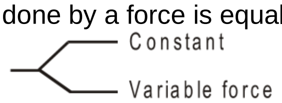
WORK, POWER AND ENERGY

1. WORK

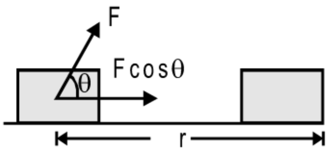
Whenever a force acting on a body displaces it, work is said to be done by the force.
Work done by a force is equal to scalar product of force applied and displacement of the body.

Force 

1.1 Work done by a constant force :

If the direction and magnitude of a force applying on a body is constant, the force is said to be constant.
Work done by a constant force,
W = Force × component of displacement along force = displacement × component of force along displacement.

If a $\vec{F}$ force is acting on a body at an angle θ to the horizontal and the displacement $\vec{r}$ is along the horizontal, the work done will, be $W = (F\cos\theta)\, r$
$= F\,(r\cos\theta)$



In vector from, $W = \vec{F}.\vec{r}$

If $\vec{F} = \hat{i}F_x + \hat{j}F_y + \hat{k}F_z$ and $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$, the work done will be, $W = F_x \,.\, x + F_y \,.\, y + F_z \,.\, z$

Note : The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

1.2 Work done by multiple forces:

If several forces act on a particle, then we can replace $\vec{F}$ in equation $W = \vec{F}.\vec{S}$ by the net force where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

$\therefore$ $W = [\Sigma\vec{F}]\,.\,\vec{S}$ ...(i)

This gives the work done by the net force during a displacement of the particle.
We can rewrite equation (i) as :

$$W = \vec{F}_1 . \vec{S} + \vec{F}_2 . \vec{S} + \vec{F}_3 . \vec{S} + ...$$

or $W = W_1 + W_2 + W_3 + ...$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

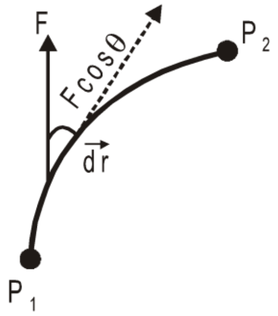
1.3 Work done by a variable force

If the force applying on a body is changing in its direction or magnitude or both, the force is said to be variable, suppose a constant force causes displacement in a body from position P_1 to position P_2. To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. It $d\vec{r}$ be small displacement of body and $\vec{F}$ be the force applying on the body, the work done by force is

$dW = \vec{F}.d\vec{r}$ ......

(i) The total work done in displacing body from P_1 to P_2 is given

by, $\int dW = \int_{P_1}^{P_2} \vec{F}.d\vec{r}$ or $W = \int_{P_1}^{P_2} \vec{F}.d\vec{r}$



If $\vec{r}_1$ and $\vec{r}_2$ be the position vectors of the points P_1 and P_2 respectively, the total work done will be -

W = $\int_{r_1}^{r_2} \vec{F}.d\vec{r}$

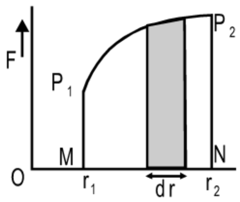
Note : When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

1.4 Calculation of work done from force displacement graph :

Suppose a body, whose initial position is r_1, is acted upon by a variable force $\vec{F}$ and consequently the body acquires its final position r_2. From position r to r + dr or for small displacement dr, the work done will be Fdr whose value will the area of the shaded strip of width dr.

The work done on the body in displacing it from position r_1 to r_2 will be equal to the sum of areas of all the such strips

Thus, total work done, W = $\sum_{r_1}^{r_2} dW$



= $\sum_{r_1}^{r_2} F.dr$

= Area of P_1P_2NM

The area between the graph between force and displacement axis is equal to the work done.

Note : To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F cos θ and r.

(i) Work is a scalar quantity

(ii) The dimensions of work : $[ML^2T^{-2}]$

(iii) Unit of work : there are two types of unit of work

(a) Absolute unit : Joule (in M.K.S), Erg (in C.G.S.) (Note : 10^7 erg = 1 joule)

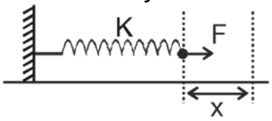
(b) Gravitational unit : Kilogram - metre (in M.K.S), Gram-cm (in C.G.S)

(Note : 1 kilogram metre = 9.8 joule = 10^5 gram cm)

1.5 Work done by spring force

Solved Examples

Example 1. Initially spring is relaxed. A person starts pulling the spring by applying a variable force F. Find out the work done by F to stretch it slowly to a distance by x.



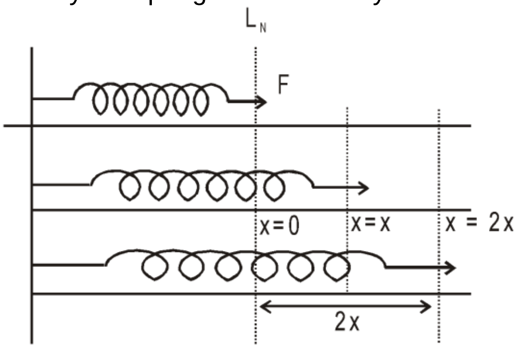
Solution : $\int dW = \int F \bullet ds = \int_0^x Kx\,dx \Rightarrow W = \left(\frac{Kx^2}{2}\right)_0^x = \frac{Kx^2}{2}$

Example 2. In the above example
(i) Where has the work gone ?
(ii) Work done by spring on wall is zero. Why?
(iii) Work done by spring force on man is _______ .

Solution : (i) It is stored in the form of potential energy in spring.
(ii) Zero, as displacement is zero.
(iii) $-\frac{1}{2}Kx^2$

Example 3. Find out work done by applied force to slowly extend the spring from x to 2x.

Solution : Initially the spring is extended by x



$W = \vec{F} \cdot \vec{ds}$

$W = \int_x^{2x} Kx.dx \Rightarrow W = \left[\frac{Kx^2}{2}\right]_x^{2x} = \frac{3}{2}Kx^2$

It can also found by difference of PE.

i.e. $U_f = \frac{1}{2}K(2x)^2 = 2kx^2 \Rightarrow U_i = \frac{1}{2}kx^2 \Rightarrow U_f - U_i = \frac{3}{2}kx^2$

1.6 Work done by internal force

Sum of internal forces is zero. But it is not necessary that work done by internal force is zero. There must be some deformation or reformation between the system to do internal work. In case of a rigid body work done by internal force is zero.

1.7 Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero

(a) Positive work : As $W = \vec{F}.\vec{r} = F\,r\cos\theta$

∴ When θ is acute (<90º) cos θ is positive. Hence work done is positive.

(i) When a body falls freely under the action of gravity θ = 0º, cos θ = + 1, therefore work done by gravity on a body, falling freely is positive.

(b) **Negative work :** When θ is obtuse (>90º), cos θ is negative. Hence work done is negative

(i) When a body is through up, its motion is opposed by gravity. The angle θ between the gravitational force $\vec{F}$ and displacement $\vec{r}$ is 180º. As cos θ = – 1, therefore, work done by gravity is negative.

(ii) When a body is moved over a rough horizontal surface, the motion is opposed by the force of friction. Hence work done by frictional force in negative. ***Note that work done by the applied force is not negative***

(iii) When a positive charge is moved closer to another positive charge, work done by electrostatic force of repulsion between the charges is negative.

(c) **Zero work :** When force $\vec{F}$ or the displacement $\vec{r}$ or both are zero, work done W, will be zero. Again when angle θ between $\vec{F}$ and $\vec{r}$ is 90º, the work done will be zero.

(i) When we fail to move a heavy stone, however hard we may try, work done by us is zero $\vec{r} = 0$

(ii) When a collie carrying some load on his head moves on horizontal platform, θ = 90º. Therefore, workdone
by the collie is zero. This is because θ = 90º

(iii) Tension in the string of simple pendulum is always perpendicular to displacement of the bob. Therefore,
work done by tension is always zero.

Note : Another way of expressing negative or positive work is that when energy is transferred to the object work done is positive and when energy is transferred from object the work done is negative and hence the
work which is a transfer of energy has same dimensions as energy.

Solved Examples

Example 4 A position dependent force $\vec{F} = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. The work done in joule will be

Solution :

$$W = \int_{x_1}^{x_2} F dx = \int_0^5 (7 - 2x + 3x^2)dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5 = 135J$$

Example 5. A uniform chain of mass M and length L is lying on a frictionless table in such a way that its 1/3 part is hanging vertically down. The work done in pulling the chain up the table is

Solution : If length x of the chain is pulled up on the table, then the length of hanging part of the chain would be $\left(\frac{L}{3} - x\right)$ and its weight would be $\frac{M}{L}\left(\frac{L}{3} - x\right)g.$ If it is pulled up further by a distance dx, the work done in pulling up.

$$= \frac{M}{L}\left(\frac{L}{3} - x\right)g\,dx \qquad \therefore \qquad w = \int_0^{L/3} \frac{M}{L}\left(\frac{L}{3} - x\right)g\,dx = \frac{MgL}{18}$$

Example 6. The work done in pulling a body of mass 5 kg along an inclined plane (angle 60º) with coefficient of friction 0.2 through 2m, will be

Solution : The minimum force with a body is to be pulled up along the inclined plane is mg (sinθ + μcosθ)

Work done, $W = \vec{F}.\vec{d}$

= Fd cos θº

= mg (sinθ + μ cosθ) × d

= 5 × 9.8 (sin 60º + 0.2 cos 60º) × 2

= 98.08 J

2. ENERGY

The energy of a body is defined as the capacity of doing work.

There are various form of energy

(i) mechanical energy	(ii) chemical energy	(iii) electrical energy	(iv) magnetic energy
(v) nuclear energy	(vi) sound energy	(vii) light energy etc	

Energy of system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another **Example**

Electric energy	$\xrightarrow{\text{Motor}}$	Mechanical energy
Mechanical energy	$\xrightarrow{\text{Generator}}$	Electrical energy
Light energy	$\xrightarrow{\text{Photocell}}$	Electrical energy
Electrical energy	$\xrightarrow{\text{Heater}}$	Heat energy
Electrical energy	$\xrightarrow{\text{Radio}}$	Sound energy
Nuclear energy	$\xrightarrow{\text{Nuclear Reactor}}$	Electrical energy
Chemical energy	$\xrightarrow{\text{Cell}}$	Electrical energy
Electrical energy	$\xrightarrow{\text{Secondary Cell}}$	Chemical energy
Heat energy	$\xrightarrow{\text{Incendence nt lamp}}$	Light

Energy is a scalar quantity
Unit : Its unit is same as that of work or torque.
In MKS : Joule, watt sec
In CGS : Erg
Note : 1 eV = 1.6×10^{-19} joule
1 KWh = 36×10^{5} joule
10^{7} erg = 1 joule
Dimension $[M^1L^2T^{-2}]$
According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other
Energy equivalent of mass m is, $E = mc^2$
Where, m : mass of the particle [in Kg]
c : velocity of light
E : equivalent energy corresponding to mass m.
In mechanis we are concerned with mechanical energy only which is of two type
(a) kinetic energy (ii) potential energy

2.1 Kinetic energy

The energy possessed by a body by virtue of its motion is called kinetic energy
If a body of mass m is moving with velocity v, its kinetic energy

KE = $\frac{1}{2}mv^2$, for translatory motion

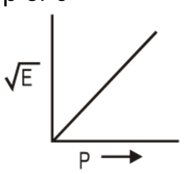
KE = $\frac{1}{2}I\omega^2$, for rotational motion

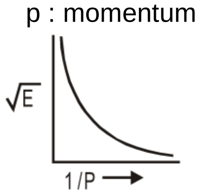
Kinetic energy is always positive

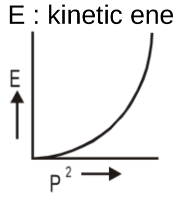
If linear momentum of body is p, KE = $\frac{p^2}{2m} = \frac{1}{2}mv^2$ - for translatory motion

If angular momentum of body is J, KE = $\frac{J^2}{2I} = \frac{1}{2}I\omega^2$ - for rotational motion

p or J $\alpha \sqrt{E}$ p : momentum E : kinetic energy







The kinetic energy of a moving body is measured by the amount of work which has been done in bringing the body from the rest position to its present moving position or
The kinetic energy of a moving body is measured by the amount of work which the body can do against the external forces before it comes to rest.
If a body performs translatory and rotational motion simultaneously, its total kinetic energy = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

3. WORK-ENERGY THEOREM

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where, W_C is the work done by all the conservative forces.
W_{NC} is the work done by all non-conservative forces.
W_{PS} is the work done by all psuedo forces.

Modified Form of Work-Energy Theorem :

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

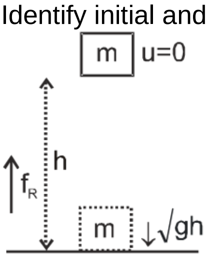
So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U \qquad W_{NC} + W_{PS} = \Delta E$$

Solved Examples

Example 7. A body of mass m when released from rest from a height h, hits the ground with speed $\sqrt{gh}$. Find work done by resistive force.

Solution : Identify initial and final state and identify all forces.

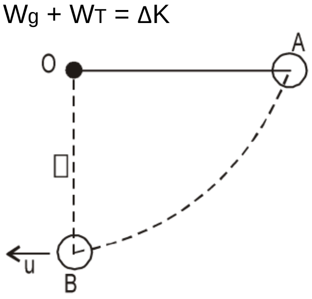


$W_g + W_{air\ res.} + W_{int\ force} = \Delta K$

$$mgh + W_{air\ res} + 0 = \frac{1}{2}m\left(\sqrt{gh}\right)^2 - 0 \quad \Rightarrow \quad W_{air\ res.} = -\frac{mgh}{2}$$

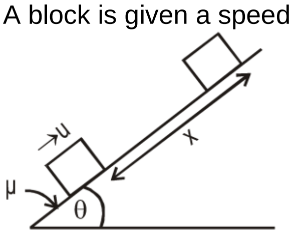
Example 8. The bob of a simple pendulum of length l is released when the string is horizontal. Find its speed at the bottom.

Solution : $W_g + W_T = \Delta K$



$$mg\ell + 0 = \frac{1}{2}mu^2 - 0 \quad \Rightarrow \quad u = \sqrt{2g\ell}$$

Example 9. A block is given a speed u up the inclined plane as shown.



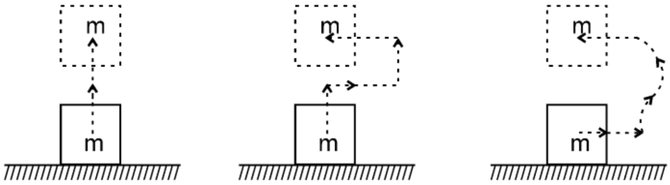
Using work energy theorem find out x when the block stops moving.

Solution : $W_g + W_f + W_N = \Delta K$

$$-mg\,x\sin\theta - \mu\,mgx\cos\theta + 0 = 0 - \frac{1}{2}mu^2 \quad \Rightarrow \quad x = \frac{u^2}{2g(\sin\theta + \mu\cos\theta)}$$

4. CONSERVATIVE FORCES

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.



Consider a body of mass m being raised to a height h vertically upwards as show in above figure. The work done is *mgh*. Suppose we take the body along the path as in (b). The work done during horizontal motion is zero. Adding up the works done in the two vertical parts of the paths, we get the result *mgh* once again. Any arbitrary path like the one shown in (c) can be broken into elementary horizontal and vertical portions. Work done along the horizontal parts is zero. The work done along the vertical parts add up to *mgh.* Thus we conclude that the work done in raising a body against gravity is independent of the path taken. It only depends upon the initial and final positions of the body. We conclude from this discussion that the force of gravity is a conservative force.

Examples of Conservative forces :

(i) Gravitational force, not only due to the Earth but in its general form as given by the universal law of gravitation, is a conservative force.

(ii) Elastic force in a stretched or compressed spring is a conservative force.

(iii) Electrostatic force between two electric charges is a conservative force.

(iv) Magnetic force between two magnetic poles is a conservative forces.

In fact, all fundamental forces of nature are conservative in nature.

Forces acting along the line joining the centres of two bodies are called central forces. Gravitational force and Electrostatic forces are two important examples of central forces. Central forces are conservative forces.

PROPERTIES OF CONSERVATIVE FORCES

(i) Work done by or against a conservative force depends only on the initial and final positions of the body.

(ii) Work done by or against a conservative force does no depend upon the nature of the path between initial and final positions of the body.

If the work done a by a force in moving a body from an initial location to a final location is independent of the path taken between the two points, then the force is conservative.

(iii) Work done by or against a conservative force in a round trip is zero. (w = 0)

If a body moves under the action of a force that does no total work during any round trip, then the force is conservative; otherwise it is non-conservative.

The concept of potential energy exists only in the case of conservative forces.

(iv) The work done by a conservative force is completely recoverable.

Complete recoverability is an important aspect of the work of a conservative force.

NON-CONSERVATIVE FORCES

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

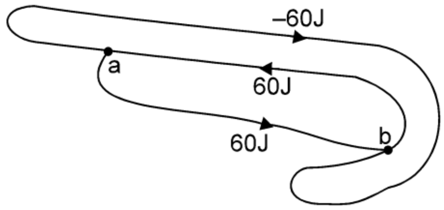
The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does no depend only on the initial and final positions. Note that the work done by frictional force in a round trip is not zero.

The velocity-dependent forces such as air resistance, viscous force etc., are non conservative forces.

S.No.	Conservative forces	Non-Conservative forces
1	Work done does not depend upon path	Work done depends on path.
2	Work done in round trip is zero.	Work done in a round trip is not zero.
3	Central in nature.	Forces are velocity-dependent and retarding in nature.
4	When only a conservative force acts within a systrem, the kinetic enrgy and potential energy can change. However their sum, the mechanical energy of the system, does not change.	Work done against a non-conservative force may be dissipated as heat energy.
5	Work done is completely recoverable.	Work done is not completely recoverable.

Solved Examples

Example 10. The figure shows three paths connecting points a and b. A single force F does the indicated work on a particle moving along each path in the indicated direction. On the basis of this information, is force F conservative?



Ans. No

Explane: For a conservative force, the work done in a round trip should be zero.

Example 11. Find the work done by a force $\vec{F} = x\hat{i} + y\hat{j}$ acting on a particle to displace it from point A (0, 0) to B(2, 3).

Solution : $dW = \vec{F}.\vec{ds} = (x\hat{i} + y\hat{j}).(dx\hat{i} + dy\hat{j})$

$$W = \int_0^2 x\,dx + \int_0^3 y\,dy = \left[\frac{x^2}{2}\right]_0^2 + \left[\frac{y^2}{2}\right]_0^3 = \frac{13}{2} \text{ units}$$

True or False

Example 12. In case of a non conservative force work done along two different paths will always be different.

Ans. False

Example 13. In case of non conservative force work done along two different paths may be different.

Ans. True

Example 14. In case of non conservative force work done along all possible paths cannot be same.

Ans. True

5. POTENTIAL ENERGY

The energy which a body has by virtue of its position or configuration in a conservation force field
Potential energy is a relative quantity.
Potential energy is defined only for conservative force field.
Potential energy of a body at any position in a conservation force field is defined as the workdone by an external agent against the action of conservation force in order to shift it from reference point.
(PE = 0) to the present position or.
Potential energy of a body in a conservation force field is equal to the work done by the body in moving from its present position to reference position.
At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.

Relationship between conservative force field and potential energy (U) $\vec{F} = -\nabla U = -\text{grad (U)}$ $= -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$

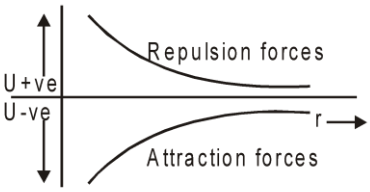
Solved Example

Example 15. $U = 3x^2$

Solution : $F_x = -\frac{dU}{dx}$ $\Rightarrow$ $\vec{F} = -6x\hat{i}$

If force varies only with one dimension then

$F = -\frac{dU}{dx}$ or $U = -\int_{x_1}^{x_2} F dx$



Potential energy may be positive or negative
(i) Potential energy is positive, if force field is repulsive in nature
(ii) Potential energy is negative, if force field is attractive in nature
If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice-versa.
If r ↑, U ↓, force field is repulsive in nature.

6. POWER

(a) The time rate of doing work is called power

(b) Power = $\frac{dw}{dt} = \vec{F}.\frac{d\vec{x}}{dt}$

In translatory motion : $P = \vec{F}.\vec{v}$

In rotational motion : $P = \vec{\tau}.\vec{\omega}$
(c) It is scalar quantity
(d) Unit :
In MKS - J/sec, watt
In CGS - erg/sec, (Note : 1KW = 10^3 watt, 1 HP = 746 watt)
(e) Dimension : $[M^1L^2T^{-3}]$
Note : Power is the rate at which applied force transfers energy

(a) Power $\bar{P} = \frac{w}{\Delta t}$ where w work is done in Δt time

(b) Instantaneous power $P = \frac{dw}{dt}$, it's value may change with time.

Solved Examples

Example 16. A pump can lift 9000 kg coal per hour from a mine of depth 120 m. Assuming its efficiency to be 75% its power will be (in watts) -

Solution : Power $= \frac{\text{work}}{\text{time}}$

Efficiency of pump $= \frac{\text{ouput power}}{\text{input power}}$; $\frac{3}{4} = \frac{\text{output power}}{P}$

$\therefore$ Output power $= \frac{3P}{4}$ $\quad \therefore \frac{3}{4}P = \frac{mgh}{t}$

$P = \frac{4}{3}\frac{mgh}{t} = \frac{4}{3} \times \frac{9000 \times 9.8 \times 120}{3600} = 3920$ W

Example 17. A person of mass 60 kg is capable of taking a 15 kg massive objets to a height of 10 m in 3 miutes. The efficiency of person is -

Solution : % Efficiency $\eta = \frac{\text{work output}}{\text{work input}} \times 100 = \frac{mgh}{(m+M)gh} \times 100$

$= \left(\frac{m}{m+M}\right) \times 100 = \left(\frac{15}{15+60}\right) \times 100 = 20\%$

Example 18. An automobile of mass m accelerates from rest. If the engine supplies constant power p, the velocity at time t is given by

Solution : Given that power = Fv = p = constant

or $m\frac{dv}{dt}v = p$ [as F = ma = $\frac{mdv}{dt}$ or $\int v\,dv = \int \frac{p}{m}dt \Rightarrow \frac{v^2}{2} = \frac{p}{m}t + C_1$

Now as initially, the body is at rest i.e. v = 0 at t = 0 so, $C_1 = 0$

$\therefore v = \sqrt{\frac{2pt}{m}}$

Example 19. In the above problem, the position (s) at time (t) is given by

Solution : By definition $v = \frac{ds}{dt}$ or $\frac{ds}{dt} = \left(\frac{2pt}{m}\right)^{1/2}$ [From (1)]

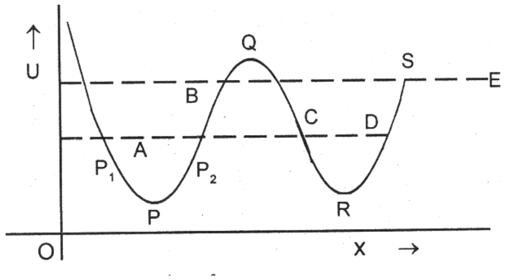
$\Rightarrow \int ds = \int \left(\frac{2pt}{m}\right)^{1/2} dt \Rightarrow s\left(\frac{2p}{m}\right)^{1/2} \frac{2}{3} t^{3/2} + C_2$

Now as t = 0, s = 0, so $C_2 = 0 \Rightarrow s = \left(\frac{8p}{9m}\right)^{1/2} t^{3/2}$

7. POTENTIAL ENERGY CURVE

A graph plotted between the PE of a particle and its displacement from the centre of force field is called PE curve

Using graph, we can predict the rate of motion of a particle at various positions.



Force on the particle is $F(x) = -\frac{dU}{dx}$

Case : I On increasing x, if U increase, force is in (–)ve x-direction i.e. attraction force.
Case : II On increasing x, if U decreases, force is in (+)ve x-direction i.e. repulsion force.

Different positions of a particle

Position of equilibrium

If net force acting on a body is zero, it is said to be in equilibrium for equilibrium

$\frac{dU}{dx} = 0$ Points P, Q, R and S are the states of equilibrium positions.

Types of equilibrium :

Stable equilibrium - When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in state equilibriums position.

Necessary conditions - $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = +ve$

Unstable equilibrium : When a particle is displaced slightly from a position and force acting on it tries to displace the practice further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition : $\frac{dU}{dx} = 0$ potential energy is max i.e. $= \frac{d^2U}{dx^2} = -ve$

Neutral equilibrium : In the neutral equilibrium potential energy is constant when a particle is displaced from its position it does not experiences any force to acting on it and continues to be in equilibrium in the displaced position, it is said to be in neutral equilibrium.

8. MECHANICAL ENERGY :

Definition: Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e., $E = K + U$

Important Points for M.E.:

1. It is a scalar quantity having dimensions $[ML^2T^{-2}]$ and SI units joule.

2. It depends on frame of reference.

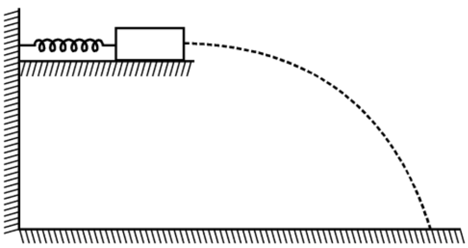
3. A body can have mechanical energy without having either kinetic energy or potential energy. However, if both kinetic and potential energies are zero, mechanical energy will be zero. The converse may or may not be true, i.e., if E = 0 either both PE and KE are zero or PE may be negative and KE may be positive such that KE + PE = 0.

4. As mechanical energy E = K + U, i.e., E - U = K. Now as K is always positive, $E - U \geq 0$,i.e., for existence of a particle in the field, $E \geq U$.

5. As mechanical energy E = K + U and K is always positive, so, if 'U' is positive 'E' will be positive. However, if potential energy U is negative, 'E' will be positive if K > |U| and E will be negative if K < |U|
i.e., mechanical energy of a body or system can be negative, and negative mechanical energy means that potential energy is negative and in magnitude it is more than kinetic energy. Such a state is called bound state, e.g., electron in an atom or a satellite moving around a planet are in bound state.

Solved Examples

Example 20. A small block of mass 100 g is pressed against a horizontal spring fixed at one end to compress the spring through 5.0 cm (figure). The spring constant is 100 N/m. When released, the block moves horizontally till it leaves the spring. Where will it hit the ground 2 m below the spring?



Solution : When block released, the block moves horizontally with speed V till it leaves the spring.

By energy conservation $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$V^2 = \frac{kx^2}{m} \Rightarrow V = \sqrt{\frac{kx^2}{m}}$

Time of flight $t = \sqrt{\frac{2H}{g}}$

So. horizontal distance travelled from the free end of the spring is = V × t

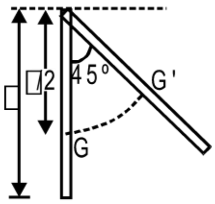
$= \sqrt{\frac{kx^2}{m}} \times \sqrt{\frac{2H}{g}} = \sqrt{\frac{100\times(0.05)^2}{0.1}} \times \sqrt{\frac{2\times 2}{10}} = 1\text{ m}$

So, At a horizontal distance of 1 m from the free end of the spring.

Example 21. A meter scale of mass m initially vertical is displaced at 45º keeping the upper end fixed, the change in PE will be-

Solution : Work = change in PE = Force × displacement

$dU = mg\frac{\ell}{2}(1-\cos\theta)$



$= mg \times \frac{1}{2}(1-\cos 45^\circ)$ $(\because \ell = 1\text{m})$ $= \frac{mg}{2}\left(1-\frac{1}{\sqrt{2}}\right)$

Example 22. If the speed of a car increases 4 times, the stopping distance for this will increase by -

Solution : Work = Change in KE

$\therefore FS = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$

$\frac{S'}{S} = \frac{v'^2}{v^2} \Rightarrow \frac{S'}{S} = 16 \Rightarrow S' = 16\,S$

Example 23. If the potential energy function for a particle is $U = a - \frac{b}{x} + \frac{c}{x^2}$ the force constant for oscillation will be.

Solution : $U = a - \frac{b}{x} + \frac{c}{x^2}$ ....(1)

$\therefore \frac{dU}{dx} = -\frac{b}{x^2} - \frac{2c}{x^3}$ ....(2)

and $\frac{d^2U}{dx^2} = \frac{1}{x^3}\left(-2b + \frac{6c}{x}\right)$ ....(3)

`

for equilibrium $\frac{dU}{dx} = 0$

$\therefore$ $x = \frac{2c}{b}$ $\therefore$ Substituting this value in (3) $\frac{d^2U}{dx^2} = \left(\frac{b}{2c}\right)^3\left[-2b + \frac{6c}{2c/b}\right] = \frac{b^4}{8c^3}$

as $\frac{d^2U}{dx^2} = K$ $\therefore$ $K = b^4/8c^3$

Miscellaneous Problem

Problem 1 : On passing through a wooden sheet a bullet looses 1/20 of initial velocity. The minimum number of sheets required to completely stop the bullet will be-

Solution : Use $v^2 = u^2 + 2as$

for a sheet of thickness s v = (19/20)u

$\left(\frac{19}{20}u\right) = u^2 + 2as$ $\Rightarrow$ $2as = (361/400)u^2 - u^2$ $a = -\left(\frac{39}{400}\right)\frac{u^2}{2s}$

suppose for n sheet v = 0 $\therefore 0^2 = u^2 + 2a(ns)$ $n = -\frac{u^2}{2as} = \frac{u^2}{2\left(\frac{39}{400}\right)\frac{u^2}{2s}\times s} \approx 11$

Problem 2 : The work done in taking out 2 lit of water using a bucket of mass 0.5 kg from a well of depth 6m will be-

Solution : W = mgh

$= (m_{bucket} + m_{water})gh$ [2 Lit water = 2 kg water]

$= (0.5 + 2.00) \times 9.8 \times 6$ $= 15 \times 9.8 = 147$ J

Problem 3 : A body has velocity 200 m/s and its kinetic energy is 200 J. The mass of the body would be

Solution : $\frac{1}{2}mv^2 = E$ or $m = \left(\frac{2E}{v^2}\right) = \frac{2\times 200}{(200)^2} = \frac{4\times 10^2}{4\times 10^4} = \frac{1}{100}$ $\therefore$ m = 0.01 kg

Problem 4 : A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = 1/2t^2$ where x is in meter and t in sec. The work done by the force in first two seconds.

Solution : Work done = change in kinetic energy

or $\frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}m\left(\frac{2t}{2}\right)^2 = \frac{1}{2}\times 8\times\left[\frac{2\times 2}{2}\right]^2 = 16\text{Joules}$

Problem 5 : A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then body would reach upto a height of

Solution : $\frac{1}{4}mgh = mgh'$ $\therefore$ $h' = \frac{h}{4} = \frac{1}{4}\times 20 = 5\text{cm}$

Problem 6 : Potential energy function describing the interaction between two atoms of a diatomic molecule is

$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

In stable equilibrium, the distance between them would be

Solution : In stable equilibrium potential energy is minimum. For minimum value of U(x)

$\frac{d}{dx}[U(x)] = 0$

or $\frac{d}{dx}\left(\frac{a}{x^{12}} - \frac{b}{x^6}\right) = 0$ or $\frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$ or $\frac{6}{x^{13}}(-2a + bx^6) = 0$

or $bx^6 - 2a = 0$ $\therefore$ $x = \left(\frac{2a}{b}\right)^{1/6}$

Problem 7 : Two electrons are at a distance of 1×10^{-12}m from each other. Potential energy (in eV) of this system would be

Solution : Potential energy of the system

$U = \frac{Kq_1q_2}{r} = \frac{9\times 10^9\times 1.6\times 10^{-19}\times 1.6\times 10^{-19}}{1\times 10^{-12}}$ $= 23.04 \times 10^{-17}$ Joule

$= \frac{23.04\times 10^{-17}}{1.6\times 10^{-19}}\text{eV} = 1.44\times 10^3\text{ eV}$

Problem 8 : Potential energy function U(r) corresponding to the central force $F = \frac{K}{r^2}$ would be

Solution : Central force is conservative. Therefore

$\vec{F}(r) = -\frac{dU}{dr}\hat{r}$ or $dU = -\vec{F}(r).\vec{dr} = -F(r)dr$

$\therefore$ $U = \int dU = -\int F(r)dr = -\int \frac{K}{2}dr = -K\int \frac{1}{r^2}dr = Kr^{-1} + C$

If at r = ∞, U = 0, then C = 0 ⇒ $U = Kr^{-1} = \frac{K}{r}$

Problem 9 : The stopping distance for a vehicle of mass m moving with speed v along level road, will be (μ is the coefficient of friction between tyres and the road)

Solution : When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = 1/2\ mv^2$ and if S is the stopping distance, work done by the friction

W = FS cos θ = m MgS cos 180º = – m MgS

So by Work-Energy theorem, $W = DK = K_f - k_i$

⇒ $-\mu MgS = 0 - 1/2\ Mv^2$ ⇒ $S = \frac{v^2}{2\mu g}$

Problem 10 : A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. The total energy of the particle is

Solution : As the particle is moving in a circle, so

$\frac{mv^2}{r} = \frac{k}{r^2}$ Now K.E = $\frac{1}{2}$ mv^2 = $\frac{k}{2r}$

Now as $F = -\frac{dU}{dr}$ P.E, $U = -\int_{\infty}^{r} F\,dr = \int_{\infty}^{r} +\left(\frac{k}{r^2}\right)dr = -\frac{k}{r}$

So total energy = U + K.E. $= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$

Negative energy means that particle is in bound state.

Problem 11 : The work done by a person in carrying a box of mass 10 kg. through a vertical height of 10 m is 4900J. The mass of the person is

Solution : Let the mass of the person is m .

Work done, W = P.E at height h above the earth surface.

= (M + m) gh or 4900 = (M + 10) 9.8 x 10 or M = 40 kg

Problem 12 : A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. The work done in keeping it vertical with one of its ends touching the ground, will be - .

Solution : As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length = l/2

Work done w = mgh = mg $\frac{\ell}{2}$ = 20 x 9.8 x $\frac{4}{2}$ = 392 J

Problem 13 : A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save

Solution : In first case, $W_1 = \frac{1}{2}\ m(v_1)^2 + mgh$

$= \frac{1}{2}\ m(12)^2 + m \times 10 \times 12$

= 72 m + 120 m

and in second case, $W_2 = mgh$

= 120 m

The percentage of energy saved $= \frac{192m - 120m}{192m} \times 100 = 38\%$

******

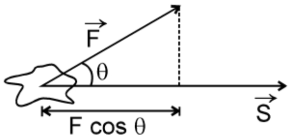
KEY CONCEPT

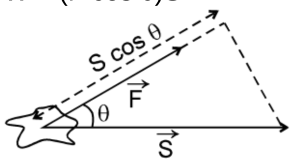
☞ **Work done by Constant Force :**

W = $\vec{F}.\vec{S}$

Since work is the dot product of two vectors therefore it is a scalar quantity.

W = FS cos θ or W = (F cos θ)S





☞ **Work Done by Multiple Forces :**

If several forces act on a particle, then we can replace $\vec{F}$ in equation W = $\vec{F}\,.\,\vec{S}$ by the net force $\Sigma\vec{F}$ where

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

$$W = [\Sigma\vec{F}]\,.\,\vec{S} \qquad ...(i)$$

$$W = \vec{F}_1\,.\,\vec{S} + \vec{F}_2\,.\,\vec{S} + \vec{F}_3\,.\,\vec{S} + ...$$

or $W = W_1 + W_2 + W_3 + ...$

So, the work done on the particle is the sum of the individual works done by all the forces acting on the particle.

☞ **Dimensions of Work :**

[Work] = [Force] [Distance] = $[MLT^{-2}]$ [L] = $[ML^2T^{-2}]$

Work has one dimension in mass, two dimensions in length and '–2' dimensions in time,

☞ **Work in Terms of Rectangular Components :**

In terms of rectangular components, $\vec{F}$ and $\vec{S}$ may be written as :

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ and $\vec{S} = S_x\hat{i} + S_y\hat{j} + S_z\hat{k}$

$\vec{F}.\vec{S} = F_xS_x + F_y S_y + F_zS_z$

☞ **Work Done by a Variable Force :**

When the magnitude and direction of a force vary in three dimensions, it can be expressed as a function of the position. For a variable force work is calculated for infinitely small displacement and for this displacement force is assumed to be constant

dW = $\vec{F}.\vec{ds}$

The total work done will be sum of infinitely small work

$$W_{A\to B} = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)\vec{ds}$$

In terms of rectangular components,

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k},\quad \vec{ds} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$W_{A\to B} = \int_{x_A}^{x_B} F_X dx + \int_{x_A}^{x_B} F_Y dy + \int_{x_A}^{x_B} F_Z dz$$

☞ **Relation Between Momentum and Kinetic Energy :**

Important Points for K.E.

1. As mass m and v^2 ($\vec{v}.\vec{v}$) are always positive, kinetic energy is always positive scalar i.e, kinetic energy can never be negative.

2. The kinetic energy depends on the frame of reference,

$K = \frac{p^2}{2m}$ and $P = \sqrt{2\,m\,K}$; P = linear momentum

Potential Energy :

In case of conservative force

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$$

i.e., $U_2 - U_1 = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = -W$, where W is work done by conservative forces

Whenever and wherever possible, we take the reference point at ∞ and assume potential energy to be zero there, i.e., If we take $r_1 = \infty$ and $U_1 = 0$ then

$$U = -\int_{\infty}^{r} \vec{F}\cdot d\vec{r} = -W$$

Types of Potential Energy :

(a) <u>Elastic Potential Energy</u>: $U = \frac{1}{2}k y^2$

where k is force constant and 'y' is the stretch or compression. Elastic potential energy is always positive.

(b) <u>Electric Potential Energy</u>: It is the energy associated with charged particles that interact via electric force. For two point charges q_1 and q_2 separated by a distance 'r',

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

As charge can be positive or negative, therefore electric potential energy can also be positive or negative.

(c) <u>Gravitational Potential Energy</u>: It is due to gravitational force. For two particles of masses m_1 and m_2 separated by a distance 'r', it is given by:

$$U = -G\frac{m_1m_2}{r}$$

which for a body of mass 'm' at height 'h' relative to surface of the earth reduces to U = mgh

Gravitational potential energy can be positive or negative.

Mechanical Energy :

Definition: Mechanical energy 'E' of an object or a system is defined as the sum of kinetic energy 'K' and potential energy 'U', i.e.,

$$E = K + U$$

Conservative Forces :

A force is said to be conservative if work done by or against the force in moving a body depends only on the initial and final positions of the body and not on the nature of path followed between the initial and final positions.

$$F = -\frac{\partial U}{\partial r}$$

Conservative Force & Potential Energy :

We know that $F = -\frac{\partial U}{\partial r}$

Types of Equilibrium :

(a) **Stable equilibrium :** When a particle is displaced slightly from a position and a force acting on it brings it back to the initial position, it is said to be in stable equilibrium position.

Necessary conditions :-

$\frac{dU}{dx} = 0$, and $= \frac{d^2U}{dx^2}$ +ve Potential energy is minimum.

(b) **Unstable Equilibrium :** When a particle is displaced slightly from a position and force acting on it tries to displace the particle further away from the equilibrium position, it is said to be in unstable equilibrium.

Condition :-

$\frac{dU}{dx} = 0$ potential energy is maximum i.e. $= \frac{d^2U}{dx^2} = -$ ve

(c) **Neutral equilibrium :** In the neutral equilibrium potential energy is constant. When a particle is displaced from its position it does not experience any force acting on it and continues to be in equilibrium in the displaced position. This is said to be neutral equilibrium.

☞ **Work-Energy Theorem :**

According to work-energy theorem, the work done by all the forces on a particle is equal to the change in its kinetic energy.

$$W_C + W_{NC} + W_{PS} = \Delta K$$

Where, W_C is the work done by all the conservative forces.

W_{NC} is the work done by all non-conservative forces.

W_{PS} is the work done by all psuedo forces.

☞ **Modified Form of Work-Energy Theorem :**

We know that conservative forces are associated with the concept of potential energy, that is

$$W_C = -\Delta U$$

So, Work-Energy theorem may be modified as

$$W_{NC} + W_{PS} = \Delta K + \Delta U$$

$$W_{NC} + W_{PS} = \Delta E$$

☞ **Power :**

Power is defined as the time rate of doing work.

The average power ($\overline{P}$ or p_{av}) delivered by an agent is given by

$$\overline{P} \text{ or } p_{av} = \frac{W}{t}$$

where W is the amount of work done in time t.

$$P = \frac{\vec{F} \cdot d\vec{S}}{dt} = \vec{F} \cdot \frac{d\vec{S}}{dt} = \vec{F} \cdot \vec{v}$$

By definition of dot product,

$$P = Fv \cos \theta$$

where θ is the smaller angle between $\vec{F}$ and . $\vec{v}$

This P is called as instantaneous power if dt is very small.