Exercise-1

Marked questions may have for revision questions.

OBJECTIVE QUESTIONS

Section (A) : Equation vs Identities and Roots of the quadratic equation

A-1. Number of values of 'p' for which the equation $(p^2 - 3p + 2)x^2 - (p^2 - 5p + 4)x + p - p^2 = 0$ possess more than two roots, is:

(1) 0 (2) 1 (3) 2 (4) 4

A-2. The roots of the equation $(x + 2)^2 = 4(x + 1) - 1$ are -

(1) ± 1 (2) $\pm i$ (3) 1, 2 (4) –1, –2

A-3. If α, β are roots of equation $x^2 + 6x + \lambda = 0$ and $3\alpha + 2\beta = -20$, then λ is equal to

(1) 16 (2) –8 (3) – 16 (4) 8

A-4. Roots of equation $\sqrt{x} = x - 2$ are

(1) 4 (2) 1,4 (3) 1 (4) –1,4

A-5. If α, β are the roots of quadratic equation $x^2 + px + q = 0$ and γ, δ are the roots of $x^2 + px - r = 0$, then $(\alpha - \gamma) . (\alpha - \delta)$ is equal to :

(1) q + r (2) q – r (3) – (q + r) (4) – (p + q + r)

A-6. If α, β are roots of the equation $px^2 + qx - r = 0$, then the value of $\frac{\alpha}{\beta^2} + \frac{\beta}{\alpha^2}$ is equal to-

(1) $-\frac{p}{qr^2}(3pr + q^2)$ (2) $-\frac{q}{pr^2}(3pr + q^2)$ (3) $-\frac{q}{pr^2}(3pr - q^2)$ (4) $\frac{q}{pr^2}(3pr + q)$

A-7. If α, β are roots of the equation $2x^2 - 35x + 2 = 0$, then the value of $(2\alpha - 35)^3 . (2\beta - 35)^3$ is equal to-

(1) 1 (2) 8 (3) 64 (4) – 64

A-8. If difference of roots of the equation $x^2 - px + q = 0$ is 1, then $p^2 + 4q^2$ equals –

(1) 2q + 3 (2) $(1 - 2q)^2$ (3) $(1 + 2q)^2$ (4) 2q – 3

A-9. If α and β are the root of $ax^2 + bx + c = 0$, then the value of $\left\{\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}\right\}$ is :

(1) $\frac{a}{bc}$ (2) $\frac{b}{ca}$ (3) $\frac{c}{ab}$ (4) $-\frac{b}{ac}$

A-10. If α and β are roots of $2x^2 - 3x - 6 = 0$, then the equation whose roots are $\alpha^2 + 2$ and $\beta^2 + 2$ will be

(1) $4x^2 + 49x - 118 = 0$ (2) $4x^2 - 49x - 118 = 0$

(3) $4x^2 - 49x + 118 = 0$ (4) $4x^2 + 49x + 118 = 0$

A-11. If α and β are roots of $x^2 - 2x + 3 = 0$, then the equation whose roots are $\frac{\alpha-1}{\alpha+1}$ and $\frac{\beta-1}{\beta+1}$ will be

(1) $3x^2 - 2x + 1 = 0$ (2) $3x^2 + 2x + 1 = 0$ (3) $3x^2 - 2x - 1 = 0$ (4) $x^2 - 3x + 1 = 0$

A-12. The roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are

(1) $\frac{c-a}{b-c}$, 1 (2) $\frac{a-b}{b-c}$, 1 (3) $\frac{b-c}{a-b}$, 1 (4) $\frac{c-a}{a-b}$, 1

A-13. The equation $x - \frac{2}{x-1} = 1 - \frac{2}{x-1}$ has

(1) No root (2) One root (3) Two equal root (4) Infinitely many roots

A-14. If roots of the equation $x^2 - bx + c = 0$ are two successive integers, then $b^2 - 4c$ equals

(1) 1 (2) 2 (3) 3 (4) 4

A-15. If α and β are roots of equation $x^2 + 2x + 4 = 0$, then $\frac{1}{\alpha^3} + \frac{1}{\beta^3} =$

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

A-16. Two real numbers α & β are such that $\alpha + \beta = 3$ and difference of α and β is 4, then α & β are the roots of the quadratic equation:

(1) $4x^2 - 12x - 7 = 0$ (2) $4x^2 - 12x + 7 = 0$

(3) $4x^2 - 12x + 25 = 0$ (4) $4x^2 + 12x + 7 = 0$

Section (B) : Theory of Equation

B-1 If roots of equation $x^3 - 5x^2 + 2x + 7 = 0$ are α, β and γ then value of $\alpha^2 + \beta^2 + \gamma^2$ is

(1) 29 (2) 21 (3) – 21 (4) – 29

B-2. If roots of equation $2x^4 - 3x^3 + 2x^2 - 7x - 1 = 0$ are α, β, γ and δ then value of $\sum \frac{\alpha+1}{\alpha}$ is equal to

(1) –3 (2) 3 (3) $\frac{11}{2}$ (4) –11

B-3 Number of real roots of equation $x^4 + x^2 - 12 = 0$ is

(1) 4 (2) 2 (3) 0 (4) 3

B-4. If two roots of the equation $x^3 - px^2 + qx - r = 0$ are equal in magnitude but opposite in sign, then:

(1) pr = q (2) qr = p (3) pq = r (4) $p^2q^2 = r$

B-5. If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, then the value of

$\left(\alpha - \frac{1}{\beta\gamma}\right)\left(\beta - \frac{1}{\gamma\alpha}\right)\left(\gamma - \frac{1}{\alpha\beta}\right)$ is

(1) $\frac{(r+1)^3}{r^2}$ (2) $-\frac{(r+1)^2}{r^2}$ (3) $-\frac{(r+1)^2}{r^3}$ (4) $-\frac{(r+1)^3}{r^2}$

B-6. If $f(x) = 2x^3 + mx^2 - 13x + n$ and 2 and 3 are roots of the equations $f(x) = 0$, then values of m and n are-
(1) 5, 30 (2) –5, 30 (3) –5, –30 (4) 5, – 30

B-7. The imaginary roots of the equation $(x^2 + 2)^2 + 8x^2 = 6x (x^2 + 2)$ are
(1) $1 \pm i$ (2) $2 \pm i$ (3) $-1 \pm i$ (4) $-2 \pm i$

Section (C) : Nature of roots

C-1. If the roots of $x^2 - 2x - 16a = 0$ are real, then
(1) $a \geq \frac{1}{4}$ (2) $a \geq \frac{1}{8}$ (3) $a \geq -\frac{1}{16}$ (4) $a \leq -\frac{1}{16}$

C-2. If a, b, c are integers and $b^2 = 4(ac + 5d^2)$, $d \in N$, then roots of the equation $ax^2 + bx + c = 0$ are
(1) Irrational (2) Rational & different
(3) Complex conjugate (4) Rational & equal

C-3. If the roots of the equation $ax^2 + x + b = 0$ be real and unequal where $a, b \in R$, then the roots of the equation $x^2 - 4\sqrt{ab}\,x + 1 = 0$ will be
(1) Rational (2) Irrational (3) Real (4) Imaginary

C-4. If one root of the equation $2x^2 - 6x + c = 0$ is $\frac{3+5i}{2}$, then the value of c will be –
(1) 7 (2) –7 (3) 17 (4) –17

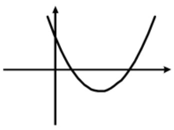
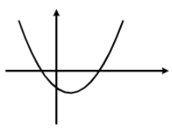
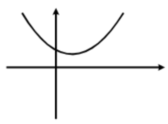
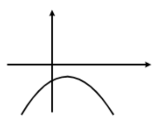
C-5. The quadratic equation with rational coefficient whose one root is $\frac{1}{2+\sqrt{5}}$, is
(1) $x^2 - 4x - 1 = 0$ (2) $\sqrt{2}\,x^2 - 4x + 1 = 0$
(3) $x^2 + 4x - 1 = 0$ (4) $x^2 + 4x + 1 = 0$

C-6. If roots of equation $x^2 + a^2 = 8x + 6a$ are real then 'a' belongs to the interval
(1) [–8,2] (2) [2,8] (3) [–2,8] (4) [–8,–2]

C-7. If the product of the roots of the equation $x^2 - 3x + k + 5 = 0$ is 7, then the roots are real for k =
(1) 2 (2) 3 (3) – 2 (4) φ

Section (D) : Graphs and range of quadratic expression

D-1. Which of the following graph represents the expression $f(x) = a x^2 + b x + c$ $(a \neq 0)$ when $a > 0$, $b < 0$ & $c < 0$?

(1)  (2)  (3)  (4) 

D-2. The expression $y = ax^2 + bx + c$ has always the same sign as of 'a' if :
(1) $4ac < b^2$ (2) $4ac > b^2$ (3) $ac = b^2$ (4) $ac < b^2$

D-3. If $a, b \in R$, $a \neq 0$ and the quadratic equation $ax^2 - bx + 1 = 0$ has imaginary roots then $a + b + 1$ is:
(1) positive (2) negative (3) zero (4) depends on the sign of b

D-4. If a and b are the non-zero distinct roots of $x^2 + ax + b = 0$, then the least value of $x^2 + ax + b$ is

(1) $\frac{3}{2}$ (2) $\frac{9}{4}$ (3) $-\frac{9}{4}$ (4) 1

D-5. Let $f(x) = x^2 + 4x + 1$, then

(1) $f(x) > 0$ for all x (2) $f(x) > 1$ when $x \ge 0$

(3) $f(x) \ge 1$ when $x \le -4$ (4) $f(x) = f(-x)$ for all x

D-6. Range of quadratic expression $f(x) = x^2 - 2x + 3 \ \forall \ x \in [0, 2]$ is

(1) [0, 1] (2)[2, 3] (3) [1, 3] (4) [2, ∞)

D-7. The equation, $\pi^x = -2x^2 + 6x - 9$ has:

(1) no solution (2) one solution (3) two solutions (4) infinite solutions

D-8. If the inequality $(m - 2)x^2 + 8x + m + 4 > 0$ is satisfied for all $x \in R$, then the least integral value of m is:

(1) 4 (2) 5 (3) 6 (4) 3

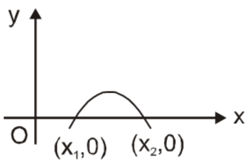
D-9. If the equation $ax^2 + 2bx - 3c = 0$ has no real roots and $\left(\frac{3c}{4}\right) < a + b$, then –

(1) $c < 0$ (2) $c > 0$ (3) $c = 0$ (4) $a < 0$

D-10. If $c < 0$ and $ax^2 + bx + c = 0$ does not have any real roots, then

(1) $a - b + c < 0$ (2) $9a + 3b + c > 0$ (3) $a + b + c > 0$ (4) All of these

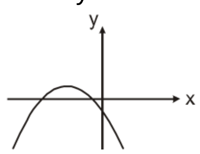
D-11. The adjoining figure shows the graph of $y = ax^2 + bx + c$, then



(1) $a > 0$ (2) $b^2 < 4ac$

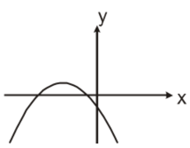
(3) $c > 0$ (4) a and b are of opposite signs

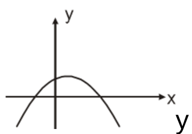
D-12. The graph of the quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure, then

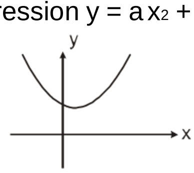


(1) $b^2 - 4ac < 0$ (2) $b < 0$ (3) $a > 0$ (4) $c > 0$

D-13. For which of the following graphs of the quadratic expression $y = ax^2 + bx + c$, the product a b c is negative

(1) 

(2) 

y (3) 

(4) All of these

D-14. $a, b, c \in R$, $a \ne 0$ and the quadratic equation $ax^2 + bx + c = 0$ has no real roots, then -

(1) $a + b + c > 0$ (2) $a(a + b + c) > 0$ (3) $b(a + b + c) > 0$ (4) $c(a + b + c) < 0$

D-15. For all real value of x, the maximum value of the expression $\frac{x}{x^2-5x+9}$ is

(1) 1 (2) 45 (3) 90 (4) $\frac{1}{11}$

D-16. If x is real, then the value of the expression $\frac{x^2+34x-71}{x^2+2x-7}$ does not exist between –

(1) – 5 and 9 (2) 5 and – 9 (3) –5 and –9 (4) 5 and 9

D-17. If x is real then the value of $\frac{x^2-2x+1}{x+1}$ will not lie between –

(1) 0 and 8 (2) –8 and 8 (3) –8 and 0 (4) – 8 and 6

Section (E) : Location of roots

E-1. If α, β are the roots of the quadratic equation $x^2 - 2p(x-4) - 15 = 0$, then the set of values of p for which one root is less than 1 & the other root is greater than 2 is:

(1) $\left(\frac{7}{3},\infty\right)$ (2) $\left(-\infty,\frac{7}{3}\right)$ (3) x ∈ R (4) $\left(-\infty,\frac{11}{4}\right)$

E-2. If both roots of the equation $x^2 - (m+1)x + (m+4) = 0$ are negative, then m equals –

(1) $-7 < m < -5$ (2) $-4 < m \le -3$ (3) $2 < m < 5$ (4) $3 \le m < 4$

E-3. If roots of $x^2 - (a-3)x + a = 0$ are such that both of them are greater than 2, then

(1) a ∈[7,9] (2) a ∈[9,10) (3) a ∈[9,7] (4) a ∈[9,12]

E-4. The real values of 'a' for which the quadratic equation $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possess roots of opposite sign is given by:

(1) a > 5 (2) 0 < a < 4 (3) a > 0 (4) a > 7

E-5. If α, β be the roots of $4x^2 - 16x + \lambda = 0$, where λ ∈ R, such that 1 < α < 2 and 2 < β < 3, then the number of integral values of λ is

(1) 5 (2) 6 (3) 2 (4) 3

E-6. If exactly one root of equation $x^2 - (p+1)x - p^2 = 0$ lie between 1 and 4 then number of integral values of p is -

(1) 4 (2) 5 (3) 7 (4) 9

E.7. If both roots of equation $x^2 + 2(a-1)x + (a+5) = 0$ lie in the interval (1,3) then complete set of values of 'a' is

(1) $\left(-\infty,-\frac{8}{7}\right)$ (2) (4,∞) (3) $\left(-\infty,-\frac{48}{3}\right)$ (4) $\left(-\frac{8}{7},-1\right]$

Section (F) : Common roots

F-1. If one of the factors of $ax^2 + bx + c$ and $bx^2 + cx + a$ is common, then :

(1) a = 0 (2) $a^3 + b^3 + c^3 = 3abc$

(3) $a = 0\ a^3 + b^3 + c^3 = 3abc$ (4) b = 0

F-2. The roots of $a_1x^2 + b_1x + c_1 = 0$ are reciprocal of the roots of the equation $a_2x^2 + b_2x + c_2 = 0$ if

(1) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (2) $\frac{b_1}{b_2} = \frac{c_1}{a_2} = \frac{a_1}{c_2}$ (3) $\frac{a_1}{a_2} = \frac{b_1}{c_2} = \frac{c_1}{b_2}$ (4) $a_1 = \frac{1}{a_2}, b_1 = \frac{1}{b_2}, c_1 = \frac{1}{c_2}$

F-3. If $x^2 - 11x + a = 0$ and $x^2 - 14x + 2a = 0$ have one common root then a is equal to –
(1) 0, – 24 (2) 0, 1 (3) 0, 24 (4) 1, 24

F-4. If both the roots of the equations $k(6x^2 + 3) + rx + 2x^2 - 1 = 0$ and $6k(2x^2 + 1) + px + 4x^2 - 2 = 0$ are common, then 2r – p is equal to –
(1) 1 (2) –1 (3) 2 (4) 0

F-5. If $x^2 + 3x + 5 = 0$ and $ax^2 + bx + c = 0$ have a common root and a, b, c ∈ N, then the minimum value of (a + b + c) is
(1) 8 (2) 9 (3) 10 (4) 7

F-6. The value of m for which one root of $x^2 - 3x + 2m = 0$ is double of one of the roots of $x^2 - x + m = 0$ is
(1) 0, – 2 (2) 0, 2 (3) 2, 4 (4) 2, –2

F-7. If the quadratic equations $ax^2 + bx + c = 0$ ($a, b, c \in R, a \neq 0$) and $x^2 + 4x + 5 = 0$ have a common root, then a, b, c must satisfy the relations:
(1) a > b > c (2) a < b < c
(3) a = k; b = 4k; c = 5k (k ∈ R, k ≠ 0) (4) a = 5b = 6c

F-8. $x^2 + x + 1$ is a factor of $ax^3 + bx^2 + cx + d = 0$, then the real root of above equation is (a, b, c, d ∈ R)
(1) (a – b)/b (2) d/a (3) (b – a)/a (4) (a – b)/a

Exercise-2

Marked questions may have for revision questions.

PART - I : OBJECTIVE QUESTIONS

1. If a,b are roots of the equation $x^2 + qx + 1 = 0$ and c,d are roots of $x^2 + px + 1 = 0$, then the value of (a – c) (b–c)(a+d) (b+d) will be
(1) $q^2 - p^2$ (2) $p^2 - q^2$ (3) $-p^2 - q^2$ (4) $p^2 + q^2$

2. In copying a quadratic equation of the form $x^2 + px + q = 0$, the coefficient of x was wrongly written as – 10 in place of – 11 and the roots were found to be 4 and 6. The roots of the correct equation are
(1) 8, 3 (2) 4, 3 (3) 6, 3 (4) 5, 6

3. If α,β are roots of the equation $(3x + 2)^2 + p(3x + 2) + q = 0$, then roots of $x^2 + px + q = 0$ are
(1) α,β (2) 3α + 2, 3β + 2 (3) $\frac{1}{3}(\alpha-2), \frac{1}{3}(\beta-2)$ (4) α – 2, β – 2

4. If α, β are the roots of $ax^2 + bx + c = 0$ and α + h, β + h are the roots of $px^2 + qx + r = 0$, then h =
(1) $\left(\frac{b}{a} - \frac{q}{p}\right)$ (2) $\frac{1}{2}\left(\frac{b}{a} - \frac{q}{p}\right)$ (3) $-\frac{1}{2}\left(\frac{a}{b} - \frac{p}{q}\right)$ (4) $-\frac{1}{2}\left(\frac{a}{b} + \frac{p}{q}\right)$

5. If α, β be the roots of the equation $(x - a)(x - b) + c = 0$ $(c \neq 0)$, then the roots of the equation $(x - c - \alpha)(x - c - \beta) = c$ are

(1) a and b + c (2) a + b and b (3) a + c and b + c (4) a – c and b – c

6. Let α, β, γ be the roots of $(x - a)(x - b)(x - c) = d$, $d \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ are :

(1) a + 1, b + 1, c + 1 (2) a, b, c (3) a – 1, b – 1, c – 1 (4) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

7. Let α, β be the roots of $x^2 + (3 - \lambda)x - \lambda = 0$. The value of λ for which $\alpha^2 + \beta^2$ is minimum, is

(1) 0 (2) 1 (3) 2 (4) 3

8. If a, b are non-zero real numbers and α, β are the roots of $x^2 + ax + b = 0$, then

(1) α^2, β^2 are the roots of $x^2 - (2b - a^2)x + a^2 = 0$

(2) $\frac{1}{\alpha}, \frac{1}{\beta}$ are the roots of $bx^2 + ax - 1 = 0$

(3) $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ are the roots of $bx^2 + (2b + a^2)x + b = 0$

(4) $(\alpha - 1), (\beta - 1)$ are the roots of the equation $x^2 + x(a + 2) + 1 + a + b = 0$

9. The values of k for which the expression $kx^2 + (k + 1)x + 2$ will be a perfect square of linear factor are

(1) $3 \pm 2\sqrt{2}$ (2) $4 \pm 2\sqrt{2}$ (3) 6 (4) 5

10. If $x^2 + (a - b)x + (1 - a - b) = 0$, $a, b \in R$ then the value of 'a' for which both roots of the equation are real and unequal $\forall\, b \in R$ is

(1) $(2, \infty)$ (2) $(3, \infty)$ (3) $(1, \infty)$ (4) $(-\infty, 1)$

11. If α, β are the real and distinct roots of $x^2 + px + q = 0$ and α^4, β^4 are the roots of $x^2 - rx + s = 0$, then the equation $x^2 - 4qx + 2q^2 - r = 0$ has always

(1) imaginary roots (2) two negative roots

(3) two positive roots (4) one positive root and one negative root

12. If a < b < c < d, then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(1) real and distinct (2) imaginary (3) real and equal (4) can't say anything

13. The values of k, for which the equation $x^2 + 2(k - 1)x + k + 5 = 0$ possess atleast one positive root, are:

(1) $[4, \infty)$ (2) $(-\infty, -1] \cup [4, \infty)$ (3) $[-1, 4]$ (4) $(-\infty, -1]$

14. The number of roots of the equation $\sqrt{x^2 - 4} - (x - 2) = \sqrt{x^2 - 5x + 6}$ is

(1) 0 (2) 1 (3) 2 (4) 3

15. If the two equations $x^2 - cx + d = 0$ and $x^2 - ax + b = 0$ have one common root and the second equation has equal roots, then 2 (b + d) =

(1) 0 (2) a + c (3) ac (4) –ac

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **STATEMENT - 1** : The nearest point from x - axis, on the curve $f(x) = x^2 - 6x + 11$ is (3, 2)

STATEMENT - 2 : If a > 0 and D < 0, then $ax^2 + bx + c > 0 \ \forall \ x \in R$.

A-2. Let α, β be the roots of $f(x) = 3x^2 - 4x + 5 = 0$.

STATEMENT-1 : The equation whose roots are 2α, 2β is given by $3x^2 + 8x - 20 = 0$.

STATEMENT-2 : To obtain, from the equation f(x) = 0, having roots α and β , the equation having roots 2α, 2β one needs to change x to $\frac{x}{2}$ in f(x) = 0.

A-3. **STATEMENT - 1** : Maximum value of $\log_{1/3}(x^2 - 4x + 5)$ is '0'.

STATEMENT - 2 : $\log_a x \le 0$ for $x \ge 1$ and 0 < a < 1.

Section (B) : MATCH THE COLUMN

B-1.

	Column – I		**Column – II**
(A)	If α, α + 4 are two roots of $x^2 - 8x + k = 0$, then possible value of k is	(p)	4
(B)	If α, β are roots of $x^2 + 2x - 4 = 0$ and $\frac{1}{\alpha}, \frac{1}{\beta}$ are roots of $x^2 + qx + r = 0$ then value of $\frac{-3}{q+r}$ is	(q)	0
(C)	If α, β are roots of $ax^2 + c = 0$, ac ≠ 0, then $\alpha^3 + \beta^3$ is equal to	(r)	12
(D)	If roots of $x^2 - kx + 36 = 0$ are Integers then number of values of k =	(s)	10

B-2. Match the following

(A)	If α, β are roots of equation $x^2 - 2x - \lambda^2 = 0$ then interval of values of $\alpha^2 + \beta^2$ is	(p)	[4,∞)

(B)	The values of x for which the equation $x^2 - x + \sin^2\alpha = 0$ have real solutions for all real values of α	(q)	$[1,\infty)$
(C)	If $-x^2 + 2x - \lambda \le 0$ for all real x then λ belongs to the interval	(r)	$(-\infty,1)$
(D)	If graph of $y = kx^2 - 2x + 1$ cut the x axis at two distinct points then k belongs to the interval	(s)	$[0,1]$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

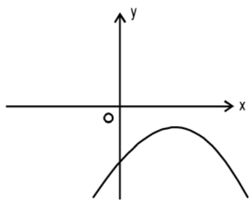
C-1 Let $a < 0$, $c < 0$ and $b < a + c$, then the equation $ax^2 + bx + c = 0$ has

(1) both negative real roots
(2) one root lies between – 1 and 0.
(3) roots are of opposite sign
(4) both positive real roots

C-2. If $f(x) = x^2 + 2(p - 3)x + 9$ and 6 lies between roots of the equation $f(x) = 0$, then

(1) $p \in \left(-\infty, -\frac{3}{4}\right)$
(2) $f(6) < 0$
(3) $6p - p^2 > 0$
(4) exactly one root lies in (0, 6)

C-3. The graph of a quadratic polynomial $y = ax^2 + bx + c$ is as shown in the figure



(1) b is greater than a + c
(2) b cannot take zero value
(3) a & c have the same sign
(4) 4a + 2 |b| + c can be positive

C-4. Let S be the set of all non-zero real numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 such that whose difference is less than 1. Which of the following intervals is(are) a subset(s) of S ?

(1) $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$
(2) $\left(-\frac{1}{\sqrt{5}}, 0\right)$
(3) $\left(0, \frac{1}{\sqrt{5}}\right)$
(4) $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

C-5. If the quadratic equations $x^2 + abx + c = 0$ and $x^2 + acx + b = 0$ have a common root, then the equation containing their other roots is/are:

(1) $x^2 + a(b + c)x - a^2bc = 0$
(2) $x^2 - a(b + c)x + a^2bc = 0$
(3) $a(b + c)x^2 + (b + c)x - abc = 0$
(4) $a(b + c)x^2 - (b + c)x + abc = 0$

C-6 If α, β are roots of $x^2 + 3x + 1 = 0$, then

(1) $(7 - \alpha)(7 - \beta) = 0$
(2) $(2 - \alpha)(2 - \beta) = 11$

(3) $\frac{\alpha^2}{3\alpha+1}+\frac{\beta^2}{3\beta+1} = -2$ (4) $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2 = 18$

C-7. Let $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$, then f(x) = 0 has

(1) exactly one real root in (2, 3) (2) exactly one real root in (3, 4)

(3) 3 different roots (4) at least one negative root

C-8. If the equations $x^2 + ax + 12 = 0$, $x^2 + bx + 15 = 0$ and $x^2 + (a + b)x + 36 = 0$ have a common positive root, then which of the following are true ?

(1) ab = 56 (2) common positive root is 3

(3) sum of uncommon roots is 21. (4) a + b = 15.

Exercise-3

Marked questions may have for revision questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If $\alpha \neq \beta$ but $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$, then the equation having the roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ is. **[AIEEE-2002(3, –1), 225]**
(1) $3x^2 + 19x + 3 = 0$ (2) $3x^2 - 19x + 3 = 0$ (3) $3x^2 - 19x - 3 = 0$ (4) $x^2 - 16x + 1 = 0$

2. The value of 'a' for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other, is : **[AIEEE-2003(3, –1), 225]**
(1) $\frac{2}{3}$ (2) $-\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $-\frac{1}{3}$

3. If $(1 - p)$ is a root of quadratic equation $x^2 + px + (1 - p) = 0$, then its roots are : **[AIEEE-2004(3, –1), 225]**
(1) 0,1 (2) –1,1 (3) 0, –1 (4) –1,2

4. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of 'q' is : **[AIEEE-2004(3, –1), 225]**
(1) $\frac{49}{4}$ (2) 12 (3) 3 (4) 4

5. If roots of the equation $x^2 - bx + c = 0$ be two consecutive integers, then $b^2 - 4c$ equals **[AIEEE-2005(3, –1), 225]**
(1) –2 (2) 3 (3) 2 (4) 1

6. The value of 'a' for which the sum of the squares of the roots of the equation $x^2 - (a - 2)x - a - 1 = 0$ assume the least value is - **[AIEEE-2005(3, –1), 225]**
(1) 1 (2) 0 (3) 3 (4) 2

7. If both the roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then 'k' lies in the interval **[AIEEE-2005(3, –1), 225]**
(1) (5, 6) (2) $(6, \infty)$ (3) $(-\infty, 4)$ (4) [4, 5]

8. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan 30° and tan 15° respectively, then the value of $2 + q - p$ is : **[AIEEE-2005(3, –1), 225]**
(1) 3 (2) 0 (3) 1 (4) 2

9. All the values of 'm' for which both roots of the equation $x^2 - 2mx + m^2 - 1 = 0$ are greater than – 2 but less than 4 lie in the interval : **[AIEEE-2006(3, –1), 165]**
(1) $m > 3$ (2) $-1 < m < 3$ (3) $1 < m < 4$ (4) $-2 < m < 0$

10. If 'x' is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is - **[AIEEE-2006 (3, –1), 165]**
(1) 41 (2) 1 (3) $\frac{17}{7}$ (4) $\frac{1}{4}$

11. If the difference between the roots of the equation $x^2 + ax + 1 = 0$ is less than , then $\sqrt{5}$ the set of possible values of 'a' is **[AIEEE-2007, (3, –1), 120]**
(1) (–3, 3) (2) $(-3, \infty)$ (3) $(3, \infty)$ (4) $(-\infty, -3)$

12. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is **[AIEEE-2008, (3, –1), 105]**
(1) 4 (2) 3 (3) 2 (4) 1

13. Sachin and Rahul attempted to solve a quadratic equation. Sachin made a mistake in writing down the constant term and ended up in roots (4, 3). Rahul made a mistake in writing down coefficient of x to get roots (3, 2). The correct roots of equation are : **[AIEEE- 2011, II, (4, –1), 120]**
(1) 6, 1 (2) 4, 3 (3) –6 , –1 (4) –4 , –3

14. Let for $a \neq a_1 \neq 0,$ $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is : **[AIEEE- 2011, II, (4, –1), 120]**

(1) 3 (2) 9 (3) 6 (4) 18

15. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, $a,b,c \in R$, have a common root, then a : b : c is **[AIEEE - 2013, (4, – 1) , 120]**

(1) 1 : 2 : 3 (2) 3 : 2 : 1 (3) 1 : 3 : 2 (4) 3 : 1 : 2

16. If $a \in R$ and the equation $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$ (where [x] denotes the greatest integer ≤ x) has no intgeral solution, then all possible values of a lie in the interval : **[JEE(Main)2014,(4, – 1), 120]**

(1) $(-2, -1)$ (2) $(-\infty, -2) \cup (2, \infty)$ (3) $(-1, 0) \cup (0, 1)$ (4) $(1, 2)$

17. Let α and β be the roots of equation $px^2 + qx + r = 0$, $p \neq 0$. If p, q ,r are in the A.P. and $\frac{1}{\alpha} + \frac{1}{\beta} = 4$, then the value of $|\alpha - \beta|$ is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{\sqrt{34}}{9}$ (2) $\frac{2\sqrt{13}}{9}$ (3) $\frac{\sqrt{61}}{9}$ (4) $\frac{2\sqrt{17}}{9}$

18. Let α and β be the roots of equation $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$, for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is equal to : **[JEE(Main) 2015, (4, – 1), 120]**

(1) 6 (2) – 6 (3) 3 (4) –3

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. For the equation $3x^2 + px + 3 = 0$, $p > 0$ if one of the roots is square of the other, then p is equal to: **[IIT-JEE-2002, Scr., (1 + 1 + 1, 0)/90]**

(A) 1/3 (B) 1 (C) 3 (D) 2/3

2. If α & β (α < β) are the roots of the equation $x^2 + bx + c = 0$, where $c < 0 < b$, then **[IIT-JEE-2002, Scr., (1 + 1 + 1, 0)/90]**

(A) $0 < \alpha < \beta$ (B) $\alpha < 0 < \beta < |\alpha|$ (C) $\alpha < \beta < 0$ (D) $\alpha < 0 < |\alpha| < \beta$

3. If b > a, then the equation $(x - a)(x - b) - 1 = 0$, has: **[IIT-JEE-2002, Scr., (1 + 1 + 1, 0)/90]**

(A) both roots in [a, b]
(B) both roots in (− ∞, a)
(C) both roots in [b, ∞)
(D) one root in (− ∞, a) & other in (b, ∞)

4. The number of solution(s) of $\log_4(x - 1) = \log_2(x - 3)$ is/are **[IIT-JEE-2002, Scr., (3, 0)/90]**

(A) 3 (B) 1 (C) 2 (D) 0

5. If $f(x) = x^2 + 2bx + 2c^2$ and $g(x) = -x^2 - 2cx + b^2$ are such that min f(x) > max g(x), then the relation between b and c, is **[IIT-JEE-2003, Scr., (3, –1)/84]**

(A) no relation (B) $0 < c < b/2$ (C) $|c| < \sqrt{2}\,|b|$ (D) $|c| > \sqrt{2}\,|b|$

6. If the quadratic expression $x^2 + 2ax - 3a + 10 > 0\ \forall\ x \in R$, then **[IIT-JEE-2004, Scr., (3, –1)/84]**

(A) $a > 5$ (B) $|a| < 5$ (C) $-5 < a < 2$ (D) $2 < a < 3$

7. If one root of the equation $x^2 + px + q = 0$ is square of other, then the relation between p, q is :

[IIT-JEE-2004, Scr., (3, –1)/84]

(A) $p^3 - q(3p - 1) + q^2 = 0$ (B) $p^3 + q(3p + 1) + q^2 = 0$

(C) $p^3 + q(3p - 1) + q^2 = 0$ (D) $p^3 - q(3p + 1) + q^2 = 0$

8. Let α, β be the roots of the equation $x^2 - px + r = 0$ and $\frac{\alpha}{2}, 2\beta$ be the roots of the equation $x^2 - qx + r = 0$. Then the value of r is **[IIT-JEE 2007, Paper-1, (3, –1)/ 81]**

(A) $\frac{2}{9}(p - q)(2q - p)$ (B) $\frac{2}{9}(q - p)(2p - q)$

(C) $\frac{2}{9}(q - 2p)(2q - p)$ (D) $\frac{2}{9}(2p - q)(2q - p)$

9. Let p and q be real numbers such that $p \neq 0$, $p^3 \neq q$ and $p^3 \neq -q$. If α and β are nonzero complex numbers satisfying $\alpha + \beta = -p$ and $\alpha^3 + \beta^3 = q$, then a quadratic equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$ as its roots is

[IIT-JEE 2010, Paper-1, (3, –1)/ 84]

(A) $(p^3 + q)x^2 - (p^3 + 2q)x + (p^3 + q) = 0$ (B) $(p^3 + q)x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

(C) $(p^3 - q)x^2 - (5p^3 - 2q)x + (p^3 - q) = 0$ (D) $(p^3 - q)x^2 - (5p^3 + 2q)x + (p^3 - q) = 0$

10. Let α and β be the roots of $x^2 - 6x - 2 = 0$, with $\alpha > \beta$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is **[IIT-JEE 2011, Paper-1, (3, –1)/ 80]**

(A) 1 (B) 2 (C) 3 (D) 4

11. A value of b for which the equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$ have one root in common is

[IIT-JEE 2011, Paper-2, (3, –1)/ 80]

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

12. The quadratic equation $p(x) = 0$ with real coefficients has purely imaginary roots. Then the equation $p(p(x)) = 0$ has **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) only purely imaginary roots (B) all real roots

(C) two real and two purely imaginary roots (D) neither real nor purely imaginary roots

13. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

[JEE (Advanced) 2016, Paper-1, (3, –1)/62]

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Answers

EXERCISE # 1

Section (A)

A-1.	(2)	**A-2.**	(2)	**A-3.**	(3)	**A-4.**	(1)	**A-5.**	(3)	**A-6.**	(2)	**A-7.**	(3)
A-8.	(3)	**A-9.**	(2)	**A-10.**	(3)	**A-11.**	(1)	**A-12**	(2)	**A-13.**	(1)	**A-14.**	(1)
A-15.	(4)	**A-16.**	(1)										

Section (B)

B-1	(2)	**B-2.**	(1)	**B-3**	(2)	**B-4.**	(3)	**B-5.**	(4)	**B-6.**	(2)	**B-7.**	(1)

Section (C) :

C-1.	(3)	**C-2.**	(1)	**C-3.**	(4)	**C-4.**	(3)	**C-5.**	(3)	**C-6.**	(3)	**C-7.**	(4)

Section (D) :

D-1.	(2)	**D-2.**	(2)	**D-3.**	(1)	**D-4.**	(3)	**D-5.**	(3)	**D-6.**	(2)	**D-7.**	(1)
D-8.	(2)	**D-9.**	(1)	**D-10.**	(1)	**D-11.**	(4)	**D-12.**	(2)	**D-13.**	(4)	**D-14.**	(2)
D-15.	(1)	**D-16.**	(4)	**D-17.**	(3)								

Section (E) :

E-1.	(2)	**E-2.**	(2)	**E-3.**	(2)	**E-4.**	(2)	**E-5.**	(4)	**E-6.**	(2)	**E.7.**	(4)

Section (F) :

F-1.	(3)	**F-2.**	(2)	**F-3.**	(3)	**F-4.**	(4)	**F-5.**	(2)	**F-6.**	(1)	**F-7.**	(3)
F-8.	(4)												

EXERCISE # 2

PART -I

1.	(2)	**2.**	(1)	**3.**	(2)	**4.**	(2)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(4)	**9.**	(1)	**10.**	(3)	**11.**	(4)	**12.**	(1)	**13.**	(4)	**14.**	(4)
15.	(3)												

PART -II

Section (A) :

A-1.	(1)	**A-2.**	(3)	**A-3.**	(1)

Section (B) :

B-1. (A) → (r), (B) → (p), (C) → (q), (D) → (s) **B-2.** (A) → p, (B) → s (C) → q (D) → r

Section (C) :

C-1	(1,2)	**C-2.**	(1,2,4,)	**C-3.**	(1,2,3)	**C-4.**	(1,4)	**C-5.**	(2,3)	**C-6**	(2,3,4)	**C-7.**	(1,2)
C-8.	(1,2,3)												

EXERCISE # 3

PART -I

1.	(2)	**2.**	(1)	**3.**	(3)	**4.**	(1)	**5.**	(4)	**6.**	(1)	**7.**	(3)
8.	(1)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(3)	**13.**	(1)	**14.**	(4)
15.	(1)	**16.**	(3)	**17.**	(2)	**18.**	(3)						

PART -II

1.	(C)	**2.**	(B)	**3.**	(D)	**4.**	(B)	**5.**	(D)	**6.**	(C)	**7.**	(A)
8.	(D)	**9.**	(B)	**10.**	(C)	**11.**	(B)	**12.**	(D)	**13.**	(C)		