HINTS & SOLUTIONS
TOPIC : CIRCULAR MOTION
EXERCISE # 1

SECTION (A)

1. Speed $v_1 = \frac{2\pi r_1}{t}$ $\quad v_2 = \frac{2\pi r_2}{t}$

$\omega_1 = \frac{v_1}{r_1} = \frac{2\pi}{t} \Rightarrow \quad \omega_2 = \frac{v_2}{r_2} = \frac{2\pi}{t} \Rightarrow \quad \omega_1 = \omega_2 \Rightarrow \frac{\omega_1}{\omega_2} = \frac{1}{1}$ **Ans.**

2. $\omega = 80$ rad/sec, $t = 5$ sec, $\omega_0 = 0$

$\theta = ?$

If α constant, then α

$\theta = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{80+0}{2}\right) 5 = 200$ rad **Ans.**

3. Speed = constant

In uniform circular motion, velocity and acceleration are constant in magnitude but direction is change. Therefore velocity and acceleration both change.

5. $V = \omega.r \Rightarrow V = 30 \times 2\pi \times \frac{1}{2} = 30\pi$

7. $w = 2\pi \times f = 2\pi \times \frac{120\,\text{rad}}{60\text{ sec.}} = 4\pi$ rad/sec.

8. Use $= w = \frac{2\pi}{T}$

10. Minute hand of a clock rotates through an angle of 2π in 60 minutes i. e. 3600 sec

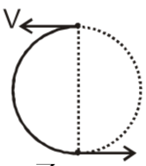
$\therefore$ Angular velocity $\omega = \frac{2\pi}{3600} = \frac{\pi}{1800}$ rad/s

11. $\omega_{second} = \frac{2\pi}{T} = \frac{2\pi}{60}$ rad/sec.

$v = \omega.r = \frac{2\pi}{60} \times 0.06$ m/s $= 2\pi$ mm/s Ans.

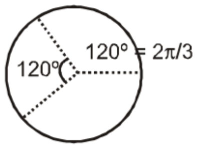
$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = \sqrt{2}\, v = 2\sqrt{2}\,\pi$ mm/s **Ans.**

12. $\Delta\vec{V} = \vec{V}_1 - \vec{V}_2 = \vec{V} - (-\vec{V}) = 2\vec{V}$



$|\Delta\vec{V}| = 2V = 2 \times 100$ km/hr $= 200$ km/hr. Ans

13. $\omega_{arg} = \langle\omega\rangle = \frac{\text{total}\,\theta}{\text{total time}}$



$= \frac{2\pi/3 + 2\pi/3}{2+1} = \frac{4\pi}{9}$ rad/sec.

14. $\theta = \frac{1}{2}\alpha t^2$ as $\omega_0 = 0 = \frac{1}{2} \times 4 \times 4^2 = 32$ rad
$\omega = \alpha.t = 4 \times 4 = 16$ rad/sec.

15. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega = \omega_0 + \alpha t = 1 + 1.5 \times 2 = 4$ rad/sec.

16. Angular velocity of particle is,

$\omega = \frac{2\pi}{T}$ or $\omega \propto \frac{1}{T}$

It simply implies that ω does not depend on mass of the body and radius of the circle.

$\therefore \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$

but time period is given same, i.e., $T_1 = T_2$

Hence, $\frac{\omega_1}{\omega_2} = \frac{1}{1}$

18. For a particle moving in a circle with constant angular speed, velocity vector is always tangent to the circle and the acceleration vector always points towards the centre of circle or is always point towards the centre of circle or is always along radius of the circle. Since, tangential vector is perpendicular to radial vector, therefore, velocity vector will be perpendicular to the acceleration vector. But in no case acceleration vector is tangent to the circle

19. When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

20. Using relation $\theta = \omega_0 t + \frac{1}{2}at^2$

$\theta_1 = \frac{1}{2}(\alpha)(2)^2 = 2\alpha$ …(i) (As $\omega_0 = 0, t = 2\,\text{sec}$)

Now using same equation for t = 4 sec, $\omega_0 = 0$

$\theta_1 + \theta_2 = \frac{1}{2}\alpha(4)^2 = 8\alpha$ …(ii)

From (i) and (ii), $\theta_1 = 2\alpha$ and $\theta_2 = 6\alpha$ $\therefore \frac{\theta_2}{\theta_1} = 3$

21. $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(2t^3 + 0.5) = 6t^2$ at t =2 s, $\omega = 6 \times (2)^2 = 24\text{rad/s}$

22. In non-uniform circular motion particle possess both centripetal as well as tangential acceleration.

SECTION (B)

1. Angular velocity of every particle of disc is same
$a_P = \omega^2 r_p$, $a_Q = \omega^2 r_Q$
$\because r_P > r_Q \Rightarrow a_P > a_Q$ **Ans.**

2. For circular motion of particle a_r not equal to zero, a_t may or may not be zero.

3. Time period = $\frac{\text{No. of revolutions}}{\text{time}} = \frac{25}{14}$ = 1.79 sec. Now angular speed

$\omega = \frac{2\pi}{T} = \frac{2\times 3.14}{1.79}$ = 3.51 rad/sec. Now magnitude of acceleration is given by

$a = \omega^2 l = (3.51)^2 \times 80 = 985.6\ \text{cm/sec}^2 = 996\ \text{cm/sec}^2$

4. In a circular motion

$a = \frac{v^2}{r}$ $\quad\therefore\quad$ $\frac{a_2}{a_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{2v_1}{v_1}\right)^2 = 4$

5. In circular motion, necessary centripetal force to the man is provided by effective weight of man.
Thus,

$m \times 9g = mr\omega^2 = mr \times 4\pi^2 n^2$ or $n = \sqrt{\frac{9g}{4\pi^2 r}}$

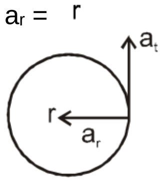
Given, r = 5m $\therefore$ $n = \sqrt{\frac{9\times 10}{4\times(3.14)^2\times 5}}$ = 0.675 rev/s

6. Rate of change of momentum is force which is in radial direction in unifom circular motion, so ans. (c)

7. $a_c = \frac{v^2}{r}$, Radius is constant in case (a) and increase in case (b). So that magnitude of acceleration is constant in case (a) and decrease in case (b).

8. $a_t = a$

$a_r = \frac{v^2}{r}$



$\vec{a} = \vec{a}_r + \vec{a}_t \Rightarrow |\vec{a}| = \sqrt{\left(\frac{v^2}{r}\right)^2 + a^2}$. Ans.

9. $F = \frac{mv^2}{R} \Rightarrow F^1 = \frac{\left(\frac{3}{2}m\right)\left(\frac{3}{2}v\right)^2}{\left(\frac{3}{2}R\right)} = \frac{9}{4}.\frac{mv^2}{R}$ F

Force increased = $\frac{F^1 - F}{F} \times 100 = \left(\frac{9}{4} - 1\right)\times 100 = \frac{5}{4}\times 100$ = 12.5 %

11. There is no relation between centripetal and tangential acceleration. Centripetal acceleration is must for circular motion but tangential acceleration may be zero.

12. $F_{C1} = F_{C2} \Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{1}{\sqrt{2}}$ **Ans.**

13. $T = m\omega^2 r$

$\Rightarrow T^1 = 2T = m\omega_1^2\, r$

$\omega_1 = \sqrt{2}\ \omega = \sqrt{2} \times 5 = \sqrt{50}$ ~ 7 rev/min **Ans.**

14. In uniform circular motion tangential acceleration remains zero but magnitude of radial acceleration remains constant.

SECTION (C)

2. Centripetal force is constant in magnitude that means speed is constant and due to change in direction velocity is variable.

3. $T = \frac{mv^2}{r} = \frac{0.5 \times (4)^2}{1} = 8N$

5. $\omega^2.r = a_r \Rightarrow \omega^2 = 9.8/20 \times 10^{-2}$, $\omega = 7$ rad/s

6. $p = mv$, & $F = mv^2/r \Rightarrow F = m\left(\frac{p}{m}\right)^2/r \Rightarrow F = p^2/mr$

7. $F = K\frac{1}{r}$ $\quad \frac{k}{r} = \frac{mv^2}{rr}$ $v = \sqrt{\frac{k}{m}}$ So, independent to r

9. Here : Mass of car m = 500 kg
Radius r = 50 m

Speed of car $\upsilon = 36$ km/hr $= \frac{36 \times 5}{18} = 10$ m/s

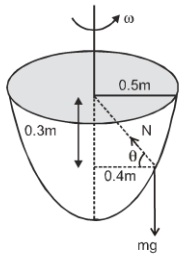
The centripetal force is given by $F = \frac{m\upsilon^2}{r} = \frac{500 \times (10)^2}{50} = 1000$ N

10. $F_{C1} = F_{C2} \Rightarrow \frac{mv_1^2}{r_1} = \frac{mv_2^2}{r_2}$

$\frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{1}{\sqrt{2}}$ **Ans.**

12. $N\cos\theta = m\omega^2.r$ ....(i)
$N\sin\theta = mg$ ....(ii)

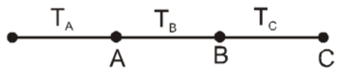
$\tan\theta = \frac{0.3}{0.4} = \frac{3}{4}$ from (i) & (ii)



$\tan\theta = \frac{mg}{m\omega^2.r} \Rightarrow \omega^2 = \frac{g}{r.\tan\theta} = \frac{10 \times 4}{0.4 \times 3} = \frac{100}{\sqrt{3}} \Rightarrow \omega = \frac{10}{\sqrt{3}}$ red/sec.

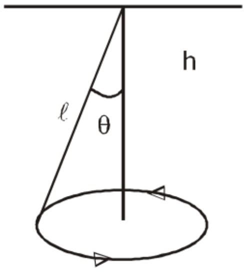
13. ω = const., for all three particles

$\omega = \frac{v}{3\ell}$



$T_C = m\omega^2 3\ell$
$T_B - T_C = m\omega^2 2\ell$
$T_B = 5\, m\omega^2\ell$
$T_A - T_B = m\omega^2\ell$
$T_A = 6\, m\omega^2\ell$
$T_C : T_B : T_A :: 3 : 5 : 6$ **Ans.**

14. $h = \ell \cos\theta$ $\quad T = 2\pi\sqrt{\dfrac{\ell\cos\theta}{g}}$



16. When train A moves form east to west

$mg - N_1 = \dfrac{m(v+\omega R)^2}{R} \Rightarrow N_1 = mg - \dfrac{m(v+\omega R)^2}{R}$

$N_1 = F_1$ When train B moves from west to east

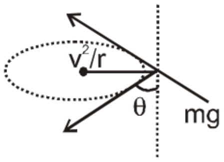
$mg - N_2 = \dfrac{m(v-\omega R)^2}{R} \Rightarrow N_2 = mg - \dfrac{m(v-\omega R)^2}{R}$

$N_2 = F_2$

$F_1 > F_2$ **Ans.**

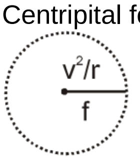
17. $V = 72 \text{ km/h} = 72 \times \dfrac{5}{18} = 20 \text{ m/s}$

$a_r = \dfrac{v^2}{r} = \dfrac{400}{80} = 50$

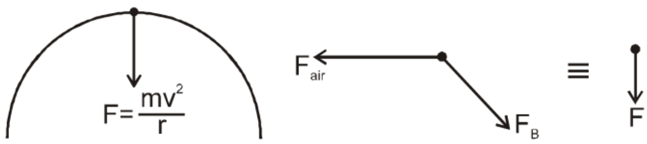


$\tan\theta = \dfrac{v^2/r}{g} = \dfrac{5}{10} = \dfrac{1}{2} \Rightarrow \theta = \tan^{-1}\left(\dfrac{1}{2}\right)$

18. Centripital force provided by friction $\mu mg > \dfrac{mv^2}{r}$



19. In uniform circular motion resultant horizontal force on the car must be towards the centre of circular path.



20. Maximum retardation $a = \mu g$

For apply brakes sharply minimum distance require to stop.

$0 = v^2 - 2\mu g s \Rightarrow s = \dfrac{v^2}{2\mu g}$ For taking turn minimum radius is

$\mu g = \dfrac{v^2}{r}, \Rightarrow r = \dfrac{v^2}{\mu g},$

here r is twice of s

so apply brakes sharply is safe for driver.

21. $T = m\omega^2.\ell$

$T_0 = m\omega_0^2.\ell_0$

$T_1 = m(2\omega_0)^2.(2\ell_0)$

$= 8 m \omega_0^2 \ell_0$

$T_1 = 8T_0$

23. Since, speed is constant throughout the motion, so it is a uniform circular motion.
Therefore, its radial acceleration

$$a = r\omega^2 = r\left(\frac{2\pi n}{t}\right)^2 = r \times \frac{4\pi^2 n^2}{t^2} = \frac{1\times 4\times \pi^2 \times (22)^2}{(44)^4} = \pi^2 \text{ m/s}^2$$

This acceleration is directed along radius of circle.

NOTE : 1. In uniform circular motion $\frac{dv}{dt} = 0$.
Thus, $a_t = 0$ and $a = a_r = r\omega^2$.

2. In accelerated circular motion $\frac{dv}{dt}$ is positive i.e. $\vec{a}_t$, is along $\hat{e}_t$ or tangential acceleration of particle is parallel to velocity $\vec{v}$ because $\vec{v} = r\omega\,\hat{e}_t$ and $\vec{a}_t = \frac{dv}{dt}\hat{e}_t$

3. In decelerated circular motion $\frac{dv}{dt}$ is negative and hence, tangential acceleration is anti-parallel to velocity $\vec{v}$.

24. Using the relation

$\frac{mv^2}{r} = \mu R$, $R = mg$ $\frac{mv^2}{r} = \mu mg$ or $v^2 = \mu rg$ or $v^2 = 0.6 \times 150 \times 10$

$\Rightarrow$ $v = 30$ m / s

25. Due to centrifugal force.

26. Tangential force (F_t) of the bead will be given by the normal reaction (N) , while centripetal force (F_c) is provided by friction (f_r). The bead starts sliding when the centripetal force is just equal to the limiting friction.

Therefore $F_t = ma = m\,\alpha\,L = N$

$\therefore$ Limiting value of friction $(fr)_{max} = \mu N = \mu m\,\alpha\,L$ ....(1)

Angular velocity at time t is $\omega = \alpha t$

$\therefore$ Centripetal force at time t will be

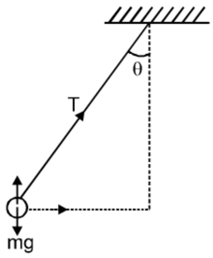
$F_c = mL\omega^2 = mL\,\alpha^2 t^2$ ....(2)

Equating equations (1) and (2), we get

$t = \sqrt{\frac{\mu}{\alpha}}$ For $t > \sqrt{\frac{\mu}{\alpha}}$, $F_c > (f_r)_{max}$ i.e. the bead starts sliding.

In the figure F_t is perpendicular to the paper inwards $\otimes$

27. $T\sin\theta = m\,L\sin\theta\,\omega^2$



$324 = 0.5 \times 0.5 \times \omega^2 \Rightarrow \omega^2 = \frac{324}{0.5\times 0.5} \Rightarrow \omega = \sqrt{\frac{324}{0.5\times 0.5}} \Rightarrow \omega = \frac{18}{0.5} = 36$ rad/sec.

SECTION (D)

1. Force is perpendicular to $\vec{v}$

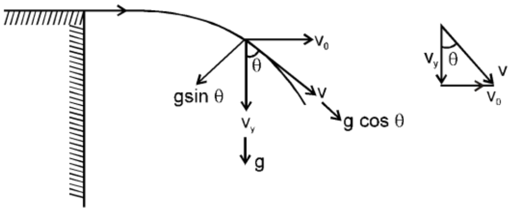
$R = \frac{v^2}{a_\perp} \Rightarrow R = \frac{mv^2}{F}$ **Ans.**

2. At t = 0, t =0
$a_\perp = g\cos\theta$,

$R = \frac{v^2}{a_\perp} = \frac{u^2}{g\cos\theta}$

3. It can be observed that component of acceleration perpendicular to velocity is

$a_c = 4\ m/s^2$ ∴ radius = $\frac{v^2}{a_c} = \frac{(2)^2}{4}$ = 1 metre.



4.
As we know :

$A_c = \frac{v^2}{R}$ (centripetal acceleration)

From figure ; $g\sin\theta = \frac{v^2}{R} \Rightarrow g \cdot \frac{v_0}{v} = \frac{v^2}{R}$ (Since ; $\sin\theta_1 = \frac{v_0}{v}$) $\Rightarrow R \propto v^3$

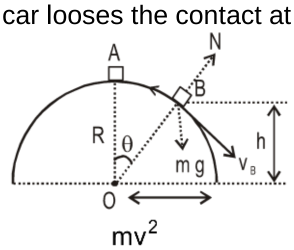
SECTION (E)

1. at lowest point

$T - mg = \frac{mv^2}{r}$

$T = mg + \frac{mv^2}{r}$

2. Let the car looses the contact at angle θ with vertical



$mg\cos\theta - N = \frac{mv^2}{R} \Rightarrow N = mg\cos\theta - \frac{mv^2}{R}$

During ascending on overbridge θ is decrease. So cos θ is increase therefore normal reaction is increase.

3. For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is -

$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR}$ **Ans.**

4. $T - mg\cos\theta = \frac{mv^2}{r}$ ....(1)

(from centripetal force) from energy conservation.

$\frac{1}{2}mu^2\ \frac{1}{2} = mv^2 + mgr(1-\cos\theta)$ (here u is speed at lowest point) from (1) and (2)

$T = \frac{mu^2}{r} + 3mg\cos\theta - 2mg$ for $\theta = 30^o$ & 60^o $\Rightarrow$ $T_1 > T_2$ **Ans.**

5. Normal reaction at highest point.

$mg - N = \frac{mv^2}{r}$ $\Rightarrow$ $N = mg - \frac{mv^2}{r}$

$R_A > R_B$ $\Rightarrow$ $N_A > N_B$ **Ans.**

7. $V = \sqrt{5kg} = \sqrt{5\times 6.4\times 10} = 4\sqrt{5} = 4 \times 2.4 = 9.6$ m/s $= 10.2$ m/s

9. $T - mg = mrw^2$

$3.7\,xg - 5g = mrw^2$

$\sqrt{\frac{3.7-g-0.5g}{0.5\times 4}} = w = \sqrt{\frac{g(3.2)}{2}} = 4$ rad/sec

12. $T - mg = mrw^2$

$3\,mg - mg = \frac{mv^2}{R}$

$2mg = \frac{mv^2}{R}$ $v^2 = 2gR$

$\frac{1}{2}mv^2 = mgR(1-\cos\theta)$

$\cos\theta = 0 = \Delta\ \theta = 90°$

13. For just slip $\Rightarrow$ $\mu mg = m\omega^2 r$

here ω is double then radius is 1/4th

$r' = 1$ cm **Ans.**

14. $mrw^2 = /mg$, $w = \sqrt{\frac{g}{r}}$, $T = 2\pi\sqrt{\frac{r}{g}} = 2\pi\sqrt{\frac{4}{10}} = 2 \times 2\sqrt{\frac{\pi^2}{10}} = 4$ Sec

15. Let v be the speed of B at lowermost position, the speed of A at lowermost position is 2v.

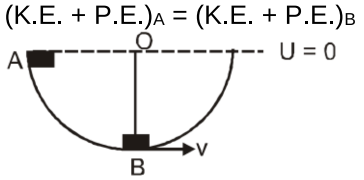
From conservation of energy

$\frac{1}{2}m(2v)^2 + \frac{1}{2}mv^2 = mg(2\ell) + mg\ell$. Solving we get $v = \sqrt{\frac{6}{5}}\,g\ell$.

16. When a string fixed with a nail, moves along a vertical circle, then the minimum horizontal velocity at the lowest point of circle is given by $\upsilon = \sqrt{5rg} = \sqrt{5\times 0.25\times 9.8} = 3.5$ m/s

17. Energy conservation between point A & B

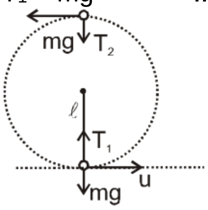
$(K.E. + P.E.)_A = (K.E. + P.E.)_B$



$0 + 0 = \frac{1}{2}mv^2 - mgR$

$V = \sqrt{2gR}$ Ans.

18. $T_1 - mg = \frac{mu^2}{\ell}$ ......(i)



$T_2 + mg = \frac{mv^2}{\ell}$ ......(ii) and $0 + \frac{1}{2}mu^2 = mg\,2\ell + \frac{1}{2}mv^2$

(i-ii) $T_1 - T_2 = \frac{m}{\ell}(u^2 - v^2) + 2mg = \frac{m}{\ell}(4g\ell) + 2\,mg$

$T_1 - T_2 = 6$ mg.

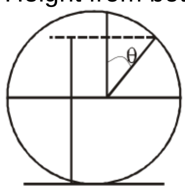
19. $mgh = \frac{1}{2}mv^2 \quad 2gh = v^2$

$V^2 > 5\,gR \qquad 2gh > 5\,Rh$

$h > \frac{5}{2}R$

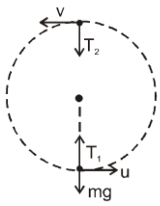
20. Particle will leave the surface at $\cos\theta = 2/3$

Height from bottam $H = R\cos\theta + R = \frac{2}{3}R + R = \frac{5R}{3}$



$= \frac{5 \times 42}{3} \qquad \frac{5 \times 21}{3} = 35$ cm

21. $\frac{T_1}{T_2} = \frac{\frac{mu^2}{\ell} + mg}{\frac{mv^2}{\ell} - mg} \qquad \frac{1}{2}mu^2 - \frac{1}{2}mv^2 = 2\,mg\ell$



$\Rightarrow \quad u^2 - v^2 = 4g\ell \quad \Rightarrow \quad u^2 = 4g\ell + v^2 \quad \Rightarrow \quad 4v^2 - 4g\ell = u^2 + g\ell$

$4v^2 - 4g\ell = 4g\ell + v^2 + g\ell \quad \Rightarrow \quad 3v^2 = 9g\ell \quad \Rightarrow \quad v = \sqrt{2g\ell} = \sqrt{3 \times 10\frac{10}{3}} = 10$ m/s

22. Maximum kinetic energy of swing should be equal to difference in potential energies to conserve energy.
From energy conservation

$\frac{1}{2}mv^2_{\,max} = mg\,(H_2 - H_1)$

Here, H_1 = minimum height of swing from earth's surface = 0.75 m

H_2 maximum height of swing from earth's surface = 2m

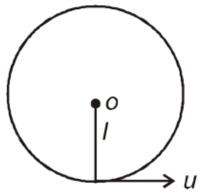
$\therefore \quad \frac{1}{2}mv^2_{\,max} = mg\,(2 - 0.75) \quad$ or $\quad v_{max} = \sqrt{2 \times 10 \times 1.25} = \sqrt{25} = 5$ m/s

23. **Key Idea :** When stone reaches a position where string is horizontal, it attains the energy partially as kinetic and partially as potential.

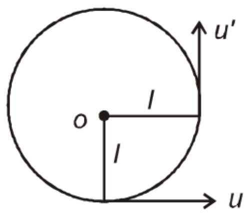
When stone is at its lowest position, it has only kinetic energy, given by

$k = \frac{1}{2} mu^2$ At the horizontal position, it has energy

$E = U + K = \frac{1}{2} mu'^2 + mg\ell$ According to conservation of energy,



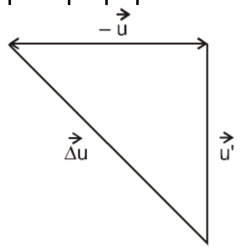
$K = E \quad \therefore \quad \frac{1}{2} mu^2 = \frac{1}{2} mu'^2 + mg\ell$ or $\frac{1}{2} mu'^2 = \frac{1}{2} mu^2 - mg\ell$



or $u'^2 = u^2 - 2g\ell$ or $u' = \sqrt{u^2 - 2g\ell}$ … (i)

So, the magnitude of change in velocity

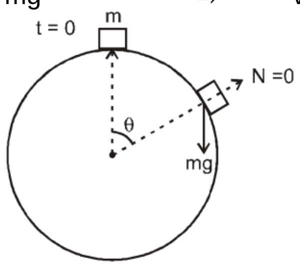
$|\Delta \vec{u}| = |\vec{u}| = \sqrt{u'^2 + u^2 + 2u'u\cos 90^\circ}$ ⇒ $|\Delta \vec{u}| = \sqrt{u'^2 + u^2} = \sqrt{2(u^2 - g\ell)}$ [from Eq. (1)]



24. For circular motion in vertical plane normal reaction is minimum at highest point and it is zero, minimum speed of motorbike is -

$mg = \frac{mv^2}{R}$ ⇒ $v = \sqrt{gR}$ **Ans.**

25.



at loose contact N = 0

$mg \cos\theta = \frac{mv^2}{R}$ ....(1) from energy conservation

$mgR(1 - \cos\theta) = \frac{1}{2} mv^2$ ....(2) from (1) & (2)

$\cos\theta = \frac{2}{3}$ ⇒ $\sin\theta = \frac{\sqrt{5}}{3}$

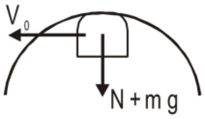
tangential acceleration = g sin θ = $\frac{\sqrt{5}g}{3}$ **Ans.**

26. (1) Difference in kinetic energy = $2mgr = 2\times1\times10\times1 = 20J$

27. Since the block rises to the same heights in all the four cases, from conservation of energy, speed of the block at highest point will be same in all four cases. Say it is V_0.

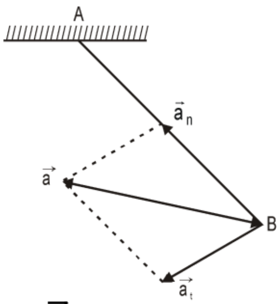
Equation of motion will be & N + mg = $\frac{mV_0^2}{R}$

or N = $\frac{mV_0^2}{R}$ – mg



R **(**the radius of curvature) in first case is minimum. Therefore, normal reaction N will be maximum in first case.

28. Net acceleration $\vec{a}$ of the bob in position B has two components.



(i) $\vec{a}_n$ = radial acceleration (towards BA)

(ii) $\vec{a}_r$ = tangential acceleration (perpendicular to BA)

Therefore, direction of $\vec{a}$ is correctly shown in option (C).

SECTION (F)

1. Here required centripetal force provide by friction force. Due to lack of sufficient centripetal force car thrown out of the road in taking a turn.

3. tan θ = $\frac{v^2}{g}$ = $\frac{h}{b}$ ⇒ h = $\frac{bv^2}{Rg}$

4. mg = mω^2 R , ω = $\sqrt{\frac{g}{R}}$

5. $\frac{v^2}{rg} = \frac{h}{l}$ ⇒ $v = \sqrt{\frac{rgh}{l}} = \sqrt{\frac{50\times1.5\times9.8}{10}} = 8.57\,m/s$

6. We know that $\tan\theta = \frac{v^2}{Rg}$ and $\tan\theta = \frac{h}{b}$

Hence $\frac{h}{b} = \frac{v^2}{Rg}$ ⇒ $h = \frac{v^2b}{Rg}$

7. The maximum velocity for a banked road with friction,

$v^2 = gr\left(\frac{\mu + \tan\theta}{1 - \mu\tan\theta}\right)$ $\Rightarrow$ $v^2 = 9.8 \times 1000 \times \left(\frac{0.5+1}{1-0.5\times 1}\right)$ $\Rightarrow$ $v = 172 m/s$

8. For critical condition at the highest point

$\omega = \sqrt{g/R}$ $\Rightarrow$ $T = \frac{2\pi}{\omega} = 2\pi\sqrt{R/g} = 2\times 3.14\sqrt{4/9.8}$ = 4 sec.

9. $\tan\theta = \frac{v^2}{Rg} = \frac{20\times 20}{20\times 10}$

$\tan\theta = 2$

$\theta = \tan^{-1}$

EXERCISE-2

1. $K = as^2 \Rightarrow as^2 = \frac{1}{2}mv^2$ $\Rightarrow$ $\sqrt{\frac{2as^2}{m}} = v \Rightarrow v = S\sqrt{\frac{2a}{m}}$

$a_t = \frac{dv}{dt} = \frac{ds}{dt}\sqrt{\frac{2a}{m}} = v\sqrt{\frac{2a}{m}}$ $\Rightarrow$ $a_t = s\sqrt{\frac{2a}{m}}\sqrt{\frac{2a}{m}}$

$\Rightarrow$ $a_t = s.\frac{2a}{m} \Rightarrow ma_t = 2as$ $\Rightarrow$ $mac = \frac{v^2}{m}$ $F_{net} = \sqrt{F_t^2 + F_c^2}$ So we get Ans No (2)

2. $F_c = mk^2rt^2$

$a_c = k^2rt^2 = \frac{v^2}{r}$ $\Rightarrow$ $v = krt$

$a_t = \frac{dv}{dt} = kr$

$F_t = mkr$ $\Rightarrow$ $P = \vec{F}.\vec{v}$ $(\because \vec{F_C}.\vec{v} = 0)$

$P = \vec{F_t}.\vec{v} = mkr \times krt = mk^2r^2t$ **Ans.**

3. Given $v_B = 0.5\sqrt{gr}$

Assume block leave the contact at C, N = 0

$\frac{mv_C^2}{r} = mg\cos\theta$ .... (1)

from energy conservation $\frac{1}{2}mv_B^2 + mgr(1 - \cos\theta) = \frac{1}{2}mv_c^2$ .... (2)

from equation (1) and (2).

$\frac{1}{2}m\left(\frac{1}{4}gr\right) + mgr(1 - \cos\theta) = \frac{1}{2}mgr\cos\theta \Rightarrow \cos\theta = \frac{3}{4}$ $\Rightarrow$ $\theta = \frac{3}{4}\cos^{-1}$ **Ans.**

4. $r = \frac{20}{\pi}$ m, a_t = constant

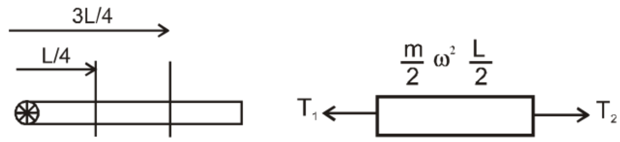
$n = 2^{nd}$ revolution

$v = 80$ m/s

$\omega_0 = 0$, $\omega_f = \frac{v}{r} = \frac{80}{20/\pi} = 4\pi$ rad/sec. $\theta = 2\pi \times 2 = 4\pi$ from 3rd equation

$\omega^2 = \omega_0^2 + 2\alpha\theta$ $\Rightarrow$ $(4\pi)^2 = 0^2 + 2\times\alpha\times(4\pi)$ $\Rightarrow$ $\alpha = 2\pi$ rad/s^2

$a_t = \alpha r = 2\pi \times \frac{20}{\pi} = 40$ m/s^2 **Ans.**

6. 

$T_1 - T_2 = \frac{M}{2}\omega^2\frac{L}{2}$

$T_1 > T_2$ **Ans.**

7. By using equation $\omega^2 = \omega_0^2 - 2\alpha\theta$

$\left(\frac{\omega_0}{2}\right)^2 = \omega_0^2 - 2\alpha(2\pi n) \Rightarrow \alpha = \frac{3}{4}\frac{\omega_0^2}{4\pi\times 36}$, (n = 36) ..(i)

Now let fan completes total n′ revolution from the starting to come to rest

$0 = \omega_0^2 - 2\alpha(2\pi n') \Rightarrow n' = \frac{\omega_0^2}{4\alpha\pi}$. Substituting the value of α from equation (i)

$n' = \frac{\omega_0^2}{4\pi}\frac{4\times 4\pi\times 36}{3\omega_0^2} = 48$ Revolution number of rotation = 48 – 36 = 12

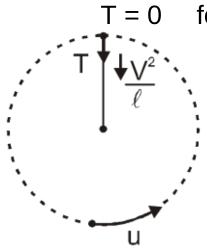
8. $\vec{T}.\vec{a}$ = | T | | a | cosθ = 0

either | T | = 0 or | a | = 0 or θ = 90º

$a = \frac{V^2}{r} \neq 0$ → for whole motion there is velocity.

So T = 0,

T = 0 for $T + mg = \frac{mV^2}{\ell} \Rightarrow T = \frac{mV^2}{\ell} - mg$



$mg\,2\ell + \frac{1}{2}mV^2 = \frac{1}{2}mu^2 \Rightarrow V^2 = u^2 - 4\,gl \Rightarrow T = \frac{mu^2}{\ell} - 5\,mg$

T = 0 or T < 0

$u \leq \sqrt{5g\ell}$ (B)

9. $\vec{T}.\vec{a}$ = | T | | a | cosθ

θ = 0º at lowest point. So $\vec{T}.\vec{a}$ = | T | | a | ≥ 0 for every value of u

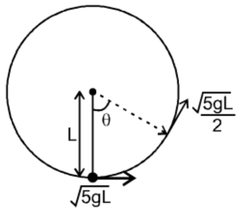
10. $\vec{T}.\vec{V}$ = | T | | V | cosθ

θ = 90º every time. So, $\vec{T}.\vec{V}$ = 0 for every value of u.

11. By energy conservation,

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\ell(1 - \cos\theta)$

$\Rightarrow V^2 = U^2 - 2g\,(L - L\cos\theta) \Rightarrow \frac{5gL}{4} = 5gL - 2gL\,(1 - \cos\theta)$



$5 = 20 - 8 + 8\cos\theta$

$\cos\theta = -\frac{7}{8}$ $\Rightarrow$ $\frac{3\pi}{4} < \theta < \pi$

12. KE = $\frac{1}{2}mv^2$ $\because$ $F = \frac{mv^2}{R}$ $\because$ $F = \frac{K}{r^2} = \frac{mv^2}{r^2}$ $\frac{1}{2}mv^2 = \frac{k}{2r}$

Potential energy U = $\int F dr = \int\left(-\frac{k}{r^2}\right)dr = \frac{k}{r}$. Total energy = U + K = $-\frac{K}{r} + \frac{K}{2r} = -\frac{K}{2r}$ E α $\frac{1}{-2r}$

EXERCISE # 3
PART - I

1. Centripetal acceleration

$a_c = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r = \left(\frac{2\pi}{0.2\pi}\right)^2 \times 5 \times 10^{-2}$ = 5 m/s² tangential acceleration is zero as constant speed so acceleration = $\sqrt{a_c^2 + a_t^2}$ = 5 m/s²

2. For banking $\tan\theta = \frac{V^2}{Rg}$ $\tan 45 = \frac{V^2}{90 \times 10} = 1$ V = 30 m/s

3. For smooth driving maximum speed of car v then

$\frac{mv^2}{R} = \mu_s mg$ $\Rightarrow$ $v = \sqrt{\mu_s Rg}$

4. $F_C = \frac{mv_1^2}{r} = \frac{2mv_2^2}{(r/2)} = \frac{4mv_2^2}{r}$ So, $v_1 = 2v_2$

5. x = 45 m 2πt, y = 4 cos (2πt)

Squiring and adding $x^2 + y^2 = 4^2$ $\Rightarrow$ R = 4 $\Rightarrow$ Circular motion

V = ωR = (2π) (4) = 8π So, Ans. is (2)

6. To complete the vertical loop, the minimum speed required at the lowest point = $\sqrt{5gR}$. So, ans is (1)

7. W_{all} = KE ↑

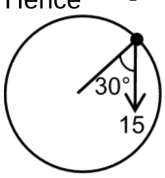
$(ma_t)(s) = \frac{1}{2}mv^2$

$(10 \times 10^{-3})(a_t)(4\pi \times 6.4 \times 10^{-2}) = 8 \times 10^{-4}$ $\Rightarrow$ a_t = 0.1 m/s m/s. Ans. will be (2)

8. For maximum speed the tendency of body is to slip up the incline

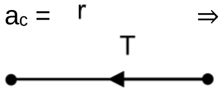
Hence $\frac{V_{max}^2}{Rg} = \frac{\tan\theta + \mu}{1 - \mu\tan\theta}$ or $V_{max} = \sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$

9.

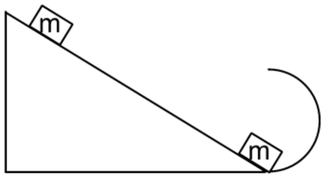


$a_c = \frac{V^2}{r}$ $\Rightarrow$ $15\cos 30° = \frac{V^2}{2.5}$ $\Rightarrow$ $V^2 = 32.73$ $\Rightarrow$ V = 5.7 m/sec **Ans.**

10.



Net force T = $\frac{mv^2}{\ell}$

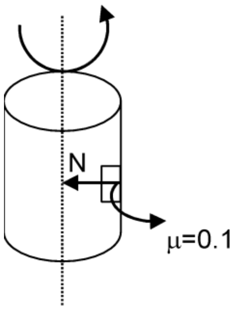
11. 

Velocity of the block at the lowest position = $\sqrt{2gh}$ to just complete the vertical circle :

$\sqrt{2gh} = \sqrt{5gr}$ ⇒ h = $\frac{5R}{2} = \frac{5\frac{D}{2}}{2}$ ⇒ h = $\frac{5D}{4}$

12. In vertical circular motion, tension in wire will be maximum at lower most point, so the wire is most likely to break at lower most point.

13. To keep the block stationary



$\mu N = mg$

N = $\frac{mg}{\mu}$ = $\frac{10 \times 10}{0.1}$ = 1000

Block is rotating about its axis ∴ N = $\frac{mV^2}{R}$ ⇒ V = $\sqrt{\frac{NR}{m}}$ = $\sqrt{\frac{1000 \times 1}{10}}$ = 10 red /sec

14. Particle attains velocity v_0 after nth round ∴ $\omega = \frac{v_o}{r}$

$\omega^2 = \omega_o^2 + 2\alpha\theta$ ⇒ ($\omega_o = 0$, ∵ particle initially at rest) ⇒ $\left(\frac{v_o}{r}\right)^2 = 2\alpha(2\pi n)$ ⇒ $\alpha = \frac{v_o^2}{4\pi n r^2}$

PART - II

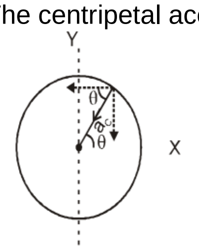
1. $S = t^3 + 5$

Linear speed of the particle

$v = \frac{dS}{dt} = 3t^2$ at t = 2 s ⇒ v = (3×2^2) m/s = 12 m/s

Linear acceleration $a_1 = \frac{dv}{dt} = 6t$ at t = 2 s, ⇒ $a_1 = 12$ m/s²

The centripetal acceleration $a_2 = \frac{v^2}{R} = \frac{12^2}{20}$ m/s² = 7.2 m/s² ∴ $a_{net} = \sqrt{a_1^2 + a_2^2} = \sqrt{12^2 + 7.2^2}$ = 14 m/s²

2. 

$a_c = -\frac{V^2}{R}\cos\theta\,\hat{i} - \frac{V^2}{R}\sin\theta\,\hat{j}$

3. They have same ω.

Centripetal acceleration = $\omega^2 r$

$\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

4. $F = \frac{k}{R^n} = m\omega^2 R$

$\omega^2 \alpha \frac{1}{R^{n+1}} \Rightarrow \therefore \quad T = \frac{2\pi}{\omega}$ So $T \alpha R^{\frac{n+1}{2}}$

5. $|\Delta\vec{V}| = 2v\sin\frac{\theta}{2} = 2v\sin 30° = 2 \times 10 \times \frac{1}{2} = 10$ m/s

6. $t = \frac{\pi}{2\omega}$