NEWTON'S LAWS OF MOTION

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

(1) may change only speed
(2) may change only direction of motion.
(3) may change both the speed and direction of motion.
(4) may change size and shape of a body

Unit of force : newton and $\frac{kg \cdot m}{s^2}$ (MKS System)

dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 newton = 10^5 dyne

Kilogram force (kgf)

The force with which earth attracts a 1kg body towards its centre is called kilogram force, thus

kgf = $\frac{\text{Force in newton}}{g}$ ($g = 9.8\ m/s^2$)

Dimensional Formula of force : $[M L T^{-2}]$

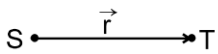
1.1 Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

[A] Gravitational Force

The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.

$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r}$$



$\vec{r}$ = position vector of test particle 'T' with respect to source particle 'S'.
G = universal gravitational constant = $6.67 \times 10^{-11}\ Nm^2/kg^2$.

(i) It is the weakest force and is always attractive.
(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.
(iii) It is independent of the nature of medium between the particles.

F = mg is the force exerted by earth on any particle of mass m near the earth surface. The value of $g = 9.81\ m/s^2 \simeq 10\ m/s^2 \simeq \pi^2\ m/s^2 \simeq 32\ ft/s^2$. It is also called acceleration due to gravity near the surface of earth.

[B] Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force
(a) These can be attractive or repulsive.
(b) These are long range forces
(c) These depend on the nature of medium between the charged particles.
(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

[C] Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

[D] Weak Force

It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.
It has been found that for two protons at a distance of 1 fermi :
$F_N:F_{EM}:F_W:F_G::1:10^{-2}:10^{-7}:10^{-38}$

2. NEWTON'S LAWS OF MOTION :

2.1 First Law of Motion

Each body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

If the net force acting on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration.
Newton's first law is sometimes called the **law of inertia** and the reference frames that it defines are called inertial reference frames.
Inertia : Inertia is the property of the body due to which body opposes the change of itself state. Inertia of a body is mearsured by mass of the body. **Inertia ∝ mass**
Heavier the body, greater is the force required to change its state and hence greater is the inertia.
Newton's law from an 1803 translation from Latin as Newton wrote
" Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

Examples of this law :

(a) A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it. The speed of bullet is very high. Due to its large inertia of motion, it cuts a clean hole through the glass. When a stone is thrown, it inertia is much lower so it cannot cut through the glass.
(b) A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.
(c) When a car rounds a curve suddenly, the person sitting inside is thrown outwards.

2.2 Second Law of Motion :

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.
Newton's law from an 1803 translation from Latin as Newton wrote
" The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."
Mathematically

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F} = m\vec{a} \quad \text{(if m = constant)}$$

where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.
Linear momentum: The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body.
Momentum $\vec{p} = m\vec{v}$
SI unit : kg m s^{-1}
Dimension : $[M L T^{-1}]$

Important points about second law

(a) The Second Law is obviously consistent with the First Law as F = 0 Implies a = 0.
(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors :

$$F_x = \frac{dp_x}{dt} = ma_x \qquad F_y = \frac{dp_y}{dt} = ma_y \qquad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(c) The Second Law of motion given above is strictly applicable to a single point mass. The force **F** in the law stand for the net external force on the particle and **a** stands for the acceleration of the particle. Any internal forces in the system are not to be included in **F.**

(d) The Second Law of motion is a local relation. What this means is that the force **F** at a point in space (location of the particle) at a certain instant of time is related to **a** at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

2.3 Third Law of Motion :

To every action, there is always an equal and opposite reaction. Newton's law from an 1803 translation from Latin as Newton wrote

"To every action there is always an equal and opposite reaction: to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

In case of two particles having linear momentum $\vec{P_1}$ and $\vec{P_2}$ and moving towards each other under mutual forces, from Newton's second law ;

$$\frac{d}{dt}\left(\vec{p_1}+\vec{p_2}\right) = \vec{F} = 0 \quad \Rightarrow \quad \frac{d\vec{p_1}}{dt} + \frac{d\vec{p_2}}{dt} = 0$$

$$\vec{F_1}+\vec{F_2} = 0 \quad \Rightarrow \quad \vec{F_2} = -\vec{F_1}$$

which is Newton's third law.

Example :

(1) In order to walk we press the gound in backward direction with our foot.

(2) When rubber ball is struck against a wall, the ball bounces back, due to the reaction of the well.

Important points about the Third Law

(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force'. A simple and clear way of stating the Third Law is as follows : Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on A by B and the force on B by A act at the same instant. Any one of them may be called action and the other reaction.

(c) Action and reaction forces act on different bodies, not on the same body. Thus if we are considering the motion of any one body (A or B), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.

However, if you are considering the system of two bodies as a whole, $\mathbf{F}_{AB}$ (force on A due to B) and $\mathbf{F}_{BA}$ (force on B due to A) are internal forces of the system (A + B). They add up to give a null force. Internal forces in a body or a system of particles thus cancel away in pairs. This is an important fact that enables the Second Law to be applicable to a body or a system of particles.

Note : I_{st} law defines force
2_{nd} law measures force
3_{rd} law gives the nature of the force (i.e. forces always exist in pairs)

Example. 2 A circket ball of mass 0.2 kg moves with a velocity of 20m/s and is brought to the rest by a player in 0.1s. Calculate average force applied by the player.

Sol. $\Delta p = p_2 - p_1 = mv - mu = 0 - (0.2 \times 20) = -4$Ns ;

Average force F = $\frac{p_2 - p_1}{t} = \frac{-4}{0.1} = -40$N

3 Classification of forces on the basis of contact :

(A) Field Force:

Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

(a) Gravitation force
(b) Electromagnetic force

(B) Contact Force:

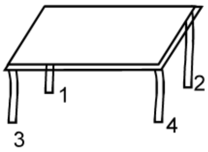
Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

Examples:

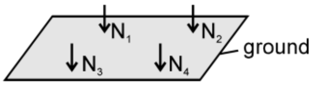
(a) Normal force (N):

It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force.

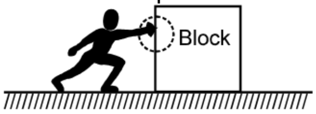
A table is placed on Earth as shown in figure



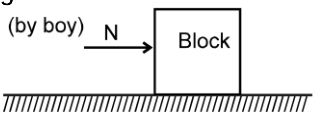
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



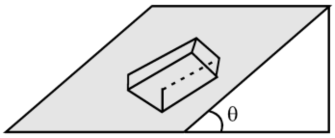
Now a boy pushes a block kept on a frictionless surface.



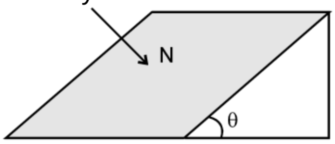
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



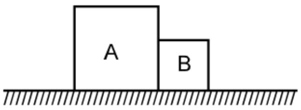
Normal force exerted by block on the surface of inclined plane is shown in figure.



Force acts perpendicular to the surface

Solved Examples

Example.1 Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

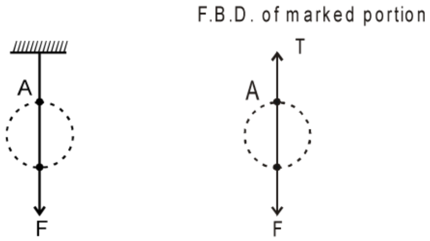


Solution: In above problem, block A does not push block B. Hence normal force exerted by A on B is zero.

Note : Normal is a dependent force, it comes in role when one surface presses the other.

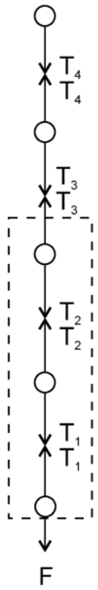
(b) Tension :

Tension in a string is a electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force ‘F’ as shown in figure, for calculating the tension at point ‘A’ we draw F.B.D. of marked portion of the string; Here string is massless.

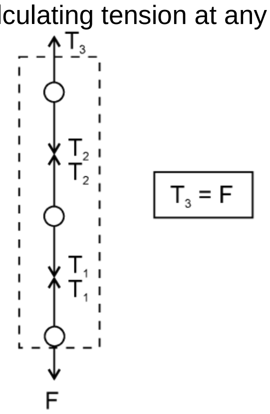


⇒ T = F

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to newton's third law.



For calculating tension at any segment, we consider two or more than two parts as a system.



Here interaction between segments are considered as internal forces, so they are not shown in F.B.D.

(C) Frictional force :

It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

4. SYSTEM :

Two or more than two objects which interact with each other form a system.

4.1 Classification of forces on the basis of boundary of system :

(A) Internal Forces:

Forces acting each with in a system among its constituents.

(B) External Forces:

Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

(C) Real Force:

Force which acts on an object due to other object is called as real force. An isolated object (far away from all objects) does not experience any real force.

5. FREE BODY DIAGRAM

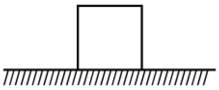
A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

5.1 Steps for F.B.D.

Step 1: Identify the object or system and isolate it from other objects clearly specify its boundary.
Step 2: First draw non-contact external force in the diagram. Generally it is weight.
Step 3: Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.
In F.B.D, internal forces are not drawn, only external are drawn.

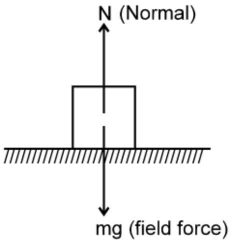
Solved Examples

Example.2 A block of mass 'm' is kept on the ground as shown in figure.

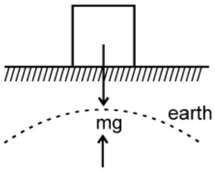


(i) Draw F.B.D. of block.
(ii) Are forces acting on block action–reaction pair.
(iii) If answer is no, draw action reaction pair.

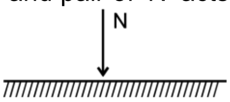
Solution: (i) F.B.D. of block



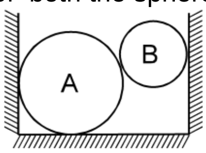
(ii) 'N' and Mg are not action-reaction pair. Since pair act on different bodies, and they are of same nature.
(iii) Pair of 'mg' of block acts on earth in opposite direction.



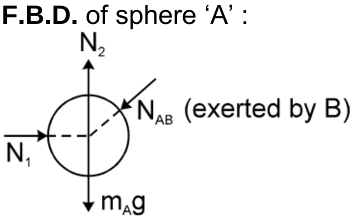
and pair of 'N' acts on surface as shown in figure.



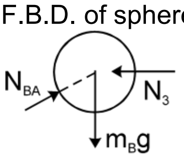
Example.3 Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Solution: **F.B.D.** of sphere 'A' :



F.B.D. of sphere 'B' :



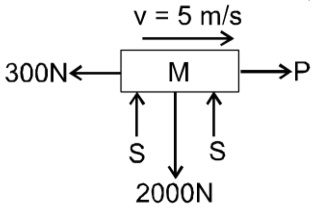
(exerted by A)

Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

Self Practice Problems

1. A bullet of 5g, travelling at a speed of 100 m/s penetrates a wooden block up to 6.0 cm. Then the average force applied by the bullet on the block is :
(1) 417 N (2) 8333N (3) 83.3N (4) Zero

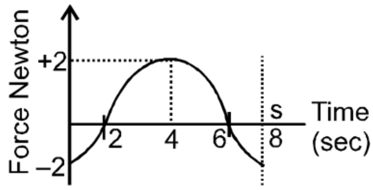
2. The forces acting on an object are shown in the fig. If the body moves horizontally at a constant speed of 5 m/s, then the values of the forces P and S are, respectively :



(1) 0N, 0N (2) 300N, 200N (3) 300N, 1000N (4) 2000N, 300N

3. A particle is moving with a constant speed along a straight line path. A force is required to :
(a) Increase its speed (b) Decrease the momentum
(c) Change the direction (d) Keep it moving with uniform velocity
(1) a, b, c, d (2) b, c, d (3) a, b, c (4) a, b, d

4. A force-time graph for a linear motion is shown in figure where the segments are circular. The linear momentum gained between zero and 8 seconds is :



(1) –2π N-s (2) 0 N.s (3) 4π N.s (4) –6π N.s

5. A particle moves in the xy plane under the action of a force F such that the value of its linear momentum (P) at any time t is, $P_x = 2$ cost, $P_y = 2$ sint. The angle θ between P and F at that time t will be :
(1) 0º (2) 30º (3) 90º (4) 180º

6. The linear momentum P of a body moving in one dimension varies with time according to the equation $P = at_3 + bt$ where a and b are positive constants. The net force acting on the body is :
(1) Proportional to t_2 (2) A constant
(3) Proportional to t (4) Inversely proportional to t

7. A thief stole a box full of valuable articles of weight w and while carrying it on his back, he jumped down a wall of height h from the ground. Before he reached the ground, he experienced a load of
(1) 2W (2) W (3) W/2 (4) Zero

8. When a horse pulls a cart, the force that helps the horse to move forward is the force exerted by :
(1) The cart on the horse (2) The ground on the horse
(3) The ground on the cart (4) The horse on the ground

9. A player catches a ball of 200g moving with a speed of 20m/s. If the time taken to complete the catch is 0.5 sec, the force exerted on the players hand is :
(1) 8N (2) 4N (3) 2N (4) 0N

Answer Key :

1.	(1)	**2.**	(3)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(1)
7.	(4)	**8.**	(2)	**9.**	(1)						

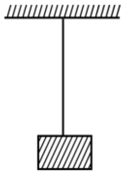
5.2 Applications of Newton's Laws

(a) To solve problems involving objects in equilibrium:

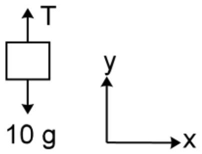
Step 1: Make a sketch of the problem.
Step 2: Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.
Step 3: Choose a convenient coordinate system and resolve all forces into rectangular conponents along x and y direction.
Step 4: Apply the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.
Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.
Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.
Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Solved Examples

Example.4 A 'block' of mass 10 kg is suspended with string as shown in figure. Find tension in the string. (g = 10 m/s^2)

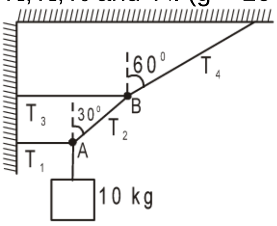


Solution: F.B.D. of block
$\Sigma F_y = 0$

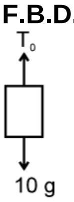


$T - 10\,g = 0$
$\therefore T = 100$ N

Example.5 The system shown in figure is in equilibrium. Find the magnitude of tension in each string, T_1, T_2, T_3 and T_4. (g = 10 m/s^{-2})



Solution: **F.B.D. of block**



$T_0 = 10\,g$
$T_0 = 100$ N

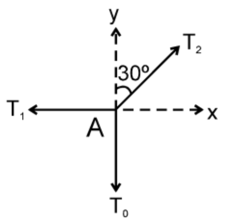
F.B.D. of point 'A'

$\Sigma F_y = 0$

$T_2 \cos 30^\circ = T_0 = 100$ N

$T_2 = \dfrac{200}{\sqrt{3}}$ N

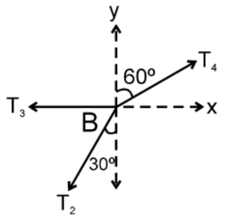
$\Sigma F_x = 0$



$T_1 = T_2 \sin 30^o = \frac{200}{\sqrt{3}} \cdot \frac{1}{2} = \frac{100}{\sqrt{3}}$ N.

F.B.D. of point of 'B'

$\Sigma F_y = 0 \Rightarrow T_4 \cos 60^{\circ} = T_2 \cos 30^{\circ}$



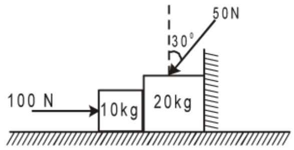
and $\Sigma F_x = 0 \Rightarrow T_3 + T_2 \sin 30^{\circ} = T_4 \sin 60^{\circ}$

∴ $T_3 = \frac{200}{\sqrt{3}}$ N, $T_4 = 200$ N

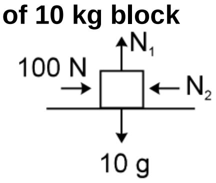
Example.6 Two blocks are kept in contact as shown in figure. Find

(a) forces exerted by surfaces (floor and wall) on blocks.

(b) contact force between two blocks.



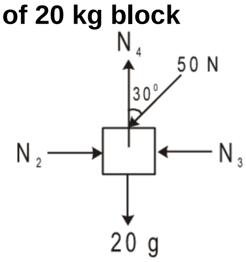
Solution: **F.B.D. of 10 kg block**



$N_1 = 10\text{ g} = 100$ N ....(1)

$N_2 = 100$ N ....(2)

F.B.D. of 20 kg block



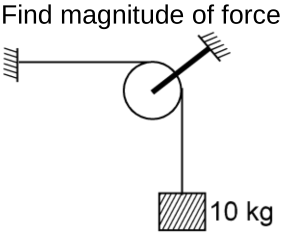
$N_2 = 50 \sin 30^o + N_3$

∴ $N_3 = 100 - 25 = 75$ N ....(3)

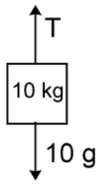
and $N_4 = 50 \cos 30^{\circ} + 20\text{ g}$

$N_4 = 243.30$ N

Example.7 Find magnitude of force exerted by string on pulley.

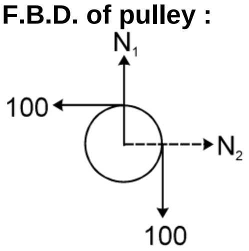


Solution: **F.B.D. of 10 kg block :**



T = 10 g = 100 N

F.B.D. of pulley :



Since string is massless, so tension in both sides of string is same.

Force exerted by string

$= \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$ N

Note : Since pulley is in equilibrium position, so net forces on it is zero.

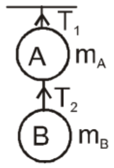
Hence force exerted by hinge on it is $100\sqrt{2}$ N.

Self Practice Problems

10. A body of weight W_1 is suspended from the ceiling of a room through a chain of weight W_2. The ceiling pulls the chain by a force :

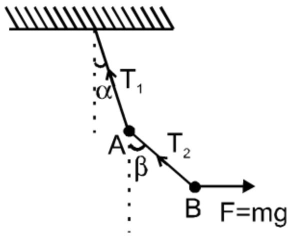
(1) W_1 (2) W_2 (3) $W_1 + W_2$ (4) $W_1 + W_2/2$

11. Two objects A and B of masses m_A and m_B are attached by strings as shown in fig. If they are given upward acceleration then the ratio of tension $T_1 : T_2$ is-



(1) $(m_A + m_B)/m_B$ (2) $(m_A + m_B)/m_A$ (3) $\frac{m_A + m_B}{m_A - m_B}$ (4) $\frac{m_A - m_B}{m_A + m_B}$

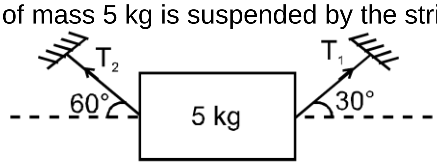
12. Two particles A and B, each of mass m, are kept stationary by applying a horizontal force F = mg on particle B as shown in fig. Then



(a) $\tan\beta = 2\tan\alpha$ (b) $2T_1 = 5T_2$ (c) $T_1\sqrt{2} = T_2\sqrt{5}$ (d) None of these

(1) a, b (2) b, c (3) c, d (4) a, c

13. A body of mass 5 kg is suspended by the string making angles 60° and 30° with the horizontal



(a) $T_1 = 25$ N (b) $T_2 = 25$ N (c) $T_1 = 25\sqrt{3}$ N (d) $T_2 = 25\sqrt{3}$ N

(1) a, b (2) a, d (3) c, d (4) b, c

Answer Key :

10.	(3)	**11.**	(1)	**12.**	(4)	**13.**	(2)

(b) To solve problems involving objects that are in accelerated motion :

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3: Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

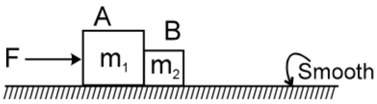
Step 4: Apply the equations $\sum F_x = m \ a_x$ and $\sum F_y = m \ a_y$.

Step 5: Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

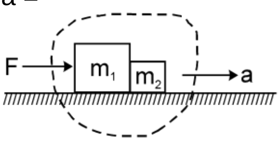
Solved Examples

Example.8 A force F is applied horizontally on mass m_1 as shown in figure. Find the contact force between m_1 and m_2.



Solution: Considering both blocks as a system to find the common acceleration.
Common acceleration

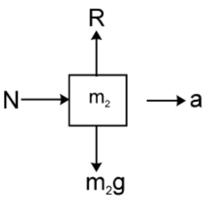
$$a = \frac{F}{(m_1 + m_2)} \qquad .....(1)$$



To find the contact force between 'A' and 'B' we draw F.B.D. of mass m_2.

F.B.D. of mass m_2

$\Sigma F_x = ma_x$



$N = m_2 . a$

$$N = \frac{m_2 F}{(m_1 + m_2)}$$

Example. 9 The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Solution. From second law of motion :

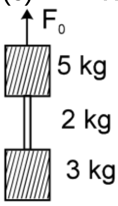
$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = 2.\frac{d}{dt}(at\hat{i} + bt^2\hat{j}) \Rightarrow \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

Example.10 A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s^2 by an external force F_0.

(a) What is F_0 ?

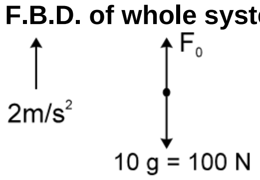
(b) What is the net force on rope ?

(c) What is the tension at middle point of the rope ? ($g = 10\ m/s^2$)



Solution. For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system



(a)

$F_0 - 100 = 10 \times 2$

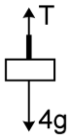
$F = 120$ N .....(1)

(b) According to Newton's second law, net force on rope.

$F = ma = (2)(2)$

$= 4$ N .....(2)

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



$T - 4g = 4.(2)$; $T = 48$ N

Self Practice Problems

14. The mass of a lift is 600 kg and it is moving upwards with a uniform acceleration of 2m/s^2. Then the tension in the cable of the lift is-

(1) 7080 N (2) 5880 N (3) 4680 N (4) zero N

15. A person standing on the floor of an elevator drops a coin. The coin reaches the floor of the elevator in a time t_1 if the elevator is stationary and in time t_2 if it is moving uniformly. Then

(1) $t_1 = t_2$ (2) $t_1 < t_2$

(3) $t_1 > t_2$ (4) $t_1 < t_2$ or $t_1 > t_2$ depending on whether the lift is going up or down.

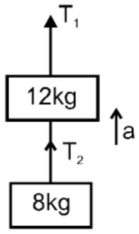
16. If the tension in the cable supporting an elevator is equal to the weight of the elevator, the elevator may be-

(A) going up with increasing speed (B) going down with increasing speed

(C) going up with uniform speed (D) going down with uniform speed

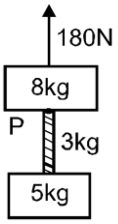
(1) A, D (2) A, B, C (3) C, D (4) A, B

17. A body of mass 8 kg is hanging from another body of mass 12 kg. The combination is being pulled up by a string with an acceleration of 2.2 m/sec^2. The tension T_1 will be



(1) 260 N (2) 240 N (3) 220 N (4) 200 N

18. Two blocks of mass 8 kg and 5 kg are connected by a heavy rope of mass 3 kg. An upward force of 180 N is applied as shown in the figure. The tension in the string at point P will be



(1) 60 N (2) 90 N (3) 120 N (4) 150 N

Answer Key :

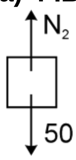
14.	(1)	**15.**	(1)	**16.**	(3)	**17.**	(2)	**18.**	(2)

Solved Examples

Example.11 A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1kg block. (All surface are smooth). Find ($g = 10\ m/s^2$)

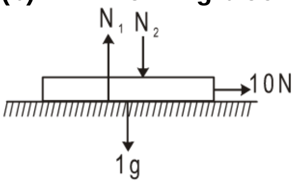
(a) Acceleration of block A and B.
(b) Force exerted by B on A.

Solution: **(a) F.B.D. of 50 kg**



$N_2 = 50\ g = 500$ N
along horizontal direction, there is no force $a_B = 0$

(b) F.B.D. of 1 kg block :



along horizontal direction

$10 = 1\ a_A.$

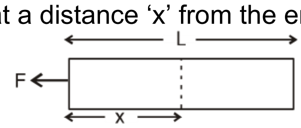
$a_A = 10\ m/s^2$

along vertical direction

$\therefore \quad N_1 = N_2 + 1g$

$= 500 + 10 = 510$ N

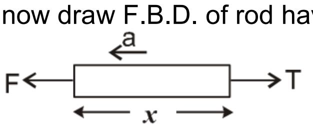
Example 12. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.



Solution : Considering rod as a system, we find acceleration of rod

$a = \frac{F}{m}$

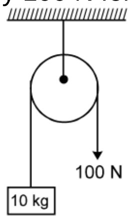
now draw F.B.D. of rod having length 'x' as shown in figure.



Using Newton's second law

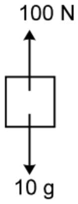
$F - T = \left(\frac{M}{L}\right) x.a \quad \Rightarrow \quad T = F - \frac{M}{L} x.\frac{F}{M} \quad \Rightarrow \quad T = F(1 - \frac{x}{L}).$

Example 13. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8\ m/s^2$)



Solution : Since string is pulled by 100 N force. So tension in the string is 100 N.

F.B.D. of 10 kg block

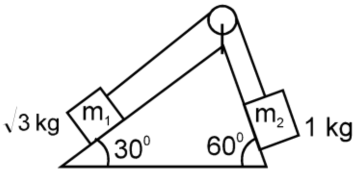


$100 - 10\,g = 10\,a$

$100 - 10 \times 9.8 = 10\,a$

$a = 0.2\ m/s^2$.

Example.14 Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :



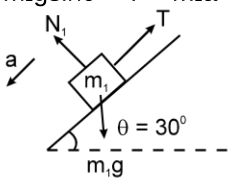
(i) acceleration of mass m_1 and m_2

(ii) tension in the string

(iii) net force on pulley exerted by string

Solution. **F.B.D. of m_1 :**

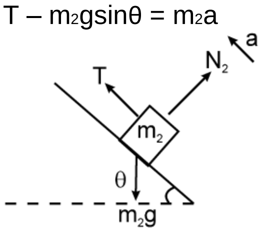
$m_1 g\sin\theta - T = m_1 a$



$\frac{\sqrt{3}}{2}g - T = \sqrt{3}\,a$ ....(i)

F.B.D. of m_2 :

$T - m_2 g\sin\theta = m_2 a$



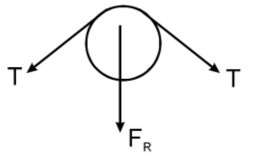
$T - 1.\frac{\sqrt{3}}{2}g = 1.a$ ....(ii)

Adding eq.(i) and (ii) we get a = 0

Putting this value in eq.(i) we get $T = \frac{\sqrt{3}g}{2}$,

F.B.D. of pulley

$F_R = \sqrt{2}\,T$



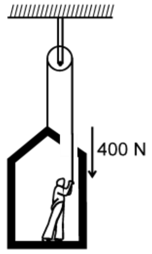
$F_R = \frac{\sqrt{3}}{2} g$

Example.15 A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

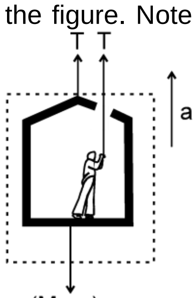
(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?



Solution. The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$2T - (M + m)g = (M + m)a$ or $a = \frac{2T-(M+m)g}{M+m}$

Here M = 60 kg; m = 15 kg; T = 400 N

$g = 10 \text{ m/s}^2$

$a = \frac{2(400)-(60+15)(10)}{60+15} = 0.67 \text{ m/s}^2$

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

$F = \frac{1}{2}(M + m)(g + a) = \frac{1}{2}(60 + 15)(10 + 1) = \mathbf{412.5\ N}$

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

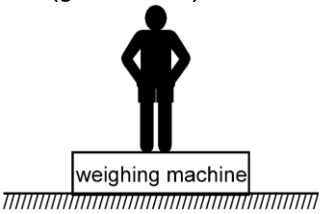
Thus, $2F - (M + m)g = 0$ or $F = \frac{(M+m)g}{2} = \frac{(60+15)(10)}{2} = \mathbf{375\ N}$

6. WEIGHING MACHINE :

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Solved Examples

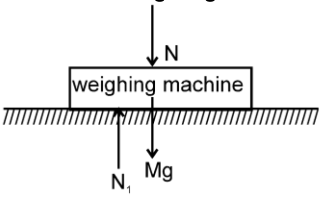
Example.16 A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine ($g = 10\ m/s^2$).



Solution. For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man F.B.D. of weighing machine

N = Mg



Here force exerted by object on upper surface is N

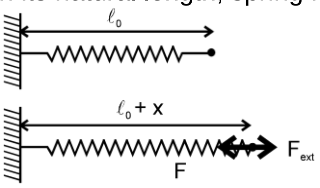
Reading of weighing machine

N = Mg = 60 × 10 N = 600 N.

7. SPRING FORCE :

Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by F = -kx, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

When spring is in its natural length, spring force is zero.



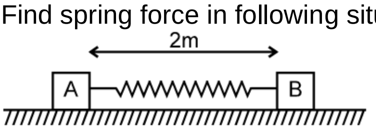
F = 0

F = – kx

Example.17 Two blocks are connected by a spring of natural length 2 m.

The force constant of spring is 200 N/m.

Find spring force in following situations :



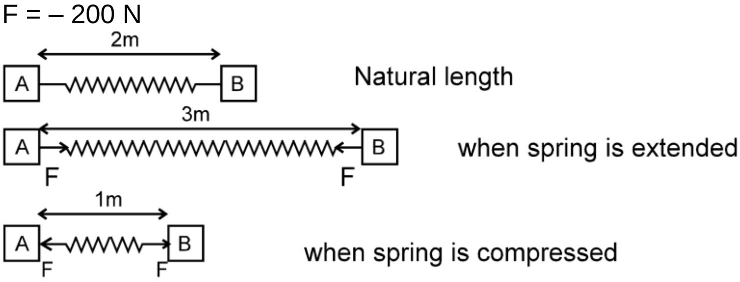
(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.

(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

Solution: (a) Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

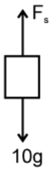
(b) In this case, change in length of spring is 1 m. So spring force is F = – Kx = – (200) . (1)

F = – 200 N



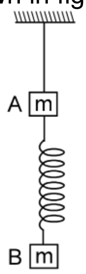
Example.18 Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10\ m/s^2$)

Solution. In this situation, spring is in extended state so spring force acts in upward direction.
Let x be the extension in the spring.
F.B.D. of 10 kg block :

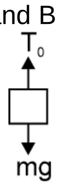


$F_s = 10g$
$Kx = 100$
$(100)x = (100)$
$x = 1m$

Example.19 Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.

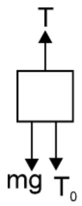


Solution. When block A and B are in equilibrium position



$T_0 = mg$

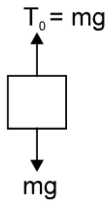
F.B.D of 'B' .... (i)



$T = mg + T_0$
$T = 2\ mg$

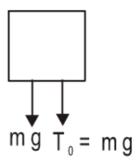
F.B.D of 'A' ....(ii)

when string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks A and B as shown in figure



$T_0 - mg = m.a_B$
$a_B = 0$

F.B.D. of 'B'

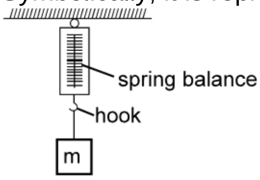


$mg + T_0 = m\ .\ a_A$
$2\ mg = m\ .\ a_A$
$a_A = 2g$ (downwards)

F.B.D. of 'A'

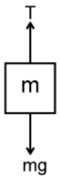
7.1 Spring Balance :

It does not measure the weight. It measures the force exerted by the object at the hook.
Symbolically, it is represented as shown in figure.



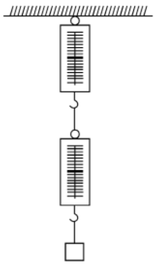
A block of mass 'm' is suspended at hook.
When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.

F.B.D. of 'm'.



mg – T = 0
T = mg
Magnitude of T gives the reading of spring balance.

Example.20 A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the :

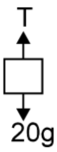


(1) reading of spring balance (1).

(2) reading of spring balance (2).

Solution: For calculating the reading, first we draw F.B.D. of 20 kg block.

F.B.D of 20 kg.



mg – T = 0

T = 20 g = 200 N

Since both balances are light so, both the scales will read 20 kg.

8. CONSTRAINED MOTION:

8.1 String Constraint :

When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

(i) Object which moves along a line can be specified by one variable.

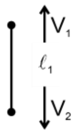
(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format.

$\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 + \ell_6 = \ell$

Step 4. Differentiate with respect to time



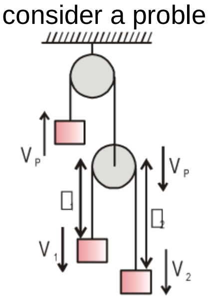
$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + ... = 0$$

$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string $\frac{d\ell_1}{dt} = V_1 + V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

Step 5. Repeat all above steps for different-different strings.

Let us consider a problem given below



Here $\ell_1 + \ell_2$ = constant

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

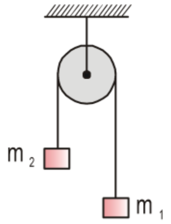
$(V_1 - V_P) + (V_P - V_2) = 0$

$V_p = \frac{V_1 + V_2}{2}$

Similarly, $a_P = \frac{a_1 + a_2}{2}$ Remember this result

Solved Examples

Example.21 Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :



(i) the acceleration of each block
(ii) the tension in the string.

Solution. The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction. If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 **F.B.D. of m_1**

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $m_1g - T = m_1a$ ....(1)

Block m_2 $-m_2g + T = m_2a_2$ ....(2)

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation?

(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

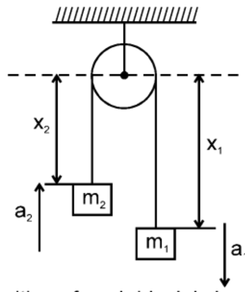
Thus, $x_1 + x_2 =$ constant

Differentiating both the sides w.r.t. time we get $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations,

therefore we differentiate it once again w.r.t. time.



Position of each block is located w.r.t. centre of the pulley

Thus $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward

(x_1 is increasing with time)

$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$

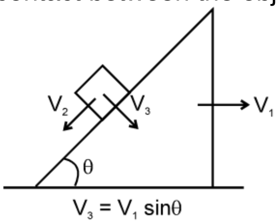
Thus $\quad a_1 - a_2 = 0 \quad$ or $\quad a_1 = a_2 = a$ (say) is the required constraint relation.
Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

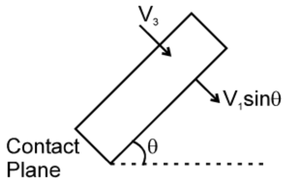
(i) $\quad a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g \qquad$ (ii) $\quad T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

8.2 Wedge Constraint :

Conditions :

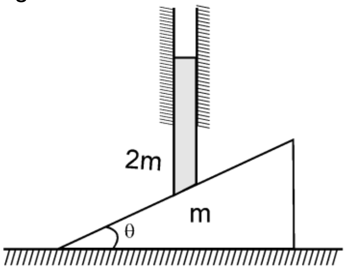
(i) There is a regular contact between two objects.
(ii) Objects are rigid.
The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



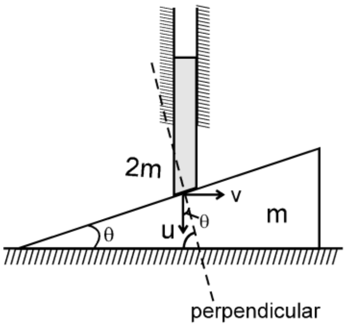


In other words,
Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Example.22 A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



Solution. Using wedge constraint.
Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.



$u \cos\theta = v \sin\theta$

$\frac{u}{v} = \tan\theta$

$u = v \tan\theta$

9. NEWTON'S LAW FOR A SYSTEM

$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

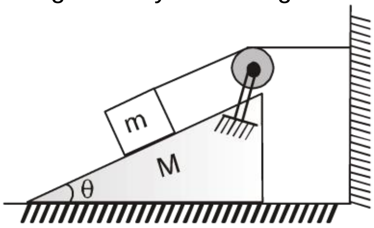
$\vec{F}_{ext} =$ Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

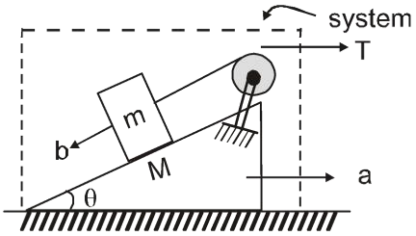
$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Solved Examples

Example.23 For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.



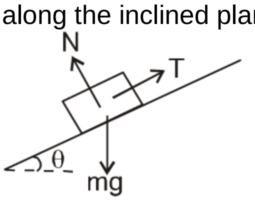
Solution. Considering block and wedge as a system and using Newton' s law for the system along x–direction



T = Ma + m (a – b cos θ) --------(i)

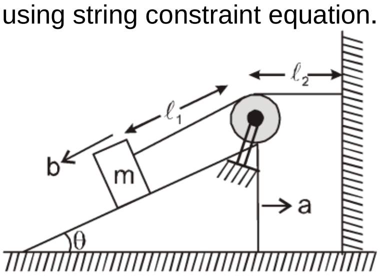
F.B.D of m

along the inclined plane



mg sin θ – T = m (b – a cos θ) -------(ii)

using string constraint equation.



$\ell_1 + \ell_2$ = constant

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

b – a = 0 ....(iii)

Solving above equations (i),(ii) & (iii) , we get

$$a = \frac{mg\sin\theta}{M + 2m(1 - \cos\theta)}$$

10. Reference Frame:

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) **Inertial reference frame:** Frame of reference which is either at rest or moving with constant velocity.

(b) **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

11. NEWTON'S LAW FOR NON INERTIAL FRAME:

$$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$$\vec{F}_{Pseudo} = -m\,\vec{a}_{Frame}$$

Magnitude of Pseudo force = mass of system × acceleration of non-inertial frame of reference .

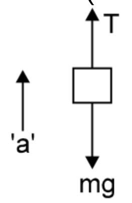
Direction of force: Opposite to the direction of acceleration of non-inertial frame of reference, (not in the direction of motion of non-inertial frame of reference)

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

Solved Examples

Example.24 A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration 'a'. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth, mass of bob of simple pendulum is m.

Solution. **F.B.D** . of bob (with respect to ground)



T – mg = ma

T = mg + ma ....(i)

With respect to boy inside the lift, the acceleration of bob is zero.

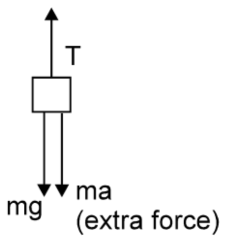
So he will write above equation in this manner.

T – mg = m. (0) .

∴ T = mg

He will tell the value of tension in string is mg. But this is 'wrong' .

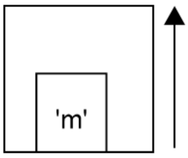
To correct his result, he makes a free body diagram in this manner, and uses Newton's second law.



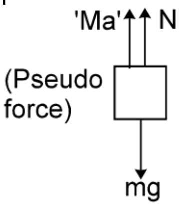
T = mg + ma ....(ii)

By using this **extra force**, equations (i) and (ii) give the same result . This **extra forc**e is called **pseudo force**. This **pseudo force** is used when a problem is solved with a accelerating frame (Non-inertial)

Example.25 A box is moving upward with retardation 'a' <g, find the direction and magnitude of " pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



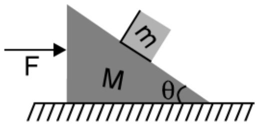
Solution. **Pseudo force** acts opposite to the direction of acceleration of reference frame.
pseudo force = ma in upward direction



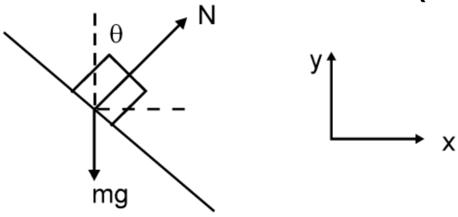
$N + ma = mg$
$N = mg - ma$

F.B.D of 'm' w.r.t. box (non–inertial)

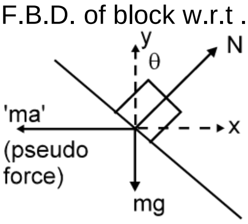
Example.26 All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.



Solution. Acceleration of (block + wedge) is a = $\frac{F}{(M+m)}$
Let us solve the problem by using both frames.
From inertial frame of reference (Ground)



F.B.D. of block w.r.t. ground (Apply real forces) :
with respect to ground block is moving with an acceleration 'a' .
$\therefore \ \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....(i)
and $F_x = ma \Rightarrow N\sin\theta = ma$ .....(ii)
From Eqs. (i) and (ii)
$a = g\tan\theta$
$\therefore \ F = (M + m)\,a \quad = (M + m)\,g\tan\theta$
From non–inertial frame of reference (Wedge) :
F.B.D. of block w.r.t . wedge (real forces + pseudo force)



w.r.t. wedge, block is stationary
$\therefore \ \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....(iii)
$\Sigma F_x = 0 \Rightarrow N\sin\theta = ma$ ....(iv)
From Eqs. (iii) and (iv) , we will get the same result
i.e. $F = (M + m)\,g\tan\theta$.

Self Practice Problems

19. A particle stays at rest as seen in a frame. We can conclude that-
(a) The frame is inertial

(b) Resultant force on the particle is zero
(c) The frame may be inertial but the resultant force on the particle is zero
(d) The frame may be none inertial but there is a nonzero resultant force.
(1) a, d (2) b,c (3) c, d (4) a, b

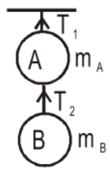
20. A particle is found to be at rest when seen from a frame s_1 and moving with a constant velocity when seen from another frame s_2. Mark out the possible options.
(a) Both the frames are inertial (b) Both the frames are non inertial
(c) s_1 is inertial and s_2 in non inhertial (d) s_1 is non inertial and s_2 is inertial
(1) a, b (2) b,c (3) a, b c (4) a, b, c, d

21. A person says that he measured the acceleration of a particle to be non zero while no force was acting on the particle -
(1) He is a liar
(2) His clock might have run slow
(3) His meter scale might have been longer than the standard
(4) He might have non inertial frame

22. A particle is observed from two frames s_1 and s_2. The frame s_2 moves with respect to s_1 with an acceleration a. Let F_1 and F_2 be the pseudo forces on the particle when seen from s_1 and s_2 respectively. Which of the following are not possible ?
(1) $F_1 = 0, F \neq 0$ (2) $F_1 \neq 0, F_2 \neq 0$ (3) $F_1 \neq 0, F_2 = 0$ (4) $F_1 = 0, F_2 = 0$

23. A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block has magnitude.
(1) mg (2) mg/cosθ (3) mg cosθ (4) mg tanθ

24. If the arrangement in fig is given a downward acceleration then the ratio of tensions T_1 and T_2 in strings is-



(1) $(m_A + m_B)/m_B$ (2) $(m_A + m_B)/m_A$ (3) $\frac{m_A - m_B}{m_A + m_B}$ (4) None of these

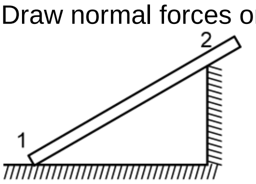
25. A smooth wedge A is fitted in a chamber hanging from a fixed celling near the earth's surface. A block B placed at the top of the wedge takes a time T to slide down the length of the wedge. If the block is placed at the top of the wedge and the cable supporting the chamber is broken at the same instant the block will-
(1) take a time longer than T to slide down the wedge
(2) take a time shorter than T to slide down the wedge
(3) remain at the top of the wedge
(4) jump off the wedge

Answer Key :

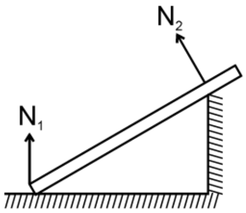
19.	(3)	**20.**	(1)	**21.**	(4)	**22.**	(4)
23.	(2)	**24.**	(1)	**25.**	(3)		

Solved Miscellaneous Examples

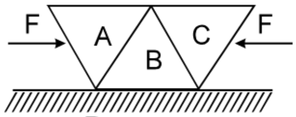
Problem 1. Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



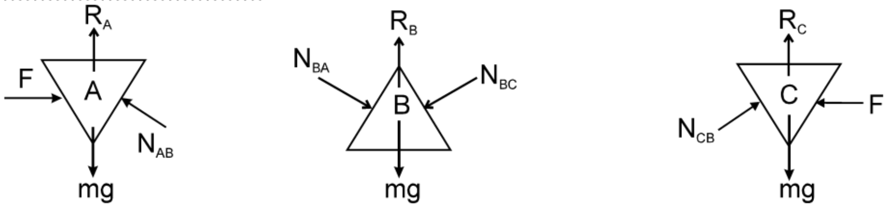
Solution : Normal force acts perpendicular to extended surface at point of contact.



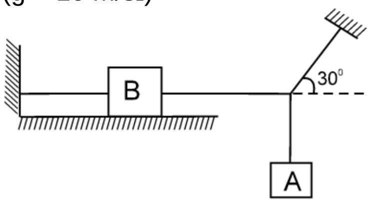
Problem 2. Three triangular blocks A,B and C of equal masses 'm' are arranged as shown in figure. Draw F.B.D. of blocks A,B and C. Indicate action–reaction pair between A,B and B,C.



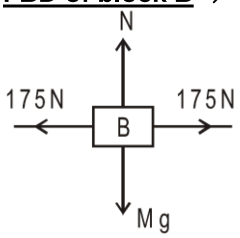
Solution :



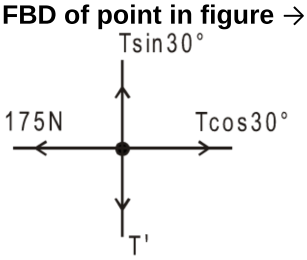
Problem 3. The breaking strength of the string connecting wall and block B is 175 N, find the magnitude of weight of block A for which the system will be stationary. The block B weighs 700 N. (g = 10 m/s2)



Solution : <u>**FBD of block B**</u> →



FBD of point in figure →



Equating forces in horizontal direction →

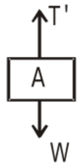
$T\cos 30° = 175$ $\qquad T = \dfrac{175 \times 2}{\sqrt{3}}$ N

In vertical direction →

$T\sin 30° = T'$

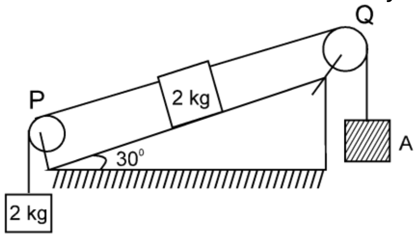
So, $T' = \dfrac{175 \times 2}{\sqrt{3}} \times \dfrac{1}{2} = \dfrac{175}{\sqrt{3}}$ N

FBD of block A →



So, **$T' = W = \frac{175}{\sqrt{3}}$ N**

Problem 4. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley Q. (g = 10 m/s^2)



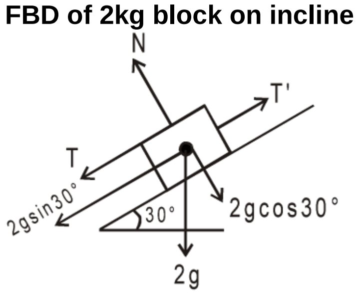
Ans. m = 3 kg, $30\sqrt{3}$ N.

Solution : From figure

FBD of 2 kg block hanging vertically →

T = 20 N .... (1)

FBD of 2kg block on incline plane →

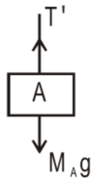


Along the plane →

$T + 2g\sin 30° = T'$

$T' = 20 + 20 \times \frac{1}{2} = 30$ N

FBD of block A →

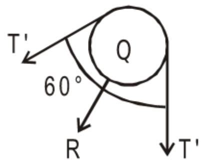


So $T' = M_A g$

$M_A = \frac{T'}{g} = \frac{30}{10} = 3$ kg

$M_A = 3$ kg

FBD of pulley Q →

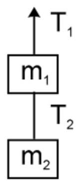


So, $R = 2T'\cos\frac{\theta}{2}$

$R = 2\times30\cos30°$

$R = 2 \times 30 \times \frac{\sqrt{3}}{2}$ ⇒ **$R = 30\sqrt{3}$ N**

Problem 5. Two blocks with masses $m_1 = 0.2$ kg and $m_2 = 0.3$ kg hang one under other as shown in figure. Find the tensions in the strings (massless) in the following situations : ($g = 10$ m/s^2)

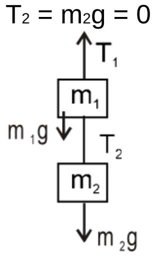


(a) the blocks are at rest
(b) they move upward at 5 m/s
(c) they accelerate upward at 2 m/s^2
(d) they accelerate downward at 2 m/s^2
(e) if maximum allowable tension is 10 N. What is maximum possible upward acceleration ?

Ans. (a) 5 N, 3 N (b) 5 N, 3 N (c) 6 N, 3.6 N (d) 4 N, 2.4 N (e) 10 m/s^2

Solution : (a) At rest $a = 0$

$T_2 = m_2g = 0.3 \times 10 = 3N$



$T_1 = m_1g + T_2$

$T_1 = 0.2 \times 10 + 3 = 5$ N

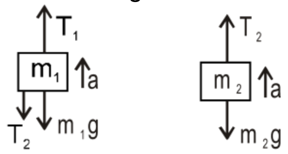
(b) same as above

$a = 0$, $T_2 = 3N$, $T_1 = 5N$

(c) $a = 2$m/s^2 ↑ (upward)

$T_2 - m_2g = m_2a$

⇒ $T_2 - m_2g = m_2a$



⇒ $T_2 - 0.3 \times 10 = 0.3 \times 2$

⇒ $T_2 = 0.6 + 3 = 3.6$ N

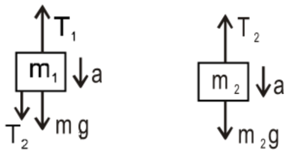
$T_1 - m_1g - T_2 = m_1a$ ⇒ $T_1 - 0.2\times 10 - 3.6 = 0.2 \times 2$ ⇒ $T_1 = 0.4 + 5.6 = 6N$

(d) $a = 2$ m/s^2 (downward)

$m_2g - T_2 = m_2a$

⇒ $0.3 \times 10 - T_2 = 0.3 \times 2$

⇒ $T_2 = 3 - 0.6 = 2.4$ N



$T_2 + m_1g - T_1 = m_1a$

$\Rightarrow 2.4 + 2 - T_1 = 0.2 \times 2$

$\Rightarrow T_1 = 4.4 - 0.4 =$ **4 N Ans.**

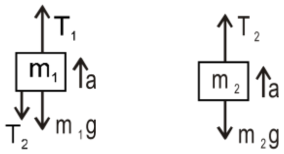
(e) Chance of breaking is of upper string means $T_1 < 10$ N

For m_1 –

$T_1 - m_1g - T_2 = m_1a$

$10 - 2 - T_2 = 0.2\,a$ .... (1)

For m_2 –



$T_2 - m_2g = m_2a$

$\Rightarrow\ T_2 - 3 = 0.3\,a$ .... (2)

Adding equation (1) and (2)

$8 - 3 = 0.5\,a$

$\Rightarrow\ a = \frac{5}{0.5} = \mathbf{10\ m/s^2}$

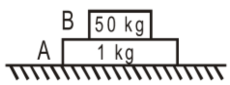
Problem 6. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 50kg block. (All surface are smooth). Find ($g = 10\ m/s^2$)

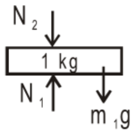
(a) Acceleration of block A and B.

(b) Force exerted by B on A.

Answer : (a) $0.2\ m/s^2$, 0 (b) 510 N

Solution : (a) **<u>FBD of A block</u>**





$N_2 = 50 \times 10 = 500$ N

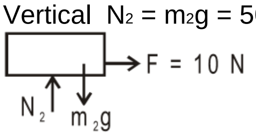
$m_1g = 1 \times 10 = 10$ N

$N_1 = N_2 + m_1g = 500 + 10 = 510$ N

$a_A = 0$ (No horizontal force)

<u>FBD of B block</u>

Vertical $N_2 = m_2g = 500$ N



Horizontal force

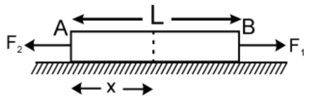
$F = ma \Rightarrow 10 = 50 \times a \Rightarrow a = \frac{1}{5}\ m/s^2$

(b) Force exerted By B on A →

$= N_2 = 500$N (Vertically downwards)

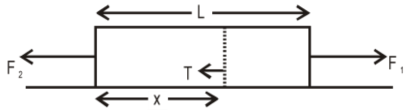
And force exerted by block A on ground $= N_1 = 510$ N

Problem 7. Two forces F_1 and F_2 (> F_1) are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',

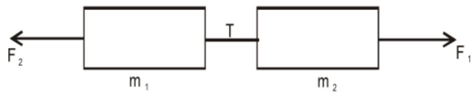


Answer : $T = F_2 - \frac{(F_2 - F_1)}{L}.x$

Solution : $a = \frac{F_2 - F_1}{m}$



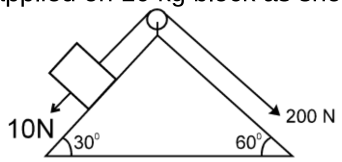
$T - F_1 = m_2a$



$\Rightarrow \quad T - F_1 = \frac{m}{L}(L-x)\frac{F_2 - F_1}{m} \qquad (m_2 = \frac{m}{L}(L-x))$

$\Rightarrow \quad T = F_1 + \left(1-\frac{x}{L}\right)(F_2 - F_1) = F + F_2 - \frac{x}{L}F_1 - (F_2 - F_1) = F_2 - \frac{x}{L}(F_2 - F_1)$

Problem 8. A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure.

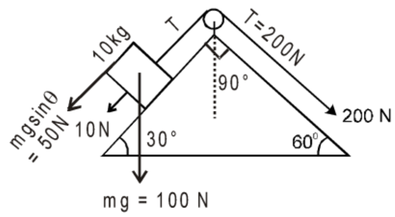


Find : (a) tension in the string.
(b) acceleration of 10 kg block.
(c) net force on pulley exerted by string

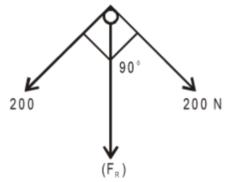
Answer : (a) 200 N, (b) 14 m/s2, (c) $200\sqrt{2}$ N

Solution : (a) T = **200 N**

(b) $T - 10 - mg\sin\theta = ma \quad \Rightarrow \quad T - 10 - 50 = 10a$



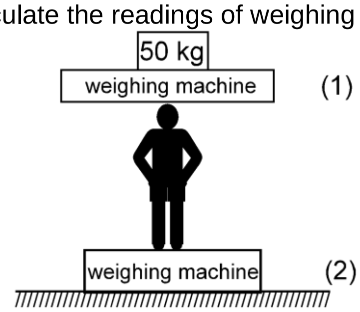
$\Rightarrow \quad 200 - 60 = 10a \quad \Rightarrow \quad a = \frac{140}{10} = \mathbf{14\ m/s^2}$



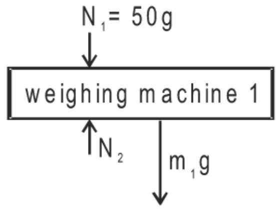
(c)

$(F_R) = \sqrt{(200)^2 + (200)^2} = \mathbf{200\sqrt{2}}$ **N Ans.**

Problem 9. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another identical weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1) . Calculate the readings of weighing machines (1) and (2) .

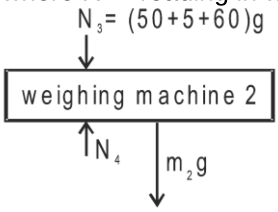


Answer : **500 N, 1150 N**



Solution :

$R_1 = N_1 = 50 \times g = 500$ N
where R_1 = reading in weighing machine 1



$R_2 = N_3$

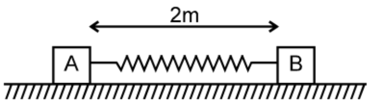
$= (50 + 5 + 60)$ g

$= 115 \times 10$

$= 1150$ N

where R_2 = reading in weighing machine 2

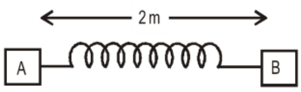
Problem 10 Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m.



Find spring force in following situations :
(a) A is kept at rest and B is displaced by 1 m in right direction.
(b) B is kept at rest and A is displaced by 1m in left direction.
(c) A is displaced by 0.75 m in right direction, and B is 0.25 m in left direction.

Answer : (a) F = 200 N, (b) 200 N, (c) 200 N

Solution : (a) Extenstion in spring = 1 m.



∴ $F_{spring} = K \times x$

$= 200 \times 1 = 200$ N

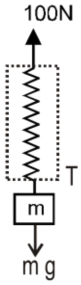
(b) Same Extension same spring force in both directions -
$F_{spring} = 200$ N.

(c) Both displacements of A of B are compressing the spring
total compressing $= 0.75 + 0.25 = 1$ m.
∴ $F_{spring} = k\,x \quad = 200 \times 1 = 200$ N.

Problem 11. If force constant of spring is 50 N/m. Find mass of the block, if it at rests in the given situation . ($g = 10$ m/s^2)

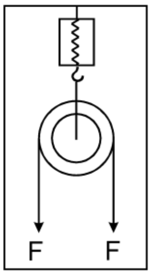
Answer m = 10 kg

Solution : T = 100 N
⇒ mg = 100 N



$m = \frac{100}{g} = 10$ kg.

Problem 12 Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.

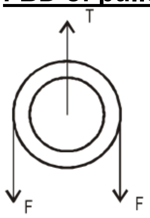


Answer : 2F

Solution : **<u>FBD of spring balance</u>**

R = T ....(i)

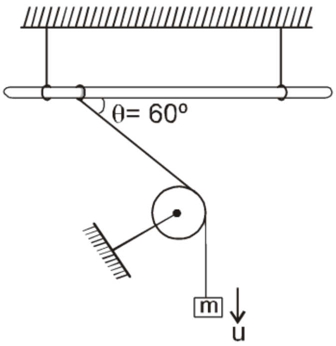
<u>FBD of pulley</u>



T = 2 F ....(ii)

R = 2F

Problem 13. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.



Answer : $V_R = \frac{u}{\cos\theta}$, $V_R = 2u$

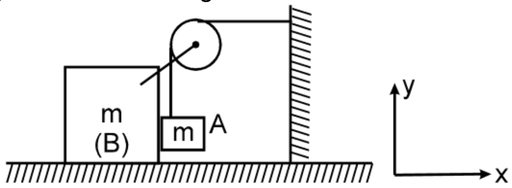
Solution : Velocity along string remains same .

$V_R \cos\theta = u$

$V_R = \frac{u}{\cos\theta}$ ⇒ $\theta = 60^o$

$V_R = 2u$

Problem 14. In the system shown in figure, the block A is released from rest. Find :



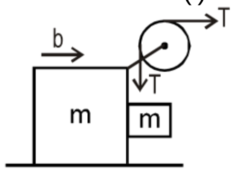
(i) the acceleration of both blocks 'A' and 'B'.
(ii) Tension in the string.
(iii) Contact force between 'A' and 'B'.

Answer : (i) $\frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$, $\frac{g}{3}\hat{i}$ (ii) $\frac{2mg}{3}$ (iii) $\frac{mg}{3}$.

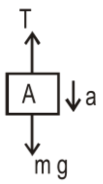
Solution : (i) Let acceleration of blocks in x & y directions are

<u>Taking both blocks as a system</u>

$T = 2mb$ .... (i)



<u>Taking A block :</u>



$mg - T = ma$ ....(ii)

From equations (i) & (ii) ;

$ma + 2\,mb = mg$

$a + 2\,b = g$ ....(iii)

From string constraint ;

$a = b$ ....(iv)

From equations (iii) & (iv);

$a = b = \frac{g}{3}$

hence, acceleration of block A

$a_A = b\hat{i} - a\hat{j}$ $a_A = \frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$

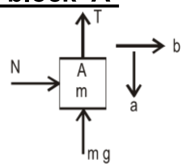
acceleration of block B

$a_B = b\hat{i} = \frac{g}{3}\hat{i}$

(ii) $T = 2mb = \frac{2mg}{3}$

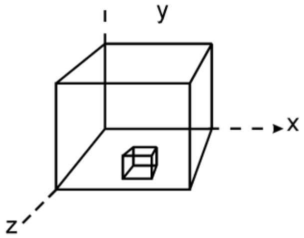
(iii) For contact force between 'A' and 'B'

FBD of block 'A'



N = mb $\qquad$ N = $\frac{mg}{3}$

Problem 15. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i}+4\hat{j}$ m/s2. Find the value of 'Pseudo force' acting on block with respect to box



Answer : $\vec{F}_{pseudo} = -m\vec{a}_{frame} = -2(-3\hat{i}+4\hat{j})$

F = $6\hat{i}-8\hat{j}$.

KEY CONCEPT

☞ **First Law of Motion :**

" Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

A particle continues to be in its original state of rest or of motion until and unless an external agent called force is not applied on it. It happens due to its property of inertia.

☞ **Second Law of Motion :**

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

☞ **Third Law of Motion :**

"To every action there is always opposed an equal and opposite reaction: to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

☞ **From Third Law of Motion :**

$\vec{F}_{AB} = -\vec{F}_{BA}$ $\Rightarrow$ $\vec{F}_{AB}$ = Force on A due to B

$\vec{F}_{BA}$ = Force on B due to A

☞ **From second law of motion :**

$F_x = \frac{dP_x}{dt} = ma_x \Rightarrow \quad F_y = \frac{dP_y}{dt} = ma_y \Rightarrow \quad F_z = \frac{dP_z}{dt} = ma_z$

☞ **Applications of Newton's Laws :**

When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3: Choose a convenient coordinate system and resolve all forces into x and y components.

Step 4: Apply the equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

Step 5: Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5 you will have enough equations to solve for all unknown quantities.

☞ **Accelerating Objects :**
To solve problems involving objects that are in accelerated motion :

Step 1: Make a sketch of the problem.

Step 2: Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3: Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

Step 4: Apply the equations $\sum F_x = m \ a_x$ and $\sum F_y = m \ a_y$.

Step 5: Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6: If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

☞ **Weighing Machine :**
A weighing machine does not measure the weight but measures the normal reaction force exerted by object on its upper surface.

☞ **Spring Force :**

$\vec{F} = -k\vec{x}$

x is extension or compression from its natural length or deformation of the spring where K = spring constant.

☞ **Spring property :**

$K \times \ell$ = constant

ℓ = Natural length of spring.

☞ If spring is cut into two in the ratio m : n then spring constant is given by

$\ell_1 = \dfrac{m\ell}{m+n}$; $k_1 = \dfrac{k(m+n)}{m} = K_m$

$\ell_2 = \dfrac{n\ell}{m+n}$ $k_2 = \dfrac{k(m+n)}{n} = k_n$

$k\ell = k_1\ell_1 = k_2\ell_2$

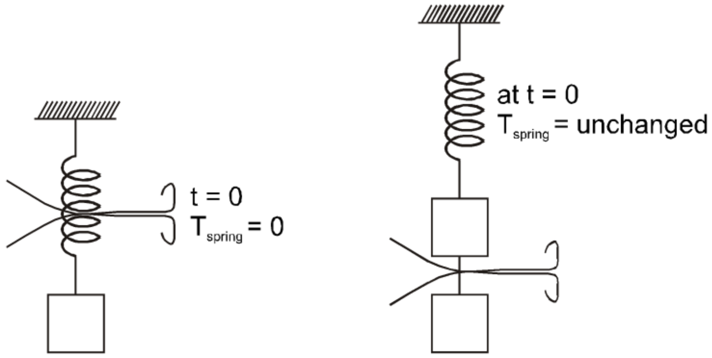
For series combination of springs

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots\ldots$$

For parallel combination of spring

$k_{eq} = k_1 + k_2 + k_3$ ....

☞ **Tension in a spring :** If spring is cut then the tension in it immediately become zero but if two ends of a spring are not free i.e. connect with a support / block at its two ends then just after the phenomenon tension remains unchanged if any change occurs



☞ **Spring Balance:**

It does not measure the weight. It measures the tension force exerted by the object at the hook.

☞ **String Constraint :**

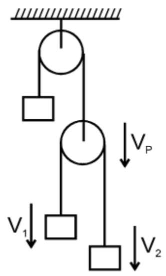
When two objects are connected through a string and if the string have the following properties :

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them. This relation is called string constraint.

☞ **Remember :**



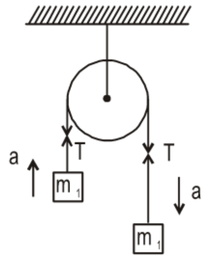
$$V_p = \frac{V_1 + V_2}{2}$$

$$a_P = \frac{a_1 + a_2}{2}$$

☞

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

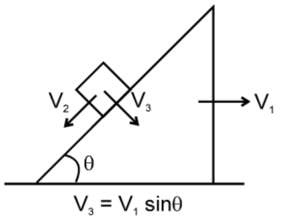
$$T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

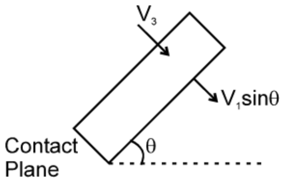


☞ **Wedge Constraint:**

Conditions :

(i) There is a regular contact between two objects.





(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.

In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

☞ **Newton's Law for a System :**

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

$\vec{F}_{ext} =$ Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and

$\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

☞ **Newton's Law for Non Inertial Frame :**

$$\acute{F}_{Real} + \acute{F}_{Pseudo} = m\vec{a}$$

Net sum of real and pseudo force is taken in the resultant force.

$\acute{a}$ = Acceleration of the particle in the non inertial frame

$$\acute{F}_{Pseudo} = -m\ \acute{a}_{Frame}$$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

Reference Frame:

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

(a) Inertial reference frame: Frame of reference either stationary or moving with constant velocity.

(b) Non-inertial reference frame: A frame of reference moving with non-zero acceleration.