Exercise-1

✍ **Marked Questions may have for Revision Questions.**

OBJECTIVE QUESTIONS

Section (A) : Equation of Tangent and Normal and angle of intersection of two curves

A-1. The equation of tangent to the curve y = 2cosx at $x = \frac{\pi}{4}$ is

(1) $y - \sqrt{2} = 2\sqrt{2}\left(x - \frac{\pi}{4}\right)$ (2) $y + \sqrt{2} = \sqrt{2}\left(x + \frac{\pi}{4}\right)$

(3) $y - \sqrt{2} = -\sqrt{2}\left(x - \frac{\pi}{4}\right)$ (4) $y - \sqrt{2} = \sqrt{2}\left(x - \frac{\pi}{4}\right)$

A-2. The equation of tangent at (–4, –4) on the curve $x^2 = -4y$ is
(1) 2x + y +4 = 0 (2) 2x – y – 12 = 0 (3) 2x + y – 4 = 0 (4) 2x – y + 4 = 0

A-3. If $x = t^2$ and y = 2t , then equation of the normal at t = 1 is
(1) x + y – 3 = 0 (2) x + y – 1 = 0 (3) x + y + 1 = 0 (4) x + y + 3 = 0

A-4. The equation of the normal to the curve $y^4 = ax^3$ at (a,a) is
(1) x + 2y = 3a (2) 3x – 4y + a = 0 (3) 4x + 3y = 7a (4) 4x – 3y = a

A-5. The curve $y - e^{xy} + x = 0$ has a vertical tangent at
(1) (1, 1) (2) (0, 1) (3) (1, 0) (4) no point

A-6.✍ If the tangent to the curve x = a (θ + sin θ), y = a (1 + cos θ) at $\theta = \frac{\pi}{3}$ makes an angle α (0 ≤ α< π) with x-axis, then α =

(1) $\frac{\pi}{3}$ (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{6}$ (4) $\frac{5\pi}{6}$

A-7. The number of tangents drawn to the curve xy = 4 from point (0, 1) is
(1) 0 (2) 1 (3) 2 (4) Infinite

A-8. Equation of the normal to the curve $y = -\sqrt{x} + 2$ at the point of its intersection with the curve $y = \tan(\tan^{-1} x)$ is
(1) 2x − y − 1 = 0 (2) 2x − y + 1 = 0 (3) 2x + y − 3 = 0 (4) none

A-9. The angle between the curves $y^2 = 4x + 4$ and $y^2 = 36(9 - x)$ is
(1) 30º (2) 45º (3) 60º (4) 90º (orthogonal curves)

A-10. The angle between the curves y = sinx and y = cosx is
(1) $\tan^{-1}\left(2\sqrt{2}\right)$ (2) $\tan^{-1}\left(3\sqrt{2}\right)$ (3) $\tan^{-1}\left(3\sqrt{3}\right)$ (4) $\tan^{-1}\left(5\sqrt{2}\right)$

A-11.✍ The subtangent, ordinate and subnormal to the parabola $y^2 = 4ax$ at a point (different from the origin) are in
(1) AP (2) GP (3) HP (4) AGP

A-12. The length of normal to the curve x = a (θ + sinθ), y = a (1– cosθ) at the point θ = π/2 is

(1) 2a (2) $\frac{a}{2}$ (3) $\sqrt{2}a$ (4) $\frac{a}{\sqrt{2}}$

Section(B) : Rate of change, Error and Approximation

B-1. The rate of change of the volume of a cone with respect to the radius of its base is -
(1) $\pi^2 h$ (2) $\frac{4}{3}\pi rh$ (3) $\frac{4}{3}\pi r^2 h$ (4) $\frac{2}{3}\pi rh$

B-2. The side of a square sheet is increasing at the rate of 4 cm per minute. The rate by which the area increasing when the side is 8 cm long is-
(1) 60 cm^2/sec. (2) 66 cm^2/sec. (3) 62 cm^2/sec. (4) 64 cm^2/sec.

B-3. The radius of an air bubble is increasing at the rate of 0.5 cm/sec. The rate by which the volume of the bubble is increasing when the radius is 1 cm, is-
(1) π cm^3/sec. (2) 3π cm^3/sec. (3) 2π cm^3/sec. (4) 4π cm^3/sec.

B-4. Water is poured into an inverted conical vessel of which the radius of the base is 2 m and height 4 m, at the rate of 77 litre/minute. The rate at which the water level is rising at the instant when the depth is 70 cm is: (use π = 22/7)
(1) 10 cm/min (2) 20 cm/min (3) 40 cm/min (4) none

B-5. A particle moves along the curve $y = x^2 + 2x$. Then the points on the curve where the x and y coordinates of the particle changing at the same rate, are
(1) $\left(\frac{-3}{4}, \frac{-1}{2}\right)$ (2) $\left(\frac{-1}{2}, \frac{-3}{4}\right)$ (3) $\left(\frac{3}{4}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{3}{4}\right)$

B-6. The point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate is-
(1) (4, 2) (2) (–4, – 2) (3) (2, 4) (4) (–2, – 4)

B-7. A man 1.5 m tall walks away from a lamp post 4.5 m high at a rate of 4 km/hr. How fast is the farther end of shadow moving on the pavement?
(1) 4 km/hr (2) 2 km/hr (3) 6 km/hr (4) 5 km/hr

B-8. A kite is 300 m high and there are 500 m of cord out. If the wind moves the kite horizontally at the rate of 5 km/hr. directly away from the person who is flying it, the rate at which the cord is being paid is
(1) 2 Km/hr (2) 4 Km/hr (3) 6 Km/hr (4) 8 Km/hr

B-9. If the radius of a sphere is measured as 8 cm with an error of 0.03 cm, then the approximate error in calculating its volume is
(1) 7.62 πcm^3 (2) 7.68 πcm^3 (3) 7.86 πcm^3 (4) 6.68 πcm^3

B-10. Using differentials, find the approximate value of $\sqrt{25.2}$.
(1) 5.02 (2) 5.01 (3) 5.03 (4) 5.04

B-11. The approximate change in the volume of a cube of side x meters caused by increasing the side by 4% is
(1) $0.06x^3 m^3$ (2) $0.09x^3 m^3$ (3) $0.12x^3 m^3$ (4) $0.15x^3 m^3$

Section(C) : Monotonicity

C-1. $f(x) = x + 1/x$, $x \neq 0$ is increasing when -
(1) $|x| < 1$ (2) $|x| > 1$ (3) $|x| < 2$ (4) $|x| > 2$

C-2. For which values of x, the function $f(x) = x^2 - 2x$ is decreasing -
(1) $x > 1$ (2) $x > 2$ (3) $x < 1$ (4) $x < 2$

C-3. Funciton $f(x) = x^3$ is
(1) Increasing in $(0, \infty)$ and decreasing in $(-\infty, 0)$ (2) Decreasing in $(0, \infty)$ and increasing in $(-\infty, 0)$
(3) Decreasing throughout (4) Increasing throughout

C-4. Function $f(x) = e^{-1/x}$ $(x > 0)$ is-
(1) Increasing (2) Decreasing (3) Not monotonic (4) None of these

C-5. The function $y = \frac{2x^2 - 1}{x^4}$ is
(1) Always increasing (2) Always decreasing
(3) Neither increasing nor decreasing (4) None of these

C-6. If $f(x) = 2x^3 - 9x^2 + 12x - 6$, then in which interval f(x) is monotonically increasing -
(1) (1, 2) (2) $(-\infty, 1)$ (3) $(2, \infty)$ (4) $(-\infty, 1)$ and $(2, \infty)$

C-7. Function $f(x) = x - \ell nx$ is decreasing, when
(1) $x \in (0, 1)$ (2) $x \in (-1, 1)$ (3) $x \in (1, \infty)$ (4) None of these

C-8. When $x \in (0, 1)$, function $f(x) = 1/\sqrt{x}$ is-
(1) Increasing (2) Decreasing
(3) Neither increasing nor decreasing (4) Constant

C-9. Let f be the function $f(x) = \cos x - \left(1 - \frac{x^2}{2}\right)$ then f(x) is strictly increasing in the interval
(1) $(-\infty, \infty)$ (2) $(-2, \infty)$ (3) $[0, \infty)$ (4) $(0, \infty)$

C-10. Function $f(x) = \frac{e^{2x} - 1}{e^{2x} + 1}$ is-
(1) Increasing (2) Decreasing
(3) Neither increasing for decreasing (4) Even function

C-11. Function $f(x) = \ell n \sin x$ is monotonically increasing when-
(1) $x \in (\pi/2, \pi)$ (2) $x \in (-\pi/2, 0)$ (3) $x \in (0, \pi)$ (4) $x \in (0, \pi/2)$

C-12. For $0 \le x \le 1$, the function $f(x) = |x| + |x - 1|$ is-
(1) Monotonically increasing (2) Monotonically decreasing
(3) Constant function (4) Identity function

C-13. For what value of 'a' the function $f(x) = x + \cos x - a$ increases
(1) 0 (2) 1 (3) –1 (4) Any value

C-14. Let $f(x) = x^3 + ax^2 + bx + 5 \sin^2 x$ be an increasing function in the set of real numbers R. Then a & b satisfy the condition:
(1) $a^2 - 3b - 15 > 0$ (2) $a^2 - 3b + 15 \le 0$ (3) $a^2 + 3b - 15 < 0$ (4) $a > 0$ & $b > 0$

C-15. The values of 'a' for which the function $f(x) = (a + 2) x^3 - 3ax^2 + 9ax - 1$ decreases for all real values of x is
(1) $(-\infty, -3]$ (2) $(-\infty, -3)$ (3) $(-\infty, -2)$ (4) $(-\infty, -3] \cup [0, \infty)$

C-16. Let the function $f(x) = \sin x + \cos x$, be defined in $[0, 2\pi]$, then f(x)
(1) increases in $(\pi/4, \pi/2)$ (2) decreases in $[\pi/4, 5\pi/4]$
(3) increases in $[0, \pi/4] \cup [\pi, 2\pi]$ (4) decreases in $[0, \pi/4) \cup (\pi/2, 2\pi]$

C-17. If $f(x) = \log (x - 2) - \frac{1}{x}$, then
(1) f(x) is M.I. for $x \in (2, \infty)$ (2) f(x) is M.I. for $x \in [-1, 2]$
(3) f(x) is M.D. for $x \in (2, \infty)$ (4) f(x) is M.D. for $x \in [-1, 2]$

C-18. The interval in which the function $f(x) = x^3$ increases less rapidly than $g(x) = 6x^2 + 15x + 5$ is :
(1) $(-\infty, -1)$ (2) $(-5, 1)$ (3) $(-1, 5)$ (4) $(5, \infty)$

C-19. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing at the point
(1) x = 3 (2) x = 1 (3) x = 2 (4) none of these

C-20. $f(x) = |x-1| + 2|x-3| - |x+2|$ is monotonically increasing at x = ?
(1) – 2, (2)0 (3) 3 (4) 5

Section(D) : Local maxima and minima

D-1. $x^3 - 3x + 4$ is minimum at -
(1) x = 1 (2) x = – 1 (3) x = 0 (4) No where

D-2. The local maximum value of $2x^3 - 9x^2 + 100$ is-
(1) 0 (2) 100 (3) 3 (4) 30

D-3. For what value of x, $x^2 \ell n(1/x)$ is maximum-
(1) $e^{-1/2}$ (2) $e^{1/2}$ (3) e (4) e^{-1}

D-4. Function $f(x) = 2x^3 - 21x^2 + 36x - 20$ has local minima at x =
(1) 2 (2) 4 (3) 6 (4) 0

D-5. Function $f(x) = -(x-1)^3 (x+1)^2$ has local minima at x =
(1) 1 (2) –1 (3) 6 (4) 0

D-6. Function $f(x) = x \,\ell nx$ has local maxima at x =
(1) x = e (2) $x = \frac{1}{e}$ (3) x= 1 (4) No local maxima

D-7. The function $f(x) = a \sin x + \frac{1}{3} \sin 3x$ has a maximum at x = π/3, then a equals-
(1) – 2 (2) 2 (3) –1 (4) 1

D-8. Function $f(x) = e^x + e^{-x}$ has -
(1) One minimum point (2) One maximum point
(3) Many extreme points (4) No extreme point

D-9. If $f(x) = 1 + 2x^2 + 4x^4 + 6x^6 + \ldots\ldots + 100x^{100}$ is a polynomial in a real variable x, then f(x) has:
(1) neither a maximum nor a minimum (2) only one maximum
(3) only one minimum (4) one maximum and one minimum

D-10. The function $f(x) = \sum_{k=1}^{5} (x-k)^2$ assumes minimum value for x given by-
(1) 5 (2) 3 (3) 5/2 (4) 2

D-11. $f(x) = \begin{cases} \tan^{-1} x, & |x| < \frac{\pi}{4} \\ \frac{\pi}{2} - |x|, & |x| \ge \frac{\pi}{4} \end{cases}$, then

(1) f(x) has no point of local maxima (2) f(x) has only one point of local maxima

(3) f(x) has exactly two points of local maxima (4) f(x) has exactly two points of local minima

D-12. The local maximum value of $x(1-x)^2$, $0 \le x \le 2$ is-

(1) 2 (2) $\frac{4}{27}$ (3) 5 (4) 2, $\frac{4}{27}$

D-13. f(3) is a maximum value of f(x) if-

(1) $f'(3) = 0, f''(3) > 0$ (2) $f'(3) = 0, f''(3) < 0$ (3) $f'(3) \ne 0, f''(3) = 0$ (4) $f'(3) < 0, f''(3) > 0$

D-14. If for a function f(x), $f'(2) = 0, f''(2) = 0, f'''(2) > 0$, then x = 2 is -

(1) A maximum point (2) A minimum point (3) An extreme point (4) Not an extreme point

D-15. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 5 & , x \le 1 \\ -2x + \log_2(b^2 - 2) & , x > 1 \end{cases}$ the set of values of b for which f(x) has greatest value at x = 1 is given by :

(1) $1 \le b \le 2$ (2) b = {1, 2}

(3) $b \in (-\infty, -1)$ (4) $\left[-\sqrt{130}, -\sqrt{2}\right) \cup \left(\sqrt{2}, \sqrt{130}\right]$

D-16. The minimum value of the function defined by f(x) = max (x, x + 1, 2 – x) is

(1) 0 (2) 1/2 (3) 1 (4) 3/2

Section(E) : Global maxima & minima

E-1. The absolute minimum and maximum values of $f(x) = x^3$, $x \in [-2, 2]$ are respectively -

(1) 6, 0 (2) 6, 2 (3) –8, 8 (4) 8, 0

E-2. The absolute maximum and minimum values of f(x) = sinx + cosx, x ∈ [0, π] are respectively

(1) $\sqrt{2}$, –1 (2) $\sqrt{2}$, 1 (3) $\sqrt{2}, -\sqrt{2}$ (4) $\sqrt{3}, \sqrt{2}$

E-3. The absolute minimum and maximum values of $f(x) = 4x - \frac{x^2}{2}$, $x \in \left[-2, \frac{9}{2}\right]$ are respectively

(1) –10, 8 (2) $\frac{63}{8}$, – 10 (3) 25, 16 (4) –10, $\frac{63}{8}$

E-4. The absolute maximum and minimum values of $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$, $x \in [0, 3]$ are respectively

(1) – 25, 39 (2) 25, –39 (3) 8, –8 (4) 8, 10

E-5. The absolute minimum and maximum values of $f(x) = \sin x + \frac{1}{2}\cos 2x$, $x \in \left[0, \frac{\pi}{2}\right]$ are respectively

(1) $\frac{3}{4}, \frac{1}{2}$ (2) $0, \frac{1}{2}$ (3) $-\frac{1}{2}, \frac{3}{4}$ (4) $\frac{1}{2}, \frac{3}{4}$

E-6. Let f(x) = x then in interval (0, 1)
(1) Local minima of f(x) exists (2) Local maxima of f(x) exists
(3) Global maxima of f(x) exists (4) Maxima or minima of f(x) does not exist

E-7. The maximum value of 5 sin θ + 3 sin (θ + π/3) + 3 is –
(1) 11 (2) 12 (3) 10 (4) 9

E-8. The minimum value of $y = x(\ell nx)^2$ is-
(1) 0 (2) 1 (3) 2 (4) None of these

E-9. If xy = 4 and x < 0 then maximum value of x + 16y is-
(1) 8 (2) – 8 (3) 16 (4) – 16

Section(F) : Application of maxima and minima

F-1. 20 is divided into two parts so that product of cube of one quantity and square of the other quantity is maximum. The parts are-
(1) 10, 10 (2) 16, 4 (3) 6, 14 (4) 12, 8

F-2. The ratio between the height of a right circular cone of maximum volume inscribed in a given sphere and the diameter of the sphere is-
(1) 2 : 3 (2) 3 : 4 (3) 1 : 3 (4) 1 : 4

F-3. A triangle with maximum area inscribed in a circle is-
(1) Right angled (2) Isosceles (3) equilateral (4) Isosceles right angled

F-4. The semi vertical angle of a right circular cone of maximum volume of a given slant height is
(1) $\cos^{-1}\sqrt{2}$ (2) $\sin^{-1}.\sqrt{2}$ (3) $\tan^{-1}\sqrt{3}$ (4) $\tan^{-1}\sqrt{2}$

F-5. The volume of the largest cylinder that can be inscribed in a sphere of radius ' r ' cm is (in cubic units)
(1) $\frac{4\pi r^3}{3\sqrt{3}}$ (2) $\frac{4\pi r^3}{3\sqrt{2}}$ (3) $\frac{4\pi r^2}{3\sqrt{2}}$ (4) $\frac{4\pi r^3}{2\sqrt{3}}$

Section(G) : Inequalities using monotonicity

G-1. For $0 < x_1 < x_2 < \frac{\pi}{2}$.
(1) $\frac{\tan x_2}{\tan x_1} < \frac{x_2}{x_1}$ (2) $\frac{\tan x_2}{\tan x_1} > \frac{x_2}{x_1}$ (3) $\frac{\tan x_2}{\tan x_1} = \frac{x_2}{x_1}$ (4) None of these

G-2. For $x \in \left(0, \frac{\pi}{2}\right)$
(1) (2sinx + tanx) > (3x)
(2) (2sinx + tanx) < (3x)
(3) $\lim_{x\to 0^+}\left[\frac{3x}{2\sin x + \tan x}\right]$ = 1, where [.] denote the GIF.
(4) Nothing can be say

G-3. The true set of real values of x for which the function, $f(x) = x \ell n x - x + 1$ is positive is

(1) $(1, \infty)$ (2) $(1/e, \infty)$ (3) $[e, \infty)$ (4) $(0, 1)$ and $(1, \infty)$

G-4.✍ If $a = (100)^{1/100}$ and $b = (101)^{1/101}$ then

(1) a = b (2) a > b (3) a < b (4) none of these

Section(H) : Rolle's theorem & LMVT

H-1.✍ The function $f(x) = x(x + 3)e^{-x/2}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$. The value of c which verifies Rolle's theorem, is

(1) 0 (2) – 1 (3) – 2 (4) 3

H-2. The Rolle's theorem is applicable in the interval $-1 \leq x \leq 1$ for the function

(1) $f(x) = x$ (2) $f(x) = x^2$ (3) $f(x) = 2x^3 + 3$ (4) $f(x) = |x|$

H-3.✍ For which interval, the function $f(x) = \frac{x^2 - 3x}{x - 1}$ satisfies all the conditions of Rolle's theorem

(1) [0, 3] (2) [–3, 0] (3) [1, 3] (4) For no interval

H-4. Rolle's theorem is not applicable to the function $f(x) = |x|$ defined on $[-1, 1]$ because

(1) f is not continuous on [–1, 1] (2) f is not differentiable on (–1, 1)

(3) $f(-1) \neq f(1)$ (4) $f(-1) = f(1) \neq 0$

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H-6. For the function $f(x) = e^x$, a = 0, b = 1, the value of c in mean value theorem will be

(1) $\ell n\ x$ (2) $\ell n\ (e - 1)$ (3) 0 (4) 1

H-7.✍ From mean value theorem $f(b) - f(a) = (b - a) f'(x_1)$; $0 < a < x_1 < b$ if $f(x) = \frac{1}{x}$, then x_1 =

(1) $\sqrt{ab}$ (2) $\frac{a+b}{2}$ (3) $\frac{2ab}{a+b}$ (4) $\frac{b-a}{b+a}$

H-8. If y = f(x) is continuous on [0,6], differentiable on (0,6), f(0) = –2 and f(6) =16 , then at some point between x = 0 and x = 6 , $f'(x)$ must be equal to

(1) – 18 (2) –3 (3) 3 (4) 14

H-9.✍ If the function $f(x) = x^3 - 6ax^2 + 5x$ satisfies the conditions of Lagrange's mean theorem for the interval [1, 2] and the tangent to the curve y = f(x) at x = 7/4 is parallel to the chord joining the points of intersection of the curve with the ordinates x = 1 and x = 2. Then the value of a is

(1) 35/16 (2) 35/48 (3) 7/16 (4) 5/16

Exercise-2

✍ Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

1.✍ The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-x/a}$ at the point

(1) (– a, 2b) (2) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (3) $\left(a, \frac{b}{e}\right)$ (4) (0, b)

2. Equation of normal drawn to the graph of the function defined as $f(x) = \frac{\sin x^2}{x}$, $x \neq 0$ and f(0) = 0 at the origin is
(1) x + y = 0 (2) x − y = 0 (3) y = 0 (4) x = 0

3. If tangents are drawn from the origin to the curve y = sin x, then their points of contact lie on the curve
(1) x – y = xy (2) x + y = xy (3) $x^2 - y^2 = x^2y^2$ (4) $x^2 + y^2 = x^2y^2$

4. Let $f(x) = \begin{cases} -x^2 & , x < 0 \\ x^2 + 8 & , x \geq 0 \end{cases}$ Equation of tangent line touching both branches of y = f(x) is
(1) y = 4x + 1 (2) y = 4x + 4 (3) y = x + 4 (4) y = x + 1

5. The point(s) on the parabola $y^2 = 4x$ which are closest to the circle,
$x^2 + y^2 - 24y + 128 = 0$ is/are:
(1) (0, 0) (2) $\left(2, 2\sqrt{2}\right)$ (3) (4, 4) (4) none

6. Minimum distance between the curves $f(x) = e^x$ & $g(x) = \ell n\, x$ is
(1) 1 (2) $\sqrt{2}$ (3) 2 (4) e

7. A curve with equation of the form $y = ax^4 + bx^3 + cx + d$ has zero gradient at the point (0, 1) and also touches the x − axis at the point (− 1, 0) then the values of x for which the curve has a negative gradient are :
(1) x > − 1 (2) x < 1 (3) x < − 1 (4) − 1 ≤ x ≤ 1

8. Let a(t) be a function of t such that $\frac{da}{dt} = 2$ for all values of t and a = 0 when t = 0.If the rate of change of distance of vertex of $y = x^2 - 2ax + a^2 + a$ from the origin with respect to t is k, then k =
(1) 2 (2) $2\sqrt{2}$ (3) $\sqrt{2}$ (4) $4\sqrt{2}$

9. The function $\frac{|x-1|}{x^2}$ is monotonically decreasing in
(1) (2, ∞) (2) (0, 1) (3) (0, 1) and (2, ∞) (4) (– ∞, ∞)

10. If $f(x) = \frac{(\sin^{-1} x + \tan^{-1} x)}{\pi} + 2\sqrt{x}$ then the range of f(x) is
(1) [–1, 1] (2) [0, 48] (3) $\left[0, \frac{15}{4}\right]$ (4) $\left[0, \frac{11}{4}\right]$

11. If f(x) is strictly increasing real function defined on R and c is a real constant, then number of solutions of f(x) = c is always equal to -
(1) 1 (2) 2 (3) 0 (4) 0 or 1

12. For what values of a does the curve $f(x) = x(a^2 - 2a - 2) + \cos x$ is always strictly monotonic decreasing ∀ x ∈ R.
(1) a ∈ R (2) $|a| < \sqrt{2}$
(3) $1 - \sqrt{2} < a < 1 + \sqrt{2}$ (4) $|a| < \sqrt{2} - 1$

13. Given that f is a real valued differentiable function such that $f(x)\, f'(x) < 0$ for all real x, it follows that
(1) f(x) is an increasing function (2) f(x) is a decreasing function
(3) |f(x)| is an increasing function (4) |f(x)| is a decreasing function

14. If g(x) is monotonically increasing and f(x) is monotonically decreasing for $x \in R$ and if (gof) (x) is defined for $x \in R$, then
(1) (gof) (x – 1) < (gof) (x + 1). (2) (gof) (x + 1) = (gof) (x – 1).
(3) (gof) (x + 1) < (gof) (x – 1). (4) None of these

15. Let f and g be two functions defined on an interval I such that $f(x) \geq 0$ and $g(x) \leq 0$ for all $x \in I$ and f is strictly decreasing on I while g is strictly increasing on I then
(1) the product function fg is strictly increasing on I
(2) the product function fg is strictly decreasing on I
(3) fog(x) is monotonically increasing on I
(4) All of these

16. If f : [1, 10] → [1, 10] is a non-decreasing function and g : [1, 10] → [1, 10] is a non-increasing function. Let h(x) = f(g(x)) with h(1) = 1, then h(2)
(1) lies in (1, 2) (2) is more than 2 (3) is equal to 1 (4) is not defined

17. Let $f(x) = (x^2 - 1)^n (x^2 + x + 1)$ then f(x) has local extremum at x = 1 when
(1) n = 5 (2) n = 7 (3) n = 3 (4) $n = 2k, k \in N$

18. If $f(x) = \dfrac{x^2-1}{x^2+1}$, for every real number, then minimum value of f(x)
(1) does not exist (2) is not attained even though f is bounded
(3) is equal to 1 (4) is equal to –1

19. If $f(x) = \begin{cases} x^2 ; & x \geq 0 \\ ax ; & x < 0 \end{cases}$. Then set of real values of 'a' such that f(x) is strictly monotonically increasing at x = 0 is
(1) $a \in R^+$ (2) $a \in R$ (3) $a \in R - \{0\}$ (4) $a \in \varphi$

20. Let $f(x) = \begin{cases} x^3 + x^2 - 10x ; & -1 \leq x < 0 \\ \sin x ; & 0 \leq x < \pi/2 \\ 1 + \cos x ; & \pi/2 \leq x \leq \pi \end{cases}$
then f(x) has
(1) local minimum at x = π/2 (2) local minima at x = –1
(3) absolute minima at x = 0, π (4) absolute maxima at x = π/2

21. If x be real, then the minimum value of $f(x) = 3^{x+1} + 3^{-(x+1)}$ is-
(1) 2 (2) 6 (3) 2/3 (4) 7/9

22. If $f(x) = \begin{cases} -\sqrt{1-x^2} , & 0 \leq x \leq 1 \\ -x , & x > 1 \end{cases}$, then
(1) Maximum of f(x) exist at x = 1 (2) Maximum of f (x) doesn't exists
(3) Maximum of $f^{-1}(x)$ exist at x = – 1 (4) Minimum of $f^{-1}(x)$ exist at x = 1

23. If $f(x) = a\,\ell n\,|x| + bx^2 + x$ has its extremum values at $x = -1$ and $x = 2$, then

(1) $a = 2, b = -1$ (2) $a = 2, b = -1/2$ (3) $a = -2, b = 1/2$ (4) none of these

24. The set of values of p for which all the points of extremum of the function $f(x) = x^3 - 3px^2 + 3(p^2 - 1)x + 1$ lie in the interval $(-2, 4)$, is:

(1) $(-3, 5)$ (2) $(-3, 3)$ (3) $(-1, 3)$ (4) $(-1, 4)$

25. A running track of 440 m is to be laid out enclosing a football field, the shape of which is a rectangle with semi circle at two opposite end. If the area of the rectangular portion is to be maximum, then the length of its sides is

(1) 120 m , $\frac{220}{\pi}$ m (2) 110 m , $\frac{\pi}{200}$ m (3) 110 m , $\frac{220}{\pi}$ m (4) 125 m , $\frac{220}{\pi}$ m

26. The radius of a right circular cylinder of greatest curved surface which can be inscribed in a given right circular cone is:

(1) one third that of the cone (2) $1/\sqrt{2}$ times that of the cone
(3) 2/3 that of the cone (4) 1/2 that of the cone

27. The maximum area of the rectangle whose sides pass through the angular points of a given rectangle of sides a and b is

(1) 2 (ab) (2) $\frac{1}{2}(a + b)^2$ (3) $\frac{1}{2}(a^2 + b^2)$ (4) none of these

28. $\lim_{x\to 0}\left[\frac{\sin x \tan x}{x^2}\right]$, (where $x \in \left(0, \frac{\pi}{2}\right)$ and [.] denotes the greatest integer function) =

(1) 0 (2) 1 (3) – 1 (4) 2

29. If the function $f(x) = x^3 - 6x^2 + ax + b$ satisfies Rolle's theorem in the interval [1,3] and $f'\left(\frac{2\sqrt{3}+1}{\sqrt{3}}\right) = 0$, then

(1) a = – 11 (2) a = – 6 (3) a = 6 (4) a = 11

30. For all real values of a_0, a_1, a_2, a_3 satisfying $a_0 + \frac{a_1}{2} + \frac{a_2}{3} + \frac{a_3}{4} = 0$, the equation $a_0 + a_1x + a_2x^2 + a_3x^3 = 0$ has a real root in the interval

(1) [0,1] (2) [–1,0] (3) [1,2] (4) [–2,–1]

31. Consider the function $f(x) = \max\{x^2, (1 - x)^2, 2x(1 - x)\}$ where $0 \le x \le 1$. Let Rolle's Theorem is applicable for f(x) on greatest interval [a, b] then a + b + c is (where c is point such that f'(c) = 0)

(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{1}{2}$ (4) $\frac{3}{2}$

32. $f : [0, 4] \to R$ is a differentiable function. Then for some $a, b \in (0, 4)$, $f^2(4) - f^2(0) =$

(1) 8f'(a) . f(b) (2) 4f'(b) f(a) (3) 2f' (b) f(a) (4) f'(b) f(a)

Comprehension # (Q.33 to Q. 35)

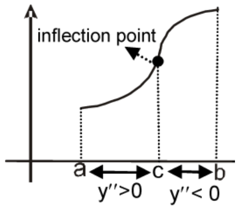
Concavity and convexity :

If $f''(x) > 0 \ \forall \ x \in (a, b)$, then the curve $y = f(x)$ is concave up (or simply concave) in (a,b) and

If $f''(x) < 0 \ \forall \ x \in (a, b)$ then the curve $y = f(x)$ is concave down (or simply convex) in (a, b).

Inflection point :

The point where concavity of the curve changes is known as point of inflection (at inflection point $f''(x)$ is equal to 0 or undefined).



33. Number of point of inflection for $f(x) = (x-1)^3 (x-2)^2$, is

(1) 1 (2) 2 (3) 3 (4) 4

34. Exhaustive set of values of 'a' for which the function $f(x) = x^4 + ax^3 + \frac{3x^2}{2} + 1$ will be concave upward along the entire real line, is :

(1) [–1,1] (2) [–2,2] (3) [0,2] (4) [0,4]

35. Area enclosed by $f(x) = \ell n (x-2) - \frac{1}{x}$, x = 6, x = 10 and x-axis is :

(1) equal to $10\ell n2 - \frac{8}{15}$ (2) less than $10\ell n2 - \frac{8}{15}$

(3) greater than $10\ell n2 - \frac{8}{15}$ (4) equal to $8\ell n2 - \frac{8}{15}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. Let f(x) = 2 + cos x for all real x.

Statement-1: For each real t, there exists a point c in [t, t + π] such that f'(c) = 0.

because

Statement-2 : f(t) = f(t + 2π) for each real t.

A-2.✍ **Statement-1:** e^{π} is bigger than π^e.

Statement-2 : $f(x) = x^{1/x}$ is a increasing function when $x \in [e, \infty)$

A-3. **Statement-1:** A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then minimum area of ΔOPQ is 32

Statement-2 : Area of triangle formed by a straight line passes through a fixed point (p ,q) and coordinate axes will be minimum then (p,q) is midpoint of intercept between coordinate axes

A-4. Let $f(x) = x^{50} - x^{20}$

Statement-1 : Global maximum of f(x) in [0, 1] is 0.

Statement-2 : x = 0 is a stationary point of f(x).

A-5. **Statement-1 :** If f(x) is increasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.

Statement-2 : If f(x) is decreasing function with concavity upwards, then concavity of $f^{-1}(x)$ is also upwards.

Section (B) : MATCH THE COLUMN

B-1. A line L : y = mx + 3 meets y - axis at E(0, 3) and the arc of the parabola $y^2 = 16x$, $0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum

Match column I with column II and select the correct answer using the code given below the lists :

	Column- I		Column - II
P.	m =	1.	$\frac{1}{2}$
Q.	Maximum area of ΔEFG is	2.	4
R.	y_0 =	3.	2
S.	y_1 =	4.	1

Codes :

	P	Q	R	S
(1)	4	1	2	3
(2)	3	4	1	2
(3)	1	3	2	4
(4)	1	3	4	2

B-2.

	Column – I		Column – II
P.	$f(x) = \frac{\sin x}{e^x}$, $x \in [0,\pi]$	1.	Conditions in Rolle's theorem are satisfied.
Q.	$f(x) = \text{sgn}((e^x - 1)\ \ell nx)$, $x \in \left[\frac{1}{2}, \frac{3}{2}\right]$	2.	Conditions in LMVT are satisfied.
R.	$f(x) = (x-1)^{2/5}$, $x \in [0,3]$	3.	At least one condition in Rolle's theorem is not satisfied.
S.	$f(x) = \begin{cases} x\left(\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right), & x \in [-1,1]-\{0\} \\ 0, & x = 0 \end{cases}$	4.	At least one condition in LMVT is not satisfied.

Codes :

	P	Q	R	S
(1)	1,2	3,4	3,4	3,4

(2)	1,2	3,4	1,2	3,4
(3)	3,4	1,2	3,4	1,2
(4)	3,4	1,2	1,2	3,4

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. The positive root of the equation, tan x – x = 0 lies in

(1) $\left(0, \frac{\pi}{2}\right)$ (2) $\left(2\pi, \frac{5\pi}{2}\right)$ (3) $\left(\pi, \frac{3\pi}{2}\right)$ (4) $\left(\frac{3\pi}{2}, 2\pi\right)$

C-2. If f(x) = $\begin{cases} 3x^2+12x-1, & -1 \le x \le 2 \\ 37-x, & 2 < x \le 3 \end{cases}$, then

(1) f(x) is increasing in [– 1, 2] (2) f(x) is continuous in [– 1, 3]
(3) f '(2) does not exist (4) f(x) has the maximum value at x = 2.

C-3. Let h(x) = f(x) – $(f(x))^2 + (f(x))^3$ ∀ x ∈ R, then

(1) h is increasing whenever f is increasing (2) h is increasing whenever f is decreasing
(3) h is decreasing whenever f is decreasing (4) nothing can be said in general

C-4. Let f(x) = $xe^{\frac{-x}{20}}$, then

(1) $40\,e^{-2} > 60e^{-3}$ (2) $5e^{\frac{-1}{4}} > 4e^{\frac{-1}{5}}$ (3) $5e^{\frac{-1}{4}} < 4e^{\frac{-1}{5}}$ (4) $40e^{-2} < 60e^{-3}$

C-5. f(x) is cubic polynomial which has local maximum at x = – 1, If f(2) = 18, f(1) = – 1 and f'(x) has local minima at x = 0, then

(1) the distance between (–1, 2) and (a, f(a)), where x = a is the point of local minima is $2\sqrt{5}$.
(2) f(x) is increasing for x ∈ [1, $2\sqrt{5}$)
(3) f(x) has local minima at x = 1
(4) the value of f(0) = 5

C-6. Which of the following are not true for polynomial function y = f(x) of degree 5, for a ∈ R.

(1) If f '(a) = 0, f ''(a) = 0, f '''(a) = 0 and f^{IV} (a) > 0 then f(x) has local minima at x = a.
(2) If f '(a) = 0, f ''(a) = 0 and f '''(a) > 0 then f(x) has local minima at x = a.
(3) If f '(a) = 0, f ''(a) = 0 and f '''(a) < 0 , then f(x) has local maxima at x = a.
(4) If f '(a) =0, f ''(a) = 0 and f '''(a) > 0, then f(x) is increasing at x = a.

C-7. A rectangular sheet of fixed perimeter with sides having their lengths in the ratio 8 : 15 is converted into an open rectangular box by folding after removing squares of equal area from all four corners. If the total area of removed squares is 100, the resulting box has maximum volume. The lengths of the sides of the rectangular sheet are

(1) 24 (2) 32 (3) 45 (4) 60

C-8. Let f, g : [–1, 2] → R be continuous function which are twice differentiable on the interval (–1, 2). Let the values of f and g at the points –1, 0 and 2 be as given in the following table :

	x = –1	x = 0	x = 2
f(x)	3	6	0
g(x)	0	1	– 1

In each of the intervals (–1, 0) and (0, 2) the function (f – 3g)'' never vanishes. Then the correct statement(s) is (are)

(1) f'(x) – 3g'(x) = 0 has exactly three solutions in (–1, 0) ∪ (0, 2)
(2) f'(x) – 3g'(x) = 0 has exactly one solution in (–1, 0)

(3) $f'(x) - 3g'(x) = 0$ has exactly one solution in (0, 2)
(4) $f'(x) - 3g'(x) = 0$ has exactly two solutions in (–1, 0) and exactly two solutions in (0, 2)

Exercise-3

✍ **Marked Questions may have for Revision Questions.**
*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The function $f(x) = \frac{x}{2} + \frac{2}{x}$ has a local minimum at **[AIEEE 2006 (3, –1), 120]**
(1) x = – 2 (2) x = 0 (3) x = 1 (4) x = 2

2. A value of c for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is **[AIEEE 2007(3, –1), 120]**
(1) $2 \log_3 e$ (2) $\frac{1}{2} \log_e 3$ (3) $\log_3 e$ (4) $\log_e 3$

3. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in **[AIEEE 2007(3, –1), 120]**
(1) (π/4, π/2) (2) (–π/2, π/4) (3) (0, π/2) (4) (–π/2, π/2)

4.✍ Suppose the cubic $x^3 - px + q = 0$ has three distinct real roots where p > 0 and q > 0. Then, which one of the following holds ? **[AIEEE 2008(3, –1), 105]**
(1) Minima at $\sqrt{\frac{p}{3}}$ and maxima at $-\sqrt{\frac{p}{3}}$ (2) Minima at $-\sqrt{\frac{p}{3}}$ and maxima at $\sqrt{\frac{p}{3}}$
(3) Minima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$ (4) Maxima at both $\sqrt{\frac{p}{3}}$ and $-\sqrt{\frac{p}{3}}$

5. Given $P(x) = x^4 + ax^3 + bx^2 + cx + d$ such that x = 0 is the only real root of P'(x) = 0. If P(–1) < P(1), then in the interval [–1, 1] **[AIEEE 2009(8, –2), 144]**
(1) P (–1) is the minimum and P(1) is the maximum of P
(2) P (–1) is not minimum but P(1) is the maximum of P
(3) P (–1) is the minimum and P(1) is not the maximum of P
(4) neither P (–1) is the minimum nor P(1) is the maximum of P

6. The shortest distance between the line y – x = 1 and the curve $x = y^2$ is **[AIEEE 2009(4, –1), 144]**
(1) $\frac{3\sqrt{2}}{8}$ (2) $\frac{2\sqrt{3}}{8}$ (3) $\frac{3\sqrt{2}}{5}$ (4) $\frac{\sqrt{3}}{4}$

7. Let f : **R** → **R** be defined by **[AIEEE 2010(8, –2), 144]**
$$f(x) = \begin{cases} k - 2x, & \text{if } x \le -1 \\ 2x + 3, & \text{if } x > -1 \end{cases}$$
If f has a local minimum at x = – 1, then a possible value of k is
(1) 0 (2) $-\frac{1}{2}$ (3) –1 (4) 1

8. Let f : **R** → **R** be a continuous function defined by $f(x) = \frac{1}{e^x + 2e^{-x}}$ **[AIEEE 2010(8, –2), 144]**

Statement -1 : $f(c) = \frac{1}{3}$, for some c ∈ R.

Statement -2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all x ∈ R.
(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.

(2) Statement-1 is true, Statement-2 is false.
(3) Statement -1 is false, Statement -2 is true.
(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

9. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x-axis, is **[AIEEE 2010 (4, –1), 144]**

(1) y = 1 (2) y = 2 (3) y = 3 (4) y = 0

10. Let f be a function defined by - **[AIEEE 2011 II(4, –1), 120]**

$$f(x) = \begin{cases} \frac{\tan x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$$

Statement - 1 : x = 0 is point of minima of f
Statement - 2 : $f'(0) = 0$.
(1) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.
(2) Statement-1 is true, statement-2 is true; statement-2 is not a correct explanation for statement-1
(3) Statement-1 is true, statement-2 is false.
(4) Statement-1 is false, statement-2 is true.

11. The shortest distance between line y – x = 1 and curve $x = y^2$ is : **[AIEEE 2011 (4, –1), 120]**

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{3\sqrt{2}}{8}$ (3) $\frac{8}{3\sqrt{2}}$ (4) $\frac{4}{\sqrt{3}}$

12. A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate (in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is : **[AIEEE 2012(4, –1), 120]**

(1) $\frac{9}{7}$ (2) $\frac{7}{9}$ (3) $\frac{2}{9}$ (4) $\frac{9}{2}$

13. Let a, b ∈ R be such that the function f given by $f(x) = \ell n\,|x| + bx^2 + ax,\ x \neq 0$ has extreme values at x = – 1 and x = 2.
Statement-1 : f has local maximum at x = – 1 and at x = 2. **[AIEEE 2012 (4, –1), 120]**
Statement-2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$.
(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

14. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1] **[AIEEE - 2013, (4, –1),120]**

(1) lies between 1 and 2 (2) lies between 2 and 3
(3) lies between –1 and 0 (4) does not exist.

15. If f and g are differentiable functions in [0, 1] satisfying f(0) = 2 = g(1), g(0) = 0 and f(1) = 6, then for some c∈]0, 1[: **[JEE(Main) 2014, (4, – 1), 120]**

(1) f'(c) = g'(c) (2) f'(c) = 2g'(c) (3) 2f'(c) = g'(c) (4) 2f'(c) = 3g'(c)

16. If x = –1 and x = 2 are extreme points of $f(x) = \alpha\log|x| + \beta x^2 + x$ then : **[JEE(Main) 2014, (4, –1), 120]**

(1) $\alpha = 2, \beta = -\frac{1}{2}$ (2) $\alpha = 2, \beta = \frac{1}{2}$ (3) $\alpha = -6, \beta = \frac{1}{2}$ (4) $\alpha = -6, \beta = -\frac{1}{2}$

17. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then **[JEE(Main) 2016, (4, – 1), 120]**

(1) (4 – π) x = πr (2) x = 2r (3) 2x = r (4) 2x = (π + 4) r

18. Consider f(x) = $\tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$. A normal to y = f(x) at x = $\frac{\pi}{6}$ also passes through the point : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\left(0, \frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6}, 0\right)$ (3) $\left(\frac{\pi}{4}, 0\right)$ (4) (0, 0)

19. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is : **[JEE(Main) 2017, (4, – 1), 120]**

(1) 12.5 (2) 10 (3) 25 (4) 30

20. The normal to the curve y(x – 2)(x – 3) = x + 6 at the point where the curve intersects the y-axis passes through the point : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(-\frac{1}{2},-\frac{1}{2}\right)$ (2) $\left(\frac{1}{2},\frac{1}{2}\right)$ (3) $\left(\frac{1}{2},-\frac{1}{3}\right)$ (4) $\left(\frac{1}{2},\frac{1}{3}\right)$

21. The radius of a circle, having minimum area, which touches the curve y = 4 – x^2 and the lines, y = |x| is

[JEE(Main) 2017, (4, – 1), 120]

(1) $2\left(\sqrt{2}+1\right)$ (2) $2\left(\sqrt{2}-1\right)$ (3) $4\left(\sqrt{2}-1\right)$ (4) $4\left(\sqrt{2}+1\right)$

22. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angels , then the value of b is :

[JEE(Main) 2018, (4, – 1), 120]

(1) 4 (2) $\frac{9}{2}$ (3) 6 (4) $\frac{7}{2}$

23. Let f(x) = $x^2 + \frac{1}{x^2}$ and g(x) = $x - \frac{1}{x}$, x ∈ R – {–1, 0, 1}. If h(x) = $\frac{f(x)}{g(x)}$, then the local minimum value of h(x) is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) – 3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The point(s) on the curve $y^3 + 3x^2 = 12y$ where the tangent is vertical, is (are)
[IIT-JEE-2002, Scr.(3, –1) /90]

(A) $\left(\pm\frac{4}{\sqrt{3}}, -2\right)$ (B) $\left(\pm\frac{\sqrt{11}}{3}, 1\right)$ (C) (0, 0) (D) $\left(\pm\frac{4}{\sqrt{3}}, 2\right)$

2. The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing is
[IIT-JEE-2002, Scr.(3, –1) /90]

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{3\pi}{2}$ (D) π

3. In [0, 1] Lagranges Mean Value theorem is NOT applicable to **[IIT-JEE-2003, Scr.(3, –1) /84]**

(A) $f(x) = \begin{cases} \frac{1}{2} - x & x < \frac{1}{2} \\ \left(\frac{1}{2} - x\right)^2 & x \geq \frac{1}{2} \end{cases}$

(B) $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(C) $f(x) = x|x|$

(D) $f(x) = |x|$

4. If f is differentiable and strictly increasing in a neighborhood of '0', then
$\lim_{x\to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)} =$ **[IIT-JEE-2004, Scr.(3, –1) /84]**

(A) 0 (B) 1 (C) – 1 (D) 2

5. If $f(x) = x^\alpha \ell nx$ and $f(0) = 0$, If Rolle's theorem can be applied to f in [0, 1] , then value of α can be
[IIT-JEE-2004, Scr.(3, –1) /84]

(A) –2 (B) –1 (C) 0 (D) 1/2

6. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **[IIT-JEE-2004, Scr.(3, –1) /84]**

(A) f(x) is a strictly increasing function (B) f(x) has a local maxima
(C) f(x) is a strictly decreasing function (D) f(x) is bounded

7. The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c - 1, e^{c-1})$ and $(c + 1, e^{c+1})$ **[IIT-JEE 2007, Paper-1, (3, –1)/ 81]**

(A) on the left of x = c (B) on the right of x = c
(C) at no point (D) at all points

8. Let the function $g : (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2 \tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is
[IIT-JEE 2008, Paper-2, (3, –1)/ 82]

(A) even and is strictly increasing in $(0, \infty)$
(B) odd and is strictly decreasing in $(-\infty, \infty)$
(C) odd and is strictly increasing in $(-\infty, \infty)$
(D) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

9.✍ The total number of local maxima and local minima of the function $f(x) = \begin{cases} (2+x)^3 & , -3 < x \leq -1 \\ x^{2/3} & , -1 < x < 2 \end{cases}$ is
[IIT-JEE 2008, Paper-1, (3, –1)/ 82]

(A) 0 (B) 1 (C) 2 (D) 3

10. The number of points in $(-\infty, \infty)$, for which $x^2 - x \sin x - \cos x = 0$, is
[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

(A) 6 (B) 4 (C) 2 (D) 0

11. If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, $f(1) = 1$, then

[JEE(Advanced) 2017, Paper-2,(3, –1)/61]

(A) $f'(1) \le 0$ (B) $f'(1) > 1$ (C) $0 < f'(1) \le \frac{1}{2}$ (D) $\frac{1}{2} < f'(1) \le 1$

Answers

EXERCISE # 1

Section (A) :

A-1.	(3)	**A-2.**	(4)	**A-3.**	(1)	**A-4.**	(3)	**A-5.**	(3)	**A-6.**	(4)	**A-7.**	(2)
A-8.	(1)	**A-9.**	(4)	**A-10.**	(1)	**A-11.**	(2)	**A-12.**	(3)				

Section(B) :

B-1.	(4)	**B-2.**	(4)	**B-3.**	(3)	**B-4.**	(2)	**B-5.**	(2)	**B-6.**	(3)	**B-7.**	(3)
B-8.	(2)	**B-9.**	(2)	**B-10.**	(1)	**B-11.**	(3)						

Section(C) :

C-1.	(2)	**C-2.**	(3)	**C-3.**	(4)	**C-4.**	(1)	**C-5.**	(3)	**C-6.**	(4)	**C-7.**	(1)
C-8.	(2)	**C-9.**	(3)	**C-10.**	(1)	**C-11.**	(4)	**C-12.**	(3)	**C-13.**	(4)	**C-14.**	(2)
C-15.	(1)	**C-16.**	(2)	**C-17.**	(1)	**C-18.**	(3)	**C-19.**	(1)	**C-20.**	(4)		

Section(D) :

D-1.	(1)	**D-2.**	(2)	**D-3.**	(1)	**D-4.**	(3)	**D-5.**	(2)	**D-6.**	(4)	**D-7.**	(2)
D-8.	(1)	**D-9.**	(3)	**D-10.**	(2)	**D-11.**	(3)	**D-12.**	(4)	**D-13.**	(2)	**D-14.**	(4)
D-15.	(4)	**D-16.**	(4)										

Section(E) :

E-1.	(3)	**E-2.**	(1)	**E-3.**	(1)	**E-4.**	(2)	**E-5.**	(4)	**E-6.**	(4)	**E-7.**	(3)
E-8.	(1)	**E-9.**	(4)										

Section(F) :

F-1.	(4)	**F-2.**	(1)	**F-3.**	(3)	**F-4.**	(4)	**F-5.**	(1)

Section(G) :

G-1.	(2)	**G-2.**	(1)	**G-3.**	(4)	**G-4.**	(2)

Section(H) :

H-1.	(3)	**H-2.**	(2)	**H-3.**	(4)	**H-4.**	(2)	**H-5.**	(2)	**H-6.**	(2)	**H-7.**	(1)
H-8.	(3)	**H-9.**	(2)										

EXERCISE # 2

PART - I

1.	(4)	**2.**	(1)	**3.**	(3)	**4.**	(2)	**5.**	(3)	**6.**	(2)	**7.**	(3)
8.	(2)	**9.**	(3)	**10.**	(4)	**11.**	(4)	**12.**	(3)	**13.**	(4)	**14.**	(3)
15.	(1)	**16.**	(3)	**17.**	(4)	**18.**	(4)	**19.**	(1)	**20.**	(3)	**21.**	(1)
22.	(1)	**23.**	(2)	**24.**	(3)	**25.**	(3)	**26.**	(4)	**27.**	(2)	**28.**	(2)
29.	(4)	**30.**	(1)	**31.**	(4)	**32.**	(1)	**33.**	(3)	**34.**	(2)	**35.**	(3)

PART -II

Section (A) :

A-1. (1) **A-2.** (2) **A-3.** (1) **A-4.** (1) **A-5.** (3)

Section (B) :

B-1. (1) **B-2.** (1)

Section (C) :

C-1. (2,3) **C-2.** (1,2,3,4) **C-3.** (1,3) **C-4.** (1,2) **C-5.** (2,3) **C-6.** (2,3)

C-7. (1,3) **C-8.** (2, 3)

EXERCISE # 3

PART - I

1. (4) **2.** (1) **3.** (2) **4.** (1) **5.** (2) **6.** (1) **7.** (3)

8. (4) **9.** (3) **10.** (2) **11.** (2) **12.** (3) **13.** (2) **14.** (4)

15. (2) **16.** (1) **17.** (2) **18.** (1) **19.** (3) **20.** (2) **21.** (3)

22. (2) **23.** (2)

PART - II

1. (D) **2.** (A) **3.** (A) **4.** (C) **5.** (D) **6.** (A) **7.** (A)

8. (C) **9.** (C) **10.** (C) **11.** (B)

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.

2. The test consists **30** questions, **4 marks** each.

3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

---

1. Number of tangents of curve $y = 4x^3 - 2x^5$ which passes through origin is
(1) 1 (2) 2 (3) 3 (4) 0

2. Value of p for which length of subtangent and subnormal is same for the curve $y = e^{px} + px$ at (0,1) is
(1) ± 1 (2) ± 2 (3) $\pm \frac{1}{3}$ (4) $\pm \frac{1}{2}$

3. If the curves $\frac{x^2}{a^2} + \frac{y^2}{4} = 1$ and $y^3 = 16x$ cut orthogonally then values of $\frac{3a^2}{2}$ is
(1) $\frac{4}{3}$ (2) 4 (3) 2 (4) 1

4. The two equal sides of an isosceles triangle with fix base 15 cm are decreasing at the rate of 2 cm / s. then the rate of change in area of triangle when the two equal sides are equal to base is
(1) $-15\sqrt{3}$ cm²/s (2) $-10\sqrt{3}$ cm²/s (3) $-20\sqrt{3}$ cm²/s (4) $-5\sqrt{3}$ cm²/s

5. If x + 4y = 14 is a normal to the curve $y^2 = \alpha x^3 - \beta$ at (2,3) then the value of $\alpha^2 + \beta^2$ is
(1) 35 (2) 53 (3) 9 (4) 45

6. Let f(x) and g(x) be differentiable for $0 \le x \le 1$ such that f(0) = 0, g(0) = 0, f(1) = 6. Let there exists a real number c in (0,1) such that f'(c) = 2g'(c) then g(1) =
(1) 1 (2) –1 (3) –2 (4) 3

7. The least value of k for which $f(x) = x^3 + x^2 + kx + 5$ is an increasing function $\forall$ x∈[1,2] is
(1) – 16 (2) –5 (3) –4 (4) –6

8. Let f(x) and g(x) be two continuous function defined from R toR such that
$f(x_1) > f(x_2)$ & $g(x_1) < g(x_2)$ $\forall$ $x_1 > x_2$ then solution set of $f(g(\alpha^2 - 2\alpha)) > f(g(3\alpha - 4))$ is
(1) (1, 4) (2) [1, 4] (3) {1, 4} (4) φ

9. If f(x) and g(x) = f(x) $\sqrt{1-2(f(x))^2}$ are strictly increasing function $\forall x \in R$ then
(1) $f(x) \ge \frac{1}{2}$ (2) $f(x) \le \frac{-1}{2}$ (3) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (4) f(x) ≥1

10. A point (a,b) on ellipse $4x^2 + 3y^2 = 12$ in first quadrant such that the area enclosed by the lines y = x , y = b, x = a and x-axis is maximum then value of 2a + b is

(1) $\frac{5}{2}$ (2) 4 (3) $\frac{7}{2}$ (4) 5

11. If f''(x) > 0 $\forall x \in R$ and f' (3) = 0 and g(x) = f ($\tan^2x$ –2 tanx + 4) for $0 < x < \frac{\pi}{2}$ then g(x) is increasing in

(1) $\left(0,\frac{\pi}{4}\right)$ (2) $\left(\frac{\pi}{6},\frac{\pi}{3}\right)$ (3) $\left(0,\frac{\pi}{3}\right)$ (4) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$

12. If $4x^4 + 9y^4 = C^6$ then the maximum value of xy is

(1) $\frac{C^2}{2\sqrt{3}}$ (2) $\frac{C^3}{2\sqrt{3}}$ (3) $\frac{C^2}{3\sqrt{2}}$ (4) $\frac{C^{3.}}{3\sqrt{2}}$

13. If $ax^2 + \frac{b}{x} \geq c$ $\forall x \in R^+$ where a > 0 and b > 0 then

(1) $27ab^2 \geq 4c^3$ (2) $27ab^3 \geq 4c^3$ (3) $27a^2b^2 \geq 4c^3$ (4) $27a^3b \geq 4c^3$

14. The angle at which the curve $y = ke^{kx}$ intersect y–axis is

(1) $\tan^{-1}(k^2)$ (2) $\cot^{-1}(k^2)$ (3) $\sin^{-1}\frac{1}{\sqrt{1+k^4}}$ (4) $\sec^{-1}\sqrt{1+k^4}$

15. If A > 0, B > 0 and A + B = $\frac{\pi}{3}$ then maximum value of (tanA. tanB) is

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\sqrt{3}$

16. If α and β are length of perpendiculars from origin to the tangent and normal to the curve $x^{2/3} + y^{2/3} = 6^{2/3}$ then value of $4\alpha^2 + \beta^2$ is

(1) 25 (2) 21 (3) 31 (4) 36

17. The distance of the point on $y = x^4 + 3x^2 + 2x$ which is nearest to the line y = 2x– 1 is

(1) $\frac{4}{\sqrt{5}}$ (2) $\frac{3}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{1}{\sqrt{5}}$

18. If x and y are two parts of 8 such that sum of their cubes is least possible then value of ab is

(1) 4 (2) 8 (3) 12 (4) 16

19. If f(x) = $\begin{cases} 3 & ,x = 0 \\ -x^2 + 3x + a & ,0 < x < 1 \\ mx + b & ,1 \leq x \leq 2 \end{cases}$, satisfy the lagrange's theorem then value of (a+b) m is

(1) 7 (2) 9 (3) 5 (4) 8

20. A quadratic polynomial y = f(x) touches the line y = x at x = 1 and passes through the point (–1,0) then value of f(2) is

(1) $\frac{5}{4}$ (2) $\frac{7}{4}$ (3) $\frac{9}{4}$ (4) $\frac{1}{4}$

21. If tangent at (α,β) to the curve $x^3 + y^3 = c^3$ meets curve again in (α_1,β_1) then $\frac{\alpha_1}{\alpha} + \frac{\beta_1}{\beta}$ =

(1) – 1 (2) 1 (3) 0 (4) 2

22. The interval in which f(x) = $\frac{x}{\ell nx}$ is decreasing is

(1) (e,∞) (2) $(-\infty,e)$ (3) $(0, e)$ (4) $(0, 3)$

23. The values of a for which $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x-1$ have a positive point of maxima is,

(1) $(-\infty,-3)$ (2) $\left(3,\frac{29}{7}\right)$ (3) $(-\infty,-3) \cup (3,\infty)$ (4) $(-\infty,-3)\cup\left(3,\frac{29}{7}\right)$

24. Minimum distance of origin from the curve $5x^2+5y^2-6xy= 4$ is

(1) $\sqrt{2}$ (2) $\frac{1}{\sqrt{2}}$ (3) 1 (4) 0

25. A long wire of length ℓ cm is bent to form a triangle with one of its angle as 60° then maximum area of Δ can be

(1) $\frac{\ell^2}{4\sqrt{3}}$ sq.cm (2) $\frac{\ell^2}{12\sqrt{3}}$ sq. cm (3) $\frac{\ell^2}{3\sqrt{3}}$ sq. cm (4) $\frac{\ell^2}{3}$ sq.cm

26. Which relation is true for $e^{\frac{-\pi}{2}} < \theta < \frac{\pi}{2}$

(1) $\cos(\ell n\theta) < \ell n(\cos\theta)$ (2) $\cos(\ell n\theta) > \ell n(\cos\theta)$

(3) $\cos(\ell n\theta) = \ell n(\cos\theta)$ (4) none of these

27. Let f and g be increasing and decreasing functions respectively from $[0,\infty)$ to $[0,\infty)$. Let h(x) = f(g(x), h(0) = 0 then h(x) – h(1) is

(1) always zero (2) always neagtive (3) always positive (4) increasing

28. A point on the curve $y = (x-2)^2$ at which the tangent is parallel to the chord joining (2,0) & (4,4) is (a,b) then a+b =

(1) 3 (2) 4 (3) 1 (4) 5

29. The maximum area of the isosceles triangle inscribed in the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ with its vertex at one end of major axis is

(1) $8\sqrt{3}$ (2) $9\sqrt{3}$ (3) $10\sqrt{3}$ (4) $\frac{3\sqrt{3}}{4}$

30. The approximate value of f(5.001) where $f(x)=x^3 - 7x^2+15$, is

(1) – 34.995 (2) 34.995 (3) –35.995 (4) 35.995

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

✍ Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

1.✍ Number of tangents drawn from the point (–1/2, 0) to the curve $y = e^{\{x\}}$. (Here { } denotes fractional part function).

(1) 2 (2) 1 (3) 3 (4) 4

2. If $f(x) = xe^{x(1-x)}$, then f(x) is

(1) increasing on $\left[-\frac{1}{2}, 1\right]$ (2) decreasing on R

(3) increasing on R (4) decreasing on $\left[-\frac{1}{2}, 1\right]$

3. The length of a longest interval in which the function $3 \sin x - 4 \sin^3 x$ is increasing is

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{2}$ (4) π

4. Let $f(x) = (1 + b^2)x^2 + 2bx + 1$ and let m(b) be the minimum value of f(x). As b varies, the range of m(b) is

(1) [0, 1] (2) $\left(0, \frac{1}{2}\right]$ (3) $\left[\frac{1}{2}, 1\right]$ (4) (0, 1]

5. If f(x) be a twice differentiable function such that $f(x) = x^2$ for x = 1, 2, 3, then]

(1) $f''(x) = 2 \quad \forall x \in [1, 3]$ (2) $f''(x) = 2$ for some $x \in (1, 3)$

(3) $f''(x) = 3 \ \forall x \in (2, 3)$ (4) $f''(x) = f'(x)$ for $x \in (2, 3)$

6. Let f, g and h be real-valued functions defined on the interval

[0, 1] by $f(x) = e^{x^2} + e^{-x^2}$, $g(x) = xe^{x^2} + e^{-x^2}$ and $h(x) = x^2e^{x^2} + e^{-x^2}$. If a, b and c denote, respectively, the absolute maximum of f, g and h on [0, 1], then

(1) a = b and c ≠ b (2) a = c and a ≠ b (3) a ≠ b and c ≠ b (4) a = b = c

7. If $f(x) = x^{\alpha} \ln x$ and f(0) = 0, If Rolle's theorem can be applied to f in [0, 1] , then value of α can be ;fn

(1) –2 (2) –1 (3) 0 (4) 1/2

8. If $P(x) = 51x^{101} - 2323x^{100} - 45x + 1035$, then P(x) = 0 has at least one root in

(1) (45, 46) (2) $(45^{1/100}, 46)$ (3) $(45^{1/50}, 46)$ (4) $(45, 46^{1/100})$

10*. The function f(x) = 2|x| + |x + 2| – ||x + 2| – 2|x|| has a local minimum or a local maximum at x =

(1) – 2 (2) $\frac{-2}{3}$ (3) 2 (4) $\frac{2}{3}$

11*. Let f: (0, ∞) → R be given by $f(x) = \int_{1/x}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$. Then

(1) f(x) is monotonically increasing on [1, ∞) (2) f(x) is monotonically decreasing on (0, 1)

(3) $f(x) + f\left(\frac{1}{x}\right) = 0$, for all $x \in (0, \infty)$ (4) $f(2^x)$ is an odd function of x on R

COMPREHENSION

Comprehension # 1 (Q.12 to 14)

If a continuous function f defined on the real line **R**, assumes positive and negative values in **R** then the equation f(x) = 0 has a root in **R**. For example, if it is known that a continuous function f on **R** is positive at some point and its minimum value is negative then the equation f(x) = 0 has a root in **R**.

Consider $f(x) = ke^x - x$ for all real x where k is a real constant.

12. The line y = x meets $y = ke^x$ for $k \le 0$ at

(1) no point (2) one point (3) two points (4) more than two points

13. The positive value of k for which $ke^x - x = 0$ has only one root is

(1) $\frac{1}{e}$ (2) 1 (3) e (4) $\log_e 2$

14. For k > 0, the set of all values of k for which $ke^x - x = 0$ has two distinct roots is

(1) $\left(0, \frac{1}{e}\right)$ (2) $\left(\frac{1}{e}, 1\right)$ (3) $\left(\frac{1}{e}, \infty\right)$ (4) (0, 1)

Comprehension # 2 (Q.15 to 16)

Let f : [0, 1] → R (the set of all real numbers) be a function. Suppose the function f is twice differentiable, f(0) = f(1) = 0 and satisfies $f''(x) - 2f'(x) + f(x) \ge e^x$, $x \in [0, 1]$.

15. Which of the following is true for 0 < x < 1 ?

(1) $0 < f(x) < \infty$ (2) $-\frac{1}{2} < f(x) < \frac{1}{2}$ (3) $-1 < f(x) < 1$ (4) $-\infty < f(x) < 0$

16. If the function $e^{-x} f(x)$ assumes its minimum in the interval [0, 1] at $x = \frac{1}{4}$, which of the following is true ?

(1) $f'(x) < f(x)$, $\frac{1}{4} < x < \frac{3}{4}$ (2) $f'(x) > f(x)$, $0 < x < \frac{1}{4}$

(3) $f'(x) < f(x)$, $0 < x < \frac{1}{4}$ (4) $f'(x) < f(x)$, $\frac{3}{4} < x < 1$

DIRECTIONS : (Q. 17)

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true. (2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true. (4) Both the statements are false.

17. **Statement 1 :** The curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{x^2}{1+a^2} + \frac{y^2}{1-b^2} = 1$ are orthogonal, for $b \in (-1,1)$.

Statement 2 : $ax^2 + by^2 = 1$ and $Ax^2 + By^2 = 1$ are orthogonal then $ab(A - B) = AB(a - b)$.

APSP Answers

PART - I

1.	(3)	**2.**	(4)	**3.**	(3)	**4.**	(2)	**5.**	(2)	**6.**	(4)	**7.**	(2)
8.	(1)	**9.**	(3)	**10.**	(2)	**11.**	(4)	**12.**	(2)	**13.**	(1)	**14.**	(2)
15.	(3)	**16.**	(4)	**17.**	(4)	**18.**	(4)	**19.**	(1)	**20.**	(3)	**21.**	(1)
22.	(3)	**23.**	(4)	**24.**	(2)	**25.**	(2)	26.	(2)	**27.**	(1)	**28.**	(2)
29.	(2)	**30.**	(1)										

PART - II

1.	(2)	**2.**	(1)	**3.**	(1)	**4.**	(4)	**5.**	(2)	**6.**	(4)	**7.**	(4)
8.	(2)	**10*.**	(1,2)	**11*.**	(1,3,4)	**12.**	(2)	**13.**	(1)	**14.**	(1)	**15.**	(4)
16.	(3)	**17.**	(1)										