MODERN PHYSICS - I

1. PHOTOELECTRIC EFFECT

1.1. Hertz's observations

The phenomenon of photoelectric emission was discovered in 1887 by Heinrich Hertz (1857–1894), during his electromagnetic wave experiments. In his experimental investigation on the production of electromagnetic waves by means of a spark discharge, Hertz observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

Light shining on the metal surface somehow facilitated the escape of free, charged particles which we now know as electrons. When light falls on a metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attraction of the positive ions in the material of the surface. After gaining sufficient energy from the incident light, the electrons escape from the surface of the metal into the surrounding space.

1.2 Hallwach's and Lenard's observations

Wilhelm Hallwachs and Phillipp Lenard investigated the phenomenon of photoelectric emission in detail during 1886–1902.

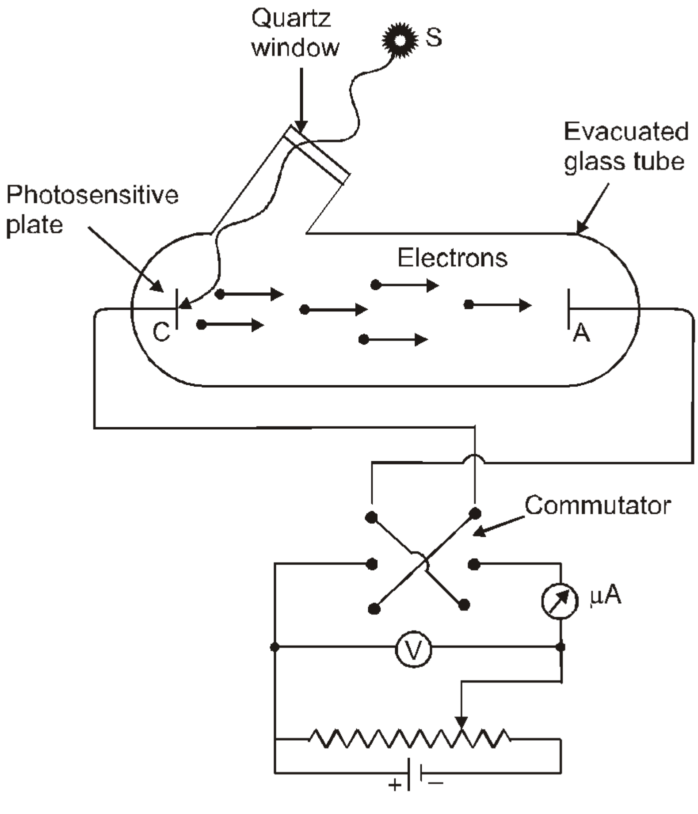


Figure: Experimental arrangement for study of photoelectric effect

Lenard (1862–1947) observed that when ultraviolet radiations were allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit figure. As soon as the ultraviolet radiations were stopped, the current flow also stopped. These observations indicate that when ultraviolet radiations fall on the emitter plate C, electrons are ejected from it which are attracted towards the positive, collector plate A by the electric field. The electrons flow through the evacuated glass tube, resulting in the current flow. Thus, light falling on the surface of the emitter causes current in the external circuit. Hallwachs and Lenard studied how this photo current varied with collector plate potential, and with frequency and intensity of incident light.

Hallwachs, in 1888, undertook the study further and connected a negatively charged zinc plate to an electroscope. He observed that the zinc plate lost its charge when it was illuminated by ultraviolet light. Further, the uncharged zinc plate became positively charged when it was irradiated by ultraviolet light. Positive charge on a positively charged zinc plate was found to be further enhanced when it was illuminated by ultraviolet light. From these observations he concluded that negatively charged particles were emitted from the zinc plate under the action of ultraviolet light.

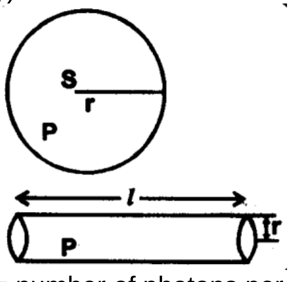
After the discovery of the electron in 1897, it became evident that the incident light causes electrons to be emitted from the emitter plate. Due to negative charge, the emitted electrons are pushed towards the collector plate by the electric field. Hallwachs and Lenard also observed that when ultraviolet light fell on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the threshold frequency. This minimum frequency depends on the nature of the material of the emitter plate.

It was found that certain metals like zinc, cadmium, magnesium, etc. responded only to ultraviolet light, having short wavelength, to cause electron emission from the surface. However, some alkali metals such as lithium, sodium, potassium, cesium and rubidium were sensitive even to visible light. All these photosensitive substances emit electrons when they are illuminated by light. After discovery of electrons, these electrons were termed as photoelectrons. The phenomenon is called photoelectric effect.

1.3 Properties of photons

- Photon is a packet of energy emitted from a source of radiation. Photons are carrier particle of electromagnetic interaction.
- Photons travel in straight lines with speed of light $c = 3 \times 10^8$ m/s.
- The energy of photons is given as $E = h\nu = \frac{hc}{\lambda} = mc^2$
  where ν is frequency, λ is wavelength, h is Planck's constant.
- The effective or motional mass of photon is given as $m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}$
- The momentum of a photon is given as $p = mc = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$
- The photon is a charge less particle of zero rest mass
- Photons are electrically neutral. They are not deflected by electric and magnetic fields.
- If E is the energy of source in joule then number of photons emitted is
  $n = \frac{\text{total energy radiated}}{\text{energy of each photon}} = \frac{E}{h\nu} = \frac{E\lambda}{hc}$
- Intensity of photons is defined as amount of energy carried per unit area per unit time. or power carried per unit area

Intesity $(I_p) = \frac{\text{Energy}}{\text{area} \times \text{time}} = \frac{\text{Power}}{\text{area}}$, $I_p = nh\nu = \frac{N}{4\pi r^2}P$



where n = number of photons per unit area per unit time
N = number of photons, P = power of source

e.g. (a) For a point source $I_p = nh\nu = \frac{N}{4\pi r^2}P$

(b) For a line source $I_p = nh\nu = \frac{N}{2\pi r\ell}P$

Solved Examples

Example 1. Find the number of photons in 6.62 joule of radiation energy of frequency 10^{12} Hz?

Solution : No. of photons n = $\frac{E}{h\nu} = \frac{6.62}{6.62 \times 10^{-34} \times 10^{12}} = 10^{22}$

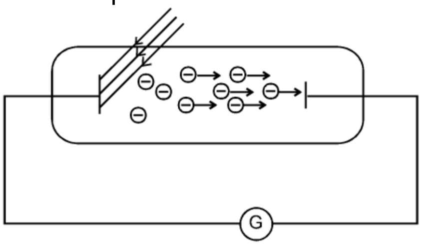
Example 2. Calculate the energy and momentum of a photon of wavelength 6600Å.

Solution : energy of photon E = $\frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6600 \times 10^{-10}} = 3 \times 10^{-19}$ J

momentum of photon p = $\frac{h}{\lambda} = \frac{6.6 \times 10^{-34}}{6600 \times 10^{-10}} = 10^{-27}$ kg m/sec

1.4 Important terms related to photoelectric effect :

When electromagnetic radiations of suitable wavelength are incident on a metallic surface then electrons are emitted, this phenomenon is called photo electric effect.



Photoelectron : The electron emitted in photoelectric effect is called photoelectron.

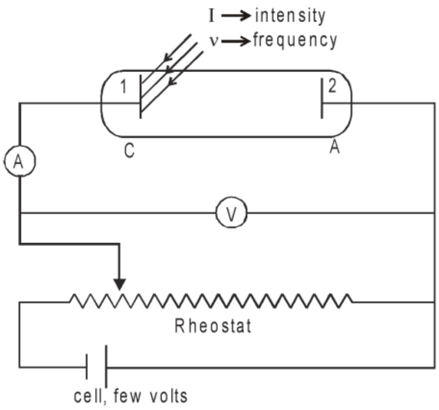
Photoelectric current : If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

Work function : The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by φ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Work functions of some photosensitive metals

Metal	Work function (ev)	Metal	Work function (eV)
Cesium	2.14	Calcium	3.2
Potassium	2.3	Copper	4.65
Sodium	2.75	Silver	4.7
Lithium	2.5	Platinum	5.65

To produce photo electric effect only metal and light is necessary but for observing it the circuit is completed. Figure shows an arrangement used to study the photoelectric effect.



Here the plate (1) is called emitter or cathode and other plate (2) is called collector or anode.

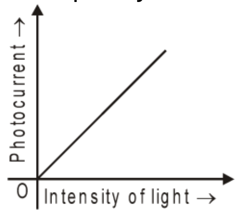
Saturation current : When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturated current, this is the maximum value of photoelectric current.

Stopping potential : Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.

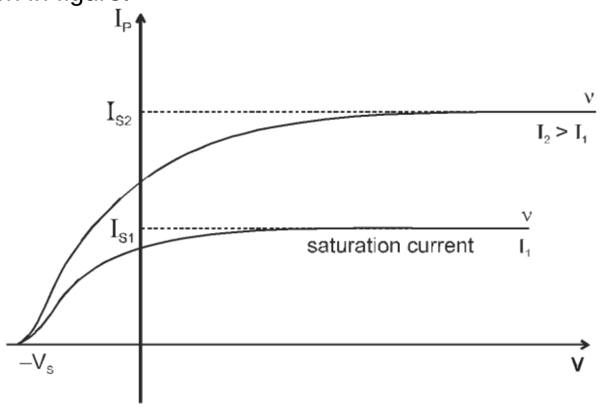
Retarding potential : Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

2. OBSERVATIONS : (MADE BY EINSTEIN)

2.1 A graph between intensity of light and photoelectric current is found to be a straight line as shown in figure. Photoelectric current is directly proportional to the intensity of incident radiation. In this experiment the frequency and retarding potential are kept constant.



2.2 A graph between photoelectric current and potential difference between cathode and anode is found as shown in figure.



In case of saturation current,
rate of emission of photoelectrons = rate of flow of photoelectrons ,

here, v_s → stopping potential and it is a positive quantity

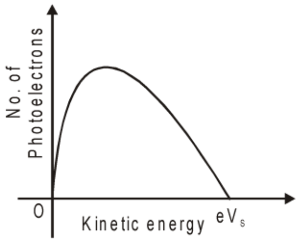
Electrons emitted from surface of metal have different energies.
Maximum kinetic energy of photoelectron on the cathode = eV_s

$$KE_{max} = eV_s$$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from

0 to $K.E_{max}$ i.e. $0 \le KE_C \le eV_s$

The energy distribution of photoelectron is shown in figure.

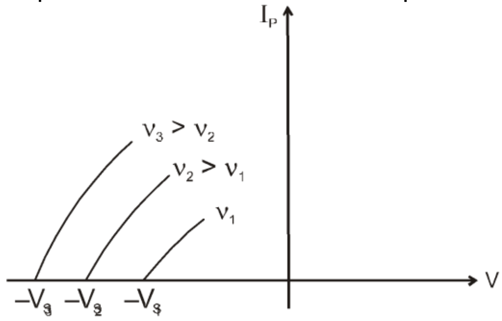


Stopping potential (V_s) $eV_s = h\nu - W$ (Work function $W = h\nu_0$)

$$V_s = \frac{h(\nu - \nu_0)}{e}$$

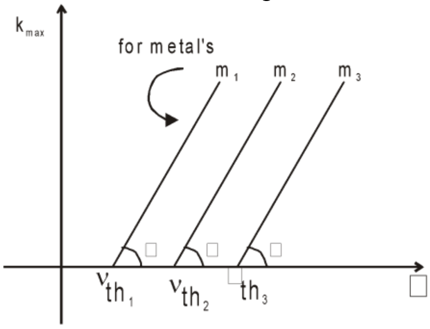
2.3 If intensity is increased (keeping the frequency constant) then saturation current is increased by same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not effected.

2.4 If light of different frequencies is used then obtained plots are shown in figure.



It is clear from graph, as ν increases, stopping potential increases, it means maximum value of kinetic energy increases.

2.5 Graphs between maximum kinetic energy of electrons ejected from different metals and frequency of light used are found to be straight lines of same slope as shown in fiugre



Graph between K_{max} and ν
m_1, m_2, m_3 : Three different metals.
It is clear from graph that there is a minimum frequency of electromagnetic radiation which can produce photoelectric effect, which is called **threshold frequency**.

ν_{th} = Threshold frequency
For photoelectric effect $\nu \geq \nu_{th}$
for no photoelectric effect $\nu < \nu_{th}$

Minimum frequency for photoelectric effect.

$$\nu_{min} = \nu_{th}$$

Threshold wavelength (λ_{th}) → The maximum wavelength of radiation which can produce photoelectric effect.

$\lambda \leq \lambda_{th}$ for photo electric effect

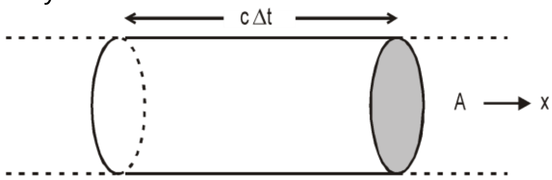
Maximum wavelength for photoelectric effect $\lambda_{max} = \lambda_{th}$.
Now writing equation of straight line from graph.
We have $K_{max} = A\nu + B$
When $\nu = \nu_{th}$, $K_{max} = 0$
and $B = -A\nu_{th}$
Hence **$[K_{max} = A(\nu - \nu_{th})]$**

and $A = \tan\theta = 6.63 \times 10^{-34}$ J-s (from experimental data)
later on 'A' was found to be 'h'.

2.6 It is also observed that photoelectric effect is an instantaneous process. When light falls on surface electrons start ejecting without taking any time.

3. THREE MAJOR FEATURES OF THE PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED IN TERMS OF THE CLASSICAL WAVE THEORY OF LIGHT.

Intensity : The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.



Consider a cylindrical volume with area of cross-section A and length c Δt along the X-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained.

$$U = u_{av}(c.\ \Delta t)A$$

The intensity is $I = \frac{U}{A\Delta t} = u_{av}\ c.$

In the terms of maximum electric field,

$$I = \frac{1}{2}\ \varepsilon_0\ E_0^2\ c.$$

If we consider light as a wave then the intensity depends upon electric field.

If we take work function W = I . A . t,

then $t = \frac{W}{IA}$

so for photoelectric effect there should be time lag because the metal has work function.

But it is observed that photoelectric effect is an instantaneous process.

Hence, light is not of wave nature.

3.1 **The intensity problem :** Wave theory requires that the oscillating electric field vector **E** of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is e**E**, this suggests that the kinetic energy of the photoelectrons should also increased as the light beam is made more intense. However observation shows that maximum kinetic energy is independent of the light intensity.

3.2 **The frequency problem :** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectrons. However observations shows that there exists for each surface a characteristic cutoff frequency ν_{th}, for frequencies less than ν_{th}, the photoelectric effect does not occur, no matter how intense is light beam.

3.3 **The time delay problem :** If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In the classical theory, the light energy is uniformly distributed over the wave front. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

Now, quantum theory solves these problems in providing the correct interpretation of the photoelectric effect.

4 PLANCK'S QUANTUM THEORY :

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light. Hence energy Q = NE,

where E = hν and N (number of photons) = 1,2,3,....

Here energy is quantized. hν is the quantum of energy, it is a packet of energy called as **photon**.

$$E = h\nu = \frac{hc}{\lambda} \quad \text{and} \quad hc = 12400 \text{ eV Å}$$

5. EINSTEIN'S PHOTON THEORY

In 1905 Einstein made a remarkable assumption about the nature of light; namely, that, under some circumstances, it behaves as if its energy is concentrated into localized bundles, later called photons. The energy E of a single photon is given by

$$E = h\nu,$$

If we apply Einstein's photon concept to the photoelectric effect, we can write

$$h\nu = W + K_{max}, \qquad \text{(energy conservation)}$$

Equation says that a single photon carries an energy $h\nu$ into the surface where it is absorbed by a single electron. Part of this energy W (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy ($h\nu$ – W) becomes the electron kinetic energy; if the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. There is complete agreement of the photon theory with experiment.

Now $IA = Nh\nu \Rightarrow N = \frac{IA}{h\nu}$ = no. of photons incident per unit time on an area 'A' when light of intensity 'I' is incident normally.

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photons or the nature of the individual photoelectric processes.

The second objection (the frequency problem) is met if K_{max} equals zero, we have

$$h\nu_{th} = W,$$

Which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If ν is reduced below ν_{th}, $h\nu$ will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory.

Hence Einstein's equation for photoelectric effect is given by

$$h\nu = h\nu_{th} + K_{max} \qquad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

Solved Examples

Example 3. In an experiment on photo electric emission, following observations were made;

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 volt.

Find : (a) Kinetic energy of photoelectrons with maximum speed.

(b) Work function and

(c) Threshold frequency;

Solution : (a) Since $V_s = 2.5$ V, $K_{max} = eV_s$

so, $K_{max} = 2.5$ eV

(b) Energy of incident photon

$$E = \frac{12400}{1980}\text{eV} = 6.26 \text{ eV} \qquad W = E - K_{max} = 3.76 \text{ eV}$$

(c) $h\nu_{th} = W = 3.76 \times 1.6 \times 10^{-19}$ J

$$\therefore \quad \nu_{th} = \frac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14} \text{Hz}$$

Example 4. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally on an area 10^{-4} m^2 of a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy (i.e. each capable photon emits one electron) calculate the number of photoelectrons liberated per second.

Solution : $E_1 = \frac{12400}{4000} = 3.1$ eV, $E_2 = \frac{12400}{4800} = 2.58$ eV $E_3 = \frac{12400}{6000} = 2.06$ eV

and $E_4 = \frac{12400}{7000} = 1.77$ eV

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

∴ Number of photoelectrons emitted per second = No. of photons incident per second)

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3} = IA\left(\frac{1}{E_1}+\frac{1}{E_2}+\frac{1}{E_3}\right) = \frac{(1.5\times10^{-3})(10^{-4})}{1.6\times10^{-19}}\left(\frac{1}{3.1}+\frac{1}{2.58}+\frac{1}{2.06}\right)$$

$= 1.12 \times 10^{12}$ **Ans.**

Example 5. A small potassium foil is placed (perpendicular to the direction of incidence of light) a distance r (= 0.5 m) from a point light source whose output power P_0 is 1.0W. Assuming wave nature of light how long would it take for the foil to soak up enough energy (= 1.8 eV) from the beam to eject an electron? Assume that the ejected photoelectron collected its energy from a circular area of the foil whose radius equals the radius of a potassium atom (1.3×10^{-10} m).

Solution : If the source radiates uniformly in all directions, the intensity I of the light at a distance r is given by

$$I = \frac{P_0}{4\pi r^2} = \frac{1.0\ \text{W}}{4\pi(0.5\ \text{m})^2} = 0.32\ \text{W/m}^2.$$

The target area A is $\pi(1.3 \times 10^{-10}\ \text{m})^2$ or $5.3 \times 10^{-20}\ \text{m}^2$, so that the rate at which energy falls on the target is given by

$$P = IA = (0.32\ \text{W/m}^2)(5.3 \times 10^{-20}\ \text{m}^2)$$
$$= 1.7 \times 10^{-20}\ \text{J/s}.$$

If all this incoming energy is absorbed, the time required to accumulate enough energy for the electron to escape is

$$t = \left(\frac{1.8\ \text{eV}}{1.7\times10^{-20}\ \text{J/s}}\right)\left(\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 17\ \text{s}.$$

Our selection of a radius for the effective target area was some-what arbitrary, but no matter what reasonable area we choose, we should still calculate a “soak-up time” within the range of easy measurement. However, no time delay has ever been observed under any circumstances, the early experiments setting an upper limit of about 10^{-9} s for such delays.

Example 6. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential is 3 V. Find the unknown wavelength.

Solution : using equation of photoelectric effect

$K_{max} = E - W$ ($K_{max} = eV_s$)

∴ $3\ \text{eV} = \frac{12400}{\lambda} - \frac{12400}{5000} = \frac{12400}{\lambda} - 2.48$ eV or $\lambda = 2262$ Å

Example 7. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35$ μm and $\lambda_2 = 0.54$ μm, it was found that the corresponding maximum velocities of photo electrons have a ratio $\eta = 2$. Find the work function of that metal.

Solution : Using equation for two wavelengths

$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W$ ....(i) $\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W$ ....(ii)

Dividing Eq. (i) with Eq. (ii), with $v_1 = 2v_2$, we have $4 = \dfrac{\dfrac{hc}{\lambda_1} - W}{\dfrac{hc}{\lambda_2} - W}$

$$3W = 4\left(\frac{hc}{\lambda_2}\right) - \left(\frac{hc}{\lambda_1}\right) = \frac{4 \times 12400}{5400} - \frac{12400}{3500} = 5.64 \text{ eV}$$

Example 8. Light described at a place by the equation E = (100 V/m) [sin (5×10^{15} s^{-1}) t + sin (8×10^{15} s^{-1})t] falls on a metal surface having work function 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons.

Solution : The light contains two different frequencies. The one with larger frequency will cause photoelectrons with largest kinetic energy. This larger frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{8 \times 10^{15} s^{-1}}{2\pi}$$

The maximum kinetic energy of the photoelectrons is

$$K_{max} = h\upsilon - W$$

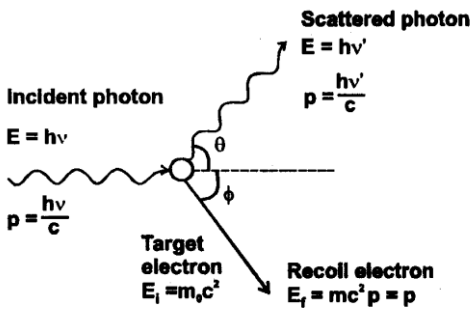
$$= (4.14 \times 10^{-15} \text{ eV-s}) \times \left(\frac{8 \times 10^{15}}{2\pi} s^{-1}\right) - 2.0 \text{ eV} = 5.27 \text{ eV} - 2.0 \text{ eV} = 3.27 \text{ eV}.$$

6. COMPTON EFFECT

The scattering of a photon by an electron in which the wavelength of scattered photon is greater than wavelength of incident photon is called Compton effect.

Conservation of energy gives $h\nu + m_0 c^2 = h\nu' + mc^2$

Conservation of momentum along y-axis gives $0 = \dfrac{h\nu'}{c}\sin\theta - p\sin\theta$



Conservation of momentum along x-axis gives $\dfrac{h\nu}{c} = \dfrac{h\nu'}{c}\cos\theta + p\cos\phi$

Compton shift $\Delta\lambda = \lambda' - \lambda = \dfrac{h}{m_0 c}(1 - \cos\theta)$

The quantity $\lambda_c = \dfrac{h}{m_0 c} = 2.42 \times 10^{12}$ m is called Compton wavelength of electron.

For maximum shift $\theta = \pi$ so $(\Delta\lambda)_{max} = \dfrac{2h}{m_0 c} = 4.48 \times 10^{-12}$ m

Compton effect shows the quantum concept and particle nature of photon.

7. de-BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, $E = mc^2$ the equivalent mass m of the photon is given by,

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c} \quad .....(i)$$

or $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ .....(ii)

Here p is the momentum of photon. By analogy de-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called de-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \quad .... (iii)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy K = qV. Combining all these relations Eq. (iii), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}} \quad \text{(de-Broglie wavelength)} \quad ....(iv)$$

7.1 de-Broglie wavelength for an electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

$$K = eV$$

Substituting the values of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda(\text{in } Å) = \sqrt{\frac{150}{V(\text{in volts})}} \approx \frac{12.27Å}{\sqrt{V}} \quad ....(v)$$

7.2 de-Broglie wavelength of a proton:

Velocity : $V_p = \sqrt{\frac{2eV}{m_p}}$ Momentum : $p_p = \sqrt{2m_p eV}$

So Wavelength is $\lambda_p = \frac{h}{\sqrt{2m_p eV}} = \frac{0.286}{\sqrt{V}}Å$

7.3 de-Broglie wavelength of a gas molecule :

Let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

K.E. = $\frac{3}{2}kT$; k = Boltzman constant $\therefore$ $\lambda_{\text{gas molecule}} = \frac{h}{\sqrt{3mkT}}$

7.4 $\lambda_{\text{neutron}} = \frac{0.286Å}{\sqrt{\text{energy in eV}}}$

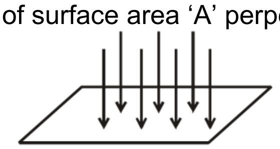
7.5 $\lambda_{\alpha\,\text{particle}} = \frac{0.101Å}{\sqrt{V}}$

7.6 $\lambda_{\text{deutron}} = \frac{0.202Å}{\sqrt{V}}$

8. FORCE DUE TO RADIATION (PHOTON)

Each photon has a definite energy and a definite linear momentum. All photons of light of a particular wavelength λ have the same energy $E = hc/\lambda$ and the same momentum $p = h/\lambda$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be 'a' and 'r' and assuming no transmission.
Assume light beam falls on surface of surface area 'A' perpendicularly as shown in figure.



For calculating the force exerted by beam on surface, we consider following cases.

Case : (I)

$a = 1, \quad r = 0$

initial momentum of the photon = $\frac{h}{\lambda}$

final momentum of photon = 0

change in momentum of photon = $\frac{h}{\lambda}$ (upward) $\Delta P = \frac{h}{\lambda}$

energy incident per unit time = IA

no. of photons incident per unit time $= \frac{IA}{h\nu} = \frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n\,\Delta P = \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c}$ (upward)

force on photons = total change in momentum per unit time = $\frac{IA}{c}$ (upward)

$\therefore$ force on plate due to photons(F) = $\frac{IA}{c}$ (downward)

pressure = $\frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case : (II)

when $r = 1, a = 0$

intial momentum of the photon $= \frac{h}{\lambda}$ (downward)

final momentum of photon $= \frac{h}{\lambda}$ (upward)

change in momentum $= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$

$\therefore$ energy incident per unit time = IA

no. of photons incident per unit time = $\frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time = $n\,.\,\Delta P = \frac{IA\lambda}{hc}\,.\,\frac{2h}{\lambda} = \frac{2IA}{C}$

force = total change in momentum per unit time

$F = \frac{2IA}{c}$ (upward on photons and downward on the plate)

pressure $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case : (III)

When $o < r < 1$ $\quad a + r = 1$

change in momentum of photon when it is reflected = $\frac{2h}{\lambda}$ (upward)

change in momentum of photon when it is absorbed = $\frac{h}{\lambda}$ (upward)

no. of photons incident per unit time $= \frac{IA\lambda}{hc}$

No. of photons reflected per unit time $= \frac{IA\lambda}{hc}\,.\,r$

No. of photon absorbed per unit time $= \frac{IA\lambda}{hc}(1 - r)$

force due to absorbed photon (F_a) $= \frac{IA\lambda}{hc}(1 - r) \cdot \frac{h}{\lambda} = \frac{IA}{c}(1 - r)$ (downward)

Force due to reflected photon (F_r) $= \frac{IA\lambda}{hc} \cdot r \frac{2h}{\lambda} = \frac{2IA\lambda}{c}$ (downward)

total force $= F_a + F_r$ (downward) $= \frac{IA}{c}(1 - r) + \frac{2IAr}{c} = \frac{IA}{c}(1 + r)$

Now pressure $P = \frac{IA}{c}(1 + r) \times \frac{1}{A} = \frac{I}{c}(1 + r)$

Solved Examples

Example 9. An electron is accelerated by a potential difference of 50 volt. Find the de-Broglie wavelength associated with it.

Solution : For an electron, de-Broglie wavelength is given by,

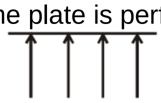
$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}} = \sqrt{3} = 1.73$ Å **Ans.**

Example 10. Find the ratio of De-Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27ºC and 127ºC respectively.

Solution : de-Broglie wavelength is given by

$\therefore \quad \frac{\lambda_{H_2}}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_{H_2}T_{H_2}}} = \sqrt{\frac{4}{2} \cdot \frac{(127+273)}{(27+273)}} = \sqrt{\frac{8}{3}}$

Example 11. A plate of mass 10 gm is in equilibrium in air due to the force exerted by light beam on plate. Calculate power of beam. Assume plate is perfectly absorbing.



Solution : Since plate is in air, so gravitational force will act on this

$F_{gravitational} = mg$ (downward)

$= 10 \times 10^{-3} \times 10$

$= 10^{-1}$ N

for equilibrium force exerted by light beam should be equal to $F_{gravitational}$

$F_{photon} = F_{gravitational}$

Let power of light beam be P

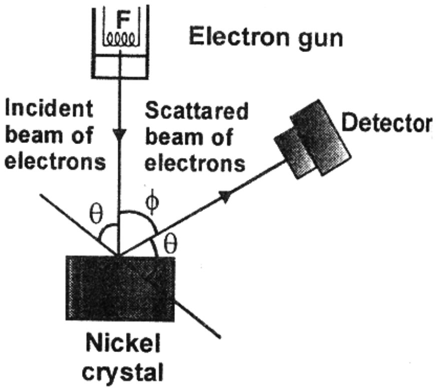
$\therefore \quad F_{photon} = \frac{P}{c} \qquad \therefore \quad \frac{P}{c} = 10^{-1}$

$P = 3.0 \times 10^{8} \times 10^{-1}$

$P = 3 \times 10^{7}$ W

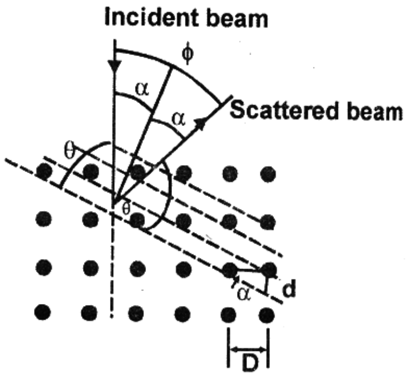
9. DAVISSON AND GERMER EXPERIMENT

- The experiment demonstrates the diffraction of electron beam by crystal surfaces
- The experiment provides first experimental evidence for wave nature of the material particles.



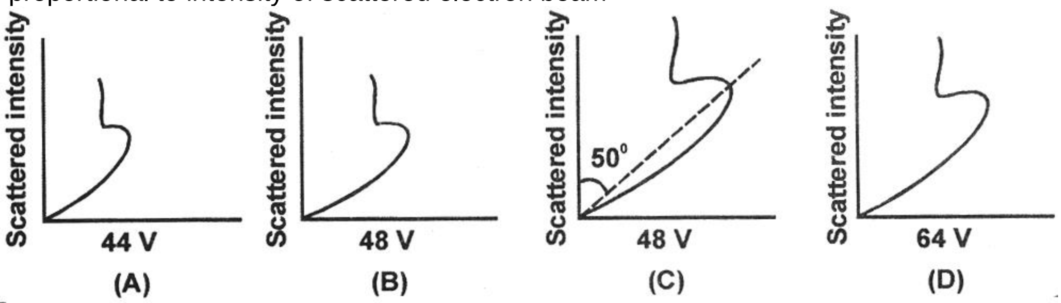
- The electrons are diffracted like X-rays. The Bragg's law of diffraction are
  D sin φ = nλ and 2d sin θ = nλ
  where D = interatomic distance and d = interplanar distance
  θ = angle between scattering plane and incident beam
  φ = scattering angle 2θ + φ = 180º

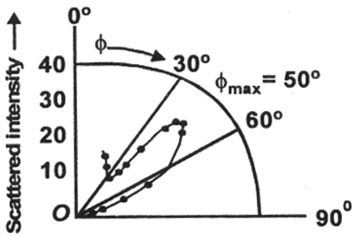
- The electrons are produced and accelerated into a beam by electron gun.
  The energy of electrons is given as $E = \frac{1}{2}mv^2 = eV$

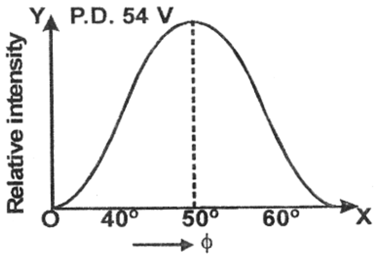


- The accelerated electron beam is made to fall on a Ni crystal. The scattered electrons are detected by a detector

- The experimental results are shown in form of polar graphs plotted between scattering angle φ and intensity of scattered electron beam at different accelerating voltage. The distance of curve from point O is proportional to intensity of scattered electron beam







IMPORTANT RESULTS

- Intensity of scattered electrons depends on scattering angle φ
- The kink at $\varphi = 50°$ is observed at all accelerating voltage
- The size of kink becomes maximum at 54 volt.
- For $\varphi = 50°$ For $\theta = \frac{180 - \phi}{2} = 65°$
  $D = \sin\varphi = n\lambda$ $2d\sin\theta = n\lambda$
  $D = 2.15Å$ & $n = 1$ (for Ni)
  $n = 1$ and $d = 0.91Å$ (for Ni)

  $\therefore \lambda = 2.15 \sin 50 = 1.65Å$
  $\lambda = 2 \times 0.91 \sin 65 = 1.65Å$
- For V = 54 volt

  de-Broglie wavelength $\lambda = \frac{12.27}{\sqrt{54}} Å = 1.67Å$

  This value of λ is in close agreement with experimental value. Thus this experiment verifies de-Broglie's hypothesis.

Solved Examples

Example 12. An electron beam of energy 10 KeV is incident on metallic foil. If the interatomic distance is 0.55Å. Find the angle of diffraction.

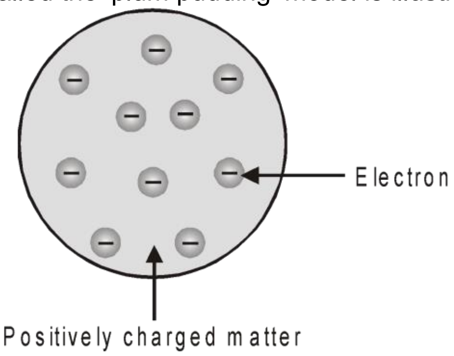
Solution : $\lambda = D\sin\varphi$ and $\lambda = \frac{12.27}{\sqrt{V}} Å$ so $\frac{12.27}{\sqrt{V}} = D\sin\phi$

$\therefore \frac{12.27 \times 10^{-10}}{\sqrt{10 \times 10^3}} = 0.55 \times 10^{-10} \sin\phi$ $\sin\varphi = \frac{12.27}{0.53 \times 100} = 0.2231$

or $\phi = \sin^{-1}(0.2231) \approx 12.89°$

10. THOMSON'S ATOMIC MODEL :

J.J. Thomson suggested that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Total charge of atom is zero and atom is electrically neutral. Thomson's model called the 'plum pudding' model is illustrated in figure.



Thomson played an important role in discovering the electron, through gas discharge tube by discovering cathode rays. His idea was taken seriously.

But the real atom turned out to be quite different.

11. RUTHERFORD'S NUCLEAR ATOM :

Rutherford suggested that; " All the positive charge and nearly all the mass were concentrated in a very small volume of nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electron static attraction between the two opposite charges being the required centripetal force for such motion.

Hence $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$ and total energy = potential energy + kinetic energy = $\frac{-kZe^2}{2r}$

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. This model suffer's from two defects

11.1 Regarding stability of atom : An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If total energy decreases then radius increases as given by above formula. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second. But atoms do not collapse. In 1913 an effort was made by Neils Bohr to overcome this paradox.

11.2 Regarding explanation of line spectrum : In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be chaning. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

12. THE BOHR'S ATOMIC MODEL

In 1913, Prof. Niel Bohr removed the difficulties of Rutherford's atomic model by the application of Planck's quantum theory. For this he proposed the following postulates

(1) An electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory.

(2) According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1 \qquad \text{or} \qquad \nu = \frac{E_2 - E_1}{h}$$

(3) Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is L = mvr for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate,

$$mvr = \frac{nh}{2\pi} \qquad (n = 1, 2, 3....)$$

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n . The value of n for each orbit is called **principal quantum number** for the orbit. Thus,

$$mv_nr_n = mvr = \frac{nh}{2\pi} \qquad ...(ii)$$

According to Newton's second law a radially inward centripetal force of magnitude F = $\frac{mv^2}{r_n}$ is needed by the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus, $$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \qquad ....(iii)$$

Solving Eqs. (ii) and (iii), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2}$$ ...(iv)

and $$v_n = \frac{e^2}{2\varepsilon_0 n h}$$ ...(v)

The smallest orbit radius corresponds to n = 1. We'll denote this minimum radius, called the **Bohr radius** as a_0. Thus,

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Substituting values of ε_0, h, p, m and e, we get

$a_0 = 0.529 \times 10^{-10}$ m = 0.529 Å ....(vi)

Eq. (iv), in terms of a_0 can be written as,

$r_n = n^2 a_0$ or $r_n \propto n^2$ ....(vii)

Similarly, substituting values of e, ε_0 and h with n = 1 in Eq. (v), we get

$v_1 = 2.19 \times 10^6$ m/s ....(viii)

This is the greatest possible speed of the electron in the hydrogen atom. Which is approximately equal to c/137 where c is the speed of light in vacuum.

Eq. (v), in terms of v_1 can be written as,

$v_n = \frac{v_1}{n}$ or $v_n \propto \frac{1}{n}$

Energy levels : Kinetic and potential energies K_n and U_n in nth orbit are given by

$K_n = \frac{1}{2} m v_n^2 = \frac{me^4}{8\varepsilon_0^2 n^2 h^2}$ and $U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2}$

(assuming infinity as a zero potential energy level)

The total energy E_n is the sum of the kinetic and potential energies.

so, $E_n = K_n + U_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$

Substituting values of m, e, ε_0 and h with n = 1, we get the least energy of the atom in first orbit, which is –13.6 eV. Hence,

$E_1 = -13.6$ eV ....(x)

and $E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2}$ eV ....(xi)

Substituting n = 2, 3, 4, ..., etc., we get energies of atom in different orbits.

$E_2 = -3.40$ eV, $E_3 = -1.51$ eV, ... $E_\infty = 0$

Solved Examples

Example 13. An α-particle with kinetic energy 10 MeV is heading towards a stationary point-nucleus of atomic number 50. Calculate the distance of closest approach.

Solution : $\because TE_A = TE_B$ $\therefore$ $10 \times 10^6 e = \frac{K \times (2e)\ (50e)}{r_0}$

$r_0 = 1.44 \times 10^{-14}$ m

$r_0 = 1.44 \times 10^{-4}$ Å

Example 14. A beam of α-particles of velocity 2.1×10^7 m/s is scattered by a gold (z = 79) foil. Find out the distance of closest approach of the α-particle to the gold nucleus. The value of charge/mass for α-particle is 4.8×10^7 C/kg.

Solution : $\frac{1}{2} m_\alpha V_\alpha^2 = \frac{K(2e)\ (Ze)}{r_0}$

$$r_0 = \frac{2K\left(\frac{2e}{m_\alpha}\right)(79\ e)}{V_\alpha^2} = \frac{2\times(9\times10^9)\ (4.8\times10^7)\ (79\times1.6\times10^{-19})}{(2.1\times10^7)^2}\ ;\ r_0 = 2.5\times10^{-14}\ \text{m}$$

12.1 Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e., one electron atoms, the nuclear charge is +ze, where z is the atomic number, equal to the number of protons in the nucleus.

The effect in the previous analysis is to replace e^2 every where by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{nmze^2} = \frac{n^2}{z} a_0 \quad \text{or} \quad r_n \propto \frac{n^2}{z} \qquad \text{....(i)}$$

where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n} \qquad \text{....(ii)}$$

where $v_1 = 2.19 \times 10^6$ m/s (speed of electron in first orbit of H)

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2} \qquad \text{....(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)

12.2 Definations valid for single electron system

(1) Ground state : Lowest energy state of any atom or ion is called ground state of the atom.
Ground state energy of H atom = –13.6 eV
Ground state energy of He^+ Ion = –54.4 eV
Ground state energy of Li^{++} Ion = –122.4 eV

(2) Excited State : State of atom other than the ground state are called its excited states.

n = 2	first excited state
n = 3	second excited state
n = 4	third excited state
$n = n_0 + 1$	n_{0th} excited state

(3) Ionisation energy (I.E.) : Minimum energy required to move an electron from ground state to n = ∞ is called ionisation energy of the atom or ion
Ionisation energy of H atom = 13.6 eV
Ionisation energy of He^+ Ion = 54.4 eV
Ionisation energy of Li^{++} Ion = 122.4 eV

(4) Ionisation potential (I.P.) : Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.
I.P of H atom = 13.6 V
I.P. of He^+ Ion = 54.4 V

(5) Excitation energy : Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.
Energy in ground state of H atom = –13.6 eV
Energy in first excited state of H-atom = –3.4 eV
I_{st} excitation energy = 10.2 eV.

(6) Excitation Potential : Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.
I_{st} excitation energy = 10.2 eV.
I_{st} excitation potential = 10.2 V.

(7) Binding energy or Separation energy : Energy required to move an electron from any state to n = ∞ is called binding energy of that state. or energy released during formation of an H-like atom/ion from n = ∞ to some particular n is called binding energy of that state.
Binding energy of ground state of H-atom = 13.6 eV

Solved Examples

Example 15. First excitation potential of a hypothetical hydrogen like atom is 15 volt. Find third excitation potential of the atom.

Solution : Let energy of ground state = E_0

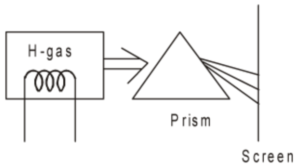
$E_0 = -13.6 Z^2$ eV and $E_n = \frac{E_0}{n^2}$ $\Rightarrow$ n = 2, $E_2 = \frac{E_0}{4}$

given $\frac{E_0}{4} - E_0 = 15 - \frac{3E_0}{4} = 15$ for n = 4, $E_4 = \frac{E_0}{16}$

third exicitation energy $= \frac{E_0}{16} - E_0 = -\frac{15}{16} E_0 = -\frac{15}{16} \cdot \left(\frac{-4 \times 15}{3}\right) = \frac{75}{4}$ eV

$\therefore$ third excitation potential is $\frac{75}{4}$ V

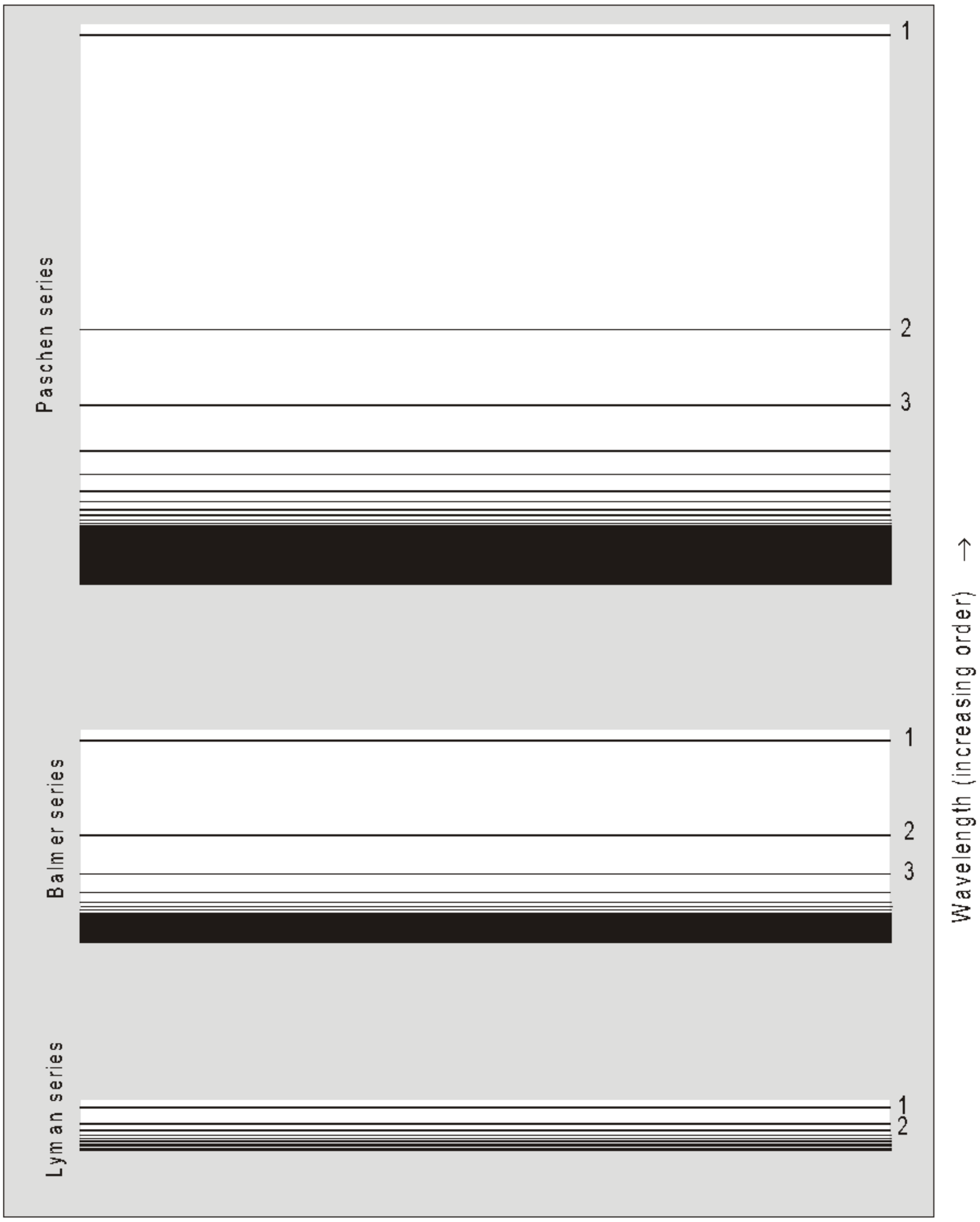
12.3 Emission spectrum of hydrogen atom :



Under normal conditions the single electron in hydrogen atom stays in ground state (n = 1). It is excited to some higher energy state when it acquires some energy from external source. But it hardaly stays there for more than 10^{-8} second. A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level. Let n_i be the initial and n_f the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

On Screen :

A photograph of spectral lines of the Lymen, Balmer, Paschen series of atomic hydrogen.

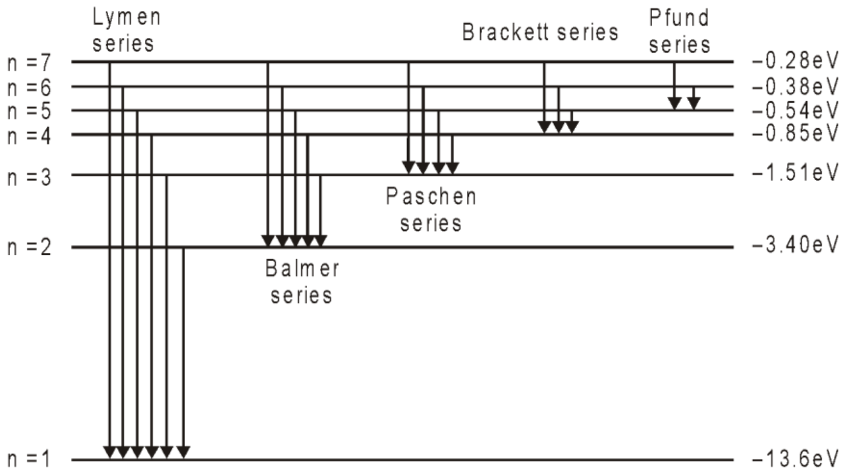


1, 2, 3..... represents the I, II & III line of Lymen, Balmer, Paschen series.

The hydrogen spectrum (some selected lines)

Name of series	Number of Line	Quantum Number			
		n_i (Lower State	n_f (Upper State)	Wavelength (nm)	Energy
Lymen	I	1	2	121.6	10.2 eV
	II	1	3	102.6	12.09 eV
	III	1	4	97	12.78 eV
	series limit	1	∞ (series limit)	91.2	13.6 eV
Balmer	I	2	3	656.3	1.89 eV
	II	2	4	486.1	2.55 eV
	III	2	5	434.1	2.86 eV
	series limit	2	∞ (series limit)	364.6	3.41 eV
Paschen	I	3	4	1875.1	0.66 eV
	II	3	5	1281.8	0.97 eV
	III	3	6	1093.8	1.13 eV
	series limit	3	∞ (series limit)	822	1.51 eV

Series limit : Line of any group having maximum energy of photon and minimum wavelength of that group is called series limit.



For the Lymen series n_f = 1, for Balmer series n_f = 2 and so on.

12.4 Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from higher energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = \frac{hc}{\lambda}$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \quad \text{(energy of emitted photon)} \quad ....(i)$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is in the red and is called H_α, the next line, in the blue-green is called H_β and so on. In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wave lengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad ....(ii)$$

Here, n = 3, 4, 5 ...., etc.
R = Rydberg constant = $1.097 \times 10^7\ m^{-1}$
and λ is the wavelength of light/photon emitted during transition,
For n = 3, we obtain the wavelength of H_α line.
Similarly, for n = 4, we obtain the wavelength of H_β line. For n = ∞, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmar series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = -\frac{Rhc}{n^2},\ n = 1, 2, 3..... \qquad ....(iii)$$

The wavelengths corresponding to other spectral series (Lymen, Paschen, (etc.) can be represented by formula similar to Balmer's formula.

Lymen Series : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, n = 2, 3, 4.....

Paschen Series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, n = 4, 5, 6.....

Brackett Series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5, 6, 7.....

Pfund Series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, n = 6, 7, 8

The Lymen series is in the ultraviolet, and the Paschen. Brackett and Pfund series are in the infrared region.

Solved Examples

Example 16. Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.

Solution : (a) H_β line of Balmer series corresponds to the transition from n = 4 to n = 2 level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$= 0.2056 \times 10^7$

∴ $\lambda = 4.9 \times 10^{-7}$ m **Ans.**

(b) $\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}}$

$= 6.12 \times 10^{14}$ Hz **Ans.**

Example 17. Find the largest and shortest wavelengths in the Lymen series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Solution : The transition equation for Lymen series is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right] \qquad n = 2, 3, ...$$

for largest wavelength, n = 2

$$\frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right) = 0.823 \times 10^7$$

∴ $\lambda_{max} = 1.2154 \times 10^{-7}$ m = 1215 Å **Ans.**

The shortest wavelength corresponds to n = ∞

$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{\infty}\right)$

or $\quad \lambda_{min} = 0.911 \times 10^{-7}$ m = 911 Å **Ans.**

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

Example 18. How may different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?

Solution : From the nth state, the atom may go to (n – 1)th state, ..., 2nd state or 1st state. So there are (n – 1) possible transitions starting from the nth state. The atoms reaching (n – 1)th state may make (n – 2) different transitions. Similarly for other lower states. The total number of possible transitions is

$(n-1) + (n-2) + (n-3) + ...2 + 1 = \frac{n(n-1)}{2}$ **(Remember)**

Example 19. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Solution : $E_1 = -13.60$ eV $\quad K_1 = -E_1 = 13.60$ eV $\quad U_1 = 2E_1 = -27.20$ eV

$E_2 = \frac{E_1}{(2)^2} = -3.40$ eV $\quad K_2 = 3.40$ eV and $U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV.

Example 20. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Solution : The force at a distance r is,

$F = -\frac{dU}{dr} = -2ar$

Suppose r be the radius of nth orbit. The necessary centripetal force is provided by the above force. Thus,

$\frac{mv^2}{r} = 2ar$

Further, the quantization of angular momentum gives,

$mvr = \frac{nh}{2\pi}$

Solving Eqs. (i) and (ii) for r, we get

$r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$ **Ans.**

Example 21. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find

(a) the allowed values of the radius 'r' of the orbit.

(b) the kinetic energy of the electron in orbit

(c) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.

(d) The total energy of the allowed energy levels.

Solution : (a) radius of circular path

$r = \frac{mv}{Be}$ ....(i)

$mvr = \frac{nh}{2\pi}$ ....(ii)

Solving these two equations, we get

r = $\sqrt{\frac{nh}{2\pi Be}}$ and v = $\sqrt{\frac{nhBe}{2\pi m^2}}$

(b) K = $\frac{1}{2}$ mv2 = $\frac{nhBe}{4\pi m}$ **Ans.**

(c) M = iA = $\left(\frac{e}{T}\right)$ (πr2) = $\frac{evr}{2}$ = $\frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}}$ = $\frac{nhe}{4\pi m}$

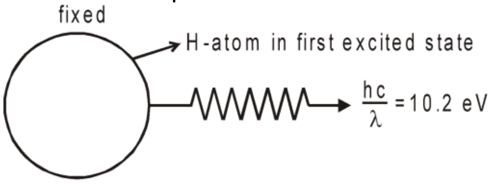
Now potential energy U = – **M . B**

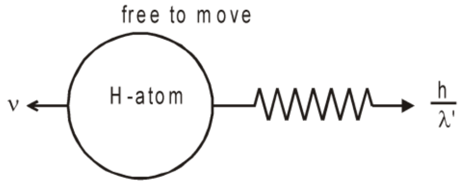
= $\frac{nheB}{4\pi m}$

(d) E = U + K = $\frac{nheB}{2\pi m}$

13. Calculation of recoil speed of atom on emission of a photon

momentum of photon = mc = $\frac{h}{\lambda}$

(a) 

(b) 

m - mass of atom

According to momentum conservation

mv = $\frac{h}{\lambda'}$ .... (i)

According to energy conservation

$\frac{1}{2}m\nu^2 + \frac{hc}{\lambda'}$ = 10.2 eV

Since mass of atom is very large than photon

hence $\frac{1}{2}m\nu^2$ can be neglected

$\frac{hc}{\lambda'}$ = 10.2 eV ⇒ $\frac{h}{\lambda'}$ = $\frac{10.2}{c}$ eV

mν = $\frac{10.2}{c}$eV ⇒ ν = $\frac{10.2}{cm}$

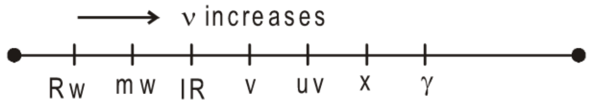
recoil speed of atom = $\frac{10.2}{cm}$

14. X-RAYS

- X-rays were discovered by *Wilhelm Roentgen* in 1895. There are also called as Roentgen rays.

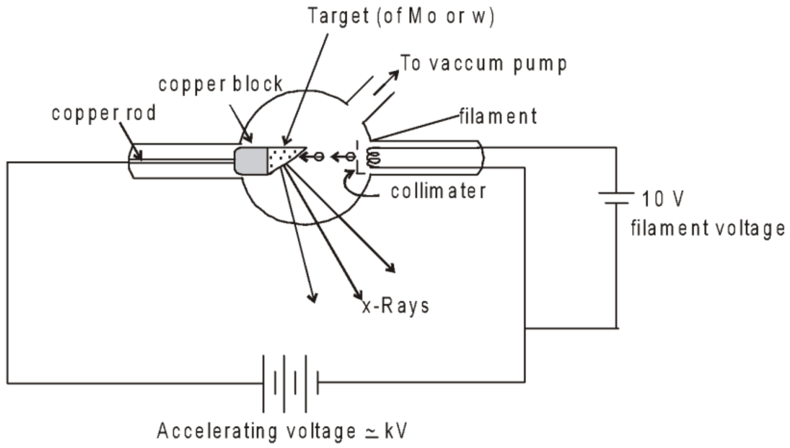
- X-rays are produced by bombarding high speed electrons on a target of high atomic weight and high melting point.
- The wavelength of X-rays lies between γ-rays and UV rays.
- The wavelength range for X-rays is 0.1Å to 10Å.
- The frequency range for X-rays is 10^{16} Hz to 10^{18} Hz.
- The energy range for X-rays is 100eV to 10000 eV.
- Hard X-rays : High frequency X-rays are called hard X-rays.
  (a) Hard X-rays have high penetration power
  (b) The wavelength range is from 0.1A to 10A.
  (c) They have high frequency 10^{18} Hz and high energy ~10^{4} eV .
- Soft X-rays : Low frequency X-rays are called soft X-rays.
  (a) These have low penetrating power
  (b) They have higher wavelength (10Å to 100Å)
  (c) They have low frequency 10^{16} Hz and low energy 10^{2} eV.

It was discovered by **ROENTGEN**. The wavelength of x-rays is found between 0.1 Å to 10 Å. These rays are invisible to eye. They are electromagnetic waves and have speed c = 3 × 10^{8} m/s in vacuum.
Its photons have energy around 1000 times more than the visible light.



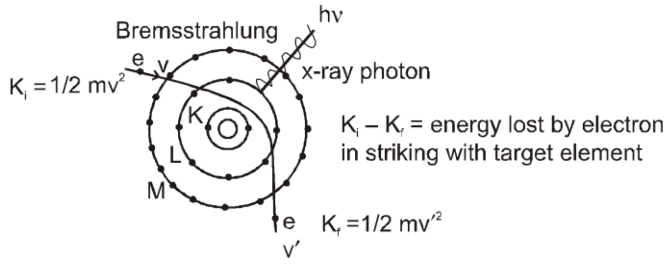
When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

14.1 Production of x-rays by coolidge tube :



The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then x-rays are produced.
When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorbe heat so that the melting point, specific heat of target should be high.
Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.
For more energetic electron, accelerating voltage is increased. For more no. of photons voltage across filament is increased. The x-ray were analysed by mostly taking their spectrum

14.2 Continuous x-ray : When high energy electrons (accelerated by coolidge tube potential) strike the target element they are deflected by coulomb attraction of nucleus & due to numerous glancing colisions with the atoms of the target, they lose energy which appears in the form of electro magneticwaves (bremsstrahlung or braking radiation) & the remaining part increases the kinetic energy of the colliding particles of the target. The energy received by the colliding particles goes into heating the target. The electron makes another collision with its remaining energy.



1. When high energetic electrons enter into target material, they are decelerated. In this process emission of energy take place. Spectrum of this energy is continuous. This is also called bremstrahlung.

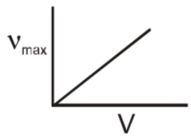
2. Continuous spectrum (ν or λ) depends upon potential difference between filament and target.

3. It does not depends upon nature of target material.

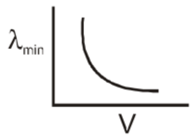
4. If V is the potential difference & ν is the frequency of emitted x-ray photon then.

Variation of frequency (ν) and wavelength (λ) of x-rays with potential difference is plotted as shown in figure :

$$eV = \frac{1}{2}mv^2 = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

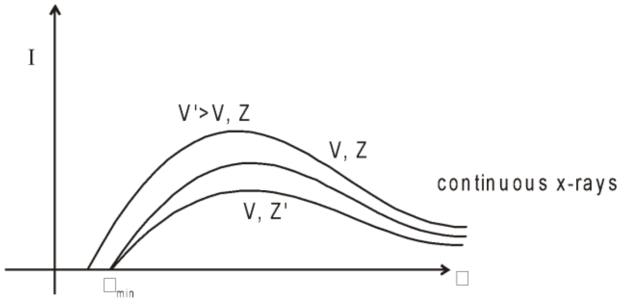


$$\nu_{max} = \frac{eV}{h}$$



$$\lambda_{min} = \frac{hc}{eV}$$

Variation of Intensity of x-rays with λ is plotted as shown in figure :



1. The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = \frac{1}{2}mv^2 = h\nu_{max} = \frac{hc}{\lambda_{min}} \Rightarrow \lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{in volts})}\ Å.$$

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

Solved Examples

Example 22. An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

Solution : Kinetic energy acquired by the electron is K = eV = 20 × 10^3 eV.

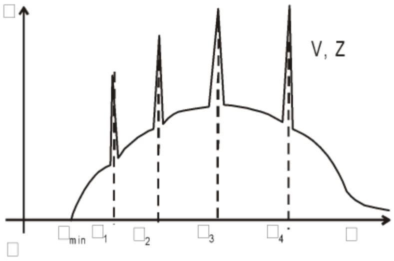
The energy of the photon = 0.05 × 20 = 10^3 eV = 10^3 eV.

Thus, $$\frac{h\nu}{\lambda} = 10^3 eV = \frac{(4.14\times10^{-15} eV - s)\times(3\times10^8 m/s)}{10^3 eV} = \frac{1242\ \ eV - nm}{10^3 eV} = 1.24\ \ nm$$

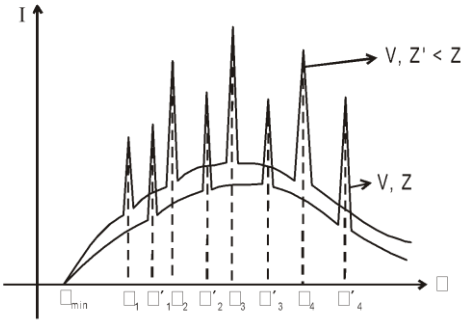
Charactristic X-rays

The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material.

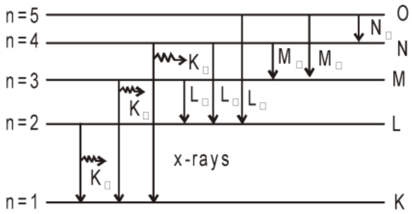
λ_1, λ_2, λ_3, λ_4, ... = charecteristic wavelength of material having atomic number Z are called **characteristic x-rays** and the spectrum obtained is called **characteristic spectrum**. If target of atomic number Z' is used then peaks are shifted.



Characteristic x-ray emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron droping from next higher level L-shell then K_α characteristic x-ray is obtained. If vaccany in K-shell is filled by an electron from M-shell, K_β line is produced and so on

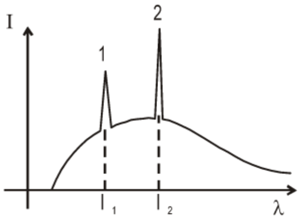


similarly L_α, L_β,.....M_α, M_β lines are produced.



Solved Examples

Example 23. Find which is K_α and K_β



Solution : $\Delta E = \frac{hc}{\lambda}$, $\lambda = \frac{hc}{\Delta E}$

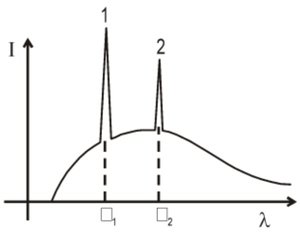
since energy difference of K_α is less than K_β

$\Delta E_{k\alpha} < \Delta E_{k\beta}$

$\lambda_{k\beta} < \lambda_{k\alpha}$

1 is K_β and 2 is K_α

Example 24. Find which is K_α and L_α



Solution : $\therefore$ $\Delta E_{K\alpha} > \Delta E_{L\alpha}$

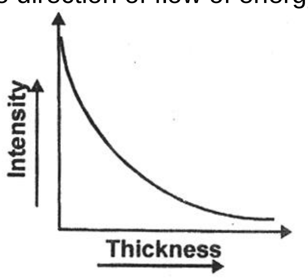
1 is K_α and 2 is L_α

14.3 PROPERTIES OF X-RAYS

- X-rays are electromagnetic waves of short wavelength which travel in straight lines with speed of light.
- They are chargeless and are not deflected in electric and magnetic fields.
- They cause fluorescence in many substances like zinc sulphide, cadmium tungstate, barium platino-cyanide.
- They produce photochemical reaction and affect a photographic plate more severely than light.
- Like light they show reflection, refraction, interference, diffraction and polarization.
- They ionize the gases through which they pass.
- When they fall on matter they produce photoelectric effect and Compton effect.
- They are highly penetrating and can pass through many solids. e.g. They pass through 1 mm thick Aluminium sheet while are absorbed by a sheet of lead of same thickness.
- The penetration power depends on applied potential difference and atomic number of cathode.
  It destroys tissues of animal bodies and white blood cells.

14.4 ABSORPTION OF X-RAYS

- The intensity of X-ray beam is defined as amount of energy carried per unit area per sec perpendicular to direction of flow of energy.



- When a beam of X-rays with incident intensity I_0 passes through material then intensity of emergent X-rays (I) is $I = I_0 e^{-\mu x}$
  where μ is absorption coefficient and x is thickness of medium
- The intensity of transmitted X-rays reduces exponentially with thickness of material.
- The absorption coefficient of the material is defined as reciprocal of thickness after which intensity of X-rays falls to $\frac{1}{e}$ times the original intensity.
  at $\mu = \frac{1}{x} \quad \Rightarrow \quad I = I_0 / e$
- The absorption coefficient depends on wavelength of X-rays (λ), the atomic number (Z) of material and density (ρ) of material.
  Absorption coefficient $\mu = CZ^4\lambda^3\rho$
  (i) $\mu \propto \lambda^3$ (ii) $\mu \propto \frac{1}{\nu^3}$ (iii) $\mu \propto Z^4$ (iv) $\mu \propto \rho$
- Best absorber of X-rays is lead while lowest absorption takes place in air.
- Half Thickness ($X_{1/2}$) : The thickness of given sheet which reduces the intensity of incident X-rays to half of its initial value is called half thickness.
  at $x = x_{1/2}$ so $x_{1/2} = \frac{0.693}{\mu} \left(\frac{I}{I_0}\right) = \left(\frac{1}{2}\right)^{x/x_{1/2}}$
- For photographing human body parts $BaSO_4$ is used.
- If number of electrons striking the target is increased, the intensity of X-rays produced also increases.
- The patients are asked to drink $BaSO_4$. solution before X-rays examination because it is a good absorber of X-rays.

Solved Examples

Example 25. The absorption coefficients of Al for soft X-rays is 1.73 per cm. Find the percentage of transmitted X-rays from a sheet of thickness 0.578 cm.

Soluton : $I = I_0 e^{-\mu x}$

so $\frac{I}{I_0} = e^{-\mu x} = e^{-1.73 \times 0.578}$ or $\frac{I}{I_0} = e^{-1} = \frac{1}{e} = \frac{1}{2.718} = 37\%$

Example 26. When X-rays of wavelength 0.5Å pass through 10 mm thick Al sheet then their intensity is reduced to one sixth. Find the absorption coefficient for Aluminium.

Solution : $\mu = \frac{2.303}{x}\log\left(\frac{I_0}{I}\right) = \frac{2.303}{10}\log_{10} 6 = \frac{2.303 \times 0.7781}{10} = 0.198/\text{mm}$

14.5 DIFFRACTION OF X-RAYS

- Diffraction of X-rays by a crystal was discovered in 1912 by Von Laue.
- The diffraction of X-rays is possible because interatomic spacings in a crystal is of the order of wavelength of X-rays.
- The diffraction of X-rays takes place according to Bragg's law
  $2d \sin\theta = n\lambda$.
- This helps to determine crystal structure and wavelength of X-rays.

14.6 X-Rays Dose :

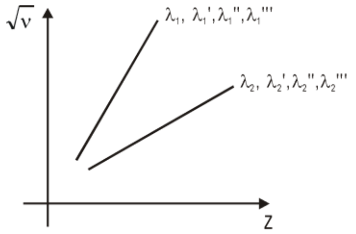
- Dose of x-ray are measured in terms of produced ions of free energy via ionisation.
- These are measured in Roentgen
- Roentgen do not measure energy but it measured ionisation power.
- Safe dose for human body per week is one Roentgen.
- One Roentgen is the amount of x-rays which emits (2.5×10^4 J) free energy through ionisation of 1 gram air at NTP.

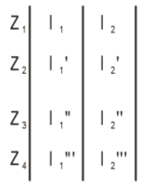
14.7 USES OF X-RAYS

- **Surgery** X-rays pass through flesh but are stopped by bones. So they are used to detect fractures, foreign bodies, diseased organs. The photograph obtained is called a radiograph.
- **Radio therapy** The X-rays can kill the diseased tissues of the body. They are used to cure skin disease, malignant tumors etc.
- **Industry** To detect defect in motor tyres, golf and tennis balls, wood and wireless valves. Used to test uniformity of insulating material and for detecting presence of pearls in oysters
- **Scientific research** Used to study structure of crystals, structure and properties of atoms, arrangement of atoms and molecules in matter.
- **Detective departments** Used at custom ports to detect goods like explosives, opium concealed in parcels without opening, detection of precious metals like gold and silver in body of smugglers.

15. MOSELEY'S LAW :

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the sqaure root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.





Wavelength of charactristic wavelengths.

Moseley's observations can be mathematically expressed as

$$\sqrt{\nu} = a(Z - b)$$

a and b are positive constants for one type of x-rays & for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} . (Z - b)$$

by using the formula $\frac{1}{\lambda} = R\,z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

b → known as screening constant or shielding effect, and (Z – b) is effective nuclear charge.

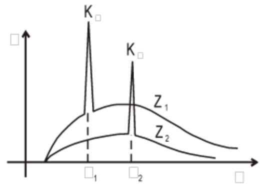
for K_α line

$n_1 = 1$, $n_2 = 2$ $\therefore \sqrt{\nu} = \sqrt{\frac{3RC}{4}}\,(Z - b)$ ⇒ $\sqrt{\nu} = a(Z - b)$

Here $a = \sqrt{\frac{3RC}{4}}$, [b = 1 for K_α lines]

Solved Examples

Example 27. Find in Z_1 and Z_2 which one is greater.



Solution : $\because \sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} . (Z - b)$

If Z is greater then ν will be greater, λ will be less. $\therefore$ $\lambda_1 < \lambda_2$ $\therefore$ $Z_1 > Z_2$.

Example 28. A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Solution : Using Moseley's law and putting c/λ for ν (and assuming b = 1), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} - a \quad \text{and} \quad \sqrt{\frac{c}{\lambda_x}} = aZ_x - a$$

Dividing yields $\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$ Substituting gives us $\sqrt{\frac{178.9\ \text{pm}}{143.5\ \text{pm}}} = \frac{Z_x - 1}{27 - 1}$.

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Example 29. Find the constants a and b in Moseley's equation $\sqrt{v} = a(Z - b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Solution : Moseley's equation is

$$\sqrt{v} = a(Z-b)$$

Thus, $\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b)$ ....(i)

and $\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b)$ ....(ii)

From (i) and (ii) $\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a\ (Z_1 - Z_2)$

or, a = $\frac{\sqrt{c}}{(Z_1 - Z_2)\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)}$

= $\frac{(3\times10^8 m/s)^{1/2}}{42-27}\left[\frac{1}{(71\times10^{-12}m)^{1/2}} - \frac{1}{(178.5\times10^{-12}m)^{1/2}}\right]$ = 5.0×10^7 $(Hz)^{1/2}$

Dividing (i) by (ii),

$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$ or, $\sqrt{\frac{178.5}{71}} = \frac{42-b}{27-b}$ or, b = 1.37

Solved Miscellaneous Problems

Problem 1. Find the momentum of a 12.0 MeV photon.

Solution : p = $\frac{E}{c}$ = 12 MeV/c.

Problem 2. Monochromatic light of wavelength 3000 Å is incident normally on a surface of area 4 cm^2. If the intensity of the light is 15×10^{-2} W/m^2, determine the rate at which photons strike the surface.

Solution : Rate at which photons strike the surface

= $\frac{IA}{hc/\lambda}$ = $\frac{6\times10^{-5} J/s}{6.63\times10^{-19} J/photon}$ = 9.05×10^{13} photon/s.

Problem 3. The kinetic energies of photoelectrons range from zero to 4.0×10^{-19} J when light of wavelength 3000 Å falls on a surface. What is the stopping potential for this light ?

Solution : $K_{max} = 4.0 \times 10^{-19}$ J × $\frac{1eV}{1.6\times10^{-19} J}$ = 2.5 eV.

Then, from $eV_s = K_{max}$, V_s = 2.5 V.

Problem 4. What is the threshold wavelength for the material in above problem ?

Solution : 2.5 eV = $\frac{12.4\times10^3 eV.Å}{3000\ Å} - \frac{12.4\times10^3 eV.Å}{\lambda_{th}}$

Solving, λ_{th} = 7590 Å.

Problem 5. Find the de Broglie wavelength of a 0.01 kg pellet having a velocity of 10 m/s.

Solution : λ = h/p = $\frac{6.63\times10^{-34} J.s}{0.01\,kg\times10\ m/s}$ = 6.63×10^{-23} Å .

Problem 6. Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1 Å, which is the size of the interatomic spacing of atoms in a crystal.

Solution : V = $\frac{h^2}{2m_0 e\lambda^2}$ = 151 V.

Problem 7. Determine the wavelength of the second line of the Paschen series for hydrogen.

Solution . $\frac{1}{\lambda} = (1.097 \times 10^{-3}\ \text{Å}^{-1})\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$ or $\lambda = 12{,}820$ Å.

Problem 8. How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the n = 5 state ?

Solution : No of possible transition from n = 5 are ${}^5C_2 = 70$

Ans. 10 photons.

Problem 9. An electron rotates in a circle around a nucleus with positive charge Ze. How is the electrons' velocity releated to the radius of its orbit ?

Solution : The force on the electron due to the nuclear provides the required centripetal force

$\frac{1}{4\pi\varepsilon_0}\frac{Ze.\ e}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 .rm}}$ **Ans.** $v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 .rm}}$.

Problem 10. A H-atom in ground state is moving with intial kinetic energy K. It collides head on with a He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.

Solution : Energy available for excitation = $\frac{4K}{5}$

Total energy required for excitation = 10.2 ev + 40.8 eV = 51.0 ev

$\therefore \quad \frac{4K}{5} = 51 \quad \Rightarrow \quad k = 63.75$ eV

Problem 11. A TV tube operates with a 20 kV accelerating potential. What are the maximum–energy X–rays from the TV set ?

Solution : The electrons in the TV tube have an energy of 20 keV, and if these electrons are brought to rest by a collision in which one X–ray photon is emitted, the photon energy is 20 keV.

Problem 12. In the Moseley relation, $\sqrt{v} = a(Z-b)$ which will have the greater value for the constant a for K_α or K_β transition ?

Solution : A is larger for the K_β transitions than for the K_α transitions.

Problem 13. A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particle.

Solution : Here the loss during the collision can only be used to excite the atoms or electrons.

So according to quantum mechanics

loss = {0, 40.8eV, 48.3eV, ..., 54.4eV} ....(1)

$E_n = -13.6\ \frac{Z^2}{n^2}$ eV

Now according to newtonion mechanics

Minimum loss = 0

maximum loss will be for perfectly inelastic collision.

let v_0 be the initial speed of neutron and v_f be the final common speed.

so by momentum conservation $mv_0 = mv_f + 4mv_f$ $v_f = \frac{v_0}{5}$

where m = mass of Neutron $\therefore$ mass of He^+ ion = 4m

so final kinetic energy of system

K.E. = $\frac{1}{2}m v_f^2 + \frac{1}{2} 4m v_f^2 = \frac{1}{2}.(5m).\frac{v_0^2}{25} = \frac{1}{5}.(\frac{1}{2}mv_0^2) = \frac{K}{5}$ maximum loss = K – $\frac{K}{5} = \frac{4K}{5}$

so loss will be $\left[0, \frac{4K}{5}\right]$ ....(2)

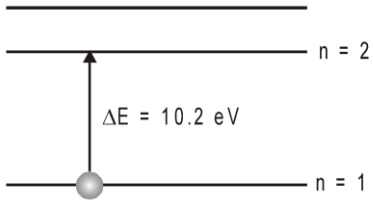
For inelastic collision there should be at least one common value other than zero in set (1) and (2) $\therefore$ $\frac{4K}{5}$ > 40.8 eV

K > 51 eV minimum value of K = 51 eV.

Problem 14. A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

Solution : Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.

From conservation of linear momentum,



p = p' or $\sqrt{2Km} = \sqrt{2K'(2m)}$

or K = 2K' ....(i)

From conservation of energy, K = K' + ΔE ....(ii)

Solving Eqs. (i) and (ii), we get ΔE = $\frac{K}{2}$

Now minimum value of ΔE for hydrogen atom is 10.2 eV.

or ΔE $\geq$ 10.2 eV

$\therefore$ $\frac{K}{2} \geq 10.2$

$\therefore$ K $\geq$ 20.4 eV

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV **Ans.**