Fundamental of Mathematics - I

Exercise-1

Marked questions may have for revision questions.

OBJECTIVE QUESTIONS

Section (A) : Representation of sets, Types of sets, subset and power set

A-1. The set of intelligent students in a class is-
(1) a null set (2) a singleton set
(3) a finite set (4) not a well defined collection

A-2. Which of the following is the empty set
(1) {x : x is a real number and $x^2 - 1 = 0$} (2) {x : x is a real number and $x^2 + 1 = 0$}
(3) {x : x is a real number and $x^2 - 9 = 0$} (4) {x : x is a real number and $x^2 = x + 2$}

A-3. The set A = {x : x ∈ R, $x^2 = 16$ and $2x = 6$} is
(1) Null set (2) Singleton set
(3) Infinite set (4) not a well defined collection

A-4. If A = {x : $-3 < x < 3$, x ∈ Z} then the number of subsets of A is –
(1) 120 (2) 30 (3) 31 (4) 32

A-5. Which of the following are true ?
(1) $[3, 7] \subseteq (2, 10)$ (2) $(0, \infty) \subseteq (4, \infty)$ (3) $(5, 7] \subseteq [5, 7)$ (4) $[2, 7] \subseteq (2.9, 8)$

A-6. The number of subsets of the power set of set A = {7, 10, 11} is
(1) 32 (2) 16 (3) 64 (4) 256

A-7. Which of the following collections is not a set ?
(1) The collection of natural numbers between 2 and 20
(2) The collection of numbers which satisfy the equation $x^2 - 5x + 6 = 0$
(3) The collection of prime numbers between 1 and 100.
(4) The collection of all intelligent women in Jalandhar.

A-8. The set A = {x : x is a positive prime < 10} in the tabular form is
(1) {1,2,3,5,7} (2) {1,3,5,7,9} (3) {2,3,5,7} (4) {1 ,3,5,7}

A-9. Which of the folowing sets is an infinite set ?
(1) Set of divisors of 24
(2) Set of all real number which lie between 1 and 2
(3) Set of all humman beings living in India.
(4) Set of all three digit natural numbers

A-10. Power set of the set A = {φ, {φ}} is :
(1) {φ, {φ}, {{φ}}} (2) {φ, {φ}, {{φ}}, A} (3) {φ, {φ}, A} (4) {{φ}, {{φ}}}

Section (B) : Operations on sets, Law of Algebra of sets

B-1. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?

(1) 3 (2) 6 (3) 9 (4) 18

B-2. Given the sets A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, then $A \cup (B \cap C)$ is

(1) {3} (2) {1, 2, 3, 4} (3) {1, 2, 4, 5} (4) {1, 2, 3, 4, 5, 6}

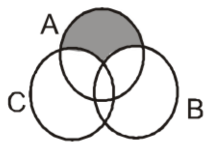
B-3. The smallest set A such that $A \cup \{1, 2\} = \{1, 2, 3, 5, 9\}$ is

(1) {2, 3, 5} (2) {3, 5, 9} (3) {1, 2, 5, 9} (4) {1, 2, 3, 5, 9}

B-4. If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then $(A \cap B) \cup (A \cap C)$ is equal to

(1) {3, 4, 10} (2) {2, 8, 10} (3) {4, 5, 6} (4) {3, 5, 14}

B-5. The shaded region in the given figure is



(1) $A \cap (B \cup C)$ (2) $A \cup (B \cap C)$ (3) $A \cap (B - C)$ (4) $A - (B \cup C)$

B-6. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7}, then $A \cap B'$ is

(1) B' (2) A (3) A' (4) B

B-7. If A = {x : x = 4n + 1, $n \leq 5$, $n \in N$} and B {3n : $n \leq 8$, $n \in N$}, then A – (A – B) is :

(1) {9, 21} (2) {9, 12} (3) {6, 12} (4) {6, 21}

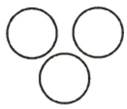
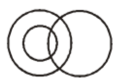
B-8. $A \cup B = A \cap B$ iff :

(1) $A \subset B$ (2) A = B (3) $A \supset B$ (4) $A \subseteq B$

B-9. If aN = {ax : $x \in N$} and $bN \cap cN = dN$, where $b, c \in N$, $b \geq 2$, $c \geq 2$ are relatively prime, then which one of the following is correct ? **[SCRA-2007, (2, –1/3)/100]**

(1) b = cd (2) c = bd (3) d = bc (4) $d_2 = bc$

B-10. Which of the following venn-diagrams best represents the sets of females, mothers and doctors ?

(1)  (2)  (3)  (4) 

Section (C) : Cardinal number Problems

C-1. Let A and B be two sets. Then

(1) $n(A \cup B) \leq n(A \cap B)$ (2) $n(A \cap B) \leq n(A \cup B)$

(3) $n(A \cap B) = n(A \cup B)$ (4) can't be say

C-2. Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then $n(A' \cap B') =$

(1) 400 (2) 600 (3) 300 (4) 200

C-3. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspaper is-

(1) at least 30 (2) at most 20 (3) exactly 25 (4) exactly 30

C-4. In a city 20 percent of the population travels by car, 50 percent travels by bus and 10 percent travels by both car and bus. Then persons travelling by car or bus is

(1) 80 percent (2) 40 percent (3) 60 percent (4) 70 percent

C-5. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families buy newspaper C, 5% families buy A and B, 3 % buy B and C and 4% buy A and C. If 2% families buy all the three news papers, then number of families which buy newspaper A only is

(1) 3100 (2) 3300 (3) 2900 (4) 1400

C-6. A class has 175 students. The following data shows the number of students obtaining one or more subjects : Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics & Physics & Chemistry 18. How many students have offered Mathematics alone ?

(1) 35 (2) 48 (3) 60 (4) 22

C-7. 31 candidates appeared for an examination, 15 candidates passed in English, 15 candidates passed in Hindi, 20 candidates passed in Sanskrit. 3 candidates passed only in English. 4. candidates passed only in Hindi, 7 candidates passed only in Sanskrit. 2 candidates passed in all the three subjects How many candidates passed only in two subjects ? **[SCRA-2005, (2, –1/3)/100]**

(1) 17 (2) 15 (3) 22 (4) 14

Comprehension (C-8 to C-10)

In a group of 1000 people, there are 750 people, who can speak Hindi and 400 people, who can speak Bengali.

C-8. Number of people who can speak Hindi only is

(1) 300 (2) 400 (3) 500 (4) 600

C-9 Number of people who can speak Bengali only is

(1) 150 (2) 250 (3) 50 (4) 100

C-10 Number of people who can speak both Hindi and Bengali is

(1) 50 (2) 100 (3) 150 (4) 200

Section (D) : Standard formulae, Polynomials & Divisional Algorithm

D-1 Sum of first 8 prime natural numbers is

(1) 59 (2) 77 (3) 76 (4) 58

D-2. If $\frac{5+3\sqrt{7}}{5-3\sqrt{7}}$ = a + b $\sqrt{7}$ then rational numbers a and b are respectively

(1) $\frac{44}{19},\frac{15}{19}$ (2) $\frac{44}{19},-\frac{15}{19}$ (3) $-\frac{15}{19},-\frac{44}{19}$ (4) $-\frac{44}{19},-\frac{15}{19}$

D-3. The number of real roots of the equation, $(x - 1)_2 + (x - 2)_2 + (x - 3)_2 = 0$ is :

(1) 0 (2) 1 (3) 2 (4) 3

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D-4. Which of the following conditions imply that the real number *x* is rational?

(i) $x^{1/2}$ is rational (ii) x^2 and x^5 are rational (iii) x^2 and x^4 are rational

(1) (i) and (ii) only (2) (i) and (iii) only (3) (ii) and (iii) only (4) (i), (ii) and (iii)

D-5. If $x + \frac{1}{x} = 2$, then $x^2 + \frac{1}{x^2}$ is equal to

(1) 0 (2) 1 (3) 2 (4) 3

D-6. If $\frac{(2+1)(2^2+1)(2^4+1)(2^8+1)}{(2^8-1)} = 4^n + 1$, then n is

(1) 4 (2) 3 (3) 2 (4) 1

D-7. If $(x + y)^2 = 2(x^2 + y^2)$ and $(x - y + \lambda)^2 = 4$, $\lambda > 0$, then λ is equal to :

(1) 1 (2) 2 (3) 3 (4) 4

D-8. If $\frac{a+3d}{a+9d} = \frac{a+d}{a+5d} = k$, then k is equal to (a, d > 0)

(1) $\frac{1}{2}$ (2) 2 (3) 6 (4) $\frac{1}{4}$

D-9 . If (x – a) is a factor of $x^3 - a^2x + x + 2$, then 'a' is equal to

(1) 0 (2) 2 (3) –2 (4) 1

D-10. The polynomials $P(x) = kx^3 + 3x^2 - 3$ and $Q(x) = 2x^3 - 5x + k$, when divided by (x – 4) leaves the same remainder, then value of k is

(1) 2 (2) 1 (3) 0 (4) –1

D-11. If $2x^3 - 5x^2 + x + 2 = (x - 2)(ax^2 - bx - 1)$, then a & b are respectively :

(1) 2, 1 (2) 2, - 1 (3) 1, 2 (4) - 1, 1/2

Section (E) : Rational Inequalities

E-1. Number of integer values of x satisfying $-5 \le x < 10$ and $0 \le x \le 15$ is

(1) 10 (2) 11 (3) 12 (4) 13

E-2. The number of positive integers satisfying the inequality $\frac{x^2-1}{2x+5} < 3$ is

(1) 10 (2) 9 (3) 8 (4) 7

E-3. The solution of the inequality $2x - 1 \le x^2 + 3 \le x - 1$ is

(1) $x \in R$ (2) $[2 - \sqrt{2}, 2 + \sqrt{2}]$ (3) $[2 - \sqrt{2}, 2]$ (4) $x \in \varphi$

E-4. The complete set of values of 'x' which satisfy the inequations : $5x + 2 < 3x + 8$ and $\frac{x+2}{x-1} < 4$ is

(1) $(-\infty, 1)$ (2) (2, 3) (3) $(-\infty, 3)$ (4) $(-\infty, 1) \cup (2, 3)$

E-5. The number of the integral solutions of $x^2 + 9 < (x + 3)^2 < 8x + 25$ is :
(1) 1 (2) 2 (3) 3 (4) 5

E-6. The complete solution set of inequality $(x - 1)(x + 3)(2x - 7)(5 - x) \le 0$ is -
(1) $(-\infty,-3] \cup \left[1,\frac{7}{2}\right] \cup [5,\infty)$ (2) $(-3,1] \cup \left[\frac{7}{2},5\right]$
(3) $(-\infty,-3] \cup (1,5) \cup (5,\infty)$ (4) $(1,3] \cup \left[\frac{7}{2},5\right]$

E-7. The complete solution set of inequality $\frac{(x+2)(x^2-2x+1)}{4+3x-x^2} \ge 0$ is
(1) $[-2, -1) \cup [1,4)$ (2) $(-\infty, -2] \cup (-1,4)$
(3) $[-2, -1) \cup (4, \infty) \cup \{1\}$ (4) $(-\infty, -2] \cup (-1,1) \cup (1,4)$

E-8. The complete solution set of the inequality $\frac{x^4-3x^3+2x^2}{x^2-x-30} \ge 0$ is:
(1) $(-\infty, -5) \cup (1, 2) \cup (6, \infty) \cup \{0\}$ (2) $(-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$
(3) $(-\infty, -5] \cup [1, 2] \cup [6, \infty) \cup \{0\}$ (4) $(-\infty, -5] \cup [1, 2] \cup [6, \infty)$

E-9. Number of integers satisfying the inequality $x^4 - 5x^2 + 4 \le 0$ is
(1) 2 (2) 3 (3) 4 (4) 5

E-10. Number of positive integral values of x satisfying the inequality
$$\frac{(x-4)^{2013}.\ (x+8)^{2014}\ (x+1)}{x^{2016}(x-2)^3\ .\ (x+3)^5\ .\ (x-6)\ (x+9)^{2012}} \le 0 \text{ is}$$
(1) 0 (2) 1 (3) 2 (4) 3

E-11. Number of non-negative integral values of x satisfying the inequality $\frac{2}{x^2-x+1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \ge 0$ is
(1) 0 (2) 1 (3) 2 (4) 3

Section (F) : Logarithm

F-1. $\frac{1}{1+\log_b a+\log_b c} + \frac{1}{1+\log_c a+\log_c b} + \frac{1}{1+\log_a b+\log_a c}$ has the value equal to
(1) abc (2) $\frac{1}{abc}$ (3) 0 (4) 1

F-2. $\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc}$ has the value equal to :
(1) 1/2 (2) 1 (3) 2 (4) 4

F-3. If $a^4 \cdot b^5 = 1$ then the value of $\log_a(a^5b^4)$ equals
(1) 9/5 (2) 4 (3) 5 (4) 8/5

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MATHEMATICS

F-4. Let $x = 2^{\log 3}$ and $y = 3^{\log 2}$ where base of the logarithm is 10, then which one of the following holds good?

(1) 2x < y (2) 2y < x (3) 3x = 2y (4) y = x

F-5. If $\log_a(ab) = x$, then $\log_b(ab)$ is equal to

(1) $\frac{1}{x}$ (2) $\frac{x}{1 + x}$ (3) $\frac{x}{1 - x}$ (4) $\frac{x}{x - 1}$

F-6. $(\log_2 10) . (\log_2 80) - (\log_2 5) . (\log_2 160)$ is equal to :

(1) $\log_2 5$ (2) $\log_2 20$ (3) $\log_2 10$ (4) $\log_2 16$

F-7. The ratio $\dfrac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}(a^2+1)^3} - 2a}{7^{4\log_{49} a} - a - 1}$ simplifies to :

(1) $a_2 - a - 1$ (2) $a_2 + a - 1$ (3) $a_2 - a + 1$ (4) $a_2 + a + 1$

F-8. $10^{\log_p(\log_q(\log_r x))} = 1$ and $\log_q\left(\log_r(\log_p x)\right) = 0$ then 'p' equals

(1) $r_{q/r}$ (2) rq (3) 1 (4) $r_{r/q}$

F-9. Which one of the following is the smallest?

(1) $\log_{10}\pi$ (2) $\sqrt{\log_{10}\pi^2}$ (3) $\left(\frac{1}{\log_{10}\pi}\right)^3$ (4) $\left(\frac{1}{\log_{10}\sqrt{\pi}}\right)$

F-10. $\log_{10}(\log_2 3) + \log_{10}(\log_3 4) + \log_{10}(\log_4 5) + ... + \log_{10}(\log_{1023} 1024)$ simplifies to

(1) a composite number (2) a prime number

(3) rational number which is not an integer (4) an integer number

F-11. The sum of all the solutions to the equation $2 \log_{10} x - \log_{10}(2x - 75) = 2$ is

(1) 30 (2) 350 (3) 75 (4) 200

F-12. If $\log_x \log_{18}\left(\sqrt{2} + \sqrt{8}\right) = \frac{1}{3}$. Then the value of 1000 x is equal to

(1) 8 (2) 1/8 (3) 1/125 (4) 125

F-13. Sum of all solutions of equation $\log_2(\log_3(x_2 - 1)) = 0$ is

(1) 4 (2) – 4 (3) 0 (4) 2

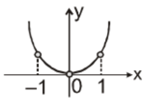
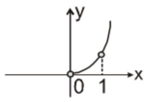
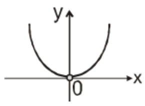
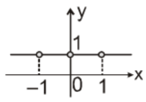
F-14. If $3^{2\log_3 x} - 2x - 3 = 0$, then the number of values of 'x' satisfying the equation is

(1) zero (2) 1 (3) 2 (4) more than 2

F-15. If $\log_2(\log_9 x + \frac{3}{2} + 8_x) = 3x$, then value of 27x is equal to

(1) $\frac{1}{27}$ 2) 27 (3) 1 (4) $\frac{1}{9}$

F-16. The correct graph of $y = x^{\log_x x^2}$ is

(1) 
(2) 
(3) 
(4) 

Section (G) : Logarithmic inequalities

G-1. The solution set of the inequality $\log_{\frac{\sqrt{3}}{2}}(x^2 - 3x + 2) \geq 2$ is

(1) $\left(\frac{1}{2}, 2\right)$
(2) $\left(1, \frac{5}{2}\right)$
(3) $\left[\frac{1}{2}, 1\right) \cup \left(2, \frac{5}{2}\right]$
(4) (1,2)

G-2. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval

(1) $(2, \infty)$
(2) (1, 2)
(3) (-2, -1)
(4) $\left(1, \frac{3}{2}\right)$

G-3. Solution set of the inequality $2 - \log_2(x^2 + 3x) \geq 0$ is :

(1) [- 4, 1]
(2) $[-4, -3) \cup (0, 1]$
(3) $(-\infty, -3) \cup (1, \infty)$
(4) $(-\infty, -4) \cup [1, \infty)$

G-4. The set of all solutions of the inequality $\left(\frac{1}{2}\right)^{x^2-2x} < \frac{1}{8}$ contains the set

(1) $(-\infty, 0)$
(2) $(-\infty, 1)$
(3) $(1, \infty)$
(4) $(4, \infty)$

G-5. Solution set of inequality $\log_3 \frac{5x+3}{7-2x} \geq 0$ is

(1) $\left[-\frac{4}{7}, \frac{7}{2}\right)$
(2) $\left[\frac{4}{7}, \frac{7}{2}\right)$
(3) $\left(-\frac{3}{5}, \frac{7}{2}\right)$
(4) $\left(-\infty, \frac{4}{7}\right] \cup \left(\frac{7}{2}, \infty\right)$

G-6 Number of integers satisfying inequality $\log_{\frac{1}{3}}\left(\frac{3x-7}{2x}\right) \geq 0$ is

(1) 6
(2) 7
(3) 4
(4) 5

Section (H) : Determinants

H-1. The value of the determinant $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ is equal to

(1) – 40
(2) 40
(3) 28
(4) 52

H-2. Value of determinant $\begin{vmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \\ 2 & -1 & -1 \end{vmatrix}$ is

(1) 20 (2) 32 (3) 22 (4) –32

H-3. The value of k for which determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix}$ vanishes, is
(1) – 3 (2) 3 (3) –2 (4) 2

Exercise-2

Marked questions may have for revision questions.

PART - I : OBJECTIVE QUESTIONS

1. Let A = $\{x : x \in R, -1 < x < 1\}$, B = $\{x : x \in R, x \le 0 \text{ or } x \ge 2\}$ and $A \cup B = R - D$, then the set D is
(1) $\{x : 1 < x \le 2\}$ (2) $\{x : 1 \le x < 2\}$ (3) $\{x : 1 \le x \le 2\}$ (4) $\{x : 1 < x < 2\}$

2. Consider the following statements:
1. $N\cup(B\cap Z) = (N\cup B)\cap Z$ for any subset B of R, where N is the set of positive integers, Z is the set of integers, R is the set of real numbers.
2. Let A = $\{n \in N : 1 \le n \le 24$, n is a multiple of 3$\}$. There exists no subset B of N such that the number of elemets in A is equal to the number of elements in B.
Which of the above statements is/are correct? **[SCRA-2011, (2, –1/3)/100]**
(1) 1 only (2) 2 only (3) Both 1 and 2 (4) Neither 1 nor 2

3. Let A_1, A_2 and A_3 be subsets of a set X. Which one of the following is correct ?
(1) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing elements of each of A_1, A_2 and A_3
(2) $A_1 \cup A_2 \cup A_3$ is the smallest subset of X containing either A_1 or $A_2 \cup A_3$ but not both
(3) The smallest subset of X containing $A_1 \cup A_2$ and A_3 equals the smallest subset of X containing both A_1 and $A_2 \cup A_3$ only if $A_2 = A_3$ **[SCRA-2009, (2, –1/3)/100]**
(4) None of these

4. Let A, B, C be distinct subsets of a universal set U. For a subset X of U, let X ' denote the complement of X in U.
Consider the following sets :
1. $((A \cap B) \cup C)' \cap B')' = B \cap C$
2. $(A' \cap B') \cap (A \cup B \cup C') = (A \cup (B \cup C))'$
Which of the above statements is/are correct ? **[SCRA-2011, (2, –1/3)/100]**
(1) 1 only (2) 2 only (3) Both 1 and 2 (4) Neither 1 nor 2

5. Let U be set with number of elements in U is 2009.
Consider the following statements **:**
I If A, B are subsets of U with n (A ∪ B) = 280, then $n(A' \cap B') = x_1^3 + x_2^3 = y_1^3 + y_2^3$
for some positive integers x_1, x_2 y_1, y_2

II If A is a subset of U with n (A) = 1681 and out of these 1681 elements, exactly 1075 elements belong to a subset B of U , then n (A – B) = $m^2 + p_1 p_2 p_3$ for some positive integer m and distinct primes p_1, p_2, p_3

Which of the statements given above is / are correct ? **[SCRA-2009, (2, –1/3)/100]**

(1) I only (2) II only (3) Both I and II (4) Neither I nor II.

6. In a class of 42 students, the number of students studying different subjects are 23 in Mathematics, 24 in Physics, 19 in Chemistry, 12 in Mathematics and Physics 9 in Mathematics and Chemistry, 7 in Physics and Chemistry and 4 in all the three subjects. The number of students who have taken exactly one subject is

(1) 15 (2) 30 (3) 22 (4) 27

7. In an examination of a certain class, at least 70% of the students failed in Physics, at least 72% failed in Chemistry, at least 80% failed in Mathematics and at least 85% failed in English. How many at least must have failed in all the four subjects ? **[SCRA-2011, (2, –1/3)/100]**

(1) 5% (2) 7%

(3) 15% (4) Cannot be determined due to insufficient data

8. If $(a + b + c)^3 = a^3 + b^3 + c^3$ then (a + b)(b + c)(c + a) is equal to :

(1) 3 (2) 1 (3) 0 (4) –1

9. $f(x) = x^5 + ax^3 + bx$. The remainder when f(x) is divided by x + 1 is '–3', then the remainder when it is divided by $x^2 - 1$ is

(1) x (2) 2x (3) 3x (4) 4x

10. Let f(x) be a polynomial function. If f(x) is divided by x–1, x+1 & x+2, then remainders are 5, 3 and 2 respectively. When f(x) is divided by $x^3 + 2x^2 - x - 2$, then remainder is :

(1) x – 4 (2) x + 4 (3) x – 2 (4) x + 2

11. If $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ is a polynomial such that when it is divided by (x – 1) and (x + 1) the remainders are 5 and 19 respectively. If f(x) is divided by (x – 2), then remainder is

(1) 8 (2) 10 (3) 5 (4) 13

12. If $\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} \le 4$, then the least and the highest values of $4x^2$ are:

(1) 0 & 81 (2) 9 & 81 (3) 36 & 81 (4) 9 & $\frac{81}{4}$

13. Sum of integers satisfying inequality $\frac{14x}{x+1} \le \frac{9x-30}{x-4}$ is

(1) 5 (2) 6 (3) 11 (4) 12

14. If $\log_a b = 2$; $\log_b c = 2$ and $\log_3 c = 3 + \log_3 a$ then (a + b + c) equals

(1) 90 (2) 93 (3) 102 (4) 243

15. The sum of the solutions of the equation $9^x - 6 \cdot 3^x + 8 = 0$ is

(1) $\log_3 2$ (2) $\log_3 6$ (3) $\log_3 8$ (4) $\log_3 4$

Fundamental of Mathematics - I

MATHEMATICS

16. The expression: $\dfrac{\left(\dfrac{x^2+3x+2}{x+2}\right)+3x-\dfrac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)}$ reduces to

(1) $\dfrac{x+1}{x-1}$ (2) $\dfrac{x^2+3x+2}{(\log_2 5)x-1}$ (3) $\dfrac{3x}{x-1}$ (4) x

17. If a, b, c are positive real numbers such that $a^{\log_3 7} = 27$; $b^{\log_7 11} = 49$ and $c^{\log_{11} 25} = \sqrt{11}$. The value of $\left(a^{(\log_3 7)^2} + b^{(\log_7 11)^2} + c^{(\log_{11} 25)^2}\right)$ equals

(1) 489 (2) 469 (3) 464 (4) 400

18. The expression $\log_p \underbrace{\sqrt[p]{\sqrt[p]{\sqrt[p]{\cdots\cdots\cdots \sqrt[p]{p}}}}}_{\text{n radical sign}}$, where p ≥ 2, p ∈ N ; n ∈ N when simplified is

(1) independent of p (2) independent of p and of n

(3) dependent on both p and n (4) positive

19. The set of values of x satisfying simultaneously the inequalities $\dfrac{\sqrt{(x-8)\quad(2-x)}}{\log_{0.3}\left(\dfrac{10}{7}\quad(\log_2 5-1)\right)} \geq 0$ and

$2_{x-3} - 31 > 0$ is :

(1) a unit set (2) an empty set

(3) an infinite set (4) a set consisting of exactly two elements.

20. If $\log_{0.5} \log_5 (x_2 - 4) > \log_{0.5} 1$, then 'x' lies in the interval

(1) $(-3, -\sqrt{5}) \cup (\sqrt{5}, 3)$ (2) $(-3, -\sqrt{5}) \cup (, 3\sqrt{5})$

(3) $(\sqrt{5}, 3\sqrt{5})$ (4) φ

21. The solution set of the inequality $\dfrac{(3^x-4^x)\cdot \ell n(x+2)}{x^2-3x-4} \leq 0$ is

(1) (–∞, 0] ∪ (4, ∞) (2) (–2, 0] ∪ (4, ∞) (3) (–1, 0] ∪ (4, ∞) (4) (–2, –1) ∪ (– 1, 0] ∪ (4, ∞)

22. If $\begin{vmatrix} x^2-2x+3 & 7x+2 & x+4 \\ 2x+7 & x^2-x+2 & 3x \\ 3 & 2x-1 & x^2-4x+7 \end{vmatrix} = ax_6 + bx_5 + cx_4 + dx_3 + ex_2 + fx + g$ then value of g is

(1) 2 (2) 1 (3) - 204 (4) –108

23. If $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$ is defined ∀x∈R, then the set of values of 'a' is

(1) [9, ∞) (2) [10, ∞) (3) [15, ∞) (4) [2, ∞)

24. Product of roots of equation $(\log_3 x)^2 - 2(\log_3 x) - 5 = 0$ is

(1) 2 (2) 3 (3) 8 (4) 9

Fundamental of Mathematics - I

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1. Let X and Y be two sets.
Statement-1 $X \cap (Y \cup X)' = \varphi$
Statement-2 If $X \cup Y$ has m elements and $X \cap Y$ has n elements then symmetric difference $X \Delta Y$ has m – n elements.

A-2. **STATEMENT 1 :** The largest prime number, that can be written as the sum of two prime numbers and as the difference of two prime numbers is 5.
STATEMENT 2 : 2 is the only even prime number and 3 is the only prime number which is divisible by 3.

A-3. **STATEMENT 1 :** When a polynomial P(x) (degree > 2) is divided by (x – 1) and (x – 2) the remainders are –1 and 1 respectively. If the same polynomial is divided by (x – 1) (x – 2) then the remainder is (2x – 3).
STATEMENT 2 : If P(x) is divided by a quadratic expression, then the remainder is either 0 or a polynomial whose degree is at most 1.

A-4. **STATEMENT 1 :** $\log_{10}(\sqrt{13}-\sqrt{12}) < \log_{0.1}(\sqrt{14}-\sqrt{13})$
STATEMENT 2 : (i) If $a > 1$, then $x > 1 \Rightarrow \log_a x > 0$ and $0 < x < 1 \Rightarrow \log_a x < 0$
(ii) If $0 < a < 1$, then $x > 1 \Rightarrow \log_a x < 0$ and $0 < x < 1 \Rightarrow \log_a x > 0$

A-5. **STATEMENT 1 :** The equation $(\log_{10} x)^2 - \log_{10} x^3 + 2 = 0$ has only one solution.
STATEMENT 2 : $\log_{10} x^2 = 2\log_{10} x$, if $x > 0$

A-6. **STATEMENT - 1** : Maximum value of $\log_{1/3}(x^2 - 4x + 5)$ is '0'.
STATEMENT - 2 : $\log_a x \le 0$ for $x \ge 1$ and $0 < a < 1$.

Section (B) : MATCH THE COLUMN

B-1. Match the set P in column one with its super set Q in column II

Column – I (set P)		**Column– II** (set Q)	
(1)	$[3^{2n} - 8n - 1 : n \in N\}$	(p)	$\{49(n-1) : n \in N$
(2)	$\{2^{3n} - 1 : n \in N\}$	(q)	$\{64(n-1) : \in N\}$
(3)	$\{3^{2n} - 1 : n \in N\}$	(r)	$\{7n : n \in N\}$
(4)	$\{2^{3n} - 7n - 1 : n \in N\}$	(s)	$\{8n : n \in N\}$

B-2.

	Column-I	**Column-II**	
(1)	When the repeating decimal 0.363636............. is written as a rational	(p)	1

	fraction in the simplest form, the sum of the numerator and denominator is		
(2)	The number of solutions of $2_{2x} – 3_{2y} = 55$, in which x and y are integers, is	(q)	0
(3)	If $\log_8 a + \log_8 b = (\log_8 a)(\log_8 b)$ and $\log_a b = 3$, then the value of 'a' is	(r)	15
(4)	If P = $3^{\sqrt{\log_3 2}} - 2^{\sqrt{\log_2 3}}$ then value of P is	(s)	16

B-3.

	Column-I		Column-II
(1)	Anti logarithm of $(0.\bar{6})$ to the base 27 has the value equal to	(p)	5
(2)	Characteristic of the logarithm of 2008 to the base 2 is		
(3)	The value of b satisfying the equation, $\log_e 2 \cdot \log_b 625 = \log_{10} 16 \cdot \log_e 10$ is	(q)	7
(4)	Number of naughts after decimal before a significant figure	(r)	9
	comes in the number $\left(\frac{5}{6}\right)^{100}$, is (Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$)	(s)	10

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. A and B are two sets such that n(A) = 3 and n(B) = 6, then

(1) minimum value of $n(A \cup B) = 6$ (2) minimum value of $n(A \cup B) = 9$

(3) maximum value of $n(A \cup B) = 6$ (4) maximum value of $n(A \cup B) = 9$

C-2 In a survery, it was found that 21 persons liked product A, 26 liked product B and 29 liked product C. If 14 persons liked products A and B, 12 liked products C and A, 13 persons liked products B and C and 8 liked all the three products then which of the following is (are) true ?

(1) The number of persons who liked the product C only = 12

(2) The number of persons who like the products A and B but not C = 6

(3) The number of persons who liked the product C only = 6

(4) The number of persons who like the products A and B but not C = 12

C-3. If $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$, then $\frac{(a^k + b^k + c^k)^{\frac{1}{k}}}{(d^k + e^k + f^k)^{\frac{1}{k}}}$ is equal to : (k ∈ N)

(1) $\frac{a}{d}$ (2) $\frac{b}{e}$ (3) $\frac{c}{f}$ (4) $\frac{b}{c}$

C-4 Let a > 2, a ∈ N be a constant. If there are just 18 positive integers satisfying the inequality $(x – a)(x – 2a)(x – a_2) < 0$ then which of the option(s) is/are correct?

(1) 'a' is composite (2) 'a' is odd

(3) 'a' is greater than 8 (4) 'a' lies in the interval (3, 11)

C-5. Let N = $\frac{\log_3 135}{\log_{15} 3} - \frac{\log_3 5}{\log_{405} 3}$. Then N is :

(1) a natural number (2) a prime number (3) a rational number (4) an integer

Fundamental of Mathematics - I

C-6. Values of x satisfying the equation $\log_{52} x + \log_{5x}\left(\frac{5}{x}\right) = 1$ are

(1) 1 (2) 5 (3) $\frac{1}{25}$ (4) 3

C-7. The equation $\log_{x^2} 16 + \log_{2x} 64 = 3$ has :

(1) one irrational solution (2) no prime solution

(3) two real solutions (4) one integral solution

C-8. The solution set of the system of equations $\log_3 x + \log_3 y = 2 + \log_3 2$ and $\log_{27}(x + y) = \frac{2}{3}$ is :

(1) {6, 3} (2) {3, 6} (3) {6, 12} (4) {12, 6}

C-9 If $\frac{1}{2} \le \log_{0.1} x \le 2$, then

(1) maximum value of x is $\frac{1}{\sqrt{10}}$ (2) x lies between $\frac{1}{100}$ and $\frac{1}{\sqrt{10}}$

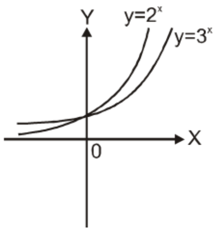
(3) minimum value of x is $\frac{1}{10}$ (4) minimum value of x is $\frac{1}{100}$

C-10. Which of the following statements are true

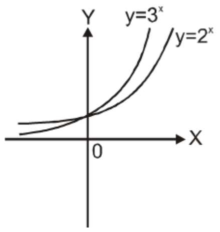
(1) $\log_2 3 < \log_{12} 10$ (2) $\log_6 5 < \log_7 8$

(3) $\log_3 26 < \log_2 9$ (4) $\log_{16} 15 > \log_{10} 11 > \log_7 6$

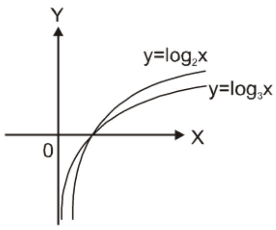
C-11 Which of the following is correct :



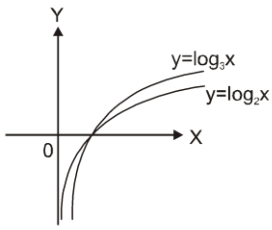
(1)



(2)



(3)



(4)

Fundamental of Mathematics - I

Exercise-3

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then **[AIEEE-2009, (4, – 1), 144]**

(1) A = C (2) B = C (3) $A \cap B = \varphi$ (4) A = B

2. Let X = {1, 2, 3, 4, 5}. The number of different ordered pairs (Y, Z) that can formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty, is : **[AIEEE-2012, (4, – 1), 120]**

(1) 5^2 (2) 3^5 (3) 2^5 (4) 5^3

3. If $X = \{4^n - 3n - 1 : n \in N\}$ and $Y = \{9(n - 1) : n \in N\}$, where N is the set of natural numbers, then $X \cup Y$ is equal to **[JEE(Main) 2014, (4, – 1), 120]**

(1) X (2) Y (3) N (4) Y – X

4. If a set contains m element and another set contains n element. If 56 is the difference between the number of subsets of both sets then find (m, n) **[BITSAT-2014]**

(1) 3, 6 (2) 6, 3 (3) 8, 3 (4) 3, 8

5. The sum of all real values of x satisfying the equation $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is **[JEE(Main) 2016, (4, – 1), 120]**

(1) – 4 (2) 6 (3) 5 (4) 3

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Marked questions may have for revision questions.

1. The number of solution(s) of $\log_4(x - 1) = \log_2(x - 3)$ is/are **[IIT-JEE-2002, Scr., (1, 0)/35]**

(A) 3 (B) 1 (C) 2 (D) 0

2. Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ell n2} = (3y)^{\ell n3}$

$3^{\ell nx} = 2^{\ell ny}$.

Then x_0 is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 6

Fundamental of Mathematics - I

Answers

EXERCISE - 1

PART- I

Section (A)

A-1.	(4)	**A-2.**	(2)	**A-3.**	(1)	**A-4.**	(4)	**A-5.**	(1)	**A-6.**	(4)	**A-7.**	(4)
A-8.	(3)	**A-9.**	(2)	**A-10.**	(2)								

Section (B)

B-1.	(2)	**B-2.**	(2)	**B-3.**	(2)	**B-4.**	(1)	**B-5.**	(4)	**B-6.**	(2)	**B-7.**	(1)
B-8.	(2)	**B-9.**	(3)	**B-10.**	(4)								

Section (C)

C-1.	(2)	**C-2.**	(3)	**C-3.**	(3)	**C-4.**	(3)	**C-5.**	(2)	**C-6.**	(3)	**C-7.**	(2)
C-8.	(4)	**C-9**	(2)	**C-10**	(3)								

Section (D)

D-1	(2)	**D-2.**	(4)	**D-3.**	(1)	**D-4.**	(1)	**D-5.**	(3)	**D-6.**	(1)	**D-7.**	(2)
D-8.	(1)	**D-9 .**	(3)	**D-10.**	(2)	**D-11.**	(1)						

Section (E)

E-1.	(1)	**E-2.**	(4)	**E-3.**	(4)	**E-4.**	(4)	**E-5.**	(4)	**E-6.**	(1)	**E-7.**	(2)
E-8.	(2)	**E-9.**	(3)	**E-10.**	(4)	**E-11.**	(4)						

Section (F) :

F-1.	(4)	**F-2.**	(2)	**F-3.**	(1)	**F-4.**	(4)	**F-5.**	(4)	**F-6.**	(4)	**F-7.**	(4)
F-8.	(1)	**F-9.**	(1)	**F-10**	(4)	**F-11.**	(4)	**F-12.**	(4)	**F-13.**	(3)	**F-14.**	(2)
F-15.	(3)	**F-16.**	(2)										

Section (G)

G-1.	(3)	**G-2.**	(1)	**G-3.**	(2)	**G-4.**	(4)	**G-5.**	(2)	**G-6**	(4)

Section (H) :

H-1.	(2)	**H-2.**	(2)	**H-3.**	(2)

EXERCISE - 2

PART- I

1.	(2)	**2.**	(1)	**3.**	(1)	**4.**	(2)	**5.**	(3)	**6.**	(3)	**7.**	(2)
8.	(3)	**9.**	(2)	**10.**	(3)	**11.**	(2)	**12.**	(1)	**13.**	(4)	**14.**	(2)
15.	(3)	**16.**	(1)	**17.**	(2)	**18.**	(1)	**19.**	(1)	**20.**	(1)	**21.**	(4)
22.	(4)	**23.**	(1)	**24.**	(4)								

PART- II

Section (A)

A-1. (1) **A-2.** (1) **A-3.** (1) **A-4.** (1) **A-5.** (3) **A-6.** (1)

Section (B)

B-1. (A) → (q), (B) → (r),(C) → (s), (D) → (p)

B-2. (A) → r, (B) → p, (C) → s, (D) → q

B-3. (A) → r, (B) → s, (C) → p, (D) → q

Section (C)

C-1. (1,4) **C-2** (1,2) **C-3.** (1,2,3) **C-4** (2,4) **C-5.** (1,2,3,4)
C-6. (1,2,3) **C-7.** (1,2,3,4) **C-8.** (1,2) **C-9** (1,2,4) **C-10.**(2,3)
C-11 (2,3)

EXERCISE - 3

PART- I

1. (2) **2.** (2) **3.** (2) **4.** (1,2) **5.** **(4)**

PART- II

1. (2) **2.** (3)