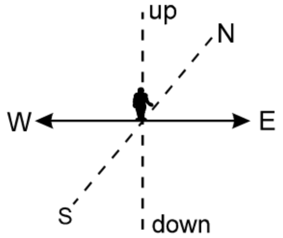
# MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

## 1. Magnet :

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. This force is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east

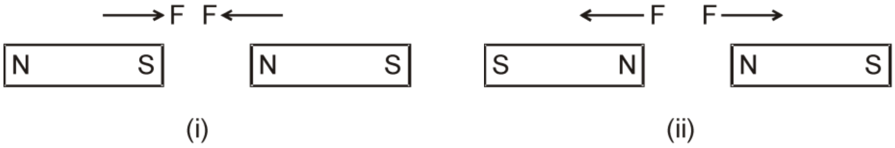



and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.

The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

### 1.1 Pole strength magnetic dipole and magnetic dipole moment :

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

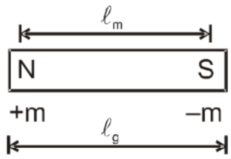



The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.

∴ they are


Know as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')




$M = m.\ell_m$ here $\ell_m$ = magnetic length of the magnet. $\ell_m$ is slightly less than $\ell_g$ (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

## 1.2 Magnetic field and strength of magnetic field.

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{B}$. It is a vector quantity.

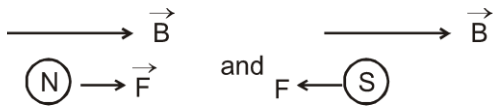
***Definition* of $\vec{B}$ :** The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \dfrac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m$^2$** (abbreviated as T and Wb/m$^2$).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



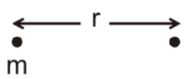

The field generated by sources does not depend on the test pole (for its any value and any sign).

**(a)** $\vec{B}$ due to various source

**(i) Due to a single pole :**

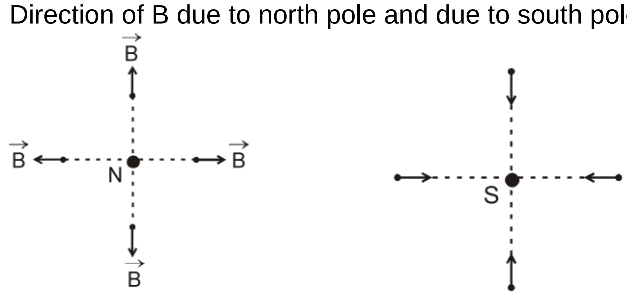
(Similar to the case of a point charge in electrostatics)

$B = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^2}$




This is magnitude

Direction of B due to north pole and due to south poles are as shown

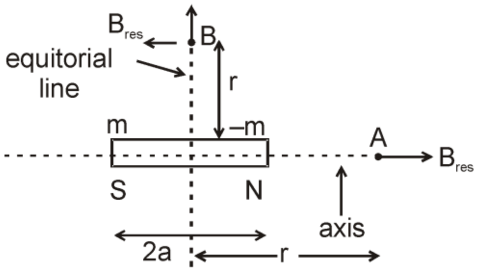



in vector form $\vec{B} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{r^3}\vec{r}$

here m is with sign and = position vector of the test point with respect to the pole.

**(ii) Due to a bar magnet :**

(Same as the case of electric dipole in electrostatics)
Independent case never found. Always 'N' and 'S' exist together as magnet.




at A (on the axis) = $\left(\frac{\mu_0}{4\pi}\right)\frac{2\vec{M}}{r^3}$ for a << r

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

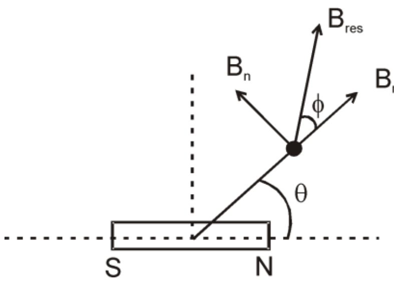
At General point :

$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

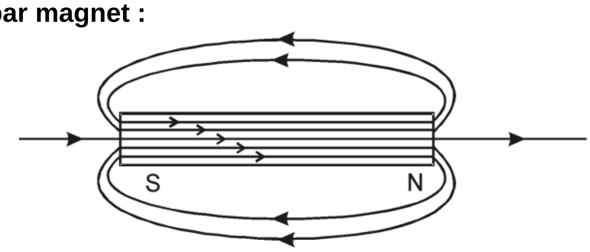
$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

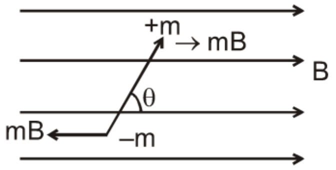
$$\tan\varphi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$




**Magnetic lines of force of a bar magnet :**



### 1.3 Magnet in an external uniform magnetic field :




(same as case of electric dipole)

$F_{res} = 0$ (for any angle)

$\tau = MB \sin\theta$

*here θ is angle between $\vec{B}$ and $\vec{M}$

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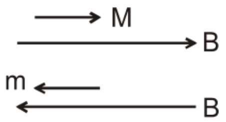
**Note :**
- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is




$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\theta = 0^o$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about θ = 0º position

$\tau = -MB\sin\theta$ ;

$I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$\alpha = -\left(\frac{MB}{I}\right)\theta$

Angular frequency of SHM

$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T}$

⇒ $T = 2\pi\sqrt{\frac{I}{MB}}$

here $I = I_{cm}$ if the dipole is free to rotate

$= I_{hinge}$ if the dipole is hinged

## Solved Examples

**Example 1.** A bar magnet having a magnetic moment of $1.0 \times 10^{-4}$ J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

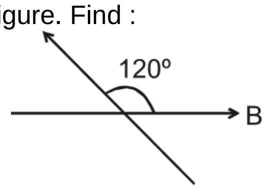
**Solution :** The work done by the external agent = change in potential energy

$= (-MB\cos\theta_2) - (-MB\cos\theta_1)$

$= -MB(\cos 60^o - \cos 0^o)$

$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^{4}\ \text{J/T})(4 \times 10^{-5}\ \text{T}) = 0.2\ \text{J}$

**Example 2.** A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :




(i) Its kinetic energy at $\theta = 90^o$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

**Solution :** (i) Apply energy conservation at $\theta = 120^o$ and $\theta = 90^o$

$$- MB \cos 120^o + 0$$
$$= - MB \cos 90^o + (K.E.)$$
$$KE = \frac{MB}{2} \quad \textbf{Ans.}$$

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0^o$. Now apply energy conservation between $\theta = 120^o$ and $\theta = 0^o$.

$$- mB \cos 120^o + 0$$
$$= -mB \cos 0^o + (KE)_{max}$$
$$(KE)_{max} = \frac{3}{2} MB \quad \textbf{Ans.}$$

The K.E. is max at $\theta = 0^o$ can also be proved by torque method. From $\theta = 120^o$ to $\theta = 0^o$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0^o$. Beyond that $\tau$ reverses its direction and then K.E. starts decreasing

∴ $\theta = 0^o$ is the orientation of M to here the maximum K.E.

(iii) Since '$\theta$' is not small.

∴ the motion is not S.H.M. but it is oscillatory and periodic amplitude is $120^o$.

### 1.4 Magnet in an External Nonuniform Magnetic Field :

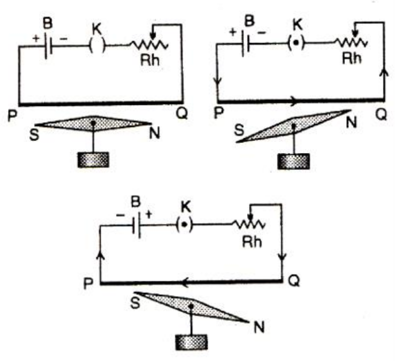
No special formula are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

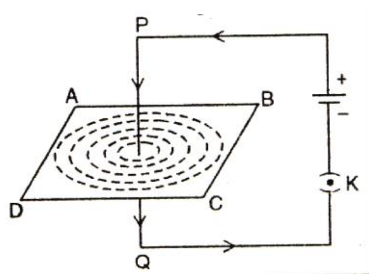
## 2. Magnetic effects of current (and moving charge)

It was observed by **<u>OERSTED</u>** that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation of displacement). This observation shows that current or moving charge produces magnetic field.

**OERSTED EXPERIMENT AND OBSERVATIONS**

(i) Oerseted performed an experiment in 1819 whose arrangement is shown in the following figure.
Following observations were noted from this experiment



(a) When no current is passed through the wire AB, the magnetic needle remains undefelcted.

(b) When current is passed through the wire AB, the magnetic needle gets deflected in a particular direction and the deflection increases as the current increases.

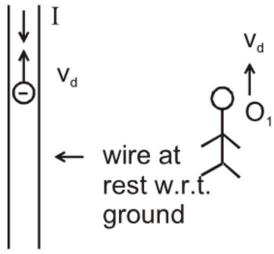
(c) When the current flowing in the wire is reversed, the magnitude needle gets deflected in the opposite direction and its deflection increases as the current increases.

(ii) Oersted concluded from this experiment that on passing a current through the conducting wire, a magnetic field is produced around this wire. As a result the magnetic needle is deflected. This phenomenon is called magnetic effect of current.

(iii) From another experiment, it is found that the magnetic lines of force due to the current flowing in the wire are in the form of concentric circles around the conducting wire.

## 2.1 Frame Dependence of $\vec{B}$.

**(a)** The motion of anything is a relative term. A charge may appear at rest by an observer (say $O_1$) and moving at same velocity with respect to observer $O_2$ and at velocity $\vec{v}_1$ with $\vec{v}_2$ respect to observers $O_3$ then $\vec{B}$ due to that charge w.r.t. $O_1$ will be zero and w.r. to $O_2$ and $O_3$ it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).




**(b)** In a current carrying wire electron move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now if some observer ($O_1$) moves with velocity $V_d$ in the direction of motion of the electrons then electrons will have zero velocity and +ve ions will have velocity $V_d$ in the downward direction w.r.t. $O_1$. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes

So, w.r.t. $O_1$ electrons will produce zero magnetic field but +ve ions will produce +ve same $\vec{B}$ due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).

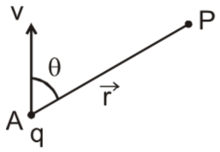
**(c)** **$\vec{B}$ due to magnet :**

$\vec{B}$ produced by the magnet does not contain the term of velocity

So, we can say that the $\vec{B}$ due magnet does not depend on frame.

## 2.2 $\vec{B}$ due to a point charge :

A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. $\vec{r}$ is the position vector of point 'P' w.r. to position of the charge. Then $\vec{B}$ at P due to q is




$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$ ; here θ = angle between $\vec{v}$ and $\vec{r}$

$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3}$ ; q with sign $\vec{B}\perp\vec{v}$ and also $\vec{B}\perp\vec{r}$.

Direction of $\vec{B}$ will be found by using the rules of vector product.

## 2.3 Biot-savart's law ($\vec{B}$ due to a wire)

It is a experimental law. A current 'i' flows in a wire (may be straight or curved). Due to 'd$\ell$' length of the wire the magnetic field at 'P' is

$$dB \propto i d\ell$$

$$\propto \frac{1}{r^2}$$

$$\propto \sin\theta$$

$$\Rightarrow \quad dB \propto \frac{i d\ell \sin\theta}{r^2}$$

$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{i d\ell \sin\theta}{r^2} \qquad \Rightarrow \qquad \vec{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\vec{d\ell}\times\vec{r}}{r^3}$$

here $\vec{r}$ = position vector of the test point w.r.t. $\vec{d\ell}$

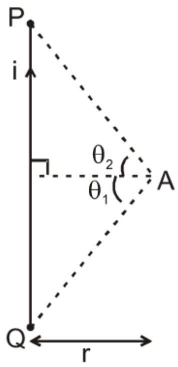
$\theta$ = angle between $\vec{d\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int \vec{dB}$

Using this fundamental formula we can derive the expression of due a long wire.

### 2.3.1 $\vec{B}$ due to a straight wire :

Due to a straight wire 'PQ' carrying a current 'i' the $\vec{B}$ at A is given by the formula

$$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

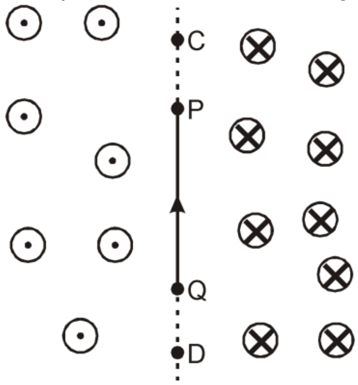



**(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)**

**Direction :**

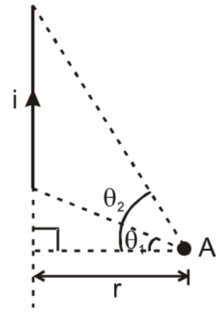
Due to every element of 'PQ' $\vec{B}$ at A is directed in wards. So its resultant is also directed inwards. It is represented by (x)

The direction of $\vec{B}$ at various points is shown in the figure shown.




At points 'C' and 'D' $\vec{B}$ = 0 (think how).

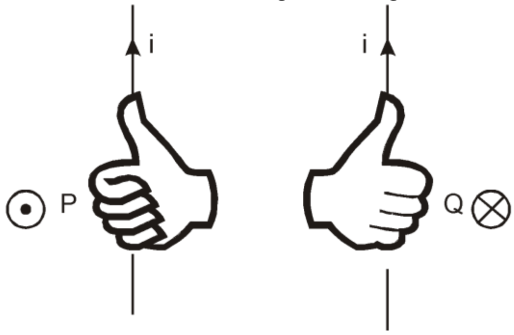
For the case shown in figure



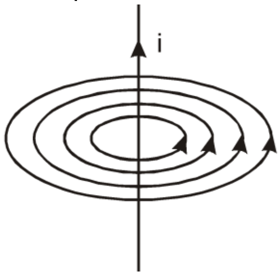

B at A = $\frac{\mu_0 i}{4\pi r}$ (sin $\theta_2$ – sin$\theta_1$) ⊗

## Shortcut for Direction :

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.

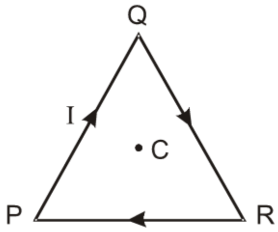



We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line given the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centres on the wire and in the plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above mentioned plane.




## Solved Examples

**Example 3.** A loop in the shape of an equilateral triangle of side ‘a’ carries a current I as shown in the figure. Find out the magnetic field at the centre ‘C’ of the triangle.

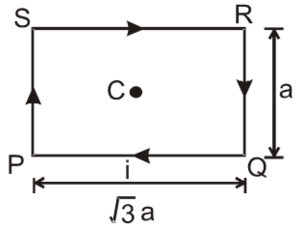



**Answer :** $\frac{9\mu_0 i}{2\pi a}$

**Solution :** $B = B_1 + B_2 + B_3 = 3B_1$

$= 3\,\dfrac{\mu_0}{4\pi} \times \dfrac{i}{\left(\dfrac{a}{2\sqrt{3}}\right)} \times (\sin 60^\circ + \sin 60^\circ) = \dfrac{9\mu_0 i}{2\pi a}$

**Example 4.** Find resultant magnetic field at 'C' in the figure shown.




**Solution :** It is clear that 'B' at 'C' due all the wires is directed ⊗. Also B at 'C' due PQ and SR is same. Also due to QR and PS is same

∴ $B_{res} = 2(B_{PQ} + B_{SP})$

$B_{PQ} = \dfrac{\mu_0 i}{4\pi \dfrac{a}{2}}$ (sin 60º + sin 60º),

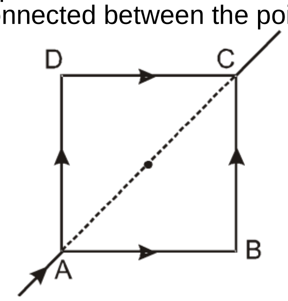
$B_{sp} = \dfrac{\mu_0 i}{4\pi \dfrac{\sqrt{3}a}{2}}$ (sin 30º + sin 30º) ⇒ $B_{res} = 2\left(\dfrac{\sqrt{3}\,\mu_0 i}{2\pi a} + \dfrac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \dfrac{4\mu_0 i}{\sqrt{3}\pi a}$

**Example 5.** Two long wires are kept along x and y axes they carry currents I & I respectively in +ve x and +ve y directions respectively. Find $\vec{B}$ at a point (0, 0, d).

**Answer :** $\dfrac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$

**Solution :** $\vec{B} = \vec{B}_1 + \vec{B}_2 = \dfrac{\mu_0}{2\pi}\dfrac{i}{d}(-\hat{j}) + \dfrac{\mu_0}{2\pi}\dfrac{i}{d}(\hat{i}) = \dfrac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$

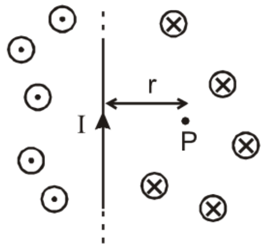
**Example 6.** Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.




**Solution :** The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

## Special case :

**(i)** If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^\circ$ and the formula of 'B' due to straight wire)



$B = \frac{\mu_0 I}{2\pi r}$ $\Rightarrow$ $B \propto \frac{I}{r}$

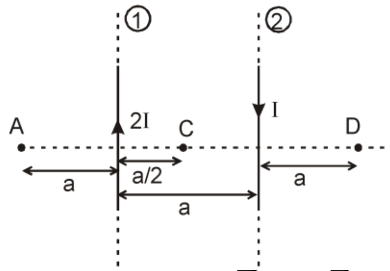
The direction of $\vec{B}$ at various is as shown in the figure. The magnetic lines of force will be concentric
circles around the wire (as shown earlier)

**(ii)** If the wire is infinitely long but 'P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At 'P'

$B = \frac{\mu_0 I}{4\pi r}$

## Solved Examples

**Example 7.** In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents 2I and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find (i) $\vec{B}$ at A, C, D
(ii) position of point on line A C D where $\vec{B}$ is O.




**Solution :** (i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B_1}$ and $\vec{B_2}$ respectively. Then

at A : $\vec{B_1}$ is $\odot$ and $\vec{B_2}$ is $\otimes$

$B_1 = \frac{\mu_0 2I}{2\pi a}$ and $B_2 = \frac{\mu_0 I}{2\pi 2a}$

$\therefore$ $B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a}$ $\odot$ **Ans.**

at C : $\vec{B_1}$ is $\otimes$ and $\vec{B_2}$ also $\otimes$

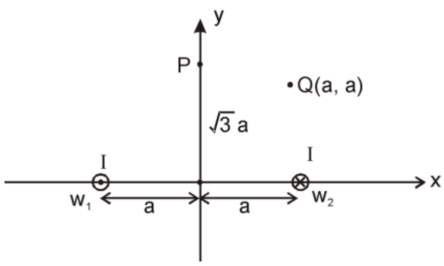
$\therefore$ $B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi \frac{a}{2}} + \frac{\mu_0 I}{2\pi \frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a}$ $\otimes$ **Ans.**

at D : $\vec{B_1}$ is $\otimes$ and $\vec{B_2}$ is $\odot$ and both are equal in magnitude.

$\therefore$ $B_{res} = 0$ **Ans.**

(ii) It is clear from the above solution that B = 0 at point 'D'.

**Example 8.** In the figure shown two long wires $W_1$ and $W_2$ each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in $W_1$ is outward and in $W_2$ it is inwards. Find the $\vec{B}$ at 'P' and 'Q'.

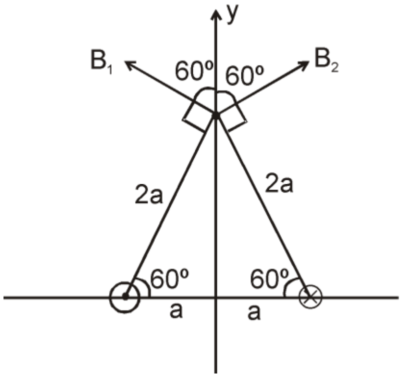


**Solution :** Let due to $W_1$ be $\vec{B}_1$ and due to $W_2$ be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2|$ = B

$B_p = 2B\cos 60^o = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$

$\therefore \quad \vec{B}_p = \frac{\mu_0 I}{4\pi a}\hat{j}$ **Ans.**

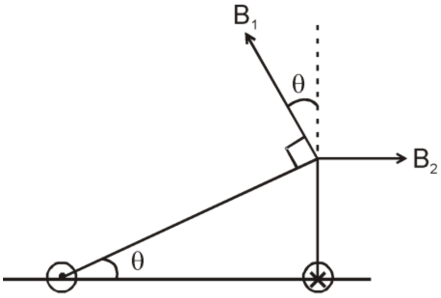



For θ $\quad B_1 = \frac{\mu_0 I}{2\pi\sqrt{5}a}$, $\Rightarrow \quad B_2 = \frac{\mu_0 I}{2\pi a}$

$\tan\theta = \frac{a}{2a} = \frac{1}{2} \quad \Rightarrow \quad \vec{B} = (B_1\cos\theta\,\hat{j}) + (B_2 - B_1\sin\theta)\,\hat{i}$

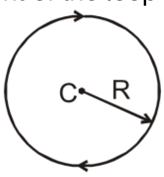
$\sin\theta = \frac{1}{\sqrt{5}} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$

$\cos\theta = \frac{2}{\sqrt{5}} \quad \Rightarrow \quad \vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$




## 2.3.2 $\vec{B}$ due to circular loop

**(a)** $\vec{B}$ **At centre :** Due to each $\vec{d\ell}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).




$\therefore \quad \vec{B}_{res}$ at 'c' is ⊗ .

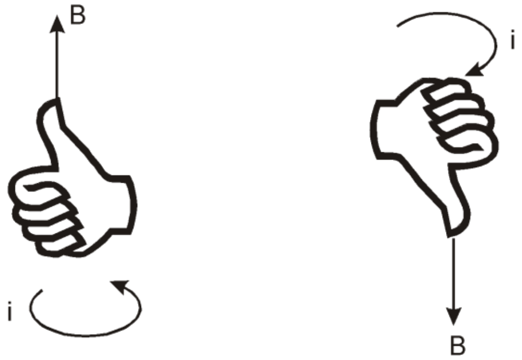
$B = \frac{\mu_0 N I}{2R}$,

N = No. of turns in the loop.

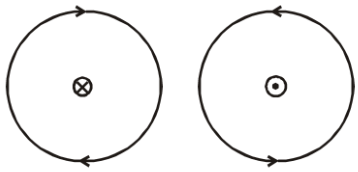
$= \frac{\ell}{2\pi R}$ ; $\ell$ = length of the loop.

N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

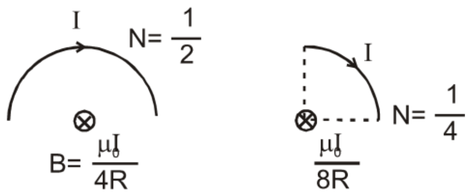
**Direction of $\vec{B}$ :** The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).




Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.



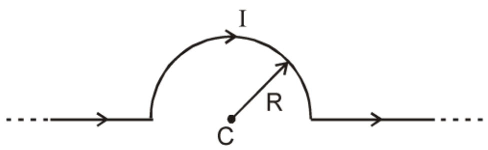
**Semicircular and Quarter of a circle :**



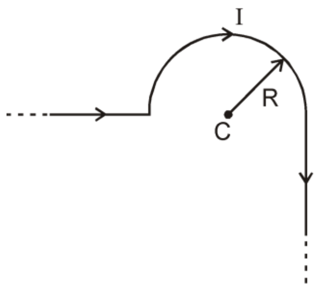

## Solved Examples

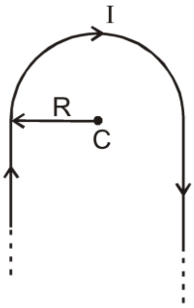
**Example 9.** Find 'B' at centre 'C' in the following cases :

(i)

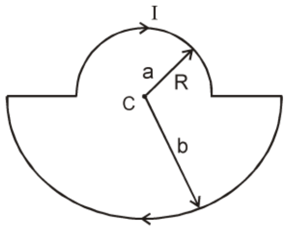



(ii)

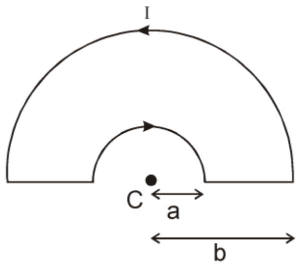




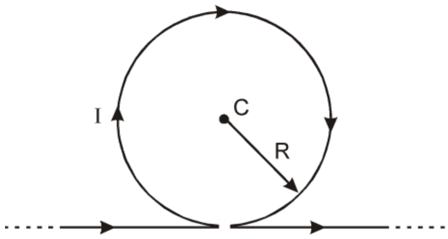

(iii)



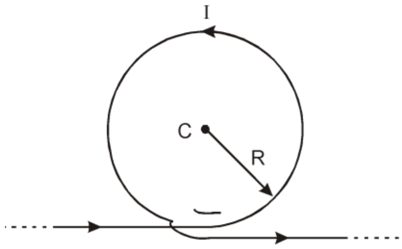

(iv)



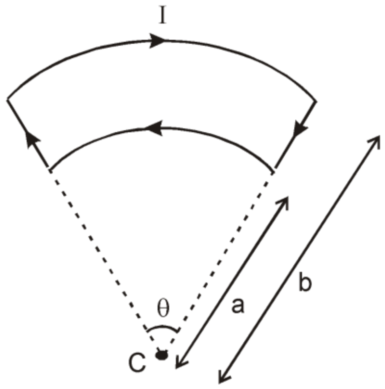

(v)




(vi)




(vii)




(viii)

**Answer :**
(i) $\frac{\mu_0 I}{4R} \otimes$
(ii) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right) \otimes$
(iii) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right) \otimes$
(iv) $\frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right) \otimes$
(v) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right) \otimes$
(vi) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right) \otimes$
(vii) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right) \odot$
(viii) $\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right) \odot$

**Solution :**

(i) $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I}{4R}$

(ii) $B = B_1 + B_2 = \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) + \left(\frac{\mu_0}{4\pi} . \frac{I}{R}\right) = \frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)$

(iii) $B = B_1 + B_2 + B_3 = 2B_1 + B_2 = \left(2 \times \frac{\mu_0}{4\pi}\frac{I}{R}\right) + \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) = \frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

(iv) $B = B_1 + B_2 = \frac{\mu_0}{2a} \times \frac{1}{2} + \frac{\mu_0}{2b} \times \frac{1}{2} = \frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right)$

(v) $B = B_1 - B_2 = \left(\frac{\mu_0 I}{2a} \times \frac{1}{2} - \frac{\mu_0 I}{2b} \times \frac{1}{2}\right) = \frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$

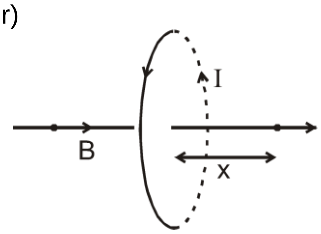
(vi) $B = B_1 - B_2 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)$

(vii) $B = B_1 + B_2 = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)$

(viii) $B = B_1 - B_2 = \frac{\mu_0 I}{2a} - \frac{\mu_0 I}{2b} \times \frac{\theta}{2\pi} = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

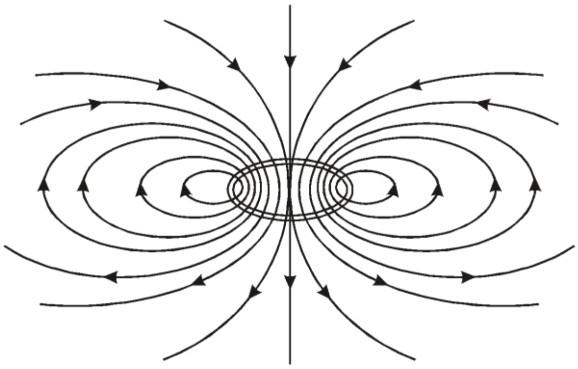
**(b) On the axis of the loop :** $B = \dfrac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}}$

N = No. of turns (integer)



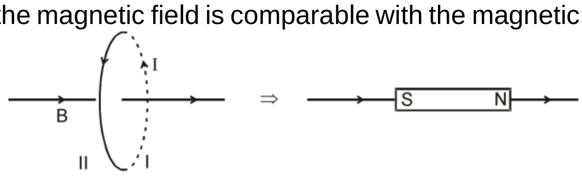

Direction can be obtained by right hand thumb rule. curl your fingers in the direction of the current then the direction of the thumb points in the direction of $\vec{B}$ at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

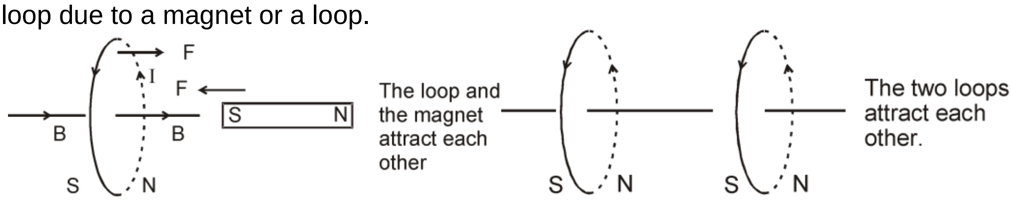


## 2.3.3 A loop as a magnet :

The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.




The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.




**Mathematically :**

$$B_{axis} = \frac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \cong \frac{\mu_0 NIR^2}{2x^3} \quad \text{for } x >> R$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\left(\frac{IN\pi R^2}{x^3}\right)$$

it is similar to $B_{axis}$ due to magnet = $2\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{x^3}$

Magnetic dipole moment of the loop

$M = IN\pi R^2$

M = INA for any other shaped loop.

Unit of M is Amp. $m^2$.

Unit of m (pole strength) = Amp. m {∵ in magnet $M = m\ell$}

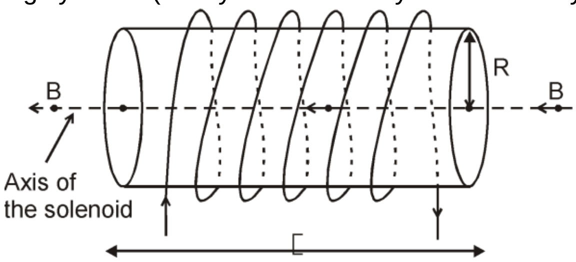
$\vec{M} = IN\vec{A}$,

$\vec{A}$ = unit normal vector for the loop.


To be determined by right hand rule which is also used to determine direction of $\vec{B}$ on the axis. It is also from 'S' side to 'N' side of the loop.

## 2.3.4 Solenoid :

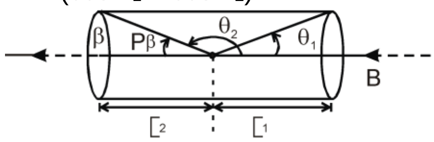
(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)




(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$




where n : number of turns per unit length.

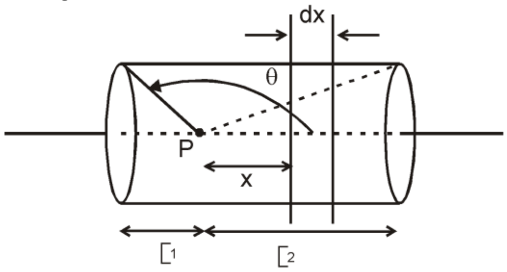
$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2 + R^2}}$ ; $\cos\beta = \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}} = -\cos\theta_2$

$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right] = \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$

**Note :** Use right hand rule for direction (same as the direction due to loop).

### Derivation :

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n : number of turns per unit length.




$$dB = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}} (ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 i R^2\, ndx}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$

**(iv) For 'Ideal Solenoid' :**

***Inside** (at the mid point)

$\ell >> R$ or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

B = [1 – (–1)]

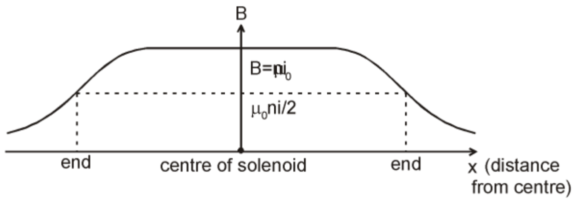
**B = $\mu_0$ni**

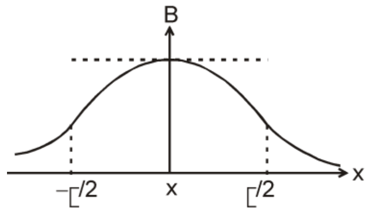
If material of the solid cylinder has relative permeability '$\mu_r$' then B = $\mu_0\mu_r$ni

At the ends **B =** $\frac{\mu_0 ni}{2}$

**(v) Comparision between ideal and real solenoid :**

**(a) Ideal Solenoid** **Real Solenoid**




## 2.4 AMPERE's circuital law :

The line integral $\oint \vec{B} \cdot \vec{d\ell}$ on a closed curve of any shape is equal to $\mu_0$ (permeability of free space) times the net current I through the area bounded by the curve.

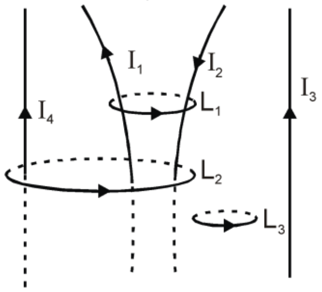
$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 \Sigma I$$

## Note :

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B} \cdot \vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current :** The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e. $\vec{B} \cdot \vec{d\ell}$ positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative.

## Solved Examples

**Example 10.** Find the values of $\oint \vec{B} \cdot \vec{d\ell}$ for the loops $L_1$, $L_2$, $L_3$ in the figure shown. The sense of $\vec{d\ell}$ is mentioned in the figure.

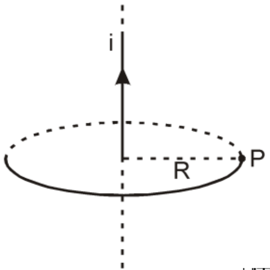


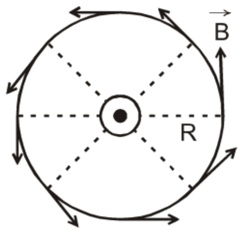

**Solution :** for $L_1$ $\oint \vec{B} \cdot \vec{d\ell} = \mu_0(I_1 - I_2)$

here $I_1$ is taken positive because magnetic lines of force produced by $I_1$ is anti clockwise as seen from top. $I_2$ produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.

for $L_2$ : $\oint \vec{B} \cdot \vec{d\ell} = \mu_0 \; (I_1 - I_2 + I_4)$ for $L_3$ : $\oint \vec{B} \cdot \vec{d\ell} = 0$ s

**Uses : 2.4.1** To find out magnetic field due to infinite current carrying wire



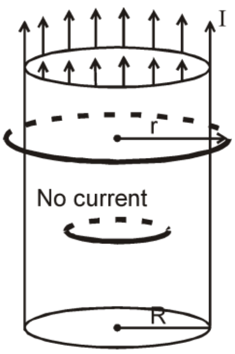

By B.S.L. $\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle.

$\oint \vec{B} \cdot \vec{d\ell} = \oint B . d\ell$ $\because \theta = 0^o$ so $B \oint d\ell = B \, 2\pi R$ ($\because$ B = const.)

Now by amperes law :

$B \, 2\pi R = \mu_0 I$ $\therefore$ $B = \dfrac{\mu_0 i}{2\pi R}$

### 2.4.2. Hollow current carrying infinitely long cylinder : (I is uniformly distributed on the whole circumference)



**(i)** for $r \geq R$

By symmetry the amperian loop is a circle.

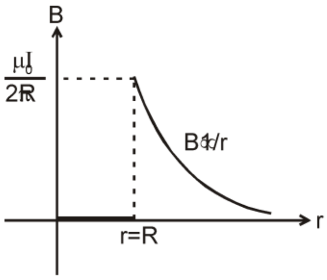
$\oint \vec{B} \cdot \vec{d\ell} = \oint B\, d\ell$ $\quad \because \quad \theta = 0$

$= B \int_0^{2\pi r} d\ell \quad \because$ B = const. $\Rightarrow$ **B =** $\frac{\mu_0 I}{2\pi r}$

**(ii)** $r < R$ $\quad = \oint \vec{B} \cdot \vec{d\ell}$

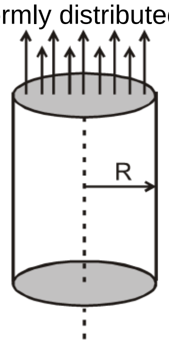
$= \oint B\, d\ell \quad = B(2\pi\, r) = 0 \quad \Rightarrow \quad$ **$B_{in}$ = 0**

**Graph :**




## 2.4.3 Solid infinite current carrying cylinder :

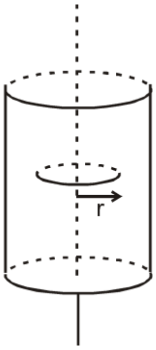
Assume current is uniformly distributed on the whole cross section area




current density $J = \frac{I}{\pi R^2}$

**Case (I) :** $r \leq R$

take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.



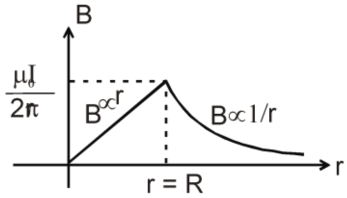

$\oint \vec{B} \cdot \vec{d\ell} = \oint B \cdot d\ell = B \oint d\ell = B \cdot 2\pi r = \mu_0 \frac{I}{\pi R^2} \pi r^2$

$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \quad \Rightarrow \quad \vec{B} = \frac{\mu_0 (\vec{J} \times \vec{r})}{2}$

**Case (II) :** $r \geq R$ $\oint \vec{B}.\vec{d\ell} = \oint B\,d\ell = B\oint d\ell = B\,.\,(2\pi r) = \mu_0\,.\,I$

$\Rightarrow$ $B = \frac{\mu_0 I}{2\pi r}$ also $\vec{B} = \frac{\mu_0 I}{2\pi r}(\hat{J}\times\hat{r}) = \frac{\mu_0 J\pi R^2}{2\pi r}$

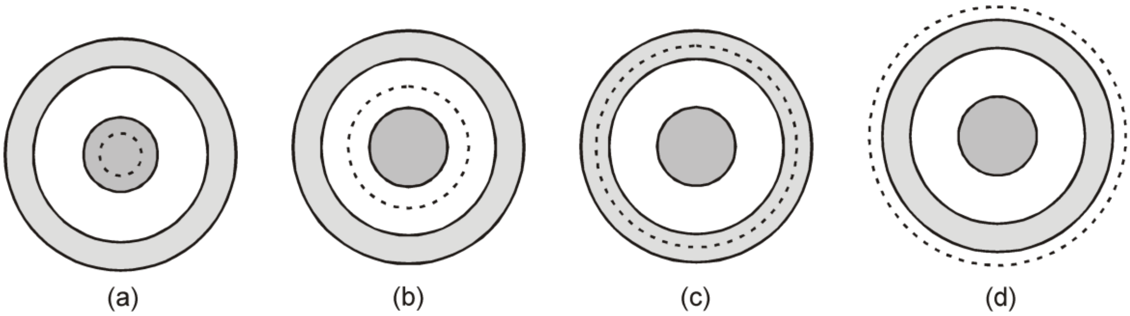
$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J}\times\vec{r})$




## Solved Examples

**Example 11.** Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current $i_0$ and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where (a) x < a, (b) a < x < b (c) b < x < c and (d) x > c. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

**Solution :**




A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.\vec{d\ell} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle in part b is $i_0$ so that

$$\frac{i_0}{\pi a^2}\,.\,\pi x^2 = \frac{i_0}{a^2}x^2$$

Ampere's law

$\oint \vec{B}.\vec{d\ell} = \mu_0 i$ gives

$$B.2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or } B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is $i_0$ so that

$B\,2\pi x = \mu_0\,i_0$ or, B = .

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$.

Thus, the current through this part is $\frac{i_0(x^2 - b^2)}{(c^2 - b^2)}$. This is in the same direction to the current $i_0$ in the inner wire. Thus, the net current enclosed by the circle is

$$i_{net} = i_0 + \frac{i_0(x^2 - b^2)}{c^2 - b^2} = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2}$$

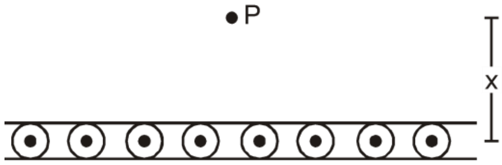
From Ampere's law,

$$B\,2\pi x = \frac{i_0(c^2 + x^2 - 2b^2)}{c^2 - b^2} \quad \text{or} \quad B = \frac{\mu_0 i_0(c^2 + x^2 - 2b^2)}{2\pi x(c^2 - b^2)}$$

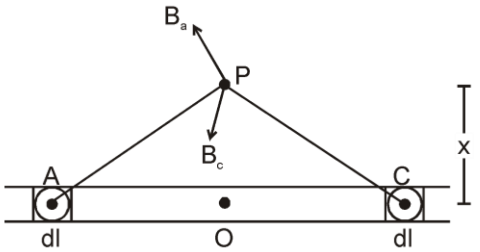
(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

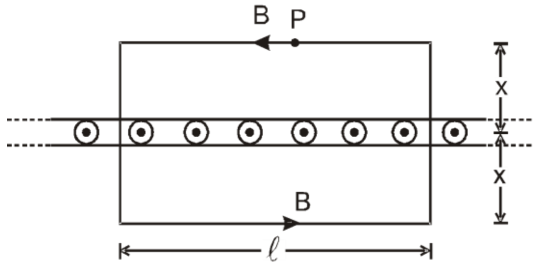
$$B\,2\pi x = \mu_0\,2i_0 \quad \text{or} \quad B = \frac{\mu_0 i_0}{\pi x}$$

**Example 12.** Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.




**Solution :** Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is $B_0$ perpendicular to AP and that due to the strip C is $B_C$ perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.



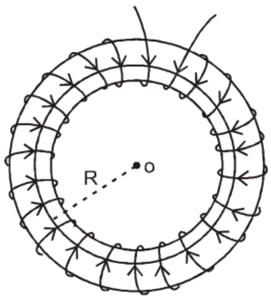

The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\ell = \mu_0 K\ell \quad \text{or,} \quad B = \frac{1}{2}\mu_0 K$$

Note that it is independent of x.

**Magnetic Field in a Toroid**

(i) Toroid is like an endless cylindrical solenoid, i.e. if a long solenoid is bent round in the form of a closed ring, then it becomes a toroid.




(ii) Electrically insulated wire is wound uniformly over the toroid as shown in the firgure.

(iii) The thickness of toroid is kept small in comparison to its radius and the number of turns is kept very large.

(iv) When a current i is passed through the toroid, each turn of the toroid produces a magnetic field along the axis at its centre. Due to uniform distribution of turns this magnetic filed has same magnitude at their centres. Thus the magnetic lines of force inside the toroid are circular.

(v) The magnetic field inside a toroid at all points is same but outside the toroid it is zero.

(vi) If total number of turns in a toroid is N and R is its radius, then number of turns per unit length of the toroid will be

$$n = \frac{N}{2\pi R}$$

(vii) The magnetic field due to toroid is determined by Ampere's law.

(viii) The magnetic field due to toroid is

$B_0 = \mu_0 ni$ or $B_0 = \mu_0 \left(\frac{N}{2\pi R}\right) i$

(ix) If a substance of permeability $\mu$ is placed inside the toroid, then

$B = \mu ni$

If $\mu_r$ is relative magnetic permeability of the substance, then

$B = \mu_r \mu_0 ni$

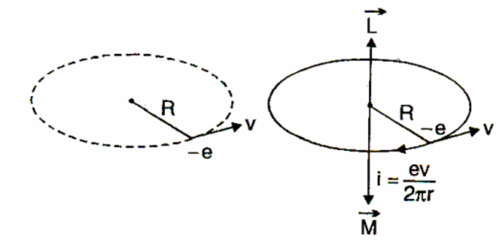
**CURRENT AND MAGNETIC FIELD DUE TO CIRCULAR MOTION OF A CHARGE**

(i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons are in the nucleus of each atom and electrons are assumed to be moving in different orbits around the nucleus.

(ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result static electric field is not observed.

(iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.

(iv) In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then




(a) current in the orbit = charge × frequency = ef

If T is the period, then $f = \frac{1}{T}$; $i = \frac{e}{T}$

(b) Magnetic field at the nucleus (centre)

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R} = \frac{\mu_0 e}{2RT}$$

(c) If the angular velocity of the electron is , then

$\omega = 2\pi f$ and $f = \frac{\omega}{2\pi}$

$\therefore$ $i = ef = \frac{e\omega}{2\pi}$ $\therefore$ $B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 e\omega}{4\pi R}$

(d) If the linear velocity of the electron is v, then

$v = R\omega = R(2\pi f)$

or $f = \left(\frac{v}{2\pi R}\right)$

$\therefore$ $i = ef = \frac{ev}{2\pi R}$ $\therefore$ $B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ev}{4\pi R^2}$

(v) Magnetic moment due to motion of electron in an orbit

$$M = iA = ef\pi R^2 = \frac{e\pi R^2}{T}$$

or $$M = \frac{e\omega\pi R^2}{2\pi} = \frac{e\omega R^2}{2}$$ or $$M = \frac{ev\pi R^2}{2\pi R} = \frac{evR}{2}$$

If the angular momentum of the electron is L, then

$$L = mvR = m\omega R^2$$

Writing M in terms of L

$$M = \frac{em\omega R^2}{2m} = \frac{emvR}{2m} = \frac{eL}{2m}$$

According to Bohr's second postulate

$$mvR = n\frac{h}{2\pi}$$

In ground state n = 1

$$L = \frac{h}{2\pi} \quad \Rightarrow \quad M = \frac{eh}{4\pi m}$$

(vi) If a charge q (or a charged ring of charge q) is moving in a circular path of radius R with a frequency f or angular velocity $\omega$, then

(a) current due to moving charge

$$i = qf = q\omega / 2\pi$$

(b) magnetic field at the centre of ring

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R} \quad \text{or} \quad B_0 = \frac{\mu_0 q\omega}{4\pi R}$$

(c) magnetic moment

$$M = i(\pi R^2) = qf\pi R^2 = \frac{1}{2}q\omega R^2$$

(vii) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity , then

(a) the magnetic field produced at its centre will be

$$B_0 = \frac{\mu_0 q\omega}{2\pi R}$$

(b) the magnetic moment of the disc will be

$$dM = (di)\,\pi x^2$$

$$= \frac{\omega}{2\pi}\, dq\, \pi x^2$$

$$= \frac{\omega q}{R^2}\, x^3 dx$$

$$\Rightarrow \quad M = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx$$

$$M = \frac{q\omega R^2}{4}$$

## 3. Magnetic force on moving charge

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$$\vec{F} = q(\vec{v} \times \vec{B})$$ Put q with sign.

$\vec{v}$ : Instantaneous velocity

$\vec{B}$ : Magnetic field at that point.

**Note :**

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because$ $\vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power P = $\vec{F} \cdot \vec{v}$ for its proof).
- Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V} \| \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

## Solved Examples

**Example 13.** A charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m$^2$ respectively.

**Solution :** $\vec{F} = q\vec{v} \times \vec{B}$ = 2 × 10$^{-6}$ ($2\hat{i} - 3\hat{j} \times 4\hat{k}$) × $(3\hat{j} - 2\hat{k})$ = 2 × 10$^{-6}$ [$-6\hat{i} + 4\hat{j} + 6\hat{k}$] N

By Newton's Law $\vec{a} = \frac{\vec{F}}{m} = \frac{2 \times 10^{-6}}{5 \times 10^{-6}} (-6\hat{i} + 4\hat{j} + 6\hat{k})$

= 0.8 ($-3\hat{i} + 2\hat{j} + 3\hat{k}$) m/s$^2$

**Example 14.** A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

**Solution :** $\because$ $\vec{F} \perp \vec{B}$

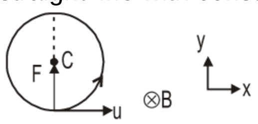
$\therefore$ $\vec{a} \perp \vec{B}$ $\therefore$ $\vec{a} \cdot \vec{B}$ = 0

$\therefore$ $(2\hat{i} + x\hat{j}) \cdot (-3\hat{i} + 2\hat{j} - 4\hat{k})$ = 0

$\Rightarrow$ –6 + 2x = 0 $\Rightarrow$ x = 3

## 3.1 Motion of charged particles under the effect of magnetic force

- Particle released if v = 0 then $f_m$ = 0
  $\therefore$ particle will remain at rest
- $\vec{V} \| \vec{B}$ here θ = 0 or θ = 180º
  $\therefore$ $F_m$ = 0 $\therefore$ $\vec{a} = 0$ $\therefore$ $\vec{V}$ = const.
  $\therefore$ particle will move in a straight line with constant velocity




- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform
  In this case $\because$ B is in z direction so the magnetic force in z-direction will be zero $\left(\because \vec{F}_m \perp \vec{B}\right)$.
  Now there is no initial velocity in z-direction.
  $\therefore$ particle will always move in xy plane.
  $\therefore$ velocity vector is always $\perp$ B $\therefore$ $F_m$ = quB = constant

now $quB = \frac{mu^2}{R} \Rightarrow R = \frac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle.

∴ path of the particle is circular.

$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$

here p = linear momentum ; k = kinetic energy

now $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$

frequency $f = qB/2\pi m$

**Note :**

- ω, f, T are independent of velocity.

## Solved Examples

**Example 15.** A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

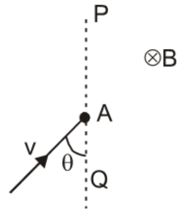
**Solution :** $R = \frac{\sqrt{2mK}}{qB}$

∴ $R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$

$T = 2\pi m/qB$

∴ $T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2$ **Ans.**

**Example 16.** A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ.

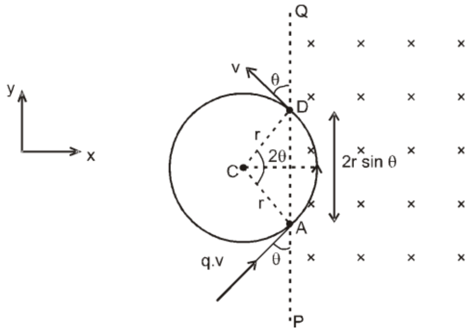



Find (i) time spent,

(ii) distance travelled in the magnetic field

(iii) impulse of magnetic force.

**Solution :** The particle will move in the field as shown




Angle subtended by the arc at the centre = 2θ

(i) Time spent by the charge in magnetic field

$\omega t = 2\theta \Rightarrow \frac{qB}{m}\, t = 2\theta \quad \Rightarrow t = \frac{m2\theta}{qB}$

(ii) Distance travelled by the charge in magnetic field :

$= r(2\theta) = \frac{mv}{qB} \,.\, 2\theta$

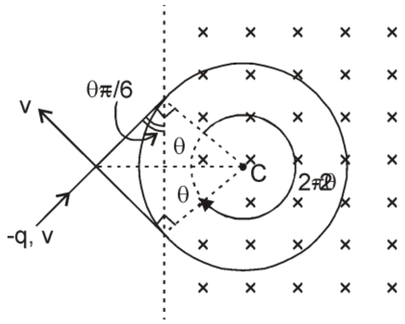
(iii) Impulse = change in momentum of the charge

$= (-mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) - (mv\sin\theta\,\hat{i} + mv\cos\theta\,\hat{j}) = -2mv\sin\theta\,\hat{i}$

**Example 17.** Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

**Solution :** (i) $2\pi - 2\theta = 2\pi - 2.\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= \omega t = \frac{qBt}{m} \Rightarrow t = \frac{5\pi m}{3qB}$



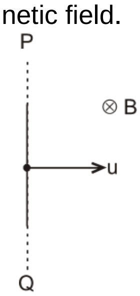

(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum

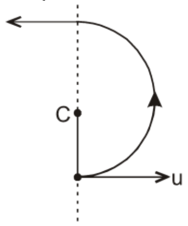
$= m(-v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j}) - m(v\sin\theta\,\hat{i} + v\cos\theta\,\hat{j})$

$= -2mv\sin\theta\,\hat{i} = -2mv\sin\frac{\pi}{6}\,\hat{i} = -mv$

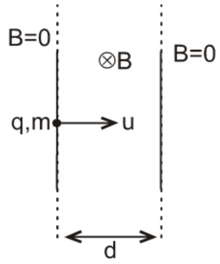
**Example 18.** In the figure shown the magnetic field on the left on ‘PQ’ is zero and on the right of ‘PQ’ it is uniform. Find the time spent in the magnetic field.




**Solution :** The path will be semicircular time spent = T/2 = πm/qB

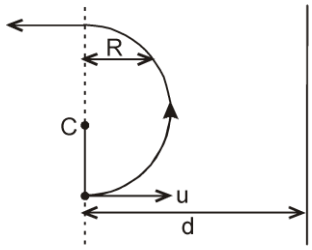


**Example 19.** A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if




(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

**Solution :**



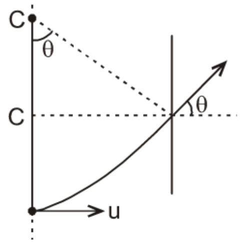

(i) $d > \frac{mu}{qB}$ means $d > R$

$\therefore \; t = \frac{T}{2} = \frac{\pi m}{qB}$

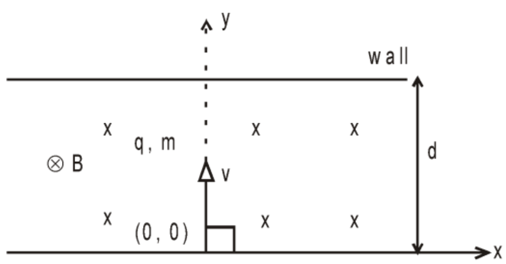
(ii) $\sin\theta = \frac{d}{R}$

$\theta = \sin^{-1}\left(\frac{d}{R}\right)$

$\omega t = \theta \;\Rightarrow\; t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$

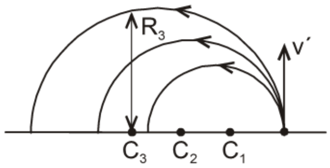



**Example 20.** What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.




**Solution :** (i) The path of the particle will be circular larger the velocity, larger will be the radius.

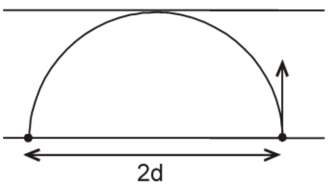
For particle not to s




strike R < d

∴ $\frac{mv}{qB} < d$ ⇒ $v < \frac{qBd}{m}$.

(ii) for limiting case $v = \frac{qBd}{m}$




R = d

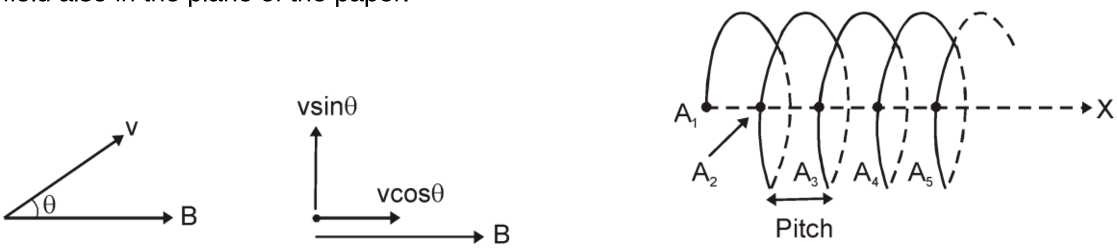
∴ coordinate = (–2d, 0, 0)

## 3.2 Helical path :

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{||}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

### Complete analysis :

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.




The particle starts from point $A_1$.

It completes its one revolution at $A_2$ and 2nd revolution at $A_3$ and so on. X-axis is the tangent to the helix points

$A_1, A_2, A_3$,..........all are on the x-axis.

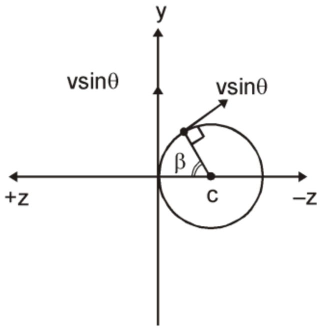
distance $A_1A_2 = A_3A_4$ = ............... = v cosθ. T = pitch

where T = Time period

Let the initial position of the particle be (0,0,0) and v sinq in +y direction. Then

in x : $F_x = 0$, $a_x = 0$, $v_x$ = constant = v cosθ, x = (v cosθ)t

**In y-z plane :**




$\beta\omega = t$

From figure it is clear that

$y = R\sin\beta$, $v_y = v\sin\theta\cos\beta$

$z = -(R - R\cos\beta)$

$v_z = v\sin\theta\sin\beta$

acceleration towards centre = $(v\sin\theta)^2/R = \omega^2 R$

$\therefore \quad a_y = -\omega^2 R\sin\beta$, $a_z = -\omega^2 R\cos\beta$

At any time : the position vector of the particle
(or its displacement w.r.t. initial position)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, x,y,z already found

velocity $\quad \vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, $v_x$, $v_y$, $v_z$ already found

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, $a_x$, $a_y$, $a_z$ already found

Radius $\quad q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R} \quad \Rightarrow \quad R = \dfrac{mv\sin\theta}{qB}$

$\omega = \dfrac{v\sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f.$

## 3.3 Charged Particle in $\vec{E}$&$\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then. Net force experienced by it is given by following equation.

$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$

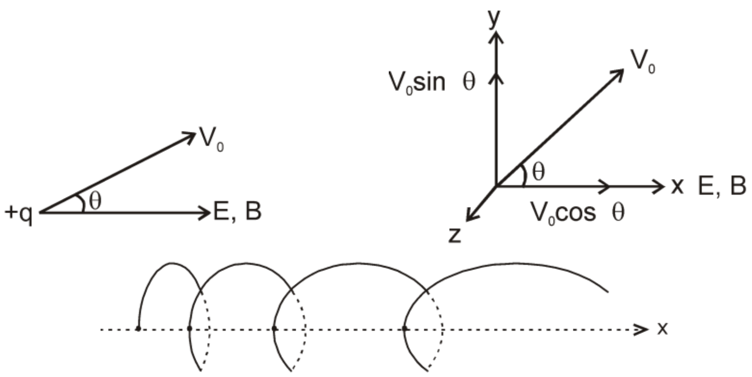

Combined force is known as Lorentz force.

$\vec{E} \,||\, \vec{B} \,||\, \vec{v}$

In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

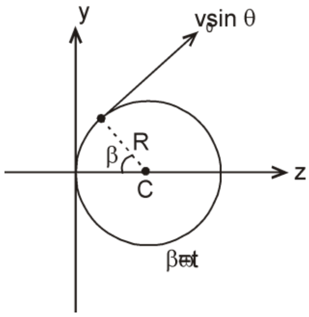
### Case(i) :

- $\vec{E} \,||\, \vec{B}$ and uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)




in x : $F_x = qE$, $a_x = \dfrac{qE}{m}$, $v_x = v_0\cos\theta + a_x t$, $x = v_0 t + \dfrac{1}{2}a_x t^2$

**in yz plane :**




$qv_0 \sin\theta\, B = m(v_0 \sin\theta)^2 / R$

$\Rightarrow \quad R = \dfrac{mv_0 \sin\theta}{qB}$,

$\omega = \dfrac{v_0 \sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f$

$\vec{r} = \{(V_0 \cos\theta)t + \dfrac{1}{2}\dfrac{qE}{m}t^2\}\,\hat{i} + R\sin\omega t\,\hat{j} + (R - R\cos\omega t)\,(-\hat{k})$

$\vec{V} = \left(V_0 \cos\theta + \dfrac{qE}{m}t\right)\hat{i} + (V_0 \sin\theta)\cos\omega t\,\hat{j} + V_0 \sin\theta \sin\omega t\,(-\hat{k})$

$\vec{a} = \dfrac{qE}{m}\,\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$

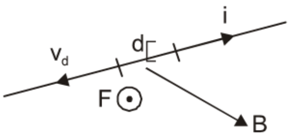
**Example 21.** A particle of charge q and mass m is projected in a uniform and constant magnetic field of strength B. The initial velocity vector $\vec{v}$ makes angle 'θ' with the $\vec{B}$. Find the distance travelled by the particle in time 't'.

**Answer :** vt

**Solution :** Speed of the particle does not change there fore distance covered by the particle is s = vt

## 3.4 Magnetic force on a current carrying wire :

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (figure). The free electrons drift with a speed $v_d$ opposite to the direction of the current. The relation between the current i and the drift speed $v_d$ is




$i = jA = nev_dA$. ....(i)

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$\vec{f} = -e\vec{v}_d \times \vec{B}$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d\, d\vec{\ell} \times \vec{B}$$

Using (i), $d\vec{F} = i\,d\vec{\ell} \times \vec{B}$ .

The quantity $i\,d\vec{\ell}$ is called a current element.

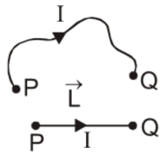
$$\vec{F}_{res} = \int d\vec{F} = \int i\,d\vec{\ell} \times \vec{B} = i\int d\vec{\ell} \times \vec{B}$$

($\because$ i is same at all points of the wire.)

If $\vec{B}$ is uniform then $\vec{F}_{res} = i(\int d\vec{\ell}) \times \vec{B}$

$$\vec{F}_{res} = i\vec{L} \times \vec{B}$$

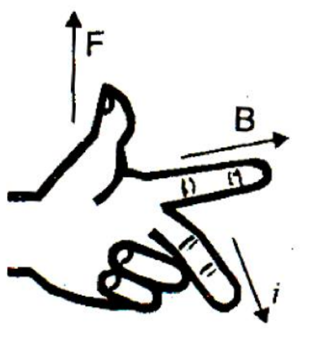
Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.




The direction of magnetic force is perpendicular to the plane of $\vec{l}$ and $\vec{B}$ according to right hand screw rule. Following two rules are used in determining the direction of the magnetic force.

(a) **Right hand palm rule :** If the right hand and the palm are stretched such that the thumb points in the direction of current and the stretched fingers in the direction of the magnetic field, then the force on the conductor will be perpendicular to the palm in the outward direction.

(b) **Fleming left hand rule :** If the thumb, fore finger and central finger of the left hand are stretched such that first finger points in the direction of magnetic field and the central finger in the direction of current, then the thumb will point in the direction of force acting on the conductor.

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**Note :** If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left.\vec{F}_{res}\right)_{magnetic}$ on it = 0 ($\because \vec{L}$ = 0).

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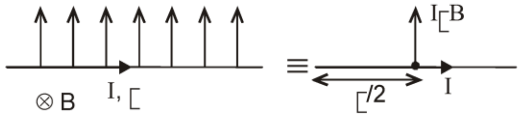
**3.5 Point of application of magnetic force :**

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.



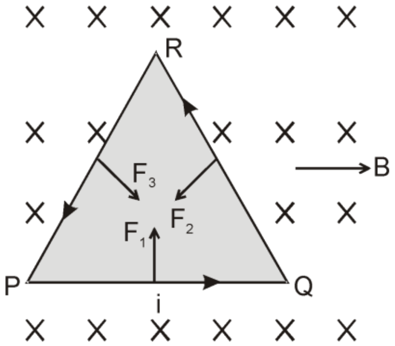

This can be used for calculation of torque.

## Solved Examples

**Example 22.** A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

**Solution :** Suppose the field and the current have directions as shown in figure. The force on PQ is

$\vec{F_1} = i\vec{\ell} \times \vec{B}$ or $F_1$ = 5.0 A × 10 cm × 2.0 T = 1.0 N

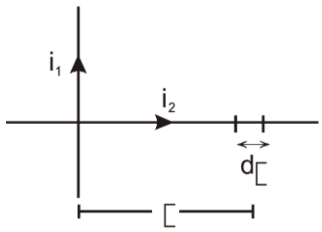



The rule of vector product shows that the force $F_1$ is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F_2}$ and $\vec{F_3}$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

**Example 23.** Two long wires, carrying currents $i_1$ and $i_2$, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance $\ell$ from the first wire.




**Solution :** The situation is shown in figure. The magnetic field at the site of $d\ell$,

due to the first wire is ,

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

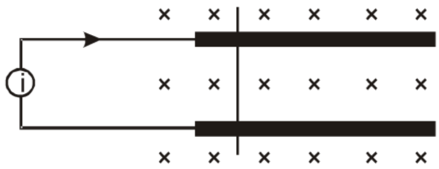
$$dF = i_2\, d\ell\, B \sin 90^o = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current $i_1$.

**Example 24.** Figure shows two long metal rails placed horizontally and parallel to each other at a separation $\ell$. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is $\mu$.

(a) What soluble the minimum value of $\mu$ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of $\mu$ is half the value found in the previous part




**Solution.** (a) The force on the wire due to the magnetic field is

$\vec{F} = i\vec{\ell} \times \vec{B}$ or $F = i\ell B$

It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If $\mu_0$ be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0$ mg. Thus,

$\mu_0 mg = i\ell B$ or $\mu_0 = \frac{i\ell B}{mg}$

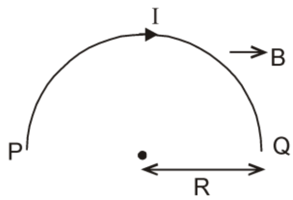
(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{i\ell B}{2mg}$, the wire will slide towards right. The frictional force by the rails is

$f = \mu mg = \frac{i\ell B}{2}$ towards left.

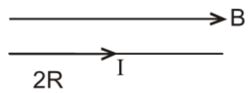
The resultant force is $i\ell B - \frac{i\ell B}{2} = \frac{i\ell B}{2}$ towards right. The acceleration will be $a = \frac{i\ell B}{2m}$.

The wire will slide towards right with this acceleration.

**Example 25.** In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force.



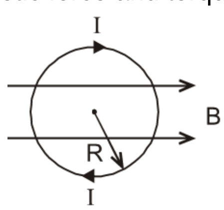

**Solution :** The wire is equivalent to




$\because$ $\theta = 0$

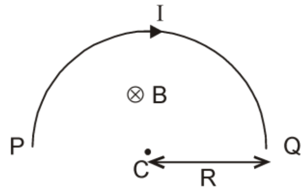
$\therefore$ $F_{res} = 0$ **Ans.**

**Example 26.** Find the resultant magnetic force and torque on the loop.



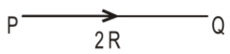

**Solution :** $\vec{F}_{res} = 0$, ($\because$ loop ) and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

**Example 27.** In the figure shown find the resultant magnetic force

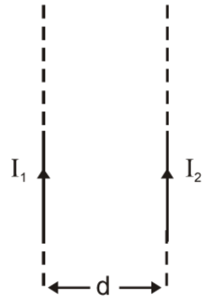



**Solution :** $\vec{F}_{nett} = I \cdot 2R \cdot B$

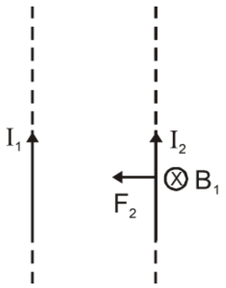
$\because \ \vec{\tau} = i\pi R^2 B(-\hat{j})$ wire is equivalent to




**Example 28.** Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2/2\pi d$. Here it is attractive also.




**Solution :**




On (2), B due to (i) is = $\frac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

$= I_2 . \frac{\mu_0 I_1}{2\pi d} . 1$ towards left it is attractive

$= \frac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

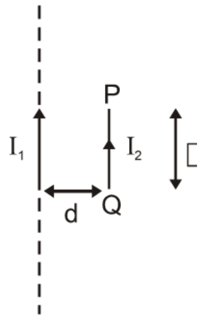
Similarly on the other wire also.

## Note :

- Definition of ampere (fundamental unit of current) using the above formula.
  If $I_1 = I_2 = 1A$, $d = 1m$ then $F = 2 \times 10^{-7}$ N
  $\therefore$ "When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of $2 \times 10^{-7}$ N on 1m length then the current is 1 ampere."

- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.

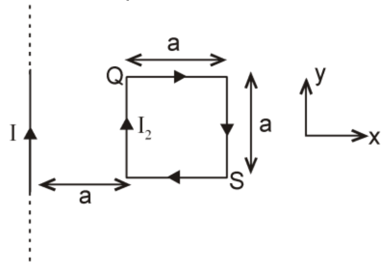
  Force per unit length on PQ = $\frac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)




(3) If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

## Solved Examples

**Example 29.** Find the magnetic force on the loop 'PQRS' due to the loop wire.

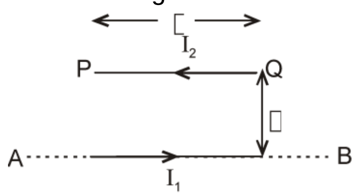



**Solution :**

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a}\, a(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i})$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$$

**Example 30.** In the figure shown the wires AB and PQ carry constant currents $I_1$ and $I_2$ respectively. PQ is of uniformly distributed mass 'm' and length '$\ell$'. AB and PQ are both horizontal and kept in the same vertical plane. The PQ is in equilibrium at height 'h'. Find




(i) 'h' in terms of $I_1$, $I_2$, $\ell$, m, g and other standard constants.

(ii) If the wire PQ is displaced vertically by small distance prove that it performs SHM. Find its time period in terms of h and g.

**Solution :** (i) Magnetic repulsive force balances the weight.

$$\frac{\mu_0 I_1 I_2}{2\pi h}\ell \quad mg \qquad \Rightarrow \qquad h = \frac{\mu_0 I_1 I_2 \ell}{2\pi mg}$$

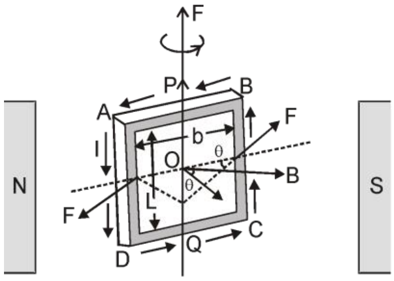
(ii) Let the wire be displaced downward by distance x(<< h).
Magnetic force on it will increase, so it goes back towards its equilibrium position. Hence it performs oscillations.

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi(h-x)}\ell - mg = \frac{mgh}{h-x} - mg = \frac{mg(h-h+x)}{h-x}$$

$$= \frac{mg}{h-x}\,x \cong \frac{mg}{h}\,x \text{ for } x << h \quad \therefore \qquad T = 2\pi\sqrt{\frac{m}{mg/h}} = 2\pi\sqrt{\frac{h}{g}}$$ **Ans.**

# 4. Torque on a current loop :

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure (A).




Axis of rotation
(A)

The arms AB and CD will experience forces B(NI)b vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.
Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

$\tau$ = F × Arm = BINL × (b sinθ)

i.e. $\tau$ = BIA sinθ with A = NLb .............(i)

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

$\vec{\tau} = \vec{M} \times \vec{B}$ [with $\vec{M} = I\vec{A} = NIA\vec{n}$ .........(ii)]

This is the required result and from this it is clear that :

(1) Torque will be minimum (= 0) when sinθ = min = 0, i.e., θ = 0º, i.e. 180º i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. (A) and (C)]

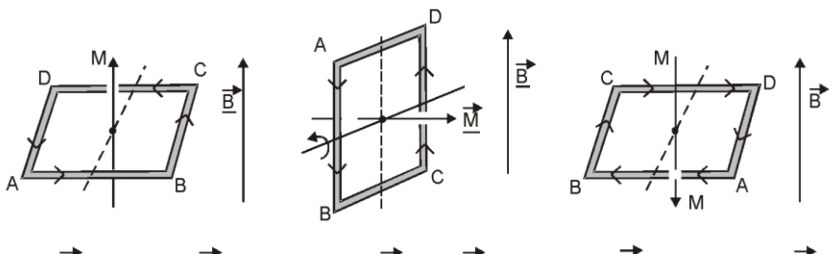
(2) Torque will be maximum (= BINA) when sinθ = max = 1, i.e., θ = 90º i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.(B)].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

U = $\vec{M} \bullet \vec{B}$ with F = $\frac{dU}{dr}$

and W = MB(1 – cosθ)

The values of U and W for different orientations of the coil in the field are shown in fig.




θ = 0º ($\vec{M}$ is parallel to $\vec{B}$)
τ = 0 = min
W = 0 = min
U = –MB = min
Stable equilibrium
(A)

θ⊥= 90º ($\vec{M}$ is to $\vec{B}$)
τ = MB = max
W = MB
U = 0
No equilibrium
(B)

θ = 180º ($\vec{M}$ is antiparallel to $\vec{B}$)
τ = 0
W = 2MB = max
U = MB = max
Unstable equilibrium
(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

## Solved Examples

**Example 31** A bar magnet having a magnetic moment of $2 \times 10^4$ $JT^{-1}$ is free to rotate in a horizontal plane. A horizontal magnetic field B= $6 \times 10^{-4}$ T exists in the space. The work done in taking the magnet slowly from a direction parallel to the field to a direction 60° from the field is

**Solution :** The work done in rotating a magnetic dipole against the torque acting on it , when placed in magnetic field is stored inside it in the form of potential energy.

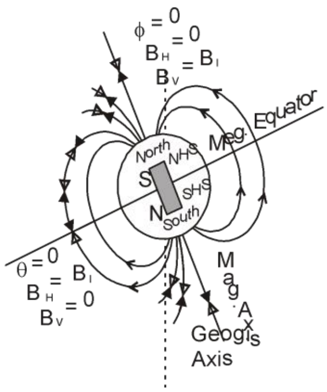
When magnetic dipole is rotated from initial position $\theta = \theta_1$ to final position $\theta = \theta_2$, then work done = MB($\cos\theta_1 - \cos\theta_2$)

$$= MB\left(1-\frac{1}{2}\right) = \frac{2\times10^4\times6\times10^{-4}}{2} = 6J$$

# 5. Terrestrial Magnetism (Earth's Magnetism) :

## 5.1 Introduction :

The idea that earth is magnetised was first suggested towards the end of the six-teenth century by Dr William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5º) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that :




**(a)** The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

**(b)** The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

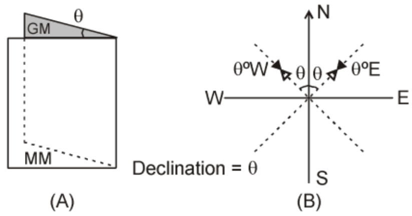
**(c)** The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

**(d)** The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

## 5.2 Elements of the Earth's Magnetism :

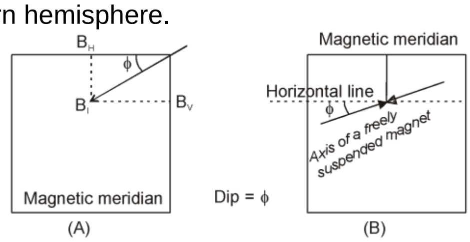
The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

**(a)** **Variation or Declination θ :** At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at θº E or θº W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10ºW means that at London the north pole of a compass needle points 10ºW, i.e., left of the geographical north.




**(b)** **Inclination or Angle of Dip φ :** It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.

Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.




Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42º.

**(c)** **Horizontal Component of Earth's Magnetic Field $B_H$ :** At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by $B_H$ and is measured with the help of a **vibration** or **deflection magnetometer**. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

If at a place magnetic field of earth is $B_I$ and angle of dip φ, then in accordance with figure (a).

$$B_H = B_I \cos \varphi$$

and $$B_V = B_I \sin \varphi \qquad ....(1)$$

so that, $$\tan \varphi = \frac{B_V}{B_H}$$

and $$I = \sqrt{B_H^2 + B_V^2} \qquad ....(2)$$

## 6. MAGNETIC CLASSIFICATION OF SUBSTANCES HYSTERESIS :

### (A) Classification of substances according to their magnetic behaviour :

All substance show magnetic properties. An iron nail brought near a pole of a bar magnet is strongly attracted by it and sticks to it, Similar is the behaviour of steel, cobalt and nickel. Such substance are called 'ferromagnetic 'substance. Some substances are only weakly attracted by a magnet, while some are repelled by it. They are called 'paramagnetic' and 'diamagnetic ' substance respectively. All substance, solids, liquids and gases, fall into one or other of these classes.

**(i)** **Diamagnetic substance :** Some substance, when placed in a magnetic field, are feebly magnetised opposite to the direction of the magnetising field. These substances when brought close to a pole of a powerful magnet, are somewhat repelled away from the magnet. They are called 'diamagnetic' substances and their magnetism is called the 'diamagnetism'.

Examples of diamagnetic substances are bisuth, zinc, copper, silver, gold, lead, water, mercury, sodium chloride, nitrogen, hydrogen, etc.

**(ii)** **Paramagnetic substances :** Some substance when placed in a magnetic field, are feebly magnetised in the direction of the magnetising field. These substance, when brought close to a pole of a powerful magnet, are attracted towards the magnet. These are called 'paramagnetic' substance and their magnetism is called 'paramagnetism'.

**(iii)** **Ferromagnetic substances :** Some substance, when placed in a magnetic field, are strongly magnetised in the direction of the magnetising field. They are attracted fast towards a magnet when brought close to either of the poles of the magnet. These are called 'ferromagnetic' substances and their magnetism is called 'ferromagnetism'.

## (B) Some important terms used in magnetism :

**MAGNETISATION AND MAGNETIC INTENSITY**

The earth abounds with a bewildering variety of elements and compounds.In addition, we have been synthesising new alloys, compounds and evenelements. One would like to classify the magnetic properties of these substances. In the present section, we define and explain certain terms which will help us to carry out this exercise.

We have seen that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and theycan give a net magnetic moment which is non-zero.

## Magnetisation

Magnetisation M of a sample to be equal to its net magnetic moment per unit volume:

$$M = \frac{m_{net}}{V}$$

M is a vector with dimensions $L^{-1}$ A and is measured in a units of A $m^{-1}$.Consider a long solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid was shown to be given by $B_0 = \mu_0 nI$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than $B_0$. The net B field in the interior of the solenoid may be expressed as

$$B = B_0 + B_m$$

where $B_m$ is the field contributed by the material core. It turns out thatthis additional field $B_m$ is proportional to the magnetisation M of the material and is expressed as $B_m = \mu_0 M$

where $\mu_0$ is the same constant (permeability of vacuum) that appears in Biot-Savart's law.

### Magnetic Intensity

It is convenient to introduce another vector field H, called the magnetic intensity, which is defined by

$$\Rightarrow \quad H = \frac{B}{\mu} \quad \Rightarrow \quad H = \frac{B_0 + B_m}{\mu}$$

where H has the same dimensions as M and is measured in units of A $m^{-1}$. Thus, the total magnetic field B is written as : $B = B_0 + B_m$

We repeat our defining procedure. We have partitioned the contributionto the total magnetic field inside the sample into two parts: one, due toexternal factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material,namely M.

$$\Rightarrow \quad H = \frac{\mu_0 H + \mu_0 M}{\mu} \quad \Rightarrow \quad H = \frac{H}{\mu_r} + \frac{M}{\mu_r} \quad \Rightarrow \quad (\mu_r - 1)H = M \quad \Rightarrow \quad M = \chi H$$

The latter quantity can be influenced by external factors. This influence is mathematically expressed as

$$M = \chi H$$

where $\chi$ , a dimensionless quantity, is appropriately called the **magnetic susceptibility**. It is a measure of how a magnetic material responds to an external field. Table lists $\chi$ for some elements. It is small and positive for materials, which are called paramagnetic. It is small and negative for materials, which are termed diamagnetic.

$$\mu_r - 1 = \chi,$$
$$\mu_r = 1 + \chi$$

is a dimensionless quantity called the relative magnetic permeability of the substance. It is the analog of the dielectric constant in electrostatics. The magnetic permeability of the substance is μ and it has the same dimensions and units as $\mu_0$ ; $\mu = \mu_0\mu_r = \mu_0 (1 + \chi)$.

$$\mu_r = (1 + \chi).$$

The three quantities $\chi$, $\mu_r$ and μ are interrelated and only one of them is independent. Given one, the other two may be easily determined.

**Table :1 Magnetic Susceptibility of Some Element At 300K**

| Diamagnetic substance | $\chi$ | Paramagnetic substance | $\chi$ |
|---|---|---|---|
| Bismuth | $-1.66 \times 10^{-5}$ | Aluminium | $2.3 \times 10^{-5}$ |
| Copper | $-9.8 \times 10^{-6}$ | Calcium | $1.9 \times 10^{-5}$ |
| Diamond | $-2.2 \times 10^{-5}$ | Chromium | $2.7 \times 10^{-4}$ |
| Gold | $-3.6 \times 10^{-5}$ | Lithium | $2.1 \times 10^{-5}$ |
| Lead | $-1.7 \times 10^{-5}$ | Magnesium | $1.2 \times 10^{-5}$ |
| Mercury | $-2.9 \times 10^{-5}$ | Niobium | $2.6 \times 10^{-5}$ |
| Nitrogen (STP) | $-5.0 \times 10^{-9}$ | Oxygen (STP) | $2.1 \times 10^{-6}$ |
| Silver | $-2.6 \times 10^{-5}$ | Platinum | $2.9 \times 10^{-4}$ |
| Silicon | $-4.2 \times 10^{-6}$ | Tungsten | $6.8 \times 10^{-5}$ |

## Solved Examples

**Example 32.** A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A. If the number of turns is 1000 per metre, calculate

(a) H (b) M (c) B (d) the magnetising current $I_m$.

**Solution :** (a) The field H is dependent of the material of the core, and is

$H = nI = 1000 \times 2.0 = 2 \times 10^3$ A/m.

(b) The magnetic field B is given by

$B = \mu_r \mu_0 H$

$= 400 \times 4\pi \times 10^{-7}$ (N/A$^2$) $\times 2 \times 10^3$ (A/m) = 1.0 T

(c) Magnetisation is given by

$M = (B - \mu_0 H)/\mu_0$

$= (\mu_r \mu_0 H - \mu_0 H) / \mu_0 = (\mu_r - 1)H = 399 \times H$

$\approx 8 \times 10^5$ A/m

(d) The magnetising current $I_M$ is the additional current that needs to be passed through the windings of the solenoid in the absence of the core which would give a B value as in the presence of the core. Thus

$B = \mu_r n_0 (I + I_M)$. Using I = 2A, B = 1 T, we get $I_M$ = 794 A.

**MAGNETIC PROPERTIES OF MATERIALS**

The discussion in the previous section helps us to classify materials as diamagnetic, paramagnetic or ferromagnetic. In terms of the susceptibility $\chi$ , a material is diamagnetic if $\chi$ is negative, para- if $\chi$ is positive and small, and ferro- if $\chi$ is large and positive.

A glance at Table 5.3 gives one a better feeling for these materials. Here $\varepsilon$ is a small positive number introduced to quantify paramagnetic materials. Next, we describe these materials in some detail.

**Table : 2**

| Diamagnetic | Paramagnetic | Ferromagnetic |
|---|---|---|
| $-1 \le \chi < 0$ | $0 < \chi < \varepsilon$ | $\chi >> 1$ |
| $0 \le \mu_r < 1$ | $1 < \mu_r < 1 + \varepsilon$ | $\mu_r >> 1$ |
| $\mu < \mu_0$ | $\mu > \mu_0$ | $\mu >> \mu_0$ |

## Diamagnetism :

Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words,unlike the way a magnet attracts metals like iron, it would repel adiamagnetic substance.

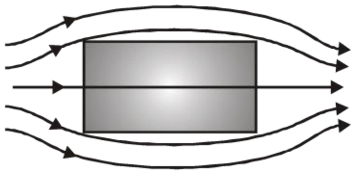


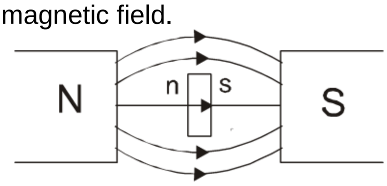
Figure shows a bar of diamagnetic material placed in an externalmagnetic field. The field lines are repelled or expelled and the field insidethe material is reduced. In most cases, as is evident from Table

1, this reduction is slight, being one part in $10^5$. When placed in a non-uniform magnetic field, the bar will tend to move from high to low field.The simplest explanation for diamagnetism is as follows. Electrons in an atom orbiting around nucleus possess orbital angular momentum.These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in thesame direction slow down and those in the opposite direction speed up.This happens due to induced current in accordance with Lenz's law whichyou will study in Electro magnetic Induction. Thus, the substance develops a net magneticmoment in direction opposite to that of the applied field and hencerepulsion.Some diamagnetic materials are bismuth, copper, lead, silicon,nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc.The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = -1$ and $\mu_r = 0$. A superconductor repels a magnet and (byNewton's third law) is repelled by the magnet. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.

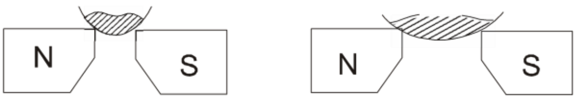
## Properties of diamagnetic substance:

Diamagnetic substance show following properties.

(i) When a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field.

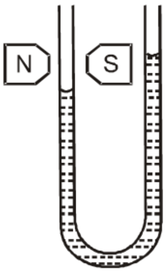



(ii) In a non-uniform magnetic field a diamagnetic substance tends to move from the stronger to the weaker part of the field.




**(a)** **(b)**

(iii) If a diamagnetic solution is poured into a U-tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm is depressed.

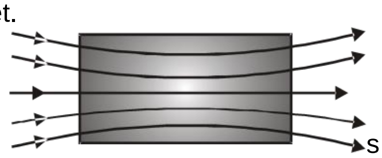



(iv) A diamagnetic gas when allowed to ascend in between the poles of a magnet spreads across the field.

(v) The susceptibility of a diamagnetic substance is independent of temperature.

**Paramagnetism**

Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.




The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random thermal motion of the atoms, no net magnetisation is seen. In the presence of an external field $B_0$, which is strong enough,and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as $B_0$. Figure shows a bar of paramagnetic material placed in an external field. The field lines gets concentrated inside the material, and the field inside is enhanced. In most cases, as is evident from Table

1, this enhancement is slight, being one part in $10^5$. When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.Some paramagnetic materials are aluminium, sodium, calcium,oxygen (at STP) and copper chloride. Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$M = C\,\frac{B_0}{T}$$

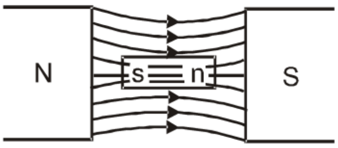
or equivalently, using Eqs $M = \chi H$ and $B_0 = \mu_0 H$

$$\chi = \frac{\mu_0}{T}$$

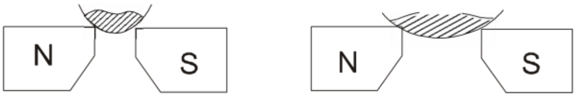
This is known as Curie's law, after its discoverer Pieree Curie (1859-1906). The constant C is called Curie's constant. Thus, for a paramagneticmaterial both $\chi$ and $\mu_r$ depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value $M_s$, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie's law is no longer valid.

## Properties of Paramagnetic Substance

(i) When a rod of paramagnetic material is suspended freely between two magnetic poles, then its axis becomes parallel to the magnetic field The poles produced at the ends of the rod are opposite to the nearer magnetic poles.

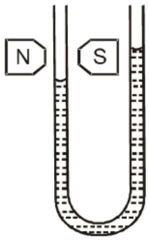



(ii) In a non-uniform magnetic field, the paramagnetic substances tend to move from weaker to stronger part of the magnetic field.




**(a)** **(b)**

(iii) If a paramagnetic solution is poured in a U-tube and one arm of the U-tube is placed between two strong poles, the level of the solution in that arm rises.



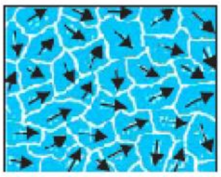

(iv) A paramagnetic gas when allowed to ascend between the pole-pieces of a magnet, spreads along the field.

(v) The susceptibility of a paramagnetic substance varies inversely as the kelvin temperature of the substance, that is,

$\chi_m \propto \frac{1}{T}$. This known as curie's law.

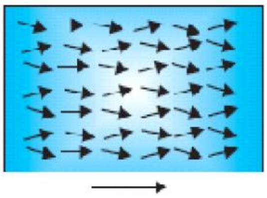
## Ferromagnetism

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. The explanation of this cooperative effect requires quantum mechanics and is beyond the scope of this textbook. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about $10^{11}$ atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Figure.



**Randomlyoriented domains,**

When we apply an external magnetic field $B_0$, the domains orient themselves in the direction of $B_0$ and simultaneously the domain oriented in the direction of $B_0$ grow in size. This existence of domains and their motion in $B_0$ are not speculations. One may observe this under a microscope after sprinkling a liquid suspension of powdered ferromagnetic substance of samples. This motion of suspension can be observed. Figure (b) shows the situation when the domains have aligned and amalgamated to form a single 'giant' domain.



**Aligned domains.**

Thus, in a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets. Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough,such materials are called soft ferromagnetic materials. There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability is >1000!

The ferromagnetic property depends on temperature. At high enoughtemperature, a ferromagnet becomes a paramagnet. The domain structuredisintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature $T_c$. Table lists the Curie temperature of certain ferromagnets. The susceptibilityabove the Curie temperature, i.e., in the paramagnetic phase isdescribed by, $\chi = \frac{C}{T - T_c}$ $(T > T_c)$

**Ferromagnetic substances :** These substances which are strongly attracted by a magnet, show all the properties of a paramagnetic substance to a much higher degree. For example, they are strongly magnetised in relatively weak magnetising field in the same direction as the field. They have relative permeabilities of the order of hundreds and thousands. Similarly, the susceptibilities of ferromagnetic have large positive values.

**Curie temperature :** Ferromagnetism decreases with rise in temperature. If we heat a ferromagnetic substance, then at a definite temperature the ferromagnetic property of the substance "suddenly" disappears and the substance becomes paramagnetic. The temperature above which a ferromagnetic substance becomes paramagnetic is called the' Curie temperature' of the substance. The curie temperature of iron is 770ºC and that of nickel is 358ºC.

## Solved Examples

**Example 33.** A domain in ferromagnetic iron is in the form of a cubeof side length 1µm. Estimate the number of iron atoms in the domainand the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its densityis 7.9 g/cm$^3$. Assume that each iron atom has a dipole momentof $9.27\times10^{-24}$ A m$^2$.

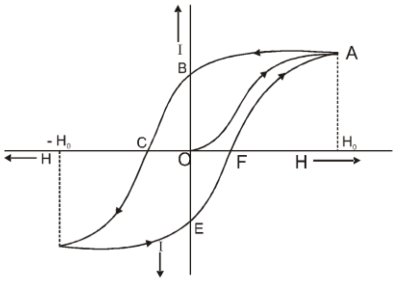
**Solution :** The volume of the cubic domain is$V = (10^{-6}\text{ m})^3 = 10^{-18}\text{ m}^3 = 10^{-12}\text{ cm}^3$ . Its mass is volume × density $= 7.9\text{ g cm}^{-3} \times 10^{-12}\text{ cm}^3 = 7.9 \times 10^{-12}$ gIt is given that Avagadro number $(6.023 \times 10^{23})$ of iron atoms have a mass of 55 g. Hence, the number of atoms in the domain is

$$N = \frac{7.9 \times 10^{-12} \times 6.023 \times 10^{23}}{55} = 8.65 \times 10^{10} \text{ atoms}$$

The maximum possible dipole moment $m_{max}$ is achieved for the(unrealistic) case when all the atomic moments are perfectly aligned.Thus,$m_{max} = (8.65 \times 10^{10})\times(9.27 \times 10^{-24}) = 8.0\times10^{-13}$A m$^2$

The consequent magnetisation is $M_{max} = m_{max}$/Domain volume = $8.0 \times 10^{-13}$ $Am^2/10^{-18}$ $m^3$ = $8.0 \times 10^5$ $Am^{-1}$

## Hysterisis




Consider that a specimen of ferromagnetic material is placed in a magnetising field, whose strength and direction can be changed. Suppose that the specimen is unmagnetised initially. When the magnetising field (H) is increased, the intensity of magnetisation (I) of the material of the specimen also increases. It is found that when the magnetising field is made zero, the intensity of magnetisation does not become zero but still has some finite value. It becomes zero only, when magnetising field is increased inreversed direction. In other words, intensity of magnetisation does not become zero on making magnetising field zero but does so a little late and this effect is called hysteresis.

**The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation and demagnetisation of a ferromagnetic material is called hysteresis.**

Fig shows the magnetisation curve of a ferromagnetic material, when it is taken over a complete cycle of magnetisation (I) is also zero. As magnetising field is increased, intensity of a mgnetisation also increases along.OA. Corresponding to point A, the intensity of magnetisation becomes maximum. The increase in value of the magnetising field beyond $H_0$ does not produce any increase in the intensity of magnetisation. In other words ,corresponding to point A, The specimen of the ferromagnetic material acquires a state of magnetic saturation. If magnetising field is now decreased slowly, intensity of magnetisation decreases but not a along the path AO.It decrease along the path AB. Corresponding to point B, magnetising field becomes zero but some magnetisation equal to OB is still left in the specimen. Here, OB gives the measure of retentivity of the material of the specimen.

**The value of the intensity of magnetisation of a material, when the magnetising field is reduced to zero, is called retentivity of the material. It is also known residual magnetism or remanetism or remanence.**

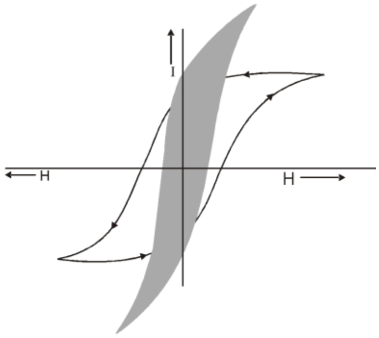
To reduce intensity of magnetisation to zero, the magnetising field has to be increased in reverse direction. As it is done so, the intensity of magnetisation decreases along BC, till it become zero corresponding to point C. Thus, to make intensity of magnetisation zero, magnetising field equal OC has to be applied in reverse direction. Here, OC gives the measure of coercivity of the material of the specimen .

**The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.**

When the magnetising field is further increased in reverse direction, intensity of magnetisation increase along CD with the increase in magnetising field. Corresponding to point D (when the magnetising field becomes-$H_0$) it again acquires a saturation value, which is symmetrical to that corresponding to point A. If the magnetising field is decreased from–$H_0$ to zero, the intensity of magnetisation follows the path DE. Finally, when magnetising field is increased field in original direction, the point A is reached via EFA. If the magnetising field is repeatedly changed between $H_0$ and – $H_0$ , the curve ABCDEFA is retraced. The curve ABCDEFA is called the hysteresis loop. It is found that the area of the hysteresis loop (I – H curve)is proportional to the net energy absorbed per unit volume by the specimen, as it taken over a complete cycle of magnetisation and demagneti-sation. The energy so absorbed by the speciment appears as the heat energy.

**Note :-** If hysteresis loop is drawn by plotting a graph between magnetic induction (B) and intensity of magnetisation, then area of the hysteresis loop is numerically equal to the work done per unit volume (or energy absorbed per unit volume) in taking the magnetic specimen over a complete cycle of magnetisation.

**COMPARISON OF HYSTERESIS LOOPS FOR SOFT IRON AND STEEL :-**




The shape of the hysteresis loop is a characteristics of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

Figure Shown hysteresis loops for soft iron and steel. Whereas the hysteresis loop for soft iron is narrow, the hysteresis loop for steel is quite wide. The following conclusion can be drawn from the study of the hysteresis loops of soft iron and steel :

1. The area of hysteresis loop for soft iron is much smaller then that for steel. Therefore, loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.

2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetisation) than steel.

3.The retentivity of soft iron is greater then that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.

4. The coercivity of steel is much larger then that of soft iron. Therefore, the residula magnetism in steel can not be destroyed that easily as in case of soft iron.

## (F) Section of magnetic materials :

The choice of a magnetic material for making permanent magnet, electromagnet, core of transformer or diaphragm of telephone ear-piece can be decided from the hysteresis curve of the material.

**(i) Permanent magnets :**

The material for a permanent magnet should have high retentivity so that the magnet is strong, and high coercivity so that the magnetisation is not wiped out by stray external fields, mechanical ill treatement and temperature changes. The hysteresis loss is immaterial because the material in this case is never put to cyclic changes of magnetisation. From these considerations permanent magnets are made of steel. The fact that the retentivity of soft iron is a little greater than that of steel is outweighed by its much smaller coercivity, which makes it very easy to demagnetise.

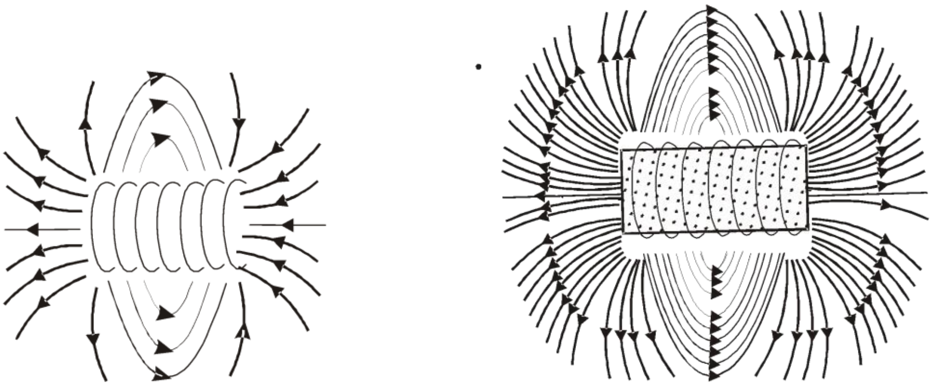
**(ii) Electromagnets :** The material for the cores of electromagnets should have high permeability (or high susceptibility), specially at low magnetising fields, and a low retentivity. Soft iron is suitable material for electromagnets).

**(iii) Transformer cores and telephone diaphragms :** In these cases the material goes through complete cycles of magnetisation continuously. The material must therefore have a low hysteresis loss to have less dissipation of energy and hence a small heating of the material (otherwise the insulation of windings may break), a high permeability (to obtain a large flux density at low field) and a high specific resistance (to reduce eddy current loses).

Soft -iron is used for making transformer cores and telephone diaphragms : More effective alloys have now been developed for transformer cores. They are permalloys, mumetals etc.

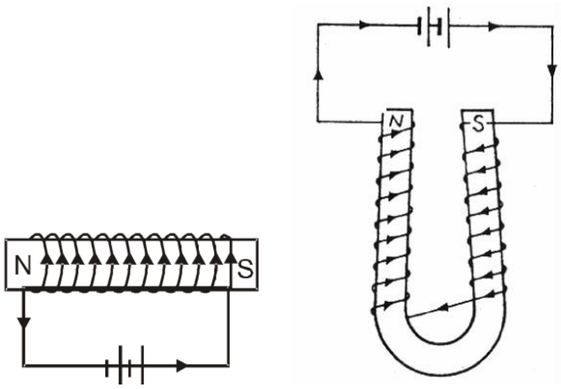
## 7. ELECTROMAGNET :

If we place a soft-iron rod in the solenoid, the magnetism of solenoid increases hundreds of time. Then the solenoid is called an ‘electromagnet’. It is a temporary magnet.



(a) (b)

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft-iron straight rod or a horse-shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.




(a) (b)

## Solved Examples

**Example 34** A magnetising field of 1600 $Am^{-1}$ produces a magnetic flux of $2.4 \times 10^{-5}$ wb in an iron bar of crosssectional area $0.2 cm^2$. Calculate permeability and susceptibility of the bar.

**Solution :** $B = \frac{\Phi}{A} = \frac{2.4 \times 10^{-5}\ Wb}{0.2 \times 10^{-4}\ m^2} = 1.2\ Wb/m^2 = 1.2\ N\ A^{-1}\ m^{-1}$.

The magnetising field (or magnetic intensity) H is 1600 $Am^{-1}$. Therefore, the magnetic permeability is given by

$$\mu = \frac{B}{H} = \frac{1.2 NA^{-1} m^{-1}}{1600\ Am^{-1}} = 7.5 \times 10^{-4}\ N/A^2.$$

Now, from the relation $\mu = \mu_0 (1 + \chi_m)$, the susceptibility is given by $\chi_m = \frac{\mu}{\mu_0} - 1$.

We known that $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$

$$\therefore \quad \chi_m = \frac{7.5 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1 = 596$$

**Example 35** The core of toroid of 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. A current of 0.6 A produces a magnetic field of 2.5 T in the core. Compute relative permeability of the core.($\mu_0 = 4\pi \times 10^{-7}$ T m $A^{-1}$).

**Solution :** The magnetic field in the empty space enclosed by the windings of a toroid carrying a current $i_0$ is $\mu_0\, n\, i_0$

where n is the number of turns per unit length of the toroid and $\mu_0$ is permeability of free space.

If the space is filled by a core of some material of permeability μ, then the field is given by

$B = \mu\, n\, i_0$

But $\mu = \mu_0 u_r$, where $\mu_r$ is the relative permeability of the core material. Thus,

$B = \mu_0\, u_r\, n i_0$

or $\mu_r = \frac{B}{\mu_0 n\, i_0}$

Here B = 2.5 T, $i_0$ = 0.7 A and $n = \frac{3000}{2\pi r}\ m^{-1}$, where r is the mean radius of the toroid

$\left(r = \frac{11+12}{2}\right.$ = 11.5 cm 11.5 × $10^{-2}$ m). Thus,

$\mu_r = \frac{2.5}{(4\pi\times10^{-7})\times(3000/2\pi\times11.5\times10^{-2})\times0.7} = \frac{2.5\times11.5\times10^{-2}}{2\times10^{-7}\times3000\times0.7}$

$\mu_r = 684.5$

## Solved Miscellaneous Problems

**Problem 1.** A charged particle of charge 2C thrown vertically upwards with velocity 10 m/s. Find the magnetic force on this charge due to earth's magnetic field. Given vertical component of the earth = 3$\mu$T and angle of dip = 37º.

**Answer :** 2 × 10 × 4 × $10^{-6}$ = 8 × $10^{-5}$ N towards west.

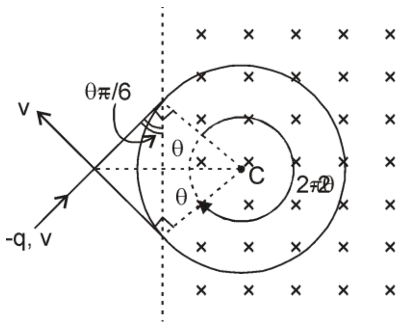
**Solution :** $\tan 37^o = \frac{B_V}{B_H} \quad \Rightarrow \quad B_H = \frac{4}{3} \times 3 \times 10^{-6}$ T

$F = q\, v\, B_H = 8 \times 10^{-5}$ N

**Problem 2.** Repeat above question if the charge is –ve and the angle made by the boundary with the velocity is $\frac{\pi}{6}$.

**Solution :** (i) $2\pi - 2\theta = 2\pi - 2.\frac{\pi}{6} = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$= \omega t = \frac{qBt}{m} \quad \Rightarrow \quad t = \frac{5\pi m}{3qB}$




(ii) Distance travelled $s = r(2\pi - 2\theta) = \frac{5\pi r}{3}$

(iii) Impulse = charge in linear momentum

$= m(-v \sin\theta\, \hat{i} + v\cos\theta\, \hat{j}) - m(v\sin\theta\, \hat{i}s + v\cos\theta\, \hat{j})$

$= -2mv\sin\theta\, \hat{i} = -2mv \sin\frac{\pi}{6}\, \hat{i} = -mv\,\hat{i}$