TOPIC : CENTRE OF MASS
EXERCISE # 1

SECTION (A)

1. Centre of mass is a point which can lie within or outside the body.

2. Self explaintory

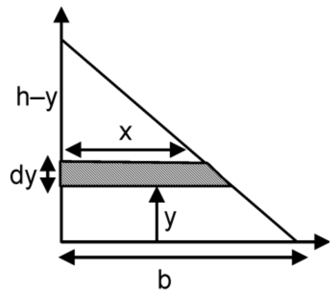
4. Centre of mass is nearer to heavier mass

5. we have $m_1 r_1 = m_2 r_2 \Rightarrow mr = 2m(3a - r) \Rightarrow r = 2a$

6. r = 1.27 Å

$r_{cm} \frac{m_1r_1 + m_2r_2}{m_1 + m_2}$. Since centre of mass cannot go beyond bond legth.

$r_{cm} \frac{0 + 35.5 \times 1.27}{35.5 + 10} = \frac{35.5 \times 1.27}{36.5}$ = 1.24 Å



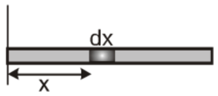
13. Let mass of strip is dm

$dm = \frac{2m}{bh} \times x\,dy \Rightarrow \frac{h}{b} = \frac{h-y}{x} \Rightarrow x = b - \frac{b}{h}y$

$$y_{cm} = \frac{\int_0^h y\,dm}{\int_0^h dm} = \frac{\int_0^h y\left(\frac{2m}{bh}\right)\left(b - \frac{by}{h}\right)dy}{\int_0^h \frac{2m}{bh}\left(b - \frac{by}{h}\right)dy} = \frac{\left(\frac{by^2}{2} - \frac{by^3}{3h}\right)_0^h}{\left(by - \frac{by^2}{2h}\right)_0^h} = \frac{h}{3} \Rightarrow y_{cm} = \frac{h}{3} \text{ by symmetry } x_{cm} = \frac{b}{3}$$

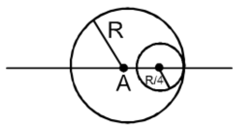
17. $X_{cm} = \frac{3\times\frac{1}{2} + 2\times 1}{6} = \frac{3.5}{6} = \frac{35}{60} = \frac{7}{12} \Rightarrow Y_{cm} = \frac{\sqrt{\frac{3}{2}}\times 3}{6} = \frac{\sqrt{3}}{4}$

18.



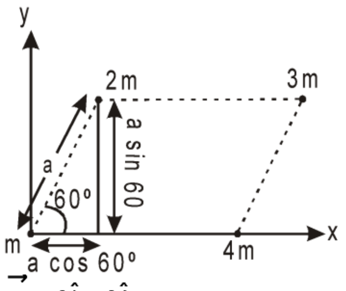
$$x_{cm} = \frac{\int dm\,x}{\int dm} = \frac{\int(\lambda dx)x}{\int \lambda dx} = \frac{\int_0^L \lambda_0 x^2 dx}{\int_0^L \lambda_0 x dx} = \frac{2L}{3}$$

19. $A_1 = \pi R^2$ $A_2 = \frac{\pi R^2}{16} \Rightarrow x_1 = 0$ $x_2 = \frac{3R}{4}$



$$x_{cen} = \frac{0 - \frac{\pi R^2}{16} \times \frac{3R}{4}}{\pi R^2 - \frac{\pi R^2}{16}} = -\frac{R}{20}$$

20. Let $m_1 = m$, $m_2 = 2m$, $m_3 = 3m$, $m_4 = 4m$



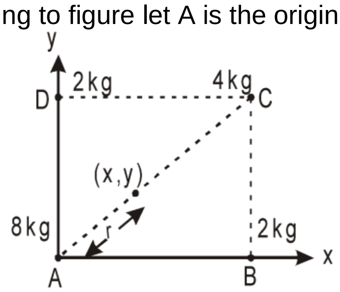
$\vec{r}_1 = 0\hat{i} + 0\hat{j}$

$\vec{r}_2 = a\cos 60\hat{i} + a\sin 60\hat{j} = \frac{a}{2}i + \frac{a\sqrt{3}}{2}j \Rightarrow \vec{r}_3 = (a + a\cos 60)\hat{i} + a\sin 60\hat{j} = \frac{3}{2}ai + \frac{a\sqrt{3}}{2}j$

$\vec{r}_4 = a\hat{i} + 0\hat{j}$ by substituting above value in the following formula

$\vec{r} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + m_4\vec{r}_4}{m_1 + m_2 + m_3 + m_4} = 0.95ai + \frac{\sqrt{3}}{4}a\hat{j}$. So the location of centre of mass $\left[0.95a.\frac{\sqrt{3}}{4}a\right]$

21. According to figure let A is the origin and co-ordinates of centre of mass be (x, y) then,

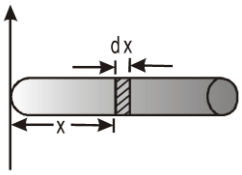


$$x = \frac{m_1x_1 + m_2x_2 + m_3x_3 + m_4x_4}{m_1 + m_2 + m_3 + m_4} = \frac{0 + 2x\frac{80}{\sqrt{2}} + 4x\frac{80}{\sqrt{2}} + 0}{16} = \frac{30}{\sqrt{2}}$$

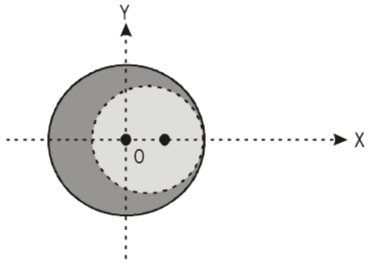
Similary y = $\frac{30}{\sqrt{2}}$ so, $r = \sqrt{x^2 + y^2}$ = 30 cm

22. Linear density of the rod varies with distance $\frac{dm}{dx} = \lambda$ (Given) ∴ dm = λdx

Position of centre of mass $x_{cm} = \frac{\int dm\ x\ x}{\int dm}$

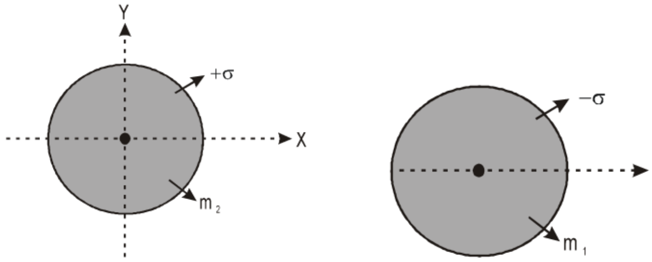


$$= \frac{\int_0^3 (\lambda\ dx) x\, x}{\int_0^3 \lambda\ dx} = \frac{\int_0^3 (2 + x) x\, x dx}{\int_0^3 (2 + x) dx} = \frac{\left[x^2 + \frac{x^3}{3}\right]_0^3}{\left[2x + \frac{x^3}{2}\right]_0^3} = \frac{9 + 9}{6 + \frac{9}{2}} = \frac{36}{21} = \frac{12}{7}m$$



23. Coordinate of centre of mass of large disc at its centre (0, 0) coordinate of centre of mass of small disc (7, 0)

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$



$m_2 = \sigma\pi(28\text{ cm})^2$ $\quad$ $m_1 = (-\sigma)\,\pi\,(21\text{ cm})^2$

$$X_{CM} = \frac{0 \times m_2 + (-\sigma)\pi(21cm)^2}{6\pi(28cm)^2 + (-\sigma)\pi(21cm)^2} = -9cm$$

$Y_{CM} = 0$

24. $X_{CM} = \frac{0 + m \times a + m \times 0}{3m} = \frac{a}{3}$ $\Rightarrow$ $Y_{CM} = \frac{0 + M \times 0 + M(a)}{3M} = \frac{a}{3}$

25. (i) $X_{CM} = \frac{0 \times m + m \times a + m \times \frac{a}{2}}{m + m + m} = \frac{a}{2}$, $\Rightarrow$ $Y_{CM} = \frac{0 \times m + 0 \times m + m \times \frac{a\sqrt{3}}{2}}{m + m + m} = \frac{a\sqrt{3}}{6}$,

26. $X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (1)(1) + 2(1/2)}{1 + 1 + 2} = 0.50$ m and

$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (1)(0) + 2(\sqrt{3}/2)}{1 + 1 + 2} = 0.43$ m

The centre of mass is at (0.50 m, 0.43 m)

27. The position vector of centre of mass

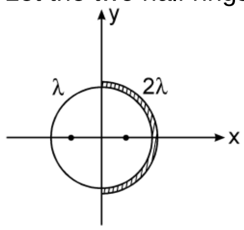
$$\vec{r} = \frac{m_1\vec{r_1} + m_1\vec{r_2}}{m_1 + m_2} = \frac{1\left(\hat{i} + 2\hat{j} + \hat{k}\right) + 3\left(-3\hat{i} - 2\hat{j} + \hat{k}\right)}{1 + 3} = \frac{1}{4}\left(-8\hat{i} - 4\hat{j} + 4\hat{k}\right) = -2\hat{i} - \hat{j} + \hat{k}$$

The centre of mass changes its position only under the translatory motion. There is no effect of rotatory motion on centre of mass of the body.

28. If mass of bigger disc is M then mass of removed disc is $\frac{M}{4}$

$$r_{CM} = \frac{M \times 0 - \frac{M}{4}R}{M - \frac{M}{4}} = \frac{R}{3} = \alpha R \quad \Rightarrow \quad \alpha = \frac{1}{3}$$

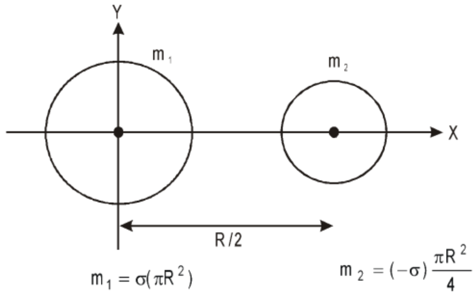
29. Let the two half rings be placed in left and right of y-axis with centre as shown in figure.



Then the coordinate of centre of mass of left and right half rings are $\left(-\frac{2R}{\pi}, 0\right)$ and $\left(\frac{2R}{\pi}, 0\right)$.

∴ x-coordinates of centre of mass of comple ring is $\dfrac{m\left(-\frac{2R}{\pi}\right)+2m\left(\frac{2R}{\pi}\right)}{3m} = \dfrac{2R}{3\pi}$

30.



Coordinate of large disc (0, 0)
Coordinate of small disc (R/2, 0)

$$X_{CM} = \frac{0+(-\sigma)\frac{\pi R^2}{4}\cdot\frac{R}{2}}{\sigma\pi R^2 - \sigma\frac{\pi R^2}{4}} = \frac{R}{6}$$, $Y_{CM} = 0$

31. Centre of mass are $r_{cm} = \frac{h}{4} = \frac{40}{4} = 10$ cm

SECTION (B)

2. In abscence of external force both move away from each other to keep the centre of mass at rest.

3. Internal forces canot change velocity but can do work.

4. If initial velocity of system is not zero then centre of mass moves with constant velocity.

5. $a_{cm} = \dfrac{m_1g+m_2g}{m_1+m_2} = g$

6. $v_{cm} = \dfrac{1\times 2+\frac{1}{2}\times 6}{1+1/2} = \dfrac{10}{3}$ m / sec.

7. vector sum of internal forces on system is zero.

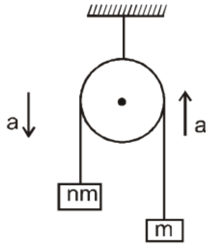
8. $V_{cm} = \dfrac{M_1V_1+M_2V_2}{M_1+M_2} = \dfrac{200\times 10\hat{i}+500(3\hat{i}+5\hat{j})}{700} = 5\hat{i}+\dfrac{25}{7}\hat{j}$

9. $V_{CM} = 0$, because internal force cannot change the velocity of centre of mass

14. by conservation of linear momentum $P_i = P_f \Rightarrow mu = \frac{m}{4}\times 0 + \frac{3m}{4}\times V \Rightarrow v = \frac{4u}{3}$

17. $a = \frac{(nm-m)}{nm+m}\,g = \frac{(n-1)}{(n+1)}\,g$

$a_1 = a_2 = a$



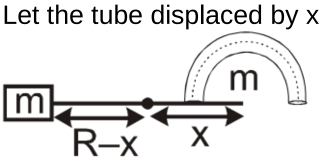
$a_{cm} = \frac{nma_1 - ma_2}{(nm+m)} = \frac{(n-1)}{(n+1)} \times a \quad \Rightarrow \quad a_{cm} = \frac{(n-1)^2}{(n+1)^2}\,g$.

18. Equation of motion

$M\vec{g} + \vec{R} = M\vec{a}_{cm}$

So, $\vec{a}_{cm} = \frac{M\vec{g} + \vec{R}}{M}$

19. Let the tube displaced by x towards left, then block will be displaced by (R – x) towards right ;



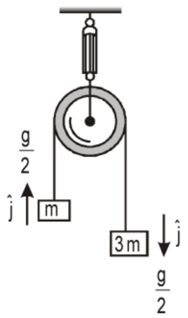
$mx = m(R-x) \quad \Rightarrow \quad x = \frac{R}{2}$

20. Net external force on box plus ball system is zero.

21. $m(L-x) + \frac{m}{3}(-x) = 0 \quad \Rightarrow \quad mL = \frac{4}{3}mx$

22. $a_{cm} = \frac{30}{(10+20)} = 1\ ms_2 \quad \Rightarrow \quad S = 0(2) + \frac{1}{2}(1)(2)_2 = 2\ m$

23. Acceleration of the system



$= \left(\frac{3m-m}{3m+m}\right) g = \frac{g}{2} \quad \Rightarrow \quad \vec{a}_{CM} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3}{m_1+m_2+m_3} = \frac{3m\frac{g}{2}\hat{j} + m\frac{g}{2}(-\hat{j})}{4m} \quad \Rightarrow \quad a_{CM} = \frac{g}{4}$

24. $\vec{a}_{CM} = \frac{\vec{F}_{ext}}{m}$

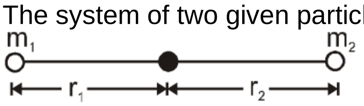
$= \frac{3\hat{i} - 2\hat{j} + 2\hat{k} + (-\hat{i}) + 2\hat{j} - \hat{k} + \hat{i} + \hat{j} + \hat{k}}{6}$

$\Rightarrow \quad \vec{a}_{CM} = \frac{1}{2}\hat{i} + \frac{1}{6}\hat{j} + \frac{1}{3}\hat{k} = \sqrt{\frac{1}{4} + \frac{1}{9} + \frac{1}{36}} = \sqrt{\frac{9+4}{36} + \frac{1}{36}} = \frac{1}{6}\sqrt{13+1} \quad \Rightarrow \quad \frac{\sqrt{14}}{6}\,ms^{-2}$

25. $\vec{V}_{CM} = \frac{m_1\vec{V}_1 + m_2\vec{V}_2}{m_1 + m_2} = \frac{10(2\hat{i} - 7\hat{j} + 3\hat{k}) + 2(-10\hat{i} + 35\hat{j} - 3\hat{k})}{12}$

$= \frac{20\hat{i} - 70\hat{j} + 30\hat{k} - 20\hat{i} + 70\hat{j} - 6\hat{k})}{12} = \frac{24\hat{k}}{12} = 2\hat{k}$

26. The system of two given particles of masses m_1 and m_2 are shown in figure.



Initially the centre of mass

$r_{CM} = \frac{m_1r_1 + m_2r_2}{m_1 + m_2}$ ....(i)

When mass m_1 moves towards centre of mass by a distance d, then let mass m_2 moves a distance d′ away from CM to keep the CM in its initial position.

So, $r_{cm} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2}$ ....(ii)

Equation Eqs. (i) and (ii), we get

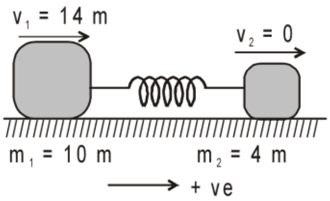
$\frac{m_1r_1 + m_2r_2}{m_1 + m_2} = \frac{m_1(r_1 - d) + m_2(r_2 + d')}{m_1 + m_2}$ ⇒ $-m_1d + m_2d' = 0$ ⇒ $d' = \frac{m_1}{m_2}d$.

Note: If both the masses are equal i.e., $m_1 = m_2$, then second mass will move a distance equal to the distance at which first mass is being displaced.

27. By the conservation of linear momentum

$(m + m)v' = m.\ 2\ v - mv$ ⇒ $2mv' = mv$ ⇒ $v' = \frac{v}{2}$

28. $v_{COM} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$



$= \frac{10 \times 14 + 4 \times 10}{10 + 4}$ = 10 m/s.

29. Mass per unit length

$= \frac{M}{L} = \frac{4}{2}$ = 2 kg/m

The mass of 0.6 m of chain = 0.6 × 2 = 1.2 kg

The centre of mass of hanging part = $\frac{0.6 + 0}{2}$ = 0.3 m

Hence, work done in pulling the chain on the table

W = mgh = 1.2 × 10 × 0.3 = 1.2 × 10 × 0.3 = 3. 6 J

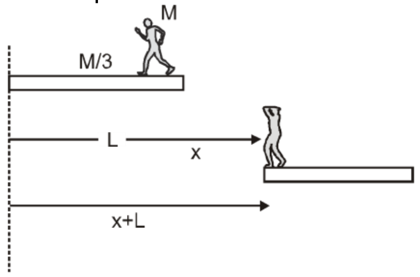
30. Before breaking, the centre of mass of system is moving under gravity. Thus, acceleration of the centre of mass is gravitational acceleration. During breaking, internal forces come into play which are not responsible for the acceleration of the centre of mass.

This indicates that, the acceleration of centre of mass remains the same (equal to gravitational acceleration).

Thus, the centre of mass of system continues its original path.

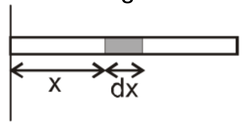
31. for $\Delta x_{cm} = 0$ ⇒ $m_1d_1 = m_2d_2$ ∴ $d_2 = \frac{m_1d_1}{m_2}$ **Ans.**

33. Initially, centre of mass of the "can + water in the can" will be at the geometrical centre of the can. As the liquid is coming out from the bottom, then centre of mass move downwards but after sometime it will move upwards and comes to its initial position. Hence, centre of mass first descends and then ascends.

34. 

$Mx + \frac{M}{3}(x + L) = 0 \Rightarrow \frac{4M}{3}x = -\frac{ML}{3} \Rightarrow x = -\frac{L}{4}$

35. Friction force between wedge and block is internal i.e. will not change motion of COM. Friction force on the wedge by ground is external and causes COM to move towards right. Gravitational force (mg) on block brings it downward hence COM comes down.

41. 

mass of small element is $dm = \lambda dx = K(x/L)^n\, dx$

$x_{CM} = \frac{\int dm.x}{\int dm} = \frac{\int_0^L K.\left(\frac{x}{L}\right)^n dx.x}{\int_0^L K.\left(\frac{x}{L}\right)^n dx} = \frac{(n+1)L}{n+2}$. So, graph shown in option (4) is correct.

SECTION (C)

1. Net external force is zero so net momentum will remain zero.

2. $\frac{k_2}{k_1} = 4 \Rightarrow \left(\frac{v_2}{v_1}\right)^2 = 4 \Rightarrow \frac{v_2}{v_1} = 2$

Then $\left(\frac{p_2 - p_1}{p_1}\right)\times 100 = \left(\frac{mv_2 - mv_1}{mv_1}\right)\times 100 = \left(\frac{v_2}{v_1} - 1\right)\times 100 = 100\%$

25. Here : Mass of the body $m_1 = m$
Velocity of the first body $u_1 = 312$ m/sec
Mass of second body at rest $m_2 = 2m$
Velocity of second body $u_2 = 0$
After combination total mass of the body
$M = m + 2m = 3m$. Now from the law of conservation of momentum
$Mv = m_1u_1 + m_2u_2$ (where $M = m + 2m = 3m$)
$3mv = m \times 3 + 2m \times 0 \Rightarrow v = 1$ km/hr

26. For bullet, $m_1 = 50$g, $u_1 = 10$ m/s $\Rightarrow$ For block, $m_2 = 950$ g, $u_2 = 0$
From law of conservation of momentum $m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$

$50 \times 10 + 950 \times 0 = (50 + 950) \times v \Rightarrow v = \frac{500}{1000} = 0.5$ m/s

Initial KE $= \frac{1}{2} m_1u_1^2 = \frac{1}{2} \times 50 \times (1000)^2 = 25 \times 10^6$ erg $\therefore$ Loss in KE $= \frac{1}{2} m_1u_1^2 - \frac{1}{2}(m_1 + m_2)\, v^2$

$\Delta K = \frac{1}{2} \times 50 \times (1000)^2 - \frac{1}{2}(50 + 950) \times (50)^2 = 25 \times 10^6 - 1250 \times 1000 = 25 \times 10^6 - 1.25 \times 10^6$

$= 23.75 \times 10^6$ erg $\therefore$ % loss in KE $= \frac{\Delta K}{K}\times 100\% = \frac{23.75\times 10^6}{25\times 10^6}\times 100 = 95.00\%$

27. $mv\sqrt{2} = 4mV_1$

$V_1 = \frac{v\sqrt{2}}{4} = \frac{v}{2\sqrt{2}}$.

28. use $m_1v_1 = m_2v_2 = P$

F.E. $= \frac{1}{2} m_1v_1^2 + \frac{1}{2} m_2v_2^2 = \frac{1}{2} m_1 \left(\frac{P}{m_1}\right)^2 + \frac{1}{2} m_2 \left(\frac{P}{m_2}\right)^2 = \frac{1}{2} \frac{P^2(m_2+m_1)}{m_1m_2}$.

29. If we treat the train as a ring of mass 'M' then its COM will be at a distance $\frac{2R}{\pi}$ from the centre of the circle. Velocity of centre of mass is :

$V_{CM} = R_{CM}.\omega = \frac{2R}{\pi}.\omega = \frac{2R}{\pi}.\left(\frac{V}{R}\right)$ $(\because \omega = \frac{V}{R})$

$\Rightarrow$ $V_{CM} = \frac{2V}{\pi}$ $\Rightarrow$ $MV_{CM} = \frac{2MV}{\pi}$ As the linear momentum of any system = MV_{CM}

$\therefore$ The linear momentum of the train = $\frac{2MV}{\pi}$ **Ans.**

30. $\frac{k_1}{k_2} = \frac{1}{2} \frac{P^2/(m)}{P^2/2(4m)} = 4/1$

31. we have $\frac{p_2 - p_1}{p_1} = 0.2$ $\Rightarrow$ $\frac{p_2}{p_1} = 1.2$

so $\left(\frac{k_2 - k_1}{k_1}\right) \times 100 = \left(\frac{k_2}{k_1} - 1\right) \times 100 = \left(\frac{p_2^2}{p_1^2} - 1\right) \times 100$ (since $k = \frac{p^2}{2m}$) $= ((1.2)^2 - 1) \times 100 = 44$ %

32. Velocity of man w.r.t. train is zero so kinetic energy of man w.r.t. train = 0

33. Kinetic energy increase due to internal energy.

34. $\vec{p}_1 + \vec{p}_2 = 0$ $\Rightarrow$ $\vec{p}_1 = -\vec{p}_2$ $\Rightarrow$ $|\vec{p}_1| = |\vec{p}_2|$

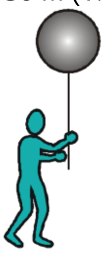
35. by conservation of linear momentum $p_i = p_f$

$\Rightarrow$ $mv = (m + M)\, u \Rightarrow$ $u = \frac{mv}{(m+M)}$

so energy of system $= \frac{1}{2} (m + M) \times u^2 = m^2v^2/2(M + m)$

36. We have $V_m + V_b = V_{rel}$ $\Rightarrow$ $V_m = V_{rel} - V_b$ by conservation of linear momentum
$mV_m - MV_b = 0$

So $m(V_{rel} - V_b) - MV_b = 0$ $\Rightarrow$ $V_b = \frac{mv_{rel}}{m+M}$



38. Equate the momenta of the system along two perpendicular axes.
Let u be the velocity and θ the direction of the third piece as shown.

Equating the momenta of the system along OA and OB to zero, we get

$m \times 30 - 3m \times v\cos\theta = 0$ ....(i)

and $m \times 30 - 3m \times v\sin\theta = 0$ ....(ii)

These give $3mv\cos\theta = 3mv\sin\theta$

or $\cos\theta = \sin\theta$ $\therefore$ $\theta = 45^\circ$

Thus, $\angle AOC = \angle BOC = 180^\circ - 45^\circ = 135^\circ$

Putting the value of θ in Eq. (i) we get

$30\,m = 3mv\cos 45^\circ = \frac{3mv}{\sqrt{2}}$ $\therefore$ $v = 10\sqrt{2}$ m/s

The third piece will go with a velocity of $10\sqrt{2}$ m/s in a direction making an angle of 135º with either piece.

39. For a exploding body, linear momentum is conseved.

From conservation of linear momentum,

$P_{initial} = P_{final}$

$0 = m_1v_1 - m_2v_2$

or $m_1v_1 = m_2v_2$ ....(i)

or $\frac{v_1}{v_2} = \frac{m_2}{m_1}$ ....(ii)

Thus, ratio of kinetic energies $\frac{E_1}{E_2} = \frac{\frac{1}{2}m_1v_1^2}{\frac{1}{2}m_2v_2^2} = \frac{m_1}{m_2} \times \left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1}$

Note: In a collision of two bodies whether it is perfectly elastic or inelastic, linear momentum is always conserved but kinetic energy need not be conserved.

40. Kinetic energy is given by

$E = \frac{1}{2}mv^2 = \frac{1}{2m}(mv)^2$

but mv = momentum of the particle = p $\therefore$ $E = \frac{p^2}{2m}$ or $p = \sqrt{2mE}$

Therefore, $\frac{p_1}{p_2} = \sqrt{\frac{m_1E_1}{m_2E_2}}$ but it is given that, $p_1 = p_2$ $\therefore m_1E_1 = m_2E_2$

or $\frac{E_1}{E_2} = \frac{m_2}{m_1}$ ....(i)

Now $m_1 > m_2$

or $\frac{m_1}{m_2} > 1$ ....(ii)

Thus, Eqs. (i) and (ii) give

$\frac{E_1}{E_2} < 1$ or $E_1 < E_2$.

41. The linear momentum of exploding part will remain conserved.

Applying conservation of linear momentum, we write

$m_1u_1 = m_2u_2$ Here, $m_1 = 18$ kg, $m_2 = 12$ kg

$\mu_1 = 6\ ms^{-1}$, $u_2 = ?$ $\therefore$ $18 \times 6 = 12u_2$ $\Rightarrow$ $u_2 = \frac{18 \times 6}{12} = 9\ ms^{-1}$

Thus, kinetic energy of 12 kg mass

$K_2 = \frac{1}{2}m_2u_2^2 = \frac{1}{2} \times 12 \times (9)^2 = 6 \times 81 = 486$ J

42. **Key Idea :** In the given problem conservation of linear momentum and energy hold good.

Conservation of momentum yields.

$m_1v_1 + m_2v_2 = 0$

or $4v_1 + 0.2\, v_2 = 0$ ....(i)

Conservation of energy yields

$\frac{1}{2}\, m_1v_1^2 + \frac{1}{2}\, m_2v_2^2 = 1050$

or $\frac{1}{2} \times 4v_1^2 + \frac{1}{2} \times 0.2 \times v_2^2 = 1050$

or $2\, v_1^2 + 0.1\, v_2^2 = 1050$ ....(ii)

Solving Eqs. (i) and (ii), we have $v_2 = 100$ m/s

44. If kinetic energy of system is zero, then momentum of system is necessarily zero.

45. By conservation of linear momentum

$P_i = P_f \Rightarrow 0 = 12 \times 4 + 4 \times v \Rightarrow v = 12$ m/s

So, kinetic energy of other mass is $\frac{1}{2}\, mv^2 = \frac{1}{2} \times 4(12)^2 = 288$ J,

46. By conservation of linear momentum

$0.5 \times 2 = (1.5)v \Rightarrow v = \frac{2}{3}$ m/s $\Rightarrow \Delta K = K_i - K_f = \frac{1}{2} \times 0.5 \times (2)^2 - \frac{1}{2} \times 1.5 \left(\frac{2}{3}\right)^2 = 0.67$ J

47. When the centre of mass remains at rest, it is possible that different individual forces do individual works though the net resultant force is zero. As work is a scalar quantity, they gets added up.

Also, $\Sigma F_{ext} = 0 \Rightarrow a_{CM} = 0$.

48. For the momentum to remain conserved all the fragments should finally move in a single plane as for three vectors to give a resultant = 0, they should form a triangle.

49. Since $\Sigma \vec{F}_{ext} = \vec{0}$ $\therefore$ Moment of system will remain conserved, equal to zero.

50. By momentum conservation $mv + M(0) = (m + M)\, v'$ $\Rightarrow$ $v' = \frac{mv}{m+M}$

51. $mV \times N = (Nm + M)\, V'$

SECTION (D)

10. by energy conservation $\frac{1}{2}\, mv^2 = \frac{1}{2}\, (2m)\left(\frac{v}{2}\right)^2 + \frac{1}{2}\, kx^2$ $\Rightarrow$ $x = \sqrt{2mK}$

SECTION (E)

3. Area of F-t curve = A = Impulse.

Impulse = dP = A = mv – 0 $\therefore v = \frac{A}{M}$.

5. $\Delta P = P_f - P_i = (-Mv) - (Mv) = -2Mv$

7. $v_1 = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2}$ $\Rightarrow$ $k_2 = \frac{1}{4}\, k_1 \Rightarrow v_2^2 = \frac{1}{4}\, v_1^2 \therefore v_2 = \frac{v_1}{2} = 5\sqrt{2}$

$|\Delta P| = |-mv_2 - (mv_1)| = m|-v_2 - v_1|$

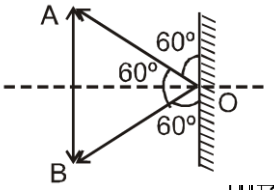
$|\Delta P| = 50 \times 10^{-3} \times \frac{3}{2} \times 10\sqrt{2} = \frac{15 \times 10^{-2}}{\sqrt{2}}$

$J = \Delta P = 1.05$N-s.

8. Impulse = $\int F\, dt$ = Area under curve = $\frac{1}{2}$ (2) (2) = 2 kg-m/sec.

9. The force imparted by the bali is equal to rate of change of linear momentum.

The vector $\vec{OA}$ represents the momentum of the ball before the collision and the vector $\vec{OB}$ that after the collision. The vector $\vec{AB}$ represents the change in momentum of the ball $\vec{\Delta p}$.



As the magnitue of $\vec{OA}$ and $\vec{OB}$ are equal the components of $\vec{OA}$ and $\vec{OB}$ along the wall are equal and in the same direction while those perpendicular to the wall are equal and opposite. Thus, the change in momentum id due only to the change in direction of the perpendicular components.

Hence $\vec{\Delta p}$ = OA sin 60º – (–OB sin 60º) = mv sin 60º + mv sin 60º = 2mv sin 60º

$= 2 \times 3 \times 100 \times \frac{\sqrt{3}}{2} = 300\sqrt{3}$ kg-m/s

The force exerted on the wall

$F = \frac{\vec{\Delta p}}{\Delta t} = \frac{300\sqrt{3}}{0.2} = 1500\sqrt{3}$ N   Note : $\vec{\Delta p}$ is directed perpendicular and away from the plate.

11. For the duration of collision

Impulse = change in momentum

Speeds of both the balls are same just before the collision with ground. ($\because v^2 = 2gh$)

For elastic collision of first ball with ground velocity of first ball is reversed

$\vec{I_1} = m\vec{v} - m\vec{u} = m(-\vec{u}) - m\vec{u} = 2m\vec{u}$ For the second ball

$\vec{I_2} = 0 - m\vec{u} = \frac{\vec{I_1}}{2}$ $\Rightarrow I_2 = \frac{I_1}{2}$

12. Impulse = change in momentum

$\frac{\pi F_0 T}{4} = mu - 0 \quad \Rightarrow \quad u = \frac{\pi F_0 T}{4m}$

13. $F_x = \frac{\Delta P_x}{\Delta t} = \frac{(Pf_x - Pi_x)}{\Delta t} = \frac{-mV\sin 60^\circ - (mV\sin 60^\circ)}{2\times 10^{-3}} = -250\sqrt{3}$ N $= 250\sqrt{3}$ N towards left

14. $\left| (m_1\vec{v}'_1 + m_2\vec{v}'_2) - (m_1\vec{v}_1 + m_2\vec{v}_2) \right|$ = | Change in momentum of the two particles |

= | External force on the system | × time interval = $(m_1 + m_2)\, g(2t_0) = 2(m_1 + m_2)\, gt_0$

SECTION (F)

3. Velocity of heavy mass donot change after collison

$\frac{v_2 - v_1}{u_2 - u_1} = -e = -1 \quad \Rightarrow \quad \frac{v_2 - v}{0 - v} = -1 \quad \Rightarrow \quad v_2 = 2v$

4. If mass = m

first ball will stop ⇒ v = 0

so k.e. = 0 (min)

In other cases there will be some kinetic energy

(K.E. can't be negative)

5. e = $\dfrac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$
For elastic collision e = 1
|Velocity of separation| = |velocity of approach|
For inelastic collision e < 1
So |velocity of separation| < |velocity of approach|

6. $v_1 = \dfrac{(m_1 - em_2)u_1}{m_1 + m_2} + \dfrac{m_2(1+e)u_2}{m_1+m_2} = \dfrac{(m - e2m)u_1}{m+2m} + \dfrac{2m(1+e)\times 0}{m+2m} = 0 \Rightarrow \quad 0 = m - e2m \quad \Rightarrow e = 1/2$

7. $mu = mv_1 + mv_2$ ....(i)
$u = v_1 + v_2$ ....(i)
$\dfrac{v_2 - v_1}{u} = e$ ......(ii) As solving have $\dfrac{v_1}{v_2} = \left(\dfrac{1-e}{1+e}\right)$.

27. $0.5 \times v_p + m \times 0 = 5.05\ v \quad \therefore \dfrac{v_f}{v_i} = \dfrac{0.05}{5} = 10^{-2} \quad \Rightarrow \dfrac{\frac{1}{2}m(v_f)^2}{\frac{1}{2}m(v_i)^2} = (10^{-2})^2 = 10^{-4}$.

28. by conservation of linear momentum $P_i = P_f \quad \Rightarrow \quad mv = (100\ m)\ u \quad \Rightarrow \quad u = v/100$

30. Kinetic energy remains conserved in elastic collision

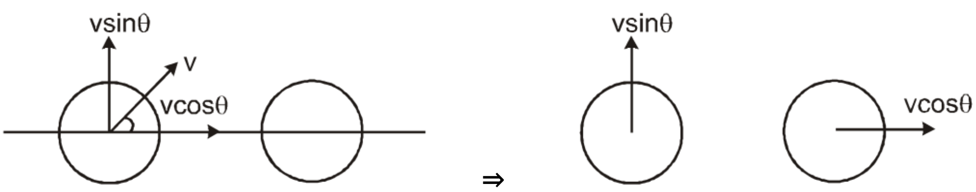
31. When $M_1 = M_2$, velocity are exchanged. So total energy of M_1 is transfred to M_2

32. By conservation of linear momentum $P_i = P_f \quad \Rightarrow \quad mv\hat{i} + mv\hat{j} = 2\ m\vec{u}$
$\Rightarrow \quad \vec{u} = \dfrac{v}{2}(\hat{i} + \hat{j}) \quad \Rightarrow \quad |\vec{u}| = \dfrac{v}{\sqrt{2}} = \dfrac{45\sqrt{2}}{\sqrt{2}} = 45$

33. e = $\dfrac{\text{velocity of seperation}}{\text{velocity of approch}}$ = 1 for elastic collision.

34. Velocities will be interchanged

35. We have $\dfrac{v_2 - v_1}{u_2 - u_1} = -e \quad \Rightarrow \quad \dfrac{4 - v_1}{4 - 6} = -1$
(velocity of heavier mass will not change) $\Rightarrow \quad 4 - v_1 = 2 \quad \Rightarrow \quad v_1 = 2$ m/s

37.


be fore collision after collision
So angle between velocity vectors is 90º

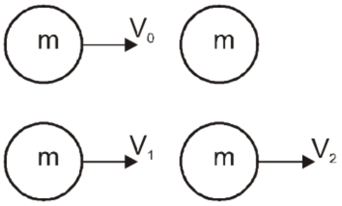
40. In an inelastic collision, the particles do not regain their shape and size completely after collision. Some fraction of mechanical energy is retained by the colliding particles in the form of deformation potential energy. Thus, the kinetic energy of particles no longer remains conserved. However, in the absence of external forces, law of conservation of linear momentum still holds good.

41. Given, $m_1 = 20$ kg, $u_1 = 10$ m/s,
$m_2 = 5$ kg, $u_2 = 0$
Using law of conservation of momentum,
$m_1u_1 + m_2u_2 = (m_1 + m_2)\, v$

$20 \times 10 + 5 \times 0 = (20 + 5) \times v \quad \therefore \quad v = \frac{200}{25} = 8$ m/s

Kinetic energy of the composite mass = $\frac{1}{2}(m_1 + m_2)\, v^2 = \frac{1}{2}(20 + 5) \times (8)^2 = 25 \times 32 = 800$ J

42. During a perfectly elastic collision both energy and momentum are conserved



43.
$V_0 = V_1 + V_2 \quad \Rightarrow \quad 2V_2 = (1 + e)\, V_0$
$eV_0 = V_2 - V_1 \qquad\quad 2V_1 = (1 - e)V_0$

So, $\frac{V_1}{V_2} = \frac{1-e}{1+e}$

47. $\Delta U = \frac{1}{2}\frac{m_1 m_2}{(m_1 + m_2)}(V_1 - V_2)^2 = \frac{100}{3}$

$(V_1 - V_2)^2 \times \frac{2m.m}{2(m+2m)} = \frac{100}{3}$ putting m = 1 kg $\Rightarrow \quad (V_1 - V_2) = 10$ m/sec.

AlternateSolution:

When deformation is maximum both the particles are moving with same velocity. So applying momentum conservation.

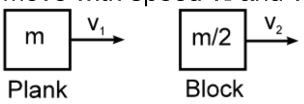
$m_1v_1 + m_2v_2 = m_1v_{1'} + m_2v_{1'} \quad \Rightarrow \quad v_1' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$

Applying energy conservation:

$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)(v_1')^2 + \Delta U_{\text{deformation}}$

$\Rightarrow \Delta U_{\text{deformation}} = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)} \times (v_1 - v_2)^2 = \frac{100}{3} \quad \Rightarrow v_1 - v_2 = 10$m/sec.

48. Let the velocities of plank and body of mass $\frac{m}{2}$
move with speed v_1 and v_2 after collision as shown.



From conservation of momentum.

$mv - \frac{m}{2}2v = mv_1 + \frac{m}{2}v_2$

or $\quad 2v_1 + v_2 = 0 \qquad$ ....(1)

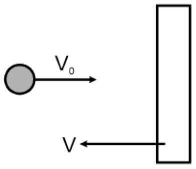
From equation of coefficient of restitution.

$e = 1 = \frac{v_2 - v_1}{v + 2v} \quad \Rightarrow \quad v_2 - v_1 = 3v \qquad$ ....(2)

Solving 1 and 2 we get

$v_1 = -v$

49.

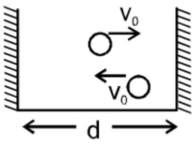


V = 5

Velocity of Seperation = Velocity of approach (collision is elastic) ∴ V – 5 = 20 + 5 ⇒ V = 30 m/s

50. $t = \frac{2d}{v_0}$ (time for succeesive collision)

$N \times t = dP = mv_0 - (-mv_0)$ ⇒ $N \times \frac{2d}{v_0} = 2mv_0$



∴ $N = \frac{mv_0^2}{d}$

52. According to Newton's Law

$e = \frac{\vec{v}_2 - \vec{v}_1}{\vec{u}_1 - \vec{u}_2}$

For elastic collision cofficient of restitution e = 1 so

$\vec{v}_2 - \vec{v}_1 = \vec{u}_1 - \vec{u}_2$ Statement - 1 is correct

Linear momentum is conserved in both elastic & non elastic collision but it's not the explanation of statement -1 so it is not the correct explanation of the statement A. **Ans. (2)**

54. Max potential energy of deformation = max. K.E. loss = $\frac{1}{2} \times 3 \times 2^2 + \frac{1}{2} \times 2 \times 3^2 = 15$ J

55. All energy is transfered to other particles.

56. $V_A = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \frac{2m_2u_1}{m_1 + m_2} = \left(\frac{10 - 38}{10 + 38}\right) 15 + \frac{2 \times 38 \times 3}{10 + 38} = -4$ m/s.

57. **(Easy)** Nothing is mentioned about coefficient of restitution. Hence the only true statement is 'their final velcities may be zero.'

60. Velocity before collision with surface is $\sqrt{2gh} = \sqrt{2g1}$

velocity after collision with surface is $e \times \sqrt{2gh} = 0.6 \times \sqrt{2 \times 10 \times 1}$

Hight reached by ball after collision $h = \frac{\left(e\sqrt{2gh}\right)^2}{2g} = 0.36$ m

SECTION (G)

1. $F = \mu \frac{dm}{dt}$ ⇒ $210 = 300 \times \frac{dm}{dt}$ ⇒ $\frac{dm}{dt} = 0.7$ kg/s.

8. Let Initial thrust of the blast be F then F – mg = ma

$F = m(g + a) = 3.5 \times 10^4 \times (10 + 10) = 7 \times 10^5$ N

9. Neglecting gravity,

$v = u\ell n \left(\frac{m_0}{m_t}\right)$;

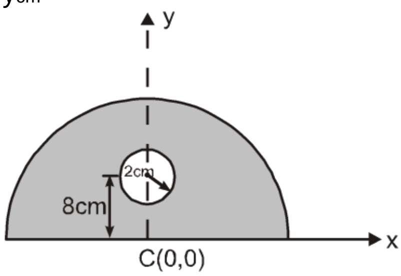
u = ejection velocity w.r.t. balloon. m_0 = initial mass m_t = mass at any time t.

$= 2\ell n \left(\frac{m_0}{m_0 / 2}\right) = 2\ell n2.$

EXERCISE # 2

1. Taking C as origin and x & y–axes as shown in figure. Due to symmetry about y–axis
$x_{cm} = 0$

$$y_{cm} = \left(\frac{m_1y_1 - m_2y_2}{m_1 - m_2}\right) = \frac{\left(\frac{\pi(6\pi)^2}{2}\right)\left(\frac{4(6\pi)}{3\pi}\right) - [\pi(2)^2(8)]}{\frac{\pi(6\pi)^2}{2} - \pi(2)^2} \qquad (m \propto \text{Area})$$



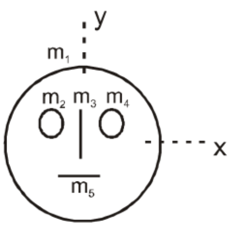
$$= \frac{8(18\pi^2 - 4)}{(18\pi^2 - 4)} = 8 \text{ cm.}$$

2. $A_1 = 2r \times r = 2r^2$ $\qquad A_2 = \frac{\pi r^2}{2}$

$x_1 = \frac{r}{2}$ $\qquad x_2 = \frac{4r}{3\pi}$

$$x_{cm} = \frac{2r^2 \times \frac{r}{2} - \frac{\pi r^2}{2} \times \frac{4r}{3\pi}}{2r^2 - \frac{\pi r^2}{2}} = \frac{r^3\left[1 - \frac{2}{3\pi}\right]}{r^2\left[\frac{4-\pi}{2}\right]} = \frac{2r}{3[4-\pi]}$$

3.



$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + m_4y_4 + m_5y_5}{m_1 + m_2 + m_3 + m_4 + m_5} \Rightarrow y_{cm} = \frac{6m(0) + m(a) + m(0) + m(a) + m(-a)}{m + m + m + m + 6m} = \frac{a}{10}.$$

4. Let at O there will be a collision. If smaller sphere moves x distance to reach at O, then bigger sphere will zmove a distance of

$$F = \frac{GM \times 5M}{(12R - x)^2} \Rightarrow a_{small} = \frac{F}{M} = \frac{G \times 5M}{(12R - x)^2}$$

$$a_{big} = \frac{F}{5M} = \frac{GM}{(12R - x)^2} \Rightarrow x = \frac{1}{2} a_{small} t^2$$

$$= \frac{1}{2} \frac{G \times 5M}{(12R - x)^2} t^2 \qquad \text{........(i)}$$

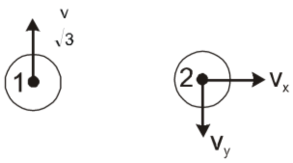
$$(9R - x) = \frac{1}{2} a_{big} t^2 = \frac{1}{2} \frac{GM}{(12R - x)^2} t^2 \qquad \text{........(ii)}$$

Thus, dividing Eq. (i) by Eq. (ii), we get $\therefore \frac{x}{9R - x} = 5 \Rightarrow x = 45R - 5x \Rightarrow 6x = 45R \Rightarrow x = 7.5R$

6. Velocity of particle after 5 s
$v = u - gt$
$v = 100 - 10 \times 5 = 100 - 50 = 50$ m/s (upwards)
Conservation of linear momentum gives
$Mv = m_1v_1 + m_2v_2$ ......(i)
Taking upward direction positive, the velocity v_1 will be negative. $\therefore v_1 = -25$ m/s, $v = 50$ m/s
Also M = 1 kg, m_1 = 400 g = 0.4 kg and $m_2 = (M - m_1) = 1 - 0.4 = 0.6$ kg
Thus, Eq. (i) becomes,
$1 - 50 = 0.4 \times (-25) + 0.6\, v_2$
or $50 = -10 + 0.65\, v_2$ or $0.6\, v_2 = 60$ or $v_2 = \frac{60}{0.6} = 100$ m/s
As v_2 is positive, therefore the other part will move upwards with a velocity 100 m/s.

7.

before collision



after collision

In x-direction
$mv + 0 = 0 + m\upsilon_x \Rightarrow \upsilon_x = \upsilon$
In y-direction
$0 + 0 = m\left(\frac{\upsilon}{\sqrt{3}}\right) - m\upsilon_y \Rightarrow \upsilon_y = \frac{\upsilon}{\sqrt{3}}$ $\therefore$ Velocity of second mass after collision
$\upsilon = \sqrt{\left(\frac{\upsilon}{\sqrt{3}}\right)^2 + \upsilon^2} = \sqrt{\frac{4}{3}\upsilon^2} \Rightarrow \upsilon' = \frac{2}{\sqrt{3}}$

9. $M_A = \rho \times \frac{4}{3}\pi r^3$ $e = \frac{1}{2}$
$M_B = \rho \times \frac{4}{3}\pi (2r)^3 = 8M_A$
$m_A v + 0 = m_A v_1 + m_B v_2$ ....(i)
$ev = v_2 - v_1$ ....(ii)
Adding (i) + (ii) $= 9v_2 = v + \frac{v}{2} = \frac{3v}{2}$
$\therefore v_1 = v_2 - \frac{v}{2} = \frac{v}{6} - \frac{v}{2} = -\frac{v}{3}$. $\therefore \frac{v_1}{v_2} = \frac{v/3}{v/6} = 2$.

10.* $\vec{P_1} = p\hat{i}$
$\vec{P_2} = -p\hat{i}$ as there is no external force so momentum will remain conserved
$\vec{P_1'} + \vec{P'_2} = \vec{P_1} + \vec{P_2}$
$\vec{P_1'} + \vec{P_2'} = 0$ Now from option
(1) $\vec{P_1'} + \vec{P_2'} = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$
(2) $\vec{P_1'} + \vec{P_2'} = (c_1 + c_2)\hat{k}$
(3) $\vec{P_1'} + \vec{P_2'} = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j}$
(4) $\vec{P_1'} + \vec{P_2'} = (a_1 + a_2)\hat{i} + 2b_1\hat{j}$ and it is given that $a_1\, b_1\, c_1$, $a_2, b_2, c_2, \neq 0$ in case of A and D it is not possible to get $\vec{P_1'} + \vec{P_2'} = 0$. Hence **Ans. (1) and (4)**

EXERCISE # 3
PART - I

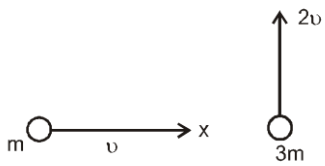
1. Apply law of conservation of linear momentum.
Momentum of first part = 1 × 12 = 12 kg ms^{-1}
Momentum of the second part = 2 × 8 = 16 kg ms^{-1}

∴ Resultant momentum = $\sqrt{(12)^2 + (16)^2}$ = 20 kg ms^{-1}
The third part should also have the same momentum.
Let the mass of the third part be M, then
4 × M = 20
M = 5 kg.

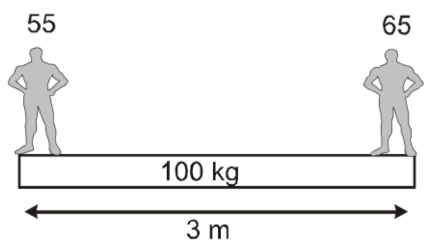
2. Here, $m_1 = m$, $m_2 = 2m$
$u_1 = 2$ m/s, $u_2 = 0$ coefficient of restitution, e = 0.5
Let v_1 and v_2 be their respective velocities after collision.
Applying the law of conservation of linear momentum, we get
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
∴ $m \times 2 + 2m \times 0 = m \times v_1 + 2m \times v_2$
or $2m = mv_1 + 2mv_2$
or $2 = (v_1 + 2v_2)$ ...(i)
By definition of coefficient of restitution,

$e = \dfrac{v_2 - v_1}{u_1 - u_2}$ or $e(u_1 - u_2) = v_2 - v_1$
$0.5(2 - 0) = v_2 - v_1$ ...(ii)
$1 = v_2 - v_1$ Solving equations (i) and (ii), we get
$v_1 = 0$ m/s, $v_2 = 1$ m/s

3. As no external force is acting on the system, the centre of mass must be at rest i.e. $v_{CM} = 0$.

4. 

From momentum conservation
$$m\upsilon\hat{i} + 3m(2\upsilon)\hat{j} = (4m)\vec{\upsilon}$$
$$\vec{\upsilon} = \frac{\upsilon}{4}\hat{i} + \frac{6}{4}\upsilon\hat{j} = \frac{\upsilon}{4}\hat{i} + \frac{3}{2}\upsilon\hat{j}$$

5. 

There is no external force so com will not shift

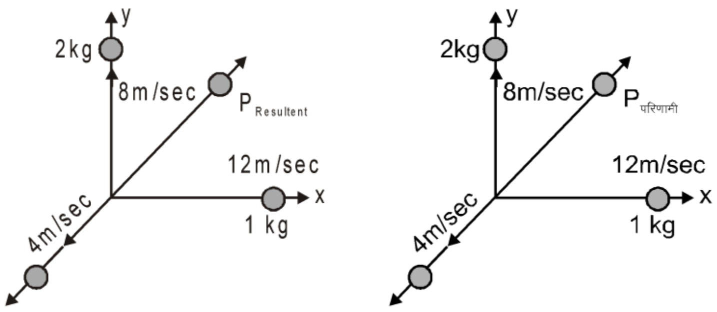
6. m_2 m_1
B → v A
u = 0 conservation of linear momentum along x direction
$m_2v = m_1v_x$
$\dfrac{m_2v}{m_1} = v_x$ along y direction $m_2 \times \dfrac{v}{2} = m_1 v_y$ ⇒ $\tan\theta = \dfrac{1}{2}$ **Ans.** 4

7. $X_{cm} = \dfrac{300 \times (0) + 500(40) + 400 \times 70}{300 + 500 + 400}$

$X_{cm} = \dfrac{500 \times 40 + 400 \times 70}{1200}$

$X_{cm} = \dfrac{50+70}{3} = \dfrac{120}{3}$ = 40 cm

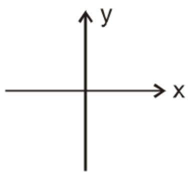
8.



$P_{resultant} = \sqrt{12^2 + 16^2} = \sqrt{144+256} = 20$ ∴ $m_3v_3 = 20$

$m_3 = \dfrac{20}{4} = 5$ kg

9.



$p_i = p_f$

$O = mv\hat{i} + mv\hat{j} + 2m\vec{v}$

$\vec{v} = -\dfrac{v}{2}\hat{i} - \dfrac{v}{2}\hat{j}$

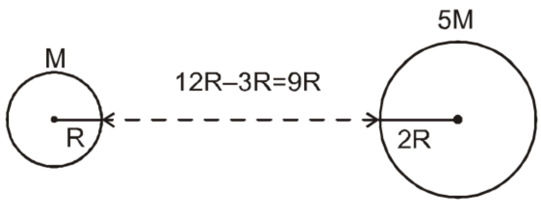
$|\vec{v}| = \dfrac{v}{\sqrt{2}}$

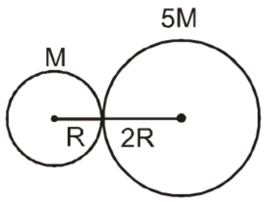
KE = $\dfrac{v}{2}mv^2 + \dfrac{v}{2}mv^2 + \dfrac{v}{2}2m\left(\dfrac{v}{\sqrt{2}}\right)^2$ = mv_2 + $\dfrac{mv^2}{2} = \dfrac{3}{2}mv^2$

10. $\dfrac{1}{2}mv_1^2 + \dfrac{1}{2}mv_2^2 = \dfrac{1}{2}m_1u_1^2 + \dfrac{1}{2}m_2u_2^2 - \varepsilon$

By energy conservation

11.





As their C.M. will remain stationery. So, (M) (x_1) = (5M) (x_2) and for tauching

$x_1 + x_2 = 9R$. So, $x_1 = 7.5$ R

12. J = 2mV cos 60 = mV **Ans.**

13. During the collision, apply moementum conservation

(0.01)(400) + 0 = (2)V + (0.01)V' where V = $\sqrt{2gh}$

V = $\sqrt{2\times10\times0.1}$

V = $\sqrt{2}$ solving V' = 120 m/sec. **Ans.**

14. Mass of balls are same and the collision is perfectly elastic, so their velocity will be interchanged.

So, $V_A = -0.3$ m/s, $V_B = 0.5$ m/s **Ans.**

15. From momentum conservation

mu + 0 = 0 + (4m) V_2 ⇒ $V_2 = \frac{u}{4}$

e = $\frac{V_2}{u} = \frac{u_4}{u} = \frac{1}{4}$ = 0.25

16. Energy transfered to B initial energy of B = zero
Final velocity of

$$V_B = \left(\frac{M_2 - M_1}{M_1 + M_2}\right)u_2 + \frac{2M_1u_1}{M_1 + M_2}$$

$M_1 = 4M$ $u_1 = u$
$M_2 = 2M$ $u_2 = 0$

$$V_B = \frac{2(4M)u}{6M} = \frac{4}{3}u \Rightarrow \frac{\frac{1}{2}M_2V_B^2}{\frac{1}{2}M_1u_1^2} = \frac{\frac{1}{2}2M\left(\frac{4}{3}\right)^2u^2}{\frac{1}{2}4Mu^2}$$ Fraction of energy lost $= \frac{8}{9}$

17. Momentum conservation $p_i = p_f$

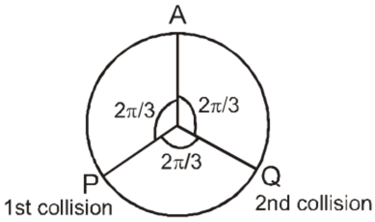
$$5m(0) = mv\hat{i} + mv(\hat{j}) + 3mv_1 \Rightarrow -(v\hat{i} + v\hat{j}) = 3v_1$$

$$v_1 = \frac{-(v\hat{i} + v\hat{j})}{3}, v_1 = \frac{\sqrt{2}v}{3}$$ Energy released is

$$E = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}3m\left(\frac{\sqrt{2}v}{3}\right)^2 \Rightarrow E = mv^2 + \frac{mv^2}{3} \Rightarrow E = \frac{4}{3}mv^2$$

PART - III

1. Since masses of particles are equal, collisons are elastic, so particles will exchange velocities after each collision. The first collision will be at a point P and second at point Q again and before third collision the particles will reach at A.



2. If initial momentum of particles is zero, then they loss all their energy in inelastic collision but here initial momentum is not zero.
Principle of conservation of momentum holds good for all collision.

3. R = $u\sqrt{\frac{2h}{g}}$ ⇒ 20 = $V_1\sqrt{\frac{2\times5}{10}}$ and 100 = $V_2\sqrt{\frac{2\times5}{10}}$ ⇒ V_1 = 20 m/s , V_2 = 100 m/sec.

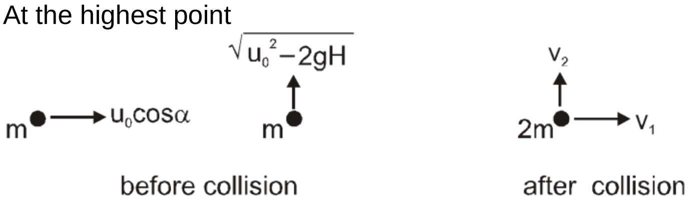
Applying momentum conservation just before and just after the collision
(0.01) (V) = (0.2)(20) + (0.01)(100) $\Rightarrow$ V = 500 m/s

4. Maximum energy loss = $\frac{P^2}{2m} - \frac{P^2}{2(m+M)}$

$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$ $\left(f = \frac{M}{m+M}\right)$

Hence Statement -1 is wrong and statement 2 is correct Hence

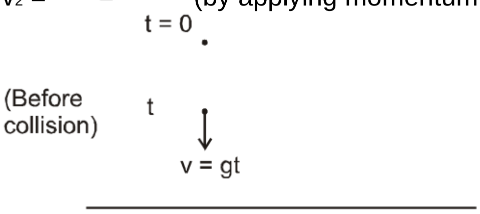
5. At the highest point



$v_1 = \frac{u_0\cos\alpha}{2}$ (by applying momentum conservation in horizontal direction)

$v_2 = \frac{u_0\cos\alpha}{2}$ (by applying momentum conservation in vertical direction) (H = $\frac{u_0^2\sin^2\alpha}{2g}$) θ = 45°

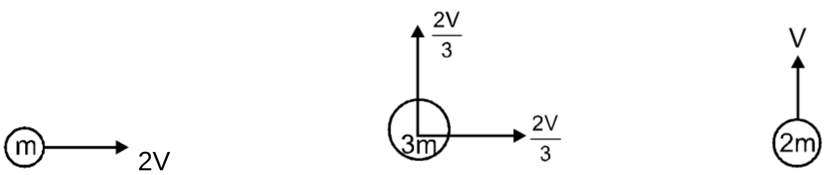
6.



K = $\frac{1}{2}mg^2t^2$

$K \propto t^2$: parabolic graph then during collision kinetic energy first decreases to elastic potential energy and then increases.
Most appropriate graph is B.

7. Just before collision Just after collision



Energy loss ΔE = $\frac{1}{2}$ m $(2V)^2$ + $\frac{1}{2}$ (2m) V^2 – $\frac{1}{2}$ (3m) 2 $\left(\frac{2V}{3}\right)^2$ = $3mV^2 - \frac{4mV^2}{3}$ = $\frac{5mV^2}{3}$ = 55.55%

8. COM of uniform solid cone of height h is at height $\frac{h}{4}$ from base, therefore from vertex its $\frac{3h}{4}$

9. **Case-I**

JBC

JAC

$2V_2 - V_1 = V$ $\Rightarrow$ $V_2 + V_1 = V$ $\Rightarrow$ $3V_2 = 2V$ $\Rightarrow$ $V_2 = \frac{2V}{3}$

$V_1 = \frac{V}{3}$ $\Rightarrow$ $P_d = \frac{\frac{1}{2}mV^2 - \frac{1}{2}mV_1^2}{\frac{1}{2}mV^2} = \frac{1-\frac{1}{9}}{1} = \frac{8}{9}$ = 0.89

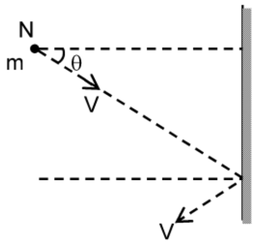
Case-II

JBC

JAC 

$12V_2 - V_1 = V \Rightarrow V_2 + V_1 = V \Rightarrow 13V_2 = 2V \Rightarrow V_2 = \frac{2V}{13}$

$V_1 = V - \frac{2V}{13} = \frac{11V}{13} \Rightarrow P_c = \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}mv_1^2}{\frac{1}{2}mv^2} = \dfrac{1-\frac{121}{169}}{1} = \frac{48}{169} = 0.28$

10. 

$F_{ovg} = 2NmV\cos\theta$

Pressure $= \dfrac{2NmV\cos\theta}{A} = \dfrac{2(10^{23})(3.32\times10^{-27})\frac{1}{\sqrt{2}}\times10^3}{2\times10^{-4}} = 2.35 \times 10^3\ N/m^2$

11. By conservation of linear momentum

$mv_0 + 0 = mv_1 + mv_2$

$v_0 = v_1 + v_2$ ....(1)

$\frac{3}{2}\left[\frac{1}{2}mv_0^2\right] = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

$\frac{3}{2}v_0^2 = v_1^2 + v_2^2$ ....(2) Solving equation (1) and (2)

$v_1 = \frac{v_0}{2}(1+\sqrt{2}) \Rightarrow v_2 = \frac{v_0}{2}(1-\sqrt{2})$

$\Rightarrow \vec{v}_{rel} = \vec{v}_1 - \vec{v}_2 \Rightarrow \frac{v_0}{2}\left[1+\sqrt{2}-1+\sqrt{2}\right] = \frac{v_0}{2}\times2\sqrt{2} = \sqrt{2}v_0$

12. $mv = (2m + M)v'$

$v' = \frac{mv}{2m+M}$

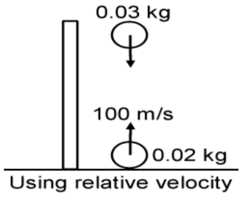
Initial energy Final energy

$\frac{1}{2}mv^2$ $\frac{1}{2}(2m+M)\left(\frac{mv}{2m+M}\right)^2$

$\frac{1}{2}mv^2$ $\frac{1}{2}\left(\frac{m^2v^2}{2m+M}\right)$

$\dfrac{\frac{1}{2}mv^2}{\frac{1}{2}\left(\frac{m^2v^2}{2m+M}\right)} = \dfrac{1}{\frac{1}{6}} = \frac{2m+M}{m} = 6$

$4m = M \Rightarrow \left[\frac{M}{m} = 4\right]$



13.

$a_{rel} = 0$

$v_{rel} = 100$

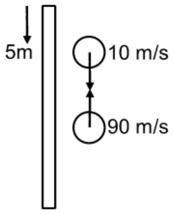
$100 = v_{rel} \times t$

$t = \frac{100}{100} = 1$ sec

$v_{bullet} = 100 - 1 \times 10 = 90$ m/s

$v_{particle} = 10 \times 1 = 10$ m/s

$S = 100 \times 1 - 1/2 \times 10 \times 1 = 95$ m

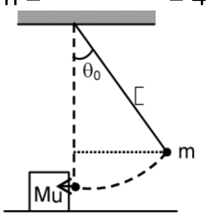


$P_i = P_f = 90 \times 0.02 - 10 \times 0.03 = 0.05V$

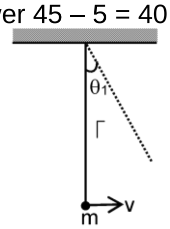
$V = 30$ m/s

$h = \frac{v^2}{2g} = \frac{900}{20} = 45$ m. So from top of tower 45 – 5 = 40 m

14.



Just before collision



Just after collision

$u = \sqrt{2g\ell(1-\cos\theta_0)}$ ....(i)

v = velocity of ball after collision

$v = \left(\frac{m-M}{m+M}\right)u$ Since ball rises up to angle θ_1

$v = \sqrt{2g\ell(1-\cos\theta_1)} = \left(\frac{m-M}{m+M}\right)u$ ....(ii) from (i) & (ii)

$\frac{m-M}{m+M} = \sqrt{\frac{1-\cos\theta_1}{1-\cos\theta_0}} = \frac{\sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right)} \Rightarrow \frac{M}{m} = \frac{\theta_0 - \theta_1}{\theta_0 + \theta_1} \Rightarrow M = \left(\frac{\theta_0 - \theta_1}{\theta_0 + \theta_1}\right)m$

15. $\mathbf{x_{cm}} = \frac{2ML + 2ML + \frac{5}{2}ML}{4M}\hat{x} \Rightarrow \mathbf{x_{cm}} = \frac{13}{8}L\hat{x}$

$y_{cm} = \frac{2ML + \frac{ML}{2} + M \times 0}{4M}\hat{y} \Rightarrow y_{cm} = \frac{5}{8}L\hat{y} \Rightarrow \vec{r}_{cm} = \frac{13}{8}L\hat{x} + \frac{5}{8}L\hat{y}$