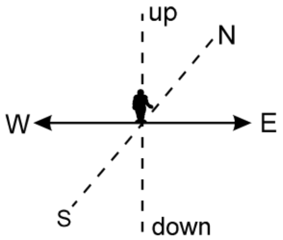
MAGNETIC EFFECT OF CURRENT AND MAGNETIC FORCE ON CHARGE OR CURRENT

1. Magnet :

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. This force is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east

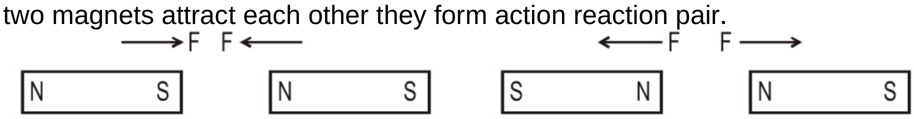


and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.

The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

1.1 Pole strength magnetic dipole and magnetic dipole moment :

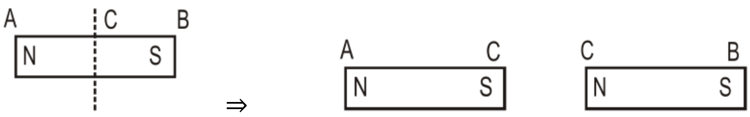
A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.



(i) (ii)

The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.

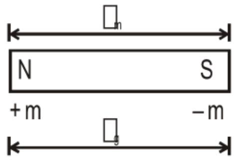
$\therefore$ they are



Know as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE

MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



M = m.ℓ_m here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].
The units of m and M will be mentioned afterwards where you can remember and understand.

1.2 Magnetic field and strength of magnetic field.

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{B}$. It is a vector quantity.

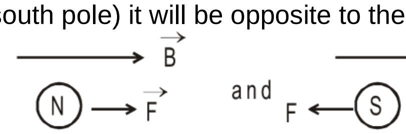
***Definition* of $\vec{B}$:** The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $\vec{B} = \frac{\vec{F}}{m}$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m^2** (abbreviated as T and Wb/m^2).

We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.

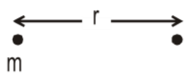


The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) $\vec{B}$ due to various source

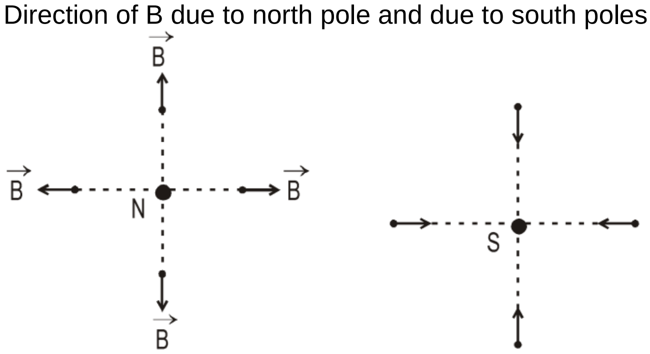
(i) Due to a single pole :

(Similar to the case of a point charge in electrostatics)



$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}.$$

This is magnitude
Direction of B due to north pole and due to south poles are as shown

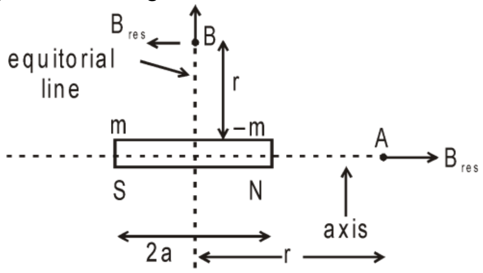


in vector form $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$

here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

(ii) Due to a bar magnet :

(Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.

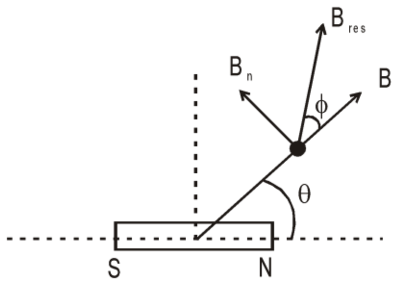


at A (on the axis) = $\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

at B (on the equatorial) = – $\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

At General point :

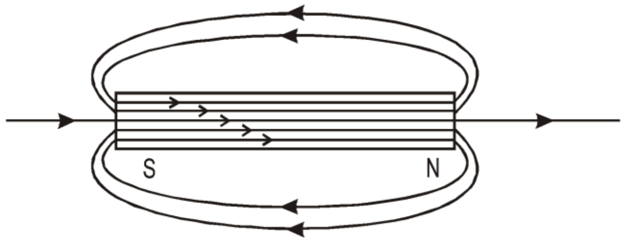
$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$ $\Rightarrow$ $B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$



B_{res} = $\frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$

tan ϕ = $\frac{B_n}{B_r} = \frac{\tan\theta}{2}$

Magnetic lines of force of a bar magnet :



Solved Examples

Example 1. Find the magnetic field due to a dipole of magnetic moment 1.2 A-m_2 at a point 1 m away from it in a direction making an angle of 60º with the dipole-axis.

Solution : The magnitude of the field is

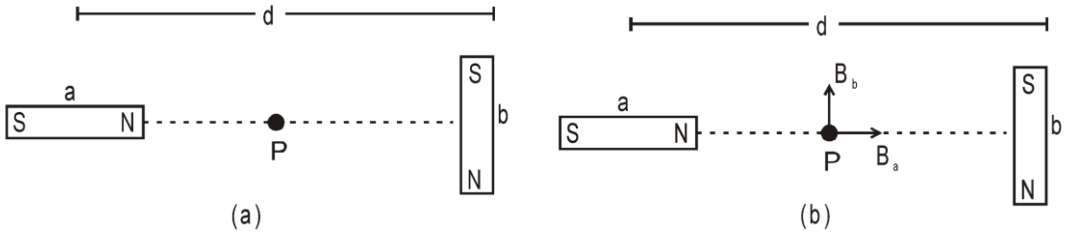
$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2\,A-m^2}{1\,m^3}\sqrt{1+3\cos^2 60^o} = 1.6 \times 10^{-7}\ T.$$

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Example 2. Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



Solution : The point P is in end-on position for the dipole a and in broadside-on position for the dipole b. The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore.

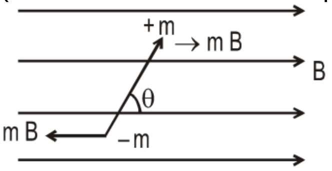
$$B = \sqrt{B_a^2 + B_b^2} = \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2+2^2}$$

$$= \frac{2\sqrt{5}\mu_0 M}{\pi d^2}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

1.3 Magnet in an external uniform magnetic field :

(same as case of electric dipole)

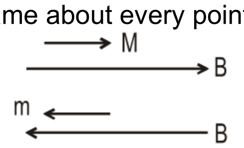


$F_{res} = 0$ (for any angle)

$\tau = MB\sin\theta$

*here θ is angle between $\vec{B}$ and $\vec{M}$

Note :
- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is



$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\theta = 0^o$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0^o$ position

$\tau = -MB \sin\theta$;

$I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

here I $= I_{cm}$ if the dipole is free to rotate

$= I_{hinge}$ if the dipole is hinged

Solved Examples

Example 3. A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60º from the field.

Solution : The work done by the external agent = change in potential energy

$= (-MB\cos\theta_2) - (-MB\cos\theta_1) = -MB(\cos 60^o - \cos 0^o)$

$= \frac{1}{2} MB = \frac{1}{2} \times (1.0 \times 10^{4}\ \text{J/T})\ (4 \times 10^{-5}\ \text{T}) = 0.2$ J

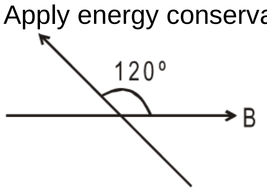
Example 4. A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :

(i) Its kinetic energy at $\theta = 90^o$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation? Periodic motion? What is its amplitude?

Solution : (i) Apply energy conservation at $\theta = 120^o$ and $\theta = 90^o$



$-MB\cos 120^o + 0$

$= -MB\cos 90^o + (K.E.)$

$KE = \frac{MB}{2}$ **Ans.**

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0^o$. Now apply energy conservation between $\theta = 120^o$ and $\theta = 0^o$.

$-mB\cos 120^o + 0$

$= -mB\cos 0^o + (KE)_{max}$

$(KE)_{max} = \frac{3}{2} MB$ **Ans.**

The K.E. is max at $\theta = 0^o$ can also be proved by torque method. From $\theta = 120^o$ to $\theta = 0^o$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0^o$. Beyond that τ reverses its direction and then K.E. starts decreasing

∴ $\theta = 0^o$ is the orientation of M to here the maximum K.E.

(iii) Since 'θ' is not small.

∴ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120º.

1.4 Magnet in an External Nonuniform Magnetic Field :

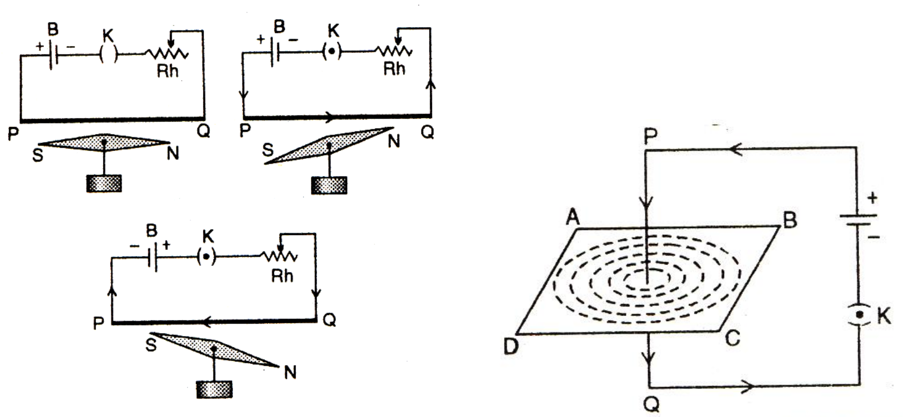
No special formula are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

2. Magnetic effects of current (and moving charge)

It was observed by **<u>OERSTED</u>** that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement (deflection or rotation of displacement). This observation shows that current or moving charge produces magnetic field.

OERSTED EXPERIMENT AND OBSERVATIONS

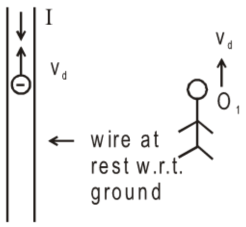
(i) Oerseted performed an experiment in 1819 whose arrangement is shown in the following figure. Following observations were noted from this experiment.



(a) When no current is passed through the wire AB, the magnetic needle remains undefelcted.
(b) When current is passed through the wire AB, the magnetic needle gets deflected in a particular direction and the deflection increases as the current increases.
(c) When the current flowing in the wire is reversed, the magnitude needle gets deflected in the opposite direction and its deflection increases as the current increases.
(ii) Oersted concluded from this experiment that on passing a current through the conducting wire, a magnetic field is produced around this wire. As a result the magnetic needle is deflected. This phenomenon is called magnetic effect of current.
(iii) From another experiment, it is found that the magnetic lines of force due to the current flowing in the wire are in the form of concentric circles around the conducting wire.

2.1 Frame Dependence of $\vec{B}$.

(a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving at same velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observers O_3 then $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r. to O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different).



(b) In a current carrying wire electron move in the opposite direction to that of the current and +ve ions (of the metal) are static w.r.t. the wire. Now if some observer (O_1) moves with velocity V_d in the direction of motion of the electrons then electrons will have zero velocity and +ve ions will have velocity V_d in the downward direction w.r.t. O_1. The density (n) of +ve ions is same as the density of free electrons and their charges are of the same magnitudes

So, w.r.t. O_1 electrons will produce zero magnetic field but +ve ions will produce +ve same due $\vec{B}$ to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).

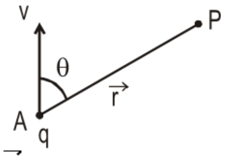
(c) **$\vec{B}$ due to magnet :**

$\vec{B}$ produced by the magnet does not contain the term of velocity

So, we can say that the $\vec{B}$ due magnet does not depend on frame.

2.2 $\vec{B}$ due to a point charge :

A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. $\vec{r}$ is the position vector of point 'P' w.r.



to position of the charge. Then $\vec{B}$ at P due to q is

$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$; here θ = angle between $\vec{v}$ and $\vec{r}$

$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3}$; q with sign $\vec{B}\perp\vec{v}$ and also $\vec{B}\perp\vec{r}$.

Direction of $\vec{B}$ will be found by using the rules of vector product.

Self Practice Problems

1. Magnetic field is produced by the flow of current in a straight wire. This phenomenon is based on-
(1) Faraday's Law (2) Maxwell's Law (3) Coulbom's Law (4) Oersted's Law

2. The field produced by a moving charged particle is-
(1) Electric (2) Magnetic
(3) Both electric and magnetic (4) Nothing can be predicted

3. The magnetic field due to a small bar magnet at a distance varies as-
(1) $\frac{1}{d^2}$ (2) $\frac{1}{d^{3/2}}$ (3) $\frac{1}{d^3}$ (4) $\frac{1}{d}$

4. A magnetic dipole of magnetic moment M is situated with its axis along the direction of a magnetic field of strength B. How much work will have to be done to rotate is through 180°?
(1) –MB (2) +MB (3) zero (4) +2MB

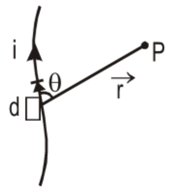
5. The magnetic field due to a small magnetic dipole of magnetic moment M, at distance r from the centre on the equatorial line is given by- (in MKS system)
(1) $\frac{\mu_0}{4\pi}\times\frac{M}{r^2}$ (2) $\frac{\mu_0}{4\pi}\times\frac{M}{r^3}$ (3) $\frac{\mu_0}{4\pi}\times\frac{2M}{r^2}$ (4) $\frac{\mu_0}{4\pi}\times\frac{2M}{r^3}$

Answer : 1. (4) **2.** (3) **3.** (3) **4.** (4) **5.** (2)

2.3 Biot-savart's law ($\vec{B}$ due to a wire)

It is a experimental law. A current 'i' flows in a wire (may be straight or curved). Due to 'dℓ' length of the wire the magnetic field at 'P' is

$dB \propto i d\ell \Rightarrow \propto \frac{1}{r^2} \Rightarrow \propto \sin\theta \Rightarrow dB \propto \frac{i d\ell \sin\theta}{r^2}$



$$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{id\ell\sin\theta}{r^2} \Rightarrow \vec{dB} = \left(\frac{\mu_0}{4\pi}\right)\frac{i\vec{d\ell}\times\vec{r}}{r^3}$$

here $\vec{r}$ = position vector of the test point w.r.t. $\vec{d\ell}$

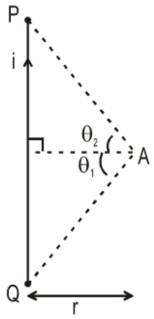
θ = angle between $\vec{d\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int \vec{dB}$

Using this fundamental formula we can derive the expression of $\vec{B}$ due a long wire.

2.3.1 $\vec{B}$ due to a straight wire :

Due to a straight wire 'PQ' carrying a current 'i' the $\vec{B}$ at A is given by the formula

$B = \frac{\mu_0 I}{4\pi r}(\sin\theta_1 + \sin\theta_2)$

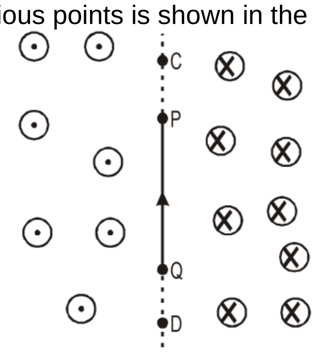


(Derivation can be seen in a standard text book like your school book or concept of physics of HCV part-II)

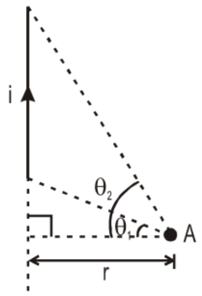
Direction :

Due to every element of 'PQ' $\vec{B}$ at A is directed in wards. So its resultant is also directed inwards. It is represented by (x)

The direction of $\vec{B}$ at various points is shown in the figure shown.

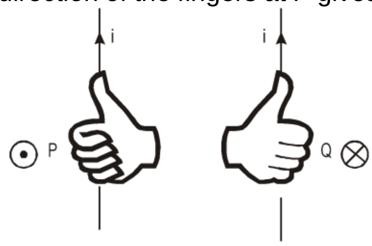


At points 'C' and 'D' $\vec{B}$ = 0 (think how). For the case shown in figure B at A = $\frac{\mu_0 i}{4\pi r}(\sin\theta_2 - \sin\theta_1)$ ⊗

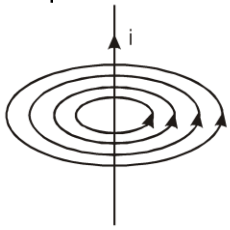


Shortcut for Direction :

The direction of the magnetic field at a point P due to a straight wire can be found by a slight variation in the right-hand thumb rule. If we stretch the thumb of the right hand along the current and curl our fingers to pass through the point P, the direction of the fingers at P gives the direction of the magnetic field there.

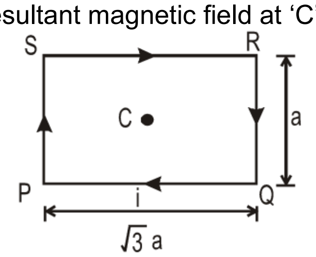


We can draw magnetic field lines on the pattern of electric field lines. A tangent to a magnetic field line given the direction of the magnetic field existing at that point. For a straight wire, the field lines are concentric circles with their centres on the wire and in the plane perpendicular to the wire. There will be infinite number of such lines in the planes parallel to the above mentioned plane.



Solved Examples

Example 5. Find resultant magnetic field at 'C' in the figure shown.

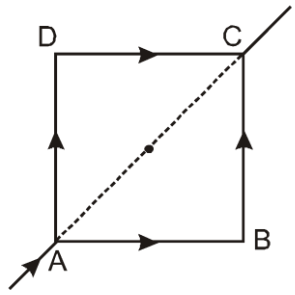


Solution : It is clear that 'B' at 'C' due all the wires is directed $\otimes$. Also B at 'C' due PQ and SR is same. Also due to QR and PS is same

$\therefore$ $B_{res} = 2(B_{PQ} + B_{SP}) \Rightarrow$ $B_{PQ} = \dfrac{\mu_0 i}{4\pi \frac{a}{2}}(\sin 60^o + \sin 60^o)$,

$B_{sp} = \dfrac{\mu_0 i}{4\pi \frac{\sqrt{3}a}{2}}(\sin 30^o + \sin 30^o) \Rightarrow$ $B_{res} = 2\left(\dfrac{\sqrt{3}\,\mu_0 i}{2\pi a} + \dfrac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \dfrac{4\mu_0 i}{\sqrt{3}\pi a}$

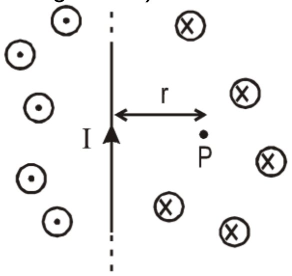
Example 6. Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points A and C.



Solution : The current will be equally divided at A. The fields at the centre due to the currents in the wires AB and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AD and BC will be zero. Hence, the net field at the centre will be zero.

Special case :

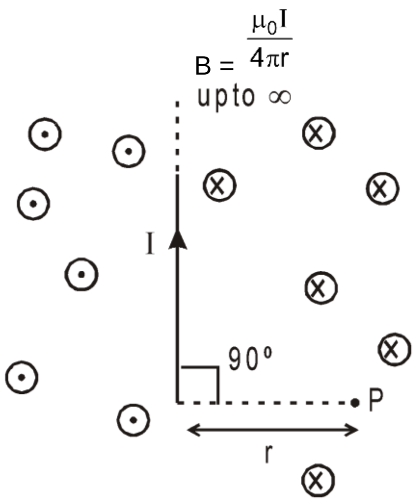
(i) If the wire is infinitely long then the magnetic field at 'P' (as shown in the figure) is given by (using $\theta_1 = \theta_2 = 90^\circ$ and the formula of 'B' due to straight wire)



$B = \frac{\mu_0 I}{2\pi r}$ $\quad\Rightarrow\quad$ $B \propto \frac{I}{r}$

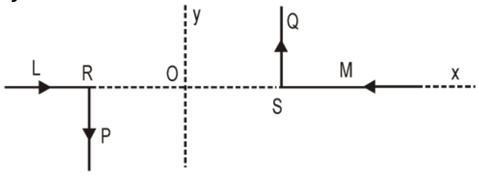
The direction of $\vec{B}$ at various is as shown in the figure.
The magnetic lines of force will be concentric circles around the wire (as shown earlier)

(ii) If the wire is infinitely long but 'P' is as shown in the figure. The direction of $\vec{B}$ at various points is as shown in the figure. At 'P'



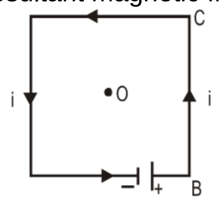
Self Practice Problems

6. A pair of stationary and infinitely long bent wires are placed in the xy plane as shown in fig. The wires carry current of i = 10 Amp. each as shown. The segment L and M are along the x-axis. The segment P and Q are parallel to the y-axis such that OS = OR = 0.02 m. The magnetic induction at the origin O is-



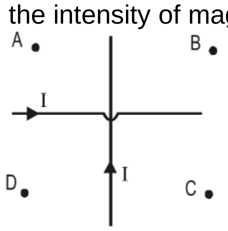
(1) 10^{-4} Wb/m^2 (2) 10^{-2} Wb/m^2 (3) 10^{-6} Wb/m^2 (4) 10^{-8} Wb/m^2

7. A current carrying wire is bent into the shape of a square coil. The magnetic field produced at the centre of coil by one arm BC is B. Then the resultant magnetic field at the centre due to all the arms will be-



(1) 4B (2) $\frac{B}{2}$ (3) B (4) $\frac{2}{B}$

8. As shown in diagram, two perpendicular wire are placed very close to each other, but they are not touching each other. The points where the intensity of magnetic field is zero are-



(1) A (2) B, D (3) A, B (4) B

9. At a distance of 10 cm. from a long straight wire carrying current, the magnetic field is 0.04. Tesla the magnetic field at the distance of 40 cm. will be-

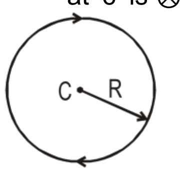
(1) 0.01 T (2) 0.02 T (3) 0.08 T (4) 0.16 T

Answer : 6. (1) **7.** (1) **8.** (2) **9.** (1)

2.3.2 $\vec{B}$ due to circular loop

(a) At centre : Due to each $\vec{d\ell}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).

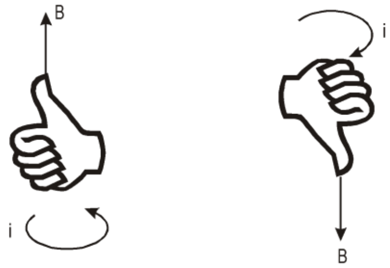
$\therefore$ $\vec{B}_{res}$ at 'c' is $\otimes$. B = $\frac{\mu_0 NI}{2R}$,



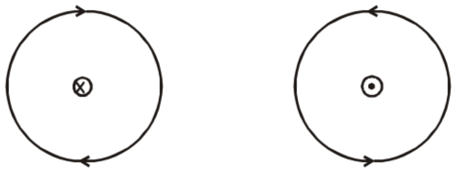
N = No. of turns in the loop = $\frac{\ell}{2\pi R}$; ℓ = length of the loop.

N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} \text{ etc.}\right)$ or integer.

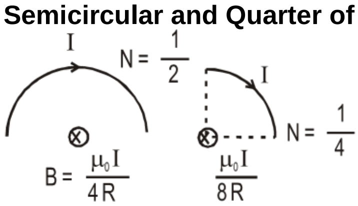
Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).



Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

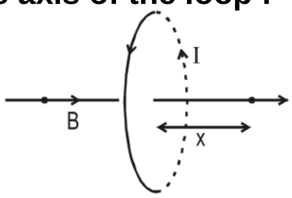


Semicircular and Quarter of a circle :



(b) On the axis of the loop :

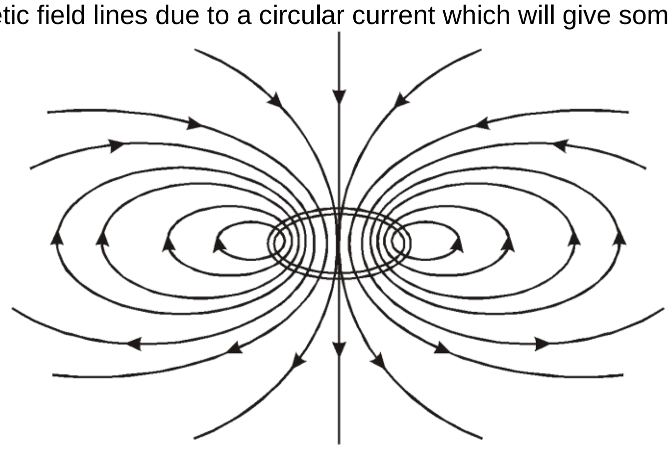
$$B = \frac{\mu_0 N I R^2}{2(R^2 + x^2)^{3/2}}$$



N = No. of turns (integer)

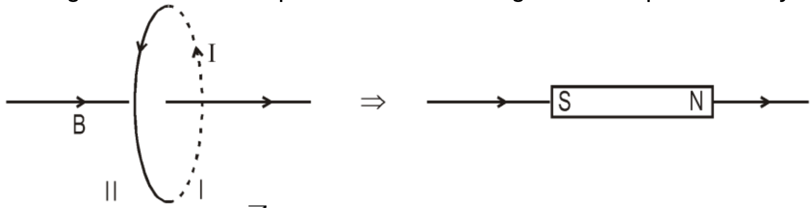
Direction can be obtained by right hand thumb rule. curl your fingers in the direction of the current then the direction of the thumb points in the direction of $\vec{B}$ at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.

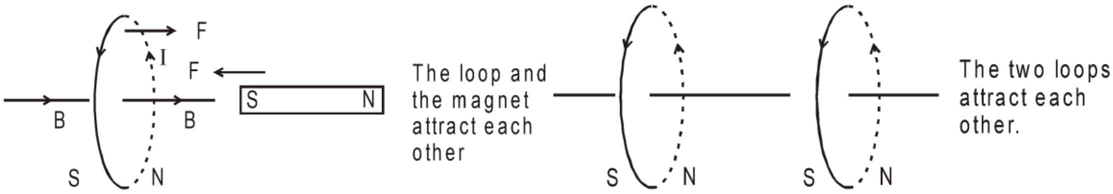


2.3.3 A loop as a magnet :

The pattern of the magnetic field is comparable with the magnetic field produced by a bar magnet.



The side 'I' (the side from which the $\vec{B}$ emerges out) of the loop acts as 'NORTH POLE' and side II (the side in which the $\vec{B}$ enters) acts as the 'SOUTH POLE'. It can be verified by studying force on one loop due to a magnet or a loop.



Mathematically :

$B_{axis} = \dfrac{\mu_0 NIR^2}{2(R^2 + x^2)^{3/2}} \cong \dfrac{\mu_0 NIR^2}{2x^3}$ for x >> R = $2\left(\dfrac{\mu_0}{4\pi}\right)\left(\dfrac{IN\pi R^2}{x^3}\right)$

it is similar to B_{axis} due to magnet = $2\left(\dfrac{\mu_0}{4\pi}\right)\dfrac{m}{x^3}$

Magnetic dipole moment of the loop

$M = IN\pi R^2$

M = INA for any other shaped loop.

Unit of M is Amp. m^2.

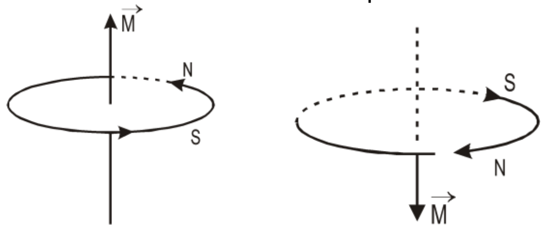
Unit of m (pole strength) = Amp. m $\{\because$ in magnet $M = m\ell\}$

$\vec{M} = IN\vec{A}$,

$\vec{A}$ = unit normal vector for the loop.

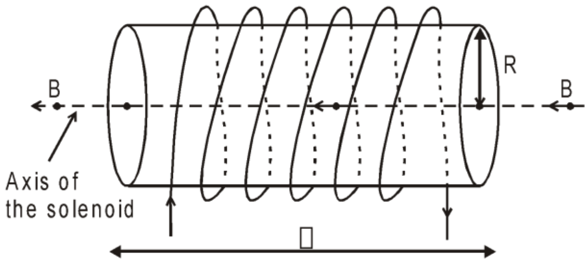
To be determined by right hand rule which is also used to determine direction of on the axis. It is also from

'S' side to 'N' side of $\vec{B}$ the loop.



2.3.4 Solenoid :

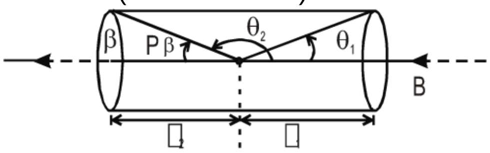
(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) $\vec{B}$ on axis (turns should be very close to each others).

$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$



where n : number of turns per unit length.

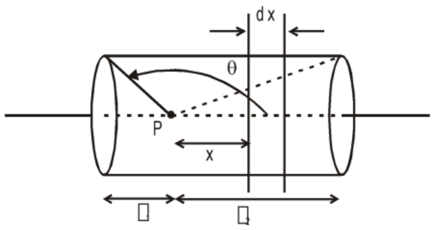
$$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} \quad ; \quad \cos\beta = \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}} = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

Note : • Use right hand rule for direction (same as the direction due to loop).

Derivation :

Take an element of width dx at a distance x from point P. [point P is the point on axis at which we are going to calculate magnetic field. Total number of turns in the element dn = ndx where n : number of turns per unit length.



$$dB = \frac{\mu_0 iR^2}{2(R^2 + x^2)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2} \frac{\mu_0 iR^2\, ndx}{2(R^2 + x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2 + R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2 + R^2}}\right] = \frac{\mu_0 ni}{2}\left[\cos\theta_1 - \cos\theta_2\right]$$

(iv) **For 'Ideal Solenoid' :**

Inside (at the mid point)

$\ell >> R$ or length is infinite

$\theta_1 \to 0$

$\theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2}[1 - (-1)]$$

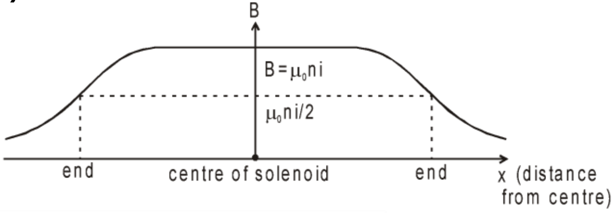
$$\mathbf{B = \mu_0 ni}$$

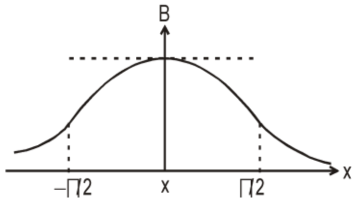
If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0\mu_r ni$

At the ends $B = \dfrac{\mu_0 ni}{2}$

(v) Comparision between ideal and real solenoid :

(a) Ideal Solenoid Real Solenoid



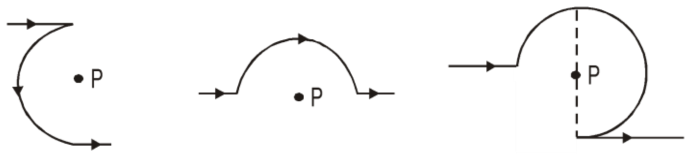


Self Practice Problems

10. The radius of a circular coil it R and it carries a current of I ampere. The intensity of magnetic field at a distance x from the centre (x >> R) will be:

(1) $B = \dfrac{\mu_0}{2}\dfrac{IR^2}{x^2}$ (2) $B = \dfrac{\mu_0}{2x^3}IR^2$ (3) $B = \dfrac{\mu_0}{2x^2}IR$ (4) $B = \dfrac{\mu_0}{2x^3}IR$

11. The magnetic field B at the centre of a circulate coil of radius r is π times that due to a long straight wire at a distance r from it, for equal currents. Fig. shows three cases; in all cases the circular part has radius r and straight ones are infinitely long. For same current the field B at the centre P in cases I, II, III has the ratio-



(I) (II) (III)

(1) $\left(-\dfrac{\pi}{2}\right) : \dfrac{\pi}{2} : \left[\dfrac{3\pi}{4} - \dfrac{1}{2}\right]$ (2) $\left(-\dfrac{\pi}{2}+1\right) : \left[\dfrac{\pi}{2}+1\right] : \left[\dfrac{3\pi}{4}+\dfrac{1}{2}\right]$

(3) $-\dfrac{\pi}{2} : \dfrac{\pi}{2} : \dfrac{3\pi}{4}$ (4) $\left(-\dfrac{\pi}{2}-1\right) : \left[\dfrac{\pi}{4}-\dfrac{1}{4}\right] : \left[\dfrac{3\pi}{4}+\dfrac{1}{2}\right]$

12. If the intensity of magnetic field at a point on the axis of current carrying coil is half of that at the centre of the coil, then the distance of that point from the centre of the coil will be-

(1) $\dfrac{R}{2}$ (2) R (3) $\dfrac{3R}{2}$ (4) 0.766 R

13. A current of 0.1 ampere flows through a coil of 100 turns and radius is 5 cm. The magnetic field at the centre of the coil will be-

(1) $4\pi \times 10^{-5}$ T (2) $8\pi \times 10^{-5}$ T (3) 4×10^{-5} T (4) 2×10^{-5} T

14. A current I flows through a circular coil of radius r the intensity of field at its centre is

(1) Proportional to r (2) Inversely proportional I

(3) Proportional to I (4) Proportional to I^2

15. In a current carrying long solenoid the field produced does not depend upon-

(1) Number of turns per unit length (2) Current flowing

(3) Radius of the solenoid (4) All of the above three

Answer : 10. (2) **11.** (1) **12.** (4) **13.** (1) **14.** (3) **15.** (3)

2.4 AMPERE's circuital law :

The line integral $\oint \vec{B} \cdot \vec{d\ell}$ on a closed curve of any shape is equal to μ_0 (permeability of free space) times the net current I through the area bounded by the curve.

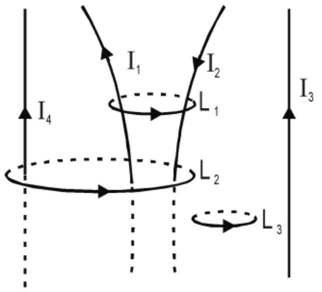
$$\oint \vec{B} \cdot \vec{d\ell} = \mu_0 \Sigma I$$

Note :

- Line integral is independent of the shape of path and position of wire with in it.
- The statement $\oint \vec{B} \cdot \vec{d\ell} = 0$ does not necessarily mean that $\vec{B} = 0$ everywhere along the path but only that no net current is passing through the path.
- **Sign of current :** The current due to which $\vec{B}$ is produced in the same sense as $\vec{d\ell}$ (i.e. $\vec{B} \cdot \vec{d\ell}$ positive will be taken positive and the current which produces $\vec{B}$ in the sense opposite to $\vec{d\ell}$ will be negative.

Solved Examples

Example 7. Find the values of $\oint \vec{B} \cdot \vec{d\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $\vec{d\ell}$ is mentioned in the figure.

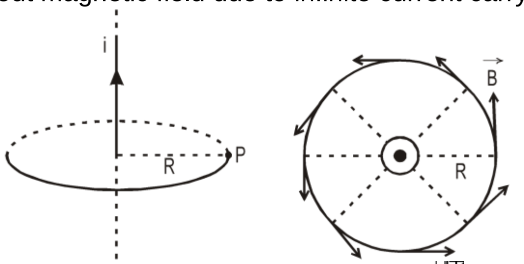


Solution : for L_1 $\oint \vec{B} \cdot \vec{d\ell} = \mu_0(I_1 - I_2)$

here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $\vec{d\ell}$ is anticlockwise as seen from top.

for L_2 : $\oint \vec{B} \cdot \vec{d\ell} = \mu_0 (I_1 - I_2 + I_4)$ for L_3 : $\oint \vec{B} \cdot \vec{d\ell} = 0$

Uses : 2.4.1 To find out magnetic field due to infinite current carrying wire

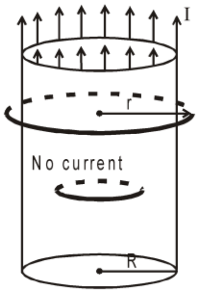


By B.S.L. $\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle.

$\oint \vec{B} \cdot \vec{d\ell}$ = $\because \theta = 0^o$ so B $\oint d\ell$ = B $2\pi R$ ($\because$ B = const.)

Now by amperes law : $B\,2\pi R = \mu_0 I$ $\therefore$ $B = \dfrac{\mu_0 i}{2\pi R}$

2.4.2. Hollow current carrying infinitely long cylinder : (I is uniformly distributed on the whole circumference)



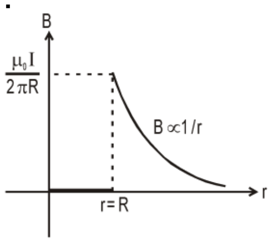
(i) for $r > R$ By symmetry the amperian loop is a circle.

$\oint \vec{B}\cdot d\vec{\ell} = \oint B\,d\ell$ $\because$ $\theta = 0$

$= B\int_0^{2\pi r} d\ell$ $\because$ B = const. $\Rightarrow$ **B =** $\dfrac{\mu_0 I}{2\pi r}$

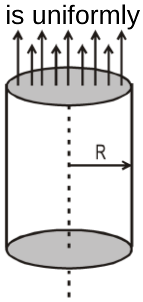
(ii) $r < R$ $= \oint \vec{B}\cdot d\vec{\ell} = \oint B\,d\ell = B(2\pi r) = 0$ $\Rightarrow$ **$B_{in} = 0$**

Graph :



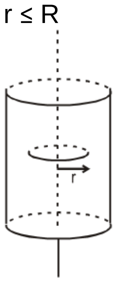
2.4.3 Solid infinite current carrying cylinder :

Assume current is uniformly distributed on the whole cross section area



current density $J = \dfrac{I}{\pi R^2}$

Case (I) : $r \le R$



take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

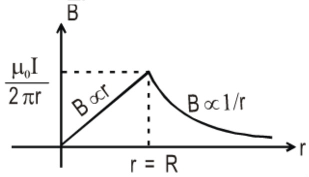
$\oint \vec{B}\cdot d\vec{\ell} = \oint B \cdot d\ell = B\oint d\ell = B \,.\, 2\pi r = \mu_0 \frac{I}{\pi R^2}\pi r^2$

$B = \frac{\mu_0 Ir}{2\pi R^2} = \frac{\mu_0 Jr}{2} \Rightarrow \vec{B} = \frac{\mu_0 (\vec{J} \times \vec{r})}{2}$

Case (II) : $r \geq R$ $\oint \vec{B}.d\vec{\ell} = \oint B\,d\ell = B\oint d\ell = B \,.\, (2\pi r) = \mu_0 \,.\, I$

$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$ also $\frac{\mu_0 I}{2\pi r}(\hat{J}\times\hat{r}) = \frac{\mu_0 J\pi R^2}{2\pi r}$

$\vec{B} = \frac{\mu_0 R^2}{2r^2}(\vec{J}\times\vec{r})$



Self Practice Problems

16. The intensity of magnetic field at a point situated at a distance r close to a long straight current carrying wire is B the intensity of field at a distance $\frac{r}{2}$ from the wire will be-

(1) $\frac{B}{2}$ (2) $\frac{B}{4}$ (3) 2B (4) 4B

17. Two straight infinitely long and thin wires are separated 0.1 m apart and carry a current of 10 Amp. each in opposite directions. The magnetic field on point at a distance 0.1 m from both the wires is-

(1) 2×10^{-5} Wb/m^2 (2) 3×10^{-5} Wb/m^2 (3) 4×10^{-5} Wb/m^2 (4) 1×10^{-5} Wb/m^2

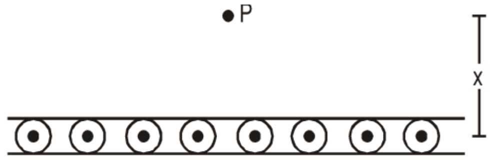
18. One ampere current is passed through a 2m long straight wire. The magnetic field in air at a point distance 3m from one end of wire on its axis will be-

(1) $\frac{\mu_0}{2\pi}$ (2) $\frac{\mu_0}{4\pi}$ (3) $\frac{\mu_0}{8\pi}$ (4) Zero

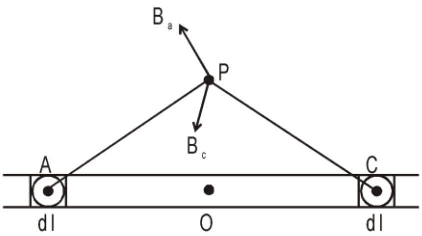
Answer : 16. (3) **17.** (1) **18.** (4)

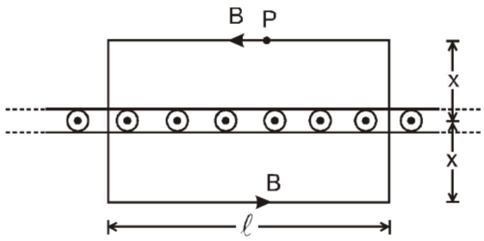
Solved Examples

Example 8. Figure shows a cross-section of a large metal sheet carrying an electric current along its surface. The current in a strip of width dl is Kdl where K is a constant. Find the magnetic field at a point P at a distance x from the metal sheet.



Solution : Consider two strips A and C of the sheet situated symmetrically on the two sides of P (figure). The magnetic field at P due to the strip A is B_0 perpendicular to AP and that due to the strip C is B_C perpendicular to CP. The resultant of these two is parallel to the width AC of the sheet. The field due to the whole sheet will also be in this direction. Suppose this field has magnitude B.



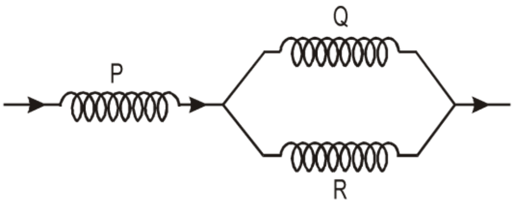


The field on the opposite side of the sheet at the same distance will also be B but in opposite direction. Applying Ampere's law to the rectangle shown in figure.

$$2B\ell = \mu_0 K\ell \qquad \text{or,} \qquad B = \frac{1}{2}\mu_0 K.$$

Note that it is independent of x.

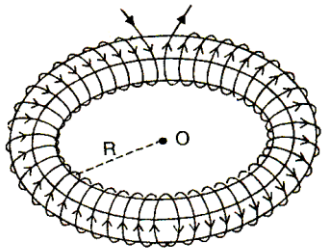
Example 9. Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Solution : As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be 1.0 T.

Magnetic Field in a Toroid

(i) Toroid is like an endless cylindrical solenoid, i.e. if a long solenoid is bent round in the form of a closed ring, then it becomes a toroid.



(ii) Electrically insulated wire is wound uniformly over the toroid as shown in the firgure.

(iii) The thickness of toroid is kept small in comparison to its radius and the number of turns is kept very large.

(iv) When a current i is passed through the toroid, each turn of the toroid produces a magnetic field along the axis at its centre. Due to uniform distribution of turns this magnetic filed has same magnitude at their centres. Thus the magnetic lines of force inside the toroid are circular.

(v) The magnetic field inside a toroid at all points is same but outside the toroid it is zero.

(vi) If total number of turns in a toroid is N and R is its radius, then number of turns per unit length of the toroid will be

$$n = \frac{N}{2\pi R}$$

(vii) The magnetic field due to toroid is determined by Ampere's law.

(viii) The magnetic field due to toroid is

$$B_0 = \mu_0 ni \qquad \text{or} \qquad B_0 = \mu_0\left(\frac{N}{2\pi R}\right)i$$

(ix) If a substance of permeability μ is placed inside the toroid, then

$$B = \mu ni$$

If μ_r is relative magnetic permeability of the substance, then

$$B = \mu_r \mu_0 ni$$

Self Practice Problems

19. A toroid has n turn density, current i then the magnetic field is-

(1) μ_0 ni (2) $\mu_0 \frac{i}{n}$ (3) Zero (4) $\mu_0 n^2 i$

20. The current on the windings on a Toroid is 2.0 A. There are 400 turns and the mean circumferential length is 40 cm. If the linside magnetic field is 1.0 T, the relative permeability is near to-

(1) 100 (2) 200 (3) 300 (4) 400

Answer : 19. (1) **20.** (4)

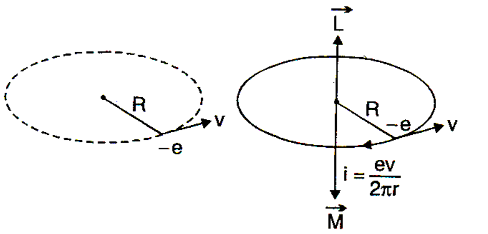
CURRENT AND MAGNETIC FIELD DUE TO CIRCULAR MOTION OF A CHARGE

(i) According to the theory of atomic structure every atom is made of electrons, protons and neutrons. protons and neutrons are in the nucleus of each atom and electrons are assumed to be moving in different orbits around the nucleus.

(ii) An electron and a proton present in the atom constitute an electric dipole at every moment but the direction of this dipole changes continuously and hence at any time the average dipole moment is zero. As a result static electric field is not observed.

(iii) Moving charge produces magnetic field and the average value of this field in the atom is not zero.

(iv) In an atom an electron moving in a circular path around the nucleus. Due to this motion current appears to be flowing in the electronic orbit and the orbit behaves like a current carrying coil. If e is the electron charge, R is the radius of the orbit and f is the frequency of motion of electron in the orbit, then



(a) current in the orbit = charge × frequency = ef

If T is the period, then $f = \frac{1}{T}$ $\therefore$ $i = \frac{e}{T}$

(b) Magnetic field at the nucleus (centre)

$$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ef}{2R} = \frac{\mu_0 e}{2RT}$$

(c) If the angular velocity of the electron is , then

$\omega = 2\pi f$ and $f = \frac{\omega}{2\pi}$

$\therefore$ $i = ef = \frac{e\omega}{2\pi}$ $\therefore$ $B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 e\omega}{4\pi R}$

(d) If the linear velocity of the electron is v, then

$v = R\omega = R(2\pi f)$

or $f = \left(\frac{v}{2\pi R}\right)$ $\therefore$ $i = ef = \frac{ev}{2\pi R}$ $\therefore$ $B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 ev}{4\pi R^2}$

(v) Magnetic moment due to motion of electron in an orbit

$$M = iA = ef\pi R^2 = \frac{e\pi R^2}{T}$$

or $M = \frac{e\omega\pi R^2}{2\pi} = \frac{e\omega R^2}{2}$ or $M = \frac{ev\pi R^2}{2\pi R} = \frac{evR}{2}$

If the angular momentum of the electron is L, then

$$L = mvR = m\omega R^2$$

Writing M in terms of L

$$M = \frac{em\omega R^2}{2m} = \frac{emvR}{2m} = \frac{eL}{2m}$$

According to Bohr's second postulate

$$mvR = n\frac{h}{2\pi}$$

In ground state n = 1

$$L = \frac{h}{2\pi} \quad \therefore \quad M = \frac{eh}{4\pi m}$$

(vi) If a charge q (or a charged ring of charge q) is moving in a circular path of radius R with a frequency f or angular velocity ω, then

(a) current due to moving charge

$$i = qf = q\omega / 2\pi$$

(b) magnetic field at the centre of ring

$B_0 = \frac{\mu_0 i}{2R} = \frac{\mu_0 qf}{2R}$ or $B_0 = \frac{\mu_0 q\omega}{4\pi R}$

(c) magnetic moment

$$M = i(\pi R^2) = qf\pi R^2 = \frac{1}{2}q\omega R^2$$

(vii) If a charge q is distributed uniformly over the surface of plastic disc of radius R and it is rotated about its axis with an angular velocity ω, then

(a) the magnetic field produced at its centre will be

$$B_0 = \frac{\mu_0 q\omega}{2\pi R}$$

(b) the magnetic moment of the disc will be

$$dM = (di)\,\pi x_2$$

$$= \frac{\omega}{2\pi}\,dq\,\pi x_2 \quad = \frac{\omega q}{R^2}\,x_3 dx$$

$\Rightarrow$ $M = \int dM = \frac{\omega q}{R^2}\int_0^R x^3 dx$ $\Rightarrow$ $M = \frac{q\omega R^2}{4}$ $\Rightarrow$ $M = \frac{q\omega R^2}{4}$

Self Practice Problems

21. A circular loop has a radius of 5 cm and it is carrying a current of 0.1 amp. Its magnetic moment is-

(1) $1.32 \times 10_{-4}$ amp. m_2 (2) $2.62 \times 10_{-4}$ amp. m_2

(3) $5.25 \times 10_{-4}$ amp. m_2 (4) $7.85 \times 10_{-4}$ amp. m_2

22. The magnetic moment of circular coil carrying current is-

(1) directly proportional to the length of the wire in the coil.

(2) inversely proportional to the length of the wire in the coil

(3) directly proportional to the square of the length of the wire in the coil

(4) inversely proportional to the square of the length of the wire in the coil.

Answer : 21. (4) **22.** (3)

3. Magnetic force on moving charge

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula :

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign.

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Note :

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because$ $\vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero. (use the formula of power
  P = $\vec{F} \cdot \vec{v}$ for its proof).
- Since the $\vec{F} \perp \vec{B}$ so work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V} \parallel \vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Solved Examples

Example 10. A charged particle of mass 5 mg and charge q = +2μC has velocity $\vec{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{j} - 2\hat{k}$ $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Solution : $\vec{F} = q\vec{v} \times \vec{B}$ = 2 × 10^{-6} $(2\hat{i} - 3\hat{j} \times 4\hat{k})$ × $(3\hat{j} - 2\hat{k})$ = 2 × 10^{-6} [–6$\hat{i}$ + 4$\hat{j}$ + 6$\hat{k}$] N

By Newton's Law $\vec{a} = \frac{\vec{F}}{m}$ = $\frac{2 \times 10^{-6}}{5 \times 10^{-6}}(-6\hat{i} + 4\hat{j} + 6\hat{k})$

= 0.8 $(-3\hat{i} + 2\hat{j} + 3\hat{k})$ m/s^2

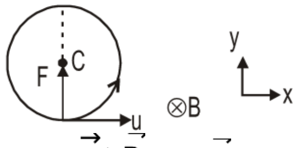
Example 11. A charged particle has acceleration $\vec{a} = 2\hat{i} + x\hat{j}$ in a magnetic field $\vec{B} = -3\hat{i} + 2\hat{j} - 4\hat{k}$. Find the value of x.

Solution : $\because$ $\vec{F} \perp \vec{B}$ $\therefore$ $\vec{a} \perp \vec{B}$ $\therefore$ $\vec{a} \cdot \vec{B}$ = 0 $\therefore$ $(2\hat{i} + x\hat{j})$. $(-3\hat{i} + 2\hat{j} - 4\hat{k})$ = 0

$\Rightarrow$ –6 + 2x = 0 $\Rightarrow$ x = 3.

3.1 Motion of charged particles under the effect of magnetic force

- Particle released if v = 0 then f_m = 0
  $\therefore$ particle will remain at rest
- $\vec{V} \parallel \vec{B}$ here θ = 0 or θ = 180º
  $\therefore$ F_m = 0 $\therefore$ $\vec{a} = 0$ $\therefore$ $\vec{V}$ = const.
  $\therefore$ particle will move in a straight line with constant velocity



- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform

In this case ∵ B is in z direction so the magnetic force in z-direction will be zero $\left(\because \vec{F}_m \perp \vec{B}\right)$.

Now there is no initial velocity in z-direction.

∴ particle will always move in xy plane. ∴ velocity vector is always $\perp \vec{B}$

∴ $F_m = quB$ = constant. now $quB = \frac{mu^2}{R}$ ⇒ $R = \frac{mu}{qB}$ = constant.

The particle moves in a curved path whose radius of curvature is same every where, such curve in a plane is only a circle. ∴ path of the particle is circular.

$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$ here p = linear momentum ; k = kinetic energy

now $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

Time period $T = 2\pi m/qB$ frequency $f = qB/2\pi m$

Note :

- ω, f, T are independent of velocity.

Solved Examples

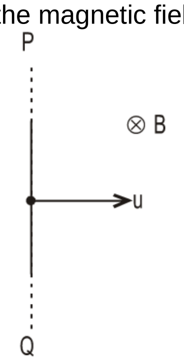
Example 12. A proton (p), α-particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods. (Neglect interaction between particles).

Solution : $R = \frac{\sqrt{2mK}}{qB}$

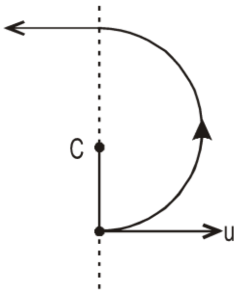
∴ $R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$ ⇒ $T = 2\pi m/qB$

∴ $T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB}$ = 1 : 2 : 2 **Ans.**

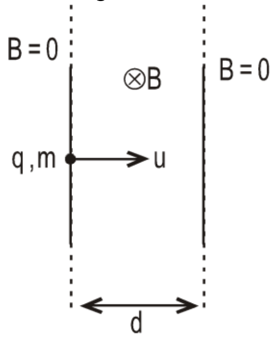
Example 13. In the figure shown the magnetic field on the left on 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Solution : The path will be semicircular time spent = $T/2 = \pi m/qB$

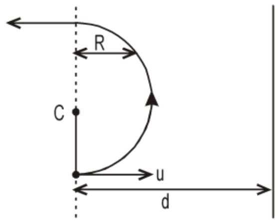


Example 14. A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if

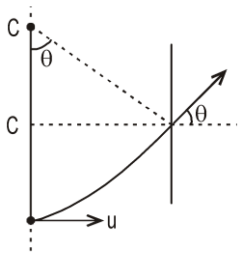


(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

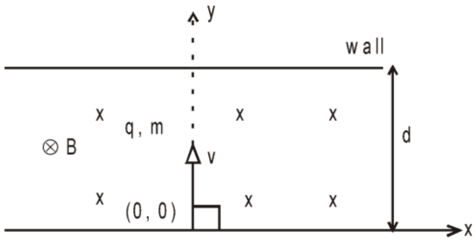
Solution : (i) $d > \frac{mu}{qB}$ means $d > R$ $\therefore$ $t = \frac{T}{2} = \frac{\pi m}{qB}$



(ii) $\sin\theta = \frac{d}{R}$ $\Rightarrow$ $\theta = \sin^{-1}\left(\frac{d}{R}\right)$ $\Rightarrow$ $\omega t = \theta$ $\Rightarrow$ $t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$

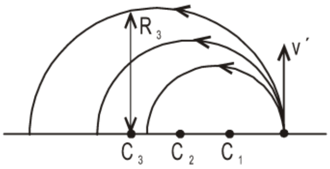


Example 15. What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



Solution : (i) The path of the particle will be circular larger the velocity, larger will be the radius.
For particle not to s
strike R < d

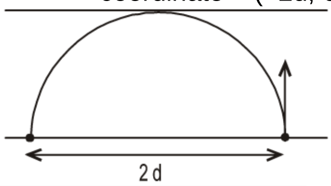
$\therefore \quad \frac{mv}{qB} < d \quad \Rightarrow \quad v < \frac{qBd}{m}$.



(ii) for limiting case $v = \frac{qBd}{m}$

R = d

$\therefore$ coordinate = (–2d, 0, 0) ℓ



Self Practice Problems

23. In a hydrogen atom, an e^- moves in Bohr's orbit of radius r = 5 x 10^{-11} m. and makes 10^{17} revolutions per second. The magnetic moment produced due to orbital motion of the e^- is-

(1) $0.40\pi \times 10^{-22}$ A-m^2 (2) $2.2\pi \times 10^{-22}$ A-m^2
(3) $2\pi \times 10^{-22}$ A-m^2 (4) None of these

24. A charged particle is moving with velocity v under the magnetic field B. The force acting on the particle will be maximum if-

(1) v and B are in the same direction (2) v and B are in the opposite direction
(3) v and B are perpendicular (4) v does not depend on the direction B.

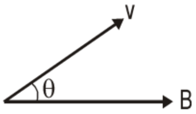
Answer : 23. (1) **24.** (3)

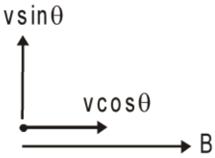
3.2 Helical path :

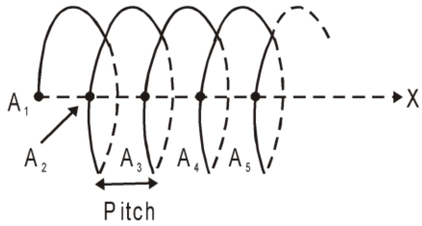
If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{||}$, parallel to the field and $v_\perp$, perpendicular to the field. The components $v_{||}$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \frac{mv_\perp}{qB}$ as given by equation. The resultant path is helix.

Complete analysis :

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.







The particle starts from point A_1.

It completes its one revolution at A_2 and 2_{nd} revolution at A_3 and so on. X-axis is the tangent to the helix points

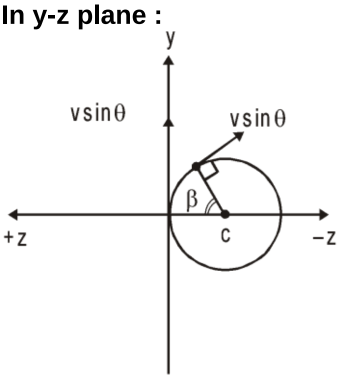
A_1, A_2, A_3,..........all are on the x-axis.

distance $A_1A_2 = A_3A_4 = ... = v\cos\theta$. T = pitch where T = Time period = $\frac{\pi 2m}{qB}$

Let the initial position of the particle be (0,0,0) and v sinq in +y direction. Then

in x : $F_x = 0$, $a_x = 0$, v_x = constant = $v\cos\theta$, $x = (v\cos\theta)t$

In y-z plane :



$\beta = \omega t$

From figure it is clear that

$y = R\sin\beta$, $v_y = v\sin\theta\cos\beta$

$z = -(R - R\cos\beta)$

$v_z = v\sin\theta\sin\beta$

acceleration towards centre = $(v\sin\theta)_2/R = \omega_2 R$ ∴ $a_y = -\omega_2 R\sin\beta$, $a_z = -\omega_2 R\cos\beta$

At any time : the position vector of the particle

(or its displacement w.r.t. initial position)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, x,y,z already found

velocity $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, v_x, v_y, v_z already found

$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$, a_x, a_y, a_z already found

Radius $q(v\sin\theta)B = \frac{m(v\sin\theta)^2}{R}$ ⇒ $R = \frac{mv\sin\theta}{qB}$

$\omega = \frac{v\sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$.

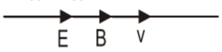
3.3 Charged Particle in $\vec{E}$&$\vec{B}$

When a charged particle moves with velocity $\vec{V}$ in an electric field $\vec{E}$ and magnetic field $\vec{B}$, then. Net force experienced by it is given by following equation.

$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B})$

Combined force is known as Lorentz force.

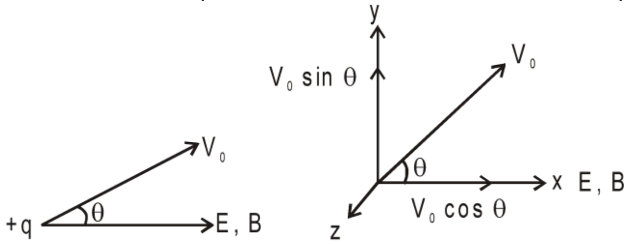
$\vec{E} \,||\, \vec{B} \,||\, \vec{v}$

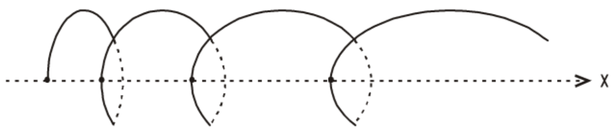


In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Case(i) :

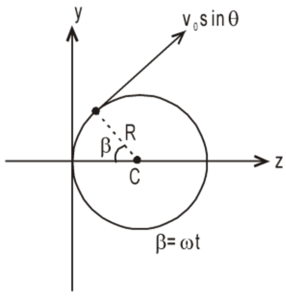
- $\vec{E} \parallel \vec{B}$ and uniform $\theta \neq 0, 180°$ ($\vec{E}$ and $\vec{B}$ are constant and uniform)





in x : $F_x = qE$, $a_x = \frac{qE}{m}$, $v_x = v_0 \cos\theta + a_x t$, $x = v_0 t + \frac{1}{2} a_x t^2$

in yz plane :



$qv_0 \sin\theta\, B = m(v_0 \sin\theta)^2 / R$

$$\Rightarrow \quad R = \frac{mv_0 \sin\theta}{qB}, \quad \omega = \frac{v_0 \sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

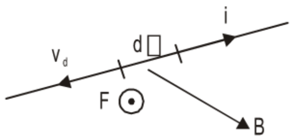
$$\vec{r} = \{(V_0 \cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\}\,\hat{i} + R\sin\omega t\,\hat{j} + (R - R\cos\omega t)\,(-\hat{k})$$

$$\vec{V} = \left(V_0 \cos\theta + \frac{qE}{m}t\right)\hat{i} + (V_0 \sin\theta)\cos\omega t\,\hat{j} + V_0 \sin\theta \sin\omega t\,(-\hat{k})$$

$$\vec{a} = \frac{qE}{m}\hat{i} + \omega^2 R[-\sin\beta\,\hat{j} - \cos\beta\,\hat{k}]$$

3.4 Magnetic force on a current carrying wire :

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire (figure). The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is



$i = jA = nev_d A$. ....(i)

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average (why average?) magnetic force

$$\vec{f} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d\, d\vec{\ell} \times \vec{B}$$

Using (i), $d\vec{F} = i\,d\vec{\ell} \times \vec{B}$.

The quantity $i\,d\vec{\ell}$ is called a *current element*.

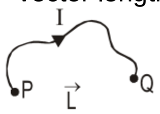
$$\vec{F}_{res} = \int d\vec{F} = \int i\,d\vec{\ell} \times \vec{B} = i \int d\vec{\ell} \times \vec{B}$$

($\because$ i is same at all points of the wire.)

If $\vec{B}$ is uniform then $\vec{F}_{res} = i\left(\int d\vec{\ell}\right) \times \vec{B}$

$$\vec{F}_{res} = i\vec{L} \times \vec{B}$$

Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.

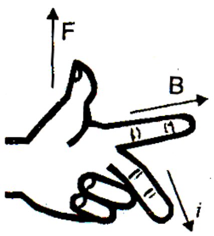


$\Rightarrow$

The direction of magnetic force is perpendicular to the plane of $\vec{I}$ and $\vec{B}$ according to right hand screw rule. Following two rules are used in determining the direction of the magnetic force.

(a) **Right hand palm rule :** If the right hand and the palm are stretched such that the thumb points in the direction of current and the stretched fingers in the direction of the magnetic field, then the force on the conductor will be perpendicular to the palm in the outward direction.

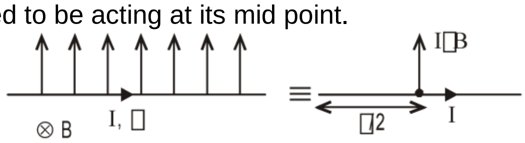
(b) **Fleming left hand rule :** If the thumb, fore finger and central finger of the left hand are stretched such that first finger points in the direction of magnetic field and the central finger in the direction of current, then the thumb will point in the direction of force acting on the conductor.



Note : If a current loop of any shape is placed in a uniform $\vec{B}$ then $\left.\vec{F}_{res}\right)_{magnetic}$ on it = 0 ($\because \vec{L}$ = 0).

3.5 Point of application of magnetic force :

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.

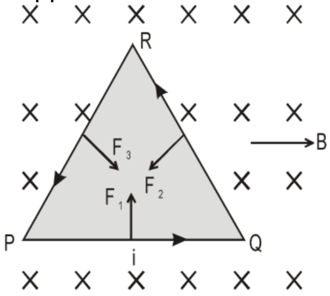


This can be used for calculation of torque.

Solved Examples

Example 16. A wire is bent in the form of an equilateral triangle PQR of side 10 cm and carries a current of 5.0 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Solution : Suppose the field and the current have directions as shown in figure. The force on PQ is



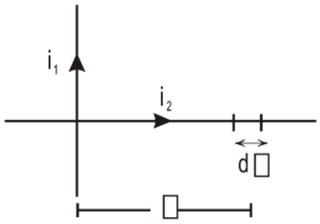
$\vec{F_1} = i\vec{\ell} \times \vec{B}$ or, $F_1 = 5.0\text{ A} \times 10\text{ cm} \times 2.0\text{ T} = 1.0\text{ N}$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

The forces $\vec{F_2}$ and $\vec{F_3}$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.

The three forces $\vec{F_1}$, $\vec{F_2}$ and $\vec{F_3}$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Example 17. Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.



Solution : The situation is shown in figure. The magnetic field at the site of $d\ell$, due to the first wire is ,

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

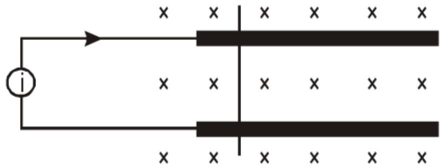
$$dF = i_2\, d\ell\, B \sin 90^\circ = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1.

Example 18. Figure shows two long metal rails placed horizontally and parallel to each other at a separation ℓ. A uniform magnetic field B exists in the vertically downward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.

(a) What soluble the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part



Solution : (a) The force on the wire due to the magnetic field is

$\vec{F} = i\vec{\ell} \times \vec{B}$ or, $F = i\ell B$

It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to μ_0 mg. Thus,

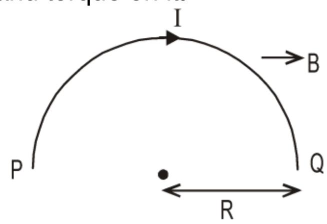
$\mu_0 mg = i\ell B$ or, $\mu_0 = \frac{i\ell B}{mg}$

(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{i\ell B}{2mg}$, the wire will slide towards right. The frictional force by the rails is

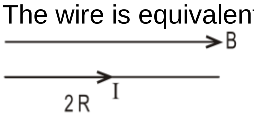
$f = \mu mg = \frac{i\ell B}{2}$ towards left.

The resultant force is $i\ell B - \frac{i\ell B}{2} = \frac{i\ell B}{2}$ towards right. The acceleration will be $a = \frac{i\ell B}{2m}$. The wire will slide towards right with this acceleration.

Example 19. In the figure shown a semicircular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.



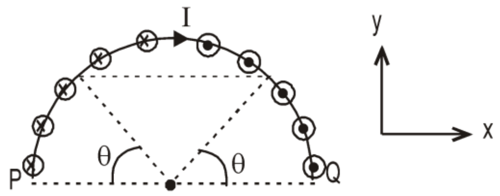
Solution : The wire is equivalent to



$\because$ $\theta = 0$ $\therefore$ $F_{res} = 0$ **Ans.**

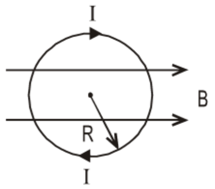
forces on individual parts are marked in the figure by ⊗ and ⊙. By symmetry their will be pair of forces forming couples.

$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta).2R\cos\theta$$



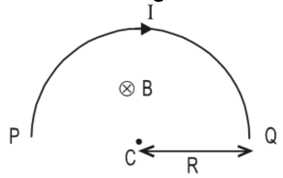
$\tau = \frac{i\pi R^2}{2}B$ $\Rightarrow$ $\vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j})$

Example 20. Find the resultant magnetic force and torque on the loop.



Solution : $\vec{F}_{res} = 0$, ($\because$ loop) and $\vec{\tau} = i\pi R^2 B(-\hat{j})$ using the above method

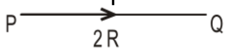
Example 21. In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.



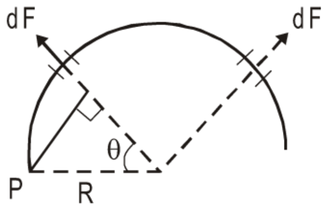
Solution : $F = I\ell_{eq}B$

$\vec{F}_{nett} = I \cdot 2R \cdot B$

$\because$ wire is equivalent to

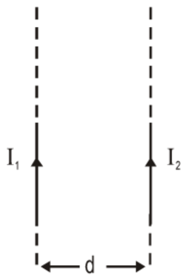


Force on each element is radially outward : $\tau_c = 0$ point about

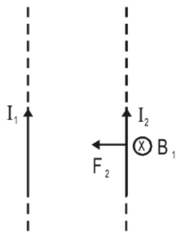


$\tau_p = f \times \ell = 2IBR_2$ **Ans.**

Example 22. Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2 / 2\pi d$. Here it is attractive also.



Solution :



On (2), B due to (i) is = $\frac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on 1m length

$= I_2 \cdot \frac{\mu_0 I_1}{2\pi d} \cdot 1$ towards left it is attractive

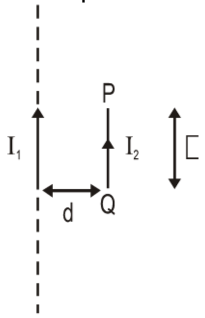
$= \frac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

Similarly on the other wire also.

Note :

- Definition of ampere (fundamental unit of current) using the above formula.
  If $I_1 = I_2 = 1A$, $d = 1m$ then $F = 2 \times 10^{-7}$ N
  ∴ "When two very long wires carrying equal currents and separated by 1m distance exert on each other a magnetic force of 2×10^{-7} N on 1m length then the current is 1 ampere."
- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.

  Force per unit length on PQ = $\frac{\mu_0 I_1 I_2}{2\pi d}$ (attractive)

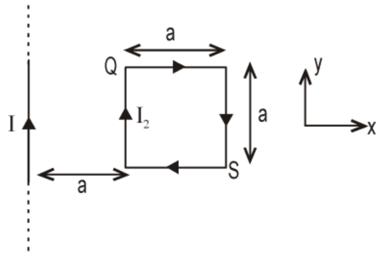


(3) If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

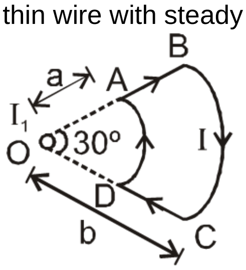
Solved Examples

Example 23. Find the magnetic force on the loop 'PQRS' due to the loop wire.

Solution : $F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a}\, a\,(-\hat{i}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{i}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{i})$



Example 24. A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and. CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30º. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



(i) The magnitude of the magnetic field due to the loop ABCD at the origin (O) is:

(1) $\frac{\mu_0 I(b-a)}{24ab}$ (2) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (3) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$ (4) zero

Solution : Magnetic field due to loop ABCD

$= \frac{\mu_0 I}{4\pi}\left(\frac{\pi}{6}\right)\times\left[\frac{1}{a}-\frac{1}{b}\right] = \frac{\mu_0 I}{24}\left[\frac{b-a}{ab}\right]$

(ii) Due to the presence of the current I_1 at the origin :
(1) The forces on AD and BC are zero.
(2) The magnitude of the net force on the loop is given by $\frac{\mu_0 I_1 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$
(3) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}$ (b – a).
(4) the forces on AB and DC are zero.

Solution : $\vec{F} = i\,(\vec{\ell}\times\vec{B})$

Magnetic field due to I_1 is parallel to AD and BC. So that force On AD and BC is zero.

Self Practice Problems

25. The force on a conductor of length ℓ placed in a magnetic field of magnitude B and carrying in current I is given by- (θ is the angle which the conductor makes with the direction of B)
(1) $I\ell B\sin\theta$ (2) $I^2\ell B^2\sin\theta$ (3) $I\ell B\cos\theta$ (4) $\frac{I^2\ell}{B}\sin\theta$

26. A charge 'q' moves in a region where electric field and magnetic field both exist, then net force on it-
(1) $q(\vec{v}\times\vec{B})$ (2) $q\vec{E}+q(\vec{v}\times\vec{B})$ (3) $q\vec{E}+q(\vec{B}\times\vec{v})$ (4) $q\vec{B}+q(\vec{E}\times\vec{v})$

27. An electron moves with speed 2×10^5 m/s along the positive x-direction in the presence of a magnetic induction $\vec{B}=\hat{i}+4\hat{j}-3\hat{k}$ (in tesla). The magnitude of the force experience by the electron in newtons is- (charge on the electron = 1.6×10^{-19} C)
(1) 1.18×10^{-13} (2) 1.28×10^{-13} (3) 1.6×10^{-13} (4) 1.72×10^{-13}

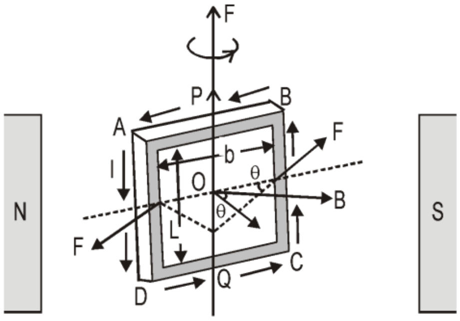
28. An electron (charge q coulomb) enters magnetic field of H weber/m^2 with velocity of v m/s in the same direction as that of the field. The force on it electron is-
(1) Hqv newtons in the direction of the magnetic filed
(2) Hqv dynes in the direction of the magnetic filed
(3) Hqv newtons at right angle to the direction of the magnetic field
(4) Zero

29. A long wire A carries a current of 10 amp. Another long wire B, which is parallel to A and separated by 0.1 m from A, carries a current of 5 amp. in the opposite direction to that in A. What is the magnitude and nature of the force experience per unit length of B- [$\mu_0 = 4\pi\times10^{-7}$ W/amp-m]
(1) Repulsive force of 10^{-4} N/m (2) Attractive force of 10^{-4} N/m
(3) Repulsive force of $2\pi\times10^{-5}$ N/m (4) Attractive force of $2\pi\times10^{-5}$ N/m

Answer : 25. (1) **26.** (2) **27.** (3) **28.** (4) **29.** (1)

4. Torque on a current loop :

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure (A).



Axis of rotation
(A)

The arms AB and CD will experience forces B(NI)b vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms AC and BD will be BINL in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

$$\tau = F \times \text{Arm} = BINL \times (b \sin\theta)$$

i.e. $\tau = BIA \sin\theta$ with $A = NLb$ ....(i)

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

$$\vec{\tau} = \vec{M} \times \vec{B} \qquad \text{[with } \vec{M} = I\vec{A} = NIA\vec{n} \text{ ....(ii)]}$$

This is the required result and from this it is clear that :

(1) Torque will be minimum (= 0) when sinθ = min = 0, i.e., θ = 0º, i.e. 180º i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. (A) and (C)]

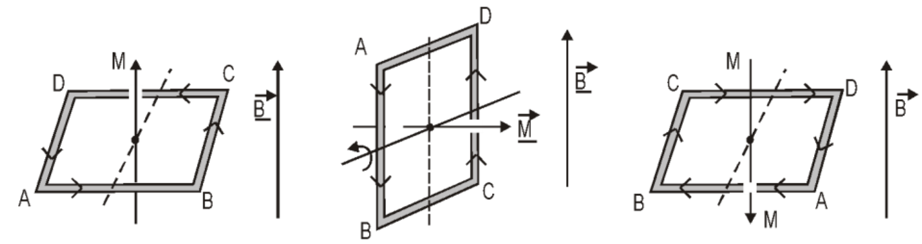
(2) Torque will be maximum (= BINA) when sinθ = max = 1, i.e., θ = 90º i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.(B)].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$$U = \vec{M} \bullet \vec{B} \qquad \text{with} \qquad F = \frac{dU}{dr}$$

and $W = MB(1 - \cos\theta)$

The values of U and W for different orientations of the coil in the field are shown in fig.



$\theta = 0^\circ$ ($\vec{M}$ is parallel to $\vec{B}$)
$\tau = 0 = \text{min}$
$W = 0 = \text{min}$
$U = -MB = \text{min}$
Stable equilibrium
(A)

$\theta = 90^\circ$ ($\vec{M}$ is $\perp$ to $\vec{B}$)
$\tau = MB = \text{max}$
$W = MB$
$U = 0$
No equilibrium
(B)

$\theta = 180^\circ$ ($\vec{M}$ is antiparallel to $\vec{B}$)
$\tau = 0$
$W = 2MB = \text{max}$
$U = MB = \text{max}$
Unstable equilibrium
(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

Solved Examples

Example 25 : A bar magnet having a magnetic moment of 2×10^4 JT^{-1} is free to rotate in a horizontal plane. A horizontal magnetic field B= 6×10^{-4} T exists in the space. The work done in taking the magnet slowly from a direction parallel to the field to a direction 60° from the field is

Solution : The work done in rotating a magnetic dipole against the torque acting on it , when placed in magnetic field is stored inside it in the form of potential energy.

When magnetic dipole is rotated from initial position $\theta = \theta_1$ to final position $\theta = \theta_2$, then work done = MB($\cos\theta_1 - \cos\theta_2$)

$$= MB\left(1-\frac{1}{2}\right) = \frac{2\times10^4\times6\times10^{-4}}{2} = 6J$$

Self Practice Problems

30. Due to flow of current in a circular loop of radius R, the magnetic induction produced at the centre of the loop is B. The magnetic moment of the loop is [μ_0 = Permeability of vacuum]

(1) $\frac{BR^3}{2\pi\mu_0}$ (2) $\frac{2\pi BR^3}{\mu_0}$ (3) $\frac{BR^2}{2\pi\mu_0}$ (4) $\frac{2\pi BR^2}{\mu_0}$

31. A current i flows in a circular coil of radius r. If the coil is placed in a uniform magnetic field B with its plane parallel to the field magnitude of torque act on coil is-

(1) Zero (2) $2\pi riB$ (3) $\pi r^2 iB$ (4) $2\pi r^2 iB$

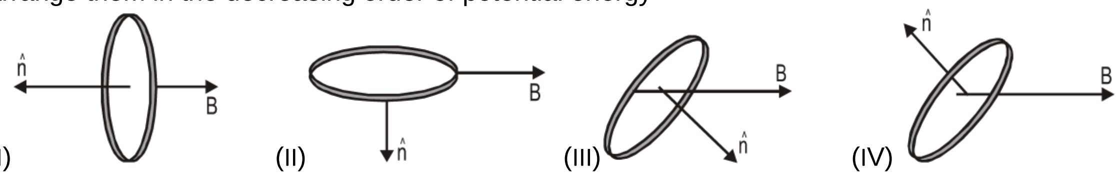
32. To double the torque acting on a rectangular coil of n turns, when placed in a magnetic field-

(1) Area of the coil and the magnetic induction should be doubled.
(2) Area and current through the coil should be doubled.
(3) Only area of coil should be doubled.
(4) Number of turns are to be halved.

33. An arbitrary shaped closed coil is made of a wire of length L and a current I ampere is flowing in it. If the plane of the coil is perpendicular to magnetic field $\vec{B}$. The force on the coil is-

(1) Zero (2) IBL (3) 2 IBL (4) $\frac{1}{2}$ IBL

34. A current carrying loop is placed in a uniform magnetic field in four different orientations, I, II, III and IV arrange them in the decreasing order of potential energy-



(I) (II) (III) (IV)

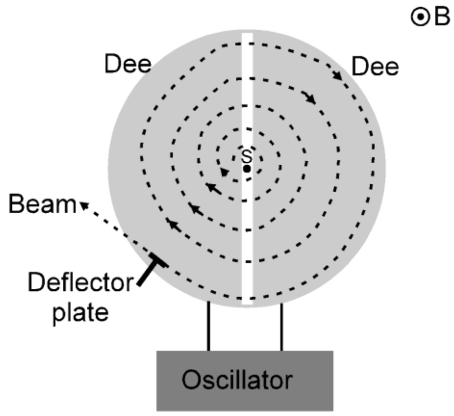
(1) I > III > II > IV (2) I > II > III > IV (3) I > IV > II > III (4) III > IV > I > II

Answer : 30. (2) **31.** (3) **32.** (3) **33.** (1) **34.** (3)

Example : 26

(Read the following passage and answer the questions. They have only one correct option)

In the given figure of a cyclotron, showing the particle source S and the dees. A uniform magnetic field is directed up from the plane of the page. Circulating protons spiral outward within the hollow dees, gaining energy every time they cross the gap between the dees.



Suppose that a proton, injected by source S at the centre of the cyclotron in Fig., initially moves toward a negatively charged dee. It will accelerate toward this dee and enter it. Once inside, it is shielded from electric field by the copper walls of the dee; that is the electric field does not enter the dee. The magnetic field, however, is not screened by the (nonmagnetic) copper dee, so the proton moves in circular path whose radius, which depends on its speed, is given by

Eq. $r = \frac{mv}{qB}$ ...(1)

Let us assume that at the instant the proton emerges into the center gap from the first dee, the potential difference between the dees is reversed. Thus, the proton again faces a negatively charged dee and is again accelerated. Thus, the proton again faces a negatively charged dee and is again accelerated. This process continues, the circulating proton always being in step, with the oscillations of the dee potential, until the proton has spiraled out to the edge of the dee system. There a deflector plate sends it out through a portal.

The key to the operation of the cyclotron is that the frequency f at which the proton circulates in the field (and that does not depend on its speed) must be equal to the fixed frequency f_{osc} of the electrical oscillator, or

$f = f_{osc}$ (resonance condition). ...(2)

This resonance condition says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field.

Combining Eq. 1 and 2 allows us to write the resonance condition as

$qB = 2\pi m f_{osc}$. ...(3)

For the proton, q and m are fixed. The oscillator (we assume) is designed to work at a single fixed frequency f_{osc}. We then "tune" the cyclotron by varying B until eq. 3 is satisfied and then many protons circulate through the magnetic field, to emerge as a beam.

(i) Ratio of radius of successive semi circular path

(1) $\sqrt{1} : \sqrt{2} : \sqrt{3} : \sqrt{4}$ ....
(2) $\sqrt{1} : \sqrt{3} : \sqrt{5}$ ....
(3) $\sqrt{2} : \sqrt{4} : \sqrt{6}$ ....
(4) 1 : 2 : 3 ....

Solution : When charge is accelerated by electric field it gains energy for first time $KE_1 = \frac{qV}{2}$

for second time $KE_2 = \frac{3}{2}qV$

for third time $KE_3 = \frac{5}{2}qV$

hence the ratio of radii are

$r_1 : r_2 : r_3 : \ldots\ldots\ldots\ldots : : \frac{\sqrt{2m\frac{qv}{2}}}{qB} : \frac{\sqrt{2m\frac{3}{2}qv}}{qB} : \ldots\ldots\ldots$

$r_1 : r_2 : r_3$ .... : : $\sqrt{1} : \sqrt{3} : \sqrt{5}$ ....

(ii) Change in kinetic energy of charge particle after every time period is :
(1) 2qV (2) qV (3) 3qV (4) None of these

Solution : In one full cycle it gets accelerated two times so change in KE = 2 qV.

(iii) If q/m for a charge particle is 10_6, frequency of applied AC is 10_6 Hz. Then applied magnetic field is:
(1) 2π tesla (2) π tesla (3) 2 tesla (4) can not be defined

Solution : $f = \frac{qB}{2\pi m}$ $\Rightarrow 10_6 = \frac{10^6 B}{2\pi}$ $\Rightarrow 2\pi T$.

(iv) Distance travelled in each time period are in the ratio of:
(1) $\sqrt{1}+\sqrt{3} : \sqrt{5}+\sqrt{7} : \sqrt{9}+\sqrt{11}$ (2) $\sqrt{2}+\sqrt{3} : \sqrt{4}+\sqrt{5} : \sqrt{6}+\sqrt{7}$
(3) $\sqrt{1} : \sqrt{2} : \sqrt{3}$ (4) $\sqrt{2} : \sqrt{3} : \sqrt{4}$

Solution : Distance travelled by particle in one time period :
$\pi(r_1 + r_2) : \pi(r_3 + r_4) : \pi(r_5 + r_6)$ ....

$$\frac{\sqrt{2m\frac{qV}{2}}}{qB} + \frac{\sqrt{2m\frac{3qV}{2}}}{qB} : \frac{\sqrt{2m\frac{5qV}{2}}}{qB} + \frac{\sqrt{2m\frac{7qV}{2}}}{qB} : \frac{\sqrt{2m\frac{9qV}{2}}}{qB} + \frac{\sqrt{2m\frac{11qV}{2}}}{qB} \ldots\ldots.$$

$S_1 : S_2 : S_3$ .... : : $(\sqrt{1}+\sqrt{3}) : (\sqrt{5}+\sqrt{7}) : (\sqrt{9}+\sqrt{11})$

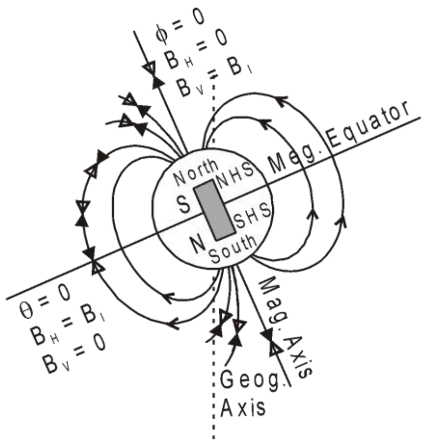
(v) For a given charge particle a cyclotron can be "tune" by :
(1) changing applied A.C. voltage only
(2) changing applied A.C. voltage and magnetic field both
(3) changing applied magnetic field only
(4) by changing frequency of applied A.C.

Solution : Frequency of A.C. depends on charge and mass only so it can be tuned by magnetic field only.

5. Terrestrial Magnetism (Earth's Magnetism) :

5.1 Introduction :

The idea that earth is magnetised was first suggested towards the end of the six-teenth century by Dr William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5º) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that:



(a) The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

(b) The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

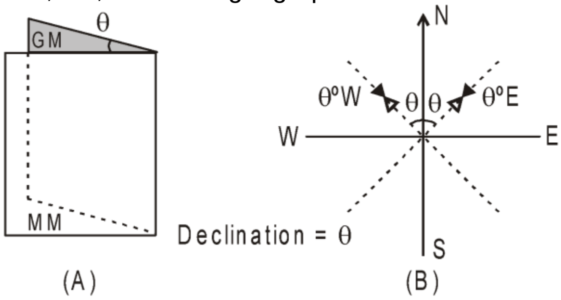
(c) The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

(d) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

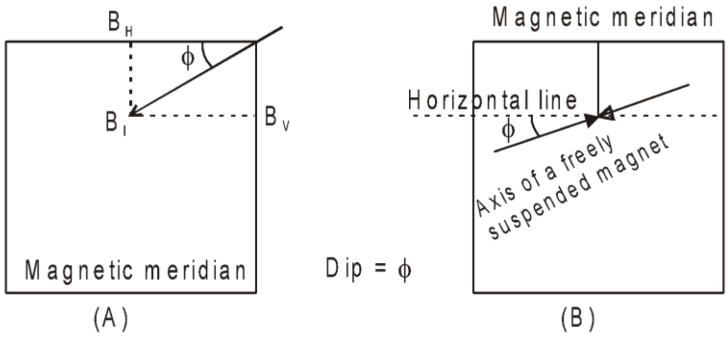
5.2 Elements of the Earth's Magnetism :

The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

(a) **Variation or Declination θ :** At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at θ^o E or θ^o W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10^oW means that at London the north pole of a compass needle points 10^oW, i.e., left of the geographical north.



(b) **Inclination or Angle of Dip φ :** It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.
Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.



Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42º.

(c) **Horizontal Component of Earth's Magnetic Field B_H :** At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a **vibration** or **deflection magnetometer**. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure (a).

$$B_H = B_I \cos\phi$$

and $$B_V = B_I \sin\phi \quad ....(1)$$

so that, $$\tan\phi = \frac{B_V}{B_H}$$

and $$I = \sqrt{B_H^2 + B_V^2} \quad ....(2)$$

Self Practice Problems

35. At a place, the magnitudes of the horizontal component and total intensity of the magnetic field of the earth are 0.3 and 0.6 oersted respectively. The value of the angle of dip at this place will be-
(1) 60° (2) 45° (3) 30° (4) 0°

36. The angle of dip at a place on the earth gives-
(1) the horizontal component of the earth's magnetic field.
(2) the location of the geographic meridian.
(3) the vertical component of the earth's field.
(4) the direction of the earth's magnetic field.

37. A bar magnet is placed north sourth with its north pole due north. The points of zero magnetic field will be in which direction from the centre of the magnet-
(1) North and south (2) East and west
(3) North east and south west (4) North west and south east

38. When the N-pole of a bar magnet points towards the south and S-pole towards the north, the null points are at the-
(1) magnetic axis (2) magnetic centre
(3) perpendicular divider of magnetic axis (4) N and S-pole

Answer : 35. (1) **36.** (4) **37.** (2) **38.** (3)

MAGNETIC SUBSTANCES & THEIR PROPERTIES :

(A) Classification of substances according to their magnetic behaviour :

All substance show magnetic properties. An iron nail brought near a pole of a bar magnet is strongly attracted by it and sticks to it, Similar is the behaviour of steel, cobalt and nickel. Such substance are called 'ferromagnetic 'substance. Some substances are only weakly attracted by a magnet, while some are repelled by it. They are called 'paramagnetic' and 'diamagnetic ' substance respectively. All substance, solids, liquids and gases, fall into one or other of these classes.

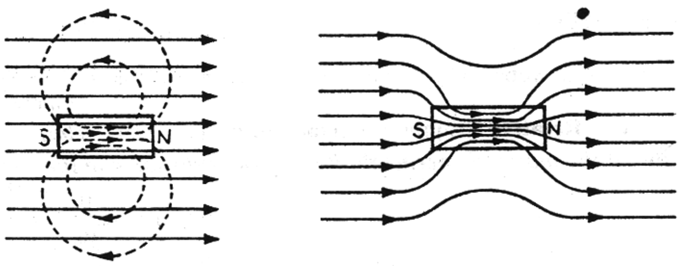
(i) **Diamagnetic substance :** Some substance, when placed in a magnetic field, are feebly magnetised opposite to the direction of the magnetising field. These substances when brought close to a pole of a powerful magnet, are somewhat repelled away from the magnet. They are called 'diamagnetic' substances and their magnetism is called the 'diamagnetism'.
Examples of diamagnetic substances are bisuth, zinc, copper, silver, gold, lead, water, mercury, sodium chloride, nitrogen, hydrogen, etc.

(ii) **Paramagnetic substances :** Some substance when placed in a magnetic field, are feebly magnetised in the direction of the magnetising field. These substance, when brought close to a pole of a powerful magnet, are attracted towards the magnet. These are called 'paramagnetic' substance and their magnetism is called 'paramagnetism'.

(iii) **Ferromagnetic substances :** Some substance, when placed in a magnetic field, are strongly magnetised in the direction of the magnetising field. They are attracted fast towards a magnet when brought close to either of the poles of the magnet. These are called 'ferromagnetic' substances and their magnetism is called 'ferromagnetism'.

(B) Some important terms used in magnetism :

(i) Magnetic induction $(\vec{B})$:
When a piece of any substance is placed in an external magnetic field, the substance becomes magnetised. The magnetism so produced in the substance is called 'induced magnetism' and this pheonomenon is called 'magnetic induction'.



(a) **(b)**

The number of magnetic lines of induction inside a magnetised substance crossing unit area normal to their direction is called the magnitude of magnetic induction, or magnetic flux density, inside the substance. It is denoted by B. Infact, magnetic induction is a vector $(\vec{B})$ whose direction at any point is the direction of magnetic line of induction at the that point.
The SI unit of magnetic induction is tesla (T) or weber /meter^2 (Wb-m^{-2}) or newton/(amper-meter) (NA^{-1}m^{-1}). The CGS unit is 'gauss'.

(ii) Intensity of magnetisation $(\vec{I})$:
The intenisity of magnetisation, or simply magnetisation of a magnetised substance represents the extent to which the substance is magnetised. It is defined as the magnetic moment per unit volume of the magnetised substance. It is denoted by I. Its SI unit is apere/meter (Am^{-1}). Numerically.

$$I = M/V.$$

In case of a bar magnet, if m be the pole-strength of the magnet , 2l its magnetic length and a its area of cross-section, then.

$$I = \frac{M}{V} = \frac{m \times 2\ell}{a \times 2\ell} = \frac{m}{a}$$

Thus, magnetisation may also be defined as pole-strength per unit area of cross-section

(iii) Magnetic Intensity or Magnetic Field strength $(\vec{H})$:

When a substance is placed in an external magnetic field, it becomes magnetised. The actual magnetic field inside the substance is the sum of the external field and the due to its magnetisation. The capability of the magnetising field to magnetise the substance is expressed by means of vector $(\vec{H})$, called the 'magnetic intensity' of the field. It is defined through the vector relation.

$$(\vec{H}) = \frac{\vec{B}}{\mu_0} - (\vec{I}),$$

Where $(\vec{B})$ is magnetic field induction inside the substance and $(\vec{I})$ is the intensity of magnetisation. μ_0 is the permeability of space.

The SI unit of $(\vec{H})$ is same as of $(\vec{I})$, that is, ampere/metre (Am^{-1}). The C.G .S. unit is 'oersted'.

(iv) Magnetic permeability (μ) :

The magnetic permeability of a substance is a measure of its conduction of magnetic lines of force through it. It is defined as the ratio of the magnetic induction $(\vec{B})$ inside the magnetised substance to the magnetic intensity $(\vec{I})$ of the magnetising field, that is,

$$\mu = \frac{\vec{B}}{\vec{H}}.$$

Numerically, $\mu = B/H$.

Its SI unit is weber/amper-metre) ($Wb\ A^{-1}\ m^{-1}$) or $\frac{\text{Newton}}{\text{Ampere}^2}$ (NA^{-2}).

(v) Relative magnetic permeability (μ_r) :

The relative magnetic permeability of a substances is the ratio of the magnetic permeability μ of the substance to the permeability of free space μ_0, that is,

$$\mu_r = \frac{\mu}{\mu_0}.$$

It is a dimensionless quantity and is equal to 1 for vacuum (by definition).

Alternatively, the relative permeability of a substance is defined as the ratio of the magnetic flux density B in the substance when placed in a magnetic field and the flux density B_0 in vacuum in the same field, that is,

$$\mu_r = \frac{B}{B_0}.$$

We can classify substances in terms of μ_r :

$\mu_r < 1$ (diamagnetic)
$\mu_r > 1$ (paramagnetic)
$\mu_r >> 1$ (ferromagnetic)

(vi) Magnetic susceptibility (χ_m) :

It is a measure of how easily a substance is magnetised in a magnetising field. For paramagnetic and diamagnetic substance, the magnetization $\vec{I}$ is directly proportional to the magnetic intensity $(\vec{H})$ of the magnetising field. That is.

$$\vec{I} = \chi_m (\vec{H})$$

The constant χ_m is called the 'magnetic susceptibility of the substance. It may be defined as the ratio of the intensity of magnetisation to the magnetic intensity of the magnetising field, that is,

$$\chi_m = \frac{I}{H}.$$

It is a pure number because I and H have same unit). Its value for vacuum is zero as there can be no magnetisation in vacuum.

We can classify substance in terms of χ_m. Substance with positive values of χ_m are paramagnetic and those with negative values of χ_m are diamagnetic. For ferromagnetic substance, χ_m is positive and very large. However, for them $\vec{I}$, is not accurately proportional to , $(\vec{H})$ and so χ_m is not strictly constant.

Relation between Relative permeability (μ_r) and magnetic susceptibility (χ_m) : When a substance is placed in a magnetising field, it becomes magnetised. The total magnetic flux density B within the substance is the flux density that would have been produced by the magnetising field in vacuum plus the flux density due to the magnetisation of the substance. If I be the intensity of magnetisatio of the substance, then, by definition, the magnetic intensity of the magnetising field is given by -

$$H = \frac{B}{\mu_0} - I$$

or But $I = \chi_m H$,where χ_m is the susceptibility of the substance.

∴ $B = \mu_0 (H + I)$.

But $I = \chi_m H$, where μ is the permeability of the substance.

∴ $\mu = \mu_0 (1 + \chi_m)$. or $\frac{\mu}{\mu_0} = 1 + \chi_m$

$\frac{\mu}{\mu_0}$ is the relative permeability μ_r, Thus

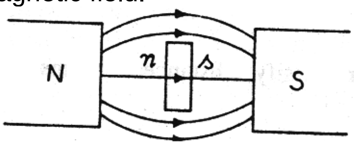
$$\boxed{\mu_r = 1 + \chi_m}$$

(C) Properties of dia, para and ferromagnetic substance :

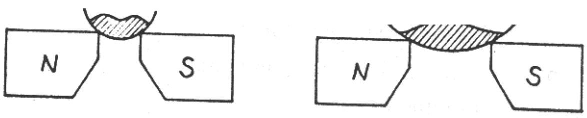
Diamagnetic substance : These substances are feebly repelled by a magnet. When placed in a magnetising field, they are feebly magnetised in a direction opposite to that of the field. Thus , the suceptibility χ_m of diamagnetic substance is negative : Further, the fulx density in a diamagnetic substance placed in a magnetising field is slightly less than in the free space. Thus, the relative permeability μ_r is less than 1.

Diamagnetic substance show following properties.

(i) When a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field.

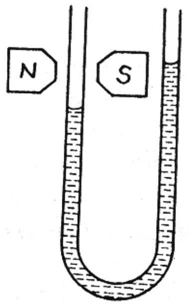


(ii) In a non-uniform magnetic field a diamagnetic substance tends to move from the stronger to the weaker part of the field.



(a) **(b)**

(iii) If a diamagnetic solution is poured into a U-tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm is depressed.



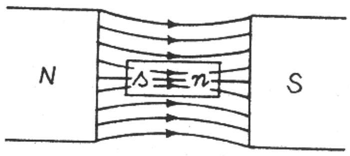
(iv) A diamagnetic gas when allowed to ascend in between the poles of a magnet spreads across the field.

(v) The susceptibility of a diamagnetic substance is independent of temperature.

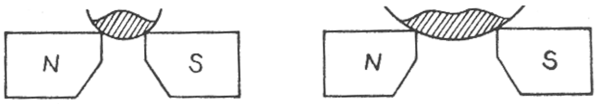
Paramagnetic substance : These substances are feebly attracted by a magnet. When placed in a magnetising field, they are feebly magnetised in the direction of the field. Thus, they have a positive susceptibility χ_m

The relative permeability μ_r for paramagnetics is slighly greater than 1 :

(i) When a rod of paramagnetic material is suspended freely between two magnetic poles, then its axis becomes parallel to the magnetic field The poles produced at the ends of the rod are opposite to the nearer magnetic poles.

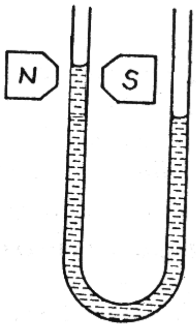


(ii) In a non-uniform magnetic field, the paramagnetic substances tend to move from weaker to stronger part of the magnetic field.



(a) **(b)**

(iii) If a paramagnetic solution is poured in a U-tube and one arm of the U-tube is placed between two strong poles, the level of the solution in that arm rises.



(iv) A paramagnetic gas when allowed to ascend between the pole-pieces of a magnet, spreads along the field.

(v) The susceptibility of a paramagnetic substance varies inversely as the kelvin temperature of the substance, that is,

$$\chi_m \propto \frac{1}{T}$$

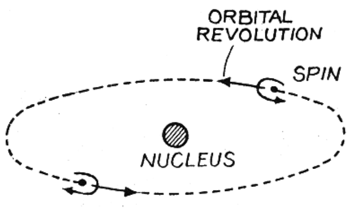
This known as curie's law.

Ferromagnetic substances : These substances which are strongly attracted by a magnet, show all the properties of a paramagnetic substance to a much higher degree. For example, they are strongly magnetised in relatively weak magnetising field in the same direction as the field. They have relative permeabilities of the order of hundreds and thousands. Similarly, the susceptibilities of ferromagnetic have large positive values.

Curie temperature : Ferromagnetism decreases with rise in temperature. If we heat a ferromagnetic substance, then at a definite temperature the ferromagnetic property of the substance "suddenly" disappears and the substance becomes paramagnetic. The temperature above which a ferromagnetic substance becomes paramagnetic is called the' Curie temperature' of the substance. The curie temperature of iron is 770ºC and that of nickel is 358ºC.

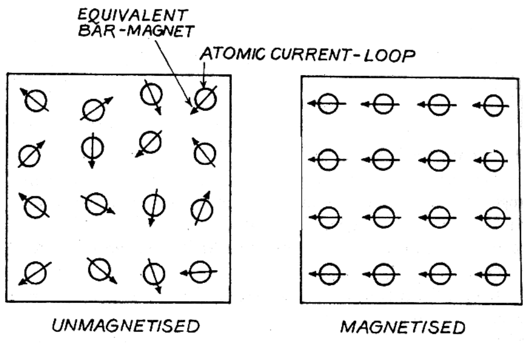
(D) Explanation of Dia-, para- and ferromagnetism on the basis of atomic model of magnetism :

The diamagnetic, paramagnetic and ferromagnetic behaviour of substance can be explained on the basis of atomic model, we known that metter is made up of atoms. Each atom of any substance has a positively charged nueleus at its centre around which electrons revolve in various discrete orbits. Each revolving electron is equivalent to a tiny current-loop (or magnetic dipole) and gives a dipole moment to the atom. Besides this, each electron "spins" about its own axis and this spin also produce a magnetic dipole moment. However, most of the magnetic moment of the atom produced by electron spin, the contribution of the orbital revolution is very small.



(i) Explanation of Diamagnetism : The property of diamagnetism is generally found is those substances whose atom, or molecules, have "even' number of electrons which form pairs. In the direction of spin of one electron is opposite to that of the other. So, the magnetic moment of one electron is neutralised by that of the other. As such, the net magnetic moment of an atom of a diamagnetic substance is zero. Diamagnetism is temperature independent.

(ii) Explanation of paramagnetism :
The property of paramagnetism is found in those substance whose atoms, or molecules, have an exces of electron spinining in the same direction. Hence atoms of paramagnetic substance have a permanent magnetic moment and behave like tiny bar-magnets. Even then the paramagnetic substances do not exhibit any magnetic effect in the absence of external magnetic field. The reason is that the atomic magnets are randomly oriented and so the magnetic moment of the bluk of the substance remains zero.



(a) (b)

Paramagnetism is temperature dependent :

Curie's Law : In 1895, Curie discovered experimentally that the magnetisation I (magnetic moment per unit volume) of a paramagnetic substance is directly proportional to the magnetic intensity H of the magnetising field and inversely proportional to the kelvin temperature T. That is -

$$I = C\left(\frac{H}{T}\right),$$

where C is constant. This equation is known as curie's law and the constant C is called the curie constant. The law, however, hold so long the ratio H/T does not become too large.

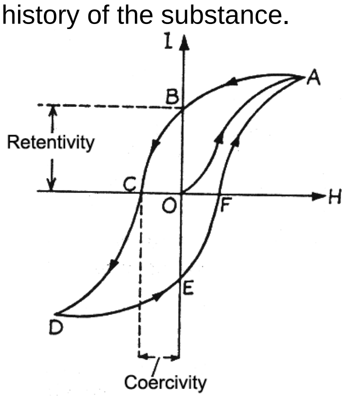
I cannot increase without limit. It approaches a maximum value corresponding to the complete alignment of all the atomic magnets constant in the substance.

Curie's law can be expressed in an alternative form. We know that the magnetic susceptibility χ_m is defined as - $\chi_m = I/H$

Making this substitution in the above expression , we get $\chi_m = C/T \Rightarrow \chi_m \propto \frac{1}{T}$.

(E) Hysteresis : Retentivity and coercivity :

Hystersis curve : When a ferromagnetic substance is placed in a magnetic field, it is magnetised by induction. If we vary magnetic intensity H of the magnetising field, the intensity of magnetisation I and the flux density B in the (ferromagnetic) substance do not vary linearly with H. In other words, the susceptibility χ_m (= I/H) and the permeability μ = (=B/H) of the substance are not constants, but vary with H and also depend upon the past history of the substance.



The variation in I with variation in H is shown in above figure. The point O represents the initial unmagnetised state of the substance (I = 0) and a zero magnetic intensity (H = 0). As H is increased, I increase (non-uniformly) along OA. At A the substance acquries a state of magnetic saturation. Any further increase in H does not produce any increase in I.

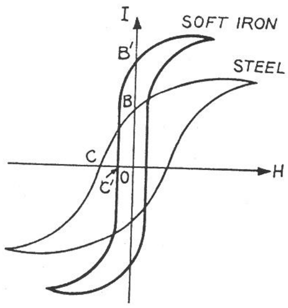
If now the magnetising field H is decreased, the magnetisation I of the substance also decrease following a new path AB (not the original path AO). Thus I lages behind H. When H becomes zero, I still has a value equal to OB. The magnetisation remaining in the substance when the magnetising field is reduced to zero is called the "residual magnetism". The power of retaining this magnetism is called the "retentivity" or the remanence of the substance. Thus , the retentivity of a substance is a measure of the magnetisation remaining in the substance when the magnetising field is removed. In figure OB represents the retentivity of the substance If now the magnetising field H is increased in the reverse direction, the magnetisation I decrease along BC, still lagging behind H, until it becomes zero at C where H equals OC. The value OC of the magnetising field is called the "coercive " or coercivity" of the substance. Thus, the coercivity of a substance is a measure of the reverse magnetising field required to destroy the residual magnetism of the substance. As H is increased beyond OC, the substance is increasingly magnetised in the opposite direction along CD, at D the substance is again magnetically saturated.

By taking H back from its maximum negative value (through zero) to its original maximum positive value, a symmetrical curve DEFA is obtained. At points B and E where the substance is magnetised in the absence of any external magnetising field , it is said to be a "permanent magnet".

It is thus found that the magnetisation I (or of B) behind H is called "hysteresis". The closed curve ABCDEFA which represents a cycle of magnetisation of the substance is known as the "hysteresis curve (or loop)" of the substance. On repeating the process, the same closed curve is traced again but the portion OA is never obtained.

(i) Hysteresis loss :
A ferromagnetic substance consist of local regions called "domains" , each of which is spontaneously magnetised. In an unmagnetised substance the directions of magnetisation in different domains are different so that, on the average, the resultant magnetisation is zero.
It can be proved that the energy lost per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop (I-Hcurve).

(ii) Difference in magnetic properties of soft iron and steel :
A comparison of the magnetic properties of ferromagnetic substance can be made by the comparison of the shapes and sizes of their hysteresis loops. In figure are shown hystersis loops of soft iron and steel for the same values of I and H. We can draw following conclusions regarding the magnetic properties of these substance from these loops :



(i) The retentivity of soft iron (OB') is greater than the retentivity of steel (OB).

(ii) The coercivity of soft iron (OC') is less than the coercivity of steel (OC).

(iii) The hysteresis loss in soft iron is smaller than that in steel because the area of the soft iron is smaller than that of steel.

(iv) Curves between magnetic flux density B and magnetising field H would reveal that the permeability of soft iron is greater than that of steel.

(F) Section of magnetic materials :

The choice of a magnetic material for making permanent magnet, electromagnet, core of transformer or diaphragm of telephone ear-piece can be decided from the hysteresis curve of the material.

(i) Permanent magnets :

The material for a permanent magnet should have high retentivity so that the magnet is strong, and high coercivity so that the magnetisation is not wiped out by stray external fields, mechanical ill treatement and temperature changes. The hysteresis loss is immaterial because the material in this case is never put to cyclic changes of magnetisation. From these considerations permanent magnets are made of steel. The fact that the retentivity of soft iron is a little greater than that of steel is outweighed by its much smaller coercivity, which makes it very easy to demagnetise.

(ii) Electromagnets : The material for the cores of electromagnets should have high permeability (or high susceptibility), specially at low magnetising fields, and a high retentivity. Soft iron is suitable material for electromagnets).

(iii) **Transformer cores and telephone diaphragms :** In these cases the material goes through complete cycles of magnetisation continuously. The material must therefore have a low hysteresis loss to have less dissipation of energy and hence a small heating of the material (otherwise the insulation of windings may break), a high permeability (to obtain a large flux density at low field) and a high specific resistance (to reduce eddy current loses).

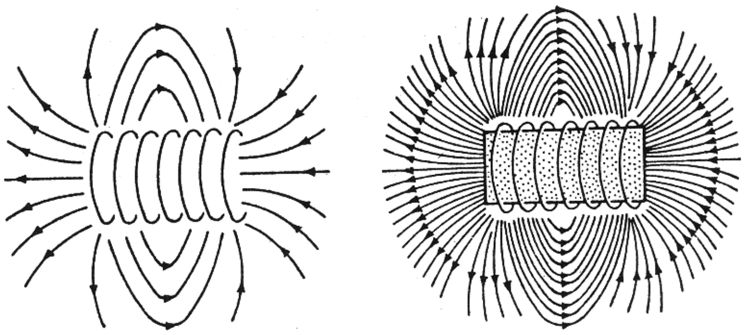
Soft -iron is used for making transformer cores and telephone diaphragms : More effective alloys have now been developed for transformer cores. They are permalloys, mumetals etc.

Comparison Chart of Dia, Para and Ferromagnetism

S.No.	Diamagnetism	Paramagnetism	Ferromagnetism
1.	Substances are feebly repelled by the magnet.	Substances are feebly attracted by the magnet.	Substances are strongly attracted by the magnet.
2.	*Magnetisation* I is small, negative, and varies linearly with field.	I is small, positive and varies linearly with field.	I is very large, positive and varies non-linearly with field.
3.	*Susceptibility* χ_m is small, negative and temperature independent.	χ_m is small, positive and varies inversely with temperature, i.e., $\chi_m \propto (1/T)$	χ_m very large, positive and temperature dependent.
4.	*Relative permeability* μ_r is slightly lesser than unity, i.e., $\mu < \mu_0$	μ_r is slightly greater than unity, i.e., $\mu < \mu_0$.	μ_r is much greater than unity, i.e., $\mu >> \mu_0$.
5.	In it lines of force are expelled from the substance, i.e., $B < B_0$.	In it lines of force are 'pulled in' by the substance, i.e., $B > B_0$	In it lines of force are 'pulled in' strongly by the substance, i.e., $B >> B_0$.
6.	It is practically independent of temperature.	It decreases with rise in temperature.	It decreases with rise in temperature and above Curie temperature becomes para.
7.	Atoms do not have any permanent dipole moment	Atoms have permanent dipole moments which are randomly oriented.	Atoms have permanent dipole moments which are organized in domains.
8.	Exhibited by solids, liquids and gases.	Exhibited by solids, liquids and gases	Exhibited by solids only, that too crystalline.
9.	Bi, Cu, Ag, Hg, Pb, water, hydrogen, He, Ne, etc. are diamagnetic.	Na, K, Mg, Mn, Al, Cr, Sn and liquid oxygen are paramagnetic	Fe, Co, Ni and their alloys are ferromagnetic.

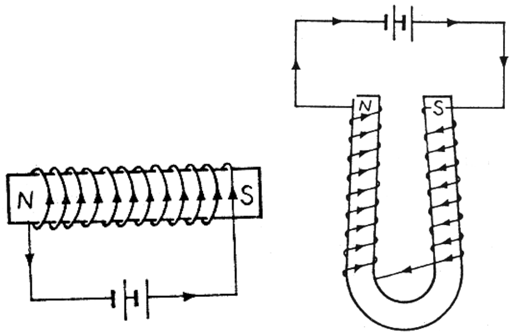
ELECTROMAGNET :

If we place a soft-iron rod in the solenoid, the magnetism of solenoid increases hundreds of time. Then the solenoid is called an 'electromagnet'. It is a temporary magnet.



(a) (b)

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft-iron straight rod or a horse-shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.



(a) (b)

Solved Examples

Example 27 : A magnetizing field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} wb in an iron bar of cross-sectional area $0.2cm^2$. Calculate permeability and susceptibility of the bar.

Solution : $B = \frac{\Phi}{A} = \frac{2.4 \times 10^{-5}\ Wb}{0.2 \times 10^{-4}\ m^2} = 1.2\ Wb/m^2 = 1.2\ N\ A^{-1}\ m^{-1}$.

The magnetising field (or magnetic intensity) H is 1600 Am^{-1}. Therefore, the magnetic permeability is given by -

$\mu = \frac{B}{H} = \frac{1.2NA^{-1}m^{-1}}{1600\,Am^{-1}} = 7.5 \times 10^{-4}\ N/A^2$.

Now, from the relation $\mu = \mu_0 (1 + \chi_m)$, the susceptibility is given by

$\chi_m = \frac{\mu}{\mu_0} - 1$.

We known that $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$. $\therefore$ $\chi_m = \frac{7.5 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1 = 596$.

Example 28. The core of toroid of 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. A current of 0.6 A produces a magnetic field of 2.5 T in the core. Compute relative permeability of the core. ($\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}).

Solution : The magnetic field in the empty space enclosed by the windings of a toroid carrying a current i_0 is $\mu_0\ n\ i_0$

where n is the number of turns per unit length of the toroid and μ_0 is permeability of free space.

If the space is filled by a core of some material of permeability μ, then the field is given by
$B = \mu n i_0$
But $\mu = \mu_0 u_r$, where μ_r is the relative permeability of the core material. Thus,

$B = \mu_0 u_r n i_0$ or $\mu_r = \dfrac{B}{\mu_0 n i_0}$

Here B = 2.5 T, i_0 = 0.7 A and $n = \dfrac{3000}{2\pi r}\ m^{-1}$, where r is the mean radius of the toroid

($r = \dfrac{11+12}{2}$ = 11.5 cm 11.5 × 10^{-2} m).

Thus, $\mu_r = \dfrac{2.5}{(4\pi\times10^{-7})\times(3000/2\pi\times11.5\times10^{-2})\times0.7} = \dfrac{2.5\times11.5\times10^{-2}}{2\times10^{-7}\times3000\times0.7}$

μ_r = 684.5

Self Practice Problems

39. A ferromagnetic material is heated above its curie temperature. Which one is a correct statement-
(1) Ferromagnetic domains are perfectly arranged.
(2) Ferromagnetic domains become random.
(3) Ferromagnetic domains are not influenced.
(4) Ferromagnetic material changes itself into diamagnetic material.

40. To protect a sensitive instrument from external magnetic jerks, it should be placed in a container made of-
(1) non magnetic substance (2) diamagnetic substance
(3) paramagnetic substance (4) ferromagnetic substance

41. The ratio of intensity of magnetisation and magnetic field intensity is known as-
(1) Permeability (2) Magnetic flux
(3) Magnetic susceptibility (4) Relative Permeability

42. If a magnetic material, moves from a stronger to weaker parts of a magnetic field, then its is known as-
(1) diamagnetic (2) paramagnetic
(3) ferromagnetic (4) anti-ferromagnetic

43. Susceptibility of a magnetic substance is found to deped on temperature and the strength of the magnetizing field. The material is a-
(1) Diamagnet (2) Ferromagnet (3) Paramagnet (4) Superconductor

44. Property possessed by ferromagnetic substance only is-
(1) attracting magnetic substance (2) hysteresis
(3) susceptibility independent of temperature (4) directional property

Answer : 39. (2) **40.** (4) **41.** (3) **42.** (1) **43.** (2) **44.** (2)

Solved Miscellaneous Problems

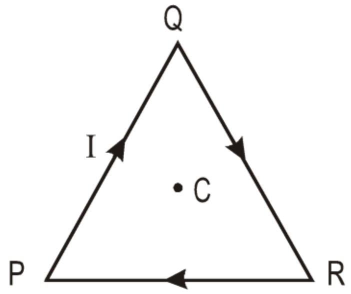
Problem 1. A bar magnet has a pole strength of 3.6 A-m and magnetic length 8 cm. Find the magnetic field at (a) a point on the axis at a distance of 6 cm from the centre towards the north pole and (b) a point on the perpendicular bisector at the same distance.

Answer : (a) 8.6 × 10^{-4} T ; (b) 7.7 × 10^{-5} T.
Solution : M = 3.6 × 8 × 10^{2} A.m^{2}

(a) $B = \dfrac{\mu_0}{4\pi}\cdot\dfrac{2Mr}{(r^2-a^2)^2}$ = 8.6 × 10^{-4} T.

(b) $B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + a^2)^{3/2}} = 7.7 \times 10^{-5}$ T

Problem 2. A loop in the shape of an equilateral triangle of side 'a' carries a current I as shown in the figure. Find out the magnetic field at the centre 'C' of the triangle.



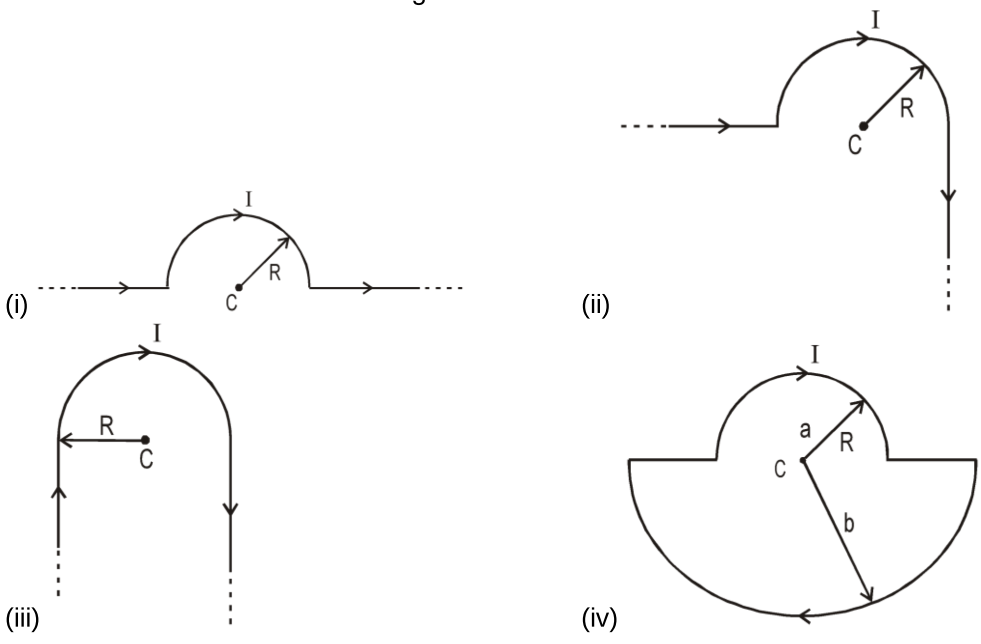
Solution : $B = B_1 + B_2 + B_3 = 3B_1$

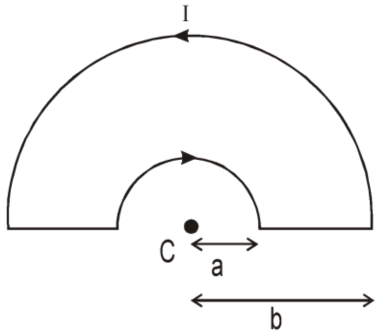
$= 3\,\frac{\mu_0}{4\pi} \times \frac{i}{\left(\frac{a}{2\sqrt{3}}\right)} \times (\sin 60° + \sin 60°) = \frac{9\mu_0 i}{2\pi a}$ **Ans.**

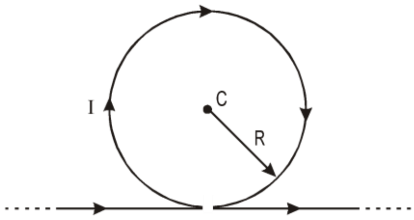
Problem 3. Two long wires are kept along x and y axes they carry currents I & I respectively in +ve x and +ve y directions respectively. Find $\vec{B}$ at a point (0, 0, d).

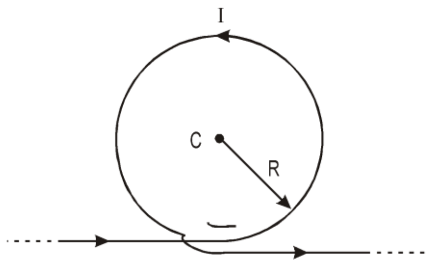
Solution : $\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0}{2\pi}\frac{i}{d}(-\hat{j}) + \frac{\mu_0}{2\pi}\frac{i}{d}(\hat{i}) = \frac{\mu_0 I}{2\pi d}(\hat{i} - \hat{j})$ **Ans.**

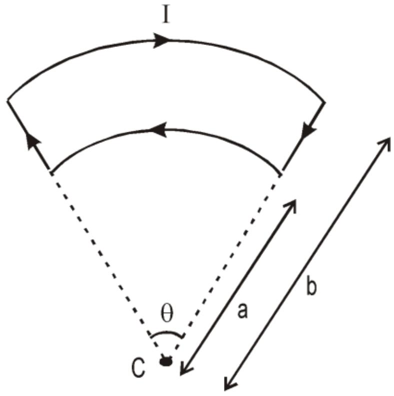
Problem 4. Find 'B' at centre 'C' in the following cases :



(v) 

(vi) 

(vii) 

(viii) 

Answer :

(i) $\frac{\mu_0 I}{4R} \otimes$ (ii) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right) \otimes$ (iii) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right) \otimes$ (iv) $\frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right) \otimes$

(v) $\frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right) \otimes$ (vi) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right) \otimes$ (vii) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right) \odot$ (viii) $\frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right) \odot$

Solution :

(i) $B = \frac{\mu_0 I}{2R} \times \frac{1}{2} = \frac{\mu_0 I}{4R}$

(ii) $B = B_1 + B_2 = \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) + \left(\frac{\mu_0}{4\pi} . \frac{I}{R}\right) = \frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)$

(iii) $B = B_1 + B_2 + B_3 = 2B_1 + B_2 = \left(2 \times \frac{\mu_0}{4\pi}\frac{I}{R}\right) + \left(\frac{\mu_0 I}{2R} \times \frac{1}{2}\right) = \frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)$

(iv) $B = B_1 + B_2 = \frac{\mu_0}{2a} \times \frac{1}{2} + \frac{\mu_0}{2b} \times \frac{1}{2} = \frac{\mu_0 I}{4}\left(\frac{1}{a}+\frac{1}{b}\right)$

(v) $B = B_1 - B_2 = \left(\frac{\mu_0 I}{2a} \times \frac{1}{2} - \frac{\mu_0 I}{2b} \times \frac{1}{2}\right) = \frac{\mu_0 I}{4}\left(\frac{1}{a}-\frac{1}{b}\right)$

(vi) $B = B_1 - B_2 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)$

(vii) $B = B_1 + B_2 = \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)$

(viii) $B = B_1 - B_2 = \frac{\mu_0 I}{2a} - \frac{\mu_0 I}{2b} \times \frac{\theta}{2\pi} = \frac{\mu_0 I\theta}{4\pi}\left(\frac{1}{a}-\frac{1}{b}\right)$

Problem 5. A thin solenoid of length 0.4 m and having 500 turns of wire carries a current 1A; then find the magnetic field on the axis inside the solenoid.

Answer : $5\pi \times 10^{-4}$ T.

Solution : $B = \mu_0 ni = \frac{\mu_0 Ni}{\ell} = 5\pi \times 10^{-4}$ T.

Problem 6. A charged particle of charge 2C thrown vertically upwards with velocity 10 m/s. Find the magnetic force on this charge due to earth's magnetic field. Given vertical component of the earth = 3μT and angle of dip = 37º.

Answer : $2 \times 10 \times 4 \times 10^{-6} = 8 \times 10^{-5}$ N towards west.

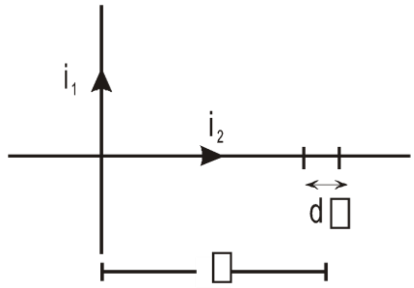
Solution : $\tan 37^o = \frac{B_V}{B_H}$ ⇒ $B_H = \frac{4}{3} \times 3 \times 10^{-6}$ T ⇒ $F = q\, v\, B_H = 8 \times 10^{-5}$ N

Problem 7. A particle of charge q and mass m is projected in a uniform and constant magnetic field of strength B. The initial velocity vector $\vec{v}$ makes angle 'θ' with the $\vec{B}$. Find the distance travelled by the particle in time 't'.

Answer : vt

Solution : Speed of the particle does not change there fore distance covered by the particle is s = vt

Problem 8. Two long wires, carrying currents i_1 and i_2, are placed perpendicular to each other in such a way that they just avoid a contact. Find the magnetic force on a small length $d\ell$ of the second wire situated at a distance ℓ from the first wire.



Solution : The situation is shown in figure. The magnetic field at the site of $d\ell$, due to the first wire is ,

$$B = \frac{\mu_0 i_1}{2\pi\ell}$$

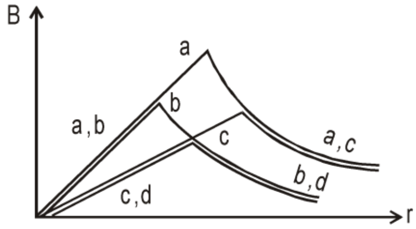
This field is perpendicular to the plane of the figure going into it. The magnetic force on the length $d\ell$ is,

$$dF = i_2\, d\ell\, B \sin 90^\circ = \frac{\mu_0 i_1 i_2 d\ell}{2\pi\ell}$$

This force is parallel to the current i_1.

Solved Examples

Example 29 : Curves in the graph shown give, as functions of radial distance r (from the axis), the magnitude B of the magnetic field (due to individual wire) inside and outside four long wires a, b, c and d, carrying currents that are uniformly distributed across the cross sections of the wires. Overlapping portions of the plots are indicated by double labels. All curves start from the origin.



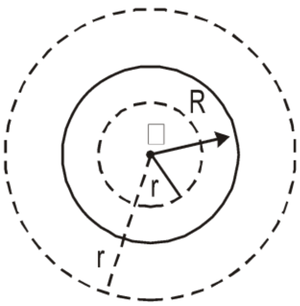
(i). Which wire has the greatest radius ?

(1) a (2) b (3) c (4) d

Sol. Inside the cylinder

$$B.2\pi r = \mu_0 . \frac{I}{\pi R^2}\pi r^2$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi R^2}.r \qquad ....(1)$$



outside the cylinder

$$B.2\pi r = \mu_0 I \qquad \therefore \qquad B = \frac{\mu_0 I}{2\pi r} \qquad ....(2)$$

Inside cylinder $B \propto r$ and outside $B \propto \frac{1}{r}$

So at the surface nature of magnetic field changes.

Hence clear from graph, wire 'c' has greatest radius.

(ii) Which wire has the greatest magnitude of the magnetic field on the surface ?

(1) a (2) b (3) c (4) d

Sol. Magnitude of magnetic field is maximum at the surface of wire 'a'.

(iii) The current density in wire a is

(1) greater than in wire c.

(2) less than in wire c.

(3) equal to that in wire c.

(4) not comparable to that of in wire c due to lack of information.

Sol. Inside the wire

$$B(r) = \frac{\mu_0}{2\pi}.\frac{I}{R^2}.r = \frac{\mu_0 Jr}{2}$$

$$\frac{dB}{dr} = \frac{\mu_0 J}{2}$$

i.e. slope $\propto J$

$\propto$ current density

It can be seen that slope of curve for wire a is greater than wire C.