# TOPIC : RIGID BODY DYNAMICS
## EXERCISE # 1

**SECTION (A)**

**1.** $\frac{d\theta}{dt} = 1.5 + 4t$

$\omega = 9.5$ rad/s

**2.** $\omega_f = \omega_i + \alpha t$

$60 = 0 + \alpha(5)$

$\alpha = 12$

$\theta = \frac{1}{2} at^2 = \frac{1}{2} \times 12 \times (5)^2 = 150$ rad

**3.** Linear speed $V = r\omega$

V depends on radius

**4.** $\theta = \omega t + \frac{1}{2} \alpha t^2 = 10$ rad

**5.** $\omega_0 = 3000$ rad/min

$\omega_0 = \frac{3000}{60}$ rad/sec = (50 rad/sec)

$t = 10$ sec

$\omega_f = 0$

$\omega_f = \omega_0 + \alpha t$

$0 = 50 - \alpha (10)$

$\alpha = 5$ rad/sec$^2$

$0 = \omega_o t + \frac{1}{2} \alpha + 2$

$0 = (50)(10) + \frac{1}{2}(-10)(10)^2$

$500 - 250 = 250$ rad

**6.** $V = \omega R$

$V = 10 \times 0.2 = 2$m /sec.

**7.** Sphere is rotating about a diameter

So, $a = \alpha R$ but, R is zero for particles on the diameter.

**SECTION (B)**

**1.** $I = mR^2 = 4$ kgm$^2$

**2.** M.I of both spheres about common tangent

$I_0 = 2\left[\frac{2}{5}mR^2 + mR^2\right] = \frac{14}{5} mR^2$

$I_0 = 7 I$

**3.** $I_x + I_y = I_z$

z axes is perpendicular to plane of body.

**8.** $I = \frac{MR^2}{2} + 2\left[\frac{3}{2}MR^2\right] = \frac{7}{2}MR^2$

**11** I = $\int dm \quad R^2 = MR^2$

**12** I = $I_{CM} + Md^2$

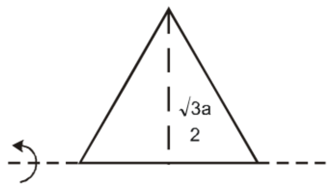
I = $\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$

I = $\frac{ML^2}{3}$

**13.** I = $\frac{ML^2}{3} + \frac{ML^2}{3} = \frac{2ML^2}{3}$

**14.** I = m $\left(\frac{\sqrt{3}a}{2}\right)^2$

I = $\frac{3ma^2}{4}$




**15.** I = $\frac{ML^2}{12} + M\left(\frac{L}{6}\right)^2 = \frac{ML^2}{9}$

**16.** $\frac{I_1}{I_2} = \frac{m_1R_1^2}{m_1R_2^2} = \frac{m_1}{m_2} \times \frac{4}{1} = \frac{2}{1}$

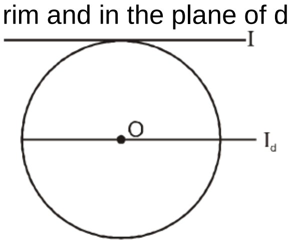
$\frac{m_1}{m_2} = \frac{1}{2}$

**17.** I = $4\left[\frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2\right] = \frac{4}{3}ML^2$

**18.** I = $\frac{2}{5}M \left(\frac{R}{2}\right)^2 + \left[\frac{2}{5}M \left(\frac{R}{2}\right)^2 + M(2R)^2\right]$

I = $\frac{21}{5}$ $MR^2$

**19.** Moment of inertia of a disc about its diameter is $I_d = \frac{1}{4}MR^2$
Now, according to perpendicular axis theorem moment of inertia of disc about a tangent passing through rim and in the plane of disc is



$I = I_d + MR^2 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$

**20.** $I = \frac{5}{4}\ MR^2$

$I' = \frac{3}{2}\ MR^2 = \frac{6}{5}\ I$

**21.** $I = \frac{7}{5}\ MR^2 \quad \Rightarrow \quad MK^2 = \frac{7}{5}\ MR^2 \quad \Rightarrow \quad K = \sqrt{\frac{7}{5}}\ R$

**22.** $I = 4 \times m \left(\frac{L}{\sqrt{2}}\right)^2 = 2\ mL^2$

$mK^2 = 2\ mL^2$

$K = \sqrt{2}\ L$

**23.** $\frac{I_{ring}}{I_{Disc}} = \frac{MR^2}{\frac{MR^2}{2}} = 2$

**24.** $I = \frac{MR^2}{2}$

$K = \frac{R}{\sqrt{2}} = 3.54$ cm

**25.** Moment of inertia of solid sphere about an axis passing through its centre of gravity.

$r = \frac{2}{5}\ mr^2$ Where m = mass of sphere R = radius of sphere

From theorem of parallel axis, moment of inertia about its tangential axis

$I = I' + mR^2 = \frac{2}{5}\ MR^2 + MR^2 = \frac{7}{5}\ MR^2$

**26.** The moment of inertia in rotational motion is equivalent to mass as in linear motion.

**27.** For disc, $I = \frac{1}{2}\ ma^2$ For ring, $I = ma^2$

For square of side 2a = $\frac{M}{12}\ [(2a)^2 + (2a)^2] = \frac{2}{3}\ Ma^2$

For square of rod of length 2a

$I = 4\left[M\frac{(2a)^2}{12} + Ma^2\right] = \frac{16}{3}\ Ma^2$ Hence, moment of inertia is maximum for square of four rods.

**28.** The moment of inertial of the given system that contains 5 particles each of mass 2 kg on the rim of circular disc of radius 0.1 m and of negligible mass is given by = MI of disc + MI of particles

Since, the mass of the disc is negligible therefore, MI of the system = MI of particles

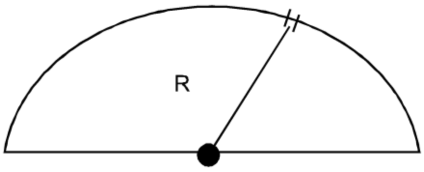
$= 5 \times 2 \times (0.1)^2 = 0.1\ kg\ m^2$

**29.** Moment of inertia of disc about a tangent and parallel to its plane,

$I = \frac{MR^2}{4} + MR^2$ ...(i)

Moment of inertia of disc about a tangent and perpendicular to its plane = $\frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$

$$\frac{I}{I_{perpendicular}} = \frac{\frac{5}{4}MR^2}{\frac{3}{2}MR^2} = \frac{5}{6} \qquad \therefore \qquad I_{perpendicular} = \frac{6}{5}I$$

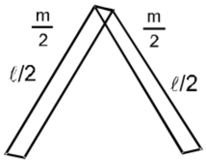
30.



$I = \int dmR^2$

$I = R^2 \int dm = R^2 M = MR^2$

31. $I_B = I_A + Md^2$

Then $I_B > I_A$

32.


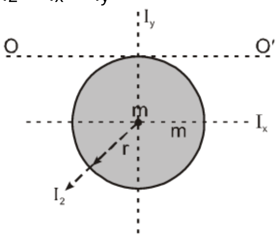

$I_0 = I_1 + I_2 \quad \Rightarrow \quad I_0 = \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} + \frac{(m/2)\left(\frac{\ell}{2}\right)^2}{3} = \frac{(m/\ell^2)}{12}$

33. $I_x + I_y = I_z$

$2I_x = I_z \quad \therefore \quad I_l = 2 \times 200 = 400$ gm cm$^2$.

34. Perpendicular axis theorem

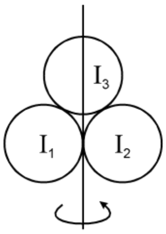
$I_2 = I_x + I_y = \frac{mr^2}{2}$




from symmetry $I_x = I_y \quad \Rightarrow \quad I_x = \frac{mr^2}{4}$ Perallel axis theorem

$I_{oo'} = I_x + mr^2 = \frac{mr^2}{4} + mr^2 = \frac{5}{4}mr^2$

35. $I = I_1 + I_2 + I_3 \quad \Rightarrow \quad I_1 = I_2 = \frac{3}{2}mr^2 \quad \Rightarrow \quad I_3 = \frac{mr^2}{2}$



$\therefore \quad I = I_1 + I_2 + I_3 = \frac{7}{2} mr^2$

Moment of inertia = $3mk^2$ where k is radius of gyration.

$3mk^2 = \frac{7}{2} mr^2 \Rightarrow \quad k = \sqrt{\frac{7}{6}}r$

**37.** Moment of inertia depends on distribution of mass about axis of rotation. Density of iron is more than that of aluminium, therefore for moment of inertia to be maximum, the iron should be far away from the axis. Thus, aluminium should beat interior and iron surrounds it.

**38.** By perpendicular axis theorem moment of inertia about any axis passing through centre and in the plane of plate will be I (by symmetry)

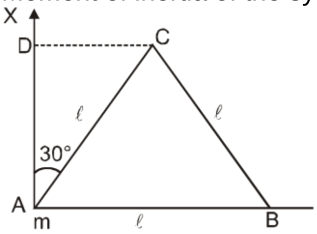
**39.** We know that M.I. of a circular wire of mass M and radius R about its diameter is $\frac{MR^2}{2}$

**40.** If a body has mass M and radius of gyration is K, then $I = MK^2$
Moment of inertia of a disc and circular ring about a tangential axis in their planes are respectively.

$I_d = \frac{5}{4}M_dR^2$ ⇒ $I_r = \frac{3}{2}M_rR^2$ but $I = MK^2$ ⇒ $K = \sqrt{\frac{I}{M}}$

∴ $\frac{K_d}{K_r} = \sqrt{\frac{I_d}{I_r} \times \frac{M_r}{M_d}}$ or $\frac{I_d}{I_r} = \sqrt{\frac{(5/4)M_dR^2}{(3/2)M_rR^2} \times \frac{M_r}{M_d}} = \sqrt{\frac{5}{6}}$ ∴ $I_d : I_r = \sqrt{5} : \sqrt{6}$

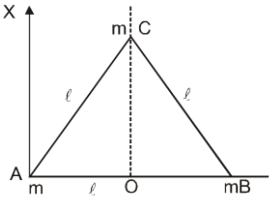
**41.** Moment of inertia of the system about AX is given by




$MI = m_Ar^2_A + m_Br^2_B + m_Cr^2_C$

$MI = m(0)^2 + m(\ell)^2 + m(\ell \sin 30°)^2 = m\ell^2 + \frac{m\ell^2}{4} = \frac{5}{4}m\ell^2$

**Alternative :** Moment of inertia of a system about a line OC perpendicular to AB, in the plane of ABC is



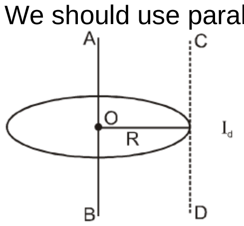

$I_{CO} = m \times 0 + m \times \left(\frac{\ell}{2}\right)^2 + m \times \left(\frac{\ell}{2}\right)^2$ ∴ $I_{CO} = \frac{m\ell^2}{4} + \frac{m\ell^2}{4} = \frac{m\ell^2}{2}$

According to parallel-axis theorem

$I_{AX} = I_{CO} + Mx^2$ where x = distance of AX from CO, M = total mass of system

$I_{AX} = \frac{m\ell^2}{2} + 3m \times \left(\frac{\ell}{2}\right)^2$ ⇒ $I_{AX} = \frac{m\ell^2}{2} + \frac{3m\ell^2}{4} = \frac{5}{4}m\ell^2$

**42.** We should use parallel axis theorem




Moment of inertia of disc passing through its centre of gravity and perpendicular to its plane is

$I_{AB} = \frac{1}{2}MR^2$ Using theorem of parallel axes, we have,

$I_{CD} = I_{AB} + MR^2 = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$

**Note :** The role of moment of inertia in the study of rotational motion is analogous to that of mass in study of linear motion.

**45.** Let same mass and same outer radii of solid sphere and hollow sphere are M and R respectively.
The moment the moment of inertia of hollow sphere (spherical shell) B about its diameter

$I_A = \frac{2}{5} MR^2$ ..................(i)

Similarly the moment of inertia of hollow sphere (spherical shell) B about its diameter

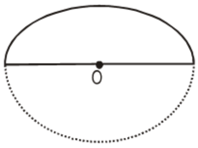
$I_B = \frac{2}{5} MR^2$ ..................(ii)

It is clear from eqs. **(i)** and **(ii)**, $I_A < I_B$

**46.** The mass of complete (circular) disc is

$M + M = 2M$

The moment of inertia of disc is $I = \frac{2Mr^2}{2}$




$= Mr^2$

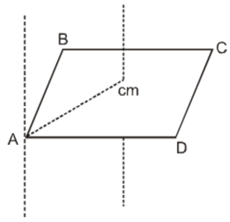
Let the moment of inertia of semicircular disc is $I_1$.
The disc may be assumed as combination of two semicircular parts.

Thus, $I_1 = I - I_1$ ∴ $I_1 = \frac{I}{2} = \frac{Mr^2}{2}$

**47.** $I = 2m(\ell/\sqrt{2})^2 + m(\sqrt{2}\ell)^2 = 3m\ell^2$ .

**48.** $I_{AC} = \frac{1}{2}\left(\frac{M\ell^2}{6}\right) = \frac{M\ell^2}{12}$ , $I_{EF} = \frac{M\ell^2}{12}$ , $I_{AC} = I_{EF}$ .

**49.** $I_A = I_{cm} + m\left(\frac{a}{\sqrt{2}}\right)^2$




$= \frac{ma^2}{6} + \frac{ma^2}{2} = \frac{2}{3} ma^2$

**SECTION (C)**

**2.** $\left(\frac{1}{2}MR^2\right)\alpha = \tau$

$\alpha = 0.25 \text{ rad/s}^2$

**4.** $\tau = I\alpha$

$I = \frac{31.4}{4\pi} = 2.5 \text{ kgm}^2$

**5.** $\ell = 3$ kg m$^2$, $\tau = 6$ Nm, t = 20 sec. From $\tau = I\alpha$

Angular accleration $\alpha = \frac{\tau}{I} = \frac{6}{3} = 2 \text{ rad/sec}^2$ ∴ Angular displacement

$= \omega_0 t + \frac{1}{2}\alpha t^2$ = 0 × 20 + $\frac{1}{2}$ (2)(20)$^2$ = 400 radian

**6.** $F = 4\hat{i} - 10\hat{j} \Rightarrow \vec{r} = (-5\hat{i} - 3\hat{j}) \Rightarrow \tau = \vec{r} \times F = (-5\hat{i} - 3\hat{j}) \times (4\hat{i} - 10\hat{j}) = 50\hat{k} + 12\hat{k} = 62\hat{k}$

**7.** $\vec{F} = 2\hat{i} + 3\hat{j} - \hat{k}$ at point (2,–3,1) torque about point (0, 0, 2)

$\vec{r} = (2\hat{i} - 3\hat{j} + \hat{k}) - 2\hat{k} \Rightarrow \vec{\tau} = \vec{r} \times \vec{F} = (2\hat{i} - 3\hat{j} - \hat{k}) \times (2\hat{i} + 3\hat{j} - \hat{k})$

$\vec{\tau} = (6\hat{i} + 12\hat{k}) \Rightarrow |\vec{\tau}| = (6\sqrt{5})$

**8.** Torque of a couple always remains constant about any point

**9.** $\tau = I\alpha$

$30 = 2 \times \alpha \Rightarrow \alpha = 15$ rad/sec$^2$

From equation of angular motion

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2} \times 15 \times 10 \times 10 = 750$ rad

**10.** $\vec{\tau} = \vec{r} \times \vec{F}$ implies that r, F and τ all are mutually perpendicular to each other. ∴ $\vec{\tau}.\vec{r} = 0$, $\vec{F}.\vec{\tau} = 0$

**SECTION (D)**

**1.** Torque about O

F × 40 + F × 80 – (F × 20 + F × 60)

In clockwise direction = F × 40

**2.** $\vec{F_1} = 2i + 3j + 4k$

$\vec{F_2} = -2i - 3j - 4k$

$\vec{r_1} = 3i + 3j + 4k$ $\vec{r_2} = i$

$\vec{\tau_1} = \vec{r_1} \times \vec{F_1} = (3\hat{i} + 3\hat{j} + 4\hat{k}) \times (2\hat{i} + 3\hat{j} + 4\hat{k})$

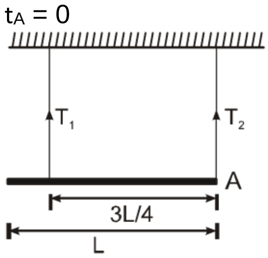
$\vec{\tau_1} = 9\hat{k} - 12\hat{j} - 6\hat{j} + 12\hat{i} + 8\hat{j} - 12\hat{i}$

$\vec{\tau_1} = -4\hat{j} + 3\hat{k}$

$\vec{\tau_2} = \vec{r_2} \times \vec{F_2} = (\hat{i}) \times (-2\hat{i} - 3\hat{j} - 4\hat{k}) = -3\hat{k} + 4\hat{j}$

$\left(\vec{\tau_1} + \vec{\tau_2} = -4\hat{i} + 3\hat{k} - 3\hat{k} + 4\hat{j} = 0\right)$ body in rotation equilibrium

**3.** $t_A = 0$

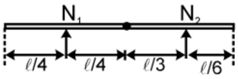



$T_1 \times \frac{3L}{4} - mg\frac{L}{2} = 0$ ....(1)

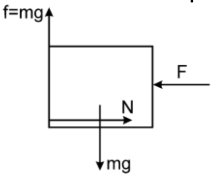
$T_1 = \frac{2mg}{3}$

$T_1 + T_2 = mg$ ....(2)

$T_2 = \frac{mg}{3}$ $\Rightarrow$ $\frac{T_1}{T_2} = \frac{2}{1}$ **Ans.**

**4.**


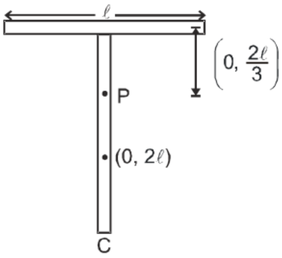

For rotational equilibrium $N_1 \times \frac{\ell}{4} = N_2 \times \frac{\ell}{6}$ $\Rightarrow$ $N_1 : N_2 = 4 : 3$

**5.**



From equilibrium,
friction = mg N = F
about centre of mass
$\tau = 0$ $\Rightarrow$ mg a = torque due to normal
$\therefore$ Normal will produce torque since F passes through centre its torque is zero.

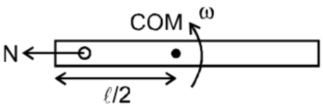
**6.** For pure translatory motion, net torque about centre of mass should be zero. Thus $\vec{F}$ is applied at centre of mass of system.

$P = \frac{0 \times \ell + \ell.2\ell}{\ell + 2\ell} = \frac{2\ell^2}{3\ell} = \frac{2\ell}{3}$




$\therefore$ $PC = \left(\ell - \frac{2\ell}{3} + \ell\right) = \frac{4\ell}{3}$

**SECTION (E)**

**1.**


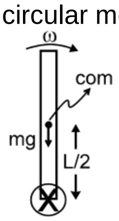

$N = \left(m\omega^2 \frac{\ell}{2}\right)$

**2.** Initial velocity of each point onthe rod is zero so angular velocity of rod is zero.
Torque about O
$T = I\,\alpha$

$20g\,(0.8) = \frac{m\ell^2}{3}\alpha$ $\Rightarrow$ $20g\,(0.8) = \frac{20\ (1.6)^2}{3}\alpha$


$\Rightarrow$ $\frac{3\ g}{3.2} = \alpha$ = angular acceleration

**3.** For the circular motion of com :

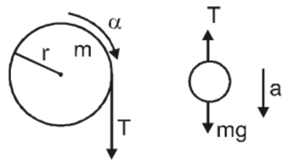


$mg = m\left(\frac{L}{2}\right)\omega^2 \Rightarrow \omega = \sqrt{\frac{2g}{L}}$

**Note :** Since the reaction at the end is zero, the gravitational force will have to provide the required centripetal force.

**5.** Beam is not at rotational equilibrium, so force exerted by the rod (beam) decrease

**6.**




There is no slipping between pulley and thread.

So, $(a = \alpha r)$ ..........(i)

For point mass :

$mg - T = ma$ ...........(ii)

Equation of torque for disc

$Tr = I.\alpha$

$Tr = \frac{mr^2}{2}.\alpha$

$T = \frac{mr\alpha}{2} = \left(\frac{mg}{2}\right)$ ...........(iii)

$mg - \frac{mg}{2} = ma \Rightarrow mg = \frac{3mg}{2} \Rightarrow a = \frac{2g}{3}$.

**SECTION (F)**

**2.** $KE = \frac{1}{2} I\omega^2$

$1500 = (1.2)\,\omega^2$

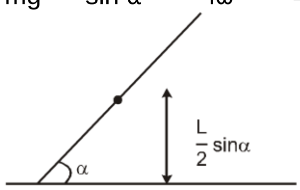
$\omega = \sqrt{\frac{3000}{1.2}}$

$\omega_f - \omega_i = \alpha t$

$\sqrt{\frac{3000}{1.2}} = 25\,t$

$t = 2$ s

**3.** $mg\frac{L}{2}\sin\alpha = \frac{1}{2}I\omega^2 \Rightarrow mg\frac{L}{2}\sin\alpha = \frac{1}{2}\frac{ML^2}{3}\omega^2$




$\omega = \sqrt{\frac{3g\sin\alpha}{L}}$

**4.** $mgh = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 \Rightarrow mgh = \frac{1}{2}mr^2\omega^2 + \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{2mgh}{I+mr^2}}$

**6.** Given $a_A = 2\alpha = 5$ m/s$^2$ $\Rightarrow \alpha = 5/2$ rad/s$^2$ $\Rightarrow a_B = 1.(\alpha) = 5/2$ m/s$^2$

**7.** Immediately after string connected to end B is cut, the rod has tendency to rotate about point A.

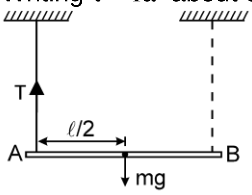
Torque on rod AB about axis passing through A and normal to plane of paper is

$$\frac{m\ell^2}{3}\alpha = mg\frac{\ell}{2} \Rightarrow \alpha = \frac{3g}{2\ell}$$

Aliter : Applying Newton's law on center of mass

mg – T =ma .....(i)

Writing τ = Iα about center of mass




$T\frac{\ell}{2} = \frac{m\ell^2}{12}\alpha$ ....(ii)

Also $a = \frac{\ell}{2}\alpha$ ....(iii) From (i) , (ii) and (iii) $\alpha = \frac{3g}{2\ell}$

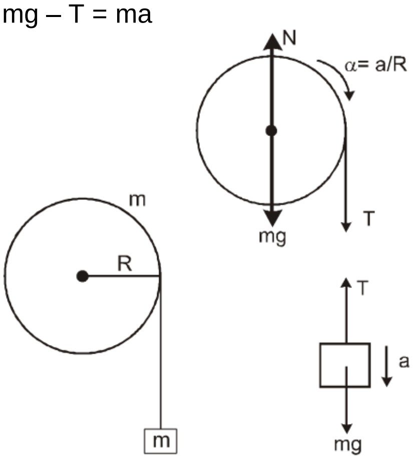
**8.** Given : I = 2kg - m$^2$, $\omega_0 = \frac{60}{60}\times 2\pi$ rad/s,

ω = 0, t = 60s

The torque required to stop the wheel's rotation is

$\tau = I\alpha = I\left(\frac{\omega_0 - \omega}{t}\right) \Rightarrow \tau = \frac{2\times 2\pi\times 60}{60\times 60} = \frac{\pi}{15}$ N-m

**10.** mg – T = ma




$TR = \frac{mR^2\alpha}{2} \Rightarrow T = \frac{mR\alpha}{2} = \frac{ma}{2} \Rightarrow mg - \frac{ma}{2} = ma \Rightarrow \frac{3ma}{2} = mg \Rightarrow a = \frac{2g}{3}$ **Ans.**

**11.** Since the work done is independent of the information about which point the rod is rotating, by work-energy theorem the kinetic energy will also be independent of the same. Hence (2)

**12.** From conservation of angular momentum (Iω = constant), angular velocity will remain half.

As, $K = \frac{1}{2}I\omega^2$ the rotational kinetic energy will become half. Hence, the correct option is (2).

**SECTION (G)**

**3.** $\frac{E_A}{E_B} = \frac{\frac{1}{2}I_A\omega_A^2}{\frac{1}{2}I_B\omega_B^2} = \frac{L_A^2}{I_A}\frac{I_B}{L_B^2} \Rightarrow \frac{L_A}{L_B} = 5$

**4.** $I_A\,\omega_A = I_B\,\omega_B$

$I_A > I_B$ therefore $\omega_A < \omega_B$

$$\frac{K_B}{K_A} = \frac{\frac{1}{2}I_B \quad \omega_B^2}{\frac{1}{2}I_A \quad \omega_A^2} = \frac{\omega_B}{\omega_A} < 1$$

**7.** $\tau = \frac{\Delta L}{\Delta t} = \frac{2}{5} = 0.4 \text{ N} - \text{m}$

**8.** Direction of Angular momentum is along the direction of angular velocity, which is an axial vector.

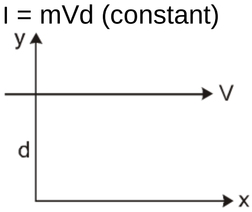
**13.** $MR^2\omega = (MR^2 + 2mR^2)\omega'$

$\omega' = \frac{\omega M}{M + 2m}$

**15.** $I\omega$ = cosntant

**16.** External torque = 0
L = constant.

**19.** $I = mVd$ (constant)




**21.** $KE = \frac{1}{2} I\omega^2 \quad \Rightarrow \quad 360 = \frac{1}{2} I (30)^2$
$I = 0.8 \text{ kg m}^2$

**22.** $L = I\omega$
$L = (mr^2)\,\omega$

$F_c = mr\omega^2 = \frac{L^2}{mr^3}$

**23.** $\vec{L} = \vec{r} \times \vec{P}$

**24.** The angular momentum of a particle of mass m moving with velocity $\upsilon$ about origin is
$J = m\upsilon \times d = m\,\upsilon\,d$ = constant

**25.** From law of conservation of angumar momentum, if no external toruque is acting upon a body rotating about an aixs, then the angular momentum of the body remains constant that is
$J = I\omega$

Also, $I = \frac{2}{5}MR^2$ for a solid sphere.

Given, $R_1 = R$ $R_2 = \frac{R}{n}$ $\therefore$ $\frac{2}{5} MR^2\omega_1 = \frac{2}{5}M\left(\frac{R}{n}\right)^2 \times \omega_2$ $\Rightarrow$ $\omega_2 = n^2\omega_1 = m^2\omega$

**26.** Angular momentum
$L = I\omega$
Kinetic energy

$K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$ {from Eq. (i)} $\therefore$ $L = \frac{2K}{\omega}$

Now $L' = \frac{2\left(\frac{K}{2}\right)}{2\omega}$ $\Rightarrow$ $L' = \frac{L}{4}$

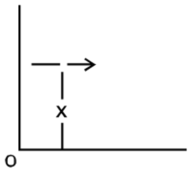
**27.** $KE_R = \frac{1}{2}I\omega^2$

$L = I\omega$

$KE_R = \frac{L^2}{2I}$.

**28.** $\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{4A_0 - A_0}{4} = \left(\frac{3A_0}{4}\right)$

**29.**




$\Rightarrow$ L = (mvx) = cont: because v = cont and x = cont.

**30.** $L = I\omega \quad \Rightarrow \quad \omega' = 2\omega$

$\frac{1}{2}\left(\frac{1}{2}I\omega^2\right) = \frac{1}{2} I'\omega'^2 \quad \Rightarrow \quad \frac{I\omega^2}{2} = I' 4\omega^2$

$I' = \left(\frac{I}{8}\right) \quad \Rightarrow \quad L' = I'\omega' = \frac{I}{8} 2\omega = \frac{I\omega}{4} = \left(L/4\right)$

**31.** External torque $\vec{\tau}_{ext} = 0$

$I_1\omega_1 = I_2\omega_2$ when he folds his arms I reduces. So, $I_1 > I_2$ then $(\omega_1 < \omega_2)$. So, (L = constant)

**32.** The direction of L is perpendicular to the line joining the bob to point C. Since this line keeps changing its orientation in space, direction of L keeps changing however as ω is constant, magnitude of L remain constant.

**Aliter :** The torque about point is perpendicular to the angular momentum vector about point C. Hence it can only change the direction of L, and not its magnitude.

**33.** The angular momentum about axis CO is the component of angular momentum about point C along the line CO. This is constant both in direction and magnitude.

**Aliter :** Torque about axis CO is zero hence L about CO is constant in both direction and magnitude.

**35.** In uniform circular motion the only force acting on the particle is centripetal (towards center). Torque of this force about the center is zero. Hence angular momentum about center remain conserved.

**37.** Conservation of angular momentum about C.O.M. of m of loop of mass m gives

$\frac{mVR}{2} = \left[\left\{m \ R^2 + m\left(\frac{R}{2}\right)^2\right\} + m\left(\frac{R}{2}\right)^2\right]\omega \quad \Rightarrow \quad V = 3\omega R \quad \Rightarrow \quad \omega = \frac{V}{3R}$ **Ans. (B)**

**SECTION (H)**

**1.** $KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 = \frac{1}{5}mV^2 \quad \Rightarrow \quad KE_{Total} = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \frac{7}{5}mV^2 \quad \Rightarrow \quad$ Ratio = $\frac{2}{7}$

**3.** $KE = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mV^2 + \frac{1}{2}mR^2\left(\frac{V}{R}\right)^2 \quad \Rightarrow \quad KE = mV^2 = 4$ Joule

**4.** $KE_t = \frac{1}{2}mV^2 \quad \Rightarrow KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}$

$\Rightarrow \left(\frac{1}{2}mR^2\right)\left(\frac{V}{R}\right)^2 = \frac{1}{4}mV^2 \qquad \frac{KE \ \ rotation}{KE \ \ Total} = \frac{1/4 \ \ mV^2}{3/4 \ \ mV^2} = \frac{1}{3}$

**6.** $mgh = \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2 \Rightarrow \quad mgh = \frac{1}{2} mV^2 + \frac{1}{2} \frac{mR^2}{2}\left(\frac{V}{R}\right)^2$

$\Rightarrow \quad mgh = \frac{1}{2} mV^2 + \frac{1}{4} mV^2 \quad \Rightarrow \quad V = \sqrt{\frac{4}{3}gh}$

**7.** The total energy of a body rolling without slipping

$K_{total} = K_{rot} + K_{trans} = \frac{1}{2} I\omega^2 + \frac{1}{2} M\upsilon^2$ but $I_{disc} = \frac{1}{2} Mr^2$ and $\upsilon = r\omega$

$\therefore K_{total} = \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 + \frac{1}{2} M(r\omega)^2 = \frac{1}{4} Mr^2\omega^2 + \frac{1}{2} Mr^2\omega^2 = \frac{3}{4} Mr^2\omega^2 = \frac{3}{4} M\upsilon^2$

**8.** The velocity of solid sphere on the bottom of inclined plane is

$\upsilon = \sqrt{\left[\frac{2gh}{(1 + I/MR^2)}\right]}$ where, I = M.I of sphere,

M = mass of sphere and R = radius of sphere

The moment of inertia so solid sphere about its diameter

$I = \frac{2}{5} MR^2 \quad \therefore \quad \upsilon = \sqrt{\left[\frac{2gh}{\left(1 + \frac{2}{5}\right)}\right]} = \sqrt{\left(\frac{10}{7}gh\right)} \quad \therefore \quad h = \frac{7\upsilon^2}{10g}$

**9.** The kinetic energy of sphere while rolling on an inclined plane is given by

$E_g$ = Translational kinetic energy + Rotational kinetic energy = $\frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$

$E_K = \frac{1}{2} \times 2 \times (0.5)^2 + \frac{1}{2} \times \frac{2}{3} \times 2 \times (0.1)^2 \times \left(\frac{0.5}{0.1}\right)^2 = 0.25 + 0.17 = 0.42$ J

**10.** In pure rolling, mechannical energy remains conserved. Therefore, when heights of inclines are equal, speed of sphere will be same in both the cases. But as acceleration down the plane, $a \propto \sin\theta$ therefore, acceleration and time of descent will be different.

**11.** Friction force is zero and impulas is pases through centre so external torque is zero and angular momentum and linear momentum remains const. so

$\Rightarrow (\omega_A = \omega)$ but due to collision linear velocity inter change.

**12.** For a body rolling without slipping, the velocity of any point P on the body is $\vec{v}_p = \vec{v}_{cm} + \vec{v}_{p,cm}$ where $\vec{v}_{p,cm}$ = Rω in direction perpendicular to line joining centre and point P.

Velocity of point A is ,

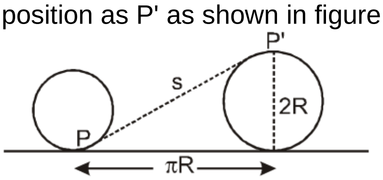
$v_A = v_{cm} + R\omega = v_{cm} + v_{cm} \quad \because (v_{cm} = R\omega) = 2v_{cm}$

velocity of point B is,

$v_B = v_{cm} - R\omega = v_{cm} - v_{cm} = 0$

Thus, the velocity of point a is $2v_{cm}$ and velocity of point B is zero.

**13.** When the wheel rolls on the ground without slipping and completes half rotation, point P takes new position as P' as shown in figure. Horizontal displacement, x = πR




Vertical displacement, y= 2R

Thus, displacement of the point P when wheel completes half rotation,

$s = \sqrt{x^2 + y^2} = \sqrt{(\pi R^2) + (2R)^2} = \sqrt{\pi^2 R^2 + 4R^2}$ but R = 1m (given) $\therefore s \sqrt{\pi^2(1)^2 + 4(1)^2} = \sqrt{\pi^2 + 4}$ m

**14.** Linear kinetic energy

$K_\ell = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\omega^2 = \frac{1}{2}I\omega^2$

Rotational kinetic energy $K_r = \frac{1}{2}I\omega^2$

Total kinetic energy $K = K_\ell + K_r = \frac{1}{2}I\omega^2 + \frac{1}{2}I\omega^2 = I\omega^2$

$\frac{K_r}{K} = \frac{1}{2}$

**16.** Moment of inertia of sphere

$I = \frac{2}{5}MR^2$ … (1)

and for pure rolling $\upsilon = R\omega$ …(2)

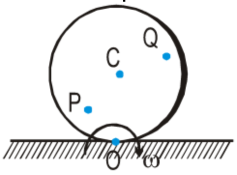
$$\frac{\text{Rotational KE}}{\text{Translational KE}} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}m\upsilon^2} \quad \text{from (1) and (2)} \quad \frac{\frac{1}{2}\times\frac{2}{5}MR^2\omega^2}{\frac{1}{2}MR^2\omega^2} = \frac{2}{5}$$

**17.** Since the inclined plane is frictionless, then there will be no rolling and the mass will only slide down

Hence acceleration $a = g\sin\theta$ is same for solid sphere, hollow sphere and ring.

$$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$$

**18.** In case of pure rolling bottommost point is the instantaneous centre of zero velocity.

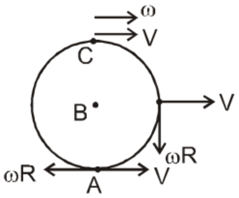



Velocity of any point on the disc, $v = r\omega$, where r is the distance of point from O.

$r_Q > r_C > r_P$ ∴ $v_Q > v_C > v_P$

Therefore, the correct option is (1).

**19.**




$\vec{V}_A = V(\hat{i}) + \omega R(-\hat{i})$; $\vec{V}_B = V\hat{i}$ ; $\vec{V}_C = V\hat{i} + \omega R\hat{i}$

$\vec{V}_C - \vec{V}_A = 2\omega R\hat{i}$

$2\left[\vec{V}_B - \vec{V}_C\right] = 2\ [\ V(\hat{i}) - V(\hat{i}) - \omega R(\hat{i})] = -2\omega R(\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = -2(\vec{V}_B - \vec{V}_C)$

so $|\vec{V}_C - \vec{V}_A| = |2(\vec{V}_B - \vec{V}_C)|$

$\vec{V}_C - \vec{V}_B = \omega R(\hat{i})$

$\vec{V}_B - \vec{V}_A = \omega R(\hat{i})$

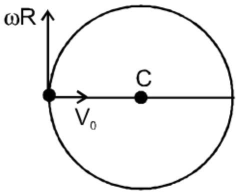
$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

Hence $\vec{V}_C - \vec{V}_A = 2\omega R(\hat{i})$

$\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$ ; $4\vec{V}_B = 4V(\hat{i}) = 4\omega R\ (\hat{i})$

Hence $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B)$

**20.**



For pure rolling $\omega R = \upsilon_0$, $\upsilon = \sqrt{v_0^2 + (\omega R)^2} = \left(v_0\sqrt{2}\right)$

**21.** Since the two bodies have same mass and collide head-on elastically, the linear momentum gets interchanged.

Hence just after the collision 'B' will move with velocity '$v_0$' and 'A' becomes stationary but continues to rotate at the same initial angular velocity $\left(\frac{v_0}{R}\right)$. Hence, after collision.

$(K.E.)_B = \frac{1}{2}mv_0^2$ and $(K.E.)_A = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{2}{3}mR^2\right) \cdot \left(\frac{v_0}{R}\right)^2 \Rightarrow \frac{(K.E.)_B}{(K.E.)_A} = \frac{3}{2}$  Hence (4).

**Note :** Sphere 'B' will not rotate, because there is no torque on 'B' during the collision as the collision is head-on.

**22.** $a = \left(\frac{g\sin\theta}{1+\frac{k^2}{R^2}}\right)$ For solid sphere $\Rightarrow I_{cm} = \frac{2}{5}mk^2 \Rightarrow k_s = R\sqrt{2/5}$

For hollow sphere = $\frac{2}{3}mR^2 = mk^2 \Rightarrow k_H = r\sqrt{2/3}$

So, $k_s < k_H$ then $a_s > a_H$ (so speed of solid sphere is greater then hollow sphere )

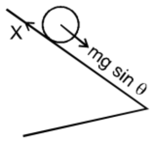
**23.** $a = \left(\frac{g\sin\theta}{1+\frac{k^2}{R^2}}\right)$

For solid sphere $\Rightarrow I_{cm} = \frac{2}{5}mk^2 \Rightarrow k_s = R\sqrt{2/5}$

For hollow sphere = $\frac{2}{3}mR^2 = mk^2 \Rightarrow k_H = r\sqrt{2/3}$

So, $k_s < k_H$ then $a_s > a_H$ (so speed of solid sphere is greater then hollow sphere)

**24.**




$mg\sin\theta - f = ma$

$a = \frac{mg\sin\theta - f}{m}$. a is equal for each body so all the object will reach at same time.

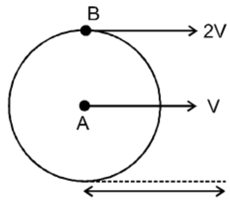
**25.** Let velocity of c.m. of sphere be v. The velocity of the plank = 2v.

Kinetic energy of plank = $\frac{1}{2} \times m \times (2v)^2 = 2mv^2$

Kinetic energy of cylinder = $\frac{1}{2}mv^2 + \frac{1}{2} + \left(\frac{1}{2}mR^2\omega^2\right) = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right) = \frac{3}{2}.\frac{1}{2}mv^2$

$\therefore \frac{\text{K.E. of plank}}{\text{K.E. of sphere}} = \frac{2mv^2}{\frac{3}{4}mv^2} = \frac{8}{3}$.

**26.** The horizontal shift of end x will be double the shift of centre of spool. Hence centre travels by $\frac{S}{2}$.

**27.** 


When A point travels $\ell$ distance then B point $2\ell$ so, $2\ell$ length of string passes through the hand of the boy .

**SECTION (I)**

**1.** Total KE = $\frac{1}{2}MV^2 + \frac{1}{2}I\omega^2 = \frac{3}{4}MV^2$

$\Delta KE = \frac{3}{4}M(4V^2 - V^2) = \frac{9}{4}MV^2$

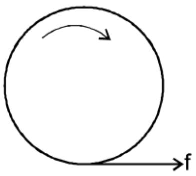
**2.** $a = \frac{g \sin\theta}{1+\left(\frac{K}{R}\right)^2} \Rightarrow$ K (solid sphere) = $\sqrt{\frac{2}{5}}\,R \Rightarrow$ K (solid cylinder) = $\frac{R}{\sqrt{2}} \Rightarrow \frac{a_{ss}}{a_{sc}} = \frac{15}{14}$

**3.** Both spheres reach the ground due to vertical component of velocity. As initial component of velocity of both spheres is zero, therefore both will reach the earth silmultaneously. The time taken being given by:

$h = u_y t + \frac{1}{2}gt^2 = 0 + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$

**4.** There is no relative motion between sphere and plank so friction force is zero then no any change in motion of sphere and plank.

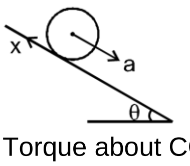
**5.** Due to linear velocity body will more forword before pure rolling.

**6.** 


Friction will act in forward direction so body will always move in forward direction.

**7.** Disc in pure rolling and external force zero after smooth surface pure rolling continue.

**8.** mg sin θ component is always down the plane whether it is rolling up or rolling down. Therefore, for no slipping, sense of angular acceleration should also be same in both the cases. Therefore, force of friction f always act upwards.

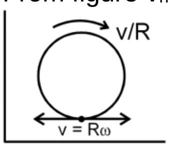
**9.** 


Torque about COM

$f.R = I \cdot \alpha \quad (a = \alpha R) \Rightarrow F.R = \frac{mR^2}{2}a = \left(\frac{mR^2}{2}\cdot R\right) \Rightarrow \left(f = \frac{ma}{2}\right)$

**10.** As the disc is in combined rotation and translation, each point has a tangential velocity and a linear velocity in the forward direction.

From figure $v_{net}$ (for lowest point) = v – Rω = v – v = 0 and Acceleration = $\frac{v^2}{R} + 0 = \frac{v^2}{R}$

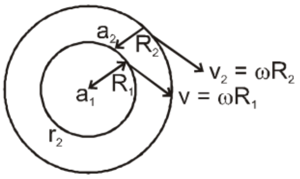



(Since linear speed is constant). Hence (4).

**11.** When a body rolls down without soliiping along an inclined plane of inclination θ, it rotates about a horizontal axis through its centre of mass and also its centre of mass moves. Therefore, rolling motion may be regarded as a rotational motion about an axis through its centre of mass plus a translational motion of the centre of mass. As it rolls down, it suffers lossin gravitational potential energy provided translational energy due to frictional force is converted into rotational energy.
**Note :** In fact, friction is needed to cause the body to roll. However the rolling friction is so small that we can use conservation of mechanical energy.

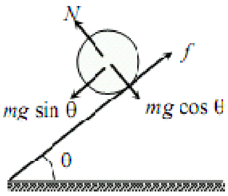
**12.** $ma_1 = \frac{mv_1^2}{R_1}$ ........ (i)

and $ma_2 = \frac{mv_2^2}{R_2}$ ........ (ii)




and $\frac{F_1}{F_2} = \frac{ma_1}{ma_2} = \frac{R_2}{mR_2^2\omega^2}$ ∴ $\frac{F_1}{F_2} = \frac{R_1}{R_2}$

**13.** $mg\sin\theta - f = ma_{CM}$ ..........(i)
$f.R = I\alpha$ ..........(ii)
$a_{CM} = R\alpha$ ..........(iii)
On solving (i),(ii) & (iii)



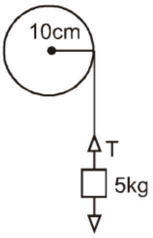

$a_{CM} = \frac{g\sin\theta}{1+\frac{I}{MR^2}}$.

**14.** a = (g tan θ) so net force along the indined plane is zero so it will continue in pure rolling with constant angular velocity.

**SECTION (J)**

**1.** As the inclined plane is smooth, the sphere can never roll rather it will just slip down.
Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

**2.** $KE = \frac{1}{2} mv^2 (1 + 1) = mv^2 = 0.4 \times 0.1^2 = 4 \times 10^{-3}$ .




**3.**

$50 - T = 5a$ (1)
$TR = I\alpha = Ia/R$ (2)

$Ia = TR^2$

Now. $5 = \frac{1}{2}\ a \times 10 \times 10$ so $a = \frac{1}{10} = 0.1$ m/sec$^2$

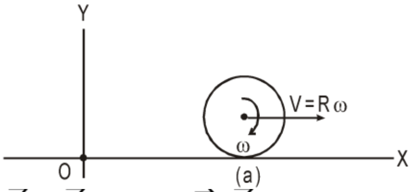
So, $50 - T = 0.5$

So, $T = 50 - 0.5 = 49.5$

So, $I \times 0.1 = \frac{49.5}{10 \times 10}$

So, $I = 4.95$ kg m$^2$ None of the answer given.

**4.** From the theorem –

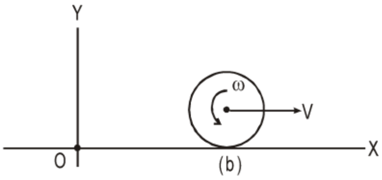



$\vec{L}_0 = \vec{L}_{com} + M\ (\vec{r} \times \vec{V})$ .......(1)

We may write

Angular momentum about O = Angular momentum about COM + Angular momentum of COM about origin

$\therefore \quad L_0 = I_\omega + MRV = \frac{1}{2}\ MR^2\,\omega + MR(R\omega)$

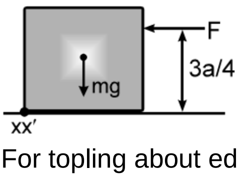



$= \frac{3}{2}\ MR^2\omega$

Note that in this case both the terms in equation (1) i.e. $\vec{L}_{com}$ and $M(\vec{r} \times \vec{V})$ have the same direction ⊗. That is why we have used $L_0 = I_\omega + MRV$. We will use $L_0 = I\ \omega \sim MRV$ if they are in opposite directions shown in figure (2).

**5.** As the inclined plane is smooth, the sphere can never roll rather it will just slip down. Hence, the angular momentum remains conserved about any point on a line parallel to the inclined plane and passing through the centre of the ball.

**SECTION (K)**

**1.** 

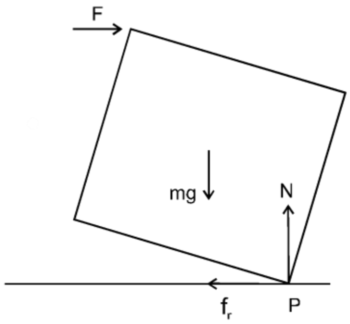

For topling about edge xx′ __________

$F_{min.}\ \frac{3a}{4} = mg\ \frac{a}{2} \quad \Rightarrow \quad F_{min.} = \frac{2mg}{3}$.

**2.** At the critical condition, normal reaction N will pass through point P. In this condition

$\tau_N = 0 = \tau_{fr}$ (About P)

the block will topple when

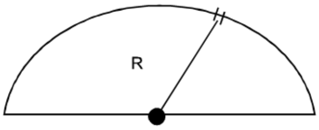



$\tau_F > \tau_{mg}$

or $\quad FL > (mg)\dfrac{L}{2} \quad \therefore \quad F > mg / 2$

# EXERCISE # 2

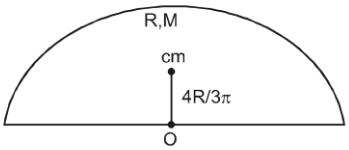
**1.**




by parellel axis theorem $I = I_{cm} + md^2$

$M r^2 = I_{CM} + M\left(\dfrac{2R}{\pi}\right)^2 \quad \Rightarrow \quad I_{CM} = Mr^2\left(1 - \dfrac{4}{\pi^2}\right)$

**2.**




$I_0 = I_{CM} + Md^2 \quad \Rightarrow \quad \dfrac{1}{2}MR^2 = I_{CM} + M\left(\dfrac{4R}{3\pi}\right)^2 \quad \Rightarrow \quad I_{CM} = MR^2\left(\dfrac{1}{2} - \dfrac{16}{9\pi^2}\right)$

**3.** Mass of the ring $M = \rho L$

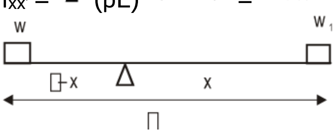
Let R be the radius of the ring. Then $L = 2\pi R$ or $R = \dfrac{L}{2\pi}$

Moment of inertia about an axis passing through O, and parallel to XX' will be $I_0 = \dfrac{1}{2} MR^2$

Therefore, moment of inertia about XX' (from parallel axis theorem) will be given by :

$I_{xx'} = \dfrac{3}{2}(\rho L)\left(\dfrac{L^2}{4\pi^2}\right) = \dfrac{3\rho L^3}{8\pi^2}$

**4.**




weight of object = w

$w(\ell - x) = w_1 x$ ...........(i)

If weight is kept in another pan then :

$w_2(\ell - x) = wx$ ...........(ii)

By (i) & (ii)

$\dfrac{w}{w_2} = \dfrac{w_1}{w} \quad \Rightarrow \quad w^2 = w_1 w_2 \quad \Rightarrow \quad w = \sqrt{w_1 w_2}$ .

**5.** **key Idea :** The net moment about point of contact between ground and ladder should ge zero. Let (as shown in figure) AB be a ladder and F be the horizontal force to keep it from slipping. w is the weight of man. Suppose $N_1$ and $N_2$ be normal reactions of ground and wall respectively.

In horizontal equlibrium,

$N_1 = F$

In vertical equilibrium,.

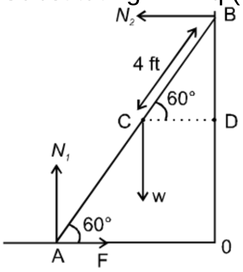
$N_1 = w$

Taking moments about A;

Clockwise torque = Anticlockwise torque $N_1 \times CD = N_2 \times OB$ but in ΔAOB, $\sin 60° = \frac{OB}{AB}$

In ΔAOB, $\cos 60° = \frac{CD}{BC}$ ⇒ $CD = BC \cos 60°$

Substitutingh in Eq.(i), we have $N_1 \times BC \cos 60° = N_2 \times AB \sin 60°$




⇒ $w \times BC \times \frac{1}{2} = F \times AB \times \frac{\sqrt{3}}{2}$. Given : w = 150 pouns, AB = 20 ndr., BC = 4 mft.

∴ $150 \times 4 \times \frac{1}{2} = F \times 20 \times \frac{\sqrt{3}}{2}$ ⇒ $F = \frac{150 \times 4 \times \sqrt{3}}{20 \times 3}$ = 17.3 pound

**6.** Net external torque on the system is zero. Therefore, angular momentum is conserved. Forces acting on the system are only conservative. Therefore, total mechanical energy of the system is also conserved.

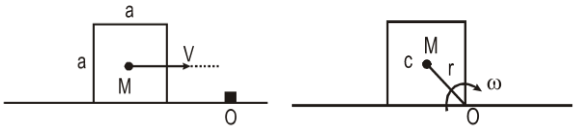
**7.** $\vec{\tau} \times \vec{L}$ then $\vec{\tau} \parallel \vec{L}$ so $|\vec{L}|$ may increase

**8.** Here, $u = V_0$, $\omega_0 = -\frac{V_0}{2R}$ At pure rolling $V = V_0 - \left(\frac{F_f}{m}\right) t$

& $\frac{V}{R} = -\frac{V_0}{2R} + \left(\frac{F_f}{m.R}\right) t$ (In pure rolling $V = R\omega$) ($\alpha = \frac{\tau}{I} = \frac{F_f.R}{mR^2}$)

⇒ $V_0 - V = V + \frac{V_0}{2}$ ⇒ $2V = \frac{V_0}{2}$ ⇒ $V = \frac{V_0}{4}$ **Ans.**

**9.**




Net torque about O is zero.

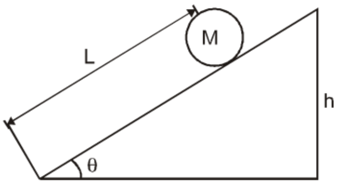
Therefore, angular momentum (L) about point O will be conserved,

or $L_i = L_f$

$MV\left(\frac{a}{2}\right) = I_0\omega = (I_{com} + Mr^2)\omega = \left\{\left(\frac{Ma^2}{6}\right) + M\left(\frac{a^2}{2}\right)\right\}\omega = \frac{2}{3} Ma^2 \omega$ ⇒ $\omega = \frac{3V}{4a}$

**10.** The linear acceleration of centre of mass will be $a = \frac{F}{m}$ wherever the force is applied. hence, the acceleration will bes ame whatever the value of h may be

**11.** The situation is shown in the figure.




Potential energy of cylinder at the top will be converted into rotational kinetic energy and translational kinetic energy. So, energy conservation gives,

Mgh = $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ = $\frac{1}{2}mv^2 + \frac{1}{2}\frac{MR^2}{2}\frac{v^2}{R^2}$ $\left(\because\ I_{cylinder} = \frac{MR^2}{2}\right)$ So, Mgh = $\frac{1}{2}Mv^2 + \frac{1}{4}mv^2$

or Mgh = $\frac{3}{4}Mv^2$ or $v^2 = \frac{4}{3}gh$ or $v = \sqrt{\frac{4}{3}gh}$

**12.** Mass of disc (X), mx = πR2tρ
Where ρ = density of material of disc

∴ IX = $\frac{1}{2}$ mXR2 = $\frac{1}{2}$ R2tρR2

IX = $\frac{1}{2}$ πρR4 .......(i) Again mass of disc (Y)

my = π = (4R)2 $\frac{t}{4}$ ρ= 4πR2 tρ and IY = $\frac{1}{2}$ mY (4R2) = $\frac{1}{2}$ 4πR2 tρ.16R2

⇒ IY = 32πtρR4 ......(ii) ∴ $\frac{I_Y}{I_X} = \frac{32\pi t\rho R^4}{\frac{1}{2}\pi\rho t R^4}$ = 64 ∴ IY = 64 Ix

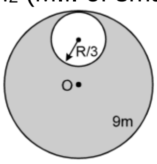
**13.** Mass of the whole disc = 4 M

Moment of inertia of the disc about the given axis = $\frac{1}{2}$ (4M)$R^2$ = 2M$R^2$

∴ Moment of inertia of quarter section of the disc = $\frac{1}{4}$ (2M$R^2$) = $\frac{1}{2}$ M$R^2$

These type of questions are often asked in objective. Students generally err in taking mass of the whole disc. They take it M instead of 4 M.

**14.** $I_0 = I_1 - I_2$ where $I_1$ = (M.I. of full disc about O)
$I_2$ (M.I. of small removed disc about O) since mass α area




$\frac{\text{mass of cut disc}}{\text{mass of total}} = \frac{\frac{R^2}{9}}{R^2} = \frac{1}{9}$ ∴ mass of cut disc = m

∴ $I_0 = \frac{(9m)R^2}{2} - m\left[\frac{\left(\frac{R}{3}\right)^2}{2} + \left(\frac{2R}{3}\right)^2\right]$ (by theorem of parallel axis.)

$= \frac{9mR^2}{2} - mR^2 = \left[\frac{1}{18} + \frac{4}{9}\right] - \frac{9mR^2}{2} = \frac{mR^2}{2} = \frac{8mR^2}{2}$ 4m$R^2$.

**15.** 2/5 M$R^2$ = 1/2 M$r^2$ + M$r^2$ ∴ 2/5 M$R^2$ = 3/2 M$r^2$

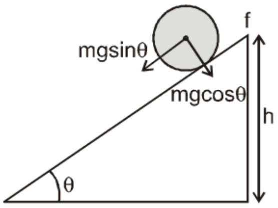
$r^2 = \frac{4}{15}R^2$ ⇒ $r = \frac{2R}{\sqrt{15}}$

**16.** If torque external = 0, then angular momentum = constant **=** $I\omega$

**17.** The acceleration of centre of mass of either cylinder

$a = \dfrac{g\sin\theta}{1+\dfrac{K^2}{R^2}}$ where K is radius of gyration.

So acceleration of centre of hollow cylinder




is less than that of solid cylinder.

Hence time taken by hollow cylinder will be more.

So statement-1 is wrong. **Ans. (4)**

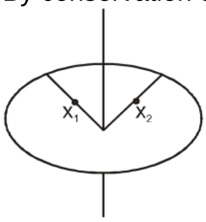
**18.** (1) Since there in no resultant external force, linear momentum of the system remains constant.

(2) Kinetic energy of the system may change.

(3) Angular momentum of the system may change as in case of couple, net force is zero but torque is not zero. Hence angular momentum of the system is not constant.

(4) Potential energy may also change.

**19.** By conservation of angular momentum




$MR^2\,\omega = \left(MR^2 + \dfrac{M}{8}\dfrac{9R^2}{25} + \dfrac{Md^2}{8}\right)\dfrac{8\omega}{9}$ $\Rightarrow$ $R^2 = \left(\dfrac{200R^2 + 9R^2 + 25d^2}{8\times 25}\right)\dfrac{8}{9}$

$225\,R^2 - 209\,R^2 = 25\,d^2$

$d = \dfrac{16R^2}{25}$ $\Rightarrow$ $d = \dfrac{4R}{5}$

**20.** Moment of Inertia about the shortest side BC is greater than the other two sides

$I_{BC} > I_{AB}$

**21.** Conserving the angular momentum :

$mua = \left[\dfrac{m(a^2+4a^2)}{12} + \dfrac{5}{4}ma^2\right]\omega$ $\Rightarrow$ $\omega = \dfrac{3}{5}\dfrac{u}{a}$ Ans.

**22.** By conservation of angular momentum about hinge O.

$L = I\,\omega$

$mv\dfrac{d}{2} = \left[\dfrac{Md^2}{12} + m\left(\dfrac{d}{2}\right)^2\right]\omega$ $\Rightarrow$ $\dfrac{mvd}{2} = \left(\dfrac{md^2}{2} + \dfrac{md^2}{4}\right)\omega$

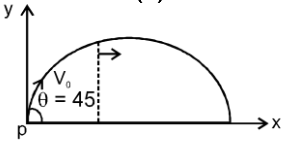
$\dfrac{mvd}{2} = \dfrac{3}{4}md^2\omega$ $\Rightarrow$ $\dfrac{2}{3}\dfrac{v}{d} = \omega$

**23.** L = Pr so log = log P + log r

y = x + logr

so (b) ans.

**24.**



$x = v_0 \cos 45^o \times t = \frac{v_0 t}{\sqrt{2}} \Rightarrow \tau = mgx = \frac{mgv_0 t}{\sqrt{2}} = \frac{dL}{dt} \Rightarrow L = \frac{mgv_0}{\sqrt{2}} \int_0^{v_0/g} t \; dt = \frac{mv_0^3}{2\sqrt{2}\;g}$

**25.** In rolling without slippping, total energy of ball is the sum of its translational and rotational energy.
Kinetic energy of rotation is

$K_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}MK^2\frac{v^2}{R^2}$ where K is radius of gyration.

Kinetic energy of translation is $K_{trans.} = \frac{1}{2}Mv^2$

Thus, total energy $E = K_{rot.} + K_{trans.} = \frac{1}{2}MK^2\frac{v^2}{R^2} + \frac{1}{2}Mv^2 = \frac{1}{2}Mv^2\left(\frac{K^2}{R^2}+1\right) = \frac{1}{2}\frac{Mv^2}{R^2}(K^2+R^2)$

Hence, $\frac{K_{rot.}}{K_{trans.}} = \frac{\frac{1}{2}MK^2\frac{v^2}{R^2}}{\frac{1}{2}\frac{Mv^2}{R^2}(K^2+R^2)} = \frac{K^2}{K^2+R^2}$

**26.** As no external torque is applied to the system, the angular momentum of the system remains conserved.
∴ $L_i = L_f$ According to given problem,
$I_t\omega_i = (I_t + I_b)\omega_f$

or $\omega_f = \frac{I_t \;\omega_i}{(I_t + I_b)}$ .........(i)

Initial energy, $E_i = \frac{1}{2}I_t\omega_i^2$ .........(ii)

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\omega_f^2$ .........(iii)

Substituting the value of $\omega_f$ from equation (i) in equation (iii) we get

Final energy, $E_f = \frac{1}{2}(I_t + I_b)\left(\frac{I_t\omega_i}{I_t + I_b}\right)^2$

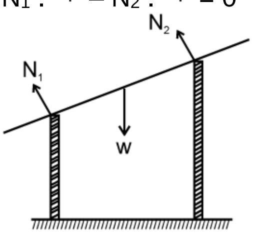
$= \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)}$ ........(iv)

Loss of energy, $\Delta E = E_i - E_f = \frac{1}{2}I_t\omega_i^2 - \frac{1}{2}\frac{I_t^2\omega_i^2}{(I_t + I_b)}$ (Using (ii) and (iv))

$= \frac{\omega_i^2}{2}\left(I_t - \frac{I_t^2}{(I_t + I_b)}\right) = \frac{\omega_i^2}{2}\left(\frac{I_t^2 + I_bI_t - I_t^2}{(I_t + I_b)}\right) = \frac{1}{2}\frac{I_bI_t}{(I_t + I_b)}\omega_i^2$

**27.** Balancing torque about the centre of the rod :

$N_1 . \frac{\ell}{4} - N_2 . \frac{\ell}{4} = 0$




$\Rightarrow$ $N_1 = N_2$.

## EXERCISE # 3
### PART - I

**3.** Mass of the disc = 9M

Mass of removed portion of disc = M

The moment of inertia of the complete disc about an axis passing through its centre O and perpendicular to its plane is $I_1 = \frac{9}{2}MR^2$

Now, the moment of inertia of the disc with removed portion $I_2 = \frac{1}{2}M\left(\frac{R}{3}\right)^2 = \frac{1}{18}MR^2$


Therefore, moment of inertia of the remaining portion of disc about O is

$I = I_1 - I_2 = 9\frac{MR^2}{2} - \frac{MR^2}{18} = \frac{40MR^2}{9}$

**4.** When angular acc. ($\alpha$) is zero than torque on the wheel becomes zero

$\theta(t) = 2t^3 - 6t^2$

$\frac{d\theta}{dt} = 6t^2 - 12t \Rightarrow \frac{d^2\theta}{dt^2} = 12t - 12 = 0 \Rightarrow t = 1$ Sec.

**6.** It's always in axial direction so

**7.** at maximum compression the solid cylinder will stop

so loss in K.E. of cylinder = gain in P.E. of spring

$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}kx^2 \Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\left(\frac{V}{R}\right)^2 = \frac{1}{2}kx^2$

$\Rightarrow \frac{3}{4}mv^2 = \frac{1}{2}kx^2 \Rightarrow \frac{3}{4} \times 3 \times (4)^2 = \frac{1}{2} \times 200 \quad x^2$

$\Rightarrow \frac{36}{100} = x^2 \Rightarrow x = 0.6$ m

**8.** $F_1 x + F_2 x = F_3 x \Rightarrow F_3 = F_1 + F_2$

**9.** using angular momentum conservation

$L_i = 0$

$L_t = mvR - I\omega$

$mvR = I.\omega$

$\omega = \left(\frac{1}{2}\right)$

$(v + \omega R)t = 2\pi R$

$\left(1 + \frac{1}{2} \times 2\right) = 2\pi \times 2$

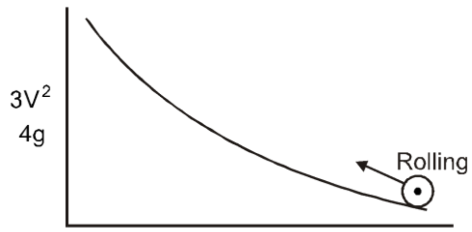
$t = 2\pi$ sec.

**10.** $I = I_{cm} + md^2$

d is maximum for point B so

$I_{max}$ about B

**11.**

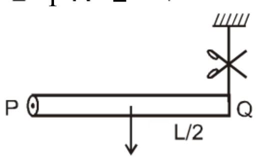



$\frac{1}{2}I\omega^2 + 0 + \frac{1}{2}mv^2 = mg \times 3v^2 \frac{3v^2}{4g}$

$\frac{1}{2}I\omega^2 = \frac{3}{4}mv^2 - \frac{1}{2}mv^2 = \frac{mv^2}{2}\left(\frac{3}{2} - 1\right)$

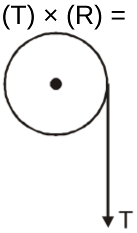
$\frac{1}{2}I\frac{V^2}{R^2} = \frac{mv^2}{4} \Rightarrow I = \frac{1}{2}mR^2$ Disc

**12.**



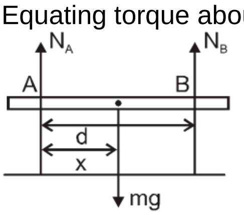

$I = mg\frac{L}{2} = I\alpha = \frac{mL^2}{3}\alpha \Rightarrow a = \alpha = \frac{3g}{2L}$

**13.** (T) × (R) = $\left(\frac{MR^2}{2}\right)(\alpha)$




T = $\left(\frac{MR}{2}\right)(\alpha)$ = $\left(\frac{50\times0.5}{2}\right)$ (2 × 2π) = 157N

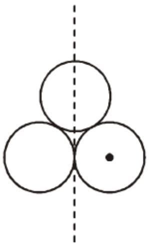
**14.** Equating torque about center of mass




$N_A$ x = $N_B$ (d-x)

$N_A + N_B = mg$ ⇒ Solving $N_A = \frac{W(d-x)}{d}$

**15.** $I_{diameter} = \frac{2}{3}MR^2$



$I_{tengential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$. So $I_{total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right)\times 2 = 4MR^2$ $= \frac{12}{3}MR^2 = 4MR^2$

**16.** K.E. = $\frac{1}{2}$ I $\omega^2$

I is min. about the centre of mass

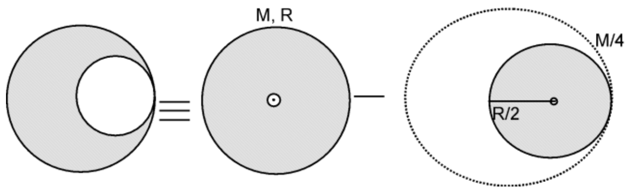
So. ($m_1$) (x) = ($m_2$) (L–x)

x = $\frac{m_2L}{m_1+m_2}$

**17.** If $\text{If } \vec{L} = \text{constant then } \vec{\tau} = 0$

$\text{so } \vec{r}\times\vec{F} = 0 \Rightarrow$ should be parallel to $\vec{r}$ so coefficient should be in same ratio.So $\frac{\alpha}{2} = \frac{3}{-6} = \frac{6}{-12}$

So α = – 1 **Ans (4)**

**18.** 


$I_1 = \frac{MR^2}{2}$

$I_2 = \frac{\left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2}{2} + \left(\frac{M}{4}\right)\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{32}$ $\Rightarrow$ $I_{net} = I_1 - I_2 = \frac{MR^2}{2} - \frac{3MR^2}{32} = \frac{13MR^2}{32}$ So, answer is 3.

**19.** Time does not depend on mass, else

$t \propto \sqrt{\left(1 + \frac{k^2}{R^2}\right)}$ $\Rightarrow$ $\frac{k^2}{R^2}$ is least for sphere and hence least time is taken by sphere

**20.** The angular speed of disc increases with time, and hence centripetal acceleration

also $a_{net} = \sqrt{a_t^2 + a_c^2}$

$a_c = \frac{\upsilon^2}{R}$

$\upsilon$ = tangential speed

R = Radius = 0.5 m

V = 2m/s at t =2 $\Rightarrow$ $a_c$ = 8m/s ; $a_t = R\alpha = (0.5)(2)$ $\Rightarrow$ $a_{net} = \sqrt{8^2 + 1^2} \simeq 8$

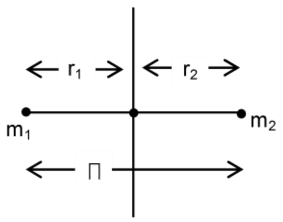
**21.** $KE_A = KE_B$

$\frac{1}{2} I_A \omega_A^2 = \frac{1}{2} I_B \omega_B^2$ $\Rightarrow$ since $I_B > I_A$ so $\omega_B < \omega_A$

$\frac{1}{2} L_A\omega_A = \frac{1}{2} L_B\omega_B$ $\Rightarrow$ $L_B > L_A$ **Ans.**

**22.** KE of sphere = $\frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 = \frac{1}{5} mR^2\omega^2$

KE of cylinder = $\frac{1}{2}\left(\frac{mR^2}{2}\right)(2\omega)^2 = mR^2\omega^2$ So, $\frac{KE_{sphere}}{KE_{cylinder}} = \frac{1}{5}$ **Ans.**

**23.** 


$I = m_1 r_1^2 + m_2 r_2^2 = m_1 \left(\frac{m_2}{m_1 + m_2}\ell\right)^2 + m_2 \left(\frac{m_1}{m_1 + m_2}\ell\right)^2 = \frac{m_1 m_2 (m_1 + m_2)\ell^2}{(m_1 + m_2)^2} = \frac{m_1 m_2 \ell^2}{(m_1 + m_2)}$ **Ans.**

**24.** $KE_t = \frac{1}{2}mV^2$

$KE_{rotation} = \frac{1}{2}I_{cm}\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{V}{R}\right)^2 = \frac{1}{5}mV^2$ $\Rightarrow$ $\frac{kE_t}{KE_t + KE_r} = \frac{\frac{1}{2}mV^2}{\frac{1}{2}mV^2 + \frac{1}{5}mV^2} = \frac{5}{7}$

**25.** $\tau_{ext} = 0$, so Angular momentum will remain conserved.

**26.** Measured diameter of ball = Main scale reading + Circular scale reading × least count
= 5 mm + (25) × (0.01 mm) = 5.25 mm
Actual diameter = measured diameter – zero error = (5.25 mm) – (– 0.04 mm) = 5.29 mm

**27.** $\omega_0$ = 3 rpm = $3 \times \frac{2\pi}{60} \text{rad/sec} = \frac{\pi}{10}$

$\omega^2 = \omega_0^2 + 2 \propto \theta$

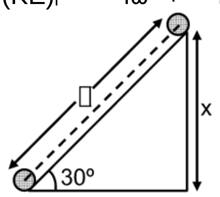
$0^2 = \left(\frac{\pi}{10}\right)^2 + 2(\alpha)(2\pi ⊠ 2\pi)$ $\Rightarrow$ $\alpha = -\frac{1}{800} \text{rad/sec}^2$

$I = \frac{mR^2}{2} = \frac{(2)\left(\frac{4}{100}\right)^2}{2} = \frac{16}{10^4}$ $\Rightarrow$ $\tau = I\alpha = \left(\frac{16}{10^4}\right) \times \left(-\frac{1}{800}\right) = -2 \times 10^{-6}$ N.m

**28.** work done = ΔKE

$(KE)_i = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{3}{4}mv^2 = \frac{3}{4} \times 100 \times (20 \times 10^{-2})^2 = \frac{3}{4} \times 100 \times 400 \times 10^{-4} = 3J$

**29.**




By energy conservation $\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right) = mgx = mg\ell \sin\theta$

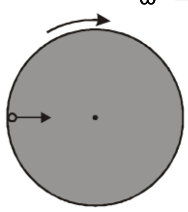
So $\ell = \frac{v^2(1 + K^2/R^2)}{2g\sin\theta}$ $\Rightarrow$ $L = \frac{(4)^2 \times \left(1 + \frac{1}{2}\right)}{2 \times 10 \times \frac{1}{2}}$ = 2.4 meter

## PART - II

**1.** $\frac{1}{2}I\omega^2 = mgh$

$\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 = mgh$ $h = \frac{\omega^2 \ell^2}{6g}$

**2.**



From angular momentum conservation about vertical axis passing through centre. When insect is coming from circumference to center. Moment of inertia first decrease then increase. So angular velocity inecrease then decrease.

**3.** To reverse the direction $\int \tau d\theta = 0$ (work done is zero)

$\tau = (20t - 5t^2)\, 2 = 40t - 10t^2$

$\alpha = \frac{\tau}{I} = \frac{40t - 10t^2}{10} = 4t - t^2 \quad \Rightarrow \quad \omega = \int_0^t \alpha dt = 2t^2 - \frac{t^3}{3}$

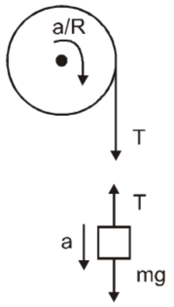
$\omega$ is zero at $2t^2 - \frac{t^3}{3} = 0$

$t^3 = 6t^2 \quad \Rightarrow \quad t = 6$ sec.

$\theta = \int \omega \; dt = \int_0^6 (2t^2 - \frac{t^3}{3})dt \quad \Rightarrow \quad \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6 = 216 \left[\frac{2}{3} - \frac{1}{2}\right] = 36$ rad.

No of revolution $\frac{36}{2\pi}$ Less than 6

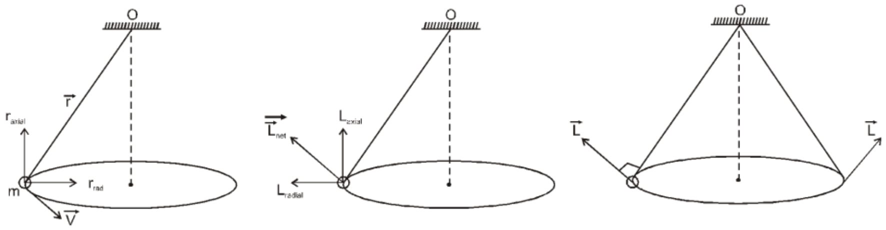
**4.**




$mg - T = ma$ .....(1)

$T.R = mR^2 \frac{a}{R}$ .....(2) $\Rightarrow \quad \frac{g}{2} = a$

**5.**



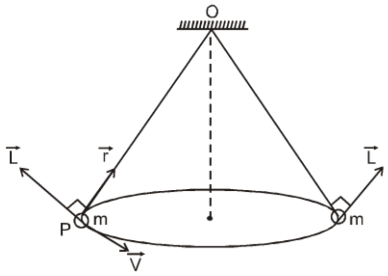
Angular momentum of the pendulum about the suspension point 'O' is

$\vec{L} = m \; (\vec{r} \times \vec{v})$

Then can be resolved into two components, radial component $r_{rad}$, and axial component $r_{axial}$. Due to $r_{rad}$, L will be axial and due to $r_{axial}$, L will be radially outwards as shown.

So net angular momentum will be as shown in figure whoose value will be constant ($|L| = mv\ell$). But its direction will change as shown in the figure.

**Short Solution**

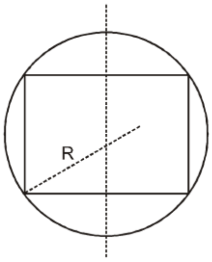



Angular momentum of the pendulum about the suspension point 'O' will have a constant magnitude : (L) = mv (op) but its direction will change as shown in the figure.

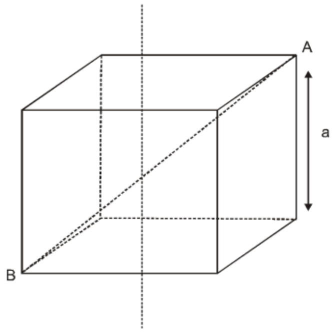
**6.** AB = 2R

$a\sqrt{3} = 2R$

$a = \frac{2R}{\sqrt{3}}$




Mass of cube = $\frac{M}{\frac{4}{3}\pi R^3} \times \left(\frac{2R}{\sqrt{3}}\right)^3 = \frac{3M}{4\pi R^3} \cdot \frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$




Moment of inertia of cube about given axis is = $\frac{ma^2}{6} = \frac{2M}{\sqrt{3}\pi} \cdot \frac{4R^2}{3} \cdot \frac{1}{6} = \frac{4MR^2}{9\sqrt{3}\pi}$

**7.** From C to D

$\vec{L}_0 = mv\left[\frac{R}{\sqrt{2}} + a\right] \hat{k}$

from B to C

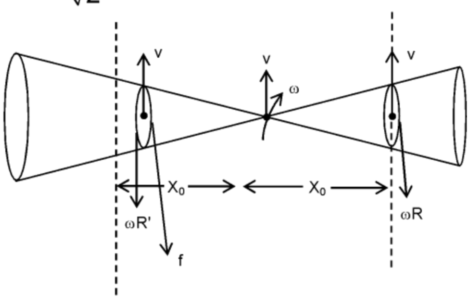
$\vec{L}_0 = mv\left[\frac{R}{\sqrt{2}} + a\right] \hat{k}$

from D to A

$\vec{L}_0 = \frac{mv}{\sqrt{2}}R\left(-\hat{k}\right)$

from A to B

$\vec{L}_0 = \frac{mv}{\sqrt{2}}R\left(-\hat{k}\right)$




**8.**

At distance $x_0$ from O v = ωR

distance less than $X_0$ v > ωR

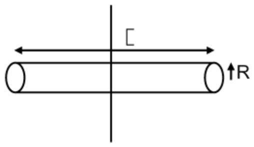
Initially, there is pure rolling at both the contacts. As the cone moves forward slipping at AB will start in forward direction as radius at left contact decreases.

Thus the cone will start turning towards left. As it moves further slipping at CD will start in backward direction which will also turn the cone towards left.

At distance $x_0$ from O $\quad v = \omega R$

Distance less than $X_0$ $\quad v > \omega R$

**9.** $I = \frac{M\ell^2}{12} + \frac{MR^2}{4}$

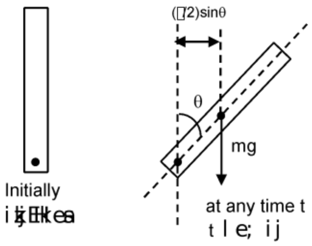



$= \frac{M\ell^2}{12} + \frac{M}{4} \times \frac{M}{\rho\pi\ell}$ $\Rightarrow$ $M = (\pi R^2 \ell)\rho$ $\Rightarrow$ $\frac{M}{\rho\pi\ell} = R^2$

$\frac{dI}{d\ell} = \frac{M}{12}(2\ell) - \frac{M^2}{4\rho\pi}\left(\frac{1}{\ell^2}\right) = 0$ $\Rightarrow$ $\frac{\ell}{6} = \frac{M}{4\rho\pi\ell^2}$

$\ell^3 = \frac{3M}{2\rho\pi} = \frac{3}{2\rho\pi} \times \pi R^2 \ell \rho$ $\Rightarrow$ $\frac{\ell^2}{R^2} = \frac{3}{2}$ $\Rightarrow$ $\frac{\ell}{R} = \sqrt{\frac{3}{2}}$

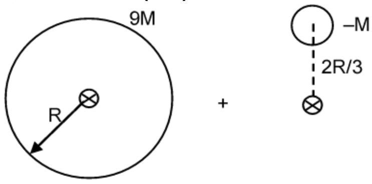
**10.**




$mg\sin\theta\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$ $\Rightarrow$ $\frac{3g}{2\ell}\sin\theta = \alpha$

**11.** $I_p = I_0 + 7M(3R)^2 = \left(\frac{MR^2}{2} + 6\left(\frac{MR^2}{2} + M(2R)^2\right)\right) + 7M(3R)^2 = \frac{181}{2}MR^2$

**12.**




[Using negative mass concept]

$$I = \frac{9MR^2}{2} - \left[\frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] = MR^2\left[\frac{9}{2} - \frac{1}{18} - \frac{4}{9}\right] = 4MR^2$$

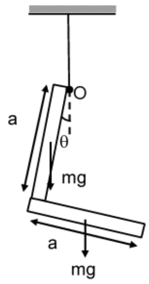
**13.** From Dimension analysis $I_0 = kMa^2$

Now for small lamina

$I' = k\frac{M}{4}\left(\frac{a}{2}\right)^2 = \frac{kMa^2}{16}$ $\Rightarrow$ $I' = \frac{I_0}{16}$

So moment of inertia of remaining part $I - I' = \frac{15I_0}{16}$

**14.** Lets considered mass of each rod is m for equilibrium the torque about point O should be zero.

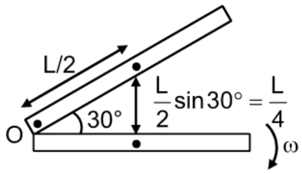



Torque balance about O

$mg\frac{a}{2}\sin\theta = mg\left(\frac{a}{2}\cos\theta - a\sin\theta\right)$

$\tan\theta = \frac{1}{3} \Rightarrow \tan^{-1}\left(\frac{1}{3}\right)$

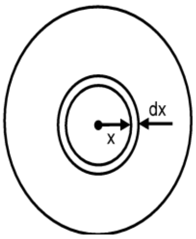
**15.**




$\frac{1}{2}I\omega^2 = Mg\frac{L}{4}$

$\Rightarrow \frac{1}{2}\frac{ML^2}{3}\omega^2 = Mg\frac{L}{4} \Rightarrow \omega^2 = \frac{3}{2}\frac{g}{L} = \frac{3\times 10}{2\times\frac{1}{2}} = 30 \Rightarrow \omega = \sqrt{30}$ rad/s

**16.**

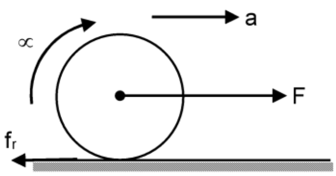



Net torque = $\sum$ Torque on ring

$\int d\tau = \int_0^R \mu \times \frac{F}{\pi R^2} \times 2\pi x.\, xdx$

$\tau = \frac{2\mu FR}{3}$

**17.**




$F - f_r = ma$ ........(1)

$f_rR = I\alpha = \frac{mR^2}{2}\alpha$ ……..(2)

for pure rolling

$a = \alpha R$ ……..(3)

from (1)(2) and (3)

$$F - \frac{mR\alpha}{2} = m\alpha R$$

$\Rightarrow \quad F = \frac{3}{2}mR\alpha \quad \Rightarrow \quad \alpha = \frac{2F}{3mR}$

**18.** $5M_o g\ell - 4M_o g\ell = [2M_o (2\ell)^2 + 5M_o (\ell^2)]\alpha$

$M_o g\ell = (13M_o\ell^2)\alpha$

$\alpha = \left(\frac{g}{13\ell}\right)$

**19.** $I_0$ = (M.I. of A & B) + (M.I. of C)

$I_0 = \left(\frac{MR^2}{4} + MR^2\right) \times 2 + \frac{MR^2}{2} = \frac{5MR^2}{2} + \frac{MR^2}{2} = 3MR^2$

**20.** Taking torque about the point of contact

$40 \times [1] = [mr^2 + mr^2]\alpha$

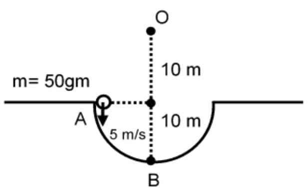
$40 = 2m \times \frac{1}{4}\alpha \quad \Rightarrow \quad 40 = 2 \times 5 \times \frac{1}{4}\alpha. \quad \therefore \quad \alpha = 16 \text{ rad/s}^2$

**21.** $\vec{\tau} = \vec{r} \times \vec{F} \quad \Rightarrow \quad \tau = rF\sin\theta \quad \Rightarrow \quad 2.5 = 5 \times 1 \times \sin\theta \quad \Rightarrow \quad \theta = 30°$

**22.** $T_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = (2\hat{i} + 3\hat{j}) \times F(\hat{k}) + 6\hat{j} \times \left(\frac{F}{2}(-\hat{i}) + \frac{F\sqrt{3}}{2}(-\hat{j})\right)$

$= 2F(-\hat{j}) + 3F(\hat{i}) + 3F(\hat{k}) + 0 = (3\hat{i} - 2\hat{j} + 3\hat{k})F$

**23.** Applying conservation of energy




$mgR = \frac{1}{2}mV_2^2 - \frac{1}{2}mV_1^2$

$V_2 = \sqrt{2gr + V_1^2} = \sqrt{2 \times 10 \times 10 + 25}$

$V_2 = 15$ m/s

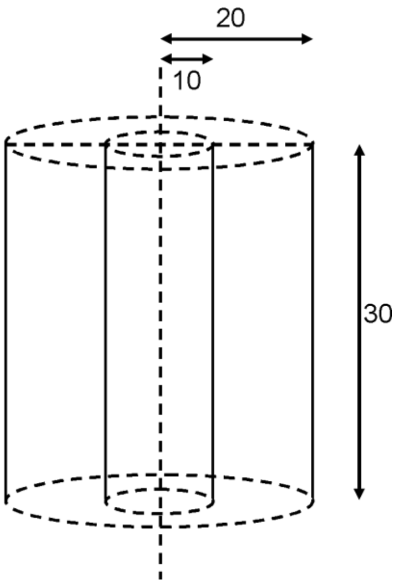
$L_0 = mV_B r = 20 \times 10^{-3} \times 20 \times 15$

$L_0 = 6 \text{ kgm}^2/\text{s}$

**24.** $I_x = I_{cm} + mx^2$

$I_x = \frac{2}{5}mR^2 + mx^2 \quad \Rightarrow$ Parabola opening upward

**25.**




$$\frac{m(20^2 + 10^2)}{2} = mk^2 \Rightarrow K = \sqrt{\frac{400+100}{2}} \qquad K = \sqrt{250}$$

$K = 5\sqrt{10}$ cm