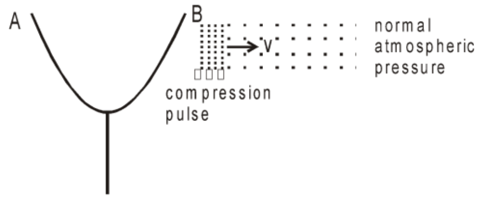
SOUND WAVES

1. PROPAGATION OF SOUND WAVES :

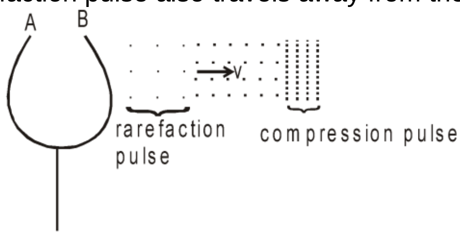
Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker. Being a mechanical wave, sound needs a medium having properties of inertia and elasticity for its propagation. Sound waves propagate in any medium through a series of periodic compressions and rarefactions of pressure, which is produced by the vibrating source.

Consider a tuning fork producing sound waves.

When Prong B moves outward towards right it compresses the air in front of it, causing the pressure to rise slightly. The region of increased pressure is called a compression pulse and it travels away from the prong with the speed of sound.



After producing the compression pulse, the prong B reverses its motion and moves inward. This drags away some air from the region in front of it, causing the pressure to dip slightly below the normal pressure. This region of decreased pressure is called a rarefaction pulse. Following immediately behind the compression pulse, the rarefaction pulse also travels away from the prong with the speed of sound.



If the prongs vibrate in SHM, the pressure variations in the layer close to the prong also varies simple harmonically and hence increase in pressure above normal value can be written as

$$\delta P = \delta P_0 \sin \omega t$$

where δP_0 is the maximum increase in pressure above normal value.

As this disturbance travel towards right with wave velocity v, the excess pressure at any position x at time t will be given by

$$\delta P = \delta P_0 \sin \omega(t - x/v) \qquad (1.1)$$

Using $p = \delta P$, $p_0 = \delta P_0$, the above equation of sound wave can be written as :

$$p = p_0 \sin \omega(t - x/v) \qquad (1.2)$$

Solved Examples

Example 1. The equation of a sound wave in air is given by

$p = (0.02\ ) \sin [(3000\ )\ t - (9.0\ )\ x\ ]$, where all variables are in S.I. units.

(a) Find the frequency, wavelength and the speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Solution : (a) Comparing with the standard form of a travelling wave

$$p = p_0 \sin [\omega\ (t - x/v)]$$

we see that ω = 3000 s^{-1}. The frequency is

$$f = \frac{\omega}{2\pi} = \frac{3000}{2\pi} \text{ Hz}$$

Also from the same comparison, ω/v = 9.0 m^{-1}.

or, $$v = \frac{\omega}{9.0 m^{-1}} = \frac{3000 s^{-1}}{9.0 m^{-1}} \frac{1000}{3} \text{ m/s}^{-1}$$

The wavelength is $$\lambda = \frac{v}{f} = \frac{1000/3 m/s}{3000/2\pi Hz} = \frac{2\pi}{9} m$$

(b) The pressure amplitude is p_0 = 0.02 N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

2. FREQUENCY AND PITCH OF SOUND WAVES

FREQUENCY :

Each cycle of a sound wave includes one compression and one rarefaction, and frequency is the number of cycles per second that passes by a given location. This is normally equal to the frequency of vibration of the (tuning fork) source producing sound. If the source, vibrates in SHM of a single frequency, sound produced has a single frequency and it is called a pure tone..

However a sound source may not always vibrate in SHM (this is the case with most of the common sound sources e.g. guitar string, human vocal card, surface of drum etc.) and hence the pulse generated by it may not have the shape of a sine wave. But even such a pulse may be considered to obtained by superposition of a large number of sine waves of different frequency and amplitudes. We say that the pulse contain all these frequencies.

AUDIBLE FREQUENCY RANGE FOR HUMAN :

A normal person hears all frequencies between 20 & 20 KHz. This is a subjective range (obtained experimentally) which may vary slightly from person to person. The ability to hear the high frequencies decreases with age and a middle-age person can hear only upto 12 to 14 KHz.

INFRASONIC SOUND :

Sound can be generated with frequency below 20 Hz called **infrasonic sound.**

ULTRASONIC SOUND :

Sound can be generated with frequency above called **infrasonic sound.**

Even through humans cannot hear these frequencies, other animals may. For instance Rhinos communicate through infrasonic frequencies as low as 5Hz, and bats use ultrasonic frequencies as high as 100 KHz for navigating.

PITCH :

Frequency as we have discussed till now is an objective property measured its units of Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called **Pitch.** A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound

Solved Examples

Example 2. A wave of wavelength 4 mm is produced in air and it travels at a speed of 300 m/s. Will it be audible ?

Solution : From the relation $v = \nu\lambda$, the frequency of the wave is

$$\nu = \frac{v}{\lambda} = \frac{300 m/s}{4 \times 10^{-3} m} = 75000 \text{ Hz}.$$

This is much above the audible range. It is an ultrasonic wave and will not be audible to humans, but it will be audible to bats.

3. PRESSURE WAVE AND DISPLACEMENT WAVE :

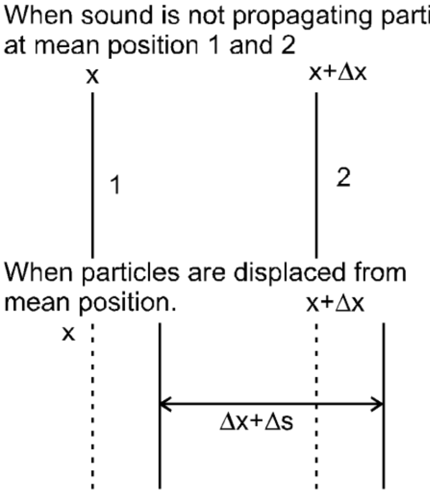
We can describe sound waves either in terms of excess pressure (equation 1.1) or in terms of the longitudinal displacement suffered by the particles of the medium w.r.t. mean position.

If $s = s_0 \sin \omega(t - x/v)$ represents a sound wave where,

s = displacement of medium particle from its mean position at x,

$s = s_0 \sin(\omega t - kx)$ ...(3.1)

Change in volume = $\Delta V = (\Delta x + \Delta s)A - \Delta x A = \Delta s A$



$$\frac{\Delta V}{V} = \frac{\Delta s A}{\Delta x A} = \frac{\Delta s}{\Delta x}$$

$$\Delta P = -\frac{B\Delta V}{V}$$

$$\Delta P = -\frac{B\Delta s}{\Delta x}$$

$$dp = -\frac{Bds}{dx}$$

$dp = -B(-k s_0)\cos(\omega t - kx)$

$dp = Bks_0 \cos(\omega t - kx)$

$dp = (dp)_{max} \cos(\omega t - kx)$

$p = p_0 \sin(\omega t - kx + \pi/2)$ .... (3.2)

where p = dp = variation in pressure at position x and

$p_0 = Bks_0$ = maximum pressure variation

Equation 3.2 represents that same sound wave where, P is excess pressure at position x, over and above the average atmospheric pressure

and pressure amplitude p_0 is given by $P_0 = BKs_0$ .....(3.3)

(B = Bulk modulus of the medium, K = angular wave number)

· Note from equation (3.1) and (3.2) that the displacement of a medium particle and excess pressure at any position are out of phase by . Hence a displacement maxima corresponds to a pressure minima and vice-versa.

Solved Examples

Example 3. A sound wave of wavelength 40 cm travels in air. If the difference between the maximum and minimum pressures at a given point is 2.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Solution : The pressure amplitude is

$$p_0 = \frac{2.0 \times 10^{-3} N/m^2}{2} = 10^{-3} N/m^2.$$

The displacement amplitude s_0 is given by $p_0 = B\,k\,s_0$

or $$s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\pi B} = \frac{10^{-3} N/m^2 \times (40 \times 10^{-2} m)}{2 \times \pi \times 14 \times 10^4 N/m^2} = Å = 6.6\ Å$$

4. SPEED OF SOUND WAVES

4.1 Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \quad ....(4.1)$$

where Y = young's modulus of elasticity and ρ = density.

4.2. Velocity of sound waves in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \quad ....(4.2)$$

where, ρ = density of the medium and B = Bulk modulus of the medium given by,

$$B = -V \quad .....(4.3)$$

Newton's formula : Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

PV = constant

$$\Rightarrow \frac{dP}{dV} = \frac{-P}{V}$$

and hence B = P using equ. ...(4.3)

and thus he obtained for velocity of sound in a gas,

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$$ where M = molar mass

the density of air at 0º and pressure 76 cm of Hg column is ρ = 1.293 kg/m^3. This temperature and pressure is called standard temperature and pressure at STP. Speed of sound in air is 280 m/s. This value is some what less than messured speed of sound in air 332 m/s then laplace suggested the correction.

Laplace's correction : Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence PV^{γ} = constant

$$\Rightarrow \frac{dP}{dV} = -\gamma \frac{P}{V}$$

where, $B = -V \frac{dP}{dV} = \gamma P$

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \quad .... (4.4)$$

4.3 Factors affecting speed of sound in atmosphere.

(a) Effect of temperature : as temperature (T) increases velocity (v) increases.

$v \propto$

For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

$$v = \sqrt{\frac{\gamma R}{M}} \times T^{1/2}$$

$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\Delta v = \left(\frac{1}{2}\frac{v}{T}\right)\Delta T$$

$$\Delta v = (0.6)\Delta T$$

(b) **Effect of pressure :**

The speed of sound in a gas is given by $v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$

(c) **Effect of humidity :** With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

So at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

Solved Examples

Example 4. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Solution : $v = \sqrt{\frac{\gamma RT}{M}}$

since temperature, T is constant,

$$\therefore \quad \frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$\Rightarrow$ $v(H_2) = 4 \times 450 = 1800$ m/s **Ans.**

Aliter : The speed of sound in a gas is given by $u = \sqrt{\frac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32 g whereas the same volume of hydrogen has a mass of 2 g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases,

$$\frac{f_{(hydrogen)}}{f_{(oxygen)}} = \sqrt{\frac{\rho_{(oxygen)}}{\rho_{(hydrogen)}}}$$

or $f_{(hydrogen)} = 4f_{(oxygen)}$

$= 4 \times 450$ m/s $= 1800$ m/s. **Ans.**

5. INTENSITY OF SOUND WAVES :

Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to do work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom created by a supersonic jet, can even cause glass panes of windows to crack.

The amount of energy carried per unit time by a wave is called its power and power per unit area held perpendicular to the direction of energy flow is called intensity.

For a sound wave travelling along positive x-axis described by the equation.

$$s = s_0 \sin(\omega t - kx + \varphi)$$

$$P = p_0 \cos(\omega t - kx + \varphi)$$

$$\frac{\delta s}{\delta t} = \omega s_0 \cos(\omega t - kx + \varphi)$$

Instantanious power $P = F.v = pA\frac{\delta s}{\delta t}$

$P = p_0 \cos(\omega t - kx + \varphi)\, A\omega s_0 \cos(\omega t - kx + \varphi)$

$P_{average} = \langle P \rangle$

$= p_0 A\omega s_0 \langle \cos^2(\omega t - kx + \varphi)\rangle$

$= \frac{p_0 \omega s_0 A}{2}$ $\Rightarrow$ $v = \sqrt{\frac{B}{\rho}}$

$B = \rho v^2$ $\Rightarrow$ $p_0 = Bks_0 = = \rho v^2 ks_0$

$P_{average} = \frac{1}{2}\omega p_0 A\left(\frac{p_0}{\rho v^2 k}\right) = \frac{p_0^2 A}{2\rho V} = \frac{pAv\omega^2 s_0^2}{2}$

maximum power $= P_{max} = \frac{p_0^2 A}{\rho V} = (pA)\, v\, v^2_{p,max} = pAv\omega^2 s_0^2$

Total energy transfer $= P_{av} \times t = \frac{\rho A v \omega^2 s_0^2}{2} \times t$

Average intensity = average power / area

the average intensity at position x is given by

$$\langle I \rangle = \frac{1}{2}\frac{\omega^2 s_0^2 B}{v} = \frac{P_0^2 v}{2B} \quad .... (5.1)$$

Substituting $B = \rho v^2$, intensity can also be expressed as

$$I = \frac{P_0^2}{2\rho v} \quad .... (5.2)$$

Note :

☞ If the source is a point source then $I \propto \frac{1}{r^2}$ and $s_0 \propto \frac{1}{r}$ and $s = \frac{a}{r}\sin(\omega t - kr + \theta)$

☞ If a sound source is a line source then $I \propto \frac{1}{r}$ and $s_0 \propto \frac{1}{\sqrt{r}}$ and

$s = \frac{a}{\sqrt{r}}\sin(\omega t - kr + \theta)$

Solved Examples

Example 5. The pressure amplitude in a sound wave from a radio receiver is 2.0×10^{-3} N/m^2 and the intensity at a point is 10^{-6} W/m^2. If by turning the “Volume” knob the pressure amplitude is increased to 3×10^{-3} N/m^2, evaluate the intensity.

Solution : The intensity is proportional to the square of the pressure amplitude.

Thus, $\frac{I'}{I} = \left(\frac{p'_0}{p_0}\right)^2$

or $I' = \left(\frac{p'_0}{p_0}\right)^2 I = \left(\frac{3}{2.0}\right)^2 \times 10^{-6}$ W/m^2 $= 2.25 \times 10^{-16}$ W/m^2.

Example 6. A microphone of cross-sectional area 0.40 cm^2 is placed in front of a small speaker emitting π W of sound output. If the distance between the microphone and the speaker is 2.0 m, how much energy falls on the microphone in 5.0 s ?

Solution : The energy emitted by the speaker in one second is π J. Let us consider a sphere of radius 2.0 m centered at the speaker. The energy π J falls normally on the total surface of this sphere in one second. The energy falling on the area 0.4 cm^2 of the microphone in one second

$$= \frac{0.4 \text{ cm}^2}{4\pi(2.0\text{m})^2} \times \pi \text{ J} = 2.5 \times 10^{-6}\text{J}.$$

The energy falling on the microphone in 5.0 is 2.5×10^{-6} J × 5 = 12.5 μJ.

Example 7. Find the amplitude of vibration of the particles of air through which a sound wave of intensity 8.0×10^{-6} W/m^2 and frequency 5.0 kHz is passing. Density of air = 1.2 kg/m^3 and speed of sound in air = 330 m/s.

Solution : The relation between the intensity of sound and the displacement amplitude is

$$I = \frac{\omega^2 s_0^2 B}{2v}, \quad \text{where } B = \rho v^2 \text{ and } \omega = 2\pi\nu$$

$$\Rightarrow \quad I = 2\pi^2 s_0^2 \nu^2 \rho_0 v \quad \text{or} \quad s_0^2 = \frac{I}{2\pi^2\nu^2\rho_0 v} = \frac{8.0 \times 10^{-6}\ \text{W/m}^2}{2\pi^2 \times (25.0 \times 10^6 \text{s}^{-2}) \times (1.2\text{kg/m}^3) \times (330\text{m/s})}$$

or s_0 = 6.4 nm.

6. LOUDNESS :

Audible intensity range for humans :

The ability of human to perceive intensity at difference frequency is different. The perception of intensity is maximum at 1000 Hz and perception of intensity decreases as the frequency decreases or increases from 1000 Hz.

- For a 1000 Hz tone, the smallest sound intensity that a human ear can detect is 10^{-12} watt./m^2. On the other hand, continuous exposure to intensities above 1W/m^2 can result in permanent hearing loss.
- The overall perception of intensity of sound to human ear is called **loudness**.
- Human ear do not percives loudness on a linear intensity scale rather it percives loudness on logarithmic intensity scale.

For exampel;

If intensity is increased 10 times human ear does not perceive10 times increase in loudness. It roughly perceived that loudness is doulbed where intensity increased by 10 times. Hence it is prudent to define a logaritimic scale for intensity.

DECIBEL SCALE :

The logarithmic scale which is used for comparing to sound intensity is called **decible scale**.

The intensity level β described in terms of decibels is defined as

$$\beta = 10 \log \left(\frac{I}{I_0}\right) \text{ (dB)}$$

Here I_0 is the threshold intensity of hearing for human ear

i.e. $I = 10^{-12}$ watt/m^2.

- In terms of decible threshold of human hearing is 1 dB
- Note that intensity level β is a dimensionless quantity and is not same as intensity expressed in W/m^2.

Solved Examples

Example 8. If the intensity is increased by a factor of 20, by how many decibels is the intensity level increased.

Solution : Let the initial intensity by I and the intensity level be β_1 and when the intensity is increased by 20 times, the intensity level increases to β_2.

Then $\beta_1 = 10 \log (I / I_0)$

and $\beta_2 = 10 \log (20I / I_0)$

Thus, $\beta_2 - \beta_1 = 10 \log (20I / I)$

= 10 log 20

= 13 dB.

Example 9. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

Solution : $\beta_1 = 10 \log \frac{I_1}{I_0}$

$\beta_2 = 10 \log \frac{I_2}{I_0}$ ⇒ $\beta_2 - \beta_1 = 10 \log \frac{I_2}{I_1}$

or $10 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$ ⇒ $\frac{I_2}{I_1} = 10^1 = 10$

for point source $I \propto \frac{1}{r^2}$ ⇒ $\frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}$ **Ans.**

Example 10. The sound level at a point is increased by 40 dB. By what factor is the pressure amplitude increased ?

Solution : The sound level in dB is

$\beta = 10 \log_{10} \left(\frac{I}{I_0}\right)$.

If β_1 and β_2 are the sound levels and I_1 and I_2 are the intensities in the two cases,

$\beta_2 - \beta_1 = 10 \left[\log_{10}\left(\frac{I_2}{I_0}\right) - \log_{10}\left(\frac{I_1}{I_0}\right)\right]$

or $40 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$ or $\frac{I_2}{I_1} = 10^4$

As the intensity is proportional to the square of the pressure amplitude,

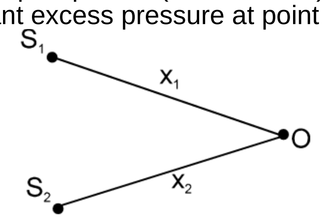
we have $\frac{p_{02}}{p_{01}} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10000} \approx 100$.

7. INTERFERENCE OF SOUND WAVES :

If $p_1 = p_{m1} \sin(\omega t - kx_1 + \theta_1)$

and $p_2 = p_{m2} \sin(\omega t - kx_2 + \theta_2)$

resultant excess pressure at point O is



$p = p_1 + p_2$

⇒ $p = p_0 \sin(\omega t - kx + \theta)$

where, $p_0 = \sqrt{p_{m_1}^2 + p_{m_2}^2 + 2p_{m_1}p_{m_2}\cos\phi}$, $\varphi = |k(x_1 - x_2) + (\theta_1 - \theta_2)|$ ...(7.1)

(i) For constructive interference

$\varphi = 2n\pi$ ⇒ $p_0 = p_{m1} + p_{m2}$

(ii) For destructive interference

$\varphi = (2n+1)\pi$ ⇒ $p_0 = |p_{m1} - p_{m2}|$

If φ is only due to path difference, then $\varphi = \frac{2\pi}{\lambda}\Delta x$, and

Condition for constructive interference : $\Delta x = n\lambda$, $n = 0, \pm 1, \pm 2$

Condition for destructive interference : $\Delta x = (2n+1)\frac{\lambda}{2}$, $n = 0, \pm 1, \pm 2$

from equation (6.1)

$$P_0^2 = P_{m_1}^2 + P_{m_2}^2 + 2P_{m_1}P_{m_2}\cos\phi$$

Since intensity, $I \propto$ (Pressure amplitude)^2, we have, for resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$ ....(7.2)

If $I_1 = I_2 = I_0$

$$I = 2I_0\ (1 + \cos\varphi) \Rightarrow I = 4I_0 \cos^2\frac{\phi}{2}$$ .....(7.3)

Hence in this case,

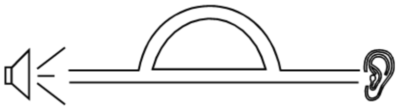
for constructive interference : $\varphi = 0, 2\pi, 4\pi....$ and $I_{max} = 4I_0$

and for destructive interference : $\varphi = \pi, 3\pi...$ and $I_{min} = 0$

7.1 **Coherence :** Two sources are said to be coherent if the phase difference between them does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

Solved Examples

Example 11. Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.



Solution : The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $p_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $p_2 = \pi\ 10$ cm = 31.4 cm before they meet again and travel to the receiver.

The path difference between the two waves received is, therefore.

$\Delta p = p_2 - p_1 = 31.4$ cm – 20 cm = 11.4 cm

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{330\ m/s}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

or, $\Delta p = n.\frac{v}{\nu}$ $\Rightarrow$ $\nu = \frac{n.v}{\Delta p}$ $\Rightarrow$ $\frac{n.342}{(0.114)} = 3000\ n$

Thus, the frequencies within the specified range which cause maximum of intensity are

3000 × 1, 3000 × 2 and 3000 × 3 Hz

Example 12. A source emitting sound of frequency 165 Hz is placed in front of a wall at a distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air = 330 m/s.

Solution : The situation is shown in figure. Suppose the detector is placed at a distance of x meter from the source. The direct wave received from the source travels a distance of x meter. The wave reaching the detector after reflection from the wall has travelled a distance of $2[(2)^2 + x^2/4]^{1/2}$ metre. The path difference between the two waves is

$$\Delta = \left\{2\left[(2)^2 + \frac{x^4}{4}\right]^{1/2} - x\right\}$$ metre.

Constructive interference will take place when $\Delta = \lambda, 2\lambda, ...$ The minimum distance x for a maximum corresponds to

$\Delta = \lambda$ ....(i)

The wavelength is $\lambda = \frac{\upsilon}{v} = \frac{330\ m/s}{165\ s^{-1}} = 2$ m.

Thus, by (i) $2\left[(2)^2 + \frac{x^2}{4}\right]^{1/2} - x = 2$

or, $\left[4 + \frac{x^2}{4}\right]^{1/2} = 1 + \frac{x}{2}$ or, $4 + \frac{x^2}{4} = 1 + \frac{x^2}{4} + x$

or, 3 = x.

Thus, the detector should be placed at a distance of 3 m from the source. Note that there is no abrupt phase change.

8. REFLECTION OF SOUND WAVES :

Reflection of sound waves from a rigid boundary (e.g. closed end of an organ pipe) is analogous to reflection of a string wave from rigid boundary; reflection accompanied by an inversion i.e. an abrupt phase change of π. This is consistent with the requirement of displacement amplitude to remain zero at the rigid end, since a medium particle at the rigid end can not vibrate. As the excess pressure and displacement corresponding to the same sound wave vary by $\pi/2$ in term of phase, a displacement minima at the rigid end will be a point of pressure maxima. This implies that the reflected pressure wave from the rigid boundary will have same phase as the incident wave, i.e., a compression pulse is reflected as a compression pulse and a rarefaction pulse is reflected as a rarefaction pulse.

On the other hand, reflection of sound wave from a low pressure region (like open end of an organ pipe) is analogies to reflection of string wave from a free end. This point corresponds to a displacement maxima, so that the incident & reflected displacement wave at this point must be in phase. This would imply that this point would be a minima for pressure wave (i.e. pressure at this point remains at its average value), and hence the reflected pressure wave would be out of phase by π with respect to the incident wave. i.e. a compression pulse is reflected as a rarefaction pulse and vice-versa.

9. LONGITUDINAL STANDING WAVES :

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering waves are given by

$p_1 = p_0 \sin(\omega t - kx)$

and $p_2 = p_0 \sin(\omega t + kx + \varphi)$

then the equation. of the resultant standing wave would be given by

$$p = p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

$$\Rightarrow \quad p = p'_0 \sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{.... (9.1)}$$

This is equation of SHM* in which the amplitude p'_0 depends on position as

$$p'_0 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \qquad \text{....(9.2)}$$

Points were pressure remains permanently at its average value; i.e. pressure amplitude is zero is called a pressure node, and the condition for a pressure node would be given by

$$p'_0 = 0$$

i.e. $\cos\left(kx + \frac{\phi}{2}\right) = 0$

i.e. $kx + \frac{\phi}{2} = 2n\pi \pm \frac{\pi}{2}$, n = 0, 1, 2,..... ..... (9.3)

Similarly points where pressure amplitude is maximum is called a pressure antinode and condition for a pressure antinode would be given by

$$p'_0 = \pm 2p_0$$

i.e. $\cos\left(kx + \frac{\phi}{2}\right) = \pm 1$

or $\left(kx + \frac{\phi}{2}\right) = n\pi$, n = 0, 1, 2,..... ....(9.4)

* Note that a pressure node in a standing wave would correspond to a displacement antinode; and a pressure anti-node would correspond to a displacement node.

* (when we label eqn (9.1) as SHM, what we mean is that excess pressure at any point varies simple-harmonically. If the sound waves were represented in terms of displacement waves, then the equation of standing wave corresponding to (9.1) would be

$$s = s'_0 \cos(\omega t + \frac{\phi}{2}) \quad \text{where} \quad s'_0 = 2s_0 \sin(kx + \frac{\phi}{2})$$

This can be easily observed to be an equation of SHM. It represents the medium particles moving simple harmonically about their mean position at x.)

Solved Examples

Example 13. A certain organ pipe resonates in its fundamental mode at a frequency of 1 kHz in air. What will be the fundamental frequency if the air is replaced by hydrogen at the same temperature ? (take molar mass of air = 29 g)

Solution : Suppose the speed of sound in hydrogen is v_h and that in air is v_a. The fundamental frequency of an organ pipe is proportional to the speed of sound in the gas contained in it. If the fundamental frequency with hydrogen in the tube is ν, we have

$$\frac{\nu}{1000\text{ Hz}} = \frac{v_h}{v_a} = \sqrt{\frac{M_{Air}}{M_{H_2}}}$$ (Since both air and H_2 are diatomic, y is same for both)

or $$\frac{\nu}{1\text{kHz}} = \sqrt{\frac{29}{2}} \Rightarrow \nu = \sqrt{\frac{29}{2}}\text{ kHz.}$$ **Ans.**

Example 14. A tube open at only one end is cut into two tubes of non equal lengths. The piece open at both ends has of fundamental frequency of 450 Hz and of 675 Hz. What is the 1st overtone frequency of the original tube.

Solution : $450 = \frac{v}{2\ell_1} \qquad 675 = \frac{v}{4\ell_2}$

length of original tube = $(\ell_1 + \ell_2)$ ∴ its first obtained frequency,

$$\nu_1 = \frac{3v}{(\ell_1+\ell_2)} = \frac{3v}{\frac{v}{900}+\frac{v}{675\times 4}} = \frac{3(2700\times 900)}{2700+900} = 2700 \times \frac{3}{4} = 2025\text{ Hz.}$$

Example 15. The range audible frequency for humans is 20 Hz to 20,000 Hz. If speed of sound in air is 336 m/s. What can be the maximum and minimum length of a musical instrument, based on resonance pipe.

Solution : For an open pipe, $f = \frac{v}{2\ell} n \Rightarrow \ell = \frac{v}{2f}.n$

Similarly for a closed pipe,

$$\ell = \frac{v}{4f}(2n+1)$$

$$\ell_{min} = \frac{v}{4f_{max}}(2n+1)_{min} = \frac{336}{4\times 20000} = 4.2\text{ mm}$$

$$\ell_{max} = \frac{v}{2f_{min}} n_{max} = \frac{v}{2\times 20} n_{max} = 8.4\text{ (m)} \times n_{max}$$

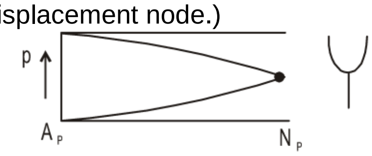
clearly there is no upper limit on the length of such an musical instrument.

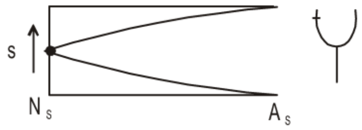
10. VIBRATION OF AIR COLUMNS :

Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air-column. To determine the natural frequency of the air-column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column.
In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

10.1 Closed organ pipe

(In the diagram, A_p = Pressure antinode, A_s = displacement antinode, N_p = pressure node, N_s = displacement node.)

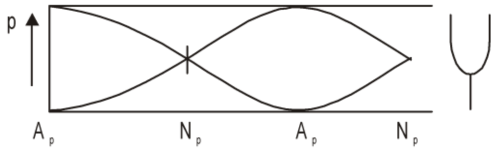


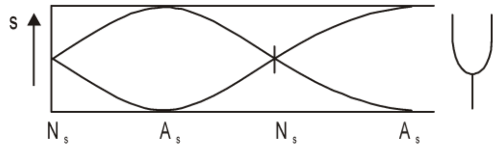


Fundamental mode :

The smallest frequency (largest wavelength) that satisties the boundary condition for resonance (i.e. displacement node at left end and antinode at right end is $\lambda_0 = 4\ell$, where ℓ = length of closed pipe the corresponding frequency.

$$\nu_0 = \frac{v}{\lambda} = \frac{v}{4L} \text{ is called the fundamental frequency.} \qquad ..(10.1)$$





First Overtone : Here there is one node and one antinode apart from the nodes and antinodes at the ends.

$$\lambda_1 = \frac{4\ell}{3} = \frac{\lambda_0}{3}$$

and corresponding frequency,

$$\nu_1 = \frac{v}{\lambda_1} = 3\nu_0$$

This frequency is 3 times the fundamental frequency and hence is called the 3rd harmonic.

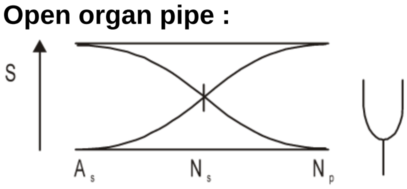
nth overtone :

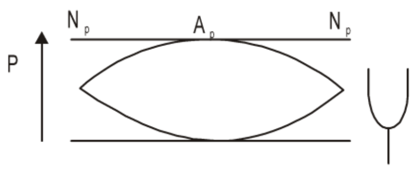
In general, the nth overtone will have n nodes and n antinodes between the two nodes. The corresponding wavelength is

$$\lambda_n = \frac{4\ell}{2n+1} = \frac{\lambda_0}{2n+1} \quad \text{and} \quad \nu_n\ (2n+1)\nu_0 \quad ....(10.2)$$

This corresponds to the $(2n + 1)^{th}$ harmonic. Clearly only odd harmonic are allowed in a closed pipe.

10.2 Open organ pipe :





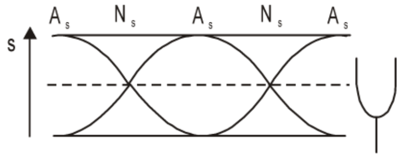
Fundamental mode :

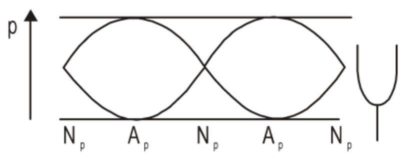
The smallest frequency (largest wave length) that satisfies the boundary condition for resonance (i.e. displacement antinodes at both ends) is,

$$\lambda_0 = 2\ell$$

corresponding frequency, is called the fundamental frequency

$$\nu_0 = \frac{v}{2\ell} \qquad \text{....(10.3)}$$





1st Overtone : Here there is one displacement antinode between the two antinodes at the ends.

$$\lambda_1 = \frac{2\ell}{2} = \lambda = \frac{\lambda_0}{2}$$

and, corresponding frequency

$$\nu_1 = \frac{v}{\lambda_1} = 2\nu_0$$

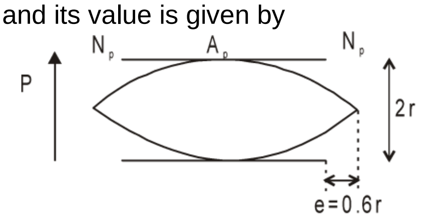
This frequency is 2 times the fundamental frequency and is called the 2nd harmonic.

nth overtone : The nth overtone has n displacement antinodes between the two antinode at the ends.

$$\lambda_n = \frac{2\ell}{n+1} = \frac{\lambda_0}{n+1} \quad \text{and} \quad \nu_n = (n + 1)\,\nu_0 \qquad \text{....(10.4)}$$

This correspond to $(n + 1)^{th}$ harmonic: clearly both even and odd harmonics are allowed in an open pipe.

10.3 **End correction :** As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by



$$e = 0.6\ r$$

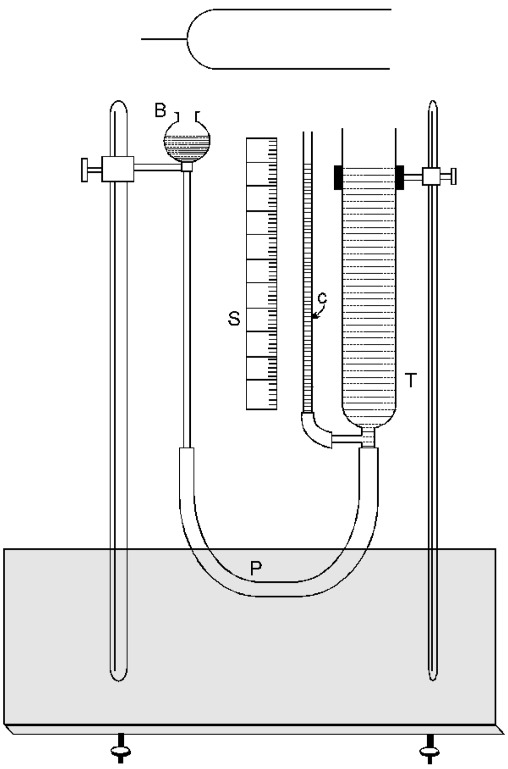
where r = radius of the organ pipe.

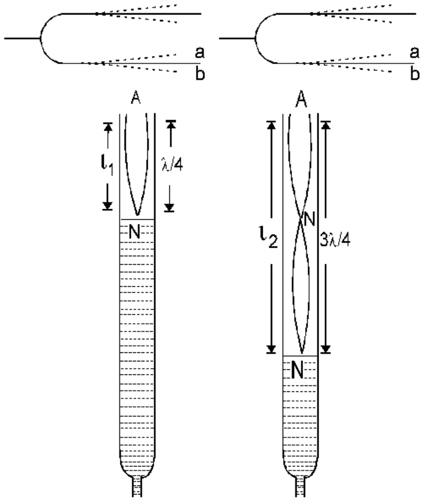
with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \quad \text{and} \quad f_0 = \frac{v}{2(\ell + 1.2r)} \qquad \text{....(10.5)}$$

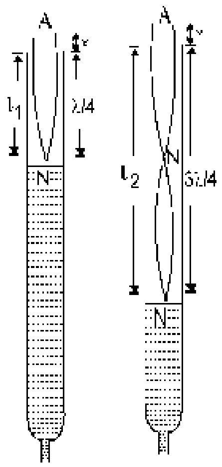
Resonance Tube

Construction : The resonance tube is a tube T (Fig.) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air–column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.





Determination of the speed of sound in air by resonance tube - First of all the water reservoir B is raised untill the water level in the tube T rises almost to the top of the tube. Then the pinch–cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube .The pinch–cock P is opened slowly so that the water level in T falls and the length of the air–column increases. At a particular length of air–column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.



(i) For first resonance $\ell_1 = \lambda/4$ ....(1)

(ii) For second resonance $\ell_2 = 3\lambda/4$ ....(2)

Substract Eq. (2) from Eq. (1)

$$\ell_2 - \ell_1 = \lambda/2$$

$$\lambda = 2(\ell_2 - \ell_1)$$

If the frequency of the fork be n and the temperature of the air-column be t^oC, then the speed of sound at t^oC is given by

$$v_t = f\lambda = 2f(\ell_2 - \ell_1)$$

The speed of sound wave at 0^oC $\quad v_0 = (v_t - 0.61\ t)$ m/s.

End Correction - In the resonance tube, the antinode is not formed exactly at the open but slightly outside at a distanc x. Hence the length of the air -column in the first and second states of resonance are (ℓ_1 + x) and (ℓ_2 + x) then

(i) For first resonance $\ell_1 + x = \lambda/4$ ....(1)

(ii) For second resonance $\ell_2 + x = 3\lambda/4$ ....(2)

Substract Eq. (2) from Eq. (1)

$\ell_2 - \ell_1 = \lambda/2$

$\lambda = 2(\ell_2 - \ell_1)$

Put the value of λ in Eq. (1) $\ell_1 + x = \frac{2(\ell_2 - \ell_1)}{4}$

⇒ $\ell_1 + x = \frac{\ell_2 - \ell_1}{2}$

$x = \frac{\ell_2 - 3\ell_1}{2}$

Solved Examples

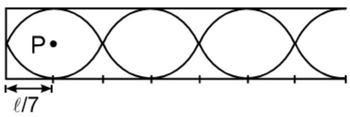
Example 16. A tuning fork is vibrating at frequency 100 Hz. When another tuning fork is sounded simultaneously,
4 beats per second are heard. When some mass is added to the tuning fork of 100 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Solution : $|f - 100| = 4$ ⇒ $f = 95$ or 105
when 1st tuning fork is loaded its frequency decreases and so does beat frequency
⇒ $100 > f$
⇒ $f = 95$ Hz.

Example 17. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Solution. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \frac{7\lambda}{4}$ or $\lambda = \frac{4\ell}{7}$ or $\frac{\lambda}{4} = \frac{\ell}{7}$



Hence the amplitude at P at a distance $\frac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point

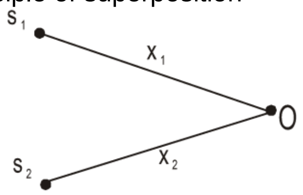
11. INTERFERENCE IN TIME : BEATS

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.
If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,

$p_1 = p_0 \sin(2\pi f_1 t - kx_1 + \theta_1)$

$p_2 = p_0 \sin(2\pi f_2 t - kx_2 + \theta_2)$

By principle of superposition



$p = p_1 + p_2 = 2p_0 \cos\{\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\} \sin\{\pi(f_1 + f_2)t + \frac{\phi_1 + \phi_2}{2}\}$

i.e., the resultant sound at point O has frequency $\left(\frac{f_1 + f_2}{2}\right)$ while pressure amplitude p'_0 (t) variex with time as

$p'_0(t) = 2p_0 \cos\left\{\pi(f_1 - f_2)t + \frac{\phi_1 - \phi_2}{2}\right\}$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\frac{f_1 - f_2}{2}\right)$.
Hence sound intensity will vary with a frequency $f_1 - f_2$.
This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 - f_2 \quad \Rightarrow \quad T_B = \frac{1}{f_1 - f_2} \qquad (11.1)$$

IMPORTANT POINTS :

(i) The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.
(ii) Beat phenomenon can be used for determining an unknown frequency by sounding it together with a source of known frequency.
(iii) If the arm of a tuning fork is waxed or loaded, then its frequency decreases.
(iv) If arm of tuning fork is filed, then its frequency increases.

Solved Examples

Example 18. Two strings X and Y of a sitar produces a beat of frequency 4Hz. When the tension of string Y is slightly increased, the beat frequency is found to be 2Hz. If the frequency of X is 300Hz, then the original frequency of Y was 1995
(1*) 296 Hz (2) 298 Hz (3) 302 Hz (4) 304 Hz.

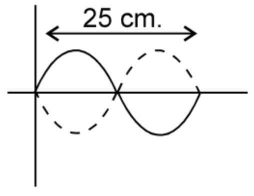
Answer. **(1)**

Example 19. A string 25 cm long fixed at both ends and having a mass of 2.5 g is under tension. A pipe closed from one end is 40 cm long. When the string is set vibrating in its first overtone and the air in the pipe in its fundamental frequency, 8 beats per second are heard. It is observed that decreasing the tension in the string decreases the beat frequency. If the speed of sound in air is 320 m/s. Find tension in the string.

Solution :

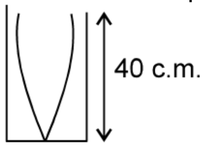
$\mu = \frac{2.5}{25} = 0.1\ g/cm = 10^{-2}\ Kg/m$

I^{st} overtone



λ_s = 25 cm = 0.25 m $\Rightarrow$ $f_s = \frac{1}{\lambda_s}\sqrt{\frac{T}{\mu}}$

pipe in fundamental freq



λ_p = 160 cm = 1.6 m $\Rightarrow$ $f_p = \frac{V}{\lambda_p}$

$\because$ by decreasing the tension , beat freq is decreased

$\therefore$ $f_s > f_p$ $\Rightarrow$ $f_s - f_p = 8$

$\Rightarrow \frac{1}{0.25}\sqrt{\frac{T}{10^{-2}}} - \frac{320}{1.6} = 8$ $\Rightarrow$ T = 27.04 N

Solved Examples

Example 20. The wavelength of two sound waves are 49cm and 50 cm respectively. If the room temperature is 30 °C then the number of beats produced by them is approximately (velocity of sound in air at 0^0C = 332 m/s).

(1) 6 (2) 10 (3*) 14 (4) 18

Answer. **(3)**

Solution : $v = 332\sqrt{\frac{303}{273}}$ $\Rightarrow$ Beat frequency = $f_1 - f_2 = v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$

$= 332\sqrt{\frac{303}{273}}\left(\frac{1}{49} - \frac{1}{50}\right) \times 100 \cong 14$ **Ans.**

12. DOPPLER'S EFFECT

When there is relative motion between the source of a sound/light wave & an observer along the line joining them, the actual frequency observed is different from the frequency of the source. This phenomenon is called Doppler's Effect. If the observer and source are moving towards each other, the observed frequency is greater than the frequency of the source. If the observer and source move away from each other, the observered frequency is less than the frequency of source.

(v = velocity of sound wrt. ground. , c = velocity of sound with respect to medium, v_m = velocity of medium, v_O = velocity of observer, v_s = velocity of source.)

(a) Sound source is moving and observer is stationary :

If the source emitting a sound of frequency f is travelling with velocity v_s along the line joining the source and observer,

observed frequency, $f' = \left(\frac{v}{v - v_s}\right)\cdot n$ f ...(12.1)

and Apparent wavelength $\lambda' = \lambda\left(\frac{v - v_s}{v}\right)$ ...(12.2)

* In the above expression, the positive direction is taken along the velocity of sound, i.e. from source to observer. Hence v_s is positive if source is moving towards the observer, and negative if source is moving away from the observer.

(b) Sound source is stationary and observer is moving with velocity v_0 along the line joining them :

The source (at rest) is emitting a sound of frequency 'f' travelling with velocity 'v' so that wavelength is $\lambda = v/f$, i.e. there is no change in wavelength. How ever since the observer is moving with a velocity v_0 along the line joining the source and observer, the observed frequency is

$$f' = f\left(\frac{v - v_0}{v}\right) \quad ...(12.3)$$

* In the a bove expression, the positive direction is taken along the velocity of sound, i.e. from source to observer. Hence v_O is positive if observer is moving away from the source, and negative if observer is moving towards the source.

(c) The source and observer both are moving with velocities v_s and v_0 along the line joining them :

The observed frequency, $f' = f\left(\frac{v - v_0}{v - v_s}\right)$ ...(12.4)

and Apparent wavelength $\lambda' = \lambda\left(\frac{v - v_s}{v}\right)$ ...(12.5)

* In the above expression also, the positive direction is taken along the velocity of sound, i.e. from source to observer.

* In all of the above expression from equation 12.1 to 12.5, v stands for velocity of sound with respect to ground.
If velocity of sound with respect to medium is c and the medium is moving in the direction of sound
from source to observer with speed v_m, $v = c + v_m$, and if the medium is moving opposite to the direction of sound from observer to source with speed v_m, $v = c - v_m$

Solved Examples

Example 21. A source of sound wave of frequency n travels with velocity v towards a large vertical plane wall. Sound is reflected from the wall. Speed of sound in medium is u (u >> v). Then what is the frequency of relfected wave.

Solution : Frequency of wave incident on wall is :

$n_1 = n\left(\frac{u}{u - v}\right)$ same frequency is refelcted.

Example 22. In above example, If an observer is also moving towards wall with velocity v behind the source. Then calculate the number of beats heard.

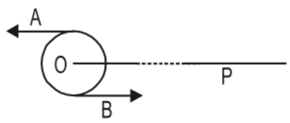
Solution : Frequency heard by observer directly from source is $n_2 = n\left(\frac{u + v}{u + v}\right) = n$
and frequency heard by observer of the wave reflected from wall.

$n_3 = n_1\left(\frac{u + v}{u}\right) = n\left(\frac{u + v}{u - v}\right)$

Number of beats = $n_3 - n_2 = n\left(\frac{u + v}{u - v}\right) - n = n\left(\frac{2v}{u - v}\right) = \frac{2nv}{u - v}$

Example 23. A whistle of frequency 540 Hz is moving in a circle of radius 2 ft at a constant angular speed of 15 rad/s. What are the lowest and height frequencies heard by a listener standing at rest, a long distance away from the centre of the circle? (velocity of sound in air is 1100 ft/sec.)

Solution : The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by



$v_S = \omega R$

where, R is radius of the circle.
At points A and B, the velocity v_S of whistle is parallel to line OP; i.e., with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP).Observer, therefore, listens maximum frequency when source is at B moving towards observer:

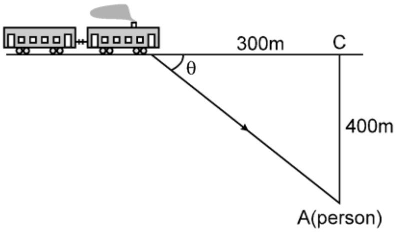
$f_{max} = f\dfrac{v}{v - v_s}$

where, v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer:

$$f_{min} = \frac{f}{v + v_s}$$

For f = 540 Hz, v_s = 2 ft × 15 rad/s = 30 ft/s, and v = 1100 ft/s,
we get (approx.)
f_{max} = 555 Hz and f_{min} = 525 Hz.

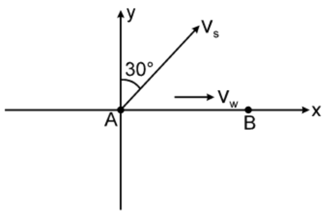
Example 24. A train approaching a railway crossing at a speed of 72 km/h sounds a short whistle at frequency 640 Hz when it is 1 km away from the crossing. The speed of sound in air is 330 m/s. A road intersects the crossing perpendicularly. What is the frequency heard by a person standing on the road at the distance of 1732 m from the crossing.



Solution : The observer A is at rest with respect to the air and the source is travelling at a velocity of 72 km/h i.e., 20 m/s.As is clear from the figure, the person receives the sound of the whistle in a direction BA making an angle θ with the track where tan θ = $\dfrac{1732}{1000} = \sqrt{3}$, i.e. θ = 60º. The component of the velocity of the source (i.e., of the train) along this direction is 20cos θ = 10 m/s. As the source is approaching the person with this component, the frequency heard by the observer is

$\upsilon' = \dfrac{v}{v - u\cos\theta}\,\upsilon = \dfrac{330}{330 - 10} \times 640$ Hz = 660 Hz.

Example 25. In the figure shown a source of sound of frequency 510 Hz moves with constant velocity v_s = 20 m/s in the direction shown. The wind is blowing at a constant velocity v_w = 20 m/s towards an observer who is at rest at point B. The frequency detected by the observer corresponding to the sound emitted by the source at initial position A, will be (speed of sound relative to air = 330 m/s)



(1) 485 Hz (2) 500 Hz
(3) 512 Hz (4*) 525 Hz

Solution : **(4)**

Apparent frequency

$n' = n\dfrac{(u + v_w)}{(u + v_w - v_s\cos 60°)} = \dfrac{510\,(330 + 20)}{330 + 20 - 20\cos 60°}$

$= 510 \times \dfrac{350}{340}$ = 525 Hz **Ans.**