Exercise-1

Section (A) : Coordinate system, Distance formula, Section formula

A-1. **Sol.** $PQ = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)} = \sqrt{4+9-6} = \sqrt{7}$

A-2. **Sol.** The points are (0, 1) and (x, –3)
Distance between them = 5
$\sqrt{x^2 + 4^2} = 5$
x = ± 3

A-3. **Sol.** A(–2, 2), B (8, –2), C(–4, –3)
AB = $\sqrt{116}$, BC = $\sqrt{145}$, CA = $\sqrt{29}$
Now $AB^2 + AC^2 = BC^2$
Δ is a right angle Δ at A

A-4 **Sol.** (a, –b), (0, 0), (–a, b) and $(ab - b^2)$
For rectangle, parallelogram, square diagonal bisect each other but for all the 4-coordinates no pair bisecting the other pair hence it does not represent rectangle, square, parallelogram.

A-5. **Sol.** AB = $\sqrt{4+9} = \sqrt{13}$
BC = $\sqrt{36+16} = 2\sqrt{13}$
CD = $\sqrt{4+9} = \sqrt{13}$
AD = $\sqrt{36+16} = 2\sqrt{13}$
AC = $\sqrt{64+1} = \sqrt{65}$
BD = $\sqrt{16+49} = \sqrt{65}$
its rectangle

A-6. **Sol.** In parallelogram diagonal bisect each other
$\frac{3-6}{2} = \frac{x-2}{2} \Rightarrow x = -1$
$\frac{5-4}{2} = \frac{y+1}{2} \Rightarrow y = 0 \Rightarrow P \equiv (-1, 0)$

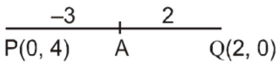
A-7. **Sol.** Point is $\left(\frac{5\lambda+3}{\lambda+1}, \frac{6\lambda+4}{\lambda+1}\right)$
If x-axis divide put y = 0
$\frac{6\lambda+4}{\lambda+1} = 0 \Rightarrow \lambda = -\frac{2}{3}$

A-8. **Sol.** Let required point are P & Q

$P\left(\frac{9+2\times0}{1+2}, \frac{1\times12+2\times0}{1+2}\right)$ divides in 1 : 2

P ≡ (3, 4)
Q divides in 2 : 1
Hence $Q\left(\frac{2\times9+1\times0}{2+1}, \frac{2\times12+1+0}{2+1}\right) \equiv Q\,(6, 8)$

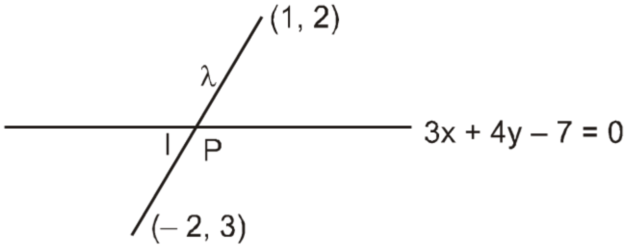
A-9. **Sol.**



$A \equiv \left[\frac{-6}{-1}, \frac{8}{-1}\right] = [6, -8]$

A-10. **Sol.** Mid point of AB is midpoint PQ.

A-11. **Sol.**



Let division point is P.

coordinate of P $\left(\frac{-2\lambda+1}{\lambda+1}, \frac{3\lambda+2}{\lambda+1}\right)$

P lies on given line

$3\left(\frac{-2\lambda+1}{\lambda+1}\right) + 4\left(\frac{3\lambda+2}{\lambda+1}\right) - 7 = 0$

$\Rightarrow \quad -6\lambda + 3 + 12\lambda + 8 - 7\lambda - 7 = 0$

$\Rightarrow \quad -\lambda + 4 = 0$

$\Rightarrow \quad \lambda = 4$

A-12. **Sol.** Let k .1

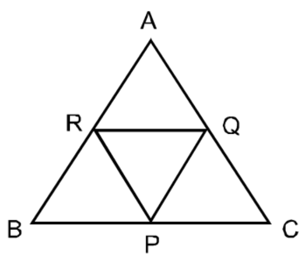
$\frac{5k+2}{k+1} = 4$

$\therefore k = 2$

Harmonic conjagate is $\left(\frac{2\times2-1\times2}{2-1}, \frac{2\times5-1\times2}{2-1}\right) \equiv (2,8)$

Section (B) : Area of triangle, Locus, Change of origin, Slope of line, Collinearty

B-1. **Sol.**



Area of the triangle formed by joining the mid points of the sides of the triangle = $\frac{1}{4}$ (area of the triangle)

$= \frac{1}{4} \times \frac{1}{2}\begin{vmatrix} 2 & 1 & 1 \\ -2 & 3 & 1 \\ 4 & -3 & 1 \end{vmatrix}$

$= \frac{1}{4} \times 6 = 1.5$ sq.units

B-2. Sol.

B-3. Sol. $h = \frac{20\cos\theta + 15}{5} = 4\cos\theta + 3$

$k = \frac{20\sin\theta}{5} = 4\sin\theta$

Locus is $\left(\frac{h-3}{4}\right)^2 + \left(\frac{k}{4}\right)^2 = 1$

$(x-3)^2 + y^2 = 16$

B-5. Sol. Let point (h,k)

Distance from y-axis = |h|

------------------- x ----- = (k)

$\therefore h^2+k^2 = 3$

$x^2 + y^2 = 3$

B-6. Sol. x = x+h, y = y +k

9 = – 3+h, 5= 9+k

h = 12, k =– 4

∴ (12, – 4)

B-7. Sol. x = x+h & y = y+k

$(x+h)^2 + 4(x+h) + 8(y+k) - 2 = 0$

$x^2 + (2h+4)x + 8y + (h^2 + 4h + 8k - 2) = 0$

$\therefore 2h + 4 = 0$ & $h^2 + 4h + 8k - 2 = 0$

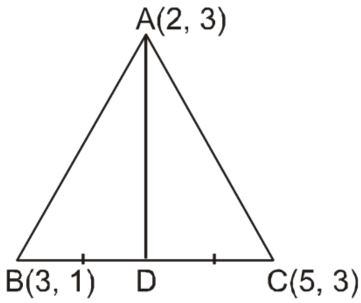
h = – 2 4– 8 + 8k – 2 = 0

$k = \frac{6}{8} = \frac{3}{4}$

$\left(-2, \frac{3}{4}\right)$

B-8. Sol. D is the mid point of BC, $D = \left[\frac{3+5}{2}, \frac{1+3}{2}\right] = [4, 2]$

Slope of AD $= \frac{3-2}{2-4} = -\frac{1}{2}$



B-9. **Sol.** Slope) = $\frac{k+1-3}{k^2-5} = \frac{1}{2}$

⇒ $k = 1 \pm \sqrt{2}$

B-10. **Sol.** Since A, B, C are collinear

Slope of AB = Slope of BC   A(k, 2 – 2k)   B(1 – k, 2k)   C(– k– 4, 6 – 2k)

$\frac{2-2k-2k}{k-1+k} = \frac{2k-6+2k}{1-k+k+4}$

⇒ $\frac{2-4k}{2k-1} = \frac{4k-6}{5}$

⇒ 10 – 20k = (4k – 6)(2k – 1)

⇒ (4k – 6)(2k – 1) + 10(2k – 1) = 0

⇒ k = , – 1

Section (C) : Various forms of straight line , Point and line, Angle between two lines

C-1. **Sol.** Points are (0, 0) and (a cos θ , a sin θ)

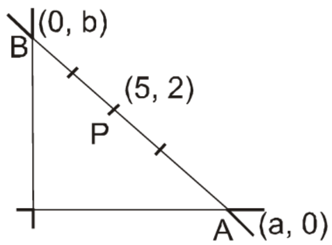
$\therefore (y-0) = \frac{a\sin\theta - 0}{a\cos\theta - 0}(x-0)$

$y = x \tan\theta$

C-2. **Sol.** If P bisect the line AB, then a = 10 and b = 4

Equation of the line is $\frac{x}{10} + \frac{y}{4} = 1$

2x + 5y = 20



C-3. **Sol.** Let line is y = mx + c, then m = tan 45° = 1 and c = 1

C-5. **Sol.** Slope of such line is ± 1

C-6. **Sol.** x cos α + y sin α = p

x cos 30º + y sin 30º = 4

$\sqrt{3}x + y = 8$

C-7. **Sol.** Point (–4, 5) does not lie on the diagonal 7x–y+8= 0, so point will lie on the other diagonal also diagonals are perpendicular

∴ Slope of other diagonal = –1/7

∴ Equation of other diagonal is

$y-5 = -\frac{1}{7}(x+4) \Rightarrow 7y+x=31.$

C-8. **Sol.** Mid point ≡ (3, 2). Equation is 2x–y–4 = 0.

C-9. **Sol.** Here $\tan\theta = \frac{1}{5}$

∴ $\tan 2\theta = \frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2} = \frac{5}{12}$

∴ required line $y = \frac{5x}{12}$

C-10. Sol. Any line ⊥ to $2x - 3y = 5$ has slope $= -\frac{3}{2}$

equation of line is $(y + 1) = -\frac{3}{2}(x - 1)$

$2y + 2 = -3x + 3$

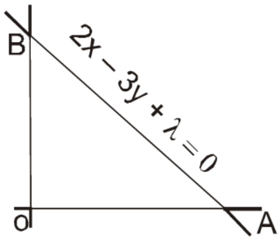
$3x + 2y = 1$

C-11. Sol. Any line ⊥ to $2x - y + 3 = 0$ has slope $= -\frac{1}{2}$

Slope of line through (4, 3) and (2, λ) is $\frac{\lambda - 3}{2 - 4} = \frac{\lambda - 3}{-2}$

$\frac{\lambda - 3}{-2} = -\frac{1}{2} \quad \Rightarrow \quad \lambda = 4$

C-12. Sol.



Any line parallel to $2x - 3y = 4$ is $2x - 3y + \lambda = 0$

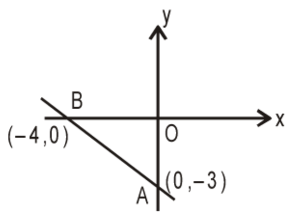
$A \equiv \left[-\frac{\lambda}{2}, 0\right]$, $B \equiv \left[0, \frac{\lambda}{3}\right]$

area of $\Delta AOB = \frac{1}{2}\left|\left(\frac{\lambda}{2}\right)\left(\frac{\lambda}{3}\right)\right| = 12$

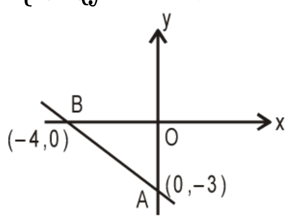
$\lambda_2 = 144 \quad \Rightarrow \quad \lambda = 12, -12$

line is $2x - 3y + 12 = 0$ and $2x - 3y - 12 = 0$

C-13. Sol. area of $\Delta OAB = \frac{1}{2} \times 3 \times 4 = 6$ sq. units

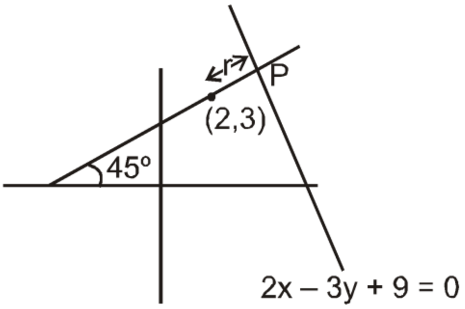


ΔOAB dk {ks=kQy $= \frac{1}{2} \times 3 \times 4 = 6$



C-14. Sol. Slope of DE = $\frac{7-3}{5-1}$ =1
⇒ Slope of AB = 1

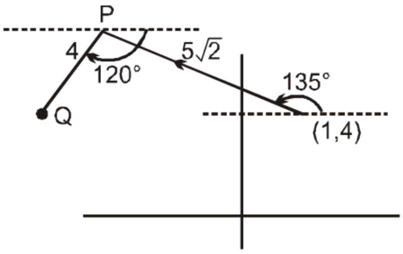
C-15. Sol.



Let coordinates of point P by parametric form
P(2 + r cos 45º, 3 + r sin 45º)
It satisfies the line 2x – 3y + 9 = 0

$\therefore 2\left(2+\frac{r}{\sqrt{2}}\right) - 3\left(3+\frac{r}{\sqrt{2}}\right) + 9 = 0$ ⇒ $r = 4\sqrt{2}$

C-16. **Ans.** $\left(-6,\ 9-2\sqrt{3}\right)$
Sol.



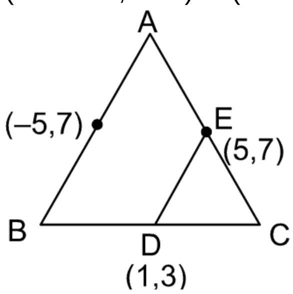
For point P $\frac{x-1}{\cos 135^\circ} = \frac{y-4}{\sin 135^\circ} = 5\sqrt{2}$ ⇒ x = – 4, y= 9
For point Q
$\frac{x+4}{\cos(-120^\circ)} = \frac{y-9}{\sin(-120^\circ)} = 4$ ⇒ x = – 6, y = $9-2\sqrt{3}$
∴ Q$\left(-6,\ 9-2\sqrt{3}\right)$.

C-17. Sol. Here (x_1,y_1) = (2,3), θ = 30º, the equation of the line is
$\frac{x-2}{\cos 30^\circ} = \frac{y-3}{\sin 30^\circ}$

⇒ $\frac{x-2}{\frac{\sqrt{3}}{2}} = \frac{y-3}{\frac{1}{2}}$

⇒ x – 2 = $\sqrt{3}$ (y – 3)
⇒ x – $\sqrt{3}$ y = 2 – 3$\sqrt{3}$
Points on the line at a distance 4 from P (2,3) are
$(x_1 \pm r\cos\theta\ ,\ y_1 \pm r\sin\theta)$
⇒ (2 ± 4 cos30º, 3 ± 4 sin30º)

⇒ $(2 \pm 2\sqrt{3}, 3 \pm 2) \Rightarrow (2 + 2\sqrt{3}, 5)$ or $(2 - 2\sqrt{3}, 1)$



Hence equation of AB is $y - 7 = (x+5) \Rightarrow x - y + 12 = 0$

C-18. Sol. Line is $L_1 = 3x - 8y - 7 = 0$

(i)	(0, –1)	$L_1 > 0$	
	(0, 0)	$L_1 < 0$	opp side
(ii)	(4, –3)	$L_1 > 0$	
	(0, 1)	$L_2 < 0$	opp side
(iii)	(–3, –4)	$L_1 > 0$	
	(1, 2)	$L_2 < 0$	opp side
(iv)	(–1, –1)	$L_1 < 0$	
	(3, 7)	$L_2 < 0$	Same side ,

C-19. Sol. condition for (x_1, y_1) & (x_2, y_2) lying on the same side w.r.t. $ax + by + c = 0$

$\dfrac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0 \quad \Rightarrow \quad \dfrac{1}{a^2 + ab + 1} > 0$

$a^2 + ab + 1 > 0$ ... (i)

It is quadratic in a

∴ (i) will be true $\forall\, a \in R$, if

$b^2 - 4 < 0 \quad \Rightarrow \quad b \in (-2, 0) \cup (0, 2)$

but $b > 0 \quad \Rightarrow \quad b \in (0, 2)$

C-20. Sol. $2x + 3y = 5 \qquad m_1 = -\dfrac{2}{3}$

$3x - 2y = 7 \qquad m_2 = \dfrac{3}{2}$

$m_1 m_2 = -1$.

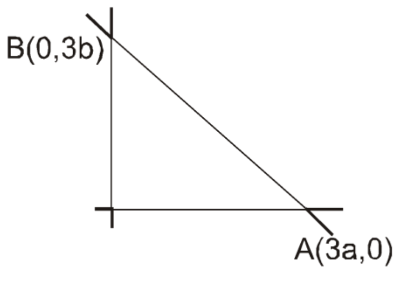
Hence they are ⊥ angle between them is 90º

Section (D) : Centroid, Circumcenter Orthocenter, Incenter, Excenter

D-1. Sol. AB : $bx + ay = 3ab$

A : (3a, 0)

B : (0, 3b)

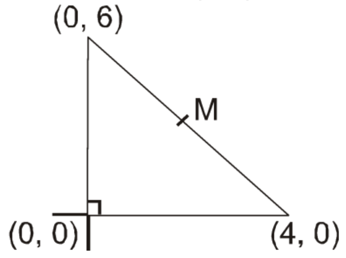


Centroid of ΔOAB $\equiv \left(\dfrac{3a}{3}, \dfrac{3b}{3}\right) \equiv (a, b)$

$\Delta OAB \equiv \left(\frac{3a}{3}, \frac{3b}{3}\right) \equiv (a, b)$

D-2. **Sol.** In a right triangle circumcentre is the mid point of the hypotenuse

$\therefore M \equiv \left(\frac{4+0}{2}, \frac{0+6}{2}\right) \equiv (2, 3)$



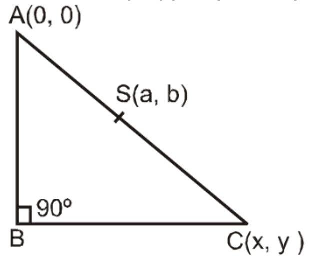
D-3. **Sol.** Since in ΔABC, B is orthocentre. Hence ∠B = 90º

Circumcentre is S(a, b)

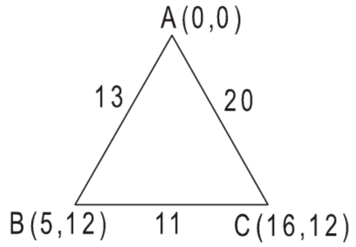
$\frac{x+0}{2} = a \Rightarrow x = 2a$

$\frac{y+0}{2} = b \Rightarrow y = 2b$

Hence, C(x, y) ≡ (2a, 2b)



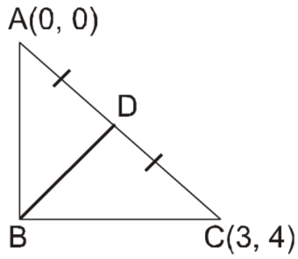
D-4. **Sol.** $I \left(\frac{0\times11+5\times20+16\times13}{13+20+11}, \frac{0\times11+12\times20+13\times12}{13+20+11}\right) \equiv (7, 9)$



D-5. **Sol.** In ΔABC right angle at B we have

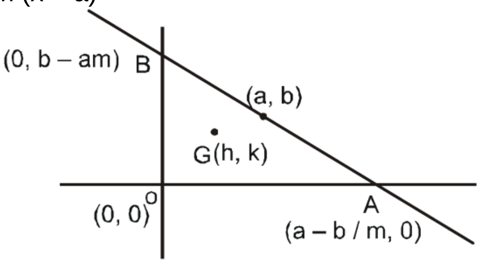
$BD = AD = DC = \frac{AC}{2}$

Hence $BD = \frac{5}{2}$



D-6. **Sol.** equation of line AB

y – b = m (x – a)



$\therefore$ $G\left(\dfrac{a-\frac{b}{m}}{3}, \dfrac{b-am}{3}\right)$ $\Rightarrow$ $h = \dfrac{a-\frac{b}{m}}{3}$, $k = \dfrac{b-am}{3}$

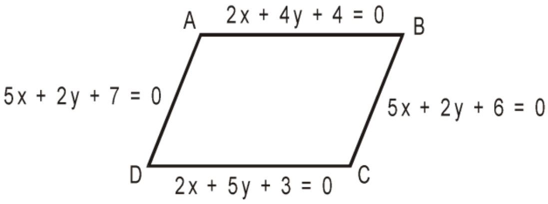
on eleminating 'm' we get required locus

bh + ak – 3hk = 0

$\Rightarrow$ bx + ay – 3xy = 0

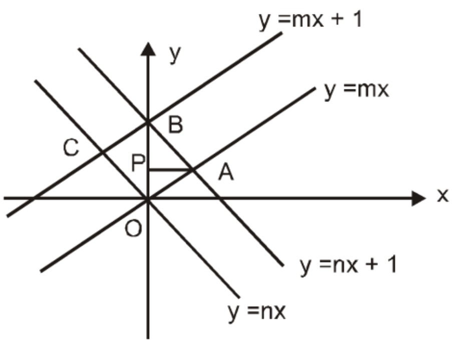
Section (E) : Distance between parallel lines, Foot of the perpendicular image of a point and Area of parallelogram

E-1. **Sol.** Distance between lines AB & CD = Distance between lines AD and BC



$\Rightarrow$ ABCD is a rhombus, also side AD is not perpendicular to DC hence not a square.

E-2.



Sol.

In parallelogram OABC

B(0,1) and point A in the point of intersection of y = mx and y = nx + 1

$\Rightarrow$ $x = \dfrac{1}{m-n}$ and $y = \dfrac{m}{m-n}$

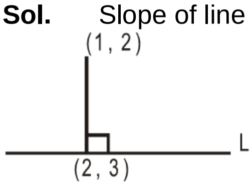
Now area of parallelogram = 2 (ΔOAB)

$= \left| 2\left(\frac{1}{2}\times 1\times \frac{1}{m-n}\right) \right|$

$= \frac{1}{|m-n|}$

E-4. Sol. Image of A(1, 2) in line mirror y = x is (2, 1)
Image of B(2, 1) in y = 0 (x – axes) is (2, – 1)
Hence, $\alpha = 2, \beta = -1$

E-5. Sol. Slope of line L is = – 1 and its equation is y – 3 = – 1(x – 2)



E-6. Sol.

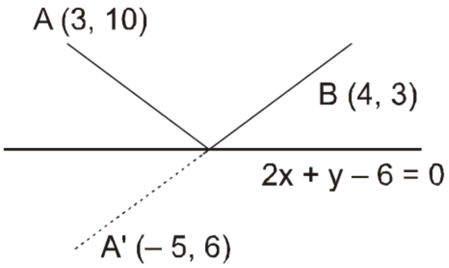


Image of A(3, 10) in 2x + y – 6 = 0

$\frac{x-3}{2} = \frac{y-10}{1} = -2\left(\frac{6+10-6}{2^2+1^2}\right)$

$\frac{x-3}{2} = \frac{y-10}{1} = -4$

A' = (– 5, 6)

Equation of A'B is $y - 3 = \left(\frac{6-3}{-5-4}\right)(x-4)$

$y - 3 = -\frac{1}{3}(x-4)$

$3y - 9 = -x + 4$

$\Rightarrow x + 3y - 13 = 0$

Section (F) : Angle bisectors, concurrent lines and family of lines

F-1. Sol. after making constant terms positive, equation of lines are

$3x - 4y + 7 = 0$ ... (i)

$-12x - 5y + 2 = 0$ ... (ii)

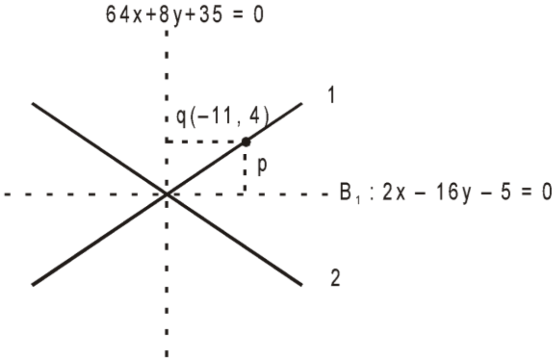
$\because$ $a_1a_2 + b_1b_2 = -36 + 20 < 0$

$\therefore$ equation of acute angle bisector is

$\frac{3x-4y+7}{5} = +\frac{-12x-5y+2}{13}$

$\Rightarrow$ $11x - 3y + 9 = 0$

F-2. **Sol.**



$p = \left|\frac{-22-64-5}{\sqrt{2^2+(-16)^2}}\right| = \frac{91}{\sqrt{260}}$

$q = \left|\frac{-64\times 11+8\times 4+35}{\sqrt{64^2+8^2}}\right| = \frac{637}{2\sqrt{260}}$

$p < q$ Hence $2x - 16y - 5 = 0$ is acute angle bisector

F-3. **Sol.** $\begin{vmatrix} p-r & q-r & r-p \\ q-r & r-p & p-q \\ r-p & p-q & q-r \end{vmatrix} = \begin{vmatrix} 0 & q-r & r-p \\ 0 & r-p & p-q \\ 0 & p-q & q-r \end{vmatrix} = 0$

Hence the lines are concurrent.

Aliter : Since sum of the coefficient of x, y and the constant term is zero, hence the lines are concurrent.

F-4. **Sol.** (d) $t = -\left(\frac{a^2+16}{a}\right) = -\left(a+\frac{16}{a}\right)$

F-5. **Sol.** (a) $x + y = 10$

$2x + y = 18$

and $4x - 3y = 26$

are equations of three lines respectively

Solving of equation (i) and (ii), we get x = 8 and y = 2

Put the values of x and y in equation (iii),

L. H.S = $4x - 3y = 4 \times 8 - 3 \times 2 = 32 - 6 = 26$ = R.H.S.

∴Point (8, 2) lies on line $4x - 3y = 26$, so these three lines are concurrent. Hence, these three equations have one and only one solution

F-6. **Sol.** $ax + by + c = 0$

$\frac{3a}{4} + \frac{b}{2} + c = 0$

compare both $(x, y) \equiv \left(\frac{3}{4}, \frac{1}{2}\right)$

Hence given family passes through $\left(\frac{3}{4}, \frac{1}{2}\right)$

F-7. **Sol.** Equation of the line through the point of intersection of the lines y = 3 and x + y = 0 is

$x + y + \lambda (y - 3) = 0$

this is parallel to the line $2x - y = 4$

$\frac{-1}{1+\lambda} = 2$

$-1 = 2 + 2\lambda \qquad \lambda = -\frac{3}{2}$

$2(x + y) - 3(y - 3)$

$2x - y + 9 = 0$

F-8. **Sol.** $x(a + 2b) + y (a + 3b) = a + b$

$a(x + y) + b (2x + 3y) = a + b$

$x + y = 1$ and $2x + 3y = 1$

$P \equiv (2, -1)$

F-9. **Sol.** Let the line be $\frac{x}{a} + \frac{y}{b} = 1$

given $\frac{1}{a} + \frac{1}{b} = \frac{1}{p}$

$\frac{x}{a} + \frac{y}{b} = \frac{p}{a} + \frac{p}{b}$

$\frac{1}{a}(x - p) + \frac{1}{b}(y - p) = 0$

fixed point is (p, p)

F-10. **Sol.** The lines passing throug the intersection of the lines

$ax + 2by + 3b + \lambda(bx - 2ay - 3a) = 0$

$\Rightarrow \quad (a + b\lambda) x + (2b - 2a\lambda)y + 3b - 3\lambda a = 0$

$y\left(2b + \frac{2a^2}{b}\right) + 3b\frac{3a^2}{b} = 0$,

$y\left(\frac{2b^2 + 2a^2}{b}\right) + 3b\frac{3a^2}{b} = 0, \quad \left(\frac{2b^2 + 2a^2}{b}\right) = \left(\frac{3b^2 + 3a^2}{b}\right)$

$y = \frac{-3\left(a^2 + b^2\right)}{2\left(b^2 + a^2\right)} = \frac{-3}{2}$, $y = -\frac{3}{2}$

Section (G) : Pairs of lines and homogenization

G-1. **Sol.** Given equation is $4x^2 - 24xy + 11y^2 = 0$

$4x^2 - 22xy + 2xy + 11y^2 = 0$

$2x(x - 11y) - 2y(x - 11y) = 0 \qquad \Rightarrow (2x - 2y)(x - 11y) = 0$

$2x - 2y = 0$ or $x - 11y = 0$, $\therefore y = x$ or $y = \frac{x}{11}$

G-2. **Sol.** $m_1 + m_2 = -10$

$m_1m_2 = \frac{a}{1}$

given $m_1 = 4m_2 \Rightarrow \qquad m_2 = -2, m_1 = -8,$

$a = 16$

G-3. **Sol.** $\sqrt{3}\, x^2 - 4xy + \sqrt{3}\, y^2 = 0$

Pair of angle bisectors are $\frac{x^2 - y^2}{\sqrt{3} - \sqrt{3}} = \frac{xy}{(-2)}$

$\Rightarrow \quad x^2 - y^2 = 0$

$\Rightarrow \quad y^2 - x^2 = 0$

G-4. Sol. Equation $ax^2 + (a + b)xy + by^2 + x + y = 0$ can be written as $(ax + by + 1)$ $(x + y) = 0$

G-5. Sol. (A) Comparing given equation with

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ we get

$a = 1, \; h = -1/2, b = -6, g = \frac{7}{2}, \; f = \frac{31}{2}, \; c = -18$

Now angle between the lines

$\theta = \tan^{-1}\left|\frac{2\sqrt{\left(\frac{-1}{2}\right)^2 - 1\times(-6)}}{1+(-6)}\right| = \tan^{-1}\left|\frac{2\sqrt{\left(\frac{1}{4}\right)+6}}{-5}\right|$

$= \tan^{-1}\left|\frac{\left(2\times\frac{5}{2}\right)}{-5}\right| \tan^{-1}|-1| = \tan^{-1}(1) = \frac{\pi}{4}$ or 45°

G-6. Sol. $2x^2 + kxy - 3y^2 - x - 4y - 1 = 0$ represent a pair of lines then D = 0

$2x^2 + kxy - 3y^2 - x - 4y - 1 = 0 \quad D = 0$

$a = 2, b = -3, h = \frac{k}{2}, g = -\frac{1}{2}, f = -2, c = -1$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$6 + 2[-2]\left[-\frac{1}{2}\right]\left[\frac{k}{2}\right] = 2.4 - \frac{3}{4} - \frac{k^2}{4}$

$6 + k = 8 - \frac{3}{4} - \frac{k^2}{4}$

$-(k^2 + 3) = 4(k + 6) - 32$

$-k^2 - 3 = 4k - 8$

$-k^2 - 4k + 5 = 0$

$k^2 + 4k - 5 = 0$

$k = -5, 1$

G-7. Sol. To represent pair of straight lines $\begin{vmatrix} 2 & -1 & -3 \\ -1 & -1 & 3 \\ -3 & 3 & c \end{vmatrix} = 0 \quad \Rightarrow \quad c = 3$

G-8. Sol. Homogenize given curve with given line

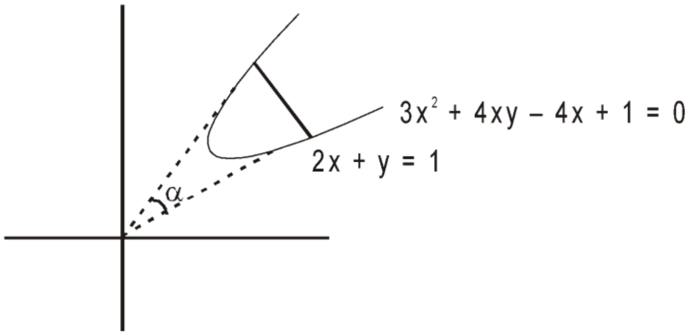
$3x^2 + 4xy - 4x(2x + y) + 1(2x + y)^2 = 0$

$3x^2 + 4xy - 8x^2 - 4xy + 4x^2 + y^2 + 4xy = 0$

$-x^2 + 4xy + y^2 =$

coeff. x^2 + coeff. $y^2 = 0$

Hence angle is 90º



G-9. **Sol.** Lines represented by given equation are x + y + a = 0 and x + y – 9a = 0

$\therefore$ distance between these parallel lines is = $\frac{10a}{\sqrt{2}} = 5\sqrt{2}a$

Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

Single choice type

1. **Sol.** $\frac{3a}{8} = -4$

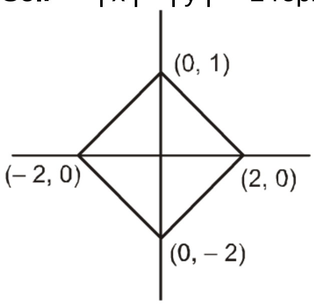
$a = -\frac{32}{3}$

$\frac{5b}{8} = 3$ $\quad$ $b = \frac{24}{5}$

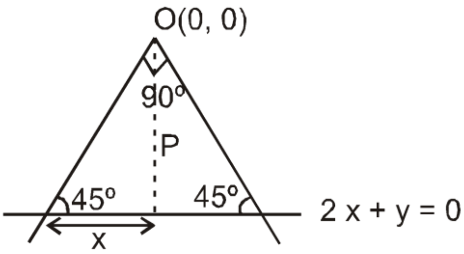
$-\frac{3x}{32} + \frac{5y}{24} = 1 \Rightarrow -9x + 20y = 96$

$\Rightarrow 9x - 20y + 96 = 0$

2. **Sol.** | x | + | y | = 2 represent a square of side = $2\sqrt{2}$ Hence area = 8



3. **Sol.**

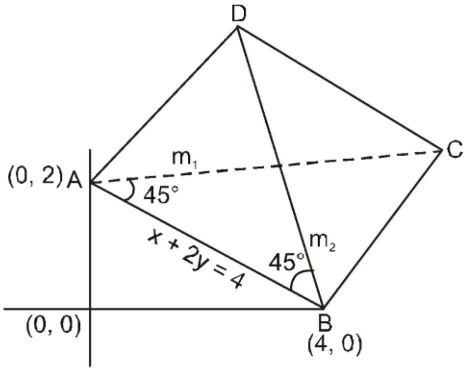


p = $\left|\frac{0+0-a}{\sqrt{5}}\right| = \frac{a}{\sqrt{5}}$

tan 45º = $\frac{p}{x}$ ⇒ p = x

Hence area = $\frac{1}{2}(2x)(p) = px = p^2 = a^2/5$

4. **Sol.**



tan 45° = $\left|\frac{m+\frac{1}{2}}{1-\frac{m}{2}}\right|$ ⇒ $\pm 1 = \frac{2m+1}{2-m}$ ⇒ m = $\frac{1}{3}$, – 3

∴ Equation of AC AC

y – 2 = $\frac{1}{3}$(x) ⇒ x – 3y + 6 = 0 .... (i)

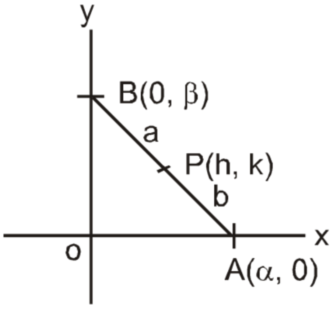
Equation of BD BD

y = – 3 (x – 4) ⇒ 3x + y – 12 = 0 .... (ii)

From (i) & (ii)

x = 3 & y =3

5. **Sol.**



By geometry

$\alpha^2 + \beta^2 = (a + b)^2$ ....(i)

By section formula

$h = \frac{a\alpha}{a+b} \Rightarrow \alpha = \frac{h(a+b)}{a}$

$k = \frac{b\beta}{a+b} \Rightarrow \beta = \frac{k(a+b)}{b}$

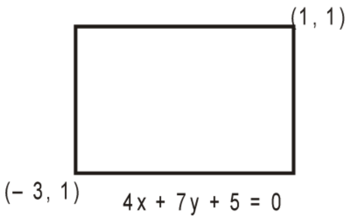
Put value of α and β in (i)

$\frac{h^2(a+b)^2}{a^2} + \frac{k^2(a+b)^2}{b^2} = (a + b)^2 \Rightarrow \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1$

Locus of p is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

6. **Sol.** Required point is foot of perpendicular from (0, 0) on the given line which is $\frac{\alpha - 0}{3} = \frac{\beta - 0}{4} = \frac{-(-1)}{25}$

7. **Sol.**



Line ⊥ to 4x + 7y + 5 = 0 is

$7x - 4y + \lambda = 0$

It passes through (– 3, 1) and (1, 1)

$-11 - 4 + \lambda = 0 \Rightarrow \lambda = 25$

$7 - 4 + \lambda = 0 \Rightarrow \lambda = -3$

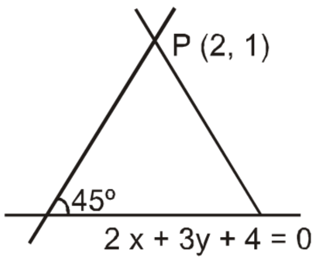
Hence lines are 7x – 4y + 25 = 0, 7x – 4y – 3 = 0

line || to 4x + 7y + 5 = 0 passing through (1, 1) is 4x + 7y + λ = 0

$\Rightarrow \lambda = -11$

$\Rightarrow 4x + 7y - 11 = 0$

8. **Sol.**



Let slope of required line is m

Now, y – 1 = m(x – 2)

$\tan 45^\circ = \left|\frac{m + \frac{2}{3}}{1 - \frac{2m}{3}}\right| = \left|\frac{3m+2}{3-2m}\right|$

$\Rightarrow \quad \frac{3m+2}{3-2m} = \pm 1 \quad \Rightarrow \quad 3m + 2 = \pm (3 - 2m)$

$\Rightarrow \quad m = \frac{1}{5}, -5$

Hence, $y - 1 = \frac{1}{5}(x - 2) \Rightarrow x - 5y + 3 = 0$

$y - 1 = -5(x - 2) \Rightarrow 5x + y - 11 = 0$

9. Sol. Area of parallogram = $\left|\frac{(3-0)(1-0)}{\begin{vmatrix} 2 & -1 \\ 1 & -1 \end{vmatrix}}\right| = 3$

10. Sol.

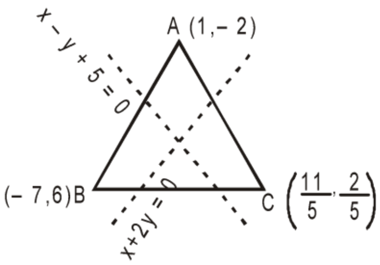


Image of A in x – y + 5 = 0 is

$\frac{x-1}{1} = \frac{y+2}{-1} = -2\left(\frac{1+2+5}{2}\right) = -8$

$x = -7, y = 6$

Image of A(1, – 2) in x + 2y = 0

$\frac{x-1}{1} = \frac{y+2}{2} = -2\left(\frac{1-4}{5}\right) = \frac{6}{5}$

$x = \frac{11}{5}, y = \frac{2}{5}$

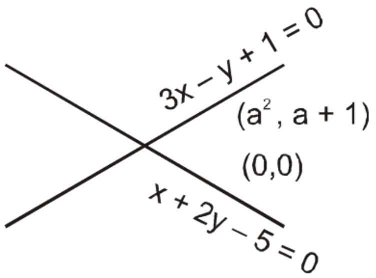
Hence equation of BC is $\quad y - 6 = \frac{(6-2/5)}{(-7-11/5)}(x + 7)$

$y - 6 = \frac{28}{-28}(x + 7)$

$y - 6 = \frac{-14}{23}(x + 7)$

$\Rightarrow \quad 14x + 23y - 40 = 0$

11. Sol.



Origin, R(a^2, a + 1) lies same side w.r.t. to given lines

$a^2 + 2a + 2 - 5 < 0 \Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a + 3)(a - 1) < 0$

$\Rightarrow a \in (-3, 1)$

$3a^2 - (a + 1) + 1 > 0 \Rightarrow 3a^2 - a > 0$

$\Rightarrow a(3a - 1) > 0$

$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$

take intersection we get $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

12. Sol. Let line be x + 7y + λ = 0

Distance of this line from (1, – 1) is = $\left|\frac{1-7+\lambda}{\sqrt{50}}\right|$.

As per question $\left|\frac{1-7+\lambda}{\sqrt{50}}\right| = 1. \Rightarrow \lambda = 6 \pm 5\sqrt{2}$

13. Sol. Any point on the line x + y = 4 can be taken as (t, 4 – t) the ⊥ distance of the point (t, 4 – t) from the line 4x + 3y = 10 is 1

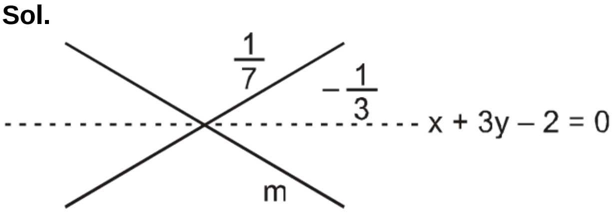
$\Rightarrow \left|\frac{4t + 3(4-t) - 10}{5}\right| = 1$

$\left|\frac{t+2}{5}\right| = 1$

$\Rightarrow |t + 2| = 5 \Rightarrow t = 3$ and $t + 2 = -5 \Rightarrow t = -7$

P ≡ (3, 1) and Q (–7, 11)

14. Sol.



point of intersection of x + 3y – 2 = 0 and x – 7y + 5 = 0 is $\left(-\frac{1}{10}, \frac{7}{10}\right)$

$\left(\frac{-\frac{1}{3} - m}{1 - \frac{m}{3}}\right) = -\left(\frac{-\frac{1}{3} - \frac{1}{7}}{1 - \frac{1}{21}}\right)$

$\Rightarrow \quad \frac{-1-3m}{3-m} = \frac{10}{20} = \frac{1}{2}$

$\Rightarrow \quad -2 - 6m = 3 - m$

$\Rightarrow \quad m = -1$

Hence required equation

$y - \frac{7}{10} = -1\left(x + \frac{1}{10}\right)$

$\Rightarrow \quad 10y - 7 = -10x - 1$

$\Rightarrow \quad 10x + 10y = 6 \Rightarrow 5x + 5y = 3$

15. **Sol.** By parametric form Q $\left(4 + \frac{11}{2\sqrt{2}}\cos\theta, 1 + \frac{11}{2\sqrt{2}}\sin\theta\right)$

it lies on $3x - y = 0 \quad \Rightarrow \quad 12 + \frac{33}{2\sqrt{2}}\cos\theta - 1 - \frac{11}{2\sqrt{2}}\sin\theta = 0$

$\Rightarrow \quad 1 + \frac{3}{2\sqrt{2}}\cos\theta - \frac{\sin\theta}{2\sqrt{2}} = 0 \quad \Rightarrow \quad 3\cos\theta - \sin\theta = -2\sqrt{2}$

squaring both sides

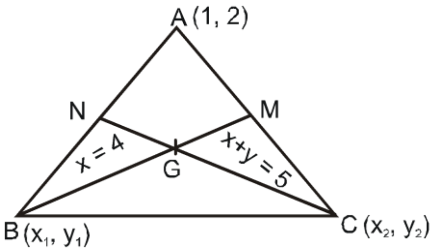
$9\cos^2\theta + \sin^2\theta - 6\sin\theta\cos\theta = 8(\sin^2\theta + \cos^2\theta) \quad \Rightarrow \quad \cos^2\theta - 6\sin\theta\cos\theta - 7\sin^2\theta = 0$

$7\tan^2\theta + 6\tan\theta - 1 = 0 \quad \Rightarrow \quad \tan\theta = -1, \frac{1}{7}$.

Hence required line are $x + y = 5$, $x - 7y + 3 = 0$

16. **Sol.** If H is orthocentre of triangle ABC, then orthocentre of triangle BCH is point A

17. **Sol.**



$x_1 + y_1 = 5$ ... (i)

$x_2 = 4$ ... (ii)

co - ordinates of G are ≡ (4, 1)

$\Rightarrow \quad \frac{1 + x_1 + x_2}{3} = 4$ ....(iii)

and $\frac{y_1 + y_2 + 2}{3} = 1$ ... (iv)

solving above equations, we get B & C.

18. **Sol.** The lines will pass through (4, 5) & parallel to the bisectors between them

$\frac{3x - 4y - 7}{5} = \pm \frac{12x - 5y + 6}{13}$

by taking + sign, we get $21x + 27y + 121 = 0$

Now by taking – sign, we get $99x - 77y - 61 = 0$

so slopes of bisectors are

$-\frac{7}{9}$, $\frac{9}{7}$

Equation of lines are

$y - 5 = -\frac{7}{9}(x - 4)$

and $y - 5 = \frac{9}{7}(x - 4)$

$\Rightarrow$ $7x + 9y = 73$ and $9x - 7y = 1$

19. **Sol.** By family of lines required line is :

$(x + y - 5) + \lambda(x - y + 3) = 0$

equation of other line is $\frac{x}{-2} + \frac{y}{-3} = 1 \Rightarrow 3x + 2y + 6 = 0$

Both are $\perp$ hence $m_1m_2 = -1$

$\Rightarrow$ $\left(\frac{\lambda+1}{1-\lambda}\right)\left(\frac{-3}{2}\right) = -1$

$\Rightarrow$ $3\lambda + 3 = -2 + 2\lambda \Rightarrow \lambda = -5$

Hence required line $-4x = 6y - 20 = 0$

$2x - 3y + 10 = 0$

$2x - 3y + 10 = 0$

20. **Sol.** $a^2 + 9b^2 - 4c^2 = 6ab$

then $a^2 + 9b^2 - 6ab = 4c^2$

$(a - 3b)^2 = (2c)^2$

$a - 3b = 2c$ and $a - 3b = -2c$

line $ax + by + c = 0$ is concurrent at

$ax + by + \left(\frac{a-3b}{2}\right) = 0$ and $ax + by + \left(\frac{3b-a}{2}\right) = 0$

$x = -\frac{1}{2}$; $y = \frac{3}{2}$ and $x = \frac{1}{2}$; $y = -\frac{3}{2}$

P $\left(-\frac{1}{2}, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, -\frac{3}{2}\right)$

21. **Sol.** $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$

$\Delta = 0$

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$12.2.k + 2.\left(-\frac{5}{2}\right)\left(\frac{11}{2}\right)(-5) - 12\left(\frac{25}{4}\right) - 2\left(\frac{121}{4}\right) - k\,(25) = 0$

$\Rightarrow$ $k = 2$

22. **Sol.** $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x + 4\sqrt{2}\,y + 1 = 0$

$(x + \sqrt{2}\,y + p)(x + \sqrt{2}\,y + q) = 0$

$p + q = 4$

$pq = 1$

Distance between || lines is $\left|\frac{p-q}{\sqrt{3}}\right| = \frac{\sqrt{(p+q)^2 - 4pq}}{\sqrt{3}} = \frac{\sqrt{16-4}}{\sqrt{3}} = 2$

23. **Sol.** Let equations of lines represented by the line pair $xy - 3y^2 + y - 2x + 10 = 0$ are

$y + c_1 = 0$, $x - 3y + c_2 = 0$

lines ⊥ to these lines and passing through origin are
$x = 0$, $y = -3x$
Joint equation
$x(3x + y) = 0$
$\Rightarrow \quad xy + 3x^2 = 0$

24. Sol. Homogenize $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ by $x + ky = 1$
$$5x^2 + 12xy - 6y^2 + 4x(x + ky) - 2y(x + ky) + 3(x + ky)^2 = 0$$
it is equally inclined with x-axes hence coeff. xy = 0
$$12 + 4k - 2 + 6k = 0$$
$$k = -1$$

PART - II : MISCELLANEOUS QUESTIONS

A-1. Ans. (1)
Sol. $ax^3 + bx^2y + cxy^2 + dy^3 = 0$
since this is homogeneous equation of degree 3 therefore it represents three straight lines passing through origin
$ax^3 + bx^2y + cxy^2 + dy^3 = (y - m_1x)(y - m_2x)(y - m_3x)$
or put y = mx in given equation we get
$m^3d + cm^2 + bm + a = 0$ ...(i)
$m_1 + m_2 + m_3 = \frac{-c}{d}$
$m_1m_2 + m_2m_3 + m_3m_1 = \frac{+b}{d}$
$m_1m_2m_3 = \frac{-a}{d}$
given two lines are perpendicular hence $m_1m_2 = -1 \Rightarrow m_3 = a/d$
put $m_3 = \frac{a}{d}$ in equation (i) we get $a^2 + ac + bd + d^2 = 0$

A-2. Ans. (1)
Sol. S_1 is true because given quadrilateral is a rhombus.
S_2 is also standard rule but S_2 does not explains S_1.

A-3. Ans. (3)
Sol. S_2 is standard result.
equation of angle bisectors of lines given in S_1 are
$\frac{3x + 4y + 2}{5} = \pm \frac{4x + 3y - 2}{5} \quad \Rightarrow \quad x - y = 0$ and $7x + 7y - 24 = 0$

A-4. Ans. (1)
Sol. equation of line $y - 2 = m(x - 8)$ where $m < 0$
$\Rightarrow \quad P \equiv \left(8 - \frac{2}{m},\ 0\right)$ and $Q \equiv (0, 2 - 8m)$
Now $OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$
$= 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times 8 \ (-m)} \geq 18$

Section (B) : MATCH THE COLUMN

B-1. Ans. (A) → p ; (B) → q ; (C) → s ; (D) → s

Sol.(A) AH $\perp$ BC. $\Rightarrow \left(\frac{k}{h}\right)\left(\frac{3-2}{-2-5}\right) = -1$

4k = 7h **DIAGRAM**

BH $\perp$ AC. $\Rightarrow \left(\frac{0+1}{0-5}\right)\left(\frac{K-3}{H+2}\right) = -1$

K – 3 = 5(h + 2)

$\Rightarrow$ 7h –12 = 20h + 40

13h = –52

h = –4 $\therefore$ k = –7

$\therefore$A(–4, –7)

(B) x + y –4 = 0

4x + 3y– 10 = 0

Let (h, 4 –h) be the point on(i)

Then $\left|\frac{4h+3(4-h)-10}{5}\right| = 1$

i.e. h + 2 = ±5 i.e. h = 3; h = –7

$\therefore$ reuired point is either (3, 1) or (–7, 11)

(C) orthocentre of the triangle is the point of intersection of the lines

i .e., (–1, 2)

(D) Since a, b, c are in A.P.

$\therefore$ $b = \frac{a+c}{2}$

$\therefore$ the family of lines is ax + $\frac{a+c}{2}$ y = c

i.e. $a\left(x+\frac{y}{2}\right)+c\left(\frac{y}{2}-1\right) = 0$

$\therefore$ point of concurrency is (–1, 2)

B-2. **Ans.** (A) → (s), (B) → (p, q), (C) → (r), (D) → (p, q, s)

Sol. For concurrency

(A) $\begin{vmatrix} 1 & 3 & -5 \\ 3 & -k & -1 \\ 5 & 2 & -12 \end{vmatrix}$ = 12 k + 2 – 3 (– 3i) – 5 (6 + 5k) = 0

$\Rightarrow$ – 13k + 2 + 93 – 30 = 0

$\Rightarrow$ – 13k + 65 = 0 $\Rightarrow$ k = 5

(B) For L_1 & L_2 to be parallel,

$\frac{1}{3} = \frac{3}{-k}$ $\Rightarrow$ k = – 9.

Also, $\frac{3}{5} = \frac{-k}{2}$ for L_2 , L_3 to be parallel

$\Rightarrow$ $k = -\frac{6}{5}$

(C) They form a triangle when lines are non-concurrent & non-parallel.

Hence k = $\frac{5}{6}$ from the given options.

(D) L_1, L_2, L_3 will not from a triangle when they are concurrent or any two of them are parallel.

C-1. Sol. $PA^2 = PB^2 = AB^2 = 8a^2$, D being equilateral.

$PA^2 = PB^2 \Rightarrow 4ah + 4ak = 0$

$\therefore$ $h = -k$

$PA^2 = 8a^2 \Rightarrow (h - a)^2 + (k - a)^2 = 8a^2$.

Put $k = -h$ $\therefore$ $2(h^2 + a^2) = 8a^2$

$\therefore$ $h = a\sqrt{3}, -a\sqrt{3}$ $\therefore$ $k = -a\sqrt{3}, a\sqrt{3}$

$\therefore$ The vertex should be $(a\sqrt{3}, -a\sqrt{3})$

or $(-a\sqrt{3}, a\sqrt{3})$ i.e., (2) and (3)

C-2. Sol. The first two lines are clearly perpendicular. Also angle between 2nd and 3rd is

$$\tan\theta = \left|\frac{(-7)-\left(-\frac{3}{4}\right)}{1+(-7)\left(-\frac{3}{4}\right)}\right| = 1 \qquad \therefore \quad \theta = 45°$$

Hence the 3rd angle is also 45°.

$\therefore$ Δ is isosceles as well as right-angled.

C-3. Sol. Triangle is obtuse so circumcentre and orthocentre lies outside the triangle.

C-4. Sol. Obvious

Exercise-3

*** Marked Questions may have more than one correct option.**

1. Sol. $2x + 11y - 5 = 0$ ...(1)

$4x - 3y - 2 = 0$ ...(2)

$24x + 7y - 20 = 0$ ...(3)

Point of intersection of (1) & (2) satisfies (3)

Hence lines are concurrent

Now Bisector of (1) & (3) $\left(\frac{4x-3y-2}{5}\right) = \pm\left(\frac{24x+7y-20}{25}\right)$

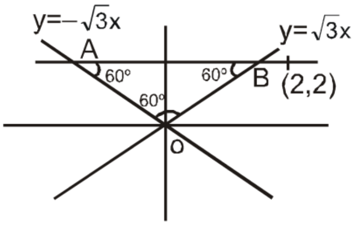
$\Rightarrow (20x - 15y - 10) = \pm (24x + 7y - 20)$

$\Rightarrow 4x + 22y - 10 = 0 \Rightarrow 2x + 11y - 5 = 0$

$44x - 8y - 30 = 0.$

Hence line (1) is bisector of (2) & (3)

2. Sol.



$y - 2 = m(x - 2)$

$$\tan 60° = \left|\frac{m-\sqrt{3}}{1+m\sqrt{3}}\right|$$

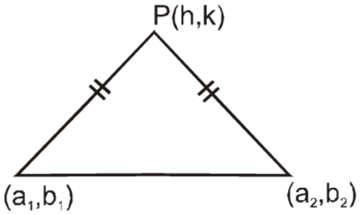
$\frac{m-\sqrt{3}}{1+m\sqrt{3}} = \pm\sqrt{3} \Rightarrow m - \sqrt{3} = \pm(\sqrt{3} + 3m)$

$\Rightarrow m = -\sqrt{3}, 0$

Hence, $y - 2 = 0$

$y - 2 = -\sqrt{3}(x - 2)$.

3. **Sol.**



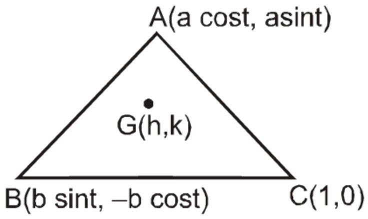
$(h - a_1)^2 + (k - b_1)^2 = (h - a_2)^2 + (k - b_2)^2$

$2h(a_1 - a_2) + 2k(b_1 - b_2) + \left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right) = 0$

compare with $(a_1 - a_2)x + (b_1 - b_2)y + c = 0$

$c = \frac{\left(a_2^2 + b_2^2 - a_1^2 - b_1^2\right)}{2}$.

4. **Sol.**



$3h - 1 = a\cos t + b\sin t$

$3k = a\sin t - b\cos t$

squaring and add. (Locus)

$(3x - 1)^2 + 9y^2 = a^2 + b^2$

5. **Sol.** $G\left(\frac{h}{3}, \frac{k-2}{3}\right)$

$\Rightarrow \frac{2h}{3} + (k - 2) = 1 \Rightarrow 2h + 3k = 9$

Locus $2x + 3y = 9$.

6. **Sol.** Let equation of line is $\frac{x}{a} + \frac{y}{b} = 1$

it passes through (4, 3) $\frac{4}{a} + \frac{3}{b} = 1$

sum of intercepts is –1 $\Rightarrow a + b = -1 \Rightarrow a = -1 - b$

$\Rightarrow \frac{4}{-1-b} + \frac{3}{b} = 1$

$\Rightarrow 4b - 3 - 3b = -b - b^2$

$\Rightarrow b^2 + 2b - 3 = 0$

$\Rightarrow b = -3, 1$

b = 1, a = –2 $\quad \frac{x}{-2} + \frac{y}{1} = 1$

b = –3, a = 2 $\quad \frac{x}{2} + \frac{y}{-3} = 1$.

7. Sol. Pair $6x^2 - xy + 4cy^2 = 0$ has its one line $3x + 4y = 0$

$\Rightarrow y = \frac{-3x}{4} \quad 6x^2 + \frac{3x^2}{4} + 4c\,\frac{9x^2}{16} = 0$

$\Rightarrow 24x^2 + 3x^2 + 9cx^2 = 0$

$\Rightarrow c = -3$.

8. Sol. $ax + 2by + 3b = 0$

$bx - 2ay - 3a = 0$

$$\frac{x}{-6ab+6ab} = \frac{y}{3b^2+3a^2} = \frac{1}{-2a^2-2b^2}$$

Hence point of intersection (0, –3/2)

Line parallel to x-axis y = –3/2.

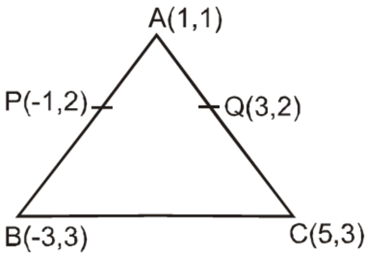
9. Sol. $\because$ a, b, c are in H.P. $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

$\Rightarrow \frac{1}{a} - \frac{2}{b} + \frac{1}{c} = 0$

given line $\frac{x}{a} + \frac{y}{b} + \frac{1}{c} = 0$

Clearly line passes through (1, –2).

10. Sol. Centroid is $\left(1, \frac{7}{3}\right)$



11. Sol. Let equation of line is

$\frac{x}{a} + \frac{y}{b} = 1$.

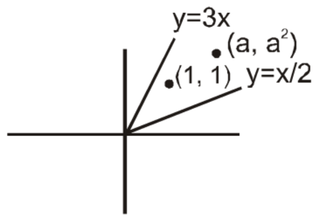
By section formula

$\frac{a}{2} = 3 \Rightarrow a = 6$

$\frac{b}{2} = 4 \Rightarrow b = 8$

$\frac{x}{6} + \frac{y}{8} = 1 \Rightarrow 4x + 3y = 24$.

12. Sol.



Since (1, 1) and (a, a^2) Both lies same side with respect to both lines

$a - 2a^2 < 0 \Rightarrow 2a^2 - a > 0$

$\Rightarrow a(2a - 1) > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$

$3a - a^2 > 0 \Rightarrow a^2 - 3a < 0 \Rightarrow a \in (0, 3)$

Hence after taking intersection $a \in \left(\frac{1}{2}, 3\right)$.

13. **Sol.** AB = $\sqrt{(h-1)^2 + (k-1)^2}$

BC = 1

AC = $\sqrt{(h-2)^2 + (k-1)^2}$

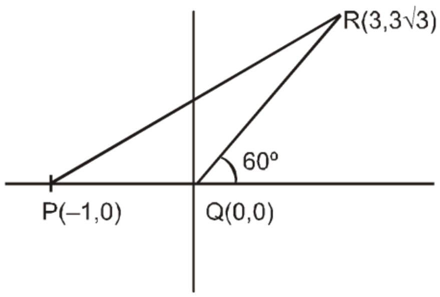
$AB^2 + BC^2 = AC^2 \Rightarrow (h - 1)^2 + (k - 1)^2 + 1 = (h - 2)^2 + (k - 1)^2$

$\Rightarrow 2h = 2 \Rightarrow h = 1$

Area of ΔABC = $\frac{1}{2}\sqrt{(h-1)^2 + (k-1)^2} \times 1 = 1$

$(K - 1)^2 = 4 \Rightarrow k - 1 = \pm 2 \Rightarrow k = 3, -1.$

14. **Solution**



The line segment QR makes an angle 60º with the positive direction of x-axis.
hence bisector of angle PQR will make 120º with +ve direction of x-axis.
Its equation

$y - 0 = \tan 120^\circ (x - 0)$

$y = -\sqrt{3}x$

$x\sqrt{3} + y = 0$

15. **Sol.** Bisector of x = 0 and y = 0 is either y = x or y = –x
If y = x is Bisector, then

$mx^2 + (1 - m^2)x^2 - mx^2 = 0$

$\Rightarrow m + 1 - m^2 - m = 0 \Rightarrow m^2 = 1 \Rightarrow m = \pm 1.$

16. **Sol.** Slope of PQ = $\frac{1}{1-k}$

Hence equation of line ⊥ to PQ line

$y - \frac{7}{2} = (k-1)\left(x - \frac{(1+k)}{2}\right)$

Put x = 0

$y = \frac{7}{2} + \frac{(1-k)\ (1+k)}{2} = -4$

$7 + (1 - k^2) = -8 \Rightarrow k^2 = 16 \Rightarrow k = \pm 4.$

Hence possible answer = –4.

17. **Sol.** $p(p^2 + 1)\, x - y + q = 0$

$(p^2 + 1)^2 x + (p^2 + 1)\, y + 2q = 0$ are perpendicular

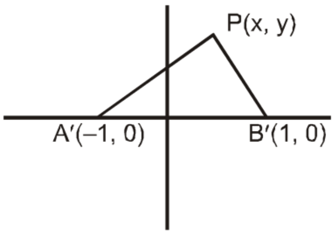
for a common line

⇒ lines are parallel

⇒ slopes are equal

$\therefore \quad \frac{p(p^2+1)}{1} = -\frac{(p^2+1)^2}{(p^2+1)} \quad \Rightarrow \quad p = -1$

18. **Sol.**



$\therefore \quad \frac{PA'}{PB'} = \frac{3}{1}$

$\therefore \quad (x + 1)^2 + y^2 = 9((x - 1)^2 + y^2)$

$x^2 + 2x + 1 + y^2 = 9x^2 + 9y^2 - 18x + 9$

$8x^2 + 8y^2 - 20x + 8 = 0$

$x^2 + y^2 - \frac{10}{4}x + 1 = 0$

$\therefore$ circumcentre $\left(\frac{5}{4},\ 0\right)$.

19. **Ans.** **(3)**

Sol. $\frac{x}{5} + \frac{y}{b} = 1$

$\frac{13}{5} + \frac{32}{b} = 1 \quad \Rightarrow \quad \frac{32}{b} = -\frac{8}{5} \quad \Rightarrow \quad b = -20$

$\frac{x}{5} - \frac{y}{20} = 1 \quad \Rightarrow \quad 4x - y = 20$

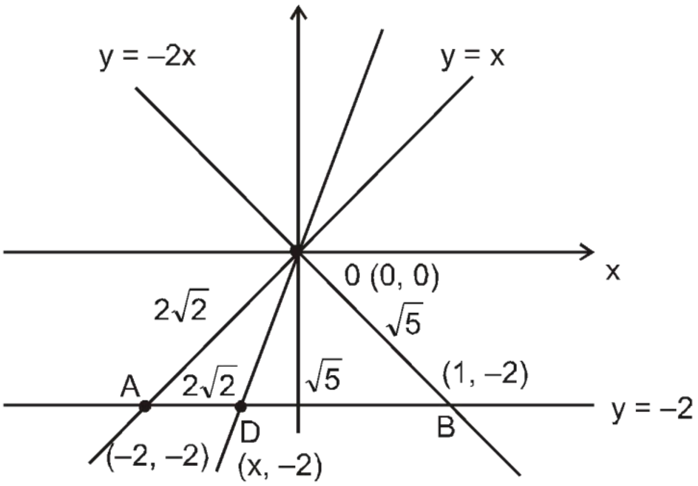
Line K has same slope $\Rightarrow -\frac{3}{c} = 4$

$c = -\frac{3}{4} \quad \Rightarrow \quad 4x - y = -3$

distance $= \frac{23}{\sqrt{17}}$

Hence correct option is (3)

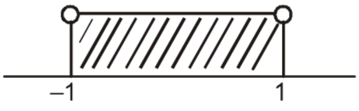
20.
Sol. **(3)**



∴ AD : DB = $2\sqrt{2} : \sqrt{5}$
∵ OD is angle bisector
of angle AOB
∴ St : 1 true
St. 2 false (obvious) **Ans.**

21. **Sol.** **(2)**
x + y = |a|
ax – y = 1
if a > 0
x + y = a
ax – y = 1
----
x(1 + a) = 1 + a as x = 1
y = a – 1
It is in the first quadrant
so a – 1 ≥ 0
a ≥ 1
a ∈ [1, ∞)
If a < 0
x + y = – a
ax – y = 1
+
----
x(1 + a) = 1 – a

$x = \frac{1-a}{1+a} > 0 \Rightarrow \frac{a-1}{a+1} < 0$

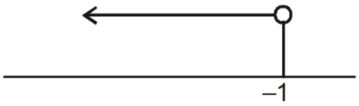


.............(1)

$y = -a - \frac{1-a}{1+a}$

$= \frac{-a-a^2-1+a}{1+a} > 0$

$-\left(\frac{a^2+1}{a+1}\right) > 0 \Rightarrow \frac{a^2+1}{a+1} < 0$

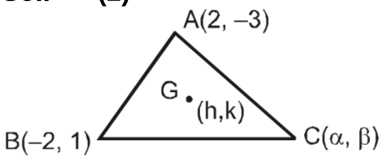


.............(2)

from (1) and (2) a ∈ {φ}

$a \in [1, \infty)$

22. Sol. (2)



$\alpha = 3h$

$\beta - 2 = 3k$

$\beta = 3k + 2$

third vertex on the line $2x + 3y = 9$

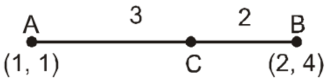
$2\alpha + 3\beta = 9$

$2(3h) + 3(3k + 2) = 9$

$2h + 3k = 1$

$2x + 3y - 1 = 0$

23. Sol.



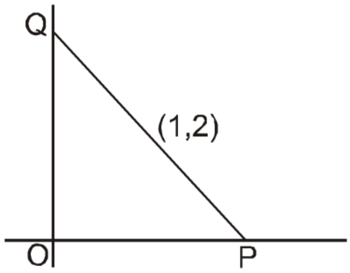
$\therefore \quad C\left(\frac{8}{5}, \frac{14}{5}\right)$

Line $2x + y = k$ passes $C\left(\frac{8}{5}, \frac{14}{5}\right)$

$\frac{2\times 8}{5} + \frac{14}{5} = k$

$k = 6$

24.
Sol.



$(y - 2) = m(x - 1)$

$OP = 1 - \frac{2}{m}$

$OQ = 2 - m$

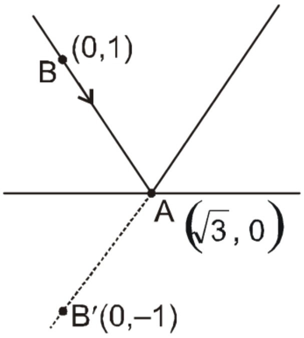
Area of $\Delta POQ = \frac{1}{2}(OP)(OQ) = \frac{1}{2}\left(1 - \frac{2}{m}\right)(2 - m)$

$= \frac{1}{2}\left[2 - m - \frac{4}{m} + 2\right]$

$= \frac{1}{2}\left[4 - \left(m + \frac{4}{m}\right)\right]$

$m = -2$

25. Sol. (2)



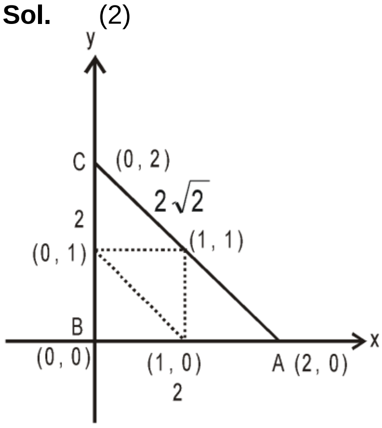
Take any point B(0, 1) on given line
Equation of AB'

$y - 0 = \frac{-1-0}{0-\sqrt{3}}\left(x-\sqrt{3}\right)$

$-\sqrt{3}y = -x + \sqrt{3}$

$x - \sqrt{3}y = \sqrt{3} \quad \Rightarrow \quad \sqrt{3}y = x - \sqrt{3}$

26. **Sol.** (2)

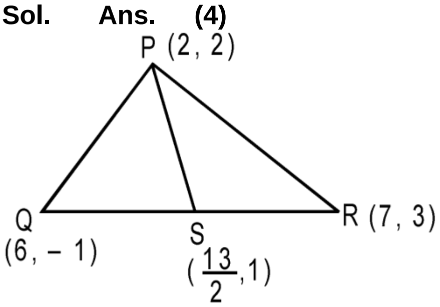


x - coordinate of incentre = $\frac{2\times 0 + 2\sqrt{2}.0 + 2.2}{2+2+2\sqrt{2}}$

$= \frac{2}{2+\sqrt{2}}$

$= 2 - \sqrt{2}$

7. **Sol.** **Ans.** **(4)**



Slope of PS = $\frac{2-1}{2-\frac{13}{2}} = \frac{-2}{9}$

Hence equation of line through (1, –1) & parallel to PS is:

$(y+1) = \frac{-2}{9}(x-1)$

$9y + 2x + 7 = 0$

28. Sol. Ans. (1)

$4ax + 2ay + c = 0$

$5bx + 2by + d = 0$

$\frac{x}{2ad - 2bc} = \frac{y}{5bc - 4ad} = \frac{1}{8ab - 10ab}$

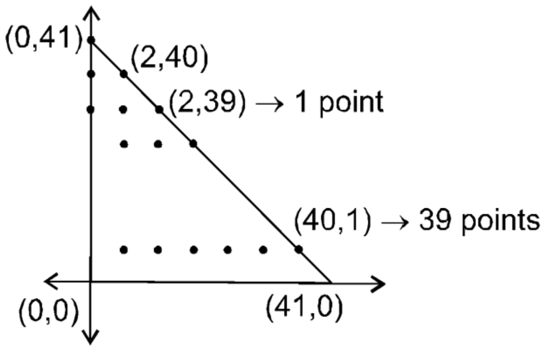
$\Rightarrow \quad x = \frac{bc - ad}{ab}, \quad y = \frac{4ad - 5bc}{2ab}$

In fourth quadrant point equidistant from axis will have sum of x & y co-ordinate = 0

$\Rightarrow \quad \frac{2bc - 2ad}{2ab} + \frac{4ad - 5bc}{2ab} = 0$

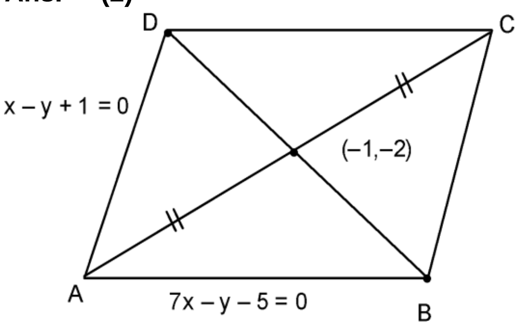
$\Rightarrow \quad 2ad - 3bc = 0$

29. Sol.



$1 + 2 + \ldots\ldots 39 = \frac{39}{2}(39 + 1) = 780$

30. Ans. (2)



Sol.

On solving equation of AB & AD

vertex A(1, 2)

$\because$ P is mid point of AC. Hence vertex C is (–3, –6).

So equation of other two sides are $7x - y + 15 = 0$ and $x - y - 3 = 0$.

Hence other vertices are $\left(\frac{1}{3}, -\frac{8}{3}\right)$ and $\left(-\frac{7}{3}, -\frac{4}{3}\right)$

31. Ans. (4)

Sol. $\begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$

$k(k-2) - 5(-3k-2) - k(-3k-k) = \pm 56$

$k^2 - 2k + 15k + 10 + 3k^2 + k^2 = \pm 56$

$5k^2 + 13k + 10 \pm 56 = 0$

$5k^2 + 13k + 66 = 0$ or $5k^2 + 13k - 46 = 0$

No solution or $k = \dfrac{-13 \pm \sqrt{169+920}}{10}$

$k = \dfrac{-13 \pm 33}{10} \Rightarrow k = 2$ or $k = -\dfrac{46}{10}$ (which is not an integer

∴ vertices A(2, – 6), B (5,2), C (–2,2)

Equation of altitude dropped from vertex A is

$x = 2$ ..... (i)

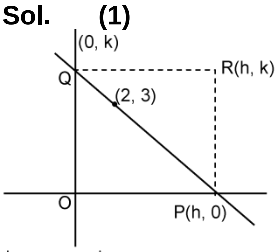
Equation of altitude dropped from vertex C is

$3x + 8y - 10 = 0$ ....(ii)

solving both (i) and (ii)

orthocentre $\left(2, \dfrac{1}{2}\right)$

32. Sol. (1)



$\begin{vmatrix} 0 & k & 1 \\ 2 & 3 & 1 \\ h & 0 & 1 \end{vmatrix} = 0$

$-(2-h) + 1(-3h) = 0$

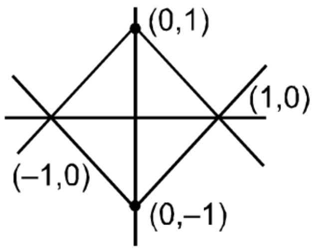
$-2y + xy - 3x = 0$

$3x + 2y = xy$ **Ans.**

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. Sol.

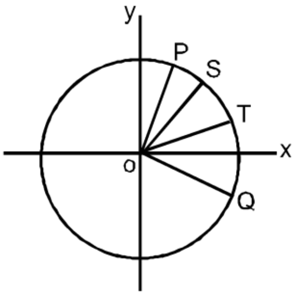


$y = |x| - 1$

$y = -|x| + 1$

Region is clearly square with vertices at the point (1,0), (0,1), (–1,0), (0,–1). So, its area = $\sqrt{2} \times \sqrt{2}$ = 2.

2. **Sol.**



Let ∠XOS = α and ∠XOT = $\frac{\alpha}{2}$

let p(cos θ , sin θ), then ∠TOP = θ – $\frac{\alpha}{2}$

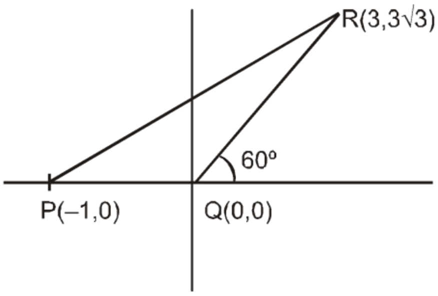
let Q be the image of P in OT. Then ∠QOT = θ – $\frac{\alpha}{2}$

∴ ∠QOX = θ – α

∴ ∠XOQ = α – θ

∴ Q is image of P in the line whose slope is tan $\frac{\alpha}{2}$

3. **Solution**

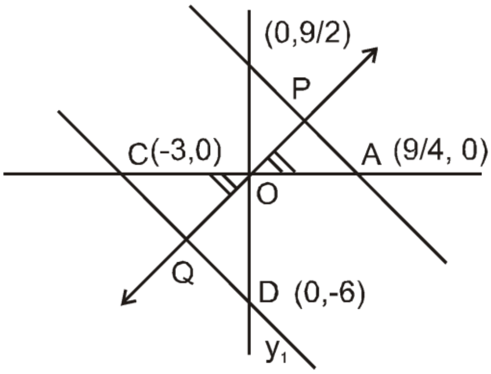


The line segment QR makes an angle 60º with the positive direction of x-axis.
hence bisector of angle PQR will make 120º with +ve direction of x-axis.
Its equation

y – 0 = tan120º (x – 0)

y = –

4. **Sol.**



as ΔOPA ~ ΔOQC

$\therefore \quad \frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

5. **Ans.** $x - 3y + 5 = 0$

Sol. The line y = mx meets the given lines in P$\left(\frac{1}{m+1}, \frac{m}{m+1}\right)$ and Q$\left(\frac{3}{m+1}, \frac{3m}{m+1}\right)$. Hence equation of L_1 is

$y - \frac{m}{m+1} = 2\left(x - \frac{1}{m+1}\right) \quad \Rightarrow \quad y - 2x - 1 = -\frac{3}{m+1}$ ....(i)

and that of L_2 is $y - \frac{3m}{m+1} = -3\left(x - \frac{3}{m+1}\right) \quad \Rightarrow \quad y + 3x - 3 = \frac{6}{m+1}$ ....(ii)

Form (i) and (ii) $\frac{y-2x-1}{y+3x-3} = -\frac{1}{2} \quad \Rightarrow \quad x - 3y + 5 = 0$; which is a straight line

6. **Ans.** 18

Sol. equation of line $y - 2 = m(x - 8)$ where $m < 0$

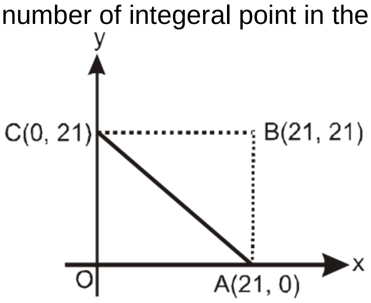
$\Rightarrow \quad P \equiv \left(8 - \frac{2}{m}, 0\right)$ and $Q \equiv (0, 2 - 8m)$

Now $\quad OP + OQ = \left|8 - \frac{2}{m}\right| + |2 - 8m|$

$= 10 + \frac{2}{(-m)} + 8(-m) \geq 10 + 2\sqrt{\frac{2}{-m} \times 8 \quad (-m)} \geq 18$

7. **Sol.** The number of integral points that lie in the interior of square OABC is 20 × 20. These points are (x, y) where x, y = 1, 2, ..., 20. Out of these 400 points 20 lie on the line AC. Out of the remaining exactly half lie in ΔABC.

$\therefore$ number of integeral point in the triangle OAC = $\frac{1}{2}$ [20 × 20 – 20] = 190

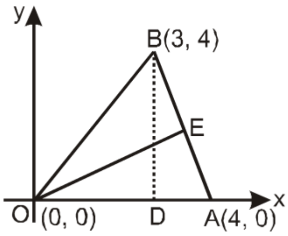


Alternative Solution

There are 19 points that lie in the interior of ΔABC and on the line x = 1, 18 point that lie on the line x = 2 and so on. Thus, the number of desired points is

$19 + 18 + 17 + ... + 2 + 1 = \frac{20 \times 19}{2} = 190.$

8. **Sol.**



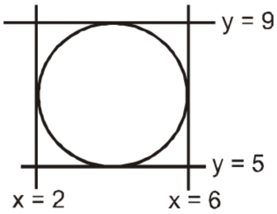
Refer Figure
Equation of altitude BD is x = 3.

slope of AB is $\frac{4-0}{3-4}$ = – 4
∴ slope of OE is 1/4
Equation of OE is

$y = \frac{1}{4}x.$

Lines BD and OE meets at (3, 3/4)

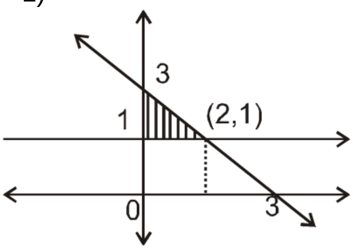
9. **Sol.**



The lines given by $x^2 - 8x + 12 = 0$ are x = 2 and x = 6.
The lines given by $y^2 - 14y + 45 = 0$ are y = 5 and y = 9
Centre of the required circle is the centre of the square.
∴ Required centre is

$\left(\frac{2+6}{2}, \frac{5+9}{2}\right)$ = (4, 7).

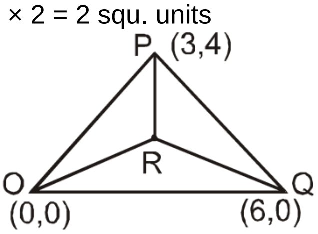
10. **Sol.** $x^2 - y^2 + 2y = 1$
x = ± (y – 1)



Bisector of above lines are x = 0, y = 1
so Area between x = 0, y = 1 and x + y = 3

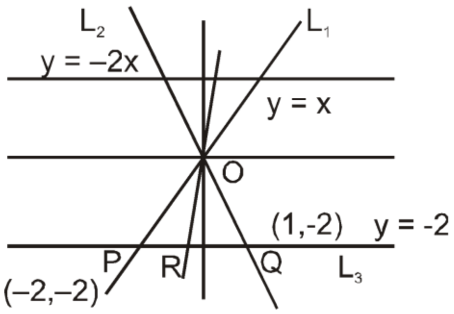
$= \frac{1}{2} \times 2 \times 2 = 2$ squ. units so Area between x = 0, y = 1 and x + y = 3

$= \frac{1}{2} \times 2 \times 2 = 2$ squ. units



11. **Sol.**

R is centroid hence R ≡ $\left(3, \frac{4}{3}\right)$

12. **Sol.** 

$\frac{PR}{RQ} = \frac{OP}{OQ}$

$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$

but statement – 2 is false

∴ Ans. (3)

13. **Sol.** Let slope of line L = m

∴ $\left|\frac{m-(-\sqrt{3})}{1+m(-\sqrt{3})}\right| = \tan 60^\circ = \sqrt{3} \Rightarrow \left|\frac{m+\sqrt{3}}{1-\sqrt{3}m}\right| = \sqrt{3}$

taking positive sign, m + $\sqrt{3}$ = $\sqrt{3}$ – 3m

m = 0

taking negative sign

m + $\sqrt{3}$ + $\sqrt{3}$ – 3m = 0

m = $\sqrt{3}$

As L cuts x-axis ⇒ m = $\sqrt{3}$

so L is y + 2 = $\sqrt{3}$ (x –3)

14. **Sol.** **(A) or (C) or Bonus**

As a > b > c > 0 ⇒ a – c > 0 and b > 0

⇒ a – c > 0 and b > 0

⇒ a + b – c > 0 ⇒ option (A) is correct

Further a > b and c > 0

⇒ a – b > 0 and c > 0

⇒ a – b > 0 and c > 0

⇒ a – b + c > 0 ⇒ option (c) is correct

Aliter

(a – b)x + (b – a)y = 0

⇒ x = y

⇒ Point of intersection $\left(\frac{-c}{a+b}, \frac{-c}{a+b}\right)$

Now $\sqrt{\left(1+\frac{c}{a+b}\right)^2+\left(1+\frac{c}{a+b}\right)^2} < 2\sqrt{2}$

⇒

⇒ a + b – c > 0

PART - I : PRACTICE TEST PAPER

1. Sol. Area = $\frac{1}{2}$ (a) (b) = $\frac{1}{2}$ ab

2. Sol. Area = $\frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{3}\right) \times 4 = \frac{1}{3}$

3. Sol. y-intercept = $-\frac{8}{a}$

x-intercept = 2

$\frac{-8}{a} = 3 \times 2 \quad \Rightarrow \quad a = -\frac{4}{3}$

4. Sol. Point of intersection of lines = $\left(\frac{2}{3}, \frac{2}{3}\right)$

distance = $\sqrt{(1-\frac{2}{3})^2 + (2-\frac{2}{3})^2} = \sqrt{\frac{1}{9} + \frac{16}{9}} = \frac{\sqrt{17}}{3}$

5. Sol. $\frac{4-a}{3+2} = -\frac{2}{5} \Rightarrow a = 6$

6. Sol. Slope of AB = $\frac{-1}{4}$

slope BC = – 2

Angle between AB and BC

$= \tan^{-1}\left(\frac{-\frac{1}{4}+2}{1+(-1/4)(-2)}\right) = \tan^{-1}\frac{7}{6}$

7. Sol. Let point be (α,β)

$\alpha + \beta = 4$ ....(1)

$\left|\frac{4\alpha + 3\beta - 10}{5}\right| = 1$

⇒ $4\alpha + 3\beta = 15$ ....(2)

$4\alpha + 3\beta = 5$ ....(3)

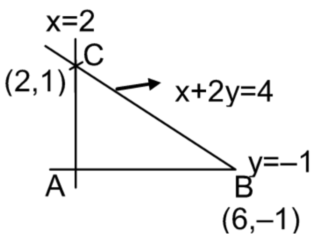
by (1) & (2) (3,1)

by (1) & (3) (–7, 11)

8. Sol. orthocentre will be point of intersection of

$4x - 7y + 10 = 0$ and $7x + 4y = 15$

$65x - 65 = 0 \quad \Rightarrow \quad x = 1, y = 2$



9. Sol.
circum centre is mid point of BC

10. Sol. $x\cos\theta - y\sin\theta = \lambda = a\cos_4\theta - \sin_4\theta = a\cos 2\theta$

11. Sol. $4x + 3y - 11 = 0$

$4x + 3y - \frac{15}{2} = 0$

$D = \left|\frac{11 - \frac{15}{2}}{5}\right| = \frac{7}{10}$

12. Sol. Let point be (α,β)

So $\left|\frac{\alpha - 2\beta + 1}{\sqrt{5}}\right| = \sqrt{5}$ $\Rightarrow$ $\alpha - 2\beta + 1 = \pm 5$

$\alpha - 2\beta = 4$ ...(1)

$\alpha - 2\beta = -6$ ...(2)

and $\left|\frac{2\alpha + 3\beta - 1}{\sqrt{13}}\right| = \sqrt{13}$ $\Rightarrow$ $\alpha + 3\beta - 1 = \pm 13$

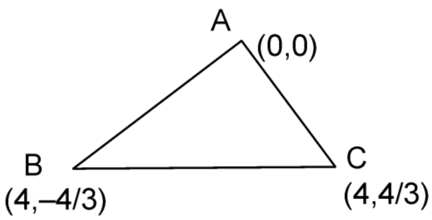
$2\alpha + 3\beta = 14$ ...(3)

$2\alpha + 3\beta = -12$ ...(4)

(1) and (3) and (1) & (4) gives two points
(2) and (3) and (2) & (4) gives two points
$\Rightarrow$ hence total 4 points are possible

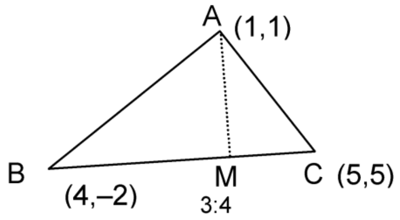
13. Sol. Check by option

15.



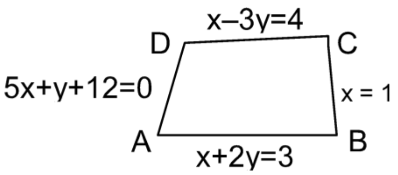
$AB = \frac{4\sqrt{10}}{3}$, $AC = \frac{4\sqrt{10}}{3}$, $BC = \frac{8}{3}$
Hence ABC is an isoscles triangle

16. Sol.



$M \equiv \left(\frac{31}{7}, 1\right)$

equation of AM $y - 1 = 0\,(x - 1) \Rightarrow y - 1 = 0$
so perpendicular form C to AM is
$x - 5 = 0$



17. **Sol.**
A= (–3,3), B = (1,1), C = (1,–1), D= (–2,–2)
Slope of AC = $\frac{3+1}{-3-1} = -1$
Slope of BD = $\frac{3}{3} = 1$
Hence Angle is 90°

18. Sol. Fixed point is centroid ≡ (1, 1)

19. **Sol.** $x_2\,(x - y) + (x - y) = 0$
$x - y = 0$ or $x_2 + 1 = 0$
only a straight line

20. **Sol.** Let ortho centre be (h,k)
$\left(\frac{k}{h}\right) \times \left(\frac{3+1}{-2-5}\right) = -1 \quad \Rightarrow \quad 4k = 7h \quad ...(1)$
and $\left(\frac{0+1}{0-5}\right) \times \left(\frac{k-3}{h+2}\right) = -1 \quad \Rightarrow \quad k - 3 = 5h + 10$
$\Rightarrow \quad 5h - k = -13 \quad ...(2)$
by (1) & (2) h=– 4, k = – 7

21. Sol. required locus
$\frac{3x+4y-11}{5} = \frac{-(12x+5y+2)}{13}$
$\Rightarrow \quad 99x + 77y - 133 = 0$

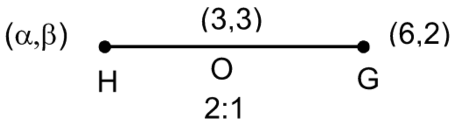
22. **Sol.** $L_1 = 2x + 3y - 4$
L_1 gives – ve sign for (–6,2)
$L_2 = 6x + 9y + 8$
L_2 gives – ve sign
below both the lines

23. **Sol.** P = $\sqrt{5}$, $\Rightarrow \frac{P}{\ell} = \sin 60 \Rightarrow l = \frac{\sqrt{5}}{\sqrt{3}} \times 2 = \sqrt{\frac{20}{3}}$

24. **Sol.** $\frac{3x+4y-1}{5} = -\frac{(12x-5y-2)}{13}$

99x + 27y = 23

25. Sol.



$\frac{12+\alpha}{3} \Rightarrow \alpha = -3$

$\frac{4+\beta}{3} = 3$

$\Rightarrow$ $\beta = 5$

26. **Sol.** $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

27. Sol. equation of line passing through (2,3) and parallel to the line x – y = 4 is

x – y = λ ⇒ λ = – 1

intersection point of line x – y + 1 = 0 and 3x + 2y = 17 is (3, 4)

distance = $\sqrt{2}$

29. **Sol.** $\frac{x-4}{5} = \frac{y+13}{1} = \frac{-2(20-13+6)}{26}$

⇒ x = –1 and y = – 14

30. **Sol.** x + 1 = 4 ⇒ x = 3

y – 2 = 5 ⇒ y = 7

PART - II : PRACTICE QUESTIONS

1. Sol. $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$

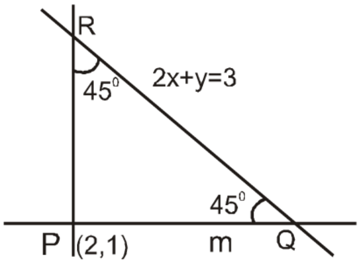
x_1, x_2, x_3 and y_1, y_2, y_3 are in G.P. of common ratio r.

$x_2 = x_1 r$, $x_3 = x_1 r^2$, $y_2 = y_1 r$, $y_3 = y_1 r^2$

Area of triangle ABC = $\frac{1}{2}\begin{vmatrix} x_1 & x_1 r & x_1 r^2 \\ y_1 & y_1 r & y_1 r^2 \\ 1 & 1 & 1 \end{vmatrix} = 0$

A, B & C are collinear.

2. Sol.



Let m be the slope of PQ then

$\tan 45^0 = \left|\frac{m-(-2)}{1+m(-2)}\right|$

$\Rightarrow \quad 1 = \left|\frac{m+2}{1-2m}\right|$

$\Rightarrow \quad \pm 1 = \frac{m+2}{1-2m}$

$\Rightarrow \quad m + 2 = 1 - 2m$ or $-1 + 2m = m + 2$

$m = -\frac{1}{3}$ or $m = 3$

PR makes 45^0 with PQ

equation of PQ $\quad y - 1 = -\frac{1}{3}(x - 2)$

$\Rightarrow \quad x + 3y - 5 = 0$

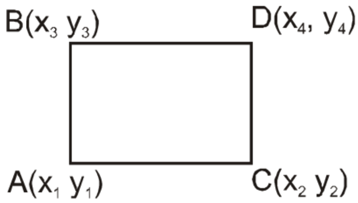
equation of PR is $y - 1 = 3(x - 2)$

$\Rightarrow \quad 3x - y - 5 = 0$

combined equation of PQ and PR $(x + 3y - 5)(3x - y - 5) = 0$

$\Rightarrow \quad 3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

3. Sol.



(x_1, y_1), $(x_2\ y_2)$, (x_3, y_3) and (x_4, y_4)

given $\sum_{i=1}^{4}(x_i^2 + y_i^2) \le 2(x_1 x_3 + x_2 x_4 + y_1 y_2 + y_3 y_4)$

$x_1^2 + y_1^2 + x_2^2 + y_2^2 + x_3^2 + y_3^2 + x_4^2 + y_4^2 \le 2(x_1 x_3 + x_2 x_4 + y_1 y_2 + y_3 y_4)$

$(x_1^2 + x_3^2 - 2x_1x_3) + (x_2^2 + x_4^2 - 2x_2x_4) + (y_1^2 + y_2^2 - 2y_1y_2) + (y_3^2 + y_4^2 - 2y_3y_4) \le 0$

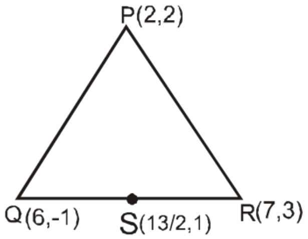
$(x_1 - x_3)^2 + (x_2 - x_4)^2 + (y_1 - y_2)^2 + (y_3 - y_4)^2 \le 0$

Only possible when $x_1 = x_3$; $x_2 = x_4$

$y_1 = y_2$; $y_3 = y_4$

hence it is a rectangle

4. **Sol.**



S is the mid point of Q and R

$S\left(\frac{7+6}{2}, \frac{3-1}{2}\right) = \left(\frac{13}{2}, 1\right)$

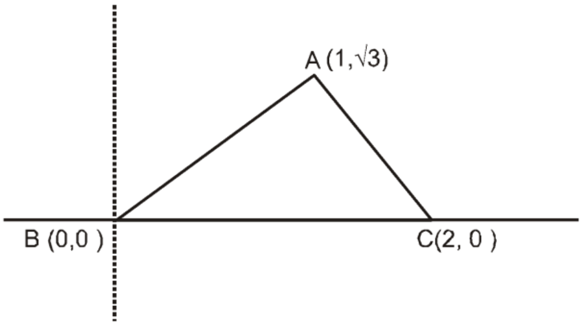
slope of PS $= m = \frac{2-1}{2-13/2} = \frac{-2}{9}$

equation of line passing through (1, –1) and parallel to PS is

$y + 1 = \frac{-2}{9}(x - 1) \quad \Rightarrow \quad 2x + 9y + 7 = 0$

5. $\left(1, \frac{1}{\sqrt{3}}\right)$

Sol.



BC = 2

AB = 2

AC = 2

Hence ABC is an equilateral triangle. In equilateral triangle incentre coincides with centroid. Thus

$I \equiv \left(\frac{0+2+1}{3}, \frac{0+0+\sqrt{3}}{3}\right) \equiv \left(1, \frac{1}{\sqrt{3}}\right)$

6. **Sol.** $c_1 \to ac_1$

$$\frac{1}{a}\begin{vmatrix} a^2x - aby - ac & bx + ay & cx + a \\ abx + a^2y & -ax + by - c & cy + b \\ acx + a^2 & cy + b & -ax - by + c \end{vmatrix}$$

$c_1 \to c_1 + bc_2 + cc_3$

$\Delta = \frac{1}{a}\begin{vmatrix} (a^2+b^2+c^2)x & ay+bx & cx+a \\ (a^2+b^2+c^2)y & by-c-ax & cy+b \\ (a^2+b^2+c^2) & b+cy & -ax-by+c \end{vmatrix} = \frac{1}{a}\begin{vmatrix} x & ay+bx & cx+a \\ y & by-c-ax & b+cy \\ 1 & b+cy & c-ax-by \end{vmatrix}$

as $a_2 + b_2 + c_2 = 1$

$C_2 \to C_2 - bC_1,\ C_3 \to C_3 - cC_1$

$\Delta = \frac{1}{a}\begin{vmatrix} x & ay & a \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$

$R_1 \to x R_1$

$= \frac{1}{ax}\begin{vmatrix} x^2 & axy & ax \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$

$R_1 \to R_1 + yR_2 + R_3$

$\Delta = \frac{1}{ax}\begin{vmatrix} x^2+y^2+1 & 0 & 0 \\ y & -c-ax & b \\ 1 & cy & -ax-by \end{vmatrix}$

$\Rightarrow \quad \Delta = (x_2 + y_2 + 1)(ax + by + c)$

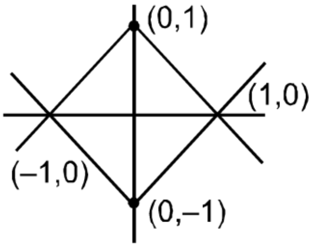
Given $\Delta = 0 \quad \Rightarrow \quad ax + by + c = 0$ which represent a straight line

7. **Sol.** The x-coordinate of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is $x = \frac{5}{3+4m}$

For x being an integer 3 + 4m should be divisor of 5
i.e. 1, –1, 5 or –5

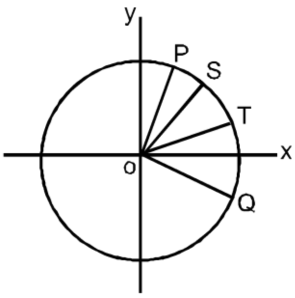
$3 + 4m = 1 \quad \Rightarrow \quad m = -\frac{1}{2}$ (Not integer)
$4m + 3 = -1 \quad \Rightarrow \quad m = -1$ (Interger)
$3 + 4m = 5 \quad \Rightarrow \quad m = -\frac{1}{2}$ (Not an integer)
$3 + 4m = -5 \quad \Rightarrow \quad m = -2$ (integer)
$\therefore$ there are two integral value of m

8. **Sol.**



$y = |x| - 1$
$y = -|x| + 1$
Region is clearly square with vertices at the point (1,0), (0,1), (–1,0), (0,–1). So,
its area = $\sqrt{2} \times \sqrt{2} = 2$.

9. **Sol.**



Let ∠XOS = α and ∠XOT = $\frac{\alpha}{2}$

let p(cos θ , sin θ), then ∠TOP = θ – $\frac{\alpha}{2}$

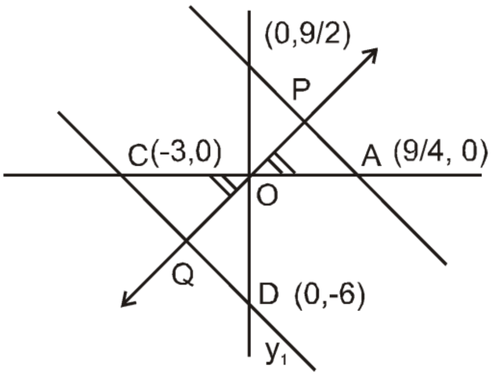
let Q be the image of P in OT. Then ∠QOT = θ – $\frac{\alpha}{2}$

∴ ∠QOX = θ – α

∴ ∠XOQ = α – θ

∴ Q is image of P in the line whose slope is tan $\frac{\alpha}{2}$

10. **Sol.**



as ΔOPA ~ ΔOQC

∴ $\frac{OP}{OQ} = \frac{OA}{OC} = \frac{9/4}{3} = \frac{3}{4}$

11. **Sol.** P ≡ (– sin (β – α) , – cos β)

Q ≡ (cos (β – α), sin β)

R ≡ (cos (β – α + θ) , sin (β – θ)) $0 < \alpha, \beta, \theta < \frac{\pi}{4}$

$x_R = \cos(\beta - \alpha)\cos\theta - \sin(\beta - \alpha)\sin\theta$

⇒ $x_R = x_Q \cdot \cos\theta + x_P \cdot \sin\theta$

$y_R = \sin\beta\cos\theta - \cos\beta\sin\theta$

⇒ $y_R = y_Q \cdot \cos\theta + y_P \cdot \sin\theta$

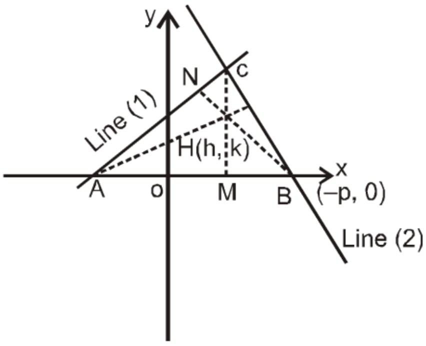
For P, Q, R to be collinear

sin θ + cos θ = 1

⇒ $\sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ ⇒ not possible for the given interval θ ∈

⇒ non collinear

12. Sol.



$(1 + p) x – py + p (1 + p) = 0$ ......(1)
$(1 + q) x – qy + q(1 + q) = 0$ ......(2)
on solving (1) and (2), we get $C(pq, (1 + p) (1 + q))$
∴ equation of altitude CM passing through C and perpendicular to AB is $x = pq$ ....(3)

∵ slope of line (2) is = $\left(\frac{1+q}{q}\right)$

∴ slope of altitude BN (as shown in figure) is = $\frac{-q}{1+q}$

∴ equation of BN is $y – 0 = \frac{-q}{1+q}(x + p)$

⇒ $y = \frac{-q}{(1+q)}(x + p)$ .... (4)
Let orthocentre of triangle be H(h, k) which is the point of intersection of (3) and (4)
∴ on solving (3) and (4), we get
$x = pq$ and $y = – pq$ ⇒ $h = pq$ and $k = –pq$
∴ $h + k = 0$
∴ locus of H(h, k) is $x + y = 0$

13. Sol. (i) By family of lines $(2x + 3y – 1) + \lambda (3x – 4y – 6) = 0$ it passes through (3, 2)

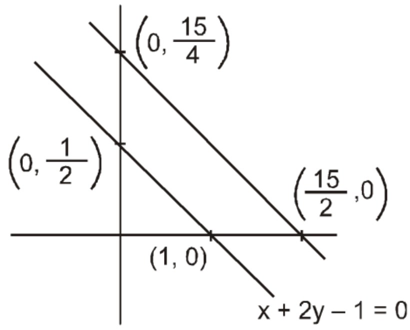
$(6 + 6 – 1) + \lambda (9 – 8 – 6) = 0$ ⇒ $\lambda = \frac{11}{5}$

$(2x + 3y – 1) + \frac{11}{5}(3x – 4y – 6) = 0$ ⇒ $43x – 29y = 71$

(ii) By family of lines $(x + 2y + 3) + \lambda (3x + 4y + 7) = 0$
This is ⊥ to $x – y + 8 = 0$

Hence $m1m2 = – 1$ ⇒ $-\frac{(3\lambda+1)}{(4\lambda+2)}(1) = – 1$ ⇒ $3\lambda + 1 = 4\lambda + 2$
⇒ $\lambda = – 1$ ⇒ $(x + 2y + 3) – (3x + 4y + 7) = 0$
⇒ $–2x – 2y – 4 = 0$ ⇒ $x + y + 2 = 0$

14. Sol.



Point P $\left(1+\frac{t}{\sqrt{2}},\ 2+\frac{t}{\sqrt{2}}\right)$ lies between given line

Hence $L_1(P) = \left(1+\frac{t}{\sqrt{2}}\right) + 2\left(2+\frac{t}{\sqrt{2}}\right) - 1 > 0$

$5 + \frac{3t}{\sqrt{2}} - 1 > 0 \Rightarrow t > -\frac{4\sqrt{2}}{3}$

Now, $L_2(P) = 2\left(1+\frac{t}{\sqrt{2}}\right) + 4\left(2+\frac{t}{\sqrt{2}}\right) - 15 < 0 \Rightarrow$ $10 + \frac{6t}{\sqrt{2}} - 15 < 0 \Rightarrow t < \frac{5\sqrt{2}}{6}$

Hence $t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$.

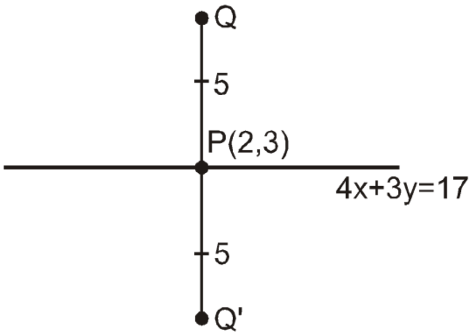
15. Sol. (i) After reflection about line y = x position of point will be (1, 4)

(ii) After this step (4, 4)

(iii) $h = 4\sqrt{2}\cos 150°$, $k = 4\sqrt{2}\sin 150°$

$h = -2\sqrt{6}$, $k = 2\sqrt{2}$

16. Sol.

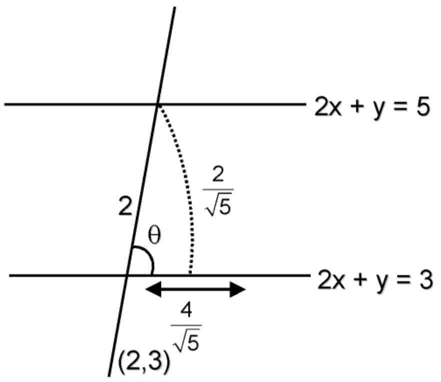


For Q and Q'

$$\frac{x-2}{\frac{4}{5}} = \frac{y-3}{\frac{3}{5}} = \pm 5$$

Q(6, 6) and Q'(– 2, 0).

17. Sol.



$\tan\theta = ½$ **Made Diagram (2,3) replace (213)**

Let slope of line passing through
(2, 3) is m. Hence y – 3 = m (x – 2)

Now, $\tan\theta = \left|\frac{m-(-2)}{1+m(-2)}\right| = \frac{1}{2} \Rightarrow \frac{m+2}{1-2m} = \pm\frac{1}{2}$

$\Rightarrow 2m + 4 = \pm (1–2m) \Rightarrow m = -\frac{3}{4}$, not defined
Hence equation of line 3x + 4y = 18, x = 2

18. **Sol.** The lines will pass through (4, 5) & parallel to the bisectors between them $\frac{3x-4y-7}{5} = \pm \frac{12x-5y+6}{13}$

by taking + sign, we get 21x + 27y + 121 = 0. Now by taking – sign, we get 99x – 77y – 61 = 0

so slopes of bisectors are $-\frac{7}{9}, \frac{9}{7}$

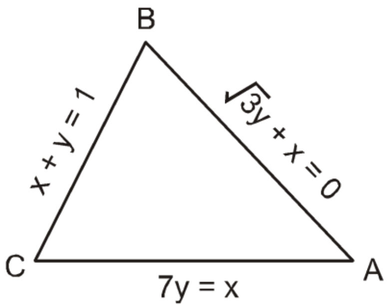
Equation of lines are $y - 5 = -\frac{7}{9}(x-4)$ and $y - 5 = \frac{9}{7}(x-4)$
$\Rightarrow$ 7x + 9y = 73 and 9x – 7y = 1

19. **Sol.** Take A(0, 0), B(a, 0), C(a , a) and D(0, a) then M(a, a/2) and P(a/2, a)

$\Delta AMP = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ a & a/2 & 1 \\ a/2 & a & 1 \end{vmatrix} = \frac{3a^2}{8}$; $\Delta MCP = \frac{a^2}{8}$ $\Rightarrow$ $\Delta ABM = \Delta ADP = \frac{a^2}{4}$

Area of quad. AMCP = $\frac{3a^2}{8} + \frac{a^2}{8} = \frac{a^2}{2}$

20. **Sol.**



Let slope of given lines $m_1 = \frac{1}{7}$, $m_2 = \frac{-1}{\sqrt{3}}$, $m_3 = -1$

Hence interior angle of triangle

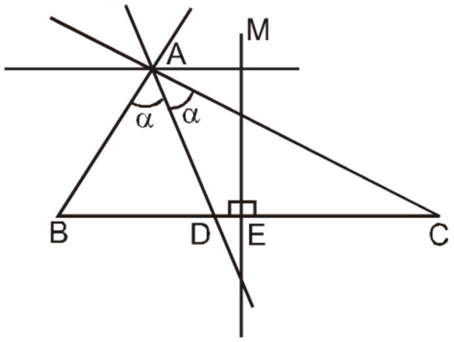
tan A = $\frac{m_1 - m_2}{1 + m_1 m_2} = \frac{\frac{1}{7} + \frac{1}{\sqrt{3}}}{1 - \frac{1}{7\sqrt{3}}} = \frac{\sqrt{3}+7}{7\sqrt{3}-1} > 0$

tan B = $\frac{m_2 - m_3}{1 + m_2 m_3} = \frac{-\frac{1}{\sqrt{3}}+1}{1+\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}-1}{\sqrt{3}+1} > 0$ $\Rightarrow$ tan C = $\frac{m_3 - m_1}{1 + m_2 m_1} = \frac{-1-\frac{1}{7}}{1-\frac{1}{7}} = \frac{-8}{6} < 0$

Hence angle C is obtuse therefore circumcentre and orthocentre lies outside the triangle.

21.

Sol.



P lies on line AD; Q lies on EM

(where E is midpoint of BC) $\Rightarrow$ $n(P \cup Q) = 2$

22. **Sol.**

AP, AB, AQ are in H.P.

AP – $\frac{AB}{2}$, AB – $\frac{AB}{2}$, AQ – $\frac{AB}{2}$ are in G.P.

MP, MB, MQ are in G.P.