Exercise-1

✍ Marked Questions can be used as Revision Questions.

*** Marked Questions may have more than one correct option.**

PART - I : FUNCTION & DIFFERENTIATION

Section (A) : Trigonometry and Function

A-1. f(x) = cos x + sin x. Find f(π/2)

A-2. If f(x) = 4x + 3. Find f(f(2))

Objective Questions

A-3. $f(x) = \log x^3$ and g (x) = log x. Which of the following statement is / are true -
(A) f(x) = g(x) (B) 3f(x) = g(x) (C) f(x) = 3g(x) (D) $f(x) = (g(x))^3$

A-4. tan15° is equilvalent to :
(A) $(2-\sqrt{3})$ (B) $(5+\sqrt{3})$ (C) $\left(\frac{5-\sqrt{3}}{2}\right)$ (D) $\left(\frac{5+\sqrt{3}}{2}\right)$

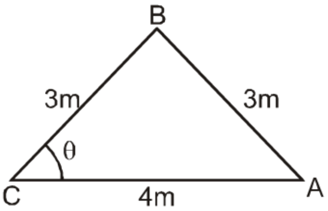
A-5. $\sin^2\theta$ is equivalent to :
(A) $\left(\frac{1+\cos\theta}{2}\right)$ (B) $\left(\frac{1+\cos 2\theta}{2}\right)$ (C) $\left(\frac{1-\cos 2\theta}{2}\right)$ (D) $\left(\frac{\cos 2\theta-1}{2}\right)$

A-6. sinA . sin(A + B) is equal to
(A) $\cos^2A \cdot \cos B + \sin A \sin^2 B$
(B) $\sin^2A \cdot \frac{1}{2}\cos B + \cos 2A.\sin B$
(C) $\sin^2A .\cos B + \frac{1}{2}\sin 2A \cdot \sin B$
(D) $\sin^2A \cdot \sin B + \cos A \cos^2 B$

A-7*.✍ $-\sin\theta$ is equivelent to :
(A) $\cos\left(\frac{\pi}{2}+\theta\right)$ (B) $\cos\left(\frac{\pi}{2}-\theta\right)$ (C) $\sin(\theta-\pi)$ (D) $\sin(\pi+\theta)$

A-8*.✍ If $x_1 = 8\sin\theta$ and $x_2 = 6\cos\theta$ then
(A) $(x_1 + x_2)_{max} = 10$
(B) $x_1 + x_2 = 10\sin(\theta + 37°)$
(C) $x_1x_2 = 24\sin 2\theta$
(D) $\frac{x_1}{x_2} = \frac{4}{3}\tan\theta$

A-9*.✍ θ is angle between side CA and CB of triangle, shown in the figure then θ is given by :
(A) $\cos\theta = \frac{2}{3}$
(B) $\sin\theta = \frac{\sqrt{5}}{3}$
(C) $\tan\theta = \frac{\sqrt{5}}{2}$
(D) $\tan\theta = \frac{2}{3}$



Section (B) : Differentiation of Elementry Functions

Find the derivative of given functions w.r.t. corresponding independent variable.

B-1. $y = x^2 + x + 8$

B-2. y = tan x + cot x

Find the first derivative & second derivative of given functions w.r.t. corresponding independent variable.

B-3. y = sin x + cos x

B-4. $y = \ell nx + e^x$

Section (C) : Differentiation by Product rule

Find derivative of given functions w.r.t. the independent variable x.

C-1. $y = e^x \ell nx$

C-2. $y = \sin x \cos x$

Section (D) : Differentiation by Quotient rule

Find derivative of given functions w.r.t. the independent variable.

D-1. $y = \dfrac{2x+5}{3x-2}$

D-2. $y = \dfrac{\ell nx}{x}$

D-3. $y = (\sec x + \tan x)(\sec x - \tan x)$

D-4. Suppose u and v are functions of x that are differentiable at x = 0 and that
$u(0) = 5$, $u'(0) = -3$, $v(0) = -1$, $v'(0) = 2$
Find the values of the following derivatives at x = 0.

(a) $\dfrac{d}{dx}(uv)$ (b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$ (c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$ (d) $\dfrac{d}{dx}(7v - 2u)$

Section (E) : Differentiation by Chain rule

Find $\dfrac{dy}{dx}$ as a function of x

E-1. $y = \sin 5x$

E-2. $y = 2\sin(\omega x + \varphi)$ where ω and φ constants

E-3. $y = (4 - 3x)^9$

Section (F) : Differentiation of Implicit functions

Find $\dfrac{dy}{dx}$

F-1. $(x + y)^2 = 4$

F-2. $x^2y + xy^2 = 6$

Section (G) : Differentiation as a rate measurement

G-1. Suppose that the radius r and area $A = \pi r^2$ of a circle are differentiable functions of t.Write an equation that relates $\dfrac{dA}{dt}$ to $\dfrac{dr}{dt}$.

G-2. Suppose that the radius r and surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t. Write an equation that relates $\dfrac{ds}{dt}$ to $\dfrac{dr}{dt}$.

Section (H) : Maxima & Minima

H-1. Particle's position as a function of time is given by $x = -t^2 + 4t + 4$ find the maximum value of position co-ordinate of particle.

H-2. Find the values of function $2x^3 - 15x^2 + 36x + 11$ at the points of maximum and minimum

Section (I) : Miscellaneous

Given y = f(u) and u = g(x) , find $\dfrac{dy}{dx}$

I-1. $y = 2u^3$, $u = 8x - 1$

I-2. $y = \sin u$, $u = 3x + 1$

I-3. $y = 6u - 9$, $u = (1/2)x^4$

I-4. $y = \cos u$, $u = -\dfrac{x}{3}$

PART - II : INTEGRATION

Section - (A) : Integration of elementry functions

Find integrals of given functions

A-1. $x^2 - 2x + 1$

A-2. $\sqrt{x} + \frac{1}{\sqrt{x}}$

A-3. $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$

A-4. $\sec^2 x$

A-5. $\csc^2 x$

A-6. $\sec x \tan x$

A-7. $\frac{1}{3x}$

Section (B) : Integration by substitution method

Integrate by using the substitution suggested in bracket.

B-1. $\int x \sin(2x^2)\,dx$, (use, $u = 2x^2$)

B-2. $\int \sec 2t \tan 2t\,dt$, (use, $u = 2t$)

Integrate by using a suitable substitution

B-3. $\int \frac{3}{(2-x)^2}\,dx$

B-4. $\int \sin(8z-5)\,dz$

Section (C) : Definite integration

C-1. $\int_{-4}^{-1} \frac{\pi}{2}\,d\theta$

C-2. $\int_{\sqrt{2}}^{5\sqrt{2}} r\,dr$

C-3. $\int_0^1 e^x\,dx$

Section (D) : Calculation of area

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0, b]

D-1. $y = 2x$

D-2. $y = \frac{x}{2} + 1$

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval $[0, \pi]$

D-3. $y = \sin x$

D-4. $y = \sin^2 x$

Objective Questions

D-5. $I = \int_0^{2\pi} \sin(\theta + \phi).d\theta$ where φ is a constant. Then value of I :

(A) may be positive
(B) may be negative
(C) may be zero
(D) Always zero for any value of φ

D-6*. If $x_1 = 3\sin\omega t$ and $x_2 = 4\cos\omega t$ then

(A) $\frac{x_1}{x_2}$ is independent of t
(B) Average value of $\langle x_1^2 + x_2^2 \rangle$ from t = 0 to $t = \frac{2\pi}{\omega}$ is 12.5
(C) $\left(\frac{x_1}{3}\right)^2 + \left(\frac{x_2}{4}\right)^2 = 1$
(D) Average value of $\langle x_1.x_2 \rangle$ from t = 0 to $t = \frac{2\pi}{\omega}$ is zero

D-7*. $I = \int_0^{\pi} \sin(\theta + \phi)d\theta$, where φ is non zero constant then the value of I :

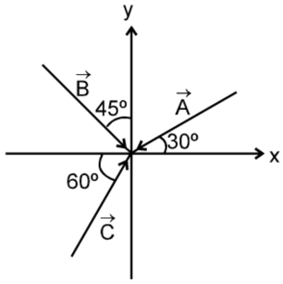
(A) may be positive
(B) may be negative

(C) may be zero (D) always zero if $\varphi = \frac{\pi}{4}$

PART - III : VECTOR

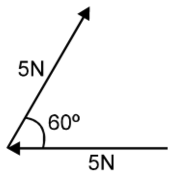
Section (A) : Definition of vector & angle between vectors

A-1. Vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ are shown in figure. Find angle between



(i) $\vec{A}$ and $\vec{B}$, (ii) $\vec{A}$ and $\vec{C}$, (iii) $\vec{B}$ and $\vec{C}$.

A-2. The forces, each numerically equal to 5 N, are acting as shown in the Figure. Find the angle between forces?



A-3. Rain is falling vertically downwards with a speed 5 m/s. If unit vector along upward is defined as $\hat{j}$, represent velocity of rain in vector form.

Section (B) : Addition of Vectors

B-1. A man walks 40 m North, then 30 m East and then 40 m South. Find the displacement from the starting point?

B-2. A vector of magnitude 30 and direction eastwards is added with another vector of magnitude 40 and direction Northwards. Find the magnitude and direction of resultant with the east.

B-3. Two vectors $\vec{a}$ and $\vec{b}$ inclined at an angle θ w.r.t. each other have a resultant $\vec{c}$ which makes an angle β with $\vec{a}$. If the directions of $\vec{a}$ and $\vec{b}$ are interchanged, then the resultant will have the same
(A) magnitude (B) direction
(C) magnitude as well as direction (D) neither magnitude nor direction

B-4. Two vectors $\vec{A}$ and $\vec{B}$ lie in a plane. Another vector $\vec{C}$ lies outside this plane (this plane is not parallel to the plane containing $\vec{A}$ and $\vec{B}$). The resultant $\vec{A}+\vec{B}+\vec{C}$ of these three vectors
(A) can be zero (B) cannot be zero
(C) lies in the plane of $\vec{A}$ & $\vec{B}$ (D) lies in the plane of $\vec{A}$ & $\vec{A}+\vec{B}$

B-5. The vector sum of the forces of 10 N and 6 N can be
(A) 2 N (B) 8 N (C) 18 N (D) 20 N.

B-6. A set of vectors taken in a given order gives a closed polygon. Then the resultant of these vectors is a
(A) scalar quantity (B) pseudo vector (C) unit vector (D) null vector.

B-7. The vector sum of two force P and Q is minimum when the angle θ between their positive directions, is
(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) π.

B-8. The vector sum of two vectors $\vec{A}$ and $\vec{B}$ is maximum, then the angle θ between two vectors is -

(A) 0º (B) 30° (C) 45° (D) 60°

B-9. Given : $\vec{C} = \vec{A} + \vec{B}$. Also, the magnitude of $\vec{A}$, $\vec{B}$ and $\vec{C}$ are 12, 5 and 13 units respectively. The angle between $\vec{A}$ and $\vec{B}$ is

(A) 0º (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π.

B-10. If $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$ and θ is the angle between $\vec{P}$ and $\vec{Q}$, then
(A) θ = 0º (B) θ = 90º (C) P = 0 (D) Q = 0

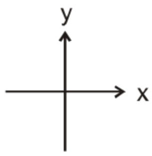
B-11. The sum and difference of two perpendicular vectors of equal lengths are
(A) of equal lengths and have an acute angle between them
(B) of equal length and have an obtuse angle between them
(C) also perpendicular to each other and are of different lengths
(D) also perpendicular to each other and are of equal lengths.

Section (C) : Resolution of Vectors

C-1. Find the magnitude of $3\hat{i} + 2\hat{j} + \hat{k}$?

C-2. If $\vec{A} = 3\hat{i} + 4\hat{j}$ then find $\hat{A}$

C-3. What are the x and the y components of a 25 m displacement at an angle of 210º with the x-axis (anti clockwise)?



C-4. One of the rectangular components of a velocity of 60 km h^{-1} is 30 km h^{-1}. Find other rectangular component?

C-5. If $0.5\hat{i} + 0.8\hat{j} + C\hat{k}$ is a unit vector. Find the value of C

C-6. The rectangular components of a vector are (2, 2). The corresponding rectangular components of another vector are (1, $\sqrt{3}$). Find the angle between the two vectors

C-7. The x and y components of a force are 2 N and – 3 N. The force is

(A) $2\hat{i} - 3\hat{j}$ (B) $2\hat{i} + 3\hat{j}$ (C) $-2\hat{i} - 3\hat{j}$ (D) $3\hat{i} + 2\hat{j}$

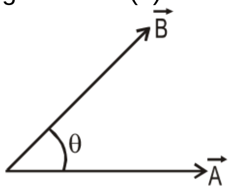
C-8. The vector joining the points A (1, 1, –1) and B (2, –3, 4) and pointing from A to B is -

(A) $-\hat{i} + 4\hat{j} - 5\hat{k}$ (B) $\hat{i} + 4\hat{j} + 5\hat{k}$ (C) $\hat{i} - 4\hat{j} + 5\hat{k}$ (D) $-\hat{i} - 4\hat{j} - 5\hat{k}$.

Section (D) : Products of Vectors

D-1. If $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = 2\hat{i} + \hat{j}$ find (a) $\vec{A} \cdot \vec{B}$ (b) $\vec{A} \times \vec{B}$

D-2. If $|\vec{A}| = 4$, $|\vec{B}| = 3$ and θ = 60° in the figure. Find (a) $\vec{A} \cdot \vec{B}$ (b) $|\vec{A} \times \vec{B}|$



D-3. Three non zero vectors $\vec{A}$, $\vec{B}$ & $\vec{C}$ satisfy the relation $\vec{A} \cdot \vec{B} = 0$ & $\vec{A} \cdot \vec{C} = 0$. Then $\vec{A}$ can be parallel to

(A) $\vec{B}$ (B) $\vec{C}$ (C) $\vec{B} \cdot \vec{C}$ (D) $\vec{B} \times \vec{C}$

D-4.*✍ The magnitude of scalar product of two vectors is 8 and that of vector product is $8\sqrt{3}$. The angle between them is :
(A) 30º (B) 60º (C) 120º (D) 150º

Exercise-2

✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

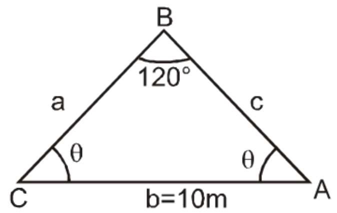
PART - I : FUNCTION & DIFFERENTIATION

1.✍ If $f(x) = \frac{x-1}{x+1}$ then find f {f(x)}

2. $y = f(x) = \frac{2x-3}{3x-2}$. Find f(y)

Objective Questions

3.✍ For a triangle shown in the figure, side CA is 10 m, angle ∠ A and angle ∠ C are equal then :



(A) side a = side c = 10m
(B) side a ≠ side c
(C) side a = side c = $\frac{10\sqrt{3}}{3}$m
(D) side a = side c = $\frac{10}{\sqrt{2}}$m

4*. If $y_1 = A\sin\theta_1$ and $y_2 = A\sin\theta_2$ then
(A) $y_1 + y_2 = 2A\sin\left(\frac{\theta_1+\theta_2}{2}\right)\cos\left(\frac{\theta_1-\theta_2}{2}\right)$
(B) $y_1 + y_2 = 2A\sin\theta_1 \sin\theta_2$
(C) $y_1 - y_2 = 2A\sin\left(\frac{\theta_1-\theta_2}{2}\right)\cos\left(\frac{\theta_1+\theta_2}{2}\right)$
(D) $y_1 . y_2 = -2A^2 \cos\left(\frac{\pi}{2}+\theta_1\right).\cos\left(\frac{\pi}{2}-\theta_2\right)$

5*. Which of following are true
(A) sin37° + cos37° = sin53° + cos53°
(B) sin37° – cos37° = cos53° – sin53°
(C) tan37° + 1 = tan 53° – 1
(D) tan37° × tan53° = 1

6*.✍ If $R^2 = A^2 + B^2 + 2AB\cos\theta$, if |A| = |B| then value of magntitude of R is equivalent to :
(A) $2A\cos\theta$ (B) $A\cos\frac{\theta}{2}$ (C) $2A\cos\frac{\theta}{2}$ (D) $2B\cos\frac{\theta}{2}$

Find the first derivative and second derivative of given functions w.r.t. the independent variable x.

7.✍ $y = \ell nx^2 + \sin x$

8. $y = \sqrt[7]{x} + \tan x$

Find derivative of given functions w.r.t. the corresponding independent variable.

9. $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$

10.✍ $r = (1 + \sec\theta)\sin\theta$

Find derivative of given functions w.r.t. the respective independent variable .

11. $y = \frac{\cot x}{1 + \cot x}$

12.✍ $\frac{\ell nx + e^x}{\tan x}$

Find $\frac{dy}{dx}$ as a function of x

13. $y = \sin^3 x + \sin 3x$

14. $\sin^2 (x^2 + 1)$

15.✍ $q = \sqrt{2r - r^2}$, find $\frac{dq}{dr}$

Find $\frac{dy}{dx}$

16.✍ $x^3 + y^3 = 18\,xy$

17. The radius r and height h of a circular cylinder are related to the cylinder's volume V by the formula $V = \pi r^2 h$.

(a) If height is increasing at a rate of 5 m/s while radius is constant, Find rate of increase of volume of cylinder.

(b) If radius is increasing at a rate of 5 m/s while height is constant, Find rate of increase of volume of cylinder.

(c) If height is increasing at a rate of 5 m/s and radius is increasing at a rate of 5 m/s, Find rate of increase of volume of cylinder.

18.✍ Find two positive numbers x & y such that x + y = 60 and xy is maximum.

19. A sheet of area 40 m^2 in used to make an open tank with a square base, then find the dimensions of the base such that volume of this tank is maximum.

PART - II : INTEGRATION

Find integrals of given functions.

1. $\int x^{-3}(x+1)\,dx$

2. $\int (1 - \cot^2 x)\,dx$

3.✍ $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta$

Integrate by using the substitution suggested in bracket

4. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\,dy$, (use , $u = y^4 + 4y^2 + 1$)

5. $\int \frac{dx}{\sqrt{5x+8}}$

(a) Using $u = 5x + 8$ (b) Using $u = \sqrt{5x+8}$

Integrate by using suitable substitution.

6.✍ $\int \sqrt{3-2s}\,ds$

7.✍ $\int \sec^2(3x+2)\,dx$

8. $\int \csc\left(\frac{\upsilon - \pi}{2}\right)\cot\left(\frac{\upsilon - \pi}{2}\right)d\upsilon$

9. $\int \frac{6\cos t}{(2+\sin t)^3}\,dt$

10. $\int_{\pi}^{2\pi} \theta\,d\theta$

11.✍ $\int_{0}^{\sqrt[3]{7}} x^2\,dx$

12. $\int_0^{\sqrt{\pi}} x \sin x^2 dx$ **13.** $\int_0^1 \frac{dx}{3x+2}$

Use a definite integral to find the area of the region between the given curve and the x–axis on the interval [0,b],

14. $y = 3x^2$

PART - III : VECTOR

SUBJECTIVE QUESTIONS

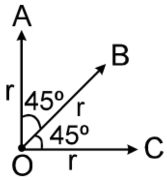
1. Vector $\vec{A}$ points N – E and its magnitude is 3 kg ms^{-1} it is multiplied by the scalar λ such that λ = –4 second. Find the direction and magnitude of the new vector quantity. Does it represent the same physical quantity or not ?

2. A force of 30 N is inclined at an angle θ to the horizontal. If its vertical component is 18 N, find the horizontal component & the value of θ.

3. Two vectors acting in the opposite directions have a resultant of 10 units. If they act at right angles to each other, then the resultant is 50 units. Calculate the magnitude of two vectors.

4. The angle θ between directions of forces $\vec{A}$ and $\vec{B}$ is 90º where A = 8 dyne and B = 6 dyne. If the resultant $\vec{R}$ makes an angle α with $\vec{A}$ then find the value of 'α' ?

5. Find the resultant of the three vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ each of magnitude r as shown in figure?



6. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = \hat{i} + \hat{j} + 2\hat{k}$ then find out unit vector along $\vec{A} + \vec{B}$

7. The x and y components of vector $\vec{A}$ are 4m and 6m respectively. The x, y components of vector $\vec{A} + \vec{B}$ are 10m and 9m respectively. Find the length of $\vec{B}$ and angle that $\vec{B}$ makes with the x axis.

OBJECTIVE QUESTIONS

Single choice type

8. A vector is not changed if

(A) it is displaced parallel to itself
(B) it is rotated through an arbitrary angle
(C) it is cross-multiplied by a unit vector
(D) it is multiplied by an arbitrary scalar.

9. If the angle between two forces increases, the magnitude of their resultant

(A) decreases
(B) increases
(C) remains unchanged
(D) first decreases and then increases

10. A car is moving on a straight road due north with a uniform speed of 50 km h^{-1} when it turns left through 90º. If the speed remains unchanged after turning, the change in the velocity of the car in the turning process is

(A) zero
(B) $50\sqrt{2}$ km h^{-1} S-W direction
(C) $50\sqrt{2}$ km h^{-1} N-W direction
(D) 50 km h^{-1} due west.

11. Which of the following sets of displacements might be capable of bringing a car to its returning point?

(A) 5, 10, 30 and 50 km
(B) 5, 9, 9 and 16 km

(C) 40, 40, 90 and 200 km (D) 10, 20, 40 and 90 km

12. When two vector $\vec{a}$ and $\vec{b}$ are added, the magnitude of the resultant vector is always
(A) greater than (a + b) (B) less than or equal to (a + b)
(C) less than (a + b) (D) equal to (a + b)

13. If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$, then the angle between $\vec{A}$ and $\vec{B}$ is
(A) 0º (B) 60º (C) 90º (D) 120º.

14. Given : $\vec{a} + \vec{b} + \vec{c} = 0$. Out of the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ two are equal in magnitude. The magnitude of the third vector is $\sqrt{2}$ times that of either of the two having equal magnitude. The angles between the vectors are:
(A) 90º, 135º, 135º (B) 30º, 60º, 90º (C) 45º, 45º, 90º (D) 45º, 60º, 90º

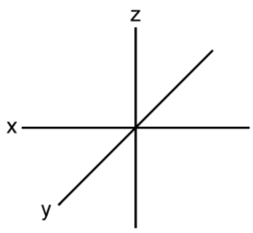
15. Vector $\vec{A}$ is of length 2 cm and is 60º above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60º below the x-axis in the fourth quadrant. The sum $\vec{A} + \vec{B}$ is a vector of magnitude -
(A) 2 along + y-axis (B) 2 along + x-axis (C) 1 along – x axis (D) 2 along – x axis

16. Six forces, 9.81 N each, acting at a point are coplanar. If the angles between neighboring forces are equal, then the resultant is
(A) 0 N (B) 9.81 N (C) 2 × 9.81 N (D) 3 × 9.81 N.

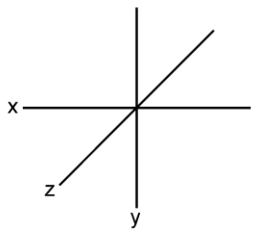
17. A vector $\vec{A}$ points vertically downward & $\vec{B}$ points towards east, then the vector product $\vec{A} \times \vec{B}$ is
(A) along west (B) along east (C) zero (D) along south

More than one choice type

18. Which of the arrangement of axes in Fig. can be labelled "right-handed coordinate system" ? As usual, each axis label indicates the positive side of the axis.



(i)



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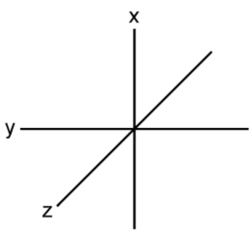
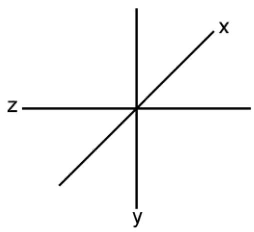


(iii)



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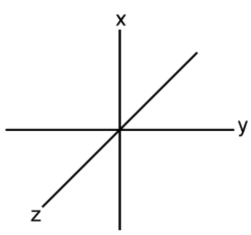

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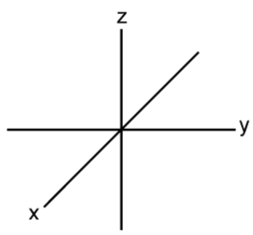


(vi)

(A) (i), (ii) (B) (iii), (iv) (C) (vi) (D) (v)

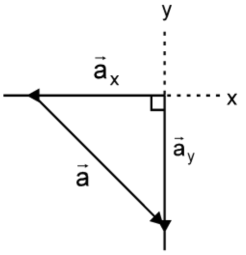
19. Which of the following is a true statement?
(A) A vector cannot be divided by another vector

(B) Angular displacement can either be a scalar or a vector.

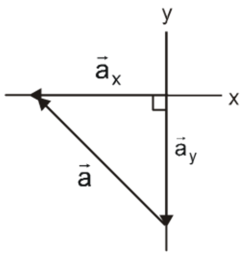
(C) Since addition of vectors is commutative therefore vector subtraction is also commutative.

(D) The resultant of two equal forces of magnitude F acting at a point is F if the angle between the two forces is 120º.

20. In the Figure which of the ways indicated for combining the x and y components of vector a are proper to determine that vector?



(i)



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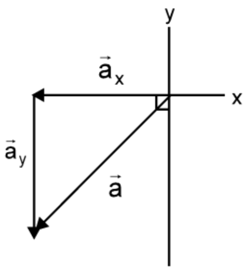

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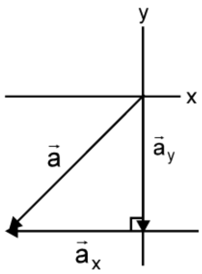

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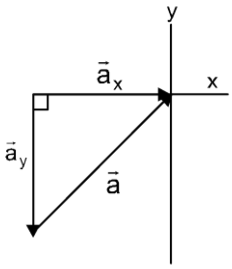
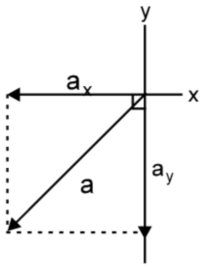


(v)



(vi)

(A) (iii) (B) (iv) (C) (vi) (D) (i), (ii) and (v).

21. Let $\vec{a}$ and $\vec{b}$ be two non-null vectors such that $|\vec{a}+\vec{b}| = |\vec{a}-2\vec{b}|$. Then the value of $\frac{|\vec{a}|}{|\vec{b}|}$ may be :

(A) $\frac{1}{4}$ (B) $\frac{1}{8}$ (C) 1 (D) 2

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

PART - I : MATCH THE COLUMN

1.✍ Match the integrals (given in column-II) with the given functions (in column - I)

Column - I	**Column - II**
(A) $\int \sec x \tan x dx$	(p) $-\frac{\operatorname{cosec} Kx}{K} + C$
(B) $\int \operatorname{cosec} Kx \cot Kx \, dx$	(q) $-\frac{\cot Kx}{K} + C$

(C) $\int \operatorname{cosec}^2 Kx\,dx$ (r) $\sec x + C$

(D) $\int \cos Kx\,dx$ (s) $\frac{\sin Kx}{K} + C$

2. Match the statements given in column-I with statements given in column - **II**

Column - I	**Column - II**
(A) if $|\vec{A}| = |\vec{B}|$ and $|\vec{A} + \vec{B}| = |\vec{A}|$ then angle between $\vec{A}$ and $\vec{B}$ is	(p) 90°
(B) Magnitude of resultant of two forces $|\vec{F_1}|$ = 8N and $|\vec{F_2}|$ = 4 N may be	(q) 120°
(C) Angle between $\vec{A} = 2\hat{i} + 2\hat{j}$ & $\vec{B} = 3\hat{k}$ is	(r) 12 N
(D) Magnitude of resultant of vectors $\vec{A} = 2\hat{i} + \hat{j}$ & $\vec{B} = 3\hat{k}$ is	(s) $\sqrt{14}$

PART - II : COMPREHENSION

Comprehension-1

A particle is moving along positive x-axis. Its position varies as $x = t^3 - 3t^2 + 12t + 20$, where x is in meters and t is in seconds.

1. Initial velocity of the particle is.

(A) 1 m/s (B) 3 m/s (C) 12 m/s (D) 20 m/s

2. Initial acceleration of the particle is

(A) Zero (B) 1 m/s^2 (C) – 3m/s^2 (D) – 6 m/s^2

3. Velocity of the particle when its acceleration zero is

(A) 1 m/s (B) 3 m/s (C) 6 m/s (D) 9 m/s

Comprehension-2

Two forces $\vec{F_1} = 2\hat{i} + 2\hat{j}$ N and $\vec{F_2} = 3\hat{j} + 4\hat{k}$ N are acting on a particle.

4. The resultant force acting on particle is :

(A) $2\hat{i} + 5\hat{j} + 4\hat{k}$ (B) $2\hat{i} - 5\hat{j} - 4\hat{k}$ (C) $\hat{i} - 3\hat{j} - 2\hat{k}$ (D) $\hat{i} - \hat{j} - \hat{k}$

5. The angle between $\vec{F_1}$ & $\vec{F_2}$ is :

(A) $\theta = \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$ (B) $\theta = \cos^{-1}\left(\frac{3}{5\sqrt{2}}\right)$ (C) $\theta = \cos^{-1}\left(\frac{2}{3\sqrt{5}}\right)$ (D) $\theta = \cos^{-1}\left(\frac{\sqrt{3}}{5}\right)$

6. The component of force $\vec{F_1}$ along force $\vec{F_2}$ is :

(A) $\frac{5}{6}$ (B) $\frac{5}{3}$ (C) $\frac{6}{5}$ (D) $\frac{5}{2}$

PART - III : ASSERTION / REASON

1. **Statement-1 :** A vector is a quantity that has both magnitude and direction and obeys the triangle law of addition.

Statement-2 : The magnitude of the resultant vector of two given vectors can never be less than the magnitude of any of the given vector.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

2. **Statement-1 :** If the rectangular components of a force are 8 N and 6N, then the magnitude of the force is 10N.
Statement-2 : If $|\vec{A}|=|\vec{B}|=1$ then $|\vec{A}\times\vec{B}|^2+|\vec{A}.\vec{B}|^2=1$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

3. **Statement-1 :** If three vectors $\vec{A}$, $\vec{B}$ **and** $\vec{C}$ satisfy the relation $\vec{A}\cdot\vec{B}=0$ & $\vec{A}\cdot\vec{C}=0$ then the vector $\vec{A}$ is parallel to $\vec{B}\times\vec{C}$.
Statement-2 : $\vec{A}\perp\vec{B}$ and $\vec{A}\perp\vec{C}$ and $\vec{B}\times\vec{C}\neq 0$ hence $\vec{A}$ is perpendicular to plane formed by $\vec{B}$ and $\vec{C}$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

4. **Statement-1 :** The minimum number of non-zero vectors of unequal magnitude required to produce zero resultant is three.
Statement-2 : Three vectors of unequal magnitude which can be represented by the three sides of a triangle taken in order, produce zero resultant.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5. **Statement-1 :** The angle between the two vectors $(\hat{i}+\hat{j})$ and $(\hat{k})$ is $\frac{\pi}{2}$ radian.
Statement-2 : Angle between two vectors $(\hat{i}+\hat{j})$ and $(\hat{k})$ is given by θ = $\cos^{-1}\left(\frac{\vec{A}.\vec{B}}{AB}\right)$.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

6. **Statement-1 :** Distance is a scalar quantity.
Statement-2 : Distance is the length of path transversed.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

PART - IV : TRUE / FALSE

1. State True or False

(i) f (x) = – f ′ (x) for some function f.

(ii) f(x) = f ′ (x) for some function f .

(iii) If $\vec{A}$ & $\vec{B}$ are two force vectors then $\vec{A}.\vec{B} = \vec{B}.\vec{A}$

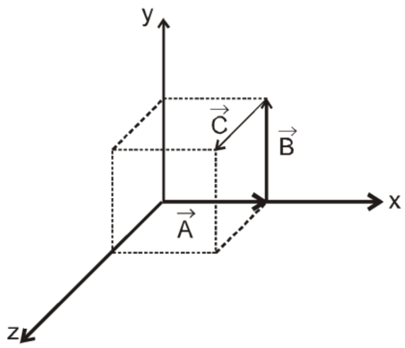
(iv) If $\vec{A}$ & $\vec{B}$ are two non-zero force vectors then $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$

(v) If the vector product of two non-zero vectors vanishes, the vectors are collinear.

PART - V : FILL IN THE BLANKS

Fill in the blanks

1. The magnitude of sum of three vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ representing the sides of a cube of length A is equal to ....



2. If $\vec{A} = 3\hat{i} + 4\hat{j}$ and $\vec{B} = 7\hat{i} + 24\hat{j}$, then the vector having the same magnitude as $\vec{B}$ and parallel to $\vec{A}$ is ....

3. If $\vec{A} \parallel \vec{B}$ then $\vec{A} \times \vec{B} =$ ....

4. The magnitude of area of the parallelogram formed by the adjacent sides of vectors $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{k}$ is ....

5. Sum of two opposite vector to each other is a .... vector.

6. The unit vector along vector $\hat{i} + \hat{j} + \hat{k}$ is ....

7. If $\vec{A}$ is .... to $\vec{B}$, then $\vec{A}.\vec{B} = 0$

8. The vector $\vec{A} = \hat{i} + \hat{j}$, where $\hat{i}$ and $\hat{j}$ are unit vectors along x-axis and y-axis respectively, makes an angle of .... degree with x-axis.

9. If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$, then $\vec{A}.(\vec{B} \times \vec{C}) =$ ....

Answers

EXERCISE-1

PART - I

Section (A) :

A-1.	1	**A-2.**	47	**A-3.**	(C)
A-4.	(A)	**A-5.**	(C)	**A-6.**	(C)
A-7.	(ACD)	**A-8.**	(ABCD)	**A-9.**	(ABC)

Section (B) :

B-1. $\frac{dy}{dx} = 2x + 1$ **B-2.** $\sec^2 x - \operatorname{cosec}^2 x$

B-3. $\frac{dy}{dx} = \cos x - \sin x$, $\frac{d^2y}{dx^2} = -\sin x - \cos x$

B-4. $\frac{dy}{dx} = \frac{1}{x} + e^x$, $\frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$

Section (C) :

C-1. $e^x \ell nx + \frac{e^x}{x}$ **C-2.** $\cos^2 x - \sin^2 x$

Section (D) :

D-1. $y' = \frac{-19}{(3x-2)^2}$ **D-2.** $\frac{1}{x^2} - \frac{\ell nx}{x^2}$

D-3. $\frac{dy}{dx} = 0$

D-4. (a) 13, (b) – 7, (c) $\frac{7}{25}$, (d) 20

Section (E) :

E-1. $5\cos 5x$ **E-2.** $2\omega\cos(\omega x + \varphi)$

E-3. $\frac{dy}{dx} = -27(4-3x)^8$

Section (F) :

F-1. $\frac{dy}{dx} = -1$

F-2. $\frac{-2xy - y^2}{x^2 + 2xy}$

Section (G) :

G-1. $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. **G-2.** $\frac{ds}{dt} = 8\pi r \frac{dr}{dt}$

Section (H) :

H-1. 8 **H-2.** $y_{max} = 39$, $y_{min} = 38$

Section (I) :

I-1. $\frac{dy}{dx} = 48(8x-1)^2$

I-2. $3\cos(3x+1)$ **I-3.** $12x^3$.

I-4. $\frac{dy}{dx} = -\frac{1}{3}\sin\frac{x}{3}$

PART - II

Section (A) :

A-1. $\frac{x^3}{3} - x^2 + x + c$ **A-2.** $\frac{2\sqrt{x^3}}{3} + 2\sqrt{x} + c$

A-3. $\frac{3x^{4/3}}{4} + \frac{3x^{2/3}}{2} + c$

A-4. $\tan x + c$ **A-5.** $-\cot x + c$

A-6. $\sec x + c$ **A-7.** $\frac{1}{3}\ell nx + c$

Section (B) :

B-1. $-\frac{1}{4}\cos(2x^2) + C$ **B-2.** $\frac{1}{2}\sec 2t + C$

B-3. $\frac{3}{2-x} + C$ **B-4.** $-\frac{\cos(8z-5)}{8} + C$

Section (C) :

C-1. $\frac{3\pi}{2}$ **C-2.** 24 **C-3.** e – 1

Section (D) :

D-1. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right– endpoint values : Area = $\int_0^b 2x\,dx = b^2$ units

D-2. $\frac{b^2}{4} + b = \frac{b(4+b)}{4}$ units

D-3. 2 units **D-4.** π/2 units **D-5.** (D)

D-6. (BCD) **D-7.** (ABC)

PART - III

Section (A) :

A-1. (i) 105º, (ii) 150º, (iii) 105º.

A-2. 120º **A-3.** $\vec{V}_R = -5\hat{j}$

Section (B) :

B-1. 30 m East **B-2.** 50, 53º with East

B-3. (A) **B-4.** (B) **B-5.** (B)

B-6. (D) **B-7.** (D) **B-8.** (A)

B-9. (C) **B-10.** (D) **B-11.** (D)

Section (C) :

C-1. $\sqrt{14}$ **C-2.** $\frac{3\hat{i} + 4\hat{j}}{5}$

C-3. – 25 cos 30º and – 25 sin 30º

C-4. $30\sqrt{3}$ km h^{-1}. **C-5.** $\pm\frac{\sqrt{11}}{10}$

C-6. 15º. **C-7.** (A) **C-8.** (C)

Section (D) :

D-1. (a) 3, (b) $-\hat{i} + 2\hat{j} - \hat{k}$

D-2. (a) 6, (b) $6\sqrt{3}$ **D-3.** (D)

D-4. (BC)

EXERCISE-2

PART - I

1. $-\frac{1}{x}$ **2.** x **3.** (C)
4. (AC) **5.** (ABD) **6.** (CD)

7. $\frac{dy}{dx} = \frac{2}{x} + \cos x$, $\frac{d^2y}{dx^2} = \frac{-2}{x^2} - \sin x$

8. $\frac{dy}{dx} = \frac{x^{-\frac{6}{7}}}{7} + \sec^2 x$, $\frac{d^2y}{dx^2} = \frac{-6}{49} x^{\frac{-13}{7}} + 2\tan x \sec^2 x$

9. $\frac{dy}{dx} = 1 + 2x + \frac{2}{x^3} - \frac{1}{x^2}$

10. $\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$ **11.** $\frac{-\csc^2 x}{(1+\cot x)^2}$

12. $\frac{\tan x\left(e^x + \frac{1}{x}\right) - \sec^2 x(e^x + \ell n x)}{\tan^2 x}$

13. $3\sin^2 x \cos x + 3\cos 3x$

14. $4x \sin(x^2 + 1)\cos(x^2 + 1)$

15. $\frac{1-r}{\sqrt{2r - r^2}}$ **16.** $\frac{dy}{dx} = \frac{18y - 3x^2}{3y^2 - 18x}$

17. (a) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} = 5\pi r^2$

(b) $\frac{dV}{dt} = 2\pi hr \frac{dr}{dt} = 10\pi rh$

(c) $\frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi hr \frac{dr}{dt} = 5\pi r^2 + 10\pi rh$

18. x = 30 & y = 30

19. $x = \sqrt{\frac{40}{3}}$ m

PART - II

1. $-\frac{1}{x} - \frac{1}{2x^2} + C$

2. $2x + \cot x + C$

3. $-\cos\theta + \theta + C$

4. $(y^4 + 4y^2 + 1)^3 + C$

5. $\left[\frac{2}{5}\sqrt{5x+8}\right] + C$

6. $-\frac{1}{3}(3 - 2s)^{3/2} + C$

7. $\frac{1}{3}\tan(3x + 2) + C$

8. $-2\csc\left(\frac{\upsilon - \pi}{2}\right) + C$

9. $\frac{-3}{(2+\sin t)^2} + C$ **10.** $\frac{3\pi^2}{2}$

11. $\frac{7}{3}$ **12.** 1

13. $\frac{1}{3}\ell n\frac{5}{2} = \ell n\left(\frac{5}{2}\right)^{\frac{1}{3}}$

14. Using n subintervals of length $\Delta x = \frac{b}{n}$ and right–end point values :Area = $\int_0^b 3x^2\,dx = b^3$

PART - III

1. No it does not represent the same physical quantity.

2. 24N; 37° approx

3. P = 40 ; Q = 30 **4.** 37º

5. $r(1 + \sqrt{2})$ **6.** $\frac{4\hat{i} + 5\hat{j} + 2\hat{k}}{\sqrt{45}}$

7. $3\sqrt{5}$, $\tan^{-1}\frac{1}{2}$ **8.** (A)

9.	(A)	**10.**	(B)	**11.**	(B)
12.	(B)	**13.**	(D)	**14.**	(A)
15.	(B)	**16.**	(A)	**17.**	(D)
18.	(ABC)	**19.**	(ABD)	**20.**	(ABC)
21.	(CD)				

EXERCISE-3

PART - I

1. (A) – r, (B) – p, (C) – q, (D) – s

2. (A) – q, (B) – r, (C) – p, (D) – s

PART - II

1.	(C)	**2.**	(D)	**3.**	(D)
4.	(A)	**5.**	(B)	**6.**	(C)

PART - III

1.	(C)	**2.**	(B)	**3.**	(D)
4.	(A)	**5.**	(A)	**6.**	(A)

PART - IV

1. (i) T (ii) T (iii) T
(iv) F (v) T

PART - V

1. $(\sqrt{3})A$ **2.** $15\hat{i}+20\hat{j}$.

3. Null vector **4.** 16 units

5. Negative, Positive or Zero

6. $\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$.

7. Perpendicular. **8.** 45^{o}

9. Zero.