Circle

Exercise-1

Marked questions may have for revision questions.

*** Marked Questions may have more than one correct option.**

OBJECTIVE QUESTIONS

Section (A) : Equation of circle, Intercepts on axes

A-1. The length of the diameter of the circle $x^2 + y^2 - 4x - 6y + 4 = 0$ is -
(1) 9 (2) 3 (3) 4 (4) 6

A-2. Which of the following is the equation of a circle ?
(1) $x^2 + 2y^2 - x + 6 = 0$ (2) $x^2 - y^2 + x + y + 1 = 0$
(3) $x^2 + y^2 + xy + 1 = 0$ (4) $3(x^2 + y^2) + 5x + 1 = 0$

A-3. The radius of the circle passing through the points (0, 0), (1, 0) and (0, 1) is-
(1) 2 (2) $1/\sqrt{2}$ (3) $\sqrt{2}$ (4) 1/2

A-4. If (x, 3) and (3, 5) are the extremities of a diameter of a circle with centre at (2, y). Then the value of x and y are-
(1) x = 1, y = 4 (2) x = 4, y = 1 (3) x = 8, y = 2 (4) None of these

A-5. If the equation $px^2 + (2 - q)xy + 3y^2 - 6qx + 30y + 6q = 0$ represents a circle, then the values of p and q are-
(1) 2, 2 (2) 3, 1 (3) 3, 2 (4) 3, 4

A-6. The centres of the circles $x^2 + y^2 - 6x - 8y - 7 = 0$ and $x^2 + y^2 - 4x - 10y - 3 = 0$ are the ends of the diameter of the circle
(1) $x^2 + y^2 - 5x - 9y + 26 = 0$ (2) $x^2 + y^2 + 5x - 9y + 14 = 0$
(3) $x^2 + y^2 + 5x - y - 14 = 0$ (4) $x^2 + y^2 + 5x + y + 14 = 0$

A-7. The circle $x^2 + y^2 - 4x - 4y + 4 = 0$
(1) touches x-axis only (2) touches both axes
(3) passes through the origin (4) touches y-axis only

A-8. The equation of the circle passing through the point (2,1) and touching y-axis at the origin is
(1) $x^2 + y^2 - 5x = 0$ (2) $2x^2 + 2y^2 - 5x = 0$
(3) $x^2 + y^2 + 5x = 0$ (4) $x^2 - y^2 - 5x = 0$

A-9. A circle touches both the axes and its centre lies in the fourth quadrant. If its radius is 1 then its equation will be -
(1) $x^2 + y^2 - 2x + 2y + 1 = 0$ (2) $x^2 + y^2 + 2x - 2y - 1 = 0$
(3) $x^2 + y^2 - 2x - 2y + 1 = 0$ (4) $x^2 + y^2 + 2x - 2y + 1 = 0$

A-10. The equation of a circle passing through (3, –6) and touching both the axes is
(1) $x^2 + y^2 - 6x + 6y + 9 = 0$ (2) $x^2 + y^2 + 6x - 6y + 9 = 0$
(3) $x^2 + y^2 + 30x - 30y + 225 = 0$ (4) $x^2 + y^2 + 30x + 30y + 225 = 0$

A-11. The equation of the circle with centre on x-axis , radius 5 and passing through the point (2,3) is
(1) $x^2 + y^2 + 4x - 21 = 0$, $x^2 + y^2 - 12x + 11 = 0$ (2) $x^2 + y^2 + 4x + 21 = 0$, $x^2 + y^2 - 12x + 11 = 0$
(3) $x^2 + y^2 - 4x - 21 = 0$, $x^2 + y^2 + 12x + 11 = 0$ (4) $x^2 + y^2 + 5x - 21 = 0$, $x^2 + y^2 - 12x - 11 = 0$

A-12. Equation of line passing through mid point of intercepts made by circle $x^2 + y^2 - 4x - 6y = 0$ on co-ordinate axes is
(1) $3x + 2y - 12 = 0$ (2) $3x + y - 6 = 0$ (3) $3x + 4y - 12 = 0$ (4) $3x + 2y - 6 = 0$

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A-13. The intercepts made by the circle $x^2 + y^2 – 5x – 13y – 14 = 0$ on the x-axis and y-axis are respectively
(1) 9, 13 (2) 5, 13 (3) 9, 15 (4) none

A-14. The circle described on the line joining the points (0, 1), (a, b) as diameter cuts the x−axis in points whose abscissa are roots of the equation:
(1) $x^2 + ax + b = 0$ (2) $x^2 − ax + b = 0$ (3) $x^2 + ax − b = 0$ (4) $x^2 − ax − b = 0$

A-15. The parametric equations of the circle $4x^2 + 4y^2 = 25$ is
(1) $x = \frac{5}{2}\cos\theta$, $y = \frac{3}{2}\sin\theta$ (2) $x = \frac{5}{2}\cos\theta$, $y = \frac{5}{2}\sin\theta$
(2) $x = \frac{7}{2}\cos\theta$, $y = \frac{7}{2}\sin\theta$ (4) $x = \frac{1}{2}\cos\theta$, $y = \frac{1}{2}\sin\theta$

Section (B) : Power and position of a point and line, Tangents and normal

B-1. The point $(\lambda, 1 + \lambda)$ lies inside the circle $x^2 + y^2 = 1$, then $\lambda \in$
(1) (–1, 0) (2) (–2, 0) (3) (–3, 2) (4) (0, 2)

B-2. The line $3x + 5y + 9 = 0$ w.r.t. the circle $x^2 + y^2 – 4x + 6y + 5 = 0$ is
(1) chord (2) diameter (3) tangent (4) None

B-3. Circle $x^2 + y^2 – 4x – 8y – 5 = 0$ will intersect the line $3x – 4y = m$ in two distinct points, if-
(1) $– 10 < m < 5$ (2) $9 < m < 20$ (3) $– 35 < m < 15$ (4) None of these

B-4. Find the co-ordinates of point on line $x + y = – 13$, nearest to the circle $x^2 + y^2 + 4x + 6y – 5 = 0$
(1) (– 6, – 7) (2) (– 15, 2) (3) (– 5, – 6) (4) (– 7, – 6)

B-5. The co−ordinate of the point on the circle $x^2 + y^2 − 12x − 4y + 30 = 0$, which is farthest from the origin are:
(1) (9, 3) (2) (8, 5) (3) (12, 4) (4) None

B-6. Radius of the circle with centre (3, –1) and cutting a chord of length 6 on the line $2x – 5y + 18 = 0$ is
(1) $\sqrt{29}$ (2) $\sqrt{38}$ (3) $\sqrt{37}$ (4) $\sqrt{41}$

B-7. Line $3x + 4y = 25$ touches the circle $x^2 + y^2 = 25$ at the point -
(1) (4, 3) (2) (3, 4) (3) (– 3, – 4) (4) None of these

B-8. The length of chord $x + y – 1 = 0$ w.r.t. circle $x^2 + y^2 – 6x – 8y = 0$ is
(1) $\sqrt{7}$ (2) $2\sqrt{7}$ (3) 49 (4)

B-9. Find equation of tangent to the circle $x^2 + y^2 – 30x + 6y + 109 = 0$ at (4,–1)
(1) $11x + 2y – 46 = 0$ (2) $11x – 2y – 46 = 0$ (3) $11x + 2y + 46 = 0$ (4) $11x – 3y – 46 = 0$

B-10. The equation of a circle which touches both axes and the line $3x – 4y + 8 = 0$ and whose centre lies in the third quadrant is
(1) $x^2 + y^2 – 4x + 4y – 4 = 0$ (2) $x^2 + y^2 – 4x + 4y + 4 = 0$
(3) $x^2 + y^2 + 4x + 4y + 4 = 0$ (4) $x^2 + y^2 – 4x – 4y – 4 = 0$

B-11. The condition so that the line $(x + g) \cos\theta + (y + f) \sin \theta = k$ is a tangent to $x^2 + y^2 + 2gx + 2fy + c = 0$ is
(1) $g^2 + f^2 = c + k^2$ (2) $g^2 + f^2 = c^2 + k$ (3) $g^2 + f^2 = c^2 + k^2$ (4) $g^2 + f^2 = c + k$

B-12. The tangent lines to the circle $x^2 + y^2 − 6x + 4y = 12$ which are parallel to the line $4x + 3y + 5 = 0$ are given by:
(1) $4x + 3y − 7 = 0$, $4x + 3y + 15 = 0$ (2) $4x + 3y − 31 = 0$, $4x + 3y + 19 = 0$
(3) $4x + 3y − 17 = 0$, $4x + 3y + 13 = 0$ (4) none of these

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B-13. The tangent to the circle $x^2 + y^2 = 5$ at the point (1, –2) also touches the circle $x^2 + y^2 - 8x + 6y + 20 = 0$ at

(1) (–2, 1) (2) (–3, 0) (3) (–1, –1) (4) (3, –1)

B-14. The equation of the normal to the circle $x^2 + y^2 = 2x$, which is parallel to the line $x + 2y = 3$ is

(1) $x + 3y = 7$ (2) $x + 2y = 1$ (3) $x + 2y = 2$ (4) $x + 2y = 5$

B- 15. The equation of normal to the circle $x^2 + y^2 - 4x + 4y - 17 = 0$ which passes through (1, 1) is

(1) $3x + y - 4 = 0$ (2) $x - y = 0$ (3) $x + y = 0$ (4) None

B-16. The normal at the point (3, 4) on a circle cuts the circle at the point (–1, –2). Then the equation of the circle is

(1) $x^2 + y^2 + 2x - 2y - 13 = 0$ (2) $x^2 + y^2 - 2x - 2y - 11 = 0$
(3) $x^2 + y^2 - 2x + 2y + 12 = 0$ (4) $x^2 + y^2 - 2x - 2y + 14 = 0$

Section (C) : Pair of tangents (Joint equation and length of tangent) Director circle, chord of contact, chord with given middle point, and chord joining two point

C-1. The number of tangents that can be drawn from the point (8, 6) to the circle $x^2 + y^2 - 100 = 0$ is

(1) 0 (2) 1 (3) 2 (4) None

C-2. A line segment through a point P cuts a given circle in 2 points A & B, such that PA = 16 & PB = 9, then the length of tangent from point P to the circle is

(1) 7 (2) 25 (3) 12 (4) None of these

C-3. The equation of the tangents drawn from the origin to the circle $x^2 + y^2 - 2rx - 2hy + h^2 = 0$ are

(1) $x = 0, y = 0$ (2) $(h^2 - r^2)x - 2rhy = 0, x = 0$
(3) $y = 0, x = 4$ (4) $(h^2 - r^2)x + 2rhy = 0, x = 0$

C-4. The length of the tangent drawn from the point (4, –1) to the circle $2x^2 + 2y^2 = 1$ is

(1) $\sqrt{\frac{17}{2}}$ (2) $\sqrt{33}$ (3) $\sqrt{\frac{33}{2}}$ (4) $\sqrt{2}$

C-5. If the length of tangent drawn from the point (5, 3) to the circle $x^2 + y^2 + 2x + ky + 17 = 0$ is 7, then k =

(1) – 6 (2) – 4 (3) 4 (4) 13/ 2

C-6. The length of the tangent drawn from any point on the circle $x^2 + y^2 + 2gx + 2fy + p = 0$ to the circle $x^2 + y^2 + 2gx + 2fy + q = 0$ is :

(1) $\sqrt{q - p}$ (2) $\sqrt{p - q}$ (3) $\sqrt{q + p}$ (4) none

C-7. Two perpendicular tangents to the circle $x^2 + y^2 = a^2$ meet at P. Then the locus of P has the equation-

(1) $x^2 + y^2 = 2a^2$ (2) $x^2 + y^2 = 3a^2$ (3) $x^2 + y^2 = 4a^2$ (4) None of these

C-8. The angle between the two tangents from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ equals

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{2}$ (4) None

C-9. The equation of the diameter of the circle $(x - 2)^2 + (y + 1)^2 = 16$ which bisects the chord cut off by the circle on the line $x - 2y - 3 = 0$ is

(1) $x + 2y = 0$ (2) $2x + y - 3 = 0$ (3) $3x + 2y - 4 = 0$ (4) none

C-10. The co-ordinates of the middle point of the chord cut off on $2x - 5y + 18 = 0$ by the circle $x^2 + y^2 - 6x + 2y - 54 = 0$ are

(1) (1, 4) (2) (2, 4) (3) (4, 1) (4) (1, 1)

C-11. The distance between the chords of contact of tangents to the circle; $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin & the point (g, f) is :

(1) $\sqrt{g^2 + f^2}$ (2) $\dfrac{\sqrt{g^2 + f^2 - c}}{2}$ (3) $\dfrac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (4) $\dfrac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

C-12. The locus of the mid point of a chord of the circle $x^2 + y^2 = 4$ which subtends a right angle at the origin is:
(1) $x + y = 2$ (2) $x^2 + y^2 = 1$ (3) $x^2 + y^2 = 2$ (4) $x + y = 1$

C-13. The locus of the centers of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:
(1) $2x - 5y + 11 = 0$ (2) $2x + 5y - 11 = 0$ (3) $2x + 5y + 11 = 0$ (4) none

Section (D) : Common tangents, common chord and orthogonality

D-1. Consider the circles $x^2 + (y - 1)^2 = 9$, $(x - 1)^2 + y^2 = 25$. They are such that-
(1) each of these circles lies outside the other (2) one of these circles lies entirely inside the other
(3) these circles touch each other (4) they intersect in two points

D-2. Number of common tangents of the circles $(x + 2)^2+(y-2)^2 = 49$ and $(x - 2)^2 + (y + 1)^2 = 4$ is:
(1) 0 (2) 1 (3) 2 (4) 3

D-3. The equation of the common tangent to the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$ at their point of contact is
(1) $12x + 5y + 19 = 0$ (2) $5x + 12y + 19 = 0$
(3) $5x - 12y + 19 = 0$ (4) $12x - 5y + 19 = 0$

D-4. Find the equations to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$
(1) $x = 0$, $3x + 4y = 10$, $y = 4$, $3y = 4x$. (2) $x = 0$, $3x - 4y = 10$, $y = 4$, $3y = 4x$.
(3) $x = 0$, $3x + 4y = 10$, $y = 4$, $3y + 4x = 0$ (4) $x = 0$, $3x - 4y = 10$, $y = 4$, $3y - 4x = 0$

D-5. The angle of intersection of two circles is 0^o if -
(1) they are separate (2) they intersect at two points
(3) they intersect only at a single point (4) it is not possible

D-6. The area of the triangle formed by the tangents from the point (4, 3) to the circle $x^2 + y^2 = 9$ and the line joining their point of contact is:
(1) $\dfrac{192}{25}$ (2) 192 (3) 25 (4) 250

D-7 Find the length of direct common tangent of circle $(x - 1)^2 + (y - 2)^2 = 4$ and $(x - 5)^2 + (y -2)^2 = 1$
(1) $\sqrt{14}$ (2) $\sqrt{15}$ (3) $\sqrt{5}$ (4) $\sqrt{7}$

D-8. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:
(1) 18 (2) 20 (3) 16 (4) 12

D-9. If the two circles, $x^2 + y^2 + 2g_1x + 2f_1y = 0$ & $x^2 + y^2 + 2g_2x + 2f_2y = 0$ touch each other then:
(1) $f_1 g_1 = f_2 g_2$ (2) $\dfrac{f_1}{g_1} = \dfrac{f_2}{g_2}$ (3) $f_1 f_2 = g_1 g_2$ (4) none

D-10. If the circle C_1: $x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the co-ordinates of the centre of C_2 are:
(1) $\left(\pm\dfrac{9}{5}, \pm\dfrac{12}{5}\right)$ (2) $\left(\pm\dfrac{9}{5}, \mp\dfrac{12}{5}\right)$ (3) $\left(\pm\dfrac{12}{5}, \pm\dfrac{9}{5}\right)$ (4) $\left(\pm\dfrac{12}{5}, \mp\dfrac{9}{5}\right)$

D-11. The locus of the centre of the circle which bisects the circumferences of the circles

$x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:

(1) a straight line (2) a circle (3) a parabola (4) none of these

D-12. Two circles whose radii are equal to 4 and 8 intersect at right angles. The length of their common chord is:

(1) $\frac{16}{\sqrt{5}}$ (2) 8 (3) $4\sqrt{6}$ (4) $\frac{8\sqrt{5}}{5}$

D-13. The circumference of the circle $x^2 + y^2 - 2x + 8y - q = 0$ is bisected by the circle $x^2 + y^2 + 4x + 12y + p = 0$, then $p + q$ is equal to:

(1) 25 (2) 100 (3) 10 (4) 48

D-14 If the circles $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then k is

(1) 2 or $-\frac{3}{2}$ (2) -2 or $-\frac{3}{2}$ (3) 2 or $\frac{3}{2}$ (4) -2 or $\frac{3}{2}$

D-15. Equation of the circle cutting orthogonally the three circles $x^2 + y^2 - 2x + 3y - 7 = 0$, $x^2 + y^2 + 5x - 5y + 9 = 0$ and $x^2 + y^2 + 7x - 9y + 29 = 0$ is

(1) $x^2 + y^2 - 16x - 18y - 4 = 0$ (2) $x^2 + y^2 - 7x + 11y + 6 = 0$

(3) $x^2 + y^2 + 2x - 8y + 9 = 0$ (4) None of these

D-16. The equation of the circle which passes through the origin has its centre on the line $x + y = 4$ and cuts the circle $x^2 + y^2 - 4x + 2y + 4 = 0$ orthogonally, is-

(1) $x^2 + y^2 - 2x - 6y = 0$ (2) $x^2 + y^2 - 6x - 3y = 0$

(3) $x^2 + y^2 - 4x - 4y = 0$ (4) None of these

Section (E) : Radical axis and family of circle

E-1. The locus of the centre of the circle which bisects the circumferences of the circles $x^2 + y^2 = 4$ & $x^2 + y^2 - 2x + 6y + 1 = 0$ is:

(1) a straight line (2) a circle (3) a parabola (4) none of these

E-2. The circle $x^2 + y^2 = 4$ cuts the circle $x^2 + y^2 + 2x + 3y - 5 = 0$ in A & B. Then the equation of the circle on AB as a diameter is:

(1) $13(x^2 + y^2) - 4x - 6y - 50 = 0$ (2) $9(x^2 + y^2) + 8x - 4y + 25 = 0$

(3) $x^2 + y^2 - 5x + 2y + 72 = 0$ (4) None of these

E-3. The equation of a circle passing through points of intersection of the circles $x^2 + y^2 + 13x - 3y = 0$ and $2x^2 + 2y^2 + 4x - 7y - 25 = 0$ and point (1,1) is

(1) $4x^2 + 4y^2 - 30x - 10y - 25 = 0$ (2) $4x^2 + 4y^2 + 30x - 10y - 25 = 0$

(3) $4x^2 + 4y^2 - 17x - 10y + 25 = 0$ (4) $4x^2 + 4y^2 - 30x + 10y - 25 = 0$

E-4. Find equation of circle passing through the point (4,4) and touching the line $x + y - 2 = 0$ at (1,1)

(1) $x^2 + y^2 - 5x + 5y + 8 = 0$ (2) $x^2 + y^2 - 5x - 5y + 8 = 0$

(3) $x^2 + y^2 + 5x - 5y + 8 = 0$ (4) $x^2 + y^2 - 5x - 5y - 8 = 0$

E-5. Find equation of circle passing through the points (1,1) and (3,3) and whose centre lies an x-axis

(1) $x^2 + y^2 + 8x + 6 = 0$ (2) $x^2 + y^2 - 8x - 6 = 0$

(3) $x^2 + y^2 - 8x + 6 = 0$ (4) $x^2 + y^2 - 8x - 8 = 0$

Exercise-2

Marked questions may have for revision questions.

*** Marked Questions may have more than one correct option.**

PART - I : OBJECTIVE QUESTIONS

1. The equation of circle which touches x & y axis and whose perpendicular distance of centre of circle from $3x + 4y + 11 = 0$ is 5 is (Given that circle lies in I[st] quadrant)

(1) $x^2 + y^2 + 4x + 4y + 4 = 0$ (2) $x^2 + y^2 - 4x - 4y + 4 = 0$

(3) $x^2 + y^2 - 4x - 4y + 8 = 0$ (4) $x^2 + y^2 - 4x - 4y - 4 = 0$

2. The equation to the circle which touches the axis of x at a distance 3 from the origin and intercepts a distance 6 on the axis of y is

(1) $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 3 = 0$ (2) $x^2 + y^2 \pm 6y \pm 6x + 9 = 0$

(3) $x^2 + y^2 \pm 6\sqrt{2}y \pm 6x + 9 = 0$ (4) $x^2 + y^2 \pm 6\sqrt{3}y \pm 6x + 9 = 0$

3. If $y = 2x$ is a chord of the circle $x^2 + y^2 - 10x = 0$, then the equation of a circle with this chord as diameter is -

(1) $x^2 + y^2 - 2x - 4y = 0$. (2) $x^2 + y^2 - 2x + 4y = 0$.

(3) $x^2 + y^2 - 2x - 8y = 0$. (4) $x^2 + y^2 + 2x + 4y = 0$.

4. Two thin rods AB & CD of lengths 2a & 2b move along OX & OY respectively, when 'O' is the origin. The equation of the locus of the centre of the circle passing through the extremities of the two rods is:

(1) $x^2 + y^2 = a^2 + b^2$ (2) $x^2 - y^2 = a^2 - b^2$ (3) $x^2 + y^2 = a^2 - b^2$ (4) $x^2 - y^2 = a^2 + b^2$

5. The equation of the circle which touches both the axes and the line $\frac{x}{3} + \frac{y}{4} = 1$ and lies in the first quadrant is $(x - c)^2 + (y - c)^2 = c^2$ where c is

(1) 1,6 (2) 2, 4 (3) 4, 6 (4) 6, 8

6. Equations of circles which pass through the points (1, –2) and (3, – 4) and touch the x-axis is

(1) $x^2 + y^2 + 6x + 2y + 9 = 0$ (2) $x^2 + y^2 + 10x + 20y + 25 = 0$

(3) $x^2 + y^2 + 6x + 4y + 9 = 0$ (4) none

7. If $\left(a, \frac{1}{a}\right)$, $\left(b, \frac{1}{b}\right)$, $\left(c, \frac{1}{c}\right)$ & $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, abcd is equal to:

(1) 4 (2) 16 (3) 1 (4) None

8. The value of 'c' for which the set,

$\{(x, y)\square x^2 + y^2 + 2x \le 1\} \cap \{(x, y)\square x - y + c \ge 0\}$ contains only one point is :

(1) $(-\infty, -1] \cup [3, \infty)$ (2) $\{-1, 3\}$ (3) $\{-3\}$ (4) $\{-1\}$

9. If from any point P on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2\alpha + (g^2 + f^2)\cos^2\alpha = 0$ (where $\alpha \in \left(0, \frac{\pi}{2}\right)$, then the angle between the tangents is:

(1) α (2) 2α (3) $\frac{\alpha}{2}$ (4) none

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10. If α, β are the angles between the two tangents drawn from (0, 0) and (8, 6) respectively to the circle $x^2 + y^2 - 14x + 2y + 25 = 0$, then $\alpha - \beta =$

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) 0^o (4) $\frac{\pi}{4}$

11. The set of values of p for which the power of a point (2, 5) is negative with respect to a circle $x^2 + y^2 - 8x - 12y + p = 0$ which neither touches the axes nor cuts them are

(1) (36, 57) (2) (36, 47) (3) (37, 47) (4) (16, 47)

12. Two lines through (2, 3) from which the circle $x^2 + y^2 = 25$ intercepts chords of length 8 units have equations

(1) $2x + 3y = 13$, $x + 5y = 17$ (2) $y = 3$, $12x + 5y = 39$
(3) $x = 2$, $9x - 11y = 51$ (4) none of these

13. The chords of contact of the pair of tangents drawn from each point on the line $2x + y = 4$ to the circle $x^2 + y^2 = 1$ pass through the point

(1) (1, 2) (2) $\left(\frac{1}{2}, \frac{1}{4}\right)$ (3) (2, 4) (4) none

14. The locus of the mid points of the chords of the circle $x^2 + y^2 + 4x - 6y - 12 = 0$ which subtend an angle of $\frac{\pi}{3}$ radians at its circumference is:

(1) $(x - 2)^2 + (y + 3)^2 = 6.25$ (2) $(x + 2)^2 + (y - 3)^2 = 6.25$
(3) $(x + 2)^2 + (y - 3)^2 = 18.75$ (4) $(x + 2)^2 + (y + 3)^2 = 18.75$

15. If tangent at (1, 2) to the circle c_1: $x^2 + y^2 = 5$ intersects the circle c_2: $x^2 + y^2 = 9$ at A & B and tangents at A & B to the second circle meet at point C, then the co−ordinates of C are:

(1) (4, 5) (2) $\left(\frac{9}{15}, \frac{18}{5}\right)$ (3) (4, −5) (4) $\left(\frac{9}{5}, \frac{18}{5}\right)$

16. A point A(2, 1) is outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ & AP, AQ are tangents to the circle. The equation of the circle circumscribing the triangle APQ is:

(1) $(x + g)(x - 2) + (y + f)(y - 1) = 0$ (2) $(x + g)(x - 2) - (y + f)(y - 1) = 0$
(3) $(x - g)(x + 2) + (y - f)(y + 1) = 0$ (4) None

17. The length of the tangents from any point on the circle $15x^2 + 15y^2 - 48x + 64y = 0$ to the two circles $5x^2 + 5y^2 - 24x + 32y + 75 = 0$ and $5x^2 + 5y^2 - 48x + 64y + 300 = 0$ are in the ratio

(1) 1 : 2 (2) 2 : 3 (3) 3 : 4 (4) None of these

18. From the point A (0, 3) on the circle $x^2 + 4x + (y - 3)^2 = 0$ a chord AB is drawn & extended to a point M such that AM = 2 AB. The equation of the locus of M is :

(1) $x^2 + 8x + y^2 = 0$ (2) $x^2 + 8x + (y - 3)^2 = 0$
(3) $(x - 3)^2 + 8x + y^2 = 0$ (4) $x^2 + 8x + 8y^2 = 0$

19. Tangents are drawn to the circle $x^2 + y^2 = 12$ at the points where it is met by the circle $x^2 + y^2 - 5x + 3y - 2 = 0$. The point of intersection of these tangents is

(1) $\left(6, -\frac{18}{5}\right)$ (2) (1, 2) (3) (1, – 2) (4) (6, 3)

20. If the length of a common internal tangent to two circles is 7, and that of a common external tangent is 11, then the product of the radii of the two circles is:

(1) 36 (2) 9 (3) 18 (4) 4

Circle

21. The equation(s) to the common tangents of the circles $x^2 + y^2 - 2x - 6y + 9 = 0$ and $x^2 + y^2 + 6x - 2y + 1 = 0$ is

(1) $x = 0, y = 4$ (2) $3x + 4y = 10$ (3) $3y = 4x$ (4) all of these

22. The circle $x^2 + y^2 - 2x - 3ky - 2 = 0$ passes through two fixed points whose coordinates are

(1) $\left(1 \pm \sqrt{3}\ ,\ 0\right)$ (2) $\left(-1 \pm \sqrt{3}\ ,\ 0\right)$ (3) $\left(-\sqrt{3} \pm 2,\ 0\right)$ (4) none of these

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** Number of circles through the three points A(3, 5), B(4, 6), C(5, 7) is 1

Statement-2 : Through three non collinear points in a plane, one and only one circle can be drawn.

A-2. **Statement-1 :** If three circles which are such that their centres are non-collinear, then exactly one circle exists which cuts the three circles orthogonally.

Statement-2 : Radical axis for two intersecting circles is the common chord.

Section (B) : MATCH THE COLUMN

B-1.

	Column - I	**Column - II**
(A)	Number of common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 + 6x - 6y + 2 = 0$ is	(p) 0
(B)	Number of indirect common tangents of the circles $x^2 + y^2 - 4x - 10y + 4 = 0$ & $x^2 + y^2 - 6x - 12y - 55 = 0$ is	(q) 1
(C)	Number of common tangents of the circles $x^2 + y^2 - 2x - 4y = 0$ & $x^2 + y^2 - 8y - 4 = 0$ is	(r) 2
(D)	Number of direct common tangents of the circles $x^2 + y^2 + 2x - 8y + 13 = 0$ & $x^2 + y^2 - 6x - 2y + 6 = 0$ is	(s) 3

B-2.

	Column-I	**Column-II**
(A)	The length of the common chord of two circles of radii 3 and 4 units which intersect orthogonally is $\frac{k}{5}$, then k equals to	(p) 1
(B)	The circumference of the circle $x^2 + y^2 + 4x + 12y + p = 0$ is bisected by the circlc $x^2 + y^2 - 2x + 8y - q = 0$, then p + q is equal to	(q) 24
(C)	Number of distinct chords of the circles	(r) 32

$2x(x-\sqrt{2}) + y(2y-1) = 0$ is passing through the point $\left(\sqrt{2}, \frac{1}{2}\right)$ and are bisected by x-axis is

(D) One of the diameters of the circles circumscribing the rectangle ABCD is $4y = x + 7$. If A and B are the points $(-3,4)$ and $(5,4)$ respectively, then the area of the rectangle is equal to — (s) 36

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. Equations of circles which pass through the points $(1, -2)$ and $(3, -4)$ and touch the x-axis is :
(1) $x^2 + y^2 + 6x + 2y + 9 = 0$ (2) $x^2 + y^2 + 10x + 20y + 25 = 0$
(3) $x^2 + y^2 - 6x + 4y + 9 = 0$ (4) None

C-2. A rectangle ABCD is inscribed in the circle $x^2 + y^2 + 3x + 12y + 2 = 0$. If the co-ordinates of A and B are $(3, -2)$ and $(-2, 0)$, then the other two vertices of the rectangle are :
(1) $(-6, -10)$ (2) $(-1, -12)$ (3) $(1, 12)$ (4) $(6, 10)$

C-3. A chord AB of circle $x^2 + y^2 = a^2$ touches the circle $x^2 + y^2 - 2ax = 0$. Locus of the point of intersections of tangents at A and B is :
(1) $x^2 + y^2 = (x - a)^2$ (2) $x^2 + y^2 = (y - a)^2$ (3) $x^2 = a(a - 2y)$ (4) $y^2 = a(a - 2x)$

C-4. If a circle passes through the points of intersection of the co-ordinate axes with the lines $\lambda x - y + 1 = 0$ and $x - 2y + 3 = 0$, then the value of λ is :
(1) 2 (2) 1/3 (3) 6 (4) 3

Exercise-3

Marked questions may have for revision questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The greatest distance of the point P (10, 7) from the circle $x^2 + y^2 - 4x - 2y - 20 = 0$ is- **[AIEEE 2002 (3–1), 225]**
(1) 10 unit (2) 15 unit (3) 5 unit (4) None of these

2. The equation of the tangent to the circle $x^2 + y^2 + 4x - 4y + 4 = 0$ which make equal intercepts on the positive coordinate axes, is- **[AIEEE 2002 (3–1), 225]**
(1) $x + y = 2$ (2) $x + y = 2\sqrt{2}$ (3) $x + y = 4$ (4) $x + y = 8$

3. If the chord $y = mx + 1$ of the circle $x^2 + y^2 = 1$ subtends an angle of measure 45º at the major segment of the circle, then value of m is **[AIEEE 2002 (3–1), 225]**
(1) $2 \pm \sqrt{2}$ (2) $-2 \pm \sqrt{2}$ (3) $-1 \pm \sqrt{2}$ (4) none of these

4. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is **[AIEEE 2002 (3–1), 225]**
(1) $4 \le x^2 + y^2 \le 64$ (2) $x^2 + y^2 \le 25$ (3) $x^2 + y^2 \ge 25$ (4) $3 \le x^2 + y^2 \le 9$

5. The centre of the circle passing through (0, 0) and (1, 0) and touching the circle $x^2 + y^2 = 9$ is **[AIEEE 2002 (3–1), 225]**
(1) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (2) $\left(\frac{1}{2}, -\sqrt{2}\right)$ (3) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{3}{2}\right)$

Circle

6. The equation of circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is : **[AIEEE 2002 (3–1), 225]**
(1) $x^2 + y^2 = a^2$ (2) $x^2 + y^2 = 4a^2$ (3) $x^2 + y^2 = 16a^2$ (4) $x^2 + y^2 = 9a^2$

7. If the two circles $(x - 1)^2 + (y - 3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then- **[AIEEE 2003 (3–1), 225]**
(1) $2 < r < 8$ (2) $r < 2$ (3) $r = 2$ (4) $r > 2$

8. The lines $2x - 3y = 5$ and $3x - 4y = 7$ are diameters of a circle having area as 154 sq unit. Then, the equation of the circle is- **[AIEEE 2003 (3–1), 225]**
(1) $x^2 + y^2 + 2x - 2y = 62$ (2) $x^2 + y^2 + 2x - 2y = 47$
(3) $x^2 + y^2 - 2x + 2y = 47$ (4) $x^2 + y^2 - 2x + 2y = 62$

9. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = 4$ orthogonally, then the locus of its centre is - **[AIEEE 2004 (3–1), 225]**
(1) $2ax + 2by + (a^2 + b^2 + 4) = 0$ (2) $2ax + 2by - (a^2 + b^2 + 4) = 0$
(3) $2ax - 2by + (a^2 + b^2 + 4) = 0$ (4) $2ax - 2by - (a^2 + b^2 + 4) = 0$

10. A variable circle passes through the fixed point A(p, q) and touches x-axis. The locus of the other end of the diameter through A is- **[AIEEE 2004 (3–1), 225]**
(1) $(x - p)^2 = 4qy$ (2) $(x - q)^2 = 4py$ (3) $(y - p)^2 = 4qx$ (4) $(y - q)^2 = 4px$

11. If the lines $2x + 3y + 1 = 0$ and $3x - y - 4 = 0$ lie along diameters of a circle of circumference 10π, then the equation of the circle is- **[AIEEE 2004 (3–1), 225]**
(1) $x^2 + y^2 - 2x + 2y - 23 = 0$ (2) $x^2 + y^2 - 2x - 2y - 23 = 0$
(3) $x^2 + y^2 + 2x + 2y - 23 = 0$ (4) $x^2 + y^2 - 2x - 2y - 23 = 0$

12. The intercept on the line $y = x$ by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is- **[AIEEE 2004 (3–1), 225]**
(1) $x^2 + y^2 - x - y = 0$ (2) $x^2 + y^2 - x + y = 0$ (3) $x^2 + y^2 + x + y = 0$ (4) $x^2 + y^2 + x - y = 0$

13. If the circles $x^2 + y^2 + 2ax + cy + a = 0$ and $x^2 + y^2 - 3ax + dy - 1 = 0$ intersect in two distinct points P and Q, then the line $5x + by - a = 0$ passes through P and Q for - **[AIEEE 2005 (3, –1), 120]**
(1) exactly two values of a (2) infinitely many values of a
(3) no value of a (4) exactly one value of a

14. If a circle passes through the point (a, b) and cuts the circle $x^2 + y^2 = p^2$ orthogonally, then the equation of the locus of its centre is **[AIEEE 2005 (3, –1), 120]**

(1) $x^2 + y^2 - 3ax - 4by + (a^2 + b^2 - p^2) = 0$ (2) $2ax + 2by - (a^2 - b^2 + p^2) = 0$
(3) $x^2 + y^2 - 2ax - 3by + (a^2 - b^2 - p^2) = 0$ (4) $2ax + 2by - (a^2 + b^2 + p^2) = 0$

15. Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre, is : **[AIEEE 2006 (3, –1), 120]**
(1) $x^2 + y^2 = 1$ (2) $x^2 + y^2 = \frac{27}{4}$ (3) $x^2 + y^2 = \frac{9}{4}$ (4) $x^2 + y^2 = \frac{3}{2}$

16. If the lines $3x - 4y - 7 = 0$ and $2x - 3y - 5 = 0$ are two diameters of a circle of area 49π square units, the equation of the circle is : **[AIEEE 2006 (3, –1), 120]**
(1) $x^2 + y^2 + 2x - 2y - 62 = 0$ (2) $x^2 + y^2 - 2x + 2y - 62 = 0$
(3) $x^2 + y^2 - 2x + 2y - 47 = 0$ (4) $x^2 + y^2 + 2x - 2y - 47 = 0$

17. Consider a family of circles which are passing through the point (–1, 1) and are tangent to x-axis. If (h, k) are the coordinates of the centre of the circles, then the set of values of k is given by the interval **[AIEEE 2007 (3, –1), 120]**
(1) $0 < k < 1 < 2$ (2) $k \geq 1/2$ (3) $-1/2 \leq k \leq 1/2$ (4) $k \leq 1/2$

18. The point diametrically opposite to the point P(1, 0) on the circle $x^2 + y^2 + 2x + 4y - 3 = 0$ is **[AIEEE 2008 (3, –1), 105]**

(1) (3, –4) (2) (–3, 4) (3) (–3, –4) (4) (3, 4)

19. If P and Q are the points of intersection of the circles $x^2 + y^2 + 3x + 7y + 2p - 5 = 0$ and $x^2 + y^2 + 2x + 2y - p^2 = 0$, then there is a circle passing through P, Q and (1, 1) for : **[AIEEE 2009 (4, –1), 144]**

(1) all except one value of p (2) all except two values of p
(3) exactly one value of p (4) all values of p

20. Three distinct points A, B and C are given in the 2-dimensional coordinate plane such that the ratio of the distance of any one of them from the point (1, 0) to the distance from the point (–1, 0) is equal to 1 : 3. Then the circumcentre of the triangle ABC is at the point **[AIEEE 2009 (4, –1), 144]**

(1) (0, 0) (2) $\left(\frac{5}{4}, 0\right)$ (3) $\left(\frac{5}{2}, 0\right)$ (4) $\left(\frac{5}{3}, 0\right)$

21. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if **[AIEEE 2010 (4, –1), 144]**

(1) $-35 < m < 15$ (2) $15 < m < 65$ (3) $35 < m < 85$ (4) $-85 < m < -35$

22. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2 (c > 0)$ touch each other if : **[AIEEE 2011, I, (4, –1), 120]**

(1) $2|a| = c$ (2) $|a| = c$ (3) $a = 2c$ (4) $|a| = 2c$

23. The equation of the circle passing through the point (1, 0) and (0, 1) and having the smallest radius is - **[AIEEE 2011, II, (4, –1), 120]**

(1) $x^2 + y^2 - 2x - 2y + 1 = 0$ (2) $x^2 + y^2 - x - y = 0$
(3) $x^2 + y^2 + 2x + 2y - 7 = 0$ (4) $x^2 + y^2 + x + y - 2 = 0$

24. The length of the diameter of the circle which touches the x-axis at the point (1, 0) and passes through the point (2, 3) is : **[AIEEE-2012, (4, –1)/120]**

(1) $\frac{10}{3}$ (2) $\frac{3}{5}$ (3) $\frac{6}{5}$ (4) $\frac{5}{3}$

25. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point **[AIEEE - 2013, (4, – 1)]**

(1) (–5, 2) (2) (2, – 5) (3) (5, – 2) (4) (–2, 5)

26. Let C be the circle with centre at (1, 1) and radius = 1. If T is the circle centred at (0, y), passing through origin and touching the circle C externally, then the radius of T is equal to : **[JEE(Main) 2014, (4, – 1), 120]**

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{\sqrt{3}}{\sqrt{2}}$ (4) $\frac{\sqrt{3}}{2}$

27. Locus of the image of the point (2, 3) in the line $(2x - 3y + 4) + k(x - 2y + 3) = 0$, $k \in R$, is a **[JEE(Main) 2015, (4, – 1), 120]**

(1) straight line parallel to x-axis (2) straight line parallel to y-axis
(3) circle of radius $\sqrt{2}$ (4) circle of radius $\sqrt{3}$

28. The number of common tangents to the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 + 6x + 18y + 26 = 0$, is **[JEE(Main) 2015, (4, – 1), 120]**

(1) 1 (2) 2 (3) 3 (4) 4

29. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on : **[JEE(Main) 2016, (4, – 1), 120]**

(1) an ellipse which is not a circle (2) a hyperbola
(3) a parabola (4) a circle

30. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (– 3, 2), then the radius of S is : **[JEE(Main) 2016, (4, – 1), 120]**

(1) $5\sqrt{3}$ (2) 5 (3) 10 (4) $5\sqrt{2}$

Circle

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is **[IIT-JEE - 2002, Scr, (3, – 1), 90]**

(A) 4 (B) $2\sqrt{5}$ (C) 5 (D) $3\sqrt{5}$

2. If a > 2b > 0, then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is **[IIT-JEE 2002, Scr, (3, – 1), 90]**

(A) $\frac{2b}{\sqrt{a^2-4b^2}}$ (B) $\frac{\sqrt{a^2-4b^2}}{2b}$ (C) $\frac{2b}{a-2b}$ (D) $\frac{b}{a-2b}$

3. The centre of circle inscribed in a square formed by lines $x^2 - 8x + 12 = 0$ and $y^2 - 14y + 45 = 0$ is **[IIT- 2003,Scr, (3, – 1), 84]**

(A) (4, 7) (B) (7, 4) (C) (9, 4) (D) (4, 9)

4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord to the circle with centre (2, 1), then the radius of the circle is **[IIT-JEE - 2004, Scr, (3, – 1), 84]**

(A) 3 (B) 2 (C) 3/2 (D) 5

5. Tangents are drawn from the point (17, 7) to the circle $x^2 + y^2 = 169$.

STATEMENT-1 : The tangents are mutually perpendicular. **[IIT-JEE - 2007, Paper-1, (3,–1), 162]**

because

STATEMENT-2 : The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$.

(A) Statement-1 is True, Statement-2 is True ; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True ; Statement-2 **is NOT** a correct explanation for Statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

6. Let a and b be non-zero real numbers. Then, the equation $(ax^2 + by^2 + c)(x^2 - 5xy + 6y^2) = 0$ represents **[IIT-JEE - 2008, Paper -1, (3,– 1), 82]**

(A) four straight lines, when c = 0 and a, b are of the same sign

(B) two straight lines and a circle, when a = b, and c is of sign opposite to that of a

(C) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a

(D) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a

7. Consider $L_1 : 2x + 3y + p - 3 = 0$ **[IIT-JEE - 2008, Paper-2, (3, – 1), 81]**

$L_2 : 2x + 3y + p + 3 = 0$

where p is a real number, and $C : x^2 + y^2 + 6x - 10y + 30 = 0$

STATEMENT -1 : If line L_1 is a chord of circle C, then line L_2 is not always a diameter of circle C

and

STATEMENT-2 :If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(A) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is** a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True ; STATEMENT-2 **is NOT** a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True

8. Tangents drawn from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is

[IIT-JEE - 2009, Paper-1, (3, –1), 80]

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$
(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

9. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point **[IIT-JEE - 2011, Paper-2, (3, –1), 80]**

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

10. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is **[IIT-JEE 2012, PAPER- 1, (3, –1)/70]** (A) $20(x^2 + y^2) - 36x + 45y = 0$ (B) $20(x^2 + y^2) + 36x - 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$ (D) $36(x^2 + y^2) + 20x - 45y = 0$

Comprehension (Q. No. 11 to 12)

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point P($\sqrt{3}$, 1). A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$. **[IIT-JEE 2012, PAPER- 2, (3, –1)/66]**

11. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{2}y = 6$

12. A possible equation of L is

(A) $x - \sqrt{3}y = 1$ (B) $x + \sqrt{3}y = 1$ (C) $x - \sqrt{3}y = -1$ (D) $x + \sqrt{3}y = 5$

13*. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is (are) **[JEE (Advanced) 2013, Paper-2, (3, –1)/60]**

(A) $x^2 + y^2 - 6x + 8y + 9 = 0$ (B) $x^2 + y^2 - 6x + 7y + 9 = 0$
(C) $x^2 + y^2 - 6x - 8y + 9 = 0$ (D) $x^2 + y^2 - 6x - 7y + 9 = 0$

14*. A circle S passes through the point (0, 1) and is orthogonal to the circles $(x - 1)^2 + y^2 = 16$ and $x^2 + y^2 = 1$. Then **[JEE (Advanced) 2014, Paper-1, (3, 0)/60]**

(A) radius of S is 8 (B) radius of S is 7
(C) centre of S is (–7, 1) (D) centre of S is (–8, 1)

15. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$
(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

16. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s) **[JEE (Advanced) 2016, Paper-1, (4, –2)/62]**

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

Circle

Answers

EXERCISE # 1

Section (A) :

A-1.	(4)	**A-2.**	(4)	**A-3.**	(2)	**A-4.**	(1)	**A-5.**	(3)	**A-6.**	(1)	**A-7.**	(2)
A-8.	(2)	**A-9.**	(1)	**A-10.**	(1)	**A-11.**	(1)	**A-12.**	(4)	**A-13.**	(3)	**A-14.**	(2)
A-15.	(2)												

Section (B) :

B-1.	(1)	**B-2.**	(2)	**B-3.**	(3)	**B-4.**	(1)	**B-5.**	(1)	**B-6.**	(2)	**B-7.**	(2)
B-8.	(2)	**B-9.**	(2)	**B-10.**	(3)	**B-11.**	(1)	**B-12.**	(2)	**B-13.**	(4)	**B-14.**	(2)
B- 15.	(1)	**B-16.**	(2)										

Section (C) :

C-1.	(2)	**C-2.**	(3)	**C-3.**	(2)	**C-4.**	(3)	**C-5.**	(2)	**C-6.**	(1)	**C-7.**	(1)
C-8.	(3)	**C-9.**	(2)	**C-10.**	(1)	**C-11.**	(3)	**C-12.**	(3)	**C-13.**	(1)		

Section (D) :

D-1.	(2)	**D-2.**	(2)	**D-3.**	(2)	**D-4.**	(1)	**D-5.**	(3)	**D-6.**	(1)	**D-7**	(2)
D-8.	(1)	**D-9.**	(2)	**D-10.**	(2)	**D-11.**	(1)	**D-12.**	(1)	**D-13.**	(3)	**D-14**	(1)
D-15.	(1)	**D-16.**	(3)										

Section (E) :

E-1.	(1)	**E-2.**	(1)	**E-3.**	(2)	**E-4.**	(2)	**E-5.**	(3)

EXERCISE # 2

PART-I

1.	(2)	**2.**	(3)	**3.**	(1)	**4.**	(2)	**5.**	(1)	**6.**	(2)	**7.**	(3)
8.	(4)	**9.**	(2)	**10.**	(3)	**11.**	(2)	**12.**	(2)	**13.**	(2)	**14.**	(2)
15.	(4)	**16.**	(1)	**17.**	(1)	**18.**	(2)	**19.**	(1)	**20.**	(3)	**21.**	(4)
22.	(1)												

PART-II

Section (A) :

1. (3) **2.** (1)

Section (B) :

B-1. (A) → s ; (B) → p ; (C) → q ; (D) → r **B-2.** A → q ; B → s ; C → p ; D → r

Section (C) :

C-1.	(2,3)	**C-2.**	(1,2)	**C-3.**	(1,4)	**C-4.**	(1,2)

EXERCISE # 3

PART-I

1.	(2)	**2.**	(2)	**3.**	(4)	**4.**	(2)	**5.**	(2)	**6.**	(2)	**7.**	(1)
8.	(3)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(1)	**13.**	(3)	**14.**	(4)
15.	(3)	**16.**	(3)	**17.**	(2)	**18.**	(3)	**19.**	(1)	**20.**	(2)	**21.**	(1)
22.	(2)	**23.**	(2)	**24.**	(1)	**25.**	(3)	**26.**	(2)	**27.**	(3)	**28.**	(3)
29.	(3)	**30.**	(1)										

PART-III

1.	(C)	**2.**	(A)	**3.**	(A)	**4.**	(A)	**5.**	(A)	**6.**	(B)	**7.**	(C)
8.	(B)	**9.**	(D)	**10.**	(A)	**11.**	(D)	**12.**	(A)	**13*.**	**(AC)**	**14*.**	**(BC)**
15.	(A,B,C)	**16.**	(A,C)										

Circle

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.
2. The test consists **30** questions, **4 marks** each.
3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. An acute angle ΔPQR is inscribed in the circle $x^2+y^2=25$. If Q and R have co-ordinates (3,4) & (–4,3) respectively then $\angle QPR$ =
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

2. The equation of a circle which passes through (1,0) and (0,1) and has its radius as small as possible is $x^2 + y^2 - gx - fy + c = 0$ where g, f $\in$ W (set of whole numbers) then g + f+ c =
(1) 0 (2) 1 (3) 2 (4) 3

3. The equation of circle which is touched by line y = x, has its centre on the positive direction of the x-axis and cut off a chord of length of 2 units along the line $\sqrt{3}y - x = 0$ is
(1) $x^2 + y^2 + 4x + 2 = 0$ (2) $x^2 + y^2 + 4x - 2 = 0$ (3) $x^2 + y^2 - 4x - 2 = 0$ (4) $x^2 + y^2 - 4x + 2 = 0$

4. The set of all values of α for which the point (α–1, α +1) lies in the larger segment of the circle $x^2 + y^2 - x - y - 6 = 0$ made by the chord x + y – 2 = 0 is
(1) [– 1, 1] (2) (–1, 1) (3) (–1, 0) (4) (0, 1)

5. The circle $x^2 + y^2 - 6x - 10y + \lambda = 0$ neither touches nor intersect the coordinate axis and the point (1,4) lies inside the circle then maximum integral value of λ can be
(1) 26 (2) 27 (3) 28 (4) 29

6. The number of integral values of λ for which $x^2 + y^2 + \lambda x + (1-\lambda) y + 5 = 0$ is the equation of a circle whose radius does not exceed 5 are
(1) 31 (2) 29 (3) 28 (4) 27

7. Value of k for which four distinct points (2k,3k) , (1,0), (0,1), (0,0) lies on a circle is
(1) 0 (2) 1 (3) $\frac{5}{13}$ (4) $\frac{13}{7}$

8. If one end of the diameter is (1,2) and other end lies on the line 2x + y = 5 then the locus of centre of circle is ax + by – 9 = 0 then a + b =
(1) 5 (2) 6 (3) 7 (4) 3

9. If a line is drawn through a fixed point P (10,7) to cut the circle $x^2+y^2 - 4x - 2y - 20 = 0$ at A and B then the value of PA.PB is
(1) $5\sqrt{3}$ (2) 75 (3) 49 (4) 7

10. If the length of tangent drawn from the point (5,3) to the circle $x^2+y^2 + 2x+ky +17 = 0$ be 7 then value of k is

(1) 4 (2) – 4 (3) – 3 (4) 3

11. If the chord of contact of tangents drawn from a point on the circle $x^2+y^2 = a^2$ to the circle $x^2+y^2 = b^2$ touches the circle $x^2+y^2 = c^2$ then

(1) 2b = a + c (2) $b = \frac{2ac}{a+c}$ (3) $b^2 = ac$ (4) 2b = a + 3c

12. The circles $x^2+y^2 + 2a'x + 2b'y + c' = 0$ and $2x^2+2y^2 +2ax + 2by + c = 0$ intersect orthogonally if $\lambda(aa' + bb') = \mu c + \delta c'(\lambda, \mu, \delta \in N)$ then minimum value of $\lambda + \mu + \delta$ is

(1) 4 (2) 3 (3) 5 (4) 7

13. If the equation of circle which cuts the three circles $x^2+y^2-3x-6y+ 14 = 0$, $x^2+y^2-x - 4y + 8 = 0$ and $x^2+y^2 + 2x - 6y + 9 = 0$ orthogonally is $x^2+y^2+ gx - 4y + \lambda = 0$ then value of $g + 5\lambda$ is

(1) – 1 (2) 2 (3) 4 (4) 3

14. The length of common chord of circles $x^2+y^2+2x+6y = 0$ & $x^2+y^2- 4x - 2y - 6 = 0$ is $\frac{\alpha\sqrt{106}}{\beta}$ where α and β are coprime then

(1) α – β = 3 (2) α – β = 2 (3) α + β = 7 (4) α + β = 5

15. The equation of the smallest circle passing through the intersection of the line x+y = 1 and the circle $x^2+y^2 = 9$ is

(1) $x^2+y^2-x-y- 8 = 0$ (2) $x^2+y^2-x-y+ 8 = 0$ (3) $x^2+y^2+x-y- 8 = 0$ (4) $x^2+y^2+x+y+ 8 = 0$

16. Equation of chord of circle $x^2+y^2-3x-4y- 4 = 0$ which passes through the origin such that origin divides it in the ratio 4:1 is ax+by = 0 (a,b∈N) then minimum (a+b) =

(1) 30 (2) 31 (3) 32 (4) 33

17. Two congruent circles with centres at (2,3) and (5,6) which intersect at right angle have radius equal to

(1) $2\sqrt{2}$ (2) 3 (3) 4 (4) $\sqrt{3}$

18. ABCD is a square of unit area. A circle is tangent to two sides of ABCD and passes through exactly one of its vertex, the radius of the circle can be

(1) $2-\sqrt{2}$ (2) $\sqrt{2}-1$ (3) $2+\sqrt{3}$ (4) $\frac{1}{\sqrt{2}}$

19. If the chord y = mx+1 of circle $x^2+y^2 = 1$ subtends an angle 45° at the major segment of the circle then value of m can be

(1) 2 (2) –2 (3) – 1 (4) $\sqrt{2}$

20. The straight line $x\cos\theta+y\sin\theta = 2$ will touch the circle $x^2+y^2-2x = 0$ if

(1) $\theta = n\pi, n \in I$ (2) $\theta = (2n+1)\pi, n \in I$ (3) $\theta = 2n\pi, n \in I$ (4) $\theta = (4n-1)\frac{\pi}{2}, n \in I$

21. If the conics whose equations are $S \equiv x^2\sin^2\theta+2hxy+y^2\cos^2\theta +32x+16y+19 = 0$ & $S' \equiv x^2\cos^2\theta+2h'xy + y^2\sin^2\theta+16x+32y+19 = 0$ intersects in four concylic points then

(1) h+h' = 0 (2) h = h' (3) h + h' =1 (4) h+h'=2

22. If the line ax+by = 2 is a normal to the circle $x^2+y^2- 4x - 4y = 0$ and a tangent to the circle $x^2+y^2=1$ then a –b = (a>b)

(1) $\sqrt{6}$ (2) $\sqrt{7}$ (3) $\sqrt{5}$ (4) $\sqrt{8}$

23. The equation of circle whose radius is 5 and which touches the circle $x^2+y^2-2x-4y-20=0$ at the point (5,5) is

(1) $x^2+y^2+18x+16y-120 = 0$ (2) $x^2+y^2-18x-16y-120 = 0$
(3) $x^2+y^2-18x-16y+120 = 0$ (4) $x^2+y^2-18x+16y+120 = 0$

24. If a circle passes through the point (a,b) and cut the circle $x^2+y^2=k^2$ orthogonally then equation of locus of its centre is

(1) $2ax + 2by = a^2+b^2+k^2$
(2) $ax + by = a^2+b^2+k^2$
(3) $x^2 + y^2 + 2ax+2by+k^2= 0$
(4) $x^2 + y^2 + 2ax-2by+a^2 + b^2 - k^2 = 0$

25. If common chord of the circle C with centre at (2,1) and radius r and the circle $x^2+y^2-2x-6y+ 6= 0$ is a diameter of 2nd circle then value of r is

(1) 3 (2) 2 (3) $\frac{3}{2}$ (4) 1

26. If the circle $x^2+y^2-6x- 4y +9 = 0$ bisect the circumference of the circle $x^2+y^2-(\lambda+4) x- (\lambda+2)y +5\lambda+3 = 0$, then λ is equal to

(1) – 1 (2) 1 (3) 2 (4) 4

27. Radical centre of three circles $x^2+y^2 =9$, $x^2+y^2-2x-2y-5= 0$ & $x^2+y^2+ 4x+6y -19 = 0$ lies on $y = mx$ then m =

(1) – 1 (2) $-\frac{2}{3}$ (3) $-\frac{3}{4}$ (4) 1

28. The area of the triangle formed by the tangent at the point (α,β) on the circle $x^2+y^2 =r^2$ where $r^4-15r^2+56 = 0$ and coordinate axis can be

(1) $\frac{49}{\alpha\beta}$ (2) $\frac{14}{\alpha\beta}$ (3) $\frac{16}{\alpha\beta}$ (4) $\frac{32}{\alpha\beta}$

29. If ℓ, m,n denote the length of intercepts made by circle $x^2+y^2-8x + 10y + 16 = 0$ on x-axis, y-axis and line $y = -x$ respectively , then $\ell^2+10m^2+26n^2 =$

(1) 2906 (2) 2908 (3) 2910 (4) 2900

30. A polygon of nine sides each of length 2 inscribed in a circle, then diameter of circle is

(1) $\operatorname{cosec}\frac{\pi}{9}$ (2) $\sin\frac{\pi}{9}$ (3) $2\operatorname{cosec}\frac{\pi}{9}$ (4) $2\sin\frac{\pi}{9}$

Practice Test (JEE-Main Pattern)
OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $pq \neq 0$) are bisected by the x-axis, then

(1) $p^2 = q^2$ (2) $p^2 = 8q^2$ (3) $p^2 < 8q^2$ (4) $p^2 > 8q^2$

2*. Let L_1 be a straight line passing through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ?

(1) $x + y = 0$ (2) $x - y = 0$ (3) $x + 7y = 0$ (4) $x - 7y = 0$

3. Let PQ and RS be tangents at the extremities of diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2r equals

(1) $\sqrt{PQ \cdot RS}$ (2) $\frac{PQ + RS}{2}$ (3) $\frac{2PQ + RS}{PQ + RS}$ (4) $\frac{\sqrt{PQ^2 + RS^2}}{2}$

4. Let AB be a chord of the circle $x^2 + y^2 = r^2$ subtending a right angle at the centre. Then, locus of the centroid of the triangle PAB as P moves on the circles is

(1) a parabola (2) a circle
(3) an ellipse (4) a pair of straight line

5. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is

(1) 4 (2) $2\sqrt{5}$ (3) 5 (4) $3\sqrt{5}$

6. If $a > 2b > 0$, then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is

(1) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (2) $\frac{\sqrt{a^2 - 4b^2}}{2b}$ (3) $\frac{2b}{a - 2b}$ (4) $\frac{b}{a - 2b}$

7. A circle is given by $x^2 + (y - 1)^2 = 1$. Another circle C touches it externally and also the x-axis, then the locus of its centre is

(1) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$ (2) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \le 0\}$
(3) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$ (4) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

8. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is

(1) 4 (2) 6 (3) 8 (4) 3

9. Let ABCD be a quadrilateral with area 18, with side AB parallel to the side CD and AB = 2 CD. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral ABCD touching all the sides, then its radius is

(1) 3 (2) 2 (3) $\frac{3}{2}$ (4) 1

10*. A straight line through the vertex P of a triangle PQR intersects the side QR at the point S and the circumcircle of the triangle PQR at the point T. If S is not the centre of the circumcircle, then

(1) $\frac{1}{PS} + \frac{1}{ST} < \frac{2}{\sqrt{QS \times SR}}$ (2) $\frac{1}{PS} + \frac{1}{ST} > \frac{2}{\sqrt{QS \times SR}}$
(3) $\frac{1}{PS} + \frac{1}{ST} < \frac{4}{QR}$ (4) $\frac{1}{PS} + \frac{1}{ST} > \frac{4}{QR}$

Comprehension # 1 (Q. No. 11 to 13)

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F, respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$. Further, it is given that the origin and the centre of C are on the same side of the line PQ.

11. The equation of circle C is

(1) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$ (2) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$
(3) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$ (4) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

12. Points E and F are given by

(1) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$, $\left(\sqrt{3}, 0\right)$ (2) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$, $\left(\sqrt{3}, 0\right)$

(3) $\left(\frac{\sqrt{3}}{2} \frac{3}{2}\right)$, $\left(\frac{\sqrt{3}}{2} \frac{1}{2}\right)$ (4) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

13. Equations of the sides QR, RP are

(1) $y = \frac{2}{\sqrt{3}}x + 1$, $y = -\frac{2}{\sqrt{3}}x - 1$ (2) $y = \frac{1}{\sqrt{3}}x$, $y = 0$

(3) $y = \frac{\sqrt{3}}{2}x + 1$, $y = -\frac{\sqrt{3}}{2}x - 1$ (4) $y = \sqrt{3}x$, $y = 0$

Comprehension # 2 (Q. No. 14 to 16)

Two circles are $S_1 \equiv (x + 3)^2 + y^2 = 9$; $S_2 \equiv (x - 5)^2 + y^2 = 16$ with centres C_1 & C_2

14. A direct common tangent is drawn from a point P on x-axis which touches S_1 & S_2 at Q & R, respectively. Find the ratio of area of ΔPQC_1 & ΔPRC_2.

(1) 3 : 4 (2) 9 : 16 (3) 16 : 9 (4) 4 : 3

15. From point 'A' on S_2 which is nearest to S_1, a variable chord is drawn to S_1. The locus of mid point of the chord is

(1) circle (2) Diameter of S_1

(3) Arc of a circle (4) chord of S_1 but not diameter

16. Locus of Q.15 cuts the circle S_1 at B & C, then line segment BC subtends an angle on the major arc BC of circle S_1 which is

(1) $\cos^{-1}\frac{3}{4}$ (2) $\tan^{-1}(3\sqrt{7})$ (3) $\cos^{-1}\left(\frac{4}{3}\right)$ (4) $\cot^{-1}\left(\frac{\sqrt{7}}{3}\right)$

APSP Answers

PART - I

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(2)	**5.**	(3)	**6.**	(1)	**7.**	(3)
8.	(2)	**9.**	(2)	**10.**	(2)	**11.**	(3)	**12.**	(3)	**13.**	(4)	**14.**	(3)
15.	(1)	**16.**	(2)	**17.**	(2)	**18.**	(1)	**19.**	(3)	**20.**	(3)	**21.**	(1)
22.	(2)	**23.**	(3)	**24.**	(1)	**25.**	(1)	**26.**	(4)	**27.**	(4)	**28.**	(4)
29.	(2)	**30.**	(1)										

PART - II

1.	(4)	**2*.**	(2,3)	**3.**	(1)	**4.**	(2)	**5.**	(3)	**6.**	(1)	**7.**	(4)
8.	(3)	**9.**	(2)	**10*.**	(2,4)	**11.**	(4)	**12.**	(1)	**13.**	(4)	**14.**	(2)
15.	(3)	**16.**	(1)										