TOPIC : FLUID MECHANICS
EXERCISE # 1

SECTION (A)

1. (b) Difference of pressure between sea level and the top of hill

$\Delta P = (h_1 - h_2) \times \rho_{Hg} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g$ …(i)

and pressure difference due to h meter of air

$\Delta P = h \times \rho_{air} \times g$ …(ii)

By equating (i) and (ii) we get

$h \times \rho_{air} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g$

$\therefore h = 25 \times 10^{-2} \left(\frac{\rho_{Hg}}{\rho_{air}}\right) = 25 \times 10^{-2} \times 10^4 = 2500\,m$ ∴ Height of the hill = 2.5 km.

2. Pressure at bottom of the lake = P0 + hρg

Pressure at half the depth of a lake = P0 + $\frac{h}{2}\rho g$

According to given condition

$P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g)$ $\frac{1}{3}$ $P_0 = \frac{1}{6}h\rho g$

$h = \frac{2P_0}{\rho g} = \frac{2 \times 10^5}{10^3 \times 10}$ = 20m.

3. $F = P \times A = hdgA = 0.4 \times 900 \times 10 \times 2 \times 10^{-3} = 7.2N$

4. P = hρf i.e pressure does not depend upon the area of bottom surface.

5. $P_1V_1 = P_2V_2 \Rightarrow (P_0 + h\rho g)\frac{4}{3}\pi r^3 = P_0 \frac{4}{3}\pi(2r)^3$

Where, h = depth of lake ⇒ $h\rho g = 7P0 \Rightarrow h = 7 \times \frac{H\rho g}{\rho g} = 7H$.

6. $P_1V_1 = P_2V_2 \Rightarrow (P_0 + h\rho g)\, V = P_0 \times 3V$

$h\rho g = 2P_0 \Rightarrow h = \frac{2 \times 75 \times 13.6 \times g}{\frac{13.6}{10} \times g}$ =15m

7. $h = \frac{p}{\rho g} \therefore h \propto \frac{1}{g}$ (P and ρ are constant)

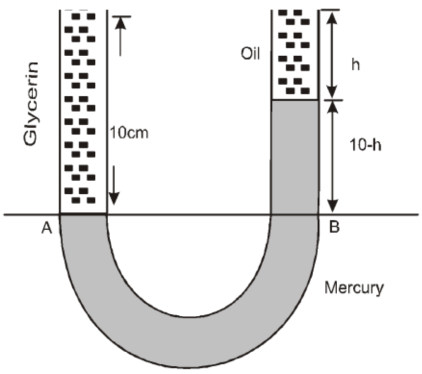
If value of g decreased by 2% then h will increase by 2%

8. $h = \frac{p}{\rho g} \therefore h \propto \frac{1}{g}$. If lift moves upward with some acceleration then effective g increases. So the value of h decreases.i.e reading will be less than 76 cm.

9. Total pressure at (near) bottom of the liquid

P = P0 + hρg

as air is continuously pumped out from jar (container), P0 decreases and hence P decreases.

10. 

At the condition of equilibrium
Pressure to point A = Pressure at point B
$P_A = P_B \Rightarrow 10 \times 1.3 \times g = h \times 0.8 \times g + (10 - h) \times 13.6 \times g$. By solving we get h = 9.7 cm

11. Pressure = hρg i.e pressure at the bottom is independent of the area of the bottom of the tank. It depends on the height of water upto which the tank is filled with water. As in both the tanks, the levels of water are the same, pressure at the bottom is also the same.

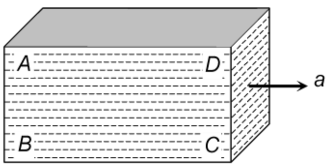
13. Use $P_1V_1 = P_2V_2$

$$(P_0 + \rho gh)\frac{4}{3}\pi r^3 = P_0 \times \frac{4}{3}\pi(2r)^3$$

$$P_0 + \rho gh = 8P_0$$

$$\rho gh = 7P_0$$

$$h = \frac{7P_0}{\rho g}$$ Ans. (4) is correct

14. 

Due to acceleration towards right, there will be a pseudo force in a left direction. So the pressure will be more on rear side (Points A and B) in comparison with front side (Point D and C).
Also due to height of liquid column pressure will be more at the bottom (points B and C) in comparison with top (point A and D).
So overall maximum pressure will be at point B and minimum pressure will be at point D.

16. As the both points are at the surface of liquid and these points are in the open atmosphere. So both point possess similar pressure and equal to 1 atm. Hence the pressure difference will be zero.

17. $\rho g(H - h)$
because pressure varies with height.

18. F = [ρgh] [A] = (1000) (10) (6) (10) (8).

19. $$\frac{m_1 g}{A_1} = \frac{m_2 g}{A_2}$$
Solving, $m_2 = 3.75$ kg.

20. $$\rho_x + \rho_w g\left(\frac{175}{100}\right) - \rho_{Hg} g\left(\frac{112}{100}\right) + \rho_w g\left(\frac{75}{100}\right) - \rho_{Hg}\left(\frac{88}{100}\right) - \rho_w g\left(\frac{62}{100}\right) = \rho_y$$
Solving $P_x - P_y = 253200\ N/m^2$ (g = 10 m/sec²)

22. Due to zero reaction force on plane of a satellite. We feel weightlessness in the satellite.

SECTION (B)

1. Weight of the bowl = mg = Vρg = $\frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$

Where D = Outer diameter,
d = Inner diameter
ρ = Density of bowl

Weight of the liquid displaced by the bowl = Vρg = $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g$

Where σ is the density of the liquid.

For the flotation $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g \Rightarrow \left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3\right] 2 \times 10^4$

By solving we get d = 0.98 m.

2. Specific gravity of alloy = $\frac{\text{density of substance}}{\text{density of water}}$

$= \frac{\text{Mass of alloy}}{\text{Volume of alloy} \times \text{density of water}} = \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}\right) \times \rho w} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1/\rho w} + \frac{m_2}{\rho_2/\rho w}} = \frac{m_1 + m_2}{\frac{m_1}{s_1} + \frac{m_2}{s_2}}$

$\left[\text{As specific gravities of substance} = \frac{\text{density of substance}}{\text{density of water}}\right]$

3. If two different bodies A and B are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})A}{(f_{in})B} = \frac{1/2}{2/3}\ \frac{3}{4}$

4. There will be no change in thelevel, because volume of water displaud will be same in both case to balance its weight.

5. There will be no change in the level, because volume of water displaced will be same in both case to balance its weight.

6. Water level will fall because after throwing balls into water, balls sink. Initially their weight was balanced when they were in the boat. So much volume was replaced. Now they are in water, so less volume will be replaced. Hence level of water will fall down.

7. Since, up thrust (F) = V σ g i.e F∝V

8. Vρg = $\frac{V}{2}$ σg ∴ ρ $\frac{\sigma}{2}$ (σ = density of water)

9. Level of water will fall because glass ball will sink after melting ice.

11. Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

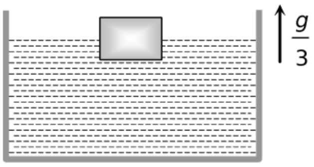
According to principle of floatation weight of whole sphere = upthrust on the sphere

$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g \Rightarrow R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$

$\Rightarrow R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \Rightarrow \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1} \Rightarrow \frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1}$

$\Rightarrow \frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1 - \rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2} \Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1 - 0.3}{2.4 - 1}\right) \times \frac{2.4}{0.3} = 4$

12. Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right)V$ i.e. it depends upon the densities of the block and liquid.
So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.



13. Apparent weight $= V(\rho - \sigma)g = \frac{M}{\rho}(\rho - \sigma)g$

$= M\left(1 - \frac{\sigma}{\rho}\right)g = 2.1\left(1 - \frac{0.8}{10.5}\right)g = 1.94\,g$ N

= 1.94 Kg-wt

14. Tension in spring T = upthrust – weight of sphere
$= V\sigma g - V\rho g = V\eta\rho g - V\rho g$ s
$= (\eta - 1)V\rho g = (\eta - 1)mg.$

15. When body (sphere) is half immersed, then
upthrust = weight of sphere
$\Rightarrow \frac{V}{2} \times \rho_{liq} \times g = V \times \rho \times g \quad \therefore \rho = \frac{\rho_{liq}}{2}$
When body (sphere) is fully immersed then,
Upthrust = wt. of sphere + wt. of water poured in sphere
$\Rightarrow V \times \rho_{liq} \times g = V \times \rho \times g + V' \times \rho_{liq} \times g \quad \Rightarrow V \times \rho_{liq} = \frac{V \times \rho_{liq}}{2} + V' \times \rho_{liq} \Rightarrow V' = \frac{V}{2}$

16. If two different bodies A and B are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})_A}{(f_{in})_B} = \frac{1/2}{2/3} = \frac{3}{4}$

17. For the floatation $V_0 d_0 g = V_{in}\, d\, g$
$\Rightarrow \quad V_{in} = V_0 \frac{d_0}{d}$
$\therefore \quad V_{out} = V_0 - V_{in} = V_0 - V_0 \frac{d_0}{d} = V_0\left[\frac{d - d_0}{d}\right] \quad \Rightarrow \frac{V_{out}}{V_0} = \frac{d - d_0}{d}$

18. Volume of ice = $\frac{M}{\rho}$, volume of water = $\frac{M}{\sigma}$
$\therefore \quad$ Change in volume = $\frac{M}{\rho} - \frac{M}{\sigma} = M\left[\frac{1}{\rho} - \frac{1}{\sigma}\right]$

19. $\rho Vg = 60$ ....(i)
$(\rho - \sigma)Vg = 40$ ....(ii)
but specific gravity = $\frac{\rho_0}{\rho_i}$

dividing (i) & (ii) $\frac{\rho_0}{\rho_i} = 3$.

20. $W_{app} = mg - F_B$
$W_{app} = \rho Vg - \rho_w Vg$
$= (\rho - \rho_w)\, Vg$
$= (7\rho_w - \rho_w)\, Vg = 6\, \rho_w\, Vg$

21. $[36 - \rho_\ell v_l]g = [48 - \rho_\ell\, v_2]g$

$$\left[36 - \rho_i\left(\frac{36}{9}\right)\right]g = \left[48 - \rho_i\left(\frac{36}{\rho_0}\right)\right]g$$

Solving, $\rho_0 = 3$.

22. If centre of buoyancy is above the centre ofgravity thebody will be in stable equilibrium.

23. Effective weight W' = m (g – a) which is less than actual weight mg, so the length of spring decreases.

24. When body (sphere) is half immersed, then
upthrust = weight of sphere

$\Rightarrow \quad \frac{V}{2} \times \rho_{liq} \times g = V \times \rho \times g \therefore \rho = \frac{\rho_{liq}}{2}$

When body (sphere) is fully immersed then, Upthrust = wt. of sphere + wt. of water poured in sphere

$\Rightarrow \quad V \times \rho_{liq} \times g = V \times \rho \times g + V' \times \rho_{liq} \times g$

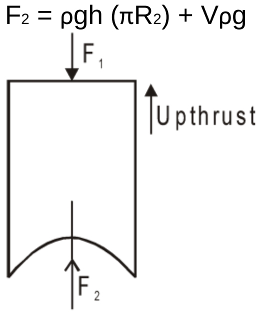
$\Rightarrow \quad V \times \rho_{liq} = \frac{V \times \rho_{liq}}{2} + V' \times \rho_{liq} \qquad \Rightarrow V' = \frac{V}{2}$

27. Let the total volume of ice-berg is V and its density is ρ. If this ice-berg floasts in water with volume V_{in} inside it then $V_{in}\, \sigma g = V\, \sigma g$

$\Rightarrow \quad V_{in} = \left(\frac{\rho}{\sigma}\right) V \qquad$ or $\qquad V_{out} = V - V_{in} = \left(\frac{\sigma - \rho}{\sigma}\right) V$

$\Rightarrow \quad \frac{V_{out}}{V} = \left(\frac{\sigma - \rho}{\sigma}\right) = \frac{1000 - 900}{1000} = \frac{1}{10} \qquad \therefore \quad V_{out} = 10\%$ of V

28. $[F_{lower} - F_{upper}]$ by $_{liquid}$ = Upthrust
$F_2 - F_1$ = upthrust
$\therefore \quad F_2 = F_1$ + upthrust
$F_2 = \rho gh\, (\pi R_2) + V\rho g$



or $\quad F_2 = \rho g(V + \pi R_2 h)$

In this problem, we did not take the force due to air pressure on the cylinder. This is because force due to air pressure is cancelled. At top and bottom of the cylinder the force due to air pressure is equal and opposite.

29. ℓ will decrease because the block moves up. h will decrease because the coin will displace the volume of water (V_1) equal to its own volume when it is in the water whereas when it is on the block it will displace the volume of water (V_2) whose weight is equal to weight of coin and since density of coin is greater than the density of water $V_1 < V_2$.

30. $\frac{\text{Relative density of body}}{\text{Relative density of water}} = \frac{\text{volume of inserted part of body}}{\text{total volume of body}}$

$\frac{d}{1} = \frac{2V}{3V}$ ⇒ $d = \frac{2}{3}$

Again using the same formula

$\frac{1}{4} = \frac{2}{3d}$ ⇒ $d = \frac{8}{3}$ gm/c.c.

31. Here area is uniform, so portion of immersed height = $\frac{1}{3}$

(Here : the ratio of density of body and density of water = $\frac{1}{3}$)

Therefore, fraction of exposed height will $1 - \frac{1}{3} = \frac{2}{3}$

32. Volume of raft is given by

$V = \frac{m}{d} = \frac{120}{600} = 0.2\ m^3$

If the raft is immersed fully in water. Then weight which can be put on the raft = Vd_w

$= 0.2 \times 10^3$ m= 200 kg = 200 kg – 120 kg = 80 kg

33. **Key Idea:** Force applied on the body will be equal to upthrust for vertical oscillations.,

Let block is displaced through x m, then weight of displaced water or upthrust (upwards)

$= -Ax\rho g$

where A is area of cross-section of the block and ρ is its density. This must be equal to force (= ma) applied, where m is mass of the block and a is acceleration.

∴ $ma = -Ax\rho g$ or $a = -\frac{A\rho g}{m}x = -\omega^2 x$

This is the equation of simple harmonic motion.

Time period of oscillation

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{A\rho g}}$ ⇒ $T \propto \frac{1}{\sqrt{A}}$

34. The time period of simple pendulum in air

$T = t_0 = 2\pi\sqrt{\left(\frac{\ell}{g}\right)}$ .... **(i)**

ℓ, being the length of simple pendulum.

In water, effective weight of bob

w' = weight of bob in air – upthrust

⇒ $\rho V g_{eff} = mg - m'g$

$= \rho Vg - \rho' Vg = (\rho - \rho')Vg$

where ρ = density of bob,

ρ' = density of water

∴ $g_{eff} = \left(\frac{\rho - \rho'}{\rho}\right)g = \left(1 - \frac{\rho'}{\rho}\right)g$ ∴ $t = 2\pi\sqrt{\left[\left(1 - \frac{\rho'}{\rho}\right)g\right]}$ **.............(ii)**

Thus $\frac{t}{t_0} = \sqrt{\frac{1}{\left[\left(1-\frac{\rho'}{\rho}\right)\right]}} = \sqrt{\left(\frac{1}{1-\frac{1000}{(4/3)\times 1000}}\right)}$

$= 2 \Rightarrow t = 2\,t_0$

35. Since solid ball floats in between the two liquids hence $\rho_1 < \rho_3 < \rho_2$

36. For equilibrium, weight should be balanced by buoyant force.
density of oil < density of water and ball should be in between oil and water.

37. $\Delta U = mgh$
$\Delta U = (\sigma_b - \sigma_\ell)\,Vgh$

38. $\rho\ell\left(\frac{4v}{5}\right)g + \left(\frac{4v}{5}\right)g = \rho_m(V)g$
on solving $\rho_m = 3.5 \times 10^3$.

39. $\rho_{mix} = \frac{m_1+m_2+m_3}{3V} = \frac{V(d+2d+3d)}{3V} = 2d.$

40. $\rho_{mix} = \frac{3m}{V_1+V_2+V_3} = \frac{3m}{\frac{m}{d}+\frac{m}{2d}+\frac{m}{3d}} = \frac{3\times 6}{11}d = \frac{18}{11}d$

41. Since not touching,
So $R = F_b = \rho_l(vg) = 40g.$

SECTION (C)

1. For streamline flow, Reynold's number $N_R \propto \frac{r\rho}{\eta}$ should be less. For less value of N_R, radius and density should be small and viscosity should be high.

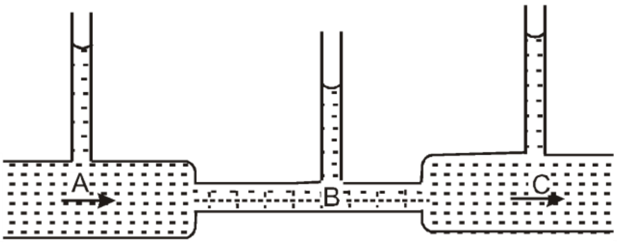
2. dA = 2cm and dB = 4cm ∴ rA = 1 cm and rB = 2cm From equation of continuity, av= constant

∴ $\frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$

3. This happens in accordance with equation of continuity and this equation was derived on the principle of conservation of mass and it is true in every case, either tube remain horizontal or vertical.

4. $a_1 v_1 v = a_2 v_2$
$4.20 \times 5.18 = 7.60 \times v_2 \Rightarrow$ 2.86 m/s

5. As cross-section areas of both the tubes A and C are same and tube is horizontal. Hence according to equation of continuity $v_A = v_C$ and therfore according to Bernoulli's theorem $P_A = P_C$ i.e height of liquid is same in both the tubes A and C.



6. From the Bernoulli's theorem

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2} \times 1.3 \times [(120)^2 - (90)^2] = 4095$ N / m^2 or Pascal

7. $v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20$ m/s

8. Pressure at the bottom of tank $P = h\rho g = 3 \times 10^5 \frac{N}{m^2}$

Pressure due to liquid column $P_1 = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$ and

velocity of water $v = \sqrt{\frac{2p_1}{\rho}} = \sqrt{\frac{2\times2\times10^5}{10^3}} = \sqrt{400}$ m/s

10. If velocities of water at entry and exit points are v_1 and v_2, then according to equation of continuity,

$A_1 v_1 = A_2 v_2 \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

11. Using Bernoulis theorem. High pressure → law velocity and low pressure → High velocity.

13. Time taken for the level to fall from H to H' $t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$

According to problem- the time taken for the level to fall from h to $\frac{h}{2}$ $t_1 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{h} - \sqrt{\frac{h}{2}}\right]$

and similarly time taken for the level to fall from $\frac{h}{2}$ to zero

$t_2 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{h}{2}} - 0\right] \therefore \frac{t_1}{t_2} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - 0} = \sqrt{2} - 1.$

14. Time required to emptied the tank $t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$ $\therefore \frac{t_2}{t_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{4h}{h}} = 2$ $\therefore t_2 = 2t$

6. According to equation of continuity the volume of liquid flowing through the tube in unit time remains constant i.e. $A_1v_1 = A_2v_2$, hence option (a) is correct

According to Bernoulli's theorem,

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \Rightarrow h\rho g = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$

$\therefore v_2^2 - v_1^2 = 2gh$

Hence option (c) is correct.

Also, according to Bernoulli's theorem option (d) is correct

18. Upthrust – weight of body = apparent weight

$VDg - Vdg = Vda$,

Where a = retardation of body ∴ $a = \left(\frac{D-d}{d}\right)g$

The velocity gained after fall from h height in air, $v = \sqrt{2gh}$

Hence, time to come in rest,

$t = \frac{v}{a} = \frac{\sqrt{2gh} \times d}{(D-d)g} = \sqrt{\frac{2h}{g}} \times \frac{d}{(D-d)}$

19. $t = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H_1} - \sqrt{H_2}\right]$

Now, $T_1 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{\frac{H}{\eta}}\right]$ and $T_2 = \frac{A}{a}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{H}{\eta}} - \sqrt{0}\right]$

According to problem $T_1 = T_2$ ∴ $\sqrt{H} - \sqrt{\frac{H}{\eta}} = \sqrt{\frac{H}{\eta}} - 0$ ⇒ $\sqrt{H} = 2\sqrt{\frac{H}{\eta}} \Rightarrow \eta = 4$

29. R = vt

R = $\sqrt{2gD}\ \sqrt{\frac{2(H-D)}{g}}$

R = $2\sqrt{D(D-h)}$.

30. In previous Question $\frac{dR}{dh}$ = 0 ⇒ h = $\frac{H}{2}$.

31. from equation of continuity,

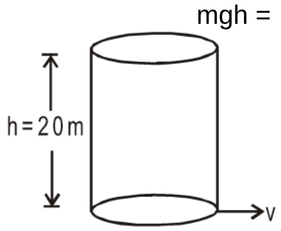
(A × 3) = (A × 1.5) + (1.5 A × V) ⇒ V = 1 m/s2

32. From continuity equation, velocity at cross-section (1) is more than that at cross-section (2). Hence; $P_1 < P_2$

33. When cross-section of duct is decreased, the velocity of water increased and in accordance with Bernoulli's theorem, the pressure P decreased at that place.

36. As P.E. = K.E.

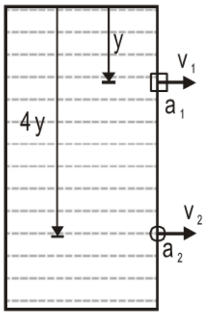
mgh = $\frac{1}{2}$ mv2



v = $\sqrt{2gh}$ = $\sqrt{2 \times 10 \times 20}$ [Here : g = 10 m/s2]

= 20 m/s

38. Velocity of efflux at a depth h is given by V = Volume of water following out per second from both the holes are equal



∴ $a_1V_1 = a_2V_2$

or $(L^2)\sqrt{2g(y)} = \pi R^2 \sqrt{2g(4y)}$

or $R = \frac{L}{\sqrt{2\pi}}$

39. Apply continuity equation $A_1V_1 = A_2V_2$ and Bernauli theorum $\frac{p_0}{\rho} + \frac{v^2}{2} + gh$ = constant at the top and at the hole.

40. As the stream falls down, its speed will increase and cross-section area will decrease.
Thus it will become narrow.
Similarly as the stream will go up, speed will decrease and cross-section area will increase.
Thus it will become broader.
Hence statement-1 is correct and statement-2 is correct explanation also.

41. According to rule for flowing of liquid product of area and velocity is same

$A_1v_1 = A_2v_2$

$$\frac{v_1}{v_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_2^2} = \left(\frac{r_2}{r_1}\right)^2 = \frac{4}{9} = 4 : 9$$

42. Bernoulli's equation for flowing liquid be written as

$p + \frac{1}{2}\rho v^2 + \rho gh$ = constant … (i)

Here, p = pressure energy per unit volume of liquid
ρ = density of liquid (water)
h = height of liquid column
v = velocity of liquid
and g = acceleration due to gravity Dividing Eq. (i) by ρg, we have

$\frac{\rho}{\rho g} + \frac{v^2}{2g} + h$ = constant

It this expression $\frac{v^2}{2g}$ is velocity head and $\frac{p}{\rho g}$ is pressure head.
It is given that,
velocity head = pressure head

ie. $\frac{v^2}{2g} = \frac{p}{\rho g}$

or $v^2 = \frac{2p}{\rho}$

or $v^2 = \frac{2 \times 13.6 \times 10^3 \times 40 \times 10^{-2} \times 9.8}{10^3}$

∴ v = 10.32 m/s

43. Let height of water column in the tank be h. Total pressure (p) = atmospheric pressure (p_0) + pressure due to water column in tank (p')

∴ $p' = p - p_0 = 3 - 1 = 2$ atm

or $h\rho g = 2 \times 10^5$

or $h \times 10^3 \times 10 = 2 \times 10^5$

or $h = 20$ m

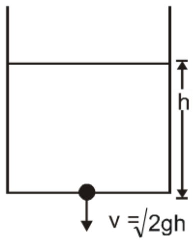
Hence, velocity of water coming from hole ie, velocity of efflux is

$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20}$

$= \sqrt{400}$ ms^{-1}

44. $V = \dfrac{20 \times (0.2)^2}{(0.1)^2} = 80$ cm/sec.

45.



$\beta = \alpha \times \sqrt{2gh}$ so $h = \dfrac{1}{2g} \cdot \dfrac{\beta^2}{\alpha^2}$

46. Bernoulli's theorem for unit mass of liquid

$\dfrac{P}{\rho} + \dfrac{1}{2}u^2$ = constant

As the liquid starts flowing, it pressure energy decreases

$\dfrac{1}{2}u^2 = \dfrac{P_1 - P_2}{\rho}$ ⇒ $\dfrac{1}{2}u^2 = \dfrac{3.5 \times 10^5 - 3 \times 10^5}{10^3}$

$= \dfrac{2 \times 0.5 \times 10^5}{10^3}$ ⇒ $u^2 = 100$ ⇒ $u = 10$ m/s

EXERCISE # 2

1. (i) $W_{apparent} = W_{air} - \rho V g$

Let ρ_1 is the density of liquid at 20°C

then $40 = 50 - V\rho_1 g$

⇒ $10 = V\rho_1 g$ ....(i)

And ρ_2 be the density of liquid at 70°C

then $45 = 50 - V\rho_2 g$

⇒ $5 = V\rho_2 g$ ...(ii)

Dividing Eq.(i) by Eq. (ii), we get

Density ratio $\dfrac{\rho_1}{\rho_2} = \dfrac{2}{1}$

(ii) $\rho_2 = \dfrac{M}{V_1(1 + \gamma\Delta\theta)} = \dfrac{\rho_1}{(1 + \gamma\Delta\theta)}$

⇒ $\dfrac{\rho_1}{\rho_2} = 1 + \gamma\Delta\theta$

As $\dfrac{\rho_1}{\rho_2} = 2$ ∴ $2 = 1 + \gamma\Delta\theta$

$\gamma\Delta\theta = 1$

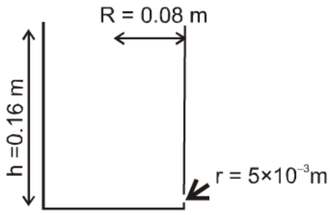
$\gamma = \frac{1}{\Delta\theta} = \frac{1}{(70-20)} = 0.02/°C$

2. $a = a_0(\hat{i} - \hat{j} + \hat{k})$
As there is no gravity; the pressure difference will be only due to the acceleration.
All points other than point 'B', are acted upon by a pseudo force.
Hence, at point 'B' pressure developed is zero.

3. Initially $\left[\frac{0+\rho g a}{2}\right](1)^2 + \left[\frac{\rho g a + 3\rho g a}{2}\right](a^2) = 2.5\,\rho g a^3$

$= \left[\frac{0+2\rho g a}{2}\right](2a^2) = 2\rho g a^2$ = difference $= 0.5\,\rho g a^3$ [which is one fifth of initial]

4.



$A_1 v_1 = A_2 v_2$

$\pi R^2\, dh/dt = \pi r^2 v$ ....(i)

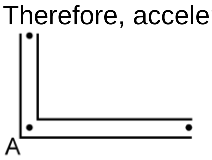
$v = \sqrt{2gh}$ ....(ii)

from equation (ii) put the value of v in equation (i)

$-\pi R^2\, dh/dt = \pi r^2\sqrt{2gh} \Rightarrow -\int \frac{R^2 dh}{r^2\sqrt{2gh}} = \int dt - \frac{R^2}{r^2\sqrt{2g}}\int_h^0 \frac{dh}{\sqrt{h}} = \int_0^t dt$

on solving t = 46.26 second.

5. No sliding ⇒ pure rolling
Therefore, acceleration of the tube = 2a (since COM of cylinders are moving at 'a')

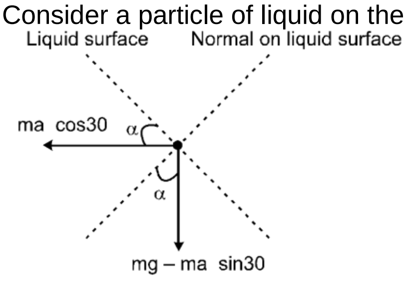


$P_A = P_{atm} + \rho(2a)L$ (From horizontal limb)

Also ; $P_A = P_{atm} + \rho g H$ (From vertical limb) ⇒ $a = \frac{gH}{2L}$ **Ans.**

6. $y = \frac{\omega^2 r^2}{2y}$
Put values and get y = 2cm.

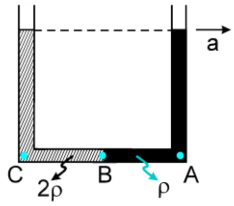
7. Acceleration of the container on smooth inclined plane is $a = g\sin\theta = 5\ ms^{-2}$.
Consider a particle of liquid on the liquid surface



$\tan\alpha = \frac{a\cos\theta}{g - a\sin\theta} = \frac{5\cos 30}{10 - 5\sin 30} = \frac{5(\sqrt{3}/2)}{10 - 5(1/2)} = \frac{5\sqrt{3}/2}{15/2}$

$\tan\alpha = \frac{1}{\sqrt{3}} = \tan 30 \qquad \alpha = 30$

8. For the given situation, liquid of density 2 ρ should be behind that of ρ.
From right limb :



$P_A = P_{atm} + \rho gh$

$P_B = P_A + \rho a \frac{\ell}{2} = P_{atm} + \rho gh + \rho a \frac{\ell}{2}$

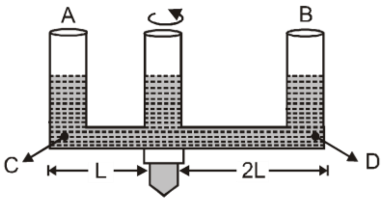
$P_C = P_B + (2\rho) a \frac{\ell}{2} = P_{atm} + \rho gh + \frac{3}{2} \rho a \ell$ .... (1)

But from left limb :

$P_C = P_{atm} + (2\rho) gh$ .... (2)

From (1) and (2) :

$P_{atm} + \rho gh + \frac{3}{2} \rho a \ell = P_{atm} + 2\rho gh \quad \Rightarrow \quad h = \frac{3a}{2g} \ell$ **Ans.**

9. 

at points C and D
due to centrifugal force the liquid will rise in both sides of tube.

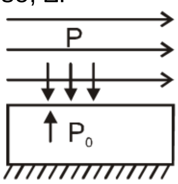
11. $F_{thrust} = \rho a v^2$
$F_{net} = F_1 - F_2 = a\rho[2g(h_1 - h_2)] = a\rho(2gh)$.

12. When we move from centre to circumference, the velocity of liquid goes on decreasing and finally becomes zero.

EXERCISE # 3
PART - I

1. $P + \frac{1}{2}\rho v^2 = P_0 + 0$

so, $\Delta P = \frac{1}{2}\rho v^2$



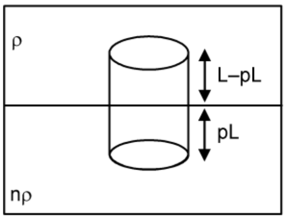
$F_{ner} = \frac{1}{2} \times 1.2 \times 40 \times 40 \times 250 \text{ N} = 2.4 \times 10^5 \text{ N}$

2. Power = $\vec{F}.\vec{V} = P A \vec{V} = \rho g h A V$

= 13.6 ×10^3 × 10 × 150 × 10^{-3} × 0.5 × 10^{-3} / 60 watt = $\frac{102}{60}$ watt = 1.70watt

3. Volume inflow rate = volume anflow rate

$\pi R^2 V = n\pi r^2$ (v) ⇒ $v = \frac{\pi R^2 V}{n\pi r^2} = \frac{VR^2}{nr^2}$

4. 

wt of body = upthrust by the two liquids

If A = Area of section then

(d A.L) g = [ρA (L - pL) + nρ ApL] g

On solving ⇒ d = (1+ (n - 1)p) ρ

5. Pressure on both sides are equal

$P_1 = P_2$

$h_{oil}\, s_{oil}\, g = h_{water}\, s_{water}\, g$

$s_{oil} = \frac{[65+65]1000}{[65+65+10]}$ = 928

6. $V = \sqrt{2gh} = \sqrt{2\times10\times2} = 2\times3.14 = 6.25 m/sec$

$\frac{d(vol)}{dt} = AV = (2\times10^6)\times6.25 = 12.6\times10^{-6}$

7. $P_o + \rho_w g(15\text{ cm}) = P_o + \rho_{oil} g(20\text{ cm})$

$\rho_w g(15\text{ cm}) = \rho_{oil} g(20\text{ cm})$

$\rho_{oil} = \frac{\rho_w .15}{20} = \frac{1000\times15}{20}$ = 750 kg/m³

PART - II

1. Diameter = 8 × 10^{-3} m

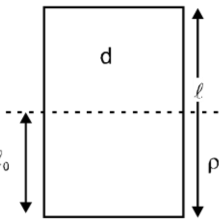
v = 0.4 m/s

$v = \sqrt{u^2+2gh} = \sqrt{(0.4)^2+2\times10\times0.2}$ = 2 m/s

$A_1v_1 = A_2v_2$

$\pi\left(\frac{8\times10^{-3}}{4}\right)^2\times0.4 = \pi\times\frac{d^2}{4}\times2$

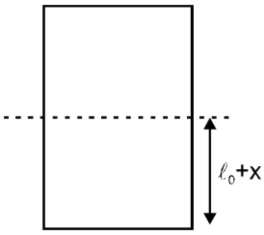
d = 3.6 × 10^{-3} m.

2. 

At equilibrium

$F_b = mg$

$\rho A\ell_0 g = dA\ell g$ ....(i)



Restoring force,

$F = mg - F_b'$

$F = mg - \rho A(\ell_0 + x)g$

$dA\ell a = dA\ell g - \rho A\ell_0 g - \rho gAx$

$a = -\frac{\rho g}{d\ell} x$

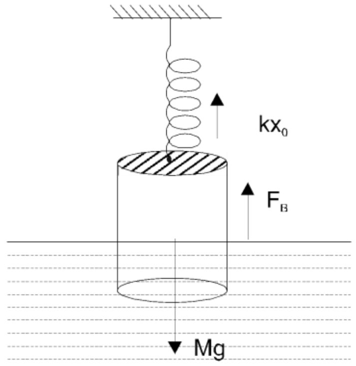
$\omega = \sqrt{\frac{\rho g}{d\ell}}$

$T = 2\pi\sqrt{\frac{\ell d}{\rho g}}$ ....(i)

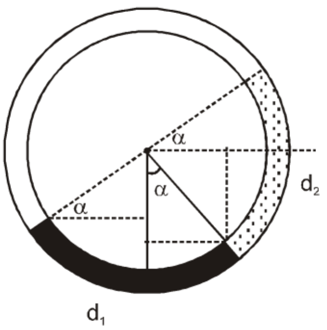
3. $kx_0 + F_B = Mg$

$kx_0 + \sigma \frac{L}{2} Ag = Mg$

$x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k}$



$= \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$



4.

$R\sin\alpha\, d_2 + R\cos\alpha\, d_2 + R(1 - \cos\alpha)d_1$

$= R(1 - \sin\alpha)\, d_1$

$(\sin\alpha + \cos\alpha)\, d_2 = d_1(\cos\alpha - \sin\alpha)$

$\Rightarrow \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$

5. $Q = 0.74\ m^3/\ min$

$= \frac{0.74}{60} m^3/sec = AV = \pi \times (0.02)^2 \times V$

$\Rightarrow$ $V = 9.81$ m/s

$h = \frac{V^2}{2g} = 4.8$ m

6. $(Ax\ \rho g) = F_{res}$

$(\pi r^2 \rho g)\ x = Fres$

$\omega^2 = \frac{\pi r^2 \rho g}{m}$

$\omega = \sqrt{\frac{\pi r^2 \rho g}{\rho v}}$

$\omega = r\sqrt{\frac{\pi g}{v}}$

$\omega = 2.5\times10^{-2}\sqrt{\frac{3.14\times10}{310\times10^{-6}}}$

$= 2.5 \times 10^{-2}\sqrt{\frac{3.14\times10^7}{3.10\times10^{-2}}}$

$= 2.5\times10^{-2}\sqrt{10^5}$

$= 2 \times 10^{-2} \times 10^2 \sqrt{10}$

$= 2.5\sqrt{10}$ rad/s

$\Rightarrow 2\pi f = 2.5\sqrt{10} = 2.5\pi$

$f = \frac{2.5}{2} = 1.25\ sec^{-1}$

7. Mass per unit time = 3Av

Force due to momentum loss = $\frac{1}{4}\rho Av \times v^2$

Force due to bounce back = $\frac{1}{4}\rho Av \times 2v^2$

Pressure = $\frac{\frac{\rho Av^2}{4} \times \frac{\rho Av^2}{2}}{A} = \frac{3}{4}\rho v^2$

8. $\delta = \frac{\rho_0 Vg \times L}{Ay}$

$\delta' = \frac{(\rho_0 - \rho_L)vg \times L}{Ay}$

$\Rightarrow \frac{\delta'}{\delta} = \frac{\rho_0 - \rho_L}{\rho_0} = \frac{8-2}{8}$

$\delta' = 3$ mm