf is induced in it. According to Lenz's law this induced emf opposes the rotation of the armature i.e., the emf induced works opposite to the emf applied by the external electric source and opposes the motion of the armature. This induced emf is called back emf. Greater is the speed of armature coil, greater is the back emf.
- At the time of start of the motor back emf is almost zero and the current flowing in the motor in maximum. As the speed of the armature coil increases, back emf also increases. When the coil increases, back emf also increases. When the coil attains maximum speed, the induced emf becomes constant and current reduced to minimum.

- Back emf is directly proportional to the angular velocity ω of rotation of armature and the magnetic field B, i.e., for constant magnetic field back emf. $e \propto \omega$ or $e = K\omega$ where K is a constant.
- If E is applied emf, e is the back emf and R is the resistance of the coil (armature), then the current flowing through the coil will be

  $i = \frac{E - e}{R}$ or $E = e + iR$ but $e = K\omega$

  $\therefore \quad i = \frac{E - K\omega}{R}$

- In the beginning, i.e. at the time of start of the motor $\omega = 0$ $\therefore i = \frac{E}{R}$

  In this case current will be maximum.
- As armature coil is made from copper wire so its resistance is very small. When motor starts running, a very heavy current passes through the armature coil in the beginning. Due to which motor may get burnt. To prevent the motor from burning at the time of start a special variable resistance is connected in series with the armature, which is called starter.
- High resistance is connected in series with the armature coil with the help of starter at the time of start of the motor. As the motor starts picking up speed, the resistance is gradually reduced till it becomes zero.
- Starter is used in a high power motors but not in the low power motors because its coil starts rotating with a very high speed in a short time
- Power of electric motor = ie
- Efficiency of motor

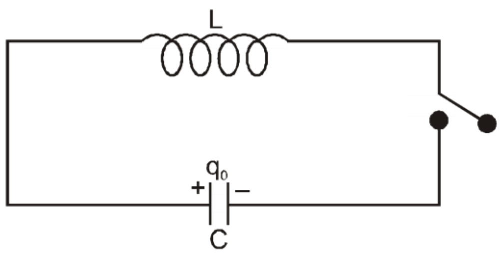
  $\eta = \frac{\text{Work done by the motor}}{\text{Energy taken from the electric source by the motor}} = \frac{W}{P} \times 100\%$

  or $\eta = \frac{\text{Back emf}}{\text{Applied emf}} \times 100\% = \frac{e}{E} \times 100\%$

  Generally the efficiency of the motor is from 80% to 90%.

12. L-C OSCILLATION

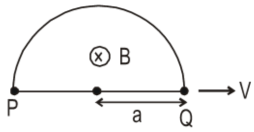
When a charged capacitor C having an initial charge q_0 is discharged through an inductance L, the charge and current in the circuit start oscillating simple harmonically. If the resistance of the circuit is zero, no energy is dissipated as heat, we also assume an idealized situation in which energy is not radiated away from the circuit. The total energy associated with the circuit is constant. Frequency of oscillation is given by



$\omega = \frac{1}{\sqrt{LC}} \frac{\text{rad}}{\text{sec}}$ or $f = \frac{1}{2\pi\sqrt{LC}}$ Hz

Solved Miscellaneous Problems

Problem 1. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalent circuit of each branch.

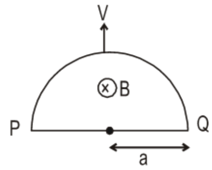


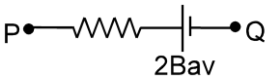
Solution : Here $\vec{v} \parallel \vec{\ell}$

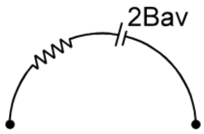
so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

Induced emf = 0

Problem 2. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalence of each branch.

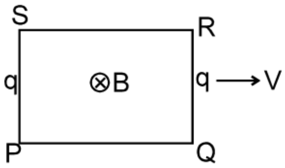




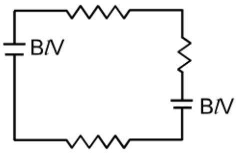


Solution : Induced emf = 2Bav

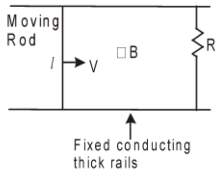
Problem 3. Figure shows a rectangular loop moving in a uniform magnetic field .Show the electrical equivalence of each branch.



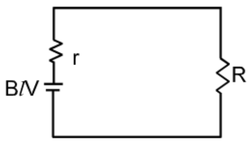
Solution :



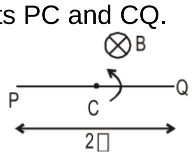
Problem 4. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R.A uniform magnetic field B is present normal to the plane of rod and rails .Show the electrical equivalence of each branch.



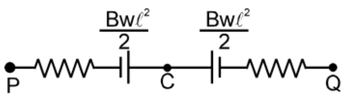
Solution :



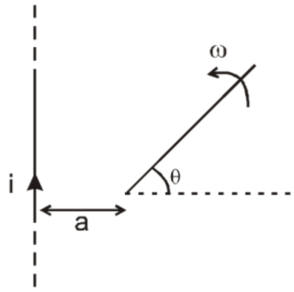
Problem 5. A rod PQ of length 2ℓ is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced emf between P Q and PC. Draw the circuit diagram of parts PC and CQ.



Solution : $emf_{PQ} = 0$; $emf_{PC} = \frac{B\omega\ell^2}{2}$



Problem 6. A rod of length ℓ is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.

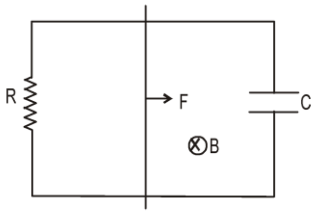


Solution :

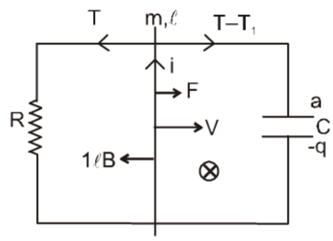
$$E = \int \frac{\mu_0 i}{2\pi \ (a + r\cos\theta)} \times (r\omega) \,.\, (dr)$$

$$E = \frac{\mu_0 \omega i}{2\pi} \int_0^{\ell} \frac{r}{a + r\cos\theta} \, dr \quad \Rightarrow E = \frac{\mu_0 i \omega}{2\pi \cos\theta}\left[\ell - \frac{a}{\cos\theta}\ell n\left(\frac{a + \ell\cos\theta}{a}\right)\right]$$

Problem 7. Find the velocity of the moving rod at time t if the initial velocity of the rod is zero and a constant force F is applied on the rod. Neglect the resistance of the rod. Capacitor is initially uncharged.



Solution : At any time t, let the velocity of the rod be v .



Applying Newtons law: $F - ilB = ma$ .....(1)

Also $B\,l\,v = i_1 R = \frac{q}{c}$

Applying Kcl, $i = i_1 + \frac{dq}{dt} = \frac{B\ell V}{R} + \frac{d}{dt}(BlvC)$

or $i = \frac{B\ell V}{R} + B\ell C\, a$

Putting the value of i in eq (1), $F - \frac{B^2\ell^2 V}{R} = (m + B^2\ell^2 C)a = (m + B^2\ell^2 C)\frac{dv}{dt}$

$$(m + B^2\ell^2 C)\frac{dv}{F - \frac{B^2 l^2 v}{R}} = dt$$

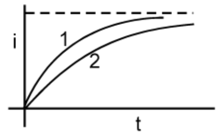
Integrating both sides, and solving we get $v = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{tB^2\ell^2}{R(m + CB^2\ell^2)}}\right)$

Problem 8. A rod PQ of length ℓ is rotating about end P, with an angular velocity ω. Due to centrifugal forces the free electrons in the rod move towards the end Q and an emf is created. Find the induced emf.

Solution : The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state $m_e\omega^2 x = e\,E$.

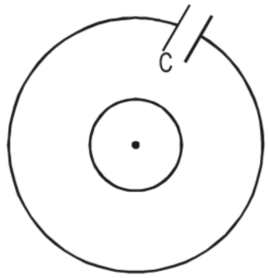
$$V_P - V_Q = \int_{x=0}^{x=\ell} \bar{E}.d\bar{x} = \int_0^{\ell} \frac{m_e\omega^2 x}{e} dx = \frac{m_e\omega^2\ell^2}{2e}$$

Problem 9. Which of the two curves shown has less time constant.



Solution : Curve1

Example 10. If the current in the inner loop changes according to i = $2t^2$ then, find the current in the capacitor as a function of time.



Solution :

$$M = \frac{\mu_0}{2b}\pi a^2$$

$$|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right) \text{ in smaller coil}\right]$$

$$\Rightarrow \quad e = \frac{\mu_0}{2b}\pi a^2 \ (4t) \qquad = \frac{2\mu_0 \pi a^2 t}{b}$$

Applying KVL :-

$$+e - \frac{q}{c} - iR = 0$$

$$\frac{2\mu_0 \pi a^2 t}{b} - \frac{q}{c} - iR = 0$$

differentiate wrt time :-

$$\frac{2\mu_0 \pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$$

on solving it

$$i = \frac{2\mu_0 \pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$$