Exercise-1

Marked Questions may have for Revision Questions.

OBJECTIVE QUESTIONS

Section (A) : Matrix, Trace of matrix, Algebra of matrices, transpose of a matrix, symmetric and skew symmetric matrix

A-1. Sol. It is a 18 elements matrices. Possible orders are 1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6 and 6 × 3.
∴ Number of possible orders is 6.

A-2. Sol. a = 2i – j
$a_{11} = 2 – 1 = 1$, $a_{21} = 4 – 1 = 3$, $a_{31} = 2 \times 3 – 1 = 5$
$a_{12} = 2 – 2 = 0$, $a_{22} = 2.2 – 2 = 2$, $4_{32} = 2 \times 3 – 2 = 4$.

∴ $A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ 5 & 4 \end{bmatrix}$

A-3. Sol. $\begin{bmatrix} x-y & 1 & z \\ 2x-y & 0 & w \end{bmatrix} = \begin{bmatrix} -1 & 1 & 4 \\ 0 & 0 & 5 \end{bmatrix}$
on comparison
x – y = – 1
2x – y = 0
z = 4
w = 5
Hence x = 1, y = 2, z = 4, w = 5

A-4. Sol. $\begin{bmatrix} x^2+x & x \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ -x+1 & x \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x^2+x & x-1 \\ -x+4 & x+2 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 5 & 1 \end{bmatrix}$

On comparing
$x_2 + x = 0 \Rightarrow x = 0, –1$ x – 1 = – 2 ⇒ x = – 1
– x + 4 = 5 ⇒ x = – 1 x + 2 = 1 ⇒ x = – 1
Hence the value of x is – 1.

A-5. Sol. $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$I \cos\theta + J \sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

A-6. Sol. Matrix A has order (3 × 1) and Matrix B has order (3 × 3).
So multiplication AB is not possible.

A-7. Sol. $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow [1 + 4x + 3 \;\; 2 + 5x + 2 \;\; 3 + 6x + 5] \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 4+4x & 5x+4 & 6x+8 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 0$

$\Rightarrow 4 + 4x + 10x + 8 + 18x + 24 = 0$

$\Rightarrow 32x + 36 = 0$

$\Rightarrow x = -\frac{9}{8}$

A-8. **Sol.** $(A + B)^2 = (A + B) . (A + B) = A^2 + BA + AB + B^2$ (in general $AB \neq BA$).

A-9. **Sol.** (1) $(A + B)^2 = (A + B) (A + B) = A^2 + BA + AB + B^2$

(2) $(A - B)^2 = (A - B) (A - B) = A^2 - BA - AB + B^2$

(3) $AB = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2+1 & 8-1 \\ -1-2 & -4+2 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ -3 & -2 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 4 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2-4 & -1+8 \\ -2-1 & 1+2 \end{bmatrix} = \begin{bmatrix} -2 & 7 \\ -3 & 3 \end{bmatrix}$

so $AB \neq BA$

A-10_ **Sol.** $A^2 = \begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} \lambda & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \lambda^2 - 1 & \lambda + 2 \\ -\lambda - 2 & -1 + 4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

so, $\lambda^2 - 1 = 8 \Rightarrow \lambda = \pm 3$, $\lambda + 2 = 5 \Rightarrow \lambda = 3$

so λ can be only 3

A-11. **Sol.** Method -1

$A^2 = A.A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$

$\Rightarrow \quad A^3 = A^2 . A = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$\Rightarrow \quad A^3 = \begin{bmatrix} 7 & -12 \\ 3 & -5 \end{bmatrix}$

A-12._ Sol. $A^2 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = -I$

$A^3 = -A$

$A^4 = -A^2 = I$

$A^5 = A$

Now

(1) $A^3 - I = -A - I$

$A(A - I) = A^2 - A = -I - A$

(2) $A^3 + I = -A + I$

$A(A^3 - I) = A(-A - I) = -A^2 - A = I - A$

(3) $A^2 + I = -I + I = 0$

$A (A^2 - I) = A (-I - I) = -2AI = -2A$

$A^2 + I \neq A(A^2 - I)$

(4) $A^4 - I = I - I = 0$

$A^2 + I = -I + I = 0$

A-13. Sol. $(I - A)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$= \left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix}\right)\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}1 & +\tan\frac{\alpha}{2}\\ -\tan\frac{\alpha}{2} & 1\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}\cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \tan\frac{\alpha}{2}\cos\alpha\\ -\tan\frac{\alpha}{2}\cos\alpha + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha\end{bmatrix}$

$= \begin{bmatrix}\cos\alpha + 2\sin\frac{\alpha}{2}\sin\alpha & \sin\frac{\alpha}{2}\left(-2\cos\frac{\alpha}{2} + \frac{\cos\alpha}{\cos\frac{\alpha}{2}}\right)\\ \sin\alpha\left(-\frac{\cos\alpha}{\cos\frac{\alpha}{2}} + 2\cos\frac{\alpha}{2}\right) & \cos\alpha + 2\sin^2\frac{\alpha}{2}\end{bmatrix}$

$= \begin{bmatrix}1 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} + \begin{bmatrix}0 & -\tan\frac{\alpha}{2}\\ \tan\frac{\alpha}{2} & 0\end{bmatrix} = I + A$

A-14. Sol. $A = \begin{bmatrix}2 & -1\\ -7 & 4\end{bmatrix}$, $B = \begin{bmatrix}4 & 1\\ 7 & 2\end{bmatrix}$

$A^T = \begin{bmatrix}2 & -7\\ -1 & 4\end{bmatrix}$, $B^T = \begin{bmatrix}4 & 7\\ 1 & 2\end{bmatrix}$

$B^TA^T =$ is an identity matrix.

A-15_ Sol. $A^T = \begin{bmatrix}5 & -3 & 6\\ -3 & 7 & -4\\ 6 & -4 & 8\end{bmatrix} = A$

so A is a symmetric matrix

A-16. Sol. Trace of A = $a_{11} + a_{22} + a_{33}$

For skew symmetric matrix $a_{11} = a_{22} = a_{33} = 0$

Trace of A = 0

A-17. Sol. The elements of main diagonal of skew symmetric matrix are all zero but not necessarily for symmetric matrix.

$\frac{A + A'}{2}$ is symmetric matrix.

$\frac{A - A'}{2}$ is skew symmetric matrix.

A-18. **Sol.** $A = \begin{vmatrix} 1^2-1^2 & 1^2-2^2 & 1^2-3^2 \\ 2^2-1^2 & 2^2-2^2 & 2^2-3^2 \\ 3^3-1^2 & 3^2-2^2 & 3^2-3^2 \end{vmatrix} = \begin{vmatrix} 0 & -3 & -8 \\ 3 & 0 & -5 \\ 8 & 5 & 0 \end{vmatrix}$

So A is skew symmetric matrix

A-19. **Sol.** A' = A and B' = B

(ABA)' = A'B'A' = ABA

$\therefore$ ABA is symmetric matrix.

Section (B) : Minors, Cofactors, Expansion of determinant, Properties of determinant, Summation, Differentiation, Multiplication of determinants, Property $|kA| = k^n|A|$ and $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$

B-1. **Sol.** $M_{21} = 2$, $M_{22} = 1$

B-2. **Sol.** $M_{21} = \begin{vmatrix} -3 & 2 \\ 5 & 2 \end{vmatrix} = -16$, $M_{22} = \begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = -4$, $M_{23} = \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} = 14$

B-3. **Sol.** $\begin{vmatrix} 0 & 1 & \sec\alpha \\ \tan\alpha & -\sec\alpha & \tan\alpha \\ 1 & 0 & 1 \end{vmatrix}$

$M_{31} = \begin{vmatrix} 1 & \sec\alpha \\ -\sec\alpha & \tan\alpha \end{vmatrix} = 1$

$\tan\alpha + \sec^2\alpha = 1$ $\quad (\because 1 + \tan^2\alpha = \sec^2\alpha)$

$\tan\alpha = \tan^2\alpha$

$\tan\alpha(1 + \tan\alpha) = 0 \quad \Rightarrow \tan\alpha = 0, \tan\alpha = -1$

$\alpha = n\pi$ or $\alpha = n\pi - \frac{\pi}{4}$, $n \in I$

B-4. **Sol.** $C_{11} = C_{12} \Rightarrow (-1)^{1+1}\begin{vmatrix} 3 & x \\ 3 & 3 \end{vmatrix} = (-1)^{2+2}\begin{vmatrix} x & 3 \\ 2 & 3 \end{vmatrix}$

$\Rightarrow 9 - 3x = 3x - 6 \Rightarrow -6x = -15 \Rightarrow x = \frac{5}{2}$

B-5 **Sol.** $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$

$1(c^2 - ab) - a(c - a) + b(b - c) = 0$

$a^2 + b^2 + c^2 - ab - bc - ca = 0$

$a = b = c$

$\therefore \sin^2A + \sin^2B + \sin^2c = 3\sin^2 60° = \frac{9}{4}$

B-6_ **Sol.** $\begin{vmatrix} \cos x & \sin x & \text{six} \\ \sin x & \cos x & \sin x \\ \sin x & \sin x & \cos x \end{vmatrix} = 0$

$\Rightarrow \cos^3x + \sin^3x + \sin^3x - 3\sin^2x\cos x = 0$

$\Rightarrow (\cos x + \sin x + \sin x)(\cos^2x + \sin^2x + \sin^2x - \cos x\sin x - \cos x\sin x - \sin^2x) = 0$

$\Rightarrow \cos x = -2\sin x$ or $\cos x = \sin x$

$\tan x = -\frac{1}{2}$ $\tan = 1 \Rightarrow x = \pi/4$

$x = -\tan^{-1}\left(-\frac{1}{2}\right)$ $\therefore$ two solutions

B-7. **Sol.** $\Delta = \begin{vmatrix} -x & a & b \\ b & -x & a \\ a & b & -x \end{vmatrix}$

$= (a + b - x)\begin{vmatrix} 1 & a & b \\ 1 & -x & a \\ 1 & b & -x \end{vmatrix}$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$= (a + b - x)\begin{vmatrix} 1 & a & b \\ 0 & -(x+a) & a-b \\ 0 & b-a & -(x+b) \end{vmatrix}$

$= (a + b - x)\{(x + a)(x + b) + (a - b)^2\}$

If a = b then x = a, –b, (a + b)

B-8. **Sol.** α, β, γ are roots of $x^3 + px + q = 0$

$\therefore$ $\alpha + \beta + \gamma = 0$

on $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$, Applying $C_1 \to C_1 + C_2 + C_3$

$(\alpha + \beta + \gamma)\begin{vmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{vmatrix} = 0$

B-9. **Sol.** $\begin{vmatrix} 15-2x & 11 & 10 \\ 11-3x & 17 & 16 \\ 7-x & 14 & 13 \end{vmatrix}$

Applying $C_2 \to C_2 - C_3$

$\begin{vmatrix} 15-2x & 1 & 10 \\ 11-3x & 1 & 16 \\ 7-x & 1 & 13 \end{vmatrix} = 0$

Applying $R_1 \to R_1 - R_3$ & $R_2 \to R_2 - R_3$

$\begin{vmatrix} 8-x & 0 & -3 \\ 4-2x & 0 & 3 \\ 7-x & 1 & 13 \end{vmatrix} = 0$ $\Rightarrow -1[(8 - x)3 + (4 - 2x)] = 0$ $\Rightarrow 9x = 36$ $\Rightarrow x = 4$

B-10 **Sol.** $x_1, x_2, x_3, \ldots, x_{13}$ are in A.P.

$\Rightarrow$ $x_1 = a$
$x_2 = a + d$
$x_3 = a + 2d$
$x_{13} = a + 12d$
Now

$$\begin{vmatrix} e^{x_1} & e^{x_4} & e^{x_7} \\ e^{x_4} & e^{x_7} & e^{x_{10}} \\ e^{x_7} & e^{x_{10}} & e^{x_{13}} \end{vmatrix} = \begin{vmatrix} e^{a} & e^{a+3d} & e^{a+6d} \\ e^{a+3d} & e^{a+6d} & e^{a+9d} \\ e^{a+6d} & e^{a+9d} & e^{a+12d} \end{vmatrix}$$

$$= e^a . e^{a+3d} . e^{a+6d} \begin{vmatrix} 1 & e^{3d} & e^{6d} \\ 1 & e^{3d} & e^{6d} \\ 1 & e^{3d} & e^{6d} \end{vmatrix} = 0$$

B-11. **Sol.** $\Delta = \begin{vmatrix} \sin\theta & 1 & 0 \\ 1 & \cos\phi & -\cos\theta \\ \sin\phi & 0 & 1 \end{vmatrix}$

$\Delta = \sin\theta(\cos\varphi + 0) - (1 + \cos\theta\sin\varphi)$
$= \sin\theta\cos\varphi - \cos\theta\sin\varphi - 1$
$= \sin(\theta - \varphi) - 1$
$\therefore$ $\Delta_{min} = -1 - 1 = -2$
$\therefore$ $|(\Delta_{min})| = 2$

B-12_ Sol. $\begin{vmatrix} \sqrt{13}+\sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{26} & 5 & \sqrt{10} \\ 3+\sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

$$= 5\begin{vmatrix} \sqrt{13} & 2 & 1 \\ \sqrt{26} & \sqrt{5} & \sqrt{2} \\ \sqrt{65} & \sqrt{3} & \sqrt{5} \end{vmatrix} + 5\begin{vmatrix} \sqrt{3} & 2 & 1 \\ \sqrt{15} & \sqrt{5} & \sqrt{2} \\ 3 & \sqrt{3} & \sqrt{5} \end{vmatrix}$$

$$= 0 + 5\sqrt{3}\begin{vmatrix} 1 & 2 & 1 \\ \sqrt{5} & \sqrt{5} & \sqrt{2} \\ \sqrt{3} & \sqrt{3} & \sqrt{5} \end{vmatrix} = 5\sqrt{3}\begin{vmatrix} 1 & 1 & 1 \\ \sqrt{5} & 0 & \sqrt{2} \\ \sqrt{3} & 0 & \sqrt{5} \end{vmatrix}$$

$$= -5\sqrt{3}\left(5-\sqrt{6}\right) = 5\sqrt{3}\left(\sqrt{6}-5\right)$$

B-13_ **Sol.** Put x = – 1

$$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12$$

$\Rightarrow$ a = 24

B-14. **Sol.** $\begin{vmatrix} b_1+c_1 & c_1+a_1 & a_1+b_1 \\ b_2+c_2 & c_2+c_2 & a_2+b_2 \\ b_3+c_3 & c_3+a_3 & a_3+b_3 \end{vmatrix}$

applying $C_1 \rightarrow C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(a_1+b_1+c_1) & c_1+a_1 & a_1+b_1 \\ 2(a_2+b_2+c_2) & c_2+a_2 & a_2+b_2 \\ 2(a_3+b_3+c_3) & c_3+a_3 & a_3+b_3 \end{vmatrix}$$

Taking two common,

Applying $C_1 \to C_2 - C_1$ & $C_3 \to C_2 - C_1$

$$= 2\begin{vmatrix} a_1+b_1+c_1 & -b_1 & -c_1 \\ a_2+b_2+c_2 & -b_2 & -c_2 \\ a_3+b_3+c_3 & -b_3 & -c_3 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$

$$= 2\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

B-15. Sol. $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N+1 \end{vmatrix}$

$$\Rightarrow \sum_{n=1}^{N} U_n = \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & (2N+1) & (2N+1) \\ \left[\frac{N(N+1)}{2}\right]^2 & 3N^2 & (3N+1) \end{vmatrix}$$

$$= \frac{N(N+1)(2N+1)}{2}\begin{vmatrix} 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N+1 \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2$

$$\frac{N(N+1)(2N+1)}{2}\begin{vmatrix} \frac{2}{3} & 0 & 4 \\ \frac{1}{3} & 1 & 1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N+1 \end{vmatrix} = 2\sum_{n=1}^{N} n^2$$

B-16_ Ans. (1)

Sol. $y'(x) = \begin{vmatrix} \cos x & -\sin x & \cos x - \sin x \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$

$y''(x) = \begin{vmatrix} -\sin x & -\cos x & -\sin x - \cos x \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$

$y''(x) + y = \begin{vmatrix} 0 & 0 & 1 \\ 23 & 17 & 13 \\ 1 & 1 & 1 \end{vmatrix}$

B-17. **Sol.** $|3\,AB| = |A|\,.\,|3B|_{3\times 3}$
$= (-1)\,.\,3^3\,|B|$
$= -81$

B-18. **Sol.** $\det(-A) = (-1)^n \det(A)$ where n is order of square matrix.

Section (C) : Cramer rule

C-1. **Sol.** Here $\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & -3 \\ 2 & 5 & -\lambda \end{vmatrix}$

system has unique solution if $\Delta \neq 0$ and at least one of Δ_x, Δ_y, Δ_z is non-zero.

$\Delta = 1(-2\lambda + 15) - 1(-\lambda + 6) - 1(5 - 4) \neq 0 \Rightarrow -2\lambda + 15 + 1 - 6 - 1 \neq 0$

$\Rightarrow -\lambda + 8 \neq 0 \Rightarrow \lambda \neq 8$

C-2. **Sol.** $\Delta = \begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$

For no solution of system $\Delta = 0$ and at least one of the Δ_x, Δ_y, Δ_z is non zero.

At $\Delta = 0$, $\lambda = -2$

C-3. **Sol.** $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & p & 2 \\ 1 & 4 & \mu \end{vmatrix}$, $\Delta_x = \begin{vmatrix} 4 & 2 & 3 \\ 3 & p & 2 \\ 3 & 4 & \mu \end{vmatrix}$,

$\Delta_y = \begin{vmatrix} 1 & 4 & 3 \\ 1 & 3 & 2 \\ 1 & 3 & \mu \end{vmatrix}$, $\Delta_z = \begin{vmatrix} 1 & 2 & 4 \\ 1 & p & 3 \\ 1 & 4 & 3 \end{vmatrix}$,

For infinite no. of solution $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ $\Rightarrow \mu = 2,\ p = 4$

C-5. **Sol.** Here $\Delta = \begin{vmatrix} a & b & b \\ b & a & b \\ b & b & a \end{vmatrix}$

Homogeneous system has non-trivial solution $\Delta = 0$.

$(a + 2b) \begin{vmatrix} 1 & b & b \\ 1 & a & b \\ 1 & b & a \end{vmatrix} = 0$

$\Rightarrow (a + 2b) \begin{vmatrix} 1 & b & b \\ 0 & a-b & 0 \\ 0 & 0 & a-b \end{vmatrix} = 0$

$\Rightarrow (a + 2b)(a - b)^2 = 0$

Here $a \neq b \therefore (a + 2b) = 0$

C-6. **Sol.** For non-trivial solution

$$\begin{vmatrix} (\alpha+a) & \alpha & \alpha \\ \alpha & \alpha+b & \alpha \\ \alpha & \alpha & \alpha+c \end{vmatrix} = 0$$

Taking α as common from each row

$$\Rightarrow \alpha^3 \begin{vmatrix} 1+\frac{a}{\alpha} & 1 & 1 \\ 1 & 1+\frac{b}{\alpha} & 1 \\ 1 & 1 & 1+\frac{c}{\alpha} \end{vmatrix} = 0$$

Applying $C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$ and expanding

$C_1 \to C_1 - C_3$, $C_2 \to C_2 - C_3$

$$\Rightarrow \alpha^3 \left[\frac{ab}{\alpha^2} + \frac{bc}{\alpha^2} + \frac{ac}{\alpha^2} + \frac{abc}{\alpha^3}\right] = 0$$

$$\Rightarrow \frac{1}{\alpha} = -\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

C-7. Sol. $\begin{vmatrix} a^3 & (a+1)^3 & (a+2)^3 \\ a & (a+1) & (a+2) \\ 1 & 1 & 1 \end{vmatrix}$ = 0 for non - zero solution

$C_1 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\begin{vmatrix} a^3 & (a+1)^3 - a^3 & (a+2)^3 - a^3 \\ a & 1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$\Rightarrow 2((a + 1)^3 - a^3) - ((a + 2)^3 - a^3) = 0$

$\Rightarrow 2\,(3a^2 + 3a + 1) = 6a^2 = 12a + 18$

$\Rightarrow - 6a + 2 = 12\ a = 8$

$\Rightarrow - 6 = 6a$

$\Rightarrow a = -1$

C-8. Sol. The equation are

$2x + y + z = 15$

$3x + 2y + 4z = 37$

$x + y + z = 12$

Applying creamer's rule

$\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 2 & 4 \\ 1 & 1 & 1 \end{vmatrix} = -2$; $\Delta_1 = \begin{vmatrix} 15 & 1 & 1 \\ 37 & 2 & 4 \\ 12 & 1 & 1 \end{vmatrix} = -6$

$\Delta_2 = \begin{vmatrix} 2 & 15 & 1 \\ 3 & 37 & 4 \\ 1 & 12 & 1 \end{vmatrix} = -8$; $\Delta_3 = \begin{vmatrix} 2 & 1 & 15 \\ 3 & 2 & 37 \\ 1 & 1 & 12 \end{vmatrix} = -10$

So $x = \frac{\Delta_1}{\Delta} = \frac{-6}{-2} = 3$; $y = \frac{-8}{-2} = 4$; $z = \frac{-10}{-2} = 5$

Section (D) : Adjoint of Matrix, Inverse of matrix and their properties, solution of system of linear equations by matrix method, Cayley-Hamilton theorem

D-1. Sol. $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$

Matrix formed by cofactors of A = C = $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

Transpose of Matrix C = C_T = $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

Adjoint of matrix A = C_T = adj A = $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

D-2. Sol. $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Matrix formed by Cofactors of A= C = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore$ Adj A = C_T = $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ = A'

D-3. Sol. kA adj (kA) = |kA| I_n

kA adj (kA) = k^n |A| I_n

kA adj (kA) = k^n A adj A

Pre-multiplying A^{-1}

adj (kA) = k^{n-1} adj A

D-4. Sol. we know that A. adj A = |A| . I and |adj A| = $|A|^{n-1}$

$\therefore$ adj A. (adj adj A) = |adj A| I

Pre-multiplying A

$\Rightarrow$ A. adj A. (adj adj A) = A $|A|^{n-1}$ I

$\Rightarrow$ |A| . I (adj adj A) = A $|A|^{(n-1)}$ $\Rightarrow$ (adj adj A) = A $|A|^{(n-2)}$

D-5. Sol. $\because$ A is square matrix of order 3

$\therefore$ |adj A| = $|A|^2$

$\therefore$ |adj(adj A)| = $|\text{adj } A|^2 = (|A|^2)^2 = |A|^4$

D-6._ Sol. |A| = 6

adj A = $\begin{bmatrix} 1 & -2 & 4 \\ 4 & 1 & 1 \\ -1 & k & 0 \end{bmatrix}$

$\therefore$ |adjA| = –1(–2 – 4) – k (1 – 16) + 0 = 6 + 15k

but |adjA| = $|A|^2$

$\therefore$ 6 + 15k = 36 $\Rightarrow$ 15k = 30

$\Rightarrow$ k = 2

D-7._ Sol. 125 $|A|^2$ = 5

$|A| = \pm \frac{1}{5}$

D-8._ **Sol.** $|A - \lambda I| = 0$

$\Rightarrow \left| \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 3 \\ 2 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow 2 - 2\lambda - \lambda + \lambda^2 - 6$

$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$

D-9. **Sol.** AB = AC

Pre-multiplying $A^{-1} \Rightarrow B = C$ hence A must be invertible matrix.

D-10._ Sol. $|A| = ad - bc$

If A is invertible then $|A| \neq 0$

$\Rightarrow ad \neq bc$

$\Rightarrow$ (i) $ad = 0$ and $bc = 1$

or (ii) $ad = 1$ and $bc = 0$

$\Rightarrow$ (a, d, b, c) = (0, 0, 1, 1), (0, 1, 1, 1), (1, 0, 1, 1), (1, 1, 0, 0), (1, 1, 0, 1), (1, 1, 1, 0)

$\Rightarrow$ 6 matrices

D-11. Sol. $|A| \neq 0$ and $|B| \neq 0 \quad \Rightarrow \quad |AB| \neq 0$

$\therefore$ AB is non singular

D-12. Sol. $|F| \neq 0$ and rFkk $|G| \neq 0$

As we knew $[AB]^{-1} = B^{-1} A^{-1}$

$\therefore \quad [F(\alpha)\, G(\beta)]^{-1} = [G(\beta)]^{-1}[B(\alpha)]^{-1}$

D-13. Sol. $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$(AB)^{-1} = B^{-1}A^{-1}$

$\therefore$ Matrix formed by Cofactors of B = $\begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix}$

$\therefore$ Transpose of formed by Cofactors of B = adjB = $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

$\therefore B^{-1} = \dfrac{\text{adj}B}{|B|} \Rightarrow B^{-1} = \dfrac{\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}}{1}$

$\therefore B^{-1} A^{-1} = \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

D-14. Sol. adj $A = A^{-1}\ |A| = \dfrac{1}{2}\begin{vmatrix} 1 & -1 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1/2 & -1/2 & 0 \\ 0 & -1 & 1/2 \\ 0 & 0 & -1/2 \end{vmatrix}$

$\because A^{-1} = \dfrac{\text{adj}\ A}{|A|}$, $|A^{-1}| = \dfrac{1}{|A|}$ and $|A| = \dfrac{1}{2}$

D-15. Sol. det $(B^{-1} AB)$ Since det (AB) = det A.det B

= det B^{-1} . det AB

= det B^{-1} . det A. det B

$= \dfrac{1}{\det B}$ det A det B = det A

D-16. Sol. $A^2 = I$

Pre-multiplying A^{-1}

$A = A^{-1}I$

$\therefore A^{-1} = A$

D-17. **Sol.** $A = \begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$A = BX \Rightarrow B^{-1}A = B^{-1}(BX) \Rightarrow B^{-1}A = X$

$X = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$

$X = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 3 & -5 \end{bmatrix}$

D.18_ **Sol.** A is invertible as $|A| \neq 0$

Now, $AX = 0 \Rightarrow A^{-1}(AX) = 0$

$\Rightarrow (A^{-1}A)X = 0$ or $IX = 0$ or $X = 0$

$\therefore$ x = y = z = 0 is the only solution of the system of equations.

D-19. **Sol.** $\Delta = \begin{vmatrix} 4 & -5 & -2 \\ 5 & -4 & 2 \\ 2 & 2 & 8 \end{vmatrix}$

|A| = 0, singular

And (adj A) B = O system is inconsistent

D-20. **Sol.** For unique solution co-efficient matrix should be non-singular.

For $(\text{adj } A)^{-1}$ to exist matrix A should be non singular.

D-21 **Sol.** $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \Rightarrow A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}$

$\frac{1}{6}(A^2 + cA + dI) = \frac{1}{6}\left(\begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix} + \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} + \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}\right)$

$\therefore A^{-1} = \frac{1}{6}\begin{bmatrix} 1+c+d & 0 & 0 \\ 2 & -1+c+d & 5+c \\ 0 & -2c-10 & 4c+d+14 \end{bmatrix}$

$\therefore 1 + c + d = 6$

$5 + c = -1$

$\Rightarrow c = -6, d = 11$

D-22. Sol. Characterstic equation of matrix A is

$|A - \lambda I| = 0$

$\begin{bmatrix} -4-\lambda & -1 \\ 3 & 1-\lambda \end{bmatrix} = 0$

$(-4 - \lambda)(1 - \lambda) + 3 = 0$

$\lambda^2 + 3\lambda - 1 = 0$

by cayley hamilton theorem

$A^2 + 3A - I = 0$

Now $(A^{2016} - 2A^{2015} - A^{2014}) = A^{2014}(A^2 - 2A - I) = A^{2014}(I - 3A - 2A - I) = -5A^{2015}$

so, $|A^{2016} - 2A^{2015} - A^{2014}| = |-5A^{2015}| = (-5)^2 |A|^{2015} = -25$ $(\because |A| = -1)$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1. Sol. AB = O

$\Rightarrow \begin{bmatrix} \cos^2\theta\cos^2\phi + \sin\theta\sin\phi\cos\theta\cos\theta & \cos^2\theta\cos\phi\sin\phi\sin^2\phi\cos\theta\sin\theta \\ \cos^2\phi\cos\theta\sin\theta + \sin^2\theta\cos\phi\sin\theta & \cos\theta\sin\theta\cos\phi\sin\phi + \sin^2\theta\sin\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi\cos(\theta-\phi) & \cos\theta\sin\phi(\cos\theta-\phi) \\ \cos\phi\sin\theta\cos(\theta-\phi) & \sin\theta\sin\phi\cos(\theta-\phi) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \cos(\theta - \varphi)$

$\Rightarrow \begin{bmatrix} \cos\theta\cos\phi & \cos\theta\sin\theta \\ \cos\phi\sin\theta & \sin\theta\sin\phi \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \cos(\theta - \varphi) = 0 \Rightarrow \theta - \varphi = (2n+1)\frac{\pi}{2}$

2. Sol. A = 3I

$\therefore$ AB = 3IB ($\because$ IB = B)

AB = 3B

3. Sol. $X = \begin{vmatrix} 3 & -4 \\ 1 & -1 \end{vmatrix} \Rightarrow X^2 = \begin{vmatrix} 5 & -8 \\ 2 & -3 \end{vmatrix}$

For n = 2 option (1), (2), (3) are not satisified. Hence option (4) is correct.

4. Sol. $A^2 = A$

$\Rightarrow$ $A^{-1} A^2 = A^{-1}A$ $\Rightarrow$ A = I

$\therefore$ $(I + A)^{10} = (I + I)^{10} = (2I)^{10}$

= 1024 I = I + kI = (k + 1) I

$\therefore$ k + 1 = 1024 $\Rightarrow$ k = 1023

5. Sol. $(A + B)(A - B) = A^2 + BA - AB + B^2 \neq A^2 - B^2$

6. **Sol.** $A^T = A^2 - 2A \Rightarrow \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

$\begin{bmatrix} 3a & c+2b \\ b+2c & 3d \end{bmatrix} = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix}$

$\Rightarrow$ $3a = a^2 + bc$, $c + 2b = 2b$, $b + 2c = 2c$ and $3d = bc + d^2$

$\Rightarrow$ c = 0, b = 0, a = 0,3 and d = 0,3

$\Rightarrow$ (a, d) = (0, 0), (0, 3), (3, 0), (3,3)

but a + d = 2

so no such matrix is possible ,

7. Sol. A = A' is symmetric

(1) (AA')' = (A')'A' = AA' (so AA' is symmetric)

(3) (A'A)' = A'(A')' = A'A (so A'A is also symmetric)

8. Sol. $A^T = -A$

$(A^n)^T = (AAA \text{-------} A)^T = (A^T A^T A^T \text{-------} A^T) = (A^T)^n$ for all n ∈ N

$(-A)^n = (-1)^n A^n$

$(A^n)^T = \begin{cases} A^n & \text{if } n \text{ is even} \\ -A^n & \text{if } n \text{ is odd} \end{cases}$

9. **Sol.** Obviosuly the skew symmetric matrix of odd order is always singular.
Let A is skew symmetric matrix of order 3

$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow |A| = -a(bc) + b(ac) = -abc + abc = 0$

Let B is skew symmetric matrix of 2^{nd} order.

$B = \begin{bmatrix} 0 & -b \\ b & 0 \end{bmatrix} \Rightarrow |B| = 0 + b^2 = b^2$

10. **Sol.** $\begin{bmatrix} a^2+1 & ab & ac \\ ba & b^2+1 & bc \\ ca & cb & c^2+1 \end{bmatrix} = \frac{1}{abc}\begin{bmatrix} a(a^2+1) & a^2b & a^2c \\ b^2a & b(b^2+1) & b^2c \\ c^2a & c^2b & c(c^2+1) \end{bmatrix}$

$= \frac{abc}{abc}\begin{bmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{bmatrix}$

Applying $R_1 \rightarrow R_1 + R_2 + R_3$

$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix} = (a^2 + b^2 + c^2+1)$

Applying $C_2 \rightarrow C_2 - C_1$ & $C_3 \rightarrow C_3 - C_1$

$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 0 & 0 \\ b^2 & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix} = (a^2 + b^2 + c^2 + 1)$

11. **Sol.** $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$

$= a^2b^2c^2 \begin{vmatrix} (1+x) & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$

Applying $C_1 \rightarrow C_2 + C_3$

$a^2b^2c^2(3+x)\begin{vmatrix} 1 & 1 & 1 \\ 1 & (1+x) & 1 \\ 1 & 1 & (1+x) \end{vmatrix}$

Applying $R_1 \rightarrow R_1 - R_2$, $R_2 \rightarrow R_2 - R_3$

$a^2b^2c^2(3+x)\ x^2$

Which is divisible by x^2

12. Sol. $\Delta = \begin{vmatrix} 1+a^2+a^4 & a+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$

$= \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$= (a-b)^2 (b-c)^2 (c-a)^2$

13. Sol. $\Delta = \begin{vmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix}$

$\Delta = \sin^2\theta\cos\theta \begin{vmatrix} \cos\phi & \sin\phi & \cot\theta \\ \cos\phi & \sin\phi & -\tan\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$

$\Delta = \sin^2\theta\cos\theta \begin{vmatrix} 0 & 0 & \cot\theta+\tan\theta \\ \cos\phi & \sin\phi & -\tan\theta \\ -\sin\phi & \cos\phi & 0 \end{vmatrix}$

$\Delta = \sin\theta$

14. Sol. $\begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix}$

$= \dfrac{1}{\sin\phi\cos\phi} \begin{vmatrix} \cos(\theta+\phi) & -\sin(\theta+\phi) & \cos^2\phi \\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi \\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi \end{vmatrix}$

Applying $R_1 \to R_1 + R_2 + R_3$

$= \dfrac{1}{\sin\phi\cos\phi} \begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi \\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi \end{vmatrix}$

$= \begin{vmatrix} 0 & 0 & 2\cos^2\phi \\ \sin\theta & \cos\theta & \sin\phi \\ -\cos\theta & \sin\theta & \cos\phi \end{vmatrix} = 2\cos^2\varphi\,(\sin^2\theta + \cos^2\theta) = 2\cos^2\varphi$

15. Sol. $\begin{vmatrix} a & b & ax+b \\ b & c & bx+c \\ ax+b & bx+c & c \end{vmatrix} = 0$

Applying $C_3 \to C_3 - C_2$

$\begin{vmatrix} a & b & ax \\ b & c & bx \\ ax+b & bx+c & -bx \end{vmatrix} = 0$

$\Rightarrow x\begin{vmatrix} a & b & a \\ b & c & b \\ ax+b & bx+c & -b \end{vmatrix} = 0$

Applying $C_1 \to C_1 - C_3$

$\Rightarrow x\begin{vmatrix} 0 & b & a \\ 0 & c & b \\ ax+2b & bx+c & -b \end{vmatrix} = 0$

$\Rightarrow x(ax + 2b)(b^2 - ac) = 0$

$\therefore$ Non zero root of equation $x = -\frac{2b}{a}$

16 Sol. Let a = b = c = 1

$\begin{vmatrix} 4 & 1 & 1 \\ 1 & 4 & 1 \\ 1 & 1 & 4 \end{vmatrix} = k.27$

$54 = 27k \quad \Rightarrow \quad k = 2$

Alter : $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$

Applying $C_1 \to C_2 - C_3$, $C_2 \to C_2 - C_3$

$(a + b + c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ c-b-a & c-a-b & (a+b)^2 \end{vmatrix}$

Applying $R_3 \to R_3 - (R_1 + R_2)$

$(a + b + c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ 0 & c+a-b & b^2 \\ -2b & -2a & 2ab \end{vmatrix}$

$= \frac{(a+b+c)^2}{ab}\begin{vmatrix} a(b+c-a) & 0 & a^2 \\ 0 & b(c+a-b) & b^2 \\ -2ab & -2ab & 2ab \end{vmatrix}$

$2ab\,(a + b + c)^2 \begin{vmatrix} (b+c-a) & 0 & a \\ 0 & (c+a-b) & b \\ -1 & -1 & 1 \end{vmatrix} = 2abc\,(a + b + c)^3$

17. Sol. $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x^2-1) \end{vmatrix}$

$= x^2(x - 1)\begin{vmatrix} 1 & x & x+1 \\ 2 & x-1 & x+1 \\ 3 & x-2 & x+1 \end{vmatrix}$

Applying $C_3 \to C_3 - C_2$ we get ls

$f(x) = x^2(x-1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 2 \\ 3 & x-2 & 3 \end{vmatrix} = 0$

$\Rightarrow f'(x) = 0$

Thus , $f'(5) = 0$

18. Sol. $x + 2y - 3z = p$

$2x + 6y - 11z = q$

$x - 2y + 7z = r$

$D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 6 & -11 \\ 1 & -2 & 7 \end{vmatrix}$

$= 1(42 - 22) - 2(14+11) - 3(-4-6)$

$= 20 - 50 + 30 = 0$

$D_1 = \begin{vmatrix} p & 2 & -3 \\ q & 6 & -11 \\ r & -2 & 7 \end{vmatrix} = p(20) - 2(7q + 11r) - 3(-2q - 6r)$

$= 20p - 14q - 22r + 6q + 18r$

$= 20p - 8q - 4r = 4(5p - 2q - r)$

If $D_1 = 0$, then there are infinite solutions which confirm at least one solution.

$\therefore 5p - 2q - r = 0$

19. Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$

$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda + \mu - 16$

$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 7)$

$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = (\mu - 10)$

For unique solution $\Delta \neq 0 \therefore \lambda \neq 3$

20. Sol. $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$

$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda + \mu - 16$

$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 7)$

$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = (\mu - 10)$

For infinite no. of solution $\Delta = 0$, $\Delta_x = 0$, $\Delta_y = 0$, $\Delta_z = 0$.

$\therefore \lambda = 3, \mu = 10$

21. **Sol.** $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = \lambda - 3$

$\Delta_x = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & \lambda \end{vmatrix} = 2\lambda + \mu - 16$

$\Delta_y = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & \lambda \end{vmatrix} = 4(\lambda - \mu + 7)$

$\Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = (\mu - 10)$

For no solution $\Delta = 0$, & at least are of Δ_x, Δ_y, Δ_z is non-zero.

$\therefore \lambda = 3, \mu \neq 10$

22. **Sol.** $3x + ky - 2z = 0$

$x + ky - 3z = 0$

$2x + 2y - 4z = 0$

$\Delta = \begin{vmatrix} 3 & k & -2 \\ 1 & k & -3 \\ 2 & 3 & -4 \end{vmatrix} = 0$

$\therefore k = \frac{33}{2}$

23._ **Sol.** $\Delta = 0 \Rightarrow \begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0 \Rightarrow 2ac = ab + bc$

24._ **Sol.**

$\begin{vmatrix} 1 & 3 & 7 \\ -1 & 4 & 7 \\ \sin 3\theta & \cos\theta & 2 \end{vmatrix} = 0$

$1(8 - 7\cos 2\theta) - 3(-2 - 7\sin 3\theta) + 7(-\cos 2\theta - 4\sin 3\theta) = 0$

$8 - 7\cos 2\theta + 6 + 21\sin 3\theta - 7\cos 2\theta - 28\sin 3\theta = 0$

$-7\sin 3\theta - 14\cos 2\theta + 14 = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2(1 - 2\sin^2\theta) - 2 = 0$

$3\sin\theta - 4\sin^3\theta + 2 - 4\sin^2\theta - 2 = 0$

$-\sin\theta(4\sin^2\theta + 4\sin\theta - 3) = 0$

$-\sin\theta(4\sin^2\theta + 6\sin\theta - 2\sin\theta - 3) = 0$

$-\sin\theta\,(2\sin\theta - 1)\,(2\sin\theta + 3) = 0$

$\sin\theta = 0,\ \sin\theta = \frac{1}{2}$

$\theta = 0,\ \pi,\ \frac{\pi}{6},\ \frac{5\pi}{6}$

25. Sol. $\Delta = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$\Delta = \begin{vmatrix} a-1 & 1-b & 0 \\ 0 & b-1 & 1-c \\ 1 & 1 & c \end{vmatrix} = 0$

$\Rightarrow (a-1)[(b-1)c - (1-c) + (1-b)(1-c)] = 0$

$\Rightarrow \frac{c}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 0$

$\Rightarrow \frac{1}{1-c} + \frac{1}{1-b} + \frac{1}{1-a} = 1$

26. Sol. $|A| + |A^T| = 2|A| \neq 0$

27. Sol. $A'A = I$

$|A - I| = |A - A'A|$

$\Rightarrow \quad |A - I| = |A|\,|I - A'|$

$\Rightarrow \quad |A - I| = -1\,.\,|A' - I|$

$\Rightarrow \quad |A - I| = -|A - I|$

$\Rightarrow \quad |A - I| = 0$

28. Sol. $A\,(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$

$A\,(\text{adj } A) = 10 \begin{bmatrix} 1 & 0 \\ 1 & 10 \end{bmatrix}$

$A\,(\text{adj } A) = |A|\,I_n$

$\therefore |A| = 10$

29. Sol. $A(\text{Adj } A) = |A|\ I_3 = kI_3 \Rightarrow k = |A| = 8$

30._ Sol. $A_2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A_3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \Rightarrow A_{10} = \begin{bmatrix} 1 & 10 \\ 0 & 1 \end{bmatrix}$

$\text{adj } A_{10} = \begin{bmatrix} 1 & 0 \\ -10 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 10 & 100 \\ 0 & 10 \end{bmatrix} + \begin{bmatrix} 1 & -10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 90 \\ 0 & 11 \end{bmatrix}$

$b_1 + b_2 + b_3 + b_4 = 22 + 90 = 112.$

31. **Sol.** Taking $C_3 \to C_3 - (C_1\alpha - C_2)$
we get

$|A| = \begin{vmatrix} a & b & 0 \\ b & c & 0 \\ 2 & 1 & -2\alpha+1 \end{vmatrix} = (1 - 2\alpha)(ac - b^2)$

$\therefore$ non-invertible if $\alpha = \frac{1}{2}$ and if a, b, c are in G.P.

32. **Sol.** $AB = C \Rightarrow \det(A)\det(B) = \det(C) \Rightarrow \det(B) = -1$

33. **Sol.** For nth order determinant $\Delta = |C| = D^{n-1}$

(1) For 3rd order determinant $\Delta = D^{3-1} = D^2$ ... (1)

(2) From (1) if D = 0 then $\Delta = 0$

(3) $D = 9 = 3^2$

$\Delta = (3^2)^2 = 3^4$ (Δ is not a perfect cube)

34. **Sol.** $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$A^{-1} = \frac{\text{adj}\ (A)}{|A|} =$

$A^3 = A^2 . A = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$

$\Rightarrow A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} = A^{-1}$

35. **Sol.** $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$; $A^3 = 5A^2 - 6AI + I^2$

$A^2 = \begin{bmatrix} 3 & 3 & 7 \\ 1 & 4 & 4 \\ 3 & 1 & 6 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix}$

$5A^2 - 6A + I = \begin{bmatrix} 10 & 9 & 23 \\ 5 & 9 & 14 \\ 9 & 5 & 19 \end{bmatrix} = A^3$

36. **Sol.** $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $bc \neq 0$

Characteristic equation is $|A - xI| = 0$

$\begin{vmatrix} a-x & b \\ c & d-x \end{vmatrix} = 0$

$(a - x)(d - x) - bc = 0$

$x^2 - x(a + d) + ad - bc = 0$

On comparing with the given equation $x^2 + k = 0$

$a + d = 0, k = ad - bc = |A|$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

A-1. **Ans.** **(1)**

Sol. If we interchange any two rows of a determinant in set B, its value becomes –1, hence it becomes a member of set C.

$\Rightarrow$ Number of elements in set B is equal to number of elements in set C.

$\Rightarrow$ Statement - 1 is true.

Also $B \cap C = \varphi \subset A \Rightarrow$ statement-2 is true.

But statement-2 is not a correct explanation of statement-1.

A-2. **Ans.** **(2)**

Sol. $A = \begin{vmatrix} 2 & 1+2i \\ 1-2i & 7 \end{vmatrix} \Rightarrow |A| = 14 - (1 - 4i^2) = 9$

satement 1 is true and obviously statement 2 is false.

A-3. **Ans.** **(1)**

Sol. $B = \begin{vmatrix} x & c & b \\ -c & x & a \\ b & -a & x \end{vmatrix}$

Transpose matrix formed by cofactors of B = $\begin{bmatrix} a^2+x^2 & ab-cx & ac+bx \\ ab+cx & b^2+x^2 & bc-ax \\ ac-bx & bc+ax & c^2+x^2 \end{bmatrix}$

$\Rightarrow$ adj B = A

$\Rightarrow |adjB| = |A|$

$\Rightarrow |B|^2 = |A|$

A-4. **Ans.** **(2)**

Sol. adj(AB) = (adj B) (adj A)

so statement 2 is false

A-5. **Ans.** **(1)**

Sol. $A^2 - 5A + 7I = 0$

$\Rightarrow A^2 - 5A = -7I$

$\Rightarrow |A^2 - 5A| = |-7I|$

$\Rightarrow |A|\,|A-5I| = 7$

so |A| can not be zero |A|

$A^2 - 5A + 7I = 0$ $\quad |A| \neq 0$

$\Rightarrow A - 5I = -7A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{7}(5I - A)$
Hence statement 1 is true
Now $A^3 - 2A^2 - 3A + I = A(A^2) - 2A^2 - 3A + I$
$= A(5A - 7I) - 2A^2 - 3A + I$
$= 3A^2 - 10A + I$
$= 5A - 20I \quad = 3((5A - 7I) - 10A + I$
$= 5(A - 4I)$
Statement 2 also correct

Section (B) : MATCH THE COLUMN

B-1. **Ans.** (A) → r ; (B) → q ; (C) → p ; (D) → q
Sol. From the first two equation
$x = z, y = 3 - 2z$
putting this in the last equation, we get
$(\lambda - 5)z = m - 9$
If $\lambda \neq 5$, the system of equation has unique solution and hence consistent when $\lambda = 5, m \neq 9$, the system has no solution
When $\lambda = 5$, $m = 9$, the system has infinite number of solution and hence consistent.

B-2. **Ans.** (A) → q ; (B) → r ; (C) → q ; (D) → s
Sol. By using $R_1 \to R_1 + R_3$ followed by $C_1 \to C_1 - C_3$, we get
$f(x) = 4\cot x \operatorname{cosec}^2 x$
$\Rightarrow f\left(\frac{\pi}{4}\right) = 8$ and $f\left(\frac{\pi}{2}\right) = 0$
$f'(x) = -4\operatorname{cosec}^4 x - 8\cot^2 x \operatorname{cosec}^2 x$
$\Rightarrow f'\left(\frac{\pi}{4}\right) = -32$ and $f'\left(\frac{\pi}{3}\right) = -\frac{32}{3}$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1 **Sol.** A'A = 1

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a^2+b^2+c^2 & ab+bc+ca & ab+bc+ca \\ ab+bc+ca & a^2+b^2+c^2 & ab+bc+ca \\ ab+bc+ca & ab+bc+ca & a^2+b^2+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a^2 + b^2 + c^2 = 1$, $ab + bc + ca = 0$
we know (ge tkurs gS fd)
$a^3 + b^3 + c^3$ abc $= (a + b = c)(a^2 + b^2 + c^2 - ab - bc - ca)$
$a^3 + b^3 + c^3 = (a + b + c)(1 - 0 - 3) +$
$a^3 + b^3 + c^3 = (a + b + c) + 3$
Now $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1 + 0$
$\therefore a^3 + b^3 + c^3 = 1 + 3 = 4$

C-2. **Sol.** $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

$C_1 \rightarrow C_1 + C_2 + C_3$

$\Rightarrow$ (sinx + 2cosx) $\begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

R_1 $R_2 - R_1$, R_3 $R_3 - R_2$

$\Rightarrow$ (sinx + 2cosx) $\begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & \cos x - \sin x & \sin x - \cos x \end{vmatrix} = 0$

$\Rightarrow$ (sinx + 2cosx) $(\sin x - \cos x)^2 = 0$

$\Rightarrow$ tanx = – 2 , tanx = 1

$\Rightarrow$ $x = \frac{\pi}{4}$ in $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

C-3. Sol. For the given homogeneous system to have non zero solution determinant of coefficient matrix should be zero, i.e.,

$\begin{vmatrix} 1 & -k & 1 \\ k & -1 & -1 \\ 1 & +1 & -1 \end{vmatrix} = 1(1 + 1) + k(-k + 1) -1(k + 1) = 0$

$\Rightarrow$ $2 - k^2 + k - k - 1 = 0$

$\Rightarrow$ $k^2 = 1$

$\Rightarrow$ $k = \pm 1$

C-4. Sol. Let $A = [a]_{3\times3}$

adj A = $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

|adj A| = 1(3 – 7) – 4(6 – 7) + 4(2 – 1) = 4

$\Rightarrow |A|^{3-1} = 4$

$\Rightarrow |A|^2 = 4$

$\Rightarrow |A| = \pm 2$

C-5. Sol. (i) Let $M^T = M$ then $(N^TMN)^T = N^TM^TN = N^TMN$ $\Rightarrow$ N^TMN is symmetric

Let $M^T = -M$ then $(N^TMN)^T = N^TM^TN = -N^TMN$ $\Rightarrow$ N^TMN is skew-symmetric

(ii) $(MN - NM)^T = (MN)^T - (NM)^T = N^TM^T - M^TN^T = NM - MN = -(MN - NM)$

$\Rightarrow$ (MN – NM) is skew-symmetric.

C-6. Sol. M = $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$

(1) $\begin{bmatrix} a \\ b \end{bmatrix}$ & [b c] are transpose.

So $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} b \\ c \end{bmatrix}$ is given $\Rightarrow$ a = b = c

M = $\begin{bmatrix} a & a \\ a & a \end{bmatrix}$ $\Rightarrow$ | M | = 0 A is wrong.

(2) [b c] & $\begin{bmatrix} a \\ b \end{bmatrix}$ are transpose.

So a = b = c B is wrong

(3) $M = \begin{bmatrix} a & 0 \\ 0 & c \end{bmatrix}$ ⇒ $|M| = ac \neq 0$ C is correct

(4) $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ given $ac \neq \lambda_2$. D is correct

(C, D) are correct.

(4) $M = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ given $ac \neq \lambda_2$ D lgh gS

C-7. **Sol.** $MN = NM$ & $M_2 - N_4 = 0$

$(M - N^2)(M + N^2) = 0$

$M - N^2 = 0$ Not Possible | $M + N^2 = 0$, $M - N^2 \neq 0$ | $|M + N^2| = 0$, $|M - N^2| = 0$

In any case $|M + N^2| = 0$

(1) $|M_2 + MN_2| = |M|\,|M + N_2|$
$= 0$
(A) is correct

(2) If $|A| = 0$ then $AU = 0$ will have ∞ solution.
Thus $(M_2 + MN_2)\,U = 0$ will have many 'U'
(B) is correct

(3) Obvious wrong.

(4) If $AX = 0$ & $|A| = 0$ then X can be non zero.
(D) is wrong

C-8. **Sol.** **(3)** $(x_4 Z_3 - Z_3 X_4)_T = (X_4Z_3)_T\,(Z_3 X_4)_T$
$= (Z_T)_3(X_T)_4 - (X_T)_4(Z_T)_3$
$= Z_3X_4 - X_4Z_3$
$= -(X_4Z_3 - Z_3X_4)$

(4) $(X_{23} + Y_{23})_T = -X_{23} - Y_{23}$ ⇒ $X_{23} + Y_{23}$ is skew–symmetric

C-9. **Sol.** $R_3 \rightarrow R_3 - R_2,\ R_2 \rightarrow R_2 - R_1$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 5+2\alpha & 5+4\alpha & 5+6\alpha \end{vmatrix} = -648\alpha$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ 3+2\alpha & 3+4\alpha & 3+6\alpha \\ 2 & 2 & 2 \end{vmatrix} = -648\alpha$$

$C_3 \rightarrow C_3 - C_2,\ C_2 \rightarrow C_2 - C_1$

$$\begin{vmatrix} (1+\alpha)^2 & \alpha(2+3\alpha) & \alpha(2+5\alpha) \\ 3+2\alpha & 2\alpha & 2\alpha \\ 2 & 0 & 0 \end{vmatrix} = -648\alpha$$

⇒ $2\alpha_2(2+3\alpha) - 2\alpha_2(2+5\alpha) = -324\alpha$

⇒ $-4\alpha_3 = -324\alpha$ ⇒ $\alpha = 0, \pm 9$

Exercise-3

Marked Questions may have for Revision Questions.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** Since $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Now $AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$

and $BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$

If $AB = BA \Rightarrow a = b$

Hence, AB = BA is possible for infinitely many B's.

2. **Sol.** Given, $A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow A^2 - B^2 = A^2 - B^2 + AB - BA \Rightarrow AB = BA$

3. **Sol.** Since, $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}$

$\therefore A^2 = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 25 & 25\alpha + 5\alpha^2 & 10\alpha + 25\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{bmatrix}$

$\Rightarrow |A^2| = \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 & 5\alpha^2 \\ 0 & \alpha^2 & 5\alpha^2 + 25\alpha \\ 0 & 0 & 25 \end{vmatrix}$

$= 25 \begin{vmatrix} 25 & 25\alpha + 5\alpha^2 \\ 0 & \alpha^2 \end{vmatrix} = 625\alpha^2$

But $|A|^2 = 25$

$\therefore 625\alpha^2 = 25 \Rightarrow |\alpha| = \frac{1}{5}$.

4. **Sol.** $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$

$C_1 \to C_1 - C_2$, $C_2 \to C_2 - C_3$

$D = \begin{vmatrix} 0 & 0 & 1 \\ -x & x & 1 \\ 0 & -y & 1+y \end{vmatrix} = xy$

$\therefore$ D is divisible by both x and y

5. **Sol.** The system of equations $x - cy - bz = 0$, $cx - y + az = 0$ and $bx + ay - z = 0$ have non-trivial solution if $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0 \Rightarrow 1(1 - a^2) + c(-c - ab) - b(ca + b) = 0 \Rightarrow a^2 + b^2 + c^2 + 2abc = 1$

6. **Sol.** As det (A) = ± 1, A^{-1} exists and $A^{-1} = \frac{1}{\det (A)}(\text{adj } A) = \pm (\text{adj } A)$

All entries in adj (A) are integers.

∴ A^{-1} has integer entries.

7. **Sol.** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

∴ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $(\because A^2 = I)$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow b(a + d) = 0, c(a + d) = 0$

and $a^2 + bc = 1$, $bc + d^2 = 1 \Rightarrow a = 1, d = -1, b = c = 0$

If $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, then $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$A \neq I, A \neq -I$

det A = –1 (Statement I is true)

Statement II tr (A) = 1 – 1 = 0, Statement II is false.

8. **Sol.** $|\text{adj } A| = |A|^{n-1} = |A|$, for n = 2

adj (A).adj(adj (A)) = |adj (A)| I ⇒ A.adj (A).adj (A)) = |adj(A)|A

⇒ |A|.adj(adj (A)) = |A| A ⇒ adj(adj (A)) = A

9. **Sol.** $\begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & b+1 & c-1 \\ a-1 & b-1 & c+1 \\ a & -b & c \end{vmatrix}^{1} = \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^n \begin{vmatrix} a+1 & a-1 & a \\ b+1 & b-1 & -b \\ c-1 & c+1 & c \end{vmatrix}$

$= \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix} + (-1)^{n+1} \begin{vmatrix} a+1 & a & a-1 \\ b+1 & -b & b-1 \\ c-1 & c & c+1 \end{vmatrix}$

$C_2 \leftrightarrow C_3$

$= (1 + (-1)^{n+2}) \begin{vmatrix} a & a+1 & a-1 \\ -b & b+1 & b-1 \\ c & c-1 & c+1 \end{vmatrix}$

This is equal to zero only if n + 2 is odd i.e. n is an odd integer.

10. **Ans.** **(3)**

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \Rightarrow$ $\det (A) = a_1 (b_2c_3 - c_2b_3) - a_2 (b_1c_3 - c_1b_3) + a_3 (b_1c_2 - c_1b_2)$

$= a_1b_2c_3 - a_1c_2b_3 + a_2c_1b_3 - a_2 b_1c_3 + a_3b_1c_2 - a_3 c_1b_2$

if any of the terms is non-zero, then det (A) will be non-zero and all the element of that term will be unity

Now there are 6 elements remaining out of which any one can be unity.

Hence number of non-singular matrices = $\underbrace{{}^6C_1}_{\text{choosing any one triplet}} \times \underbrace{{}^6C_1}_{\text{choosing any one element}}$
Hence correct option is (3)

11. **Ans.** **(2)**

Sol. Let A = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; $A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & b(a+d) \\ c(a+d) & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow$ a + d = 0 and $a^2 + bc = 1$
$\Rightarrow$ Tr(A) = 0
Statement-1 is true
Statement-2 |A| = ad – bc = – a^2 – bc = – 1
Statement-1 is true statement-2 is false.
Hence correct option is (2)

12. **Ans.** **(3)**
Sol. Equation (2) – equation (1) $\Rightarrow$ $x_1 + x_2 = 0$
(3) – 2(1) $\Rightarrow$ $x_1 + x_2 = -5$
No solution
Hence correct option is (3)

13. **Sol.** **(2)**
A' = A , B' = A
P = A(BA)
P' = (A(BA))' = (BA)' A' = (A'B') A' = (AB) A = A(BA)
$\therefore$ A(BA) is symmetric
similarly (AB) A is symmetric
Statement(2) is correct but not correct explanation of statement (1).

14. **Sol.** **(2)**

$\Delta = \begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0 \quad \Rightarrow \quad 8 - k(k-2) - 2(2k-8) = 0$

$\Rightarrow 8 - k^2 + 2k - 4k + 16 = 0 \quad \Rightarrow \quad -k^2 - 2k + 24 = 0$
$\Rightarrow k^2 + 2k - 24 = 0 \quad \Rightarrow \quad (k+6)(k-4) = 0 \quad \Rightarrow \quad k = -6, 4$
Number of values of k is 2

15. **Sol.** $H^2 = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = \begin{bmatrix} \omega^2 & 0 \\ 0 & \omega^2 \end{bmatrix}$

If $H^k = \begin{bmatrix} \omega^k & 0 \\ 0 & \omega^k \end{bmatrix}$, then $H^{k+1} = \begin{bmatrix} \omega^{k+1} & 0 \\ 0 & \omega^{k+1} \end{bmatrix}$

So by mathematical induction,

$H^{70} = \begin{bmatrix} \omega^{70} & 0 \\ 0 & \omega^{70} \end{bmatrix} = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix} = H$

16. **Sol.** **(1)**
x – ky + z = 0
kx + 3y – kz = 0
3x + y – z = 0
this system will have non trivial solution if (non-trival solution)

$\begin{vmatrix} 1 & -k & 1 \\ k & 3 & -k \\ 3 & 1 & -1 \end{vmatrix} = 0$

$1(-3 + k) + k(-k + 3k) + 1(k - 9) = 0$
$k - 3 + 2k^2 + k - 9 = 0$
$2k^2 + 2k - 12 = 0$
$k^2 + k - 6 = 0$
$k = -3, k = 2$
so the system of equations will have only trivial solution when $k \in R - \{2, -3\}$

17. **Sol.** **(4)**
Statement-1 : Determinant of a skew sysmmetric matrix of odd order is zero
Statement-2 : $\det(A^T) = \det(A)$
$\det(-A) = (-1)^n \det(A)$ where A is a n × n order matrix

18 **Sol.** $A(u_1 + u_2) = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$; $u_1 + u_2 = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}$ adj A ; $|A| = 1$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}$ Now $u_1 + u_2 = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

19. **Sol.** Subtracting $P^3 - P^2Q = Q^3 - Q^2P \Rightarrow P^2(P - Q) + Q^2(P - Q) = 0$
$(P^2 + Q^2)(P - Q) = 0$
If $|P^2 + Q^2| \neq 0$ then $P^2 + Q^2$ is invertible $\Rightarrow P - Q = O$ contradiction
Hence $|P^2 + Q^2| = 0$

20. **Sol.** **(2)**
$\frac{k+1}{k} = \frac{8}{k+3} \neq \frac{4k}{3k-1}$ $\Rightarrow$ $k^2 + 4k + 3 = 8k$ $\Rightarrow$ $k^2 - 4k + 3 = 0$
$k = 1, 3$
If $k = 1$ then $\frac{8}{1+3} \neq \frac{4.1}{2}$ False
And If $k = 3$ then $\frac{8}{6} \neq \frac{4.3}{9-1}$ True
therefore k = 3/ Hence only one value of k.

21. **Sol.** **(2)**
$|P| = 1(12 - 12) - \alpha(4 - 6) + 3(4 - 6) = 2\alpha - 6$
$|P| = |A|^2 = 16 \Rightarrow 2\alpha - 6 = 16 \Rightarrow \alpha = 11.$

22. **Sol.** **Ans.** **(1)**

$\begin{vmatrix} 1+1+1 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \times \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix}^2$

$= (1 - \alpha)^2 (1 - \beta)^2 (\alpha - \beta)^2$

23. Sol. Ans. (4)

$BB^T = B(A^{-1}A^T)^T$
$= B(A^T)^T (A^{-1})^T$
$= BA(A^{-1})^T$
$= A^{-1}A^TA(A^{-1})^T$
$= A^{-1}AA^T(A^{-1})^T$
$= IA^T(A^{-1})^T$
$= A^T(A^{-1})^T$
$= A^T(A^T)^{-1}$
$= I$

24. Ans. (4)

Sol. $AA^T = 9I$

$\Rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = 9 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow$ $a + 4 + 2b = 0,\ 2a + 2 - 2b = 0,\ a^2 + 4 + b^2 = 9$

$\Rightarrow$ $a = -2,\ b = -1.$

25. Ans. (3)

Sol. $\begin{vmatrix} \lambda - 2 & 2 & -1 \\ 2 & -3 - \lambda & 2 \\ -1 & 2 & -\lambda \end{vmatrix} = 0$

$\Rightarrow$ $(\lambda - 2)(3\lambda + \lambda^2 - 4) - 2(-2\lambda + 2) - 1(4 - 3 - \lambda) = 0$

$\Rightarrow$ $(\lambda - 2)(\lambda^2 + 3\lambda - 4) + 4(\lambda - 1) + (\lambda - 1) = 0$

$\Rightarrow$ $(\lambda - 1)(\lambda^2 + 2\lambda - 8 + 5) = 0$ $\Rightarrow$ $(\lambda - 1)(\lambda^2 + 2\lambda - 3) = 0$

Two elements $(\lambda - 1)^2 (\lambda + 3) = 0$

26. Ans. (3)

Sol. $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) - 1(\lambda + 1) = 0$

$\lambda + 1 + \lambda^3 - \lambda - \lambda - 1 = 0$

$\lambda^3 - \lambda = 0$

Three values

27. Ans. (1)

Sol. $|A|I = AA^T$

$\Rightarrow (10a + 3b) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$\Rightarrow 25a^2 + b^2 = 10a + 3b$ & $15a - 2b = 0$ & $10a + 3b = 13$

$\Rightarrow 10a + \dfrac{3.15a}{2} = 13$ $\Rightarrow 65a = 2 \times 13$

$\Rightarrow a = \dfrac{2}{5}$ $\Rightarrow 5a = 2$

$\Rightarrow 2b = 6$ $\Rightarrow b = 3$

$\therefore \quad 5a + b = 5$

28. Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = -(a-1)^2 = 0$

$\Rightarrow a = 1$

For a = 1 we have first two planes co-incident

$x + y + z = 1$

$ax + by + z = 0$

For no solution these two are parallel

$\frac{1}{a} = \frac{1}{b} \frac{1}{1} \Rightarrow a = 1 , b = 1$

29. Ans. (2)

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$A^2 - 3A - 10I = 0$

$A^2 = 3A + 10I$

$3A^2 + 12A = 3(3A + 10I) + 12A = 21A + 30I$

$21A + 30I = \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix} + \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} = \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

$\text{adj}(21A + 30I) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Marked Questions may have for Revision Questions.

1. Sol. Given that $\omega = -\frac{1}{2} = -\frac{1}{3} + i\frac{\sqrt{3}}{2}$

$\omega^2 = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$

Also $1 + \omega + \omega_2 = 0$ and $\omega_3 = 1$

Now given det. is

$D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} = 3\omega(\omega - 1)$

2. Sol. $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$A_2 = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\alpha_2 = 1$, ... (1) $\qquad \alpha + 1 = 5$ ....(2)

$\alpha = \pm 1 \qquad \alpha = 4$

(1) and (2) not satisfied simultaneously so no real solution of α.

3. **Sol.** $D = \begin{vmatrix} 2 & -1 & -1 \\ 1 & -2 & 1 \\ 1 & 1 & \lambda \end{vmatrix}$

$= 2(-2\lambda - 1) + (\lambda - 1) - (3)$

$= -4\lambda - 2 + \lambda - 1 - 3$

$= -3\lambda - 6$ for D = 0

$\lambda = -2$

$D_3 = \begin{vmatrix} 2 & -1 & 12 \\ 1 & -2 & 4 \\ 1 & 1 & 4 \end{vmatrix}$

$= 2(-8 - 4) - (0) + 12 \times 3$

$= 12 \neq 0$

For no solution D = 0 and atleast one of $D_1, D_2, D_3 \neq 0$

$\lambda = 2$

4. **Sol.** $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^T Q^{2005} P$

We observe that $Q = PAP^T$

$Q^2 = (PAP^T)(PAP^T) = PA(P^TP)AP^T = PA(IA)P^T = PA^2P^T$

Proceeding in the same way, we get

$Q^{2005} = PA^{2005}P^T$

Also $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

And proceeding in the same way

$A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

$P^TP = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, $X = P^TQ^{2005}P = P^T(PA^{2005}P^T)P = (P^TP)A^{2005}(P^TP) = I A^{2005} I$

$= A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

5. **Sol.** $|A^n| = |A|^n$; $|A^3| = 125$

$|A|^3 = 125 \Rightarrow \begin{vmatrix} \alpha & 2 \\ 2 & \alpha \end{vmatrix} = 5$

$\alpha^2 - 4 = 5 \Rightarrow \alpha \equiv \pm 3$

6. **Sol.** $D = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 0$; $D_1 = \begin{vmatrix} -1 & -2 & 3 \\ k & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = 3 - k$

$D_2 = \begin{vmatrix} 1 & -1 & 3 \\ -1 & k & -2 \\ 1 & 1 & 4 \end{vmatrix} = k - 3$; $D_3 = \begin{vmatrix} 1 & -2 & -1 \\ -1 & 1 & k \\ 1 & -3 & 1 \end{vmatrix} = k - 3$

for $k \neq 3$ $D_1 \neq 0, D_2 \neq 0, D_3 \neq 0$

⇒ Statement- 1 is true and Statement 2 is also true. and its correct explanation of statement-1

∴ option A is correct

7. Sol. Data inconsistent

A 3 × 3 non-singular matrix cannot be skew-symmetric

However considering M, N matrices as even order, we obtain correct answer.

$M^2 N^2 (M^T N)^{-1} (MN^{-1})^T = M^2N^2 N^{-1} (M^T)^{-1} (N^{-1})^T M^T \Rightarrow -M^2 N^2 N^{-1} M^{-1} N^{-1} M$

$\Rightarrow -M^2 NM^{-1} N^{-1} M \Rightarrow -MNN^{-1} M \Rightarrow -M^2$

8. Sol. Ans (D)

Given $P = [a]_{3\times3}$ $b = 2^{i+j} a$

$Q = [b]_{3\times3}$

$P = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ $|P| = 2$; $Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} 4a_{11} & 8\ a_{12} & 16\ a_{13} \\ 8a_{21} & 16\ a_{22} & 32\ a_{23} \\ 16a_{31} & 32\ a_{32} & 64\ a_{33} \end{bmatrix}$

Determinant of $Q = \begin{vmatrix} 4a_{11} & 8\ a_{12} & 16\ a_{13} \\ 8a_{21} & 16\ a_{22} & 32\ a_{23} \\ 16a_{31} & 32\ a_{32} & 64\ a_{33} \end{vmatrix} = 4 \times 8 \times 16 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 2a_{21} & 2\ a_{22} & 2\ a_{23} \\ 4a_{31} & 4\ a_{32} & 4\ a_{33} \end{vmatrix}$

$= 4 \times 8 \times 16 \times 2 \times 4 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 2^2 . 2^3 . 2^4 . 2^1 . 2^2 . 2^1 = 2^{13}$

9. Sol. Ans. (D)

$P^T = 2P + I \Rightarrow (P^T)^T = (2P + I)^T \Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I \Rightarrow 3P = -3I \Rightarrow P = -I$

$\Rightarrow PX = -IX = -X$

10. Ans. (B)

Sol. $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 1 & 0 & 0 \\ 8 & 1 & 0 \\ 16(1+2) & 8 & 1 \end{bmatrix}, P^3 = \begin{bmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 16(1+2+3) & 12 & 1 \end{bmatrix}$

$P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 4\times50 & 1 & 0 \\ 16(1+2+...+50) & 4\times50 & 1 \end{bmatrix}$, $P^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix}$ $\because P^{50} - Q = I$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 200 & 1 & 0 \\ 20400 & 200 & 1 \end{bmatrix} - \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 200 - q_{21} = 0 \Rightarrow q_{21} = 200$

$20400 - q_{31} = 0 \Rightarrow q_{31} = 20400$ and $200 - q_{32} = 0 \Rightarrow q_{32} = 200$

$\therefore \frac{q_{31} + q_{32}}{q_{21}} = \frac{20400 + 200}{200} = 103$

11. Ans. (A)

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 5$

Case - I : Five (1's) and four (0's)

$^9C_5 = 126$

Case - II : One (2) and one (1)

$^9C_2 \times 2! = 72$ ∴ Total = 198

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

1. **Sol.** 1 × 1 matrix [0] i.e.1

2 × 2 matrix $\begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix}$ i.e 2 × 1 =2

3 × 3 matrix i.e $\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ 2 × 2 × 2 = 8

Total = 1 + 2 + 8 = 11

2. **Sol.** Given $\begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} - \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix} = 0$

$\Rightarrow abc\begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} - \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix} = 0$

$\Rightarrow abc(a-b)(b-c)(c-a)(ab+bc+ca) - (a-b)(b-c)(c-a)(a+b+c) = 0$

$\Rightarrow abc(ab+bc+ca) = a+b+c$ $\quad \therefore a \neq b \neq c$

$\Rightarrow \dfrac{abc(ab+bc+ca)}{a+b+c} = 1$ **∴ Ans: 3**

3. **Sol.** Here $A^2 = A \Rightarrow$ $a^2 + bc = a$, $b(a+d) = b$

$c(a+d) = c$, $bc + d^2 = d$

$\therefore abcd \neq 0$ $\quad \therefore a + d = 1$ $\Rightarrow$ $bc = ad$

$\Rightarrow$ $bc - ad = 0$

$\Rightarrow$ $|A| = 0$ Ans.

4. **Sol.** Here $AA' = A'A \Rightarrow a = b$

5. **Sol.** Let $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ = $(a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1) - (a_1b_3c_2 + a_2b_1c_3 + a_3b_2c_1)$

$\because$ each element of Δ is either 0 or 2 , therefore the value of Δ cannot exceed 24

$\Delta = 24 \Rightarrow a_1b_2c_3 + a_3b_1c_2 + a_2b_3c_1 = 24 = 8 + 8 + 8$

$\Rightarrow a_1 = a_2 = a_3 = 2,\ b_1 = b_2 = b_3 = 2,\ c_1 = c_2 = c_3 = 2$

but in this case $\Delta = 0$

$\therefore \Delta = \begin{vmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{vmatrix} = 0(0-4) - 2(0-4) + 2(4-0) = 0 + 8 + 8 = 16$

6. **Sol.** Here $\Delta = \begin{vmatrix} abc & b^2c & c^2b \\ abc & c^2a & ca^2 \\ abc & a^2b & b^2a \end{vmatrix} = 0$

$\Rightarrow -a^2b^2c^2(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow (a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$

$\Rightarrow a + b + c = 0 \qquad \because a \neq b \neq c \neq 0$

7. **Sol.** Here $\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) = abc\left(1+\frac{ab+bc+bc}{abc}\right)$ = abc + (ab+bc+ca)

Now $a+b+c = -3$

$ab+bc+ca = 4$

$abc = 1$

$\therefore \quad \Delta = -1 + 4 = 3$

8. **Sol.** $\Delta = \begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = \begin{vmatrix} a^2 & b^2 & c^2 \\ 4a & 4b & 4c \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix}$ $R_2 \to R_2 - R_3$

$= 4\begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ $R_3 \to R_3 - R_1 + 2R_2 = -4(a-b)(b-c)(c-a)$

9. **Sol.** Let $\Delta = \begin{vmatrix} x^3-1 & 0 & x-x^4 \\ 0 & x-x^4 & x^3-1 \\ x-x^4 & x^3-1 & 0 \end{vmatrix}$ = $(|A|)^2 = (-2)^2 = 4$ as it is a cofactor determinant of A

10. **Sol.** $\frac{dy}{dx} = \begin{vmatrix} \cos x & -\sin x & \cos x \\ \cos x & -\sin x & \cos x \\ 2x & 3 & 4 \end{vmatrix} + \begin{vmatrix} \sin x & \cos x & \sin x \\ -\sin x & -\cos x & -\sin x \\ 2x & 3 & 4 \end{vmatrix} + \begin{vmatrix} \sin x & \cos x & \sin x \\ \cos x & -\sin x & \cos x \\ 2 & 0 & 0 \end{vmatrix}$

$= 0 + 0 + 2(\cos^2 x + \sin^2 x) = 2$

11. **Sol.** $\Delta = \begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & a+b & c \end{vmatrix} = 0 \Rightarrow \frac{a^2+b^2+c^2}{a}\begin{vmatrix} 1 & b-c & c+b \\ 0 & c & -a-b \\ 0 & c+a & -b \end{vmatrix} = 0$ $\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$

$\Rightarrow$ a+b+c=0 $\therefore$ x = 1, y = 1

12. **Sol.** use $C_1 \to C_1 + C_2 + C_3$ in each then we get x + 9 = 0 $\Rightarrow$ x = –9

13. **Sol.** Given can be written as

$$p + \frac{q}{\lambda} + \frac{r}{\lambda^2} + \frac{5}{\lambda^3} + \frac{t}{\lambda^4} = \begin{vmatrix} 1+3/\lambda & 1-\frac{1}{\lambda} & 1+\frac{3}{\lambda} \\ 1+\frac{1}{\lambda^2} & \frac{2}{\lambda}-1 & 1-\frac{3}{\lambda} \\ 1-\frac{3}{\lambda^2} & 1+\frac{4}{\lambda} & 3 \end{vmatrix}$$

Taking limit $\lambda \to \infty$

$$p = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 3 \end{vmatrix} = -4$$

14. **Sol.** For consistency we have,

$\begin{vmatrix} 1 & 1 & -1 \\ 2 & -1 & -c \\ -b & 3b & -c \end{vmatrix} = 0 \Rightarrow c = \frac{5b}{4b+3} < 1 \Rightarrow \frac{5b-4b-3}{4b+3} < 0 \Rightarrow \frac{b-3}{4b+3} < 0 \Rightarrow b \in \left(-\frac{3}{4}, 3\right)$

15. **Sol.** $\Delta = \begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -i+\omega-1 & -1 \end{vmatrix} = \begin{vmatrix} 0 & 1+\omega+\omega^2 & 0 \\ 1-i & -1 & \omega^2-1 \\ -1 & -i+\omega-1 & -1 \end{vmatrix} = 0$ $(R_1 \to R_1 + R_3 - R_2)$

16. **Sol.** AB = B $\Rightarrow \begin{bmatrix} ap+bq \\ cp+dq \end{bmatrix} = \begin{bmatrix} p \\ q \end{bmatrix}$

$\Rightarrow$ ap + bq = p

cp + dq = q

Eliminating p and q we get

$\Rightarrow$ ad – bc – (a+d) + 1 = 0

$\Rightarrow$ ad – bc – 3 + 1 = 0

$\Rightarrow$ ad – bc = 2 $\Rightarrow$ |A| = 2

17. **Sol.** $\therefore$ A is non singular $\therefore$ |A| ≠ 0

$\therefore$ AB – BA = A $\Rightarrow$ AB = A + BA = A(I + B)

$\Rightarrow$|A| |B| = |A| |I + B| $\Rightarrow$ |B| = |I + B|

Similarly |B| = |B – I| $\therefore$ |B – I| + |B – I| = 6

18. **Sol.** Let A = diag(a_1 a_2 a_3 ....a_n)

A_3 = diag $(a_1^3, a_2^3, a_3^3, ...a_n^3)$

$\therefore$ A_3 = A $\therefore$ $a_{13} = a_1$ $\Rightarrow$ a_1= 0, 1, – 1

$\therefore$ Total Number of diagonal matrix = 3_n

19. **Sol.** AB = A(adjA) = |A| I_3 = $-2I_3$

$\therefore$(AB + $3I_3$) = |$-2I_3$ + $3I_3$| = |I_3| = 1

20. **Sol.** Here $|A| = xyz - (8x + 3z + 4y) + 28 = 60 - 20 + 28 = 68$

$\therefore |A.\text{adj}A| = |A|\,|\text{adj}A| = |A|\,.\,|A|^2 = 68^3$

21. **Sol.** Let A = diagonal (a, b, c), B is any other square matrix of order 3

$\therefore$ AB = BA $\therefore$ a = b = c

given a + b+ c = 12 $\therefore$ a = b = c = 4

$\therefore$ |A| = abc = 4. 4. 4 = 64 $\therefore$ $|A|^{\frac{1}{2}} = 8$

22. **Sol.** Here $A^2 = 3A - 2I$ $\therefore A^8 = 255A - 254I$

$\therefore \lambda = 255$, $\mu = -254$ $\therefore \lambda + \mu = 1$

23. **Sol.** Here $f(x) = 2\sin^2 x + 2\cos^2 x = 2 \therefore f'(x) = 0$

$$\int_0^{\pi/2}(f(x) + f'(x))dx = \int_0^{\pi/2} 2dx = 2 \times \frac{\pi}{2} = \pi$$

24. **Sol.** A + B = AB

$\Rightarrow$ $I_n - A - B + AB = I_n$ $\Rightarrow (I_n - A)(I_n - B) = I_n$

$\Rightarrow$ $(I_n - A)^{-1} = (I_n - B) \therefore (I_n - B)(I_n - A) = I_n$

$\Rightarrow$ $I_n - B - A + BA = I_n$ $\Rightarrow$ A + B = BA

$\therefore$ AB = BA

25. **Sol.** $\therefore$ $B = -A^{-1}BA$ $\therefore AB = -BA$

$\Rightarrow$ AB + BA = 0

$\therefore$ $(A + B)^2 + A^2 + AB + BA + B^2 = A^2 + B^2$

26. **Sol.** For non trivial solution we have

$$\Delta = \begin{vmatrix} \lambda & \lambda+1 & \lambda-1 \\ \lambda+1 & \lambda & \lambda+2 \\ \lambda-1 & \lambda+2 & \lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda = -\frac{1}{2}$$

27. **Sol.** $a + b = \sum_{k=1}^{9}(a_k + b_k) = \sum_{k=1}^{9}\left(k^{10}c_k + (10-k)^{10}c_k\right) = 10\sum_{k=1}^{9}{}^{10}C_k = 10(2^{10} - 2) = 10220$

28. **Sol.** $|A - \lambda I| = \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0 \Rightarrow (-\lambda + 2)\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$(\lambda - 2)\ [(\lambda^2 - 1) - (-\lambda - 1) + 1(1 + \lambda)] = 0$

$\Rightarrow$ $(\lambda - 2)(\lambda^2 + 2\lambda + 1) = 0$ $\Rightarrow$ $\lambda = 2, -1, -1$

29. **Sol.** $\Delta = \begin{vmatrix} \sqrt{6} & 2i & 3+\sqrt{6} \\ \sqrt{12} & \sqrt{3}+\sqrt{8}i & 3\sqrt{2}+\sqrt{6}i \\ \sqrt{18} & \sqrt{2}+\sqrt{12}i & \sqrt{27}+2i \end{vmatrix}$

$= \sqrt{6}\begin{vmatrix} 1 & 2i & 3+\sqrt{6} \\ \sqrt{2} & \sqrt{3}+2i\sqrt{2} & 3\sqrt{2}+\sqrt{6}\ i \\ \sqrt{3} & \sqrt{2}+2\sqrt{3}i & 3\sqrt{3}+2\ i \end{vmatrix}$ $R_2 \to R_2 - \sqrt{2}\ R_1$, & $R_3 \to R_3 - \sqrt{3}\ R_1$

$$= \sqrt{6}\begin{vmatrix} 1 & 2i & 3+\sqrt{6} \\ 0 & \sqrt{3} & \sqrt{6}i-2\sqrt{3} \\ 0 & \sqrt{2} & 2i-3\sqrt{2} \end{vmatrix} = -6$$

30. Ans. (3)

Sol. As PQ = kI $\Rightarrow$ $Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(\text{adj}P)\, I$ $\Rightarrow$ $Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha-4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8}$ $\Rightarrow$ $\frac{k}{(20+12\alpha)}(-3\alpha-4) = \frac{-k}{8}$ $\Rightarrow$ $2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3$ $\Rightarrow$ $\alpha = -1$

also]kh $|Q| = \frac{k^3|I|}{|P|}$ $\Rightarrow$ $\frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20 + 12\alpha) = 2k$ $\Rightarrow$ $8 = 2k$ $\Rightarrow$ $k = 4$

(1) incorrect

(2) correct

(3) $|P(\text{adj}Q)| = |P|\,|\text{adj}Q| = P|\,|Q|^2 = 2^2(2^3)^2 = 2^9$ correct

(4) $|Q(\text{adj}P)| = |Q|\,|\text{adj}P| = |Q|\,|P|^2 = 2^3(2^3)^2 = 2^9$ incorrect

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

Marked Questions may have for Revision Questions.

1. Sol. $f(x) = \begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$

$= \sin x\,(x^2 - x) - \cos x\,(x^3 - 2x^2) + \tan x\,(x^3 - 2x^3)$

$= (x^2 - x)\sin x - x^2(x - 2)\cos x - x^3 \tan x$

$\frac{f(x)}{x^2} = \left(1 - \frac{1}{x}\right)\sin x - (x - 2)\cos x - x\tan x$

∴ $\lim_{x\to 0}\frac{f(x)}{x^2} = 0 - 1 - 0 + 2 - 0 = 1.$

2. Ans. (2)

Sol. Make $C_1 \to C_1 + C_3$ we get

$C_1 \to C_1 + C_3$

determinant $= f(\theta) = \begin{bmatrix} 1 & \tan\theta+\sec^2\theta & 3 \\ 0 & \cos\theta & \sin\theta \\ 0 & -4 & 3 \end{bmatrix}$

$= 3\cos\theta + 4\sin\theta$

⇒ $f'(\theta) = 0$ ⇒ $-3\sin\theta + 4\cos\theta = 0$

⇒ $\tan\theta = \frac{4}{3}$ ⇒ $\theta = \tan^{-1}\frac{4}{3}$

⇒ $f(\theta)$ is ↑ for $\left[0,\tan^{-1}\frac{4}{3}\right]$

and ↓ for $\left[\tan^{-1}\frac{4}{3},\frac{\pi}{2}\right]$

max $f(\theta)$ is at $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

⇒ max $f(\theta) = 3\left(\frac{3}{5}\right)+4\left(\frac{4}{5}\right)=5$

minimum $f(\theta)$ is at $\theta = 0$

⇒ min $f(\theta) = 3$

3. Sol. det (A) = $\begin{vmatrix} 2k-1 & 2\sqrt{k} & 2\sqrt{k} \\ 2\sqrt{k} & 1 & -2k \\ -2\sqrt{k} & 2k & -1 \end{vmatrix}$ $C_2 \to C_2 - C_3$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 2\sqrt{k} & 1+2k & -2k \\ -2\sqrt{k} & 2k+1 & -1 \end{vmatrix}$ $R_2 \to R_2 - R_3$

$= \begin{vmatrix} 2k-1 & 0 & 2\sqrt{k} \\ 4\sqrt{k} & 0 & 1-2k \\ -2\sqrt{k} & 2k+1 & -1 \end{vmatrix} = (2k+1)^3$

∵ B is a skew-symmetric matrix of odd order therefore det(B) = 0

Now det (adj A) + det (adj B) = 10^6

⇒ $\{(2k+1)^3\}^2 + 0 = 10^6$ ⇒ $2k + 1 = 10$, as $k > 0$ ⇒ $k = 4.5$ ⇒ $[k] = 4$

4. Sol. Let xyz = t

$t\sin3\theta - y\cos3\theta - z\cos3\theta = 0$ ....(1)

$t\sin3\theta - 2y\sin3\theta - 2z\cos3\theta = 0$ ....(2)

$t\sin3\theta - y(\cos3\theta + \sin3\theta) - 2z\cos3\theta = 0$ ....(3)

$y_0 . z_0 \neq 0$ hence homogeneous equation has non-trivial solution

$$D = \begin{vmatrix} \sin 3\theta & -\cos 3\theta & -\cos 3\theta \\ \sin 3\theta & -2\cos 3\theta & -2\cos 3\theta \\ \sin 3\theta & -(\cos 3\theta + \sin 3\theta) & -2\cos 3\theta \end{vmatrix} = 0$$

$\Rightarrow$ $\sin 3\theta \cos 3\theta (\sin 3\theta - \cos 3\theta) = 0$

$\Rightarrow$ $\sin 3\theta = 0$ or $\cos 3\theta = 0$ or $\tan 3\theta = 1$

Case - I $\sin 3\theta = 0$

From equation (2)

$z = 0$ not possible

Case - II $\cos 3\theta = 0$, $\sin 3\theta \neq 0$

$t. \sin 3\theta = 0$ $\Rightarrow$ $t = 0$ $\Rightarrow$ $x = 0$

From equation (2)

$y = 0$ not possible

Case- III $\tan 3\theta = 1$

$\Rightarrow$ $3\theta = n\pi + \frac{\pi}{4}$, $n \in I$

$\Rightarrow$ $x. y. z \sin 3\theta = 0$ $\Rightarrow$ $\theta = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$

$\Rightarrow$ $x = 0$, $\sin 3\theta \neq 0$ $\Rightarrow$ $\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}$

Hence 3 solutions

5. **Sol.** $A = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix}$

Adj.A = $\begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$ $\Rightarrow$ $|A| = -17$ $\Rightarrow$ $A^{-1} = \frac{\text{AdjA}}{|A|}$

$A^{-1} = \frac{1}{17}\begin{bmatrix} 1 & 5 & 1 \\ 8 & 6 & -9 \\ 10 & -1 & -7 \end{bmatrix}$ $\Rightarrow$ $\begin{bmatrix} 3x+0+3z \\ 2x+y+0 \\ 4x+0+2z \end{bmatrix} = \begin{bmatrix} 8+2y \\ 1+z \\ 4+3y \end{bmatrix}$

$\Rightarrow$ $3x + 3z = 8 + 2y$ $\Rightarrow$ $2x + y = 1 + z$ $\Rightarrow$ $4x + 2z = 4 + 3y$

By Solving $x = 1$, $y = 2$ & $z = 3$

6. **Sol.** $x.x^2 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 0 & 2 & 6x^3-1 \\ 0 & 6 & 2x^3-2 \end{vmatrix}$ $\Rightarrow$ $x^3(12x^3 + 2) = 10$

$6x^6 + x^3 - 5 = 0$ $\Rightarrow$ $6x^6 + 6x^3 - 5x^3 - 5 = 0$

$(6x^3 - 5)(x^3 + 1) = 0$ $\Rightarrow$ $x^3 = -1$, $x^3 = \frac{5}{6}$

$x = -1$, $x = \left(\frac{5}{6}\right)^{1/3}$ so two solutions $= 2$

7. **Sol.** $\sum_{r=0}^{m} D_r = \begin{vmatrix} \sum_{r=0}^{m}(2r-1) & \sum_{r=0}^{m} {}^mC_r & \sum_{r=0}^{m} 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$

$\sum_{r=0}^{m}(2r-1) = m^2 - 1$, $\sum_{r=0}^{m} {}^mc_r = 2^m$, $\sum_{r=0}^{m} 1 = m+1$ $= \begin{vmatrix} m^2-1 & 2^m & m+1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(m^2) & \sin^2(m) & \sin^2(m+1) \end{vmatrix} = 0$

8. **Sol.** $ax + 2y = \lambda$

$3x - 2y = \mu$

(1) $a = -3$ gives

$\lambda = -\mu$

or ;k $\lambda + \mu = 0$ not for all λ, μ

(2) $a \neq -3 \Rightarrow \Delta \neq 0$ where $\Delta = \begin{vmatrix} a & 2 \\ 3 & -2 \end{vmatrix} = -2a - 6$

$\therefore$ (B) is correct

(3) correct

(4) if $\lambda + \mu \neq 0$

$\Rightarrow -3x + 2y = \lambda$ ....(1)

& $3x - 2y = \mu$ ....(2)

inconsistent $\Rightarrow$ (D) correct

9. **Sol.** AB = A

Premultiplying by B

BAB = BA

BB = B

$B^2 = B$

$\Rightarrow$ B is idempotent

similarly on post mutliplying by A

$ABA = A^2$

$AB = A^2$

$A = A^2$

$\Rightarrow$ A is idempotent

10. **Sol.** For orthogonal matrix AA' = I

$\Rightarrow \begin{vmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{vmatrix} \begin{vmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow \begin{vmatrix} 4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2 \\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

$\Rightarrow 4\beta^2 + \gamma^2 = 1,\ 2\beta^2 - \gamma^2 = 0,\ -2\beta^2 + \beta^2 = 0,\ \alpha^2 - \beta^2 - \gamma^2 = 0,\ \alpha^2 + \beta^2 + \gamma^2 = 1$

$\therefore \alpha = \pm\frac{1}{\sqrt{2}},\ \beta = \pm\frac{1}{\sqrt{6}},\ \gamma = \pm\frac{1}{\sqrt{3}}$

11. **Sol.** $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} \Rightarrow A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ ⇒ A is nilpotent of order 3.

12. **Sol.** Let $U_1 = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ then $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a \\ 2a+b \\ 3a+2b+c \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

⇒ a = 1, b = – 2, c = 1

∴ $U_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ Similarly, $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$, $U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

∴ $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \Rightarrow |U| = 3$

13. **Sol.** $U_{-1} = \frac{1}{3}\begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$ ⇒ Sum of the elements = 0

14. **Sol.** $\begin{bmatrix} 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 0 \end{bmatrix}\begin{bmatrix} 7 \\ -8 \\ -5 \end{bmatrix} = [5]$

17. **Sol.** **(Q.No. 15 to 17)**

a + 8b + 7c = 0 .... (i)

9a + 2b + 3c = 0 .... (ii)

a + b + c = 0 .... (iii)

$\Delta = \begin{vmatrix} 1 & 8 & 7 \\ 9 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 1.(-1) - 8(6) + 7(7) = 0$

Let C = λ

∴ a + 8b = –7λ

a + b = –λ

$\Rightarrow b = \lambda\frac{-6}{7}$ & $a = \frac{-\lambda}{7}$

∴ $(a, b, c) \equiv \left(\frac{-\lambda}{7}, \frac{-6\lambda}{7}, \lambda\right)$ where λ ∈ R

15. P(a, b, c) lies on the plane 2x + y + z = 1

∴ $\frac{-2\lambda}{7} - \frac{6\lambda}{7} + \lambda = 1 \Rightarrow \frac{-\lambda}{7} = 1 \Rightarrow \lambda = -7$

∴ 7a + b +c = 7 + 6 – 7 = 6

16. a = 2 ⇒ λ = –14

∴ b = 12 & c = –14

Now $\frac{3}{\omega^a} + \frac{1}{\omega^b} + \frac{3}{\omega^c} = \frac{3}{\omega^2} + \frac{1}{\omega^{12}} + 3.\omega^{14} = 3\omega + 1 + 3\omega_2 = 3(\omega + \omega_2) + 1 = -2$

17. b = 6 ⇒ λ = –7

⇒ a = 1 & c = – 7

now $ax^2 + bx + c = 0$ ⇒ $x^2 + 6x - 7 = 0$

⇒ x = –7 , 1

$$\therefore \sum_{n=0}^{\infty}\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)^n = \sum_{n=0}^{\infty}\left(\frac{6}{7}\right)^n$$

$$= 1 + \frac{6}{7} + \left(\frac{6}{7}\right)^2 + \ldots\ldots\ \infty = \frac{1}{1-\frac{6}{7}} = 7$$