Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

OBJECTIVE QUESTIONS

Section (A) : Equation of SHM

A-1. For a particle executing simple harmonic motion, the acceleration is proportional to
(1) displacement from the mean position (2) distance from the mean position
(3) distance travelled since t = 0 (4) speed

A-2. The displacement of a particle in simple harmonic motion in one time period is
(1) A (2) 2A (3) 4A (4) zero

A-3. The distance moved by a particle in simple harmonic motion in one time period is
(1) A (2) 2A (3) 4A (4) zero

A-4. The time period of a particle in simple harmonic motion is equal to the time between consecutive appearance of the particle at a particular point in its motion. This point is
(1) the mean position
(2) an extreme position
(3) between the mean position and the positive extreme
(4) between the mean position and the negative extreme.

A-5. Two SHM's are represented by $y = a \sin (\omega t - kx)$ and $y = b \cos (\omega t - kx)$. The phase difference between the two is :
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{3\pi}{4}$

A-6. The average acceleration in one time period in a simple harmonic motion is
(1) $A\omega^2$ (2) $A\omega^2/2$ (3) $A\omega^2/\sqrt{2}$ (4) zero

A-7. A particle moves on the X-axis according to the equation $x = A + B \sin \omega t$. The motion is simple harmonic with amplitude
(1) A (2) B (3) A + B (4) $\sqrt{A^2 + B^2}$

Section (B) : Energy

B-1. A body executing SHM passes through its equilibrium. At this instant, it has
(1) maximum potential energy (2) maximum kinetic energy
(3) minimum kinetic energy (4) maximum acceleration

B-2. The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is
(1) $\sqrt{2}A$ (2) $\frac{A}{2}$ (3) $\frac{A}{\sqrt{2}}$ (4) $\sqrt{\frac{2}{3}}A$

B-3✍. A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3} J. The equation of motion of this particle when the initial phase of oscillation is 45° can be given by
(1) $0.1 \cos\left(4t + \frac{\pi}{4}\right)$ (2) $0.1 \sin\left(4t + \frac{\pi}{4}\right)$ (3) $0.4 \sin\left(t + \frac{\pi}{4}\right)$ (4) $0.2 \sin\left(\frac{\pi}{2} + 2t\right)$

B-4✍. The average energy in one time period in simple harmonic motion is
(1) $\frac{1}{2} m\omega^2 A^2$ (2) $\frac{1}{4} m\omega^2 A^2$ (3) $m\omega^2 A^2$ (4) zero

B-5. A particle executes simple harmonic motion with a frequency ν. The frequency with which the kinetic energy oscillates is
(1) ν/2 (2) ν (3) 2ν (4) zero

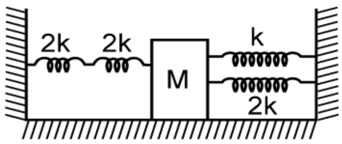
Section (C) : Spring Mass System

C-1. A force of 6.4 N stretches a vertical spring by 0.1 m. The mass that must be suspended from the spring so that it oscillates with a period of (π/4) sec is :
(1) (π/4)kg (2) 1 kg (3) (1/π) kg (4) 10 kg

C-2. A spring mass system oscillates with a frequency ν. If it is taken in an elevator slowly accelerating upward, the frequency will
(1) increase (2) decrease (3) remain same (4) become zero

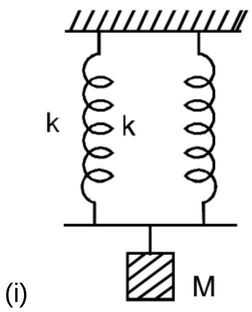
C-3. A spring-mass system oscillates in a car. If the car accelerates on a horizontal road, the frequency of oscillation will
(1) increase (2) decrease (3) remain same (4) become zero.

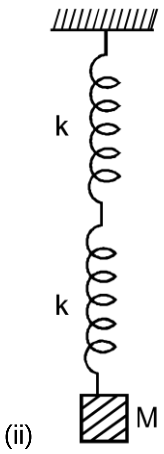
C-4. Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system.



(1) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$
(3) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

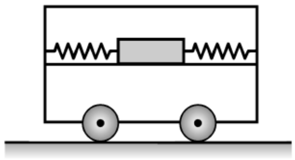
C-5. Two springs A and B having force constants k each are arranged (i) in parallel and (ii) in series. A mass M is attached to two arrangements separately. If time period in case (i) is T_1 and in case (ii) is T_2., then ratio $\frac{T_1}{T_2}$ is





(1) 1 (2) 2 (3) 0.5 (4) 0.25

C-6. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is :



(1) $T = 2\pi\sqrt{\frac{M\ \ m}{2k}}$ (2) $T = 2\pi\sqrt{\frac{(M+m)}{kmM}}$

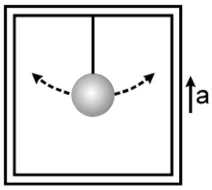
(3) $T = 2\pi\sqrt{\frac{m\ \ M}{2k(M+m)}}$ (4) $T = 2\pi\frac{(M+m)^2}{k}$

Section (D) : Simple Pendulum

D-1. A pendulum clock that keeps correct time on the earth is taken to the moon. It will run

(1) at correct rate (2) 6 times faster (3) $\sqrt{6}$ times faster (4) $\sqrt{6}$ times slower

D-2. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(1) $\frac{T}{4}$ (2) 4 T (3) $\frac{2}{\sqrt{5}}T$ (4) $\frac{\sqrt{5}}{2}T$

D-3. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitudes is decreased by 5%?

(1) 6 % (2) 3 % (3) 1.5 % (4) 0 %

D-4. A wall clock uses a vertical spring-mass system to measure the time. Each time the mass reaches an extreme position, the clock advances by a second. The clock gives correct time at the equator. If the clock is taken to the poles it will

(1) run slow (2) run fast

(3) stop working (4) give correct time.

D-5. A pendulum clock keeping correct time is taken to high altitudes,

(1) it will keep correct time

(2) its length should be increased to keep correct time

(3) its length should be decreased to keep correct time

(4) it cannot keep correct time even if the length is changed.

Section (E) : Compound Pendulum & Torsional Pendulum

E-1. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically ab ove the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then maintain the orientation. If the sphere is then released, its time period of the oscillation will be :

(1) π second (2) $\sqrt{2}\,\pi$ second (3) 2π second (4) 4π second

Section (F) : Superposition of SHM

F-1. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

(1) the resulting motion is uniform circular motion.

(2) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.

(3) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.

(4) the two S.H.M. will cancel each other.

F-2. The motion of a particle is given by $x = A \sin \omega t + B \cos \omega t$. The motion of the particle is

(1) not simple harmonic (2) simple harmonic with amplitude A + B

(3) simple harmonic with amplitude (A + B)/2 (4) simple harmonic with amplitude $\sqrt{A^2 + B^2}$

F-3. A simple harmonic motion is given by y = 5 (sin $3\pi t$ + $\sqrt{3}$ cos $3\pi t$). What is the amplitude of motion if y is in m ?
(1) 100 cm (2) 5 m (3) 200 cm (4) 1000 cm

Exercise-2

Marked Questions may have for Revision Questions.

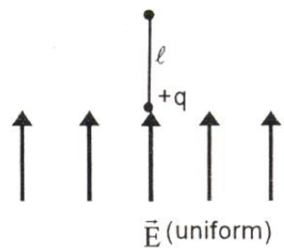
PART - I : OBJECTIVE QUESTION

1. A student says that he had applied a force F = $-k\sqrt{x}$ on a particle and the particle moved in simple harmonic motion. He refuses to tell whether k is a constant or not. Assume that he has worked only with positive x and no other force acted on the particle.
(1) As x increases k increases (2) As x increases k decreases
(3) As x increases k remains constant. (4) The motion cannot be simple harmonic.

2. Equation of SHM is x = 10 sin $10\pi t$. Find the distance between the two points where speed is 50π cm/sec. x is in cm and t is in seconds.
(1) 10 cm (2) 20 cm (3) 17.32 cm (4) 8.66 cm.

3. A particle moves on the X-axis according to the equation x = $x_0 \sin^2 \omega t$. The motion is simple harnomic
(1) with amplitude x_0 (2) with amplitude $2x_0$ (3) with time period $\frac{2\pi}{\omega}$ (4)with time period $\frac{\pi}{\omega}$

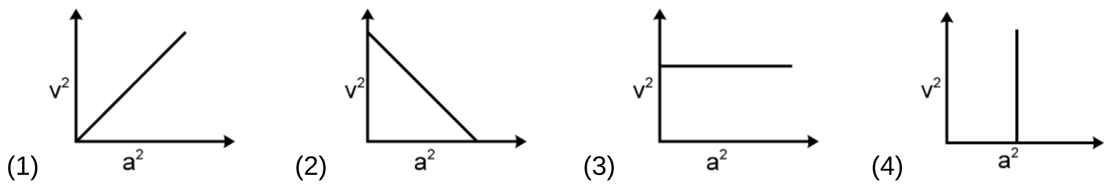
4. A simple pendulum with its bob (mass m) charged with +q oscillates in a uniform electric field E, as shown in the figure the period of oscillation shall be-



(1) $2\pi\left(\frac{\ell}{g}\right)^{1/2}$ (2) $2\pi\left(\frac{\ell}{g+qE/m}\right)^{1/2}$
(3) $2\pi\left(\frac{\ell}{g-qE/m}\right)^{1/2}$ (4) $2\pi\left(\frac{\ell}{gE/m}\frac{q}{}\right)^{1/2}$

5. Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement is half of their amplitude. The phase-difference between them is
(1) 0° (2) 120° (3) 180° (4) 135°

6. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity ν is



7. The total mechanical energy of a spring - mass system in simple harmonic motion is E = $\frac{1}{2}$m $\omega^2 A^2$. Suppose the oscillating particle is replaced by another particle of double the mass while the amplitude A remains the same. The new mechanical energy will

(1) become 2 E (2) become E/2 (3) become $\sqrt{2}$ E (4) remain E

8. In a simple harmonic motion

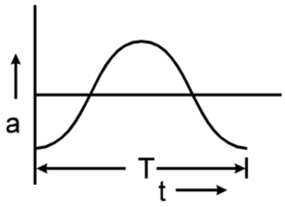
(1) the potential energy is always equal to the kinetic energy

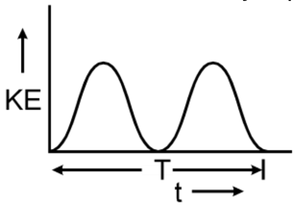
(2) the potential energy is never equal to the kinetic energy

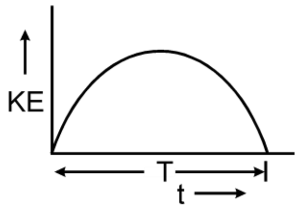
(3) the average potential energy in any time interval is equal to the average kinetic energy in that time interval

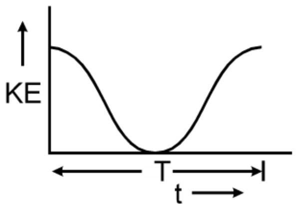
(4) the average potential energy in one time period is equal to the avrage kinetic energy in this period.

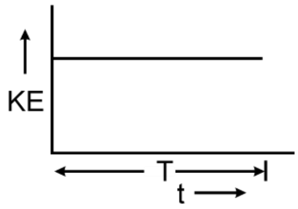
9. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time peirod. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(1)


(2)


(3)


(4)


10. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

(1) remain T (2) become 2T (3) become T/2 (4) become T/$\sqrt{2}$

11. Two bodies A and B of equal mass are suspended from two separate massless springs of spring cosntant k_1 and k_2 respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of A to that of B is

(1) k_1/k_2 (2) $\sqrt{k_1/k_2}$ (3) k_2/k_1 (4) $\sqrt{k_2/k_1}$

12. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is

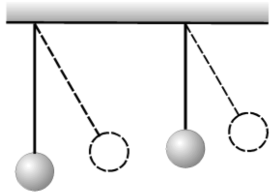
(1) $2\pi\sqrt{\frac{m\ (2k)}{k^2}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{m\ (2k)}{k^2}}$

(3) $2\pi\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{m}{k+k}}$

13. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half :

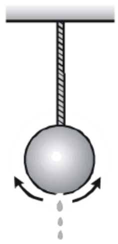
(1) 1 kg (2) 2 kg (3) 3 kg (4) 4 kg

14. Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) ($g = \pi^2$):



(1) 1 vibration (2) 3 vibrations
(3) 4 vibrations (4) 5 vibrations

15. A simple pendulum has a hollow bob filled with a liquid of density ρ. If the liquid drains out of a small hole in the bottom of the bob, then frequency of oscillations



(1) goes on increasing
(2) goes on decreasing
(3) remains same
(4) first decreases and then increases

16. The position vector of a particle from origin is given by $\vec{r} = A(\hat{i}\cos\omega t + \hat{j}\sin\omega t)$. The motion of the particle is
(1) simple harmonic (2) on a straight line
(3) on a circle (4) with constant acceleration

17. The displacement of a particle executing periodic motion is given by $y = 4\cos^2(0.5t)\sin(1000t)$. The given expression is composed by minimum :
(1) nil SHM (2) four SHMs (3) three SHMs (4) one SHM

PART - II : MISCELLANEOUS QUESTIONS

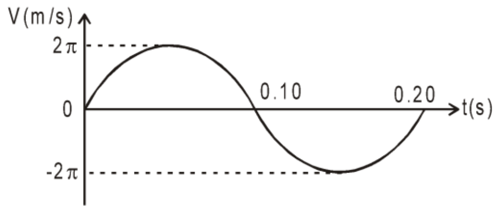
Section (A) : Assertion / Reasoning

A-1. **STATEMENT-1 :** Kinetic energy of SHM at mean position is equal to potential energy at ends for a particle moving in SHM.
STATEMENT-2 : Total energy in SHM is conserved.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

A-2. **STATEMENT-1 :** A particle is moving along x-axis. The resultant force F acting on it is given by $F = -ax - b$. Where a and b are both positive constants. The motion of this particle is not SHM.
STATEMENT-2 : In SHM resultant force must be proportional to the displacement from mean position.
(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

Section (B) : Match the column

B-1. A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown.



Correctly match the required information in the left column with the values given in the right column. (use $\pi^2 = 10$)

	Column-I	Column-II	
(1)	The mass of block in kg	(P)	– 0.20
(2)	The x-coordinate of block at t = 0 in metres	(Q)	– 200
(3)	The acceleration of block at t = 0.10 s in m/s^2	(R)	0.20
(4)	The block's maximum kinetic energy in Joule	(S)	4.0

Section (C) : One or More Than One Options Correct

C-1. Which of the following quantities are always non-positive in a simple harmonic motion along a straight line?

(1) $\vec{F}.\vec{a}$ (2) $\vec{v}.\vec{r}$ (3) $\vec{a}.\vec{r}$ (4) $\vec{F}.\vec{r}$

C-2. The quantities which are always Zero for a particle performing linear SHM :

(1) $\vec{a}\times\vec{F}$ (2) $\vec{r}\times\vec{v}$ (3) $\vec{r}\times\vec{a}$ (4) $\vec{r}\times\vec{F}$

C-3. The displacement (in m) of a particle of mass 100 g from its equilibrium position is given by the equation: $y = 0.05 \sin 3\pi\,(5t + 0.4)$

(1) the time period of motion is $\frac{1}{30}$ sec

(2) the time period of motion is $\frac{1}{7.5}$ sec

(3) the maximum acceleration of the particle is $11.25\pi^2\ m/s^2$

(4) the force acting on the particle is zero when the displacement is 0.05 m.

C-4. The displacement of a particle executing SHM is given by $x = 0.01 \sin 100\pi(t + 0.05)$. The time period is in seconds and amplitude of the particle is in meters

(1) Time period of the particle is 0.02 sec. (2) Amplitude of the particle is 0.01 m

(3) Time period of the particle is 0.01 sec. (4) Amplitude of the particle is 0.02 m

C-5. If a SHM is given by $y = (\sin \omega t + \cos \omega t)$ m, which of the following statements are true?

(1) The amplitude is 1m

(2) The amplitude is $(\sqrt{2})$ m

(3) Time is considered from y = 1 m

(4) Time is considered from y = 0 m

Exercise-3

✍ **Marked Questions may have for Revision Questions.**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. In a simple harmonic oscillator, at the mean position: **[AIEEE 2002; 3/225, –1]**

(1) kinetic energy is minimum, potential energy is maximum

(2) both kinetic and potential energies are maximum

(3) kinetic energy is maximum, potential energy is minimum
(4) both kinetic and potential energies are minimum

2. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes 5T/3. Then the ratio of m/M is : **[AIEEE 2002; 3/225, –1]**
(1) 3/5 (2) 25/9 (3) 16/9 (4) 5/3

3. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is :**[AIEEE 2003; 3/225, –1]**
(1) 11% (2) 21% (3) 42% (4) 10%

4. The displacement of a particle varies according to the relation x = 4 (cos πt + sin πt). The amplitude of the particle is : **[AIEEE 2003; 3/225, –1]**
(1) –4 (2) 4 (3) $4\sqrt{2}$ (4) 8

5. A body executes simple harmonic motion. The potential energy (P.E.), the kinetic energy (K.E.) and total energy (T.E.) are measured as function of displacement x. Which of the following statements is true? **[AIEEE 2003; 3/225, –1]**
(1) K.E. is maximum when x = 0 (2) T.E. is zero when x = 0
(2) K.E. is maximum when x is maximum (4) P.E. is maximum when x = 0

6. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m^3. What relationship between t and t_0 is true? **[AIEEE 2004; 3/225, –1]**
(1) $t = t_0$ (2) $t = t_0/2$ (3) $t = 2t_0$ (4) $t = 4t_0$

7. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then : **[AIEEE 2004; 3/225, –1]**
(1) $T = t_1 + t_2$ (2) $T^2 = t_1^2 + t_2^2$ (3) $T^{-1} = t_1^{-1} + t_2^{-1}$ (4) $T^{-2} = t_1^{-2} + t_2^{-2}$

8. The total energy of a particle, executing simple harmonic motion is : **[AIEEE 2004; 3/225, –1]**
(1) ∝ x (2) ∝ x^2 (3) independent of x (4) ∝ $x^{1/2}$
Where x is the displacement from the mean position.

9. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to : **[AIEEE 2004; 3/225, –1]**
(1) $\frac{m}{\omega_0^2 - \omega^2}$ (2) $\frac{1}{m(\omega_0^2 - \omega^2)}$ (3) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (4) $\frac{m}{\omega_0^2 + \omega^2}$

10. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then : **[AIEEE 2004; 3/225, –1]**
(1) $\omega_1 = \omega_2$
(2) $\omega_1 > \omega_2$
(3) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large
(4) $\omega_1 < \omega_2$

11. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2} + \alpha x = 0$, its time period is:**[AIEEE 2005; 3/225, –1]**
(1) $\frac{2\pi}{\alpha}$ (2) $\frac{2\pi}{\sqrt{\alpha}}$ (3) $2\pi\alpha$ (4) $2\pi\sqrt{\alpha}$

12. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would : **[AIEEE 2005; 3/225, –1]**
(1) first increase and then decrease to the original value
(2) first decrease and then increase to the original value
(3) remain unchanged
(4) increase towards a saturation value

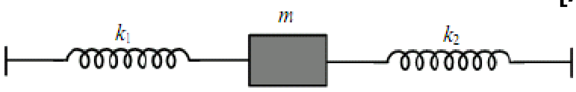
13. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is : **[AIEEE 2006; 3/165, –1]**
(1) 100 s (2) 0.01 s (3) 10 s (4) 0.1 s

14. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω.The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : **[AIEEE 2006; 3/165, –1]**
(1) at the highest position of the platform (2) at the mean position of the platform
(3) for an amplitude of $\frac{g}{\omega^2}$ (4) for an amplitude of $\frac{g^2}{\omega^2}$

15. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos\pi t$ metres. The time at which the maximum speed first occurs is :
[AIEEE 2007; 3/120, –1]
(1) 0.5 s (2) 0.75 s (3) 0.125 s (4) 0.25 s

16. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as , $a = A \cos(\omega t + \delta)$, then : **[AIEEE 2007; 3/120, –1]**
(1) $A = x_0$, $\delta = -\pi/4$ (2) $A = x_0\omega^2$, $\delta = -\pi/4$ (3) $A = x_0\omega^2$, $\delta = -\pi/4$ (4) $A = x_0\omega^2$, $\delta = 3\pi/4$

17. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes: **[AIEEE 2007; 3/120, –1]**



(1) $f/2$ (2) $f/4$ (3) 4f (4) $2f$

18. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end (for the one position) is :
[AIEEE 2007; 3/120, –1]
(1) $\pi^2 ma^2 \upsilon^2$ (2) $\frac{1}{4} ma^2 \upsilon^2$ (3) $4\pi^2 ma^2 \upsilon^2$ (4) $2\pi^2 ma^2\upsilon^2$

19.* If x,v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time?
[AIEEE 2007; 3/120, –1]
(1) $\frac{aT}{x}$ (2) $aT + 2\pi v$ (3) $\frac{aT}{v}$ (4) $a^2T^2 + 4\pi^2v^2$

20. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is : **[AIEEE - 2011, 4/120, –1]**
(1) $\frac{M}{M+m}$ (2) $\frac{M+m}{M}$ (3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M+m}{M}\right)^{1/2}$

21. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x - axis. Their mean position is separated by distance X_0 ($X_0 > A$). If the maximum separation between them is ($X_0 + A$), the phase difference between their motion is : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

22. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (2) $2\pi\sqrt{\frac{\ell \rho}{dg}}$ (3) $2\pi\sqrt{\frac{\ell d}{(\rho - d)g}}$ (4) $2\pi\sqrt{\frac{\ell \rho}{(\rho - d)g}}$

23. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the averatge life time of the pendulum is (assuming damping is small) in seconds :
[AIEEE 2012; 4/120, –1]

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

24. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals. **[JEE(Main)-2013; 4/120, –1]**

(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

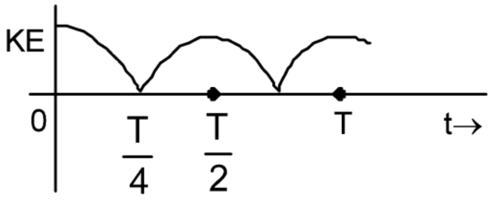
25. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE(Main)-2014; 4/120, –1]**

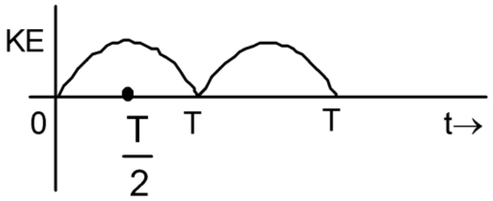
(1) amplitude of motion is 3a
(2) time period of oscillations is 8τ
(3) amplitude of motion is 4a
(4) time period of oscillations is 6τ

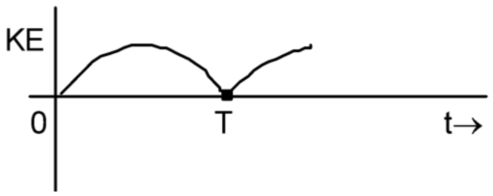
26. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at distance $\frac{2A}{3}$ from equilibrium position . The new amplitude of the motion is.
[JEE(Main)-2016; 4/120, –1]

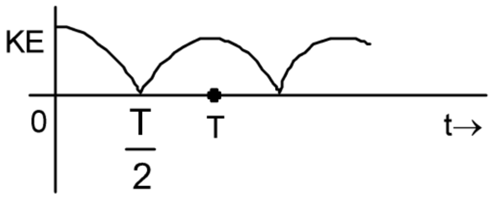
(1) 3A (2) A$\sqrt{3}$ (3) $\frac{7A}{3}$ (4) $\frac{A}{3}\sqrt{41}$

27. A particle is executing simple harmonic motion with a time period T. At time = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like : **[JEE(Main)-2017; 4/120, –1]**

(1) 

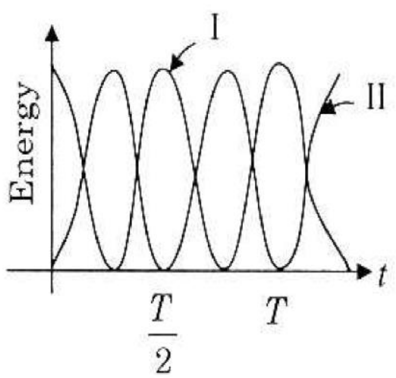
(2) 

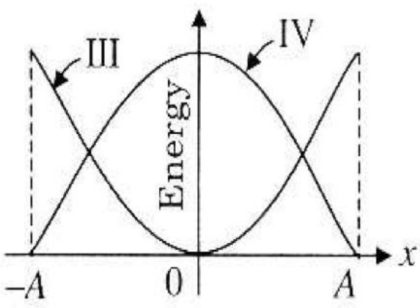
(3) 

(4) 

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. For a particle executing SHM the displacement x is given by x = A cos ωt. Identify the graphs which represents the variation of potential energy (P.E.) as a function of time t and displacement x :
[JEE (Scr.) 2003, 3/84,–1]





(A) I and III (B) II and IV (C) II and III (D) I and IV

2. A simple pendulum has time period T_1. The point of suspension is now moved upward according to the relation $y = kt^2$ ($k = 1\ m/s^2$) where y is the vertical displacement, the time period now becomes T_2. The ratio of $\left(\frac{T_1}{T_2}\right)^2$ is : ($g = 10\ m/s^2$) **[JEE (Scr.) 2005, 3/84,–1]**

(A) $\frac{5}{6}$ (B) $\frac{6}{5}$ (C) 1 (D) $\frac{4}{5}$

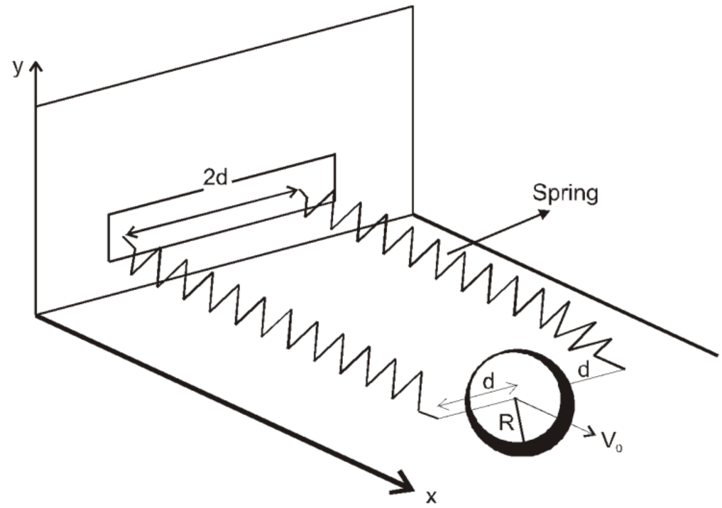
3*. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM **[JEE 2006, 5/184,–1]**

(A) for any value of A,B and C (except C = 0)
(B) If A = – B, C = 2B , amplitude = $|B\sqrt{2}|$
(C) If A = B ; C = 0
(D) If A = B ; C = 2B , amplitude = |B|

Comprehension # 1

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. **[JEE-2008, 3×4/163]**

Figure :



4. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) –kx (B) –2kx (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

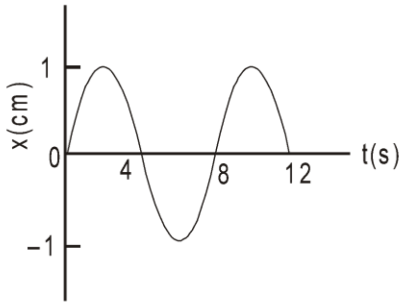
5. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

6. The maximum value of V_0 for which the disk will roll without slipping is

(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

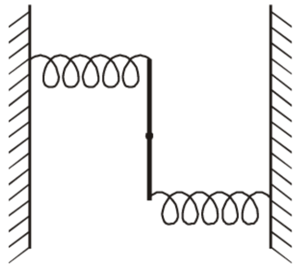
7. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is **[IIT-JEE 2009, 3/160, –1]**



(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (B) $\frac{-\pi^2}{32}$ cm/s^2

(C) $\frac{\pi^2}{32}$ cm/s^2 (D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

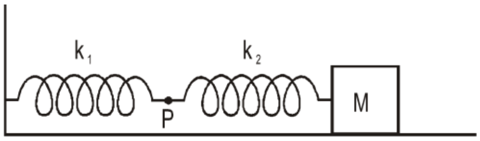
8. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is : **[IIT-JEE 2009, 3/160, –1]**



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

9. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[JEE 2009, 3/160, –1]**

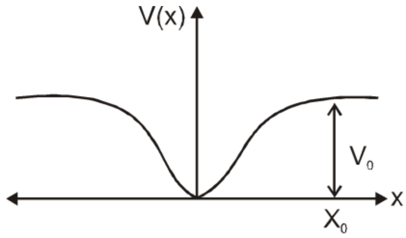


(A) $\frac{k_1A}{k_2}$ (B) $\frac{k_2A}{k_1}$

(C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

Comprehension : 2

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)**[JEE 2010; 3*3/160, –1]**



10. If the total energy of the particle is E, it will perform periodic motion only if :

(A) E < 0 (B) E > 0 (C) $V_0 > E > 0$ (D) $E > V_0$

11. For periodic motion of small amplitude A, the time period T of this particle is proportional to :

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

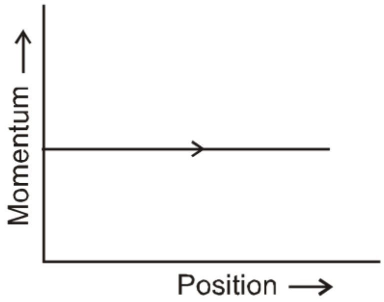
12. The acceleration of this particle for $|x| > X_0$ is :

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) zero

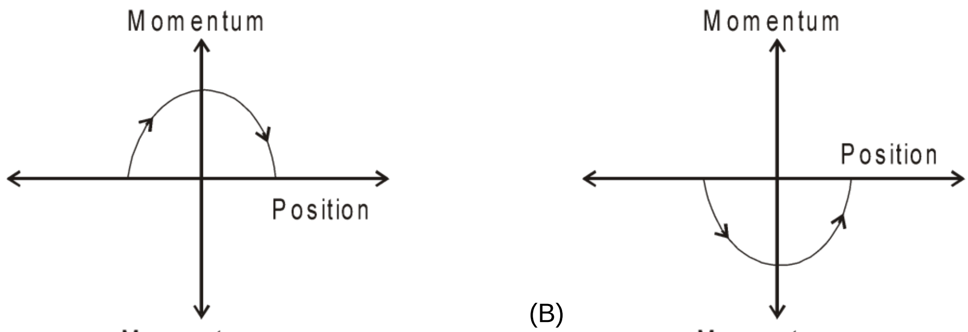
Paragraph for Question Nos. 13 to 15

Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum. upwards (or to right) is positive and downwards (or to left) is negative.

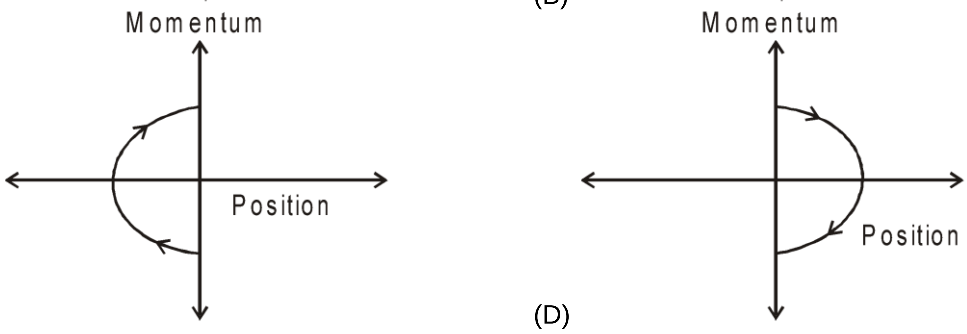


[JEE 2011, 3×3/160, –1]

13. The phase space diagram for a ball thrown vertically up from ground is : **[JEE 2011; 3/160, –1]**



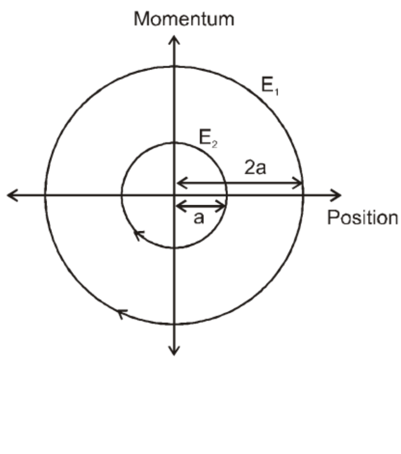
(A) (B)



(C) (D)

14. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

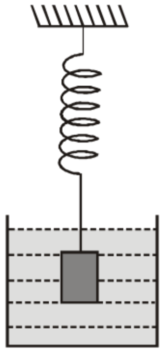
[JEE 2011; 3/160,–1]

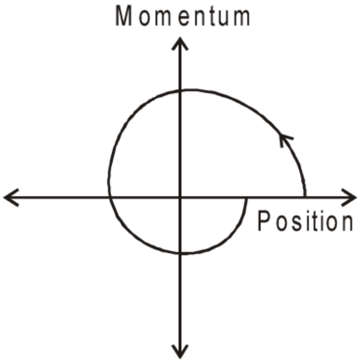


(A) $E_1 = \sqrt{2}\, E_2$ (B) $E_1 = 2E_2$

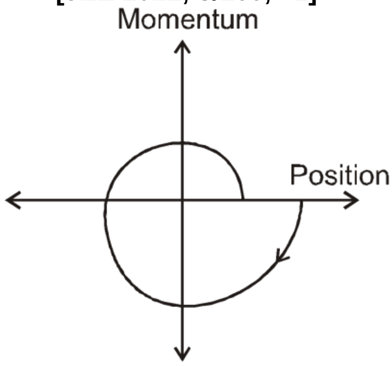
(C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

15. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is : **[JEE 2011; 3/160, –1]**

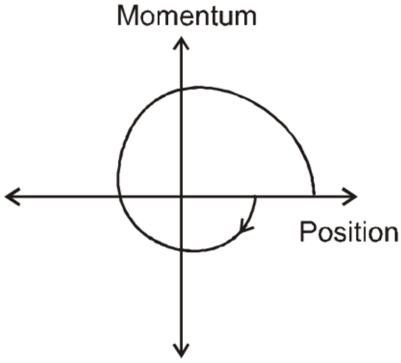




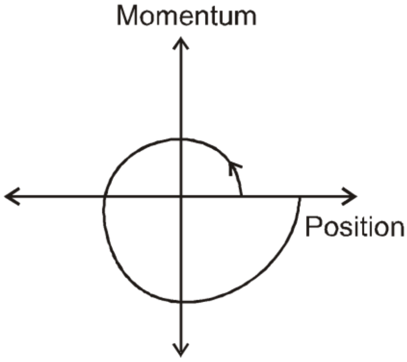
(A)



(B)



(C)

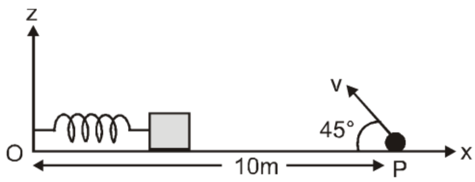


(D)

16. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \varphi)$ brings the mass to a complete rest. The values of B and φ are **[JEE 2011, 3/160, –1]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

17. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3} rad/s$. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45º as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1s, the value of v is (take $g = 10 \ m/s^2$):



[IIT-JEE-2012, Paper-1; 3/70, –1]

(A) $\sqrt{50} m/s$ (B) $\sqrt{51} \ m/s$

(C) $\sqrt{52} \ m/s$ (D) $\sqrt{53} \ m/s$

18*. A particle of mass m is attached to one end of a mass–less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is $0.5 \ u_0$, it collies elastically with a rigid wall. After this collision : **[JEE(Advanced) 2013; 4/60, –1]**

(A) the speed of the particle when it returns to its equilibrium position is u_0 .

(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes througout the equilibrium position for the second time is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

19. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0 ; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (< M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M ? **[JEE(Advanced) P-1, 2016; 4/60, –1]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases

Answers

EXERCISE # 1

Section (A)

A-1.	(1)	**A-2.**	(4)	**A-3.**	(3)
A-4.	(2)	**A-5.**	(1)	**A-6.**	(4)
A-7.	(2)				

Section (B)

B-1.	(2)	**B-2.**	(3)	**B-3.**	(2)
B-4.	(1)	**B-5.**	(3)		

Section (C)

C-1.	(2)	**C-2.**	(3)	**C-3.**	(3)
C-4.	(2)	**C-5.**	(3)	**C-6.**	(3)

Section (D)

D-1.	(4)	**D-2.**	(3)	**D-3.**	(4)
D-4.	(4)	**D-5.**	(3)		

Section (E)

E-1. (4)

Section (F)

F-1.	(2)	**F-2.**	(4)	**F-3.**	(4)

EXERCISE # 2

PART - I

1.	(1)	**2.**	(3)	**3.**	(4)
4.	(3)	**5.**	(2)	**6.**	(2)
7.	(4)	**8.**	(4)	**9.**	(1)
10.	(4)	**11.**	(4)	**12.**	(3)
13.	(3)	**14.**	(4)	**15.**	(4)
16.	(3)	**17.**	(3)		

PART - II

Section (A)

A-1. (4) **A-2.** (4)

Section (B)

B-1. (1 → R); (2 → P); (3 → Q); (4 → S)

Section (C)

C-1.	(3,4)	**C-2.**	(1,2,3,4)	**C-3.**	(2,3)
C-4.	(1,2)	**C-5.**	(2,3)		

EXERCISE # 3

PART - I

1.	(3)	**2.**	(3)	**3.**	(4)
4.	(3)	**5.**	(1)	**6.**	(3)
7.	(2)	**8.**	(3)	**9.**	(2)
10.	(1)	**11.**	(2)	**12.**	(1)
13.	(2)	**14.**	(3)	**15.**	(1)
16.	(4)	**17.**	(4)	**18.**	(1)
19.	(1,4)	**20.**	(4)	**21.**	(2)
22.	(1)	**23.**	(4)	**24.**	(3)
25.	(4)	**26.**	(3)	**27.**	(1)

PART - II

1.	(A)	**2.**	(B)	**3.**	(A,B,D)
4.	(D)	**5.**	(D)	**6.**	(C)
7.	(D)	**8.**	(C)	**9.**	(D)
10.	(C)	**11.**	(B)	**12.**	(D)
13.	(D)	**14.**	(C)	**15.**	(B)
16.	(B)	**17.**	(A)	**18.**	(A,D)
19.	(A,B,D)				