Indefinite Integration

MATHEMATICS

Exercise-1

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

OBJECTIVE QUESTIONS

Section (A) : Integration using standard Integral :

A-1. $\int (e^{a\ell nx} + e^{x\ell na})$ dx, where x > 0, a > 0

(1) $x^{a+1} + \frac{a^x}{\ell na}$ + c (2) $\frac{x^{a+1}}{a+1}$ + a^x ℓn a + c (3) $\frac{x^{a+1}}{a+1} + \frac{a^x}{\ell na}$ + c (4) None of these

A-2. If f'(x) = x^2 + 5 and f(0) = –1, then f(x) =

(1) x^3 + 5x – 1 (2) x^3 + 5x + 1 (3) $\frac{1}{3}x^3$ + 5x – 1 (4) $\frac{1}{3}x^3$ + 5x + 1

A-3. $\int \frac{\cos 2x}{\cos x}$ dx is equal to

(1) 2 sin x – ℓn (sec x + tan x) + c (2) 2 sin x – ℓn (sec x – tan x) + c

(3) 2 sin x + ℓn (sec x + tan x) + c (4) None of these

A-4. $\int$ sin x. cos x. cos 2x. cos 4x. cos 8x. cos 16 x dx is equal to

(1) $\frac{\sin 16x}{1024}$ + c (2) – $\frac{\cos 32x}{1024}$ + c (3) $\frac{\cos 32x}{1096}$ + c (4) – $\frac{\cos 32x}{1096}$ + c

A-5. $\int \sqrt{1-\sin 2x}\, dx$ where x ∈ (0, π/4) is equal to

(1) – sinx + cosx + c (2) sinx – cosx + c (3) tanx + secx + c (4) sinx + cosx + c

A-6. $\int \frac{1+\cos^2 x}{\sin^2 x} dx$ =

(1) –cot x – 2x + c (2) –2cot x – 2x + c (3) –2cot x – x + c (4) –2cot x + x + c

A-7. $\int \frac{dx}{\tan x + \cot x}$ =

(1) $\frac{\cos 2x}{4} + c$ (2) $\frac{\sin 2x}{4} + c$ (3) $-\frac{\sin 2x}{4} + c$ (4) – $\frac{\cos 2x}{4} + c$

A-8. $\int \tan^{-1}\sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$ dx , where $0 < x < \frac{\pi}{2}$. is equal to

(1) $2x^2$ + c (2) x^2 + c (3) $\frac{x^2}{2}$ + c (4) 2x + c

A-9. $\int x^{51}(\tan^{-1} x + \cot^{-1} x) dx$ =

(1) $\frac{x^{52}}{52}$ $(\tan^{-1}x + \cot^{-1}x) + c$

(2) $\frac{x^{52}}{52}$ $(\tan^{-1}x - \cot^{-1}x) + c$

(3) $\frac{\pi x^{52}}{102} + \frac{\pi}{2} + c$

(4) $\frac{x^{52}}{52} + \frac{\pi}{2} + c$

Section (B) : Integration by substitution

B-1. $\int \frac{a^{\sqrt{x}}}{\sqrt{x}}$ dx is equal to

(1) $\frac{a^{\sqrt{x}}}{\sqrt{x}} + c$ (2) $\frac{2a^{\sqrt{x}}}{\ell n\, a} + c$ (3) $2a^{\sqrt{x}} . \ell n\, a + c$ (4) none of these

B-2. $\int 5^{5^{5^x}} . 5^{5^x} . 5^x$ dx is equal to

(1) $\frac{5^{5^x}}{(\ell n 5)^3} + c$ (2) $5^{5^{5^x}} (\ell n\, 5)^3 + c$ (3) $\frac{5^{5^{5^x}}}{(\ell n\, 5)^3} + c$ (4) none of these

B-3. If $\int \frac{2^x}{\sqrt{1-4^x}}$ dx $= K \sin^{-1} (2^x) + C$, then K is equal to

(1) $\ell n\, 2$ (2) $\frac{1}{2} \ell n\, 2$ (3) $\frac{1}{2}$ (4) $\frac{1}{\ell n\, 2}$

B-4. $\int \tan^3 2x \sec 2x\, dx$ is equal to :

(1) $\frac{1}{3}\sec^3 2x - \frac{1}{2}\sec 2x + c$

(2) $-\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c$

(3) $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c$

(4) $\frac{1}{3}\sec^3 2x + \frac{1}{2}\sec 2x + c$

B-5. $\int \frac{\cos 2x}{(\sin x + \cos x)^2}$ dx is equal to

(1) $\frac{-1}{\sin x + \cos x} + c$

(2) $\ell n (\sin x + \cos x) + c$

(3) $\ell n (\sin x - \cos x) + c$

(4) $\ell n (\sin x + \cos x)^2 + c$

B-6. The value of $\int \frac{\ell n\left(1+\frac{1}{x}\right)}{x(x+1)}$ dx is

(1) $-\frac{1}{2}\left(\ell n\left(1+\frac{1}{x}\right)\right)^2 + C$

(2) $\frac{1}{2}\left(\ell n\left(1+\frac{1}{x}\right)\right)^2 + C$

(3) $-\frac{1}{2}\left(\ell n\left(1-\frac{1}{x}\right)\right)^2 + C$ (4) $-\frac{1}{3}\left(\ell n\left(1+\frac{1}{x}\right)\right)^2 + C$

B-7. The value of $\int \frac{d(x^2+1)}{\sqrt{(x^2+2)}}$ is equal to

(1) $\sqrt{(x^2-2)}+C$ (2) $2\sqrt{(x^2+2)}+C$ (3) $2\sqrt{(x^2+3)}+C$ (4) $\sqrt{(x^2+2)}+C$

B-8. The value of $\int \frac{\sqrt{\tan x}}{\sin x \cos x}$ dx is equal to

(1) $2\sqrt{\tan x}$ + C (2) $2\sqrt{\cot x}$ + C (3) $\frac{\sqrt{\tan x}}{2}$ + C (4) $\sqrt{\tan x}$ + C

B-9. If y = $\int \frac{dx}{(1+x^2)^{3/2}}$ and y = 0 when x = 0, then value of y when x = 1, is:

(1) $\sqrt{\frac{2}{3}}$ (2) $\sqrt{2}$ (3) $3\sqrt{2}$ (4) $\frac{1}{\sqrt{2}}$

Section (C) : Integration by parts

C-1. $\int (x-1)\, e^{-x}$ dx is equal to

(1) $-xe^x + C$ (2) $xe^x + C$ (3) $-xe^{-x} + C$ (4) $xe^{-x} + C$

C-2. $\int e^{\tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)$ dx is equal to

(1) $x e^{\tan^{-1}x}$ + c (2) $x^2 e^{\tan^{-1}x}$ + c (3) $\frac{1}{x} e^{\tan^{-1}x}$ + c (4) none of these

C-3. $\int e^{\tan\theta}(\sec\theta - \sin\theta)\, d\theta$ is equal to

(1) $-e^{\tan\theta}\sin\theta + c$ (2) $e^{\tan\theta}\sin\theta + c$ (3) $e^{\tan\theta}\sec\theta + c$ (4) $e^{\tan\theta}\cos\theta + c$

C-4. $\int (x\, e^{\ell n \sin x} - \cos x)$ dx is equal to:

(1) x cos x + c (2) sin x − x cos x + c

(3) $-e^{\ell n x}\cos x + c$ (4) sin x + x cos x + c

C-5. $\int [f(x)g''(x) - f''(x)g(x)]$ dx is equal to

(1) $\frac{f(x)}{g'(x)}$ (2) f'(x) g(x) – f(x) g'(x)

(3) f(x) g'(x) – f'(x) g(x) (4) f(x) g'(x) + f'(x) g'(x)

C-6. If $\int e^{3x} \cos 4x \, dx = e^{3x} (A \sin 4x + B \cos 4x) + C$ then:

(1) 4A = 3B (2) 2A = 3B (3) 3A = 4B (4) 4A + 3B + 1 = 0

C-7. If $F(x) = \int \frac{x + \sin x}{1 + \cos x} dx$ and F(0) = 0, then the value of F(π/2) is

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) π

C-8. Let $F(x) = \int e^{\sin^{-1} x} \left(1 - \frac{x}{\sqrt{1 - x^2}}\right) dx$ and F(0) = 1, If $F(1/2) = \frac{k\sqrt{3}\, e^{\pi/6}}{\pi}$, then the value of k is

(1) π (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{2}$

C-9. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \left(x + \sqrt{x}\right) dx$ is equal to :

(1) $2e^{\sqrt{x}} [\sqrt{x} - x + 1] + c$ (2) $2e^{\sqrt{x}} [x - 2\sqrt{x} + 1] + c$

(3) $2e^{\sqrt{x}} [x - \sqrt{x} + 1] + c$ (4) $2e^{\sqrt{x}} (x + \sqrt{x} + 1) + c$

C-10. $\int \sec^3 x dx$ is equal to

(1) $\frac{\sec x \tan x}{2} + \frac{1}{2} \ell n |\sec x + \tan x| + C$ (2) $\frac{\sec x \tan x}{2} - \frac{1}{2} \ell n |\sec x + \tan x| + C$

(3) $\frac{\sec x \tan x}{2} + \frac{1}{2} \ell n |\sec x - \tan x| + C$ (4) $\sec x \tan x + \ell n |\sec x + \tan x| + C$

Section (D) : Algebraic Integral

D-1. The value of $\int \frac{dx}{x^2 + x + 1}$ is equal to

(1) $\frac{\sqrt{3}}{2} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$ (2) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

(3) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$ (4) $\frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + C$

D-2. $\int \frac{dx}{2x^2 + x + 1}$ equals

(1) $\frac{1}{\sqrt{7}} \tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right) + c$ (2) $\frac{1}{2\sqrt{7}} \tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right) + c$

(3) $\frac{1}{2} \tan^{-1}\left(\frac{4x+1}{\sqrt{7}}\right) + c$ (4) None of these

Indefinite Integration

D-3. If $\int \frac{x+1}{x^2+x+3}dx = a\,\ell n\,|x^2+x+3| + \frac{1}{\sqrt{b}}\tan^{-1}\left(\frac{2x+c}{\sqrt{11}}\right) + k$, then

(1) $a + b = 23/2$ (2) $c = 3$ (3) $b + c = 11$ (4) $c = 2$

D-4. If $\int \frac{2x+3}{x^2-5x+6}dx = A\ln|x-3| + B\ln|x-2| + C$, then $A + B =$

(1) 16 (2) 0 (3) 2 (4) 4

D-5. The value of $\int \sqrt{\frac{e^x-1}{e^x+1}}\,dx$ is equal to

(1) $\ell n\left(e^x+\sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x) + C$ (2) $\ell n\left(e^x+\sqrt{e^{2x}-1}\right) + \sec^{-1}(e^x) + C$

(3) $\ell n\left(e^x-\sqrt{e^{2x}-1}\right) - \sec^{-1}(e^x) + C$ (4) None of these

D-6. The value of $\int \frac{dx}{x\sqrt{1-x^3}}$ is equal to

(1) $\frac{1}{3}\ell n\left|\frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1}\right| + C$ (2) $\frac{1}{3}\ell n\left|\frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1}\right| + C$

(3) $\frac{1}{3}\ell n\left|\frac{1}{\sqrt{1-x^3}}\right| + C$ (4) $\frac{1}{3}\ell n\,|1-x^3| + C$

D-7. If $\int \frac{dx}{x^4+x^3} = \frac{A}{x^2} + \frac{B}{x} + \ell n\left|\frac{x}{x+1}\right| + C$, then

(1) $A = \frac{1}{2}, B = 1$ (2) $A = 1, B = -\frac{1}{2}$ (3) $A = -\frac{1}{2}, B = 1$ (4) $A = -\frac{1}{2}, B = \frac{1}{2}$

D-8. $\int \frac{1}{(x+1)(x+2)}dx =$

(1) $\ell n\left|\frac{x+2}{x+1}\right| + c$ (2) $\ell n|x+1| + \ell n|x+2| + c$

(3) $\ell n\left|\frac{x+1}{x+2}\right| + c$ (4) None of these

D-9. $\int \frac{x^4+4}{x^2-2x+2}dx$ is equal to

(1) $\frac{x^3}{2} + x^2 + 2x + c$ (2) $\frac{x^3}{3} + x^2 + 2x + c$ (3) $\frac{x^3}{3} + \frac{x^2}{2} + x + c$ (4) $\frac{x^3}{3} + x^2 - 2x + c$

D-10. $\int \frac{dx}{(x^2+1)(x^2+4)} =$

(1) $\frac{1}{3}\tan^{-1}x - \frac{1}{3}\tan^{-1}\frac{x}{2} + c$ (2) $\frac{1}{3}\tan^{-1}x + \frac{1}{3}\tan^{-1}\frac{x}{2} + c$

(3) $\frac{1}{3}\tan^{-1}x - \frac{1}{6}\tan^{-1}\frac{x}{2} + c$ (4) $\tan^{-1}x - 2\tan^{-1}\frac{x}{2} + c$

D-11. $\int \frac{1-x^7}{x\left(1+x^7\right)}dx$ is equal to

(1) $\ell n|x| + \frac{2}{7}\ell n|1+x^7| + c$ (2) $\ell n|x| - \frac{2}{7}\ell n|1-x^7| + c$

(3) $\ell n|x| - \frac{2}{7}\ell n|1+x^7| + c$ (4) $\ell n|x| + \frac{2}{7}\ell n|1-x^7| + c$

D-12. $\int \frac{x^2+2}{x^4+4}dx$ is equal to

(1) $\frac{1}{2}\tan^{-1}\left(\frac{x^2+2}{2x}\right) + c$ (2) $\frac{1}{2}\tan^{-1}\left(\frac{x^2-2}{2x}\right) + c$

(3) $\frac{1}{2}\tan^{-1}\left(\frac{2x}{x^2-2}\right) + c$ (4) $\frac{1}{2}\tan^{-1}(x^2+2) + c$

D-13. $\int \frac{1}{x(x^n+1)}dx$ is equal to

(1) $\frac{1}{n}\ell n\left(\frac{x^n}{x^n+1}\right) + c$ (2) $\frac{1}{n}\ell n\left(\frac{x^n+1}{x^n}\right) + c$ (3) $\ell n\left(\frac{x^n}{x^n+1}\right) + c$ (4) none of these

D-14. $\int \frac{1}{x^2(x^4+1)^{3/4}}dx$ is equal to

(1) $\left(1+\frac{1}{x^4}\right)^{1/4} + c$ (2) $(x^4+1)^{1/4} + c$ (3) $\left(1-\frac{1}{x^4}\right)^{1/4} + c$ (4) $-\left(1+\frac{1}{x^4}\right)^{1/4} + c$

D-15. $\int \frac{dx}{(x+1)\sqrt{x-2}}$ is equal to

(1) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{x-2}{3}\right) + c$ (2) $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x-2}}{3}\right) + c$

(3) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\sqrt{\frac{x-2}{3}}\right) + c$ (4) None of these

Section (E) : Integration of trigonometric functions, Reduction formulae, Miscellaneous

E-1. $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} =$

(1) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$ (2) $\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\tan x}{\sqrt{5}}\right) + c$

(3) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$ (4) None of these

E-2. Antiderivative of $\frac{\sin^2 x}{1+\sin^2 x}$ with respect to x is:

(1) $x - \frac{\sqrt{2}}{2}\ \text{arctan}\left(\sqrt{2}\tan x\right) + c$ (2) $x - \frac{1}{\sqrt{2}}\ \text{arctan}\left(\frac{\tan x}{\sqrt{2}}\right) + c$

(3) $x - \sqrt{2}\ \text{arctan}\left(\sqrt{2}\tan x\right) + c$ (4) $x - \sqrt{2}\ \text{arctan}\left(\frac{\tan x}{\sqrt{2}}\right) + c$

E-3. If $\int \frac{1}{1+\sin x}\,dx = \tan\left(\frac{x}{2}+a\right) + b$, then

(1) $a = -\frac{\pi}{4}$, $b \in R$ (2) $a = \frac{\pi}{4}$, $b \in R$ (3) $a = \frac{5\pi}{4}$, $b \in R$ (4) none of these

E-4. $\int \frac{dx}{5+4\cos x} = k\tan^{-1}\left(m\tan\frac{x}{2}\right) + C$ then:

(1) k = 2/3 (2) m = 4/3 (3) k = 1/3 (4) m = 2/3

E-5. $\int \frac{dx}{\sin x + \sqrt{3}\cos x} =$

(1) $\frac{1}{4}\ell n\left|\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right|+C$ (2) $\ell n\left|\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right|+C$

(3) $\frac{1}{2}\ell n\left|\sec\left(x+\frac{\pi}{6}\right)+\tan\left(x+\frac{\pi}{6}\right)\right|+C$ (4) $\frac{1}{2}\ell n\left|\sec\left(x-\frac{\pi}{6}\right)+\tan\left(x-\frac{\pi}{6}\right)\right|+C$

E-6. The value of $\int \frac{2\sin x + 3\cos x}{2\cos x + 3\sin x}dx$

(1) $\frac{12}{13}\ell n\,|2\cos x + 3\sin x| + \frac{5}{13}x + C$ (2) $\frac{5}{13}\ell n\,|2\cos x - 3\sin x| + \frac{12}{13}x + C$

(3) $\frac{5}{13}\ell n\,|2\cos x + 3\sin x| + \frac{12}{13}x + C$ (4) $\frac{5}{13}\ell n\,|2\cos x + 3\sin x| - \frac{12}{13}x + C$

E-7. $\int \sin^3 x\cos^3 x\,dx$ is equal to

(1) $\frac{1}{2}\sin_4 x + \cos_6 x + c$ (2) $\frac{1}{4}\sin_4 x - \frac{1}{6}\sin_6 x + c$

(3) $\sin_{-4} x + \cos_6 x + c$ (4) $\cos_6 x + \sin_6 x + c$

E-8. If $\int \frac{dx}{\sqrt{\sin^3 x \cos^5 x}} = a\sqrt{\cot x} + b\sqrt{\tan^3 x}$ + c where c is an arbitrary constant of integration then the values of 'a' and 'b' are respectively:

(1) -2 & $\frac{2}{3}$ (2) 2 & $-\frac{2}{3}$ (3) 2 & $\frac{2}{3}$ (4) none of these

E-9. $\int \frac{1}{\sqrt{\sin^3 x \cos x}}\,dx =$

(1) $\frac{2}{\sqrt{\tan x}} + C$ (2) $\frac{-2}{\sqrt{\tan x}} + C$ (3) $\frac{-3}{\sqrt{\tan x}} + C$ (4) $\frac{-5}{\sqrt{\tan x}} + C$

E-10. If $\int \sqrt{\frac{\cos^3 x}{\sin^{11} x}}\,dx = -2\left(A\tan^{\frac{-9}{2}} x + B\tan^{\frac{-5}{2}} x\right) + C$, then

(1) $A = \frac{1}{9}$, $B = \frac{-1}{5}$ (2) $A = \frac{1}{9}$, $B = \frac{1}{5}$ (3) $A = -\frac{1}{9}$, $B = \frac{1}{5}$ (4) $A = -\frac{1}{9}$, $B = -\frac{1}{5}$

E-11. The value of $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$ is equal to

(1) $\sqrt{2}\left(\sqrt{\cos x} + \frac{1}{5}\tan^{5/2} x\right) + C$ (2) $\sqrt{2}\left(\sqrt{\tan x} + \frac{1}{5}\tan^{5/2} x\right) + C$

(3) $\sqrt{2}\left(\sqrt{\tan x} - \frac{1}{5}\tan^{5/2} x\right) + C$ (4) $\sqrt{2}\left(\sqrt{\cos x} - \frac{1}{5}\tan^{5/2} x\right) + C$

E.12. The value of $\int \frac{\sin x + \cos x}{5\sin 2x + 7}dx$

(1) $\frac{1}{4\sqrt{5}}\,.\,\ell n\left|\frac{\sin x - \cos x + \sqrt{\frac{12}{5}}}{\sin x - \operatorname{cox} - \sqrt{\frac{12}{5}}}\right| + C$ (2) $\frac{1}{4\sqrt{5}}\,.\,\ell n\left|\frac{\sin x - \cos x + \sqrt{\frac{12}{5}}}{\sin x - \operatorname{cox} - \sqrt{\frac{12}{5}}}\right| + C$

(3) $\frac{1}{2\sqrt{15}}\,.\,\ell n\left|\frac{\sin x - \cos x + \sqrt{\frac{12}{5}}}{\sin x - \operatorname{cox} - \sqrt{\frac{12}{5}}}\right| + C$ (4) $\frac{1}{4\sqrt{5}}\,.\,\ell n\left|\frac{\sin x + \cos x + \sqrt{\frac{12}{5}}}{\sin x + \operatorname{cox} - \sqrt{\frac{12}{5}}}\right| + C$

E-13. The value of $\int \frac{\sin x - \cos x}{26 + \sin 2x}dx$

(1) $-\frac{1}{5}\tan^{-1}\frac{(\sin x + \cos x)}{5} + C$ (2) $\frac{1}{5}\tan^{-1}\frac{(\sin x + \cos x)}{5} + C$

(3) $-\frac{1}{5}\tan^{-1}\frac{(\sin x - \cos x)}{5} + C$ (4) $\frac{1}{5}\tan^{-1}\frac{(\sin x - \cos x)}{5} + C$

E-14. Integrate $\frac{1}{1-\cot x}$

(1) $\frac{1}{2}\log|\sin x-\cos x|+\frac{1}{2}x+C$

(2) $\frac{1}{2}\log|\sin x+\cos x|+\frac{1}{2}x+C$

(3) $\frac{1}{2}\log|\sin x+\cos x|-\frac{1}{2}x+C$

(4) $\frac{1}{2}\log|\sin x-\cos x|-\frac{1}{2}x+C$

E-15._ The value of $\int \cos^6 x dx$ is

(1) $\frac{5}{16}\left[x+\cos x\sin x\right]-\frac{5}{24}\cos^3 x\sin x+\frac{1}{6}\cos^5 x \sin x + C$

(2) $\frac{5}{16}\left[x+\cos x\sin x\right]+\frac{5}{24}\cos^3 x\sin x-\frac{1}{6}\cos^5 x \sin x + C$

(3) $\frac{5}{8}\left[x+\cos x\sin x\right]+\frac{5}{12}\cos^3 x\sin x+\frac{1}{3}\cos^5 x \sin x + C$

(4) $\frac{5}{16}\left[x+\cos x\sin x\right]+\frac{5}{24}\cos^3 x\sin x+\frac{1}{6}\cos^5 x \sin x+C$

E-16. The reduction formula of $I_n = \int \cot^n x\, dx$ is

(1) $I_n = \frac{\cot^{n-1} x}{n-1} - I_{n-2}$, $n \geq 2$

(2) $I_n = -\frac{\cot^{n-1} x}{n-1} - I_{n-2}$, $n \geq 2$

(3) $I_n = -\frac{\cot^{n-1} x}{n-1} + I_{n-2}$, $n \geq 2$

(4) $I_n = \frac{\cot^{n-1} x}{n-1} + I_{n-2}$, $n \geq 2$

Indefinite Integration

MATHEMATICS

Exercise-2

*** Marked Questions may have more than one correct option.**

PART - I : OBJECTIVE QUESTIONS

1. The value of $\int \frac{\ell n\left(\frac{x-1}{x+1}\right)}{x^2-1}$ dx is equal to

(1) $\frac{1}{2}\ \ell n^2 \frac{x-1}{x+1}$ + C (2) $\frac{1}{4}\ \ell n^2 \frac{x-1}{x+1}$ + C (3) $\frac{1}{2}\ \ell n^2 \frac{x+1}{x-1}$ + C (4) $\frac{1}{4}\ \ell n^2 \frac{x+1}{x-1}$ + C

2. The value of $\int \frac{\ell n(\tan x)}{\sin x\ \cos x}$ dx is equal to

(1) $\frac{1}{2}\ \ell n^2 (\cot x)$ + C (2) $\frac{1}{2}\ \ell n^2 (\sec x)$ + C (3) $\frac{1}{2}\ \ell n^2 (\sin x)$ + C (4) $\frac{1}{2}\ \ell n^2 (\cos x)$ + C

3. If f(x) = $\int \frac{2\sin x - \sin 2x}{x^3}$ dx, where x ≠ 0, then $\lim\limits_{x \to 0}$ f '(x) has the value

(1) 0 (2) 1 (3) 2 (4)

4. If $\int (x^9 + x^6 + x^3)(2x^6 + 3x^3 + 6)^{1/3}$ dx = $\frac{1}{a}$ $(2x^9 + 3x^6 + 6x^3)^{4/3}$ + c, then 'a' is equal to

(1) $\frac{1}{6}$ (2) $\frac{1}{24}$ (3) 24 (4) 6

5. $\int 2^{mx} . 3^{nx}$ dx when m, n ∈ N is equal to:

(1) $\frac{2^{mx} + 3^{nx}}{m\ell n2 + n\ell n3}$ + c (2) $\frac{(mn) . 2^x . 3^x}{m\ell n2 + n\ell n3}$ + c (3) $\frac{2^{mx} . 3^{nx}}{\ell n\left(2^m . 3^n\right)}$ + c (4) none of these

6. $\int \frac{dx}{\sin x . \sin(x+\alpha)}$ is equal to

(1) cosec α $\ell n\left|\frac{\sin x}{\sin(x+\alpha)}\right|$ + C (2) cosec α $\ell n\left|\frac{\sin(x+\alpha)}{\sin x}\right|$ + C

(3) cosec α $\ell n\left|\frac{\sec(x+\alpha)}{\sec x}\right|$ + C (4) cosec α $\ell n\left|\frac{\sec x}{\sec(x+\alpha)}\right|$ + C

7. $\int (\sin 2x - \cos 2x)$ dx = $\frac{1}{\sqrt{2}}$ sin (2x – a) + b, then

(1) a = $\frac{5\pi}{4}$, b ∈ R (2) a = – $\frac{5\pi}{4}$, b ∈ R (3) a = $\frac{\pi}{4}$, b ∈ R (4) none of these

8. $\int [1 + \tan x . \tan(x+\alpha)]$ dx is equal to

Indefinite Integration

(1) $\cos\alpha \cdot \ell n\left|\dfrac{\sin x}{\sin(x+\alpha)}\right| + C$ (2) $\tan\alpha \cdot \ell n\left|\dfrac{\sin x}{\sin(x+\alpha)}\right| + C$

(3) $\cot\alpha \cdot \ell n\left|\dfrac{\sec(x+\alpha)}{\sec x}\right| + C$ (4) $\cot\alpha \cdot \ell n\left|\dfrac{\cos(x+\alpha)}{\cos x}\right| + C$

9. $\int 4\sin x\cos\dfrac{x}{2}\cos\dfrac{3x}{2}\,dx$ is equal to

(1) $\cos x - \dfrac{1}{2}\cos 2x + \dfrac{1}{3}\cos 3x + c$ (2) $\cos x - \dfrac{1}{2}\cos 2x - \dfrac{1}{3}\cos 3x + c$

(3) $\cos x + \dfrac{1}{2}\cos 2x + \dfrac{1}{3}\cos 3x + c$ (4) $\cos x + \dfrac{1}{2}\cos 2x - \dfrac{1}{3}\cos 3x + c$

10. $\int \dfrac{x^2+\cos^2 x}{1+x^2}\,\text{cosec}^2 x\,dx$ is equal to:

(1) $-\tan^{-1}x + \cot x + c$ (2) $2\tan^{-1}x + c$ (3) $-\tan^{-1}x - \dfrac{\text{cosec}\,x}{\sec x} + c$ (4) none of these

11. $\int \sqrt{\dfrac{x-1}{x+1}}\cdot\dfrac{1}{x^2}\,dx$ is equal to

(1) $\sin^{-1}\dfrac{1}{x} + \dfrac{\sqrt{x^2-1}}{x} + c$ (2) $\dfrac{\sqrt{x^2-1}}{x} + \cos^{-1}\dfrac{1}{x} + c$

(3) $\sec^{-1}x - \dfrac{\sqrt{x^2-1}}{x} + c$ (4) $\tan^{-1}\sqrt{x^2+1} - \dfrac{\sqrt{x^2-1}}{x} + c$

12. If $\int \dfrac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\,dx = Ax + B\,\ell n\,|9e^{2x}-4| + C$, then

(1) A + 18B = 16 (2) 18B – A = 19 (3) A –18B = 17 (4) A + 18B = 32

13. If $0 < x < \pi$, then $\int \dfrac{dx}{\sqrt{\sin^3 x\sin(x-\alpha)}}$ is equal to

(1) $\sqrt{\cos\alpha+\sin\alpha\cot x} + c$ (2) $2\,\text{cosec}\,\alpha\sqrt{\cos\alpha-\sin\alpha\cot x} + c$

(3) $-2\,\text{cosec}\,\alpha\sqrt{\cos\alpha+\sin\alpha\cot x} + c$ (4) None of these

14. If $\int \dfrac{\cos 4x+1}{\cot x-\tan x}\,dx = A\cos 4x + B$; where A & B are constants, then

(1) A = − 1/4 & B may have any value (2) A = − 1/8 & B may have any value

(3) A = − 1/2 & B = − 1/4 (4) none of these

15. If $\int \tan^4 x\,dx = a\tan^3 x + b\tan x + \varphi(x)$, then

(1) $a = \dfrac{1}{3}$ (2) b = – 1 (3) $\varphi(x) = x + c,\ c \in R$ (4) All of these

16. $\int \dfrac{dx}{1+\sin 2x-\cos 2x}$ is equal to

(1) $\ell n\,|\,(1+\cot x)\,| + c$ (2) $\sin^2 x + \cos x + c$

(3) $-\frac{1}{2}\ \ell n\,|\,(1+\cot x) + c$ (4) none of these

17. $\int \frac{\sin^2 x}{\cos^6 x}dx$ is equal to

(1) $\tan x + \frac{\tan^5 x}{5} + c$ (2) $\frac{\tan^3 x}{3} + \frac{\tan^5 x}{5} + c$

(3) $\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$ (4) $\tan x + \frac{\tan^3 x}{3} + c$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** $\int (\sin x)^5 \cos x\, dx = \frac{\sin^6 x}{6} + C$

Statement-2 : $\int (f(x))^n f'(x)\, dx = \frac{(f(x))^{n+1}}{n+1} + C,\ n \in I$

A-2. **Statement-1 :** If $x > 0,\ x \neq 1$ then $\int (\log_x e - (\log_x e)^2)dx = x \log_x e + C$

Statement-2 : $\int (\log_x e - (\log_x e)^2)dx = e^x f(x) + C$ and $e^t = x$ iff $t = \ell n x$

Section (B) : MATCH THE COLUMN

B-1. If $I = \int \frac{dx}{a + b\cos x}$, where a, b > 0 and a + b = u, a – b = v, then match the following column

Column – I	Column – II
(A) v = 0	(p) $I = \frac{1}{\sqrt{uv}}\ \ell n\left|\frac{\sqrt{u}+\sqrt{v}\ \tan\frac{x}{2}}{\sqrt{u}-\sqrt{v}\ \tan\frac{x}{2}}\right| + C$

(B) $v > 0$ (q) $I = \frac{2}{\sqrt{uv}} \tan^{-1}\left(\sqrt{\frac{v}{u}} \tan\frac{x}{2}\right) + C$

(C) $v < 0$ (r) $I = \frac{1}{\sqrt{-uv}} \ell n \left|\frac{\sqrt{u} + \sqrt{-v} \tan\frac{x}{2}}{\sqrt{u} - \sqrt{-v} \tan\frac{x}{2}}\right| + C$

(s) $\frac{2}{u} \tan \frac{x}{2} + C$

Indefinite Integration

MATHEMATICS

Section (C) : One or More Than One Options Correct

C-1. If $f(x) = \dfrac{(2x+(1+x^2)3x^4)}{1+2x^2+x^4}$, $g(x) = \dfrac{x^2e^{x^3}}{x^2+1}$ and $\int f(x)dx = g(x) + \varphi(x)$, $\varphi(0) = 1$, then

(1) $\varphi(2) = 1$ (2) $\varphi'(1) = 0$ (3) $\varphi(x)$ is even (4) $\varphi(x)$ is odd

C-2. $\displaystyle\int \frac{(2x-1)}{x^4-2x^3+x+1}dx$ is equal to :

(1) $\dfrac{1}{\sqrt{3}}\tan^{-1}\left[\dfrac{2x^2-2x-1}{\sqrt{3}}\right]+C$

(2) $\dfrac{1}{\sqrt{3}}\cot^{-1}\left[\dfrac{1+2x-2x^2}{\sqrt{3}}\right]+C$

(3) $\dfrac{2}{\sqrt{3}}\tan^{-1}\left(\dfrac{2(x^2-x)-1}{\sqrt{3}}\right)+C$

(4) $\dfrac{-2}{\sqrt{3}}\cot^{-1}\left(\dfrac{2x^2-2x-1}{\sqrt{3}}\right)+C$

C-3. If $\displaystyle\int \frac{\sin 2x}{\sin^4 x+\cos^4 x}dx = a\cot^{-1}(b\tan^2 x) + C$, then :

(1) a = 1, b = 1 (2) a = –1, b = 1 (3) a + b = 2 (4) a + b = 0

Exercise-3

Marked Questions may have for Revision Questions.

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. $\displaystyle\int \frac{dx}{x(x^n+1)}$ is equal to- **[AIEEE 2002, (4, –1), 225]**

(1) $\dfrac{1}{n}\log\left|\dfrac{x^n}{x^n+1}\right| + c$ (2) $\dfrac{1}{n}\log\left|\dfrac{x^n+1}{x^n}\right| + c$ (3) $\log\left|\dfrac{x^n}{x^n+1}\right| + c$ (4) None of these

2. If $\displaystyle\int \frac{\sin x}{\sin(x-\alpha)}dx = Ax + B\log\sin(x-\alpha) + c$, then value of (A, B) is- **[AIEEE 2004, (4, –1), 225]**

(1) $(\sin\alpha, \cos\alpha)$ (2) $(\cos\alpha, \sin\alpha)$ (3) $(-\sin\alpha, \cos\alpha)$ (4) $(-\cos\alpha, \sin\alpha)$

3. $\displaystyle\int \frac{dx}{\cos x-\sin x}$ is equal to- **[AIEEE 2004, (4, –1), 225]**

(1) $\dfrac{1}{\sqrt{2}}\log\left|\tan\left(\dfrac{x}{2}-\dfrac{\pi}{8}\right)\right| + c$

(2) $\dfrac{1}{\sqrt{2}}\log\left|\cot\left(\dfrac{x}{2}\right)\right| + c$

(3) $\dfrac{1}{\sqrt{2}}\log\left|\tan\left(\dfrac{x}{2}-\dfrac{3\pi}{8}\right)\right| + c$

(4) $\dfrac{1}{\sqrt{2}}\log\left|\tan\left(\dfrac{x}{2}+\dfrac{3\pi}{8}\right)\right|+c$

4. $\displaystyle\int \left\{\frac{(\ell n x-1)}{1+(\ell n x)^2}\right\}^2 dx$ is equal to- **[AIEEE 2005 (3, 0), 225]**

Indefinite Integration

MATHEMATICS

(1) $\frac{x}{(\ell n x)^2+1}$ + c (2) $\frac{xe^x}{1+x^2}$ + c (3) $\frac{x}{x^2+1}$ + c (4) $\frac{\ell nx}{(\ell nx)^2+1}$ + c

5. $\int \frac{dx}{\cos x+\sqrt{3}\sin x}$ is equal to **[AIEEE 2007 (3, –1), 120]**

(1) $\frac{1}{2}\ell n \tan\left(\frac{x}{2}+\frac{\pi}{12}\right)$ + c (2) $\frac{1}{2}\ell n \tan\left(\frac{x}{2}-\frac{\pi}{12}\right)$ + c

(3) $\ell n \tan\left(\frac{x}{2}+\frac{\pi}{12}\right)$ + c (4) $\ell n \tan\left(\frac{x}{2}-\frac{\pi}{12}\right)$ + c

6. The value of $\int \frac{\sin x\,dx}{\sin\left(x-\frac{\pi}{4}\right)}$ is- **[AIEEE 2008 (3, –1), 105]**

(1) $x + \ell n\left|\cos\left(x-\frac{\pi}{4}\right)\right|$ + c (2) $x - \ell n\left|\sin\left(x-\frac{\pi}{4}\right)\right|$ + x

(3) $x + \ell n\left|\sin\left(x-\frac{\pi}{4}\right)\right|$ + c (4) $x - \ell n\left|\cos\left(x-\frac{\pi}{4}\right)\right|$ + c

7. If the integral $\int \frac{5\tan x}{\tan x-2}$ dx = x+a ℓn |sin x – 2 cos x| + k, then a is equal to: **[AIEEE-2012, (4, –1)/120]**

(1) – 1 (2) – 2 (3) 1 (4) 2

8. If $\int f(x)dx=\psi(x)$, then $\int x^5 f(x^3)dx$ is equal to **[AIEEE - 2013, (4, – 1)]**

(1) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^2\psi(x^3)dx\right]+C$ (2) $\frac{1}{3}x^3\psi(x^3)-3\int x^3\psi(x^3)dx+C$

(3) $\frac{1}{3}x^3\psi(x^3)-\int x^2\psi(x^3)dx+C$ (4) $\frac{1}{3}\left[x^3\psi(x^3)-\int x^3\psi(x^3)dx\right]+C$

9. The integral $\int\left(1+x-\frac{1}{x}\right)e^{x+\frac{1}{x}}dx$ is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $(x + 1)\,e^{x+\frac{1}{x}}$ + c (2) $-x\,e^{x+\frac{1}{x}}$ + c (3) $(x – 1)\,e^{x+\frac{1}{x}}$ + c (4) $x\,e^{x+\frac{1}{x}}$ + c

10. The integral $\int \frac{dx}{x^2\left(x^4+1\right)^{3/4}}$ equals **[JEE(Main) 2015, (4, – 1), 120]**

(1) $\left(\frac{x^4+1}{x^4}\right)^{1/4}$ + c (2) $(x^4 + 1)^{1/4}$ + c (3) $-(x^4 + 1)^{1/4}$ + c (4) $-\left(\frac{x^4+1}{x^4}\right)^{1/4}$ + c

11. Let $I_n = \int \tan^n x\,dx$, (n > 1). If $I_4 + I_6 = a\tan^5x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to **[JEE(Main) 2017, (4, – 1), 120]**

(1) $\left(-\frac{1}{5},1\right)$ (2) $\left(\frac{1}{5},0\right)$ (3) $\left(\frac{1}{5},-1\right)$ (4) $\left(-\frac{1}{5},0\right)$

12. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to: **[JEE(Main) 2018, (4,–1), 120]**

(1) $\frac{1}{1+\cot^3 x} + C$ (2) $\frac{-1}{1+\cot^3 x} + C$ (3) $\frac{1}{3(1+\tan^3 x)} + C$ (4) $\frac{-1}{3(1+\tan^3 x)} + C$

(where C is a constant of integration)

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Let $f(x) = \int e^x (x-1)(x-2)\, dx$ then f decreases in the interval: **[IIT-JEE 2000, Scr, (1, 0), 35]**

(A) $(-\infty, 2)$ (B) $(-2, -1)$ (C) $(1, 2)$ (D) $(2, +\infty)$

2. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}} dx$ is equal to **[IIT-JEE 2006, (3, –1), 184]**

(A) $\frac{\sqrt{2x^4-2x^2+1}}{x^2} + C$ (B) $\frac{\sqrt{2x^4-2x^2+1}}{x^3} + C$

(C) $\frac{\sqrt{2x^4-2x^2+1}}{x} + C$ (D) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2} + C$

3. Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \ge 2$ and $g(x) = \underbrace{(f \circ f \circ \ldots \circ f)}_{f \text{ occurs } n \text{ times}}(x)$. Then $\int x^{n-2} g(x)\, dx$ equals **[IIT-JEE 2007, Paper-2, (3, – 1), 81]**

(A) $\frac{1}{n(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}} + K$ (B) $\frac{1}{(n-1)}\left(1+nx^n\right)^{1-\frac{1}{n}} + K$

(C) $\frac{1}{n(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}} + K$ (D) $\frac{1}{(n+1)}\left(1+nx^n\right)^{1+\frac{1}{n}} + K$

4. Let F(x) be an indefinite integral of $\sin^2 x$. **[IIT-JEE 2007, Paper-1, (3, – 1), 81]**

STATEMENT-1 : The function F(x) satisfies $F(x + \pi) = F(x)$ for all real x.

because

STATEMENT-2 : $\sin^2(x + \pi) = \sin^2 x$ for all real x.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5. Let $I = \int \frac{e^x}{e^{4x}+e^{2x}+1} dx$, $J = \int \frac{e^{-x}}{e^{-4x}+e^{-2x}+1} dx$. Then, for an arbitrary constant C, the value of J – I is equal to : **[IIT-JEE 2008, Paper-2, (3, – 1), 81]**

(A) $\frac{1}{2}\ell n\left|\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right| + C$ (B) $\frac{1}{2}\ell n\left|\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right| + C$

(C) $\frac{1}{2}\ell n\left|\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right| + C$ (D) $\frac{1}{2}\ell n\left|\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right| + C$

6. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$ **[IIT-JEE 2012, Paper-1, (3, –1), 70]**

(B) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}-\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(C) $\frac{-1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

(D) $\frac{1}{(\sec x+\tan x)^{11/2}}\left\{\frac{1}{11}+\frac{1}{7}(\sec x+\tan x)^2\right\}+K$

Answers

EXERCISE # 1

Section (A)

A-1. (3) **A-2.** (3) **A-3.** (1) **A-4.** (2) **A-5.** (4) **A-6.** (3) **A-7.** (4)
A-8. (3) **A-9.** (1)

Section (B)

B-1. (2) **B-2.** (3) **B-3.** (4) **B-4.** (3) **B-5.** (2) **B-6.** (1) **B-7.** (2)
B-8. (1) **B-9.** (4)

Section (C)

C-1. (3) **C-2.** (1) **C-3.** (4) **C-4.** (3) **C-5.** (3) **C-6.** (3) **C-7.** (1)
C-8. (4) **C-9.** (3) **C-10.** (1)

Section (D)

D-1. (2) **D-2.** (4) **D-3.** (1) **D-4.** (3) **D-5.** (1) **D-6.** (1) **D-7.** (3)
D-8. (3) **D-9.** (2) **D-10.** (3) **D-11.** (3) **D-12.** (2) **D-13.** (1) **D-14.** (4)
D-15. (3)

Section (E)

E-1. (3) **E-2.** (1) **E-3.** (1) **E-4.** (1) **E-5.** (4) **E-6.** (3) **E-7.** (2)
E-8. (1) **E-9.** (2) **E-10.** (2) **E-11.** (2) **E.12.** (2) **E-13.** (1) **E-14.** (1)
E-15. (4) **E-16.** (2)

EXERCISE # 2

PART - I

1. (2,4) **2.** (1,3,4) **3.** (2) **4.** (3) **5.** (3) **6.** (1) **7.** (2)
8. (3) **9.** (2) **10.** (3) **11.** (3) **12.** (1,2) **13.** (2) **14.** (2)
15. (4) **16.** (3) **17.** (2)

PART - II

Section (A) :

A-1. **(2)** **A-2.** **(1)**

Section (B) :

B-1. (A) → (s) ; (B) → (q) ; (C) → (r)

Section (C) :

C-1. (1,2,3) **C-2.** (3,4) **C-3.** (2,4)

EXERCISE # 3

PART - I

1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(1)	**5.**	(1)	**6.**	(3)	**7.**	(4)
8.	(3)	**9.**	(4)	**10.**	(4)	**11.**	(2)	**12.**	(4)				

PART - II

1.	(C)	**2.**	(D)	**3.**	(A)	**4.**	(D)	**5.**	(C)	**6.**	(C)

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self t esting amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

1. If $\int \frac{1}{(x+2)(x^2+1)}dx = \alpha \ell n\ (1+x^2) + \beta \tan^{-1}x + \gamma \log_e|x + 2| + C$, then

(1) $\alpha = -\frac{1}{10}, \beta = -\frac{2}{5}, \gamma = \frac{1}{5}$ (2) $\alpha = \frac{1}{10}, \beta = -\frac{2}{5}, \gamma = -\frac{1}{5}$

(3) $\alpha = -\frac{1}{10}, \beta = +\frac{2}{5}, \gamma = \frac{1}{5}$ (4) $\alpha = \frac{1}{10}, \beta = \frac{2}{5}, \gamma = -\frac{1}{5}$

2. $\int e^x\left(\ell n(4x+1)+\frac{16}{(4x+1)^2}\right)dx$ is equal to

(1) $e^x\left(\ell n(4x+1)-\frac{4}{4x+1}\right)+C$ (2) $e^x\left(\ell n(4x+1)\right)+C$

(3) $e^x\left(\ell n(4x+1)+\frac{1}{4x+1}\right)+C$ (4) $e^x\left(\ell n(4x+1)+\frac{4}{4x+1}\right)+C$

Indefinite Integration

3. $\int\left(1+2x^2+\frac{1}{x}\right).e^{x^2-\frac{1}{x}}dx$ is equal to

(1) $(2x + 1)\ e^{\left(x^2-\frac{1}{x}\right)} + C$ (2) $(2x - 1)\ e^{\left(x^2-\frac{1}{x}\right)} + C$

(3) $xe^{\left(x^2-\frac{1}{x}\right)} + C$ (4) $-xe^{\left(x^2-\frac{1}{x}\right)} + C$

4. $\int e^{\sin^2 x}.\sin x(\cos x+\cos^3 x)dx$ is equal to

(1) $\frac{1}{2}e^{\sin^2 x}(3-\sin^2 x)+C$ (2) $e^{\sin^2 x}(3\cos^2 x+2\sin^2 x)+C$

(3) $e^{\sin^2 x}(2\cos^2 x+3\sin^2 x)+C$ (4) $\frac{1}{2}e^{\sin^2 x}\left(1-\frac{1}{2}\cos^2 x\right)+C$

5. If $\int\frac{1}{\cos^3 x\sqrt{2\sin 2x}}dx$ = $(\tan x)^A + C(\tan x)^B + k$,where k is a constant of integration, then A + B + C is equal to

(1) $\frac{7}{10}$ (2) $\frac{16}{5}$ (3) $\frac{27}{10}$ (4) $\frac{21}{5}$

6. $\int\frac{1}{5+4\cos x}dx$ = $a\tan^{-1}\left(b\tan\frac{x}{2}\right)$ + C, then

(1) $a = \frac{2}{3}$, $b = -\frac{1}{3}$ (2) $a = \frac{2}{3}$, $b = \frac{1}{3}$ (3) $a = -\frac{2}{3}$, $b = \frac{1}{3}$ (4) $a = -\frac{2}{3}$, $b = -\frac{1}{3}$

7. If $\int\frac{(\sqrt{x})^5}{(\sqrt{x})^7+x^6}dx=\alpha\ell n\left(\frac{x^\beta}{x^\beta+1}\right)$ + C, then value of α and β are respectively are

(1) $\frac{5}{2}$ and 2 (2) $\frac{2}{5}$ and $\frac{5}{2}$ (3) $\frac{5}{2}$ and $\frac{2}{5}$ (4) 2 and $\frac{5}{2}$

8. $\int x^3 d(\tan^{-1}x)$ is equal to

(1) $\frac{x^2}{2}+\frac{1}{2}\ell n(1+x^2)+C$ (2) $-\frac{x^2}{2}-\frac{1}{2}\ell n(1+x^2)+C$

(3) $-\frac{x^2}{2}+\frac{1}{2}\ell n(1+x^2)+C$ (4) $\frac{x^2}{2}-\frac{1}{2}\ell n(1+x^2)+C$

9. If $I = \int e^{-x}\ell n(e^x+1)\,dx$, then I equals

(1) $x + (e^{-x} + 1)\ \ell n(e^x+1) + C$ (2) $x + (e^x + 1)\ \ell n(e^x+1) + C$

(3) $x - (e^{-x} + 1)\ \ell n(e^x+1) + C$ (4) none of these

10. $\int\frac{(f(x)g'(x)-f'(x)g(x))}{f(x).g(x)}(\ell n(g(x))-\ell n(f(x)))dx$ is equal to

(1) $\ell n\left(\frac{g(x)}{f(x)}\right)+C$ (2) $\frac{g(x)}{f(x)}\ell n\left(\frac{g(x)}{f(x)}\right)+C$ (3) $\frac{1}{2}\left(\ell n\left(\frac{g(x)}{f(x)}\right)\right)^2+C$ (4) $\ell n\left(\frac{g(x)}{f(x)}\right)^2+C$

11. If $f(x) = \int \frac{x^2+\sin^2 x}{1+x^2}.\sec^2 x\, dx$ and f(0) = 0, then f(1) is equal to

(1) $\frac{\pi}{4}-\tan 1$ (2) $\frac{\pi}{4}-\tan\frac{\pi}{4}$ (3) $\tan 1 - \frac{\pi}{4}$ (4) $\frac{\pi}{4}-1$

12. $I = \int \frac{1}{1+\cos^2 x} d(\cos x)$ is equal to

(1) $\ell n(1+\cos^2 x)+C$ (2) $-\tan^{-1}(\cos x) + C$ (3) $-\cot^{-1}(\cos x) + C$ (4) $-\ell n(1+\cot^2 x)+C$

13. $\int e^{\tan^{-1} x}(1+x+x^2)d(\cot^{-1} x)$ is equal to

(1) $-e^{\tan^{-1} x}+C$ (2) $e^{\tan^{-1} x}+C$ (3) $xe^{\tan^{-1} x}+C$ (4) $-xe^{\tan^{-1} x}+C$

14. If $I_n = \int (\ell nx)^n dx$, then $I_5 + 5I_4$ is equal to

(1) $\frac{(\ell nx)^5}{x}$ (2) $x(\ell nx)^2$ (3) $x(\ell nx)^5$ (4) $x(\ell nx)^4$ a

15. $\int \frac{\cos x.\operatorname{cosec}^2 x}{(1+\sin^5 x)^{4/5}}dx$ is equal to

(1) $-(1 + \sin^5 x)^{1/5} + C$ (2) $-\sin x(1 + \sin^5 x)^{1/5} + C$

(3) $-\frac{(1+\sin^5 x)^{1/5}}{\sin x}+C$ (4) $-\frac{(1+\sin^{-5} x)^{1/5}}{\sin x}+C$

16. If $\int x^{-11}(1+x^4)^{-1/2}dx = \frac{t^5}{a}-\frac{t^3}{b}-\frac{t}{c}+k$, where $t = \sqrt{1+x^{-4}}$ and k is constant of integration, then (c – b – a) is equal to

(1) 10 (2) 11 (3) 12 (4) 15

17. If $f'(x) = \frac{1}{\sqrt{x^2+1}-x}$ and $f(0) = -\frac{\sqrt{2}+1}{2}$, then f(1) is equal to

(1) $1+\sqrt{2}$ (2) 1 (3) $\ell n(1+\sqrt{2})$ (4) $\ell n\ (\sqrt{\sqrt{2}+1})$

18. Let $\vec{a} = f(x)\hat{i}+f'(x)\hat{j}$ and $\vec{b} = f''(x)\hat{i}-f'(x)\hat{j}$ where f(x) is differentiable everywhere with $f'(x) \neq 0$ and f(0) = 1, f '(0) = 2, if $\vec{a}.\vec{b}=0$, then f(x) is

(1) $x^2 + 2x + 1$ (2) $2e^x - 1$ (3) e^{2x} (4) $4e^{x/2} - 3$

19. Let g(x) be the primitive of $\frac{3x+2}{\sqrt{x-9}}$ with respective to x. If g(13) = 132, then the value of g(10) is

(1) 66 (2) 60 (3) 248 (4) 0

Indefinite Integration

MATHEMATICS

20. $\int\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)dx$, where x∈ $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ is equal to

(1) $-4\cos\left(\frac{x}{2}\right)+C$ (2) $4\sin\left(\frac{x}{2}\right)+C$ (3) $\sin\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)+C$ (4) $-\sin\frac{x}{2}+C$

21. $\int\frac{\operatorname{cosec}^2 x-2017}{\cos^{2017}x}dx$ is equal to

(1) $-\frac{\operatorname{cosec}x}{(\cos x)^{2016}}+C$ (2) $\frac{\cot x}{(\cos x)^{2017}}+C$ (3) $-\frac{\cot x}{(\cos x)^{2016}}$ (4) $\frac{\tan x}{(\cos x)^{2017}}+C$

22. If $xf(x) = 3(f(x))^2 + 2$, then $\int\frac{2x^2-12xf(x)+f(x)}{(6f(x)-x)(x^2-f(x))^2}\ dx$ is equal to

(1) $\frac{1}{x^2-f(x)}+C$ (2) $\frac{1}{x^2+f(x)}+C$ (3) $\frac{1}{x+f(x)}+C$ (4) $\frac{1}{x-f(x)}+C$

23. If $I_{m,n}=\int\cos^m x.\sin nx dx$, then $(7I_{4,3}-4I_{3,2})$ is

(1) $-\cos^2 x + C$ (2) $-\cos 3x.\cos^4 x + C$ (3) constant (4) $\cos 7x - \cos^4 x + C$

24. If $\int f(x)dx=g(x)$ and $f^{-1}(x)$ is differentiable, then $\int f^{-1}(x)dx$ is equal to

(1) $xf^{-1}(x) + C$
(2) $xf^{-1}(x) - g(f^{-1}(x)) + C$
(3) $f^{-1}(x) + C$
(4) $xf^{-1}(x) - g(x) + C$

25. $\int\sin(2017x).\sin^{2015}x\,dx$ is equal to

(1) $-\frac{1}{2016}\sin(2016x).(\sin x)^{2016}+C$
(2) $\frac{\sin(2016x)(\sin x)^{2016}}{2016}+C$
(3) $\frac{(\sin x)^{2016}\cos(2016x)}{2016}+C$
(4) $\frac{(\sin x)^{2014}\sin(2017x)}{2014}+C$

26. $\int\frac{(x+1)^2}{x(x^2+1)}dx$ is equal to

(1) $\ell n|x|+C$
(2) $\ell n|x|+2\tan^{-1}x+C$
(3) $\ell n\left(\frac{1}{1+x^2}\right)+C$
(4) $\ell n|x(x^2+1)+C$

27. $\int x^{27}(6x^2+5x+4)(1+x+x^2)^6\,dx$ is equal to

(1) $\frac{x^4}{7}.(1+x+x^2)^7$ (2) $\frac{x^{28}(1+x+x^2)^7}{7}+C$ (3) $\frac{x^{28}(1+x+x^2)^7}{28}+C$ (4) $\frac{x^{28}(1+x+x^2)^6}{6}+C$

28. If $\int\frac{1}{(1+\sqrt{x})^{2017}}dx=2\left(\frac{1}{\alpha(1+\sqrt{x})^{\alpha}}-\frac{1}{\beta(1+\sqrt{x})^{\beta}}\right)+C$, where α, β > 0, then α – β is equal to

(1) 1 (2) –1 (3) – 2 (4) 2

29. If $\int \frac{\sin\left(\frac{\pi}{4}-x\right)}{2+\sin 2x}dx$ = Atan^{-1}(f(x)) + B, where A and B are constants, then the range of Af(x) is

(1) [0, 1] (2) [–1, 0] (3) $\left[-\sqrt{2}, \sqrt{2}\right]$ (4) [–1, 1]

30. $\int \frac{e^x(x-2)}{x(x^2+e^x)}dx$ is equal to (where x > 0)

(1) $\ell n\left(1+\frac{e^x}{x^2}\right)$ + C (2) $\ell n\left(-\frac{1}{2}+\frac{e^x}{x^2}\right)$ + C (3) $\ell n\left(2+\frac{e^x}{x^2}\right)$ + C (4) $\ell n\left(x+\frac{e^x}{x^2}\right)$ + C

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. The value of $\int \frac{\ell n\,|x|}{x\sqrt{1+\ell n\,|x|}}$ dx equals :

(1) $\frac{2}{3}\sqrt{1+\ell n\,|x|}$ $(\ell n|x| - 2)$ + C (2) $\frac{2}{3}\sqrt{1+\ell n\,|x|}$ $(\ell n|x| + 2)$ + C

(3) $\frac{1}{3}\sqrt{1+\ell n\,|x|}$ $(\ell n|x| - 2)$ + C (4) $2\sqrt{1+\ell n\,|x|}$ $(3\,\ell n|x| - 2)$ + C

2. $\int \sqrt{\frac{1-\cos x}{\cos\alpha - \cos x}}$ dx, where 0 < α < x < π, is equal to

(1) $2\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)$ + c (2) $\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)$ + c

(3) $2\sqrt{2}\,\ell n\left(\cos\frac{\alpha}{2}-\cos\frac{x}{2}\right)$ + c (4) $-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right)$ + c

3. The value of $\int \frac{\cos^3 x}{\sin^2 x+\sin x}$ dx is equal to :

(1) $\ell n |\sin x| + \sin x + C$ (2) $\ell n |\sin x| - \sin x + C$

(3) $-\ell n |\sin x| - \sin x + C$ (4) $-\ell n |\sin x| + \sin x + C$

4. The value of $2\int \sin x \,.\, \text{cosec}4x \, dx$ is equal to

(1) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$

(2) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$

(3) $\frac{1}{2\sqrt{2}} \ell n \left|\frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x}\right| - \frac{1}{4} \ell n \left|\frac{1+\sin x}{1-\sin x}\right| + C$

(4) none of these

5. The value of $\int \{1+2\tan x(\tan x+\sec x)\}^{1/2} dx$ is equal to

(1) $\ell n |\sec x (\sec x - \tan x)| + C$ (2) $\ell n |\text{cosec}\, x (\sec x + \tan x)| + C$

(3) $\ell n |\sec x (\sec x + \tan x)| + C$ (4) $\ell n |(\sec x + \tan x)| + C$

6. The value of $\int \frac{\sin^8 x - \cos^8 x}{1-2\sin^2 x \cos^2 x} dx$ is :

(1) $\frac{1}{2} \sin 2x + C$ (2) $-\frac{1}{2} \sin 2x + C$ (3) $-\frac{1}{2} \sin x + C$ (4) $-\sin^2 x + C$

7. $\int \frac{x^3-1}{x^3+x} dx$ is equal to

(1) $x - \ell n |x| + \ell n (x^2 + 1) - \tan^{-1}x + C$ (2) $x - \ell n |x| + \frac{1}{2} \ell n (x^2 + 1) - \tan^{-1}x + C$

(3) $x + \ell n |x| + \frac{1}{2} \ell n (x^2 + 1) + \tan^{-1}x + C$ (4) none of these

8. $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$ is equal to

(1) $\sqrt{x}\sqrt{1-x} - 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + c$ (2) $\sqrt{x}\sqrt{1-x} + 2\sqrt{1-x} + \cos^{-1}\left(\sqrt{x}\right) + c$

(3) $\sqrt{x}\sqrt{1-x} - 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + c$ (4) $\sqrt{x}\sqrt{1-x} + 2\sqrt{1-x} - \cos^{-1}\left(\sqrt{x}\right) + c$

9. $\int \frac{1}{[(x-1)^3(x+2)^5]^{1/4}} dx$ is equal to

(1) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}$ +C (2) $\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{1/4}$ + C (3) $\frac{1}{3}\left(\frac{x-1}{x+2}\right)^{1/4}$ + C (4) $\frac{1}{3}\left(\frac{x+1}{x-1}\right)^{1/4}$ + C

10. If $\int \frac{\left(\frac{1}{x}+\frac{1}{x^2}\right)(x-1)}{\left(\frac{1}{x^4}+\frac{1}{x^2}\right)\sqrt{(x^4-x^3+x^2)(x^4+x^3+x^2)}}dx$ = $\sec^{-1}(f(x))$ + C then

(1) Maximum value of f(x) is –2 when x < 0 (2) Minimum value of f(x) is 2 when x > 0

(3) f(0) is not defined (4) f(e) < f(π)

11. Primitive of $\frac{3x^4-1}{\left(x^4+x+1\right)^2}$ w.r.t. x is -

(1) $\frac{x}{x^4+x+1}$ + C (2) $\frac{x}{x^4+x^2+1}$ + C (3) $\frac{x^2}{x^4+x+1}$ + C (4) None of these

12. $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}}$ dx is equal to

(1) $\frac{\sqrt{2x^4-2x^2+1}}{x^2}$ + C (2) $\frac{\sqrt{2x^4-2x^2+1}}{x^3}$ + C

(3) $\frac{\sqrt{2x^4-2x^2+1}}{x}$ + C (4) $\frac{\sqrt{2x^4-2x^2+1}}{2x^2}$ +C

13. The value of $\int \frac{1+x^4}{(1-x^4)^{3/2}}$ dx is equal to

(1) $\frac{2}{\sqrt{\frac{1}{x^2}-x^2}}$ – C (2) $\frac{1}{\sqrt{\frac{1}{x^2}-x^2}}$ + C (3) $\frac{1}{\sqrt{\frac{1}{x^2}+x^2}}$ + C (4) $\frac{1}{\sqrt{\frac{1}{x^2}-x}}$ + C

APSP Answers

PART - I

1.	(3)	**2.**	(1)	**3.**	(3)	**4.**	(1)	**5.**	(2)	**6.**	(2)	**7.**	(2)
8.	(4)	**9.**	(3)	**10.**	(3)	**11.**	(3)	**12.**	(3)	**13.**	(4)	**14.**	(3)
15.	(3)	**16.**	(4)	**17.**	(4)	**18.**	(3)	**19.**	(2)	**20.**	(2)	**21.**	(1)
22.	(1)	**23.**	(2)	**24.**	(2)	**25.**	(2)	**26.**	(2)	**27.**	(2)	**28.**	(1)
29.	(4)	**30.**	(1)										

PART - II

1.	(1)	**2.**	(4)	**3.**	(2)	**4.**	(1)	**5.**	(3)	**6.**	(2)	**7.**	(2)
8.	(1)	**9.**	(1)	**10.**	(1234)	**11.**	(4)	**12.**	(4)	**13.**	(2)		