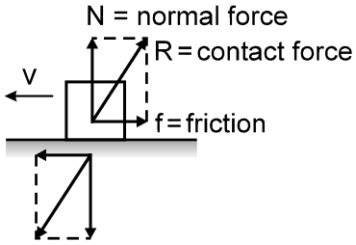
FRICTION

1. CONTACT FORCE

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it. In the figure the perpendicular component to the contact surface is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

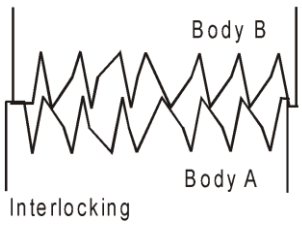
Therefore if R is contact force then

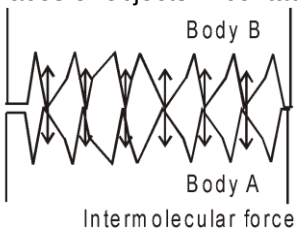
$$R = \sqrt{f^2 + N^2}$$

2. REASONS FOR FRICTION

(i) Inter-locking of extended parts of one object into the extended parts of the other object.

(ii) Bonding between the molecules of the two surfaces or objects in contact.





3. FRICTION FORCE IS OF TWO TYPES.

a. Kinetic b. Static

(a) Kinetic Friction Force

Kinetic friction exists between two contact surfaces only when there is **relative motion** between the two contact surfaces. It stops acting when relative motion between two surfaces ceases.

DIRECTION OF KINECTIC FRICTION ON AN OBJECT

It is opposite to the relative velocity of the object considered with respect to the other object in contact.

Note that its direction is not opposite to the force applied it is opposite to the relative motion of the body considered which is in contact with the other surface.

MAGNITUDE OF KINETIC FRICTION

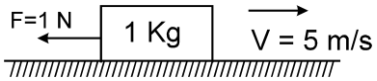
The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.**

Solved Examples

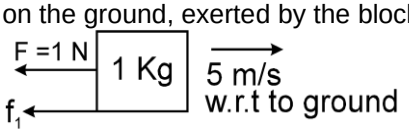
Example 1. Find the direction of kinetic friction force



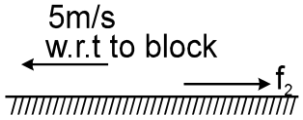
(a) on the block, exerted by the ground.
(b) on the ground, exerted by the block.

Solution :

(a)



(b)



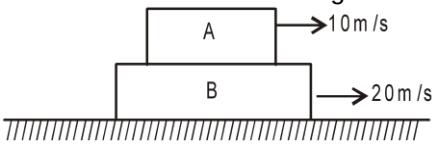
where f_1 and f_2 are the friction forces on the block and ground respectively.

Example 2. In above example correct relation between magnitude of f_1 and f_2 is

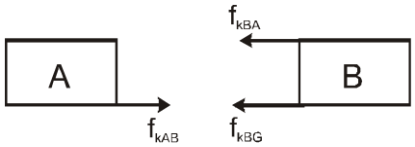
(A) $f_1 > f_2$ (B) $f_2 > f_1$
(C) $f_1 = f_2$ (D) not possible to decide due to insufficient data.

Solution : By Newton's third law the above friction forces are action-reaction pair, equal but opposite to each other in direction. Hence (C).
Also note that the direction of kinetic friction has nothing to do with applied force F.

Example 3. All surfaces as shown in the figure are rough. Draw the friction force on A & B

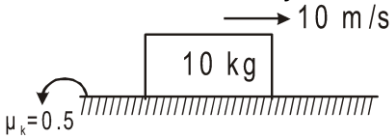


Solution :

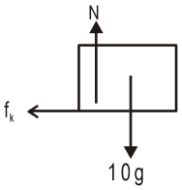


Kinetic friction acts in such a way so as to reduce relative motion.

Example 4. Find out the distance travelled by the blocks shown in the figure before it stops.



Solution :



$N - 10\,g = 0$

$N = 100$ N

$f_k = \mu_k N$

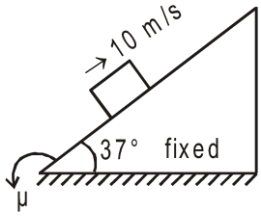
$f_k = 0.5 \times 100 = 50$ N

$f_k = ma$

$50 = 10\,a \Rightarrow a = 5$ $\quad\therefore v_2 = u_2 + 2as$

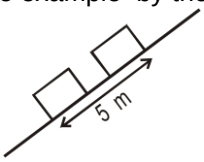
$0_2 = 10_2 + 2(-5)(S)$ $\quad\therefore S = 10$ m

Example 5. Find out the distance travelled by the block on incline before it stops. Initial velocity of the block is 10 m/s and coefficient of friction between the block and incline is $\mu = 0.5$.



Solution : $N = mg\cos37°$
$\therefore mg\sin 37° + \mu N = ma$
$a = 10\ m/s^2$ down the incline
Now $v^2 = u^2 + 2as$
$0 = 10^2 + 2(-10)\ S$
$\therefore S = 5\ m$

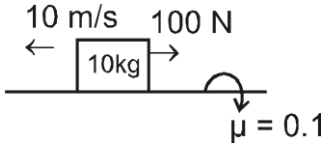
Example 6. Find the time taken in the above example by the block before it stops.



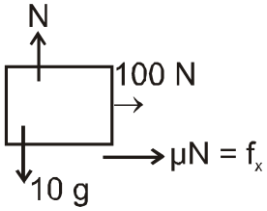
Solution : $a = g\sin 37° + \mu g\cos 37°$
$\therefore a = 10\ m/s^2$ down the incline

$\therefore S = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad 5 = \frac{1}{2} \times 10 \times t^2$
$\therefore t = 1sec.$

Example 7. A block is given a velocity of 10 m/s and a force of 100 N in addition to friction force is also acting on the block. Find the retardation of the block?



Solution : As there is relative motion
$\therefore$ kinetic friction will act to reduce this relative motion.
$f_k = \mu N = 0.1 \times 10 \times 10 = 10\ N$
$100 + 10 = 10a$
$a = 11\ m/s^2$



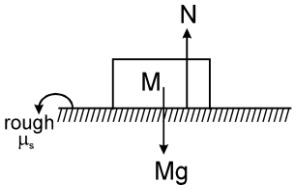
(b) STATIC FRICTION

It exists between the two surfaces when there is tendency of relative motion but no relative motion occurs along the two contact surfaces.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Solved Examples

Example 8. What is value of static friction force on the block?



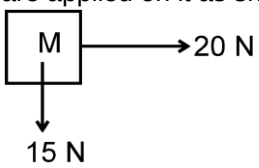
Solution : In horizontal direction as acceleration is zero.
Therefore $\Sigma F = 0$.
$\therefore f = 0$

Direction of static friction force :
The static friction force on an object is opposite to its impending motion relative to the surface.
Following steps should be followed in determining the direction of static friction force on an object.
(i) Draw the free body diagram with respect to the other object on which it is kept.
(ii) Include pseudo force also if contact surface is accelerating.
(iii) Decide the resultant force and the component parallel to the surface of this resultant force.
(iv) The direction of static friction is opposite to the above component of resultant force.

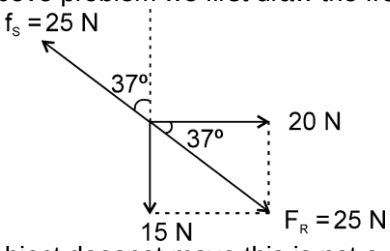
Note : Here once again the static friction is involved when there is no relative motion between two surfaces.

Solved Examples

Example 9. In the following figure an object of mass M is kept on a rough table as seen from above. Forces are applied on it as shown. Find the direction of static friction if the object does not move.



Solution : In the above problem we first draw the free body diagram to find the resultant force.



As the object doesnot move this is not a case of limiting friction. The direction of static friction is opposite to the direction of the resultant force F_R as shown in figure by f_s. Its magnitude is equal to 25 N.

4. MAGNITUDE OF KINETIC AND STATIC FRICTION

Kinetic friction :
The magnitude of the kinetic friction is proportional to the normal force acting between the two bodies. We can write

$$f_k = \mu_k N$$

where N is the normal force. **The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.** If the surfaces are smooth μ_k will be small, if the surfaces are rough μ_k will be large. It also depends on the materials of the two bodies in contact.

Static friction :

The magnitude of static friction is equal and opposite to the external force exerted, till the object at which force is exerted is at rest. **This means it is a variable and self adjusting force. However it has a maximum value called limiting friction.**

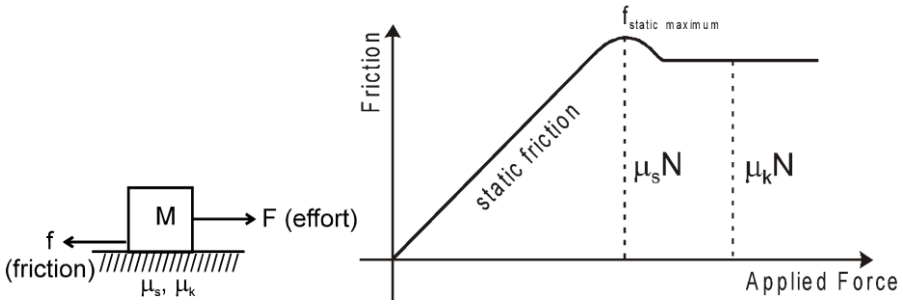
$$f_{max} = \mu_s N$$

The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0, f_s, f_{smax}$$

Here μ_s and μ_k are proportionality constants. μ_s is called coefficient of static friction and μ_k is called coefficient of kinetic friction. They are dimensionless quantities independent of shape and area of contact . It is a property of the two contact surfaces.

NOTE :- $\mu_s > \mu_k$ for a given pair of surfaces. If not mentioned then $\mu_s = \mu_k$ can be taken. Value of μ can be from 0 to ∞.



Following table gives a rough estimate of the values of coefficient of static friction between certain pairs of materials. The actual value depends on the degree of smoothness and other environmental factors. For example, wood may be prepared at various degrees of smoothness and the friction coefficient will vary.

Material	μ_s	Material	μ_s
Steel and steel	0.58	Copper and copper	1.60
Steel and brass	0.35	Teflon and teflon	0.04
Glass and glass	1.00	Rubber tyre on dry concrete road	1.0
Wood and wood	0.35		
Wood and metal	0.40	Rubber tyre on wet concrete road	0.7

(c) Rolling Friction : When a body (say wheel) rolls on a surface the resistance offered by the surface is called rolling friction.

Rolling friction forces arise as, for example, a rubber tyre rolls on pavement, primarily because the tyre deforms as the wheel rolls. The sliding of molecules against each other within the rubber causes energy to be lost.

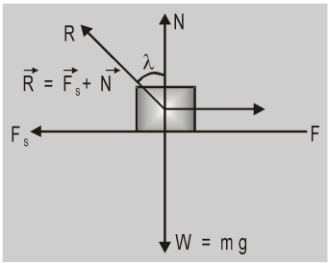
The velocity of the point of contact with respect to the surface remains zero.

The rolling friction is negligible in comparison to static or kinetic friction which may be present simultaneously i.e., $\mu_R < \mu_K < \mu_S$

ANGLE OF FRICTION

The angle of friction is the angle which the resultant of limiting friction FS and normal reaction N makes with the normal reaction. It is represented by λ, Thus from the figure.

$$\tan\lambda = \frac{F_S}{N} \quad (\because Fs = \mu N) \qquad \text{or} \qquad \frac{F_S}{N} = \tan\theta \quad \tan\lambda = \mu$$

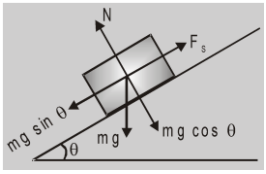


For smooth surfaces, $\lambda = 0$ (zero)

ANGLE OF REPOSE (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will just begin to slide down this angle is called angle of repose (θ).

FS = mg sinθ and N = mg cosθ



So, or $\mu = \tan\theta$

Relation between angle of friction (λ) and angle of repose (θ)

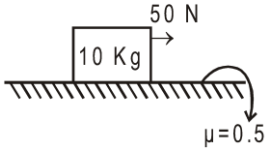
We know that $\tan\lambda = \mu$ and $\mu = \tan\theta$

hence $\tan\lambda = \tan\theta$ or **$\theta = \lambda$**

Thus, **angle of repose = angle of friction**

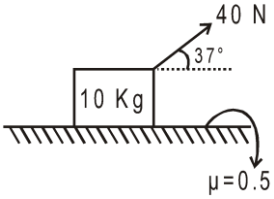
Solved Examples

Example 10. Find acceleration of block. Initially the block is at rest.



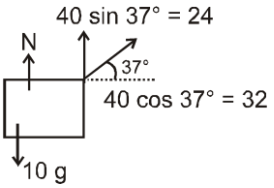
Solution : zero

Example 11. Find out acceleration of the block. Initially the block is at rest.



Solution : N + 24 – 100 = 0 for vertical direction

∴ N = 76 N



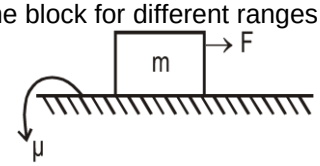
Now $0 \le f_s \le \mu_s N$

$0 \le f_s \le 76 \times 0.5$

$0 \le f_s \le 38$ N

∴ 32 < 38 Hence f = 32 ∴ acceleration of block is zero.

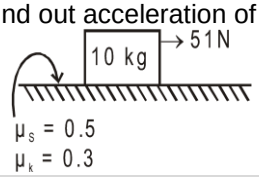
Example 12. Find out acceleration of the block for different ranges of F.



Solution : $0 \le f \le \mu_s N$ ⇒ $0 \le f \le \mu_s mg$

a = 0 if $F \le \mu_s mg$ ⇒ $a = \frac{F - \mu Mg}{M}$ if $F > \mu Mg$

Example 13. Find out acceleration of the block. Initially the block is at rest.

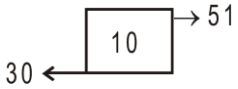


Solution : $0 \le f_s \le \mu_s N \quad \Rightarrow \quad 0 \le f_s \le 50$

Now $51 > 50$

$\therefore$ Block will move but if the block starts moving then kinetic friction is involved.

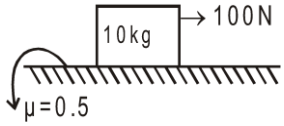
$f_k = \mu_k N = 0.3 \times 100 = 30$ N



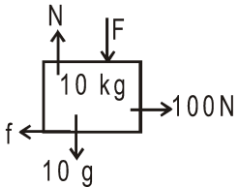
$\therefore 51 - 30 = 10\ a$

$\therefore \ a = 2.1\ m/s^2$

Example 14. Find out the minimum force that must be applied on the block vertically downwards so that the block doesn't move.



Solution :



$100 - f_s = 0$

$\therefore \quad f_s = 100$ ..... (1)

$F + 10\ g = N \ \Rightarrow \ N = 100 + F$ ..... (2)

Now $0 \le f_s \le \mu N$

$100 \le 0.5\ N$

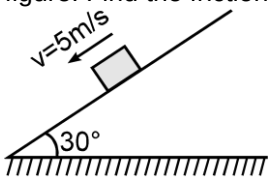
$100 \le 0.5\ [100 + F]$

$200 \le 100 + F$

$F \ge 100$ N

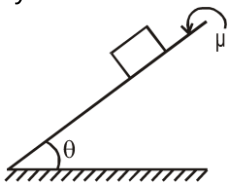
$\therefore$ Minimum F = 100 N

Example 15. A particle of mass 5 kg is moving on rough fixed inclined plane with constant velocity of 5 m/s as shown in the figure. Find the friction force acting on a body by plane.

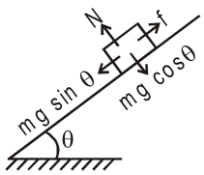


Solution : $f_k = \mu_k N = \mu_k\ mg\ \cos 37^0 = mg\ \sin 30^0 = 5\ (10) \left(\frac{1}{2}\right) \quad \Rightarrow \quad f_k = 25$ N

Example 16. The angle of inclination is slowly increased. Find out the angle at which the block starts moving.



Solution :



$0 \le f \le \mu_s N$

$mg \sin\theta > f_{smax}$

$mg \sin\theta > \mu N$

$mg \sin\theta > \mu\ mg \cos\theta$

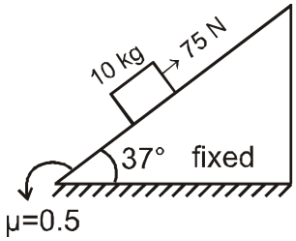
$\therefore \tan\theta > \mu$

$\theta = \tan^{-1}\mu$

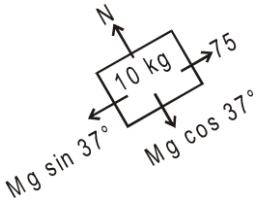
for $\tan\theta \le \mu$ no sliding on inclined plane.

This method is used for finding out the value of μ practically.

Example 17. Find out the acceleration of the block. If the block is initially at rest.



Solution : (FBD of the block excluding friction) $N = 10\,g\cos 37° = 80$ N



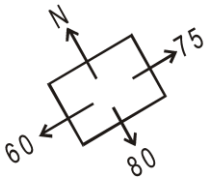
Now $0 \le f_s \le \mu N$

$0 \le f_s \le 0.5 \times 80 \quad \therefore \quad f_s \le 40$ N

We will put value of f in the last i.e. in the direction opposite to resultant of other forces. f acts down the incline and its value is of = 75 – 60 = 15 N

So acceleration is zero

Example 18. In the above problem how much force should be added to 75 N force so that block starts to move up the incline.



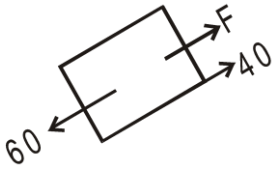
Solution : $\therefore\ 60 + 40 = 75 + f_{extra}$

$\therefore\ f_s = 25$ N

Example 19. In the above problem what is the minimum force by which 75 N force should be replaced with so that the block does not move.

Solution : In this case the block has a tendency to move downwards.

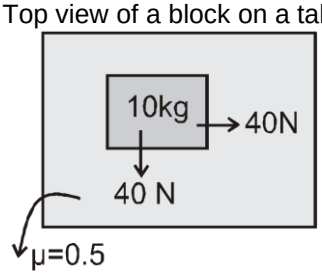
Hence friction acts upwards.



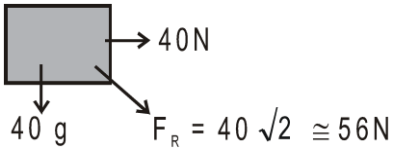
$\therefore F + 40 = 60$

$\therefore F = 20$ N

Example 20. Top view of a block on a table is shown ($g = 10\ m/s^2$) . Find out the acceleration of the block.



Solution : Now $f_s \le \mu N \quad \therefore f_s \le 50$

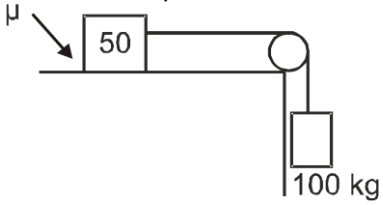


$F_R > f_{smax}$

Hence the block will move.

$a = \dfrac{40\sqrt{2}-50}{10} = \left(4\sqrt{2}-5\right)$ m/s^2

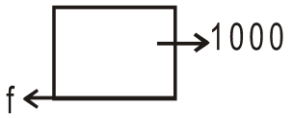
Example 21. Find minimum μ so that the blocks remain stationary.



Solution : T = 100 g = 1000 N

∴ f = 1000 to keep the block stationary

Now f_{max} = 1000



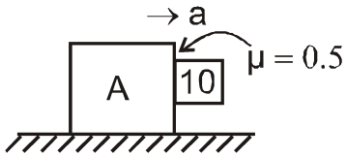
μN = 1000

μ = 2

Can μ be greater than 1 ?

Yes 0 < μ ≤ ∝

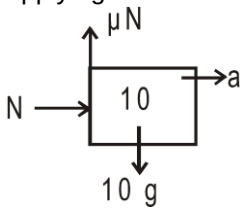
Example 22. Find out minimum acceleration of block A so that the 10 kg block doesn't fall.



Solution : Applying NL in horizontal direction

N = 10 a ..... (1)

Applying NL in vertical direction

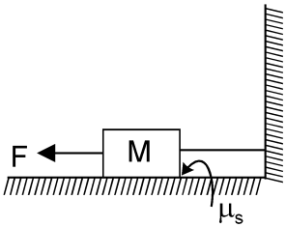


10 g = μ N .....(2)

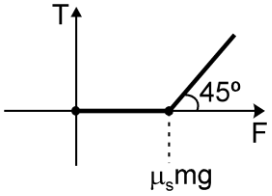
10 g = μ 10 a from (1) & (2)

∴ a = 20 m/s^2

Example 23. In the following figure force F is gradually increased from zero. Draw the graph between applied force F and tension T in the string. The coefficient of static friction between the block and the ground is μ_s.

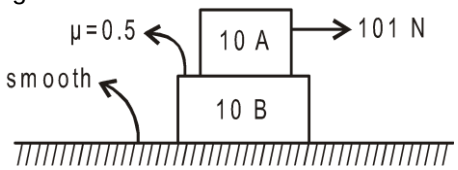


Solution : As the external force F is gradually increased from zero it is compensated by the friction and the string bears no tension. When limiting friction is achieved by increasing force F to a value till $\mu_s mg$, the further increase in F is transferred to the string.



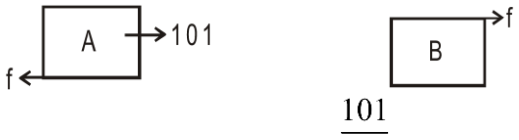
TWO BLOCK PROBLEMS

Example 24. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



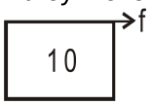
Solution : f_{max} = 50 N

$\therefore$ f ≤ 50 N



$$\frac{101}{20}$$

(i) If they move together a = $\frac{101}{20}$ = 5.05 m/s^2



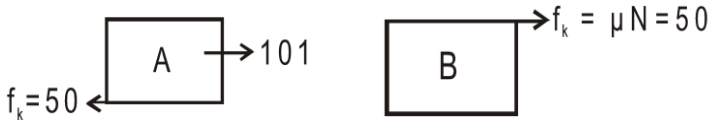
(ii) Check friction on B

f = 10 × 5.05 = 50.5 (required)

50.5 > 50 (therefore required > available)

Hence they will not move together.

(iii) Hence they move separately so kinetic friction is involved.



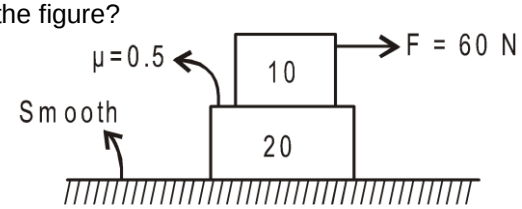
$$\frac{50}{10}$$

$$\frac{101-50}{10}$$

$\therefore$ for $a_B = \frac{50}{10}$ = 5.1 m/s^2 $\Rightarrow$ $a_A = \frac{101-50}{10}$ = 5 m/s^2

Also $a_A > a_B$ as force is applied on A.

Example 25. Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution :

Move Together

a = 2 m/s^2

Check friction on 20 kg.

f = 20 x 2

f = 40 (which is required)

40 < 50 (therefore required < available) $\therefore$ will move together.

Move Separately

No need to calculate.

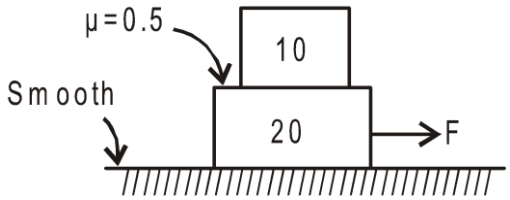
Example 26. In above example find maximum F for which two blocks will move together.

Solution : Observing the critical situation where friction becomes limiting.

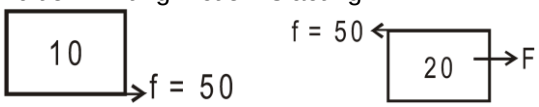
∴ F – f_{max} = 10 a ..... (1)

f_{max} = 20 a ..... (2) ∴ F = 75 N

Example 27. Initially the system is at rest. find out minimum value of F for which sliding starts between the two blocks.



Solution : At just sliding condition limiting friction is acting.



F – 50 = 20 a ..... (1)

f = 10 a ..... (2)

50 = 10 a

∴ a = 5 m/s_2

hence F = 50 + 20 × 5 = 150 N

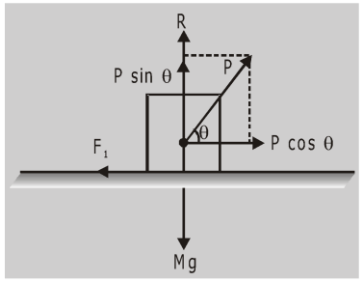
∴ F_{min} = 150 N

It is Easier to Pull Than to Push a Body. Why -

Let a force P be applied to pull a body of weight Mg (fig).

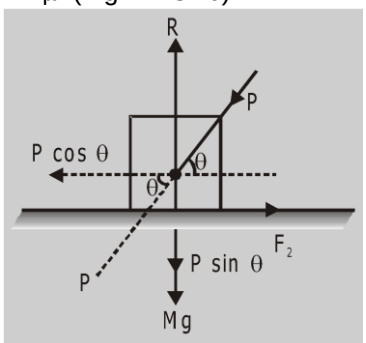
Applied force is resolved into two components : P cosθ and P sinθ

The normal reaction, R = (Mg – P sinθ)



Now, kinetic force of friction is given by

$F_1 = \mu_k R = \mu_k (Mg – P \sin\theta)$ .....(1)



On the other hand, when the same force is applied to push a body of weight Mg, than normal reaction,

R = (Mg + P sinθ) .....(2)

∴ Kinetic force of friction is $F_2 = \mu_k R = \mu_k (mg + P \sin\theta)$

From eqn (1) and (2), it is clear and $F_2 > F_1$

That is, force of friction in case of push is more than that in case of pull.

Hence, it is easier to pull than to push the body.

Friction is a Necessary Evil :

Friction is a necessary evil. It means it has advantage as well as disadvantages. In other words, friction is not desirable but without friction, we cannot think of survival.

Disadvantages :

(i) A significant amount of energy of a moving object is wasted in the form of heat energy to overcome the force of friction.

(ii) The force of friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.

(iii) The efficiency of machines decreases due to the presence of force of friction.

(iv) The force of friction causes lot of wear and tear in the moving parts of a machine.

(v) Sometimes, a machine gets burnt due to the friction force between different moving parts.

Advantages :

(i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.

(ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to write on the glazed paper as there is not force of friction.

(iii) The force of friction between the tyres of a vehicle and the road helps the vehicle to stop when brake is applied. In the absence of friction, the vehicle skid off the road when brake is applied.

(iv) Moving belts remain on the rim of a wheel because of friction.

(v) The force of friction between a chalk and the black board helps us to write on the board.

Thus, we observe that irrespective of various disadvantages of the friction, it is very difficult to part with it. So, friction is a necessary evil.

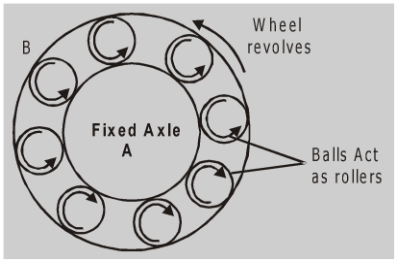
METHODS OF REDUCING FRICTION

As friction causes the wastage of energy so it becomes necessary to reduce the friction. Friction can be reduced by the following methods.

(i) Polishing the surface. We know, friction between rough surface is much more than between the polished surfaces. So we polish the surface to reduce the friction. The irregularities on the surface are filled with polish and hence the friction decreases.

(ii) Lubrication. To reduce friction, lubricants like oil or greese are used. When the oil or greese is put in between the two surfaces, the irregularities remain apart and do not interlock tightly. Thus, the surface can move over each other with less friction between them.

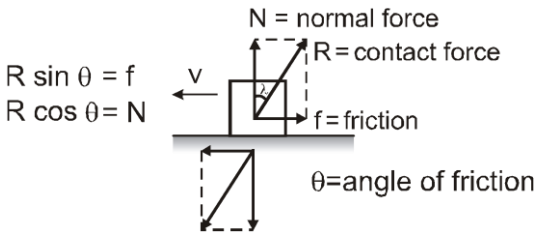
(iii) By providing the streamlined shape. When a body (e.g. bus, train, aeroplane etc.) moves with high speed, air resistance (friction) opposes its motion. The effect of air resistance on the motion of the objects (stated above) is decreased
by providing them a streamlined shape.



(iv) Converting sliding friction into rolling friction. Since rolling friction is much less than the sliding friction, so we convert the sliding friction into rolling friction. This is done by using ball bearings arrangement. Ball bearings are placed in between the axle (A) and hub (B) of the wheel as shown in figure. The ball bearing tend to roll round the axle as the wheel turns and as such the frictional force is diminished.

Key Concept

- Part of the contact force that is tangential to the surface is called friction force. Microscopically friction force because of attraction between molecules of the two surfaces.
- **Friction force is of two type :**
  (A) Kinetic friction
  (B) Static friction



- **Kinetic friction force :**
  Kinetic friction exists between two surfaces (in case of block), or two points (in case of sphere), or two line (in case of cylinder) when there is relative motion. It stops acting when relative motion ceases to exist.
  Direction of Kinetic Friction.
  It is opposite to the relative velocity of contact surfaces.
  Note : Its direction is not opposite to the force applied it is opposite to the motion of the body considered which is in contact with the other surface.
  Kinetic friction $f_k = \mu_k N$
  The proportionality constant μ_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.

- **Static friction :**
  When two surfaces in contact have relative velocity zero, but there is tendency of relative motion then the friction force acting will be static.

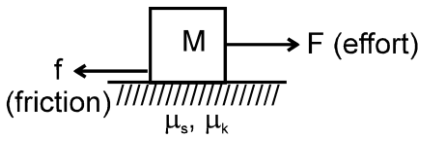
  Direction of static friction
  If there is tendency of sliding between the contact surfaces, it will act in such a direction to prevent sliding. Static friction is variable and self adjusting force. It can adjust its value upto a limit which is called limiting friction force (f_{smax}).

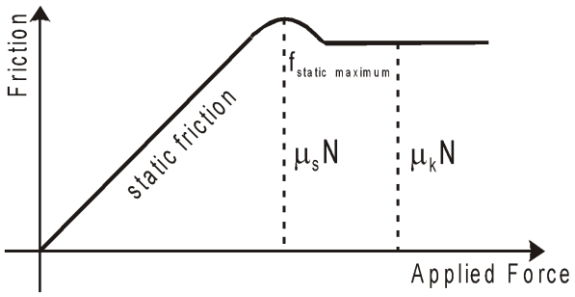
$$f_{smax} = \mu_s N$$

  Here μ_s is the coefficient of static friction which depends on the nature of two contact surfaces.
  The actual force of static friction may be smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest.

$$0 \quad f_s \quad f_{smax}$$

  Following steps should be followed in determining the direction of static friction force on an object.
  (i) Draw the free body diagram with respect to the other object on which it is kept.
  (ii) Include pseudo force also if contact surface is accelerating.
  (iii) Decide the resultant force and the component parallel to the surface of this resultant force.
  (iv) The direction of static friction is opposite to the above component of resultant force.





  Here μ_s and μ_k are dimensionless quantities independent of shape and area of contact. It is a property of the two contact surfaces. In general $\mu_s > \mu_k$ for a given pair of surfaces. If it is not mentioned separately the $\mu_s = \mu_k$ can be taken.

μ_s and μ_k can also be represented as angles. If θ_s and θ_k are angles of static friction and kinetic friction respectively, then

$\theta_s = \tan^{-1} \mu_s$

$\theta_k = \tan^{-1} \mu_k$

θ_s is also called angle of repose.

- **Rolling Friction :**

When a body rolls on a surface the resistance offered by the surface is called as rolling friction.

Rolling friction is less than sliding friction

In sliding motion, elevation collide. This introduces friction. In rolling motion, elevations are crossed over. This avoids friction.

Rolling friction zero in ideal case :

In ideal rolling the contact with cylindrical surface of body and lower surface must be along a straight line. Elevations must be crossed over and no friction be present.

But no rolling is ideal. Due to deformation of moving cylindrical surface (wheel of a loaded truck), or deformation of lower surface (mud street), contact becomes over a flat surface. This introduces sliding, which causes friction.