GEOMETRICAL OPTICS

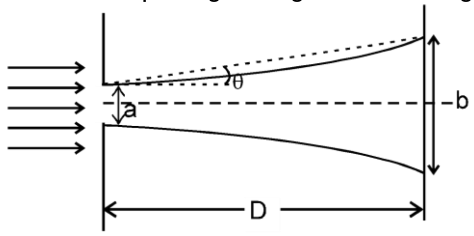
INTRODUCTION :

Blue lakes, ochre deserts, green forest, and multicolored rainbows can be enjoyed by anyone who has eyes with which to see them. But by studying the branch of physics called **optics**, which deals with the behavior of light and other electromagnetic waves, we can reach a deeper appreciation of the visible world. A knowledge of the properties of light allows us to understand the blue color of the sky and the design of optical devices such as telescopes, microscopes, cameras, eyeglasses, and the human eye. The same basic principles of optics also lie at the heart of modern developments such as the laser, optical fibers, holograms, optical computers, and new techniques in medical imaging.

1. CONDITION FOR RECTILINEAR PROPAGATION OF LIGHT

(only for information not in jee syllabus)

Some part of the optics can be understood if we assume that light travels in a straight line and it bends abruptly when it suffers reflection or refraction.
The assumption that the light travels in a straight line is correct if
(i) the medium is isotropic, i.e. its behavior is same in all directions and (ii) the obstacle past which the light moves or the opening through which the light moves is not very small.



Consider a slit of width ‘a’ through which monochromatic light rays pass and strike a screen, placed at a distance D as shown.

It is found that the light strikes in a band of width ‘b’ more than ‘a’. This bending is called **diffraction**.
Light bends by (b-a)/2 on each side of the central line .It can be shown by wave theory of light that

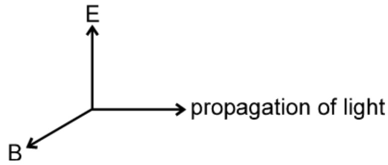
$\sin\theta = \frac{\lambda}{a}$ ....(A),
where θ is shown in figure.
This formula indicates that the **bending is considerable only when a $\underset{-}{\sim}$ λ** . Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.

It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$ .If this condition is fulfilled, light is said to move rectilinearly. In most of the situations including geometrical optics the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted.
$\frac{b-a}{2D} \approx \frac{\lambda}{a}$
Thus geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A **ray** of light is the straight line path of transfer of light energy. Arrow represents the direction of propagation of light.
Figure shows a ray which indicates light is moving from A to B.

2. PROPERTIES OF LIGHT

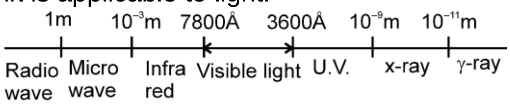
(i) Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.



(iii) Light carries energy and momentum.

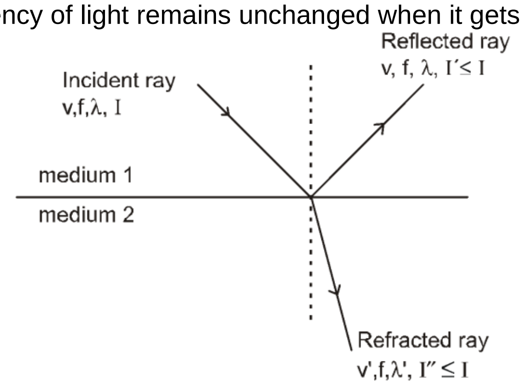
(iv) The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

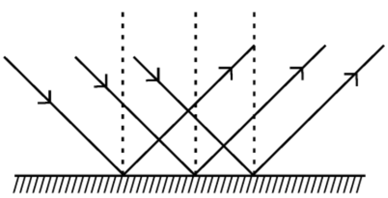


3. REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called *Reflection of Light.*

(a) Regular Reflection:

When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.



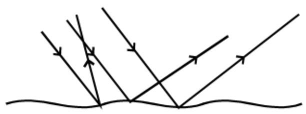
Regular Reflection

(b) Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite

possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position.
Such a reflection is called as **diffused reflection**.

For example reflection from a wall, from a news paper etc. This is why you can not see your face in news paper and in the wall.

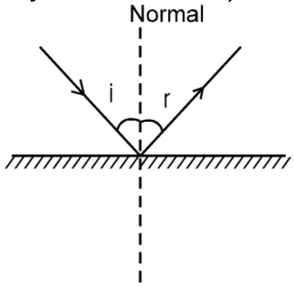


Diffused Reflection

3.1 Laws of Reflection

(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection).** This condition can be expressed mathematically as $\vec{R}\cdot(\vec{I}\times\vec{N}) = \vec{N}\cdot(\vec{I}\times\vec{R}) = \vec{I}\cdot(\vec{N}\times\vec{R}) = 0$ where $\vec{I}$, $\vec{N}$ and $\vec{R} \neq$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively.

(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.

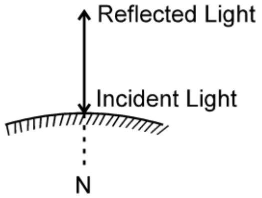


$\angle i = \angle r$

Special Cases :

Normal Incidence : In case light is incident normally,

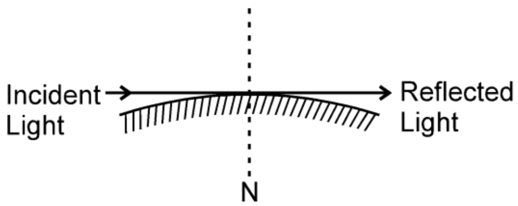
$i = r = 0$



$\delta = 180^\circ$

Note : We say that the ray has retraced its path.

Grazing Incidence : In case light strikes the reflecting surface tangentially,
$i = r = 90$
$\delta = 0^\circ$ or 360°

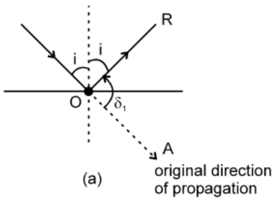


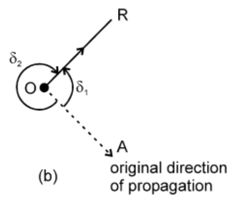
Note : In case of reflection speed (magnitude of velocity) of light remains unchanged but in Grazing incidence velocity remains unchanged.

Solved Example

Example 1. Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Solution :





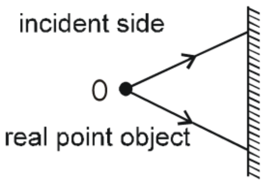
From figure (b) it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 (= $2\pi - \delta_1$) clockwise.

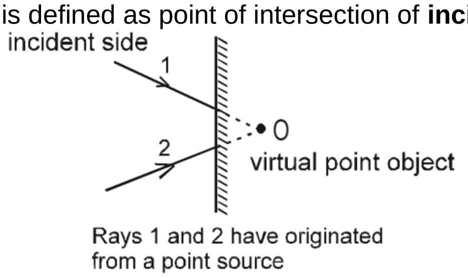
From figure (a) $\delta_1 = \pi - 2i$.

$\therefore \quad \delta_2 = \pi + 2i$.

3.2 Object and Image

(a) Object (O) : Object is defined as point of intersection of **incident** rays.

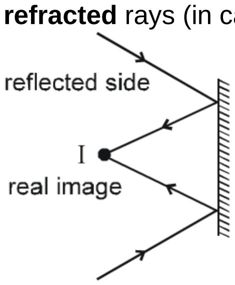


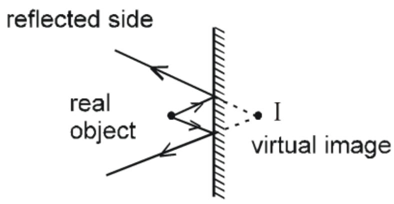


Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side.

Note : An object is called **real** if it lies on incident side otherwise it is called **virtual**.

(b) Image (I) : Image is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction).

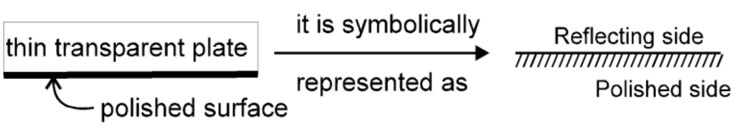




Note : An image is called **real** if it lies on reflected or refracted side otherwise it is called **virtual.**

4. PLANE MIRROR

Plane mirror is formed by polishing one surface of a plane thin glass plate .It is also said to be silvered on one side.

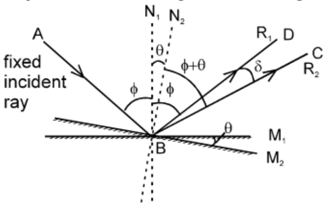


PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Solved Example

Example 2. For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.



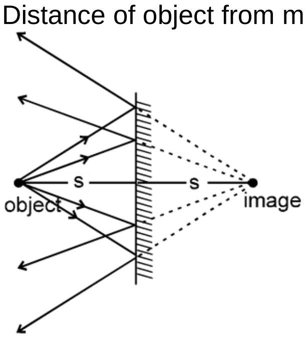
Solution : See figure M_1, N_1 and R_1 indicate the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray respectively. From figure it is clear that ABC = $2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

Note : Keeping the mirror fixed if the incident ray is rotated by angle θ then reflected ray rotates by same angle in the opposite direction of rotation.

4.1 Point object

Characteristics of image due to Reflection by a Plane Mirror :

(i) Distance of object from mirror = Distance of image from the mirror.



(ii) All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

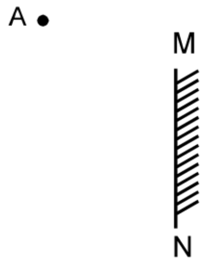
(iii) The line joining a point object and its image is normal to the reflecting surface.

(iv) For a real object the image is virtual and for a virtual object the image is real

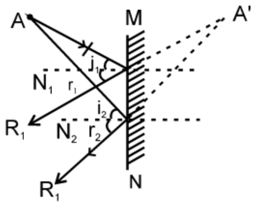
(v) The region in which observer's eye must be present in order to view the image is called **field of view**.

Solved Examples

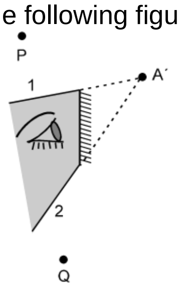
Example 3. Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer’s eye must be present in order to view the image. (This region is called **field of view**).



Solution : See figure, consider any two rays emanating from the object. N_1 and N_2 are normals ;
$i_1 = r_1$ and $i_2 = r_2$



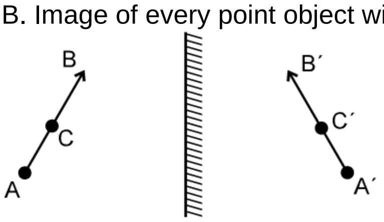
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A´. To obtain the region in which reflected rays are present, join A´ with the ends of mirror and extend. The following figure shows this



region as shaded. In figure there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.

4.2 Extended object :

An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images

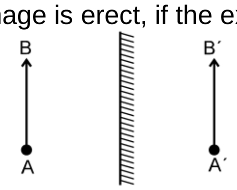


will be formed. A´ will be image of A, C´ will be image of C, B´ will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.

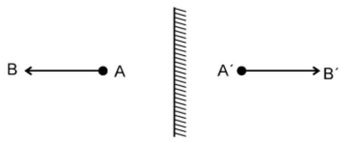
Properties of image of an extended object, formed by a plane mirror :

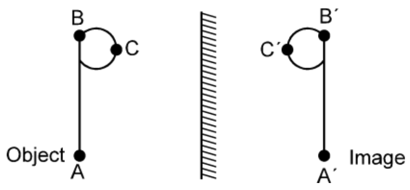
(1) Size of extended object = size of extended image.

(2) The image is erect, if the extended object is placed parallel to the mirror.

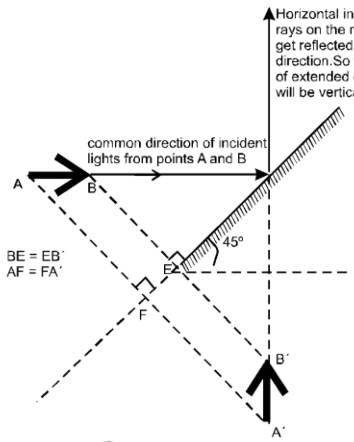


(3) The image is inverted if the extended object lies perpendicular to the plane mirror.





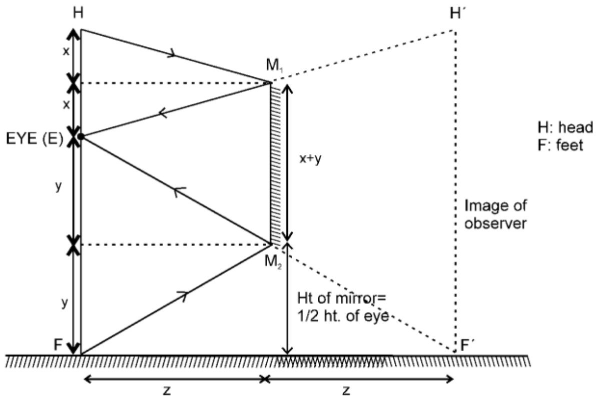
(4) If an extended horizontal object is placed infront of a mirror inclined 45° with the horizontal, the image formed will be vertical. See figure.



Solved Example

Example 4. Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

Solution : See the following figure. It is self explanatory if you consider lengths 'x' and 'y' as shown in figure.



Aliter :

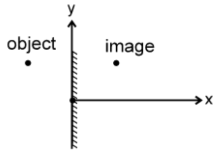
ΔE M_1, M_2 and ΔE H´F´ are similar

$\therefore \quad \frac{M_1M_2}{H'F'} = \frac{z}{2z}$ or $M_1 M_2$ = H´ F´ / 2 = HF / 2

4.3 Relation between velocity of object and image :

From mirror property : $x_{im} = - x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror. Similarly others have meaning.



Differentiating w.r.t time , we get

$v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z}$,

$\Rightarrow$ for x axis $v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$

but for y axis and z axis $v_{iG} - v_{mG} = (v_{oG} - v_{mG})$ or $v_{iG} = v_{oG}$.

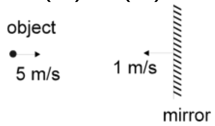
here: v_{iG} = velocity of image with respect to ground.

Solved Examples

Example 5 An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.

Solution : Take ► as + direction. $v_i - v_m = v_m - v_0$

$v_i - (-1) = (-1) - 5$



$v_i = -7$m/s.

$\Rightarrow$ 7 m/s and direction towards left.

Example 6. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

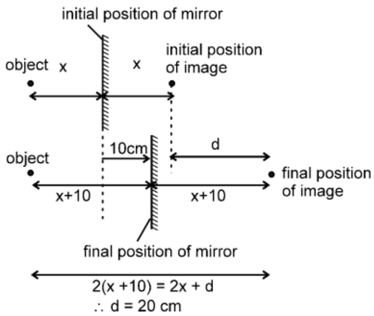
Solution : We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$

or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$.

In this Q. $\Delta x_o = 0$; $\Delta x_m = 10$ cm.

Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

or

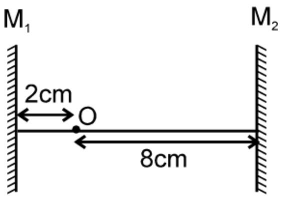


4.4 Images formed by two plane mirrors :

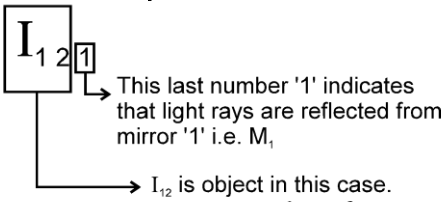
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

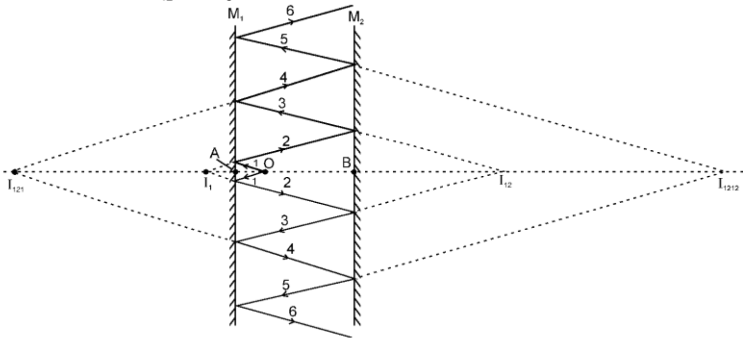
Solved Examples

Example 7. Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



Solution : To understand how images are formed see the following figure and table. You will require to know what symbols like I_{121} stands for. See the following diagram.



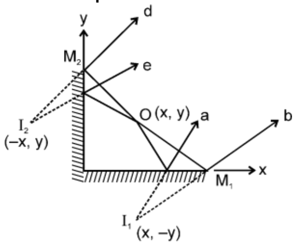


Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	AO = 2cm	AI_1 = 2 cm
Rays 2	M_2	Rays 3	I_1	I_{12}	BI_1 = 12 cm	BI_{12} = 12 cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{12} = 22cm	AI_{121} = 22cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{121} =32cm	BI_{1212}=32cm

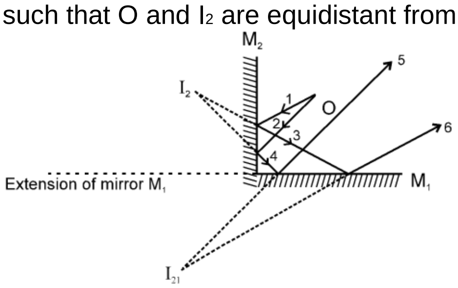
Similarly images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Example 8. Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

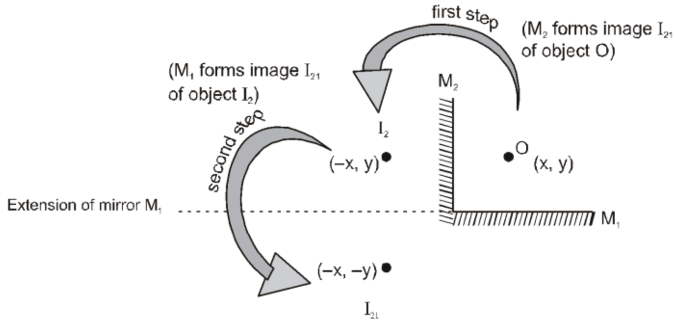
Solution : Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at (x, –y), such that O and I_1 are equidistant



from mirror M_1. These rays donot form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at (–x, y), such that O and I_2 are equidistant from mirror M_2.



Now consider those rays which strike mirror M_2 first and then the mirror M_1.



For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.

Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21}, are equidistant from M_1. To summarize see the following figure

For rays reflecting first from M_1 and then from M_2, first image I_1 (at (x, –y)) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at (–x, –y)) will be formed.

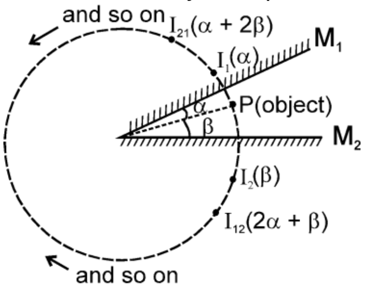
I_{12} and I_{21} coincide.

∴ A total of three images are formed

4.5 Locating all the images formed by two plane mirrors

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha+\beta$ as shown in figure.

Point P is an object kept such that it makes angle α with



mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1 ,denoted by I_1 ,will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2 ,denoted by I_2 ,will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.

Now I_2 will act as an object for M_1 which is at an angle (α+2β) from M_1 . Its image will be formed at an angle (α+2β) on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this process will stop. Hint: The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360^o}{\theta}$ = even number; number of image = $\frac{360^o}{\theta} - 1$

(ii) If $\frac{360^o}{\theta}$ = odd number; number of image = $\frac{360^o}{\theta} - 1$,
if the object is placed on the angle bisector.

(iii) If $\frac{360^o}{\theta}$ = odd number; number of image = $\frac{360^o}{\theta}$,
if the object is not placed on the angle bisector.

(iv) If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

1. What should be the angle between two plane mirrors so that whatever be the angle of incidence, the incident ray and the reflected ray from the two mirrors be parallel to each other
(1) 60° (2) 90° (3) 120° (4) 175°

2. A light bulb is placed between two plane mirrors inclined at an angle of 60°. The number of images formed are
(1) 6 (2) 2 (3) 5 (4) 4

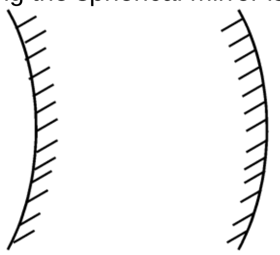
3. A ray of light incidents on a plane mirror at an angle of 30°. The deviation produced in the ray is
(1) 30° (2) 60° (3) 90° (4) 120°

4. A man runs towards mirror at a speed of 15m/s. What is the speed of his image
(1) 7.5 m/s (2) 15 m/s (3) 30 m/s (4) 45 m/s

Ans. **1.** (2) **2.** (3) **3.** (4) **4.** (2)

5. SPHERICAL MIRRORS

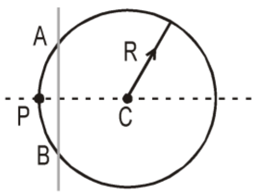
Spherical Mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as (a) Concave mirror, if the



Concave mirror Convex mirror

side towards center of curvature is shining and (b) Convex mirror if the side away from the center of curvature is shining.

5.1 Important terms related with spherical mirrors :



A spherical shell with the center of curvature, pole aperture and radius of curvature identified

(a) Center of Curvature (C) :
The center of the sphere from which the spherical mirror is formed is called the Center of curvature of the mirror. It is represented by C and is indicated in figure.

(b) Pole (P) :
The center of the mirror is called as the Pole. It is represented by the point P on the mirror APB in figure.

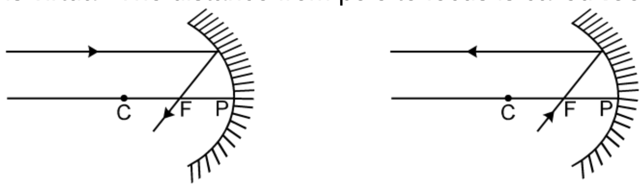
(c) Principal Axis :
The Principal Axis is a line which is perpendicular to the plane of the mirror and passes through the pole. The Principal Axis can also be defined as the line which joins the Pole to the Center of Curvature of the mirror.

(d) Aperture (A) :

The aperture is the segment or area of the mirror which is available for reflecting light. In figure. APB is the aperture of the mirror.

(e) **Principle focus (F) :**

It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In concave mirror it is real and in the convex mirror it is virtual. The distance from pole to focus is called **focal length.**



Concave mirror

Convex mirror

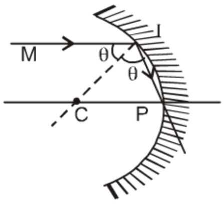
Solved Examples

Example 9. Find the angle of incidence of ray for which it passes through the pole, given that MI || CP.

Solution : ∠ MIC = ∠ CIP = θ

MI || CP ∠ MIC = ∠ ICP = θ

CI = CP

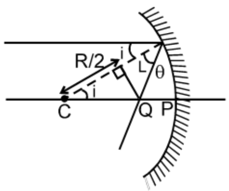


∠ CIP = ∠CPI = θ

∴ In ΔCIP all angle are equal

$3\theta = 180^\circ \Rightarrow \theta = 60^\circ$

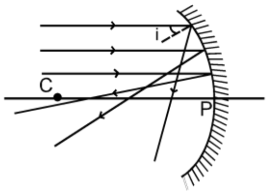
Example 10. Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if i → 0.



Solution : $\cos i = \frac{R}{2CQ} \Rightarrow CQ = \frac{R}{2\cos i}$

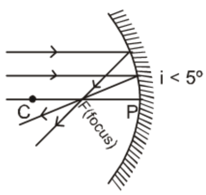
As i increases cos i decreases.

Hence CQ increases



If i is a small angle cos i 1

∴ CQ = R/2

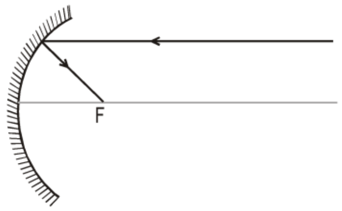


So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.

6.1 Ray tracing :

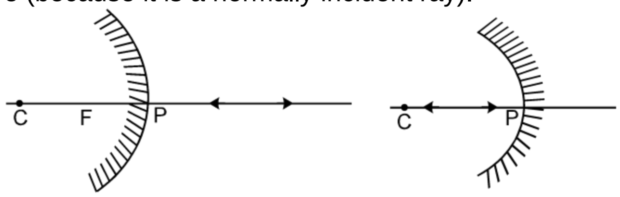
Following facts are useful in ray tracing.

(i) If the incident ray is parallel to the principle axis, the reflected ray passes through the focus.

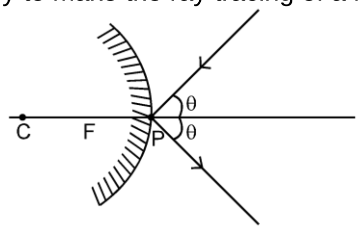


(ii) If the incident ray passes through the focus, then the reflected ray is parallel to the principle axis.

(iii) Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).



(iv). It is easy to make the ray tracing of a ray incident at the pole as shown in below.



6.2 Sign Convention

We are using co–ordinate sign convention.

(i) Take origin at pole (in case of mirror) or at optical centre (in case of lens)
Take X axis along the Principle Axis, taking **positive direction along the incident light.**
u, v, R and **f** indicate the x coordinate of object, image, centre of curvature and focus respectively.

(ii) y-coordinates are taken positive above Principle Axis and negative below Principle Axis'
h_1 and **h_2** denote the y coordinate of object and image respectively.

Note : This sign convention is used for reflection from mirror, refraction through flat or curved surfaces or lens.

6.3 Formulae for Reflection from spherical mirrors :

(a) Mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}=\frac{1}{f}$

X-coordinate of centre of Curvature and focus of Concave mirror are negative and those for Convex mirror are positive. In case of mirrors since light rays reflect back in X-direction, therefore **-ve sign of v indicates real image and +ve sign of v indicates virtual image.**

Solved Examples

Example 11. Figure shows a spherical concave mirror with its pole at (0, 0) and principle axis along x axis. There is a point object at (–40 cm, 1cm), find the position of image.

Solution : According to sign convention,

$u = -40$ cm
$h_1 = +1$ cm
$f = -5$ cm.

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$\Rightarrow \quad \frac{1}{v} + \frac{1}{-40} = \frac{1}{-5}$; $v = \frac{-40}{7}$ cm. ; $\frac{h_2}{h_1} = \frac{-v}{u}$

$\Rightarrow \quad h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(-\frac{40}{7}\right)\times 1}{-40} = -\frac{1}{7}$ cm.

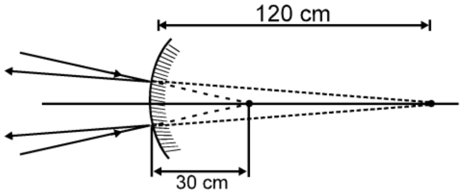
$\therefore$ The position of image is $\left(\frac{-40}{7}\text{cm}, -\frac{1}{7}\text{cm}\right)$

Example 12. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

Solution : In this case $u = +30$

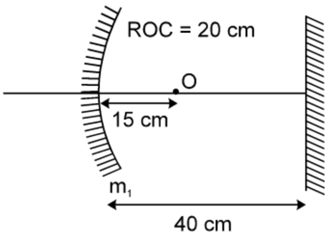
$\Rightarrow \quad v = +120$

$\therefore \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{120} + \frac{1}{30}$



f = 24 cm

Example 13. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution : **I reflection :**

Focus of mirror $= -10$ cm $\Rightarrow$ $u = -15$ cm

Applying mirror formula :

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = -30$ cm.

For II reflection on plane mirror :

$u = -10$ cm $\quad \therefore \quad v = 10$ cm

For III reflection on curved mirror again :

$u = -50$ cm ; $f = -10$ cm

Applying mirror formula :

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; $v = -12.5$ cm.

(b) **Lateral magnification** (or transverse magnification) denoted by m is defined as m= and is related as m $= \frac{h_2}{h_1}$ .From the definition of m positive sign of m indicates erect image and negative sign indicates inverted image.

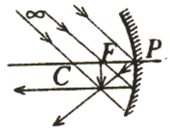
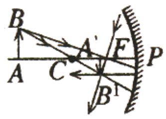
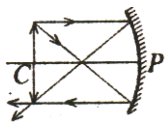
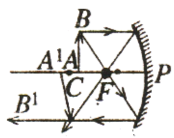
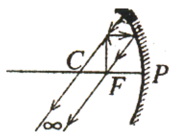
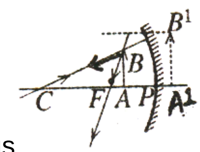
(c) In case of successive reflection from mirrors, the overall lateral magnification is given by $\mathbf{m_1 \times m_2 \times m_3}$ ......, where m_1 , m_2 etc. are lateral magnifications produced by individual mirrors.
$\mathbf{h_1}$ and $\mathbf{h_2}$ denote the y coordinate of object and image respectively.

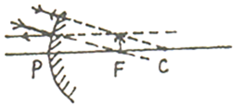
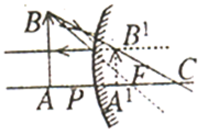
Note : Using (5.3.a**) and (**5.3.b**) the following conclusions can be made** (check yourself)**.**

Nature of Object	**Nature of Image**	**Inverted or erect**
Real	Real	Inverted
Real	Virtual	Erect
Virtual	Real	Erect
Virtual	Virtual	Inverted

(d) From (**5.3.a**) and (**5.3.b**); we get m= $\frac{f}{f-u} = \frac{f-v}{f}$ ....(just a time saving formula)

IMAGE FORMED BY THE CONCAVE MIRROR

S.No.	Position of object	Ray diagram	Position of image	Nature of image	Size of image
1.	At infinity		at focus	real and inverted	very small
2.	Between infinity and centre of cruvature		between focus and centre of curvature	real and inverted	small
3.	At centre of curvature		at centre of curvature	real and inverted	equal to object size
4.	Between focus and centre of curvature		between centre of curvature and infinity	real and inverted	enlarged
5.	At focus		at infinity	real and inverted	very large
6.	Between pole and focus		between poles and focus	virtual and erect	enlarged

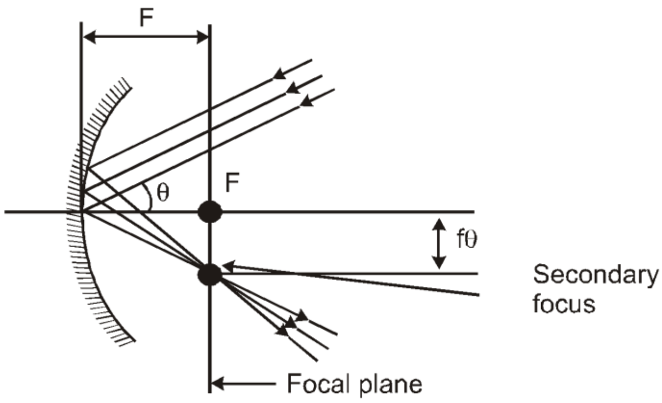
7.	At infinity		at focus	virtual and erect	very small
8.	Between pole and infinity		between focus & pole	virtual and erect	small

FOCAL PLANE

A plane passing through focus and perpendicular to principal axis is called focal plane

SECONDARY FOCUS

Paraxial rays which are parallel to each other but not parallel to principal axis will also meet at a single point in focal plane after reflection from spherical mirror (or refraction from lens). That point is known an secondary focus.



Solved Examples

Example 14. An extended object is placed perpendicular to the principle axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Solution : u = – 15 cm f = – 10 cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we get, v = – 30 cm

∴ m = $-\frac{v}{u}$ = – 2.

Aliter : m = $\frac{f}{f-u}$ = $\frac{-10}{-10-(-15)}$ = – 2

Example 15. A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 cm then find the nature of radius of curvature of the mirror.

Solution : Person will see his face only when the image is virtual. Virtual image of real object is erect.

Hence m = 2

∴ $\frac{-v}{u}$ = 2 ⇒ v = 40 cm

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; f = – 40 cm or R = 80 cm.

Aliter : $m = \frac{f}{f-u}$ ⇒ $2 = \frac{f}{f-(-20)}$

⇒ f = – 40 cm or R = 80 cm

Example 16. An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm then the image become triple its size. Find the nature and ROC of the mirror.

Solution : Since the images formed on screen it is real. Real object and real image implies concave mirror.

Applying $m = \frac{f}{f-u}$ or $-2 = \frac{f}{f-(u)}$ ....(1)

After shifting $-3 = \frac{f}{f-(u+5)}$ ....(2)

[Why u + 5 ? , why not u – 5 : In a concave mirror are size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x coordinate will increase]

From (1) & (2) we get,

f = – 30 cm or R = 60 cm

(d) Velocity of image

(i) Object moving perpendicular to principal axis :

From the relation in 5.3.b,

we have

$$\frac{h_2}{h_1} = -\frac{v}{u} \quad \text{or} \quad h_2 = -\frac{v}{u} \cdot h_1$$

If a point object moves perpendicular to the principle axis, x coordinate of both the object & the image become constant. On differentiating the above relation w.r.t. time , we get,

$$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$

Here, $\frac{dh_1}{dt}$ denotes velocity of object perpendicular to the principle axis and $\frac{dh_2}{dt}$ denotes velocity of image perpendicular to the principle axis.

(ii) Object moving along principal axis :

On differentiating the mirror formula with respect to time we get $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$, where $\frac{dv}{dt}$ is the velocity of image along Principle axis $\frac{du}{dt}$ and is the velocity of object along principle axis. Negative sign implies that the image , in case of mirror, always moves in the direction opposite to that of object. This discussion is for velocity with respect to mirror and along the x axis.

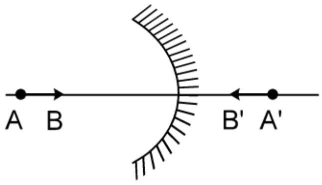
(iii) Object moving at an angle with the principal axis : Resolve the velocity of object along and perpendicular to the principle axis and find the velocities of image in these directions separately and then find the resultant.

(e) **Optical power of a mirror (in Diopters) =** $\frac{1}{f}$
f = focal length with sign and in meters.

(f) If object lying along the principle axis is not of very small size, the **longitudinal magnification** = $\frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

(g) **If the size of object is very small as compared to its distance from Pole then**

On differentiating the mirror formula we get $\frac{dv}{du}$ **=** $-\frac{v^2}{u^2}$ **:**
Mathematically 'du' implies small change in position of object and 'dv' implies corresponding small change in position of image. If a small object lies along principle axis, du may indicate the size of object and dv the size of its image along Principle axis (Note that the focus should not lie in between the initial and final points of object). In this case $\frac{du}{dv}$ is called longitudinal magnification. **Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.**



Solved Example

Example 17. A point object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principle axis. Find
(a) the position of image
(b) If object is shifted 1 mm towards the mirror along principle axis find the shift in image. Explain the result.

Solution : (a) u = – 60 cm
f = – 10cm

$$v = \frac{fu}{u-f} = \frac{-10\ (-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

(b) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating , we get $dv = -\frac{v^2}{u^2}\, du = -\left(\frac{-12}{-60}\right)^2 [1 \text{ mm}] = -\frac{1}{25}$ mm
[∵ du = 1mm; sign of du is +ve because it is shifted in +ve direction defined by signconvention.]

- –ve sign of dv indicates that the image will shift towards negative direction.
- The sign of v is negative. Which implies the image is formed on negative side of pole. (A) and (B) together imply that the image will shift away from pole.
  Note that differentials dv and du denote small changes only.

(h) **Newton's Formula**: **XY = f** ^{2}

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

In case of spherical mirrors if object distance (x) and image distance (y) are measured from focus instead of pole, u = –(f + x) and v = – (f + y),

by $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ we can write – $\frac{1}{(f+y)} - \frac{1}{(f+x)} = -\frac{1}{f}$

on solving $xy = f^2$ This is Newton's formula.

Self Practice Problems

5. A dimenished virtual image can be formed only in
(1) Plane mirror (2) A concave mirror (3) A convex mirror (4) Concave-parabolic mirror

6. An object 5 cm tall is placed 1m from a concave spherical mirror which has a radius of curvature of 20 cm. The size of the image is
(1) 0.11 cm (2) 0.50 cm (3) 0.55 cm (4) 0.60 cm

7. In a concave mirror experiment, an object is placed at a distance x_1 from the focus and the image is formed at a distance x_2 from the focus. The focal length of the mirror would be
(1) x_1x_2 (2) $\sqrt{x_1x_2}$ (3) $\frac{x_1+x_2}{2}$ (4) $\sqrt{\frac{x_1}{x_2}}$

8. Given a point source of light, which of the following can produce a parallel beam of light
(1) Convex mirror (2) Concave mirror
(3) Concave lens (4)Two plane mirrors inclined at an angle of 90°

Ans. **5.** (3) **6.** (3) **7.** (2) **8.** (2)

7. REFRACTION OF LIGHT

When the light changes its medium some changes occurs in its properties the phenomenon is known as refraction.

If the light is incident at an angle (0 < i < 90) then it deviates from its actual path. It is due to change in speed of light as light passes from one medium to another medium.

If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

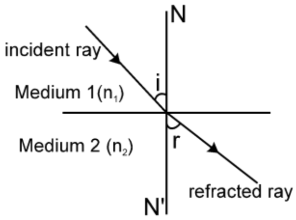
Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum. $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called denser (rarer) medium.

7.1 Laws of Refraction

(a) The incident ray , the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\text{Sin } i}{\text{Sin } r}$ **=** Constant for any pair of media and for light of a given wave length. This is known as *Snell's*



Law.

Also, $\frac{\text{Sin } i}{\text{Sin } r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

For applying in problems remember

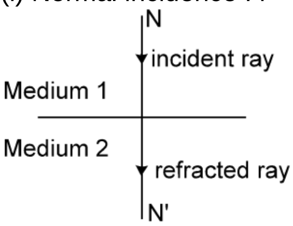
$n_1 \sin i = n_2 \sin r$

$\frac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

C = speed of light in air (or vacuum) = 3×10^8 m/s.

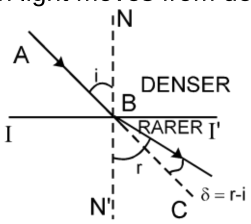
Special cases :

(i) Normal incidence : i = 0



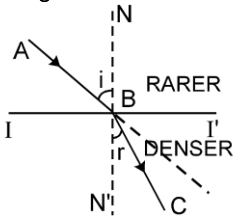
from snell's law : r = 0

(ii) When light moves from denser to rarer medium it bends away from normal.



r > i

(iii) When light moves from rarer to denser medium it bends towards the normal.



Note :

(i) Higher the value of R.I., denser (optically) is the medium.

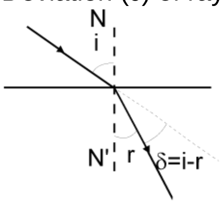
(ii) Frequency of light does not change during refraction.

(iii) Refractive index of the medium relative to vacuum = $\sqrt{\mu_r \in_r}$

n_{vacuum} = 1 ; $n_{air} \simeq 1$; n_{water} (average value) = 4/3 ; n_{glass} (average value)= 3/2

7.2 Deviation of a Ray Due to Refraction

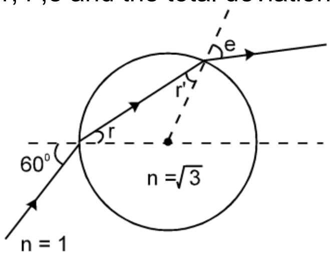
Deviation (δ) of ray incident



at $\angle$ i and refracted at $\angle$ r
is given by $\boldsymbol{\delta = |i - r|}$ **.**

Solved Examples

Example 18. A light ray is incident on a glass sphere at an angle of incidence 60° as shown. Find the angles r, r',e and the total deviation after two refractions.



Solution : Applying Snell's law $1\sin 60^\circ = \sin r$ $\Rightarrow$ $r = 30^\circ$

From symmetry $r' = \sqrt{3}$ $r = 30^\circ$.
Again applying snell's law at second surface $1\sin e = \sin r$
$\Rightarrow$ $e = 60^\circ$
Deviation at first surface = $i - r = 60^\circ - 30^\circ = 30^\circ$
Deviation at second surface = $e - r' = 60^\circ - 30^\circ = 30^\circ$
Therefore total deviation = 60° .

Example 19. Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in figure.

Solution : Snell's Law

$\mu_w \sin\theta_w = \mu_a \sin\theta_a$; $\frac{4}{3}\times\frac{3}{5} = 1$ $\sin\theta_a$; $\sin\theta_a = \frac{4}{5}$; $\theta_a = \sin^{-1}\frac{4}{5}$

Example 20. Find the speed of light in medium 'a' if speed of light in medium 'b' is where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45º and 60º respectively with the normal.

Solution : Snell's Law

$\mu_a \sin\theta_a = \mu_b \sin\theta_b$; $\frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b$.

$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ$. ; $v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$

7.3 Principle of Reversibility of Light Rays

(a) A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.

(b) A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus the incident and refracted rays are mutually reversible.

8. REFRACTION THROUGH A PARALLEL SLAB

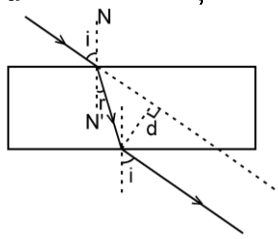
When light passes through a parallel slab, having same medium on both sides, then

(a) Emergent ray is parallel to the incident ray.

Note : Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

(b) Light is shifted laterally, given by (student should be able to derive it)

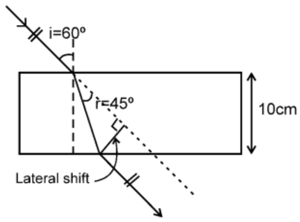
d = $\dfrac{t\ \sin(i-r)}{\cos r}$ **;** t = thickness of slab



Solved Example

Example 21. Find the lateral shift of light ray while is passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60º and the angle of refraction in glass is 45º.

Solution : d = $\dfrac{t\sin\ (i-r)}{\cos r}$ = $\dfrac{10\sin\ (60^\circ-45^\circ)}{\cos 45^\circ}$



= $\dfrac{10\sin\ 15^\circ}{\cos 45^\circ}$ = $10\sqrt{2}$ sin 15º.

8.1 Apparent Depth and shift of Submerged Object

At near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

d′ = $\dfrac{d}{n_{relative}}$ and $v'=\dfrac{v}{n_{relative}}$

where $n_{relative} = \dfrac{n_i\ (\text{R.I. of medium of incidence})}{n_r\ (\text{R. I. of medium of refraction})}$

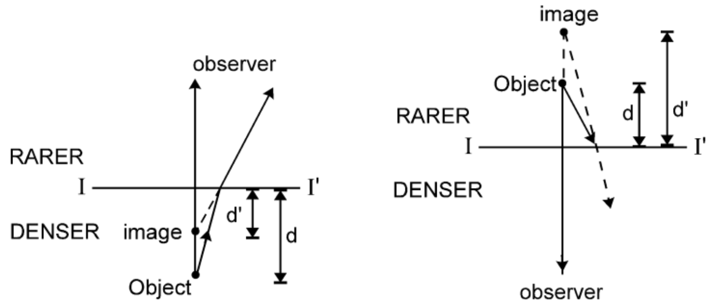
d = distance of object from the interface = real depth

d’ = distance of image from the interface = apparent depth

v = velocity of object perpendicular to interface relative to surface.

v´ = velocity of image perpendicular to interface relative to surface.

This formula can be easily derived using snell’s law and applying the condition of nearly normal incidence.... (try it or see in text book).



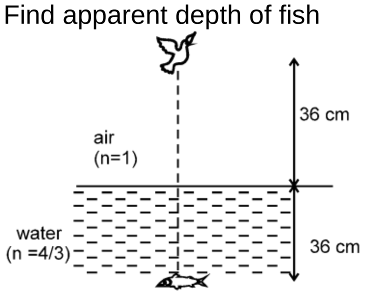
Apparent shift = $d\left(1-\dfrac{1}{n_{rel}}\right)$

Solved Examples

Example 22. An object lies 100 cm inside water .It is viewed from air nearly normally. Find the apparent depth of the object.

Solution : $d' = \dfrac{d}{n_{relative}} = \dfrac{\frac{100}{4/3}}{1} = 75$ cm

Example 23. See the figure
(i) Find apparent height of the bird
(ii) Find apparent depth of fish



(iii) At what distance will the bird appear to the fish.
(iv) At what distance will the fish appear to the bird
(v) If the velocity of bird is 12 cm/sec downward and the fish is 12 cm/sec in upward direction, then find out their relative velocities with respect to each other.

Solution :
(i) $d'_B = \dfrac{\frac{36}{1}}{\left(\frac{4}{3}\right)} = \dfrac{36}{3/4} = 48$ cm

(ii) $d'_F = \dfrac{36}{4/3} = 27$ cm

(iii) For fish : $d_B = 36 + 48 = 84$ cm

(iv) For bird : $d_F = 27 + 36 = 63$ cm.

(v) Velocity of fish with respect to bird = $12 + \left(\dfrac{\frac{12}{4/3}}{1/1}\right) = 21$ cm/sec.

Velocity of bird with respect to fish = $12 + \left(\dfrac{\frac{12}{3/4}}{1/1}\right) = 28$ cm/sec.

Example 24. See the figure. Find the distance of final

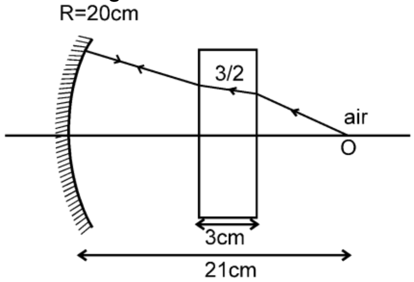


image formed by mirror

Solution : Shift = $3\left(1-\frac{1}{3/2}\right)$

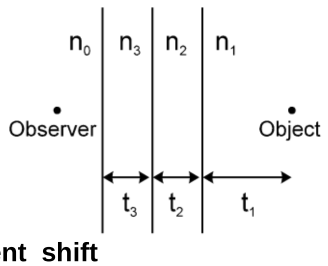
For mirror object is at a distance = 21 – $3\left(1-\frac{1}{3/2}\right)$ = 20 cm

∴ Object is at the centre of curvature of mirror. Hence the light rays will retrace and image will formed on the object itself.

8.2 Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of R. I. n_0)

Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\,rel}} + \frac{t_2}{n_{2\,rel}} + \frac{t_3}{n_{3\,rel}} + \ldots\ldots\ldots + \frac{t_n}{n_{n\,rel}}$$



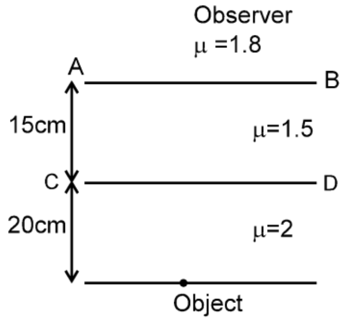
Apparent shift

$$= t_1\left[1-\frac{1}{n_{1\,rel}}\right] + t_2\left[1-\frac{1}{n_{2\,rel}}\right] + \ldots\ldots\ldots + \left[1-\frac{n}{n_{n\,rel}}\right]t_n$$

Where 't' represents thickness and 'n' represents the R.I. of the respective media, relative to the medium of observer. (i.e. $n_{1rel} = n_1/n_0$, $n_{2\,rel} = n_2/n_0$ etc.)

Solved Example

Example 25. See figure. Find the apparent depth of object seen below surface AB.



Solution : $D_{app} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)}$ = 18 + 18 = 36 cm.

Self Practice Problems

9. Monochromatic light is refracted from air into the glass of refractive index μ. The ratio of the wavelength of incident and refracted waves is
(1) 1 : μ (2) 1 : μ^2 (3) μ : 1 (4) 1 : 1

10. The index of refraction of diomond is 2.0, velocity of light in diamond in cm/second is approximately
(1) 6×10^{10} (2) 3.0×10^{10} (3) 2×10^{10} (4) 1.5×10^{10}

11. A beam of light is converging towards a point I on a screen. A plane glass plate whose thickness in the direction of the beam = t, refractive index = μ, is introducted in the path of the beam. The convergence point is shifted by
(1) $t\left(1-\frac{1}{\mu}\right)$ away (2) $t\left(1+\frac{1}{\mu}\right)$ away (3) $t\left(1-\frac{1}{\mu}\right)$ nearer (4) $t\left(1+\frac{1}{\mu}\right)$ nearer

12. When light travels from air to water and from water to glass again from glass to CO_2 gas and finally through air. The relation between their refractive indices will be given by
(1) ${}_an_\omega \times {}_\omega n_{gl} \times {}_{gl}n_{gas} \times {}_{gas}n_a = 1$ (2) ${}_an_\omega \times {}_\omega n_{gl} \times {}_{gas}n_{gl} \times {}_{gl}n_a = 1$
(3) ${}_an_\omega \times {}_\omega n_{gl} \times {}_{gl}n_{gas} = 1$ (4) There is no such relation

13. When light enters from air to water, then its
(1) Frequency increases and speed decreases
(2) Frequency is same but the wavelength is smaller in water than in air
(3) Frequency is same but the wavelength in water is greater than in air
(4) Frequency decreases and wavelength is smaller in water than in air

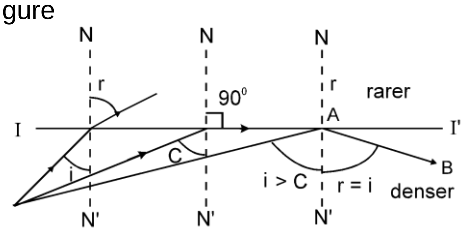
14. A mark at the bottom of a liquid appears to rise by 0.1 m. The depth of the liquid is 1m. The refractive index of the liquid is
(1) 1.33 (2) $\frac{9}{10}$ (3) $\frac{10}{9}$ (4) 1.5

Ans. **9.** (3) **10.** (4) **11.** (1) **12.** (1) **13.** (2) **14.** (3)

9 . CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION (T. I. R.)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90º. When angle in denser medium is more then critical angle the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the figure



O = Object

NN′ = Normal to the interface

II′ = Interface

C = Critical angle;

AB = reflected ray due to T. I. R.

When i = C then r = 90°

$$\therefore C = \sin^{-1}\frac{n_r}{n_d}$$

9.1 Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > c).

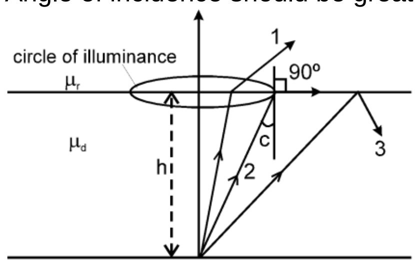


Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d . Subscript r &d stand for rarer and denser medium respectively.

In the figure ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance (C.O.I.)**. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object can not be seen from the rarer medium. Radius of C.O.I can be easily found.

Solved Examples

Example 26. Find the max. angle that can be made in glass medium (μ = 1.5) if a light ray is refracted from glass to vacuum.

Solution : 1.5 sin C = 1 sin 90°, where C = critical angle.

sin C = 2/3

C = $\sin^{-1}$ 2/3

Example 27. Find the angle of refraction in a medium (μ = 2) if light is incident in vacuum, making angle equal to twice the critical angle.

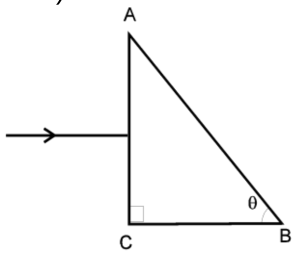
Solution : Since the incident light is in rarer medium. Total Internal Reflection can not take place.

$C = \sin^{-1}\frac{1}{\mu} = 30^\circ \quad \Rightarrow \quad \therefore \quad i = 2C = 60^\circ$

Applying Snell's Law. 1 sin 60° = 2 sin r

$\sin r = \frac{\sqrt{3}}{4} \quad \Rightarrow \quad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$.

Example 28. What should be the value of angle θ so that light entering normally through the surface AC of a prism (n=3/2) does not cross the second refracting surface AB.



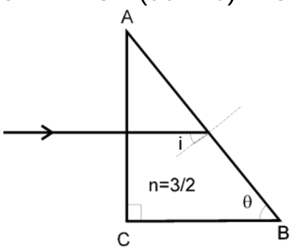
Solution : Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

i = 90-θ

For the required condition:

90° – θ > C

or sin (90° – θ) > sinC



or $\cos\theta > \sin C = \frac{1}{3/2} = \frac{2}{3}$ or $\theta < \cos^{-1}\frac{2}{3}$.

Example 29. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod.

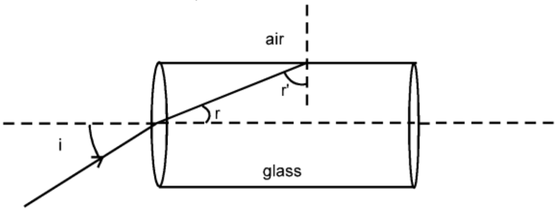
Solution : It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle ....(A)

Angle r' is minimum when r is maximum i.e. C(why ?).Therefore the minimum value of r'is 90-C.

From condition (A) :

90° – C > C or C < 45°

sin C < sin 45° ; $\frac{1}{n} < \frac{1}{\sqrt{2}}$ or n> $\sqrt{2}$.



Self Practice Problems

15. A cut diamond sparkles because of its

(1) Hardness
(2) High refractive index
(3) Emission of light by the diamond
(4) Absorption of light by the diamond

16. Critical angle of light passing from glass to air is minimum for

(1) Red (2) Green (3) Yellow (4) Violet

17. For total internal reflection to take place, the angle of incidence i and the refractive index μ of the medium must satisfy the inequality

(1) $\frac{1}{\sin i} < \mu$ (2) $\frac{1}{\sin i} > \mu$ (3) $\sin i < \mu$ (4) $\sin i > \mu$

18. Critical angle is that angle of incidence in the denser medium for which the angle of refraction in rarer medium is

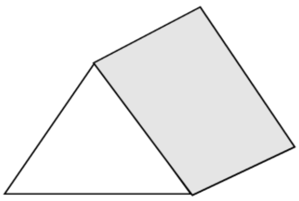
(1) 0° (2) 57° (3) 90° (4) 180°

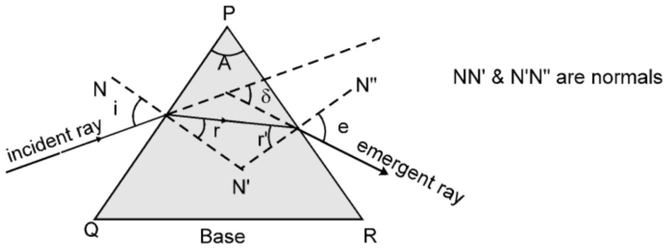
Ans. **15.** (2) **16.** (4) **17.** (1) **18.** (3)

10. CHARACTERISTICS OF A PRISM

(a) **A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism :**

3-D view





Refraction through a prism:

View from one side

(b) PQ and PR are refracting surfaces.

(c) ∠QPR = A is called refracting angle or the angle of prism (also called Apex angle).

(d) δ = angle of deviation

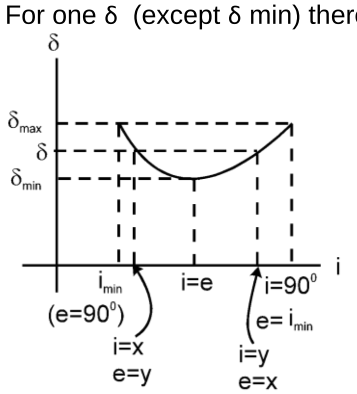
(e) For refraction of a monochromatic (single wave length) ray of light through a prism;

$\delta = (i + e) - (r_1 + r_2)$ and $r_1 + r_2 = A$

∴ $\boldsymbol{\delta = i + e - A.}$

(f) Variation of δ versus i (shown in diagram).

For one δ (except δ min) there are two values



of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light

Note : (i) For application of above result medium on both sides of prism must be same.

(ii) Based on above graph we can also derive following result, which says that i and e can be interchanged for a particular deviation in other words there are two angle of incidence for a given deviation (except minimum deviation).

i	r_1	r_2	e	δ
θ_1	θ_2	θ_4	θ_4	θ_5
θ_4	θ_3	θ_2	θ_1	θ_5

(g) There is one and only one angle of incidence for which the angle of deviation is minimum.

(h) When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that

$\delta_{min} = 2i_{min} - A$

where i_{min} = angle of incidence for minimum deviation, and r = A/2.

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1)\,A$ (for small values of $\angle A$)

(i) For a thin prism ($A \le 10_o$) and for small value of i, all values of

$$\boldsymbol{\delta = (n_{rel} - 1)\,A} \qquad \text{where } n_{rel} = \frac{n_{prism}}{n_{surrounding}}$$

Solved Examples

Example 30. Refracting angle of a prism A = 60º and its refractive index is, n = 3/2, what is the angle of incidence i to get minimum deviation. Also find the minimum deviation. Assume the surrounding medium to be air (n = 1).

Solution : For minimum deviation,

$r_1 = r_2 = \; = 30^o$.

applying snell's law at I surface

$1 \times \sin i = \sin 30^o \quad \Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right) \quad \Rightarrow \quad \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - \frac{\pi}{3}$

Example 31. See the figure Find the deviation caused by a prism having refracting angle 4º and refractive index $\frac{3}{2}$.

Solution : $\delta = (\frac{3}{2} - 1) \times 4_0 = 2_0$

11. DISPERSION OF LIGHT

The angular splitting of a ray of white light into a number of components and spreading in different directions is called *Dispersion of Light.* [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

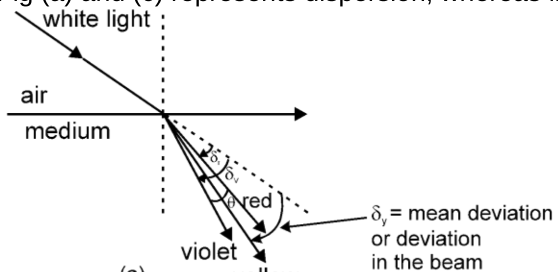
Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $n(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

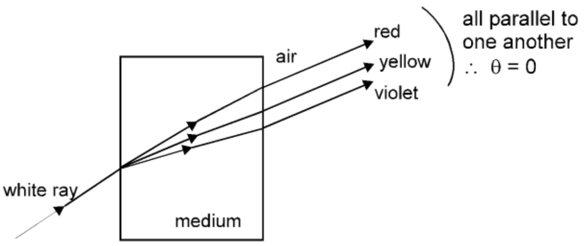
Note : Such phenomenon is not exhibited by sound waves.

Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.** $\theta = \delta_v - \delta_r$ (Fig. (a))

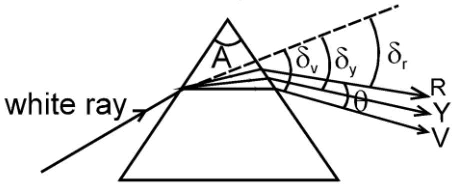
Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



(a)



(b)



(c)

For prism of small 'A' and with small 'i' :

$\theta = \delta_v - \delta_r = (n_v - n_r)A$

Deviation of beam (also called mean deviation)

$\delta = \delta_y = (n_y - 1)A$

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

Solved Examples

Example 32. The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Solution : Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)\,A = (1.632 - 1.613) \times 5^\circ = 0.095^\circ$.

Note : Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

Dispersive power (ω) of the medium of the material of prism is given by: $\omega = \frac{n_v - n_r}{n_y - 1}$

- ω is the property of a medium.

  For small angled prism ($A \le 10^\circ$) with light incident at small angle i :

  $$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

  [$n_y = \frac{n_v + n_r}{2}$ if n_y is not given in the problem]

- $n - 1$ = refractivity of the medium for the corresponding colour.

Example 33. Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively. Find
(a) the ref. index for yellow colour, approximately
(b) Dispersive power of the medium.

Solution : (a) $\mu_r \simeq = \dfrac{\mu_v + \mu_R}{2} = \dfrac{1.50 + 1.60}{2}$ 1.55

(b) $\omega = \dfrac{\mu_v - \mu_R}{\mu_r - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

11.1 Dispersion without deviation (Direct Vision Combination)

The condition for direct vision combination is :

$$\left[n_y - 1\right] A = \left[n'_y - 1\right] A' \Leftrightarrow \left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

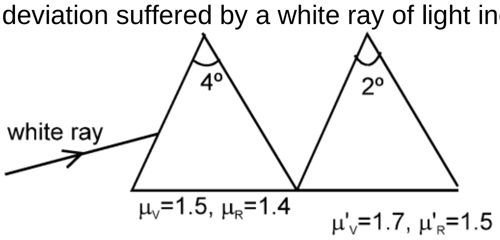
Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.

11.2 Deviation without dispersion (Achromatic Combination)

Condition for achromatic combination is: $(n_v - n_r) A = (n'_v - n'_r) A'$

Solved Examples

Example 34. If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution : Both prisms will turn the light rays towards their bases and hence in same direction. Therefore turnings caused by both prisms are additive. Total angular dispersion

$= \theta + \theta' = (\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A'$

$= (1.5 - 1.4) 4^o + (1.7 - 1.5)2^o = 0.8°$

Total deviation

$= \delta + \delta'$

$= \left(\dfrac{\mu_V + \mu_R}{2} - 1\right) A + \left(\dfrac{\mu'_V + \mu'_R}{2} - 1\right) A' = \left(\dfrac{1.5 + 1.4}{2} - 1\right) 0.4^o + \left(\dfrac{1.7 + 1.5}{2} - 1\right) 0.2^o$

$= (1.45 - 1)\ 0.4^o + (1.6 - 1)\ 0.2^o$

$= 0.45 \times 0.4^o + 0.6 \times 0.2^o$

$= 1.80 + 1.2 = 3.0^o$ Ans.

Example 35. Two thin prisms are combined to form an achromatic combination. For I prism A = 4º, μ_R = 1.35, μ_Y = 1.40, μ_v = 1.42. for II prism μ'_R= 1.7, μ'_Y = 1.8 and μ'_R = 1.9 find the prism angle of II prism and the net mean deviation.

Solution : Condition for achromatic combination.

$\theta = \theta'$

$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A'$

$$\therefore A' = \frac{(1.42-1.35)4^\circ}{1.9-1.7} = 1.4^\circ$$

$\delta_{Net} = \delta \sim \delta' = (\mu_Y - 1)A \sim (\mu'_Y - 1)A' = (1.40 - 1)\ 4^\circ \sim (1.8 - 1)\ 1.4^\circ = 0.48^\circ$.

Example 36. A crown glass prism of angle 5_0 is to be combined with a flint prism in such a way that the mean ray passes undeviated. Find (a) the angle of the flint glass prism needed and (b) the angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.5, 1.6 and 1.7 respectively for crown glass and 1.8,2.0 and 2.2 for flint glass.

Solution : The deviation produced by the crown prism is

$\delta = (\mu - 1)A$

and by the flint prism is :

$\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is :

$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$. ....(1)

(a) If the net deviation for the mean ray is zero,

$(\mu - 1)A = (\mu' - 1)A'$.

or, $$A' = \frac{(\mu-1)}{(\mu'-1)} A = \frac{1.6-1}{2.0-1} \times 5^0 = 3_0$$

(b) The angular dispersion produced by the crown prism is :

$\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is,

$\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is,

$(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A$

$= (1.7 - 1.5) \times 5_0 - (2.2 - 1.8) \times 3_0$

$= -0.2_0$.

The angular dispersion has magnitude 0.2_0 .

Self Practice Problems

19. Formula for dispersive power is (where symbols have their usual meanings) or If the refractive indices of crown glass for red, yellow and violet colours are respectively , , and , then the dispersive power of this glass would be

(1) $\frac{\mu_\upsilon - \mu_y}{\mu_r - 1}$ (2) $\frac{\mu_\upsilon - \mu_r}{\mu_y - 1}$ (3) $\frac{\mu_\upsilon - \mu_y}{\mu_y - \mu_r}$ (4) $\frac{\mu_\upsilon - \mu_r}{\mu_y} - 1$

20. Angle of minimum deviation for a prism of refractive index 1.5 is equal to the angle of prism. The angle of prism is (cos 41° = 0.75)

(1) 62° (2) 41° (3) 82° (4) 31°

21. In the formation of primary rainbow, the sunlight rays emerge at minimum deviation from rain-drop after

(1) One internal reflection and one refraction (2) One internal reflections and two refractions
(3) Two internal reflections and one refraction (4) Two internal reflections and two refractions

22. Dispersive power depends upon

(1) The shape of prism (2) Material of prism (3) Angle of prism (4) Height of the prism

Ans. **19.** (2) **20.** (3) **21.** (2) **22.** (2)

12. SPECTRUM

(Only for your knowledge and not of much use for JEE)

Ordered pattern produced by a beam emerging from a prism after refraction is called *Spectrum*.

12.1 Types of spectrum:

(a) Line spectrum:
Due to source in atomic state.

(b) Band spectrum:
Due to source in molecular state.

(c) Continuous spectrum:
Due to white hot solid.

12.2 In Emission spectrum

Bright colours or lines, emitted from source are observed.

The spectrum emitted by a given source of light is called emission spectrum. It is a wavelength-wise distribution of light emitted by the source. The emission spectra are given by incandescent solids, liquids and gases which are either burned directly as a flame (or a spark) or burnt under low pressure in a discharge tube.

12.3 In Absorption spectrum

Dark lines indicates frequencies absorbed.

When a beam of light from a hot source is passed through a substance (at a lower temperature), a part of the light is transmitted but rest of it is absorbed. With the help of a spectrometer, we can know the fraction of light absorbed corresponding to each wavelength. The distribution of the wavelength absorption of light by a substance is called an absorption spectrum. Every substance has its own characteristic absorption spectrum.

12.4 Spectrometer

Consists of a collimator (to collimate light beam), prism and telescope. It is used to observe the spectrum and also measure deviation.

13. REFRACTION AT SPHERICAL SURFACES

For paraxial rays incident on a spherical surface separating two media:

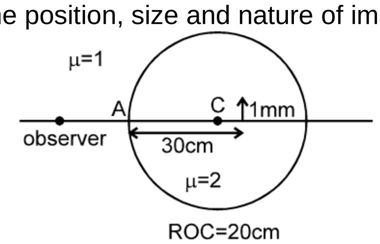
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \text{.................... (A)}$$

where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by **m =** $\frac{v-R}{u-R} = \left(\frac{v/n_2}{u/n_1}\right)$

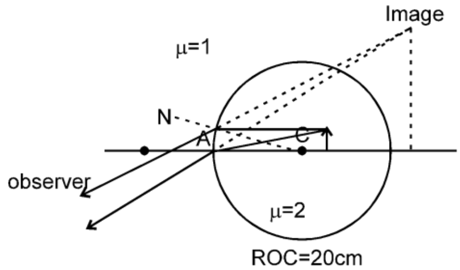
Solved Example

Example 37. Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram .



Solution : For refraction near point A, u = – 30 ; R = – 20; n_1 = 2 ; n_2 = 1.
Applying refraction formula

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \quad \Rightarrow \quad \frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$



v = – 60 cm

m = $\frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)}$ = 4

∴ h_2 = 4 mm.

Special case:
Refraction at plane Surfaces

Putting R = in the formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, we get;

v = $\frac{n_2 u}{n_1}$

The same sign of v and u implies that the object and the image are always on the same side of the interface separating the two media. If we write the above formula as

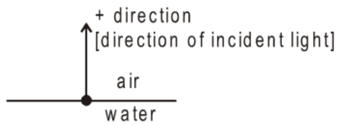
v = $\frac{u}{n_{rel}}$,

it gives the relation between the apparent depth and real depth, as we have seen before.

Example 38. Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen near normally from air.

Solution : Put R = in the formula of the Refraction at Spherical Surfaces we get,

v = $\frac{un_2}{n_1}$ ⇒ u = – 10 cm

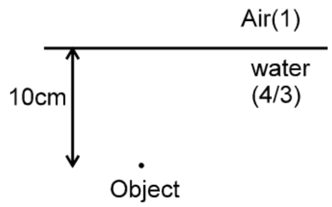


$n_1 = \frac{4}{3}$ ⇒ n_2 = 1 ⇒ v = – $\frac{10 \times 1}{4/3}$ = – 7.5 cm

negative sign implies that the image is formed in water.

Aliter:

$$d_{app} = \frac{d_{real}}{\mu_{rel}} = \frac{10}{4/3} = \frac{30}{4} = 7.5 \text{ cm.}$$

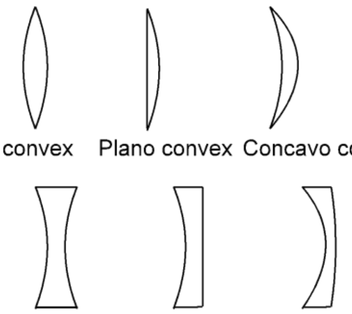


14. THIN LENS

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends. One surface of a convex lens is always convex . Depending on the other surface a convex lens is categorized as

(a) biconvex or convexo convex , if the other surface is also convex,
(b) Plano convex if the other surface is plane and
(c) Concavo convex if the other surface is concave.

Similarly concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.



For a spherical, thin lens *having the same medium on both sides*:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{.........(a),}$$

where $n_{rel} = \frac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x coordinates of the centre of curvature of the 1^{st} surface and 2^{nd} surface respectively.

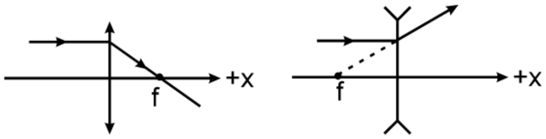
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \rightarrow \text{Lens Maker's Formula................(b)}$$

Lens has two Focii:

If $u = \infty$, then $\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$ $\Rightarrow$ $v = f$

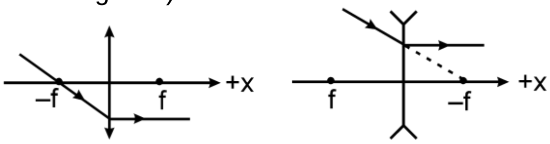
$\Rightarrow$ If incident rays are parallel to principal axis then its refracted ray will cut the principal axis at ‘f’.
It is called 2^{nd} focus.

In case of converging lens it is positive and in case of diverging lens it is negative.



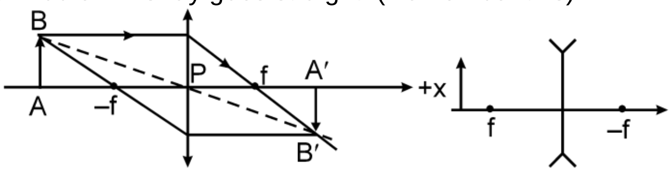
If v = ∞ that means $\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$ ⇒ u = – f

If incident rays cuts principal axis at – f then its refracted ray will become parallel to the principal axis. It is called 1st focus. In case of converging lens it is negative (∵ f is positive) and in the case of diverging lens it positive (∵ f is negative)



use of – f & + f is in drawing the ray diagrams.

Notice that the point B, its image B′ and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight. (Remember this)



From the relation $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ it can be seen that the second focal length depends on two factors.

(A) The factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is
- **(i)** **Positive** for all types of **convex** lenses and
- **(ii)** **Negative** for all types of **concave** lenses.

(B) The factor (n_{rel} − 1) is
- **(i)** **Positive** when **surrounding medium is rarer than the medium of lens.**
- **(ii)** **Negative** when **surrounding medium is denser than the medium of lens.**

(C) So a lens is **converging** if **f is positive** which happens when **both the factors (A) and (B) are of same sign**.

(D) And a lens is **diverging** if **f is negative** which happens when the **factors (A) and (B) are of opposite signs**.

(E) Focal length of the lens depends on medium of lens as well as surrounding.

(F) It also depends on wavelength of incident light. Incapability of lens to focus light rays of various wavelengths at single point is known as **chromatic aberration**.

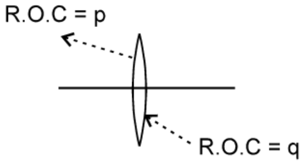
Solved Examples

Example 39. Find the behavior of a concave lens placed in a rarer medium.

Solution : Factor (A) is **negative** , because the lens is **concave**.
Factor (B) is **positive** , because the **lens is placed in a rarer medium**.
Therefore the focal length of the lens, which depends on the product of these factors , is negative and hence the lens will behave as diverging lens.

Example 40. Show that the factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ (and therefore focal length) does not depend on which surface of the lens light strike first.

Solution : Consider a convex lens of radii of curvature p and q as shown.



CASE 1: Suppose light is incident from left side and strikes the surface with radius of curvature p, first.

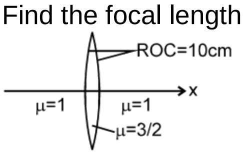
Then $R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{p}-\frac{1}{-q}\right)$

CASE 2: Suppose light is incident from right side and strikes the surface with radius of curvature q, first.

Then $R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{q}-\frac{1}{-p}\right)$

Though we have shown the result for biconvex lens , it is true for every lens.

Example 41. Find the focal length of the lens shown in the figure.

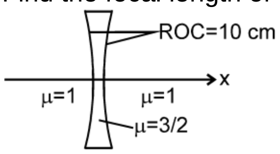


converging lens

Solution : $\because \quad \frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$\Rightarrow \quad \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10}-\frac{1}{(-10)}\right) \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{2} \times \frac{2}{10} \quad \Rightarrow \quad f = +10$ cm.

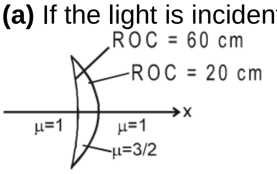
Example 42. Find the focal length of the lens shown in figure



Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{-10}-\frac{1}{10}\right)$ $f = -10$ cm

Example 43. Find the focal length of the lens shown in figure

(a) If the light is incident from left side.



(b) If the light is incident from right side.

Solution : (a) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{-60}-\frac{1}{-20}\right)$ $f = 60$ cm

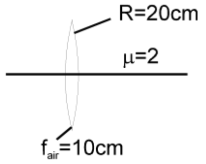
(b) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{20}-\frac{1}{60}\right)$ $f = 60$ cm

Example 44. Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss about the position of the image formed .

Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$ [$\because$ $R_1 = R_2$]

$\frac{1}{v} - \frac{1}{u} = 0$ or $v = u$ i.e. rays pass without appreciable bending.

Example 45. Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index μ=2. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length



Solution : Let radius of I surface be R_1 and refractive index of lens be μ.
Let parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{V_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \quad ...(1)$$

At II surface $\frac{2}{V} - \frac{\mu}{V_1} = \frac{2-\mu}{-20}$ ...(2)

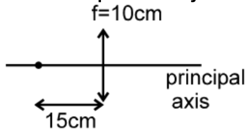
Adding (1) and (2)

$$\frac{\mu}{V_1} - \frac{1}{\infty} + \frac{2}{V} - \frac{\mu}{V_1} = \frac{\mu-1}{R_1} + \frac{2-\mu}{-20}$$

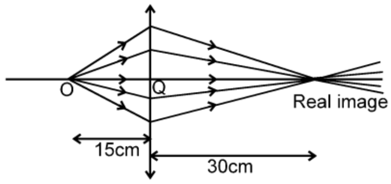
$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu-1}{20} - \frac{2-\mu}{20} = \frac{1}{f} \text{ (in air)} + \frac{1}{20} - \frac{2}{20}$$

$\Rightarrow$ v = 40 cm $\Rightarrow$ f = 40 cm

Example 46. Figure shown a point object and a converging lens. Find the final image formed.

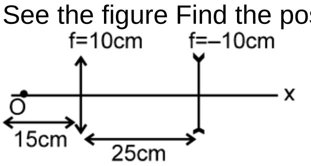


Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$



v = + 30 cm

Example 47. See the figure Find the position of final image formed.



Solution : For converging lens

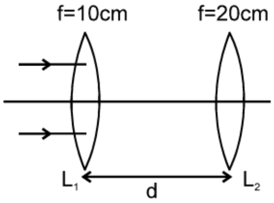
$u = -15$ cm, $f = 10$ cm $\qquad v = \frac{fu}{f+u} = 30$ cm

For diverging lens

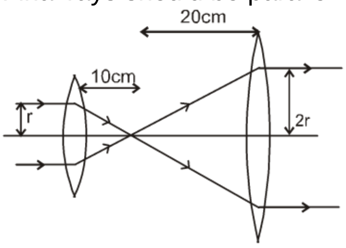
$u = 5$ cm

$f = -10$ cm $\qquad v = \frac{fu}{f+u} = 10$ cm

Example 48. Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.



Solution : Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2.



$d = 10 + 20 = 30$ cm

Here the diameter of ray beam becomes wider.

14.2 Transverse magnification (m)

Transverse magnification (m) of (of dimension perpendicular to principal axis) is given by

$$m = \frac{v}{u}$$

If the lens is thick or/and the medium on both sides is different, then we have to apply the formula given for refraction at spherical surfaces step by step.

Solved Example

Example 49. An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30 cm.

(i) Find the lateral magnification produced by the lens.

(ii) Find the height of the image.

(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Solution : Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and $m = \frac{v}{u}$

we get $m = \frac{f}{f+u}$ ± ....(A) $\therefore$ $m = \frac{+20}{+20+(-30)} = \frac{+20}{-10} = -2$

–ve sign implies that the image is inverted.

(ii) $\frac{h_2}{h_1} = m$ $\qquad \therefore$ $h_2 = \frac{-f}{(f+u)^2}$ $mh_1 = (-2)(2) = -4$ cm

(iii) Differentiating (A) we get

dm = $\frac{-f}{(f+u)^2}$ du = $\frac{-(20)}{(-10)^2}$ (0.1) = $\frac{-2}{100}$ = –.02

Note that the method of differential is valid only when changes are small.

Alternate method :

u (after displacing the object)

= –(30 + 0.1) = – 29.9 cm

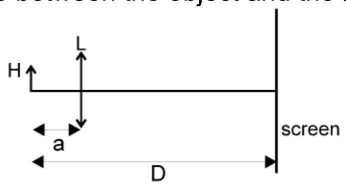
Applying the formula

m = $\frac{f}{f+u}$; m = $\frac{20}{20+(-29.9)}$ = – 2.02 ∴ change in 'm' = – 0.02.

Since in this method differential is not used, this method can be used for changes, small or large.

14.3 Displacement method to find focal length of converging lens :

Fix an object of small height H and a screen at a distance D from object (as shown in figure). Move a converging lens from the object towards the screen. Let a sharp image forms on the screen when the distance between the object and the lens is 'a'. From lens formula we have



$\frac{1}{D-a} - \frac{1}{-a} = \frac{1}{f}$ or $a^2 - Da + fD = 0$ ...(A)

This is quadratic equation and hence two values of 'a' are possible. Call them a_1 and a_2. Thus a, and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

∴ $a_1 + a_2 = D$ ⇒ $a_1a_2 = fD$

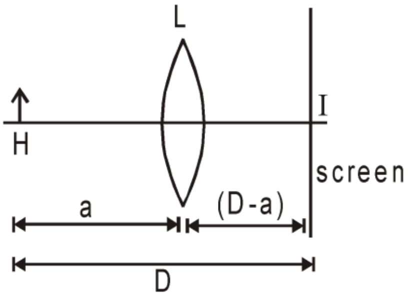
Also $(a_1 - a_2) = \sqrt{(a_1+a_2)^2 - 4a_1a_2} = \sqrt{D^2 - 4fD}$ = d (suppose).

'd' physically means the separation between the two position of lens.

The focal length of lens in terms of D and d.

so, $a_1 - a_2 = \sqrt{(a_1+a_2)^2 - 4a_1a_2}$

$\sqrt{D^2 - 4fD}$ = d ⇒ $\boxed{f = \frac{D^2 - d^2}{4D}}$



condition, d = 0, i.e. the two position coincide

f = $\frac{D^2}{4D}$ ∴ $\boxed{D = 4f}$

Roots of the equation $a^2 - Da + fD = 0$, become imaginary if

$b^2 - 4ac < 0. \quad = D^2 - 4fD < 0 \quad = D(D - 4f) < 0 = \boxed{D - 4f < 0}$

for real value of a in equation $a^2 - Da + fD = 0$

$b^2 - 4ac \geq 0. \quad = D^2 - 4fD \geq 0.$

so, $\quad D \geq 4f \quad \Rightarrow \quad D_{min} = 4f$

Lateral magnification in displacement method:

if m_1 and m_2 be two magnifications in two positions (In the displacement method)

$$m_1 = \frac{v_1}{u_1} = \frac{(D - a_1)}{-a_1} \qquad m_2 = \frac{v_2}{u_2} = \frac{D - a_2}{-a_2} = \frac{a_1}{-(D - a_1)}$$

So $\quad m_1 m_2 = \dfrac{(D - a_1)}{-a_1} \times \dfrac{a_1}{-(D - a_1)} = 1.$

If image length are h_1 and h_2 in the two cases,

then $m_1 = -\dfrac{h_1}{H}$; $\quad m_2 = -\dfrac{h_2}{H}$; $m_1 m_2 = 1$

$\therefore \dfrac{h_1 h_2}{H^2} = 1; \quad h_1 h_2 = H^2$; $H = \sqrt{h_1 \; h_2}$

Self Practice Problems

23. The radius of curvature for a convex lens is 40 cm, for each surface. Its refractive index is 1.5. The focal length will be

(1) 40 cm (2) 20 cm (3) 80 cm (4) 30 cm

24. A thin lens focal length f_1 and its aperture has diameter d . It forms an image of intensity I. Now the central part of the aperture upto diameter d/2 is blocked by an opaque paper. The focal length and image intensity will change to

(1) $\dfrac{f}{2}$ and $\dfrac{I}{2}$ (2) f and $\dfrac{I}{4}$ (3) $\dfrac{3f}{4}$ and $\dfrac{I}{2}$ (4) f and $\dfrac{3I}{4}$

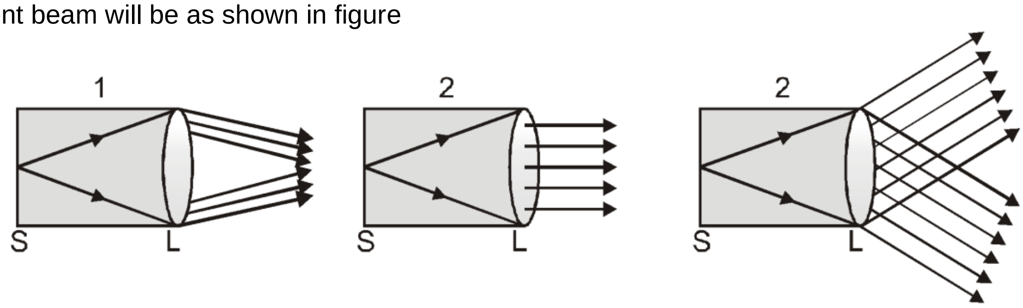
25. A lens of power +2 diopters is placed in contact with a lens of power –1 diopter. The combination will behave like

(1) A convergent lens of focal length 50 cm
(2) A divergent lens of focal length 100 cm
(3) A convergent lens of focal length 100 cm
(4) A convergent lens of focal length 200 cm

26. If in a plano - canvex lens, the radius of curvature of the convex surface is 10 cm and the focal length of the lens is 30 cm, then the refractive index of the material of lens will be

(1) 1.5 (2) 1.66 (3) 1.33 (4) 3

27. The silt of a collimator is illuminated by a source as shown in the adjoining figures. The distance between the silt S and the collimating lens L is equal to the focal length of the lens. The correct direction of the emergent beam will be as shown in figure



(1) 1 (2) 3 (3) 2 (4) None of the figures

28. A convex lens makes a real image 4 cm long on a screen. When the lens is shifted to a new position without disturbing the object, we again get a real image on the screen which is 16 cm tall. The length of the object must be

(1) 1/4 cm (2) 8 cm (3) 12 cm (4) 20 cm

29. A thin convex lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in liquid of refractive index 4/3, its focal length will be

(1) 15 cm (2) 10 cm (3) 30 cm (4) 60 cm

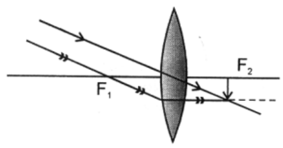
Ans. **23.** (1) **24.** (4) **25.** (3) **26.** (3) **27.** (3) **28.** (2) **29.** (4)

IMAGE FORMATION BY A LENS

Position of Object | **Details of Image** | **Figure**

At infinity

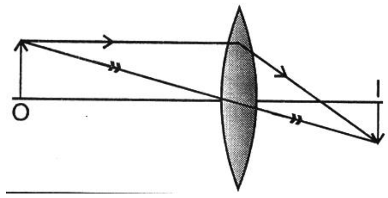
Real, Inverted Diminished



(m <<–1) At F

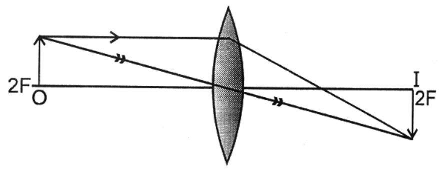
Between and 2F

Real inverted Diminished
(m < –1) between F and 2F



At 2F

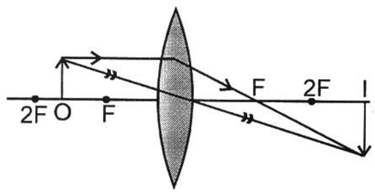
Real inverted equal



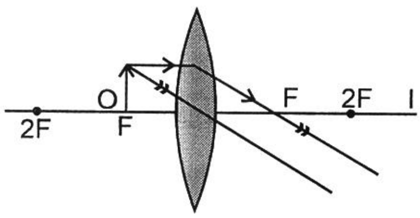
m = –1 At 2F

Between 2F and F

Real, inverted Enlarged



(m > – 1) between 2F and



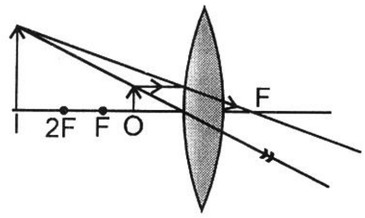
At F

Real inverted Enlarged
(m >> –1) At inifinity

Between Focus &

Virtual, erect enlarged

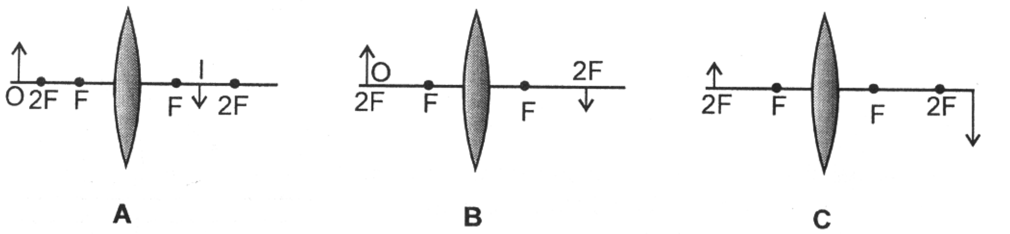
Pole (m > +1) between



∞ and Object on same side

(b) For real extended object, if the image formed by a single lens in inverted (i.e. m is negative) it is always real and the lens is convergent i.e., convex. In this situation if the size of image is -

Smaller than object	Equal to object	Larger than object
Object between and 2F	Object is at 2F	Object between 2F and F
Image is between F and 2F	Image is at 2F	Image is between 2F and



15. COMBINATION OF LENSES

The equivalent focal length of thin lenses in contact is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}...$$

where f_1 , f_2 , f_3 are focal lengths of individual lenses.

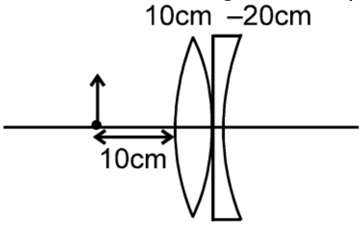
If two converging lenses are separated by a distance d and the incident light rays are parallel to the common principal axis ,then the combination behaves like a single lens of focal length given by the relation

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$$

and the position of equivalent lens is $\frac{-d \quad F}{f_1}$ with respect to 2nd lens

Solved Examples

Example 50. Find the lateral magnification produced by the combination of lenses shown in the figure.

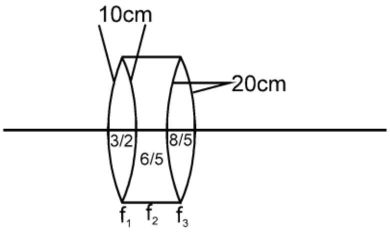


Solution : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$ ⇒ f = + 20

∴ $\frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$ ⇒ $\frac{1}{v} = \frac{1}{20} - \frac{1}{10} = \frac{-1}{20}$ = – 20 cm

∴ m = = 2

Example 51. Find the focal length of equivalent system.



Solution :

$$\frac{1}{f_1} = \left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$$

$$\frac{1}{f_2} = \left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right) = \frac{1}{5} \times \left(\frac{-30}{10\times 20}\right) = \frac{-3}{100}$$

$$\frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{3}{50}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{10} + \frac{-3}{100} + \frac{3}{50} \qquad f = \frac{100}{13}$$ **Ans.**

Self Practice Problems

30. Two similar plano-convex lenses are combined together in three different ways as shown in the adjoining figure. The ratio of the focal lengths in three cases will be

(1) 2 : 2 : 1 (2) 1 : 1 : 1 (3) 1 : 2 : 2 (4) 2 : 1 : 1

31. A concave and convex lens have the same focal length of 20 cm and are put into contact to form a lens combination. The combination is used to view an object of 5 cm length kept at 20 cm from the lens combination. As compared to the object, the image will be

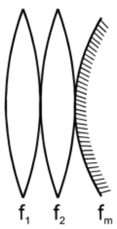
(1) Magnified and inverted (2) Reduced and erect
(3) Of the same size as the object and erect (4) Of the same size as the object but inverted

Ans. **30.** (2) **31.** (3)

16. COMBINATION OF LENS AND MIRROR

The combination of lens and mirror behaves like a mirror of focal length ‘f’ given by

$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}$$



If lenses are more then one, ‘f’ is given by

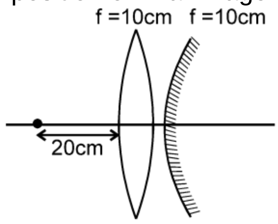
$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\sum \frac{1}{f_\ell}\right)$$

For the following figure

'f' is given by $\frac{1}{f} = \frac{1}{F_m} - 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$

Solved Example

Example 52. Find the position of final image formed. (The gap shown in figure is of negligible width)



Solution : $\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10} \Rightarrow$ $f_{eq} = -10$ cm

$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10} \Rightarrow$ $v = -20$ cm

Hence image will be formed on the object itself

Self Practice Problems

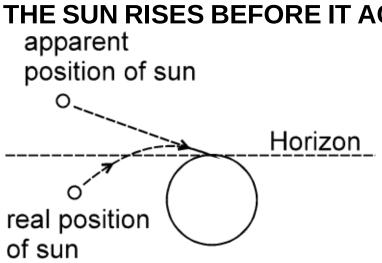
32. A plano - convex lens of focal length 20 cm silvered at the plane surface will behave as convergent mirror of focal length.

(1) 20 cm (2) 30 cm (3) 40 cm (4) 10 cm

Ans. **32.** (4)

Some interesting facts about light :

(1) THE SUN RISES BEFORE IT ACTUALLY RISES AND SETS AFTER IT ACTUALLY SETS :



The atmosphere is less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height. Due to this the light rays bend as they move in the earth's atmosphere

(2) THE SUN IS OVAL SHAPED AT THE TIME OF ITS RISE AND SET :

The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) THE STARS TWINKLE BUT NOT THE PLANETS :

The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

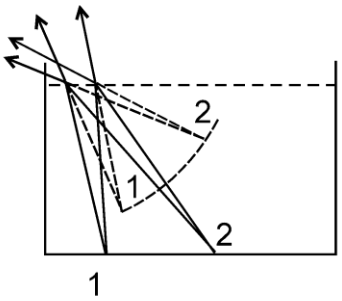
(4) GLASS IS TRANSPARENT, BUT ITS POWDER IS WHITE :

When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) GREASED OR OILED PAPER IS TRANSPARENT, BUT PAPER IS WHITE :

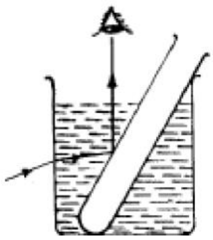
The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased, very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

(6) AN EXTENDED WATER TANK APPEARS SHALLOW AT THE FAR END :



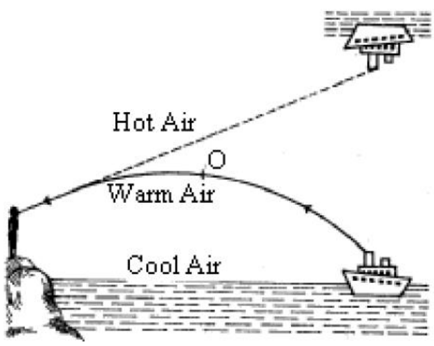
(7) A TEST TUBE OR A SMOKED BALL IMMERSED IN WATER APPEARS SILVERY WHITE WHEN VIEWED FROM THE TOP :

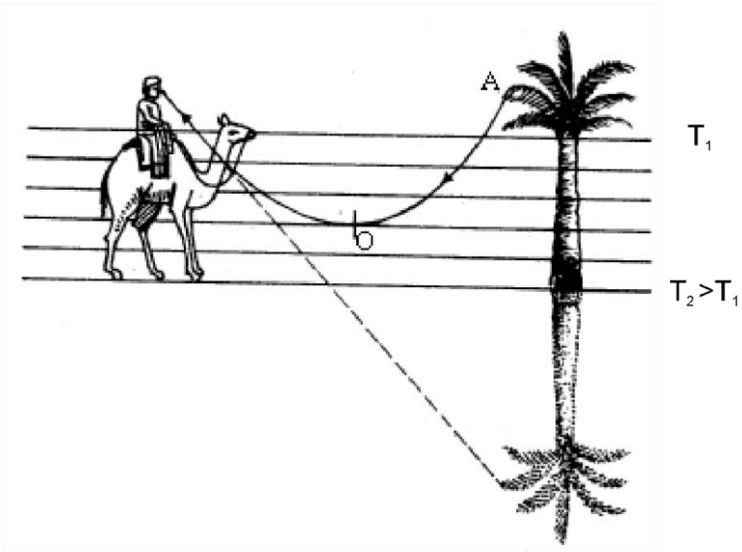
This is due to Total internal reflection



(8) SHIPS HANG INVERTED IN THE AIR IN COLD COUNTRIES AND TREES HANG INVERTED UNDERGROUND IN DESERTS:

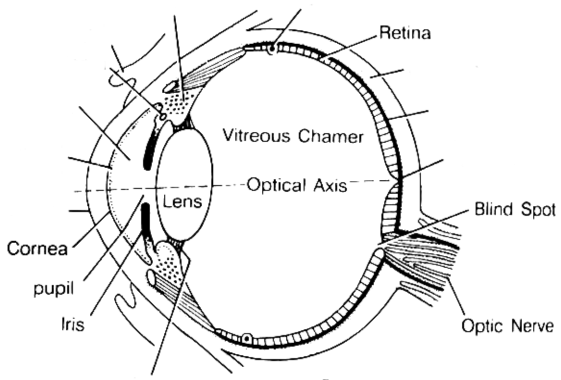
This is due to Total internal reflection





17. STRUCTURE OF EYE :

Light enters the eye through a curved front surface, the corner. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is released, the focal length is about 2.5 cm and (for a normal eye) objects at infinity are in sharp focus on the retinas. When the object is brought closer to the eye, in order to maintain the same image-lens distance (2.5 cm), the focal length of the eye-lens becomes shorter by the action of the ciliary muscles. This property of the eye in called accommodation. If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred.



Human Eye

The closest distance for which the lens can focus light on the retina is called the least distance of distinct vision, or the near point. The standard value (for normal vision) taken here is 25 cm. (Often the near point is given the symbol D.)

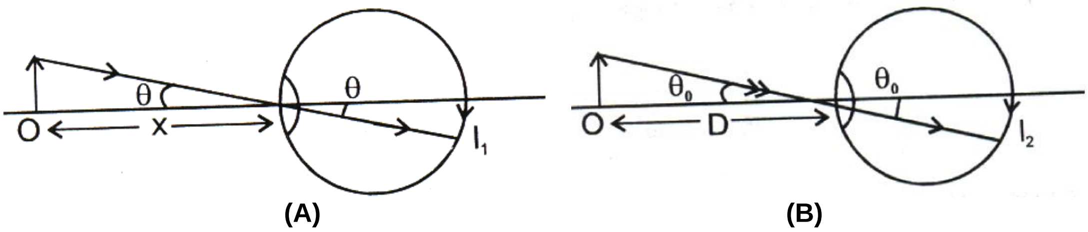
18. DEFECTS OF VISION

Regarding eye it is nothing that:

(1) In eye convex eye-lens forms real inverted and diminished image at the retina by changing its convexity (the distance between eye lens and retina is fixed)

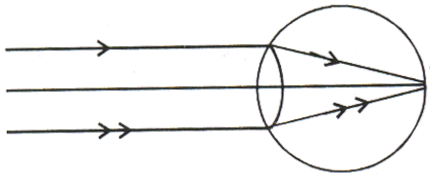
(2) The human eye is most sensitive to yellow green light having wavelength 5550 A_0 and least to violet (4000 A_0) and red (7000 A_0)

(3) The size of an object as perceived by eye depends on its visual-angle when object is distant its visual angle and hence image I_1 at retina is small (it will appear small) and as it is brought near to the eye its visual angle and hence size of image I_2 will increase.



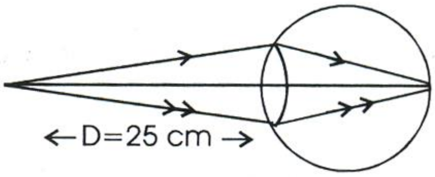
(4) The far and near point for normal eye are usually taken to be infinity and 25 cm respectively ie., normal eye can see very distant object clearly but near objects only if they are at distance greater than 25 cm from the eye. The ability of eye to see objects from infinite distance to 25 cm from it is called Power of accommodation.

(5) If object is at infinity i.e., parallel beam of light enters the eye is least strained and said to be relaxed or unstrained. However, if the object is at least distance of distint vision (L.D.D.V] i.e., D (=25 cm) eye is under maximum strain and visual angle is maximum.



Relaxed or normal eye

(A)



Maximum strained eye

(B)

(6) The limit of resolution of eye is one minute ie., two object will not be visible distintely to the eye if the angle substanded by them on the eye is lesser than one minute.

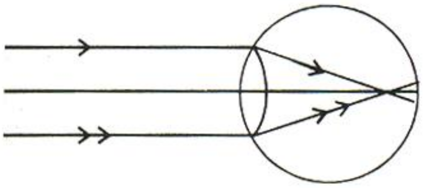
(7) The persistance of vision is (1/10) sec i.e., If time interval between two consecutive light pulses is lesser than 0.1 sec eye cannot distinguish them separately. This fact is taken into account in motion pictures.

In case of eye following are the common defects of vision.

18.1 MYOPIA

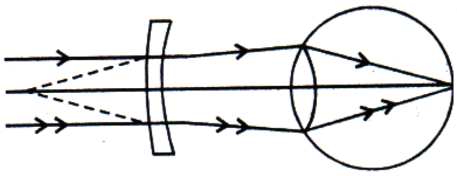
[or short-sightendness or near - sightendness]

In it distant objects are not clearly visible. i.e. Far Point is at a distance lesser than Infinity and hence image of distant object is formed before the retina.



Defective-eye

(A)



Corrected-eye

(B)

This defect is (i.e., negative focal length or power) which forms the image of distant object at the far point of patient - eye [which is lesser than] so that in this case from lens formula we have

$$\frac{1}{-F.P} - \frac{1}{-(\text{distance of object})} = \frac{1}{f} = P$$

And if the object is at $P = \frac{1}{f} = \frac{1}{-F.P.}$ .... (1)

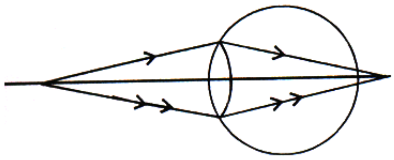
where F.P is farther point of eye

18.2 HYPERMETROPIA

[Or Long-sightendness or far-sightendness]

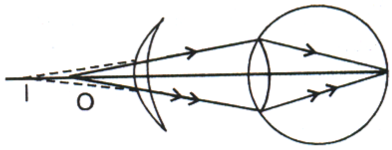
In it near object are not clearly visible i.e., Near Point is at a distance greater than 25 cm and hence image of near object is formed behind the retina.

This defect is remedied by using spectacles having convergent lens (i.e., positive focal length of power) which the image of near objects at the Near Point of the



Defective-eye

(A)



Corrected-eye

(B)

Patient-eye (which is more than 25 cm). So that in this case from lens formula we have

$$\frac{1}{-\text{N.P.}}\ \frac{1}{-\text{F.P.}}-\frac{1}{-(\text{distance of object})}=\frac{1}{f}=P$$

If object is placed at D = 25 cm = 0.25 m
where N.P is near point of eye

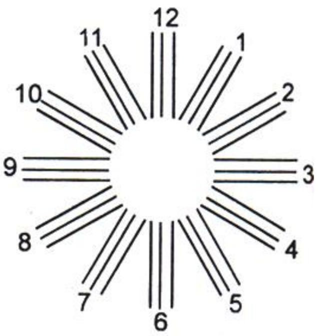
$$P=\frac{1}{f}=\left[\frac{1}{0.25}-\frac{1}{\text{N.P.}}\right] \qquad ......(2)$$

18.3 PRESBYOPIA

In this both near and far objects are not clearly visible i.e., far point is lesser than infinity and near point greater than 25 cm. It is an old age disease as at old age ciliary muscles lose their elasticity and so can not change the focal length of eye-lens effectively and hence eye loses its power of accommodation.

18.4 ASTIGMATISM

In it due to imperfect spherical nature of eye-lens, the focal length of eye lens is two orthogonal directions becomes different and so eye cannot see object in two orthogonal directions clearly simultaneously. This defect is directional and is remedied by using cylinderical lens in particular direction. If in the spectacle of a person suffering from astigmatism, the lens is slightly rotated the arrangement will get spoiled.



Self Practice Problems

33. For a normal eye , the least distance of distinct vision is
(1) 0.25 m (2) 0.50 m (3) 25 m (4) Infinite

34. For the myopic eye, the defect is cured by
(1) Convex lens (2) Concave lens (3) Cylindrical lens (4) Toric lens

35. Lens used to remove long sightedness (hypermetropia) is
or
A person suffering from hypermetropia requires which type of spectacle lenses
(1) Concave lens (2) Plano- concave lens
(3) Convexo- concave lens (4) Convex lens

36. Image formed on the retina is
(1) Real and inverted (2) Virtual and erect (3) Real and erect (4) Virtual and inverted

Ans. **33.** (1) **34.** (2) **35.** (4) **36.** (1)

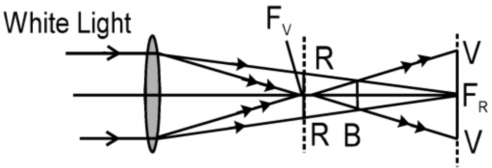
Example 53 :

Chromatic Aberration

The image of a white object in white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration and arises due to the fact that focal length of a lens is different for different colours. As R.I. μ of lens is maximum for violet while minimum for red, violet is focused nearest to the lens while red farthest from it as shown in figure.

As a result of this, in case of convergent lens if a screen is placed at F_V centre of the image will be violet and focused while sides are red and blurred. While at F_R, reverse is the case, i.e., centre will be red and focused while sides violet and blurred. The difference between f_V and f_R is a measure of the longitudinal chromatic aberration (L.C.A), i.e.,

L.C.A. = $f_R - f_V = -df$ with $df = f_V - f_R$ ....(1)



However, as for a single lens,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...........(2)}$$

$$\Rightarrow -\frac{df}{f^2} = d\mu\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \text{...........(3)}$$

Dividing Eqn. (3) by (2) ;

$$-\frac{df}{f} = \frac{d\mu}{(\mu - 1)} = \omega \qquad \left[\omega = \frac{d\mu}{(\mu - 1)}\right] = \text{dispersive power} \quad \text{........(4)}$$

And hence, from Eqns. (1) and (4),

L.C.A. = $-df = \omega f$

Now, as for a single lens neither f nor ω can be zero, we cannot have a single lens free from chromatic aberration.

Condition of Achromatism :

In case of two thin lenses in contact

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \text{i.e.,} \quad -\frac{dF}{F^2} = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2}$$

The combination will be free from chromatic aberration if dF = 0

i.e., $\frac{df_1}{f_1^2} + \frac{df_2}{f_2^2} = 0$

which with the help of Eqn. (4) reduces to

$$\frac{\omega_1 f_1}{f_1^2} + \frac{\omega_2 f_2}{f_2^2} = 0 \qquad \text{i.e.,} \quad \frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \quad \text{...........(5)}$$

This condition is called condition of achromatism (for two thin lenses in contact) and the lens combination which satisfies this condition is called achromatic lens**,** from this condition, i.e., from Eqn. (5) it is clear that in case of achromatic doublet :

(1) The two lenses must be of different materials.

Since, if $\omega_1 = \omega_2$, $\frac{1}{f_1} + \frac{1}{f_2} = 0$ i.e., $\frac{1}{F} = 0$ or $F = \infty$

i.e., combination will not behave as a lens, but as a plane glass plate.

(2) As ω_1 and ω_2 are positive quantities, for equation (5) to hold, f_1 and f_2 must be of opposite nature, i.e. if one of the lenses is converging the other must be diverging.

(3) If the achromatic combination is convergent,

$f_C < f_D$ and as $-\frac{f_C}{f_D} = \frac{\omega_C}{\omega_D}$, $\omega_C < \omega_D$

i.e., in a convergent achromatic doublet, convex lens has lesser focal length and dispersive power than the divergent one.

Self Practice Problems

37. Chromatic aberration in the formation of images by a lens arises because :

(1) of non-paraxial rays. (2) the radii of curvature of the two sides are not same.

(3) of the defect in grinding. (4) the focal length varies with wavelength.

Sol. From passage, (4) is correct.

38. Chromatic aberration of a lens can be corrected by :

(1) providing different suitable curvatures of its two surfaces.

(2) proper polishing of its two surfaces.

(3) suitably combining it with another lens.

(4) reducing its aperture.

Sol. From passage, (3) is correct.

39. A combination is made of two lenses of focal lengths f and f' in contact ; the dispersive powers of the materials of the lenses are ω and ω'. The combination is achromatic when :

(1) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = 2f$ (2) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = f/2$

(3) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -f/2$ (4) $\omega = \omega_0$, $\omega' = 2\omega_0$, $f' = -2f$

Sol. From points (2) and (3) of passage :

f and f' must be of opposite sign.

Also $\omega_C < \omega_D$ and $f_C < f_D$

which is satisfied only by (4).

40. The dispersive power of crown and flint glasses are 0.02 and 0.04 respectively. An achromatic converging lens of focal length 40 cm is made by keeping two lenses, one of crown glass and the other of flint glass, in contact with each other. The focal lengths of the two lenses are :

(1) 20 cm and 40 cm (2) 20 cm and –40 cm

(3) –20cm and 40 cm (4) 10 cm and –20cm

Sol. $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} = \frac{1}{2}$ .... (1)

$\Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40}$ ....(2)

After solving (1) & (2)

$f_1 = 20$ cm $f_2 = -40$ cm.

41. Chromatic aberration in a spherical concave mirror is proportional to :

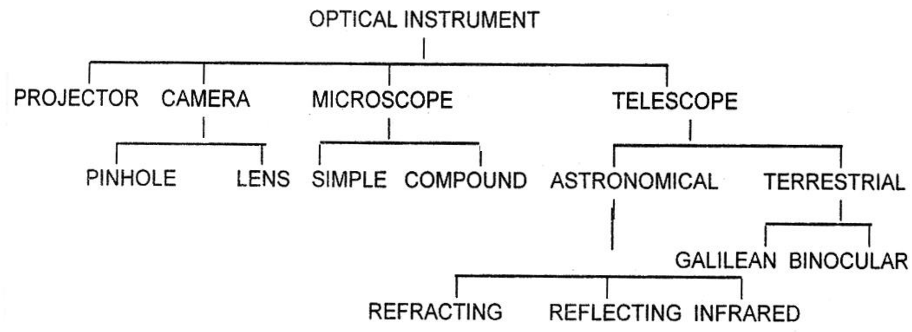
(1) f (2) f_2 (3) $1/f$ (4) None of these

Sol. Chromatic aberration doesn't occur in case of spherical mirrors.

Ans. **37.** (4) **38.** (3) **39.** (4) **40.** (2) **41.** (4)

OPTICAL INSTRUMENTS

Definition :
Optical instruments are used primarily to assist the eye in viewing an object.
Types of Instruments :
Depending upon the use, optical instruments can be categorised in the following way :

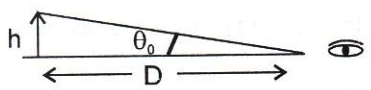


1. MICROSCOPE

It is an optical instrument used to increase the visual angle of neat objects which are too small to be seen by naked eye.

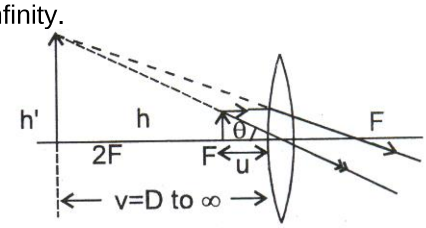
1.1. SIMPLE MICROSCOPE

It is also known as magnifying glass or simply magnifier and consists of a convergent lens with object between its focus and optical centre and eye close to it. The image formed by it is erect, virtual enlarged and on same side of lens between object and infinity.



unadded-eye

(A)



eye with Instrument

(B)

The magnifying power (MP) or angular magnification of a simple microscope (or an optical instrument) is defined as the ratio of visual angle with instrument to the maximum visual angle for clear vision when eye is unadded (i.e., when the object is at least distance of distinct vision)
i.e., magnifying power is M.P

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max.visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

If an object of size h is placed at a distance u (< D) from the lens and its image size h' is formed at a distance V (D) from the eye

$$\theta = \frac{h'}{v} = \frac{h}{u} \qquad \text{with} \qquad \theta_0 = \frac{h}{D}$$

So magnifying power MP = $\frac{\theta}{\theta_0} = \frac{h}{u} \times \frac{D}{h} = \frac{D}{u}$ ...... (1)

Now there are two possibilities

(a_1) If their image is at infinity [Far point]

In this situation from lens formula -

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have $\frac{1}{\infty}-\frac{1}{-u}=\frac{1}{f}$ i.e., u = f So $MP=\frac{D}{u}=\frac{D}{f}$ ...... (2)

As here u is maximum [as object is to be with in focus], MP is minimum and as in this situation parallel beam of light enters the eye, eye is least strained and is said to be normal, relaxed or unstrained.

(a_2) If the image is at D [Near point]

In this situation as v = D, from lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have $\frac{1}{-D}-\frac{1}{-u}=\frac{1}{f}$

i.e., $\frac{D}{u}=1+\frac{D}{f}$ So $MP=\frac{D}{u}=\left[1+\frac{D}{f}\right]$ ...... (3)

As the minimum value of v for clear vision is D, in this situation u is minimum and hence this is the maximum possible MP of a simple microscope and as in this situation final image is closest to eye, eye is under maximum strain.

SPECIAL POINTS

(1) Simple magnifier is an essential part of most optical instruments (such as microscope or telescope) in the form of eye piece or ocular.

(2) The magnifying power (MP) have no unit. It is different from power of a lens which is expressed in diopter (D) and is equal to the reciprocal of focal length in metre.

(3) With increase in wavelength of light used, focal length of magnifier will increase and hence its MP will decrease.

Solved Example

Example 53. A man with normal near point (25 cm) reads a book with small print using a magnifying a thin convex lens of focal length 5 cm. (a) What is the closest farest distance at which he can read the book when viewing through the magnifying glass? (b) What is the maximum and minimum MP possible using the above simple microscope?

Solution : (a) As for normal eye far and near point are and 25 cm respectively, so for magnifier and . However, for a lens as

$\frac{1}{\upsilon}-\frac{1}{u}=\frac{1}{f}$ i.e., $u=\frac{f}{\left(\frac{f}{\upsilon}\right)-1}$

So u will be minimum when

i.e., $(u)_{min}=\frac{5}{\left(\frac{-5}{25}\right)-1}=-\frac{25}{6}=-4.17\,cm$

And u will be maximum when

So, the closest and farest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) An in case of simple magnifier $MP=\left(\frac{D}{u}\right)$. So MP will be minimum when u = max = 5 cm

i.e., $(MP)_{min}=\frac{-25}{-5}=5 \qquad \left[=\frac{D}{f}\right]$

And MP will be maximum when u = min = $\left(\frac{25}{6}\right)$cm

i.e., $(MP)_{max}=\frac{-25}{-\left(\frac{25}{6}\right)}=6\left[=1+\frac{D}{f}\right]$

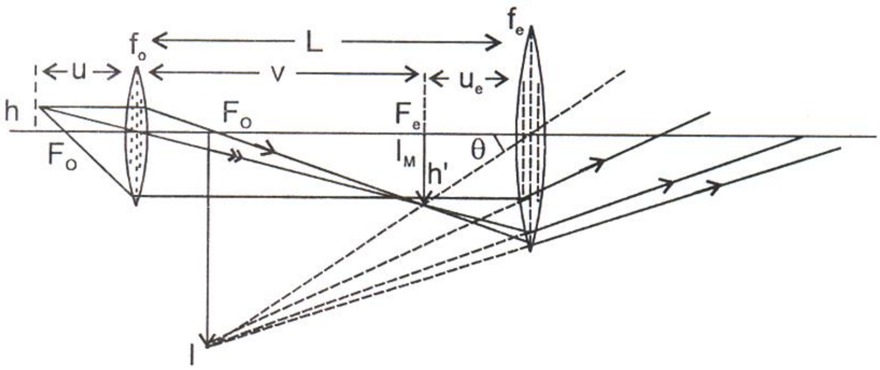
1.2. COMPOUND-MICROSCOPE

CONSTRUCTION

It consists of two convergent lenses of short focal lengths and apertures arranged co-axially lens (of focal length f_0) facing the object is called objective or field lens while the lens (of focal length f_e) facing the eye, eye-piece or ocular. The objective has a smaller aperture and smaller focal length than eye-piece. The separation between objective and eye-piece can be varied.

IMAGE FORMATION :

The object is placed between F and 2F of objective so the image IM formed by objective (called intermediate image) is inverted, real enlarged and at a distance greater than f_0 on the otherside of the lens. This image IM acts as object for eye-piece and is with in its focus. So eye-piece forms final image I which is erect, virtual and enlarged with respect to intermediate image I_M. So the final image I with respect to object is inverted, virtual, enlarged and at a distance D to from eye on the same side of eye-piece as I_M. This all is shown in figure.



Magnifying power (MP)

Magnifying power of an optical instrument is defined as

$$MP = \frac{\text{Visual angle with instrument}}{\text{Max.Visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

If the size of object is h and least distance of distinct vision is D.

$$\theta_0 = \left[\frac{h}{u_e}\right] \times \left[\frac{D}{h}\right] = \left[\frac{h'}{h}\right]\left[\frac{D}{u_e}\right]$$

But for objective

$$m = \frac{I}{O} = \frac{v}{u} \text{ i.e., } \frac{h'}{h} = -\frac{v}{u} \quad \text{[as u is positive]}$$

So,

$$MP = -\frac{v}{u}\left[\frac{D}{u_e}\right]$$

with length of tube

$$L = v + u_e, \qquad \text{......(1)}$$

now there are two possibilities

(b_1) If the final image is at infinity (far point):

This situation is called normal adjustment as in this situation eye is least strained or relaxed. In this situation as for eye-piece $v = \infty$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \text{ i.e., } \quad u_e = f_e = \text{maximum}$$

Substitution this value of in equation (1), we have

$$MP = -\frac{v}{u}\left[\frac{D}{f_e}\right] \text{ with } L = v + f_e \qquad \text{..... (2)}$$

A microscope is usually considered to operate in this mode unless state otherwise. In this mode as u_e is maximum MP is minimum for a given microscope.

(b_2) If the final image is at D (near point) :

In this situation as for eye-piece v = D

$$\frac{1}{-D}-\frac{1}{-u_e}=\frac{1}{f_e} \quad \text{i.e.,} \quad \frac{1}{u_e}=\frac{1}{D}\left[1+\frac{D}{f_e}\right]$$

Subsittuting this value of u_e in equation (1), we have

$$MP=-\frac{v}{u}\left[1+\frac{D}{f_e}\right] \quad \text{with} \quad L=v+\frac{f_eD}{f_e+D} \qquad \text{..... (3)}$$

In this situation as u_e is minimum MP is maximum and eye is most strained.

Solved Example

Example 54. The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of object from the objective, when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power.

Solution : Given f_0 = 2 cm, f_e = 5 cm

$|v_o| + |u_e| = 20$ cm

$\therefore$ $v_e = -25$ cm

From lens formula $\frac{1}{f_e}=\frac{1}{v_o}-\frac{1}{u_e}$ $\Rightarrow$ $\frac{1}{u}=\frac{1}{v_e}-\frac{1}{f_e}=-\frac{1}{25}-\frac{1}{5}$

$\therefore$ $u_e = -\frac{25}{6}$ cm

Distance of real image from objective

$v_o = 20 - |u_e| = 20 - \frac{25}{6} = \frac{120-25}{6}=\frac{95}{6}$ cm

Now $\frac{1}{f_o}=\frac{1}{v_o}-\frac{1}{u_o}$ given $\frac{1}{u_o}=\frac{1}{v_o}-\frac{1}{f_o}=\frac{1}{(95/6)}-\frac{1}{2}$

i.e., $\frac{1}{u_o}=\frac{6}{95}-\frac{1}{2}=\frac{12-95}{190}=-\frac{83}{190}$ $\therefore$ $u_o = -\frac{190}{83} = -2.3$ cm

Magnifying power $M = -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) = -\frac{95/6}{(190/83)}\left(1+\frac{25}{3}\right) = -41.5$

Self Practice Problems

42. The focal lengths of the objective and eye lens of a microscope are 1 cm and 5cm respectively If the magnifying power for the relaxed eye is 45, then the length of the tube is

(1) 30 cm (2) 25 cm (3) 15cm (4) 12 cm

43. In a compound microsocope magnification will be large ,if the focal length of the eye piece is

(1) Large (2) Smaller
(3) Equal to that of objective (4) Less than that of objective

44. Magnifying power of a simple microscope is (when final image is formed at D= 25 cm from eye)

(1) $\frac{D}{f}$ (2) $1+\frac{D}{f}$ (3) $1+\frac{f}{D}$ (4) $1-\frac{D}{f}$

45. If in compound microscope m_1 and m_2 be the linear magnification of the objective lens and eye lens respectively then magnifying power of the compound microscope will be

(1) $m_1 - m_2$ (2) $\sqrt{m_1 + m_2}$ (3) $(m_1 + m_2)/2$ (4) $m_1 \times m_2$

42. (3) **43.** (2) **44.** (2) **45.** (4)

2. TELESCOPE

2.1 ASTRONOMICAL TELESCOPE

It is an optical instrument used to increase the visual angle of distant large objects such as a star a planet or a cliff etc. Astronomical telescope consists of two converging lens. The one facing the object is called objective or field-lens and has large focal length and aperture. The distance between the two lenses is adjustable.

As telescope is used to see distant objects, in it object is between and 2F of objective and hence image formed by objective is real, inverted, and diminished and is between F and 2F on the other side of it. This image is (called intermediate image) acts as object for eye-piece and shifting the position of eye-piece is brought with in its focus. So final image I, with respect to intermediate image is erect, virtual, enlarged and at a distance D to from the eye. This in turns implies that final image with respect to object is inverted, enlarged and at a distance D to from the eye.

Magnifying Power (MP)

Magnifying Power of a telescope is defined as

$$MP = \frac{\text{Visual angle with instrument}}{\text{Visual angle for unadded eye}} = \frac{\theta}{\theta_0}$$

But from figure.

$$\theta_0 = \left(\frac{y}{f_0}\right) \quad \text{and} \quad \theta = \left(\frac{y}{-u_e}\right)$$

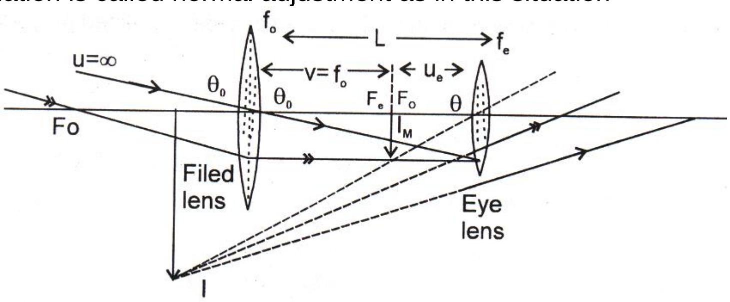
So $MP = \frac{\theta}{\theta_0} = -\left[\frac{f_0}{u_e}\right]$ with length of tube

$$L = (f_0 + u_e) \qquad \text{.... (1)}$$

Now there are two possibilities

(d_1) If the final image is at infinity (far point)

This situation is called normal adjustment as in this situation



eye is least strained or relaxed. In this situation as for eye-piece v = ∞

$$\frac{1}{-\infty} - \frac{1}{u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad u_e = f_e$$

So, substituting this value of u_e in equation (1) we have

$$MP = -\left(\frac{f_0}{f_e}\right) \quad \text{and} \quad L = (f_0 + f_e)$$

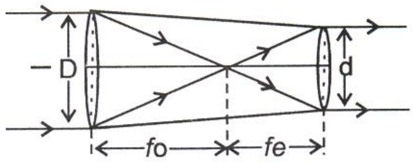
Usually telescope operates in this mode unless stated other wise. In this mode as u_e is maximum for a given telescope MP is minimum while length of tube maximum.

(d_2) If the final image is at D (near point)

In this situation as for eye-piece v = D

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \quad \text{i.e.,} \quad \frac{1}{-u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

So substituting this value of u_e in Equation (1), we have



$$MP = \frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ with } \quad L = f_0 + \frac{f_e D}{f_e + D} \quad \text{...... (3)}$$

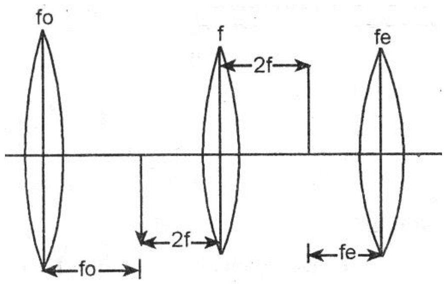
In this situation u_e is minimum so for a given telescope MP is maximum while length of tube minimum and eye is most strained. In case of a telescope if object and final image are at infinity and total light entering the telescope leaves it parallel to its axis as shown in figure.

$$\frac{f_0}{f_e} = \frac{\text{Aperture of object}}{\text{Aperture of eye piece}} \quad \text{i.e.,} \quad MP = \frac{f_0}{f_e} = \frac{D}{d} \quad \text{..... (4)}$$

2.2. TERRESTRIAL TELESCOPE

Uses a thrd lens in between objective and eyepieces so as to form final image erect. This lens simply invert the image formed by objective without affecting the magnification.

Length of tube $L = f_0 + f_e + 4f$



2.3 GALILEO'S TELESCOPE

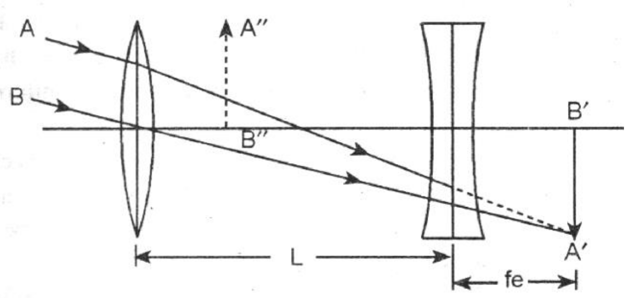
Convex lens as objective.

Concave lens as eyepiece.

Field of view is much smaller

∵ eyepiece lens in concave.

(i) $M = \frac{f_0}{f_e}\left[1 - \frac{f_e}{v_e}\right]$



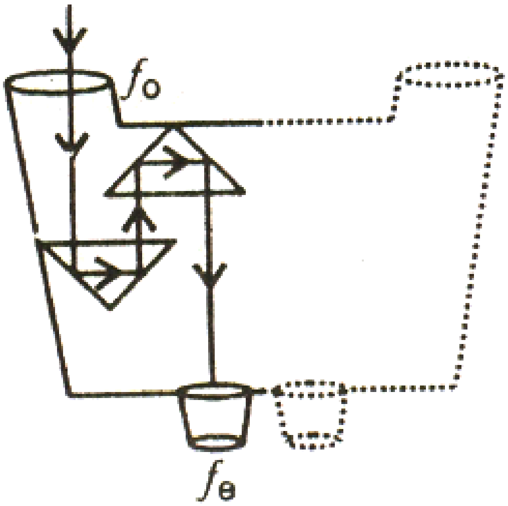
(ii) $M = \frac{f_0}{f_e}$

Final image is at α $\quad L = f_0 - f_e$

(iii) $M = \frac{f_0}{f_e}\left[1 - \frac{f_e}{D}\right]$ Final image is at D. $L = f_0 - u_e$

2.4 BINOCULAR

In this telescope as intermediate image is outside the tube, the telescope cannot be used for making measurements. If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called **'binocular'** . In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms (fig.)which provide intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, i.e., binocular vision given proper three-dimensional (3-D) image.



Binocular

Solved Examples

Example 55. A telescope consists of two convex lens of focal length 16 cm and 2 cm. What is angular magnification of telescope for relased eye? What is the separation between the lenses? If object subtends an angle of 0.5º on the eye, what will be angle subtended by its image ?

Solution : Angular magnification $M = \frac{\alpha}{\beta} = \frac{F}{f} = \frac{16}{2} = 8$ cm

Separation between lenses = F + f = 16 + 2 = 18 cm

Here $\alpha = 0.5^o$

∴ Angular subtended by image $\beta = M\alpha = 8 \times 0.5^o = 4^o$

Example 56. The magnifying power of the telescope if found to be 9 and the separation between the lenses is 20 cm for relased eye. What are the focal lengths of component lenses ?

Solution : Magnification $M = \frac{F}{f}$

Separaton between lenses

d = F + f

Given $\frac{F}{f} = 9$ i.e., F = 9f ......(1)

and $F + f = 20$ ......(2)

Putting value of F from (1) in (2), we get

$9f + f = 20 \Rightarrow 10f = 20 \Rightarrow \frac{20}{10} = 2\text{cm}$

$\therefore$ $F = 9f = 9 \times 2 = 18$ cm

$\therefore$ $F = 18$ cm, $f = 2$ cm

Self Practice Problems

46. The magnifying power of a telescope can be increased by
(1) Increasing focal length of the system
(2) Fitting eye piece of high power
(3) Fitting eye piece of low power
(4) Increasing the distance of objects

47. A simple telescope, consisting of an objective of focal length 60 cm and a single eye lens of focal length 5 cm is focussed on a distant object is such a way that parallel rays comes out from the eye lens. If the object subtends an angle 2° at the objective, the angular width of the image
(1) 10° (2) 24° (3) 50° (4) 1/6°

48. If the telescope is reversed i.e., seen from the objective side
(1) Object will appear very small
(2) Object will appear very large
(3) There will be no effect on the image formed by the telescope
(4) Image will be slightly greater than the earlier one

49. The aperture of a telescope is made large, because
(1) To increase the intensity of image
(2) To decrease the intensity of image
(3) To have greater magnification
(4) To have lesser resolution

50. The magnifying power of a telescope is 9. When it is adjusted for parallel rays, the distance between the objective and the eye-piece is found to be 20 cm. The focal length of the two lenses are
(1) 18 cm, 2 cm (2) 11 cm, 9 cm (3) 10 cm, 10 cm (4) 15 cm, 5 cm

51. A reflecting telescope utilizes
(1) A concave mirror (2) A convex mirror (3) A prism (4) A plano-convex lens

Ans. **46.** (2) **47.** (2) **48.** (1) **49.** (1) **50.** (1) **51.** (1)

3. **COMPARISION BETWEEN COMPOUND - MICROSCOPE & ASTRONOMICAL-TELESCOPE**

S.No.	**Compound - Microscope**	**Astronomical - Telescope**
1.	It is used to increase visual angle of near tiny object.	It is used to increase visual angle of distant large objects.
2.	In it field and eye lense both are convergent, of short focal lengh and aperture.	In it field lens is of large focal length and aperture while eye lens of short focal length and aperature and both are convergent.
3.	Final image is inverted. virtual and enlarged and at a distance D to ∞ from the eye.	Final image in inverted, virtual and enlarged at a distance D to ∞ from the eye
4.	MP does not change appreciably if field and eye lens are intercharged [MP ~ $(LD/f_0 f_e)$]	MP becomes $(1/m_2)$ times of its initial value if field and eye-lenses are interchanged as MP ~$[f_0/f_e]$
5.	MP is increased by decreasing the focal of length of both the lenses viz. find and length eye lens.	MP is increased by increasing the focal length field of field lens (and decreasing the focal of eye lens.)

6.	RP is increased by decreasing the wavelength of light used.	RP is increased by increasing the aperture of objective.

SCATTERING OF LIGHT

(i) When light from some source (ga, sun, stars) enters the earth atmosphere then it gets reflected in various direction by the particles of dust, smoke and gas molecules. The phenomenon of this diffuse reflection is known an scattering This was initially suggested by Tindal.

(ii) According to Rayleigh, the intensity (I) of scattered light is inversely proportional to the fourth power of wavelength of light. i.e. λ

i.e. $I \propto \frac{1}{\lambda^4}$

That is the reason why red light (λ more) gets scattered minimum and violet light (λ less) gets scattered maximum

($I_R = 16I_V$)

(iii) Consequences of scattering of light -

(a) Appearance of blue colour of sky.

(b) The danger signals are made red.

(c) Appearance of black colour of sky in the absence of atmosphere.

(d) Appearance of red colour of sun at sun rise and sun set.

Luminous bodies — The bodies which emit light themselves are known as luminous bodies.

RAINBOW

(a) The seven coloured curved strip formed as a result of dispersion of light through water droplets which keep suspended in the atmosphere after rains, is known as rainbow.

(b) The rainbow is of two types -

(i) Primary rainbow (ii) Secondary rainbow

(c) Difference between primary and secondary rainbow —

	Primary rainbow	Secondary rainbow
(i)	In this, the incident ray of light undergoes total internal reflection once,	It this the incident ray undergoes total internal reflection twice
(ii)	The order of the colours is from red to violet	The order of colours is from violet to red.
(iii)	It is formed by one total internal reflection and dispersion.	It is formed by two internal reflections and dispersion.

Limit of resolution and resolving power of optical instruments –

(i) Resolving power –

(a) The ability of an optical instrument to produce separate diffraction patterns of two nearby objects is known as resolving power.

(b) The ability of an optical instrument to show two closely lying objects or spectral lines as separate, is known as its resolving power.

(ii) Limit of resolution – The reciprocal of resolving power is defined as the limit of resolution.

(iii) Rayleigh's limit of resolution – The distance between two object points, when the central maximum of diffraction pattern of one coincides with the first minimum of diffraction pattern of another, is defined as the Rayleigh's limit of resolution.

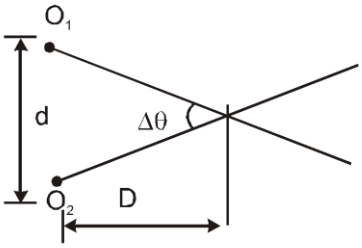
Resolving power (R.P.) of telescope

$$R.P = \frac{a}{1.22\ \lambda} = \frac{1}{\Delta\theta} = \frac{D}{d}$$

a = diameter of the aperture of objective

λ = wavelength of light used

Δθ = Limit of resolution



d = distance of two objects
D = distance of objects from objective lens
The resolving power of an electron microscope is 4 × 10$_3$ times that of an ordinary microscope.

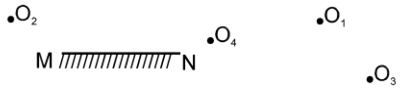
Resolving power (R.P.) of mcroscope

R.P. = $\frac{2\mu \sin\theta}{1.22 \quad \lambda}$

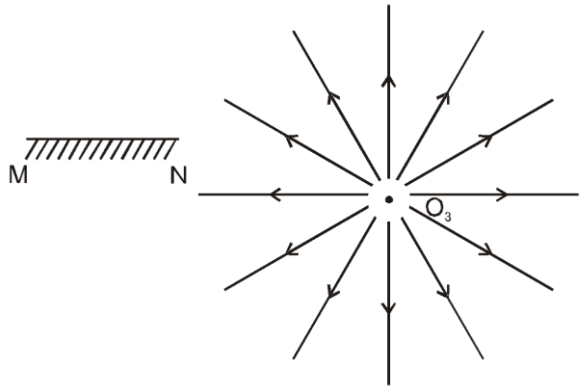
μ = refractive index of medium.

SOLVED MISCELLANEOUS PROBLEMS

Problem 1. See the following figure. Which of the object(s) shown in figure will not form its image in the mirror.

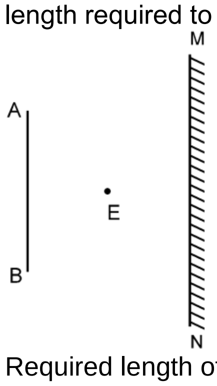


Solution :

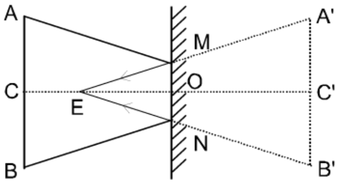


no ray from O3 is incident on reflecting surface of the mirror, so its image is not formed.

Problem 2. Figure shows an object AB and a plane mirror MN placed parallel to object. Indicate the mirror length required to see the image of object if observer's eye is at E.



Solution : Required length of mirror = MN.



Δ MNE & ΔA'B'E are similar

$\dfrac{MN}{OE} = \dfrac{A'B'}{C'E} \Rightarrow MN = \dfrac{A'B'}{2} = \dfrac{AB}{2}$.

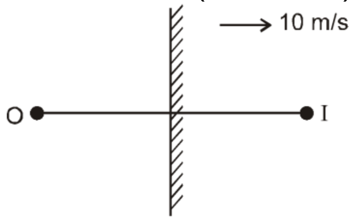
Problem 3. An object is kept fixed in front of a plane mirror which is moved by 10 m/s away from the object, find the velocity of the image.

Solution :

$\vec{V}_{IM} = -\vec{V}_{OM}$

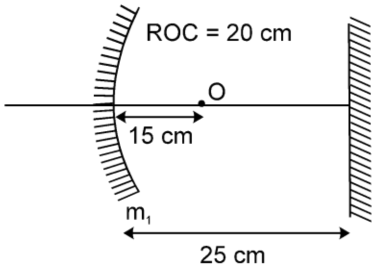
$\vec{V}_{I,G} - \vec{V}_{M,G} = -\vec{V}_{O,G} + \vec{V}_{M,G}$

$\vec{V}_{M,G} = \dfrac{\vec{V}_{I,G} + \vec{V}_{O,G}}{2}$ $(\because \vec{V}_{O,G} = 0) \Rightarrow \dfrac{\vec{V}_{I,G}}{2} = 10\hat{i}$ m/s



$\vec{V}_{I,G} = 20\hat{i}$ m/s

Problem 4. Find the position of final image after three successive reflections taking first reflection on m_1



Solution : 1st reflection :

u = – 15cm

f = –10 cm

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f} = \dfrac{-3+2}{30} = -\dfrac{1}{30}$

v = – 30 cm

2nd reflection at plane mirror :

u = 5 cm

v = – 5 cm

for **3rd** reflection on curved mirror again :

u = – 20 cm

$v = \dfrac{uf}{u-f} = \dfrac{(-20)\times(-10)}{-20+10} = \dfrac{200}{-10} = -20$ cm

Problem 5. A coin is placed 10 cm in front of a concave mirror. The mirror produces a real image that has diameter 4 times that of the coin. What is the image distance.

Solution : $m = \frac{d_2}{d_1} = -\frac{v}{u}$

We have, u = 10 cm (virtual object) as real image is formed

v = – mu

= –4 ×10 cm

= – 40 cm **Ans.**

Problem 6. A small statue has a height of 1 cm and is placed in front of a spherical mirror . The image of the statue is inverted and is 0.5cm tall and located 10 cm in front of the mirror. Find the focal length and nature of the mirror.

Solution : We have $m = \frac{h_2}{h_1} = -\frac{0.5}{1} = -0.5$

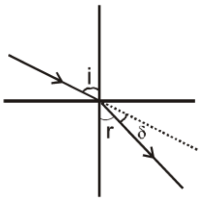
v = – 10 cm (real image)

But $m = \frac{f-v}{f}$ $-0.5 = \frac{f+10}{f}$ $\Rightarrow$ $f = \frac{-20}{3}$ cm

so, concave mirror. **Ans.**

Problem 7. A light ray deviates by 30° (which is one third of the angle of incidence) when it gets refracted from vacuum to a medium. Find the refractive index of the medium.

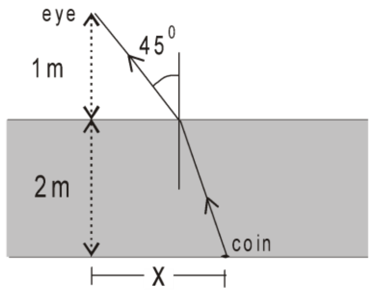
Solution : δ = i – r



$\Rightarrow$ $\frac{i}{3} = i - r = 30^o$. $\Rightarrow$ $i = 90^o$ $\Rightarrow$ $2i = 3r$

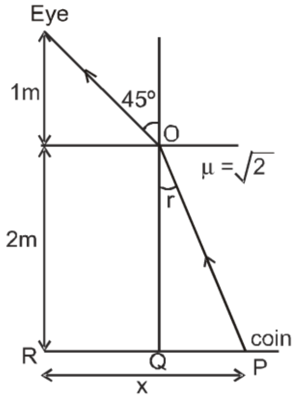
$\therefore$ $r = \frac{2\ i}{3} = 60^o$ So, $\mu = \frac{\sin 90^o}{\sin 60^o} = \frac{1}{\sqrt{3/2}} = \frac{2}{\sqrt{3}}$ **Ans.**

Problem 8. A coin lies on the bottom of a lake 2m deep at a horizontal distance x from the spotlight (a source of thin parallel beam of light) situated 1 m above the surface of a liquid of refractive index $\mu = \sqrt{2}$ and height 2m. Find x.



Solution : $\sqrt{2} = \frac{\sin\ 45^o}{\sin\ r}$ $\Rightarrow \sin r = \frac{1}{2}$ $\Rightarrow$ $r = 30^o$

x = RQ + QP = 1m + 2tan30° m = $\left(1+\frac{2}{\sqrt{3}}\right)$ m **Ans.**

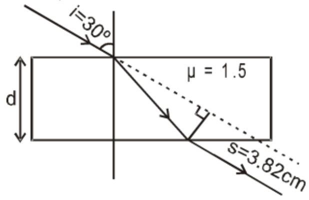


Problem 9. A ray of light falls at an angle of 30º onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 3.82 cm?

[Given : $\sin^{-1}\left(\frac{1}{3}\right) = 19.5^\circ$; $\cos 19.5^\circ = 0.94$; $\sin 10.5^\circ = 0.18$]

Solution : Using $s = \frac{d\sin(i-r)}{\cos r}$ $\Rightarrow$ $d = \frac{3.82\times\cos r}{\sin(30^\circ - r)}$ .... (1)

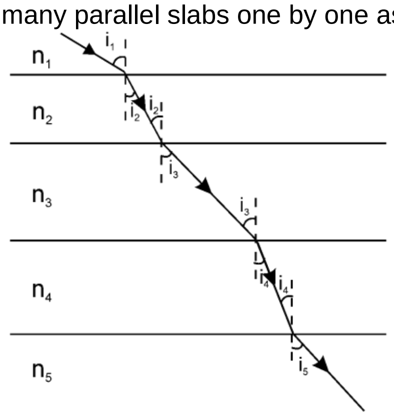
Also, $1.5 = \frac{\sin 30^\circ}{\sin r}$ $\Rightarrow$ $\sin r = \frac{1}{3}$



so, $r = 19.5^\circ$

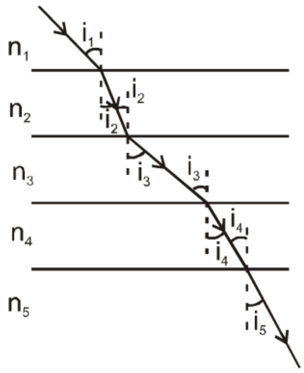
So, $d = \frac{3.82\times\cos 19.5^\circ}{\sin(30^\circ - 19.5^\circ)} = \frac{3.82\times 0.94}{\sin 10.5^\circ} = \frac{3.82\times 0.94}{0.18}$ = 19.948 cm ≈ 0.2 cm

Problem 10. A light passes through many parallel slabs one by one as shown in figure.



Prove that $n_1\sin i_1 = n_2\sin i_2 = n_3\sin i_3 = n_4\sin i_4$ =.............**[Remember this].** Also prove that if $n_1 = n_4$ then light rays in medium n_1 and in medium n_4 are parallel.

Solution : We have, $\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1}$



⇒ $n_1 \sin i_1 = n_2 \sin i_2$ ....(i)

Similarly $n_2 \sin i_2 = n_3 \sin i_3$ so on

so, $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 =$ ....

$n_1 \sin i_1 = n_4 \sin i_4 \Rightarrow \sin i_1 = \sin i_4$ $(\because n_1 = n_2)$

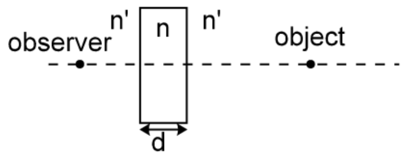
so, $i_1 = i_4$

Hence, light rays in medium n_1 and in medium n_4 are parallel.

Problem 11. An object lies 90 cm in air above water surface .It is viewed from water nearly normally. Find the apparent height of the object.

Solution : $d' = \dfrac{d}{n_{rel}} = \dfrac{d}{n_i / n_r} = \dfrac{\dfrac{90\ \text{cm}}{1}}{4/3} = \dfrac{90 \times 4}{3}$ cm = 120 cm **Ans.**

Problem 12. Prove that the shift in position of object due to parallel slab is given by shift $= d\left(1 - \dfrac{1}{n_{rel}}\right)$ where $n_{rel} = \dfrac{n}{n'}$.



Solution : Because of the ray refraction at the first surface, the image of O is formed at I_1 . For this refraction, the real depth is AO = x and apparent depth is AI_1.

Thus : $AI_1 = \dfrac{AO}{n_i / n_r} = \dfrac{AO}{n'/n} = \dfrac{n\ (AO)}{n'}$

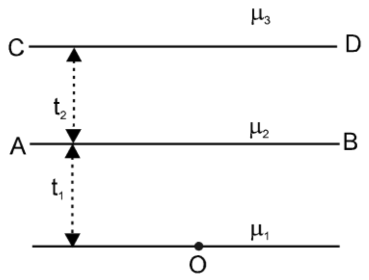
The point I_1 acts as the object for the refraction of second surface. Due to this refraction, the image of I_1 is formed at I_2. Thus,

$BI_2 = \dfrac{(BI_1)}{(n/n')} = \dfrac{n'}{n}$ (BI_1) = n'/n $(AB + AI_1)$ = $\dfrac{n'}{n}\left[d + \dfrac{n}{n'}(AO)\right] = \dfrac{n'}{n}$ d + AO.

Net shift = OI_2 = BO – BI_2

$= d + (AO) - \dfrac{n'}{n} d - AO = d\left(1 - \dfrac{n'}{n}\right) = d\left(1 - \dfrac{1}{n_{rel}}\right)$ where $n_{rel} = n_{rel} = \dfrac{n}{n'}$.

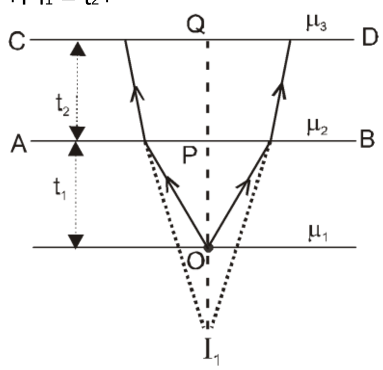
Problem 13. Find the apparent depth of object O below surface AB, seen by an observer in medium of refractive index μ_2



Solution : $d_{app.} = \dfrac{t_1}{\mu_1/\mu_2}$

Problem 14. In above question what is the depth of object corresponding to incident rays striking on surface CD in medium μ_2.

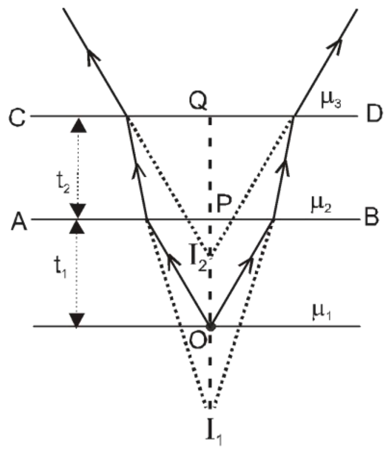
Solution : Depth of the object corresponding to incident ray striking on the surface CD in medium μ_2 = $t_2 + PI_1 = t_2 + \dfrac{t_1}{\mu_1/\mu_2}$



Problem 15. In above question if observer is in medium μ_3, what is the apparent depth of object seen below surface CD.

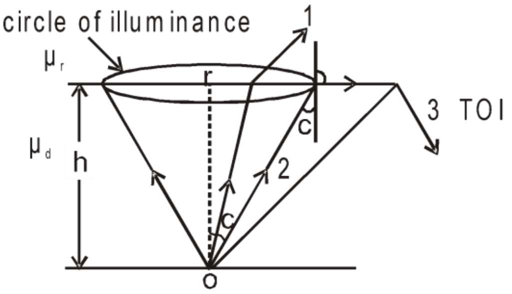
Solution : If the observer is in medium μ_3. apparent depth below surface CD = QI_2.

$$= \sum \frac{t_i}{(n_{rel})_i} = \frac{t_2}{\mu_2/\mu_3} + \frac{t_1}{\mu_1/\mu_3}$$



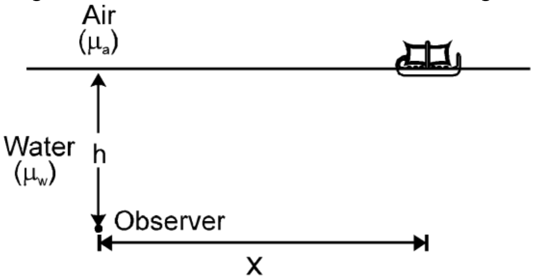
Problem 16. Find the radius of circle of illuminance, If a luminous object is placed at a distance h from the interface in denser medium.

Solution : $\tan c = \frac{r}{h}$. $\therefore$ $r = h \tan c$.

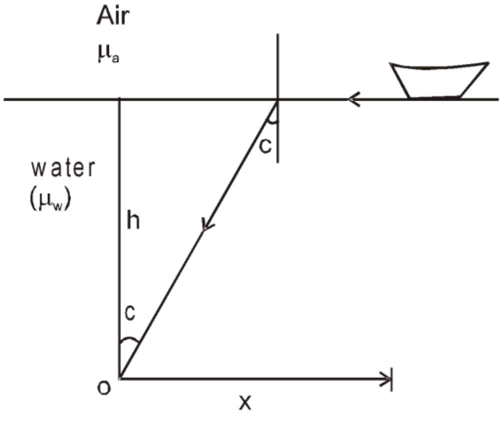


But $C = \sin^{-1}\frac{1}{(\mu_d/\mu_r)}$ so, $r = h\tan\left[\sin^{-1}\frac{1}{(\mu_d/\mu_r)}\right] = h.\frac{\mu_r}{\sqrt{\mu_d^2-\mu_r^2}}$

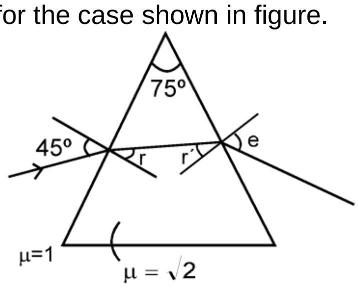
Problem 17. A ship is sailing in river. An observer is situated at a depth h in water (μ_w). If x >> h, find the angle made from vertical, of the line of sight of ship.



Solution : $C = \sin^{-1}\left(\frac{\mu_a}{\mu_w}\right)$



Problem 18. Find r, r´, e, δ for the case shown in figure.



Solution : Here $\theta = 180° - 75° = 105°$

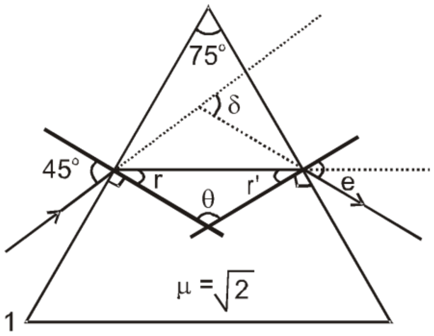
$\sin 45° = \sqrt{2} \sin r$

$\therefore r = \sin^{-1} \frac{1}{2} = 30°$.

$r' = 180° - (r + \theta)$

$= 180° - 30° - 105° = 45°$

$\sin e = \sqrt{2} \sin r'$

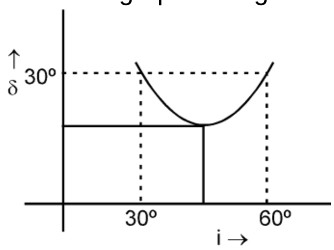


$\therefore \sin e = \sqrt{2} \times \sin 45° = 1$

$\therefore \boxed{e = 90°}$

So, $\delta = i + e - A = 45° + 90° - 75° = 60°$.

Problem 19. From the graph of angle of deviation δ versus angle of incidence i, find the prism angle



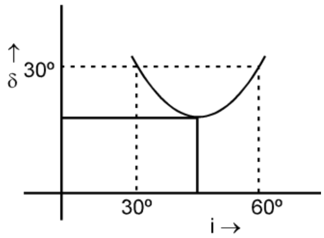
Solution : From the graph ;

$\delta = i + e - A$.

$30° = 30° + 60° - A$

$\therefore \boxed{A = 60°}$

use the result : If i and e are interchanged



then we get same value of δ

Problem 20. Find the focal length of a plano-convex lens with R_1 = 15 cm and $R_2 = \infty$. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5-1)\left(\frac{1}{15} - \frac{1}{\infty}\right). \quad = 0.5 \times \frac{1}{15}$

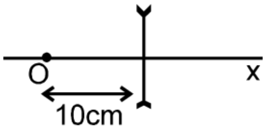
$\therefore \quad f = 30$ cm .

Problem 21. Find the focal length of a concavo-convex lens (positive meniscus) with R_1 = 15 cm and R_2 = 25 cm. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{25}\right) = 0.5\left(\frac{10-6}{150}\right)$.

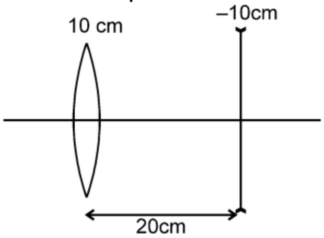
$\therefore f = \frac{300}{4} = 75$ cm

Problem 22. Figure shows a point object and a diverging lens. Find the final image formed.



Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{-10} + \frac{1}{(-10)} = -\frac{2}{10}$.= V = – 5 cm

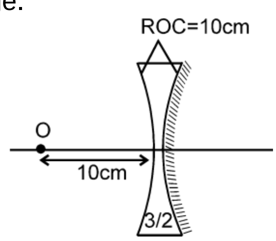
Problem 23. Find the equivalent focal length of the system for paraxial rays parallel to axis.



Solution : $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{10} + \frac{1}{-10} - \frac{20}{10(-10)} = \frac{1}{5}$

$\Rightarrow$ f_{eq} = 5 cm

Problem 24. See the figure Find the equivalent focal length of the combination shown in the figure and position of image.



Solution : For the woreme lens

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right) = -\frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$

Ago, Fm = R/2 = $\frac{10}{2}$ = 5 cm

$\therefore \frac{1}{f_{eq}} = \frac{1}{F_m} - 2\frac{1}{f} = \frac{1}{5} + 2 \times \frac{1}{10} = \frac{2}{5}$ $\boxed{f_{eq} = 2.5 \text{ cm}}$ **Ans.**