

Exercise-1

Marked Questions may have for Revision Questions.

* Marked Questions may have more than one correct option.

OBJECTIVE QUESTIONS

Section (A) : Sine Rule

- A-1** In a ΔABC , $a \sin(B - C) + b \sin(C - A) + c \sin(A - B) =$
 (1) 0 (2) $a + b + c$ (3) a (4) b
- A-2** In a ΔABC , $\frac{a^2 \sin(B - C)}{\sin A} + \frac{b^2 \sin(C - A)}{\sin B} + \frac{c^2 \sin(A - B)}{\sin C} =$
 (1) abc (2) $a + b + c$ (3) $a^2 + b^2 + c^2$ (4) 0
- A-3** The angles of a ΔABC are in A.P. (order being A, B, C) and it is being given that $b : c = \sqrt{3} : \sqrt{2}$, then $\angle A$ is equal to
 (1) 45° (2) 75° (3) 60° (4) 30°
- A-4.** If $\cos A + \cos B = 4 \sin^2\left(\frac{C}{2}\right)$, then sides a, c, b of the triangle ABC are in
 (1) G.P. (2) A.P. (3) H.P. (4) A.G.P.
- A-5.** If in a ΔABC , $\frac{\sin A}{\sin C} = \frac{\sin(A - B)}{\sin(B - C)}$, then a^2, b^2, c^2 are in
 (1) G.P. (2) H.P. (3) A.P. (4) A.G.P.
- A-6.** In a ΔABC , $A : B : C = 3 : 5 : 4$. Then $a + b + c \sqrt{2}$ is equal to
 (1) $2b$ (2) $2c$ (3) $3b$ (4) $3a$
- A-7.** If in a ΔABC , $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$, then the triangle is :
 (1) right angled (2) isosceles (3) equilateral (4) obtuse angled
- A-8.** In a ΔABC $\frac{bc \sin^2 A}{\cos A + \cos B \cos C}$ is equal to
 (1) $b^2 + c^2$ (2) bc (3) a^2 (4) $a^2 + bc$

Section (B) : Cosine Rule, projection formula

- B-1** In a ΔABC , $2(bc \cos A + ca \cos B + ab \cos C)$ is equal to
 (1) $a^2 + b^2 + c^2$ (2) $a + b + c$ (3) abc (4) $\sin A \sin B \sin C$
- B-2** In a ΔABC , $(a - b)^2 \cos^2 \frac{C}{2} + (a + b)^2 \sin^2 \frac{C}{2}$ is equal to
 (1) b^2 (2) a^2 (3) c^2 (4) abc

- B-3** In a ΔABC , $b^2 \sin 2C + c^2 \sin 2B$ is equal to
 (1) $2bc \cos A$ (2) $bc \sin A$ (3) $bc \cos A$ (4) $2bc \sin A$
- B-4** In a ΔABC , $\left(\frac{c - a \cos B}{b - a \cos C} \right) \sin C$ is equal to
 (1) $\cos A$ (2) $\cos B$ (3) $\sin B$ (4) $a \cos A$
- B-5.** In a triangle ABC, for any angle θ , $b \cos (A - \theta) + a \cos (B + \theta)$ is equal to
 (1) $b \sin \theta$ (2) $c \sin \theta$ (3) $a \cos \theta$ (4) $c \cos \theta$.
- B-6.** If in a triangle ABC, the altitude AM be the bisector of $\angle BAD$, where D is the mid point of side BC, then $(b^2 - c^2)$ is equal to
 (1) a^2 (2) $a^2/2$ (3) ab (4) bc
- B-7.** In a ΔABC , $2 \left[a \sin^2 \frac{C}{2} + c \sin^2 \frac{A}{2} \right] = c + a - b$.
 (1) $c + a + b$ (2) $c - a - b$ (3) $c + a - b$ (4) $c - a + b$
- B-8.** If in a triangle ABC, $(a + b + c)(b + c - a) = k \cdot bc$, then :
 (1) $k < 0$ (2) $k > 6$ (3) $0 < k < 4$ (4) $k > 4$
- B-9.** In a triangle ABC, $a : b : c = 4 : 5 : 6$. Then $3A + B$ equals to :
 (1) $4C$ (2) 2π (3) $\pi - C$ (4) π
- B-10.** The distance between the middle point of BC and the foot of the perpendicular from A is :
 (1) $\frac{-a^2 + b^2 + c^2}{2a}$ (2) $\frac{b^2 - c^2}{2a}$ (3) $\frac{b^2 + c^2}{\sqrt{bc}}$ (4) $\frac{b^2 + c^2}{2a}$

Section (C) : Napier formulae, Area of Triangle

- C-1.** In a triangle ABC, $4\Delta (\cot A + \cot B + \cot C)$ is equal to
 (1) $a^2 - b^2 + c^2$ (2) $a^2 + b^2 + c^2$ (3) $a^2 b^2 c^2$ (4) abc
- C-2** If in a ΔABC , $a = 6$, $b = 3$ and $\cos(A - B) = 4/5$, then its area is equal to
 (1) 6 sq. unit (2) 12 sq. unit (3) 9 sq. unit (4) 18 sq. unit
- C-3.** If in a triangle ABC, $\angle A = 30^\circ$ and the area of triangle is $\frac{\sqrt{3} a^2}{4}$, then
 (1) $C = 4B$ (2) $C = B$. (3) $C = 2B$. (4) $C = 3B$.
- C-4.** In a ΔABC , $A = \frac{2\pi}{3}$, $b - c = 3\sqrt{3}$ cm and area $(\Delta ABC) = \frac{9\sqrt{3}}{2}$ cm². Then 'a' is
 (1) $6\sqrt{3}$ cm (2) 9 cm (3) 18 cm (4) 7 cm

Section (D) : Half Angle formulae

- D-1** In a ΔABC , $\frac{\cos^2 \frac{A}{2}}{a} + \frac{\cos^2 \frac{B}{2}}{b} + \frac{\cos^2 \frac{C}{2}}{c}$ is equal to

(1) $\frac{s}{abc}$

(2) $\frac{2s^2}{abc}$

(3) $\frac{3s^2}{abc}$

(4) $\frac{s^2}{abc}$

- D-2.** In a ΔABC , $4 \left(bc \cos^2 \frac{A}{2} + ca \cos^2 \frac{B}{2} + ab \cos^2 \frac{C}{2} \right)$ is equal to
 (1) $a + b + c$ (2) $a^2 + b^2 + c^2$ (3) $(a + b + c)^2$ (4) $abc(a + b + c)$

- D-3.** In a ΔABC , $\left(\frac{2abc}{a+b+c} \right) \cos \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}$ is equal to
 (1) 2Δ (2) Δ (3) $\Delta/2$ (4) Δ/abc

- D-4.** If the sides a, b, c of a triangle are in A.P., then the value of $\tan \frac{A}{2} + \tan \frac{B}{2}$ is equal to
 (1) $\frac{1}{3} \cot \frac{B}{2}$ (2) $\frac{3}{2} \cot \frac{B}{2}$ (3) $\frac{2}{3} \cot \frac{B}{2}$ (4) $\frac{1}{2} \cot \frac{B}{2}$

- D-5.** If in a triangle ABC , $b \cos^2 \frac{A}{2} + a \cos^2 \frac{B}{2} = \frac{3}{2} c$, then a, c, b are :
 (1) in A.P. (2) in G.P. (3) in H.P. (4) None

- D-6.** In a ΔABC if $b + c = 3a$, then $\cot \frac{B}{2} \cdot \cot \frac{C}{2}$ has the value equal to:
 (1) 4 (2) 3 (3) 2 (4) 1

- D-7.** If in a ΔABC , $\Delta = a^2 - (b - c)^2$, then $\tan A$ is equal to
 (1) $15/16$ (2) $8/15$ (3) $8/17$ (4) $1/2$

- D-8.** If in a ΔABC , $\angle A = \frac{\pi}{2}$, then $\tan \frac{C}{2}$ is equal to
 (1) $\frac{a-c}{2b}$ (2) $\frac{a-b}{2c}$ (3) $\frac{a-c}{b}$ (4) $\frac{a-b}{c}$

- D-9.** In ΔABC , $\frac{2ab}{(a+b+c)\Delta} \cdot \cos^2 \frac{C}{2}$ is equal to
 (1) $\frac{s-a}{\Delta}$ (2) $\frac{s-b}{\Delta}$ (3) $\frac{a+b+c}{\Delta}$ (4) $\frac{s-c}{\Delta}$

Section (E) : Circumradius and Inradius

- E-1.** In ΔABC , $Rr(\sin A + \sin B + \sin C)$ is equal to
 (1) Δ (2) 2Δ (3) 3Δ (4) $\Delta/2$

- E-2.** In ΔABC , $a \cos B \cos C + b \cos C \cos A + c \cos A \cos B$ is equal to
 (1) $\frac{\Delta}{2R}$ (2) $\frac{2\Delta}{R}$ (3) $\frac{R}{\Delta}$ (4) $\frac{\Delta}{R}$

E-3. In ΔABC , $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca}$ is equal to

- (1) $\frac{1}{Rr}$ (2) $\frac{1}{2Rr}$ (3) $\frac{2}{Rr}$ (4) $\frac{3}{2Rr}$

E-4. In ΔABC , $\cos^2 \frac{A}{2} + \cos^2 \frac{B}{2} + \cos^2 \frac{C}{2}$

- (1) $2 + \frac{r}{2R}$ (2) $1 + \frac{r}{2R}$ (3) $1 + \frac{r}{R}$ (4) $2 + \frac{r}{R}$

E-5. In ΔABC , $a \cot A + b \cot B + c \cot C$

- (1) $R + r$ (2) $2R + r$ (3) $R + 2r$ (4) $2(R + r)$

E-6. If R denotes circumradius, then in ΔABC , $\frac{b^2 - c^2}{2aR}$ is equal to

- (1) $\cos(B - C)$ (2) $\sin(B - C)$ (3) $\cos B - \cos C$ (4) $\sin(B + C)$

E-7. In a ΔABC , the value of $\frac{a \cos A + b \cos B + c \cos C}{a + b + c}$ is equal to:

- (1) $\frac{r}{R}$ (2) $\frac{R}{2r}$ (3) $\frac{R}{r}$ (4) $\frac{2r}{R}$

E-8. In a triangle ABC , if $a : b : c = 3 : 7 : 8$, then $R : r$ is equal to

- (1) $2 : 7$ (2) $7 : 2$ (3) $3 : 7$ (4) $7 : 3$

E-9. In a ΔABC , $a = 1$ and the perimeter is six times the arithmetic mean of the sines of the angles. Then measure of $\angle A$ is

- (1) $\frac{\pi}{3}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

Section (F) : Length of Median, angle bisector, altitude

F-1. If α, β, γ are the respective altitudes of a triangle ABC , $\left(\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} \right)$ is equal to

- (1) $\frac{a^2 + b^2 + c^2}{\Delta^2}$ (2) $\frac{a^2 + b^2 + c^2}{4 \Delta^2}$ (3) $\frac{a^2 + b^2 + c^2}{2\Delta^2}$ (4) $\frac{a + b + c}{4\Delta^2}$

F-2. If α, β, γ are the respective altitudes of a triangle ABC , $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is equal to

- (1) $\frac{s - c}{\Delta}$ (2) $\frac{s - a}{\Delta}$ (3) $\frac{a + b - c}{\Delta}$ (4) $\frac{a + b - c}{2\Delta}$

F-3. In a ΔABC , if $AB = 5$ cm, $BC = 13$ cm and $CA = 12$ cm, then the distance of vertex 'A' from the side BC is (in cm)

(1) $\frac{25}{13}$

(2) $\frac{60}{13}$

(3) $\frac{65}{12}$

(4) $\frac{144}{13}$

- F-4.** If AD, BE and CF are the medians of a $\triangle ABC$, then $(AD^2 + BE^2 + CF^2) : (BC^2 + CA^2 + AB^2)$ is equal to
 (1) 4 : 3 (2) 3 : 2 (3) 3 : 4 (4) 2 : 3

- F-5.** In an acute angled triangle ABC, AP is the altitude. Circle drawn with AP as its diameter cuts the sides AB and AC at D and E respectively, then length DE is equal to

(1) $\frac{\Delta}{2R}$

(2) $\frac{\Delta}{3R}$

(3) $\frac{\Delta}{4R}$

(4) $\frac{\Delta}{R}$

- F-6.** If ' ℓ ' is the length of median from the vertex A to the side BC of a $\triangle ABC$, then

(1) $4\ell^2 = b^2 + 4ac \cos B$

(2) $4\ell^2 = a^2 + 4bc \cos A$

(3) $4\ell^2 = c^2 + 4ab \cos C$

(4) $4\ell^2 = b^2 + 2c^2 - 2a^2$

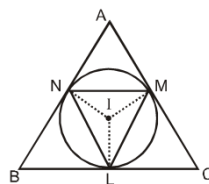
Exercise-2

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PART - I : OBJECTIVE QUESTIONS

- If in a triangle ABC, right angle at B, $s - a = 3$ and $s - c = 2$, then
 (1) $a = 2, c = 3$ (2) $a = 3, c = 4$ (3) $a = 4, c = 3$ (4) $a = 6, c = 8$
- If in a triangle ABC, the line joining the circumcentre and incentre is parallel to BC, then $\cos B + \cos C$ is equal to :
 (1) 0 (2) 1 (3) 2 (4) $\frac{1}{2}$
- If the incircle of the $\triangle ABC$ touches its sides at L, M and N as shown in the figure and if x, y, z be the circumradii of the triangles MIN, NIL and LIM respectively, where I is the incentre, then the product xyz is equal to :



- (1) $R r^2$ (2) $r R^2$ (3) $\frac{1}{2} R r^2$ (4) $\frac{1}{2} r R^2$
- A triangle is inscribed in a circle. The vertices of the triangle divide the circle into three arcs of length 3, 4 and 5 units. Then area of the triangle is equal to:
 (1) $\frac{9\sqrt{3}(1+\sqrt{3})}{\pi^2}$ (2) $\frac{9\sqrt{3}(\sqrt{3}-1)}{\pi^2}$ (3) $\frac{9\sqrt{3}(1+\sqrt{3})}{2\pi^2}$ (4) $\frac{9\sqrt{3}(\sqrt{3}-1)}{2\pi^2}$
- In a triangle ABC, $B = 60^\circ$ and $C = 45^\circ$. Let D divides BC internally in the ratio 1 : 3, then value of $\frac{\sin \angle BAD}{\sin \angle CAD}$ is
 (1) $\sqrt{\frac{2}{3}}$ (2) $\frac{1}{\sqrt{3}}$ (3) $\frac{1}{\sqrt{6}}$ (4) $\frac{1}{3}$

6. With usual notations, if in a ΔABC , $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13}$, then which of the following is false
 (1) $\cos A = \frac{1}{5}$ (2) $\cos B = \frac{19}{35}$ (3) $\cos C = \frac{5}{7}$ (4) $\sin C = \frac{19}{35}$
7. Let a , b and c be the sides of a ΔABC . If a^2 , b^2 and c^2 are the roots of the equation $x^3 - Px^2 + Qx - R = 0$, where P , Q & R are constants, then find the value of $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c}$ in terms of P , Q and R .
 (1) $\frac{P}{2\sqrt{R}}$ (2) $\frac{P}{2\sqrt{Q}}$ (3) $\frac{P}{\sqrt{R}}$ (4) $\frac{R}{2\sqrt{P}}$
8. In a ΔABC , $(b-c) \cot \frac{A}{2} + (c-a) \cot \frac{B}{2} + (a-b) \cot \frac{C}{2}$ is equal to
 (1) 0 (2) abc (3) $a+b+c$ (4) $(a+b+c)^2$
- 9.. A regular hexagon & a regular dodecagon are inscribed in the same circle. If the side of the dodecagon is $(\sqrt{3}-1)$, if the side of the hexagon is $\sqrt[4]{k}$, then find value of k .
 (1) $\sqrt{2}$ (2) $2\sqrt{2}$ (3) $\sqrt{2}+1$ (4) $\sqrt{2}-1$
10. If H is the orthocentre of a triangle ABC , then the radii of the circle circumscribing the triangles BHC , CHA and AHB are respectively equal to :
 (1) R, R, R (2) $\sqrt{2} R, \sqrt{2} R, \sqrt{2} R$
 (3) $2R, 2R, 2R$ (4) $\frac{R}{2}, \frac{R}{2}, \frac{R}{2}$
11. Let f , g , h be the lengths of the perpendiculars from the circumcentre of the ΔABC on the sides BC , CA and AB respectively. If $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = \lambda \frac{abc}{fgh}$, then the value of ' λ ' is:
 (1) $1/4$ (2) $1/2$ (3) 1 (4) 2
12. AA_1 , BB_1 and CC_1 are the medians of triangle ABC whose centroid is G . If points A , C_1 , G and B_1 are concyclic, then
 (1) $2b^2 = a^2 + c^2$ (2) $2c^2 = a^2 + b^2$ (3) $2a^2 = b^2 + c^2$ (4) $3a^2 = b^2 + c^2$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

- (1) Both the statements are true.
 (2) Statement-I is true, but Statement-II is false.
 (3) Statement-I is false, but Statement-II is true.
 (4) Both the statements are false.

- A-1. **Statement-1** : In a triangle ABC , the harmonic mean of the three exradii is three times the inradius.
Statement-2 : In any triangle ABC , $r_1 + r_2 + r_3 = 4R$.

A-2. Statement-1 : If R be the circumradius of a ΔABC , then circumradius of its excentral $\Delta I_1 I_2 I_3$ is $2R$.

Statement-2 : If circumradius of a triangle be R, then circumradius of its pedal triangle is $\frac{R}{2}$.

Section (B) : MATCH THE COLUMN

B-1. Match the column

Column-I		Column-II
(A) In a ΔABC , $2B = A + C$ and $b^2 = ac$.	(p)	8
Then the value of $\frac{a^2(a+b+c)}{3abc}$ is equal to		
(B) In any right angled triangle ABC, the value of $\frac{a^2 + b^2 + c^2}{R^2}$ is always equal to (where R is the circumradius of ΔABC)	(q)	1
(C) In a ΔABC if $a = 2$, $bc = 9$, then the value of $2R\Delta$ is equal to	(r)	5
(D) In a ΔABC , $a = 5$, $b = 3$ and $c = 7$, then the value of $3 \cos C + 7 \cos B$ is equal to	(s)	9

B-2 Match the column

Column - I		Column - II
(A) In a ΔABC , $a = 4$, $b = 3$ and the medians AA_1 and BB_1 are mutually perpendicular, then square of area of the ΔABC is equal to	(p)	27
(B) In any ΔABC , minimum value of $\frac{r_1 r_2 r_3}{r^3}$ is equal to	(q)	7
(C) In a ΔABC , $a = 5$, $b = 4$ and $\tan \frac{C}{2} = \sqrt{\frac{7}{9}}$, then side 'c' is equal to	(r)	6
(D) In a ΔABC , $2a^2 + 4b^2 + c^2 = 4ab + 2ac$, then value of $(8 \cos B)$ is equal to	(s)	11

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If in a ΔABC , $a = 5$, $b = 4$ and $\cos(A - B) = \frac{31}{32}$, then
 (1) $c = 6$ (2) $\sin A =$ (3) area of $\Delta ABC =$ (4) $c = 8$

C-2. Which of the following holds good for any triangle ABC?

- | | |
|---|---|
| (1) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$ | (2) $\frac{\sin A}{a} + \frac{\sin B}{b} + \frac{\sin C}{c} = \frac{3}{2R}$ |
| (3) $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ | (4) $\frac{\sin 2A}{a^2} = \frac{\sin 2B}{b^2} = \frac{\sin 2C}{c^2}$ |

C-3. In a triangle ABC, right angled at B, then

$$(1) r = \frac{AB + BC - AC}{2} \quad (2) r = \frac{AB + AC - BC}{2} \quad (3) r = \frac{AB + BC + AC}{2} \quad (4) R = \frac{s - r}{2}$$

C-4. If in a triangle ABC, $\cos A \cos B + \sin A \sin B \sin C = 1$, then the triangle is

- (1) isosceles (2) right angled (3) equilateral (4) None of these

C-5. The product of the distances of the incentre from the angular points of a ΔABC is:

$$(1) 4 R^2 r \quad (2) 4 R r^2 \quad (3) \frac{(abc)R}{s} \quad (4) \frac{(abc)r}{s}$$

C-6. Three equal circles of radius unity touches one another. Radius of the circle touching all the three circles is :

$$(1) \frac{2 - \sqrt{3}}{\sqrt{3}} \quad (2) \frac{\sqrt{3} - \sqrt{2}}{\sqrt{2}} \quad (3) \frac{2 + \sqrt{3}}{\sqrt{3}} \quad (4) \frac{\sqrt{3} + \sqrt{2}}{\sqrt{2}}$$

C-7 In a triangle ABC, with usual notations the length of the bisector of internal angle A is :

$$(1) \frac{2bc \cos \frac{A}{2}}{b + c} \quad (2) \frac{2bc \sin \frac{A}{2}}{b + c} \quad (3) \frac{abc \operatorname{cosec} \frac{A}{2}}{2R(b + c)} \quad (4) \frac{2\Delta}{b + c} \cdot \operatorname{cosec} \frac{A}{2}$$

Exercise-3

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PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side 'a', is :

[AIEEE - 2002 (3, -1), 225]

$$(1) a \cot \left(\frac{\pi}{n} \right) \quad (2) \frac{a}{2} \cot \left(\frac{\pi}{2n} \right) \quad (3) a \cot \left(\frac{\pi}{2n} \right) \quad (4) \frac{a}{4} \cot \left(\frac{\pi}{2n} \right)$$

2. If in a triangle ABC, $a \cos^2 \left(\frac{C}{2} \right) + c \cos^2 \left(\frac{A}{2} \right) = \frac{3b}{2}$, then the sides a, b and c :

[AIEEE - 2003 (3, -1), 225]

- (1) are in A.P. (2) are in G.P. (3) are in H.P. (4) satisfy $a + b = c$.

3. In a triangle ABC, medians AD and BE are drawn. If $AD = 4$, $\angle DAB = \frac{\pi}{6}$ and $\angle ABE = \frac{\pi}{3}$, then the area of the ΔABC is :

[AIEEE - 2003 (3, -1), 225]

$$(1) \frac{8}{3} \quad (2) \frac{16}{3} \quad (3) \frac{32}{3\sqrt{3}} \quad (4) \frac{64}{3}$$

4. The sides of a triangle are $\sin \alpha$, $\cos \alpha$ and $\sqrt{1 + \sin \alpha \cos \alpha}$ for some $0 < \alpha < \frac{\pi}{2}$. Then the greatest angle of the triangle is :

[AIEEE - 2004 (3, -1), 225]

- (1) 60° (2) 90° (3) 120° (4) 150°

5. In a triangle ABC, let $\angle C = \pi/2$, if r is the inradius and R is the circumradius of the triangle ABC, then $2(r+R)$ equals : **[AIEEE - 2005 (3, 0), 225]**

- (1) $c + a$ (2) $a + b + c$ (3) $a + b$ (4) $b + c$

6. If in a ΔABC , the altitudes from the vertices A,B,C on opposite sides are in H.P., then $\sin A, \sin B, \sin C$ are in: **[AIEEE - 2005 (3, 0), 225]**

- (1) HP (2) Arithmetico-Geometric Progression
(3) AP (4) GP

7. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A **false** statement among the following is **[AIEEE - 2010 (4, -1), 144]**

- (1) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$. (2) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$.

- (3) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$. (4) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$.

8. ABCD is a trapezium such that AB and CD are parallel and $BC \perp CD$. If $\angle ADB = \theta$, $BC = p$ and $CD = q$, then AB is equal to : **[AIEEE - 2013, (4, -1), 360]**

- (1) $\frac{(p^2 + q^2) \sin \theta}{p \cos \theta + q \sin \theta}$ (2) $\frac{p^2 + q^2 \cos \theta}{p \cos \theta + q \sin \theta}$ (3) $\frac{p^2 + q^2}{p^2 \cos \theta + q^2 \sin \theta}$ (4) $\frac{(p^2 + q^2) \sin \theta}{(p \cos \theta + q \sin \theta)^2}$

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. If I_n is the area of n sided regular polygon inscribed in a circle of unit radius and O_n be the area of the polygon circumscribing the given circle, prove that **[IIT-JEE-2003, Main., (4)/60]**

$$I_n = \frac{O_n}{2} \left(1 + \sqrt{1 - \left(\frac{2I_n}{n} \right)^2} \right)$$

2. If the angles of a triangle are in the ratio $4 : 1 : 1$, then the ratio of the longest side to the perimeter is— **[IIT-JEE-2003, Scr., (3, -1)/84]**

- (A) $\sqrt{3} : (2 + \sqrt{3})$ (B) $1 : \sqrt{3}$ (C) $1 : 2 + \sqrt{3}$ (D) $2 : 3$

3. If a, b, c are the sides of a triangle such that $a : b : c = 1 : \sqrt{3} : 2$, then ratio $A : B : C$ is equal to — **[IIT-JEE-2004, Scr., (3, -1)/84]**

- (A) $3 : 2 : 1$ (B) $3 : 1 : 2$ (C) $1 : 2 : 3$ (D) $1 : 3 : 2$

4. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is **[IIT-JEE - 2005, Scr- (3, - 1), 84]**

- (A) $4 + 2\sqrt{3}$ (B) $6 + 4\sqrt{3}$ (C) $12 + \frac{7\sqrt{3}}{4}$ (D) $3 + \frac{7\sqrt{3}}{4}$

5. If a, b, c denote the lengths of the sides of a triangle opposite to angles A, B, C respectively of a $\triangle ABC$, then the correct relation among a, b, c, A, B and C is given by – **[IIT-JEE-2005, Scr., (3, -1)/ 84]**
- (A) $(b + c) \sin \left(\frac{B + C}{2} \right) = a \cos \frac{A}{2}$ (B) $(b - c) \cos \frac{A}{2} = a \sin \left(\frac{B - C}{2} \right)$
 (C) $(b - c) \cos \frac{A}{2} = 2a \sin \left(\frac{B - C}{2} \right)$ (D) $(b - c) \sin \left(\frac{B - C}{2} \right) = a \cos \frac{A}{2}$
6. Circles with radii 3, 4 and 5 touch each other externally. If P is the point of intersection of tangents to these circles at their points of contact, find the distance of P from the points of contact. **[IIT-JEE-2005, Main., (2)/60]**
7. Given an isosceles triangle, whose one angle is 120° and radius of its incircle is $\sqrt{3}$ unit. Then the area of the triangle in sq. units is **[IIT-JEE-2006, Main., (3, -1)/184]**
- (A) $7 + 12\sqrt{3}$ (B) $12 - 7\sqrt{3}$ (C) $12 + 7\sqrt{3}$ (D) 4π
- 8.* Internal bisector of $\angle A$ of triangle ABC meets side BC at D . A line drawn through D perpendicular to AD intersects the side AC at E and the side AB at F . If a, b, c represent sides of $\triangle ABC$, then **[IIT-JEE-2006, Main., (5, -1)/184]**
- (A) AE is HM of b and c (B) $AD = \frac{2bc}{b+c} \cos \frac{A}{2}$
 (C) $EF = \frac{4bc}{b+c} \sin \frac{A}{2}$ (D) the triangle AEF is isosceles
9. Let ABC and ABC' be two non-congruent triangles with sides $AB = 4$, $AC = AC' = 2\sqrt{2}$ and angle $B = 30^\circ$. Find the absolute value of the difference between the areas of these triangles. **[IIT-JEE 2009, Paper-2, (4, -1), 80]**
- 10*. In a triangle ABC with fixed base BC , the vertex A moves such that $\cos B + \cos C = 4 \sin^2 \frac{A}{2}$. If a, b and c denote the lengths of the sides of the triangle opposite to the angles A, B and C respectively, then **[IIT-JEE 2009, Paper-2, (4, -1), 80]**
- (A) $b + c = 4a$ (B) $b + c = 2a$
 (C) locus of points A is an ellipse (D) locus of point A is a pair of straight lines
11. If the angle A, B and C of a triangle are in arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of the expression $\frac{a}{c} \sin 2C + \frac{c}{a} \sin 2A$ is **[IIT-JEE 2010, Paper-1, (3, -1), 84]**
- (A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$
12. Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a, b and c denote the lengths of the sides opposite to A, B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is (are) **[IIT-JEE 2010, Paper-1, (3, 0), 84]**
- (A) $-(2 + \sqrt{3})$ (B) $1 + \sqrt{3}$ (C) $2 + \sqrt{3}$ (D) $4\sqrt{3}$

13. Consider a triangle ABC and let a , b and c denote the lengths of the sides opposite to vertices A, B and C respectively. Suppose $a = 6$, $b = 10$ and the area of the triangle is $15\sqrt{3}$. If $\angle ACB$ is obtuse and if r denotes the radius of the incircle of the triangle, then r^2 is equal to **[IIT-JEE 2010, Paper-2, (3, 0), 79]**
14. Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a , b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals **[IIT-JEE 2012, Paper-2, (3, -1), 66]**
- (A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$
- 15.* In a triangle PQR, P is the largest angle and $\cos P = \frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are) **[JEE (Advanced) 2013, Paper-2, (3, -1)/60]**
- (A) 16 (B) 18 (C) 24 (D) 22
16. In a triangle the sum of two sides is x and the product of the same two sides is y . If $x^2 - c^2 = y$, where c is the third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is **[JEE (Advanced) 2014, Paper-2, (3, -1)/60]**
- (A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$ (C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$
17. In a triangle XYZ, let x , y , z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s = x + y + z$. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then **[JEE (Advanced) 2016, Paper-1, (4, -2)/62]**
- (A) area of the triangle XYZ is $6\sqrt{6}$
- (B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$
- (C) $\sin \frac{X}{2} \sin \frac{Y}{2} \sin \frac{Z}{2} = \frac{4}{35}$
- (D) $\sin^2 \left(\frac{X+Y}{2} \right) = \frac{3}{5}$

Answers

EXERCISE # 1

Section (A)

A-1. (1) A-2. (4) A-3. (2) A-4. (2) A-5. (3) A-6. (3) A-7. (3)
A-8. (3)

Section (B)

B-1. (1) B-2. (3) B-3. (4) B-4. (3) B-5. (4) B-6. (2) B-7. (3)
B-8. (3) B-9. (4) B-10. (2)

Section (C)

C-1. (2) C-2. (3) C-3. (1) C-4. (2)

Section (D)

D-1. (4) D-2. (3) D-3. (2) D-4. (3) D-5. (1) D-6. (3) D-7. (2)
D-8. (4) D-9. (4)

Section (E)

E-1. (1) E-2. (4) E-3. (2) E-4. (1) E-5. (4) E-6. (2) E-7. (1)
E-8. (2) E-9. (3)

Section (F)

F-1. (2) F-2. (1) F-3. (2) F-4. (3) F-5. (4) F-6. (2)

EXERCISE # 2

PART-I

1. (2) 2. (2) 3. (3) 4. (1) 5. (3) 6. (4) 7. (1)
8. (1) 9. (1) 10. (1) 11. (1) 12. (3)

PART-II

Section (A) :

A-1. (3) A-2. (1)

Section (B) :

B-1. (A) → (q), (B) → (p), (C) → (s), (D) → (r)

B-2. (A) → (s), (B) → (p), (C) → (r), (D) → (q)

Section (C) :

C-1. (1,2) C-2. (1,2) C-3. (1,4) C-4. (1,2) C-5. (2,4)
C-6. (1,3) C-7. (1,3,4)

EXERCISE # 3

PART-I

1. (2) 2. (1) 3. (3) 4. (3) 5. (3) 6. (3) 7. (2)
8. (1)

PART-II

2. (A) 3. (C) 4. (B) 5. (B) 6. ($\sqrt{5}$) 7. (C)
8.* (ABCD) 9. (BC) 10.* (B,C) 11. (D) 12. (B) 13. 3 14. (C)
15.* (B, D) 16. (B) 17. (A,C,D)