TOPIC : WAVE ON A STRING
EXERCISE # 1

SECTION (A)

8. By defination

9. $y_1 = a\sin(\omega t - kx)$
$y_2 = a\cos(\omega t - kx)$

Phase difference $\Delta\phi = \frac{\pi}{2}$

SECTION (B)

1. $v_{max} = A\omega = 4v$ $A = 4.\frac{v}{\omega}$ $\Rightarrow$ $A = 4\,.\,\frac{\lambda}{2\pi}$ $\lambda = \frac{\pi}{2}.A$

2. $\because$ $\omega = \frac{2\pi}{0.01}$, $K = \frac{2\pi}{0.3}$

$\frac{\omega}{v} = k$ $\Rightarrow$ $v = \frac{\omega}{k} = \frac{2\pi}{0.01} \times \frac{0.3}{2\pi}$

$v = 30$ units

3. $y = 3\cos\left(\frac{x}{4} - 10t - \frac{\pi}{2}\right)$

maximum particle velocity = $A\omega = 3 \times 10 = 30$ m/s

4. $y = a\sin\pi\,(40t - x) = a\sin 40\pi\left(t - \frac{x}{40}\right)$

$y = a\sin\omega\left(t - \frac{x}{v}\right)$ wave velocity = 40

5. $\frac{T}{4} = 0.17$ sec

$T = 0.17 \times 4 = 0.68$ sec.

$f = \frac{1}{T} = \frac{1}{0.68} = \frac{100}{68} = 1.47$ Hz

6. $y = y_0\sin\frac{2\pi}{\lambda}(vt - x)$

given

$A\omega = 2 \times v$

$A\,.\,\frac{\omega}{v} = 2$

$A\,.\,\frac{2\pi}{\lambda} = 2$

$\lambda = \pi A$

$\lambda = \pi y_0$

8. $y = a\sin\pi\left[\frac{t}{2} - \frac{x}{4}\right]$

$\Rightarrow$ $v = 2$

distance travelled by wave in t = 8 seconds

$d = v \times t$

$2 \times 8 = 16$ units

10. $\omega = 2\pi \times f = 2\pi \times \frac{1}{0.04}$

$f = \frac{100}{4} = 25$ Hz

acceleration $= -\omega^2 y$

maximum acceleration $= -\omega^2 A = \left(\frac{2\pi}{0.04}\right)^2 \times 3 = 7.5 \times 10^4$ cm/s^2

19. Equation of wave is

$y = a \sin\left(400\pi t - \frac{\pi x}{0.85}\right)$ Comparing this equation with $y = a \sin\left(\omega t - \frac{2\pi x}{\lambda}\right)$

$\omega = 400\pi$, or $2\pi n = 400\pi$ $\Rightarrow$ $n = 200$

and $\lambda = 0.85 \times 2 = 1.7$

Velocity $v = n\lambda = 1.7 \times 200 = 340$ m/s

20. Given equation is

$y = 25 \cos(2\pi t - px)$ …(1)

Standard equation is

$y = A \cos(\omega t - kx)$ …(2)

Comparing (1) and (2),

Amplitude $A = 25$, $\omega = 2\pi$, $k = \pi$

$\therefore$ Frequency $n = \frac{\omega}{2\pi} = \frac{2\pi}{2\pi} = 1$ Hz

22. $v = 10$ m/s, $f = 100$ Hz

phase difference between two particles at $\Delta x = 2.5$ cm

$\lambda = \frac{10}{100} = \frac{1}{10}$m $= 10$ cm

$D\phi = \frac{2\pi}{\lambda}.\Delta x = \frac{2\pi}{10} \times 2.5 = \frac{\pi}{2}$

23. The sign between two terms in argument of sine will define its direction.

Writing the given wave equation

$y = 0.25 \sin(10\pi x - 2\pi t)$ ....(i)

The minus (–) between ($10\pi x$) and ($2\pi t$) implies that the wave is travelling along positive x direction.

Now comparing Eq. (i) with standard wave equation

$y = a \sin(kx - \omega t)$ ....(ii)

We have

$a = 0.25$ m, $\omega = 2\pi$, $k = 10\pi$ m

$\therefore$ $\frac{2\pi}{T} = 2\pi$ $\Rightarrow$ $f = 1$ Hz

Also, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{10\pi} = 0.2$ m

Therefore, the wave is travelling along +ve x direction with frequency 1 Hz and wavelength 0.2 m.

24. $\frac{2\pi}{\lambda} = \alpha$ $\alpha = \frac{2\pi}{0.05} = 25\pi$ $\frac{2\pi}{T} = \beta = 2$ $T = \pi$

25. As $f_1 = f$, $f_2 = \frac{f}{2}$, $f_3 = f$

$\therefore \omega_1 = 2\pi f$ $\Rightarrow \omega_3 = 2\pi f$ and $\omega_2 = \pi f$

26. Satisfy the standard equation of wave

27. Here, displacement equation is

$y = 0.03 \sin \pi (2t - 0.01 x)$ ...(i)

The standard equation is

$y = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ ...(ii)

Comparing the given Eq. (i) with standard Eq. (ii), we have

$\frac{2}{\lambda} = 0.01$ or $\lambda = \frac{2}{0.01} = 200$ m.

29. The given waves are

$y_1 = 10^{-6} \sin[100t + (x/50) + 0.5]$ m

and $y_2 = 10^{-6} \cos[100t + (x/50)]$ m

$\Rightarrow$ $y_2 = 10^{-6} \sin [100t + (x/50) + \frac{\pi}{2}]$m

$\left[\because \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta\right]$

Hence, the phase difference between the waves is

$\Delta\phi = \left(\frac{\pi}{2} - 0.5\right)\text{rad} = \left(\frac{3.14}{2} - 0.5\right)\text{rad} = (1.57 - 0.5)$ rad $= (1.07)$ rad

Note : The given waves are sine and cosine function so there are plane progressive harmonic waves.

30. $y = 10 \sin\pi(0.02x - 2.00t)$

$\frac{\partial y}{\partial t} = -20\pi \cos \pi(0.02x - 2.00t)$

$\left(\frac{\partial y}{\partial t}\right)_{max} = 20\pi = 63 \text{ units}$

SECTION (C)

1. Ratio of amplitudes

$A_1 : A_2 = 10 : 5 = 2 : 1$

3. $y_2 = a \cos\omega t + y_1 = a \sin\left(\omega t + \frac{\pi}{6}\right)$

$y_2 = a \sin\left(\omega t + \frac{\pi}{6}\right)$

$\phi = \omega t + \frac{\pi}{2} - \omega t + \frac{\pi}{6} = \frac{\pi}{2} - \frac{\pi}{6}$

$A = \sqrt{a^2 + a^2 + 2a.a.\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)} = a\sqrt{1+1+1} = \sqrt{3}a$

4. $y_1 = 4\sin\omega t$, $y_2 = 3\sin\left(\omega t + \frac{\pi}{2}\right)$

$A_R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos 90}$

$= \sqrt{4^2 + 3^2 + 2 \times 3 \times 4.\cos 90} = 5$

5. $A_R = \sqrt{A_1^2 + A_2^2 + 2A_1.A_2 \cos\Delta\phi}$

$A_R = A = A_1 = A_2$ ⇒ $\Delta\phi = 120°$

7. y = 10 sin (ax + bt)

81% of energy is reflected

Reflected energy = $\frac{81}{100}$ × Energy incidented

$A_{Ref}^2 = \frac{81}{100}$ × Energy incidented

$A_{ref} = \frac{9}{10}.A = \frac{9}{10}\times 10 = 9$

Equation of reflected wave is

x = 9 sin(ax – bt)

8. $A = \sqrt{a_1^2 + a_2^2 + 2a_1.a_2.\cos 90°} = \sqrt{a_1^2 + a_2^2}$

11. $\frac{v_1}{v_2} = \frac{\sqrt{\frac{T}{\mu_1}}}{\sqrt{\frac{T}{\mu_2}}} = \sqrt{\frac{\mu_2}{\mu_1}} = \sqrt{\frac{300/2}{150/1}}$ ⇒ $\frac{v_1}{v_2} = \frac{1}{1}$

SECTION (D)

1. $T_1 : T_2 : T_3 : T_4 = 1 : 4 : 9 : 16$

$f = \sqrt{T}$

Ratio of fundamental frequencies is

$f_1 : f_2 : f_3 : f_4 = 1 : 2 : 3 : 4$

2. $f = \frac{1}{2\ell}.\sqrt{\frac{T}{\mu}}$

$\frac{df}{f} = \frac{1}{2\ell^2}\sqrt{\frac{T}{\mu}}\cdot\frac{d\ell}{f}$ ⇒ $\frac{df}{f} = \frac{1}{2\ell^2}\sqrt{\frac{T}{\mu}}\cdot\frac{d\ell}{\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}} = \left(\frac{d\ell}{\ell}\right)$

$\left(\frac{df}{f}\right) = \left(\frac{d\ell}{\ell}\right)$ ⇒ % change in frequency = 1%

3. $f \propto \sqrt{T}$ ⇒ To double the frequency tension should be increased 4 times.

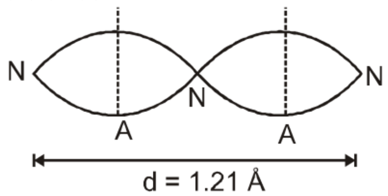
4. Wave is pulted at 25 cm from one end. this point becomes antinode.

No. of loops = 2 = p

$f = \frac{p}{2\ell}.\sqrt{\frac{T}{\mu}} = \frac{2}{2\times 1}\sqrt{\frac{20}{5\times 10^{-4}}} = 1\times\sqrt{4\times 10^4}$ = 200 Hz

5. 3 = Nodes, 2 = antinodes

distance = 1.21 Å



No. of loops = 2
$\lambda = 1.21$ Å

6. f = $\frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{T}{\pi r^2.\ell^2.\rho}}$ ⇒ $f \propto \sqrt{\frac{T}{r^2}}$

$\frac{f_1 r_1}{\sqrt{T_1}} = \frac{f_2 r_2}{\sqrt{T_2}}$ ⇒ $f_1 = f_2$ $\frac{f_1 \times 2}{2} = \frac{f_2 \times 1}{1}$ ⇒ $f_1 : f_2 = 1 : 1$

7. n = $\frac{1}{2\ell}\sqrt{\left(\frac{T}{\mu}\right)}$ ∴ T = $(2n\ell)^2 \times \frac{M}{\ell}$ = $\left(2\times\frac{1}{4}\times 2\right)^2 \times \frac{80}{2}$ = 40 N

8. $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{(1+2)^2}{(1-2)^2} = \frac{9}{1}$

10. y = 0.15 sin 5x . cos300t
kx = 5x

$\frac{2\pi}{\lambda} = 5$ $\lambda = \frac{2\pi}{5} = \frac{2\times 3.14}{5}$ = 0.4 × 3.14 = 1.256 m

11. ℓ = 110 cm
no. of loops = 3
$f_1 : f_2 : f_3 = 1 : 2 : 3$

13. T = 10N,
∵ $f \propto \sqrt{T}$ to produce double frequency tension should be 4 times = 40 N

14. $\frac{\pi x}{20} = \frac{2\pi x}{\lambda}$

λ = 40 separation between two consecutive nodes = $\frac{\lambda}{2}$ = 20 cm

15. y = acos(kx – ωt)
Equation of other wave to produce stationary wave, y= 0 at x = 0 is y = –a cos(kx + ωt)

17. y = 3cos $\left(\frac{\pi}{4} - 10t - \frac{\pi}{2}\right)$ maximum velocity of particle = Aω = 3 × 10 = 30 units

18. k = π
k = $\frac{2\pi}{\lambda} = \pi$
λ = 2 cm

19. Velocity of standing wave is not difined.

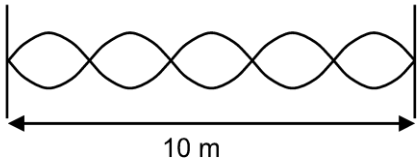
21. ∵ $f \propto \sqrt{T}$ ⇒ new frequency = $\sqrt{2}$ n

23. Frequency] n = $\frac{1}{2\ell}\sqrt{\frac{T}{m}}$
$n \propto \frac{\sqrt{T}}{\ell}$

Ratio of tensions

$$\frac{T_2}{T_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{\ell_2}{\ell_1}\right)^2 \quad \Rightarrow \quad (2)^2 \times \left(\frac{3}{4}\right)^2 = \frac{9}{4}$$

24. As standing waves are produced in the string and the string is vibrating in 5 segments, it can be shown as



$\therefore \; 5\frac{\lambda}{2} = 10 \quad \Rightarrow \quad \lambda = 4$ m

Given the velocity of the wave v = 20 m/s $\quad \therefore$ Frequency $\upsilon = \frac{v}{\lambda} = \frac{20}{4} = 5\,s^{-1} = 5\,Hz$

25. $\ell = 50$ cm $\quad f_0 = 270$ Hz

to produce f = 1000 Hz $\quad \ell = ?$

$f_0 = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad f_0 \times \ell_0 = \frac{1}{2}\sqrt{\frac{T}{\mu}}$

$\Rightarrow \quad f_0\ell_0 = f_1\ell_1 = f_2\ell_2$

$270 \times 50 = 1000 \times \ell_0$

$\ell = \frac{27 \times 5}{10} = \frac{135}{10} = 13.5$ cm

26. $\phi = 100$ Hz. distance of node from fixed end $= \frac{\lambda}{2} = 10$ cm

$\lambda = 20$ cm speed of waves $= \lambda \times f = 100 \times 20 = 2000$ cm/s = 20 m/s

28. n_x mode of vibration = n $\quad \therefore$ antinodes $\quad$ (n + 1) nodes

29. $(n + 1)\frac{v}{2\ell} = 420 \quad$ ......(1)

$\frac{nv}{2\ell} = 315 \quad$ ......(2)

(1) – (2) $\frac{V}{\mu} = 105$ Hz $\quad f_{min} = 105$ Hz

30. y = a sin ωt cos Kx

$y = \frac{1}{2}$ (2a sin ωt cos Kx) $\quad \therefore$ Amplitude of component wave is $\frac{a}{2}$

31. From law of length, the frequency of vibrating string is inversely proportional to its length, i.e.,

$n \propto \frac{1}{\ell}$

or $\quad n\ell$ = constant (say k)

or $\quad n\ell = k \;$ or $\ell = \frac{k}{n}$

The segments of string of length ℓ_1, ℓ_2, ℓ_3..... have frequencies n_1, n_2, n_3,....

Total length of string is ℓ

So, $\quad \ell = \ell_1 + \ell_2 + \ell_3 + ...$

$\therefore \quad \frac{k}{n} = \frac{k}{n_1} + \frac{k}{n_2} + \frac{k}{n_3} + ...$

or $\quad \frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3} + ...$

EXERCISE # 2

1. As $\frac{5\lambda}{2} = 20 \Rightarrow \lambda = 8$ cm

$K = \frac{2\pi}{\lambda} = \frac{314}{4}$

$\omega = KV \; \frac{2\pi}{8 \times 10^{-2}} \times 350 = 27500 \quad \therefore \quad y = 0.05 \sin\left(\frac{314}{4}x - 27500t\right)$

2. $y = \frac{1}{1+x^2}$ (t = 0)

$y = \frac{1}{1+(x-2v)^2}$ (t = 2)

Now comparing

$x - 2v = x - 1$

$v = 0.5 \frac{m}{sec}$

3. $V_{P_{max}} = A\omega = Y_0\, 2\pi f = 4V_\omega$

$Y_0\, 2\pi f = 4 \frac{2\pi f}{\frac{2\pi}{\lambda}} \quad \therefore \quad \lambda = \frac{\pi Y_0}{2}$

4. Put α, β, A, x and t in the equation

$\frac{2\pi}{\lambda} = 0.56\ cm^{-1}$

$2\pi f = 12\ sec^{-1}$

$\alpha x + \beta t + \frac{\pi}{6} = \frac{12.56 \times 180}{3.14} + 30 = 750^o$

$y = 7.5$ cm $\sin 750^o = 3.75$ cm.

$v = \frac{dy}{dt} = Ab \cos\left(\alpha x + \beta t + \frac{\pi}{6}\right)$

$= 7.5 \times 12 \times \frac{\sqrt{3}}{2} = 77.94$ cm/sec.

5. $\omega = 2\pi f = 4\pi\ sec^{-1}$

$K = \frac{2\pi}{\lambda} = 2\pi\ m^{-1}$

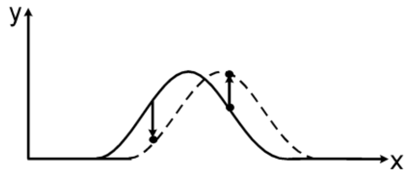
$\therefore y = 0.5 \cos(2\pi x + 4\pi t)$

6. $V_{CD} = \sqrt{\frac{3.2g}{8 \times 10^{-3}}} = \sqrt{4000} \cong 63 \frac{m}{sec}$

$V_{AB} = \sqrt{\frac{6.4g}{10 \times 10^{-3}}} \cong \sqrt{6400} \cong 79 \frac{m}{sec}$

7. $R_A = \frac{V}{V_A}$, $R_B = \frac{V}{V_B}$

as $V_A > V_B$, $R_A < R_B$

8. 

Dotted shape shows pulse position after a short time interval. Direction of the velocities are decided according to direction of displacements of the particles.

at x = 1.5 slope is +ve

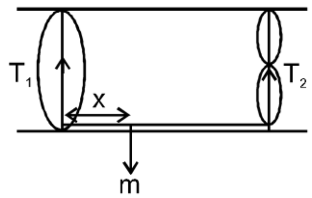
at x = 2.5 slope is –ve

9. By defination

10. $\frac{\lambda_1}{2} = \ell \Rightarrow \lambda_1 = 2\ell$

$\lambda_2 = \ell \quad \Rightarrow \therefore \frac{\lambda_1}{\lambda_2} = 2$

$\frac{v_1/f}{v_2/f} = 2$



$\frac{v_1}{v_2} = 2 = \sqrt{\frac{T_1/\mu}{T_2/\mu}} \quad \Rightarrow \quad \frac{T_1}{T_2} = 4$ ——(1)

Now moment about P : $T_1 x = T_2 (\ell - x)$

$\ell - x = 4x \qquad x = \ell/5$

11. $\frac{I_1}{I_2} = \frac{1}{16} = \frac{A_1^2}{A_2^2} \quad \Rightarrow \quad \frac{A_1}{A_2} = \frac{1}{4}$

12. $7I_0 = I_0 + 9I_0 + 2 \times I_0 \times 3 . \cos \Delta\phi$

$-3I_0 = -6I_0 . \cos\Delta\phi$

$\cos\Delta\phi = \frac{1}{2} = \cos 60°$

13. $\frac{A_1}{A_2} = \frac{1}{3}$, $\frac{I_{max}}{I_{min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2} = \frac{16}{4} = \frac{4}{1}$

14. Amplitude varies between 0 and 2A

15. Path difference is λ between B and G.

16. By defination

17. $p_0 = A_0^2 \omega_0^2 \mu v$

$\frac{p_0}{2} = A^2 \omega^2 \mu v$ ∴ $2 = \frac{A_0^2 \omega_0^2}{A^2\omega^2}$

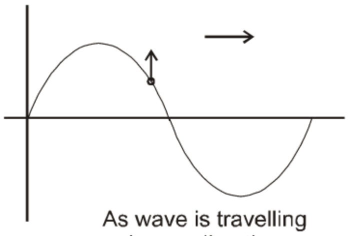
∴ As, $\omega = \omega_0$ (frequency remains the same) ∴ $A = \frac{A_0}{\sqrt{2}}$

18. $I = \frac{1}{2} \rho A^2 \omega^2 V$ put values

19. $V = \omega \sqrt{A^2 - y^2}$

$V_P = 2\pi f \sqrt{A^2 - y^2}$

$V_P = 2\pi \left(\frac{V}{\lambda}\right) \sqrt{A^2 - y^2} = \frac{2\pi}{0.5} \times 0.1 \sqrt{(0.1)^2 - (0.05)^2}$



$V_P = \frac{\sqrt{3}\pi}{50}$ j m/s

20. $\frac{\lambda}{2} \times n = 1$

22. $\frac{2\pi}{\lambda} = \frac{\pi}{4}$ $\lambda = 8$

24. As x = 0 is node ⇒ standing wave should be y = 2a sin kx sin ωt

25. $\frac{n}{2\ell}\sqrt{\frac{T}{\mu}} = 350$ and $\frac{n+1}{2\ell}\sqrt{\frac{T}{\mu}} = 420$

∴ $\frac{n}{n+1} = \frac{350}{420}$ ⇒ n = 5 ∴ $\frac{5\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\lambda}{5}$

$\frac{v}{f} = \frac{2\ell}{5}$ ⇒ $\frac{v}{2\ell} = \frac{f}{5}$ ⇒ $f' = \frac{f}{5} = 70$ Hz

26. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{T}{\rho.(A\ell)} \times \frac{1}{\ell^2}}$ ⇒ $f = \frac{1}{2}\sqrt{\frac{T}{\rho.\pi r^2 \ell^2}}$ ⇒ $f_1 : f_2 = 1 : 2$

27. The freqeuncy of vibrations of string is

$n = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

Given, $n_A = 2n_B$ ∴ $\frac{1}{2\pi}\sqrt{\frac{g}{\ell_A}} = 2.\frac{1}{2\pi}\sqrt{\frac{g}{\ell_B}}$ or $\frac{1}{\ell_A} = \frac{4}{\ell_B}$ or $\ell_B = 4\ell_A$

It is obvious from Eq. (i), the frequency of vibrations of strings does not depend on their mass.

28. **Key Ideal :** The standard wave equation is

y = a sin (ωt – kx)

The given wave equation is

$y = a \sin\left(100t - \frac{x}{10}\right)$

Compare it with the standard wave equation, we obtain

$\omega = 100$, $k = \frac{1}{10}$

Velocity of the wave,

$v = \frac{\omega}{k} = \frac{100}{\frac{1}{10}} = 100 \times 10 = 1000$ m/s

29. The frequency of vibrating wire is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$, where T is the tension in the wire.

We have n =

Here, m = mass per unit length = $\pi r^2 d$

$\therefore \quad n = \frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$

or $\quad n \propto \frac{1}{r}\left(\frac{T}{d}\right)^{1/2} \quad \therefore \quad \frac{n_1}{n_2} = \frac{r_2}{r_1}\left(\frac{T_1}{T_2} \times \frac{d_2}{d_1}\right)^{1/2}$

We have given,

$\frac{T_1}{T_2} = \frac{1}{2}$, $\frac{d_1}{d_2} = 2$, $\frac{r_1}{r_2} = \frac{1}{2}$

$\therefore \quad \frac{n_1}{n_2} = \frac{2}{1}\left(\frac{1}{2} \times \frac{1}{2}\right)^{1/2}$

or $\quad \frac{n_1}{n_2} = \frac{2}{1} \times \frac{1}{2} = 1 \quad$ or $\quad n_2 = n_1 = n$

30. **Key Ideal:** The expression of travelling wave is sine or cosine function of $\omega t \pm kx$.

The general expression of travelling wave can be written as

$y = A \sin(\omega t \pm kx)$ ...(i)

For travelling wave along positive x-axis we should use minus (–) sign only

$\therefore \quad y = A \sin(\omega t - kx)$

but $\quad \omega = \frac{2\pi v}{\lambda}$ and $k = \frac{2\pi}{\lambda}$

So, $\quad y = A \sin \frac{2\pi}{\lambda}(vt - x)$ ...(ii)

Given, A = 0.2 m , v = 360 m/s , λ = 60 m

Substituting in Eq. (ii), we have

$y = 0.2 \sin \frac{2\pi}{60}(360t - x)$ or $y = 0.2 \sin 2\pi\left(6t - \frac{x}{60}\right)$

31. $K = 2\pi = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = 1$

Minimum length = $\frac{\lambda}{2} = \frac{1}{2}$m

32. $\frac{I_1}{I_2} = \frac{9}{1}$

$\frac{A_1}{A_2} = \frac{3}{1}$

$$\frac{I_{max}}{I_{min}} = \frac{4^2}{2^2} = \frac{4}{1}$$

33. $\mu = 1.3 \times 10^{-4}$ $T = ?, k = 1, \omega = 30$

$v = \sqrt{\frac{T}{\mu}}$

$k = \frac{\omega}{v}$, $v = \frac{\omega}{k} = 30$

$T = v^2 \times \mu$

$= (30)^2 \times 1.3 \times 10^{-4} = 900 \times 1.3 \times 10^{-4}$

$= 1.17 \times 10^{-1} = 0.117$ N

34. $y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + s\, kx)$

$\frac{\lambda}{2} = -2a \cos \omega t \times \sin kx$

$\Rightarrow$ $y_1 + y_2 = 0$ at $x = 0$

36. $\eta = \frac{2}{2\ell}\sqrt{\frac{T}{\mu}} = 100$ Hz

37. $f = \frac{1}{2\ell}\sqrt{\frac{T}{\rho A}}$ $\therefore \frac{f_1}{f_2} = \dfrac{\frac{1}{2L}\sqrt{\frac{T}{\rho}\pi 4r^2}}{\frac{1}{4L}\sqrt{\frac{T}{\rho\pi r^2}}} = \frac{1}{1}$

38. $V_{vel.} = 10 + 10 = 20 \frac{m}{sec}$

when string is flate $v = f\lambda$

$20 = \frac{1}{\Delta t}\lambda$

$\lambda = 20\,\Delta t = 10$ m.

39. Energy $\propto A^2 \omega^2$

$\therefore \frac{E_1}{E_2} = \frac{A^2\omega^2}{A^2(2\omega)^2}$

$\therefore$ $E_2 = 4E_1$

40. After 2 sec.

$D = D_1 + D_2 = (2 \times 2) - (2 \times 2) = 0$

As their amplitude is same

$\therefore$ P.E. = 0 $\therefore$ purely kinetic

41. $f = \frac{5}{2L}\sqrt{\frac{9g}{\mu}}$ now $f_1 = \frac{3}{2L}\sqrt{\frac{mg}{\mu}}$

as $f = f_1$ $\Rightarrow$ $m = 25$ Kg

EXERCISE # 3

PART - I

1. Find the parameters and put in the general wave equation.

Here, $A = 2$ cm direction = +ve x direction

$v = 128$ ms^{-1} and $5\lambda = 4$

Now, $k = \frac{2\pi}{\lambda} = \frac{2\pi \times 5}{4}$ = 7.85 and $v = \frac{\omega}{k}$ = 128 ms^{-1} ⇒ ω = v × k = 128 × 7.85 = 1005

As, y = A sin (kx – ωt)

∴ y = 2sin (7.85 x – 1005 t)

= (0.02)m sin (7.85 x – 1005 t)

2. The given wave euqation is

$y = A\sin(\omega t - kx)$

Wave velocity, $v = \frac{\omega}{k}$ ...(i)

Particle velocity, $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$

Maximum particle velocity, $(v_p)_{max} = A\omega$ ...(ii)

According to the given question

$v = (v_p)_{max}$

$\frac{\omega}{k} = A\omega$ (Using (i) and (ii))

$\frac{1}{k} = A$ or $\frac{\lambda}{2\pi} = A \left(\because k = \frac{2\pi}{\lambda}\right)$

λ = 2πA

3. $\Delta\phi = \phi_1 - \phi_2 = \frac{\pi}{2} - 0.57$

= 1 radian

4. Fundamental frequency is given by

$$\nu = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

$$\nu \propto \frac{1}{\ell}$$

Here $\ell = \ell_1 + \ell_2 + \ell_3$

so $\frac{1}{\nu} = \frac{1}{\nu_1} + \frac{1}{\nu_2} + \frac{1}{\nu_3}$

5. $y = 3\sin\frac{\pi}{2}(50t - x)$ $y = 3\sin\left(25\pi t - \frac{\pi}{2}x\right)$

Wave velocity $v = \frac{\omega}{k} = \frac{25\pi}{\pi/2}$ = 50 m/sec.

$v_P = \frac{\partial y}{\partial t} = 75\pi\cos\left(25\pi t - \frac{\pi}{2}x\right)$

$v_{p\ max} = 75\pi$

then $\frac{v_{p_{max}}}{v} = \frac{75\pi}{50} = \frac{3\pi}{2}$

6. Y = A sin (ωt – kx + ϕ)

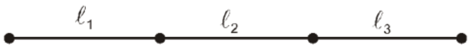
$\omega = 2\pi f = \frac{2\pi}{\pi} = 2$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi} = 1$

Y = 1 sin (2t – x + ϕ)

Alternate: $P \propto T_3$

$P \propto (PV)_3$
$P_2V_3 = C$
$PV_{3/2} = C$
$\gamma = 3/2.$

7. 

$n_1 = \frac{1}{2\ell_1}\sqrt{\frac{T}{\mu}}$

$n_2 = \frac{1}{2\ell_2}\sqrt{\frac{T}{\mu}}$

$n_3 = \frac{1}{2\ell_3}\sqrt{\frac{T}{\mu}}$

$n = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$\ell = \ell_1 + \ell_2 + \ell_3$

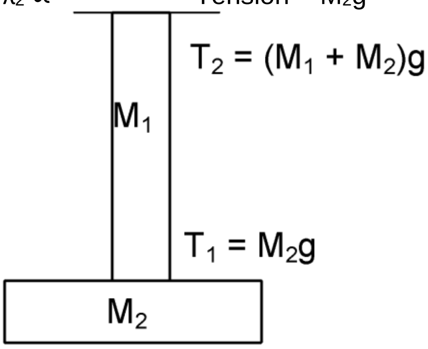
$\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

8. Fundamental frequency = highest common factor = 105Hz

9. $\lambda \propto v \propto \sqrt{\frac{T}{m/\ell}}$

$\lambda_1 \propto \sqrt{M_2}$ Tension = M_2g

$\lambda_2 \propto \sqrt{M_2 + M_1}$ Tension = M_2g



$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{M_1 + M_2}{M_2}}$

PART - III

1. By equation

$f = \frac{1}{0.04}$ and $\lambda = 0.5 \Rightarrow$ $V = \frac{1}{0.04} \times 0.5 = \frac{25}{2}$

by $V = \sqrt{\frac{T}{\mu}}$ $\Rightarrow$ $\left(\frac{25}{2}\right)^2 = \frac{T}{0.04} \Rightarrow T = \frac{625}{4} \times 0.04$

T = 6.25 N

2. $y(x,t) = e^{-[\sqrt{a}x + \sqrt{b}t]^2}$

It is transverse type $y(x,t) = e^{-(ax+bt)^2}$

Speed v = $\frac{\sqrt{b}}{\sqrt{a}}$

and wave is moving along –x direction.

3. $Y = A\sin(\omega t - kx) + A\sin(\omega t + kx)$

$Y = 2A \sin\omega t \cos kx$ standing wave

For nodes $\cos kx = 0$

$\frac{2\pi}{\lambda}.x = (2n + 1)\frac{\pi}{2}$ $\therefore$ $x = \frac{(2n+1)\lambda}{4}$, n = 0,1,2,3,........

4. Since, $I \propto A^2 \omega^2$

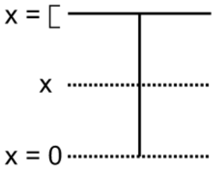
$I_1 \propto (2a)^2 \omega^2$

$I_2 \propto a^2 (2\omega)^2$

$I_1 = I_2$

Intensity depends on frequency also.

5. Let mass per unit length be λ.



$T = \lambda g x$ $v = \sqrt{\frac{T}{\lambda}} = \sqrt{gx}$

$v^2 = gx$,

$a = \frac{vdv}{dx} = \frac{g}{2}$

$\ell = \frac{1}{2}\frac{g}{2}t^2$ $\Rightarrow t = \sqrt{\frac{4\ell}{g}} = 2\sqrt{2}\text{ sec}$

6. $v = \sqrt{T/\mu} = \sqrt{M\frac{g}{\mu}}$

$\frac{\sqrt{g^2 + a^2}}{g} = \left(\frac{60.5}{60}\right)^2$

$1 + \frac{1}{2}\frac{a^2}{g^2} = 1 + \frac{1}{60}$ using by binomial approximation $\Rightarrow a = \frac{g}{\sqrt{30}}$ $\Rightarrow$ closest answer $a = \frac{g}{5}$

7. $V = F\lambda$

$\sqrt{\frac{T}{\mu}} = F\lambda$

$\sqrt{\frac{8}{5\times 10^{-3}}} = 100\lambda$

$\lambda = 40$ cm

$\frac{\lambda}{2} = 20\text{cm}$

8. k = 9 ω = 450 $\therefore$ $v = \frac{\omega}{k}$ = 50 m/s

$v = \sqrt{\frac{T}{\mu}}$ $\therefore$ $T = \mu v^2 = 50^2 \times 5 \times 10^{-3} = 12.5N$

9. $v = \frac{\omega}{k} = \frac{50}{2} = 25 m/s$

As ωt & kx both have +ve sing, wave is traveling to the –ve x-axis.