TOPIC : CAPACITANCE
EXERCISE # 1

SECTION (A)

1. $Q_t = Q_1 + Q_2 = 150\mu C$

$\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{1}{2}$ ⇒ $Q_1' = 50\mu C$

$Q_2' = 100\mu C$

$25\mu C$ charge will flow from smaller to bigger sphere .

2. Charge is flow untill potential are equal and in charge flow energy is decrease

$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$ ⇒ $Q_1R_2 = Q_2R_1$.

3. The charge on the capacitot remains constants

Capacitance $C = \frac{\varepsilon_0 A}{d}$ d ↑ C ↓

Energy $U = \frac{1}{2}\frac{Q^2}{C}$ C ↓ U ↑

Potential $V = \frac{Q}{C}$ C ↓ V ↑

4. Isolated capacitor ⇒ Q = constant

sepration d increase ⇒ C = decrease

Q = CV ⇒ V = increase

14. Capacitance of spherical conductor= $4\pi\epsilon_0 a$ where a is radius of conductor

Therefore $C = \frac{1}{9\times10^9} = \frac{1}{9}\times10^{-9}$ $= 0.11 \times 10^{-9}$ F = 1.1×10^{-10} F **Ans.**

15. $C = 12\ \mu F$

$d' = \frac{d}{2}$

$A' = 2A$

$\frac{C'}{C} = \frac{A'}{A}\frac{d}{d'} = 2 \times 2 = 4$ ⇒ $C' = 4C = 4 \times 1C = 48\ \mu F$

16. $C = \frac{\varepsilon_0(\pi R^2)}{x} = 4\pi\varepsilon_0 R$ ⇒ $x = R/4$

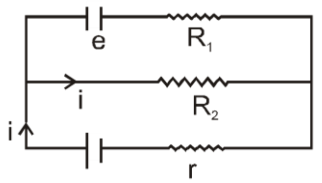
SECTION (B)

1. $W = V_f - V_i = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_1^2 = \frac{1}{2}C(40^2 - 20^2)$ W = 600 C

$W_1 = \frac{1}{2}C(50^2 - 40^2) = \frac{900}{2}C$ $W_1 = \frac{900}{2}.\frac{W}{600} = \frac{3}{4}W$ **Ans**

2. Charge on capacitor = CV = capacitance × (voltage across it)

In steady state, there will be no current through capacitor.



voltage across $C = iR_2 = \frac{E\,R_2}{R_2+r}$. Charge on capacitor $= CiR_2 = \frac{CER_2}{R_2+r}$

3. As battery is disconnected, charge remains constant in the work process.
Work done = final potential energy – initial potential energy

$$= \frac{Q^2}{2C'} - \frac{Q^2}{2C^o}$$

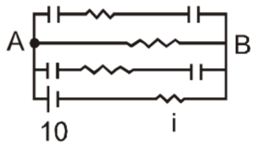
$$= \frac{Q^2}{2}\left\{\frac{1}{C'} - \frac{1}{C^o}\right\}$$

Where , $Q = CV = \frac{A\varepsilon_o V}{d}$ $\quad C = \frac{A\varepsilon_o}{d}$ $\quad C' = \frac{A\varepsilon_o}{2d}$

Now, work done $= \frac{\varepsilon_o AV^2}{2d}$ Ans. is (4)

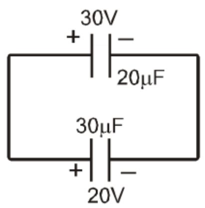
14. If S_1 is closed and S_2 is open then, condenser C is fully charged at potential V.

18. Charge on each capacitor will be same. In steady state current through capacitor will be zero



current in steady state $= i = \frac{10}{5}$ = 2 amp
potential across AB = iR = 2 × 4 = 8 V.
Potential across each capacitor = 4 V
on each plate Q = C V = 3 × 4 = 12 μC

19.



Common potential $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{600+600}{20+30}$ = 24 V

20. $W = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C}$

$$= \frac{1}{2}\times\frac{(8\times10^{-18})^2}{100\times10^{-6}} = \frac{1}{2}\times\frac{64\times10^{-36}}{100\times10^{-6}} = 32 \times 10^{-32}\,J$$

21. **Key Idea :** On removing the battery after charging, the charge stored in the capacitor remains constant.
When a capacitor is charged by connecting a battery across its plates, the initial energy stored,

$$U = \frac{q^2}{2C}$$

When the battery is disconnected, then the charge remains constant i.e., q = constant.
Now another identical capacitor is connected across it i.e., the capacitors are connected in parallel, so the equivalent capacitance

$C_{eq} = C_1 + C_2 = C + C = 2C$

Thus, final energy stored by the system of capacitors,

$$U' = \frac{q^2}{2C_{eq}} = \frac{q^2}{2(2C)} = \frac{1}{2}U \quad \therefore \quad U' = \frac{U}{2}$$

22. **Key Idea :** Energy stored between the plates of a capacitor is equal to $\frac{1}{2}\frac{Q^2}{C}$

Energy stored, $U = \frac{1}{2}\frac{Q^2}{C}$ but $\sigma = \frac{Q}{A}$ and $C = \frac{\varepsilon_0 A}{d}$

$\therefore$ $U = \frac{1}{2}\frac{(\sigma A)^2}{(\varepsilon_0 A / d)}$ or $U = \frac{A\sigma^2 d}{2\varepsilon_0}$

or $U = \frac{1}{2}\left(\frac{\sigma}{\varepsilon_0}\right)^2 \times \varepsilon_0 Ad$ or $U = \frac{1}{2}E^2\varepsilon_0 Ad$

Energy stored per unit volume i.e., energy density is thus given by

$$u = \frac{U}{V} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{V}{d}\right)^2 = \frac{1}{2}\frac{\varepsilon_0 V^2}{d^2}$$

Note : $\frac{1}{2}$ $\varepsilon_0 E_2$ is also a force on a conductor per unit area which is every where along the outward drawn normal to the surface.

23. The common potential difference across two capacitors connected in parallel

$V' = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$ Here, $V_1 = V$, $V_2 = 0$ $\therefore$ $V' = \frac{C_1 V}{C_1 + C_2}$

25. The two condensers in the circuit are in parallel order, hence

$$C' = C + \frac{C}{2} = \frac{3C}{2}$$

the work done in charging the equivalent capacitor is stored in the form of potential energy.

Hence, $W = U = \frac{1}{2}$ $C' V_2$ $\frac{1}{2}\left(\frac{3C}{2}\right)$ $= V_2 = \frac{3}{4}$ CV_2

28. Energy stored by any system of capacitors is $= \frac{1}{2} C_{net} V_2$ where V is source voltage
Thus, n capacitors are connected in parallel,

Therefore $C_{net} = nC$ $\therefore$ $E_{net} = \frac{1}{2}$ nCV_2

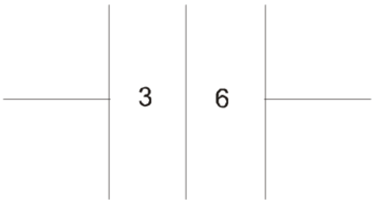
29. Work done by battery = (CV) V = CV_2
Energy stored in capacitor = CV_2

$$\frac{\text{Energy stored in capacitor}}{\text{Work done by battery}} = \frac{\frac{1}{2}CV^2}{CV^2} = \frac{1}{2}$$

Correct choice is (4)

30. $C' = \dfrac{A\epsilon_0}{\frac{d_1}{3}+\frac{d_2}{6}} = \dfrac{A\epsilon_0}{\frac{d}{9}+\frac{2d}{18}} = \dfrac{18A\epsilon_0}{4d}$

C = 9 PF



C´ = 40.5 PF

Correct choice is (3).

31. $U = \frac{1}{2}CV^2$

$V = \sqrt{\dfrac{2V}{C}} = \sqrt{\dfrac{2\times 0.16}{2\times 10^{-6}}} = \sqrt{\dfrac{16}{10^{-4}}} = 400\ \text{V}$

32. $W = \frac{1}{2}CV^2 = \frac{1}{2} \times 6 \times 10^{-6} \times (50)^2$

$= 3 \times 10^{-4} \times 25$

$= 75 \times 10^{-4}$ J

$= 7.5 \times 10^{-3}$ J

33. ΔU = decrease in potential energy

$= U_i - U_f$

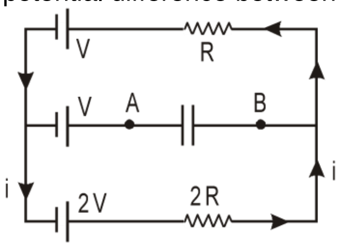
$= \frac{1}{2}C(V_1^2 + V_2^2) - \frac{1}{2}(2C)\left(\frac{V_1+V_2}{2}\right)^2 = \frac{1}{4}C(V_1 - V_2)^2$

34. When capacitor is fully charged, it draws no current. When no current in the circuit, potential difference across the cell = emf of the cell = potential difference across the capacitor.

37. In steady state condition, no current will flow through the capacitor C. Current in the outer circuit,

$i = \dfrac{2V - V}{2R + R} = \dfrac{V}{3R}$

potential difference between A and B -



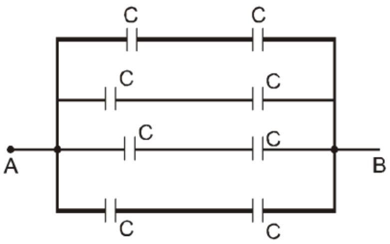
$V_A - V + V + iR = V_B$

$\therefore V_B - V_A = iR = \left(\frac{V}{3R}\right) R = \frac{V}{3}$

In this problem charge stored in the capacitor can also be asked which is equal to $q = C\frac{V}{3}$ with positive charge on B side and negative on A side because $V_B > V_A$.

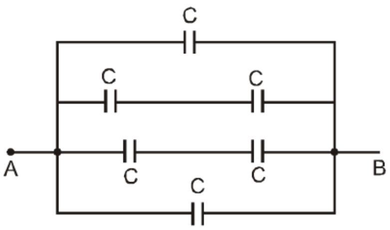
SECTION (C)

1.



$C_{eq} = \frac{4C}{2} = 2C.$

2.

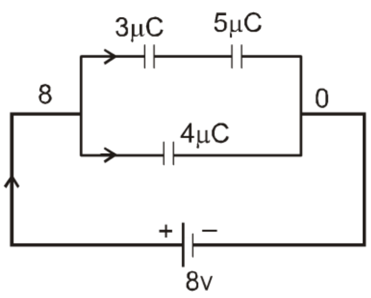


$C_{eq} = C + \frac{2C}{2} + C = 3C.$

3. $\frac{1}{C_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ $\Rightarrow$ $C_1 = 1\ \mu F$, $C_2 = 2 + 1 = 3\ \mu F$

$C_{eq} = 1\mu F.$

4.

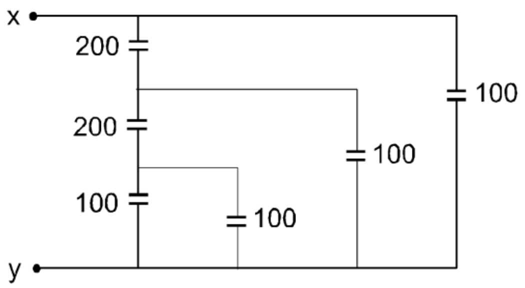


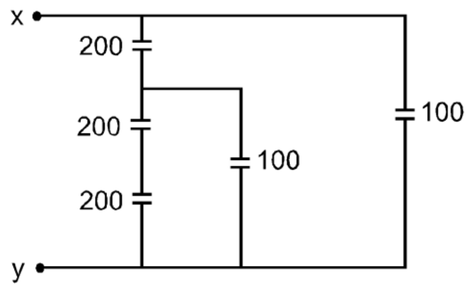
$C_{eq} = \frac{15}{8} + 4 = \frac{47}{8}\ \mu F$

$\frac{q}{3} + \frac{q}{5} = 8$ $\Rightarrow$ $q = 15\mu C$

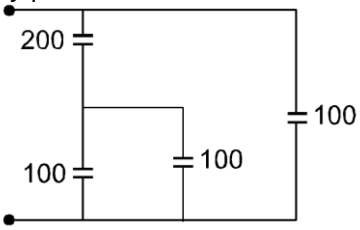
Charge on 2μF

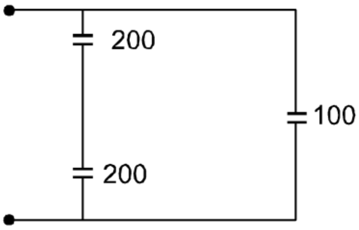
$\frac{q_1}{2} = \frac{15 - q_1}{3}$ $\Rightarrow$ $q_1 = \frac{30}{5} = 6.0\mu C$ **Ans.**

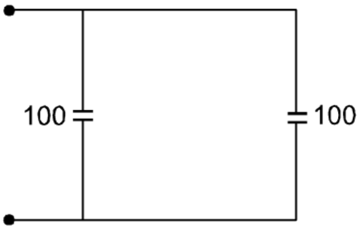
6. 

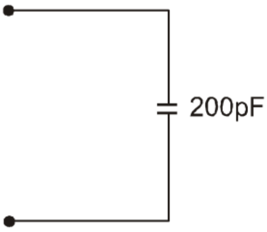
⇒ 

solving by parallel series combinations,

⇒ 

⇒ 

⇒ 

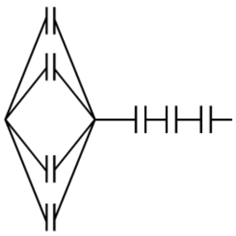
⇒ 

C_{eq} = 200 pF Ans.

7. Combining two capacitors in series combination

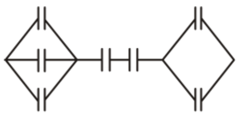
$\frac{1}{C} = \frac{1}{1} + \frac{1}{1}$ ⇒ C = 0.5

Now, forming one capacitors in parallel with this combination, total capacity = 0.5 + 1 = 1.5

33. 

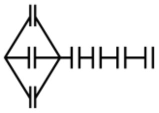
$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{8} + \frac{3\times4}{2\times4}$

$\frac{1}{C_{eq}} = \frac{13}{8}$ ⇒ $C_{eq.} = \frac{8}{13}$ μF



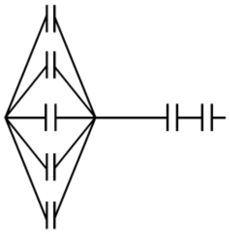
$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$

$\frac{1}{C_{eq}} = \frac{17}{12}$ ⇒ $C_{eq.} = \frac{12}{17}$



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} \times 4$$

$$\frac{1}{C_{eq}} = \frac{13}{6} \Rightarrow C_{eq} = \frac{6}{13}\ \mu F$$



$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{C_{eq}} = \frac{10}{11}\ \mu F$$

37. **Key Idea :** Charge on a capacitor is the product of capacitance and potential difference across it.
The charge flowing through C_4 is

$q_4 = C_4 \times V = 4\ CV$

The series combination of C_1, C_2 and C_3 gives

$$\frac{1}{C'} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C}$$

$$= \frac{6+3+2}{6C} = \frac{11}{6C}$$

$$\Rightarrow \quad C' = \frac{6C}{11}$$

Now, C' and C_4 form parallel combination giving

$C'' = C' + C_4$

$$= \frac{6C}{11} + 4C = \frac{50C}{11}$$

Net charge q = C'' V

$$= \frac{50}{11}\ CV$$

Total charge flowing through C_1, C_2, C_3 will be

$q' = q - q_4$

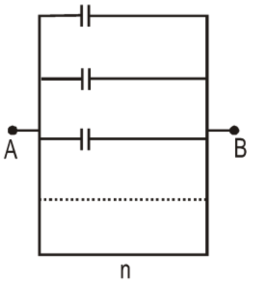
$$= \frac{50}{11}CV - 4CV = \frac{6CV}{11}$$

Since, C_1, C_2 and C_3 are in series combination hence, charge flowing through these will be same.

$$q_2 = q_1 = q_3 = q' = \frac{6CV}{11}$$

Thus, $$\frac{q_2}{q_4} = \frac{6CV/11}{4CV} = \frac{3}{22}$$

38. Capacitance between any two adjacent plates is 'C'



Capacitor consists of n plates, then there are (n -1) combinations joined in parallel.

∴ $C_{AB} = (n - 1)C$

39. $C_1 = 3\mu F$ $V_1 = 300v$

$C_2 = 5\mu F$ $V_2 = 500v$

$$V_C = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} = \frac{3 \times 300 + 5 \times 500}{3 + 5} = \frac{900 + 2500}{8}$$

$$\frac{3400}{8} = 425 \text{ V}$$

40. $$\frac{1}{2} = \left(\frac{1/C_2}{1/C_2 + 1/C_2 + 1/C_3}\right)^{V_0} = \left(\frac{1/3}{1/2 + 1/3 + 1/6}\right)^6 = \frac{(1/3)}{\left(\frac{3+2+1}{6}\right)}.6 = 2V$$

41. Seeing in the given circuit C_1 and C_2 are connected in parallel. Hence, their equivalelnt capacitance

$C_{eq} = C_1 + C_2 = 5 + 10$

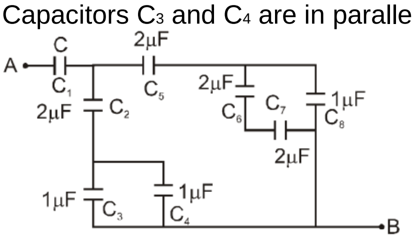
$= 15\ \mu F$

As C_{eq} and C_3 are connected in series, Hence, resultant capacitance between P and Q is given by

$$\frac{1}{C_{PQ}} = \frac{1}{C_{eq}} + \frac{1}{C_3} = \frac{1}{15} + \frac{1}{4} = \frac{19}{60}$$

$$C_{PQ} = \frac{60}{19} = 3.2\ \mu F$$

42. Capacitors C_3 and C_4 are in parallel, therefore their resultanct capacitance,



Now, capacitors C_2 and C' are in series, therefore their resultant capacitance,

$$\frac{1}{C''} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} \qquad C'' = 1\ \mu F$$

Capacitors C_6 and C_7 are in series, therefore their resultanct capacitance,

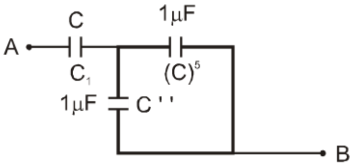
$$\frac{1}{C'''} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1 \qquad C''' = 1\ \mu F.$$

Now, C''' and C_8 are in parallel, therefore their resultant capacitance,

$C'''' = 1 + 1 = 2\ \mu F.$

Now, C'''' and C_5 are in series. Therefore, their resultant capacitance,

$$\frac{1}{(C)^{5'}} = \frac{1}{2} + \frac{1}{2}$$ $(C)_5' = 1\ \mu F$.



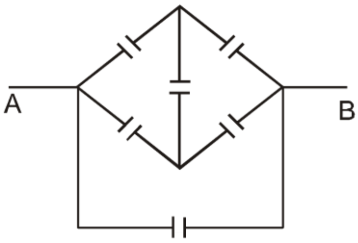
Now, C'' and $(C)_5'$ are in parallel. Therefore, their resultant capacitance,

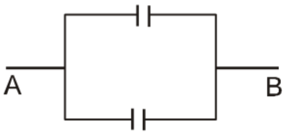
$(C)_6' = 1 + 1 = 2\ \mu F$

Now, C_1 and $(C)_6'$ are in series and their resultant capacitance is given 1 μF.

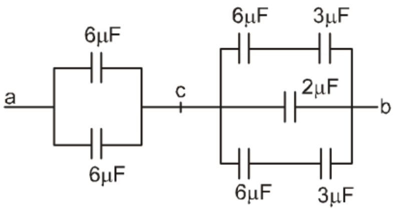
$\therefore \quad \frac{1}{1} = \frac{1}{2} + \frac{1}{C} \qquad \therefore \quad \frac{1}{C} = \frac{1}{1} - \frac{1}{2} = \frac{1}{2}$

C = 2 μF

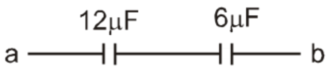
43. 

≡ 

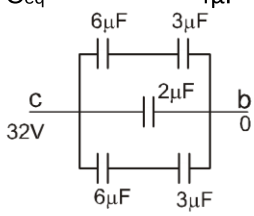
$C_{eq} = 2C$

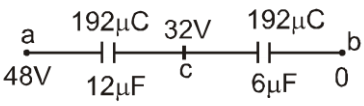
44. **(i)** 

$C_{ac} = 6 + 6 = 12\mu F$ $\quad C_{cb} = 2\left(\frac{6.3}{6+3}\right) + 2 = 6\mu F$

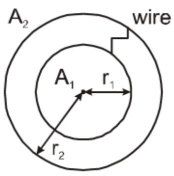


$C_{eq} = \frac{12 \times 6}{12 + 6} = 4\mu F$ **Ans**



(ii) 

Charge on 2μF capacitor ⇒ Q = CV Q = 2 × 32 = 64μC **Ans**

45. 

All given charge of A_1 goes to A_2

Therefore $C = 4\pi\epsilon_0 r_2$

46. N = 8

$R = N_{1/3}r$

$= 8_{1/3}r$

R = 2r

$\frac{C}{c} = \frac{R}{r} = 2$

47. $C_{AB} = (C/2) \| C_{Bridge} \| (C/2) = 2C$

SECTION (D)

1. (i) **(2)** at t_0 ; $q = q_0 = 60\ \mu C$

(ii) **(3)** $q = q_0 e^{-t/RC} = 60 \times 10^{-6}\ e^{-100\times10^{-6}/10\times10^{-6}\times10} = \frac{60}{e}\ \mu C = 22\ \mu C.$

(iii) **(1)** $q = q_0 e^{-t/RC} = 60 \times 10^{-6}\ e^{-1\times10^{-3}/10\times10^{-6}\times10} = \frac{60}{e^{10}}\ \mu C = 0.003\ \mu C.$

2. (i) **(1)** $i_0 = \frac{V}{R} = \frac{6}{24} = 0.25$ A

(ii) **(2)** $i = i_0\ e^{-t/RC} = 0.25\ e^{-1} = \frac{0.25}{e} = 0.09$ A.

3. (i) **(3)** $i = \frac{6}{1}\ e^{-12\times10^{-6}/12\times10^{-6}\times1}$ amp

$= \frac{6}{E} = 2.21$ amp.

(ii) **(2)** $P = V.i = 6 \times 2.21 = 13.2$ W

(iii) **(4)** $\frac{dH}{dt} = i^2R$

$= (2.21)^2 \times 1$

$= 4.88$ W

(iv) **(4)** $P_{battery} = P_{Heat} + P_C$

$P_C = P_{battery} - P_{Heat}$

$= 13.2 - 4.87$

$= 8.37$ W.

4. **(3)** $q = \frac{q_1}{2} = \frac{4\times10^{-6}\times6}{2}\left(1 - e^{-\frac{0.2\times10^{-3}}{4\times10^{-6}\times25}}\right)$

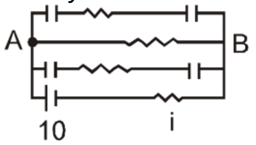
$q = 12(1 - e^{-2})\mu C$

$= 12\ \frac{(e^2-1)}{e^2}\ \mu C$

$= 10.37\ \mu C.$

7. If S_1 is closed and S_2 is open then, condenser C is fully charged at potential V.

8. Charge on each capacitor will be same. In steady state current through capacitor will be zero current in steady state = $i = \frac{10}{5} = 2$ amp potential across AB = iR = 2 × 4 = 8 V.



Potential across each capacitor = 4 V on each plate   Q = C V = 3 × 4 = 12 μC

9. (i) **(B)** at t_0 ; $q = q_0 = 60\ \mu C$

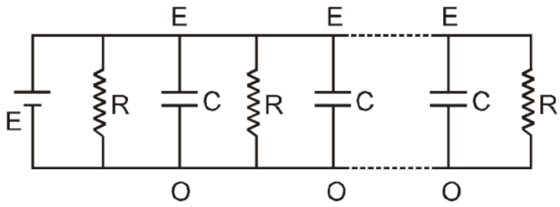
(ii) **(C)** $q = q_0 e^{-t/RC} = 60 \times 10^{-6}\ e^{-100\times10^{-6}/10\times10^{-6}\times10} = \frac{60}{e}\ \mu C = 22\ \mu C.$

(iii) **(A)** $q = q_0 e^{-t/RC} = 60 \times 10^{-6}\ e^{-1\times10^{-3}/10\times10^{-6}\times10} = \frac{60}{e^{10}}\ \mu C = 0.003\ \mu C.$

10. $\log i = \frac{-t}{RC} + \log\left(\frac{E}{R}\right)$; Compare with $y = mx + c$

$\Rightarrow$ $|m| = \frac{1}{RC}$;$c = \log\left(\frac{E}{R}\right)$ as R increases, |m| and C both decreases so (B) is the correct option.

11.



$Q_{first} = Q_{lost} = CE$

Ratio $= \frac{Q_{first}}{Q_{lost}} = 1.$

SECTION (E)

1. $C' = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C.$

2. **(3)** Q = constant

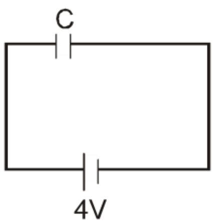
New capacitance = KC (increases)

$V' = \frac{V}{K}$ (decreases)

$U' = \frac{Q^2}{2CK}$ (decreases)

$E = \frac{Q}{A\epsilon_0}$ $\Rightarrow$ $E' = \frac{Q}{KA\epsilon_0}$ (decreases)

3.



Here, Potential difference on the capacitor will depend on emf of battery i.e., 4V

4. Charge or battery = Q = CV = 4 C

Now charge remains same, as battery is disconnected new capacitance = C´ = KC = 8C

$C'V' = Q \qquad V' = \frac{Q}{C} = \frac{4C}{8C} = \frac{1}{2}V$ (1)

5. $U_0 = \frac{1}{2}CV^2$ (given) Now energy = $U' = \frac{1}{2}C'V^2$

$C' = CK$

$U' = \frac{1}{2}CV^2K = U_oK$ **Ans. is (1)**

6. Now, charge remains same on the plates.

$U_0 = \frac{Q^2}{2C}$ (given)

Now energy = $U' = \frac{Q^2}{2C'} = \frac{Q^2}{2CK} = \frac{U_o}{K}$ (3)

33. No change in the energy of the system. Hence, not net work done by the system is zero.
Correct choice is (4)

34. Conceptual

35. In the arrangement shown two capacitors each of area A and separation d/2 are in series.

$\therefore \quad C_1 = \frac{K_2\varepsilon_0A}{d/2} = \frac{2K_1\varepsilon_0A}{d}$

$C_2 = \frac{K_2\varepsilon_0A}{d/2} = \frac{2K_2\varepsilon_0A}{d}$

$\therefore$ Effective capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1\varepsilon_0A} + \frac{d}{2K_2\varepsilon_0A} = \frac{d}{2\varepsilon_0A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$\therefore \quad C = \frac{2\varepsilon_0A}{d}\left(\frac{K_1+K_2}{K_1+K_2}\right)$$

36. As insulator plate is passed between the plates of the capacitor, its capacity increases first and then decreases as the plate slips out. As a result, positive charge on plate A increases first and then decreases, hence current in outer circuit flows from B to A and then from A to B.

37. Aluminium is a metal, so when we insert an aluminium foil, equal and opposite charges appear on its two surfaces. Since it is of negligible thickness, it will not affect the capacitance.
Alternative : From the formula,

$$C = \frac{\varepsilon_0A}{d - t + \frac{t}{K}}$$

$\Rightarrow$ Here, K = ∞ and t $\to$ 0

so, $C = \dfrac{\varepsilon_0 A}{d+0} = \dfrac{\varepsilon_0 A}{d} = C_0$

38. $V_{C_2} = V_{C_2} = V$

$C_1 = C$

$C_2 = KC$

$q_1 = C_1 V_{C_1} = CV$

$q_2 = C_2 V_{C_2} = KCV$

$q_1 < q_2$.

39. $E = \dfrac{\sigma}{\varepsilon_0} = 2.1 \times 10^{-5}$

$E' = \dfrac{\sigma}{\varepsilon_0 K} = 1 \times 10^{-5}$ $\therefore$ $\dfrac{E}{E'} = \dfrac{\sigma}{\varepsilon_0} \times \dfrac{\varepsilon_0 K}{\sigma} = \dfrac{2.1 \times 10^{-5}}{1 \times 10^{-5}}$ or $K = 2.1 \approx 2$

40. $C = \dfrac{\epsilon_0 A}{d}$

$d' = 2d$, $K = 3$

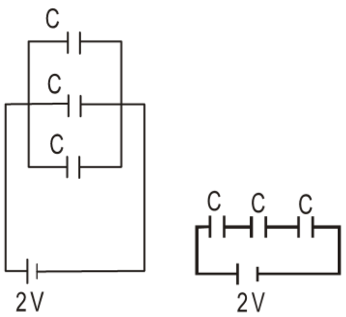
$C' = \dfrac{3}{2}$ C.

EXERCISE # 2

1. $V_2 = \dfrac{(1/C_2)}{1/C_1 + 1/C_2} V_0 = \left(\dfrac{1/6}{1/6 + 1/2}\right) 100 = 25$ V

2. $V_1 = V_0 - V_2 = 100 - 25 = 75$ V

3.



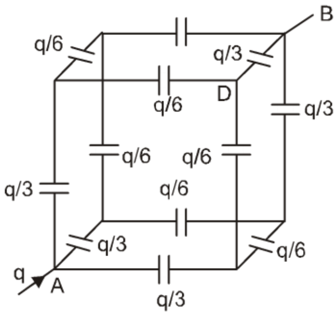
$q_1 + q_2 + q_3 + = 1.8\ \mu C = (3C)2 = 1.8\ \mu C$

$C = 0.3 \mu F$

$C_{eq} = \dfrac{C}{3} = 0.1\ \mu F$

$Q = C_{eq} V = 0.1 \times 2 = 0.2\ \mu C$

4.



Due to symetric charge distribution for loop ACDB

$V_A - \frac{q}{3C} - \frac{q}{6C} - \frac{q}{3C} = V_B$ $\Rightarrow$ $V_A - V_B = \frac{5q}{6C}$ $V_A - V_B = \frac{q}{C_{eq}}$ $\Rightarrow$ $C_{eq} = \frac{6C}{5}$ **Ans**

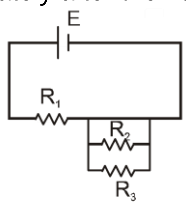
5. $\frac{dq}{dt} = C\frac{dV}{dt}$

$\frac{dV}{dt} = 0.2$ V/s

$V = \left(\frac{dV}{dt}\right)\Delta t$ 123456

$\Delta t = 100$ s

6. Immediately after the key is closed, capacitor be have like a conduetory wire, therefore.



$i = \frac{E}{R_1}$ **Ans**

After a long time interval, capacitor be have like a open circuit. Therefore.

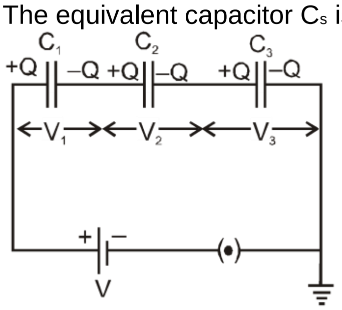
7. $q_3 = \frac{C_3}{C_2 + C_3}.Q$

$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80 = 48\ \mu C$

EXERCISE # 3

PART - I

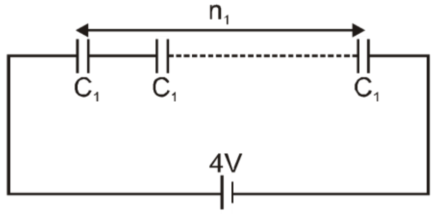
1. In series arrangement charge on each plate of each capacitor has same magnitude.
The potential difference is distributed inversely in the ratio of capacitors ie.
$V = V_1 + V_2 + V_3$.......... Here, $V = 3V$
The equivalent capacitor C_s is given by



$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots\ldots\ldots\ldots$$

$C_s = \frac{C}{3}$

2. A series combination of n_1 capacitors each of capacitance C_1 are connected to 4V source as shown in the figure.



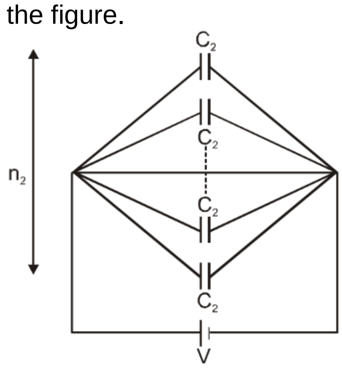
Total capacitance of the series combination of the capacitors is

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_1} + \frac{1}{C_1} +$ .... upto n_1 terms = $\frac{n_1}{C_1}$ or $C_s = \frac{C_1}{n_1}$

Total energy stored in a series combination of the capacitors is

$u_s = \frac{1}{2}C_s(4V)^2 = \frac{1}{2}\left(\frac{C_1}{n_1}\right)(4V)^2$ (using (i))...(ii)

A parallel combination of n_2 capacitors each of capacitance C_2 are connected to V source as shown in the figure.



Total capacitance of the parallel combination of capacitors is

$C_p = C_2 + C_2 + \ldots.. +$ upto n_2 terms $= n_2C_2$

or $C_p = n_2C_2$ ...(iii)

Total energy stored in a parallel combination of capacitors is

$u_p = \frac{1}{2}C_pV^2$

$= \frac{1}{2}(n_2C_2)(V)^2$ (Using (iii))....(iv)

According to the given problem,

$U_s = U_p$

Substituting the values of u_s and u_p from equations (ii) and (iv), we get

$\frac{1}{2}\frac{C_1}{n_1}(4V)^2 = \frac{1}{2}(n_2C_2)(V)^2$ or $\frac{C_1 16}{n_1} = n_2C_2$ or $C_2 = \frac{16C_1}{n_1n_2}$

3. $U = \frac{1}{2}cv^2$

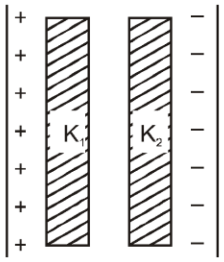
$U = \frac{1}{2}\left(\frac{A\epsilon_0}{d}\right)(Ed)^2 = \frac{1}{2}A\epsilon_0E^2d$

4. $U = \frac{1}{2}CV^2$

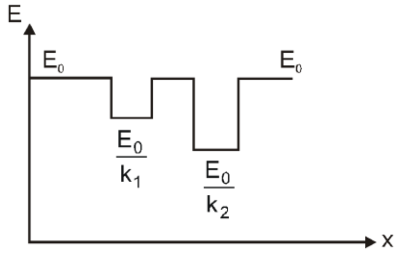
$V = E.d.$

$C = \frac{A\varepsilon_0}{d}$

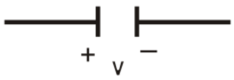
$U = \frac{1}{2}\frac{A\varepsilon_0}{d}(Ed)^2 = \frac{1}{2}\frac{A\varepsilon_0 E^2 d}{d}$



5.



6.



$u_i = \frac{1}{2}cv^2 = \frac{Q^2}{2C}$

Q = cv

$u_f = \frac{Q^2}{2f} = \frac{Q^2}{2kc} = \frac{C^2V^2}{2KC} = \left(\frac{u_i}{k}\right)$

$\Delta u = u_f - u_i = \frac{1}{2}cv^2\left\{\frac{1}{k} - 1\right\}$ (3) ans

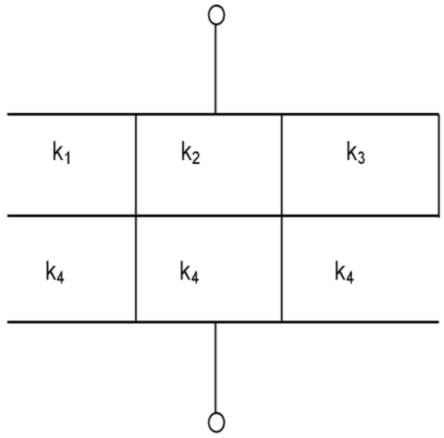
As the capacitor is isolated, so charge will remain conserved

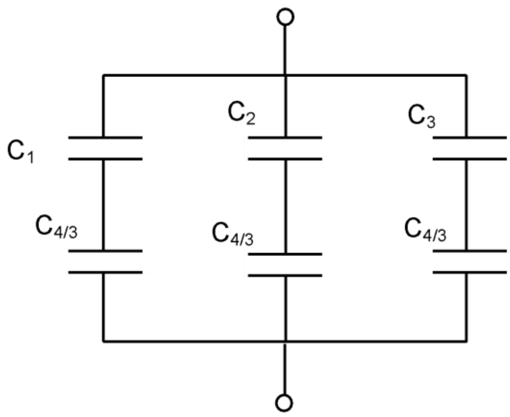
(4) pd = $\frac{Q}{C_f} = \frac{Q}{KC} = \frac{V}{K}$

7. Initial energy stored in the 2 F capacitor is = $\frac{1}{2}$ (2)V2 = V2 J

Energy less = $\frac{C_1C_2}{2(C_1 + C_2)}$ (V1 - V2)2 = $\frac{(2\mu)(8\mu)}{2(2\mu + 8\mu)}$ (V - 0)2

Eloss = $\frac{5}{4}$ V2 J ⇒ % loss = $\frac{5/4\,V^2}{V^2}$ × 100 = 80%





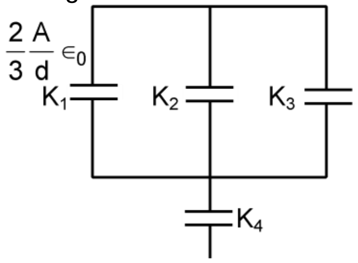
8.

$$\frac{1}{C_1}+\frac{3}{C_4}=\frac{3d}{2k_1\varepsilon_0 A}+\frac{3d}{2k_4\varepsilon_0 A}=\frac{3d}{2\varepsilon_0 A}\left\{\frac{1}{k_1}+\frac{1}{k_4}\right\} \Rightarrow C_{eq}=\frac{K\varepsilon_0 A}{d}=\frac{2\varepsilon_0 A}{3d}\left\{\frac{k_1k_4}{k_2+k_4}+\frac{k_2k_4}{k_2+k_4}+\frac{k_3k_4}{k_3+k_4}\right\}$$

$$k=\frac{2}{3}\left\{\frac{k_1k_4}{k_1+k_4}+\frac{k_2k_4}{k_2+k_4}+\frac{k_3k_4}{k_3+k_4}\right\}$$

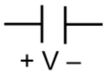
Alter :

Wrong solution seems to be correct as per the options given.



$= \frac{2A}{3}\frac{\in_0}{d}(k_1+k_2+k_3)=C_A=\frac{A\in_0}{d}k_4=C_B$

$$\frac{1}{C_{eq.}}=\frac{1}{C_A}+\frac{1}{C_B} \Rightarrow \frac{d}{a\varepsilon_0 K_{eq}}=\frac{3d}{2A\varepsilon_0 (k_1+k_2+k_3)}\frac{d}{2A\varepsilon_0 k_4} \Rightarrow \frac{1}{k_{eq}}=\frac{3}{2}(k_1+k_2+k_3)+\frac{1}{2k_4}$$

9. Initially 

$U_1=\frac{1}{2}CV^2$

On connecting $V_f=\frac{CV}{C+C}=\frac{V}{2}$

Final energy $=2\times\frac{1}{2}C\times\left(\frac{V}{2}\right)^2=\frac{CV^2}{4}$

So, energy decreases by a factor of 2.

10 $F=\frac{Q^2}{2A\varepsilon_0}=$ Independent of the distance between the plates

11. $E_i=\frac{1}{2}CV^2$, $E_f=\frac{1}{2}.C.\left(\frac{V}{2}\right)^2+\frac{1}{2}.C.\left(\frac{V}{2}\right)^2=2\times\frac{1}{2}.C.\frac{V^2}{4}=\frac{CV^2}{4}=\frac{E_i}{2}$

% loss = $\dfrac{E_i - E_F}{E_i} \times 100$ = $\dfrac{\frac{1}{2}Cv^2 - \frac{1}{4}CV^2}{\frac{1}{2}CV^2} \times 100$ = 50%

PART - II

1. $U_0 = \dfrac{q_0^2}{2C}$ $\quad U = \dfrac{q_0^2 e^{-2t_1/\tau}}{2C} = \dfrac{U_0}{2} = \dfrac{q_0^2}{4C}$ $\quad \Rightarrow \quad e^{-2t_1/\tau} = \dfrac{1}{2}$

$t_1 = \dfrac{\tau}{2}\ \ell n2$ ....(1)

and o $q = q_0 e^{-t_2/\tau}$

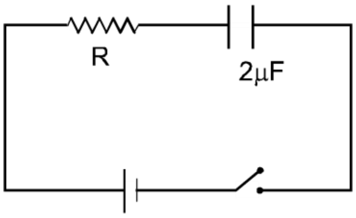
$\dfrac{q_0}{4} = q_0 ,\ e^{-t_2/\tau}$

$e^{-t_2/\tau} = \dfrac{1}{4}$

$t_2 = 2\tau \ln 2$ ....(2)

$\dfrac{t_1}{t_2} = \dfrac{1}{4}$

2.



$v = 200(1 - e^{-t/t})$

$120 = 200(1 - e^{-t/\tau})$

$e^{-t/\tau} = \dfrac{200 - 120}{200} = \dfrac{80}{200}$

$t/\tau = \log(2.5) = 0.4$

$5 = (0.4) \times R \times 2 \times 10^{-6}$

$\Rightarrow \quad R = \dfrac{5}{(0.4) \times 2 \times 10^{-6}}$

$= \quad R = 2.7 \times 10^{6}$ **Ans.**

3. Time constant for parallel combination = 2RC

Time constant for series combination = $\dfrac{RC}{2}$

In first case :

$V = V_0 e^{-\frac{t_1}{2RC}} = \dfrac{V_0}{2}$ ....(i)

In second case

$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \dfrac{V_0}{2}$ ....(ii)

From (i) & (ii),

$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)}$ $\Rightarrow$ $t_2 = \frac{t_1}{4} = \frac{10}{4}$ = 2.5 sec.

4. $Q = c\varepsilon_0\ e^{-t/cR}$

$4\varepsilon = 4\varepsilon_0\ \varepsilon^{-t/\tau}$

$\varepsilon = \varepsilon_0\ \varepsilon^{-t/\tau}$

When $t = 0 \Rightarrow \varepsilon_0 = 25$

$\varepsilon = \varepsilon_0 = 25$

when $t = 200 \Rightarrow \varepsilon = 5$

$5 = 25\ e^{-\frac{200}{\tau}}$

$\ln 5 = \frac{200}{\tau}$

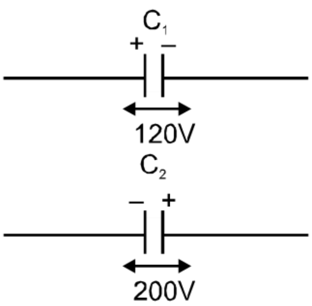
$\tau = \frac{200}{\ell n5} = \frac{200}{\ell n10 - \ell n2} = \frac{200}{\ell n10 - 0.693}$

Alternative :

Time constant is the time in which 63% discharging is completed.

So remaining charge = 0.37 × 25 = 9.25 V

Which time in 100 < t < 150 sec.

5. 

For potential to be made zero, after connection

$120C_1 = 200\ C_2$

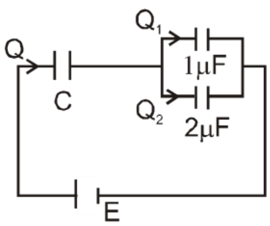
$\Rightarrow$ $3C_1 = 5C_2$

Ans. (2)

6. Electric field inside dielectric $\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

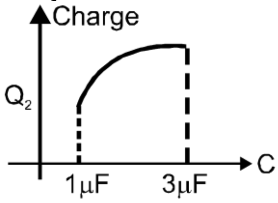
$= 6 \times 10^{-7}\ C/m^2$

7. 

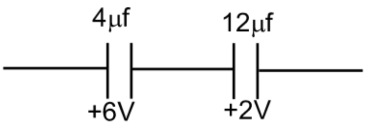
$Q_2 = \frac{2}{2+1}\ Q = \frac{2Q}{3}$

$Q = E\left(\frac{C \times 3}{C+3}\right)$

$\Rightarrow Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$

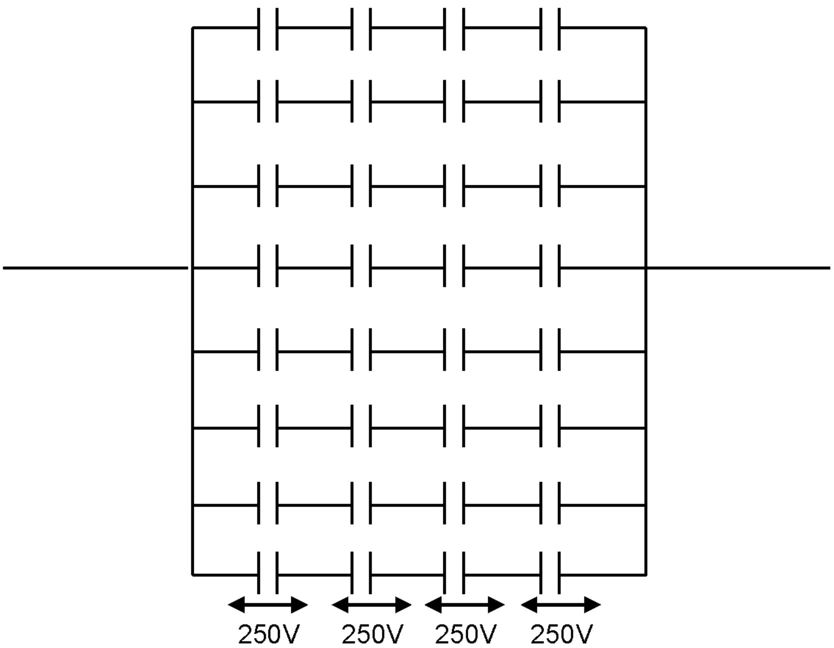


8.



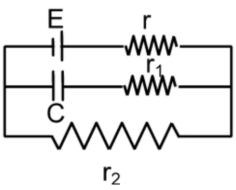
$Q_1 = 24MC$
$Q_2 = 18_MC$
$Q = 42MC$
$E = 10^7 \times 42 \times 10^{-6}$
$E = 420$ N/C

9.

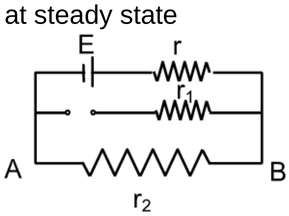


Minimum no. of capacitors required = 32

10.



at steady state



current in the circuit

$I = \dfrac{E}{r + r_2}$

potential difference across AB = Ir_2

$= \dfrac{Er_2}{r + r_2}$

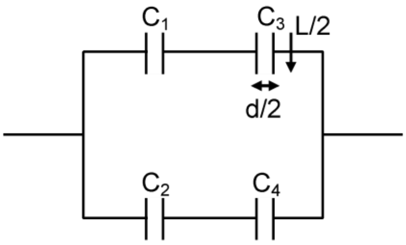
charge on capacitor = Q = $C(\Delta V)_{AB}$

$Q = \dfrac{CEr_2}{r + r_2}$

11. $Q_{cap} = KC_0V$

$\left|Q_{polarised}\right| = \left|Q_{cap}\left(1 - \dfrac{1}{k}\right)\right| = (90 \times 10^{-12})\ (20) \left(\dfrac{5}{3}\right)\left(1 - \dfrac{3}{5}\right)$ Coulomb

$= 1200 \times 10^{-12}$ Coulomb = 1.2 nc

12. 

$C_1 = \dfrac{K_1 \frac{A}{2} \varepsilon_0}{\frac{d}{2}} = \dfrac{K_1 A \varepsilon_0}{d}$

$C_2 = \dfrac{K_2 A \varepsilon_0}{d}$

$C_3 = \dfrac{K_3 A \varepsilon_0}{d}$

$C_4 = \dfrac{K_4 A \varepsilon_0}{d}$

$C_{eq} = \dfrac{C_1 C_3}{C_1 + C_3} + \dfrac{C_2 C_4}{C_2 + C_4}$

$C_{eq} = K_{eq} \dfrac{A\varepsilon_0}{d} = \left[\dfrac{K_1 K_3}{K_1 + K_3} + \dfrac{K_2 K_4}{K_2 + K_4}\right] \dfrac{A\varepsilon_0}{d}$

$K_{eq} = \left[\dfrac{K_1 K_3}{K_1 + K_3} + \dfrac{K_2 K_4}{K_2 + K_4}\right]$

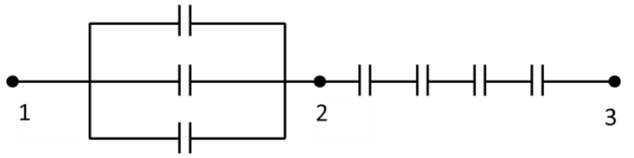
According to this all options are incorrect

13. $C = \dfrac{E_0 A}{d}$

$C_1 = \dfrac{K_1 E_0 A}{3d} = \dfrac{K_1 C}{3}$ $\Rightarrow$ $C_2 = \dfrac{K_2 C}{3}$ $\Rightarrow$ $C_3 = \dfrac{K_3 C}{3}$

$C_{eq} = K_{eq} = \dfrac{C}{3}(K_1 + K_2 + K_3)$ $\Rightarrow$ $K_{eq} = \dfrac{K_1 + K_2 + K_3}{3} = \dfrac{36}{3} = 12$

14. $W = -(U_f - U_i) = -\left(\frac{(\varepsilon C)^2}{2KC} - \frac{(\varepsilon C)^2}{2C}\right) = \frac{\varepsilon^2 C}{2}\left(\frac{K-1}{K}\right) = \frac{10^2 \times 12 \times 10^{-12}}{2}\left(\frac{5.5}{6.5}\right) = 508\text{pJ}$



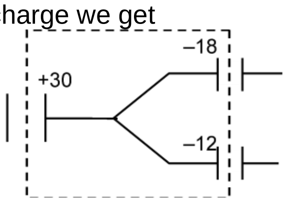
15.

$C_{12} = 6\ \mu F$

$C_{23} = \frac{2}{4} = \frac{1}{2}\ \mu F$

$\Rightarrow \quad C_{13} = \frac{6 \times \frac{1}{2}}{6 + \frac{1}{2}} = \frac{3}{\frac{13}{2}} = \frac{6}{13}\ \mu F$

16. Applying the concept of charge conservation on isolated plates of 10μF, 6μF & 4μF and distribuiting the charge we get



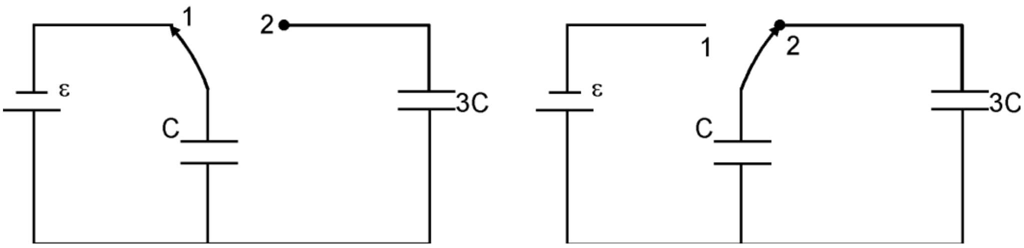
$= \frac{6}{(6+4)} \times 30$ mc = + 18 μc

17. $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0} = 100\,V/m$

$Q = 100 \times A\,\varepsilon_0$

$\Rightarrow \quad Q = 100 \times 1 \times 8.85 \times 10^{-12}\,C = 8.85 \times 10^{-10}\,C$

18. Energy dissipated when switch is thrown from 1 to 2.



$Q = CV$

$V' = \frac{CV+0}{C+3C} = \frac{V}{4}$

$H = \frac{1}{2}CV^2 - \left[\frac{1}{2}C\times\left(\frac{V}{4}\right)^2 + \frac{1}{2}3C\left(\frac{V}{4}\right)^2\right]$

$\Rightarrow \quad H = \frac{1}{2}CV^2\left[1-\left\{\frac{1}{16}+\frac{3}{16}\right\}\right]$

$= \frac{1}{2}CV^2 \times \frac{3}{4} = \frac{3}{8}CV^2 = \frac{3}{8} \times \frac{Q^2}{C} = \frac{3}{8}\frac{Q^2}{C}$