Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A): Matrix, Trace of matrix, Algebra of matrices, transpose of a matrix, symmetric and skew symmetric matrix

A-1. The number of different possible orders of matrices having 18 identical elements is
(1) 3 (2) 1 (3) 6 (4) 4

A-2. A 3 × 2 matrix whose elements are given by $a_{ij} = 2i - j$ is
(1) $\begin{bmatrix} 1 & 2 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 \\ 3 & -2 \\ 5 & 4 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

A-3. If $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$, then x + y + z + w =
(1) 10 (2) 8 (3) 9 (4) 12

A-4. $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$ then x is equal to -
(1) – 1 (2) 0 (3) 1 (4) No value of x

A-5. If I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, J = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and B = $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B =
(1) Icosθ + Jsinθ (2) Icosθ – Jsinθ (3) Isinθ + Jcosθ (4) – Icosθ + Jsinθ

A-6. If A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and B = $\begin{bmatrix} -5 & 4 & 0 \\ 0 & 2 & -1 \\ 1 & -3 & 2 \end{bmatrix}$, then
(1) AB = $\begin{bmatrix} -5 & 8 & 0 \\ 0 & 4 & -2 \\ 3 & -9 & 6 \end{bmatrix}$ (2) AB = [– 2 – 1 4]
(3) AB = $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ (4) AB does not exist

A-7. If $\begin{bmatrix} 1 & x & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ = O then x =
(1) 2 (2) –2 (3) $\frac{5}{2}$ (4) – $\frac{9}{8}$

A-8. If A and B are square matrices of order 2, then $(A + B)^2 =$

(1) $A^2 + 2AB + B^2$ (2) $A^2 + AB + BA + B^2$ (3) $A^2 + 2BA + B^2$ (4) $A^2 + B^2$

A-9. Which relation is true for $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix}$

(1) $(A + B)^2 = A^2 + 2AB + B^2$ (2) $(A - B)^2 = A^2 - 2AB + B^2$

(3) $AB = BA$ (4) $AB \neq BA$

A-10. If $A = \begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix}$ and $A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ then λ is equal to

(1) ± 3 (2) 2 (3) 3 (4) -3

A-11. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then $= \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

(1) A^2 (2) A^3 (3) A^4 (4) A^5

A-12. If $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$, then which one of the following statements is **not** correct ?

(1) $A^3 - I = A(A - I)$ (2) $A^3 + I = A(A^3 - I)$ (3) $A^2 + I = A(A^2 - I)$ (4) $A^4 - I = A^2 + I$

A-13. If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ then $(I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} =$

(1) $I + A$ (2) $I + 2A$ (3) $2A$ (4) $I + 3A$

A-14. $A = \begin{bmatrix} 2 & -1 \\ -7 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 1 \\ 7 & 2 \end{bmatrix}$ then $B^T A^T$ is

(1) a null matrix (2) an identity matrix

(3) scalar, but not an identity matrix (4) such that $\text{Tr}(B^T A^T) = 4$

A-15. Matrix $A = \begin{bmatrix} 5 & -3 & 6 \\ -3 & 7 & -4 \\ 6 & -4 & 8 \end{bmatrix}$ is a

(1) diagonal matrix (2) skew symmetric matrix

(3) symmetric matrix (4) lower triangular matrix

A-16. If A is a skew- symmetric matrix, then trace of A is

(1) 1 (2) – 1 (3) 0 (4) 4

A-17. Which one of the following is wrong ?

(1) The elements on the main diagonal of a symmetric matrix are all zero

(2) The elements on the main diagonal of a skew - symmetric matrix are all zero

(3) For any square matrix A, $\frac{1}{2}(A + A')$ is symmetric matrix

(4) For any square matrix A, $\frac{1}{2}(A - A')$ is skew - symmetric matrix

A-18. Let $A = [a_{ij}]_{n \times n}$ where $a_{ij} = i^2 - j^2$. Then A is :

(1) skew-symmetric matrix. (2) symmetric matrix

(3) null matrix (4) unit matrix

A-19. If A and B are symmetric matrices, then ABA is
(1) symmetric matrix (2) skew-symmetric matrix (3) diagonal matrix (4) scalar matrix

Section (B) : Minors, Cofactors, Expansion of determinant, Properties of determinant, Summation, Differentiation, Multiplication of determinants, Property $|kA| = k^n|A|$ and $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$

B-1. The sum of the minors of all elements in the second row of determinant $\begin{vmatrix} 1 & 2 \\ -3 & 4 \end{vmatrix}$ is
(1) 2 (2) 0 (3) 3 (4) 4

B-2. The sum of the minors of all elements in the second row of determinant $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$ is
(1) –6 (2) 6 (3) 0 (4) 4

B-3. If the minor of three-one element (i.e. M_{31}) in the determinant $\begin{vmatrix} 0 & 1 & \sec\alpha \\ \tan\alpha & -\sec\alpha & \tan\alpha \\ 1 & 0 & 1 \end{vmatrix}$ is 1 then the value of α $(0 \le \alpha \le \pi)$ is
(1) 0 (2) $\frac{3\pi}{4}$ (3) π (4) all of these

B-4. If in the determinant $\begin{vmatrix} x & 3 & 3 \\ 3 & 3 & x \\ 2 & 3 & 3 \end{vmatrix}$, $C_{11} = C_{22}$, where C_{ij} is cofactor of element a_{ij} then x =
(1) 2 (2) –2 (3) $\frac{5}{2}$ (4) $-\frac{9}{8}$

B-5 In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then $\sin^2A + \sin^2B + \sin^2C$ is :
(1) $\frac{3\sqrt{3}}{2}$ (2) $\frac{9}{4}$ (3) $\frac{5}{4}$ (4) 2

B-6. The number of distinct real roots of the equation, $\begin{vmatrix} \cos x & \sin x & \text{six} \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$ in the interval $\left[-\frac{\pi}{4}\ \frac{\pi}{4}\right]$ is :
(1) 4 (2) 1 (3) 2 (4) 3

B-7. Let a, b > 0 and $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$, then
(1) a + b – x is a factor of Δ (2) $x^2 + (a + b)x + a^2 + b^2 - ab$ is a factor of Δ
(3) Δ = 0 has three real roots if a = b (4) all of these

B-8. If α, β & γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is-

(1) p (2) q (3) $p^2 - 2q$ (4) 0

B-9. The non − zero roots of the equation $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix} = 0$ are

(1) 2 (2) 4 (3) 1 (4) $\frac{11}{2}$

B-10 If $x_1, x_2, x_3, \ldots, x_{13}$ are in A.P. then the value of $\begin{vmatrix} e^{x_1} & e^{x_4} & e^{x_7} \\ e^{x_4} & e^{x_7} & e^{x_{10}} \\ e^{x_7} & e^{x_{10}} & e^{x_{13}} \end{vmatrix}$ is -

(1) 9 (2) 27 (3) 0 (4) 1

B-11. The absolute value of minimum value of the determinant $\begin{vmatrix} \sin\theta & 1 & 0 \\ 1 & \cos\phi & -\cos\theta \\ \sin\phi & 0 & 1 \end{vmatrix}$ is

(1) 2 (2) –2 (3) $\frac{5}{2}$ (4) $-\frac{9}{8}$

B-12 The value of the determinant $\begin{vmatrix} \sqrt{13} & +\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} & +\sqrt{26} & 5 & \sqrt{10} \\ 3 & +\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$ is equal to :

(1) $5\sqrt{3}\,(\sqrt{6} - 5)$ (2) $5\sqrt{3}\,(\sqrt{6} - \sqrt{5})$ (3) $5(\sqrt{3} - 5)$ (4) $\sqrt{3}\,(\sqrt{6} - \sqrt{5})$

B-13. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax - 12$, then 'a' is equal to :

(1) 24 (2) –12 (3) –24 (4) 12

B-14. The determinant $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+a_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix} =$

(1) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (2) $2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (3) $3\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ (4) $2\begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix}$

B-15. If $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$, then $\sum_{n=1}^{N} U_n$ is equal to

(1) $2\sum_{n=1}^{N} n$ (2) $2\sum_{n=1}^{N} n^2$ (3) $\frac{1}{2}\sum_{n=1}^{N} n^2$ (4) 0

B-16. If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$, $x \in R$, then $\frac{d^2y}{dx^2} + y$ is equal to :
(1) 6 (2) 4 (3) –10 (4) 0

B-17. if A and B are square matrices of order 3 such that |A| = – 1, |B| = 3, then |3AB| is equal to
(1) – 9 (2) – 81 (3) – 27 (4) 81

B-18. If A is a square matrix of order 3, then the true statement is (where I is unit matrix).
(1) det (− A) = − det A (2) det A = 0
(3) det (A + I) = 1 + det A (4) det (2A) = 2 det A

Section (C) : Cramer rule

C-1. The system of linear equations x + y − z = 6, x + 2y − 3z = 14 and 2x + 5y − λz = 9 (λ ∈ R) has a unique solution if
(1) λ = 8 (2) λ ≠ 8 (3) λ = 7 (4) λ ≠ 7

C-2. The system of equation − 2x + y + z = 1, x − 2y + z = −2, x + y + λz = 4 will have no solution if
(1) λ = − 2 (2) λ = −1 (3) λ = 3 (4) λ = 2

C-3. If the system of equations x + 2y + 3z = 4, x + py + 2z = 3, x + 4y + μz = 3 has an infinite number of solutions, then :
(1) p = 2, μ = 3 (2) p = 2, μ = 4 (3) 3 p = 2 μ (4) p = 4, μ = 2

C-4. The system of linear equations x – y + z = 1, x + y – z = 3, x – 4y + 4z = α has :
(1) a unique solution when α = 2 (2) a unique solution when α ≠ 2
(3) an infinite number of solutions, when α = 2 (4) an infinite number of solutions, when α = – 2

C-5. If a ≠ b, then the system of equations ax + by + bz = 0, bx + ay + bz = 0, bx + by + az = 0 will have a non-trivial solution if
(1) a + b = 0 (2) a + 2b = 0 (3) 2a + b = 0 (4) a + 4b = 0

C-6. If a, b, c are non zeros, then the system of equations
(α + a) x + αy + αz = 0, αx + (α + b)y + αz = 0, αx + αy + (α + c)z = 0 has a non-trivial solution if
(1) $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$ (2) $\alpha^{-1} = a + b + c$
(3) α + a + b + c = 1 (4) α = a + b + c

C-7. The value of a for which system of equations, $a^3x + (a + 1)^3y + (a + 2)^3z = 0$,
ax + (a + 1) y + (a + 2) z = 0, x + y + z = 0, has a non−zero solution is:
(1) −1 (2) 0 (3) 1 (4) 2

C-8. A man has to distribute three types of chocolates A,B,C to students of three schools E, F, G. IF he distribute x, y, z chocolates to 2, 1, and 1 students of "E" corrospondingly y, 3, 2 and 4 students of school "F" corrosponding and 1, 1 and 1 students of school "G". If he distributed total 15 chocolates in school E, 37 in school F and 12 in school "G" then find x, y, and z ?
(1) x = 3 ; y = 4 ; z = 5 (2) x = 1 ; y = 2 ; z = 5 (3) x = 1 ; y = 2 ; z = 0 (4) x = 3 ; y = –4 ; z = 5

Section (D) : Adjoint of Matrix, Inverse of matrix and their properties, solution of system of linear equations by matrix method, Cayley-Hamilton theorem

D-1. If A = $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, then adj A =

(1) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & -2 \\ -2 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

D-2. If A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then adj A =

(1) A' (2) I (3) O (4) A^2

D-3. If A is a square matrix of order n × n and k is a scalar, then adj (kA) is equal to

(1) k adj A (2) k^n adj A (3) k^{n-1} adj A (4) k^{n+1} adj A

D-4. If A is square matrix of order n then adj(adj A) =

(1) $|A|^{n-1}$ A (2) $|A|^{n-2}$ A (3) $|A|^{n-2}$ (4) $|A|^n$ A

D-5. If A is square matrix of order 3 then |adj(adj A)| =

(1) |A| (2) $|A|^2$ (3) $|A|^3$ (4) $|A|^4$

D-6. If for a matrix A, |A| = 6 and adj A = $\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$, then k is equal to :

(1) –1 (2) 0 (3) 1 (4) 2

D-7. If A is a 3 × 3 matrix such that |5.adjA| = 5, then |A| is equal to :

(1) $\pm \frac{1}{5}$ (2) $\pm \frac{1}{25}$ (3) ± 1 (4) ± 5

D-8. Given A = $\begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$; I = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ If A – λ I is a singular matrix then

(1) $\lambda \in \varphi$ (2) $\lambda^2 - 3\lambda - 4 = 0$ (3) $\lambda^2 + 3\lambda + 4 = 0$ (4) $\lambda^2 - 3\lambda - 6 = 0$

D-9. From the matrix equation AB = AC, we conclude B = C provided :

(1) A is singular matrix (2) A is non−singular matrix
(3) A is symmetric matrix (4) A is a square matrix

D-10. Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, be a 2 × 2 matrix where a,b,c,d take the values 0 or 1 only. The number of such matrices which have inverses is:

(1) 8 (2) 7 (3) 6 (4) 5

D-11. If A, B are two n × n non-singular matrices, then

(1) AB is non-singular matrix (2) AB is singular matrix
(3) $(AB)^{-1} = A^{-1} B^{-1}$ (4) $(AB)^{-1}$ does not exist

D-12. If F (α) = $\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and G (β) = $\begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$, then $[F(\alpha)\,G(\beta)]^{-1}$=
(1) F (α) − G(β) (2) − F (α) − G (β) (3) $[F(\alpha)]^{-1}[G(\beta)]^{-1}$ (4) $[G(\beta)]^{-1}[F(\alpha)]^{-1}$

D-13. If A^{-1} = $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ & B = $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, then $(AB)^{-1}$
(1) $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (2) $\begin{bmatrix} 9 & 3 & 5 \\ 2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} 9 & 3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & -2 \end{bmatrix}$

D-14. If A^{-1} = $\begin{bmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$, then
(1) | A | = 2 (2) A is singular matrix
(3) Adj. A = $\begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{bmatrix}$ (4) A is skew symmetric matrix

D-15. If B is a non-singular matrix and A is a square matrix, then det $(B^{-1}AB)$ is equal to
(1) det (A^{-1}) (2) det (B^{-1}) (3) det (A) (4) det (B)

D-16. If A is a square matrix such that $A^2 = I$, then A^{-1} =
(1) 2A (2) A (3) O (4) A + I

D-17. Let A = $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and X be a matrix such that A = BX, then X is equal to
(1) $\frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (2) $\frac{1}{2}\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$ (3) $\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 4 \\ 3 & 5 \end{bmatrix}$

D.18. If A = $\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ and |A| ≠ 0, then the system of equations $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3z = 0$ has
(1) only one solution (2) infinite number of solutions
(3) no solution (4) more than one but finite number of solutions

D-19. The system of equations
$$\begin{aligned} 4x - 5y - 2z &= 2 \\ 5x - 4y + 2z &= 3 \\ 2x + 2y + 8z &= 1 \end{aligned}$$
(1) has a unique solution (2) has infinite solutions
(3) has two solutions (4) is inconsistent

D-20. Identify the correct statement(s)
(1) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is singular
(2) If system of n simultaneous linear equations has a unique solution, then coefficient matrix is

non singular

(3) If A^{-1} exists, $(\text{adj } A)^{-1}$ may or may not exist

(4) $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 0 \end{bmatrix}$, then F(x) . F(y) = F(x – y)

D-21 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$, then the value of c and d are

(1) –6, – 11 (2) 6, 11 (3) – 6, 11 (4) 6, – 11

D-22. If $A = \begin{bmatrix} -4 & 1 \\ 3 & 1 \end{bmatrix}$, then the determinant of the matrix $(A^{2016} - 2A^{2015} - A^{2014})$ is

(1) 2014 (2) 2016 (3) –175 (4) –25

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. If AB = O for the matrices

$A = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ and $B = \begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ then $\theta - \varphi$ is

(1) an odd multiple of $\frac{\pi}{2}$ (2) an odd multiple of π

(3) an even multiple of $\frac{\pi}{2}$ (4) 0

2. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$ and AB is equal to kB, then the value of k is

(1) 2 (2) 3 (3) 0 (4) 6

3. If $X = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then value of X^n is, (where n is natural number)

(1) $\begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ (2) $\begin{bmatrix} 2+n & 5-n \\ n & -n \end{bmatrix}$ (3) $\begin{bmatrix} 3^n & (-4)^n \\ 1^n & (-1)^n \end{bmatrix}$ (4) $\begin{bmatrix} 2n+1 & -4n \\ n & -(2n-1) \end{bmatrix}$

4. Suppose A is a matrix such that $A^2 = A$ and $(I + A)^{10} = I + kA$, then k =

(1) 1023 (2) 1024 (3) 1047 (4) 2048

5. Which of the following is incorrect

(1) $A^2 - B^2 = (A + B)(A - B)$ (2) $(A^T)^T = A$

(3) $(AB)^n = A^nB^n$, where A, B commute (4) $(A - I)(I + A) = O \Leftrightarrow A^2 = I$

6. Let S be the set of all real matrices, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that a + d = 2 and $A^T = A^2 - 2A$. Then S

(1) has exactly two elements. (2) has exactly four elements.

(3) is an empty set. (4) has exactly one element.

7. If A is a square matrix, then

(1) AA' is symmetric matrix
(2) AA' is skew - symmetric matrix
(3) A'A is skew - symmetric matrix
(4) A_2 is symmetric matrix

8. If A is a skew - symmetric matrix and n is an even positive integer, then A_n is

(1) a symmetric matrix
(2) a skew-symmetric matrix
(3) a diagonal matrix
(4) a scalar matrix

9. If A is a skew–symmetric matrix of odd order then det(A) is

(1) 1 (2) 8 (3) 0 (4) –1

10. If $D = \begin{vmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$ then D =

(1) $1 + a_2 + b_2 + c_2$ (2) $a_2 + b_2 + c_2$ (3) $(a + b + c)_2$ (4) $a_2 + b_2 + c_2 - 1$

11. The determinant $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ is divisible by

(1) 1 + x (2) $(1 + x)_2$ (3) x_2 (4) $x_2 + 1$

12. $\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$ is equal to

(1) $(a - b)_2 (b - c)_2 (c - a)_2$
(2) 2(a – b) (b – c) (c – a)
(3) 4(a – b) (b – c) (c – a)
(4) $(a + b + c)_3$

13. Let $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$, then

(1) Δ is independent of θ
(2) Δ is indepedent of φ
(3) Δ is a constant
(4) Δ is depedent of φ

14. The value of the determinant $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$ is

(1) 0 (2) independent of θ (3) independent of φ (4) independent of θ & φ both

15. The non – zero roots of the equation $\Delta = \begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & c \end{vmatrix} = 0$ are

(1) ab (2) $-\frac{b}{a}$ (3) $-\frac{2b}{a}$ (4) $\frac{2b}{a}$

16 If $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = k\,abc\,(a+b+c)^3$ then the value of k is

(1) 2 (2) –2 (3) $\frac{5}{2}$ (4) $-\frac{9}{8}$

17. If f(x) = $\begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x^2-1) \end{vmatrix}$ then f '(5) is equal to

(1) 5050 (2) 100 (3) 0 (4) –100

18. Let p, q, r be real numbers such that p + q + r ≠ 0. The system of linear equations x + 2y – 3z = p, 2x + 6y – 11z = q , x – 2y + 7z = r has at least one solution if :

(1) 5p + 2q – r = 0 (2) 5p – 2q – r = 0 (3) 5p + 2q + r = 0 (4) 5p – 2q + r = 0

19. For what values of λ, μ the simultaneous equations

x + y + z = 6; x + 2 y + 3 z = 10 & x + 2 y + λ z = μ have a unique solution;

(1) λ = 3 (2) λ ≠ 3, μ ∈ R (3) λ ≠ 3, μ ≠ 2 (4) λ = 3, μ = 2

20. For what values of λ, μ the simultaneous equations

x + y + z = 6; x + 2 y + 3 z = 10 & x + 2 y + λ z = μ have infinite number of solutions

(1) λ = 3, μ = 5 (2) λ = 3, μ = 10 (3) λ ≠ 3, μ ≠ 2 (4) λ = 3, μ = 2

21. For what values of λ, μ the simultaneous equations

x + y + z = 6; x + 2 y + 3 z = 10 & x + 2 y + λ z = μ have no solution

(1) λ = 3, μ = 10 (2) λ = 3, μ ≠ 10 (3) λ ≠ 3, μ ≠ 2 (4) λ = 4, μ = 2

22. The value of ' k ' for which the set of equations 3x + ky − 2z = 0, x + ky + 3z = 0, 2 x + 3 y − 4 z = 0 has a non − trivial solution over the set of rational is:

(1) $\frac{33}{2}$ (2) $\frac{31}{2}$ (3) 16 (4) 15

23. If the system of linear equations, x + 2ay + az = 0, x + 3by + bz = 0 and x + 4cy + cz = 0 has a non-zero solution, then a, b, c satisfy

(1) 2ac = ab + bc (2) 2ab = ac + bc (3) 2b = a + c (4) b^2 = ac

24. If the system of linear equations :

x + 3y + 7z = 0, – x + 4y + 7z = 0, (sin3θ) x + (cos2θ)y + 2z = 0

has a non-trivial solution, then the number of values of θ lying in the interval [0, π], is

(1) one (2) two (3) three (4) more than three

25. If the system of equations ax + y + z = 0,. x + by + z = 0 and x + y + cz = 0, where a, b, c ≠ 1, has a non−trivial solution, then the value of $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c}$ is

(1) 0 (2) 1 (3) –1 (4) 2

26. If A is a non−singular matrix and A^T denotes the transpose of A, then:

(1) $|A| \neq |A^T|$ (2) $|A. A^T| \neq |A|^2$ (3) $|A^T A| \neq |A^T|^2$ (4) $|A|+|A^T| \neq 0$

27. If A' denotes transpose of matrix A, A' A = I and det A = 1, then det (A – I) must be equal to
(1) 0 (2) – 1 (3) 1 (4) 2

28. For any 2 × 2 matrix A, if A (adjA) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then |A| =
(1) 10 (2) 100 (3) 0 (4) 1000

29. If A = $[a_{ij}]_{3\times3}$ is a scalar matrix with $a_{11} = a_{22} = a_{33} = 2$ and A(adjA) = kI_3 then k is
(1) 1 (2) 8 (3) 0 (4) –1

30. Let A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$. If $10A^{10} + \text{adj}(A^{10}) = B$, then $b_1 + b_2 + b_3 + b_4$ is equal to
(1) 91 (2) 92 (3) 111 (4) 112

31. Matrix $\begin{bmatrix} a & b & (a\alpha - b) \\ b & c & (b\alpha - c) \\ 2 & 1 & 0 \end{bmatrix}$ is non invertible if
(1) α = 1 (2) a, b, c are in A.P. (3) a, b, c are in G.P. (4) a, b, c are in H.P.

32. If A and B are two invertible matrices such that AB = C and |A| = 2, |C| = – 2,then det(B) is
(1) 1 (2) 8 (3) 0 (4) –1

33. If D is a determinant of order three and Δ is a determinant formed by the cofactors of determinant D ; then
(1) $\Delta = D^2$ (2) $\Delta = D^3$
(3) if D = 9, then Δ is perfect cube (4) Δ = D

34. If A = $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then value of A^{-1} is equal to :
(1) A (2) A^2 (3) A^3 (4) A^4

35. If A = $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then (5A – I) (A – I) =
(1) A^2 (2) A^3 (3) A^4 (4) A

36. If A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ (where bc ≠ 0) satisfies the equations $x^2 + k = 0$, then
(1) a – d = 0 (2) k = – |A| (3) k = |A| (4) k = a + d

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1. Let A be set of all determinants of order 3 with entries 0 or 1, B be the subset of A consisting of all determinants with value 1 and C be the subset of A consisting of all determinants with value –1. Then
STATEMENT -1 : The number of elements in set B is equal to number of elements in set C.
and
STATEMENT-2 : $(B \cap C) \subseteq A$

A-2. **Statement 1 :** If A = $\begin{bmatrix} 2 & 1+2i \\ 1-2i & 7 \end{bmatrix}$ then det(A) is real.
Statement 2 : If A = $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, a_{ij} being complex numbers then det(A) is always real.

A-3. **Statement 1 :** If A = $\begin{bmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+xc & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{bmatrix}$ and B = $\begin{bmatrix} x & c & -b \\ -c & x & a \\ b & -a & x \end{bmatrix}$, then $|A| = |B|^2$.
Statement 2 : If A_c is cofactor matrix of a square matrix A of order n then $|A_c| = |A|^{n-1}$.

A-4. Let A and B be two 2 × 2 matrices.
Statement - 1 : A(adj A) = | A | I_2
Statement - 2 : adj(AB) = (adj A) (adj B)

A-5. Let A be a 3 × 3 matrix such that $A^2 - 5A + 7I = 0$.
Statement – I : $A^{-1} = \frac{1}{7}(5I - A)$.
Statement – II : The polynomial $A^3 - 2A^2 - 3A + I$ can be reduced to $5(A - 4I)$.

Section (B) : MATCH THE COLUMN

B-1. The system of equations
x + y + z = 3
x + 2y + 3z = 6
x + 3y + λz = m

	Column -I		**Column -II**
(A)	has unique solution	(p)	λ = 5, m = 9
(B)	has no solution	(q)	λ = 5, m ≠ 9
(C)	has infinite number of solutions	(r)	λ ≠ 5, m ≠ 9
(D)	is inconsistent	(s)	λ ∈ R

B-2. Let f(x) = $\begin{vmatrix} 2\cot x & -1 & 0 \\ 1 & \cot x & -1 \\ 0 & 1 & 2\cot x \end{vmatrix}$ then

	Column -I		**Column -II**
(A)	$3f'\left(\frac{\pi}{3}\right)$	(p)	– 16

(B) $f\left(\frac{\pi}{4}\right)$ (q) -32

(C) $f'\left(\frac{\pi}{4}\right)$ (r) 8

(D) $f\left(\frac{\pi}{2}\right)$ (s) 0

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1 If $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, where a, b, c are real positive numbers, $abc = 1$ and $A^T A = I$, then

(1) $ab + bc + ca = 0$ (2) $a^2 + b^2 + c^2 = 1$ (3) $a^3 + b^3 + c^3 = 4$ (4) $a + b + c = 2$

C-2. If $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then tan x is

(1) 0 (2) –2 (3) 1 (4) 3

C-3. If the system of equation $x - ky - z = 0$, $kx - y - z = 0$, $x + y - z = 0$ has a non-zero solution, then the possible values of k are

(1) 1 (2) 2 (3) 0 (4) –1

C-4. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)

(1) –2 (2) –1 (3) 1 (4) 2

C-5. For 3×3 matrices M and N, which of the following statement(s) is (are) **NOT** correct ?

(1) $N^T M N$ is symmetric or skew symmetric, according as M is symmetric or skew symmetric

(2) M N – N M is skew symmetric for all symmetric matrices M and N

(3) M N is symmetric for all symmetric matrices M and N

(4) (adj M) (adj N) = adj(MN) for all invertible matrices M and N

C-6. Let M be a 2 × 2 symmetric matrix with integer entries. Then M is invertible if

(1) the first column of M is the transpose of the second row of M

(2) the second row of M is the transpose of first column of M

(3) M is a diagonal matrix with nonzero entries in the main diagonal

(4) the product of entries in the main diagonal of M is not the square of an integer

C-7. Let M and N be two 3 × 3 matrices such that MN = NM. Further, if $M \ne N^2$ and $M^2 = N^4$, then

(1) determinant of $(M^2 + MN^2)$ is 0

(2) there is a 3 × 3 non-zero matrix U such that $(M^2 + MN^2)U$ is the zero matrix

(3) determinant of $(M^2 + MN^2) \ge 1$

(4) for a 3 × 3 matrix U, if $(M^2 + MN^2)U$ equals the zero matrix then U is the zero matrix

C-8. Let X and Y be two arbitrary, 3 × 3, non-zero, skew-symmetric matrices and Z be an arbitrary 3 × 3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric ?

(1) $Y^3Z^4 - Z^4Y^3$ (2) $X^{44} + Y^{44}$ (3) $X^4Z^3 - Z^3X^4$ (4) $X^{23} + Y^{23}$

C-9. Which of the following values of α satisfy the equation $\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648\alpha$?

(1) – 4 (2) 9 (3) – 9 (4) 4

Exercise-3

Marked Questions may have for Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let A = $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and B = $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, a, b $\in$ N. Then, **[AIEEE 2006, (3, –1), 120]**
(1) there exist more than one but finite number of B's such that AB = BA
(2) there exists exactly one B such that AB = BA
(3) there exists infinitely many B's such that AB = BA
(4) there cannot exist any B such that AB = BA

2. If A and B are square matrices of size n × n such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true ? **[AIEEE 2006, (3, –1), 120]**
(1) AB = BA (2) either A or B is a zero matrix
(3) either A or B is an identity matrix (4) A = B

3. Let A = $\begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals - **[AIEEE 2007 (3, –1), 120]**
(1) 5^2 (2) 1 (3) $\frac{1}{5}$ (4) 5

4. If D = $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for x ≠ 0, y ≠ 0, then D is **[AIEEE 2007 (3, –1), 120]**
(1) divisible by y but not x (2) divisible by neither x nor y
(3) divisible by both x and y (4) divisible by x but not y

5. Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that x = cy + bz, y = az + cx and z = bx + ay. Then $a^2 + b^2 + c^2 + 2abc$ is equal to **[AIEEE 2008 (3, –1), 105]**
(1) 2 (2) –1 (3) 0 (4) 1

6. Let A be a square matrix all of whose entries are integers. Then, which one of the following is true ? **[AIEEE 2008 (3, –1), 105]**
(1) If det (A) = ± 1, then A^{-1} exists but all its entries are not necessarily integers
(2) If det (A) ≠ ± 1, then A^{-1} exists but all its entries are non-integers
(3) If det (A) = ± 1, then A^{-1} exists and all its entries are integers
(4) If det (A) = ± 1, then A^{-1} need not exist

7. Let A be a 2 × 2 matrix with real entries. Let I be the 2 × 2 identity matrix. Denote by tr(A), the sum of diagonal entries of A. Assume that $A^2 = I$. **[AIEEE 2008 (3,–1),105]**
Statement-1 If A ≠ I and A ≠ – I, then det (A) = – 1.
Statement-2 If A ≠ I and A ≠ – I, then tr (A) ≠ 0.
(1) Statement-1 is false, statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statment-2 is a correct explanation for statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1.
(4) Statement-1 is true, statement-2 is false.

8. Let A be a 2 × 2 matrix.
Statement-1 : adj(adj (A)) = A.
Statement-2 : |adj A| = |A| **[AIEEE 2009 (4, –1), 144]**
(1) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is false.

(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

9. Let a, b, c be such that b(a + c) ≠ 0. If $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ (-1)^{n+2}a & (-1)^{n+1}b & (-1)^{n}c \end{vmatrix}$ = 0. Then the value of 'n' is - **[AIEEE 2009 (4, –1), 144]**
(1) zero (2) any even integer (3) any odd integer (4) any integer

10. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is **[AIEEE 2010 (8, –2), 144]**
(1) 5 (2) 6 (3) at least 7 (4) less than 4

11. Let A be a 2 × 2 matrix with non-zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix.
Tr(A) = sum of diagonal elements of A and |A| = determinant of matrix A. **[AIEEE 2010 (4, –1), 144]**
Statement -1 : Tr(A) = 0
Statement -2 : |A| = 1
(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

12. Consider the system of linear equations : **[AIEEE 2010 (4, –1), 144]**
$x_1 + 2x_2 + x_3 = 3$, $2x_1 + 3x_2 + x_3 = 3$, $3x_1 + 5x_2 + 2x_3 = 1$
The system has
(1) exactly 3 solutions (2) a unique solution (3) no solution (4) infinite number of solutions

13. Let A and B be two symmetric matrices of order 3. **[AIEEE 2011, I, (4, –1), 120]**
Statement-1 : A(BA) and (AB)A are symmetric matrices.
Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.
(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1.
(3) Statement-1 is true, Statement-2 is false.
(4) Statement-1 is false, Statement-2 is true.

14. The number of values of k for which the linear equations **[AIEEE 2011, I, (4, –1), 120]**
4x + ky + 2z = 0 , kx + 4y + z = 0 , 2x + 2y + z = 0 posses a non-zero solution is :
(1) 3 (2) 2 (3) 1 (4) zero

15. If ω ≠ 1 is the complex cube root of unity and matrix H = $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to - **[AIEEE 2011, I, (4, –1), 120]**
(1) 0 (2) – H (3) H^2 (4) H

16. If the trivial solution is the only solution of the system of equations **[AIEEE 2011, II, (4, –1), 120]**
x – ky + z = 0 , kx + 3y – kz = 0 , 3x + y – z = 0 then the set of all values of k is :
(1) R – {2, –3} (2) R – { 2 } (3) R – { –3 } (4) {2, –3}

17. **Statement - 1 :** Determinant of a skew-symmetric matrix of order 3 is zero.
Statement - 2 : For any matrix A, det $(A)^T$ = det(A) and det (–A) = – det(A).
Where det (B) denotes the determinant of matrix B. Then : **[AIEEE 2011, II, (4, –1), 120]**
(1) Both statements are true (2) Both statements are false

(3) Statement-1 is false and statement-2 is true. (4) Statement-1 is true and statement-2 is false

18 Let A = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$. If u_1 and u_2 are column matrices such that $Au_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $Au_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, then $u_1 + u_2$ is equal to : **[AIEEE-2012, (4, –1)/120]**

(1) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ (2) $\begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$ (3) $\begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix}$ (4) $\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

19. Let P and Q be 3 × 3 matrices P ≠ Q. If $P^3 = Q^3$ and $P^2Q = Q^2P$, then determinant of $(P^2 + Q^2)$ is equal to :

[AIEEE-2012, (4, –1)/120]

(1) – 2 (2) 1 (3) 0 (4) – 1

20. The number of values of k, for which the system of equations : **[AIEEE - 2013, (4, – 1) 120]**
$(k + 1)x + 8y = 4k$, $kx + (k + 3)y = 3k – 1$ has no solution, is

(1) infinite (2) 1 (3) 2 (4) 3

21. If P = $\begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and |A| = 4, then α is equal to :

[AIEEE - 2013, (4, – 1) 120]

(1) 4 (2) 11 (3) 5 (4) 0

22. If α, β ≠ 0 and $f(n) = \alpha^n + \beta^n$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = K(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$, then K is equal to **[JEE(Main) 2014, (4, – 1), 120]**

(1) 1 (2) –1 (3) αβ (4) $\frac{1}{\alpha\beta}$

23. If A is an 3 × 3 non-singular matrix such that AA' = A'A and $B = A^{-1}A'$, then BB' equals :

[JEE(Main) 2014, (4, – 1), 120]

(1) B^{-1} (2) $(B^{-1})'$ (3) I + B (4) I

24. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ is a matrix satisfying the equation $AA^T = 9I$, where I is 3 × 3 identity matrix, then the ordered pair (a, b) is equal to : **[JEE(Main) 2015, (4, – 1), 120]**

(1) (2, – 1) (2) (–2, 1) (3) (2, 1) (4) (–2, – 1)

25. The set of all value of λ for which the system of linear equations :
$2x_1 – 2x_2 + x_3 = \lambda x_1$, $2x_1 – 3x_2 + 2x_3 = \lambda x_2$, $– x_1 + 2x_2 = \lambda x_3$ has a non-trivial solution,

[JEE(Main) 2015, (4, – 1), 120]

(1) is an empty set (2) is a singleton
(3) contains two elements (4) contains more than two elements

26. The system of linear equations x + λy – z = 0, λx –y –z = 0, x +y – λz = 0 has a non-trivial solution for :

[JEE(Main) 2016, (4, – 1), 120]

(1) Exactly one value of λ. (2) Exactly two values of λ.
(3) Exactly three values of λ. (4) Infinitely many values of λ.

27. If A = $\begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and A adj A = A A^T, then 5a + b is equal to **[JEE(Main) 2016, (4, – 1), 120]**
(1) 5 (2) 4 (3) 13 (4) – 1

28. It S is the set of distinct values of 'b' for which the following system of linear equations
x + y + z = 1 **[JEE(Main) 2017, (4, –1), 120]**
x + ay + z = 1
ax + by + z = 0 has no solution, then S is :
(1) an empty set (2) an infinite set
(3) a finite set containing two or more elements (4) a singleton

29. If A = $\begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj ($3A^2$ + 12A) is equal to **[JEE(Main) 2017, (4, –1), 120]**
(1) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (2) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$ (3) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (4) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Let ω = $-\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is:
[IIT-JEE 2002, Scr, 3, – 1), 90]
(A) 3 ω (B) 3 ω (ω − 1) (C) 3 ω^2 (D) 3 ω (1 − ω)

2. If A = $\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then value of α for which A^2 = B is :
[IIT-JEE 2003, Scr, (3,–1), 84]
(A) 1 (B) – 1 (C) 4 (D) no real values

3. The value of λ for which the system of equations 2x – y – z = 12, x – 2y + z = 4, x + y + λz = 4 has no solution is **[IIT-JEE 2004, Scr, (3, – 1), 84]**
(A) 3 (B) – 3 (C) 2 (D) – 2

4. If P = $\begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, A = $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP^T and x = $P^TQ^{2005}P$, then x is equal to
[IIT-JEE 2005 Scr, (3, – 1), 84]
(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$ (C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

5. If for the matrix A = $\begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$; $|A^3|$ = 125, then the value of α is **[IIT-JEE 2005 Main, (3 – 1), 84]**
(A) ± 1 (B) ± 4 (C) ± 3 (D) ± 2

6. Consider the system of equations **[IIT-JEE 2008, Paper-1, (3, – 1), 163]**
x – 2y + 3z = – 1, – x + y – 2z = k, x – 3y + 4z = 1
STATEMENT - 1 : The system of equations has no solution for k ≠ 3
and
STATEMENT-2 : The determinant $\begin{vmatrix} 1 & 3 & -1 \\ -1 & -2 & k \\ 1 & 4 & 1 \end{vmatrix}$ ≠ 0, for k ≠ 3.
(A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1
(C) STATEMENT-1 is True, STATEMENT-2 is False
(D) STATEMENT-1 is False, STATEMENT-2 is True

7. Let M and N be two 2n × 2n non-singular skew-symmetric matrices such that MN = NM. If P^T denotes the transpose of P, then $M^2 N^2 (M^T N)^{-1} (MN^{-1})^T$ is equal to **[IIT-JEE 2011, Paper-1, (4, 0), 80]**
(A) M^2 (B) $-N^2$ (C) $-M^2$ (D) MN

8. Let P = $[a_{ij}]$ be a 3 × 3 matrix and let Q = $[b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for 1 ≤ i, j ≤ 3. If the determinant of P is 2, then the determinant of the matrix Q is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**
(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

9. If P is a 3 × 3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that **[IIT-JEE 2012, Paper-2, (3, –1), 66]**
(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D) PX = – X

10. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix}$ and I be the identity matrix of order 3. If Q = $[q_{ij}]$ is a matrix such that $P^{50} - Q = I$, then $\frac{q_{31} + q_{32}}{q_{21}}$ equals **[JEE (Advanced) 2016, Paper-2 (3, –1)/62]**
(A) 52 (B) 103 (C) 201 (D) 205

11. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5 ? **[JEE(Advanced) 2017, Paper-2,(3, –1)/61]**
(A) 198 (B) 162 (C) 126 (D) 135

Answers

EXERCISE - 1

Section (A):

A-1.	(3)	**A-2.**	(4)	**A-3.**	(4)	**A-4.**	(1)	**A-5.**	(1)	**A-6.**	(4)	**A-7.**	(4)
A-8.	(2)	**A-9.**	(4)	**A-10.**	(3)	**A-11.**	(2)	**A-12.**	(3)	**A-13.**	(1)	**A-14.**	(2)
A-15.	(3)	**A-16.**	(3)	**A-17.**	(1)	**A-18.**	(1)	**A-19.**	(1)				

Section (B) :

B-1.	(3)	**B-2.**	(1)	**B-3.**	(4)	**B-4.**	(3)	**B-5**	(2)	**B-6.**	(3)	**B-7.**	(4)
B-8.	(4)	**B-9.**	(2)	**B-10**	(3)	**B-11.**	(1)	**B-12**	(1)	**B-13.**	(1)	**B-14.**	(2)
B-15.	(2)	**B-16.**	(1)	**B-17.**	(2)	**B-18.**	(1)						

Section (C) :

C-1.	(2)	**C-2.**	(1)	**C-3.**	(4)	**C-4.**	(4)	**C-5.**	(2)	**C-6.**	(1)	**C-7.**	(1)
C-8.	(1)												

Section (D) :

D-1.	(1)	**D-2.**	(1)	**D-3.**	(3)	**D-4.**	(2)	**D-5.**	(4)	**D-6.**	(4)	**D-7.**	(1)
D-8.	(2)	**D-9.**	(2)	**D-10.**	(3)	**D-11.**	(1)	**D-12.**	(4)	**D-13.**	(1)	**D-14.**	(3)
D-15.	(3)	**D-16.**	(2)	**D-17.**	(1)	**D.18.**	(1)	**D-19.**	(4)	**D-20.**	(2)	**D-21**	(3)
D-22.	(4)												

EXERCISE - 2

PART - I

1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(1)	**5.**	(1)	**6.**	(3)	**7.**	(1)
8.	(1)	**9.**	(3)	**10.**	(1)	**11.**	(3)	**12.**	(1)	**13.**	(2)	**14.**	(2)
15.	(3)	**16**	(1)	**17.**	(3)	**18.**	(2)	**19.**	(2)	**20.**	(2)	**21.**	(2)
22.	(1)	**23.**	(1)	**24.**	(4)	**25.**	(2)	**26.**	(4)	**27.**	(1)	**28.**	(1)
29.	(2)	**30.**	(4)	**31.**	(3)	**32.**	(4)	**33.**	(1)	**34.**	(3)	**35.**	(2)
36.	(3)												

PART - II

Section (A) :

A-1.	(1)	**A-2.**	(2)	**A-3.**	(1)	**A-4.**	(2)	**A-5.**	(1)

Section (B) :

B-1. (A) → r ; (B) → q ; (C) → p ; (D) → q **B-2.** (A) → q ; (B) → r ; (C) → q ; (D) → s

Section (C) :

C-1	(1,2,3)	**C-2.**	(2,3)	**C-3.**	(1,4)	**C-4.**	(1,4)	**C-5.**	(3,4)	**C-6.**	(3,4)	**C-7.**	(1,2)
C-8.	(3,4)	**C-9.**	(2,3)										

EXERCISE - 3

PART - I

1.	(3)	**2.**	(1)	**3.**	(3)	**4.**	(3)	**5.**	(4)	**6.**	(3)	**7.**	(4)
8.	(1)	**9.**	(3)	**10.**	(3)	**11.**	(2)	**12.**	(3)	**13.**	(2)	**14.**	(2)
15.	(4)	**16.**	(1)	**17.**	(4)	**18**	(4)	**19.**	(3)	**20.**	(2)	**21.**	(2)
22.	(1)	**23.**	(4)	**24.**	**(4)**	**25.**	(3)	**26.**	(3)	**27.**	(1)	**28.**	(4)
29.	(2)												

PART - II

1.	(B)	**2.**	(D)	**3.**	(D)	**4.**	(A)	**5.**	(C)	**6.**	(A)	**7.**	(C)
8.	(D)	**9.**	(D)	**10.**	(B)	**11.**	(A)						