

Exercise-1

▣ Marked Questions can be used as Revision Questions.

* Marked Questions may have more than one correct option.

ONLY ONE OPTION CORRECT TYPE

Section (A) : Kinematics

- A-1.** If angular displacement of a particle moving on a curved path be given as, $\theta = 1.5t + 2t^2$, where t is in sec, the angular velocity at $t = 2$ sec, will be
 (1) 1.5 (2) 2.5 (3) 9.5 (4) 8.5
- A-2.** A wheel starts rotating from rest (with constant α) and attains an angular velocity of 60 rad/sec in 5 seconds. The total angular displacement in radians will be
 (1) 60 (2) 80 (3) 100 (4) 150
- A-3.** All the particles of a rigid body in a rotatory motion have:
 (1) equal linear and angular velocity
 (2) linear velocity equal, but angular velocity unequal
 (3) equal angular velocity, but unequal linear velocity
 (4) both linear and angular velocities unequal
- A-4.▣** A fan is running at 3000 rpm. It is switched off. It comes to rest by uniformly decreasing its angular speed in 10 seconds. The total number of revolution in this period.
 (1) 150 (2) 250 (3) 350 (4) 300
- A-5.** A block hangs from a string wrapped on a disc of radius 20 cm free to rotate about its axis which is fixed in a horizontal position. If the angular speed of the disc is 10 rad/s at some instant, with what speed is the block going down at that instant ?
 (1) 4 m/s (2) 3 m/s (3) 2 m/s (4) 5 m/s

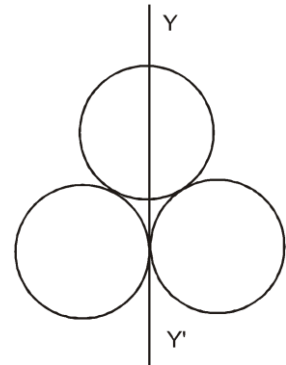
Section (B) : Moment of inertia

- B-1.** A stone of mass 4kg is whirled in a horizontal circle of radius 1m and makes 2 rev/sec. The moment of inertia of the stone about the axis of rotation is
 (1) $64 \text{ kg} \times \text{m}^2$ (2) $4 \text{ kg} \times \text{m}^2$ (3) $16 \text{ kg} \times \text{m}^2$ (4) $1 \text{ kg} \times \text{m}^2$
- B-2.▣** Two solid spheres of same mass and radius are in contact with each other. If the moment of inertia of a sphere about its diameter is I , then the moment of inertia of both the spheres about the tangent at their common point would be
 (1) $3I$ (2) $7I$ (3) $4I$ (4) $5I$
- B-3.▣** From the theorem of perpendicular axes. If the lamina is in X- Y plane
 (1) $I_x - I_y = I_z$ (2) $I_x + I_z = I_y$ (3) $I_x + I_y = I_z$ (4) $I_y + I_z = I_x$
- B-4.** The moment of inertia of a body depends upon
 (1) mass only (2) angular velocity only
 (3) distribution of particles only (4) mass and distribution of mass about the axis
- B-5.** A wheel of mass 10 kg has a moment of inertia of 160 kg m^2 about its own axis, the radius of gyration will be
 (1) 10 m (2) 8 m (3) 6 m (4) 4 m
- B-6.** The moment of inertia of a uniform ring of mass M and radius r about a tangent lying in its own plane is

- (1) $2Mr^2$ (2) $\frac{3}{2} Mr^2$ (3) Mr^2 (4) $\frac{1}{2} Mr^2$

B-7. Three rings each of mass M and radius R are arranged as shown in the figure. The moment of inertia of the system about YY' will be

- (1) $3 MR^2$
(2) $\frac{3}{2} MR^2$
(3) $5 MR^2$
(4) $\frac{7}{2} MR^2$

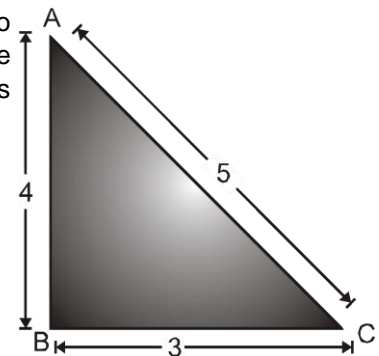


B-8. Radius of gyration of a body depends on

- (1) Mass and size of body (2) Mass distribution and axis of rotation
(3) Size of body (4) Mass of body

B-9. ABC is a triangular plate of uniform thickness. The sides are in the ratio shown in the figure. I_{AB} , I_{BC} , I_{CA} are the moments of inertia of the plate about AB, BC, CA respectively. Which one of the following relations is correct

- (1) I_{CA} is maximum
(2) $I_{AB} > I_{BC}$
(3) $I_{BC} > I_{AB}$
(4) $I_{AB} + I_{BC} = I_{CA}$



B-10. The moment of inertia of a uniform semicircular wire of mass M and radius R about a line perpendicular to the plane of the wire through the centre is

- (1) MR^2 (2) $\frac{1}{2} MR^2$ (3) $\frac{1}{4} MR^2$ (4) $\frac{2}{5} MR^2$

B-11. The moment of inertia of a uniform semicircular wire of mass M and radius r about a line perpendicular to the plane of the wire and passing through the centre of mass of the system

- (1) $Mr^2 \left(1 - \frac{4}{\pi^2}\right)$ (2) $Mr^2 \left(1 + \frac{4}{\pi^2}\right)$ (3) $Mr^2 \left(1 - \frac{\pi^2}{4}\right)$ (4) $Mr^2 \left(1 + \frac{\pi^2}{4}\right)$

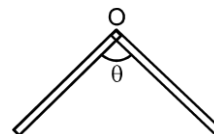
B-12. Find moment of inertia of a semi circular disc about axis passing through center of mass and perpendicular to its plane.

- (1) $MR^2 \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)$ (2) $MR^2 \left(\frac{1}{2} + \frac{16}{9\pi^2}\right)$
(3) $MR^2 \left(\frac{1}{2} - \frac{9\pi^2}{16}\right)$ (4) $MR^2 \left(\frac{1}{2} + \frac{9\pi^2}{16}\right)$

B-13. Let I_A and I_B be moments of inertia of a body about two axes A and B respectively, The axis A passes through the centre of mass of the body but B does not.

- (1) $I_A < I_B$ (2) If $I_A < I_B$, the axes are parallel.
(3) If the axes are parallel, $I_A < I_B$ (4) If the axes are not parallel, $I_A \geq I_B$.

B-14. A thin rod of length L and mass M is bend at the middle point O as shown in figure. Consider an axis passing through two middle point O and perpendicular to the plane of the bent rod. Then moment of inertia about this axis is :



- (1) $\frac{2}{3} mL^2$ (2) $\frac{1}{3} mL^2$ (3) $\frac{1}{12} mL^2$ (4) $\frac{1}{24} mL^2$

B-15. The moment of inertia of a uniform circular disc about its diameter is 200 gm cm^2 . Then its moment of inertia about an axis passing through its center and perpendicular to its circular face is

- (1) 100 gm cm^2 (2) 200 gm cm^2 (3) 400 gm cm^2 (4) 1000 gm cm^2

Section (C) : Torque

C-1. The product of moment of inertia and angular acceleration is

- (1) force (2) torque (3) angular momentum (4) rotational kinetic energy

C-2. A disc of radius 2m and mass 200kg is acted upon by a torque 100N-m . Its angular acceleration would be

- (1) 1 rad/sec^2 (2) 0.25 rad/sec^2 (3) 0.5 rad/sec^2 (4) 2 rad/sec^2

C-3. Rotational power in rotational motion is

- (1) $\vec{\omega} \cdot \vec{\tau}$ (2) $\vec{\omega} \times \vec{\tau}$ (3) $\vec{\tau} \cdot \vec{\alpha}$ (4) $\vec{\tau} \times \vec{\alpha}$

C-4. A force $\vec{F} = 4\hat{i} - 10\hat{j}$ acts on a body at a point having position vector $-5\hat{i} - 3\hat{j}$ relative to origin of co-ordinates on the axis of rotation . The torque acting on the body is :

- (1) $38\hat{k}$ (2) $-25\hat{k}$ (3) $62\hat{k}$ (4) none of these

C-5. A force $F = 2\hat{i} + 3\hat{j} - \hat{k}$ acts at a point (2, -3, 1). Then magnitude of torque about point (0, 0, 2) will be:

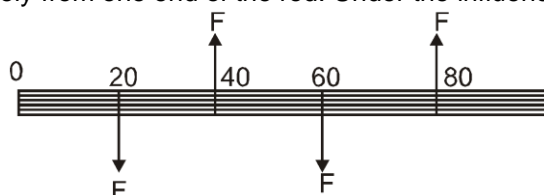
- (1) 6 (2) $3\sqrt{5}$ (3) $6\sqrt{5}$ (4) none of these

C-6. In case of torque of a couple if the axis is changed by displacing it parallel to itself, torque will :aa

- (1) Increase (2) Decrease (3) Remain constant (4) None of these

Section (D) : Rotational Equilibrium

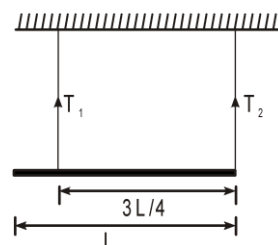
D-1. Four equal and parallel forces are acting on a rod (as shown in figure) at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces the rod :



- (1) Is at rest (2) Experiences a torque
(3) Experiences a linear motion (4) Experiences a torque and also a linear motion

- D-2.▲** A uniform rod of mass m and length L is suspended with two massless strings as shown in the figure. If the rod is at rest in a horizontal position the ratio of tension in the two strings T_1/T_2 is:

(1) 1: 1
 (2) 1: 2
 (3) 2: 1
 (4) 4: 3

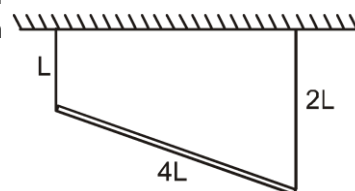


- D-3.** Two persons of equal height are carrying a long uniform wooden beam of length ℓ . They are at distance $\ell/4$ and $\ell/6$ from nearest ends of the rod. The ratio of normal reactions at their heads is:

(1) 2 : 3 (2) 1 : 3 (3) 4 : 3 (4) 1 : 2

- D-4▲** A uniform rod of length $4L$ and mass M is suspended from a horizontal roof by two light strings of length L and $2L$ as shown. Then the tension in the left string of length L is

(1) $\frac{Mg}{2}$ (2) $\frac{Mg}{3}$
 (3) $\frac{3}{5} Mg$ (4) $\frac{Mg}{4}$

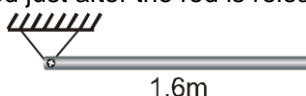


Section (E) : Rotation about Fixed axis ($\tau_H = I_H \alpha$)

- E-1.▲** One end of a uniform rod of mass m and length ℓ is clamped. The rod lies on a smooth horizontal surface and rotates on it about the clamped end at a uniform angular velocity ω . The force exerted by the clamp on the rod has a horizontal component

(1) $m\omega^2 \ell$ (2) zero (3) mg (4) $\frac{1}{2} m\omega^2 \ell$

- E-2.▲** The uniform rod of mass 20 kg and length of 1.6 m is pivoted at its end and swings freely in the vertical plane. Angular acceleration of rod just after the rod is released from rest in the horizontal position.



(1) $\frac{15g}{16}$ (2) $\frac{17g}{16}$ (3) $\frac{16g}{15}$ (4) $\frac{g}{15}$

- E-3.▲** A body is rotating with constant angular velocity about a vertical axis fixed in an inertial frame. The net force on a particle of the body not on the axis is

(1) horizontal and skew with the axis (2) vertical
 (3) horizontal and intersecting the axis (4) none of these.

- E-4.▲** One end of a uniform rod having mass m and length ℓ is hinged. The rod is placed on a smooth horizontal surface and rotates on it about the hinged end at a uniform angular velocity ω . The force exerted by the hinge on the rod has a horizontal component

(1) $m\omega^2 \ell$ (2) zero (3) mg (4) $\frac{1}{2} m\omega^2 \ell$

Section (F) : Rotation about Fixed Axis (Energy conservation)

F-1. The rotational kinetic energy is :

- (1) $\frac{1}{2}m\omega^2$ (2) $\frac{1}{2}I\alpha^2$ (3) $\frac{1}{2}I\omega^2$ (4) $\frac{1}{2}mv^2$

F-2.▲ A uniform metre stick is held vertically with one end on the floor and is allowed to fall. The speed of the other end when it hits the floor assuming that the end at the floor does not slip :

- (1) $\sqrt{4g}$ (2) $\sqrt{3g}$ (3) $\sqrt{5g}$ (4) \sqrt{g}

Section (G) : Angular Momentum & its conservation

G-1.▲ The moments of inertia of two rotating bodies A and B are I_A and I_B ($I_A > I_B$) and their angular momentum are equal. If their kinetic energies be K_A and K_B , respectively, then

- (1) $\frac{K_A}{K_B} > 1$ (2) $\frac{K_B}{K_A} > 1$ (3) $\frac{K_A}{K_B} = 1$ (4) $\frac{K_A}{K_B} = \frac{1}{2}$

G-2. When a body rotates about an axis the quantity which remains same for all its particles, is

- (1) linear velocity (2) angular velocity (3) linear acceleration (4) angular momentum

G-3. The time rate of change of angular momentum is called

- (1) angular velocity (2) angular acceleration (3) force (4) torque

G-4. The torque applied to a ring revolving about its own axis so as to change its angular momentum by 2 J-s. in 5 s, is

- (1) 10N-m (2) 2.5 N-m (3) 0.1N-m (4) 0.4 N-m

G-5. When a mass is rotating in a plane about a fixed point, its angular momentum is directed along

- (1) radius (2) the tangent to the orbit
(3) a line perpendicular to the plane of rotation (4) none of the above

G-6*. The motion of planets in the solar system is an example of the conservation of

- (1) Mass (2) Linear momentum
(3) Angular momentum (4) Energy

G-7.▲ The angular momentum of a system of particles is conserved

- (1) When no external force acts upon the system
(2) When no external torque acts on the system
(3) When no external impulse acts upon the system
(4) When axis of rotation remains same

G-8.▲ The angular speed of a body changes from ω_1 to ω_2 without applying a torque but due to change in its moment of inertia. The ratio of radii of gyration in the two cases is

- (1) $\sqrt{\omega_2} : \sqrt{\omega_1}$ (2) $\sqrt{\omega_1} : \sqrt{\omega_2}$ (3) $\omega_1 : \omega_2$ (4) $\omega_2 : \omega_1$

G-9.▲ If the radius of the earth is suddenly contracts to half of its present value, then the duration of day will be of

- (1) 6 hours (2) 12 hours (3) 18 hours (4) 24 hours

G-10.▲ If the earth is treated as a sphere of radius R and mass M. Its angular momentum about the axis of rotation with period T is

- (1) $\frac{\pi MR^3}{T}$ (2) $\frac{MR^2\pi}{T}$ (3) $\frac{2\pi MR^2}{5T}$ (4) $\frac{4\pi MR^2}{5T}$
- G-11.** A man is standing at the edge of a circular plate which is rotating with a constant angular speed about a perpendicular axis passing through the centre. If the man walks towards the axis along the radius, its angular velocity.
 (1) Decreases (2) Remains constant (3) Increases (4) Information is incomplete
- G-12.** A thin circular ring of mass M and radius R is rotating about its axis with a constant angular velocity ω . Two objects each of mass m , are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity
 (1) $\frac{\omega M}{M + m}$ (2) $\frac{\omega(M - 2m)}{M + 2m}$ (3) $\frac{\omega M}{M + 2m}$ (4) $\frac{\omega(M + 2m)}{M}$
- G-13.** What remains constant in the field of central force
 (1) Potential energy (2) Kinetic energy (3) Angular momentum (4) Linear momentum
- G-14.** A constant torque acting on a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 sec. the magnitude of this torque is :
 (1) $4A_0$ (2) A_0 (3) $3A_0/4$ (4) $12A_0$
- G-15.** A particle moves with a constant velocity parallel to the X-axis. Its angular momentum with respect to the origin.
 (1) is zero (2) remains constant (3) goes on increasing (4) goes on decreasing.
- G-16.** A particle performs uniform circular motion with an angular momentum L . If the frequency of particle's motion is doubled and its kinetic energy is halved, the angular momentum becomes :
 (1) $2L$ (2) $4L$ (3) $L/2$ (4) $L/4$
- G-17.** A person sitting firmly over a rotating stool has his arms stretched. If he folds his arms, his angular momentum about the axis of rotation
 (1) increases (2) decreases (3) remains unchanged (4) doubles

Section (H) : Combined Translational + Rotation Motion (Kinematics)

- H-1.** A ring of mass 1 kg and diameter 1m is rolling without slipping on a plane road with a speed 2m/s. Its kinetic energy would be
 (1) 1 joule (2) 4 joule (3) 2 joule (4) 0.5 joule
- H-2.** A solid cylinder starts rolling from a height h on an inclined plane. At some instant t , the ratio of its rotational K.E. and the total K.E. would be
 (1) 1 : 2 (2) 1 : 3 (3) 2 : 3 (4) 1 : 1
- H-3.** The centre of a wheel rolling without slipping in a plane surface moves with speed v_0 . A particle on the rim of the wheel at the same level as the centre will be moving at speed
 (1) zero (2) v_0 (3) $\sqrt{2}v_0$ (4) $2v_0$
- H-4.** A body is given translational velocity and kept on a surface that has sufficient friction. Then:
 (1) body will move forward before pure rolling
 (2) body will move backward before pure rolling
 (3) body will start pure rolling immediately
 (4) none of these

Section (I) : Combined translational & Rotational Motion (Dynamics)

- I-1.** The velocity of centre of mass of a disc rolling with out slipping on an inclined plane changes from V to $2V$ at any instant of time. If M is the mass of disc then increases in its kinetic energy will be
 (1) $(9/4) MV^2$ (2) $MV^2/2$ (3) MV^2 (4) $3MV^2$
- I-2.** A solid spherical ball rolls on a table without slipping. Then the fraction of its total energy associated with rotation is
 (1) $2/5$ (2) $3/5$ (3) $2/7$ (4) $3/7$
- I-3.▲** A sphere of mass 0.5 kg and diameter 1 m rolls without sliding with a constnat velocity of 5 m/s , calculate what is the ratio of the rotational K.E. to the total kinetic energy of the sphere
 (1) $\frac{7}{10}$ (2) $\frac{5}{7}$ (3) $\frac{2}{7}$ (4) $\frac{1}{2}$
- I-4.** A solid homogeneous sphere is moving on a rough horizontal surface partly rolling and partly sliding. During this kind of motion of the sphere
 (1) Total kinetic energy is conserved
 (2) The angular momentum of the sphere about the point of contact with the plane is conserved
 (3) Only the rotational kinetic energy about the centre of mass is conserved
 (4) Angular momentum about the centre of mass is conserved
- I-5.▲** A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. The smallest kinetic energy at the bottom of the incline will be achieved by
 (1) the solid sphere (2) the hollow sphere
 (3) the disc (4) all will achieve same kinetic energy.
- I-6.** A hollow sphere and a solid sphere having same mass and same radii are rolled down (without sliding) a rough inclined plane.
 (1) The hollow sphere reaches the bottom first
 (2) The solid sphere reaches the bottom with greater speed.
 (3) The solid sphere reaches the bottom with greater kinetic energy
 (4) The two spheres will reach the bottom with same linear momentum
- I-7.** A solid sphere, a hollow sphere and a disc, all having same mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are same and not sufficient to allow pure rolling. Least time will be taken in reaching the bottom by
 (1) the solid sphere (2) the hollow sphere (3) the disc (4) all will take same time.

Section (J) : conservation of angular momentum (combined translation & rotational motion)

- J-1.** A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:
 (1) conserved about every point
 (2) conserved about the point of contact only
 (3) conserved about the centre of the sphere only

(4) conserved about any point on a fixed line parallel to the inclined plane and passing through the centre of the ball.

J-2. A smooth sphere A is moving on a frictionless horizontal plane with angular speed ω and centre of mass velocity v . It collides elastically and head on with an identical sphere B at rest. Neglect friction everywhere. After the collision, their angular speeds are ω_A & ω_B respectively. Then:

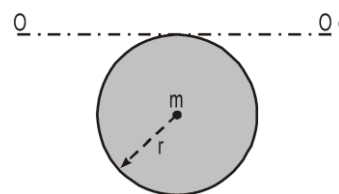
- (1) $\omega_A < \omega_B$ (2) $\omega_A = \omega_B$ (3) $\omega_A = \omega$ (4) $\omega_B = \omega$

Exercise-2

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Moment of inertia of a uniform disc about OO' is:

- (1) $\frac{3mr^2}{2}$ (2) $\frac{mr^2}{2}$
 (3) $\frac{5mr^2}{2}$ (4) $\frac{5mr^2}{4}$

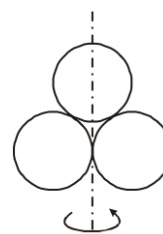


2. Three identical rods, each of length l , are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is .

- (1) $\frac{l}{\sqrt{3}}$ (2) $\frac{l}{\sqrt{2}}$ (3) $\frac{l}{\sqrt{5}}$ (4) $\frac{l}{\sqrt{7}}$

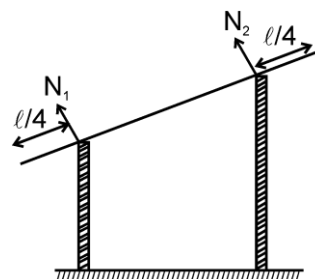
3. Three rings each of mass m and radius r are so placed that they touch each other. The radius of gyration of the system about the axis as shown in the figure is :

- (1) $\sqrt{\frac{6}{5}} r$ (2) $\sqrt{\frac{5}{6}} r$
 (3) $\sqrt{\frac{6}{7}} r$ (4) $\sqrt{\frac{7}{6}} r$



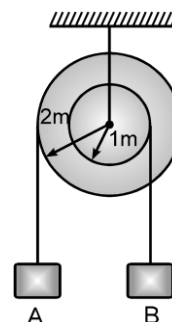
4. A uniform rod of length l is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then

- (1) $N_1 > N_2$
 (2) $N_1 < N_2$
 (3) $N_1 = N_2$
 (4) N_1 and N_2 would be in the vertical directions.



5. In the pulley system (stick to each other) shown, if radii of the bigger and smaller pulley are 2 m and 1 m respectively and the acceleration of block A is 5 m/s^2 in the downward direction, then the acceleration of block B will be:

- (1) 0 m/s^2
 (2) 5 m/s^2
 (3) 10 m/s^2
 (4) $5/2 \text{ m/s}^2$

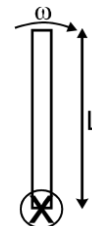


6. A uniform thin rod of mass 'm' and length ℓ is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut :

- (1) $\frac{g}{2\ell}$ (2) $\frac{g}{\ell}$ (3) $\frac{3g}{2\ell}$ (4) $\frac{2g}{\ell}$

7. A uniform rod hinged at its one end is allowed to rotate in vertical plane. Rod is given an angular velocity ω in its vertical position as shown in figure. The value of ω for which the force exerted by the hinge on rod is zero in this position is :

- (1) $\sqrt{\frac{g}{L}}$ (2) $\sqrt{\frac{2g}{L}}$ (3) $\sqrt{\frac{g}{2L}}$ (4) $\sqrt{\frac{3g}{L}}$

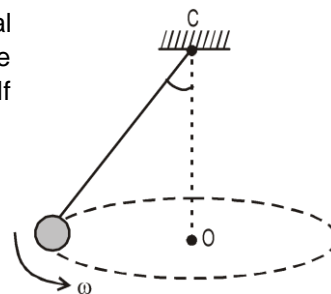


8. If $\vec{\tau} \times \vec{L} = 0$ for a rigid body, where $\vec{\tau}$ = resultant torque and \vec{L} = angular momentum about a point and both are non - zero. Then :

- (1) \vec{L} = constant (2) $|\vec{L}|$ = constant
(3) $|\vec{L}|$ will increase (4) $|\vec{L}|$ may increase

9. A conical pendulum consists of a simple pendulum moving in a horizontal circle as shown. C is the pivot, O the centre of the circle in which the pendulum bob moves and ω the constant angular velocity of the bob. If \vec{L} is the angular momentum about point C, then

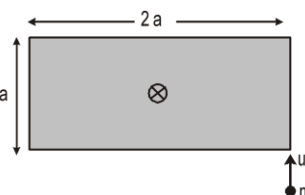
- (1) \vec{L} is constant
(2) only direction of \vec{L} is constant
(3) only magnitude of \vec{L} is constant
(4) none of the above.



10. In the above problem if \vec{L} is the angular momentum about the axis CO, then,
(1) \vec{L} is constant (2) only direction of \vec{L} is constant
(3) only magnitude of \vec{L} is constant (4) none of the above.

11. A uniform rectangular plate of mass m which is free to rotate about the smooth vertical hinge passing through the centre and perpendicular to the plate, is lying on a smooth horizontal surface. A particle of mass m moving with speed 'u' collides with the plate and sticks to it as shown in figure. The angular velocity of the plate after collision will be :

- (1) $\frac{12u}{5a}$ (2) $\frac{12u}{19a}$
(3) $\frac{3u}{2a}$ (4) $\frac{3u}{5a}$



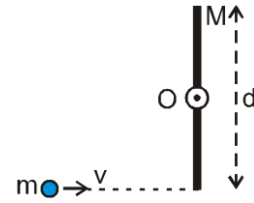
12. A rod can rotate about a fixed vertical axis. The mass is non-uniformly distributed along the length of the rod. A horizontal force of constant magnitude and always perpendicular to the rod is applied at the end. Which of the following quantity (after one rotation) will not depend on the information that through which end the axis passes ? (Assuming initial angular velocity to be zero)

- (1) angular momentum (2) kinetic energy
(3) angular velocity (4) none of these

Rigid Body Dynamics

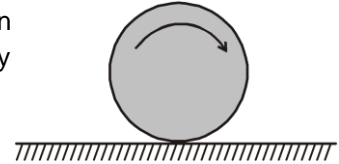
13. A particle of mass m is moving horizontally at speed v perpendicular to a uniform rod of length d and mass $M = 6m$. The rod is hinged at centre O and can freely rotate in horizontal plane about a fixed vertical axis passing through its centre O . The hinge is frictionless. The particle strikes and sticks to the end of the rod. The angular speed of the system just after the collision :

- (1) $2v/3d$ (2) $3v/2d$
(3) $v/3d$ (4) $2v/d$



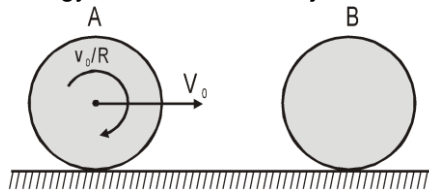
14. A body of mass m and radius r is rotated with angular velocity ω as shown in the figure & kept on a surface that has sufficient friction then the body will move :

- (1) backward first and then move forward
(2) forward first and then move backward
(3) will always move forward
(4) none of these



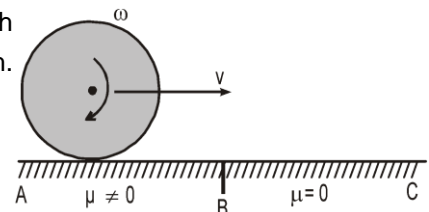
15. A hollow smooth uniform sphere A of mass ' m ' rolls without sliding on a smooth horizontal surface. It collides elastically and headon with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is :

- (1) 5 : 2 (2) 1 : 1 (3) 2 : 3 (4) 3 : 2



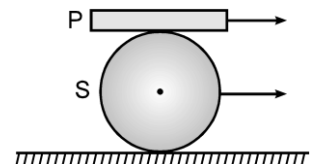
16. As shown in the figure, a disc of mass m is rolling without slipping with a angular velocity ω . The portion AB is rough and BC is smooth. When it crosses point B disc will be in :

- (1) translational motion only
(2) pure rolling motion
(3) rotational motion only
(4) none of these



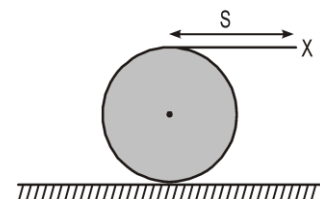
17. A plank P is placed on a solid cylinder S, which rolls on a horizontal surface. The two are of equal mass. There is no slipping at any of the surfaces in contact. The ratio of the kinetic energy of P to the kinetic energy of S is:

- (1) 1 : 1 (2) 2 : 1
(3) 8 : 3 (4) 1 : 4



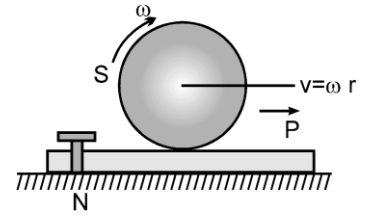
18. A large spool of rope lies on the ground as shown in the fig. The end, labelled X is pulled a distance S in the horizontal direction. The spool rolls without slipping. The centre of mass of the spool moves a distance

- (1) $2S$ (2) S
(3) $\frac{S}{2}$ (4) $\frac{S}{4}$



Rigid Body Dynamics

19. A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin N. If N is suddenly removed:



- (1) S will begin to slip on P
- (2) P will begin to move backwards
- (3) the speed of S will decrease and its angular velocity will increase
- (4) there will be no change in the motion of S and P will still be at rest.

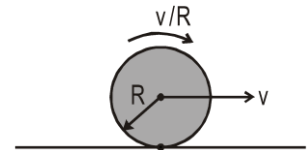
20. A solid sphere and a solid cylinder having the same mass and radius, roll down the same incline. The ratio of their acceleration will be

- (1) 15 : 14
- (2) 14 : 15
- (3) 5 : 3
- (4) 3 : 5

21. A uniform ring of radius R is given a back spin of angular velocity $V_0/2R$ and thrown on a horizontal rough surface with velocity of center to be V_0 . The velocity of the centre of the ring when it starts pure rolling will be

- (1) $V_0/2$
- (2) $V_0/4$
- (3) $3V_0/4$
- (4) 0

22. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in figure. At any instant, for the lower most point of the disc



- (1) Velocity is v, acceleration is zero
- (2) Velocity is zero, acceleration is zero

- (3) velocity is v, acceleration is $\frac{v^2}{R}$

- (4) velocity is zero, acceleration is $\frac{v^2}{R}$

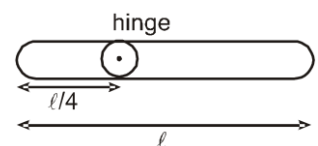
23. A metre stick is held vertically with one end on the floor and is allowed to fall. The speed of the other end when it hits the floor assuming that the end at the floor does not slip :

- (1) $\sqrt{4g}$
- (2) $\sqrt{3g}$
- (3) $\sqrt{5g}$
- (4) \sqrt{g}

24. Two men support a uniform horizontal rod at its two ends. If one of them suddenly lets go, the force exerted by the rod on the other man will:

- (1) remain unaffected
- (2) increase
- (3) decrease
- (4) become unequal to the force exerted by him on the beam.

25. A uniform rod is hinged as shown in the figure and is released from a horizontal position. The angular velocity of the rod as it passes the vertical position is:



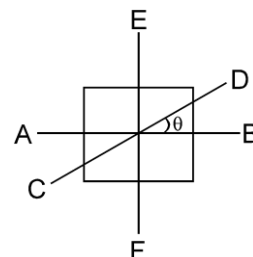
- (1) $\sqrt{\frac{12g}{3l}}$
- (2) $\sqrt{\frac{2g}{3l}}$
- (3) $\sqrt{\frac{24g}{7l}}$
- (4) $\sqrt{\frac{3g}{7l}}$

26. A particle is projected at time $t = 0$ from a point P on the ground with a speed V_0 , at an angle of 45° to the horizontal. What is the magnitude of the angular momentum of the particle about P at time $t = v_0/g$.

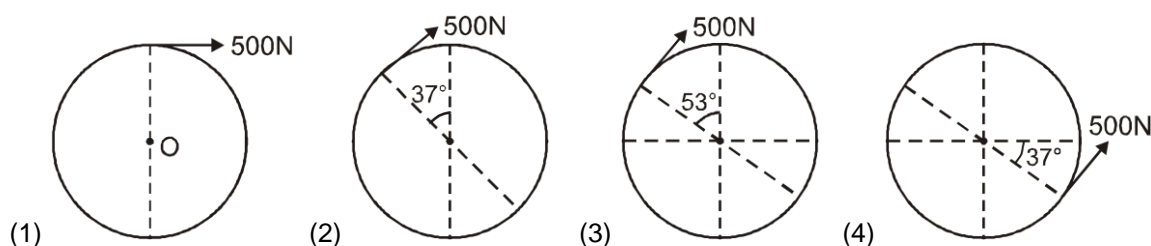
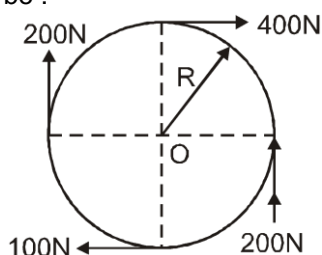
(1) $\frac{mv_0^2}{2\sqrt{2}g}$ (2) $\frac{mv_0^3}{\sqrt{2}g}$ (3) $\frac{mv_0^2}{\sqrt{2}g}$ (4) $\frac{mv_0^3}{2\sqrt{2}g}$

27. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB as shown in figure. The moment of inertia of the plate about the axis CD is then equal to

(1) I (2) $I \sin^2 \theta$
(3) $I \cos^2 \theta$ (4) $I \cos^2 (\theta/2)$



28. Four forces tangent to the circle of radius 'R' are acting on a wheel as shown in the figure. The resultant equivalent one force system will be :



29. The angular momentum of a particle about origin is varying as $L = 4t + 8$ (SI units) when it moves along a straight line $y = x - 4$ (x, y in meters). The magnitude of force acting on the particle would be :

(1) 1 N (2) 2 N (3) $\sqrt{2}$ N (4) $\sqrt{3}$ N

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : Assertion/Reasoning

- A-1. **STATEMENT-1** : If two different axes are at same distance from centre of mass of a rigid body, then moment of inertia of the given rigid body about both axis will always be same.

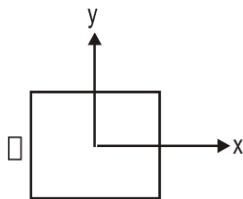
STATEMENT-2 : From parallel axis theorem $I = I_{cm} + md^2$, where all terms have usual meaning.

- (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1.
(3) Statement-1 is True, Statement-2 is False
(4) Statement-1 is False, Statement-2 is True

- A-2. STATEMENT-1 :** A rigid disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then the acceleration of lowest point on the disc is zero.
STATEMENT-2 : For a rigid disc rolling without slipping on a fixed rough horizontal surface, the velocity of the lowest point on the disc is always zero.
- (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
 - (2) Statement-1 is True, Statement-2 is True; Statement-2 is **NOT** a correct explanation for Statement-1
 - (3) Statement-1 is True, Statement-2 is False
 - (4) Statement-1 is False, Statement-2 is True.
- A-3. STATEMENT-1 :** A uniform disc rolls without slipping on a fixed rough horizontal surface with uniform angular velocity. Then all the points on disc at same distance from axis of disc have same magnitude of acceleration (from ground frame).
STATEMENT-2 : In the frame of centre of mass of a uniform disc rolling without slipping on a rough horizontal surface with uniform angular velocity, all the points on the disc rotating in circle of same radii have same magnitude of centripetal acceleration.
- (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.
 - (2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
 - (3) Statement-1 is True, Statement-2 is False
 - (4) Statement-1 is False, Statement-2 is True

Section (B) : Match the Column

- B-1.** Four identical rods, each of mass m and length ℓ are joined to form a rigid square frame. The frame lies in the X-Y plane, with its centre at the origin and the sides parallel to the x and y axis. its moment of inertia about :-



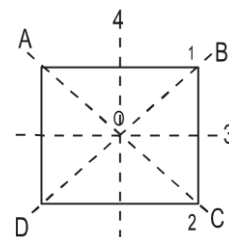
- Column I**
- (1) An axis parallel to z-axis and passing through a corner
 - (2) One side
 - (3) The x-axis
 - (4) The z-axis

- Column II**
- (p) $5/3 m\ell^2$
 - (q) $2/3 m\ell^2$
 - (r) $4/3 m\ell^2$
 - (s) $10/3 m\ell^2$

Section (C) : One or More Than One Options Correct

- C-1.** A sphere is rotating uniformly about an axis passing through its centre then:
- (1) The particles on the surface of the sphere do not have any angular acceleration.
 - (2) The particles on the axis do not have any linear acceleration
 - (3) Different particles on the surface have same angular speeds.
 - (4) All the particles on the surface have same linear speed

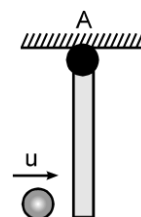
- C-2.** The moment of inertia of a thin uniform square plate ABCD of uniform thickness about an axis passing through the centre O and perpendicular to the plate is - where I_1 , I_2 , I_3 , and I_4 are respectively the moments of inertia about axes 1, 2, 3, and 4 which are in the plane of the plate.



- (1) $I_1 + I_2$
- (2) $I_3 + I_4$
- (3) $I_1 + I_3$
- (4) $I_1 + I_2 + I_3 + I_4$

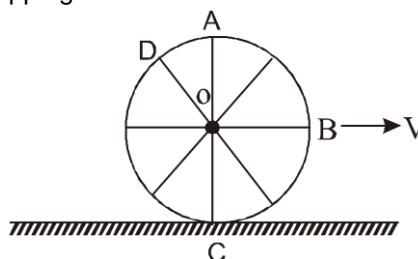
- C-3.** In absence of external forces on a rigid system, which of the following quantities must remain constant?
- (1) angular momentum
 - (2) linear momentum
 - (3) moment of inertia
 - (4) kinetic energy

- C-4.** In the given figure a ball strikes a rod elastically and rod is smoothly hinged at point A. Then which of the statement(s) is/are correct for the collision?



- (1) linear momentum of system (ball + rod) is conserved
- (2) angular momentum of system about hinged point A is conserved
- (3) initial KE of the system is equal to final KE of the system
- (4) linear momentum of ball is conserved.

- C-5.** Consider a disc rolling without slipping on a horizontal surface at a linear speed V as shown in figure



- (1) the speed of the particle A is $2V$
- (2) the speed of B, C and D are all equal to V
- (3) the speed of C is zero and speed of B is $\sqrt{2}V$
- (4) the speed of O is less than the speed of B

- C-6.** When a bicycle is in motion on a rough horizontal plane, the force of friction exerted by the plane on the two wheels is such that it acts :

- (1) In the backward direction on the front wheel and in the forward direction on the rear wheel, when the speed increases.
- (2) In the forward direction on the front wheel and in the backward direction on the rear wheel, when brakes are applied on rear wheel.
- (3) In the backward direction on both front and the rear wheels, when brakes are applied on both wheels.
- (4) In the forward direction on both the front and the rear wheels

- C-7.** A horizontal disc rotates freely about a vertical fixed axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc coaxially. After some time. the two rotate with a common angular velocity:

- (1) some friction exists between the disc and the ring before achieving common angular velocity
- (2) the angular momentum of the 'disc plus ring' about axis of rotation is conserved
- (3) the final common angular velocity is $2/3^{\text{rd}}$ of the initial angular velocity of the disc
- (4) The final common angular velocity is $1/3^{\text{rd}}$ of the initial angular velocity of the disc

Exercise-3

* Marked Questions may have more than one correct option.

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Initial angular velocity of a circular disc of mass M is ω_1 . Then two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc ? **[AIEEE 2002]**
 (1) $\left(\frac{M+m}{M}\right)\omega_1$ (2) $\left(\frac{M+m}{m}\right)\omega_1$ (3) $\left(\frac{M}{M+4m}\right)\omega_1$ (4) $\left(\frac{M}{M+2m}\right)\omega_1$
2. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. The maximum acceleration down the plane is for (no rolling) : **[AIEEE 2002]**
 (1) solid sphere (2) hollow sphere (3) ring (4) all same
3. Moment of inertia of a circular wire of mass M and radius R about its diameter is : **[AIEEE 2002]**
 (1) $\frac{MR^2}{2}$ (2) MR^2 (3) $2MR^2$ (4) $\frac{MR^2}{4}$
4. A thin and circular disc of mass M and radius R is rotating in a horizontal plane about an axis passing through its centre and perpendicular to its plane with an angular velocity ω . If the another disc of same dimensions but of mass $M/4$ is placed gently on the first disc co-axially, then the new angular velocity of the system is **[AIEEE 2002]**
 (1) $\frac{5}{4}\omega$ (2) $\frac{2}{3}\omega$ (3) $\frac{4}{5}\omega$ (4) $\frac{3}{2}\omega$
5. A circular disc X of radius R is made from an iron plate of thickness t , and another disc Y of radius $4R$ is made from an iron plate thickness $t/4$. Then the relation between the moment of inertia I_X and I_Y is : **[AIEEE 2003]**
 (1) $I_Y = 32I_X$ (2) $I_Y = 16I_X$ (3) $I_Y = I_X$ (4) $I_Y = 64 I_X$
6. A particle performing uniform circular motion has angular momentum L . If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is : **[AIEEE 2003]**
 (1) $L/4$ (2) $2L$ (3) $4L$ (4) $L/2$
7. Let \vec{F} be the force acting on a particle having position vector \vec{r} and $\vec{\tau}$ be the torque of this force about the origin. Then : **[AIEEE 2003]**
 (1) $\vec{r} \cdot \vec{F} = 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$ (2) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} = 0$
 (3) $\vec{r} \cdot \vec{\tau} \neq 0$ and $\vec{F} \cdot \vec{\tau} \neq 0$ (4) $\vec{r} \cdot \vec{\tau} = 0$ and $\vec{F} \cdot \vec{\tau} = 0$
8. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same which one of the following will not be affected ? **[AIEEE 2004]**
 (1) Moment of inertia (2) Angular momentum
 (3) Angular velocity (4) Rotational kinetic energy
9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that : **[AIEEE 2004]**
 (1) $I_A = I_B$ (2) $I_A > I_B$ (3) $I_A < I_B$ (4) $I_A / I_B = d_A / d_B$
 where d_A and d_B are their densities.

10. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of

the ring, $\frac{F_1}{F_2}$ is :

[AIEEE 2005]

- (1) $\frac{R_2}{R_1}$ (2) $\left(\frac{R_1}{R_2}\right)^2$ (3) 1 (4) $\frac{R_1}{R_2}$

11. The moment of inertia of uniform semicircular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is :

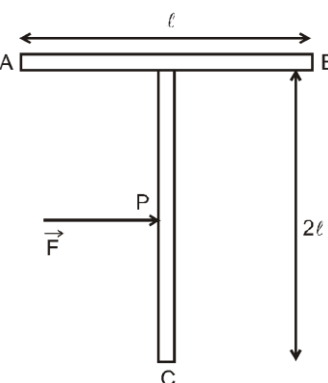
[AIEEE 2005]

- (1) $\frac{1}{4} Mr^2$ (2) $\frac{2}{5} Mr^2$ (3) Mr^2 (4) $\frac{1}{2} Mr^2$

12. A 'T' shaped object with dimensions shown in the figure, is lying on a smooth floor. A force \vec{F} is applied at the point P parallel to AB , such that the object has only the translational motion without rotation. Find the location of P with respect to C :

[AIEEE 2005]

- (1) $\frac{2}{3} \ell$ (2) $\frac{3}{2} \ell$
(3) $\frac{4}{3} \ell$ (4) ℓ



13. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω . Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' :

[AIEEE-2006]

- (1) $\frac{\omega m}{(m+2M)}$ (2) $\frac{\omega(m+2M)}{m}$ (3) $\frac{\omega(m-2M)}{(m+2M)}$ (4) $\frac{\omega m}{(m+M)}$

14. Four point masses, each of value m , are placed at the corners of a square $ABCD$ of side ℓ . The moment of inertia about an axis passing through A and parallel to BD is :

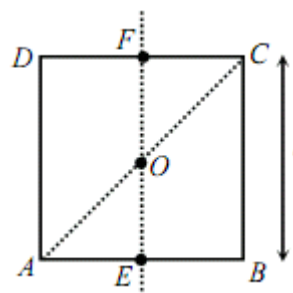
[AIEEE-2006]

- (1) $m\ell^2$ (2) $2m\ell^2$ (3) $\sqrt{3} m\ell^2$ (4) $3m\ell^2$

15. For the given uniform square lamina $ABCD$, whose centre is O ,

[AIEEE-2007; 3/120]

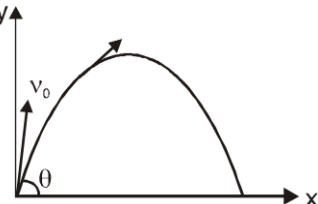
- (1) $\sqrt{2} I_{AC} = I_{EF}$
(2) $I_{AD} = 2I_{EF}$
(3) $I_{AC} = I_{EF}$
(4) $I_{AC} = \sqrt{2} I_{EF}$



16. A round uniform body of radius R , mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then its acceleration is :

[AIEEE-2007; 3/120]

- (1) $\frac{g \sin \theta}{1 + I/MR^2}$ (2) $\frac{g \sin \theta}{1 + MR^2/I}$ (3) $\frac{g \sin \theta}{1 - I/MR^2}$ (4) $\frac{g \sin \theta}{1 - MR^2/I}$

17. Angular momentum of the particle rotating with a central force is constant due to : [AIEEE-2007; 3/120]
- (1) constant force (2) constant linear momentum
(3) zero torque (4) constant torque
18. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is : [AIEEE-2008, 3/105]
- (1) $\frac{1}{12} ma^2$ (2) $\frac{7}{12} ma^2$ (3) $\frac{2}{3} ma^2$ (4) $\frac{5}{6} ma^2$
19. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω . Its centre of mass rises to a maximum height of : [AIEEE-2009, 4/144, -1]
- (1) $\frac{1}{6} \frac{\ell \omega}{g}$ (2) $\frac{1}{2} \frac{\ell^2 \omega^2}{g}$ (3) $\frac{1}{6} \frac{\ell^2 \omega^2}{g}$ (4) $\frac{1}{3} \frac{\ell^2 \omega^2}{g}$
20. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin \theta}{g}$, the angular momentum of the particle is [AIEEE-2010, 4/144, m-1]
- 
- (1) $-mg v_0 t^2 \cos \theta \hat{j}$ (2) $mg v_0 t \cos \theta \hat{k}$
(3) $-\frac{1}{2} mg v_0 t^2 \cos \theta \hat{k}$ (4) $\frac{1}{2} mg v_0 t^2 \cos \theta \hat{i}$
- where \hat{i} , \hat{j} and \hat{k} are unit vectors along x, y and z-axis respectively.
21. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : [AIEEE-2011, 4/120, -1]
- (1) remains unchanged (2) continuously decreases
(3) continuously increases (4) first increases and then decreases
22. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R . Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m , if the string does not slip on the pulley, is : [AIEEE - 2011, 4/120, -1]
- (1) $\frac{3}{2}g$ (2) g (3) $\frac{2}{3}g$ (4) $\frac{g}{3}$
23. A pulley of radius $2m$ is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2 , the number of rotations made by the pulley before its direction of motion if reversed, is : [AIEEE - 2011, 4/120, -1]
- (1) less than 3 (2) more than 3 but less than 6
(3) more than 6 but less than 9 (4) more than 9

24. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is : [AIEEE 2011, 11 May; 4, -1]

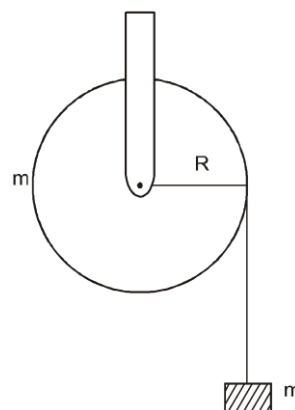
(1) zero (2) $\frac{mv^3}{\sqrt{2}g}$ (3) $\frac{\sqrt{3}}{16} \frac{mv^3}{g}$ (4) $\frac{\sqrt{3}}{2} \frac{mv^2}{g}$

25. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ? [JEE-Main 2013, 4/120, -1]

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

26. A mass 'm' supported by a massless string wound around a uniform hollow cylinder of mass m and radius R . If the string does not slip on the cylinder, with what acceleration will the mass fall on release ? [JEE-Main 2014, 4/120, -1]

(1) $\frac{2g}{3}$
(2) $\frac{g}{2}$
(3) $\frac{5g}{6}$
(4) g



27. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with a angular speed ω rad/s about the vertical. About the point of suspension : [JEE-Main 2014, 4/120, -1]

- (1) angular momentum is conserved.
(2) angular momentum changes in magnitude but not in direction
(3) angular momentum changes in direction but not in magnitude.
(4) angular momentum changes both in direction and magnitude.

28. Form a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is [JEE(Main)-2015; 4/120, -1]

(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$ (3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

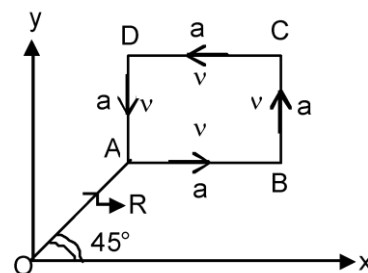
- 29.* A particle of mass m is moving along the side of square of side 'a' with a uniform speed v in the x-y plane as shown in the figure :
Which of the following statements is false for the angular momentum about the origin ? [JEE Main 2016 ; 4/120, -1]

(1) $\vec{L} = mv \left[\frac{R}{\sqrt{2}-a} - a \right] \hat{k}$ when the particle is moving from C to D.

$$\vec{L} = mv \left[\frac{R}{\sqrt{2}-a} + a \right] \hat{k} \quad \text{when the particle is moving from B to C.}$$

$$\vec{L} = \frac{mv}{\sqrt{2}} R \hat{k} \quad \text{when the particle is moving from D to A.}$$

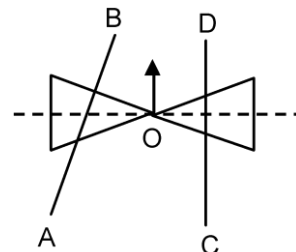
$$\vec{L} = -\frac{mv}{\sqrt{2}} R \hat{k} \quad \text{when the particle is moving from A to B.}$$



30. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :

[JEE Main 2016 ; 4/120, -1]

- (1) turn right
- (2) go straight
- (3) turn left and right alternately
- (4) turn left.



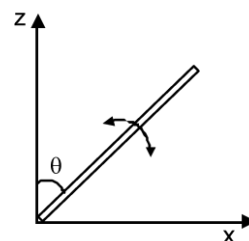
31. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I . What is the ratio l/R such that the moment of inertia is minimum ? [JEE(Main)-2017, 4/120, -1]

- (1) $\frac{3}{\sqrt{2}}$
- (2) $\sqrt{\frac{3}{2}}$
- (3) $\frac{\sqrt{3}}{2}$
- (4) 1

32. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is.

[JEE(Main)-2017, 4/120, -1]

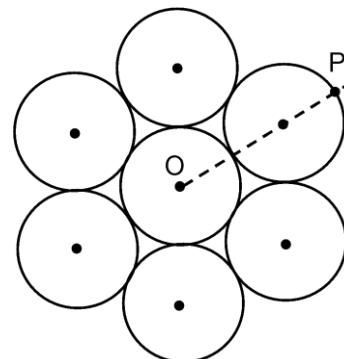
- (1) $\frac{2g}{3l} \cos \theta$
- (2) $\frac{3g}{2l} \sin \theta$
- (3) $\frac{2g}{3l} \sin \theta$
- (4) $\frac{3g}{2l} \cos \theta$



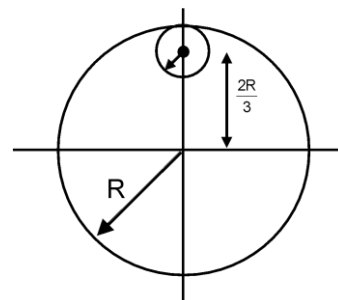
33. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is :

[JEE(Main)-2018, 4/120, -1]

- (1) $\frac{73}{2} MR^2$
- (2) $\frac{181}{2} MR^2$
- (3) $\frac{19}{2} MR^2$
- (4) $\frac{55}{2} MR^2$



34. From a uniform circular disc of radius R and mass $9M$, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is :



[JEE(Main)-2018, 4/120, -1]

- (1) $10MR^2$ (2) $\frac{37}{9}MR^2$
 (3) $4MR^2$ (4) $\frac{40}{9}MR^2$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

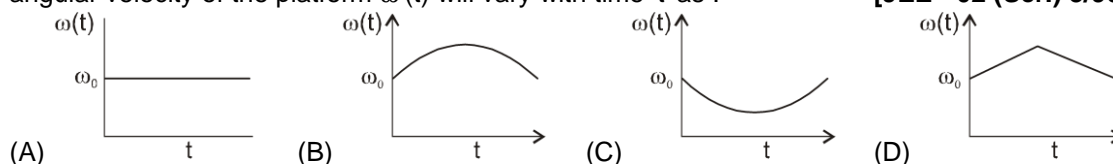
1. A cylinder rolls up an inclined plane, reaches some height and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are :

[JEE - 02 (Scr.) 3/90]

- (A) Up the incline while ascending and down the incline while descending
 (B) Up the incline while ascending as well as descending
 (C) Down the incline while ascending and up the incline while descending
 (D) Down the incline while ascending as well as descending.

2. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now the platform is given an angular velocity ω_0 . When the tortoise moves along a chord of the platform with a constant velocity (with respect to the platform) the angular velocity of the platform $\omega(t)$ will vary with time t as :

[JEE - 02 (Scr.) 3/90]



3. A particle is in uniform circular motion in a horizontal plane. Its angular momentum is constant when the origin is taken at :

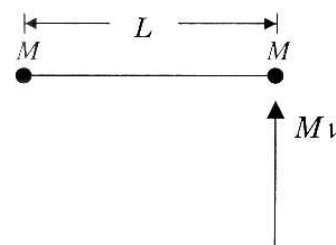
[JEE Sc. 2003' 3/84]

- (A) centre of the circle (B) any point on the circumference of the circle
 (C) any point inside the circle (D) any point outside the circle

4. Two particles, each of mass M , are connected by a rod of negligible mass and length L . The system is lying on a horizontal frictionless surface. An impulse Mv , perpendicular to the rod, is given at one end of the rod as shown in the figure. The angular velocity acquired by the rod is

[JEE Sc. 2003' 3/84]

- (A) $\frac{4v}{L}$ (B) $\frac{2v}{L}$
 (C) $\frac{v}{L}$ (D) $\frac{v}{4L}$



5. A platform is revolving in horizontal plane about a fixed axis and a boy is sitting at centre. The initial kinetic energy of system is K . If the boy stretches his arms then moment of inertia of system becomes double. Final kinetic energy of system is :

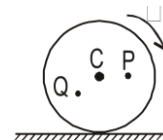
[JEE Sc. 2004' 3/84]

- (A) K (B) $\frac{K}{2}$ (C) $\frac{K}{4}$ (D) $2K$

6. A disc is moving without slipping on ground then the relation between magnitude of velocity of points P, C and Q is [distance CP = CQ]

[JEE Sc. 2004' 3/84]

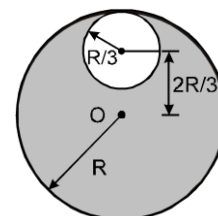
- (A) $Q > C > P$ (B) $P > C > Q$
(C) $C > Q > P$ (D) All will be same



7. A disc has mass $9m$. A hole of radius $\frac{R}{3}$ is cut from it as shown in the figure. The moment of inertia of the remaining part about an axis passing through the centre 'O' of the disc and perpendicular to the plane of the disc is :

[JEE Scr. 2005 , 3/84]

- (A) $8 mR^2$ (B) $4 mR^2$ (C) $\frac{40}{9} mR^2$ (D) $\frac{37}{9} mR^2$



8. A particle moves in circular path with decreasing speed. Which of the following is correct

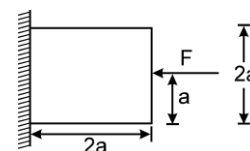
[JEE Scr. 2005 , 3/84]

- (A) \vec{L} is constant
(B) only direction of \vec{L} is constant
(C) acceleration \vec{a} is towards the centre
(D) it will move in a spiral and finally reach the centre

9. A block of mass m is held fixed against a wall by applying a horizontal force F . Which of the following option is incorrect.

[JEE Scr. 2005 , 3/84]

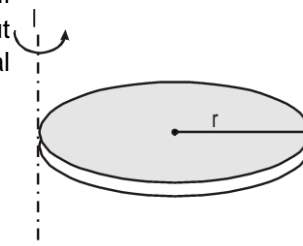
- (A) friction force = mg
(B) normal will not produce torque
(C) F will not produce torque
(D) normal reaction = F



10. A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t . If its moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I , then the value of r is equal to :

[JEE 2006 , 3/184]

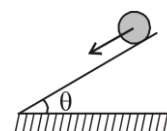
- (A) $\frac{2}{\sqrt{15}} R$ (B) $\frac{2}{\sqrt{5}} R$
(C) $\frac{3}{\sqrt{15}} R$ (D) $\frac{\sqrt{3}}{\sqrt{15}} R$



- 11.* A solid sphere is in pure rolling motion on an inclined surface having inclination θ .

[JEE 2006 , 5/184]

- (A) frictional force acting on sphere is $f = \mu mg \cos \theta$.
(B) f is dissipative force.
(C) friction will increase its angular velocity and decreases its linear velocity.
(D) If θ decreases, friction will decrease.



Comprehension

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and $2I$ respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire

potential energy of a spring compressed by a distance x_1 . Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2 . Both the discs rotate in the clockwise direction. [JEE-2007, 12/162]

12. The ratio x_1/x_2 is

- (A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

13. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t . The average frictional torque on one disc by the other during this period is

- (A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

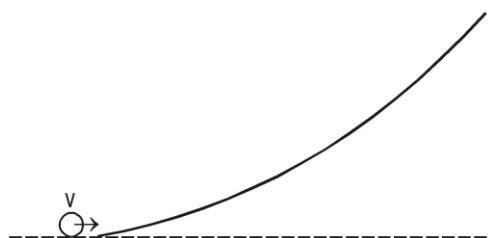
14. The loss of kinetic energy during the above process is

- (A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

15. A small object of uniform density rolls up a curved surface with an initial velocity v . It reaches up to a

maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is

[JEE-2007, 3/162]



- (A) ring (B) solid sphere (C) hollow sphere (D) disc

16. **STATEMENT – 1**

If there is no external torque on a body about its centre of mass, then the velocity of the center of mass remains constant. [JEE-2007, 3/162]

because

STATEMENT – 2

The linear momentum of an isolated system remains constant.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

17. **STATEMENT -1**

Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first. [JEE-2008, 3/163]

and

STATEMENT -2

By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is a correct explanation for STATEMENT -1

(B) STATEMENT -1 is True, STATEMENT -2 is True; STATEMENT -2 is NOT a correct explanation for STATEMENT -1

(C) STATEMENT -1 is True, STATEMENT -2 is False

(D) STATEMENT -1 is False, STATEMENT -2 is True.

18.▲ If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009, 4/160, -1]**

(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

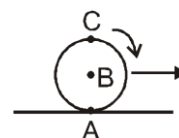
19*.▲ A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[JEE 2009, 4/160, -1]**

(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4|\vec{V}_B|$



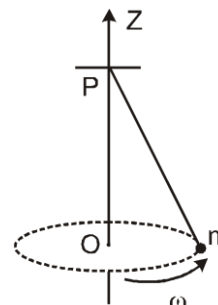
20.▲ A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x - y plane with centre at O and constant angular speed ω . If the angular momentum of the system, calculated about O and P are denoted by \vec{L}_O and \vec{L}_P respectively, then **[IIT-JEE-2012, Paper-1; 3/70, -1]**

(A) \vec{L}_O and \vec{L}_P do not vary with time.

(B) \vec{L}_O varies with time while \vec{L}_P remains constant.

(C) \vec{L}_O remains constant while \vec{L}_P varies with time.

(D) \vec{L}_O and \vec{L}_P both vary with time.



21.▲ Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement (s) is (are) correct? **[IIT-JEE-2012, Paper-2; 4/66]**

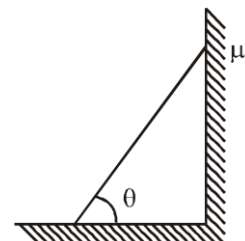
(A) Both cylinders P and Q reach the ground at the same time

(B) Cylinder P has larger linear acceleration than cylinder Q.

(C) Both cylinder Q reaches the ground with same translational kinetic energy.

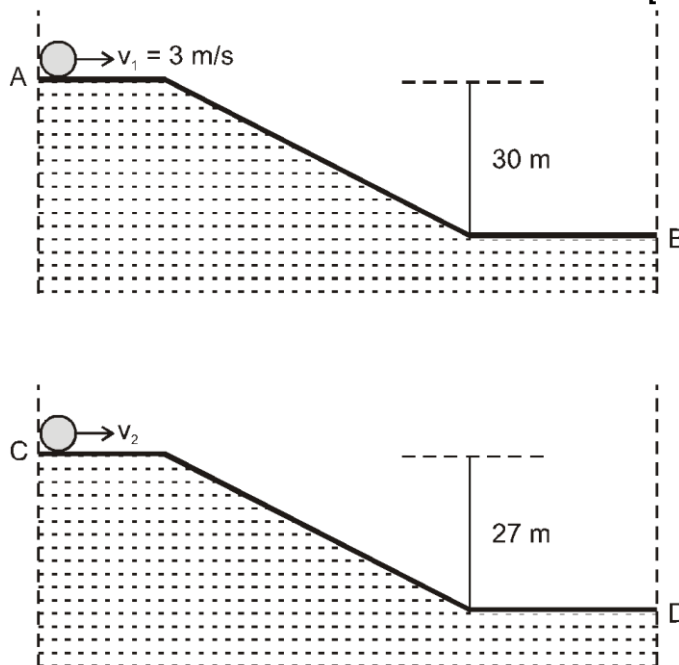
(D) Cylinder Q reaches the ground with larger angular speed.

22.▲ In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2 . The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2 . If the ladder is about to slip, then **[JEE (Advanced)-2014, P-1, 3/60]**

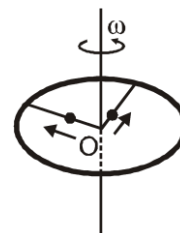


- (A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan \theta = \frac{mg}{2}$ (B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan \theta = \frac{mg}{2}$
- (C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1 + \mu_1 \mu_2}$ (D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan \theta = \frac{mg}{2}$

23. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 , respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3 \text{ m/s}$, then v_2 in m/s is $(g = 10 \text{ m/s}^2)$ [JEE(Advanced) 2015 ; 4/88]



24. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O . These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}R$ from O . At this instant the distance of the other mass from O is :



[JEE(Advanced) 2015 ; P-1, 4/88, -2]

- (A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$
- (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

25. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B , respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is : [JEE(Advanced) 2015 ; P-2 4/88]

26. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height h ($< l$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction

of the floor on the stick. The ratio $\frac{h}{l}$ and the frictional force f at the bottom of the stick are ($g = 10 \text{ ms}^{-2}$)

- (A) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$ (B) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$ (C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} \text{ N}$ (D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} \text{ N}$

27. The position vector \vec{r} of particle of mass m is given by the following equation

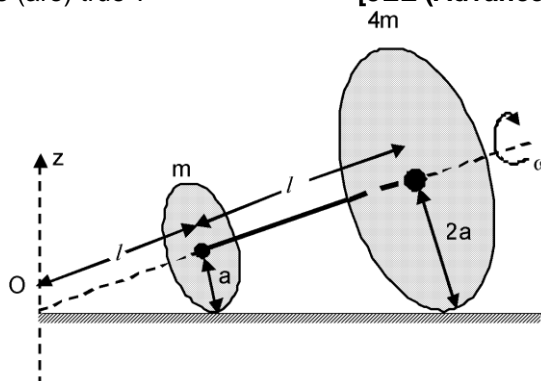
$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$$

Where $\alpha = \frac{10}{3} \text{ ms}^{-3}$, $\beta = 5 \text{ m s}^{-2}$ and $m = 0.1 \text{ kg}$. At $t = 1 \text{ s}$, which of the following statement(s) is (are) true about the particle.

- (A) The velocity \vec{v} is given by $\vec{v} = (10\hat{i} + 10\hat{j}) \text{ ms}^{-1}$
 (B) The angular momentum \vec{L} with respect to the origin is given by $\vec{L} = -(5/3)\hat{k} \text{ N ms}$
 (C) The force \vec{F} is given by $\vec{F} = (\hat{i} + 2\hat{j}) \text{ N}$
 (D) The torque with respect to the origin is given by $\vec{\tau} = -\frac{20}{3}\hat{k} \text{ Nm}$.

28. Two thin circular discs of mass m and $4m$, having radii of a and $2a$, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω . The angular momentum of the entire assembly about the point 'O' is (see the figure). Which of the following statement (s) is (are) true ?

[JEE (Advanced) 2016 ; P-2, 4/62, -2]



- (A) The magnitude of the z-component of \vec{L} is $55 ma^2 \omega$.
 (B) The magnitude of angular momentum of the assembly about its centre of mass is $17 ma^2 \frac{\omega}{2}$.
 (C) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81 ma^2 \omega$.
 (D) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\frac{\omega}{5}$.

Answers

EXERCISE-1

Section (A)

- A-1. (3) A-2. (4) A-3. (3)
 A-4. (2) A-5. (3)

Section (B)

- B-1. (2) B-2. (2) B-3. (3)
 B-4. (4) B-5. (4) B-6. (2)
 B-7. (4) B-8. (2) B-9. (3)

B-10. (1)	B-11. (1)	B-12. (1)
B-13. (3)	B-14. (3)	B-15. (3)
Section (C)		
C-1. (2)	C-2. (2)	C-3. (1)
C-4. (3)	C-5. (3)	C-6. (3)
Section (D)		
D-1. (2)	D-2. (3)	D-3. (3)
D-4. (A)		
Section (E)		
E-1. (4)	E-2. (1)	E-3. (3)
E-4. (4)		
Section (F)		
F-1. (3)	F-2. (2)	
Section (G)		
G-1. (2)	G-2. (2)	G-3. (4)
G-4. (4)	G-5. (3)	G-6. (1,3,4)
G-7. (2)	G-8. (1)	G-9. (1)
G-10. (4)	G-11. (3)	G-12. (3)
G-13. (3)	G-14. (3)	G-15. (2)
G-16. (4)	G-17. (3)	
Section (H)		
H-1. (2)	H-2. (2)	H-3. (3)
H-4. (1)		
Section (I)		
I-1. (1)	I-2. (3)	I-3. (3)
I-4. (2)	I-5. (2)	I-6. (2)
I-7. (4)		
Section (J)		
J-1. (4)	J-2. (3)	

EXERCISE-2

PART - I

1. (4)	2. (2)	3. (4)
4. (3)	5. (4)	6. (3)
7. (2)	8. (4)	9. (3)

10. (1)	11. (4)	12. (2)
13. (1)	14. (3)	15. (4)
16. (2)	17. (3)	18. (3)
19. (4)	20. (1)	21. (2)
22. (4)	23. (2)	24. (3)
25. (3)	26. (4)	27. (1)
28. (3)	29. (3)	

PART – II

Section (A)

A-1. (4)	A-2. (4)	A-3. (1)
-----------------	-----------------	-----------------

Section (B)

B-1. (1 \rightarrow s), (2 \rightarrow p), (3 \rightarrow q), (4 \rightarrow r)

Section (C)

C-1. (1,2,3)	C-2. (1,2,3)	C-3. (1,2,3,4)
C-4. (2,3)	C-5. (1,3,4)	C-6. (1,2,3)
C-7. (1,2,4)		

EXERCISE-3

PART - I

1. (3)	2. (4)	3. (1)
4. (3)	5. (4)	6. (1)
7. (4)	8. (2)	9. (3)
10. (4)	11. (4)	12. (3)
13. (1)	14. (4)	15. (3)
16. (1)	17. (3)	18. (3)
19. (3)	20. (3)	21. (4)
22. (3)	23. (2)	24. (3)
25. (3)	26. (2)	27. (3)
28. (3)	29. (1, 3)	30. (4)
31. (2)	32. (2)	33. (2)
34. (3)		

PART – II

1. (B)	2. (B)	3. (A)
4. (C)	5. (B)	6. (B)
7. (B)	8. (B)	9. (B)
10. (A)	11. (C,D)	12. (C)
13. (A)	14. (B)	15. (D)
16. (D)	17. (D)	18. (A)
19. (B)	20. (C)	21. (D)
22. (C,D)	23. 7	24. (D)
25. 6	26. (D)	27. (A,B,D)
28. (B,D)		