Exercise-1

✍ **Marked Questions may have for Revision Questions.**

*** Marked Questions may have more than one correct option.**

OBJECTIVE QUESTIONS

Section (A) : Definition of limit, LHL & RHL, Indeterminate forms

A-1. If $f(x) = \begin{cases} 4x, & x < 0 \\ 1, & x = 0 \\ 3x^2, & x > 0 \end{cases}$, then $\lim_{x\to 0} f(x)$ equals

(1) 0 (2) 1 (3) 3 (4) Does not exist

A- 2.✍ The value of $\lim_{x\to 2}\left\{\frac{x}{2}\right\}$, where {.} represents fractional part function, is

(1) 0 (2) 1 (3) –1 (4) Limit does not exists

A-3. Which of the following limit exists ?

(1) $\lim_{x\to 0}\left(\frac{1}{x}\right)$ (2) $\lim_{x\to 0}\left(\frac{1}{x^2}\right)$ (3) $\lim_{x\to 0}\left(2^{1/x}\right)$ (4) $\lim_{x\to \pi/2}(\sin x)$

A-4. Which of the following limits exists -

(1) $\lim_{x\to 0} x\,|x|$ (2) $\lim_{x\to 1/4} [x]$ (3) $\lim_{x\to 0} x \sin 1/x$ (4) All of the above

A-5.✍ The value of $\lim_{x\to \pi}$ sgn [tan x], where [.] represents greatest integer function, is

(1) 0 (2) 1 (3) –1 (4) Limit does not exists

A-6. $\lim_{x\to 0} \sin^{-1}(\sec x)$ is equal to

(1) $\frac{\pi}{2}$ (2) 1 (3) 0 (4) Limit not exist

A-7. $\lim_{x\to 0} \frac{x}{|x|+x^2}$ equals-

(1) 1 (2) – 1 (3) 0 (4) Does not exist

A-8. $\lim_{x\to 0^-} \frac{[x]}{x}$ (where [.] denotes greatest integer function) is

(1) an indeterminate form $\frac{0}{0}$ (2) equal to 1

(3) equal to 0 (4) not an indeterminate form

A-9. $\lim\limits_{x\to-\infty} \sqrt{x^2+1} + x$ is

(1) an indeterminate form (2) equal to 1

(3) not an indeterminate form (4) equal to 2

A-10. $\lim\limits_{x\to1} (1 - x + [x - 1] + [1 - x])$ is equal to (where [ ] denotes greatest integer function)

(1) 0 (2) 1 (3) -1 (4) does not exist

A-11. If [x] = greastest integer ≤ x, then $\lim\limits_{x\to2} (-1)^{[x]}$ is equal to -

(1) 1 (2) – 1 (3) ± 1 (4) limit doesn't exist

A-12. Let f(x) = $\dfrac{|x+\pi|}{\sin x}$, then $\lim\limits_{x\to-\pi} f(x) =$

(1) -1 (2) 1 (3) 0 (4) does not exist

Section (B) : Factorisation, Rationalisation, Use of standard limits, use of substitution

B-1. $\lim\limits_{x\to1} \dfrac{x-1}{2x^2-7x+5}$ equals -

(1) 1/3 (2) –1/3 (3) 1/2 (4) –1/2

B-2. $\lim\limits_{x\to0} (x^2 - 9)\left(\dfrac{1}{x+3}+\dfrac{1}{x-3}\right)$ equals -

(1) 2 (2) 4 (3) 0 (4) Does not exist

B-3. $\lim\limits_{x\to1} \dfrac{\sqrt{7x+2}-3}{\sqrt{5x-1}-\sqrt{6x-2}}$

(1) $\dfrac{7}{3}$ (2) $-\dfrac{7}{3}$ (3) $-\dfrac{14}{3}$ (4) $\dfrac{7}{6}$

B-4. $\text{Lim}\limits_{x\to a} \dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}$ equal to :

(1) $\dfrac{2}{3}$ (2) $\dfrac{2}{\sqrt{3}}$ (3) 2 (4) $\dfrac{2}{3\sqrt{3}}$

B-5. $\lim\limits_{x\to1} \dfrac{(3x-4)(\sqrt{x}-1)}{2x^2+x-3} =$

(1) $\dfrac{-1}{10}$ (2) $\dfrac{1}{10}$ (3) $\dfrac{-1}{8}$ (4) $\dfrac{1}{8}$

B-6. $\lim\limits_{x\to4}\left(\dfrac{x^{3/2}-8}{x-4}\right) =$

(1) $\dfrac{3}{2}$ (2) 3 (3) $\dfrac{2}{3}$ (4) $\dfrac{1}{3}$

B-7. $\lim\limits_{x\to 0} \dfrac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$ is equal to

(1) $\sqrt{2}$ (2) $\dfrac{\sqrt{2}}{8}$ (3) 0 (4) $\dfrac{1}{\sqrt{2}}$

B-8. The value of $\lim\limits_{x\to a} \dfrac{\sqrt{x-b}-\sqrt{a-b}}{x^2-a^2}$ (a > b) equals

(1) $\dfrac{1}{4a}$ (2) $\dfrac{1}{a\sqrt{a-b}}$ (3) $\dfrac{1}{2a\sqrt{a-b}}$ (4) $\dfrac{1}{4a\sqrt{a-b}}$

B-9. $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\tan 5x}$

(1) $\dfrac{5}{3}$ (2) $-\dfrac{5}{3}$ (3) $\dfrac{3}{5}$ (4) $-\dfrac{3}{5}$

B-10. $\lim\limits_{x\to 0} \dfrac{\tan mx}{\ell n(1-5x)}$

(1) $\dfrac{m}{5}$ (2) $\dfrac{5}{m}$ (3) $-\dfrac{m}{5}$ (4) $-\dfrac{5}{m}$

B-11. $\lim\limits_{x\to a} \dfrac{(x)^{5/3}-(a)^{5/3}}{x-a}$ =

(1) $a^{2/3}$ (2) $\dfrac{5}{3}a^{5/3}$ (3) $\dfrac{5}{3}a^{2/3}$ (4) $\dfrac{3}{5}a^{2/3}$

B-12. $\lim\limits_{x\to 0^+} \dfrac{\sqrt{x}\tan x}{\left(e^x-1\right)^{3/2}}$ equals

(1) 0 (2) 1 (3) 1/2 (4) 2

B-13. $\lim\limits_{x\to 0} \dfrac{e^{\alpha x}-e^{\beta x}}{x}$ equal to -

(1) $\alpha\beta$ (2) $\alpha + \beta$ (3) $\alpha - \beta$ (4) $\dfrac{\alpha}{\beta}$

B-14. $\lim\limits_{x\to 0} \dfrac{(4^x-1)^3}{\sin\left(\dfrac{x}{p}\right)\ \ell n\left(1+\dfrac{x^2}{3}\right)}$ is equal to

(1) $9\,p\,(\ell n\, 4)$ (2) $3\,p\,(\ell n\, 4)^3$ (3) $12\,p\,(\ell n\, 4)^3$ (4) $27\,p\,(\ell n\, 4)^2$

B-15. $\lim\limits_{x\to 0} \dfrac{2^x-1}{(1+x)^{1/2}-1}$ =

(1) $\log_e 2$ (2) $\log_e 4$ (3) $\log_e \sqrt{2}$ (4) $\log_e 8$

B-16. $\underset{x\to 0}{\ell im} \dfrac{\sin\ \ 4x}{1-\sqrt{(1-x)}}$ equals -

(1) 4 (2) 8 (3) 10 (4) 12

B-17. $\underset{x\to 0}{\ell im} \dfrac{x\left(1-\sqrt{1-x^2}\right)}{\sqrt{1+x^2}\,(\sin^{-1}x)^3}$ equals

(1) 0 (2) 1 (3) 1/2 (4) 1/4

B-18. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x} =$

(1) – 6 (2) – 2 (3) 2 (4) 1

B-19. $\underset{x\to 0}{\ell im} \dfrac{e^{\alpha x}-e^{\beta x}}{\sin\alpha x-\sin\beta x}$ equals

(1) 0 (2) $\alpha-\beta$ (3) – 1 (4) 1

B-20. The value of $\underset{x\to 0}{\ell im} \dfrac{\sec 4x-\sec 2x}{\sec 3x-\sec x}$ is

(1) 1 (2) 0 (3) 3/2 (4) ∞

B-21. $\underset{x\to 0}{\ell im} \dfrac{\ell n(1+\tan x)}{x} =$

(1) 0 (2) 1 (3) – 1 (4) Limit doesn't exists

B-22. $\underset{x\to 0}{\ell im} \dfrac{\ell n(2+x)+\ell n 0.5}{x}$ is equal to

(1) $\dfrac{1}{2}$ (2) $\dfrac{3}{2}$ (3) $-\dfrac{1}{2}$ (4) 1

B-23 $\underset{x\to 1}{\ell im} \dfrac{\ell n(1-\ell n x)}{\ell n x^2} =$

(1) $-\dfrac{1}{2}$ (2) $\dfrac{1}{2}$ (3) –1 (4) Limit doesn't exists

B-24. $\underset{x\to 0}{\ell im} \dfrac{e^{\tan x}-e^{x}}{\tan x-x}$ is equal to

(1) $\dfrac{1}{2}$ (2) 1 (3) $-\dfrac{1}{2}$ (4) – 1

B-25. $\underset{x\to 2}{\ell im} \dfrac{(x+6)^{1/3}-2}{2-x} =$

(1) $\dfrac{1}{12}$ (2) $-\dfrac{1}{12}$ (3) $\dfrac{1}{2}$ (4) does not exist

Section (C) : Infinite limits

C-1. The value of $\lim\limits_{x\to\infty} \dfrac{\sqrt{3x^2-1}-\sqrt{2x^2-1}}{4x+3}$ is -

(1) $\dfrac{1}{4}(\sqrt{3}-\sqrt{2})$ (2) $\dfrac{1}{4}(\sqrt{3}+\sqrt{2})$ (3) $(\sqrt{3}-\sqrt{2})$ (4) $\dfrac{1}{2}(\sqrt{3}-\sqrt{2})$

C-2. The value of $\lim\limits_{x\to\infty} \dfrac{\sqrt{(x^2-1)}}{2x+1}$ is -

(1) 1 (2) 0 (3) –1 (4) 1/2

C-3. The value of $\lim\limits_{n\to\infty} \left(\dfrac{1}{1-n^4}+\dfrac{8}{1-n^4}+.......+\dfrac{n^3}{1-n^4}\right)$ is -

(1) 1 (2) 0 (3) $-\dfrac{1}{4}$ (4) $-\dfrac{1}{2}$

C-4. $\lim\limits_{n\to\infty} \dfrac{1+5+5^2+.......+5^{n-1}}{1-25^n} =$

(1) 0 (2) –1 (3) 1 (4) ∞

C-5. If n ∈ N, then $\lim\limits_{n\to\infty} \dfrac{(n+2)!+(n+3)!}{(n+4)!} =$

(1) 0 (2) ∞ (3) 1 (4) $\dfrac{1}{2}$

C-6. $\lim\limits_{n\to\infty} \left[\sqrt{x}\left(\sqrt{x+c}-\sqrt{x}\right)\right] =$

(1) c/2 (2) 2c (3) 1/c (4) c

C-7. $\lim\limits_{n\to\infty} \left(\sqrt{(x+a)(x+b)}-x\right) =$

(1) $\dfrac{a-b}{2}$ (2) $\dfrac{a-b}{3}$ (3) $\dfrac{a+b}{2}$ (4) $\dfrac{a+b}{3}$

C-8. $\lim\limits_{n\to\infty} \dfrac{5^{n+1}+3^n-2^{2n}}{5^n+2^n+3^{2n+3}} =$

(1) 5 (2) 3 (3) 1 (4) zero

C-9. $\lim\limits_{x\to-\infty} \dfrac{x^4\sin\left(\dfrac{1}{x}\right)+x^2}{1+|x|^3} =$

(1) 0 (2) 1 (3) – 1 (4) ∞

C-10. $\lim\limits_{x\to\infty} \sqrt{\dfrac{x-\sin x}{x+\cos^2 x}} =$

(1) – 1 (2) 1 (3) 0 (4) does not exist

C-11. $\lim\limits_{x\to\infty}\left(\dfrac{(x+1)^{10}+(x+2)^{10}+.....+(x+100)^{10}}{x^{10}+10^{10}}\right)=$

(1) 10^2 (2) 10^3 (3) ∞ (4) 10^4

Section (D) : Use of expansion, L-Hospital rule

D-1. $\lim\limits_{x\to0}\dfrac{e^x+e^{-x}-2\cos x}{x\sin x}=$

(1) 1 (2) 2 (3) –1 (4) – 2

D-2. $\lim\limits_{x\to0}\dfrac{1+\sin\ x-\cos\ x+\log_e\ (1-x)}{x^3}=$

(1) 1/2 (2) –1/2 (3) 0 (4) 1

D-3. $\lim\limits_{x\to1}\dfrac{1+\log_e\ x\ -x}{1-2x+x^2}=$

(1) 1 (2) – 1 (3) –1/2 (4) 1/2

D-4. The value of $\lim\limits_{h\to0}\left(\dfrac{1}{h(8+h)^{1/3}}-\dfrac{1}{2h}\right)$ is-

(1) 1/12 (2) –4/3 (3) –16/3 (4) –1/48

D-5. If $\lim\limits_{x\to0}\dfrac{axe^x-\ b\ \ln\ (1+x)\ +\ cxe^{-x}}{x^2\sin x}=2$, then a + b + c =

(1) 12 (2) 24 (3) 36 (4) –12

D-6. $\lim\limits_{x\to\pi/2}$ tan x $\log_e$ sin x =

(1) 0 (2) 1 (3) – 1 (4) $\dfrac{1}{2}$

D-7. The value of $\lim\limits_{x\to1}$ sec $\dfrac{\pi}{2x}$ $\log_e$ x is -

(1) $\pi/2$ (2) $2/\pi$ (3) $-\pi/2$ (4) $-2/\pi$

D-8. $\lim\limits_{x\to1}\dfrac{x+x^2+.....+x^n-n}{x-1}=$

(1) n (2) 0 (3) $\dfrac{n^2}{2}$ (4) $\dfrac{n(n+1)}{2}$

D-9. $\lim\limits_{x\to a}\dfrac{a^x-x^a}{x^x-a^a}$ = – 1, then a equals -

(1) 0 (2) 1 (3) e (4) – 1

D-10. $\lim\limits_{x\to0}\dfrac{(a+x)^2\sin(a+x)-a^2\sin\ a}{x}=$

(1) a^2 cos a + 2a sin a (2) a(cos a + 2 sin a)
(3) a^2 (cos a + 2 sin a) (4) a^2 cos a – 2a sin a

Section (E) : Limits of form ∞ – ∞ , 0º, ∞º , 1∞ , $\underset{x\to\infty}{\ell im}\frac{x}{e^x}$, $\underset{x\to\infty}{\ell im}\frac{\ell nx}{x}$, Sandwitch theorem

E-1. $\lim\limits_{x\to1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=$

(1) 1 (2) – 1 (3) 0 (4) 3

E-2. $\lim\limits_{x\to0}\left(\frac{a}{x}-\cot\frac{x}{a}\right)=$

(1) a (2) 0 (3) $\frac{1}{a}$ (4) 1

E-3. $\lim\limits_{x\to\infty}\left(\sqrt{x^2-5x+6}-x\right)=$

(1) 0 (2) ∞ (3) $\frac{5}{2}$ (4) $-\frac{5}{2}$

E-4. $\underset{x\to\infty}{\ell im}\left[\sqrt{(a^2x^2+ax+1)}-\sqrt{(a^2x^2+1)}\right]=$

(1) 1 (2) 2 (3) 0 (4) 1/2

E-5. $\underset{x\to0}{\text{Lim}}\ (1+\tan x)^{1/x}=$

(1) 1 (2) e (3) e^2 (4) $\frac{1}{e}$

E-6. $\underset{x\to0}{\text{Lim}}\left(\frac{1+x}{1-x}\right)^{\frac{1}{x}}=$

(1) 1 (2) e (3) e^2 (4) e^{-2}

E-7. $\underset{x\to\infty}{\text{Lim}}\left(1+\frac{2}{x^2}\right)^{x^2}=$

(1) 1 (2) e (3) e^2 (4) e^{-2}

E-8. $\underset{x\to0^+}{\ell im}\left(1+\tan^2\sqrt{x}\right)^{5/x}=$

(1) e^5 (2) e^2 (3) e (4) e^3

E-9. $\underset{x\to\infty}{\text{Lim}}\left(\frac{x+1}{2x+1}\right)^{x^2}=$

(1) $\frac{1}{2}$ (2) 1 (3) 0 (4) $\frac{1}{2}$ e

E-10. $\underset{x\to1}{\text{Lim}}\ (1+\log_e x)^{1/1-x}$

(1) $\frac{1}{e}$ (2) 1 (3) e (4) e^2

E-11. If α and β be the roots of $ax^2 + bx + c = 0$, then $\underset{x\to\alpha}{\ell im}\left(1+ax^2+bx+c\right)^{\frac{1}{x-\alpha}}$ is equal to

(1) a (α − β) (2) ln |a (α − β) | (3) $e^{a(\alpha-\beta)}$ (4) $e^{a|\alpha-\beta|}$

E-12. $\lim\limits_{x\to\infty}\left(\dfrac{x^2-2x+1}{x^2-4x+2}\right)^x =$

(1) 1 (2) 2 (3) e^2 (4) e

E-13. $\lim\limits_{x\to 0}(\sin x)^x =$

(1) 0 (2) 1 (3) $\dfrac{1}{2}$ (4) 2

E-14. The value of $\lim\limits_{x\to 0^+}(\operatorname{cosec} x)^{1/\ell n x}$ is

(1) 1 (2) – 1 (3) e (4) 1/e

E-15. The value of $\lim\limits_{x\to \frac{\pi^+}{2}}\dfrac{\log\left(x-\frac{\pi}{2}\right)}{\tan x}$ is -

(1) 0 (2) 1 (3) – 1 (4) 2

E-16. $\lim\limits_{x\to\infty}\dfrac{x^2}{e^x} =$

(1) 0 (2) 1 (3) – 1 (4) limit does not exists

E-17. $\lim\limits_{x\to\infty}\dfrac{\ell n x}{x^3} =$

(1) 0 (2) 1 (3) – 1 (4) limit does not exists

E-18. The value of $\lim\limits_{x\to 0^+}\dfrac{x}{5}\cdot\left[\dfrac{2}{x}\right]$ (where [.] denotes the greatest integer function) is

(1) $\dfrac{2}{5}$ (2) $-\dfrac{2}{5}$ (3) 0 (4) ∞

E-19. The value of $\lim\limits_{n\to\infty}\dfrac{[1.2x]+[2.3x]+.....+[n.(n+1)x]}{n^3}$ (where [.] denotes the greatest integer function) is

(1) x (2) $\dfrac{x}{2}$ (3) 2x (4) $\dfrac{x}{3}$

Section (F) : Continuity at a point

F-1. If f(x) = $\begin{cases} x\cos(1/x), & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at x = 0, then

(1) k > 0 (2) k < 0 (3) k = 0 (4) k ≥ 0

F-2. If f(x) = $\begin{cases} \dfrac{x^2-(a+2)x+a}{x-2} & x \neq 2 \\ 2, & x = 2 \end{cases}$ is continuous at x = 2, then a is equal to -

(1) 0 (2) 1 (3) –1 (4) 2

F-3. If f(x) = $\begin{cases} (\cos x)^{\frac{1}{x}}, x \neq 0 \\ a, \quad x = 0 \end{cases}$ is continuous at x = 0 then the value of a is-

(1) 0 (2) 1 (3) – 1 (4) e

F-4. If $f(x) = \dfrac{1-\cos 7(x-\pi)}{x-\pi}$, $(x \neq \pi)$ is continuous at $x = \pi$, then $f(\pi)$ equals-
(1) 0 (2) – 1 (3) 1 (4) 7

F-5. For function $f(x) = \begin{cases} \left(1+\dfrac{4x}{5}\right)^{1/x} & x \neq 0 \\ e^{4/5}, & x = 0 \end{cases}$, the correct statement is -
(1) $f(0_+)$ and $f(0_-)$ do not exist (2) $f(0_+) \neq f(0_-)$
(3) f(x) is continuous at x = 0 (4) $\lim\limits_{x\to 0} f(x) \neq f(0)$

F-6. If $f(x) = (\tan x \cot \alpha)^{1/(x-\alpha)}$, $x \neq \alpha$ is continuous at $x = \alpha$, then the value of $f(\alpha)$ is-
(1) $e^{2 \sin 2\alpha}$ (2) $e^{2 \operatorname{cosec} 2\alpha}$ (3) $e^{\operatorname{cosec} 2\alpha}$ (4) $e^{\sin 2\alpha}$

F-7. If function $f(x) = \left(\dfrac{\sin x}{\sin \alpha}\right)^{1/(x-\alpha)}$ for $x \neq \alpha$ where, $\alpha \neq m\pi$ $(m \in I)$ is continuous at $x = \alpha$ then -
(1) $f(\alpha) = e^{\tan \alpha}$ (2) $f(\alpha) = e^{\cot \alpha}$ (3) $f(\alpha) = e^{2 \cot \alpha}$ (4) $f(\alpha) = \cot \alpha$

F-8. Function $f(x) = \begin{cases} \dfrac{x^2-4x+3}{x^2-1} \\ 2, \end{cases}$ is-
(1) Continuous at x = 1 (2) Continuous at x = – 1
(3) Continuous at x = 1 and x = – 1 (4) Discontinuous at x = 1 and x = – 1

F-9. If $f(x) = \begin{cases} ax^2 - b & \text{when } 0 \le x < 1 \\ 2 & \text{when } x = 1 \\ x+1 & \text{when } 1 < x \le 2 \end{cases}$ is continuous at x = 1, then the most suitable values of a, b are-
(1) a = 2, b = 0 (2) a = 1, b = –1 (3) a = 4, b = 2 (4) All the above

F-10. If $f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & x<0 \\ c & x=0 \\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{bx\sqrt{x}} & x>0 \end{cases}$ is continuous at x = 0, then-

(1) a = 3/2, c =1/2, $b \in R - \{0\}$ (2) a = – 3/2, c =1/2, $b \in R - \{0\}$
(3) a = 3/2, c = –1/2, $b \in R - \{0\}$ (4) $a \in R$, c =1/2, $b \in R - \{0\}$

F-11. For function $f(x) = \dfrac{1}{1 + 2^{\cot x}}$

(1) $\lim\limits_{x\to 0^-} f(x) = 1$ (2) $\lim\limits_{x\to 0^+} f(x) = -1$ (3) f(0) = 0 (4) continuous at x= 0

F-12. Let f(x) = [x] + [–x] and m is any integer, then correct statement is,
([.] denotes greatest integer function) -

(1) $\lim_{x\to m} f(x)$ does not exist
(2) f(x) is continuous at x = m
(3) $\lim_{x\to m} f(x)$ exists
(4) $\lim_{x\to m^+} f(x) = 1$

Section (G) : Continuity in an interval, Theorems on continuity, Continuity of composite functions, intermediate mean value theorem

G-1. Let f(x) = 3 – |sin x|, then f(x) is-
(1) Everywhere continuous
(2) Everywhere discontinuous
(3) Continous only at x = 0
(4) Discontinous only at x = 0

G-2. If f(x) = $\begin{cases} \dfrac{\tan x}{\sin x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then f(x) is-
(1) Continuous everywhere
(2) Continuous nowhere
(3) Continuous at x = 0
(4) Continuous only at x = 0

G-3. If f(x) = $\begin{cases} |x|, \text{when} & x < 0 \\ x, \text{when} & 0 \le x < 1 \\ 1, \text{when} & x > 1 \end{cases}$ then f is-
(1) Continuous for every real number x
(2) Discontinuous at x = 0
(3) Discontinuous at x = 1
(4) Discontinuous at x = 0, x = 1

G-4. If f(x) = $\begin{cases} x^2+2 & , x \ge 2 \\ 1-x & , x < 2 \end{cases}$ and g(x) = $\begin{cases} 2x & , x > 1 \\ 3-x & , x \le 1 \end{cases}$, then f (g(x)) is
(1) continuous x ∈ R – {2}
(2) continuous at x ∈ R – {1}
(3) continuous at x ∈ R
(4) continuous at x ∈ R – {0,1,2}

G-5. If f(x) = $\begin{cases} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$, then it is discontinuous at -
(1) x = 0
(2) All points
(3) No point
(4) all natural numbers

G-6. If f(x) = $\begin{cases} -x^2, & x \le 0 \\ 5x-4, & 0 < x \le 1 \\ 4x^2-3x, & 1 < x < 2 \\ 3x+4, & x \ge 2 \end{cases}$, then f(x) is-
(1) Continuous at x = 0 but not at x =1
(2) Continuous at x = 2 but not at x = 0
(3) Continuous at x = 0, 1, 2
(4) Discontinuous at x = 0, 1, 2

G-7. If f(x) = $\begin{cases} \dfrac{x^2}{a} & 0 \le x < 1 \\ a, & 1 \le x < \sqrt{2} \\ \dfrac{(2b^2-4b)}{x^2}, & \sqrt{2} \le x < \infty \end{cases}$ is continuous in the interval [0, ∞) then values of a and b are respectively-

(1) 1, – 1 (2) –1, 1+ $\sqrt{2}$ (3) –1, 1 (4) 1, 1+ $\sqrt{2}$

G-8. If f(x) is continous function and g(x) is discontinuous function, then correct statement is-
(1) f(x) + g(x) is a continuous function (2) f(x) – g(x) is a continuous function
(3) f(x) + g(x) is a discontinuous function (4) f(x) g(x) is a continuous function

G-9. Function $f(x) = 4x_3 + 3x_2 + e_{\cos x} + |x - 3| + \log(a_x - 1) + x_{1/3}$ (a > 1) is discontinuous at -
(1) x = 0 (2) x = 1 (3) x = 2 (4) x = $\frac{\pi}{2}$

G-10. If f(x) = |x – 2| + $\frac{x^2 - 5x + 6}{x - 1}$ + tan x then f (x) is discontinuous in domain at
(1) Only one point (2) Two integral point (3) Three integral point (4) Allways continuous

G-11. Let f(x) = $\left|\left(x + \frac{1}{2}\right)[x]\right|$, when – 2 ≤ x ≤ 2. where [.] represents greatest integer function. Then
(1) f(x) is continuous at x = 2 (2) f(x) is continuous at x = 1
(3) f(x) is continuous at x = – 1 (4) f(x) is discontinuous at x = 0

G-12. If f(x) = $\frac{1}{(1 - x)}$ and g(x) = f[f{f(x))}], then g(x) is discontinuous at-
(1) x = 3 (2) x = 2 (3) x = 0 (4) x = 4

G-13. If y = $\frac{1}{t^2 + t - 2}$ where t = $\frac{1}{x - 1}$, then the number of points of discontinuities of y = f(x), x ∈ R is
(1) 1 (2) 2 (3) 3 (4) infinite

G-14. If f(x) = $\frac{x^2 + 1}{x^2 - 1}$ and g(x) = tan x, then fog (x) is discontinuous at x =
(1) nπ ± $\frac{\pi}{4}$, n ∈ I (2) (2n + 1) $\frac{\pi}{2}$, n ∈ I
(3) (2n – 1) $\frac{\pi}{2}$, n ∈ I (4) All of these

G-15. The equation 2 tan x + 5x – 2 = 0 has
(1) no solution in [0, π/4] (2) at least one real solution in [0, π/4]
(3) two real solution in [0, π/4] (4) None of these

Section (H) : Derivability at a point, Derivability in intervals, Relation between continuity and differentiability

H-1. If f(x)=|x| then RHD of f(x) at x = 0 is
(1) 1 (2) – 1 (3) 0 (4) $\frac{1}{2}$

H-2. If f(x) = $\begin{cases} 0, & x < 0 \\ x^2, & x \geq 0 \end{cases}$ then L.H.D. of f(x) at x = 0 is

(1) 1 (2) – 1 (3) 0 (4) 2

H-3. If f(x) = $\begin{cases} \dfrac{\tan x}{x} & ,x \neq 0 \\ 1 & ,x = 0 \end{cases}$ then f(x) at x = 0 is

(1) continuous and differentiable (2) continuous but not differentiable

(3) neither continuous nor differentiable (4) f(0) does not exists

H-4. Which of the following functions is not differentialble at x = 0

(1) x |x| (2) x^3 (3) e^{-x} (4) x + |x|

H-5. If f(x) = $\begin{cases} x^3 - 1 & x > 1 \\ x - 1 & ,x \leq 1 \end{cases}$ then at x = 1, f(x) is

(1) continuous and differentiable (2) continuous but not differentiable

(3) discontinuous and differentiable (4) nither continuous nor differentiable

H-6 If f(x) = $\begin{cases} e^x + ax & x < 0 \\ b(x-1)^2 & x \geq 0 \end{cases}$ is differentiable at x =0, then (a,b) is

(1) (– 3, 1) (2) (– 3, –1) (3) (3, 1) (4) (–3,2)

H-7. The function $f(x) = \sqrt{1 - e^{-x^2}}$ at x = 0 is

(1) continuous and differentiable (2) discontinuous

(3) differentiable (4) non differentiable

H-8. Function f(x) = $\begin{cases} x^2, & x \leq 0 \\ 1, & 0 < x \leq 1 \\ 1/x, & x > 1 \end{cases}$ is-

(1) Differentiable at x = 0, 1 (2) Differentiable only at x = 0

(3) Differentiable at only x = 1 (4) Not differentiable at x = 0, 1

H-9. If f(x) = $\begin{cases} \dfrac{x(3e^{1/x} + 4)}{2 - e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then f(x) is

(1) continuous as well differentiable at x = 0

(2) continuous but not differentiable at x = 0

(3) neither differentiable at x = 0 nor continuous at x = 0

(4) none of these

H-10. If f(x) = $x\left(\sqrt{x} - \sqrt{x+1}\right)$, then indicate the correct alternative(s):

(1) f(x) is continuous but not differentiable at x = 0

(2) f(x) is differentiable at x = 0

(3) f(x) is not differentiable at x = 0

(4) f(x) is discontinous at x = 0

H-11. Let f : R→ R be such that f(1) = 3 and f'(1) = 6. Then $\lim\limits_{x \to 0} \left(\dfrac{f(1+x)}{f(1)}\right)^{\frac{1}{x}}$ equals

(1) 1 (2) $e^{\frac{1}{2}}$ (3) e^2 (4) e^3

H-12. Which of the following is differentiable function every where -

(1) $x^2 \sin x$ (2) $x|x|$ (3) $\cos x$ (4) all of these

H-13. If $f(x) = \begin{cases} \dfrac{x}{1+e^{1/x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then the function f(x) is differentiable for-

(1) $x \in R_+$ (2) $x \in R$ (3) $x \in R - \{0\}$ (4) $x \in R - \{0,1\}$

H-14. Function $f(x) = |x-1| + |x-2|$ is differentiable in [0, 3], except at -

(1) x = 0 and x = 3 (2) x = 1 (3) x = 2 (4) x = 1 and x = 2

H-15. If $f(x) = \begin{cases} x\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} & x \neq 0 \\ 0, & x = 0 \end{cases}$, then correct statement is-

(1) f is continuous at all points except at x = 0
(2) f is continuous at every point but not differentiable at x = 0
(3) f is differentiable at every point
(4) f is differentiable only at the origin

H-16. Let $f(x) = \begin{cases} x^n \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then f(x) is continuous but not differentiable at x = 0 if -

(1) $n \in (0, 1]$ (2) $n \in [0, \infty)$ (3) $n \in (-\infty, 0)$ (4) n = 0

H-17. Number of points where $f(x) = |\, x \operatorname{sgn}(1 - x^2)\, |$ is non-differentiable is

(1) 1 (2) 2 (3) 3 (4) 4

H-18. The set of all points, where the function $\sqrt[3]{x^2 |x|}$ is differentiable is

(1) $(-\infty, 0) \cup (0, \infty)$ (2) $(-\infty, \infty)$ (3) $(0, \infty)$ (4) $(-\infty, 0)$

H-19. If $f(x) = \cos^{-1}(\cos x)$, then at the points, where f is differentiable, f'(x) equals

(1) 1 (2) – 1 (3) sgn (sinx) (4) – sgn (sinx)

Section (I) : Theorems in derivability, functional equations

I-1. If f (x) is differentiable everywhere, then:

(1) $|f|$ is differentiable everywhere (2) $|f|^2$ is differentiable everywhere
(3) $f|f|$ is not differentiable at some point (4) $f + |f|$ is differentiable everywhere

I-2. The number of points at which the function, $f(x) = |x - 0.5| + |x - 1| + \tan x$ does not have a derivative in the interval (0, 2) are:

(1) 1 (2) 2 (3) 3 (4) 4

I-3. If $f(x) = \sum_{k=1}^{n} a_k |x|^k$, where a_i's are real constants, then f(x) is

(1) continuous at x = 0 for all a_i only if $a_{2k} = 0$ (2) differentiable at x = 0 for all $a_i \in R$
(3) differentiable at x = 0 for all $a_{2k-1} = 0$ (4) none of these

I-4. Which of the following functions is differentiable at x = 0?

(1) $\cos(|x|) + |x|$ (2) $\cos(|x|) - |x|$ (3) $\sin(|x|) + |x|$ (4) $\sin(|x|) - |x|$

I-5. If f(x) be a differentiable function such that f(x+y) = f(x) +f(y) and f(1) = 2 then f'(2) is equal to
(1) 1 (2) 1/2 (3) – 1 (4) 2

I-6. If f(x) be a differentiable function such that f(x+y) = f(x).f(y) $\forall x, y \in R$ and f(1) = 2 then $\sum_{r=0}^{9} f(r) =$
(1) 1025 (2) 1023 (3) $\frac{1025}{2}$ (4) $\frac{1023}{2}$

I-7. If f(x) be a differentiable function for all positive numbers such that f(x.y) = f(x) +f(y) and f(e) = 1 then $\lim_{x \to 0} \frac{f(x+1)}{2x}$
(1) 2 (2) 1 (3) 1/2 (4) –1

I-8. Let f (x + y) = f(x) f(y) for all x, y, ∈ R, suppose that f(3) = 3 and f '(0) = 2 then f '(3) is equal to-
(1) 22 (2) 44 (3) 28 (4) 33

I-9. If f(x) is a polynomial functions satisfying the condition f(x) + f $\left(\frac{1}{x}\right)$ = f(x). f $\left(\frac{1}{x}\right)$, find f(2) if f(3) = – 80.
(1) 15 (2) 17 (3) –15 (4) 0

Exercise-2

✍ **Marked Questions may have for Revision Questions.**

*** Marked Questions may have more than one correct option.**

PART - I : OBJECTIVE QUESTIONS

1. $\lim\limits_{x\to 0} \dfrac{\sin(6x^2)}{\ell n\cos(2x^2 - x)}$ is equal to

(1) 12 (2) − 12 (3) 6 (4) − 6

2. $\lim\limits_{x\to 0^+} \dfrac{\cos^{-1}(1-x)}{\sqrt{x}}$ is equal to

(1) $\dfrac{1}{\sqrt{2}}$ (2) $\sqrt{2}$ (3) 1 (4) 0

3.✍ $\lim\limits_{n\to\infty} n\cos\left(\dfrac{\pi}{4n}\right)\sin\left(\dfrac{\pi}{4n}\right)$ has the value equal to:

(1) π/3 (2) π/4 (3) π/6 (4) π/2

4.✍ $\lim\limits_{x\to 1} \dfrac{\left(\sum\limits_{k=1}^{100} x^k\right) - 100}{x-1}$ is equal to

(1) 0 (2) 5050 (3) 4550 (4) − 5050

5. If $f(x) = \begin{cases} x^2+4, & x \le 2 \\ x+2, & x > 2 \end{cases}$ and $g(x) = \begin{cases} x^2, & x \le 2 \\ 8, & x > 2 \end{cases}$ then $\lim\limits_{x\to 2} f(x)\,g(x)$ equals-

(1) 64 (2) 32 (3) 4 (4) 16

6.✍ If $f(x) = \begin{cases} x & \text{when } x \in Q \\ -x & \text{when } x \notin Q \end{cases}$, then $\lim\limits_{x\to 0} f(x)$ equals-

(1) 0 (2) 1 (3) – 1 (4) Does not exist

7. If f is an odd function and $\lim\limits_{x\to 0} f(x)$ exists then $\lim\limits_{x\to 0} f(x)$ equals-

(1) 0 (2) 1 (3) – 1 (4) $\dfrac{1}{2}$

8.✍ $\lim\limits_{x\to\infty} \dfrac{\sqrt{x}}{\sqrt{x+\sqrt{x+\sqrt{x}}}}$ =

(1) 0 (2) 1 (3) ∞ (4) 2

9.✍ $\lim\limits_{x\to\pi/2} \left[\dfrac{x-\frac{\pi}{2}}{\cos x}\right]$ is equal to (where [.] represents greatest integer function)

(1) – 1 (2) 0 (3) – 2 (4) does not exist

10. $\lim\limits_{x\to a^-}\left(\dfrac{|x|^3}{a}-\left[\dfrac{x}{a}\right]^3\right)$ (a < 0), where [x] denotes the greatest integer less than or equal to x is

(1) $a^2 + 1$ (2) $-a^2 - 1$ (3) a^2 (4) $-a^2$

11. If $\lim\limits_{x\to 0}\dfrac{x^3}{\sqrt{a+x}\,(bx-\sin x)} = 1$, then the constants ‘a’ and ‘b’ are (where a > 0)

(1) b = 1, a = 36 (2) a = 1, b = 6 (3) a = 1, b = 36 (4) b = 1, a = 6

12. If $\lim\limits_{x\to 0}\dfrac{a+b\sin x-\cos x+ce^x}{x^3}$ exists, then the value of a + b + c is

(1) 0 (2) 1 (3) 2 (4) 10

13. If $\lim\limits_{x\to 0}\left[1+x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta$, b > 0 and $\theta \in (-\pi, \pi]$, then the value of θ is

(1) $\pm\dfrac{\pi}{4}$ (2) $\pm\dfrac{\pi}{3}$ (3) $\pm\dfrac{\pi}{6}$ (4) $\pm\dfrac{\pi}{2}$

14. If $\ell = \lim\limits_{x\to 0}\dfrac{x(1+a\cos x)-b\sin x}{x^3} = \lim\limits_{x\to 0}\dfrac{1+a\cos x}{x^2} - \lim\limits_{x\to 0}\dfrac{b\sin x}{x^3}$, where ℓ∈σσR, then

(1) (a, b) = (–1, 0) (2) a & b are any real numbers

(3) (a, b) = (1, 0) (4) (a, b) = (0, 1)

15. Let $f(x) = \dfrac{x^2 - 9x + 20}{x - [x]}$ (where [.] represents greatest integer function), then $\lim\limits_{x\to 5} f(x) =$

(1) 0 (2) 1 (3) 2 (4) does not exist

16. $\lim\limits_{x\to 1/\sqrt{2}}\dfrac{x-\cos(\sin^{-1}x)}{1-\tan(\sin^{-1}x)} =$

(1) $\dfrac{1}{\sqrt{2}}$ (2) $-\dfrac{1}{\sqrt{2}}$ (3) $\dfrac{1}{2}$ (4) $-\dfrac{1}{2}$

17. The value of $\lim\limits_{x\to\infty}(x + e^x)^{2/x}$ is-

(1) 1 (2) 2 (3) e (4) e^2

18. $\lim\limits_{x\to\infty}\dfrac{e^x\left(\left(2^{x^n}\right)^{\frac{1}{e^x}}-\left(3^{x^n}\right)^{\frac{1}{e^x}}\right)}{x^n}$, n ∈ N is equal to :

(1) 0 (2) $\ell n\dfrac{2}{3}$ (3) $\ell n\dfrac{3}{2}$ (4) $\ell n\dfrac{1}{2}$

19. The value of $\lim\limits_{x\to\infty}\dfrac{x\,\ell n\left(1+\dfrac{\ell n\,x}{x}\right)}{\ell n x}$ is

(1) 1 (2) 0 (3) – 1 (4) limit does not exist

20. $\lim\limits_{x\to 0}\left(\dfrac{1^x+2^x+3^x+.......+n^x}{n}\right)^{1/x} =$

(1) $(n!)_n$ (2) $(n!)_{1/n}$ (3) n! (4) ln (ns!)

21. $\lim\limits_{x\to 1}$ $(\log_5 5x)_{\log x\,5}$ =

(1) 1 (2) e (3) – 1 (4) does not exist

22. $\lim\limits_{\theta\to 0}\left(\left[\dfrac{n\sin\theta}{\theta}\right]+\left[\dfrac{n\tan\theta}{\theta}\right]\right)$, where [ ] is the greatest integer function and n ∈ N, is

(1) 2n (2) 2n + 1 (3) 2n – 1 (4) does not exist

23. A function f(x) is defined as f(x) = $\dfrac{\cos(\sin x)-\cos x}{x^2}$, x ≠ 0 and f(0) = a then f(x) is continuous at x = 0 if 'a' equals

(1) 0 (2) 4 (3) 5 (4) 6

24. Let f(x) = $\dfrac{1-\sin x}{(\pi-2x)^2}\cdot\dfrac{\ell n(\sin x)}{\ell n(1+\pi^2-4\pi x+4x^2)}$, x≠ $\dfrac{\pi}{2}$. The value of f$\left(\dfrac{\pi}{2}\right)$ so that the function is continuous at x = π/2 is:

(1) 1/16 (2) 1/32 (3) −1/64 (4) 1/128

25. Which of the following function(s) defined below do not have single point continuity.

(1) f(x) = $\begin{cases}1 & \text{if } x\in Q\\ 0 & \text{if } x\notin Q\end{cases}$ (2) g(x) = $\begin{cases}x & \text{if } x\in Q\\ 1-x & \text{if } x\notin Q\end{cases}$

(3) h(x) = $\begin{cases}x & \text{if } x\in Q\\ 0 & \text{if } x\notin Q\end{cases}$ (4) k(x) = $\begin{cases}x & \text{if } x\in Q\\ -x & \text{if } x\notin Q\end{cases}$

26. If f(x) = x + {− x} + [x], where [.] is the integral part & { . } is the fractional part function, then the number of points of discontinuity of f in [−2, 2] is/are

(1) 3 (2) 4 (3) 5 (4) 7

27. Let f(x) = [x] + $\sqrt{x-[x]}$, where [.] denotes the greatest integer function. Then

(1) f(x) is discontinuous on R_+ (2) f(x) is continuous on R

(3) f(x) is discontinuous on R – I (4) discontinuous at x = 1

28._ The value of λ for which $\dfrac{x^3}{4}$ – sin πx+3 = λ, x∈ [–2,2] has atleast a solution

(1) $-\dfrac{1}{2}$ (2) $\dfrac{1}{2}$ (3) $\dfrac{7}{3}$ (4) 6

29. If f(x) = $\begin{cases}ax^2-b & \text{if } |x|<1\\ -\dfrac{1}{|x|} & \text{if } |x|\ge 1\end{cases}$ is derivable at x = 1 , then the values of a + b is

(1) 0 (2) 1 (3) 2 (4) 3

30. If $f(x) = \dfrac{x}{\sqrt{x+1}-\sqrt{x}}$ be a real valued function, then

(1) f(x) is continuous, but f'(0) does not exist
(2) f(x) is differentiable at x = 0
(3) f(x) is not continuous at x = 0
(4) f(x) is not differentiable at x = 0

31. Function $f(x) = \begin{cases} x\tan^{-1}(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$ at x = 0 is-

(1) Discontinuous (2) Continuous (3) Differentiable (4) None of these

32. Let $f(x) = \begin{cases} \sin 2x, & 0 < x \le \pi/6 \\ ax+b, & \pi/6 < x < 1 \end{cases}$. If f(x) and f '(x) are continuous, then-

(1) $a = 1, b = \dfrac{1}{\sqrt{2}} + \dfrac{\pi}{6}$ (2) $a = \dfrac{1}{\sqrt{2}}, b = \dfrac{1}{\sqrt{2}}$ (3) $a = 1, b = \dfrac{\sqrt{3}}{2} - \dfrac{\pi}{6}$ (4) $a = 1, b = \dfrac{\sqrt{3}}{2} + \dfrac{\pi}{6}$

33. For what triplets of real numbers (a, b, c) with a ≠ 0 the function

$f(x) = \begin{cases} x, & x \le 1 \\ ax^2 + bx + c, & \text{otherwise} \end{cases}$ is differentiable for all real x?

(1) $\{(a, 1-2a, a) \mid a \in R, a \neq 0\}$
(2) $\{(a, 1-2a, c) \mid a, c \in R, a \neq 0\}$
(3) $\{(a, b, c) \mid a, b, c \in R, a + b + c = 1\}$
(4) $\{(a, 1-2a, 0) \mid a \in R, a \neq 0\}$

34. If the derivative of the function $f(x) = \begin{cases} ax^2 + b, & x < -1 \\ bx^2 + ax + 4, & x \ge -1 \end{cases}$ is everywhere continuous, then-

(1) a = 2, b = 3 (2) a = 3, b = 2 (3) a = – 2, b = – 3 (4) a = – 3, b = – 2

35. [x] denotes the greatest integer less than or equal to x. If f(x) = [x] [sin πx] in (−1,1), then f(x) is:

(1) continuous at x = 0
(2) continuous in (−1, 0)
(3) differentiable in (−1,1)
(4) none of these

36. The number of points at which the function f(x) = max. {a − x, a + x, b}, − ∞ < x < ∞, 0 < a < b cannot be differentiable is:

(1) 1 (2) 2 (3) 3 (4) 4

37. Let f : R → R be a function defined by f(x) = max. $\{x, x^3\}$. The set of all points where f(x) is not differentiable is

(1) {–1, 1} (2) {–1, 0} (3) {0, 1} (4) {–1, 0, 1}

38. Let f(x) be defined in [−2, 2] by $f(x) = \begin{cases} \max\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right), & -2 \le x \le 0 \\ \min\left(\sqrt{4-x^2}, \sqrt{1+x^2}\right), & 0 < x \le 2 \end{cases}$, then f(x) :

(1) is continuous at all points
(2) is not continuous at more than one point
(3) is not differentiable only at one point
(4) is not differentiable at more than one point

39. $f(x) = \sin^{-1}\left(\dfrac{1+x^2}{2x}\right)$ is:

(1) continuous but not differentiable at x = 1
(2) differentiable at x = 1
(3) neither continuous nor differentiable at x = 1

(4) continuous everywhere

40. The domain of the derivative of the function f(x) = $\begin{cases} \tan^{-1} x & , \text{ if } |x| \le 1 \\ \frac{1}{2}(|x|-1) & , \text{ if } |x| > 1 \end{cases}$ is

(1) R – {0} (2) R – {1} (3) R – {–1} (4) R – {–1, 1}

41. Lef f: R → R be any function. Define g : R → R by g(x) = |f(x)| for all x. Then g is

(1) onto if f is onto (2) one-one if f is one-one

(3) continuous if f is continuous (4) differentiable if f is differentiable

42. Let f''(x) be continuous at x = 0 and f''(0) = 4 then value of $\lim_{x\to 0} \frac{2f(x)-3f(2x)+f(4x)}{x^2}$ is

(1) 11 (2) 2 (3) 12 (4) 9

43. Let f : R → R be a function such that $f\left(\frac{x+y}{3}\right) = \frac{f(x)+f(y)}{3}$, f(0) = 0 and f'(0) = 3, then

(1) $\frac{f(x)}{x}$ is differentiable in R (2) f(x) is continuous but not differentiable in R

(3) f(x) is continuous in R (4) f(x) is bounded in R

44. Suppose that f is a differentiable function with the property that f(x + y) = f(x) + f(y) + xy and $\lim_{h\to 0} \frac{1}{h}$ f(h) = 3, then

(1) f is a linear function (2) f(x) = 3x + x_2

(3) f(x) = 3x + $\frac{x^2}{2}$ (4) f(x) = 3x – $\frac{x^2}{2}$

45. If a differentiable function f satisfies $f\left(\frac{x+y}{3}\right) = \frac{4-2(f(x)+f(y))}{3}$ ∀ x, y ∈ R, then f(x) is equal to

(1) $\frac{1}{7}$ (2) $\frac{2}{7}$ (3) $\frac{8}{7}$ (4) $\frac{4}{7}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** $\lim\limits_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{\frac{1}{x}} = e$

Statement-2 : $\lim\limits_{x\to a}(1 + f(x))^{g(x)} = e^{\lim\limits_{x\to a} f(x).g(x)}$, if $\lim\limits_{x\to a} f(x) = 0$ and $\lim\limits_{x\to a} g(x) = \infty$.

A-2. **Statement-1 :** $\lim\limits_{x\to\infty}\frac{2x^4+3x^3+7x}{3x^4+2x^2+3x}=\frac{2}{3}$.

Statement-2 : If P(x) and Q(x) are two polynomials with rational coefficients, then

$$\lim_{x\to\infty}\frac{P(x)}{Q(x)}=\frac{\text{coefficient of highest power of x in P(x)}}{\text{coefficient of highest power of x in Q(x)}}$$

A-3. **Statement - 1** f (x) = |x| cos x is not differentiable at x = 0

Statement - 2 Every absolute value function is not differentiable.

A-4. **Statement - 1** f (x) = {tan x} – [tan x] is continuous at $x = \frac{\pi}{3}$, where [.] and {.} represent greatest integral function and fractional part function respecitvely.

Statement - 2 If y = f (x) & y = g (x) are continuous at x = a then y = f (x) ± g (x) are continuous at x= a

Section (B) : MATCH THE COLUMN

B-1_.

Column-I		**Column-II**
(P) $\lim\limits_{x\to 0}\frac{\tan x-\sin x}{x^3}=$	(1)	$\frac{1}{2}$
(Q) $\lim\limits_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}}=$	(2)	1
(R) Let $G(x)=-\sqrt{25-x^2}$ .If $\lim\limits_{x\to 1}\frac{G(x)-G(1)}{x-1}=\frac{1}{2\sqrt{\lambda}}$ then value of λ =	(3)	6
(S) If $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}=e^{1/\lambda}$ then value of λ =	(4)	3

Codes :

	P	Q	R	S
(A)	1	3	2	4
(B)	4	2	3	1
(C)	1	2	3	4

(D)	4	2	3	1

B-2. **Match the column**

Column I		**Column II**
(P) $x\,|x|$	(1)	continuous in (– 1, 1)
(Q) $\sqrt{|x|}$	(2)	differentiable in (– 1, 1)
(R) $x + [x]$	(3)	strictly increasing in (– 1, 1)
(S) $|x - 1| + |x + 1|$	(4)	not differentiable at least at one point in (–1, 1)

Codes :

	P	Q	R	S
(A)	1, 2,3	1,3	3,4	1,2
(B)	1,3,4	1,4	1,4	1,2,3
(C)	1,2,3	1,4	3,4	1,2
(D)	1,3,4	1,2,3	1,4	1,3,4

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If $\lim\limits_{x \to 0} \left(\frac{a+2x}{c+x}\right)^{\frac{1}{x}} = \ell$, where a, c > 0

(1) If $\frac{a}{c} > 1$, then $\ell \to \infty$ (2) If $0 < \frac{a}{c} < 1$, then $\ell = 0$

(3) If $\frac{a}{c} = 1$, then ℓ = non zero finite (4) none of these

C-2. If $\lim\limits_{x \to 0} \frac{(\tan x - x)\,(e^x - \sin x - 1)}{x^n} = \ell$ = non zero finite, then

(1) n = 5 (2) n = 4 (3) $\ell = \frac{5}{6}$ (4) $\ell = \frac{1}{6}$

C-3. For f(x) = [$\cos^{-1}x$], which is true (where [·] denotes greatest integer function.)

(1) $\lim\limits_{x \to (\cos 2)^+} f(x) = 1$ (2) $\lim\limits_{x \to (\cos 2)^-} f(x) = 2$

(3) $\lim\limits_{x \to \cos 2} f(x) = 2$ (4) $\lim\limits_{x \to \cos 2} f(x)$ = does not exist.

C-4. If $\ell\text{im}_{x \to 0} \frac{\sin 2x + a \sin x}{x^3}$ = p (finite), then

(1) a = – 2 (2) a = –1 (3) p = – 2 (4) p = –1

C-5. Let $f(x) = \begin{cases} 1+\dfrac{2x}{a}, & 0 \le x < 1 \\ ax, & 1 \le x < 2 \end{cases}$, if $\lim\limits_{x\to1} f(x)$ exists, then values of a is/are

(1) 2 (2) –1 (3) –2 (4) 1

C-6. Let f(x) = min $\{x, x^2\}$ for every real x, then

(1) f is continuous for all x

(2) f is differentiable for all x

(3) $f'(x) = 0 \ \forall\ x > 1$

(4) f is not differentiable at x = 0, 1

C-7. $x + |y| = 2y$, then y as a function of x is

(1) defined for all real x

(2) continuous at x = 0

(3) differentiable for all x

(4) such that $\dfrac{dy}{dx} = \dfrac{1}{3}$ for x < 0.

C-8. Let $f(x) = \begin{cases} \dfrac{x^2}{2}, & 0 \le x < 1 \\ 2x^2 - 3x + \dfrac{3}{2}, & 1 \le x \le 2 \end{cases}$, then

(1) f is continuous in [0, 2]

(2) f ' is continuous in [0, 2]

(3) f '' is continuous in [0, 2]

(4) f ' is differentiable in [0, 2]

Exercise-3

✍ Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Let α and β be the distinct roots of $ax^2 + bx + c = 0$, then $\lim\limits_{x\to\alpha}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ is equal to : **[AIEEE 2005 (3, –1), 225]**

(1) $\frac{1}{2}(\alpha-\beta)^2$ (2) $-\frac{a^2}{2}(\alpha-\beta)^2$ (3) 0 (4) $\frac{a^2}{2}(\alpha-\beta)^2$

2. If f is a real-valued differentiable function satisfying $|f(x) - f(y)| \le (x - y)^2$, $x,y \in R$ and $f(0) = 0$, then f(1) equals : **[AIEEE 2005 (3, –1), 225]**

(1) 1 (2) 2 (3) 0 (4) – 1

3. Suppose f(x) is differentiable at x = 1 and $\lim\limits_{h\to 0}\frac{1}{h}f(1+h) = 5$, then f'(1) equals : **[AIEEE 2005 (3, –1), 225]**

(1) 6 (2) 5 (3) 4 (4) 3.

4. The set of points, where $f(x) = \dfrac{x}{1+|x|}$ is differentiable, is : **[AIEEE 2006, (3, –1), 120]**

(1) $(-\infty, -1) \cup (-1, \infty)$ (2) $(-\infty, \infty)$ (3) $(0, \infty)$ (4) $(-\infty, 0) \cup (0, \infty)$

5. The function $f : R - \{0\} \to R$ given by $f(x) = \dfrac{1}{x} - \dfrac{2}{e^{2x}-1}$ can be made continuous at x = 0 by defining f(0) as **[AIEEE 2007, (3, –1), 120]**

(1) 2 (2) – 1 (3) 0 (4) 1

6. Let $f : R \to R$ be a function defined by f(x) = Min {x + 1, |x| + 1}. Then which of the following is true? **[AIEEE 2007, (3, –1), 120]**

(1) f(x) ≥ 1 for all x ∈ R (2) f(x) is not differentiable at x = 1

(3) f(x) is differentiable everywhere (4) f(x) is not differentiable at x = 0

7. Let $f(x) = \begin{cases}(x-1)\sin\dfrac{1}{x-1}, & x \ne 1\\ 0, & x = 1\end{cases}$ Then which one of the following is true ? **[AIEEE 2008, (3, –1), 105]**

(1) f is differentiable at x = 0 and at x = 1 (2) f is differentiable at x = 0 but not at x =1

(3) f is differentiable at x = 1 but not at x = 0 (4) f is neither differentiable at x = 0 nor at x = 1

8. Let f(x) = x|x| and g(x) = sin x **[AIEEE 2009, (8, –2), 144]**

Statement-1 gof is differentiable at x = 0 and its derivative is continuous at that point.

Statement-2 gof is twice differentiable at x = 0.

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

9. Let f : **R** → **R** be a positive increasing function with $\lim_{x\to\infty}\frac{f(3x)}{f(x)}$ = 1. Then $\lim_{x\to\infty}\frac{f(2x)}{f(x)}$ is equal to . **[AIEEE– 2010, (8, –2), 144]**

(1) $\frac{2}{3}$ (2) $\frac{3}{2}$ (3) 3 (4) 1

10. Let f : **R** → **R** be a continuous function defined by f(x) = $\frac{1}{e^x + 2e^{-x}}$ **[AIEEE 2010 (4, –1), 144]**

Statement -1 : f(c) = $\frac{1}{3}$, for some c ∈ R.

Statement -2 : 0 < f(x) ≤ $\frac{1}{2\sqrt{2}}$, for all x ∈ R.

(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.

(2) Statement-1 is true, Statement-2 is false.

(3) Statement -1 is false, Statement -2 is true.

(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

11. $\lim_{x\to 2}\left(\frac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$ **[AIEEE– 2011, I, (4, –1), 120]**

(1) does not exist (2) equals $\sqrt{2}$ (3) equals $-\sqrt{2}$ (4) equals $\frac{1}{\sqrt{2}}$

12. Let f: $R \to [0,\infty)$ be such that $\lim_{x\to 5}$ f(x) exists and $\lim_{x\to 5}\frac{(f(x))^2-9}{\sqrt{|x-5|}} = 0$. Then $\lim_{x\to 5}$ f(x) equals : **[AIEEE– 2011, II,(4, –1), 120]**

(1) 0 (2) 1 (3) 2 (4) 3

13. The value of p and q for which the function f(x) = $\begin{cases} \frac{\sin(p+1)x + \sin x}{x} & , \quad x < 0 \\ q & , \quad x = 0 \\ \frac{\sqrt{x+x^2}-\sqrt{x}}{x^{3/2}} & , \quad x > 0 \end{cases}$ is continuous for all x in R, are: **[AIEEE 2011, I,(4, –1), 120]**

(1) p = $\frac{1}{2}$, q = $-\frac{3}{2}$ (2) p = $\frac{5}{2}$, q = $\frac{1}{2}$ (3) p = $-\frac{3}{2}$, q = $\frac{1}{2}$ (4) p = $\frac{1}{2}$, q = $\frac{3}{2}$

14. Define F(x) as the product of two real functions $f_1(x)$ = x, x ∈ R, and $f_2(x)$ = $\begin{cases} \sin\frac{1}{x} & , \quad \text{If } x \neq 0 \\ 0 & , \quad \text{If } x = 0 \end{cases}$ as follows :

F(x) = $\begin{cases} f_1(x) \,.\, f_2(x) & , \quad \text{If } x \neq 0 \\ 0 & , \quad \text{If } x = 0 \end{cases}$ **[AIEEE 2011, II,(4, –1), 120]**

Statement - 1 : F(x) is continuous on R.

Statement - 2 : $f_1(x)$ and $f_2(x)$ are continuous on R.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is true, Statement-2 is true; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

15. If function f(x) is differentiable at x = a, then $\lim_{x \to a} \frac{x^2 f(a) - a^2 f(x)}{x - a}$ is : **[AIEEE 2011, II,(4, –1), 120]**

(1) $-a^2 f'(a)$ (2) $af(a) - a^2 f'(a)$ (3) $2af(a) - a^2 f'(a)$ (4) $2af(a) + a^2 f'(a)$

16. If f : R → R is a function defined by f(x) = [x] cos $\left(\frac{2x-1}{2}\right)\pi$, where[x] denotes the greatest integer function, then f is : **[AIEEE- 2012, (4, –1), 120]**

(1) continuous for every real x.
(2) discontinuous only at x = 0.
(3) discontinuous only at non-zero integral values of x.
(4) continuous only at x = 0.

17. Consider the function, f(x) = |x – 2| + |x – 5|, x ∈ R . **[AIEEE- 2012, (4, –1), 120]**

Statement-1 : f'(4) = 0
Statement-2 : f is continuous in [2, 5], differentiable in (2, 5) and f(2) = f(5).
(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

18. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to **[AIEEE - 2013, (4, –1),360]**

(1) $-\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

19. $\lim_{x \to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$ is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) –π (2) π (3) π/2 (4) 1

20. $\lim_{x \to 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to **[JEE(Main) 2015, (4, – 1), 120]**

(1) 4 (2) 3 (3) 2 (4) $\frac{1}{2}$

21. If the function g(x) = $\begin{cases} k\sqrt{x+1} & , \quad 0 \le x \le 3 \\ mx + 2 & , \quad 3 < x \le 5 \end{cases}$ is differentiable, then the value of k+ m is; **[JEE(Main) 2015, (4, – 1), 120]**

(1) 2 (2) $\frac{16}{5}$ (3) $\frac{10}{3}$ (4) 4

22. Let p = $\lim_{x \to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to: **[JEE(Main) 2016, (4, – 1), 120]**

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

23 For x∈R, f(x) = |log2 – sinx| and g(x) = f(f(x)), then **[JEE(Main) 2016, (4, – 1), 120]**

(1) g'(0) = cos(log2)
(2) g'(0) = –cos(log2)
(3) g is differentiable at x = 0 and g'(0) = –sin(log2)
(4) g is not differentiable at x = 0

24. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$ equals **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\frac{1}{24}$ (2) $\frac{1}{16}$ (3) $\frac{1}{8}$ (4) $\frac{1}{4}$

25. For each t∈R let [t] be the greatest integer less than or equal to t. Then $\lim\limits_{x\to 0^+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+......+\left[\frac{15}{x}\right]\right)$ **[JEE(Main) 2018, (4, – 1), 120]**

(1) is equal to 120 (2) does not exist (in R) (3) is equal to 0 (4) is equal to 15

26. Let S = {t ∈ R : f(x) = |x – π|. $(e^{|x|} - 1)$ sin|x| is not differentiable at t.} Then the set S is equal to : **[JEE(Main) 2018, (4, – 1), 120]**

(1) {π} (2) {0, π} (3) φ (an empty set) (4) {0}

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The integer ' n ' for which $\lim\limits_{x\to 0} \frac{(\cos x - 1)\ (\cos x - e^x)}{x^n}$ is a finite non-zero number, is **[IIT-JEE-2002, Scr. (3, –1), 90]**

(A) 1 (B) 2 (C) 3 (D) 4

2. The domain of the derivative of the function f(x) = $\begin{cases} \tan^{-1} x & , \text{ if } |x| \le 1 \\ \frac{1}{2}(|x|-1) & , \text{ if } |x| > 1 \end{cases}$ is **[IIT-JEE 2002 , Scr, (3, –1), 90]**

(A) R – {0} (B) R – {1} (C) R – {–1} (D) R – {–1, 1}

3. Let f : R → R be such that f(1) = 3 and f'(1) = 6. Then $\lim\limits_{x\to 0}\left(\frac{f(1+x)}{f(1)}\right)^{\frac{1}{x}}$ equals **[IIT-JEE 2002, Scr, (3, –1), 90]**

(A) 1 (B) (C) e^2 (D) e^3

4. If $\lim\limits_{x\to 0} \frac{\left((a-n)nx - \tan x\right)\sin nx}{x^2}$ = 0, where n is a non-zero real number, then a is equal to **[IIT-JEE-2003, Scr. (3, –1), 84]**

(A) 0 (B) $\frac{n+1}{n}$ (C) n (D) n + $\frac{1}{n}$

5. If f(x) is differentiable and strictly increasing function, then the value of $\lim\limits_{x\to 0} \frac{f(x^2) - f(x)}{f(x) - f(0)}$ is **[IIT-JEE 2004, Scr, (3, –1), 84]**

(A) 1 (B) 0 (C) –1 (D) 2

6. For x > 0, $\lim\limits_{x\to 0} (\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x}$ is equal to **[IIT-JEE-2006, (3, –1), 184]**

(A) 0 (B) – 1 (C) 1 (D) 2

7. Let g(x) = $\frac{(x-1)^n}{\log \cos^m (x-1)}$; 0 < x < 2, m and n are integers, m ≠ 0, n > 0, and let p be the left hand derivative of |x – 1| at x = 1. If $\lim\limits_{x\to 1^+}$ g(x) = p, then **[IIT-JEE 2008, P-2, (3, –1), 82]**

(A) n = 1, m = 1 (B) n = 1, m = – 1 (C) n = 2, m = 2 (D) n > 2, m = n

8. If $\lim\limits_{x\to\infty}\left(\frac{x^2+x+1}{x+1} - ax - b\right)$ = 4, then **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) a = 1, b = 4 (B) a = 1, b = –4 (C) a = 2, b = –3 (D) a = 2, b = 3

9. Let $f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}, \quad x \in IR$, then f is **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(A) differentiable both at x = 0 and at x = 2
(B) differentiable at x = 0 but not differentiable at x = 2
(C) not differentiable at x = 0 but differentiable at x = 2
(D) differentiable neither at x = 0 nor at x = 2

Answers

EXERCISE # 1

Section (A)

A-1.	(1)	**A- 2.**	(4)	**A-3.**	(4)	**A-4.**	(4)	**A-5.**	(4)	**A-6.**	(4)	**A-7.**	(4)
A-8.	(4)							**A-9.**	(1)	**A-10.**	(3)	**A-11.**	(4)
		A-12.	(4)										

Section (B)

B-1.	(2)	**B-2.**	(3)	**B-3**.	(3)	**B-4.**	(4)	**B-5.**	(1)	**B-6**.	(2)	**B-7.**	(2)
B-8.	(4)	**B-9.**	(3)	**B-10.**	(3)	**B-11.**	(3)	**B-12.**	(2)	**B-13.**	(3)	**B-14.**	(2)
B-15.	(2)	**B-16.**	(2)	**B-17.**	(3)	**B-18.**	(4)	**B-19.**	(4)	**B-20.**	(3)	**B-21.**	(2)
B-22.	(1)	**B-23**	(1)	**B-24.**	(2)	**B-25.**	(2)						

Section (C)

C-1.	(1)	**C-2.**	(4)	**C-3.**	(3)	**C-4.**	(1)	**C-5.**	(1)	**C-6.**	(1)	**C-7.**	(3)
C-8.	(4)	**C-9.**	(3)	**C-10.**	(2)	**C-11.**	(1)						

Section (D)

D-1.	(2)	**D-2.**	(2)	**D-3.**	(3)	**D-4.**	(4)	**D-5.**	(2)	**D-6.**	(1)	**D-7.**	(2)
D-8.	(4)	**D-9.**	(2)	**D-10.**	(1)								

Section (E)

E-1.	(2)	**E-2.**	(2)	**E-3.**	(4)	**E-4.**	(4)	**E-5.**	(2)	**E-6.**	(3)	**E-7.**	(3)
E-8.	(1)	**E-9.**	(3)	**E-10.**	(1)	**E-11.**	(3)	**E-12.**	(3)	**E-13.**	(2)	**E-14.**	(4)
E-15.	(1)	**E-16.**	(1)	**E-17.**	(1)	**E-18.**	(1)	**E-19.**	(4)				

Section (F)

F-1.	(3)	**F-2.**	(1)	**F-3.**	(2)	**F-4.**	(1)	**F-5.**	(3)	**F-6.**	(2)	**F-7.**	(2)
F-8.	(4)	**F-9.**	(4)	**F-10.**	(2)	**F-11.**	(1)	**F-12.**	(3)				

Section (G)

G-1.	(1)	**G-2.**	(3)	**G-3.**	(3)	**G-4.**	(3)	**G-5.**	(1)	**G-6.**	(2)	**G-7.**	(3)
G-8.	(3)	**G-9.**	(1)	**G-10.**	(4)	**G-11.**	(4)	**G-12.**	(3)	**G-13.**	(3)	**G-14.**	(4)
G-15.	(2)												

Section (H)

H-1.	(1)	**H-2.**	(3)	**H-3.**	(1)	**H-4.**	(4)	**H-5.**	(2)	**H-6**	(1)	**H-7.**	(4)
H-8.	(4)	**H-9.**	(2)	**H-10.**	(2)	**H-11.**	(3)	**H-12.**	(4)	**H-13.**	(3)	**H-14.**	(4)
H-15.	(2)	**H-16.**	(1)	**H-17.**	(3)	**H-18.**	(1)	**H-19.**	(3)				

Section (I)

I-1.	(2)	**I-2.**	(3)	**I-3.**	(3)	**I-4.**	(4)	**I-5.**	(4)	**I-6.**	(2)	**I-7.**	(3)
I-8.	(4)	**I-9.**	(3)										

EXERCISE # 2

PART - I

1.	(2)	**2.**	(2)	**3.**	(2)	**4.**	(2)	**5.**	(2)	**6.**	(1)	**7.**	(1)
8.	(2)	**9.**	(3)	**10.**	(2)	**11.**	(1)	**12.**	(3)	**13.**	(4)	**14.**	(1)
15.	(4)	**16.**	(2)	**17.**	(4)	**18.**	(2)	**19.**	(1)	**20.**	(2)	**21.**	(2)
22.	(3)	**23.**	(1)	**24.**	(3)	**25.**	(1)	**26.**	(3)	**27.**	(2)	**28.**	(3)
29.	(3)	**30.**	(2)	**31.**	(2)	**32.**	(3)	**33.**	(1)	**34.**	(1)	**35.**	(2)
36.	(2)	**37.**	(4)	**38.**	(4)	**39.**	(3)	**40.**	(4)	**41.**	(3)	**42.**	(3)
43.	(3)	**44.**	(3)	**45.**	(4)								

PART - II

Section (A)

A-1. (3) **A-2.** (2) **A-3.** (2) **A-4.** (1)

Section (B)

B-1. (C) **B-2.** (C)

Section (C)

C-1.	(1,2,3)	**C-2.**	(1, 4)	**C-3.**	(1, 2, 4)	**C-4.**	(1, 4)	**C-5.**	(1, 2)	**C-6.**	(1, 4)	**C-7.**	(1,2,4)
C-8.	(1, 2)												

EXERCISE # 3

PART - I

1.	(4)	**2.**	(3)	**3.**	(2)	**4.**	(2)	**5.**	(4)	**6.**	(3)	**7.**	(2)
8.	(3)	**9.**	(4)	**10.**	(4)	**11.**	(1)	**12.**	(4)	**13.**	(3)	**14.**	(3)
15.	(3)	**16.**	(1)	**17.**	(3)	**18.**	(4)	**19.**	(2)	**20.**	(3)	**21.**	(1)
22.	(2)	**23**	(1)	**24.**	(2)	**25.**	(1)	**26.**	(3)				

PART - II

1.	(C)	**2.**	(D)	**3.**	(C)	**4.**	(D)	**5.**	(C)	**6.**	(C)	**7.**	(C)
8.	(B)	**9.**	(B)										