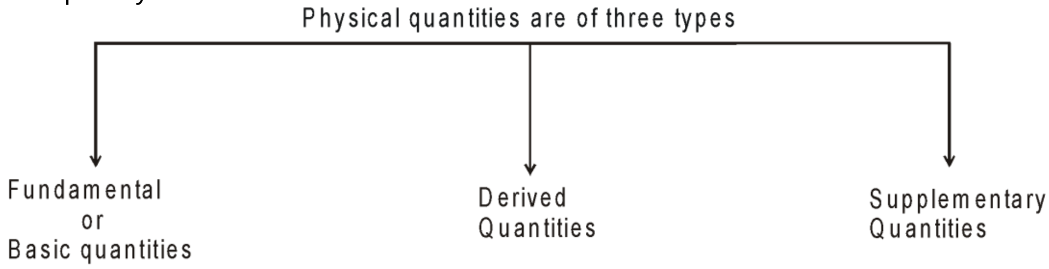
UNIT & DIMENSION

Physical Quantities :

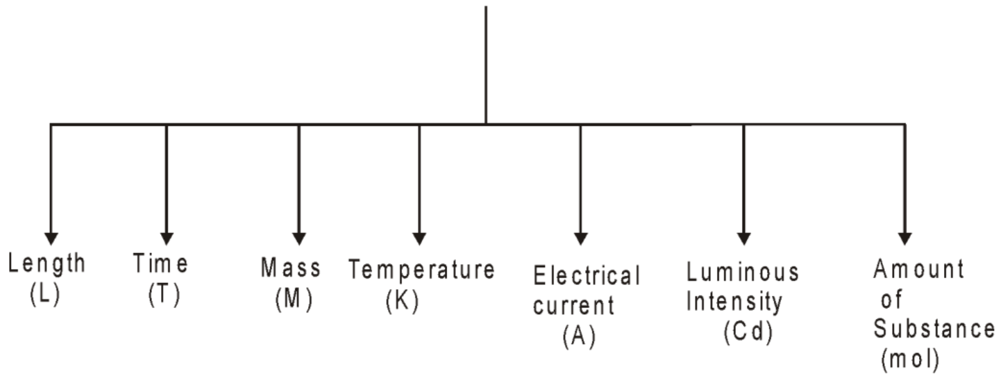
The quantities which can be measured by an instrument and by means of which we can describe the laws of physics are called physical quantities. Till class X we have studied many physical quantities
eg. length, velocity, acceleration, force, time, pressure, mass, density etc.
Those quantities which can describe the laws of physics & possible to measure are called physical quantities. A physical quantity is that which can be measured.
Physical quantity is completely specified :
If it has
Numerical value only (ratio); e.g. refractive index, dielectric constant etc.
Magnitude only (scalar); e.g. mass, charge etc.
Magnitude and Direction (vector); e.g. Displacement, torque etc.

Note: (1) There are also some physical quantities which are not completely specified even by magnitude, unit and direction. These physical quantities are called tensors. Ex. moment of Inertia.
(2) Physical quantity = Numerical value x unit

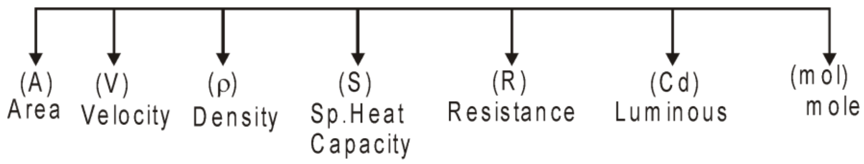


1. Fundamental (Basic) Quantities :

These are the elementary quantities which covers the entire span of physics.
Any other quantities can be derived from these.
All the basic quantities are chosen such that they should be different, that means independent of each other. (i.e., distance (d) , time (t) and velocity (v) cannot be chosen as basic quantities (because they are related as $V = \frac{d}{t}$). An International Organization named CGPM:General Conference on weight and Measures, has choosen seven physical quantities as basic or fundamental.

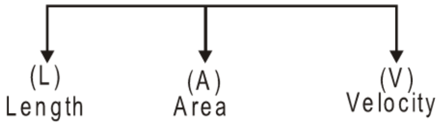


These are the elementary quantities (in our planet) that's why chosen as basic quantities.
In fact any set of independent quantities can be chosen as basic quantities by which all other physical quantities can be derived.



i.e.,

Can be chosen as basic quantities (on some other planet, these might also be used as basic quantities)



But
cannot be used as basic quantities as
Area = $(\text{Length})^2$ so they are not independent.

2. Derived Quantities :

Physical quantities which can be expressed in terms of basic quantities (M,L,T....) are called derived quantities.

i.e., Momentum $P = mv = (m)\dfrac{\text{displacement}}{\text{time}} = \dfrac{ML}{T} = M^1 L^1 T^{-1}$

For example speed = $\dfrac{\text{distance}}{\text{time}}$, Density = $\dfrac{\text{mass}}{\text{volume}}$

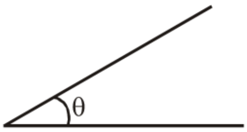
Ex.1 Which of the following sets cannot enter into the list of fundamental quantities in any system of units.
(1) Length, mass and velocity (2) Length, time and velocity
(3) Mass, time and velocity (4) Length, time and mass

Sol. The group of fundamental quantities are those quantities which do not depend upon other physical quantities in the group. But is set (2) we can predict the relation between given quantities as length = velocity × time. Hence set (2) cannot enter into the list of fundamental quantities.
Hence correct answer is (2)
Here [$M^1 L^1 T^{-1}$] is called dimensional formula of momentum , and we can say that momentum has
1 Dimension in M (mass)
1 Dimension in L (length)
and –1 Dimension in T (time)

The representation of any quantity in terms of basic quantities (M,L,T....) is called dimensional formula and in the representation, the powers of the basic quantities are called dimensions.

3. Supplementary quantities :

Besides seven fundamental quantities two supplementary quantities are also defined. They are
- Plane angle (The angle between two lines)
- Solid angle



(a) Radian : 1 radian is the angle subtended by an arc of length equal to the radius, of the centre of the circle.
(b) Steradian : It is defined as the solid angle subtended at the centre of a sphere by an area of its surface equal to the square of radius of the sphere.

Solid angle $\Omega = \dfrac{A}{R^2}$ where $A = R^2$, then $\Omega = 1$ steradian

Self Practise Problems

1. Which of the following is usually a derived quantity ?
(1) mass (2) velocity (3) length (4) time

2. A dimensionless quantity
(1) never has a unit (2) always has a unit (3) may have a unit (4) does not exist

3. A unitless quantity
(1) never has a non-zero dimension (2) always has a non-zero dimension
(3) may have a non-zero dimension (4) does not exist

4. choose the wrong statement
(1) all quantities can be expressed dimensionally in terms of the fundamental quantities

(2) a fundamental quantity cannot be represented dimensionally in terms of the rest of fundamental quantities
(3) the dimension of a fundamental quantity, in other fundamental quantities is always zero
(4) the dimension of a derived quantity is never zero in any fundamental quantity
Answer Key : 1. (2) **2.** (3) **3.** (1) **4.** (4)

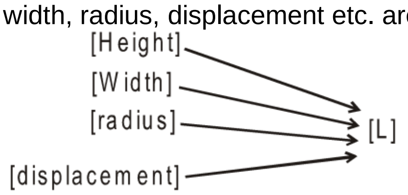
Finding Dimensions of various physical quantities :

The limit of a derived quantity in terms of necessary basic quantities is called dimensional formula and the raised powers on the basic quantities are called dimensions.
The basic units are represented as :
Mass → M Distance → L Time → T
Temperature → KElectric Current → A Luminous Intensity → Cd Amount of Substance → mol.

Note :

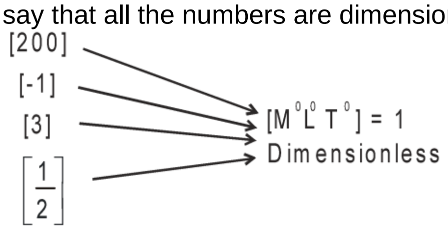
1. A physical quantity may have a number of units but their dimensions would be same,
e.g. The units of velocity are: cms^{-1}, ms^{-1}, kms^{-1}. But the dimensional formula is $M^0L^1T^1$.
2. Dimension does not depend on the unit of quantity.

Height, width, radius, displacement etc. are a kind of length. So we can say that their dimension is [L]



here [Height] can be read as "Dimension of Height"
For rectangle

Area = Length × Width
So, dimension of area is [Area] = [Length] × [Width]
= [L] × [L]
= [L^2]
For circle
Area = πr^2
[Area] = [π] [r^2] = [1] [L^2] = [L^2]
Here π is not a kind of length or mass or time so π shouldn't affect the dimension of area.
Hence its dimension should be 1 ($M^0L^0T^0$) and we can say that it is dimensionless. From similar logic we can say that all the numbers are dimensionless.



For cube
[Volume] = [Length] × [Width] × [Height]
= L × L × L = [L^3]
For sphere

Volume = $\frac{4}{3}\pi r^3$

[Volume] = $\left[\frac{4}{3}\pi\right]$ [r^3]
= (1) [L^3] = [L^3]
So dimension of volume will be always [L^3] whether it is volume of a cuboid or volume of sphere.

Dimension of a physical quantity will be same, it doesn't depend on which formula we are using for that quantity.

Density = $\frac{\text{mass}}{\text{volume}}$

[Density] $= \frac{[mass]}{[volume]} = \frac{M}{L^3} = [M^1L^{-3}]$

Velocity (v) = $\frac{displacement}{time}$

[v] = $\frac{[Displacement]}{[time]} = \frac{L}{T} = [M^0L^1T^{-1}]$

Acceleration (a) = $\frac{dv}{dt}$

[a] = $\left[\frac{dv}{dt}\right] = \frac{LT^{-1}}{T} = LT^{-2}$

Momentum (P) = mv

$[P] = [M]\ [v] = [M]\ [LT^{-1}] = [M^1L^1T^{-1}]$

Force (F) = ma

$[F] = [m]\ [a] = [M]\ [LT^{-2}]$

$= [M^1L^1T^{-2}]$ (You should remember the dimensions of force because it is used several times)

Work or Energy = force × displacement

[Work] = [force] [displacement] $= [M^1L^1T^{-2}]\ [L] = [M^1L^2T^{-2}]$

Power = $\frac{work}{time}$

[Power] = $\frac{[work]}{[time]} = \frac{M^1L^2T^{-2}}{T} = [M^1L^2T^{-3}]$

Pressure = $\frac{Force}{Area}$

[Pressure] = $\frac{[Force]}{[Area]} = \frac{M^1L^1T^{-2}}{L^2} = M^1L^{-1}T^{-2}$

1. Dimensions of angular quantities :

Angle (θ)

(Angular displacement) $\theta = \frac{Arc}{radius}$

$[\theta] = \frac{[Arc]}{[radius]} = \frac{L}{L} = [M^0L^0T^0]$ (Dimensionless)

Angular velocity (ω) = $\frac{\theta}{t}$

$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = [M^0L^0T^{-1}]$

Angular acceleration (α) = $\frac{d\omega}{dt}$

$[\alpha] = \frac{[d\omega]}{[dt]} = \frac{M^0L^0T^{-1}}{T} = [M^0L^0T^{-2}]$

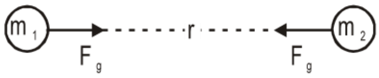
Torque = Force × Arm length

[Torque] = [force] × [arm length]

$= [M^1L^1T^{-2}] \times [L] = [M^1L^2T^{-2}]$

2. Dimensions of Physical Constants :

Gravitational Constant :



If two bodies of mass m_1 and m_2 are placed at r distance, both feel gravitational attraction force, whose value is,

Gravitational force $F_g = \frac{Gm_1m_2}{r^2}$

where G is a constant called Gravitational constant

$$[F_g] = \frac{[G][m_1][m_2]}{[r^2]}$$

$$[M^1L^1T^{-2}] = \frac{[G][M][M]}{[L^2]}$$

$$[G] = M^{-1}L^3T^{-2}$$

Specific heat capacity :

To increase the temperature of a body by ΔT, Heat required is $Q = ms\,\Delta T$

Here s is called specific heat capacity.

$$[Q] = [m]\,[s]\,[\Delta T]$$

Here Q is heat : A kind of energy so $[Q] = M^1L^2T^{-2}$

$$[M^1L^2T^{-2}] = [M]\,[s]\,[K]$$

$$[s] = [M^0L^2T^{-2}K^{-1}]$$

Gas constant (R) :

For an ideal gas, relation between Pressure (P), Volume (V), Temperature (T) and moles of gas (n) is PV = nRT where R is a constant, called gas constant.

$[P]\,[V] = [n]\,[R]\,[T]$ .... (1)

here $[P]\,[V] = \frac{[Force]}{[Area]}\,[Area \times Length]$

$= [Force] \times [length]$

$= [M^1L^1T^{-2}]\,[L^1] = M^1L^2T^{-2}$

From equation (1)

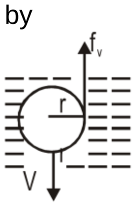
$[P]\,[V] = [n]\,[R]\,[T]$

$\Rightarrow [M^1L^2T^{-2}] = [mol]\,[R]\,[K]$

$\Rightarrow [R] = [M^1L^2T^{-2}\,mol^{-1}\,K^{-1}]$

Coefficient of viscosity :

If any spherical ball of radius r moves with velocity v in a viscous liquid, then viscous force acting on it is given by



$$F_v = 6\pi\eta rv$$

Here η is coefficient of viscosity

$$[F_v] = [6\pi]\,[\eta]\,[r]\,[v]$$

$$M^1L^1T^{-2} = (1)\,[\eta]\,[L]\,[LT^{-1}]$$

$$[\eta] = M^1L^{-1}T^{-1}$$

Planck's constant :

If light of frequency υ is falling , energy of a photon is given by

$E = h\upsilon$ Here h = Planck's constant

$[E] = [h]\,[\upsilon]$

υ = frequency = $\frac{1}{\text{Time Period}}$ $\Rightarrow$ $[\upsilon] = \frac{1}{[\text{Time Period}]} = \left[\frac{1}{T}\right]$

so $M^1L^2T^{-2} = [h]\,[T^{-1}]$

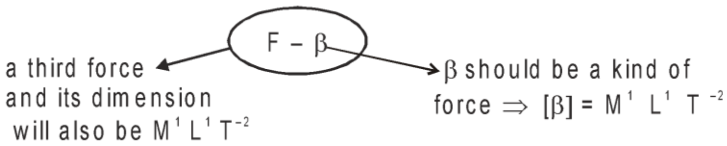
$[h] = M^1L^2T^{-1}$

3. Some special features of dimensions :

Suppose in any formula, $(L + \alpha)$ term is coming (where L is length). As length can be added only with a length, so α should also be a kind of length.

So $[\alpha] = [L]$

Similarly consider a term $(F - \beta)$ where F is force. A force can be added/substracted with a force only and give rises to a third force. So β should be a kind of force and its result $(F - \beta)$ should also be a kind of force.



Rule No. 1 : One quantity can be added / substracted with a similar quantity only and gives rise to the similar quantity.

Ex.10 $\frac{\alpha}{t^2} = Fv + \frac{\beta}{x^2}$

Find dimension formula for $[\alpha]$ and $[\beta]$ (here t = time, F = force, v = velocity, x = distance)

Sol. Since dimension of $Fv = [Fv] = [M^1L^1T^{-2}]\ [L^1T^{-1}] = [M^1L^2T^{-3}]$,

so $\left[\frac{\beta}{x^2}\right]$ should also be $M^1L^2T^{-3}$

$\frac{[\beta]}{[x^2]} = M^1L^2T^{-3}$

$[\beta] = M^1L^4T^{-3}$

and $\left[Fv + \frac{\beta}{x^2}\right]$ will also have dimension $M^1L^2T^{-3}$, so L.H.S. should also have the same dimension $M^1L^2T^{-3}$

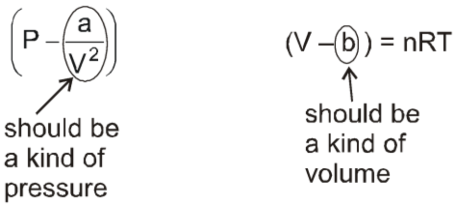
so $\frac{[\alpha]}{[t^2]} = M^1L^2T^{-3}$

$[\alpha] = M^1L^2T^{-1}$

Ex.11 For n moles of gas, Vander waal's equation is

$\left(P - \frac{a}{V^2}\right)(V - b) = nRT$

Find the dimensions of a and b, where P is gas pressure, V = volume of gas T = temperature of gas



Sol.

So $\frac{[a]}{[V^2]} = M^1L^{-1}T^{-2}$ So $[b] = L^3$

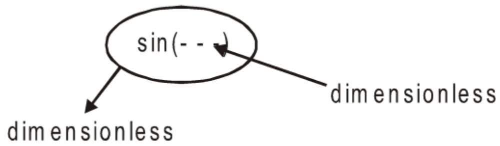
$\frac{[a]}{[V^2]} = M^{-1}L^{-1}T^{-2}$ $\Rightarrow$ $[a] = M^1L^5T^{-2}$

Rule No. 2 : **Consider a term sin(θ)**

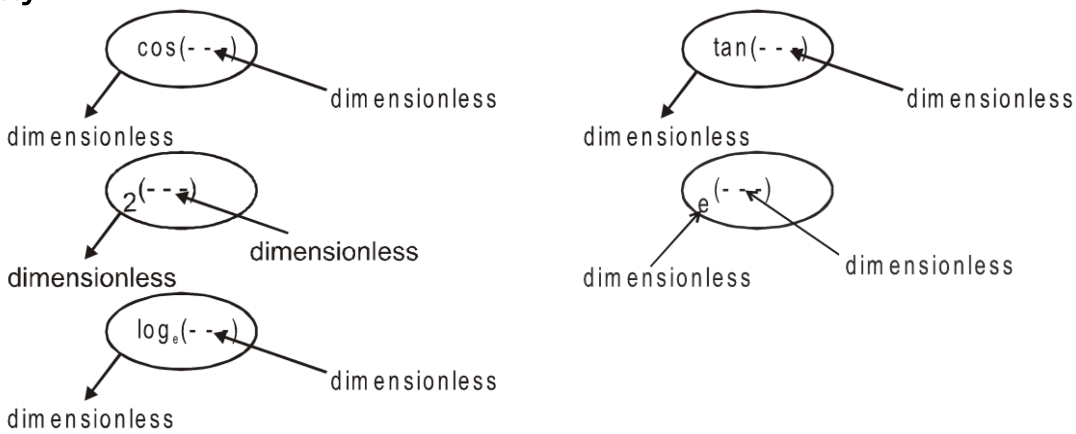
Here θ is dimensionless and sinθ $\left(\frac{\text{Perpendicular}}{\text{Hypoteneous}}\right)$ is also dimensionless.

⇒ Whatever comes in sin(......) is dimensionless and entire [sin (.......)] is also dimensionless.

⇒



Similarly :



Self Practise Problems

12. a, b are two different physical quantities with different dimensions which one of the following is correct.

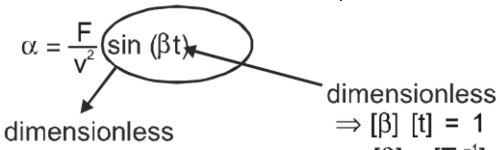
(1) a + b (2) a – b (3) a/b (4) $e^{a/b}$

Answer Key : (3)

Ex.12 $\alpha = \frac{F}{v^2}\sin(\beta t)$ (here v = velocity, F = force, t = time)

Find the dimension of α and β

Sol.



$\Rightarrow [\beta]\ [t] = 1$

$[\beta] = [T^{-1}]$

So $[\alpha] = \frac{[F]}{[v^2]} = \frac{[M^1L^1T^{-2}]}{[L^1T^{-1}]^2} = M^1L^{-1}T^0$

Ex.13 $\alpha = \frac{Fv^2}{\beta^2} \log_e \left(\frac{2\pi\beta}{v^2}\right)$ where F = force , v = velocity

Find the dimensions of α and β.

Sol.

$$\alpha = \frac{Fv^2}{\beta^2}\left(\log_e \frac{2\pi\beta}{v^2}\right)$$

dimensionless dimensionless

$\frac{[2\pi][\beta]}{[v^2]} = 1$ $\frac{[1][\beta]}{L^2T^{-2}} = 1$ $[\beta] = L^2T^{-2}$

as $[\alpha] = \frac{[F][v^2]}{[\beta^2]}$ $[\alpha] = \frac{[M^1L^1T^{-2}][L^2T^{-2}]}{[L^2T^{-2}]^2}$ $\Rightarrow$ $[\alpha] = M^1L^{-1}T^0$

4. USES OF DIMENSIONS :

(i) Conversion of one system of units into another :

(ii) To check the correctness of the formula :

(iii) We can derive a new formula roughly :

(iv) We can express any quantity in terms of the given basic quantities.

(i) Conversion of one system of units into another :

Let n_1 and n_2 be the numerical values of a given quantity Q in two unit system then.

$U_1 = M_1^aL_1^bT_1^c$ and $U_2 = M_2^aL_2^bT_2^c$ (in two systems respectively)

Therefore, By the principle nU = constant

$\Rightarrow$ $n_2U_2 = n_1U_1$

$$n_2[M_2^a\,L_2^b\,T_2^c] = n_1[M_1^a\,L_1^b\,T_1^c]$$

$\Rightarrow$ $n_2 = \frac{n_1[M_1^a\,L_1^b\,T_1^c]}{[M_2^a\,L_2^b\,T_2^c]}$ $\Rightarrow$ $\boxed{n_2 = \left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c n_1}$

(ii) To check the correctness of the formula :

If the dimensions of the L.H.S and R.H.S are same, then we can say that this equation is at least dimensionally correct. So the equation may be correct.

But if dimensions of L.H.S and R.H.S is not same then the equation is not even dimensionally correct. So it cannot be correct.

e.g. A formula is given centrifugal force $F_e = \frac{mv^2}{r}$

(where m = mass , v = velocity , r = radius)

we have to check whether it is correct or not.

Dimension of L.H.S is

$[F] = [M^1L^1T^{-2}]$

Dimension of R.H.S is

$\frac{[m]\,[v^2]}{[r]} = \frac{[M]\,[LT^{-1}]^2}{[L]} = [M^1L^1T^{-2}]$

So this eqn. is at least dimensionally correct.

thus we can say that this equation may be correct.

Ex.14 Check whether this equation is correct or not

Pressure $P_r = \frac{3\,Fv^2}{\pi^2t^2x}$ (where P_r = Pressure , F = force , v = velocity , t = time , x = distance)

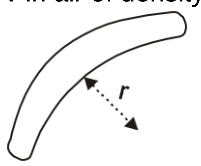
Sol. Dimension of L.H.S $= [P_r] = M^1L^{-1}T^{-2}$

Dimension of R.H.S $= \frac{[3]\,[F]\,[v^2]}{[\pi]\,[t^2]\,[x]} = \frac{[M^1L^1T^{-2}]\,[L^2T^{-2}]}{[T^2]\,[L]} = M^1L^2T^{-6}$

Dimension of L.H.S and R.H.S are not same. So the relation cannot be correct.

Sometimes a question is asked which is beyond our syllabus, then certainly it must be the question of dimensional analyses.

Ex.15 A Boomerang has mass m, surface Area A, radius of curvature of lower surface r and it is moving with velocity v in air of density ρ. The resistive force on it should be –



(1) $\frac{2\rho v A}{r^2}\log\left(\frac{\rho m}{\pi A r}\right)$ (2) $\frac{2\rho v^2 A}{r}\log\left(\frac{\rho A}{\pi m}\right)$

(3) $2\rho v^2 A\log\left(\frac{\rho A r}{\pi m}\right)$ (4) $\frac{2\rho v^2 A}{r^2}\log\left(\frac{\rho A r}{\pi m}\right)$

Sol. Only 3 is dimensionally correct.

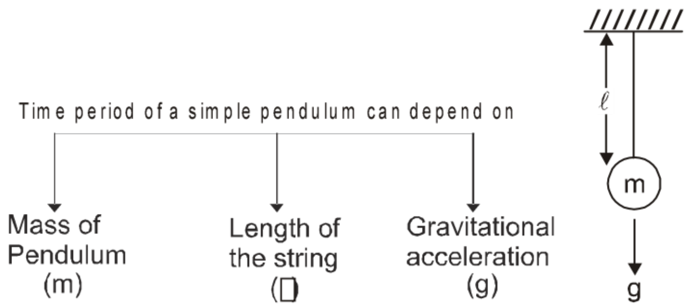
13. The dimensions of impulse are equal to that of
(1) force (2) angular momentum (3) pressure (4) linear momentum

14. The velocity of water waves may depend on their wavelength λ, the density of water ρ and the acceleration due to gravity g. The method of dimensions gives the relation between these quantities as
(1) $v^2 = K\lambda^{-1} g^{-1} \rho^{-1}$ (2) $v^2 = K g\lambda$ (3) $v^2 = K g\lambda\rho$ (4) $v^2 = k\lambda^3 g^{-1}\rho^{-1}$

Answer Key : 13. (4) **14.** (2)

(iii) We can derive a new formula roughly :
If a quantity depends on many parameters, we can estimate, to what extent, the quantity depends on the given parameters !

Ex.16



So we can say that expression of T should be in this form

$T = (\text{Some Number})\,(m)^a\,(\ell)^b\,(g)^c$

Equating the dimentions of LHS and RHS,

$M^0L^0T^1 = (1)\,[M^1]^a\,[L^1]^b\,[L^1T^{-2}]^c$

$M^0L^0T^1 = M^a\,L^{b+c}\,T^{-2c}$

Comparing the powers of M,L and T, get $a = 0,\ b + c = 0,\ -2c = 1$

so $a = 0,\ b = \frac{1}{2},\ c = -\frac{1}{2}$ so $T = (\text{some Number})\,M^0\,L^{1/2}\,g^{-1/2}$

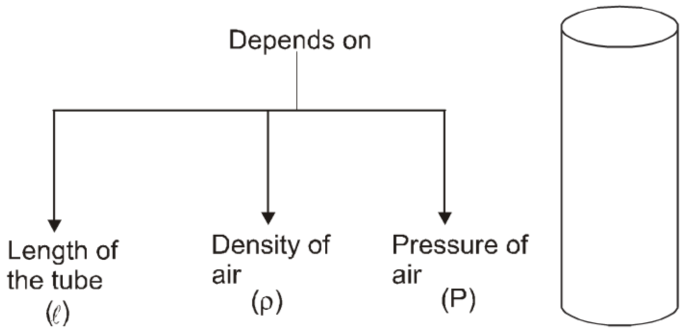
$T = (\text{Some Number})\sqrt{\frac{\ell}{g}}$

The quantity "Some number" can be found experimentally. Measure the length of a pendulum and oscillate it, find its time period by stopwatch.
Suppose for ℓ = 1m, we get T = 2 sec. so

$2 = (\text{Some Number})\sqrt{\frac{1}{9.8}}$

"Some number" = 6.28 ≈ 2π.

Ex.17 Natural frequency (f) of a closed pipe



So we can say that f = (some Number) $(\ell)^a\ (\rho)^b\ (P)^c$

$$\left[\frac{1}{T}\right] = (1)\ [L]^a\ [ML^{-3}]^b\ [M^1L^{-1}T^{-2}]^c$$

$M^oL^oT^{-1} = M^{b+c}\ L^{a-3b-c}\ T^{-2c}$

comparing powers of M, L, T

$0 = b + c$

$0 = a - b - c$

$-1 = -2c$

get $a = -1$, $b = -1/2$, $c = 1/2$

So f = (some number) $\frac{1}{\ell}\sqrt{\frac{P}{\rho}}$

(iii) We can express any quantity in terms of the given basic quantities.

Ex.18 If velocity (V), force (F) and time (T) are chosen as fundamental quantities , express (i) mass and (ii) energy in terms of V,F and T

Sol. Let M = (some Number) $(V)^a\ (F)^b\ (T)^c$

Equating dimensions of both the sides

$M^1L^0T^0 = (1)\ [L^1T^{-1}]^a\ [M^1L^1T^{-2}]^b\ [T^1]^c$

$M^1L^0T^0 = M^b\ L^{a+b}\ T^{-a-2b+c}$

get $a = -1$, $b = 1$, $c = 1$

M = (Some Number) $(V^{-1}\ F^1\ T^1)$ ⇒ $[M] = [V^{-1}\ F^1\ T^1]$

Similarly we can also express energy in terms of V , F , T

Let $[E]$ = [some Number] $[V]^a\ [F]^b\ [T]^c$

⇒ $[ML^2T^{-2}] = [M^oL^oT^o]\ [LT^{-1}]^a\ [MLT^{-2}]^b\ [T]^c$

⇒ $[M^1L^2T^{-2}] = [M^b\ L^{a+b}\ T^{-a-2b+c}]$

⇒ $1 = b$; $2 = a + b$; $-2 = -a - 2b + c$

get $a = 1$; $b = 1$; $c = 1$

∴ E = (some Number) $V^1F^1T^1$ or $[E] = [V^1][F^1][T^1]$.

Self Practise Problems

15. If force, length and time would have been the fundamental units, what would have been the dimensional formula for mass ?

(1) $F\ L^{-1}\ T^2$ (2) $F\ L\ T^{-2}$ (3) $F\ L\ T^{-1}$ (4) F

Answer Key : 15. (1)

(iv) To find out unit of a physical quantity :

Suppose we want to find the unit of force. We have studied that the dimension of force is

[Force] = $[M^1L^1T^{-2}]$

As unit of M is kilogram (kg) , unit of L is meter (m) and unit of T is second (s) so unit of force can be written as $(kg)^1\ (m)^1\ (s)^{-2}$ = kg m/s^2 in MKS system. In CGS system, unit of force can be written as $(g)^1\ (cm)^1\ (s)^{-2}$ = g cm/s^2.

Limitations of Dimensional Analysis :

From Dimensional analysis we get T = (Some Number) $\sqrt{\frac{\ell}{g}}$

so the expression of T can be :

$T = 2\sqrt{\frac{\ell}{g}}$ or $T = \sqrt{\frac{\ell}{g}}\ \sin(.....)$ or

$T = 50\sqrt{\frac{\ell}{g}}$ or $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T = \sqrt{\frac{\ell}{g}}\ \log(......)$ or $T = \sqrt{\frac{\ell}{g}} + (t_0)$

Dimensional analysis doesn't give information about the "some Number" :The dimensional constant.

This method is useful only when a physical quantity depends on other quantities by multiplication and power relations.

(i.e., $f = x^a y^b z^c$)

It fails if a physical quantity depends on sum or difference of two quantities

(i.e.$f = x + y - z$)

i.e., we cannot get the relation

$S = ut + \frac{1}{2}at^2$ from dimensional analysis.

This method will not work if a quantity depends on another quantity as sine or cosine ,logarithmic or exponential relation. The method works only if the dependence is by power functions.

We equate the powers of M,L and T hence we get only three equations. So we can have only three variable (only three dependent quantities)
So dimensional analysis will work only if the quantity depends only on three parameters, not more than that.

Ex.19 Can Pressure (P), density (ρ) and velocity (v) be taken as fundamental quantities ?

Sol. P, ρ and v are not independent, they can be related as $P = \rho v^2$,so they cannot be taken as fundamental variables.
To check whether the 'P' , 'ρ' , and 'V' are dependent or not, we can also use the following mathematical method :

$[P] = [M^1L^{-1}T^{-2}]$
$[\rho] = [M^1L^{-3}T^0]$
$[V] = [M^0L^1T^{-1}]$

Check the determinant of their powers :

$$\begin{vmatrix} 1 & -1 & -2 \\ 1 & -3 & 0 \\ 0 & 1 & -1 \end{vmatrix} = 1(3) - (-1)(-1) - 2(1) = 0,$$

So these three terms are dependent.

ONLY FOR 12TH CLASS STUDENTS

DIMENSIONS BY SOME STANDARD FORMULAE :

In many cases, dimensions of some standerd expression are asked **e.g.** find the dimension of ($\mu_0\varepsilon_0$) for this, we can find dimensions of μ_0 and ε_0, and multiply them, but it will be very lengthy process. Instead of this, we should just search a formula, where this term ($\mu_0\varepsilon_0$) comes.

It comes in c = (where $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ speed of light)

$\Rightarrow \mu_0\,\varepsilon_0 = \frac{1}{c^2}$ $[\mu_0\varepsilon_0] = \frac{1}{c^2} = \frac{1}{(L/T)^2} = L^{-2}\,T^2$

Ex.20 Find the dimensions of

(i) ε_0E^2 (ε_0 = permitivity in vaccum , E = electric field)

(ii) $\frac{B^2}{\mu_0}$ (B = Magnetic field , μ_0 = magnetic permeability)

(iii) $\frac{1}{\sqrt{LC}}$ (L = Inductance , C = Capacitance)
(iv) RC (R = Resistance , C = Capacitance)
(v) $\frac{L}{R}$ (R = Resistance , L = Inductance)
(vi) $\frac{E}{B}$ (E = Electric field , B = Magnetic field)
(vii) $G\varepsilon_0$ (G = Universal Gravitational constant , ε_0 = permitivity in vaccum)
(viii) $\frac{\phi_e}{\phi_m}$ (ϕ_e = Electrical flux ; ϕ_m = Magnetic flux)

Sol. (i) Energy density = $\frac{1}{2}\varepsilon_0 E^2$
[Energy density] = $[\varepsilon_0 E^2]$

$$\left[\frac{1}{2}\varepsilon_0 E^2\right] = \frac{[\text{energy}]}{[\text{volume}]} = \frac{M^1L^2T^{-2}}{L^3} = M^1L^{-1}T^{-2}$$

(ii) $\frac{1}{2}\frac{B^2}{\mu_0}$ = Magnetic energy density

$$\left[\frac{1}{2}\frac{B^2}{\mu_0}\right] = [\text{Magnetic Energy density}]$$

$$\left[\frac{B^2}{\mu_0}\right] = \frac{[\text{energy}]}{[\text{volume}]} = \frac{M^1L^2T^{-2}}{L^3} = M^1L^{-1}T^{-2}$$

(iii) $\frac{1}{\sqrt{LC}}$ = angular frequency of L – C oscillation

$$\left[\frac{1}{\sqrt{LC}}\right] = [\omega] = \frac{1}{T} = T^{-1}$$

(iv) RC = Time constant of RC circuit = a kind of time
$[RC] = [\text{time}] = T^1$

(v) $\frac{L}{R}$ = Time constant of L – R circuit

$$\left[\frac{L}{R}\right] = [\text{time}] = T^1$$

(vi) magnetic force $F_m = qvB$, electric force $F_e = qE$
$\Rightarrow$ $[F_m] = [F_e]$ $\Rightarrow$ $[qvB] = [qE]$

$$\left[\frac{E}{B}\right] = [v] = LT^{-1}$$

(vii) Gravitational force $F_g = \frac{Gm^2}{r^2}$, Electrostatic force $F_e = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

$$\left[\frac{Gm^2}{r^2}\right] = \left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] \Rightarrow [G\varepsilon_0] = \left[\frac{q^2}{m^2}\right] = \left[\frac{(it)^2}{m^2}\right] = A^2T^2M^{-2}$$

(viii)
$$\left[\frac{\phi_e}{\phi_m}\right] = \left[\frac{ES}{BS}\right] = \left[\frac{E}{B}\right] = [\mathbf{v}] = LT^{-1}$$

Dimensions of quantities related to Electromagnetic and Heat (only for XII and XIII students)

(i) Charge (q) :→

We know that electrical current $i = \frac{dq}{dt} = \frac{\text{a small charge flow}}{\text{small time interval}}$

$[i] = \frac{[dq]}{[dt]}$ $[A] = \frac{[q]}{t}$ $\Rightarrow [q] = [A^1 T^1]$

(ii) Permittivity in Vacuum (ε_0) :→

Electrostatic force between two charges $F_e = \frac{k q_1 q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2}$

$[F_e] = \frac{1}{[4\pi][\varepsilon_0]} \frac{[q_1][q_2]}{[r]^2}$

$M^1 L^1 T^{-2} = \frac{1}{(1)[\varepsilon_0]} \frac{[AT][AT]}{[L]^2}$

$[\varepsilon_0] = M^{-1} L^{-3} T^4 A^2$

(iii) Electric Field (E) : → Electrical force per unit charge $E = \frac{F}{q}$

$[E] = \frac{[F]}{[q]} = \frac{[M^1 L^1 T^{-2}]}{[A^1 T^1]} = M^1 L^1 T^{-3} A^{-1}$

(iv) Electrical Potential (V) :→ Electrical potential energy per unit charge $V = \frac{U}{q}$

$[V] = \frac{[U]}{[q]} = \frac{[M^1 L^2 T^{-2}]}{[A^1 T^1]} = M^1 L^2 T^{-3} A^{-1}$

(v) Resistance (R) :→

From Ohm's law $V = i R$

$[V] = [i]\,[R]$

$[M^1 L^2 T^{-3} A^{-1}] = [A^1]\,[R]$

$[R] = M^1 L^2 T^{-3} A^{-2}$

(vi) Capacitance (C) :→

$C = \frac{q}{V} \Rightarrow [C] = \frac{[q]}{[V]} = \frac{[A^1 T^1]}{[M^1 L^2 T^{-3} A^{-1}]}$

$[C] = M^{-1} L^{-2} T^4 A^2$

(vii) Magnetic field (B) :→

magnetic force on a current carrying wire $F_m = i \ell B \Rightarrow [F_m] = [i]\,[\ell]\,[B]$

$[M^1 L^1 T^{-2}] = [A^1]\,[L^1]\,[B]$

$[B] = M^1 L^0 T^{-2} A^{-1}$

(viii) Magnetic permeability in vacuum (μ_0) :→

Force /length between two wires $\frac{F}{\ell} = \frac{\mu_o}{4\pi} \frac{i_1 i_2}{r^2}$

$\frac{M^1 L^1 T^{-2}}{L^1} = \frac{[\mu_O]}{[4\pi]} \frac{[A][A]}{[L]^2} \Rightarrow [\mu_0] = M^1 L^2 T^{-2} A^{-2}$

(ix) Inductance (L) :→

Magnetic potential energy stored in an inductor $U = 1/2\, L i^2$

$[U] = [1/2]\,[L]\,[i]^2$

$[M^1 L^2 T^{-2}] = (1)\,[L]\,(A)^2$

$[L] = M^1 L^2 T^{-2} A^{-2}$

(x) Thermal Conductivity : →

Rate of heat flow through a conductor $\frac{dQ}{dt} = KA \left(\frac{dT}{dx}\right)$

$\frac{[dQ]}{[dt]} = [K]\,[A] \frac{[dT]}{[dx]}$

$$\frac{[M^1L^2T^{-2}]}{[T]} = [K]\,[L^2]\,\frac{[K]}{[L^1]}$$

$[K] = M^1L^1T^{-3}\,K^{-1}$

(xi) Stefan's Constant (σ): →

If a black body has temperature (T), then Rate of radiation energy emitted $\frac{dE}{dt} = \sigma A T^4$

$$\frac{[dE]}{[dt]} = [\sigma]\,[A]\,[T^4]$$

$$\frac{[M^1L^2T^{-2}]}{[T]} = [\sigma]\,[L^2]\,[K^4]$$

$[\sigma] = [M^1\,L^0\,T^{-3}\,K^{-4}]$

(xii) Wien's Constant : →

Wavelength corresponding to max. spectral intensity . $\lambda_m = \frac{b}{T}$ (where T = temp. of the black body)

$$[\lambda_m] = \frac{[b]}{[T]}$$

$$[L] = \frac{[b]}{[K]}$$

$[b] = [L^1K^1]$

Unit :

Measurement of any physical quantity is expressed in terms of an internationally accepted certain basic standard called unit.

For the measurement of a physical quantity a definite magnitude of quantity is taken as standard and the name given to this standard is called unit.

PROPERTIES OF UNIT

(a) The unit should be well-defined.
(b) The unit should be of some suitable size.
(c) The unit should be easily reproducible.
(d) The unit should not change with time.
(e) The unit should not change with physical condition like pressure, temperature etc.
(f) Unit should be of proper size.

SI Units : In 1971 , an international Organization "CGPM" : (General Conference on weight and Measure) decided the standard units, which are internationally accepted. These units are called SI units (International system of units)

1. SI Units of Basic Quantities :

Base Quantity	SI Units		
	Name	Symbol	Definition
Length	metre	m	The metre is the length of the path traveled by light in vacuum during a time interval of 1/299, 792, 458 of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

2. Two supplementary units were also defined :

Plane angle – Unit = radian (rad)
Solid angle – Unit = Steradian (sr)

(a) Radian : → 1 radian is the angle subtended by arc of length equal to the radius, at the centre of the circle.

(b) steradian : It is defined as the solid angle subtendid at the centre of the sphere by an arc of its surface equal to the square of radius of the sphere.

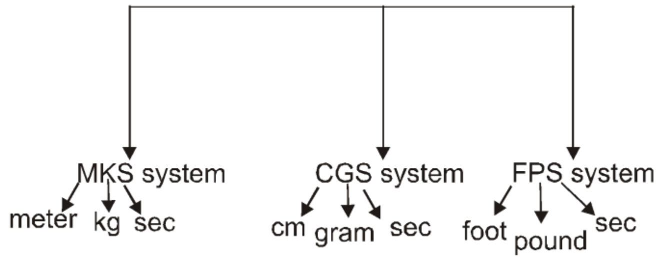
solid angle $\Omega = \frac{A}{R^2}$

when $A = R_2$

$\Omega = 1$ steradian .

3. Other classification :

If a quality involves only length, mass and time (quantities in mechanics), then its unit can be written in MKS, CGS or FPS system.



For MKS system : In this system Length, mass and time are expressed in meter, kg and sec. respectively. It comes under SI system.

For CGS system : In this system ,Length, mass and time are expressed in cm, gram and sec. respectively.

For FPS system : In this system, length, mass and time are measured in foot, pound and sec. respectively.

4. SI units of derived Quantities :

Velocity = $\dfrac{\text{displacement} \rightarrow \text{metre}}{\text{time} \rightarrow \text{second}}$

So unit of velocity will be m/s

Acceleration = $\dfrac{\text{change in velocity}}{\text{time}} = \dfrac{m/s}{s} = \dfrac{m}{s^2}$

Momentum = mv

so unit of momentum will be = (kg) (m/s) = kg m/s

Force = ma

Unit will be = (kg) × (m/s^2) = kg m/s^2 called newton (N)

Work = FS

unit = (N) × (m) = N m called joule (J)

Power = $\dfrac{\text{work}}{\text{time}}$

Unit = J / s called watt (W)

5. Units of some physical Constants :

Unit of "Universal Gravitational Constant" (G)

F = $\dfrac{G(m_1)(m_2)}{r^2}$ $\Rightarrow$ $\dfrac{kg \times m}{s^2} = \dfrac{G(kg)(kg)}{m^2}$

so unit of G = $\dfrac{m^3}{kg\ s^2}$

Unit of specific heat capacity (s) :

$Q = ms\,\Delta T$

J = (kg) (S) (K)

Unit of s = J / kg K

Unit of μ_0 :

force per unit length between two long parallel wires is: $\dfrac{F}{\ell} = \dfrac{\mu_0}{4\pi}\dfrac{i_1 i_2}{r^2}$

$\dfrac{N}{m} = \dfrac{\mu_0}{(1)}\dfrac{(A)(A)}{(m^2)}$ Unit of $\mu_0 = \dfrac{N.m}{A^2}$

6. SI Prefix :

Suppose distance between kota to Jaipur is 3000 m. so

d = 3000 m = 3 × (1000)m → kilo(k) = 3 km (here 'k' is the prefix used for 1000 (10^3))

Suppose thickness of a wire is 0.05 m   d = 0.05 m $= 5 \times 10^{-2}$ m (10^{-2} → centi(c))

= 5 cm (here 'c' is the prefix used for (10^{-2}))

Similarly, the magnitude of physical quantities vary over a wide range. So in order to express the very large magnitude as well as very small magnitude more compactly, "CGPM" recommended some standard prefixes for certain power of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Practical Units of Length

1. Light year = 9.46×10^{15} m
2. Parsec = 3.084×10^{16} m
3. Fermi = 10^{-15} m8. Acto meter = 10^{-18} m
4. Angstrom (A°) = 10^{-10} m
5. Micron/Micrometer = 10^{-6} m
6. Nano meter = 10^{-9} m
7. Picometer = 10^{-12} m
9. Astro nomical unit (A.U.) = 1.496×10^{11} m
10. Otto meter = 10^{-21} m

Some Important Practical Units

S.No. Quantity Unit

1. Mass Solar mass = 2×10^{30}
   Dalton = 1.66×10^{-27} kg
   Chander Shekhar = 1.4 times of mass of sun
2. Pressure Pascal = 1 N/m^2
   Bar = 10^5 N/m^2
3. Area barn = 10^{-28} m^2
4. Radio Activity Baquerrel
5. Radiation doze for cancer Rontgen
6. Time Shake = 10^{-8} sec

Ex.2 Convert all in meters (m) :

(i) 5 μm. (ii) 3 km (iii) 20 mm (iv) 73 pm (v) 7.5 nm

Sol. (i) 5 μm = 5×10^{-6} m

(ii) 3 km = 3×10^{3} m

(iii) 20 mm = 20×10^{-3} m

(iv) 73 pm = 73×10^{-12} m

(v) 7.5 nm = 7.5×10^{-9} m

Ex.3 F = 5 N convert it into CGS system

Sol. $F = 5 \frac{kg \times m}{s^2} = (5) \frac{(10^3 g)(100\ cm)}{s^2}$

$= 5 \times 10^5 \frac{g\ cm}{s^2}$ (in CGS system).

This unit ($\frac{g\ cm}{s^2}$) is also called dyne

Ex.4 $G = 6.67 \times 10^{-11} \frac{m^3}{kg\ s^2}$ convert it into CGS system.

Sol. $G = 6.67 \times 10^{-11} \frac{m^3}{kg\ s^2}$

$= (6.67\times10^{-11}) \frac{(100\ cm)^3}{(1000\ g)s^2}$ $= 6.67 \times 10^{-8} \frac{cm^3}{gs^2}$

Ex.5 $\rho = 2\ g/cm^3$
convert it into MKS system

Sol.
$\rho = 2\ g/cm^3$

$= (2) \frac{10^{-3}\ kg}{(10^{-2}m)^3}$

$= 2 \times 10^3\ kg/m^3$

Ex.6 V = 90 km / hour
convert it into m/s

Sol. V = 90 km / hour

$= (90) \frac{(1000\ m)}{(60 \times 60\ second)}$

$V = (90) \left(\frac{1000}{3600}\right) \frac{m}{s}$

$V = 90 \times \frac{5}{18} \frac{m}{s}$

V = 25 m/s

Point to remember : To convert km/hour into m/sec, multiply by $\frac{5}{18}$.

Ex.7 Convert 7 pm into μm

Sol. Let 7 pm = (x) μm , Now lets convert both LHS & RHS into meter

$7 \times (10^{-12})\ m = (x) \times 10^{-6}\ m$

get $x = 7 \times 10^{-6}$ So 7 pm = (7×10^{-6}) μm

Self Practise Problems

5. The unit of energy is
(1) J/s (2) watt-day (3) kilowatt (4) $g\text{-}cm/s^2$

6. In the S.I. system, the unit of temperature is
(1) degree centigrade (2) kelvin (3) degree Celsius (4) degree Fahrenheit

7. In the S.I. system the unit of energy is
(1) erg (2) calorie (3) joule (4) electron volt

8. Unit of pressure in S.I. system is
(1) atmosphere (2) dynes per square cm (3) pascal (4) bar

9. Which of the following is not a unit of time?
(1) microsecond (2) leap year (3) lunar month (4) light year

10. What will be the unit of time in that system in which the unit of length is metre unit of mass 'kg' and unit of force 'kg. wt' ?
(1) $1/\sqrt{9.8}$ sec (2) $(9.8)^2$ / sec (3) $\sqrt{9.8}$ sec (4) 9.8 sec

11. The M.K.S.A. system was first introduced by
(1) Archimedes (2) Galileo (3) Newton (4) Giorgi

Answer Key :
5. (2) **6.** (2) **7.** (3) **8.** (3) **9.** (4) **10.** (1) **11.** (4)

8. SI Derived units, named after the scientist :

S.N	Physical Quantity	SI Units			
		Unit name	Symbol of the unit	Expression in terms of other units	Expression in terms of base units
1.	Frequency $(f = \frac{1}{T})$	hertz	Hz	$\frac{\text{Oscillation}}{\text{s}}$	s^{-1}
2.	Force (F = ma)	newton	N	- - - - -	$Kg\ m / s^2$
3.	Energy, Work, Heat (W = Fs)	joule	J	Nm	$Kg\ m^2 / s^2$
4.	Pressure, stress $(P = \frac{F}{A})$	pascal	Pa	N / m^2	$Kg / m\ s^2$
5.	Power, $(\text{Power} = \frac{W}{t})$	watt	W	J / s	$Kg\ m^2 / s^3$
6.	Electric charge (q = it)	coulomb	C	- - - - - -	A s
7.	Electric Potential Emf. $(V = \frac{U}{q})$	volt	V	J / C	$Kg\ m^3 / s^3\ A$
8.	Capacitance $(C = \frac{q}{V})$	farad	F	C / V	$A\ s^4\ kg^{-1}\ m^{-2}$
9.	Electrical Resistance (V = i R)	ohm	Ω	V / A	$kg\ m^2\ s^{-3}\ A^{-2}$
10.	Electrical Conductance $(C = \frac{1}{R} = \frac{i}{V})$	siemens (mho)	S , ℧	A / V	$kg^{-1}\ m^{-2}\ s^3\ A^2$
11.	Magnetic field	tesla	T	Wb / m^2	$kg\ s^{-2}\ A^{-1}$
12.	Magnetic flux	weber	Wb	V s or J/A	$kg\ m^2 s^{-2}\ A^{-1}$
13.	Inductance	henry	H	Wb / A	$kg\ m^2\ s^{-2}\ A^{-2}$
14.	Activity of radioactive material	becquerel	Bq	$\frac{\text{Disintegration}}{\text{second}}$	s^{-1}

9. Some SI units expressed in terms of the special names and also in terms of base units:

Physical Quantity	SI Units	
	In terms of special names	In terms of base units
Torque (τ = Fr)	N m	Kg m^2 / s^2
Dynamic Viscosity ($F_v = \eta A \frac{dv}{dr}$)	Poiseiulle (P[illegible]) or Pa s	Kg / m s
Impulse ($J = F\,\Delta t$)	N s	Kg m / s
Modulus of elasticity ($Y = \frac{\text{stress}}{\text{strain}}$)	N / m^2	Kg / m s^2
Surface Tension Constant (T) ($T = \frac{F}{[illegible]}$)	N/m or J/m^2	Kg / s^2
Specific Heat capacity (s) ($Q = ms\Delta T$)	J/kg K (old unit s $\frac{cal}{g.\,^\circ C}$)	$m^2 s^{-2} K^{-1}$
Thermal conductivity (K) ($\frac{dQ}{dt} = KA\frac{dT}{dr}$)	W / m K	m kg $s^{-3} K^{-1}$
Electric field Intensity $E = \frac{F}{q}$	V/m or N/C	m kg $s^{-3} A^{-1}$
Gas constant (R) (PV = nRT) or molar Heat Capacity ($C = \frac{Q}{M\Delta T}$)	J / K mol	m^2 kg $s^{-2} k^{-1} mol^{-1}$

10. Change of Numerical value with the change of unit :

Suppose we have

$$\ell = 7\text{ cm} \xrightarrow[\text{it into metres, we get}]{\text{If we convert}} = \frac{7}{100}\text{m}$$

we can say that if the unit is increased to 100 times (cmm),

the numerical value became $\frac{1}{100}$ times $\left(7 \rightarrow \frac{7}{100}\right)$

So we can say

Numerical value $\propto \frac{1}{\text{unit}}$

we can also tell it in a formal way like the following :–

Magnitude of a physical quantity = (Its Numerical value) (unit)

= (n) (u)

Magnitude of a physical quantity always remains constant, it will not change if we express it in some other unit.

$(n)\ (u) = \text{constant}$

So $\quad n \propto \frac{1}{u} \qquad n_1u_1 = n_2u_2$

numerical value $\propto \frac{1}{\text{unit}}$

Ex.8 If unit of length is doubled, the numerical value of Area will be ....

Sol. As unit of length is doubled, unit of Area will become four times. So the numerical value of Area will became one fourth. Because numerical value $\propto \frac{1}{\text{unit}}$,

Ex.9 Force acting on a particle is 5N.If unit of length and time are doubled and unit of mass is halved than the numerical value of the force in the new unit will be.

Sol. Force = 5 $\frac{\text{kg} \times \text{m}}{\text{sec}^2}$

If unit of length and time are doubled and the unit of mass is halved.

Then the unit of force will be $\left(\frac{\frac{1}{2}\times 2}{(2)^2}\right) = \frac{1}{4}$ times

Hence the numerical value of the force will be 4 times. (as numerical value $\propto \frac{1}{\text{unit}}$)

Force = 20 units