Complex Numbers

Exercise-1

✍ **Marked Questions may have for Revision Questions.**

OBJECTIVE QUESTIONS

Section (A) : Algebra , Modulus and Conjugate of complex number

A-1. $\sqrt{-2}\sqrt{-3} =$

(1) $\sqrt{6}$ (2) $-\sqrt{6}$ (3) $i\sqrt{6}$ (4) $-i\sqrt{6}$

A-2. If n is a positive integer, then which of the following relations is false

(1) $i^{4n} = 1$ (2) $i^{4n-1} = i$ (3) $i^{4n+1} = i$ (4) $i^{-4n} = 1$

A-3. If $i^2 = -1$, then the value of $\sum_{n=1}^{200} i^n$ is

(1) 50 (2) – 50 (3) 0 (4) 100

A-4.✍ The value of $i^{1+3+5+.....+(2n+1)}$ is

(1) i if n is even, – i if n is odd (2) 1 if n is even, – 1 if n is odd
(3) 1 if n is odd, – 1 if n is even (4) 1 if n is odd, i if n is even

A-5.✍ Find the least value of n ($n \in N$), for which $\left(\frac{1+i}{1-i}\right)^n$ is real.

(1) 1 (2) 2 (3) 3 (4) 4

A-6. If $(1 - i)x + (1 + i)y = 1 - 3i$, then $(x, y) =$

(1) (2, – 1) (2) (– 2, 1) (3) (– 2, 1) (4) (2, 1)

A-7. The real part of $(1 - \cos\theta + 2i \sin\theta)^{-1}$ is

(1) $\frac{1}{3+5\cos\theta}$ (2) $\frac{1}{5-3\cos\theta}$ (3) $\frac{1}{3-5\cos\theta}$ (4) $\frac{1}{5+3\cos\theta}$

A-8. The imaginary part of $\frac{(1+i)^2}{(2-i)}$ is

(1) $\frac{1}{5}$ (2) $\frac{3}{5}$ (3) $\frac{4}{5}$ (4) $-\frac{4}{5}$

A-9.✍ If $z = 3 - 4i$, then $z^4 - 3z^3 + 3z^2 + 99z - 95$ is equal to

(1) 5 (2) 6 (3) – 5 (4) – 4

A-10.✍ The value of x and y for which the numbers $3 + ix^2y$ and $x^2 + y + 4i$ are conjugate complex of each other, can be

(1) (– 2, – 1) (2) (– 1, 2) or (– 2, 2) (3) (1, 2) or (– 1, – 2) (4) (2, –3)

A-11. $\sqrt{-8-6i} =$

(1) $1 \pm 3i$ (2) $\pm(1 - 3i)$ (3) $\pm(1 + 3i)$ (4) $\pm(3 - i)$

A-12.✍ If $(-7 - 24i)^{1/2} = x - iy$, then $x^2 + y^2 =$

(1) 15 (2) 25 (3) – 25 (4) –15

Complex Numbers

A-13. The number of solutions of the system of equations Re $(z^2) = 0$, $|z| = 2$ is
(1) 4 (2) 3 (3) 2 (4) 1

Section (B) : Representation of a complex number, Principal argument, Argument and its properties,

B-1. If z is a complex number such that $|z| = 4$ and $\arg(z) = \frac{5\pi}{6}$, then z is equal to
(1) $-2\sqrt{3} + 2i$ (2) $2\sqrt{3} + i$ (3) $2\sqrt{3} - 2i$ (4) $-\sqrt{3} + i$

B-2. Argument and modulus of $\frac{1+i}{1-i}$ are respectively
(1) $\frac{-\pi}{2}$ and 1 (2) $\frac{\pi}{2}$ and $\sqrt{2}$ (3) 0 and $\sqrt{2}$ (4) $\frac{\pi}{2}$ and 1

B-3. If $z = \frac{1-i\sqrt{3}}{1+i\sqrt{3}}$, then arg(z) =
(1) 60° (2) 120° (3) 240° (4) 300°

B-4. The modulus and amplitude of $(1+i\sqrt{3})^8$ are respectively
(1) 256 and π/3 (2) 256 and 2π/3 (3) 2 and 2π/3 (4) 256 and 8π/3

B-5. If $z_1 = 1 + i$, $z_2 = 1 + i\frac{\sqrt{3}}{2}$, $z_3 = -2 - \frac{2}{\sqrt{3}}i$ then the value of amp z_1 + amp z_2 + amp z_3 is
(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{12}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{4}$

B-6. If $z = \frac{1+i\sqrt{3}}{\sqrt{3}+i}$, then $(\bar{z})^{100}$ lies in
(1) I quadrant (2) II quadrant (3) III quadrant (4) IV quadrant

B-7. If arg (z) < 0, then arg (- z) - arg (z) =
(1) π (2) - π (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

B-8. The principal value of the arg(z) and $|z|$ of the complex number $z = 1 + \cos\left(\frac{11\pi}{9}\right) + i \sin\left(\frac{11\pi}{9}\right)$ are respectively :
(1) $\frac{11\pi}{18}$, $2\cos\frac{\pi}{18}$ (2) $-\frac{7\pi}{18}$, $2\cos\frac{7\pi}{18}$ (3) $\frac{2\pi}{9}$, $2\cos\frac{7\pi}{18}$ (4) $-\frac{\pi}{9}$, $-2\cos\frac{\pi}{18}$

B-9. The amplitude of $\sin\frac{\pi}{5} + i\left(1-\cos\frac{\pi}{5}\right)$ is
(1) π/5 (2) 2π/5 (3) π/10 (4) π/15

B-10. The argument of the complex number $\sin\frac{6\pi}{5} + i\left(1+\cos\frac{6\pi}{5}\right)$ is
(1) $\frac{6\pi}{5}$ (2) $\frac{5\pi}{6}$ (3) $\frac{9\pi}{10}$ (4) $\frac{2\pi}{5}$

Section (C) : Properties of conjugate and modulus and Triangle inequality

C-1. If z is a complex number, then $z.\bar{z} = 0$ if and only if
(1) $|z| = 1$ (2) Re(z) = 0 and Im(z) = 0

(3) Im(z) = 1 (4) Re(z) = 1

C-2.✍ If (2 + i)(2 + 2i) (2 + 3i) ...... (2 + 9i) = x + iy, then 5.8.13. ....85 =
(1) $x_2 + y_2$ (2) $x_2 - y_2$ (3) $(x_2 + y_2)_2$ (4) $(x_2 - y_2)_2$

C-3.✍ If z_1 and z_2 are any two complex numbers then $|z_1 + z_2|_2 + |z_1 - z_2|_2$ is equal to
(1) $2|z_1|_2\,|z_2|_2$ (2) $2|z_1|_2 + 2|z_2|_2$ (3) $|z_1|_2 + |z_2|_2$ (4) $2|z_1|\,|z_2|$

C-4.✍ If $|z_1 + z_2| = |z_1 - z_2|$ then the value of $|\text{amp } z_1 - \text{amp } z_2|$ is -
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) π (4) $\frac{\pi}{3}$

C-5.✍ If z_1, z_2, z_3 are complex numbers such that $|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$, then $|z_1 + z_2 + z_3|$ is :
(1) equal to 1 (2) less than 1 (3) greater than 3 (4) equal to 3

C-6.✍ If $\frac{z-i}{z+i}$ $(z \neq -i)$ is a purely imaginary number, then $z\bar{z}$ is equal to
(1) 0 (2) 1 (3) 2 (4) –1

C-7.✍ If z $(\neq -1)$ is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then |z| is equal to
(1) 1 (3) 2 (3) 3 (4) 5

C-8. Let z_1 lies on | z | = 1 and z_2 lies on | z | = 2 then maximum value of $|z_1 + z_2|$
(1) 3 (2) 1 (3) 4 (4) 5

C-9. Let z_1 lies on | z | = 1 and z_2 lies on | z | = 2 then minimum value of $|z_1 - z_2|$ is
(1) 3 (2) 1 (3) 4 (4) 5

C-10.✍ If $|z + 3| \leq 3$ then minimum and maximum values of |z + 1| are respectively
(1) 1, 5 (2) 0, 5 (3) 2, 5 (4) 1,2

C-11.✍ For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is
(1) 0 (2) 2 (3) 7 (4) 17

C-12.✍ If $|z - 2 + i| = 2$, then the greatest and least value of $|z|$ are respectively
(1) $\sqrt{5} + 2$, $\sqrt{5} - 2$ (2) $\sqrt{5} + 2$, $2 - \sqrt{5}$ (3) $\sqrt{5} + 2$, 0 (4) $\sqrt{5} - 2$, 0

C-13. If $|z_1 - 1| < 1$, $|z_2 - 2| < 2$, $|z_3 - 3| < 3$ then $|z_1 + z_2 + z_3|$
(1) is less than 6 (2) is more than 3 (3) is less than 12 (4) lies between 6 and 12

Section (D) : Geometry of complex number and Rotation theorem

D-1. Length of the line segment joining the points – 1 – i and 2 + 3i is
(1) – 5 (2) 15 (3) 5 (4) 25

D-2. The vector z = - 4 + 5i is turned counter clockwise through an angle of 180º & stretched 1.5 times. The complex number corresponding to the newly obtained vector is :

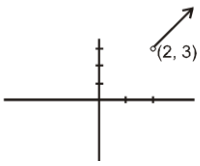
(1) $6 - \frac{15}{2}i$ (2) - $6 + \frac{15}{2}i$ (3) $6 + \frac{15}{2}i$ (4) $6 + 15i$

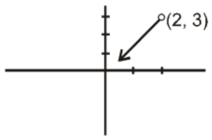
D-3. The points z_1, z_2, z_3, z_4 in the complex plane are the vertices of a parallelogram taken in order if and only if :
(1) $z_1 + z_4 = z_2 + z_3$ (2) $z_1 + z_3 = z_2 + z_4$ (3) $z_1 + z_2 = z_3 + z_4$ (4) $z_1 + z_2 + z_3 + z_4 = 0$

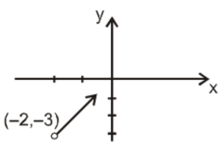
D-4. If $z = x + iy$ and $|z - 2 + i| = |z - 3 - i|$, then locus of z is
(1) $2x + 4y - 5 = 0$ (2) $2x - 4y - 5 = 0$ (3) $x + 2y = 0$ (4) $x - 2y + 5 = 0$

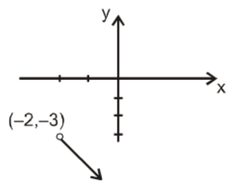
D-5. The complex number $z = x + iy$ which satisfy the equation $\left|\frac{z-5i}{z+5i}\right| = 1$ lie on :
(1) the x-axis (2) the straight line $y = 5$
(3) a circle passing through the origin (4) the y-axis

D-6. If Arg $(z - 2 - 3i) = \frac{\pi}{4}$, then the locus of z is

(1) 

(2) 

(3) 

(4) 

D-7. The equation $|z - 1|^2 + |z + 1|^2 = 2$ represents
(1) a circle of radius '1' (2) a straight line
(3) the ordered pair (0, 0) (4) set of two points

D-8. if $|z + 1|^2 + |z|^2 = 4$ **,** then the locus of z is
(1) Straight line (2) Circle with radius $\frac{7}{4}$
(3) Parabola (4) Circle with radius $\frac{\sqrt{7}}{2}$

D-9. The points of intersection of the two curves $|z - 3| = 2$ and $|z| = 2$ in an argand plane are:
(1) $\frac{1}{2}\left(7 \pm i\sqrt{3}\right)$ (2) $\frac{1}{2}\left(3 \pm i\sqrt{7}\right)$ (3) $\frac{3}{2} \pm i\sqrt{\frac{7}{2}}$ (4) $\frac{7}{2} \pm i\sqrt{\frac{3}{2}}$

D-10. If $\left|\frac{z-2}{z-3}\right| = 2$ represents a circle, then its radius is equal to
(1) 1 (2) 1/3 (3) 3/4 (4) 2/3

D-11. If $|z + 1| = \sqrt{2}\,|z - 1|$, then the locus described by the point z in the Argand diagram is a
(1) Straight line (2) Circle (3) Parabola (4) One point

D-12. Let A, B, C represent the complex numbers z_1, z_2, z_3 respectively on the complex plane. If the circumcentre of the triangle ABC lies at the origin, then the orthocentre is represented by the complex number :
(1) $z_1 + z_2 - z_3$ (2) $z_2 + z_3 - z_1$ (3) $z_3 + z_1 - z_2$ (4) $z_1 + z_2 + z_3$

Complex Numbers

D-13. If z_1, z_2, z_3, are vetices of equiletral triangle then the value of $z_1^2 + z_2^2 + z_3^2$ is
(1) $z_1 z_2 + z_2 z_3 + z_3 z_1$ (2) $z_1 z_2 - z_2 z_3 - z_3 z_1$ (3) $-z_1 z_2 - z_2 z_3 - z_3 z_1$ (4) $z_1 + z_2 + z_3$

D-14. If z_1, z_2, are roots of $z_2 - az + b = 0$ and $0, z_1, z_2$ are vertices of equitetral triangle then
(1) $a_2 + 3b = 0$ (2) $a_2 - 3b = 0$ (3) $a_2 + 3b = 1$ (4) $a + 3b = 0$

D-15. The complex numbers z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a triangle which is
(1) of area zero (2) right angled isosceles
(3) equilateral (4) obtuse angled isosceles

D-16. Points z_1 & z_2 are adjacent vertices of a regular octagon. The vertex z_3 adjacent to z_2 ($z_3 \neq z_1$) is represented by :
(1) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 + z_2)$ (2) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 - z_2)$
(3) $z_2 + \frac{1}{\sqrt{2}}(1 \pm i)(z_2 - z_1)$ (4) $z_1 + \frac{1}{\sqrt{2}}(1 \pm i)(z_1 + z_2)$

Comprehension # 2 (D-17,D-18)

ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2AC. Let the points D and M represent complex numbers 1 + i and 2 – i respecetively.

D-17. A possible representation of point A is
(1) $3 - \frac{i}{2}$ (2) $3 + \frac{i}{2}$ (3) $1 + \frac{3}{2}i$ (4) $3 - \frac{3}{2}i$

D-18. e_{iz} =
(1) $e_{-r\cos\theta}(\cos(r\cos\theta) + i\sin(r\sin\theta))$ (2) $e_{-r\cos\theta}(\sin(r\cos\theta) + i\cos(r\cos\theta))$
(3) $e_{-r\sin\theta}(\cos(r\cos\theta) + i\sin(r\cos\theta))$ (4) $e_{-r\sin\theta}(\sin(r\cos\theta) + i\cos(r\sin\theta))$

Section (E) : De moivre's theorem, cube roots and nth roots of unity

E-1-. $\frac{(\cos\theta + i\sin\theta)^4}{(\sin\theta + i\cos\theta)^5}$ is equal to
(1) $\cos\theta - i\sin\theta$ (2) $\cos 9\theta - i\sin 9\theta$ (3) $\sin\theta - i\cos\theta$ (4) $\sin 9\theta - i\cos 9\theta$

E-2. The value of $\frac{(\cos 2\theta - i\sin 2\theta)^4(\cos 4\theta + i\sin 4\theta)^{-5}}{(\cos 3\theta + i\sin 3\theta)^{-2}(\cos 3\theta - i\sin 3\theta)^{-9}}$ is
(1) $\cos 49\theta - i\sin 49\theta$ (2) $\cos 23\theta - i\sin 23\theta$ (3) $\cos 49\theta + i\sin 49\theta$ (4) $\cos 21\theta + i\sin 21\theta$

E-3. $\left[\frac{1 + \cos(\pi/8) + i\sin(\pi/8)}{1 + \cos(\pi/8) - i\sin(\pi/8)}\right]^8$ is equal to
(1) – 1 (2) 0 (3) 1 (4) 2

E-4. If $(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta) \ldots (\cos n\theta + i\sin n\theta) = 1$, then the value of θ is
(1) $4m\pi, m \in Z$ (2) $\frac{2m\pi}{n(n+1)}, m \in Z$ (3) $\frac{4m\pi}{n(n+1)}, m \in Z$ (4) $\frac{m\pi}{n(n+1)}, m \in Z$

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E-5. $\frac{\left(-1+i\sqrt{3}\right)^{15}}{(1-i)^{20}}+\frac{\left(-1-i\sqrt{3}\right)^{15}}{(1+i)^{20}}$ is equal to

(1) – 64 (2) – 32 (3) – 16 (4) $\frac{1}{16}$

E-6.✍ $\left(\frac{-1+i\sqrt{3}}{2}\right)^{20}+\left(\frac{-1-i\sqrt{3}}{2}\right)^{20}$ =

(1) 20 $\sqrt{3}i$ (2) 1 (3) $\frac{1}{2^{19}}$ (4) – 1

E-7. If ω is the cube root of unity, then $(3 + 5\omega + 3\omega_2)_2 + (3 + 3\omega + 5\omega_2)_2$ =

(1) 4 (2) 0 (3) – 4 (4) 4i

E-8. If ω (≠ 1) be a cube root of unity and $(1 + \omega_4)_n = (1 + \omega_2)_n$ then the least positive integral value of n is

(1) 3 (2) 2 (3) 4 (4) 0

E-9. The value of $(1 - \omega + \omega_2)(1 - \omega_2 + \omega_4)(1 - \omega_4 + \omega_8)$ .... to 2n factors is

(1) 2_n (2) 4_n (3) 4n (4) 2n

E-10. If 1, ω, ω_2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to-

(1) 0 (2) 1 (3) ω (4) ω_2

E-11.✍ If i = $\sqrt{-1}$, then 4 + 5 $\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{334}$ + 3 $\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^{365}$ is equal to

(1) $1-i\sqrt{3}$ (2) $-1+i\sqrt{3}$ (3) $i\sqrt{3}$ (4) – $i\sqrt{3}$

E-12. If x = a + b + c, y = aα + bβ + c and z = aβ + bα + c, where α and β are complex cube roots of unity, then xyz =

(1) $2(a_3 + b_3 + c_3)$ (2) $2(a_3 - b_3 - c_3)$ (3) $a_3 + b_3 + c_3 - 3abc$ (4) $a_3 - b_3 - c_3$

E-13.✍ If $x_2 + x + 1 = 0$ then the numerical value of;

$$\left(x+\frac{1}{x}\right)^2+\left(x^2+\frac{1}{x^2}\right)^2+\left(x^3+\frac{1}{x^3}\right)^2+\left(x^4+\frac{1}{x^4}\right)^2+......+\left(x^{27}+\frac{1}{x^{27}}\right)^2 =$$

(1) 54 (2) 36 (3) 27 (4) 18

E-13_ If ω is one of the imaginary cube root of unity then the value of expression ,
$(1 + 2\omega + 2\omega^2)^{10} + (2 + \omega + 2\omega^2)^{10}$

(1) 0 (2) 1 (3) ω (4) ω^2

E-14.✍ Let z_1 and z_2 be two nonreal complex cube roots of unity and $|z - z_1|_2 + |z - z_2|_2 = \lambda$ be the equation of a circle with z_1, z_2 as ends of a diameter then the value of λ is

(1) 4 (2) 3 (3) 2 (4) $\sqrt{2}$

E-15. If α, β, γ are cube roots of 8 , then the value of $\dfrac{a\alpha + b\beta + c\gamma}{a\beta + b\gamma + c\alpha}$ is

(1) 2ω (2) ω^2 (3) 1 (4) $2\omega^2$

E-16. The sum of roots of equation $(z - 1)^4 - 16 = 0$ is

(1) 0 (2) 4 (3) 1 – 2 i (4) 1 + 2 i

E-17. The product of all the roots of $\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)^{3/4}$ is

(1) –1 (2) 1 (3) $\dfrac{3}{2}$ (4) $-\dfrac{1}{2}$

E-18.✍ If $z_r = \cos\dfrac{2r\pi}{5} + i\sin\dfrac{2r\pi}{5}$, r = 0, 1, 2, 3, 4, ... then the value of $z_1 . z_2 . z_3 . z_4 . z_5$ is -

(1) 3 (2) 5 (3) 1 (4) 1

E-19. If α is non real and $\alpha = \sqrt[5]{1}$ then the value of $2^{|1 + \alpha + \alpha^2 + \alpha^{-2} - \alpha^{-1}|}$ is equal to

(1) 4 (2) 2 (3) 1 (4) 0

Comprehension # 1 (E-20 to E-22)

If 1, α, α^2, ...α^{n-1} are n^{th} roots of unity such that $a = \cos\dfrac{2\pi}{n} + i\sin\dfrac{2\pi}{n}$ then they satisfy following properties :

(i) $1 + \alpha + \alpha^2 + ...+ \alpha^{n-1} = 0$,

(ii) $1. \alpha . \alpha^2 ....\alpha^{n-1} = (-1)^{n-1}$

(iii) 1, α, α^2, ...α^{n-1} lie on circle with centre as origin and unit radius, being equidistant on the circumference of circle by angle $\dfrac{2\pi}{n}$.

Read the above passage and answer the following :

E-20.✍ The value of $\sum_{k=1}^{6}\left(\sin\dfrac{2\pi k}{7} - i\cos\dfrac{2\pi k}{7}\right)$ is :

(1) - 1 (2) 0 (3) - i (4) i

E-21.✍ Let z_1 and z_2 be n^{th} roots of unity which subtend a right angle at the origin. Then n must be of the form

(1) 4k + 1 (2) 4k + 2 (3) 4k + 3 (4) 4k

E-22.✍ The product of cube roots of – 1 is equal to

(1) – 2 (2) 0 (3) – 1 (4) 4

Exercise-2

✍ **Marked Questions may have for Revision Questions.**

PART - I : OBJECTIVE QUESTIONS

1. The modulus and the principal argument of the complex number z = -2 (cos 30° + i sin 30°) are respectively

(1) 2, $-\dfrac{\pi}{6}$ (2) 2, $-\dfrac{5\pi}{6}$ (3) –2, $\dfrac{\pi}{6}$ (4) 2, $\dfrac{7\pi}{6}$

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2. The modulus and the principal argument of the complex number $z = 1 + \cos\frac{18\pi}{25} + i\sin\frac{18\pi}{25}$ are respectively

(1) $2\cos\frac{9\pi}{25}, \frac{9\pi}{25}$ (2) $2\sin\frac{9\pi}{25}, \frac{9\pi}{25}$ (3) $\cos\frac{9\pi}{25}, \frac{9\pi}{25}$ (4) $2\cos\frac{9\pi}{25}, \frac{16\pi}{25}$

3. If $(a + ib)^5 = \alpha + i\beta$ then $(b + ia)^5$ is equal to

(1) $\beta + i\alpha$ (2) $\alpha - i\beta$ (3) $\beta - i\alpha$ (4) $-\alpha - i\beta$

4. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ then

(1) amp $\frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$ (2) $\frac{z_1}{z_2}$ is purely imaginary

(3) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$ (4) All of these

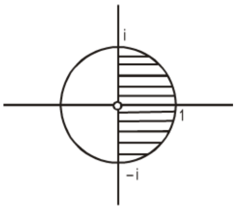
5. The region represented by $\text{Re}(z) \le 2$, $\text{Im}(z) \le 2$ and $\frac{\pi}{8} \le \arg(z) \le \frac{3\pi}{8}$ lies in

(1) 1st quadrant (2) 2nd quadrant (3) 3rd quadrant (4) 4th quadrant

6. The inequality $|z - 4| < |z - 2|$ represents :

(1) $\text{Re}(z) > 0$ (2) $\text{Re}(z) < 0$ (3) $\text{Re}(z) > 2$ (4) $\text{Re}(z) > 3$

7. The locus of z which lies in shaded region is best represented by



(1) $|z| \le 1, \frac{-\pi}{2} \le \arg z \le \frac{-\pi}{2}$ (2) $|z| = 1, \frac{-\pi}{2} \le \arg z \le 0$

(3) $|z| \ge 0, 0 \le \arg z \le \frac{\pi}{2}$ (4) $|z| \le 1, \frac{\pi}{2} \le \arg z \le \pi$

8. POQ is a straight line through the origin O . P and Q represent the complex number a + i b and c + i d respectively and OP = OQ. Then

(1) $|a + ib| = |c + id|$ (2) $a - c = b - d$

(3) $\arg(a + ib) = \arg(c + id)$ (4) $a + c = b + d$

9. If z satisfies the inequality $|z - 1 - 2i| \le 1$, then

(1) $\min(\arg(z)) = \tan^{-1}\left(\frac{3}{4}\right)$ (2) $\max(\arg(z)) = \frac{\pi}{6}$

(3) $\min(|z|) = \sqrt{5}$ (4) $\max(|z|) = \sqrt{5}$

10. O is origin and affixes of P, Q, R are respectively z, iz, z + iz. If $\Delta PQR = 200$ then the value of $|z|$ is

(1) 15 (2) 20 (3) 10 (4) 25

11. If a and b are real numbers between 0 and 1 such that the points $z_1 = a + i$, $z_2 = 1 + bi$ and $z_3 = 0$ form an equilateral triangle, then

(1) $a = b = 2 + \sqrt{3}$ (2) $a = b = 2 - \sqrt{3}$

(3) $a = 2 - \sqrt{3}$ and $b = 2 + \sqrt{3}$ (4) $a = 2 + \sqrt{3}$ and $b = 2 - \sqrt{3}$

Complex Numbers

12. Let z_1, z_2, z_3 be three vertices of an equilateral triangle circumscribing the circle $|z| = 1$. If $z_1 = \frac{1}{2} + \frac{\sqrt{3}i}{2}$ and z_1, z_2, z_3 are in anticlockwise sense then z_2 is

(1) $1+\sqrt{3}i$ (2) $\frac{1}{2} - \frac{\sqrt{3}i}{2}$ (3) 1 (4) – 1

13. If $|z_1| = |z_2| = |z_3| = 1$ and z_1, z_2, z_3 are represented by the vertices of an equilateral triangle then

(1) $z_1 + z_2 + z_3 = 0$ (2) $z_1z_2z_3 = 1$ (3) $z_1 + z_2 + z_3 = 1$ (4) $z_1z_2z_3 = 0$

14. If three complex numbers are in A.P., then they lie on

(1) A circle in the complex plane (2) A straight line in the complex plane

(3) A parabola in the complex plane (4) Can not say

15. The radius of the circle $z\bar{z} + (4 - 3i)z + (4 + 3i)\bar{z} + 5 = 0$ is

(1) $2\sqrt{5}$ (2) $\sqrt{5}$ (3) $3\sqrt{5}$ (4) $4\sqrt{5}$

16. The value of expression $\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)\left(\cos\frac{\pi}{2^2} + i\sin\frac{\pi}{2^2}\right)$ .... to ∞ is

(1) – 1 (2) 1 (3) 0 (4) 2

17. The expression $\left[\frac{1+i\tan\alpha}{1-i\tan\alpha}\right]^n - \frac{1+i\tan n\alpha}{1-i\tan n\alpha}$ when simplified reduces to :

(1) zero (2) $2\sin n\alpha$ (3) $2\cos n\alpha$ (4) $-2\cos n\alpha$

18. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\varphi = y + \frac{1}{y}$, then

(1) $x^n + \frac{1}{x^n} = 2\cos(n\theta)$ (2) $x^n + \frac{1}{x^n} = 2\sin(n\theta)$

(3) $x^n - \frac{1}{x^n} = 2\cos(n\theta)$ (4) $y^n + \frac{1}{y^n} = 2\sin(n\varphi)$

19. Let ω be the non real cube root of unity which satisfy the equation $h(x) = 0$ where $h(x) = x f(x^3) + x^2 g(x^3)$. If $h(x)$ is polynomial with real coefficient then which statement is incorrect.

(1) $f(1) = 0$ (2) $g(1) = 0$ (3) $h(1) = 0$ (4) $g(1) \neq f(1)$

20. Let ω be an imaginary root of $x^n = 1$. Then $(5 - \omega)(5 - \omega^2) \ldots (5 - \omega^{n-1})$ is

(1) 1 (2) $\frac{5^n+1}{4}$ (3) 4^{n-1} (4) $\frac{5^n-1}{4}$

21. If $1, \alpha_1, \alpha_2, \alpha_3, \alpha_4$ be the roots of $x^5 - 1 = 0$, then the value of $\frac{\omega-\alpha_1}{\omega^2-\alpha_1}\frac{\omega-\alpha_2}{\omega^2-\alpha_2}\frac{\omega-\alpha_3}{\omega^2-\alpha_3}\frac{\omega-\alpha_4}{\omega^2-\alpha_4}$ ... is . (where ω is imaginary cube root of unity.)

(1) ω (2) ω^2 (3) 2ω (2) $2\omega^2$

22. If $\alpha = e^{i2\pi/11}$ then Real $(\alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5)$ equals to :

(1) $\frac{1}{2}$ (2) 1 (3) $-\frac{1}{2}$ (4) –1

23. If $a = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$ then the quadratic equation whose roots are $\alpha = a + a^2 + a^4$ and

$\beta = a_3 + a_5 + a_6$ is

(1) $x^2 + x - 2 = 0$ (2) $x^2 - x + 2 = 0$ (3) $x^2 - x - 2 = 0$ (4) $x^2 + x + 2 = 0$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.
(2) Statement-I is true, but Statement-II is false.
(3) Statement-I is false, but Statement-II is true.
(4) Both the statements are false.

A-1. **Statement-1 :** Arg (2 + 3i) + Arg (2 – 3i) = 0 (Arg z stands for principal argument of z)
Statement-2 : Arg z + Arg $\bar{z}$ = 0, z = x + iy, ∀ x, y ∈ R (Arg z stands for principal argument of z)

A-2. **Statement-1 :** Roots of the equation $(1 + z)^6 + z^6 = 0$ are collinear.
Statement-2 : If z_1, z_2, z_3 are in A.P. then points represented by z_1, z_2, z_3 are collinear

A-3. Let z_1, z_2, z_3 represent vertices of a triangle.
Statement-1 : $\frac{1}{z_1 - z_2} + \frac{1}{z_2 - z_3} + \frac{1}{z_3 - z_1} = 0$, when triangle is equilateral.
Statement-2 : $|z_1|^2 - z_1\bar{z}_0 - \bar{z}_1 z_0 = |z_2|^2 - z_2\bar{z}_0 - \bar{z}_2 z_0 = |z_3|^2 - z_3\bar{z}_0 - \bar{z}_3 z_0$, where z_0 is circumcentre of triangle.

Section (B) : MATCH THE COLUMN

B-1. Let z_1 lies on | z | = 1 and z_2 lies on | z | = 2.

	Column – I		Column – II
(A)	Maximum value of $\lvert z_1 + z_2 \rvert$	(p)	3
(B)	Minimum value of $\lvert z_1 - z_2 \rvert$	(q)	1
(C)	Minimum value of $\lvert 2z_1 + 3z_2 \rvert$	(r)	4
(D)	Maximum value of $\lvert z_1 - 2z_2 \rvert$	(s)	5

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If z = x + iy, x y ∈ R be any complex number such that its real part be $\sum_{r=1}^{6} \cos\left(\frac{(2r-1)\pi}{13}\right)$ and imaginary part be $\sum_{p=1}^{9} \cos\frac{(2p-1)\pi}{19}$, then

(1) $0 < \arg(z) < \frac{\pi}{2}$ (2) 0 < | z | < 1 (3) $\left|z + \frac{1}{2} - \frac{i}{2}\right| = 1$ (4) z lies on x + y = 2

C-2. If amp $(z_1z_2) = 0$ and $|z_1| = |z_2| = 1$, then

(1) $z_1 + z_2 = 0$ (2) $z_1z_2 = 1$ (3) $z_1 = \bar{z}_2$ (4) $z_1 = z_2$

Complex Numbers

C-3. If $|z_1 + z_2|^2 = |z_1|^2 + |z_2|^2$ (where z_1 and z_2 are non-zero complex numbers), then

(1) $\frac{z_1}{z_2}$ is purely real

(2) $\frac{z_1}{z_2}$ is purely imaginary

(3) $z_1\bar{z}_2 + z_2\bar{z}_1 = 0$

(4) amp $\frac{z_1}{z_2}$ may be equal to $\frac{\pi}{2}$

C-4. If Z = $\frac{(1+i)(1+2i)(1+3i)......(1+ni)}{(1-i)(2-i)(3-i)......(n-i)}$, $n \in N$ then principal argument of Z can be

(1) 0

(2) $\frac{\pi}{2}$

(3) $-\frac{\pi}{2}$

(4) π

C-5. If z_1, z_2, z_3 be the vertices (in anticlockwise sense) of an equilateral triangle incribed in $| z | = k$ $(k > 0)$ and $z_1 = 1 + \sqrt{3}\,i$, then

(1) $z_3 = \bar{z}_2$

(2) z_2 is pure real

(3) $z_3 = \bar{z}_1$

(4) $z_1 + z_2 + z_3 = 0$

Complex Numbers

Exercise-3

✍ **Marked Questions may have for Revision Questions.**

* **Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. If ω is an imaginary cube root of unity, then $(1 + \omega - \omega^2)^7$ equals- **[AIEEE 2002, (3, –1), 225]**
(1) 128ω (2) -128ω (3) $128\omega^2$ (4) $-128\omega^2$

2. If $(\omega \neq 1)$ is a cubic root of unity, then $\begin{vmatrix} 1 & 1+i+\omega^2 & \omega^2 \\ 1-i & -1 & \omega^2-1 \\ -i & -1+\omega-i & -1 \end{vmatrix}$ equals- **[AIEEE 2002, (3, –1), 225]**
(1) 0 (2) 1 (3) i (4) ω

3. Let z_1 and z_2 be two roots of the equation $z^2 + az + b = 0$, z being complex. Further, assume that the origin, z_1 and z_2 form an equilateral tringle. Then : **[AIEEE 2003, (3, –1), 225]**
(1) $a^2 = b$ (2) $a^2 = 2b$ (3) $a^2 = 3b$ (4) $a^2 = 4b$

4. If z and ω are two non-zero complex numbers such that $|z\omega| = 1$, and $\arg(z) - \arg(\omega) = \frac{\pi}{2}$, then $\bar{z}\omega$ is equal to : **[AIEEE 2003, (3, –1), 225]**
(1) 1 (2) – 1 (3) i (4) – i

5. If $\left(\frac{1+i}{1-i}\right)^x = 1$, then **[AIEEE 2003, (3, –1), 225]**
(1) x = 4 n, where n is any positive integer (2) x = 2 n, where n is any positive integer
(3) x = 4 n + 1, where n is any positive integer (4) x = 2n + 1, where n is any positive integer

6. If 1, ω, ω^2 are the cube roots of unity, then $\Delta = \begin{vmatrix} 1 & \omega^n & \omega^{2n} \\ \omega^n & \omega^{2n} & 1 \\ \omega^{2n} & 1 & \omega^n \end{vmatrix}$ is equal to-
[AIEEE 2003, (3, –1), 225]
(1) 0 (2) 1 (3) ω (4) ω^2

7. Let z,w be complex numbers such that $\bar{z} + i\bar{w} = 0$ and arg zw = π. Then arg z equals :
[AIEEE 2004, (3, –1), 225]
(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{3\pi}{4}$ (4) $\frac{5\pi}{4}$

8. If $z = x - iy$ and $z^{1/3} = p + iq$, then $\left(\frac{x}{p}+\frac{y}{q}\right)\Big/(p^2+q^2)$ is equal to : **[AIEEE 2004, (3, –1), 225]**
(1) 1 (2) – 1 (3) 2 (4) – 2

9. If $|z^2 - 1| = |z|^2 + 1$, then z lies on : **[AIEEE 2004, (3, –1), 225]**
(1) the real axis (2) the imaginary axis (3) a circle (4) an ellipse

Complex Numbers

10. If the cube roots of unity are $1, \omega, \omega^2$, then the roots of the equation $(x - 1)^3 + 8 = 0$, are : **[AIEEE 2005, (3, –1), 225]**

(1) $-1, 1 + 2\omega, 1 + 2\omega^2$. (2) $-1, 1 - 2\omega, 1 - 2\omega^2$.
(3) $-1, -1, -1$. (4) $-1, -1 + 2\omega, -1 - 2\omega^2$.

11. If z_1 and z_2 are two non-zero complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$, then $\arg z_1 - \arg z_2$ is equal to: **[AIEEE 2005, (3, –1), 225]**

(1) $-\frac{\pi}{2}$ (2) 0 (3) $-\pi$ (4) $\frac{\pi}{2}$.

12. If $w = \frac{z}{z - \frac{1}{3}i}$ and $|w| = 1$, then z lies on : **[AIEEE 2005, (3, –1), 225]**

(1) a parabola (2) a straight line (3) a circle (4) an ellipse.

13. The value of $\sum_{k=1}^{10}\left(\sin\frac{2k\pi}{11} + i\cos\frac{2k\pi}{11}\right)$ is : **[AIEEE 2006 (3, –1), 120]**

(1) 1 (2) – 1 (3) – i (4) i

14. If $z^2 + z + 1 = 0$, where z is complex number, then the value of **[AIEEE 2006 (3, –1), 120]**

$\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \ldots\ldots + \left(z^6 + \frac{1}{z^6}\right)^2$ is :

(1) 54 (2) 6 (3) 12 (4) 18

15. If $|z + 4| \le 3$, then the maximum value of $|z + 1|$ is **[AIEEE 2007 (3, –1), 120]**

(1) 4 (2) 10 (3) 6 (4) 0

16. The conjugate of a complex number is $\frac{1}{i-1}$. Then, that complex number is- **[AIEEE 2008 (3, –1), 105]**

(1) $-\frac{1}{i-1}$ (2) $\frac{1}{i+1}$ (3) $-\frac{1}{i+1}$ (4) $\frac{1}{i-1}$

17. If $\left|z - \frac{4}{z}\right| = 2$, then the maximum value of $|z|$ is equal to : **[AIEEE 2009 (4, –1), 144]**

(1) $\sqrt{5} + 1$ (2) 2 (3) $2 + \sqrt{2}$ (4) $\sqrt{3} + 1$

18. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$ **[AIEEE 2010 (4, –1), 144]**

(1) – 1 (2) 1 (3) 2 (4) –2

19. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals **[AIEEE 2010 (4, –1), 144]**

(1) 1 (2) 2 (3) ∞ (4) 0

20. If $\omega (\neq 1)$ is a cube root of unity and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals **[AIEEE 2011, I, (4, –1), 120]**

(1) (0, 1) (2) (1, 1) (3) (1, 0) (4) (–1, 1)

21. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that : **[AIEEE 2011, I, (4, –1), 120]**

(1) $\beta \in (0, 1)$ (2) $\beta \in (-1, 0)$ (3) $|\beta| = 1$ (4) $\beta \in (1, \infty)$

22. If $z \neq 1$ and $\frac{z^2}{z-1}$ is real, then the point represented by the complex number z lies : **[AIEEE-2012, (4, –1)/120]**
(1) either on the real axis or on a circle passing through the origin.
(2) on a circle with centre at the origin.
(3) either on the real axis or on a circle not passing through the origin.
(4) on the imaginary axis.

23. If z is a complex number of unit modulus and argument θ, then arg $\left(\frac{1+z}{1+\bar{z}}\right)$ equals : **[AIEEE - 2013, (4, – 1) 120]**
(1) $-\theta$ (2) $\frac{\pi}{2} - \theta$ (3) θ (4) $\pi - \theta$

24. If z a complex number such that $|z| \geq 2$, then the minimum value of $\left|z + \frac{1}{2}\right|$: **[JEE(Main) 2014,(4,–1), 120]**
(1) is strictly greater than 5/2 (2) is strictly greater than 3/2 but less than 5/2
(3) is equal to 5/2 (4) lie in the interval (1, 2)

25 A complex number z is said to be unimodular if $|z| = 1$. Suppose z_1 and z_2 are complex numbers such that $\frac{z_1 - 2z_2}{2 - z_1\bar{z}_2}$ is unimodular and z_2 is not unimodular. Then the point z_1 lies on a : **[JEE(Main) 2015, (4, – 1), 120]**
(1) straight line parallel to x-axis (2) straight line parallel to y-axis
(3) circle of radius 2 (4) circle of radius $\sqrt{2}$

26. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is : **[JEE(Main) 2016, (4, – 1), 120]**
(1) $\frac{\pi}{6}$ (2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (4) $\frac{\pi}{3}$

27. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2 - 1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k$, then k is equal to: **[JEE(Main) 2017, (4, – 1), 120]**
(1) $-z$ (2) z (3) -1 (4) 1

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. For all complex numbers z_1, z_2 satisfying $|z_1| = 12$ and $|z_2 - 3 - 4i| = 5$, the minimum value of $|z_1 - z_2|$ is **[IIT-JEE-2002, Scr, (3, – 1), 90]**
(A) 0 (B) 2 (C) 7 (D) 17

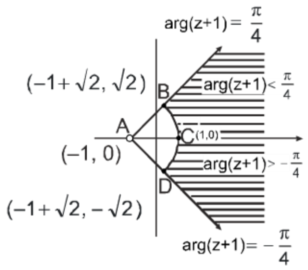
2. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$. Then the value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix}$ is: **[IIT-JEE-2002, Scr, (3, – 1), 90]**
(A) 3ω (B) $3\omega(\omega - 1)$ (C) $3\omega^2$ (D) $3\omega(1 - \omega)$

Complex Numbers

3. If |z| = 1 and $\omega = \frac{z-1}{z+1}$ (where $z \neq -1$), the Re(ω) is **[IIT-JEE-2003, Scr, (3, – 1), 84]**

(A) 0 (B) $-\frac{1}{|z+1|^2}$ (C) $\left|\frac{z}{z+1}\right| \cdot \frac{1}{|z+1|^2}$ (D) $\frac{\sqrt{2}}{|z+1|^2}$

4. The locus of z which lies in shaded region (excluding the boundaries) is best represented by
[IIT-JEE-2005, Scr, (3, –1), 84]



(A) z : |z + 1| > 2 and |arg (z + 1)| < π/4 (B) z : |z – 1| > 2 and |arg (z – 1)| < π/4
(C) z : |z + 1| < 2 and |arg (z + 1)| < π/2 (D) z : |z – 1| < 2 and |arg (z – 1)| < π/2

5.✍ a, b, c are integers, not all simultaneously equal and ω is cube root of unity (ω ≠ 1), then minimum value of $|a + b\omega + c\omega^2|$ is **[IIT-JEE-2005, Scr, (3, –1), 84)]**

(A) 0 (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

6.✍ Let ω = α + iβ, β ≠ 0 and z ≠ 1, If $\frac{\omega - \bar{\omega}z}{1-z}$ is purely real, then the set of values of z is
[IIT-JEE-2006, Main, (3, –1), 184]

(A) {z : |z| = 1} (B) {z : $\bar{z}$ = z} (C) {z : |z| ≠ 1} (D) {z : |z| = 1, z ≠ 1}

7.✍ A man walks a distance of 3 units from the origin towards the north-east (N 45° E) direction. From there, he walks a distance of 4 units towards the north-west (N 45° W) direction to reach a point P. Then the position of P in the Argand plane is **[IIT-JEE-2007, Paper-I, (3, – 1), 81]**

(A) $3e^{i\pi/4} + 4i$ (B) $(3 - 4i)e^{i\pi/4}$ (C) $(4 + 3i)e^{i\pi/4}$ (D) $(3 + 4i)e^{i\pi/4}$

8.✍ If |z| = 1 and z ≠ ± 1, then all the values of $\frac{z}{1-z^2}$ lie on **[IIT-JEE-2007, Paper-II, (3, – 1), 81]**

(A) a line not passing through the origin (B) $|z| = \sqrt{2}$
(C) the x-axis (D) the y-axis

9.✍ Let ω ≠ 1 be a cube root of unity and S be the set of all non-singular matrices of the form

$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$, where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is **[IIT-JEE-2011, Paper-2, (3, –1)/80]**

(A) 2 (B) 6 (C) 4 (D) 8

10.✍ Let z be a complex number such that the imaginary part of z is non zero and $a = z^2 + z + 1$ is real. Then a cannot take the value **[IIT-JEE 2012, PAPER- 1, (3, –1)/70]**

(A) –1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

11.*✍ Let ω be a complex cube root of unity with ω ≠ 1 and P = $[p_{ij}]$ be a n × n matrix with $p_{ij} = \omega^{i+j}$. Then $P^2 \neq 0$, when n = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) 57 (B) 55 (C) 58 (D) 56

Paragraph for Question Nos. 12 to 13

Let $S = S_1 \cap S_2 \cap S_3$, where

$S_1 = \{z \in C : |z| < 4\}$, $S_2 = \left\{z \in C : \text{Im}\left[\frac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right] > 0\right\}$ and

$S_3 : \{z \in C : \text{Re } z > 0\}$.

12.✍ Area of S = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{10\pi}{3}$ (B) $\frac{20\pi}{3}$ (C) $\frac{16\pi}{3}$ (D) $\frac{32\pi}{3}$

13.✍ $\min_{z\in S} |1-3i-z| =$ **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $\frac{2-\sqrt{3}}{2}$ (B) $\frac{2+\sqrt{3}}{2}$ (C) $\frac{3-\sqrt{3}}{2}$ (D) $\frac{3+\sqrt{3}}{2}$

14. Let $z_k = \cos\left(\frac{2k\pi}{10}\right) + i\sin\left(\frac{2k\pi}{10}\right);\ k = 1, 2, \ldots 9$.

[JEE (Advanced) 2014, Paper-2, (3, –1)/60]

	List I		**List II**
P.	For each z_k there exists a z_j such that $z_k . z_j = 1$	**1.**	True
Q.	There exists a k ∈ {1,2,....,9} such that $z_1 . z = z_k$ has no solution z in the set of complex numbers.	**2.**	False
R.	$\frac{\|1-z_1\|\|1-z_2\|.....\|1-z_9\|}{10}$ equals	**3.**	1
S.	$1 - \sum_{k=1}^{9} \cos\left(\frac{2k\pi}{10}\right)$ equals	**4.**	2

	P	Q	R	S
(A)	1	2	4	3
(B)	2	1	3	4
(C)	1	2	3	4
(D)	2	1	4	3

15. Let a, b ∈R and $a^2 + b^2 \neq 0$. Suppose $S = \left\{z \in R : z = \frac{1}{a+ibt}, t \in R, t \neq 0\right\}$, where $i = \sqrt{-1}$.

If z = x + iy and z ∈ S, then (x, y) lies on **[JEE (Advanced) 2016, Paper-2, (4, –2)/62]**

(A) the circle with radius $\frac{1}{2a}$ and centre $\left(\frac{1}{2a}, 0\right)$ for a > 0 , b ≠ 0

(B) the circle with radius $-\frac{1}{2a}$ and centre $\left(-\frac{1}{2a}, 0\right)$ for a < 0, b ≠ 0

(C) the x-axis for $a \neq 0$, $b = 0$

(D) the y-axis for $a = 0$, $b \neq 0$

16. Let a, b, x and y be real numbers such that $a - b = 1$ and $y \neq 0$. If the complex number $z = x + iy$ satisfies $\text{Im}\left(\frac{az+b}{z+1}\right) = y$, then which of the following is(are) possible value(s) of x ?

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(A) $1 - \sqrt{1+y^2}$ (B) $-1 - \sqrt{1-y^2}$ (C) $1 + \sqrt{1+y^2}$ (D) $-1 + \sqrt{1-y^2}$

Answers

EXERCISE - 1

Section (A) :

A-1.	(2)	**A-2.**	(2)	**A-3.**	(3)	**A-4.**	(4)	**A-5.**	(2)	**A-6.**	(1)	**A-7.**	(4)
A-8.	(3)	**A-9.**	(1)	**A-10.**	(1)	**A-11.**	(2)	**A-12.**	(2)	**A-13.**	(1)		

Section (B) :

B-1.	(1)	**B-2.**	(4)	**B-3.**	(3)	**B-4.**	(2)	**B-5.**	(3)	**B-6.**	(3)	**B-7.**	(1)
B-8.	(2)	**B-9.**	(3)	**B-10.**	(3)								

Section (C) :

C-1.	(2)	**C-2.**	(1)	**C-3.**	(2)	**C-4.**	(1)	**C-5.**	(1)	**C-6.**	(2)	**C-7.**	(1)
C-8.	(1)	**C-9.**	(2)	**C-10.**	(2)	**C-11.**	(2)	**C-12.**	(1)	**C-13.**	(3)		

Section (D) :

D-1.	(3)	**D-2.**	(1)	**D-3.**	(2)	**D-4.**	(1)	**D-5.**	(1)	**D-6.**	(1)	**D-7.**	(3)
D-8.	(4)	**D-9.**	(2)	**D-10.**	(4)	**D-11.**	(2)	**D-12.**	(4)	**D-13.**	(1)	**D-14.**	(2)
D-15.	(3)	**D-16.**	(3)	**D-17.**	(1)	**D-18.**	(3)						

Section (E) :

E-1-.	(4)	**E-2.**	(1)	**E-3.**	(1)	**E-4.**	(3)	**E-5.**	(1)	**E-6.**	(4)	**E-7.**	(3)
E-8.	(1)	**E-9.**	(2)	**E-10.**	(1)	**E-11.**	(3)	**E-12.**	(3)	**E-13.**	(1)	**E-13.**	(1)
E-14.	(2)	**E-15.**	(2)	**E-16.**	(2)	**E-17.**	(2)	**E-18.**	(3)	**E-19.**	(1)	**E-20.**	(4)
E-21.	(4)	**E-22.**	(3)										

EXERCISE - 2

PART - I

1.	(2)	**2.**	(1)	**3.**	(1)	**4.**	(4)	**5.**	(1)	**6.**	(4)	**7.**	(1)
8.	(1)	**9.**	(1)	**10.**	(2)	**11.**	(2)	**12.**	(4)	**13.**	(1)	**14.**	(2)
15.	(1)	**16.**	(1)	**17.**	(1)	**18.**	(1)	**19.**	(4)	**20.**	(4)	**21.**	(1)
22.	(3)	**23.**	(4)										

PART - II

Section (A) :

A-1. (2) **A-2.** (1) **A-3.** (1)

Section (B) :

B-1. (A) → (p), (B) → (q), (C) → (r), (D) → (s)

Section (C) :

C-1. (1,2,3,) **C-2.** (2,3) **C-3.** (2,3,4) **C-4.** (1,2,3,4) **C-5.** (2,3,4,)

EXERCISE - 3

PART - I

1.	(4)	**2.**	(1)	**3.**	(3)	**4.**	(4)	**5.**	(1)	**6.**	(1)	**7.**	(3)

Complex Numbers

8.	(4)	**9.**	(2)	**10.**	(2)	**11.**	(2)	**12.**	(2)	**13.**	(3)	**14.**	(3)
15.	(3)	**16.**	(3)	**17.**	(1)	**18.**	(2)	**19.**	(1)	**20.**	(2)	**21.**	(4)
22.	(1)	**23.**	(3)	**24.**	(4)	**25**	(3)	**26.**	(3)	**27.**	(1)		

PART - II

1.	(B)	**2.**	(B)	**3.**	(A)	**4.**	(A)	**5.**	(B)	**6.**	(D)	**7.**	(D)
8.	(D)	**9.**	(A)	**10.**	(D)	**11.***	(BCD)	**12.**	(B)	**13.**	(C)	**14.**	(C)
15.	(A,C,D)	**16.**	(B,D)										