Differential Equation

Exercise-1

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

OBJECTIVE QUESTIONS

Section (A) : Degree and order, Formation of diferential equation

A-1. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + y^4 = 0$ are respectively

(1) 2, 2 (2) 2, 1 (3) 1, 2 (4) 3, 2

A-2. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \frac{d^3y}{dx^3} + \left(\frac{dy}{dx}\right)^4 = y$ are respectively

(1) 2, 2 (2) 3, 2 (3) 2, 3 (4) 1, 3

A-3. The order and degree of the differential equation $r = \dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ are respectively

(1) 2, 2 (2) 2, 3 (3) 2, 1 (4) 1,4

A-4. If p and q are order and degree of differential equation $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 3x\left(\frac{dy}{dx}\right)^{1/3} + x^2y^2 = \sin x$, then :

(1) $p > q$ (2) $\frac{p}{q} = \frac{1}{2}$ (3) $p = q$ (4) $p < q$

A-5. The degree of the differential equation $\frac{d^2y}{dx^2} = \sin\left(x + \frac{dy}{dx}\right)$ is

(1) 2 (2) 3 (3) 0 (4) Not defined

A-6. The order of the differential equation whose general solution is given by

$y = (C_1 + C_2) \sin (x + C_3) - C_4\, e^{x+C_5}$ is

(1) 5 (2) 4 (3) 2 (4) 3

A-7. The order of the differential equation of family of curves $(\sin a)\, x + (\cos a)\, y = \pi$ is (where a is parameter)

(1) 1 (2) 2 (3) 3 (4) 4

A-8. The order of the differential equation of family of curves $y^2 = 4a\, e^{x+b}$ is (where a, b are parameters)

(1) 1 (2) 2 (3) 3 (4) 5

A-9. The order of the differential equation of family of curves $\ell n\, (ay) = be^x + c$ is (where a, b, c are parameters)

(1) 1 (2) 2 (3) 4 (4) 5

A-10. The order of the differential equation of family of curves $y = \tan\left(\frac{\pi}{4}+ax\right)\tan\left(\frac{\pi}{4}-ax\right) + c\,e^{bx+d}$ is (where a, b, c, d are parameters)
(1) 1 (2) 2 (3) 3 (4) 4

A-11. Degree of differential equation of family of curve $y = Ax + A^3$ is
(1) three (2) two (3) one (4) four

A-12. The differential equation for all the straight lines which are at a unit distance from the origin is
(1) $\left(y - x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$ (2) $\left(y + x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$
(3) $\left(y - x\frac{dy}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$ (4) $\left(y + x\frac{dy}{dx}\right)^2 = 1 - \left(\frac{dy}{dx}\right)^2$

A-13. The order and degree of differential equation of all tangent lines to parabola $x^2 = 4y$ is
(1) 1, 2 (2) 2, 2 (3) 3, 1 (4) 4, 1

A-14. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2 - y^2)\frac{dy}{dx} = g(x)\,y$, then g(x) equals
(1) $2x^2$ (2) $\frac{1}{2}x^2$ (3) $\frac{1}{2}x$ (4) $2x$

A-15. If differential equation of family of curves $y\,\ell n|cx| = x$, where c is an arbitrary constant, is $y' = \frac{y}{x} + \varphi\left(\frac{x}{y}\right)$, for some function φ, then φ (2) is equal to :
(1) $\frac{1}{4}$ (2) $-\frac{1}{4}$ (3) – 4 (4) 4

Section (B) : Variable separable, Homogeneous equation

B-1. If $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5, the value of x for y = 3 is
(1) e^5 (2) $e^6 + 1$ (3) $\frac{e^6+9}{2}$ (4) $\ell n6$

B-2. If φ(x) = φ'(x) and φ(1) = 2, then φ(3) equals
(1) e^2 (2) $2e^2$ (3) $3e^2$ (4) $2e^3$

B-3. If $\frac{dy}{dx} = 1 + x + y + xy$ and y (– 1) = 0, then function y is
(1) $e^{(1-x)^2/2}$ (2) $e^{(1+x)^2/2} - 1$ (3) $\ell n(1 + x) - 1$ (4) 1 + x

B-4. If y(x) is the solution of the differential equation $(x + 2)\frac{dy}{dx} = x^2 + 4x - 9$, $x \neq -2$ and y(0) = 0, then y(– 4) is equal to :
(1) 2 (2) 0 (3) –1 (4) 1

Differential Equation

B-5. The solution of differential equation $\frac{dy}{dx} = \frac{x(2\ell nx+1)}{\sin y + y\cos y}$ is -
(1) y siny = $x\ell nx$ + c (2) y siny = $x^2\ell nx$ + c (3) siny = $x^2\ell nx$ + c (4) y cosy = $x^2\ell nx$ + c

B-6. The solution of the differential equation $\frac{dy}{dx}$ – ky = 0,y(0) = 1, approaches zero when $x \to \infty$, if
(1) k = 0 (2) k > 0 (3) k < 0 (4) k ≥ 0

B-7. If y(x) is a solution of the differential equation $\frac{dy}{dx}$ + 3y = 2 , then $\lim_{x\to\infty}$ y(x) is equal to -
(1) $\frac{2}{3}$ (2) 1 (3) 0 (4) $\frac{3}{2}$

B-8. The solution of $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is
(1) $\sin^{-1} x . \sin^{-1} y = C$ (2) $\sin^{-1} x = C \sin^{-1} y$ (3) $\sin^{-1} x - \sin^{-1} y = C$ (4) $\sin^{-1}x+\sin^{-1} y = C$

B-9. The general solution of the differential equation y dy + $\sqrt{1+y^2}$ dx = 0 represents a family of :
(1) circles (2) ellipses other than circles
(3) hyperbolas (4) parabolas

B-10. The value of $\lim_{x\to\infty}$ y(x) obtained from the differential equation $\frac{dy}{dx} = y - y^2$, where y (0) = 2 is
(1) zero (2) 1 (3) ∞ (4) 2

B-11. The conic whose differential equation is $(1 + y^2)$ dx – xydy = 0 and which passess through (1, 0) is
(1) $x^2 - y^2 = 1$ (2) $2x^2 - 3y^2 = 1$ (3) $x^2 + y^2 = 1$ (4) $x^2 - y^2 = 2$

B-12. Solution of differential equation xdy – ydx = 0 represents :
(1) rectangular hyperbola (2) straight line passing through origin
(3) parabola whose vertex is at origin (4) circle whose centre is at origin

B-13. The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is
(1) $\ell n\left(\cot\frac{y}{2}\right) + 2\sin x = C$ (2) $\ell n\left(\cot\frac{y}{4}\right) + 2\sin\frac{x}{2} = C$
(3) $\ell n\left(\tan\frac{y}{2}\right) + 2\sin x = C$ (4) $\ell n\left(\tan\frac{y}{4}\right) + 2\sin\frac{x}{2} = C$

B-14. The solution of the differential equation $\frac{dy}{dx}$ = sin(x + y) + cos(x + y) is -
(1) $\ell n\left|\tan\left(\frac{x+y}{2}\right)+1\right| = x + c$ (2) ℓn (tan x) = y + c
(3) $\ell n \tan\left(\frac{x+y}{2}\right) = x + c$ (4) ℓn (cosx) = y + c

B-15. The solution of the differential equation $\frac{dy}{dx} + e^{x-y} + e^{y-x} = 1$ is -
(1) $\tan(e^{y-x}) + x = c$ (2) $\tan^{-1}(e^{y-x}) + x = c$
(3) $\tan y = x + c$ (4) $\tan x = y + c$

B-16. The solution of diferential equation $\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$ is
(1) $x^2 + y^2 - xy + x - y = c$ (2) $y - 2x + \frac{3}{8}\ \ell n\,(24y + 16x + 23) = c$
(3) $4xy + 3(x^2 + y^2) - 10(x + y) = c$ (4) $y + 2x + \frac{3}{8}\ \ell n\,(24y + 16x + 23) = c$

B-17. $f(x, y) = e^{\frac{y}{x}} + \tan\frac{y}{x}$ is homogeneous of degree
(1) 0 (2) 1 (3) 2 (4) 3

B-18. The equation of the curve satisfying $\frac{dy}{dx} = \frac{y^2 - 2xy - x^2}{y^2 + 2xy - x^2}$ and passing through (1, –1) is .
(1) $x = y + c$ (2) $y = 2x + c$ (3) $y = x$ (4) $y = -x$

B-19. Integral curve satisfying $y' = \frac{x^2 + y^2}{x^2 - y^2}$, $y(1) = 2$, has the slope of the curve at the point (1, 2), is equal to
(1) $-\frac{5}{3}$ (2) -1 (3) 1 (4) $\frac{5}{3}$

B-20. The solution of differential equation $\left[x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right] y - \left[y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right] x\,\frac{dy}{dx} = 0$ is -
(1) $y\cos\left(\frac{y}{x}\right) = c$ (2) $xy\sin\left(\frac{y}{x}\right) = c$ (3) $xy\cos\left(\frac{y}{x}\right) = c$ (4) $xy\tan\left(\frac{y}{x}\right) = c$

Section (C) : Linear differential equation, Bernoulli's equation

C-1. The solution of the equation $\frac{dv}{dt} + \frac{k}{m} v = -g$ is
(1) $v = ce^{-\frac{k}{m}t} - \frac{mg}{k}$ (2) $v = c - \frac{mg}{k} e^{-\frac{k}{m}t}$ (3) $v\, e^{-\frac{k}{m}t} = c - \frac{mg}{k}$ (4) $v\, e^{\frac{k}{m}t} = c - \frac{mg}{k}$

C-2. The solution of the differential equation $\frac{dy}{dx} = y\tan x - 2\sin x$ is
(1) $y = \cos x + c\sec x$ (2) $y = \cos x + c$ (3) $y = \sin x + c$ (4) $y = \mathrm{cosec}\,x + c$

C-3. The solution of the differential equation $(1 + x^2)\frac{dy}{dx} + 2xy = \cos x$ is
(1) $y(1 + x^2) = c + \cos x$ (2) $y(1 + x^2) = c + \sin x$
(3) $y = x + c$ (4) $y = \cos x + x^2$

C-4. The solution of the differential equation $(x + 3y^2)\frac{dy}{dx} = y, y > 0$ is

(1) $\frac{x}{y} = 3y + c$ (2) $x = 2y^3 + 3y^2 + c$ (3) $y = 3x^2 + c$ (4) $y = 3x + c$

C-5. Consider the differential equation, $ydx - (x + y^2)dy = 0$. If for $y = 1$, x takes value 1, then value of x when $y = 4$ is :

(1) 64 (2) 9 (3) 16 (4) 36

C-6. The solution of the differential equation $(1 + y + x^2y)\,dx + (x + x^3)dy = 0$ is

(1) $xy = c - \tan x$ (2) $xy = c - \text{arc}\tan x$ (3) $xy = c - x$ (4) $xy = c + \text{arc}\tan x$

C-7. The solution of differential equation $\frac{dy}{dx} = \cos x(2 - y\,\text{cosec}x)$ is

(1) $y = \tan\frac{x}{2} + \cot\frac{x}{2} + C$ (2) $y = \frac{1}{\sqrt{2}}\sec\frac{x}{2} + \sqrt{2}\cos\frac{x}{2} + C$

(3) $y = \sin x + C\,\text{cosec}x$ (4) $y = \sin x - \cos x + C$

C-8. The solution of differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$, $x > 0$ is

(1) $y\,e^{-2\sqrt{x}} + 2\sqrt{x} = C$ (2) $e^{2\sqrt{x}} + 2\sqrt{x}\,y = C$

(3) $y\,e^{2\sqrt{x}} = 2\,e^{2\sqrt{x}}.\sqrt{x} + C$ (4) $y\,e^{2\sqrt{x}} = 2\sqrt{x} + C$

C-9. If $\frac{dy}{dx} + y\tan x = \sin 2x$ and $y(0) = 1$, then $y(\pi)$ is equal to :

(1) –5 (2) 5 (3) 1 (4) –1

C-10. The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx} - \sqrt{\tan x}\right) - y = 0$, is

(1) $y\sqrt{\tan x} = \cot x + C$ (2) $y\sqrt{\cot x} = \tan x + C$ (3) $y\sqrt{\cot x} = x + C$ (4) $y\sqrt{\tan x} = x + C$

C-11. The solution of differential equation $x\frac{dy}{dx} + y = x^2y^4$ is

(1) $\frac{1}{y^3} = 3x^2 + cx^3$ (2) $3x^2 + y^3 = c$ (3) $x^2 = y^3 + c$ (4) $y^3 = x + c$

C-12. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{2}\sec x = \frac{\tan x}{2y}$, where $0 \le x < \frac{\pi}{2}$ and $y(0) = 1$, is given by

(1) $y = 1 - \frac{x}{\sec x + \tan x}$ (2) $y^2 = 1 + \frac{x}{\sec x + \tan x}$ (3) $y = 1 + \frac{x}{\sec x + \tan x}$ (4) $y^2 = 1 - \frac{x}{\sec x + \tan x}$

C-13. The solution of differential equation $2\frac{dy}{dx} = \frac{y^2 - x}{xy + y}$ is

(1) $y^2 = \ell n|x + 1| + c$ (2) $y^2 = \ell nx + c$

(3) $y^2 = e^{x^2} + c$ (4) $y^2 = -(x + 1)\,\ell n|x + 1| + c(x + 1) - 1$

Differential Equation

Section (D) : Exact differential equation, Geometrical and physical applications

D-1. The solution of $y\,dx - x\,dy + 3x^2 y^2 e^{x^3} dx = 0$ is

(1) $\frac{x}{y} + e^{x^3} = C$ (2) $\frac{x}{y} - e^{x^3} = 0$ (3) $-\frac{x}{y} + e^{x^3} = C$ (4) $\frac{x}{y} + e^{x^2} = C$

D-2. The solution of differential equation $y(x^2y + e^x)\,dx = e^x\,dy$ is

(1) $e^x = x^3y + c$ (2) $\frac{1}{y} e^x = -\frac{x^3}{3} + c$ (3) $e^x = y + c$ (4) $e^{-x} = y + c$

D-3. The solution of differential equation $2y \sin x\,dy + (y^2 \cos x + 2x)\,dx = 0$ is

(1) $y^2 \sin x = x^2 + c$ (2) $y^2 \sin x = -x^2 + c$ (3) $y^2 = \sin x + c$ (4) $y^2 = \cos x + c$

D-4. The solution of diferential equation $(2x - y + 1)\,dx + (2y - x - 1)\,dy = 0$ is

(1) $x^2 + y^2 - xy + x - y = c$ (2) $y - 2x + \frac{3}{8} \ell n\,(24y + 16x + 23) = c$

(3) $4xy + 3\,(x^2 + y^2) - 10\,(x + y) = c$ (4) $x^2 - y^2 - xy + x + y = c$

D-5. The solution of diferential equation $(2x + 3y - 5)\,dy + (3x + 2y - 5)\,dx = 0$ is

(1) $x^2 + y^2 - xy + x - y = c$ (2) $y - 2x + \frac{3}{8} \ell n\,(24y + 16x + 23) = c$

(3) $4xy + 3\,(x^2 + y^2) - 10\,(x + y) = c$ (4) $x^2 - y^2 - xy + x + y = c$

D-6. The solution of the differential equation $(x^2y^2 - 1)dy + 2x\,y^3\,dx = 0$ is

(1) $1 + x^2y^2 = cx$ (2) $1 + x^2y^2 = cy$ (3) $y = c$ (4) $y = -\frac{1}{x^2} + c$

D-7. The solution of the differential equation $ydx - (x + 2y^2)dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to :

(1) 4 (2) 3 (3) 1 (4) 2

D-8. The solution of the differential equation $\frac{ydx + xdy}{ydx - xdy} = \frac{x^2e^{xy}}{y^4}$ satisfying $y(0) = 1$, is

(1) $x^3 = 3y^3(-1 + e^{-xy})$ (2) $x^3 = 3y^3(1 - e^{-xy})$ (3) $x^3 = 3y^3(-1 + e^{xy})$ (4) $x^3 = 3y^3(1 - e^{xy})$

D-9. The solution of the differential equation $(x^2 \sin^3 y - y^2 \cos x)\,dx + (x^3 \cos y \sin^2 y - 2y \sin x)\,dy = 0$ is

(1) $x^3 \sin^3 y = 3y^2 \sin x + C$ (2) $x^3 \sin^3 y + 3y^2 \sin x = C$

(3) $x^2 \sin^3 y + y^3 \sin x = C$ (4) $2x^2 \sin y + y^2 \sin x = C$

D-10. The slope of a curve at any point is the reciprocal of twice the ordinate of that point and it passes through the point (4, 3). The equation of the curve is

(1) $x^2 = y + 5$ (2) $y^2 = x - 5$ (3) $y^2 = x + 5$ (4) $x^2 = y + 5$

D-11. The equation of the curve whose subnormal is constant 'a' is

(1) $y = ax + b$ (2) $y^2 = 2ax + b$ (3) $ay^2 - x^2 = a$ (4) $ay^2 + x^2 = a$

D-12. If the solution of $\frac{dy}{dx} = \frac{ax + 3}{2y + 1}$ represents a circle, then the value of a is

(1) 4 (2) – 2 (3) – 4 (4) 2

D-13. The equation of the curve such that the distance between the origin and the tangent at an arbitrary point is equal to the distance between the origin and the normal at the same point is

(1) $\sqrt{x^2+y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$ (2) $\sqrt{x^2-y^2} = ce^{\pm\tan^{-1}\frac{y}{x}}$

(3) $-\sqrt{x^2+y^3} = ce^{\pm\tan^{-1}\frac{y}{x}}$ (4) $\sqrt{x^2+y^2} = ce^{\pm\tan\frac{y}{x}}$

D-14. The equation of the curve which is such that the portion of the axis of x cut off between the origin and tangent at any point is proportional to the ordinate of that point is, where a is constant of proportionality

(1) $x = y(b - a\,\ell ny)$ (2) $\ell nx = by^2 + a$

(3) $x^2 = y(a - b\,\ell ny)$ (4) $y^2 = x(a - b\,\ell n\, y)$

Exercise-2

Marked questions may have for revision questions.

PART - I : OBJECTIVE QUESTIONS

1. Degree of differential equation $\left(\frac{dy}{dx}\right)^{\frac{5}{2}} - 5x\left(\frac{d^2y}{dx^2}\right)^2 + 7y = 0$ is

(1) 2 (2) 5 (3) 4 (4) 1

2. The order and degree of differential equation of family of circles touching y axis at origin, is

(1) 1,1 (2) 1,2 (3) 2,1 (4) 2,2

3. The order and degree of differential equation of family of normal to parabola $y^2 = 4x$ is

(1) 1,2 (2) 1,1 (3) 2, 1 (4) 1,3

4. If $\frac{dy}{dx} + \frac{2y}{x} = 0$, y (1) = 1, then y(2) =

(1) $\frac{1}{4}$ (2) 4 (3) $-\frac{1}{2}$ (4) $-\frac{1}{4}$

5. Solution of the differential equation $\frac{dy}{dx} + \frac{1+y^2}{\sqrt{1-x^2}} = 0$ is

(1) $\tan^{-1}y + \sin^{-1}x = c$ (2) $\tan^{-1}x + \sin^{-1}y = c$ (3) $\tan^{-1}y \,.\, \sin^{-1}x = c$ (4) $\tan^{-1}x - \sin^{-1}y = c$

6. Solution of $y - \frac{xdy}{dx} = y^2 + \frac{dy}{dx}$ is

(1) $(2x + 1)(1 + y) = c$ (2) $(1 + x)(1 + y) = cx^2$ (3) $(x + 1)(1 - y) = cy$ (4) $x - y + x^2y - 1 = c$

7. If $(x^2 + y^2)\, dy = xydx$ and y(1) = 1 and $y(x_0) = e$, then x_0 =

(1) 3e (2) $\sqrt{2}\,e$ (3) $\sqrt{3}$ (4) $\sqrt{3}\,e$

8. The solution of $(x + y + 1)\, dy = dx$ is

(1) $x + y + 2 = Ce^y$ (2) $x + y + 4 = C\,\ell ny$ (3) $\ell n(x + y + 2) = Cy$ (4) $\ell n(x - y + 2) = C + y$

9. If y(t) is solution of $(t + 1)\frac{dy}{dt} - ty = 1$, $y(0) = -1$, then $y(1) =$

(1) $\frac{1}{4}$ (2) -2 (3) $-\frac{1}{2}$ (4) $\frac{1}{2}$

10. If $y_1(x)$ is a solution of the differential equation $\frac{dy}{dx} + f(x)\,y = 0$, then a solution of differential equation $\frac{dy}{dx} + f(x)\,y = r(x)$ is

(1) $\frac{1}{y\ (x)}\int y_1(x)\ dx$ (2) $y_1(x)\int \frac{r(x)}{y_1(x)}\ dx$ (3) $\int r(x)y_1(x)\ dx$ (4) $\int (r(x))^2 y_1(x)\ dx$

11. If $y_1(x)$ and $y_2(x)$ are two solutions of $\frac{dy}{dx} + f(x)\,y = r(x)$ then $y_1(x) + y_2(x)$ is solution of :

(1) $\frac{dy}{dx} + f(x)\,y = 0$ (2) $\frac{dy}{dx} + 2f(x)\,y = r(x)$ (3) $\frac{dy}{dx} + f(x)\,y = 2\,r(x)$ (4) $\frac{dy}{dx} + 2f(x)\,y = 2r(x)$

12. Solution of differential equation $f(x)\frac{dy}{dx} = f^2(x) + f(x)\,y + f'(x)\,y$ is

(1) $y = f(x) + ce^x$ (2) $y = -f(x) + ce^x$ (3) $y = -f(x) + ce^x f(x)$ (4) $y = cf(x) + e^x$

13. Solution of $(2x - 10y^3)\frac{dy}{dx} + y = 0$ is

(1) $xy^2 = 2y^5 + c$ (2) $x = 10y^3 + cy^2$ (3) $x = 10y^3 + cy$ (4) $xy = 2y^5 + c$

14. Solution of $\sin y\frac{dy}{dx} = \cos y\,(1 - x\cos y)$ is

(1) $\sec y = x + 1 + ce^{-x}$ (2) $\sec y = x + 1 + ce^x$ (3) $\cos y = x + 1 + ce^x$ (4) $\tan y = 1 + ce^{-x}$

15. Solution of $\sec^2 y\,dy + \tan y\,dx = dx$ is

(1) $\cot y = e^x + 2x + c$ (2) $\sec y = x + 1 + ce^x$ (3) $\tan y = e^x$ (4) $\tan y = 1 + ce^{-x}$

16. If $xdy = y(dx + ydy)$, $y(1) = 1$ and $y(x_0) = -3$, then $x_0 =$

(1) $\frac{1}{4}$ (2) -15 (3) $-\frac{1}{2}$ (4) $\sqrt{3}\ e$

17. The general solution of $(2x^3 - xy^2)\,dx + (2y^3 - x^2y)\,dy = 0$ is

(1) $x^4 + x^2y^2 - y^4 = c$ (2) $x^4 - x^2y^2 + y^4 = c$ (3) $x^4 - x^2y^2 - y^4 = c$ (4) $x^4 + x^2y^2 + y^4 = c$

18. General solution of the differential equation $\frac{xdy}{x^2+y^2} + \left(1 - \frac{y}{x^2+y^2}\right)dx = 0$ is

(1) $x + \tan^{-1}\left(\frac{y}{x}\right) = c$ (2) $x + \tan^{-1}\frac{x}{y} = c$ (3) $x - \tan^{-1}\left(\frac{y}{x}\right) = c$ (4) $2x - 3\tan^{-1}\left(\frac{y}{x}\right) = c$

19. General solution of the differential equation $e^y\,dx + (xe^y - 2y)\,dy = 0$ is

(1) $xe^y - y^2 = c$ (2) $ye^x - x^2 = c$ (3) $ye^y + x = c$ (4) $xe^y - 1 = cy^2$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true. (2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true. (4) Both the statements are false.

A-1 **Statement -1 :** The relation $y = A \sin x + B \cos x$ can be represented by the differential equation $\frac{d^2y}{dx^2} + y = 0$.

Statement -2 : Solution of $\sec^2 y \frac{dy}{dx} + x \tan y = x^2$ is $\tan y = x^2 - c\ e^{x^2/2} + 2$

A-2. **Statement -1 :** The solution of D.E. $= \frac{\frac{xdy}{dx} - y}{\sqrt{x^2 - y^2}}$ mx^2 is given by $\tan^{-1} \frac{y}{x} = \frac{mx^2}{2} + c$

Statement -2 : The solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is $x(y + \cos x) = \sin x + c$

A-3. **Statement -1 :** Solution of the differential equation $y - x\frac{dy}{dx} = y^2 + \frac{dy}{dx}$ is $y = c\,(1 - y)\,(x + 1)$

Statement -2 : Differential equation = f(x) . g(y) can be solved by seperating variables. $\frac{dy}{g(y)} = f(x)\,dx$

Section (B) : MATCH THE COLUMN

B-1.

	Column I		Column I
	Sum of order and degree of differential equation is		
(A)	$\left(\frac{d^2y}{dx^2}\right) = \left(y + \left(\frac{dy}{dx}\right)^6\right)^{\frac{1}{4}}$	(p)	7
(B)	$\left(\frac{d^4y}{dx^4}\right)^3 + 3\left(\frac{d^2y}{dx^2}\right)^6 + \sin x = 2 \cos x$	(q)	5
(C)	$\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} + 4 - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = 0$	(r)	3
(D)	$\frac{dy}{dx} + y = \frac{1}{\left(\frac{dy}{dx}\right)}$	(s)	6

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. For the differential equation whose solution is $(x - h)^2 + (y - k)^2 = a^2$ (a is a constant), its
(1) order is 2 (2) order is 3 (3) degree is 2 (4) degree is 3

C-2. The equation of the curve satisfying the differential equation $y\left(\frac{dy}{dx}\right)^2 + (x - y)\frac{dy}{dx} - x = 0$ can be a
(1) circle (2) straight line (3) parabola (4) ellipse

C-3. Which of the following equation (s) is/are linear ?
(1) $\frac{dy}{dx} + \frac{y}{x} = \ell nx$ (2) $y\left(\frac{dy}{dx}\right) + 4x = 0$ (3) $(2x + y^3)\left(\frac{dy}{dx}\right) = 3y$ (4) $x\left(\frac{dy}{dx}\right)^2 + y^3\frac{d^2y}{dx^2} + 7 = e^x$

C-4. The graph of the function y = f(x) passing through the point (0,1) and satisfying the differential equation $\frac{dy}{dx} + y \cos x = \cos x$ is such that

(1) it is a constant function
(2) it is periodic
(3) it is neither an even nor an odd function
(4) it is continuous and differentiable for all x

C-5. In which of the following differential equation degree is not defined ?

(1) $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x\ell n\left(\frac{d^2y}{dx^2}\right)$
(2) $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$
(3) $x = \sin\left(\frac{dy}{dx} - 2y\right)$, $|x| < 1$
(4) $x - 2y = \ell n\left(\frac{dy}{dx}\right)$

C-6. A normal is drawn at a point P(x, y) of a curve, It meets the x-axis at Q. If PQ is of constant length k. Such a curve passing through (0, k) is :

(1) a circle with centre (0, 0)
(2) a hyperbola with eccentricity $\sqrt{2}$
(3) $x^2 + y^2 = k^2$
(4) $x^2 - y^2 = k^2$

C-7. If y(x) satisfies the differential equation $y' - y \tan x = 2x \sec x$ and y(0) = 0, then

(1) $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{8\sqrt{2}}$
(2) $y'\left(\frac{\pi}{4}\right) = \frac{\pi^2}{18}$
(3) $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{9}$
(4) $y'\left(\frac{\pi}{3}\right) = \frac{4\pi}{3} + \frac{2\pi^2}{3\sqrt{3}}$

C-8. Let y(x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If y(0) = 2, then which of the following statements is (are) true ?

(1) y(–4) = 0
(2) y(–2) = 0
(3) y(x) has a critical point in the interval (–1, 0)
(4) y(x) has no critical point in the interval (–1, 0)

Exercise-3

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The order and degree of the differential equation $\left(1+3\frac{dy}{dx}\right)^{2/3} = 4\frac{d^3y}{dx^3}$ are **[AIEEE 2002, (4, –1), 120]**

(1) $\left(1, \frac{2}{3}\right)$
(2) (3, 1)
(3) (3, 3)
(4) (1, 2)

2. The solution of the equation $\frac{d^2y}{dx^2} = e^{-2x}$ is **[AIEEE 2002, (4, –1), 120]**

(1) $\frac{e^{-2x}}{4}$
(2) $\frac{e^{-2x}}{4} + cx + d$
(3) $\frac{1}{4}e^{-2x} + cx^2 + d$
(4) $\frac{1}{4}e^{-2x} + c + d$

3. The degree and order of the differential equation of the family of all parabolas whose axis is x-axis, are respectively **[AIEEE 2003, (4, –1), 120]**

(1) 2, 1
(2) 1, 2
(3) 3, 2
(4) 2, 3

4. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$, is **[AIEEE 2003, (4, –1), 120]**

(1) $(x - 2) = ke^{\tan^{-1} y}$
(2) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$
(3) $xe^{\tan^{-1} y} = \tan^{-1} y + k$
(4) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$

5. The differential equation for the family of curves $x^2 + y^2 - 2ay = 0$, where a is an arbitrary constant, is

Differential Equation

[AIEEE 2004, (4, –1), 120]

(1) $2(x^2 - y^2)\, y' = xy$ (2) $2(x^2 + y^2)\, y' = xy$ (3) $(x^2 - y^2)\, y' = 2xy$ (4) $(x^2 + y^2)\, y' = 2xy$

6. The solution of the differential equation $y\,dx + (x + x^2y)\,dy = 0$ is- **[AIEEE 2004, (4, –1), 120]**

(1) $-\frac{1}{xy} = c$ (2) $-\frac{1}{xy} + \ell n\, y = c$ (3) $\frac{1}{xy} + \ell n y = c$ (4) $\ell n y = cx$

7. The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where $c > 0$ is a parameter , is of order and degree as follows- **[AIEEE 2005, (4, –1), 120]**

(1) order 2, degree 2 (2) order 1, degree 3 (3) order 1, degree 1 (4) order 1, degree 2

8. If $x\frac{dy}{dx} = y(\ell n\, y - \ell n x + 1)$, then the solution of the equation is **[AIEEE 2005, (4, –1), 120]**

(1) $\ell n\left(\frac{x}{y}\right) = cy$ (2) $\ell n\left(\frac{y}{x}\right) = cx$ (3) $x\ \ell n\left(\frac{y}{x}\right) = cy$ (4) $y\ \ell n\left(\frac{x}{y}\right) = cx$

9. The differential equation whose solution is $Ax^2 + By^2 = 1$, where A and B are arbitary constant, is of **[AIEEE 2006, (4, –1), 120]**

(1) first order and second degree (2) first order and first degree

(3) second order and first degree (4) second order and second degree

10. The differential equation of all circles passing through the origin and having their centres on the x-axis is- **[AIEEE 2007, (4, –1), 120]**

(1) $x^2 = y^2 + xy\frac{dy}{dx}$ (2) $x^2 = y^2 + 3xy\frac{dy}{dx}$ (3) $y^2 = x^2 + 2xy\frac{dy}{dx}$ (4) $y^2 = x^2 - 2xy\frac{dy}{dx}$

11*. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a **[AIEEE 2007, (4, –1), 120]**

(1) ellipse (2) parabola (3) circle (4) hyperbola

12. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ satisfying the condition y (1) = 1 is **[AIEEE 2008 (3, –1), 105]**

(1) $y = \log x + x$ (2) $y = x \log x + x^2$ (3) $y = xe^{(x-1)}$ (4) $y = x \log x + x$

13. The differential equation of the family of circles with fixed radius 5 units and centre on the line y = 2 is **[AIEEE 2008 (3, –1), 105]**

(1) $(x - 2)y'^2 = 25 - (y - 2)^2$ (2) $(y - 2)y'^2 = 25 - (y - 2)^2$

(3) $(y - 2)^2y'^2 = 25 - (y - 2)^2$ (4) $(x - 2)^2y'^2 = 25 - (y - 2)^2$

14. The differential equation which represents the family of curves $y = c_1, e^{c_2 x}$ where c_1 and c_2 are arbitary constants is **[AIEEE 2009 (4, –1), 144]**

(1) $y' = y^2$ (2) $y'' = y'\,y$ (3) $y.y'' = y'$ (4) $y.y'' = (y')^2$

15. Solution of the differential equation $\cos x\,dy = y(\sin x - y)\,dx$, $0 < x < \frac{\pi}{2}$ is **[AIEEE 2010 (4, –1), 144]**

(1) $y \sec x = \tan x + c$ (2) $y \tan x = \sec x + c$ (3) $\tan x = (\sec x + c)y$ (4) $\sec x = (\tan x + c)\, y$

Differential Equation

16. Let I be the purchase value of an equipment and V(t) be the value after it has been used for t years. The value V(t) depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where k > 0 is a constant and T is the total life in years of the equipment. Then the scrap value V(T) of the equipment is :

[AIEEE 2011, I, (4, –1), 120]

(1) $T^2 - \frac{1}{k}$ (2) $I - \frac{kT^2}{2}$ (3) $I - \frac{k(T-t)^2}{2}$ (4) e^{-kT}

17. If $\frac{dy}{dx} = y + 3 > 0$ and y(0) = 2, then y(ℓn2) is equal to : **[AIEEE 2011, I, (4, –1), 120]**

(1) 7 (2) 5 (3) 13 (4) –2

18. The curve that passes through the point (2, 3), and has the property that the segment of any tangent to it lying between the coordinate axes is bisected by the point of contact is given by :

[AIEEE 2011, II, (4, –1), 120]

(1) $2y - 3x = 0$ (2) $y = \frac{6}{x}$ (3) $x^2 + y^2 = 13$ (4) $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 2$

19. Consider the differential equation $y^2dx + \left(x - \frac{1}{y}\right)dy = 0$. If y (1) = 1, then x is given by :

[AIEEE 2011, II, (4, –1), 120]

(1) $4 - \frac{2}{y} - \frac{e^{\frac{1}{y}}}{e}$ (2) $3 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$ (3) $1 + \frac{1}{y} - \frac{e^{\frac{1}{y}}}{e}$ (4) $1 - \frac{1}{y} + \frac{e^{\frac{1}{y}}}{e}$

20. The population p(t) at time t of a certain mouse species satisfies the differential equation $\frac{dp(t)}{dt} = 0.5\,p(t) - 450$. If p(0) = 850, then the time at which the population becomes zero is :

[AIEEE-2012, (4, –1)/120]

(1) 2 ℓn 18 (2) ℓn 9 (3) $\frac{1}{2}\ell$n 18 (4) ℓn 18

21. At present, a firm is manufacturing 2000 items. It is estimated that the rate of change of production P w.r.t. additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is **[AIEEE - 2013, (4, –1),360]**

(1) 2500 (2) 3000 (3) 3500 (4) 4500

22. Let the population of rabbits surviving at a time t be governed by the differential equation $\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 200$. If p(0) = 100, then p(t) equals : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $600 - 500\,e^{t/2}$ (2) $400 - 300\,e^{-t/2}$ (3) $400 - 300\,e^{t/2}$ (4) $300 - 200\,e^{-t/2}$

23. Let y(x) be the solution of the differential equal $(x \log x)\frac{dy}{dx} + y = 2x \log x, (x \geq 1)$. Then y(e) is equal to

[JEE(Main) 2015, (4, – 1), 120]

(1) e (2) 0 (3) 2 (4) 2e

24. If a curve y = f(x) passes through the point (1, –1) and satisfies the differential equation, y(1 + xy) dx = xdy, then $f\left(-\frac{1}{2}\right)$ is equal to **[JEE(Main) 2016, (4, – 1), 120]**

(1) $-\frac{4}{5}$ (2) $\frac{2}{5}$ (3) $\frac{4}{5}$ (4) $-\frac{2}{5}$

25. If $(2 + \sin x)\frac{dy}{dx} + (y + 1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

[JEE(Main) 2017, (4, – 1), 120]

(1) $\frac{1}{3}$ (2) $-\frac{2}{3}$ (3) $-\frac{1}{3}$ (4) $\frac{4}{3}$

26. Let $y = y(x)$ be the solution of the differential equation

$\sin x\frac{dy}{dx} + y\cos x = 4x,\ x \in (0,\pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

[JEE(Main) 2018, (4, – 1), 120]

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$ (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. If $y(t)$ is a solution of $(1 + t)\frac{dy}{dt} - ty = 1$ and $y(0) = -1$, then $y(1)$ is equal to :

[IIT-JEE-2003, Scr. (3, –1), 84]

(A) $-\frac{1}{2}$ (B) $e + \frac{1}{2}$ (C) $e - \frac{1}{2}$ (D) $\frac{1}{2}$

2. If $y = y(x)$ and $\left(\frac{2+\sin x}{y+1}\right)\frac{dy}{dx} = -\cos x$; $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

[IIT-JEE-2004, Scr. (3, 0), 84]

(A) 1/3 (B) – 2/3 (C) 2/3 (D) – 1/3

3. Let $f(x)$ be differentiable on the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t\to x}\frac{t^2f(x)-x^2f(t)}{t-x} = 1$ for each $x > 0$. Then $f(x)$ is

[IIT-JEE-2007, Paper-1, (3, –1), 81]

(A) $\frac{1}{3x} + \frac{2x^2}{3}$ (B) $\frac{-1}{3x} + \frac{4x^2}{3}$ (C) $\frac{-1}{x} + \frac{2}{x^2}$ (D) $\frac{1}{x}$

4. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

[IIT-JEE-2007, Paper-2, (3, –1), 81]

(A) variable radii and a fixed centre at (0, 1)
(B) variable radii and a fixed centre at (0, –1)
(C) fixed radius 1 and variable centres along the x-axis
(D) fixed radius 1 and variable centres along the y-axis

5. A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be $\frac{y}{x} + \sec\left(\frac{y}{x}\right)$, $x > 0$. Then the equation of the curve is

[JEE (Advanced) 2013, Paper-1, (2, 0)/60]

(A) $\sin\left(\frac{y}{x}\right) = \ell n\, x + \frac{1}{2}$ (B) $\text{cosec}\left(\frac{y}{x}\right) = \ell n\, x + 2$

(C) $\sec\left(\frac{2y}{x}\right) = \ell n\ x + 2$ (D) $\cos\left(\frac{2y}{x}\right) = \ell n\ x + \frac{1}{2}$

6. The function y = f(x) is the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$ in (–1, 1) satisfying f(0) = 0. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) $\frac{\pi}{3} - \frac{\sqrt{3}}{2}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$ (C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

7. Let f : (0, ∞) → R be a differentiable function such that f'(x) = $2 - \frac{f(x)}{x}$ for all x ∈ (0, ∞) and f(1) ≠ 1. Then **[JEE (Advanced) 2016, Paper-1, (3, –1)/62]**

(A) $\lim\limits_{x\to0^+} f'\left(\frac{1}{x}\right) = 1$ (B) $\lim\limits_{x\to0^+} x\, f\left(\frac{1}{x}\right) = 2$ (C) $\lim\limits_{x\to0^+} x^2\, f'(x) = 0$ (D) $|f(x)| \le 2$ for all x ∈ (0, 2)

8. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$, x > 0, passes through the point (1, 3). Then the solution curve

(A) intersects y = x + 2 exactly at one point (B) intersects y = x + 2 exactly at two points

(C) intersects $y = (x + 2)^2$ (D) does NOT intersect $y = (x + 3)^2$

9. If y = y(x) satisfies the differential equation $8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx$, x > 0 and y (0) = $\sqrt{7}$, then y(256) = **[JEE(Advanced) 2017, Paper-2,(3, –1)/61]**

(A) 16 (B) 3 (C) 9 (D) 80

Differential Equation

Answers

EXERCISE # 1

Section (A)

A-1.	(1)	**A-2.**	(2)	**A-3.**	(1)	**A-4.**	(4)	**A-5.**	(4)	**A-6.**	(4)	**A-7.**	(1)
A-8.	(1)	**A-9.**	(2)	**A-10.**	(2)	**A-11.**	(1)	**A-12.**	(3)	**A-13.**	(1)	**A-14.**	(4)
A-15.	(2)												

Section (B)

B-1.	(3)	**B-2.**	(2)	**B-3.**	(2)	**B-4.**	(2)	**B-5.**	(2)	**B-6.**	(3)	**B-7.**	(1)
B-8.	(4)	**B-9.**	(3)	**B-10.**	(2)	**B-11.**	(1)	**B-12.**	(2)	**B-13.**	(4)	**B-14.**	(1)
B-15.	(2)	**B-16.**	(2)	**B-17.**	(1)	**B-18.**	(4)	**B-19.**	(1)	**B-20.**	(3)		

Section (C)

C-1.	(1)	**C-2.**	(1)	**C-3.**	(2)	**C-4.**	(1)	**C-5.**	(3)	**C-6.**	(2)	**C-7.**	(3)
C-8.	(4)	**C-9.**	(1)	**C-10.**	(3)	**C-11.**	(1)	**C-12.**	(4)	**C-13.**	(4)		

Section (D)

D-1.	(1)	**D-2.**	(2)	**D-3.**	(2)	**D-4.**	(1)	**D-5.**	(3)	**D-6.**	(2)	**D-7.**	(2)
D-8.	(2)	**D-9.**	(1)	**D-10.**	(3)	**D-11.**	(2)	**D-12.**	(2)	**D-13.**	(1)	**D-14.**	(1)

EXERCISE # 2

PART- I

1.	(3)	**2.**	(1)	**3.**	(4)	**4.**	(1)	**5.**	(1)	**6.**	(3)	**7.**	(4)
8.	(1)	**9.**	(3)	**10.**	(2)	**11.**	(3)	**12.**	(3)	**13.**	(1)	**14.**	(2)
15.	(4)	**16.**	(2)	**17.**	(2)	**18.**	(1)	**19.**	(1)				

PART- II

Section (A) :

A-1 (2) **A-2.** (3) **A-3.** (1)

Section (B)

B-1. A → s, B → p, C → q, D → r

Section (C) :

C-1.	(1,3)	**C-2.**	(1,2)	**C-3.**	(1,3)	**C-4.**	(1,2,4)	**C-5.**	(1,2)	**C-6.**	(1,3)	**C-7.**	(1,4)
C-8.	(1,3)												

EXERCISE # 3

PART- I

1.	(3)	**2.**	(2)	**3.**	(2)	**4.**	(2)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(2)	**9.**	(3)	**10.**	(3)	**11*.**	(1,4)	**12.**	(4)	**13.**	(3)	**14.**	(4)
15.	(4)	**16.**	(2)	**17.**	(1)	**18.**	(2)	**19.**	(3)	**20.**	(1)	**21.**	(3)
22.	(3)	**23.**	(3)	**24.**	(3)	**25.**	(1)	**26.**	(1)				

PART- II

1.	(A)	**2.**	(A)	**3.**	(A)	**4.**	(C)	**5.**	(A)	**6.**	(B)	**7.**	(A)
8.	(A,D)	**9.**	(B)										