Exercise-1

Marked Questions may have for Revision Questions.
*** Marked Questions may have more than one correct option.**

OBJECTIVE QUESTIONS

Section (A) : Basic Problems (Definition based, Substitution, By parts)

A-1. $\int_0^1 \frac{dx}{\sqrt{x+1}+\sqrt{x}}$ dx =

(1) $\frac{4}{3}\left(\sqrt{2}+1\right)$ (2) $\frac{4}{3}\left(\sqrt{2}-1\right)$ (3) $\frac{3}{4}\left(\sqrt{2}-1\right)$ (4) $\frac{3}{4}\left(\sqrt{2}-2\right)$

A-2. $\int_0^1 xe^x\,dx$ =

(1) 1 (2) 2 (3) 3 (4) 4

A-3. $\int_0^1 \frac{dx}{(x^2+1)(x^2+2)}$ =

(1) $\frac{\pi}{4}+\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$ (2) $\frac{\pi}{2}-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

(3) $\frac{\pi}{4}-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$ (4) $\frac{\pi}{3}-\frac{1}{\sqrt{2}}\tan^{-1}\frac{1}{\sqrt{2}}$

A-4. $\int_0^{\pi/4} \tan^2 x$ dx equals -

(1) π/4 (2) 1 + (π / 4) (3) 1 – (π / 4) (4) 1 – (π / 2)

A-5. $\int_0^2 \frac{3^{\sqrt{x}}}{\sqrt{x}}$ dx, equals-

(1) $\frac{2}{\ell n3}(3^{\sqrt{2}}-1)$ (2) 0 (3) $\frac{2\sqrt{2}}{\ell n3}$ (4) $\frac{3^{\sqrt{2}}}{\sqrt{2}}$

A-6. $\int_0^{\pi/2} \sqrt{1+\sin 2x}$ dx equals -

(1) 1/2 (2) 1 (3) 2 (4) $\frac{3}{2}$

A-7. $\int_0^{\pi/4} \frac{x\,.\sin x}{\cos^3 x}$ dx equals to :

(1) $\frac{\pi}{4}+\frac{1}{2}$ (2) $\frac{\pi}{4}-\frac{1}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{4}+1$

A-8. $\int_{\ell n\pi-\ell n2}^{\ell n\pi} \dfrac{e^x}{1-\cos\left(\frac{2}{3}e^x\right)}$ dx is equal to

(1) $\sqrt{3}$ (2) $-\sqrt{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $-\frac{1}{\sqrt{3}}$

A-9. $\int_1^2 e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$ dx equals

(1) $e\left(\frac{e}{2}-1\right)$ (2) 1 (3) e (e – 1) (4) $\frac{e}{2}$

A-10. If $\int_0^\infty e^{-x^2}$ dx $= \frac{\sqrt{\pi}}{2}$, then $\int_0^\infty e^{-ax^2}$ dx where a > 0 is :

(1) $\frac{\sqrt{\pi}}{2}$ (2) $\frac{\sqrt{\pi}}{2a}$ (3) $2\frac{\sqrt{\pi}}{a}$ (4) $\frac{1}{2}\sqrt{\frac{\pi}{a}}$

A-11. If $I_1 = \int_e^{e^2} \frac{dx}{\ell n\ x}$ and $I_2 = \int_1^2 \frac{e^x}{x}$ dx, then

(1) $I_1 = I_2$ (2) $2I_1 = I_2$ (3) $I_1 = 2I_2$ (4) $2I_1 = 3I_2$

A-12. If $\frac{d}{dx}$ f(x) = g(x) for a ≤ x ≤ b, then $\int_a^b f(x)g(x)dx$ equals to :

(1) f(2) – f(1) (2) g(2) – g(1) (3) $\frac{[f(b)]^2-[f(a)]^2}{2}$ (4) $\frac{[g(b)]^2-[g(a)]^2}{2}$

A-13 The value of $\int_5^{10}\left[\frac{x-5}{5}\right]$ dx, where [.] represents greatest integer function is

(1) 1 (2) 2 (3) 3 (4) 0

A-14. The value of $\int_{-\tan 1}^{0}\left[-\tan^{-1}x\right]$ dx, where [.] represents greatest integer function is

(1) 1 (2) 2 (3) 3 (4) 0

A-15. The value of $\frac{6}{\pi}\int_{\pi/6}^{\pi/3}[2\sin x]$ dx, where [.] represents greatest integer function is

(1) 1 (2) 2 (3) 3 (4) 0

Section (B) : Properties of definite integration

B-1. If f(x) = $\begin{cases} x & x<1 \\ x-1 & x\ge 1\end{cases}$, then $\int_0^2 x^2 f(x)$ dx is equal to :

(1) 1 (2) $\frac{4}{3}$ (3) $\frac{5}{3}$ (4) $\frac{5}{2}$

B-2. $\int_0^{\pi} |1+2\cos x|\, dx$ is equal to :

(1) $\frac{2\pi}{3}$ (2) π (3) 2 (4) $\frac{\pi}{3} + 2\sqrt{3}$

B-3. $\int_0^1 |3x-1|\, dx$ equals

(1) 5/6 (2) 5/3 (3) 10/3 (4) 5

B-4. $\int_{1/e}^{e} |\ell nx|\, dx$ equals

(1) e^{-1} –1 (2) 2 (1 –1/e) (3) 1 – 1/e (4) e – 1

B-5. $\int_0^{1.5} [x^2]\, dx$, where [.] denotes the greatest integer function, is equal to

(1) $\sqrt{2}$ – 2 (2) 2 – $\sqrt{2}$ (3) 2 + $\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

B-6. The value of $\int_{-1}^{3} (|x-2|+[x])\, dx$ is ([x] stands for greatest integer less than or equal to x)

(1) 7 (2) 5 (3) 4 (4) 3

B-7. $\int_1^4 \log_e [x]\, dx$ equals ([.] denotes greatest integer function)

(1) $\log_e 6$ (2) $\log_e 3$ (3) $\log_e 2$ (4) $\log_e 4$

B-8. Suppose for every integer n, . $\int_n^{n+1} f(x)dx = n^2$ The value of $\int_{-2}^{4} f(x)dx$ is :

(1) 16 (2) 14 (3) 19 (4) 21

B-9. The value of $\int_0^{\pi} |\cos x|\, dx$ is -

(1) 2 (2) 0 (3) 3 (4) 1

B-10. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tan x^3\, dx$ =

(1) 1 (2) $\frac{1}{2}$ (3) 2 (4) 0

B-11. $\int_{-2}^{2} x^4\, dx$ =

(1) $\frac{32}{5}$ (2) $\frac{64}{5}$ (3) $\frac{16}{5}$ (4) $\frac{8}{5}$

B-12. $\int_{-1}^{1} x^{17} \cos^4 x\, dx$ is equals to

(1) –2 (2) 2 (3) 0 (4) 1

B-13. The value of $\int_{-\pi/2}^{\pi/2}(x^3 + x\cos x + \tan^5 x + 1)dx$ is equal to

(1) 0 (2) 2 (3) π (4) $\frac{\pi}{2}$

B-14. Let f : R → R, g : R → R be continuous functions. Then the value of integeral $\int_{\ell n\lambda}^{\ell n 1/\lambda}\frac{f\left(\frac{x^2}{4}\right)[f(x)-f(-x)]}{g\left(\frac{x^2}{4}\right)[g(x)+g(-x)]}$ dx is :

(1) depend on λ (2) a non-zero constant (3) zero (4) 1

B-15. $\int_{-1}^{1}\frac{\sin x - x^2}{3-|x|}\,dx$ equals

(1) 0 (2) $2\int_0^1\frac{\sin x}{3-|x|}\,dx$ (3) $\int_0^1\frac{-2x^2}{3-|x|}\,dx$ (4) $2\int_0^1\frac{\sin x - x^2}{3-|x|}\,dx$

B-16. The value of $\int_{-1}^{1}\frac{\cot^{-1}x}{\pi}$ dx is

(1) 1 (2) 2 (3) 3 (4) 0

B.17. $\int_1^2\frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}}$ dx =

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) 0

B-18. If f(3 – x) = f(x), then $\int_1^2 x\ f(x)\ dx$ is equal to

(1) $\frac{3}{2}\int_1^2 f(2-x)\ dx$ (2) $\frac{3}{2}\int_1^2 f(x)\ dx$ (3) $\frac{1}{2}\int_1^2 f(x)\ dx$ (4) $\int_1^2 f(x)\ dx$

B-19. $\int_{2-\log 3}^{3+\log 3}\frac{\log\ (4+x)}{\log(4+x)+\log(9-x)}$ dx is equal to :

(1) cannot be evaluated (2) $\frac{5}{2}$ (3) 1 + 2 log 3 (4) $\frac{1}{2}$ + log 3

B-20. $\int_0^{\pi/2}\frac{a\sin x + b\cos x}{\sin x+\cos x}\,dx$ equal

(1) 0 (2) $(a+b)\frac{\pi}{2}$ (3) a + b (4) $(a+b)\frac{\pi}{4}$

B-21. The value of the integral $\int_0^1 x(1-x)^n\ dx$ is

(1) $\frac{1}{n+1}+\frac{1}{n+2}$ (2) $\frac{1}{(n+1)(n+2)}$ (3) $\frac{1}{n+2}-\frac{1}{n+1}$ (4) $2\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

B-22. If f(x) and g(x) are continuous functions satisfying f(x) = f(a – x) and g(x) + g(a – x) = 2, then $\int_0^a f(x)\, g(x)\, dx$ is equal to :

(1) $\int_0^a g(x)dx$ (2) $\int_0^a f(x)dx$ (3) 0 (4) f(a)g(a)

B-23. $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x}$ equals

(1) 0 (2) 1 (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

B-24. $\int_0^{2\pi} |\sin x|\, dx =$

(1) 1 (2) 2 (3) 3 (4) 4

B-25. The value of the integral $\int_0^{\pi} x \;\ell n \sin x \, dx$ is

(1) $-\frac{\pi^2}{2}\ell n2$ (2) $\frac{\pi^2}{2}\ell n2$ (3) $\pi^2 \ell n2$ (4) – $\pi^2 \ell n2$

B-26. The value of $\int_0^{\pi/2} \ell n|\tan x + \cot x|\, dx$ is equal to :

(1) π ℓn 2 (2) –π ℓn 2 (3) $\frac{\pi}{2}$ ℓn 2 (4) $\frac{\pi}{2}$ – ℓn2

B-27. $\int_0^{\pi/2} (2\ell n \sin x - \ell n \sin 2x)\, dx$ equals

(1) π ℓn 2 (2) – π ℓn 2 (3) $\frac{\pi}{2}\ell n\frac{1}{2}$ (4) – $\frac{\pi}{2}\ell n\frac{1}{2}$

Section (C) : Integration of periodic functions

C-1. $\int_0^{2\pi} |\sin 3x|\, dx$ =

(1) 1 (2) 2 (3) 3 (4) 4

C-2. If $\int_0^{11} \frac{11^x}{11^{[x]}}\, dx = \frac{k}{\log 11}$, (where [ ] denotes greatest integer function) then value of k is

(1) 11 (2) 101 (3) 110 (4) 111

C-3. $\int_{-2}^{10} \operatorname{sgn}\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right) dx$ equals ([.] denotes greatest integer function)

(1) 10 (2) 11 (3) 9 (4) 12

C-4. The value of $\int_0^{[x]} (x - [x])\, dx$ is ([.] denotes greatest integer function)

(1) [x] (2) 2 [x] (3) $\frac{[x]}{2}$ (4) 3 [x]

C-5. The value of $\int_0^{10\pi} (|\sin x| + |\cos x|)\ dx$ is

(1) 10 (2) 20 (3) 40 (4) 50

C-6. $\int_0^{2n\pi} \left(|\sin x| - \left[\left|\frac{\sin x}{2}\right|\right]\right) dx$ (where [ ] denotes the greatest integer function and $n \in I$) is equal to :

(1) 0 (2) 2n (3) $2n\pi$ (4) 4n

C-7. If I = $\int_0^{2\pi} \sin^2 x dx,$ then

(1) I = $2\int_0^{\pi/2} \sin^2 x dx$ (2) I = $4\int_0^{\pi/2} \sin^2 x dx$ (3) I = $2\int_0^{\pi/2} \cos^2 x dx$ (4) I = $8\int_0^{\pi/4} \sin^2 x dx$

Section (D) : Leibinitz theorem, Estimation of definite integrals, Definite integral as limit of sum

D-1. $\lim_{h \to 0} \frac{\int_a^{x+h} \ell n^2 t\ dt - \int_a^x \ell n^2 t\ dt}{h}$ equals to :

(1) 0 (2) $\ell n^2 x$ (3) $\frac{2\ell n x}{x}$ (4) does not exist

D-2. If $\int_a^y \cos t^2 dt = \int_a^{x^2} \frac{\sin t}{t} dt$, then the value of $\frac{dy}{dx}$ is

(1) $\frac{2\sin^2 x}{x\cos^2 y}$ (2) $\frac{2\sin x^2}{x\cos y^2}$ (3) $\frac{2\sin x^2}{x\left(1-2\sin\frac{y^2}{2}\right)}$ (4) $\frac{2\cos x^2}{x\sin y^2}$

D-3. $\frac{d}{dx}\left(\int_{f(x)}^{g(x)} \phi(t)\ dt\right)$ is equal to

(1) $\varphi(g(x)) - \varphi(f(x))$ (2) $\frac{1}{2}[\phi(g(x))]^2 - \frac{1}{2}[\phi(f(x))]^2$

(3) $g'(x)\ \varphi(g(x)) - f'(x)\ \varphi(f(x))$ (4) $\varphi'(g(x))\ g'(x) - \varphi'(f(x)\ f'(x)$

D-4. The value of the function $f(x) = 1 + x + \int_1^x (\ln^2 t + 2\ln t)\, dt$, where $f'(x)$ vanishes is:

(1) e^{-1} (2) 0 (3) $2e^{-1}$ (4) $1 + 2e^{-1}$

D-5. If $x = \int_2^{\sin t} \sin^{-1} z\ dz$ and $y = \int_n^{\sqrt{t}} \frac{\sin z^2}{z}\ dz$. then $\frac{dy}{dx}$ is equal to

(1) $\frac{\tan t}{2t^2}$ (2) $\frac{2t^2}{\tan t}$ (3) $\frac{\tan t}{t^2}$ (4) $\frac{t^2}{\tan t}$

D-6. The value of $\lim_{x \to 0} \frac{\frac{d}{dx}\int_0^{x^3} \sqrt{\cos t}\ dt}{1-\sqrt{\cos x}}$ is

(1) 0 (2) 11 (3) 10 (4) 12

D-7. Let $I = \int_0^{\pi/4} \frac{\tan x}{x} dx$, then

(1) $0 < I < \frac{\pi}{4}$ (2) $I > 1$ (3) $\frac{\pi}{4} < I < 1$ (4) None of these

D-8. If $I = \int_0^1 \frac{\tan x}{\sqrt{x}} dx$ then

(1) $I < \frac{2}{3}$ (2) $I > \frac{2}{3}$ (3) $I < \frac{5}{9}$ (4) $I < \frac{1}{3}$

D-9. Let $I = \int_1^2 \frac{dx}{\sqrt{1+x^4}}$ then

(1) $I > \frac{1}{\sqrt{2}}$ (2) $I < \frac{1}{\sqrt{17}}$ (3) $\frac{1}{\sqrt{17}} \le I \le \frac{1}{\sqrt{2}}$ (4) $I < \frac{1}{\sqrt{19}}$

D-10. $\lim_{n\to\infty} \frac{\sqrt{1} + 2\sqrt{2} + 3\sqrt{3} + ... + n\sqrt{n}}{n^{5/2}}$ is equal to

(1) $\int_0^1 x\sqrt{x}\ dx$ (2) $\frac{5}{2}$ (3) 0 (4) 1

D-11. $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{r^3}{r^4 + n^4}\right)$ equals to :

(1) $\ell n\ 2$ (2) $\frac{1}{2}\ \ell n\ 2$ (3) $\frac{1}{3}\ \ell n\ 2$ (4) $\frac{1}{4}\ \ell n\ 2$

D-12. $\lim_{n\to\infty} \left(\sin\frac{\pi}{2n}.\sin\frac{2\pi}{2n}.\sin\frac{3\pi}{2n}.......\sin\frac{(n-1)\pi}{n}\right)^{1/n}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $\frac{1}{5}$

D-13. If f(x) is integrable over [1, 2], then $\int_1^2 f(x)$ dx is equal to :

(1) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} f\left(\frac{r}{n}\right)$ (2) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=n+1}^{2n} f\left(\frac{r}{n}\right)$

(3) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r+n}{n}\right)$ (4) $\lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{2n} f\left(\frac{r}{n}\right)$

Section (E) : Reduction Formulae, Walli's formula

E-1. If $u_{10} = \int_0^{\pi/2} x^{10}$ sinx dx, then the value of $u_{10} + 90\ u_8$ is :

(1) $9\left(\frac{\pi}{2}\right)^8$ (2) $\left(\frac{\pi}{2}\right)^9$ (3) $10\left(\frac{\pi}{2}\right)^9$ (4) $9\left(\frac{\pi}{2}\right)^9$

E-2. $\int_0^{\infty} x^n e^{-x} dx$ (n is a + ve integer) is equal to

(1) n ! (2) (n – 1) ! (3) (n – 2) ! (4) (n + 1)!

E-3. The value of $\int_0^{\pi/2} \sin^6 x dx$ is

(1) $\frac{5\pi}{16}$ (2) $\frac{5\pi}{64}$ (3) $\frac{5\pi}{8}$ (4) $\frac{5\pi}{32}$

E-4. The value of $\int_0^{\pi} \sin^7 x \cos^6 x dx$ is

(1) $\frac{32}{3003}$ (2) $\frac{32}{303}$ (3) $\frac{32}{3001}$ (4) $\frac{32}{301}$

E-5. The value of $\int_0^{3} x^{5/2}\left(\sqrt{3-x}\right)^3 dx$ is $\frac{3^a\pi}{2^b}$, a,b∈N then

(1) 3a = 4b (2) 2a = 3b (3) 4a = 3b (4) a = b

E-6. $\int_0^{\pi/4} \tan^6 x \, dx =$

(1) $\frac{13}{15}+\frac{\pi}{4}$ (2) $\frac{11}{13}+\frac{\pi}{4}$ (3) $\frac{11}{13}-\frac{\pi}{4}$ (4) $\frac{13}{15}-\frac{\pi}{4}$

Section (F) : Area Under the Curves

F-1. The area bounded by curve y = ℓnx, x = 1, x = 2 and x-axis is

(1) ℓn4e (2) $\ell n \frac{4}{e}$ (3) $\ell n \frac{e}{4}$ (4) $\ell n \frac{3}{e}$

F-2. The area bounded by curve y = e^x, y = 1, y = 3 and y-axis is $\lambda \ell n3 + \mu,\ \lambda, \mu \in I$ then $\lambda + \mu$ =

(1) 5 (2) – 1 (3) 1 (4) 2

F-3. The area bounded by the curve y = tanx, x = $-\frac{\pi}{4}$, x = $\frac{\pi}{3}$, and x-axis is

(1) $\frac{\ell n2}{2}$ (2) ℓn2 (3) $\frac{2}{3}\ell n2$ (4) $\frac{3}{2}\ell n2$

F-4. The area bounded by curve y = x^3, y = – 1, y = 8 and y-axis is

(1) $\frac{45}{4}$ (2) $\frac{4}{45}$ (3) $\frac{51}{4}$ (4) $\frac{47}{4}$

F-5. The area bounded by the curves $25x^2 + 9y^2 = 225$ and 5x + 3y = 15 in first quadrant is

(1) $\frac{15}{2}\left(\frac{\pi}{2}+1\right)$ (2) $\frac{15}{2}\left(\frac{\pi}{2}+2\right)$ (3) $\frac{15}{2}\left(\frac{\pi}{2}-1\right)$ (4) $\frac{15}{2}\left(\frac{\pi}{2}-2\right)$

F-6. The area bounded by the curve xy = 4 and the line x + y = 5 is

(1) $\frac{15}{2}+\ell n4$ (2) $\frac{15}{2}+\ell n2$ (3) $\frac{15}{2}-4\ell n4$ (4) $\frac{15}{2}-\ell n2$

F-7. The area bounded by the curve $x^2 = 4y$, x-axis and the line x = 2 is

(1) 1 (2) $\frac{2}{3}$ (3) $\frac{3}{2}$ (4) 2

F-8. The area bounded by the parabola $y = 4x^2$, $x = 0$ and $y = 1$, $y = 4$ is

(1) 7 (2) $\frac{7}{2}$ (3) $\frac{7}{3}$ (4) $\frac{7}{4}$

F-9. The area bounded by the curve $y^2 = 4x$ and the line $2x - 3y + 4 = 0$ is

(1) $\frac{1}{3}$ (2) $\frac{2}{3}$ (3) $\frac{4}{3}$ (4) $\frac{5}{3}$

F-10. The area of the figure bounded by right of the line $y = x + 1$, $y = \cos x$ & x−axis is:

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{5}{6}$ (4) $\frac{3}{2}$

F-11. The area bounded by the curves y = sinx, y = cosx and y-axis in I quadrant is

(1) $\sqrt{2}$ (2) $\sqrt{2} + 1$ (3) $\sqrt{2} - 1$ (4) $\sqrt{2} + 2$

F-12. The area bounded in the first quadrant between the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and the line $3x + 4y = 12$ is:

(1) $6(\pi - 1)$ (2) $3(\pi - 2)$ (3) $3(\pi - 1)$ (4) $2(\pi - 2)$

Exercise-2

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*** Marked Questions may have more than one correct option.**

PART - I : OBJECTIVE QUESTIONS

1. The value of the integral , $\int_0^1 \frac{dx}{x^2 + 2x\cos\alpha + 1}$ where $0 < \alpha < \frac{\pi}{2}$, is equal to:

(1) $\sin\alpha$ (2) $\alpha\sin\alpha$ (3) $\frac{\alpha}{2\sin\alpha}$ (4) $\frac{\alpha}{2}\sin\alpha$

2. The value of $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}$ dx is

(1) $\frac{4+\pi}{4\sqrt{2}}$ (2) $\frac{4-\pi}{4\sqrt{2}}$ (3) $\frac{\pi}{2}$ (4) $-\frac{\pi}{2}$

3. The value of $\int_{1/e}^{\tan x} \frac{t}{1+t^2}\,dt + \int_{1/e}^{\cot x} \frac{1}{t(1+t^2)}\,dt$, where $x \in (\pi/6, \pi/3)$, is equal to :

(1) 0 (2) 2 (3) 1 (4) cannot be determined

4. If $f(x) = \begin{cases} 0 \ , & \text{where } x = \frac{n}{n+1},\ n = 1, 2, 3..... \\ 1 \ , & \text{else where} \end{cases}$, then the value of $\int_0^2 f(x)\,dx$ is -

(1) 1 (2) 0 (3) 2 (4) ∞

5. If $\int_0^{100} f(x)\,dx = a$, then $\sum_{r=1}^{100}\left(\int_0^1 f\,(r-1+x)\;dx\right) =$

(1) 100 a (2) a (3) 0 (4) 10 a

6. $\int_0^{\infty}\left[2e^{-x}\right]\,dx$ where, [.] denotes the greatest integer function, is equal to :

(1) 0 (2) $\ell n2$ (3) e^2 (4) $2e^{-1}$

7. $\int_{-\pi/2}^{\pi/2}\frac{|x|\,dx}{8\cos^2 2x+1}$ has the value :

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{12}$ (3) $\frac{\pi^2}{24}$ (4) $\frac{\pi}{12}$

8. If [x] stands for the greatest integer function, the value of $\int_4^{10}\frac{[x^2]}{[x^2-28x+196]+[x^2]}\,dx$ is :

(1) 0 (2) 1 (3) 3 (4) $\frac{3}{2}$

9. $\int_0^{\infty}\frac{\ell n(1+x^2)}{1+x^2}\,dx$ equals

(1) $\pi\ \ell n\ 2$ (2) $-\pi\ \ell n\ 2$ (3) $\frac{\pi}{2}\ell n2$ (4) $-\frac{\pi}{2}\ell n2$

10. $\int_{-1}^{1}\frac{e^x+1}{e^x-1}\,dx$ equals -

(1) $\ell n\ (e^x+1)$ (2) $\ell n\ (e^x-1)$ (3) 1 (4) 0

11. If $I_1 = \int_0^{\pi} x\ \ f(\sin^3 x+\cos^2 x)\ \ dx$ and $I_2 = \pi\ \int_0^{\pi/2} f(\sin^3 x+\cos^2 x)\ \ dx$ then

(1) $I_1 = I_2$ (2) $I_1 + I_2 = 0$ (3) $I_1 = 2I_2$ (4) $2I_1 = I_2$

12. The value of $\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ is

(1) $\frac{\pi^2}{4}$ (2) $\frac{\pi^2}{8}$ (3) $\frac{\pi^2}{16}$ (4) $\frac{3\pi^2}{16}$

13. If $\int_0^{\pi/2}\frac{\sin x}{1+\sin x+\cos x}dx = \frac{\pi}{a}-\frac{1}{2}\ell nb,\ a,b\in N$ then

(1) a+b = 4 (2) a–b = 4 (3) a+b = 6 (4) a–b = 6

14. $\int_0^{\pi}\frac{x^2\cos^4 x\sin x}{2\pi x-\pi^2}\ dx =$

(1) $\frac{2}{5}$ (2) $\frac{1}{5}$ (3) $\frac{2\pi}{5}$ (4) $\frac{\pi}{5}$

15. The value of $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}\ dx$ is

(1) π (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

16. The value of $\int_{-1}^{2} \{2x\}\ dx$ is (where function {.} denotes fractional part function)

(1) 3 (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

17. The value of $\int_0^{400\pi} \sqrt{1-\cos 2x}\ dx$ is

(1) 400 $\sqrt{2}$ (2) 200 $\sqrt{2}$ (3) 600 $\sqrt{2}$ (4) 800 $\sqrt{2}$

18. If f(x) = $\int_0^{x} (2\cos^2 3t + 3\sin^2 3t)$ dt, f(x + π) is equal to :

(1) f(x) + 2f(π) (2) f(x) + 2f $\left(\frac{\pi}{2}\right)$ (3) f(x) + 4f $\left(\frac{\pi}{4}\right)$ (4) f(x) + 2f $\left(\frac{\pi}{4}\right)$

19. Let I = $\int_0^{1} (\tan^{-1} x)^2\ dx$ then

(1) I > $\frac{\pi^2}{4}$ (2) I > $\frac{\pi^2}{8}$ (3) 0 < I < $\frac{\pi^2}{16}$ (4) I > $\frac{\pi^2}{16}$

20. Let I = $\int_{1/\sqrt{2}}^{1} e^{\cos^{-1} x} dx$ then

(1) I < 1 – $\frac{1}{\sqrt{2}}$ (2) I > (1 – $\frac{1}{\sqrt{2}}$)$e^{\pi/4}$ (3) I < (1 – $\frac{1}{\sqrt{2}}$)$e^{\pi/4}$ (4) I > 2

21. $\lim_{n\to\infty} \left(\frac{n!}{n^n}\right)^{1/n}$ equals

(1) e (2) $\frac{1}{e}$ (3) 2e (4) – e

22.. If f(x) = $\int_{x^2}^{x^3} \frac{dt}{\ell nt}$. x > 0 then

(1) f' (x) = – $\frac{1}{6\ell nx}$ (2) f is an increasing function on (0, ∞)

(3) f has minimum at x = 1 (4) f is a decreasing function on [0, ∞)

23. Value of $\int_0^{1} x^5(1-x^2)^4 dx$ is

(1) $\frac{1}{210}$ (2) $\frac{1}{211}$ (3) $\frac{\pi}{210}$ (4) $\frac{1}{310}$

24. Let $I_n = \int_0^{1} x^n \tan^{-1} x dx$ then (n+1) I_n +(n–1) I_{n-2} =

(1) $\frac{\pi}{2}+\frac{1}{n}$ (2) $\frac{\pi}{2}-\frac{2}{n}$ (3) $\frac{\pi}{2}+\frac{2}{n}$ (4) $\frac{\pi}{2}-\frac{1}{n}$

25. Let $I_n = \int_0^1 a^x x^n dx, a > 1$ then

(1) $I_5 = \frac{a}{\ell na}+\frac{5}{\ell na} I_4$ (2) $I_5 = \frac{a}{\ell na}-\frac{5}{\ell na} I_4$ (3) $I_3 = \frac{a}{\ell na}+\frac{3}{\ell na} I_2$ (4) $I_2 = \frac{a}{\ell na}+\frac{2}{\ell na} I_1$

26. The area of the closed figure bounded by y = x, y = − x & the tangent to the curve y = $\sqrt{x^2-5}$ at the point (3, 2) is:

(1) 5 (2) $\frac{15}{2}$ (3) 10 (4) $\frac{35}{2}$

27. The area bounded by the curve y = f(x), x-axis and the ordinates x = 1 and x = b is (b – 1) sin (3b + 4), b ∈ R, then f(x) =

(1) (x – 1) cos (3x + 4) (2) sin (3x + 4)
(3) sin (3x + 4) + 3(x – 1) cos (3x + 4) (4) cos (3x + 4)

28. Find the area of the region bounded by the curves $y = x^2 + 2$, y = x, x = 0 and x = 3.

(1) $\frac{21}{2}$ sq. unit (2) 22 sq. unit (3) 21 sq. unit (4) $\frac{23}{2}$ sq. unit

29. The line y = mx bisects the area enclosed by the curve $y = 1 + 4x - x^2$ & the lines $x = \frac{3}{2}$, x = 0 & y = 0. Then the value of m is:

(1) $\frac{13}{6}$ (2) $\frac{6}{13}$ (3) $\frac{3}{2}$ (4) 4

30. The area of the closed figure bounded by $y = \frac{1}{\cos^2 x}$; x = 0; y = 0 & $x = \frac{\pi}{4}$ is:

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{4}+1$ (3) 1 (4) 2

31. The area bounded by $x^2 + y^2 - 2x = 0$ & $y = \sin\frac{\pi x}{2}$ in the upper half of the circle is:

(1) $\frac{\pi}{2}-\frac{4}{\pi}$ (2) $\frac{\pi}{4}-\frac{2}{\pi}$ (3) $\pi-\frac{8}{\pi}$ (4) $\frac{\pi}{2}+\frac{4}{\pi}$

32.. The area of the region for which $0 < y < 3 - 2x - x^2$ and x > 0 is

(1) $\int_1^3 (3-2x-x^2)\ dx$ (2) $\int_0^3 (3-2x-x^2)\ dx$ (3) $\int_0^1 (3-2x-x^2)\ dx$ (4) $\int_{-1}^3 (3-2x-x^2)\ dx$

33. Area of the region R ≡ {(x, y) ; $x^2 \le y \le x$} is

(1) $\frac{1}{12}$ (2) $\frac{1}{6}$ (3) $\frac{1}{2}$ (4) $\frac{1}{3}$

34. The area bounded by the curve y = sin ax with x-axis in between any two successive points of inflection is(a > 0)

(1) $\frac{4}{a}$ (2) $\frac{2}{a}$ (3) $\frac{1}{a}$ (4) 2a

35. The area bounded by $y = 2 - |2 - x|$ and $y = \frac{3}{|x|}$ is:

(1) $\frac{4+3\ \ell n\ 3}{2}$ (2) $\frac{4-3\ \ell n\ 3}{2}$ (3) $\frac{3}{2} + \ell n\ 3$ (4) $\frac{1}{2} + \ell n\ 3$

36. Area enclosed by the curve $|x - 2| + |y + 1| = 1$ is equal to

(1) 2 Sq. unit (2) $\frac{3}{2}$ Sq. unit (3) $\frac{1}{2}$ Sq. unit (4) $\frac{5}{2}$ Sq. uinit

37. The area of the region bounded by the curves $y = |x - 1|$ and $y = 3 - |x|$ is

(1) 2 (2) 3 (3) 4 (4) 1

38. The area of the figure bounded by $|y| = 1 - x^2$ is in square units,

(1) 4/3 (2) 8/3 (3) 16/3 (4) 1

39. The area between two arms of the curve $|y| = x^3$ from x = 0 to x = 2 is

(1) 2 (2) 4 (3) 8 (4) 16

40. Area bounded by $y = x^3 - x$ and $y = x^2 + x$ is-

(1) $\frac{37}{24}$ (2) $\frac{37}{12}$ (3) $\frac{11}{24}$ (4) $\frac{37}{23}$

41. The area bounded by the curve $y = 2x^4 - x^2$, x-axis and the two ordinates corresponding to the minima of the function is

(1) $\frac{3}{120}$ (2) $\frac{5}{120}$ (3) $\frac{1}{20}$ (4) $\frac{7}{120}$

42. The area bounded by the curves $y = x\,e^x$, $y = x\,e^{-x}$ and the line x = 1, is -

(1) $\frac{2}{e}$ (2) $1 - \frac{2}{e}$ (3) $\frac{1}{e}$ (4) $1 - \frac{1}{e}$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** Let f(x) be an even function which is periodic, then $g(x) = \int_a^x f(t)\ dt$ is also periodic.

Statement-2 : If $\alpha(x)$ is a differentiable and periodic function, then $\alpha'(x)$ is also periodic.

A-2. **Statement-1 :** If {.} represents fractional part function, then $\int_0^{5.5} \{x\}\ dx = \frac{21}{8}$

Statement-2 : If [.] and {.} represent greatest integer and frational part functions respectively, then $\int_0^{t} \{x\}\ dx = \frac{[t]}{2} + \frac{\{t\}^2}{2}$

A-3. **Statement-1 :** $\int_0^{10\pi} |\cos x|\, dx = 20$

Statement-2 : $\int_a^b f(x)dx \ge 0$, then $f(x) \ge 0,\ \forall\ x \in (a, b)$

A-4. **Statement-1 :** $\int_0^{2\pi} \tan^2 x\ dx = 4\int_0^{\pi/2} \tan^2 x\ dx$

Statement-2 : $\int_0^{nT} f(x)\ dx = n\int_0^{T} f(x)\ dx$, where n is an integer and T is a period of f(x)

Section (B) : MATCH THE COLUMN

B-1. Match the column

Column – I	**Column – II**
(A) $\int_{-1}^{1} \frac{dx}{1+x^2} =$	(p) $\frac{1}{2}\ \ell n\left(\frac{2}{3}\right)$
(B) $\int_0^1 \frac{dx}{\sqrt{1-x^2}} =$	(q) $2\ \ell n\left(\frac{2}{3}\right)$
(C) $\int_2^3 \frac{dx}{1-x^2} =$	(r) $\frac{\pi}{3}$
(D) $\int_1^2 \frac{dx}{x\sqrt{x^2-1}} =$	(s) $\frac{\pi}{2}$

B-2.

Column – I		**Column – II**	
(A)	Area bounded by region $0 \le y \le 4x - x^2 - 3$ is	(p)	32/3
(B)	Area of the region enclosed by $y^2 = 8x$ and $y = 2x$ is	(q)	1/2
(C)	The area bounded by $|x| + |y| \le 1$ and $|x| \ge 1/2$ is	(r)	8/3
(D)	Area bounded by $x \le 4 - y^2$ and $x \ge 0$ is	(s)	4/3

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If a curve $y = a\sqrt{x} + bx$ passes through the point (4, 2) and the area bounded by the curve, line x = 4 and x-axis is 16 square units, then :

(1) a = –9 (2) b = 4 (3) a = 9 (4) b = –4

C-2. If f is a continuous function and $\varphi(x) = \int_0^x \left((3t+4)\int_t^3 f(u)\ du\right)dt$ and $\int_0^3 f(x)dx = 3,$ then :

(1) φ'(0) = 0 (2) φ'(0) = 12 (3) φ''(3) = 13 f(3) (4) φ''(3) = –13 f(3)

C-3. Let $f(x) = \int_{1/x}^{\sqrt{x}} (\sin(t^2))\ dt,$ then

(1) $f'(1) = 3/2$ (2) $f'(1) = 3/2 \sin 1$ (3) $\lim_{x \to 0} f'(x) = 0$ (4) $\lim_{x \to \infty} f'(x) = 0$

C-4. The equation of the tangent of slope 1 to the curve $f(x) = \int_{-1}^{x} |t|\, dt$ is/are :
(1) $y = x$ (2) $y = x - 1$ (3) $y = x + 1$ (4) $y = x + 2$

C-5. If $I_1 = \int_0^1 \frac{1+x^8}{1+x^4}\, dx$ and $I_2 = \int_0^1 \frac{1+x^9}{1+x^3}\, dx$, then :
(1) $I_1 < I_2$ (2) $I_1 > I_2$ (3) $I_1 > I_2 > 1$ (4) $I_2 < I_1 < 1$

C-6. If $I_n = \int_{-\pi}^{\pi} \frac{\sin nx}{(1+\pi^x)\sin x}\, dx$, n = 0, 1, 2, ..., then
(1) $I_n = I_{n+2}$ (2) $\sum_{m=1}^{10} I_{2m+1} = 10\pi$ (3) $\sum_{m=1}^{10} I_{2m} = 0$ (4) $I_n = I_{n+1}$

Exercise-3

Marked Questions may have for Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The value of the integral $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x}+\sqrt{x}}\, dx$ is- **[AIEEE 2006, (3, –1), 120]**
(1) $\frac{3}{2}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

2. $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2 (x + 3\pi)]\, dx$ is equal to- **[AIEEE 2006, (3, –1), 120]**
(1) $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$ (2) $\frac{\pi}{2}$ (3) $\left(\frac{\pi}{4}\right) - 1$ (4) $\frac{\pi^4}{32}$

3. $\int_0^{\pi} x f(\sin x)\, dx$ is equal to- **[AIEEE 2006, (3, –1), 120]**
(1) $\pi \int_0^{\pi} f(\sin x)\, dx$ (2) $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x)\, dx$ (3) $\pi \int_0^{\pi/2} f(\cos x)\, dx$ (4) $\pi \int_0^{\pi} f(\cos x)\, dx$

4. The value of $\int_1^a [x]\, f'(x)\, dx$, a > 1, where [x] denotes the greatest integer not exceeding x is **[AIEEE 2006, (3, –1), 120]**
(1) af(a) – {f(1) + f(2) +..... + f([a])} (2) [a] f(a) – {f(1) + f(2) +....+ f([a])}
(3) [a] f([a]) – {f(1) + f(2) +..... + f(a)} (4) af([a]) – {f(1) + f(2) +..... + f(a)}

5. Let $F(x) = f(x) + f\left(\frac{1}{x}\right)$, $f(x) = \int_1^x \frac{\log t}{1+t}\, dt$. Then F(e) equals- **[AIEEE 2007 (3, –1), 120]**
(1) (2) 0 (3) 1 (4) 2

6. The solution for x of the equation $\int_{\sqrt{2}}^{x} \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{2}$ is **[AIEEE 2007 (3, –1), 120]**
(1) $2\sqrt{2}$ (2) $-\sqrt{2}$ (3) π (4) $\frac{\sqrt{3}}{2}$

7. The area enclosed between the curves $y^2 = x$ and $y = |x|$ is **[AIEEE 2007 (3, –1), 120]**

(1) $\frac{2}{3}$ sq unit (2) 1 sq unit (3) $\frac{1}{6}$ sq unit (4) $\frac{1}{3}$ sq unit

8. Let $I = \int_0^1 \frac{\sin x}{\sqrt{x}}$ dx and $J = \int_0^1 \frac{\cos x}{\sqrt{x}}$ dx. Then, which one of the following is true ? **[AIEEE 2008 (3, –1), 105]**

(1) $I > \frac{2}{3}$ and $J > 2$ (2) $I < \frac{2}{3}$ and $J < 2$ (3) $I < \frac{2}{3}$ and $J > 2$ (4) $I > \frac{2}{3}$ and $J < 2$

9. The area of the plane region bounded by the curves $x + 2y^2 = 0$ and $x + 3y^2 = 1$ is equal to **[AIEEE 2008 (3, –1), 105]**

(1) $\frac{5}{3}$ sq unit (2) $\frac{1}{3}$ sq unit (3) $\frac{2}{3}$ sq unit (4) $\frac{4}{3}$ sq unit

10. $\int_0^{\pi} [\cot x]dx$, where [·] denotes the greatest integer function, is equal to : **[AIEEE 2009 (4, –1), 144]**

(1) 1 (2) – 1 (3) $-\frac{\pi}{2}$ (4) $\frac{\pi}{2}$

11. The area of the region bounded by the parabola $(y - 2)^2 = x - 1$, the tangent to the parabola at the point (2, 3) and the x-axis is **[AIEEE 2009 (8, –2), 144]**

(1) 6 sq unit (2) 9 sq unit (3) 12 sq unit (4) 3 sq unit

12. Let p(x) be a function defined on **R** such that p'(x) = p'(1 – x), for all x ∈ [0, 1], p(0) = 1 and p(1) = 41. Then $\int_0^1 p(x)dx$ equals **[AIEEE 2010 (8, –2), 144]**

(1) 21 (2) 41 (3) 42 (4) $\sqrt{41}$

13. The area bounded by the curves y = cos x and y = sinx between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is **[AIEEE 2010 (4, –1), 144]**

(1) $4\sqrt{2}+2$ (2) $4\sqrt{2}-1$ (3) $4\sqrt{2}+1$ (4) $4\sqrt{2}-2$

14. For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int_0^x \sqrt{t}$ sin t dt. Then f has : **[AIEEE 2011, I, (4, –1), 120]**

(1) local maximum at π and 2π. (2) local minimum at π and 2π
(3) local minimum at π and local maximum at 2π (4) local maximum at π and local minimum at 2π.

15. The value of $\int_0^1 \frac{8\ell n(1+x)}{1+x^2}$ dx is : **[AIEEE 2011, I, (4, –1), 120]**

(1) π ℓn 2 (2) $\frac{\pi}{8}$ ℓn 2 (3) $\frac{\pi}{2}$ ℓn 2 (4) ℓn 2

16. Let [.] denote the greatest integer function then the value of $\int_0^{1.5} x\ [x^2]$ dx is : **[AIEEE 2011, II, (4, –1), 120]**

(1) 0 (2) $\frac{3}{2}$ (3) $\frac{3}{4}$ (4) $\frac{5}{4}$

17. The area of the region enclosed by the curves y = x, x = e, y = $\frac{1}{x}$ and the positive x-axis is **[AIEEE 2011, I, (4, –1), 120]**

(1) $\frac{1}{2}$ square units (2) 1 square units (3) $\frac{3}{2}$ square units (4) $\frac{5}{2}$ square units

18. The area bounded by the curves $y^2 = 4x$ and $x^2 = 4y$ is : **[AIEEE 2011, II, (4, –1), 120]**

(1) $\frac{32}{3}$ (2) $\frac{16}{3}$ (3) $\frac{8}{3}$ (4) 0

19*. If g(x) = $\int_0^x \cos 4t \ dt$, then g(x + π) equals **[AIEEE-2012, (4, –1)/120]**

(1) $\frac{g(x)}{g(\pi)}$ (2) g(x) + g(π) (3) g(x) – g(π) (4) g(x) . g(π)

20. The area bounded between the parabolas $x^2 = \frac{y}{4}$ and $x^2 = 9y$ and the straight line y = 2 is : **[AIEEE-2012, (4, –1)/120]**

(1) $20\sqrt{2}$ (2) $\frac{10\sqrt{2}}{3}$ (3) $\frac{20\sqrt{2}}{3}$ (4) $10\sqrt{2}$

21. The intercepts on x-axis made by tangents to the curve, y = $\int_0^x |t| \ dt$, x ∈ R, which are parallel to the line y = 2x, are equal to : **[AIEEE - 2013, (4, – 1) 120]**

(1) ±1 (2) ±2 (3) ±3 (4) ±4

22. Statement-I : The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to π/6. **[AIEEE - 2013, (4, –1),360]**

Statement-II : $\int_a^b f(x)dx = \int_a^b f(a+b-x) \ dx$. **[AIEEE - 2013, (4, –1),360]**

(1) Statement-I is true; Statement-II is true; Statement-II is a **correct** explanation for Statement-I.
(2) Statement-I is true; Statement-II is true; Statement-II is **not** a correct explanation for Statement-I.
(3) Statement-I is true; Statement-II is false.
(4) Statement-I is false; Statement-II is true.

23. The area (in square units) bounded by the curves y = $\sqrt{x}$, 2y – x + 3 = 0, x-axis, and lying in the first quadrant is : **[AIEEE - 2013, (4, –1),360]**

(1) 9 (2) 36 (3) 18 (4) $\frac{27}{4}$

24. The integral $\int_0^{\pi} \sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}} \ dx$ equals : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $4\sqrt{3}-4$ (2) $4\sqrt{3}-4-\frac{\pi}{3}$ (3) π – 4 (4) $\frac{2\pi}{3}-4-4\sqrt{3}$

25. The area of the region described by A = {(x, y) : $x^2 + y^2 \le 1$ and $y^2 \le 1 - x$} is : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{\pi}{2} - \frac{2}{3}$ (2) $\frac{\pi}{2} + \frac{2}{3}$ (3) $\frac{\pi}{2} + \frac{4}{3}$ (4) $\frac{\pi}{2} - \frac{4}{3}$

26. The integral $\int_2^4 \frac{\log x^2}{\log x^2 + \log(36 - 12x + x^2)} dx$ is equal to **[JEE(Main) 2015, (4, – 1), 120]**

(1) 2 (2) 4 (3) 1 (4) 6

27. The area (in sq. units) of the region described by $\{(x, y); y^2 \le 2x \text{ and } y \ge 4x - 1\}$ is **[JEE(Main) 2015, (4, – 1), 120]**

(1) $\frac{7}{32}$ (2) $\frac{5}{64}$ (3) $\frac{15}{64}$ (4) $\frac{9}{32}$

28. $\lim_{n\to\infty} \left(\frac{(n+1)(n+2).......3n}{n^{2n}}\right)^{1/n}$ is equal to : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\frac{27}{e^2}$ (2) $\frac{9}{e^2}$ (3) 3 log3 – 2 (4) $\frac{18}{e^4}$

29. The area (in sq.units) of the region $\{(x,y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$ (3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

30. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3} dx$ is equal to **[JEE(Main) 2016, (4, – 1), 120]**

(1) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (2) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$ (3) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$ (4) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

where C is an arbitrary constant

31. The area (in sq. units) of the region $\{(x, y) : x \ge 0 , x + y \le 3 , x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$ is : **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\frac{59}{12}$ (2) $\frac{3}{2}$ (3) $\frac{7}{3}$ (4) $\frac{5}{2}$

32. The integral $\int_{\pi/4}^{3\pi/4} \frac{dx}{1 + \cos x}$ is equal to **[JEE(Main) 2017, (4, – 1), 120]**

(1) –2 (2) 2 (3) 4 (4) –1

33. The value of $\int_{-\pi/2}^{\pi/2} \frac{\sin^2 x}{1 + 2^x} dx$ is : **[JEE(Main) 2018, (4, – 1), 120]**

(1) 4π (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{2}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. $\lim\limits_{x \to \frac{\pi}{4}} \dfrac{\int_{2}^{\sec^2 x} f(t)\ dt}{x^2 - \frac{\pi^2}{16}}$ equals **[IIT-JEE - 2007,Paper-1, (3, – 1), 81]**

(A) $\frac{8}{\pi} f(2)$ (B) $\frac{2}{\pi} f(2)$ (C) $\frac{2}{\pi} f\left(\frac{1}{2}\right)$ (D) 4f(2)

2. Area of the region bounded by the curve $y = e^x$ and lines $x = 0$ and $y = e$ is
[IIT-JEE 2009, P-1, (4, –1), 80]

(A) e – 1 (B) $\int_{1}^{e} \ell n\ (e+1-y)\ dy$ (C) $e - \int_{0}^{1} e^x\ dx$ (D) $\int_{1}^{e} \ell n\ y\ dy$

3. The value of $\lim\limits_{x \to 0} \frac{1}{x^3}\int_{0}^{x} \frac{t\ell n\ (1+t)}{t^4+4}\ dt$ is **[IIT-JEE-2010, Paper-1 (3, –1)/84]**

(A) 0 (B) $\frac{1}{12}$ (C) $\frac{1}{24}$ (D) $\frac{1}{64}$

4. The value(s) of $\int_{0}^{1} \frac{x^4(1-x)^4}{1+x^2}\ dx$ is (are) **[IIT-JEE-2010, Paper-1 (3, 0)/84]**

(A) $\frac{22}{7} - \pi$ (B) $\frac{2}{105}$ (C) 0 (D) $\frac{71}{15} - \frac{3\pi}{2}$

5. Let f be a real-valued function defined on the interval (–1, 1) such that $e^{-x} f(x) = 2 + \int_{0}^{x} \sqrt{t^4+1}\ dt$, for all $x \in (-1, 1)$ and let f^{-1} be the inverse function of f. Then $(f^{-1})'(2)$ is equal to
[IIT-JEE-2010, Paper-2 (5, –2)/84]

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{e}$

6. The value of $\int_{\sqrt{\ell n 2}}^{\sqrt{\ell n 3}} \frac{x \sin x^2}{\sin x^2 + \sin(\ell n 6 - x^2)}\ dx$ is **[IIT-JEE 2011, Paper-1, (3, –1), 80]**

(A) $\frac{1}{4} \ell n \frac{3}{2}$ (B) $\frac{1}{2} \ell n \frac{3}{2}$ (C) $\ell n \frac{3}{2}$ (D) $\frac{1}{6} \ell n \frac{3}{2}$

7. Let the straight line x = b divide the area enclosed by $y = (1 - x)^2$, y = 0, and x = 0 into two parts R_1 $(0 \le x \le b)$ and R_2 $(b \le x \le 1)$ such that $R_1 - R_2 = \frac{1}{4}$. Then b equals
[IIT-JEE 2011, Paper-1, (3, –1), 80]

(A) $\frac{3}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

8. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$.

Let $R_1 = \int_{-1}^{2} xf(x)dx$, and R_2 be the area of the region bounded by y = f(x), x = –1, x = 2, and the x-axis. Then **[IIT-JEE 2011, Paper-2, (3, –1), 80]**

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

9. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(x^2 + \ell n \frac{\pi + x}{\pi - x}\right)\cos x \; dx$ is **[IIT-JEE 2012, Paper-2, (3, –1), 66]**

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

10. The area enclosed by the curves y = sinx + cosx and y = |cosx – sinx| over the interval $\left[0, \frac{\pi}{2}\right]$ is **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) $4\left(\sqrt{2}-1\right)$ (B) $2\sqrt{2}\left(\sqrt{2}-1\right)$ (C) $2\left(\sqrt{2}+1\right)$ (D) $2\sqrt{2}\left(\sqrt{2}+1\right)$

11. Let f : $\left[\frac{1}{2}, 1\right]$ → R (the set of all real numbers) be a positive, non-constant and differentiable function such that f'(x) < 2 f(x) and $f\left(\frac{1}{2}\right)$ = 1. Then the value of $\int_{1/2}^{1} f(x) \; dx$ lies in the interval **[JEE (Advanced) 2013, Paper-1, (2, 0)/60]**

(A) (2e – 1, 2e) (B) (e – 1, 2e – 1) (C) $\left(\frac{e-1}{2}, e-1\right)$ (D) $\left(0, \frac{e-1}{2}\right)$

12.* For a ∈ R (the set of all real numbers), a ≠ –1, $\lim_{n\to\infty} \frac{(1^a + 2^a + ... + n^a)}{(n+1)^{a-1}[(na+1)+(na+2)+...+(na+n)]} = \frac{1}{60}$. Then a = **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) 5 (B) 7 (C) $\frac{-15}{2}$ (D) $\frac{-17}{2}$;

13. The following integral $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\operatorname{cosec} x)^{17} dx$ is equal to **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) $\int_{0}^{\log(1+\sqrt{2})} 2(e^u + e^{-u})^{16} du$ (B) $\int_{0}^{\log(1+\sqrt{2})} (e^u + e^{-u})^{17} du$

(C) $\int_{0}^{\log(1+\sqrt{2})} (e^u - e^{-u})^{17} du$ (D) $\int_{0}^{\log(1+\sqrt{2})} 2(e^u - e^{-u})^{16} du$

14. Let f : [0, 2] → R be a function which is continuous on [0, 2] and is differentiable on (0, 2) with f(0) = 1. Let $F(x) = \int_{0}^{x^2} f(\sqrt{t})dt$ for x ∈ [0, 2]. If F'(x) = f'(x) for all x ∈ (0, 2), then F(2) equals **[JEE (Advanced) 2014, Paper-2, (3, –1)/60]**

(A) $e^2 - 1$ (B) $e^4 - 1$ (C) e – 1 (D) e^4

15. If $\alpha = \int_0^1 \left(e^{9x+3\tan^{-1}x}\right)\left(\frac{12+9x^2}{1+x^2}\right)dx$ where $\tan^{-1}x$ takes only principal values, then the value of $\left(\log_e|1+\alpha| - \frac{3\pi}{4}\right)$ is **[JEE (Advanced) 2015, P-2 (4, 0) / 80]**

(A) 7 (B) 8 (C) 9 (D) 10

16. Let f: R → R be a function defined by $f(x) = \begin{cases} [x], & x \le 2 \\ 0, & x > 2 \end{cases}$ where [x] is the greatest integer less than or equal to x. If $I = \int_{-1}^{2} \frac{xf(x^2)}{2+f(x+1)}\, dx$, then the value of (4I–1) is

[JEE (Advanced) 2015, P-1 (4, 0) /88]

(A) –1 (B) 1 (C) 0 (D) 4

17. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2 \cos x}{1+e^x} dx$ is equal to **[JEE (Advanced) 2016, Paper-2, (3, –1)/62]**

(A) $\frac{\pi^2}{4} - 2$ (B) $\frac{\pi^2}{4} + 2$ (C) $\pi^2 - e^{\pi/2}$ (D) $\pi^2 + e^{\pi/2}$

18. Area of the region $\left\{(x,y) \in R^2 : y \ge \sqrt{|x+3|}, 5y \le x+9 \le 15\right\}$ is equal to

[JEE (Advanced) 2016, Paper-2, (3, –1)/62]

(A) $\frac{1}{6}$ (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$

Answers

EXERCISE # 1

Section (A)

A-1.	(2)	**A-2.**	(1)	**A-3.**	(3)	**A-4.**	(3)	**A-5.**	(1)	**A-6.**	(3)	**A-7.**	(2)
A-8.	(1)	**A-9.**	(1)	**A-10.**	(4)	**A-11.**	(1)	**A-12.**	(3)	**A-13**	(4)	**A-14.**	(4)
A-15.	(1)												

Section (B)

B-1.	(3)	**B-2.**	(4)	**B-3.**	(1)	**B-4.**	(2)	**B-5.**	(2)	**B-6.**	(1)	**B-7.**	(1)
B-8.	(3)	**B-9.**	(1)	**B-10.**	(4)	**B-11.**	(2)	**B-12.**	(3)	**B-13.**	(3)	**B-14.**	(3)
B-15.	(3)	**B-16.**	(1)	**B.17.**	(2)	**B-18.**	(2)	**B-19.**	(4)	**B-20.**	(4)	**B-21.**	(2)
B-22.	(2)	**B-23.**	(4)	**B-24.**	(4)	**B-25.**	(1)	**B-26.**	(1)	**B-27.**	(3)		

Section (C)

C-1.	(4)	**C-2.**	(3)	**C-3.**	(4)	**C-4.**	(3)	**C-5.**	(3)	**C-6.**	(4)	**C-7.**	(2)

Section (D)

D-1.	(2)	**D-2.**	(2)	**D-3.**	(3)	**D-4.**	(4)	**D-5.**	(1)	**D-6.**	(4)	**D-7.**	(3)
D-8.	(2)	**D-9.**	(3)	**D-10.**	(1)	**D-11.**	(4)	**D-12.**	(3)	**D-13.**	(2)		

Section (E)

E-1.	(3)	**E-2.**	(1)	**E-3.**	(4)	**E-4.**	(1)	**E-5.**	(3)	**E-6.**	(4)

Section (F)

F-1.	(2)	**F-2.**	(3)	**F-3.**	(4)	**F-4.**	(3)	**F-5.**	(3)	**F-6.**	(3)	**F-7.**	(2)
F-8.	(3)	**F-9.**	(1)	**F-10.**	(4)	**F-11.**	(3)	**F-12.**	(2)				

EXERCISE # 2

PART - I

1.	(3)	**2.**	(2)	**3.**	(3)	**4.**	(3)	**5.**	(2)	**6.**	(2)	**7.**	(2)
8.	(3)	**9.**	(1)	**10.**	(4)	**11.**	(1)	**12.**	(3)	**13.**	(3)	**14.**	(2)
15.	(4)	**16.**	(2)	**17.**	(4)	**18.**	(2)	**19.**	(3)	**20.**	(3)	**21.**	(2)
22.	(2)	**23.**	(1)	**24.**	(4)	**25.**	(2)	**26.**	(1)	**27.**	(3)	**28.**	(1)
29.	(1)	**30.**	(3)	**31.**	(1)	**32.**	(3)	**33.**	(2)	**34.**	(2)	**35.**	(2)
36.	(1)	**37.**	(3)	**38.**	(2)	**39.**	(3)	**40.**	(2)	**41.**	(4)	**42.**	(1)

PART - II

Section (A)

A-1.	(3)	**A-2.**	(1)	**A-3.**	(2)	**A-4.**	(1)

Section (B)

B-1. (A) → (s), (B) → (s),(C) → (p), (D) → (r)

B-2. (A) → (s), (B) → (s),(C) → (q), (D) → (p)

Section (C)

C-1.	(3,4)	**C-2.**	(2,4)	**C-3.**	(2,4)	**C-4.**	(1,3)	**C-5.**	(2,4)	**C-6.**	(1,2,3)

EXERCISE # 3

PART - I

1.	(1)	**2.**	(2)	**3.**	(3)	**4.**	(2)	**5.**	(1)	**6.**	(2)	**7.**	(3)
8.	(2)	**9.**	(4)	**10.**	(3)	**11.**	(2)	**12.**	(1)	**13.**	(4)	**14.**	(4)
15.	(1)	**16.**	(3)	**17.**	(3)	**18.**	(2)	**19*.**	(2,3)	**20.**	(3)	**21.**	(1)
22.	(4)	**23.**	(1)	**24.**	(2)	**25.**	(3)	**26.**	(3)	**27.**	(4)	**28.**	(1)
29.	(1)	**30.**	(1)	**31.**	(4)	**32.**	(2)	**33.**	(2)				

PART - II

1.	(A)	**2.**	(B)	**3.**	(B)	**4.**	(A)	**5.**	(B)	**6.**	(A)	**7.**	(B)
8.	(C)	**9.**	(B)	**10.**	(B)	**11.**	(D)	**12.**	(B)	**13.**	(A)	**14.**	(B)
15.	(C)	**16.**	(C)	**17.**	(A)	**18.**	(C)						

Additional Problems For Self Practice (APSP)

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

1. $\int_0^{2/3} \frac{dx}{4+9x^2}$ equal to

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{24}$ (4) $\frac{\pi}{48}$

2. $\int_2^4 \frac{\sqrt{x^2-4}}{x}\,dx$ equal to

(1) $2\left(3\sqrt{3}-\pi\right)$ (2) $2\sqrt{3}-\pi$ (3) $\frac{2}{3}(3\sqrt{3}-\pi)$ (4) π

3. $\int_0^{\infty} \frac{x^3dx}{(1+x^2)^{9/2}}$ is equal to

(1) $\frac{2}{35}$ (2) $\frac{4}{35}$ (3) $\frac{6}{35}$ (4) $\frac{3}{35}$

4. The value of the integration $\int_{-4}^{4} (ax^3+bx+c)\ dx$ depends on

(1) a and b (2) a, b and c (3) only c (4) c and b

5. The value of $\int_{-1}^{2} \frac{|x|}{x}dx$ equal to

(1) 3 (2) 2 (3) 1 (4) –1

6. $\int_0^{\infty} \frac{dx}{1+e^x}$ is equal to

(1) $\log_e 2-1$ (2) $\log_e 2$ (3) $\log_e 4-1$ (4) $-\log_e 2$

7. $\int_{-1}^{1} x\tan^{-1}x\,dx$ is equal to

(1) $\frac{1}{2}(\pi+2)$ (2) $\frac{1}{2}(\pi-2)$ (3) $\frac{1}{4}(\pi+2)$ (4) $\frac{1}{4}(\pi-2)$

8. $\int_{\pi/4}^{3\pi/4} \frac{x}{1+\sin x}dx$ is equal to

(1) $\pi\left(\sqrt{2}-1\right)$ (2) $\pi\left(\sqrt{2}+1\right)$ (3) $2\pi\left(\sqrt{2}-1\right)$ (4) $2\pi\left(\sqrt{2}+1\right)$

9. $\int_{1}^{4} \frac{2\sqrt{x}}{\sqrt{5-x}+\sqrt{x}}dx$ is equal to

(1) $\frac{3}{2}$ (2) $\frac{5}{2}$ (3) $\frac{1}{2}$ (4) 3

10. $\int_{-\pi/2}^{\pi/2}$ $\sin^2x\cos^2x(\sin x+\cos x)\,dx$ is equal to

(1) $\frac{2}{15}$ (2) $\frac{1}{5}$ (3) $\frac{4}{15}$ (4) $\frac{1}{3}$

11. $\int_{8}^{15} \frac{dx}{(x-3)\sqrt{x+1}}$ is equal to

(1) $\ell n\frac{5}{3}$ (2) 0 (3) $-\ell n\frac{5}{3}$ (4) $\frac{1}{2}\ell n\frac{5}{3}$

12. $\int_{0}^{1} \frac{x^3}{\sqrt{1-x^8}}$ dx is equal to

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{8}$

13. $\int_{-2}^{2}$ $|1-x^2|$ dx equal to

(1) 0 (2) 1 (3) 2 (4) 4

14. $\int_{0}^{1} \ell n\left(\frac{1}{x}-1\right)dx$ is equal to

(1) 0 (2) 1 (3) $\frac{1}{2}$ (4) 2

15. If [.] denotes the greatest integer function then

$\int_{\pi}^{2\pi} [2\sin x]dx$ is equal to

(1) – π (2) –2 π (3) $\frac{-5\pi}{3}$ (4) $\frac{5\pi}{3}$

16. $\int_0^{\pi}\sqrt{\frac{1}{2}(1+\cos 2x)}$ dx is equal to

(1) 0 (2) $\frac{1}{2}$ (3) 1 (4) 2

17. If $\int_0^x f(t)dt = x + \int_x^1 tf(t)\,dt$, then f(1) is equal to

(1) $\frac{1}{2}$ (2) 0 (3) 1 (4) $-\frac{1}{2}$

18. If F(x) = $\int_{x^2}^{x^3} \ell n\,t\,dt (x > 0)$, then F'(x) is equal to

(1) $(x^3–x^2)\ell nx$ (2) $\ell n\,x$ (3) $(9x^2–4x)\ell nx$ (4) $(9x^2+4x)x$

19. Area bounded by the curve y = sinx, y-axis, and y = 1 is (in first quadrant)

(1) 1 (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{2}-1$ (4) π

20. Area bounded by the line x+2y = 5, x = 1, x = 4 and x-axis is-

(1) $\frac{7}{2}$ (2) $\frac{15}{4}$ (3) 6 (4) 4

21. Area bounded by the line y = |x+5|, coordinate axis and line x = 2 is

(1) $\frac{15}{2}$ (2) $\frac{9}{2}$ (3) 12 (4) 24

22. Area bounded by the curve y = (x–1) (x–2) (x–3) ; x-axis and ordinates x = 0, x = 3 is

(1) $\frac{9}{4}$ (2) $\frac{11}{4}$ (3) $\frac{11}{2}$ (4) $\frac{19}{4}$

23. The area between curves y = $\frac{2}{e^x + e^{-x}}$ and x-axis is

(1) 4π (2) $\frac{3\pi}{2}$ (3) 2π (4) π

24. The area between the curve x^2=4y and x=4y–2 is

(1) 9 (2) $\frac{9}{2}$ (3) $\frac{9}{4}$ (4) $\frac{9}{8}$

25. The area between the curve y = x and y = x^3 is

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) $\frac{3}{4}$ (4) 1

26. The area bounded by the curve y = sinx, y = cosx and x-axis in the 1st quadrant where $x \in \left[0, \frac{\pi}{2}\right]$

(1) $2-\sqrt{2}$ (2) $2+\sqrt{2}$ (3) $1+\sqrt{2}$ (4) $\sqrt{2}-1$

27. The area between curves $x^2 + y^2 = \pi^2$ and curve y = sinx which lies above the x-axis is

(1) $\left(\frac{\pi^3 - 8}{2}\right)$ (2) $\left(\frac{\pi^3 - 4}{2}\right)$ (3) $\left(\frac{\pi^3 - 2}{2}\right)$ (4) $\left(\frac{\pi^3 - 8}{4}\right)$

28. The value of $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{n}{n^2+nr+r^2}\right)$

(1) $\frac{\pi}{\sqrt{3}}$ (2) $\frac{\pi}{2\sqrt{3}}$ (3) $\frac{\pi}{3\sqrt{3}}$ (4) $\frac{\pi}{3}$

29. $\lim_{n\to\infty}\frac{1^p + 2^p + 3^p + \ldots.. + n^p}{n^{p+1}}$ is equal to

(1) p+1 (2) p (3) $\frac{1}{p+1}$ (4) $\frac{1}{p}$

30. $\lim_{n\to\infty}\frac{1}{1^3+n^3} + \frac{4}{2^3+n^3}$ + ...+ $\frac{1}{2n}$ is equal to

(1) $\ell n2$ (2) $\frac{1}{2}\ell n2$ (3) $\frac{1}{3}\ell n2$ (4) $\frac{1}{4}\ell n2$

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

1. The value of $5050\,\frac{\int_0^1 (1-x^{50})^{100}\,dx}{\int_0^1 (1-x^{50})^{101}\,dx}$ is

(1) 5051 (2) 5049 (3) 5055 (4) 5050

2.* Let $S_n = \sum_{k=1}^{n}\frac{n}{n^2+kn+k^2}$ and $T_n = \sum_{k=0}^{n-1}\frac{n}{n^2+kn+k^2}$ for n = 1, 2, 3, ... Then,

(1) $S_n < \frac{\pi}{3\sqrt{3}}$ (2) $S_n > \frac{\pi}{3\sqrt{3}}$ (3) $T_n < \frac{\pi}{3\sqrt{3}}$ (4) $T_n > \frac{\pi}{3\sqrt{3}}$

3. Let f : R → R be a continuous function which satisfies f(x) = $\int_0^x f(t)\,dt$. Then the value of f(ℓn 5) is

(1) 0 (2) 1 (3) 3 (4) 4

4. If f (x) is an even function and $\int_0^{\pi/2} f(\cos 2x)\cos x\,dx = k\int_0^{\pi/4} f(\sin 2x)\cos x\,dx$, then value of k is

(1) 3 (2) $\sqrt{2}$ (3) 2 (4) 4

5. If the value of $\int_0^{\pi} e^{|\cos x|}\left(2\sin\left(\frac{1}{2}\cos x\right)+3\cos\left(\frac{1}{2}\cos x\right)\right)\sin x\,dx$ is $k\left[e\cos\left(\frac{1}{2}\right)+\frac{1}{2}e\sin\left(\frac{1}{2}\right)-1\right]$ then k =

(1) $\frac{24}{5}$ (2) $\frac{12}{5}$ (3) $\frac{23}{5}$ (4) $\frac{32}{5}$

6. For any real number, let [x] denote the largest integer less than or equal to x. Let f be a real valued function defined on the interval [–10, 10] by

$$f(x) = \begin{cases} x-[x]; & \text{if } [x] \text{ is odd,} \\ 1+[x]-x; & \text{if } [x] \text{ is even} \end{cases}$$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos \pi x\,dx$ is

(1) 0 (2) 2 (3) 3 (4) 4

7*. f(x) = $\begin{cases} x, & 0 \le x \le 1 \\ 2-e^{x-1}, & 1 < x \le 2 \\ x-e, & 2 < x \le 3 \end{cases}$ and g(x) = $\int_0^x f(t)\,dt$, x ∈ [1, 3] then

(1) g(x) has no local maxima (2) g(x) has no local minima

(3) g(x) has a local maxima at x = 1 + ℓn2 (4) g(x) has a local minima at x = e

8. Let f'(x) = $\frac{192x^3}{2+\sin^4 \pi x}$ for all x ∈ R with f $\left(\frac{1}{2}\right)$ = 0. If m ≤ $\int_{1/2}^{1} f(x)\,dx$ ≤ M, then the possible values of m and M are

(1) m = 13, M = 24 (2) m = $\frac{1}{4}$, M = $\frac{1}{2}$ (3) m = – 11, M = 0 (4) m = 1, M = 12

9*. If f(x) = $\int_0^x e^{t^2}(t-2)(t-3)\,dt$ for all x ∈ (0, ∞), then

(1) f has a local maximum at x = 2

(2) f is decreasing on (2, 3)

(3) there exists some c ∈ (0, ∞) such that f''(c) = 0

(4) f has a local minimum at x = 3

10. Let f: R → R be a continuous odd function, which vanishes exactly at one point and $f(1) = \frac{1}{2}$. Suppose that $F(x) = \int_{-1}^{x} f(t)\,dt$ for all $x \in [-1, 2]$ and $G(x) = \int_{-1}^{x} t\,|f(f(t))|\,dt$ for all $x \in [-1, 2]$. If $\lim_{x\to 1}\frac{F(x)}{G(x)} = \frac{1}{14}$, then the value of $f\left(\frac{1}{2}\right)$ is.

(1) 2 (2) 5 (3) 6 (4) 7

11. The area bounded by the curve $x^2 = y$, $x^2 = -y$ and $y^2 = 4x - 3$ is

(1) 2/3 (2) 1/3 (3) 4/3 (4) 1

12. If $\begin{bmatrix} 4a^2 & 4a & 1 \\ 4b^2 & 4b & 1 \\ 4c^2 & 4c & 1 \end{bmatrix}\begin{bmatrix} f(-1) \\ f(1) \\ f(2) \end{bmatrix} = \begin{bmatrix} 3a^2+3a \\ 3b^2+3b \\ 3c^2+3c \end{bmatrix}$, f(x) is a quadratic function and its maximum value occurs at a point V. A is a point of intersection of y = f(x) with x-axis and point B is such that chord AB subtends a right angle at V. The area enclosed by f(x) and chord AB is

(1) $\frac{25}{3}$ (2) $\frac{24}{3}$ (3) $\frac{125}{3}$ (4) $\frac{2}{3}$

13. The area of the region between the curves $y = \sqrt{\frac{1+\sin x}{\cos x}}$ and $y = \sqrt{\frac{1-\sin x}{\cos x}}$ bounded by the lines x = 0 and $x = \frac{\pi}{4}$ is

(1) $\int_{0}^{\sqrt{2}-1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (2) $\int_{0}^{\sqrt{2}-1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$

(3) $\int_{0}^{\sqrt{2}+1} \frac{4t}{(1+t^2)\sqrt{1-t^2}}\,dt$ (4) $\int_{0}^{\sqrt{2}+1} \frac{t}{(1+t^2)\sqrt{1-t^2}}\,dt$

Comprehension # 1 (Q. No.14 to 16)

If $y = \int_{u(x)}^{v(x)} f(t)\,dt$, let us define $\frac{dy}{dx}$ in a different manner as $\frac{dy}{dx} = v'(x)\,f^2(v(x)) - u'(x)\,f^2(u(x))$ and the equation of the tangent at (a, b) as $y - b = \left(\frac{dy}{dx}\right)_{(a,b)}(x - a)$

14. If $y = \int_{x}^{x^2} t^2\,dt$, then equation of tangent at x = 1 is

(1) y = x + 1 (2) x + y = 1 (3) y = x – 1 (4) y = x

15. If $F(x) = \int_{1}^{x} e^{t^2/2}(1 - t^2)\,dt$, then $\frac{d}{dx}F(x)$ at x = 1 is

(1) 0 (2) 1 (3) 2 (4) – 1

16. If $y = \int_{x^3}^{x^4} \ell nt \; dt$, then $\lim_{x\to 0^+} \frac{dy}{dx}$ is

(1) 0 (2) 1 (3) 2 (4) – 1

Comprehension # 2 (Q.17 to Q.18)

Suppose we define the definite integral using the following formula $\int_a^b f(x)dx = \frac{b-a}{2}$ (f(a) + f(b)), for more accurate result for c ∈ (a, b), $F(c) = \frac{c-a}{2}(f(a) + f(c)) + \frac{b-c}{2}(f(b) + f(c))$.

When $c = \frac{a+b}{2}$, $\int_a^b f(x)dx = \frac{b-a}{4}(f(a) + f(b) + 2f(c))$.

17. $\int_0^{\pi/2} \sin x \, dx$ is equal to

(1) $\frac{\pi}{8}(1 + \sqrt{2})$ (2) $\frac{\pi}{4}(1 + \sqrt{2})$ (3) $\frac{\pi}{8\sqrt{2}}$ (4) $\frac{\pi}{4\sqrt{2}}$

18. If f(x) is a polynomial and if $\lim_{t\to a} \frac{\int_a^t f(x)dx - \frac{(t-a)}{2}(f(t)+f(a))}{(t-a)^3} = 0$ for all a.
then the degree of f(x) can atmost be

(1) 1 (2) 2 (3) 3 (4) 4

19. If f''(x) < 0 ∀ x ∈ (a, b) and c is a point such that a < c < b, and (c, f(c)) is the point lying on the curve for which F(c) is maximum, then f'(c) is equal to

(1) $\frac{f(b)-f(a)}{b-a}$ (2) $\frac{2(f(b)-f(a))}{b-a}$ (3) $\frac{2f\ \ (b)-f(a)}{2b-a}$ (4) 0

Comprehension # 3 (20 to 22)

Consider the functions defined implicitly by the equation $y^3 - 3y + x = 0$ on various intervals in the real line.

If x ∈ (–∞, –2) ∪ (2, ∞), the equation implicitly defines a unique real valued differentiable function y = f(x).

If x ∈ (–2, 2), the equation implicitly defines a unique real valued differentiable function y = g(x) satisfying g(0) = 0

20. If $f\left(-10\sqrt{2}\right) = 2\sqrt{2}$, then $f''\left(-10\sqrt{2}\right) =$

(1) $\frac{4\sqrt{2}}{7^3 3^2}$ (2) $-\frac{4\sqrt{2}}{7^3 3^2}$ (3) $\frac{4\sqrt{2}}{7^3 3}$ (4) $-\frac{4\sqrt{2}}{7^3 3}$

21. The area of the region bounded by the curve y = f(x), the x-axis, and the lines x = a and x = b, where $-\infty < a < b < -2$, is

(1) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)} dx + bf(b) - af(a)$ (2) $-\int_a^b \frac{x}{3\left((f(x))^2-1\right)} dx + bf(b) - af(a)$

(3) $\int_a^b \frac{x}{3\left((f(x))^2-1\right)} dx - bf(b) + af(a)$ (4) $-\int_a^b \frac{x}{3\left((f(x))^2-1\right)} dx - bf(b) + af(a)$

22. $\int_{-1}^{1} g'(x) \; dx =$

(1) 2g (–1) (2) 0 (3) –2g (1) (4) 2g(1)

Comprehension # 4 (Q. 23 to 25)

Consider the polynomial $f(x) = 1 + 2x + 3x^2 + 4x^3$ Let s be the sum of all distinct real roots of f(x) and let $t = |s|$

23. The real number s lies in the interval.

(1) $\left(-\frac{1}{4}, 0\right)$ (2) $\left(-11, \frac{3}{4}\right)$ (3) $\left(-\frac{3}{4}, -\frac{1}{2}\right)$ (4) $\left(0, \frac{1}{4}\right)$

24. The area bounded by the curve y = f(x) and the lines x = 0, y = 0 and x = t, lies in the interval

(1) $\left(\frac{3}{4}, 3\right)$ (2) $\left(\frac{21}{64}, \frac{11}{16}\right)$ (3) (9, 10) (4) $\left(0, \frac{21}{64}\right)$

25. The function f'(x) is

(1) increasing in $\left(-t, \frac{1}{4}\right)$ and decreasing in $\left(-\frac{1}{4}, t\right)$

(2) decreasing in $\left(-t, -\frac{1}{4}\right)$ and increasing in $\left(-\frac{1}{4}, t\right)$

(3) increasing in (–t, t)

(4) decreasing in (–t, t)

Comprehension # 5 (Q. 26 to 27)

Let $F : R \to R$ be a thrice differentiable function. Supose that F(1) = 0, F(3) = –4 and F'(x) < 0 for all $x \in (1/2, 3)$. Let f(x) = xF(x) for all $x \in R$.

26. The correct statement(s) is(are)

(1) f '(1) < 0 (2) f (2) < 0

(3) f '(x) ≠ 0 for any $x \in (1, 3)$ (4) f '(x) = 0 for some $x \in (1, 3)$

27. If $\int_1^3 x^2 F'(x)dx = -12$ and $\int_1^3 x^3 F''(x)dx = 40$, then the correct expression(s) is(are)

(1) 9f '(3) + f '(1) – 32 = 0 (2) $\int_1^3 f(x)dx = 12$

(3) 9f '(3) – f '(1) + 32 = 0 (4) $\int_1^3 f(x)dx = -12$

APSP Answers

PART - I

1.	(3)	**2.**	(3)	**3.**	(1)	**4.**	(3)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(1)	**9.**	(1)	**10.**	(3)	**11.**	(4)	**12.**	(4)	**13.**	(4)	**14.**	(1)
15.	(3)	**16.**	(4)	**17.**	(1)	**18.**	(3)	**19.**	(3)	**20.**	(2)	**21.**	(3)
22.	(2)	**23.**	(4)	**24.**	(4)	**25.**	(2)	**26.**	(1)	**27.**	(2)	**28.**	(3)
29.	(3)	**30.**	(3)										

PART - II

1.	(1)	**2.***	(1,4)	**3.**	(1)	**4.**	(2)	**5.**	(1)	**6.**	(4)	**7*.**	(3,4)
8.	(4)	**9*.**	(1,2,3,4)	**10.**	(4)	**11.**	(2)	**12.**	(3)	**13.**	(2)	**14.**	(3)
15.	(1)	**16.**	(1)	**17.**	(1)	**18.**	(1)	**19.**	(1)	**20.**	(2)	**21.**	(1)
22.	(4)	**23.**	(3)	**24.**	(1)	**25.**	(2)	**26.**	(1,2,3)	**27.**	(3,4)		