Fundamental of Mathematics - II

Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Modulus Function

A-1. Sum of the solutions of the equation $|x| + 2 = 3$ is
(1) 2 (2) 1 (3) 0 (4) None of these

A-2. The number of real roots of the equation $x^2 + 3|x| + 2 = 0$ is
(1) 0 (2) 2 (3) 3 (4) 4

A-3. If $3|3 - x| = 7$, then the product of all the possible values of x is
(1) 1/9 (2) 6 (3) 2/3 (4) 32/9

A-4. Sum of solutions of the equation $|x|^2 - |x| + 4 = 2x^2 - 3|x| + 1$ is
(1) 3 (2) 6 (3) 0 (4) None of these

A-5. For the equation $|x^2| + |x| - 6 = 0$, the roots are
(1) real and equal (2) real with sum 0 (3) real with sum – 1 (4) real with product 0

A-6. Solutions of equation $||x - 1| - 2| = 1$ are
(1) ±1, ±3 (2) ±2, ±1 (3) ±2, 0, 4 (4) ±2, 3, 1

A-7. Solution of $|4x + 3| + |3x - 4| = 12$ is
(1) $x = -\frac{7}{3}, \frac{3}{7}$ (2) $x = -\frac{5}{2}, \frac{2}{5}$ (3) $x = -\frac{11}{7}, \frac{13}{7}$ (4) $x = -\frac{13}{7}, \frac{11}{7}$

A-8. Number of real solutions of equation $|x - 3| + 2|x + 1| = 4$ is
(1) 0 (2) 1 (3) 2 (4) 3

A-9. Number of solutions of equation $|x| - 2x + 5 = 0$ is
(1) 1 (2) 0 (3) 2 (4) 3

A-10. If $\{x \in R : |x - 2| = x^2\}$, then values of x are
(1) – 1, 2 (2) 1, 2 (3) –1, – 2 (4) 1, – 2

A-11. The number of solutions of $|x + 2| = 2(3 - x)$ is
(1) 1 (2) 2 (3) 3 (4) 0

A-12. Number of solutions of the equation $x|x| = 4$ is
(1) 2 (2) 1 (3) 0 (4) None of these

A-13. Solution of the equation $|x^2 - 4x + 3| + x = 7$ is
(1) – 1,4 (2) 1,4 (3) – 1,–4 (4) 1,–4

A-14. The minimum value of $f(x) = |x - 1| + |x - 2| + |x - 3|$ is equal to
(1) 1 (2) 2 (3) 3 (4) 0

A-15. Solution of equation $||x - 1| - 2| = |x - 3|$ is
(1) $(-\infty, 1]$ (2) $[1, \infty)$ (3) $(-\infty, -1] \cup [1, \infty)$ (4) $\{1, -1\}$

Section (B) : Modulus Inequalities

B-1. Complete set of real 'x' satisfying $|x - 3| \geq 2$ is

(1) $(-\infty, -5] \cup [-1, \infty)$ (2) $(-\infty, -2] \cup [2, \infty)$ (3) $[1, 5]$ (4) $(-\infty, 1] \cup [5, \infty)$

B-2. Sum of integral solutions of inequality $||x - 2| - 3| \leq 0$ is

(1) 6 (2) –4 (3) 4 (4) –6

B-3. Number of integral solutions of inequality $|x + 3| > |2x - 1|$ is

(1) 3 (2) 4 (3) 5 (4) 2

B-4. The complete set of real ' x ' satisfying $||x - 1| - 1| \leq 1$ is

(1) [0, 2] (2) [− 1, 3] (3) [− 1, 1] (4) [1, 3]

B-5. Complete solution of inequality $||3x - 9| + 2| > 2$ is

(1) $(-\infty, \infty)$ (2) {3} (3) R – {3} (4) φ

B-6. Complete solution of inequality $\left|1+\frac{3}{x}\right| > 2$ is

(1) $(-1, 0) \cup (0, 3)$ (2) $(-\infty, -1) \cup (0, \infty)$ (3) $(-\infty, -1) \cup (3, \infty)$ (4) $(-\infty, -1)$

B-7. Number of integral solutions of inequality $|2x - 3| - |x| \leq 3$ is

(1) 6 (2) 5 (3) 4 (4) 7

Comprehension # 1 (B-8 to B-10)

Given an inequation $\left|1+\frac{3}{x}\right| > 2$ whose solution set is given by $(a, 0) \cup (0, b)$, then answer the following questions :

B-8. The value of $|a + b|$ is

(1) 1 (2) 2 (3) 3 (4) 4

B-9. If $x^3 - kx^2 + x + 2$ is divisible by $x - a$, then value of k is

(1) 0 (2) 1 (3) 3 (4) 4

B-10. If solution set for $(x + 1)^2 < (7x - 3)$ is (c, d), then (a + b + c + d) equals to

(1) 7 (2) 8 (3) 9 (4) 10

B-11. $|x-2| + |x+1| \geq 3$, then complets solution set of this inequation is :

(1) $[1, \infty)$ (2) $(-\infty, -2]$ (3) R (4) $[-2, 1]$

Section (C) : Irrational inequalities

C-1. The set of values of x satisfying inequality $\sqrt{6-x} > x - 1$ is

(1) $(-\infty, 1]$ (2) $\left(-\infty, \frac{1+\sqrt{21}}{2}\right]$ (3) $\left(-\infty, \frac{1-\sqrt{21}}{2}\right]$ (4) $[1, \infty)$

C-2. The set of values of x satisfying inequlity $x + 3 < \sqrt{2-x}$

(1) $\left(\frac{-7-\sqrt{21}}{2} - 3\right)$ (2) $\left(\frac{-7-\sqrt{21}}{2} - 3\right) \cup \left(\frac{-7+\sqrt{21}}{2}, 2\right]$

(3) $\left(\frac{-7+\sqrt{21}}{2},2\right]$ (4) $\left(\frac{-7-\sqrt{21}}{2}-3\right)\cap\left(\frac{-7+\sqrt{21}}{2},2\right]$

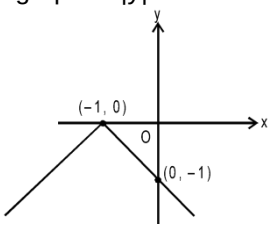
C-3. The set of values of x satisfying inequality $\sqrt{x^2-x-6} < 2x-3$ is
(1) $(-\infty, 1]$ (2) $[1, \infty)$ (3) $[2, \infty)$ (4) $[3, \infty)$

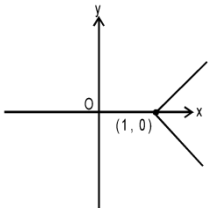
C-4. Complete set of values of x satisfying the inequality $x-3 < \sqrt{x^2+4x-5}$ is
(1) $(-\infty, -5] \cup [1, \infty)$ (2) $(-5, 3]$ (3) $[3, 5)$ (4) $(-5, 3)$

C-5. The set of values of x satisfying inequlity $4-x < \sqrt{2x-x^2}$
(1) $(-\infty, -5]$ (2) $(0, 2]$ (3) $[0, 2] \cup (4,\infty)$ (4) $x \in \varphi$

Section (D) : Transformation of curves

D-1. The graph of $|y| + x + 1 = 0$ is

(1)


(2)


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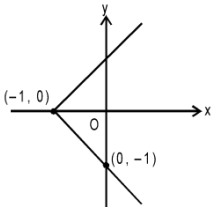

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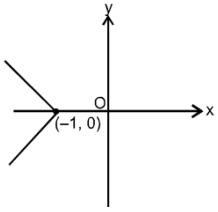
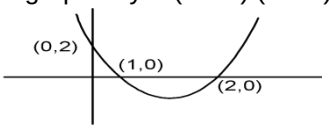
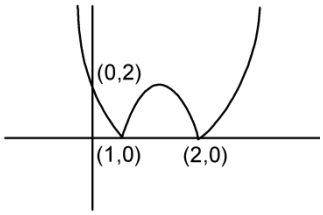
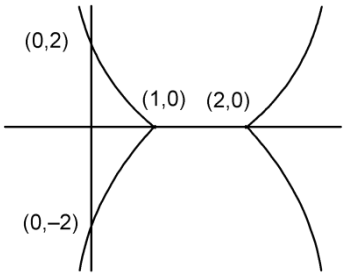


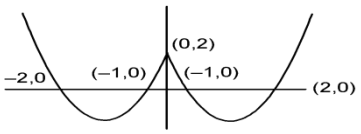
D-2. If graph of $y = (x-1)(x-2)$ is given by



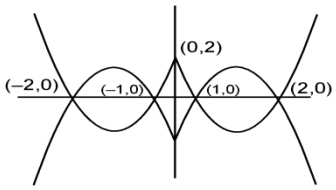
Then the graph of $|y| = |(|x|-1)(|x|-2)|$

(1)


(2)


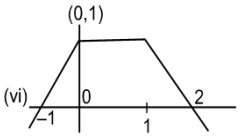


(3)

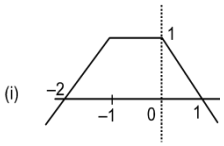


(4)

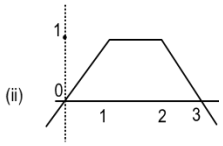
D-3. If graph of y = f(x) is



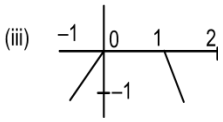
Then the graph of y = f (x + 1)



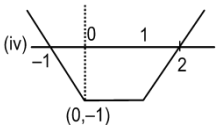
(1)



(2)

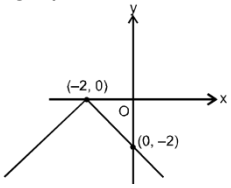


(3)

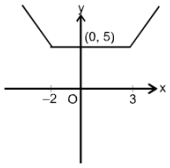


(4)

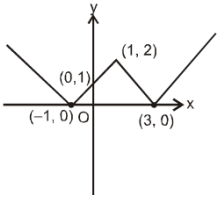
D-4. The graphs of of the functions $f_1(x) = -|x + 2|$, $f_2(x) = | | x - 1 | - 2|$, $f_3(x)=|x + 2| + |x - 3|$



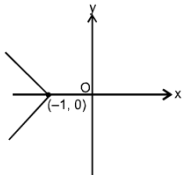
(A)



(B)



(C)

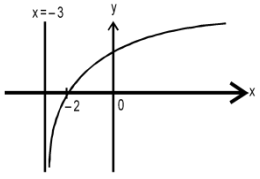


(D)

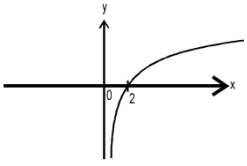
The correct order of graphs of functions $f_1(x)$, $f_2(x)$, $f_3(x)$ is

(1) BCD (2) ABC (3) ACB (4) ACD

D-5. The graphs of of the functions $f_1(x) = \ell n(x + 3)$, $f_2(x) = \cos\left(x - \frac{\pi}{4}\right)$, $f_3(x) = \sin \pi x$

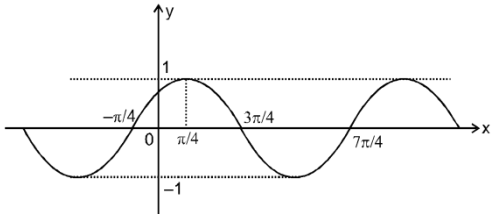


(A)

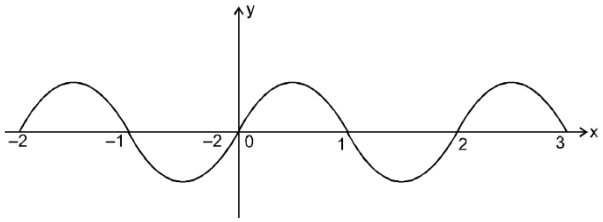


(B)

(C)



(D)



The correct order of graphs of functions $f_1(x)$, $f_2(x)$, $f_3(x)$ is

(1) BCD (2) ABC (3) ACB (4) ACD

Section (E) : Greatest Integer [·] Fractional part {·} and signum function

E-1. The value of [e] – [– π] where [.] denotes greatest integer function, is

(1) 5 (2) 6 (3) 7 (4) 8

E-2. The set of solution of inequality $– 5 \le [x + 1] < 2$, where [.] denotes greatest integer function is

(1) [–6, 1) (2) [–6, 2) (3) [–7, 1) (4) [–6, 3)

E-3. The set of solution of inequality $[x]^2 + 5[x] – 6 < 0$, where [.] denotes greatest integer function is

(1) [–2, 0) (2) [–5, 2) (3) [–5, 1) (4) [–6, 5)

E-4. The number of solutions of the equation $\{x\}^2 = – \{x\}$ is (where {.} denotes the fractional part function)

(1) 0 (2) 1 (3) 2 (4) infinite

E-5. The number of solutions of the equation $2\{x\}^2 – 5 \{x\} + 2 = 0$ is (where {.} denotes the fractional part function)

(1) 0 (2) 1 (3) 2 (4) infinite

E-6. The number of solution of the equation $\text{sgn}(x^2) = |x – 2|$ is

(1) 1 (2) 0 (3) 2 (4) 3

E-7. The complete set of values of x satisfying the equation sgn x = |1–x| is

(1) x = 2 (2) x = 0 (3) x = – 1 (4) $x \in \varphi$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. If $|x_2 – 2x – 8| + |x_2 + x – 2| = 3 | x + 2|$, then the set of all real values of x is

(1) [1, 4] ∪ {–2} (2) [1, 4] (3) [–2, 1] ∪ [4,∞) (4) (–∞, –2] ∪ [1, 4]

2. If $|x_3 – 9x_2 + 26x –24|$ is a prime number then number of possible integral value of x is

(1) 1 (2) 2 (3) 0 (4) 3

3. Number of integers satisfying the equation $|x^2 + 5x| + |x – x^2| = |6x|$ is

(1) 3 (2) 5 (3) 7 (4) 9

4. Maximum value of $f(x) = |x + 1| - 2|x - 1|$ is
(1) 3 (2) 2 (3) 1 (4) 0

5. Number of roots of equation $||x| - 2| - 2| = 0$ and $y = \frac{|x|+2}{2}$ equals to
(1) 1 (2) 2 (3) 3 (4) 4

6. Complete solution of inequality $\left|\frac{3x}{x^2-4}\right| \le 1$ is
(1) $[-4, -1]$ $[1, 4]$ (2) $(-\infty, -4]$ $(-2, 1]$ $(2, \infty)$
(3) $(-\infty, -2)$ $[-1, 2)$ $[4, \infty)$ (4) $(-\infty, -4]$ $[-1, 1]$ $[4, \infty)$

7. Number of positive integer solutions of inequality $|2x - 3| + |x + 5| \le |x - 8|$ is
(1) 7 (2) 2 (3) 1 (4) 2

8. Complete solution of inequality $\frac{|x+3|+x}{x+2} > 1$ is
(1) $(-\infty, -2) \cup (-1, \infty)$ (2) $(-\infty, -1) \cup (2, 5)$ (3) $(-2, -1)$ (4) $(-5, -2) \cup (-1, \infty)$

9. Solve the inequality $(|x| - 3)(|x| - 5) < 0$
(1) (3,5) (2) (–5,–3) (3) $(-5,-3) \cap (3,5)$ (4) $(-5,-3) \cup (3,5)$

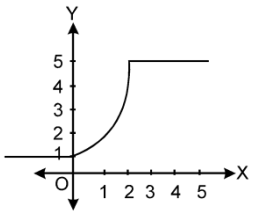
10. Complete solution of inequality $(|x - 1| - 3)(|x + 2| - 5) < 0$ is
(1) $(-\infty, -7) \cup (-2, 3) \cup (4, \infty)$ (2) $(-\infty, -4) \cup (-3, 2) \cup (7, \infty)$
(3) $(-4, -3) \cup (2, 7)$ (4) $(-7, -2) \cup (3, 4)$

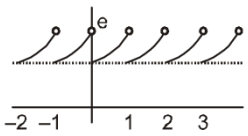
11. Find the number of all the integral solutions of the inequality $\frac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x^2-9)} \le 0$
(1) 1 (2) 2 (3) 3 (4) 4

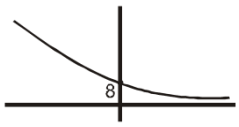
12. Complete set of solution of inequation $\sqrt{x+5} - \sqrt{x} > 1$ is $[a, b)$ (where $a, b \in R$) then $a + b$ is equal to
(1) 4 (2) 3 (3) 2 (4) 1

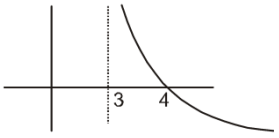
13. Complete set of solution of a inequation $\sqrt{2\sqrt{x-1}} + \sqrt{x-1} > 1$
(1) $x \in (2 - \sqrt{3}, 1)$ (2) $x \in (1, 2 + \sqrt{3})$ (3) $x \in (2 - \sqrt{3}, 2 + \sqrt{3})$ (4) $x \in (2 - \sqrt{3}, \infty)$

14. The graphs of functions $f_1(x) = \begin{cases} 1, & \text{if } x \le 0 \\ x^2 + 1, & \text{if } 0 < x < 2 \\ 5, & \text{if } x \ge 2 \end{cases}$, $f_2(x) = \log_{1/2}(x - 3)$, $f_3(x) = 2^{3-x}$ and $f_4(x) = e^{\{x\}}$ where $\{ . \}$ denote the fracitonal part function are given (not in order) as :

(1) 

(2) 

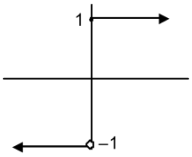
(3) 

(4) 

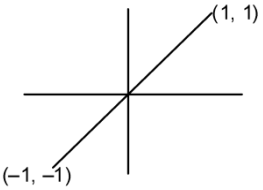
The correct order of graphs of functions $f_1(x)$, $f_2(x)$, $f_3(x)$ and $f_4(x)$ is

(1) ABCD (2) ACBD (3) CBDA (4) ADCB

15. Let graph of y = f(x) is



and graph of y = g(x) is



The graph of y = |f(x) + g(x)| is same as

(1) y = |f(x)| + |g(x)| (2) y = 2|x| (3) y = |x| (4) y = 2

16. Let $f(n) = \left[\frac{1}{2} + \frac{n}{100}\right]$, where [.] denotes the greatest integer function, then the value of $\sum_{n=1}^{151} f(n)$ is

(1) 101 (2) 102 (3) 104 (4) 103

17. The set of solution of inequality $[x]^2 = -[x]$, where [.] denotes greatest integer function is

(1) {–1, 0} (2) [–1, 0) (3) [0, 2) (4) [–1, 1)

18. The set of all values of x for which $\frac{\sqrt{-x^2+5x-6}}{\sqrt{1-2\{x\}}} \geq 0$ is (where {.} denotes the fractional part function)

(1) $\left[2, \frac{5}{2}\right) \cup \{3\}$ (2) (2, 3) (3) $\left(\frac{5}{2}, 3\right]$ (4) $\left[2, \frac{5}{2}\right) \cup \left(\frac{5}{2}, 3\right]$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement -1 :** If $|x-2|+|x-7|=|2x-9|$, then $x \le 2$ or $x \ge 7$

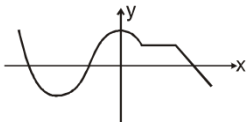
Statement -2 : $|x-a|+|x-b| = b-a$ has infinitely many solution, for $a < b$.

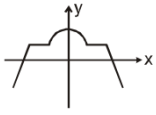
A-2 **Statement -1 :** The solution of inequality $\frac{|x-3|}{x-3} + 5 > x$ is $(-\infty, 6)$

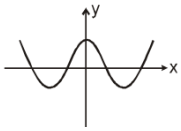
Statement -2 : : $|x| = -x$ if $x \le 0$.

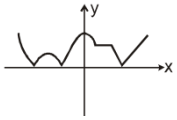
Section (B) : MATCH THE COLUMN

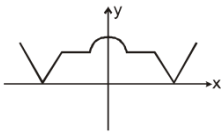
1. If y = f(x) has following graph, then match the column.



(A) $y = |f(x)|$ (p) 

(B) $y = f(|x|)$ (q) 

(C) $y = f(-|x|)$ (r) 

(D) $y = |f(|x|)|$ (s) 

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

1. The equation $|x-2|^{10x^2-1} = |x-2|^{3x}$, where $x \ne 2$ has :

(1) two positive & two negative solutions (2) four real solutions

(3) three positive & one negative solutions (4) three real solutions .

2. The simultaneous equations $y = x + 2|x|$ and $y = 4 + x - |x|$ have the solution set given by

(1) $\left(\frac{4}{3},\frac{4}{3}\right)$ (2) $\left(4,\frac{4}{3}\right)$ (3) $\left(-\frac{4}{3},\frac{4}{3}\right)$ (4) $\left(\frac{4}{3},4\right)$

3. Consider f(x) = ||x –1| – |x + 2|| = P.

(1) If P = 0 then f(x) has exactly one solution

(2) If P =1 then f(x) has exactly 2 solution

(3) If P = 3 then f(x) has infinite solution

(4) If P = 4 then f(x) has no solution

Fundamental of Mathematics - II

Answers

EXERCISE - 1

Section (A) :

A-1.	(3)	**A-2.**	(1)	**A-3.**	(4)	**A-4**	(3)	**A-5.**	(2)	**A-6.**	(3)	**A-7.**	(3)
A-8.	(2)	**A-9.**	(1)	**A-10.**	(4)	**A-11.**	(1)	**A-12.**	(2)	**A-13.**	(1)	**A-14.**	(2)
A-15.	(2)												

Section (B) :

B-1.	(4)	**B-2.**	(3)	**B-3.**	(2)	**B-4.**	(2)	**B-5.**	(3)	**B-6.**	(1)	**B-7.**	(4)
B-8.	(2)	**B-9.**	(1)	**B-10.**	(1)	**B-11.**	(3)						

Section (C) :

C-1.	(2)	**C-2.**	(2)	**C-3.**	(4)	**C-4.**	(1)	**C-5.**	(4)

Section (D) :

D-1.	(4)	**D-2.**	(4)	**D-3.**	(1)	**D-4.**	(3)	**D-5.**	(4)

Section (E) :

E-1.	(2)	**E-2.**	(1)	**E-3.**	(3)	**E-4.**	(4)	**E-5.**	(4)	**E-6.**	(3)	**E-7.**	(1)

EXERCISE - 2

PART - I

1.	(1)	**2.**	(3)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(4)	**7.**	(3)
8.	(4)	**9.**	(4)	**10.**	(4)	**11.**	(2)	**12.**	(1)	**13.**	(4)	**14.**	(4)
15.	(1)	**16.**	(3)	**17.**	(4)	**18.**	(1)						

PART - II

A-1.	(1)	**A-2**	(3)

Section (B) :

1. (A) → (r), (B) → (p), (C) → (q), (D) → (s)

Section (C) :

1.	(2,3)	**2.**	(3,4)	**3.**	(1,2,3,4)