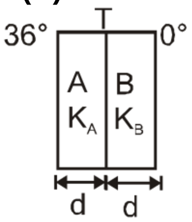
TOPIC : HEAT TRANSFER
EXERCISE # 1

SECTION (A)

1. 

$K_A = 2K_B = 2$ K

Heat current through both layers will be same

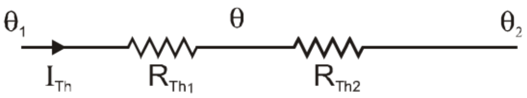
$\left(\frac{36-T}{d}\right) K_A A = \left(\frac{T-O}{d}\right) K_B A$

$(36 - T)\, 2\, K = \frac{72}{3}\, T\, K$

T = = 24

ΔT = temp diff = 36 – 24 = 12

2. $\frac{dQ}{dt} = KA\frac{\Delta T}{\ell}$ ⇒ $4000 = \frac{400\times100\times10^{-4}\times\Delta T}{0.1}$ ⇒ ΔT = 100°C

3. 

$\theta = \theta_1 - I_{th}R_{th1} = \theta_1 - \left(\frac{\theta_2-\theta_1}{R_{Th1}-R_{Th2}}\times R_{Th1}\right) = \theta_1 - \left(\frac{\theta_2-\theta_1}{\frac{K_1A}{d_1}+\frac{K_2A}{d_2}}\times\frac{K_1A}{d_1}\right)$

$= \theta_1 - \left(\frac{\theta_2-\theta_1}{\frac{K_1}{d_1}+\frac{K_2}{d_2}}\times\frac{K_1}{d_1}\right) = \frac{K_1\theta_1 d_2 + K_2\theta_2 d_1}{K_1 d_2 + K_2 d_1}$

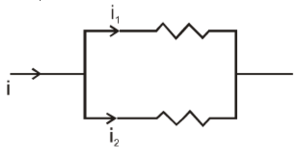
4. Utensil should have low thermal resistance $\left(R = \frac{\ell}{KA}\right)$
and low specific heat so that heat loss is less

5. $\frac{R_1}{R_2} = \frac{\frac{\ell_1}{K_1A_1}}{\frac{\ell_2}{K_2A_2}} = \frac{\frac{\ell}{K\pi(2r)^2}}{\frac{2\ell}{K\pi(3r)^2}} = \frac{9}{8}$

∵ $I = \frac{\Delta T}{R}$ ⇒ $I \propto \frac{1}{R}$

So, $\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{8}{9}$

6.



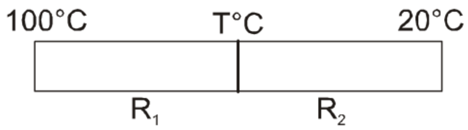
$i_1 = \frac{(100-20)}{3\times10^{-2}}$ (209) 9×10^{-2}

$i_2 = \frac{100-20}{3\times10^{-2}}$ (385) 9×10^{-2}

$i_T = i_1 + i_2$

$\frac{i_{Cu}}{i_{Al}} = \frac{i_2}{i_1} = \frac{385}{209}$

7.

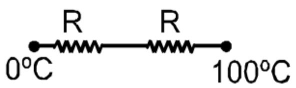


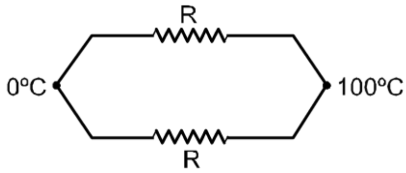
$T = 100 - IR_1 = 100 - \left(\frac{100-20}{R_1+R_2}\right)\times R_1 = 100 - \left(\frac{80}{\frac{\ell}{5KA}+\frac{\ell}{3KA}}\right)\times\frac{\ell}{5KA} = 70°C$

8. $I_{th} = \frac{\frac{\Delta T}{R} = \frac{100-0}{\ell}}{KA} = 100 \times 100 \times 100 \times 10^{-4} = 100$ W/sec $= 60 \times 100$ W/min $= 6 \times 10^3$ W/min.

9. K depends on material of metal only

10.





$\frac{Q_1}{t_1} = i_{H_1} = \frac{100-0}{2R} = \frac{50}{R}$

$i_{H2} = \frac{100}{R/2} = \frac{200}{R} = \frac{Q_2}{t_2}$

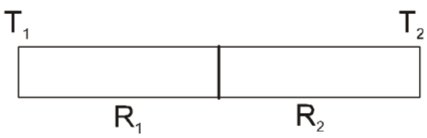
$Q_1 = Q_2 = 10$ cal.

$\frac{50}{R}\times(2) = \frac{200}{R}\times t_2$

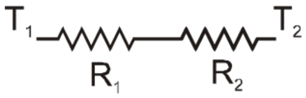
$t_2 = \frac{1}{2}$ min.

11. Thermal resistance should be equal, so $\frac{\ell}{K_1A_1} = \frac{\ell}{K_2A_2}$ ⇒ $K_1A_1 = K_2A_2$

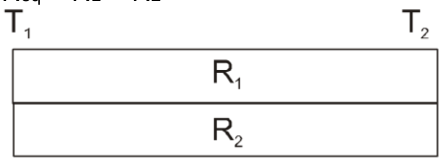
12.



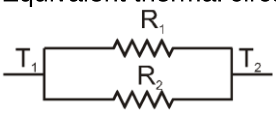
Equivalent thermal circuit



$R_{eq} = R_1 + R_2 = \dfrac{2\ell}{KA} = \dfrac{\ell}{K_1 A} + \dfrac{\ell}{K_2 A} \Rightarrow K = \dfrac{2K_1K_2}{K_1 + K_2}$

13. 

Equivalent thermal circuit



$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \Rightarrow \dfrac{K_{eq} \times 2A}{\ell} = \dfrac{KA}{\ell} + \dfrac{2KA}{\ell} \Rightarrow K_{eq} = \dfrac{3}{2}K$

14. For a rod of length L and area of cross-section A whose faces are maintained at temperatrure T_1 and T_2 respectively.

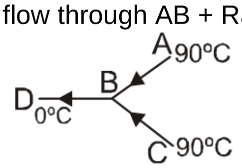
Then in steady state the rate of heat flowing from one face to the other face in time t is given by

The curved surface of rod is kept insulated from surrounding to avoid leakge of heat

$$\frac{dQ}{dt} = \frac{KA(T_1 - T_2)}{L}$$

15. Square is made of four rod of same material, so the temperature difference of second diagonal will also be100°C.

16. Let q be the temperature of the junction (say B). Thermal resistance of all the three rods is equal. Rate of heat flow through AB + Rate of heat flow through CB = Rate of heat flow through BD



$\therefore \quad \dfrac{90-\theta}{R} + \dfrac{90-\theta}{R} = \dfrac{\theta - 0}{R}$

Here R = Thermal Resistance

$\therefore \quad 3\theta = 180$ or $\theta = 60ºC$

18. Net heat given/sec = 1000 – 160

= 840 J/S

if it takes a time t then

840 t = 2000 × 4.2 × (77 – 27)

t = 500 sec = 8 min 20 sec.

19. Thermal conductivity depends on the material, not on temperature difference.

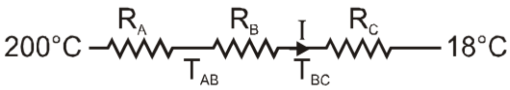
20. $Q_1 = \dfrac{\Delta T}{\left(\dfrac{\ell}{k\pi r^2}\right)}$, $Q_2 = \dfrac{\Delta T}{\left(\dfrac{2\ell}{k\pi (2r)^2}\right)} \Rightarrow \dfrac{Q_2}{Q_1} = 2$

21. Let temperature of the interface = T

$$\frac{T_1 - T}{\left(\frac{L_1}{AK_1}\right)} = \frac{T - T_2}{\left(\frac{L_2}{K_2 A}\right)}$$

$$\Rightarrow \quad T\left(\frac{L_1}{K_1} + \frac{L_2}{K_2}\right) = \frac{T_1 \; L_2}{K_2} + \frac{T_2 L_1}{K_1} \quad \Rightarrow \quad T = \frac{T_1 \; K_1 \; L_2 + T_2 \; L_1 \; K_2}{L_1 \; K_2 + L_2 \; K_1}$$

22.



$$T_{AB} = 200 - IR_A = 200 - \frac{(200-18)}{R_A + R_B + R_C} \times R_A = 200 - \frac{182}{\frac{\ell}{KA} + \frac{\ell}{2KA} + \frac{2\ell}{3KA}} \times \frac{\ell}{KA} = 116°C$$

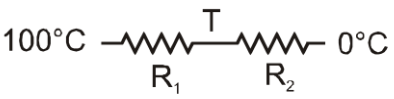
23. $$T_{BC} = 200 - IR_A - IR_B = 200 - \frac{182}{\frac{\ell}{KA} + \frac{\ell}{2KA} + \frac{2\ell}{3KA}} \times \left(\frac{\ell}{KA} + \frac{\ell}{2KA}\right) = 74°C$$

24. $$\frac{I_1}{I_2} = \frac{R_2}{R_1} = \frac{\frac{2\ell}{2K \times 2A}}{\frac{\ell}{KA}} = 2 \quad \Rightarrow I_2 = \frac{I_1}{2} = \frac{4}{2} = 2$$

25. $$\frac{dQ}{dt} = KA\frac{dT}{dx} \quad \Rightarrow \quad \frac{1}{A}\frac{dQ}{dt} = K\frac{dT}{dx}$$

$\frac{1}{A}\frac{dQ}{dt}$ is same so $\frac{dT}{dx}$ is smaller for higher K.

26.



$$T = 100 - IR_1 = 100 - \frac{\Delta T}{(R_1 + R_2)} \times R_1 = 100 - \frac{100}{\frac{\ell_1}{K_1 A} + \frac{\ell_2}{K_2 A}} \times \frac{\ell_1}{K_1 A}$$

After substituting the values T = 60°C

27. I ∝ for point source.

$$\frac{I'}{I} = \left(\frac{d}{d'}\right)^2 = \left(\frac{d}{2d}\right)^2 = \frac{1}{4} \Rightarrow I' = \frac{I}{4}$$

28. Copper has higher thermal conductivity, so lower resistance.

29. In 1st case

$$I_1 = \frac{\Delta T}{R_1 + R_2} = \frac{20}{4} \text{ cal/min}$$

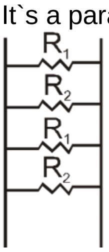
$\Rightarrow \dfrac{\Delta T}{2R}$ = 5 cal/min. (since $R_1 = R_2 = R$)

$\Rightarrow \dfrac{\Delta T}{R}$ = 10 cal/min.

In second case

$I_2 = \dfrac{\Delta T}{\frac{R_1R_2}{R_1+R_2}} \Rightarrow I_2 = \dfrac{\Delta T}{\frac{R}{2}} \Rightarrow \dfrac{20}{t} = I_2 = 2 \times 10 \Rightarrow t = 1$ min.

30. It`s a parallel Combination



$R_1 = \dfrac{d}{K_1A}$ $R_2 = \dfrac{d}{K_2A}$

$\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} +$ ....upto n_{th}

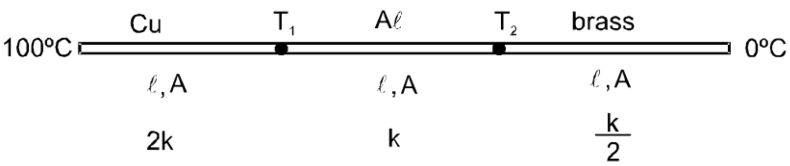
$\dfrac{1}{R_{eq}} = \dfrac{n}{2R_1} + \dfrac{n}{2R_2} = \dfrac{n}{2}\left(\dfrac{R_1+R_2}{R_1R_2}\right)$

$R_{eq} = \dfrac{2(R_1R_2)}{n(R_1+R_2)} \Rightarrow \dfrac{(d/n)}{K_{eq}(A)} = \dfrac{2\left(\frac{d}{K_1A}\right)\times\frac{d}{K_2A}}{n\frac{d}{A}\left(\frac{1}{K_1}+\frac{1}{K_2}\right)}$

$\dfrac{1}{K_{eq}} = \dfrac{2}{K_1+K_2} \Rightarrow K_{eq} = \dfrac{K_1+K_2}{2}$

31. For balanced wheatstone bridge $\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4} \Rightarrow \dfrac{K_3}{K_1} = \dfrac{K_4}{K_2} \Rightarrow K_1K_4 = K_2K_3$

32.



$R_{eq} = R_1 + R_2 + R_3$ where $R_1 = \dfrac{\ell}{(2k)A}$, $R_2 = \dfrac{\ell}{kA}$, $R_3 = \dfrac{2\ell}{kA}$

Thermal current through rods

$\dfrac{100-0}{R_{eq}} = \dfrac{100-T_1}{R_1} = \dfrac{100-T_2}{R_1+R_2} = \dfrac{T_2-0}{R_3}$

$\Rightarrow \dfrac{100-0}{\frac{7\ell}{2kA}} = \dfrac{100-T_1}{\frac{\ell}{2kA}} = \dfrac{100-T_2}{\frac{3}{2}\frac{\ell}{kA}} = \dfrac{T_2-0}{2\frac{\ell}{kA}}$

after solving $T_1 = 86°C$, $T_2 = 57°C$

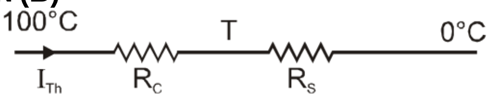
33. Thermal Resistance R = $\int_{r_1}^{r_2} \frac{dr}{k.4\pi r^2} = \frac{1}{4\pi k}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$ = $\frac{(r_2 - r_1)}{4\pi k.\ r_1\ r_2}$

Rate of heat flow = $\frac{T_2 - T_1}{R} = \frac{(T_1 - T_2)}{(r_2 - r_1)}$. $4\pi k\ r_1 r_2$

$\propto \frac{r_1\ r_2}{r_2 - r_1}$

SECTION (B)

1.



T = 0° + $I_{th}R_S$ = $\frac{100-0}{R_c + R_S} \times R_S = \frac{100}{\frac{0.18}{9KA} + \frac{0.06}{KA}} \times \frac{0.06}{KA} = 75°C$

SECTION (C)

2. Natural convection occurs due to gravity.

4. Hot air escape from ventilators while fresh heavy air enters in room through doors and windows

6. $\frac{dQ}{dt} \propto T^4$ temperature is doubled (from 300K to 600 K)

so $\frac{dQ}{dt}$ is increased sixteen times

7. By stefan's law $\frac{dQ}{dt} \propto T^4$

8. Energy emitted by sun per second $\propto T^4$.

9. for point source Intensity = $\frac{P}{4\pi r^2}$
since r is doubled, so Intensity is reduced to one fourth of initial value

10. $\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$ $\Rightarrow \lambda_2 = \frac{T_1}{T_2} \times \lambda_1 = \frac{200}{100} \times 14$ = 2.8 μm

11. $\left(\frac{E_2 - E_1}{E_1}\right) \times 100 = \left(\frac{E_2}{E_1} - 1\right) \times 100 = \left[\left(\frac{T_2^4}{T_1^4}\right) - 1\right] \times 100$

$= \left[\left(\frac{3}{2}\right)^4 - 1\right] \times 100 = \left(\frac{81-16}{16}\right) \times 100 = \frac{6500}{16} \approx 400\%$

14. Good absorber is good emitter. Black spot aborbs more when heated and emitts more when kept in dark room.

15. T λ = Constant $\nu_m = \frac{C}{\lambda_{max}}$

$\frac{T}{\nu_{max}}$ = Constant

$\frac{T_1}{\nu_1} = \frac{T_2}{\nu_2}$ $\nu_2 = \frac{T_2}{T_1} . \nu_1 = \frac{2\ T}{T} . \nu_1 = 2\nu_1$

$E = \sigma e A T^4$

$E \alpha T^4$ $\frac{E_2}{E_1} = (2)^4 = 16$

18. **Key Idea:** Amount of heat energy radiated per second by unit area of a black body is directly proportional to fourth power of absolute temperature.

According to Stefan's law,

$E \propto T^4$

or $E = \sigma T^4$

where σ is constant of proportionality and called Stefan's constant. Its value is

$5.67 \times 10^{-8}\ Wm^{-2}\ K^{-4}$

Hence $E \propto (727 + 273)^4$

⇒ $E \propto (1000)^4$

Note: If the body at temperature T is surrounded by a body at temperature T_0 then Stefan's law is $E = \sigma (T^4 - T_0^4)$

This statement is called Stefan-Batsman law.

19. Radiated energy is directly proportional to the fourth power of temperature $E \propto T^4$

$$\frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{T}{T + \frac{T}{10}}\right)^4 = \left(\frac{10}{11}\right)^4$$

$$\frac{E_1}{E_2} = \frac{10000}{14641}$$

∴ increase = $\frac{E_2 - E_1}{E_1} \times 100$ 46% (app.)

21. Wein's displacement law for a perfectly black body is -

λm T = constant = Wein's constant b

Here λ_m is the minimum wavelength corresponding to maximum intensity I.

or

From the figure

$(\lambda_m)_1 < (\lambda_m)_3 < (\lambda_m)_2$

Therefore $T_1 > T_3 > T_2$

→ objective questions based on Wein's displacement law are usually asked in IIT-JEE. Question number 34 of section I of JEE-1998 is also based on Wein's displacement law.

22. Bulb heats up by radiation process

23. From Stefan's law, energy emitted by a black body at absolute temperature T is $E = \sigma T^4 \propto T^4$

24. According to Wien's displacement law, $\lambda_m T$ = constant.
When we begin to heat the substance, initially temperature is small, so λ_m will be large. In visible region, wavelength is maximum for red colour. So, initial colour will be red.

25. From Stefan's law
$E \propto T^4$
$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{T_1}{T_2}\right)^4 \quad ...(1)$$
here E_1 = E (initial radiation say),
T_1 = T, T_2 = 2T
put the given values in eq.(1)
$$\therefore \quad \frac{E_1}{E_2} = \left(\frac{T}{2T}\right)^4 \quad \Rightarrow \quad E_2 = 16E$$

26. From Wien's displacement law
$$\lambda_m \propto \frac{1}{T}$$
or $T \propto \frac{1}{\lambda_m}$
$$\therefore \quad \frac{T'}{T} = \frac{\lambda_m}{\lambda'_m} \quad ...(1)$$
Given: T = 1000K, $\lambda_m = 1.4 \times 10^{-6}$ m,
$\lambda'_m = 2.8 \times 10^{-6}$ m
$$\therefore \quad \frac{T'}{1000} = \frac{1.4\times10^{-6}}{2.8\times10^{-6}}$$
$$\Rightarrow \quad T' = \frac{1}{2} \times 1000 = 500 \text{ K}$$

27. Here : Initial temperature
T_1 = 27°C = 300K
Final temperature T_2 = 927°C = 1200 K
According to Stefan's law, the radiant energy is
$E \propto T^4$ here $\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4}$
$$= \left(\frac{300}{1200}\right)^4 = \left(\frac{1}{4}\right)^4 = \frac{1}{256}$$
Hence, $E_1 : E_2$ = 1 : 256

28. We know that heat lost = mcθ
For a given quantity of heat, we must need a minimum mass of water for cooling the radiators due to a high value of specific heat .

29. According to weins displacement law, $\lambda_m T$ = constant or temperature $\propto \frac{1}{\lambda_m}$. it represents that greater the temperature T of an emitted star, smaller the value of wave length λ. We also know the wave length of ray depnds upon its colour. Hence, when the temperature of star increases, the wave length of star decreases. When temperature of star decreases, thewave length increases and star moves towards the red colour. Therefore, the colour of star indicates its temperature.

30. The only means of energy transfer in vacuum is radiation because, due to absence of matter convection and conduction are not possible.

31. According to Stefan's law the radiated energy

$E \propto T^4$

Hence $\frac{E_1}{E_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T}{2T}\right)^4 = \frac{1}{16}$ $\therefore$ $E_2 = 16E_1$

32. From Wien's displacement law the relation between maximum wavelength λ_m and temperature T is

$\lambda_m T$ = constant

Which implies that longeris the wavelength smaller the temperature is. Since, red colour has maximum wavelength, so its temperature will be minimum and hence, it will cool at the earliest

33. From Kirchhoff's law at a definite temperature and for a given wavelength, the ratio of the emissive power to the absorptive power for different surfaces is same.

i.e., $\frac{e_\lambda}{a_\lambda} = E_\lambda$

when red glass is heated in a dark room to a red hot state it will appear green, because according to Kirchhoff's law the emissive power of red glas will be maximum for green light.

34. Through radiation mainly

35. (a) Conduction is the process of transmission of heat in a body from the hotter part to the colder part without any bodily movement of constituent atoms or molecules of the body.
(b) In convection, the heated lighter particles move upward and colder heavier particles move downward to their place. This depends on weight and hence, on gravity.
(c) Radiation is the process of trasmission of heat from one body to another body through electromagnetic wave eventhrough vacuum, irrespective of their temperatures.
Hence, choice (2) is correct

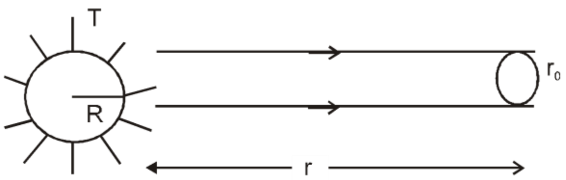
36. $60 = K(1000^4 - 500^4)$ ...(i)
$E = K(1500^4 - 500^4)$ ...(ii)

from (i) and (ii) $\frac{E}{60} = \frac{1500^4 - 500^4}{1000^4 - 500^4}$

$\Rightarrow E = 320$

40. ratio = $\frac{e\ \sigma\ (4\pi\ (2R)^2)(2T)^4}{e\ \sigma\ (4\pi\ R^2)\ T^4}$ = 64

41.



Total radiant power incident of earth = $\left(\frac{\sigma(4\pi R^2)T^4}{4\pi r^2}\right)\pi r_0^2$
(Taking sun as a block body)

42. $E \propto T^4$
$\Rightarrow E = C(1500)^4$ and $E' = C \times (3000)^4$
$\Rightarrow$ = $(2)^4 = 16$ $\Rightarrow$ $E' = 16E$

43. $\lambda_1 = \frac{b}{T_1}$ and $\lambda_2 = \frac{b}{T_2}$
$\Rightarrow \lambda_2 - \lambda_1 = b\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$
$\Rightarrow \Delta\lambda = b\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$
After putting the values $T_2 = 300K$

44. Incident intensity = $\frac{\sigma 4\pi R_S^2 T^4}{4\pi R_e^2}$ = $\left(\frac{R_s}{R_e}\right)^2$ σT^4

45. Black absorvers more, rough surfaces has more area,
So rough black surface absorbs maximum radiant energy.

46. Body at higher temperature appears blue. While body at lower temperature appears red.

47. $\lambda = \frac{b}{T}$ = $\frac{3}{600} \times 10^{-3}$ = 50000 Å

48. We have $\frac{dQ}{dt} = 400$
$\Rightarrow$ $-\frac{dT}{dt} = \frac{1}{ms}\frac{dQ}{dt}$ = $\frac{\frac{400}{60}}{1000 \times 4.2} = 5.7K/\min$

49. $\frac{\lambda_1}{\lambda_2} = \frac{T_2}{T_1}$ $\Rightarrow$ $\lambda_2 = \left(\frac{T_1}{T_2}\right)\lambda_1$

50. $\lambda_m = \frac{b}{T}$ $\Rightarrow$ $\frac{\Delta\lambda_m}{\lambda_m}$ = $\frac{\Delta T}{T}$ $\Rightarrow$ $\frac{\Delta\lambda_m}{\lambda_m} \times 100$ = $\frac{\Delta T}{T} \times 100$ = 10

51. $\frac{E_b}{E_s} = \frac{e\sigma T_b^4}{\sigma T_S^4}$ = $\frac{10}{10^6}$
$\Rightarrow$ $0.1 \times \frac{T_b^4}{T_S^4}$ = 10^{-5} $\Rightarrow$ $\frac{T_b^4}{T_S^4}$ = 10^{-4} $\Rightarrow T_b = T_S \times 10^{-1} = 6000 \times 10^{-1} = 600$ K

53. $\frac{m_1}{m_2}$ $\Rightarrow$ $\frac{\rho \frac{4}{3}\pi R^3}{\rho \frac{4}{3}\pi r^3} = \frac{8}{1}$ R = 2r

$$\frac{E_1}{E_2} = \frac{\sigma 4\pi R^2 (2000)^4}{\sigma 4\pi r^2 (1000)^4} \Rightarrow \frac{\sigma 4\pi (2r)^2 (2000)^4}{\sigma 4\pi r^2 (1000)^4} = \frac{64}{1}$$

54. As temperature increases, wavelength of radiation decreases

55. $E = \sigma AT^4 = 5.67 \times 10^{-8} \times 0.1 \times (100)^4$ J/sec. $= \frac{5.67\times10^{-8}\times0.1\times10^8\times60}{4.2}$ Cal/min. = 8.1 cal/min.

56. Energy emitted by sun per second $\propto T^4$

SECTION (D)

1. For small temperature difference, Stefan's law can be written as

$\Delta u = e\sigma A((T + \Delta T)^4 - T^4)$

or $\Delta u = e\sigma AT^4 \left[\left[1+\frac{\Delta T}{T}\right]^4 - 1\right]$ or $\Delta u = e\sigma AT^4 \times 4 \times \frac{\Delta T}{T}$

or $\Delta u \propto \Delta T$

Hence Newton's law of cooling is a special case of stefan's law.

2. The temperature falls exponentially.

3.

$$\frac{100-70}{4} = K\left(\frac{100+70}{2} - 15\right) \quad ...(i)$$

$$\frac{70-40}{t} = K\left(\frac{70+40}{2} - 15\right) \quad ...(ii)$$

Solving (1) and (2)

t = 7 minute

4.

$$\frac{80-60}{1} = K\left(\frac{80+60}{2} - 30\right) \quad ...(i)$$

$$\frac{60-50}{t} = K\left(\frac{60+50}{2} - 30\right) \quad ...(ii)$$

Solving (i) and (ii)

$t = \frac{4}{5}$ minute = 48 second

5. As temperature of liquid decreases, rate of cooling decreases. So it takes more time for same temperature change.

6. According to Kirchoff, each body emits and absorbs radiation at every temperature.

7. From Newton's cooling law

$$\frac{\theta_1 - \theta_2}{t} = K\left(\frac{\theta_1+\theta_2}{2} - 26\right)$$

Case I: $\frac{12}{10} = K \times 30$

$K = \frac{12}{10\times30} = \frac{1}{25}$

Case II :

$$\frac{50-\theta_2}{10} = K\left(\frac{50+\theta_2}{2} - 26\right)$$

$$\frac{50-\theta_2}{10} = \frac{1}{25}\left(\frac{\theta_2 - 2}{2}\right)$$

$250 - 5\theta_2 = \theta_2 - 2$

$6\theta_2 = 252$

$\theta_2 = \frac{252}{6} = 42°C$

8. Using the relation

Rate of loss of heat $\propto \theta - \theta_D$

where θ is the average temperature in the given time interval. So,

$$\frac{(60° - 50°)}{10} = K\left(\frac{60° + 50°}{2} - 25°\right) \quad ...(i)$$

$$\frac{50-\theta}{10} = K\left(\frac{50+\theta}{2} - 25\right) \quad ...(ii)$$

On solving (i) and (ii) we get θ = 41.67°C

9. The formula for rate of cooling is given by

$$\frac{d\theta}{dt} = \frac{1}{ms}\frac{dQ}{dt} = \frac{1}{ms} \times A \times K$$

As, mass = volume × density

Mass of sphere = $\frac{4}{3}\pi r^3 \times \rho$

where ρ is density

mass per unit area = $\frac{\frac{4}{3}\pi r^3 \times \rho}{4\pi r^2} = \frac{1}{3} r\rho$

Hence, rate of cooling must be proportional to 1/ rρ. (s and k are same)

Hence, ratio of rate of cooling for two spheres is = $\frac{r_2\rho_2}{r_1\rho_1}$

where $r_2 : r_1 = 2 : 1$ and $\rho_2 : \rho_1 = 1 : 2$ = $\frac{2}{1} \times \frac{1}{2}$ = 1 : 1

10. We know that the time of formation of ice in a lake from thickness y_1 to y_2 is directly proportional to

$(y_2^2 - y_1^2) \propto t_1$ ...(1)

Again, $(y_3^2 - y_2^2) \propto t_2$ ...(2)

Given : $y_1 = 0$ cm, $y_2 = 1$cm

$y_3 = 2$ cm, $t_1 = 7$ hour, $t_2 = ?$

From (1) and (2), we get

$$\frac{y_3^2 - y_2^2}{y_2^2 - y_1^2} = \frac{t_2}{t_1} \quad \text{or} \quad \frac{2^2 - 1^2}{1^2 - 0^2} = \frac{t_2}{7} \quad \text{or} \quad \frac{3}{1} = \frac{t_2}{7}$$

Hence, $t_2 = 7 \times 3 = 21$ hour

11. According to Newton's law of cooling rate of cooling is given by

$$\left(\frac{-dT}{dt}\right) = \frac{eA\sigma}{mc}(T^4 - T_0^4)$$

where c is specific heat of material.

or $\left(\frac{-dT}{dt}\right) \propto \frac{1}{c}$

ie, rate of cooling varies inversely as specific heat, from thegraph, for A rate of cooling is larger. Therefore, specific heat of A is smaller.

12. According to Newton's law of cooling,

$$\frac{dQ}{dt} \propto \Delta\theta$$

But $\frac{dQ}{dt} \propto (\Delta\theta)^n$ (given) ∴ n = 1

15. $\frac{61-59}{4} = K\left(\frac{61+59}{2} - 30\right)$ ...(i)

$\frac{51-49}{t} = K\left(\frac{51+49}{2} - 30\right)$ ...(ii)

Solving Equation (i) and (ii)

t = 6 minute

EXERCISE # 2

1. Ratio of thermal currents = I_1/I_2 = $\frac{\frac{\Delta T}{R_1}}{\frac{\Delta T}{R_2}}$ = $\frac{R_2}{R_1}$ = $\frac{\frac{\ell}{K\frac{\pi(2D)^2}{4}}}{\frac{2\ell}{K\frac{\pi D^2}{4}}}$ = $\frac{1}{8}$

3. $H = \sigma e A T^4$ $H \propto A \propto r^2$

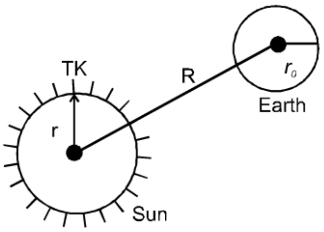
$C = \frac{\sigma e A}{ms}(4T_S^3 \Delta T)$ $C \propto \frac{A}{m} \propto \frac{r^2}{r^3} \propto \frac{1}{r}$

6. From Stefan's law, the rate at which energy is radiated by sun at its surface is

$p = \sigma \times 4\pi r^2 T^4$

[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it e = 1] The intensely of this power at earth's surface (under the assumption $R >> r_0$) is

$I = \frac{p}{4\pi R^2} = \frac{\sigma \times 4\pi r^2 T^4}{4\pi R^2} = \frac{\sigma r^2 T^4}{R^2}$

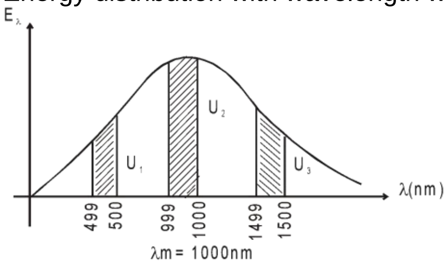


$= \frac{\sigma r^2 (t+273)^2}{R^2}$

8. Wein's displacement law is

$\lambda_m T = b$ (b = wein's constant) ∴ $\lambda_m = \frac{2.88 \times 10^6 nm-k}{2880 nm}$ =

Energy distribution with wavelength will be as follows :



From the graph it is clear that
(In fact U_2 is maximum)

9. Rate of cooling $\left(-\frac{dT}{dt}\right)$ ∝ emissivity (e)

From the graph, $\left(-\frac{dT}{dt}\right)_y > \left(-\frac{dT}{dt}\right)_x$ ∴ $e_y > e_x$

Further emissivity (e) ∝ absorptive power (1) (good absorbers are good emitters also)

∴ $a_y > a_x$

Hence the correct answer is (3).

Note : Emissivity is a pure ratio (dimensionless) while the emissive power has a unit J/s or watt.

10. $Q \propto AT^4$ and $\lambda_m \times T$ = constant. Hence,

$$Q \propto \frac{A}{(\lambda_m)^4} \quad \text{or} \quad Q \propto \frac{r^2}{(\lambda_m)^4}$$

$Q_A : Q_B : Q_C = \frac{(2)^2}{(3)^4} : \frac{(4)^2}{(4)^4} : \frac{(6)^2}{(5)^4}$

$= \frac{4}{81} : \frac{1}{16} : \frac{36}{625}$ = 0.05 : 0.0625 : 0.0576

i.e. Q_B is maximum.

Hence, the correct option is (2).

11. According to Wien's displacement law

$\lambda_m T$ = constant

∴ $T \propto \frac{1}{\lambda_{max}}$ from graph $\lambda_{max(1)} > \lambda_{max(2)} > \lambda_{max(3)}$ ∴ $T_1 < T_2 < T_3$

the material having low temperature has the graph having lower peak.

12. In configuration 1 equivalent thermal resistance is $\frac{3R}{2}$

In configuration 2 equivalent thermal resistance is $\frac{R}{3}$

Thermal Resistance ∝ time taken by heat flow from high temperature to low temperature

13. In steady state

$$I\pi R^2 = \sigma \quad (T^4 - T_0^4) \quad 4\pi R^2$$

⇒ $I = \sigma (T^4 - T_0^4)\ 4$

⇒ $T^4 - \ = 40 \times 10^8$

⇒ $T^4 - 81\times10^8 = 40 \times10^8$

⇒ $T^4 = 121 \times 10^8$
⇒ T 330 K

14. According to Wien's displacement law

λ_{m_A} T_A = $\lambda_{m_B} T_B$

Ratio of energy radiated per unit time

$$\frac{E_A}{E_B} = \frac{\sigma T_A^4 A_A}{\sigma T_B^4 A_B}$$

$$\frac{10^4 E}{E} = \frac{(\sigma)(4\pi)(400r)^2 T_A^4}{(\sigma)(4\pi)(r)^2 T_B^4}$$ C

$\left\{\frac{\lambda_B}{\lambda_A}\right\}^4 .(400)^2 = 10^4$ ⇒ $\left\{\frac{\lambda_A}{\lambda_B}\right\}^4 = 2^4$ ⇒ $\frac{\lambda_A}{\lambda_B} = 2$

18. By newton's law of cooling

$$\frac{95-90}{30} = K\left(\frac{95+90}{2} - T_0\right) \quad \text{...(i)}$$

$$\frac{55-50}{70} = K\left(\frac{55+50}{2} - T_0\right) \quad \text{...(ii)}$$

Solving (i) and (ii) $T_0 = 22.5°C$

19. By Newton's Law of cooling

$$\frac{365-361}{2} = K\left(\frac{365+361}{2} - 293\right) \quad \text{...(i)}$$

$$\frac{344-342}{t} = K\left(\frac{344+342}{t} - 293\right) \quad \text{...(ii)}$$

Solving equation (i) and (ii)
t = 1.4 min.

20. a + r + t = 1 ⇒ 0.5 + 0.5 + t = 1 ⇒ t = 0

21. Intensity recieved by earth = $\frac{\sigma 4\pi R_S^2 T^4}{4\pi d^2} = \sigma\left(\frac{R_S}{d}\right)^2 \times T^4$

⇒ $1400 = 5.67 \times 10^{-8} \left(\frac{7\times10^8}{1500\times10^8}\right)^2 \times T^4$ ⇒ T = 5800 K

23. $\frac{dT}{dt} \propto \frac{1}{ms}$

$$\frac{80-60}{8\times60} \propto \frac{A\sigma}{1\times1} \quad \text{...(i) (For Sample 1)}$$

$$\frac{80-60}{8\times60} \propto \frac{A\sigma}{1.02\times s} \quad \text{...(ii) (For Sample 2)}$$

Solving (i) and (ii)

$\frac{8\times60}{15\times60} = 1.02 \times s$ ⇒ s = 0.52

24. Rate of cooling ∝

$\Rightarrow \dfrac{\left(\dfrac{dT}{dt}\right)_A}{\left(\dfrac{dT}{dt}\right)_B} = \dfrac{m_B s_B}{m_A s_A} \Rightarrow \dfrac{6}{5} = \dfrac{\rho_B v s_B}{\rho_A v s_A} = \dfrac{3}{4} \times \dfrac{s_B}{s_A} \Rightarrow \dfrac{s_A}{s_B} = \dfrac{5}{8}$

25. Power recieved $\propto \dfrac{T^4}{r^2} \Rightarrow P \propto \dfrac{T^4}{(r)^2}$, $P' \propto \dfrac{(2T)^4}{(2r)^2} \Rightarrow = 4 \Rightarrow \dfrac{P'}{P} = \dfrac{16}{4}$ P' = 4P

26. $R_{eq} = R_1 + R_2$ (∵ slabs are in series) $= \dfrac{x}{kA} + \dfrac{4x}{2kA} = \dfrac{3x}{kA}$

Rate of heat transfer $= \dfrac{T_2 - T_1}{\left(\dfrac{3x}{kA}\right)} \Rightarrow f = \dfrac{1}{3}$

EXERCISE # 3
PART - I

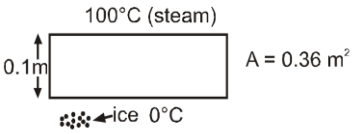
1. $Q = \sigma e A T^4$

$T = \left[\dfrac{Q}{\sigma(4\pi R^2)}\right]^{1/4}$

Here e = 1

$A = 4\pi R^2$ **Ans. (4)**

2.



Rate of heat given by steam = Rate of heat taken by ice

$$\frac{dQ}{dt} = \frac{KA(100-0)}{\ell} = m\frac{dL}{dt}$$

$$\frac{K \times 100 \times 0.36}{0.1} = \frac{4.8 \times 3.36 \times 10^5}{60 \times 60}$$

K = 1.24 J/m/s/°C

3. Wein's displacement law

$$\lambda_{\max} \propto \frac{1}{T}$$

4. $\dfrac{60-70}{5} = -K(65-T)$ $\dfrac{54-60}{5} - K(57-T)$

$$\frac{-10}{-6} = \frac{65-T}{57-T}$$

285 – 5T = 195 – 3T

90 = 2T

T = 45°

5. P – max. intensity is at violet ⇒ λ_m is minimum ⇒ temp maximum

R – max. intensity is at green ⇒ λ_m is moderate ⇒ temp moderate

Q – max. intensity is at red ⇒ λ_m is maximum ⇒ temp minimum

$T_P > T_R > T_Q$

6. As the temperature difference as well as the thermal resistance is same for both the cases, so thermal current will also be same for both the cases.

7. $\ell_2' = \ell_2 (1 + \alpha_2(\Delta\theta))$
$\ell_1' = \ell_1(1 + \alpha_1(\Delta\theta))$
$\ell_2' - \ell_1' = (\ell_2 - \ell_1) + (\alpha_2\ell_2 - \alpha_1 \ell_1)\Delta\theta$
As the length difference is independent of temperature difference hence
$\alpha_1 \ell_2 - \alpha_1\ell_1 = 0 \quad \Rightarrow \quad \alpha_2\ell_2 = \alpha_1\ell_1$

8. C.O.P. = $\dfrac{\text{Heat extracted}}{\text{effort put}} = \dfrac{T_2}{T_1 - T_2}$; ($T2 < T1$)
for 1 second analysis
$$\frac{(600)(4.2)}{\text{Effort put}} = \frac{277}{26}$$

9. $\dfrac{Mgh}{4} = mL$
$h = \dfrac{4L}{g} = 136$ km

10. $\lambda_{min}\ T = b$
$$\lambda \propto \frac{1}{T}$$
$u \propto (T)^4 \propto \dfrac{1}{(\lambda)^4}$
so
$u_1 > u_2$

11. Body at 100°C temperature has greater heat capacity than body at 0°C so final temperature will be closer to 100°C. So $T_c > 50°C$

12. $\Delta T = \Delta T_0 e^{-\lambda t}$
$T = 2Te^{-\lambda(10\ min)}$
$\Delta T' = 2Te^{-\lambda(20\ min)} = 2T\left(\dfrac{1}{2}\right)^2 = \dfrac{T}{2}$ So, $T_f = T + \dfrac{T}{2} = \dfrac{3T}{2}$

13. $\dfrac{K_1A}{d} + \dfrac{K_2A}{d} = \dfrac{K_{eq} \times 2A}{d} \Rightarrow K_{eq} = \dfrac{K_1 + K_2}{2}$

14. $P = \sigma AT^4$
$$P' = \sigma\left(\frac{A}{4}\right)(2T)^4 = 4P$$
$= 4 \times 450$ watt $= 1800$ watt.

15. $T \propto \dfrac{1}{\lambda_m} \propto \dfrac{1}{3/4}$
$$T \propto \frac{4}{3}\text{times}$$
$$\frac{dE}{dt} \propto T^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}\text{times}$$

16. $\ell'_{CU} = \ell_{CU}(1 + \alpha_{CU}\Delta T)$ ....(i)
$\ell'_{Al} = \ell_{Al}(1 + \alpha_{Al}\Delta T)$ .....(ii)
Equation (2) – equation (1)
$$\ell'_{Al} - \ell'_{CU} = \ell_{Al} + \ell_{Al}\alpha_{Al}\Delta T - (\ell_{CU} + \ell_{CU}\alpha_{CU}\Delta T)$$
$$\ell'_{Al} - \ell'_{CU} = \ell_{Al} - \ell_{CU} + (\ell_{Al}\alpha_{Al} - \ell_{CU}\alpha_{CU})\Delta T$$

When increases in length is not depend on temperature.

$\alpha_{CU}\,\ell_{CU} = \alpha_{Al}\,\ell_{Al}$

$1.7 \times 10^{-5} \times 88 = 2.2 \times 10^{-5} \times \ell_{Al}$

$\ell_{Al} = 68$ cm

17. $H = \dfrac{(k)A(T_2 - T_1)}{\ell}$ $\Rightarrow$ $(k) = (H)\left(\dfrac{\ell}{A}\right)\dfrac{1}{[T_2 - T_1]}$

$k = w\dfrac{1}{m}\dfrac{1}{k}$

$K = wm^{-1}k^{-1}$

18. Newton's cooling law

$\left(\dfrac{\theta_1 - \theta_2}{t}\right) = K\left(\dfrac{\theta_1 + \theta_2}{2} - \theta_0\right)$

$\left(\dfrac{80 - 70}{12}\right) = K\left(\dfrac{80 + 70}{2} - 25\right)$

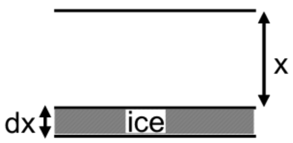
$\dfrac{10}{12} = K(50)$ $\Rightarrow$ $K = \dfrac{1}{60}$

$\left(\dfrac{70 - 60}{t}\right) = \dfrac{1}{60}\left(\dfrac{70 + 60}{2} - 25\right)$

$\dfrac{10}{t} = \dfrac{1}{60}(40)$

$t = 15$ min

19.



Heat taken = heat given

$KA\dfrac{dT}{x} = dmL/dt$

$KA\dfrac{(26 - 0)}{x} = \rho A\dfrac{dx}{dt}L$ $\Rightarrow$ $\dfrac{26K}{\rho xL} = \dfrac{dx}{dt}$

$\therefore$ Rate of increase of the thickness of ice layer = $\dfrac{26K}{\rho xL}$

PART - II

1. $\dfrac{P_1}{T_1} = \dfrac{P_2}{T_2}$. Here $P_1 = 1$, $P_2 = 2.5$, $T_1 = 300$ K.

This gives $T_2 = 750$ K.

2. The rate of heat flowing through a conductor is given by

$\dfrac{Q}{t} = -KA\left(\dfrac{d\theta}{dx}\right)$ $\Rightarrow$ $Q = -KA\left(\dfrac{d\theta}{dx}\right)t$

Where K = thermal conductivity of the material

A = area of cross-section

$\dfrac{d\theta}{dx}$ = temperature gradient, t = time

Now if we cut the rod into 4 pieces, A remains same K remains same, $\frac{d\theta}{dx}$ is said to be same so for same time t, Q is going to be same as earlier.

3. $\lambda_m T = 2.898 \times 10^{-3}$ mK

$$\lambda_m = \frac{2.9 \times 10^{-3}}{10^6} = 2.9 \times 10^{-9} m = 2.9 nm$$

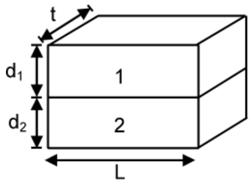
It lies in the X-ray region of the electromagnetic spectrum.

4. Let t be the width and L be the length of each conductor.

Effective thermal resistance, $R_p = \frac{R_1 R_2}{R_1 + R_2}$

$\because \quad R = \frac{L}{KA}$

Where k = thermal conductivity



$$\frac{L}{AK_{eq}} = \frac{\frac{L}{K_1A_1} - \frac{L}{K_2A_2}}{\frac{L}{K_1A_1} + \frac{L}{K_2A_2}} \Rightarrow \frac{1}{K_{eq}(A_1 + A_2)} = \frac{1}{K_2A_2 + K_1A_1} \Rightarrow \frac{K_1d_1t + K_2d_2t}{d_1t + d_2t} = \frac{K_1d_1 + K_2d_2}{d_1 + d_2}$$

PART - III

1. $$\frac{dQ}{dt} = -kA\frac{d\theta}{dx}$$

at steady state $\frac{dQ}{dt}$ = constant.

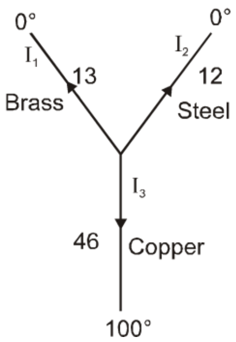
$d\theta \ \alpha - dx$

$$\int_{\theta_0}^{\theta} d\theta = -k\int_0^x dx$$

$\theta = \theta_0 - kx$

2. According to Newtons cooling law option **(3)** is correct Answer.

3.



$I_1 + I_2 + I_3 = 0$

$$\frac{K_1(T-0)}{\ell_1}+\frac{K_2(T-0)}{\ell_2}+\frac{K_3(T-100)}{\ell_3}=0 \quad \Rightarrow \quad \frac{0.12}{12}T+\frac{0.26}{13}T+\frac{0.92}{46}(T-100)=0$$

T = 40°C

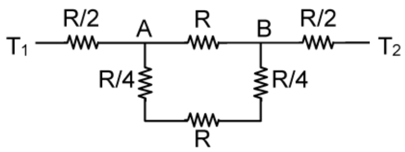
$\frac{dQ}{dt}$ through copper = $\frac{0.92\times 4}{46}$ (100 – 40) = 4.8 cal/sec.

4. $C = C_V + \frac{R}{1-n}$

$C - C_V = \frac{C_P - C_v}{1-n}$; $1 - n = \frac{C_P - C_v}{C - C_v}$

$n = 1 - \frac{C_P - C_v}{C - C_v} = \frac{C - C_P}{C - C_v}$

5.



$$T_A - T_B = \frac{\frac{T_1 - T_2}{\frac{8R}{5}}\times\frac{3R}{5}}{} = \frac{3}{8}\times 120 = 45^\circ C$$

6. $\frac{\dot{Q}}{A} = K \times \frac{1000-100}{1}$ = 90 W/m^2

7. $$\frac{\frac{X_0}{2}-\frac{X_0}{3}}{X_0-\frac{X_0}{3}} = \frac{C-0}{100-0}$$

$$\frac{1}{4} = \frac{C}{100}$$

$\Rightarrow$ C = 25°C

8. Thermal resistance $R = \frac{\ell}{KA}$

$R = \frac{\ell}{K_1\pi R^2}$

$R_2 = \frac{\ell}{K_2\pi\left(4R^2 - R^2\right)}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1}+\frac{1}{R_2}$$

$$\frac{k_{eq}\times 4\pi R^2}{\ell} = \frac{K_1\pi R^2}{\ell}+\frac{3K_2\pi R^2}{\ell}$$

$K_{eq} = \frac{K_1 + 3K_2}{4}$

9. Changing in length in both rods are same

$\Delta\ell = \alpha\ell\Delta\theta$

$\therefore \alpha_1\ell_1\Delta\theta_1 = \alpha_2\ell_2\Delta\theta_2$

$4 \times (180 - 30) = (T - 30)3$

$T = 230$