TOPIC : GRAVITATION
EXERCISE # 1

SECTION (A)

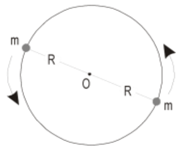
1. Weight of an object is the gravitational force exerted on the object.
For object close to surface of earth, it is approximately equal to gravitational force on the object by the earth.

3. F ∝ . If r becomes double then F reduces to

4. $F = G\frac{m_1 m_2}{r^2} = 6.675 \times \frac{1\times 1}{1^2} \times 10^{-11} = 6.675 \times 10^{-11}$ N

5. Centripetal force provided by the gravitational force of attraction between two particles

i.e. $\frac{mv^2}{R} = \frac{Gm\times m}{(2R)^2} = v = \frac{1}{2}\sqrt{\frac{Gm}{R}}$



8. $F = \frac{GM \quad (M-m)}{r^2}$

For F_{min}, ⇒ $\frac{dF}{dm} = 0$

M – 2m = 0 ⇒ $\frac{m}{M} = \frac{1}{2}$

9. Given that $m' = 4m_e$ & $R' = R_e$

$g \propto \frac{m}{R^2} \Rightarrow \frac{g^1}{g} = \frac{m' \quad R_e}{(R')^2 \quad m_e}$

g′ = 4g
w = mg′h
w = 2 × (4g) × 2 = 160 J

10. Substitute the dimensions for the quentities involved in an expression written for gravitational constant.
According to Newton's law of gravitation, the force of attraction between two masses m_1 and m_2 separated by a distance r is,

$F = \frac{Gm_1m_2}{r^2} \Rightarrow G = \frac{Fr^2}{m_1m_2}$

Substituting the dimensions for the quantities on the right hand side, we obtain

dimensions of G = $\frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

SECTION (B)

3. $g = \frac{GM}{R^2}$ or $\frac{g}{\frac{4}{3}\pi R} = \frac{GM}{\frac{4}{3}\pi R^3}$ or $\frac{3 \quad g}{4\pi RG} = \rho$

4. The acceleration due to gravity at a distance x (x < R) from centre of earth (of radius R) is

g(x) = g $\frac{x}{R}$ ∴ g $\left(\frac{R}{2}\right)$ = $\frac{g}{2}$

5. $\frac{g\left(1-\frac{2h}{R_e}\right)-g}{g}$ = –0.1 × $\frac{1}{100}$ ⇒ $-\frac{2h}{R_e}$ = $-\frac{1}{1000}$ ⇒ h = $\frac{R_e}{2000}$ = $\frac{6400}{2000}$ km = 3.2 km.

6. Time of decent t = $\sqrt{\frac{2h}{g}}$. In vacuum no other force works except gravity so time period will be exactly equal.

8. Because acceleration due to gravity increases

9. Because acceleration due to gravity decreases

12. Value of g decreases when we go from poles to equator.

15. Using g = $\frac{GM}{R^2}$ we get g_m = g / 5

16. g' = g ⇒ $\left(\frac{R}{R+h}\right)^2$ when h = R then g' = $\frac{g}{4}$

17. Same change in the value of g can be observed at a depth x and height 2x'
given d = x = 10 km ∴ h = 2x = 20 km

18. g' = g $\left(\frac{R}{R+h}\right)^2$ ⇒ $\frac{1}{\sqrt{2}}$ = $\frac{R}{R+h}$

19. g' $\left(\frac{R}{R+h}\right)^2$ = g = $\frac{g}{\left(1+\frac{h}{R}\right)^2}$

20. g ∝ p

22. g' = g $\left(a-\frac{d}{R}\right)$ ⇒ $\frac{g}{4}$ = g $\left(1-\frac{d}{R}\right)$ ⇒ d = $\frac{3}{4}R$

23. $\frac{G \quad \times 100}{x^2}$ = $\frac{G \quad \times 10^4}{(1-x)^2}$ ⇒ (1 – x)^2 = 100 × x

11x = 1 ⇒ x = $\frac{1}{11}$ m

24. If we take complete spherical shell then gravitational field intensity at P will be zero hence for the hemi spherical shell shown the intensity at P will be along c.

25. $-\left[\frac{G\times10^2}{(1/2)}+\frac{G\times10^3}{(1/2)}\right]$ = – 147 × 10^{-9} J / kg

26. T ∝ $\frac{1}{\sqrt{g}}$ i.e. $\frac{T_2}{T_1}=\sqrt{\frac{g_1}{g_2}}$

g_1 ⇒ acceleration due to gravity on earth's surface = g

g_2 ⇒ acceleration due to gravity at a height h = R from earth's surface = g/4

$\left| U\sin g\ \ g(h)=\frac{g}{\left(1+\frac{h}{R}\right)^2} \right|$ ⇒ $\frac{T_2}{T_1}=\sqrt{\frac{g}{g/4}}$ = 2

27. We know, $T^2 \propto R^3$

∴ $\left(\frac{T_2}{T_1}\right)^2=\left(\frac{R_2}{R_1}\right)^3$ ⇒ $\frac{T_2}{4}=\left(\frac{4R}{R}\right)^{3/2}$ ⇒ T_2 = 4 × 8 = 32 h.

28. Applying Kepler's third law

$T^2 \propto R^3$

Hence $\left(\frac{T_1}{T_2}\right)^2=\left(\frac{R_1}{R_2}\right)^3=\left(\frac{R}{2R}\right)^3$

$\frac{T}{T_2}=\left(\frac{1}{2}\right)^{3/2}$

$T_2 = (2)^{3/2}$ T So, T_2 = 2.8 T

29. In case of earth the gravitational field is zero at infinity as well as the the centre and the potential is minimum at the centre.

SECTION (C)

1. ΔU = m ($V_f - V_i$)

ΔU = $\left(\frac{-GM}{(4R)}-\left(\frac{-GM}{R}\right)\right)$ m = $\frac{3}{4}$ m $\left(\frac{GM}{R}\right)$ = $\frac{3}{4}$ mR $\left(\frac{GM}{R^2}\right)$ = mRg × 3/ 4

2. ΔU = $\frac{mgh}{1+\frac{h}{R}}$ = $\frac{mgnR}{1+\frac{nR}{R}}$ = $\frac{nmgR}{n+1}$

3. Potential energy of the 1 kg mass which is placed at the earth surface = –

Its potential energy at infinite = 0 ∴ Work done = change in potential energy = $\frac{GM}{R}$

6. $v_e = \sqrt{\frac{2GM}{R}}$ = 100 ⇒ $\frac{GM}{R}$ = 5000. Potential energy U = – $\frac{GMm}{R}$ = – 5000 J

7. $\frac{1}{2}mv_e^2$ = $\frac{1}{2}$ m 2gR = mgR

8. Escape velocity does not depend on the mass of the projectile

10. $v_e = v_0$, i.e. if the orbital velocity of moon is increased by factor of then it wil escape out from the gravitational field of earth.

11. $v_e = \sqrt{\frac{2GM}{R}}$ $\Rightarrow v_e \propto$ if R = constant

13. $v_e = R\sqrt{\frac{8}{3}G\pi p}$ $\therefore v_e \propto R\sqrt{p}$

15. $v = \sqrt{2gR}$ $\Rightarrow \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g_e}\times\frac{R_p}{R_e}} = \sqrt{1\times 4} = 2$

16. $v = \sqrt{2gR}$. If g and R both are doubled then v will becomes two times i.e. 11.2 × 2 = 22.4 km/s

17. $PE = \frac{-GM_e m}{R}$

$\Delta PE = PE_f - PE_i = \frac{-GM_e m}{(2R)} - \left(-\frac{GM_e m}{R}\right) = \frac{GM_e m}{2R}$ $\because \frac{GM_e}{R^2} = g = \frac{mgR}{2}$

19. $v_e = \sqrt{\frac{2GM}{R}}$ $\Rightarrow$ $v_0 = \sqrt{\frac{GM}{R}}$

$v_e = \sqrt{2}v_0$

21. From energy conservation

$PE_i + KE_i = PE_f + KE_f$

$-\left(\frac{GM_1}{d/2}+\frac{GM_2}{d/2}\right)m + \frac{1}{2}mv^2 = 0 + 0$

$V^2 = \frac{4G}{d}(m_1 + m_2)$ $\Rightarrow$ $V = \sqrt{\frac{4G(m_1+m_2)}{d}}$

24. $V_e \sqrt{gR}$

$\frac{V_p}{V_e} = \sqrt{\frac{g_p R_p}{g_e R_e}}$ $\Rightarrow$ $\frac{V_p}{V_e} = \sqrt{\frac{10g_e}{g_e}}$ $\Rightarrow$ $V_p = \sqrt{10}\,V_e$

25. The additional velocity be given to the space shuttle to get free from the influence of gravitational force.

$= \sqrt{2gR} - \sqrt{gR} = 3.28$ km/s

28. $V_e = \sqrt{2gR}$ $\Rightarrow$ $V_e = \sqrt{\frac{2GM_e R}{R^2}}$

$V_e = \sqrt{\dfrac{2G\left(\frac{4}{3}\pi R^3\rho\right)R}{R^2}}$ ⇒ $V_e \propto \sqrt{\rho R^2}$

$\dfrac{V}{v_0} = \sqrt{\dfrac{\rho(2R)^2}{\rho R^2}}$ ⇒ $V = 2V_0$

30. $V_e = \sqrt{\dfrac{2GM_e}{R^2}R}$. So, V_e does not depend on mass of the body.

33. Gravitational field is a conservative force field. In a conservative force field work done is path independent.

∴ $W_I = W_{II} = W_{III}$

34. Escape velocity $u_e = \sqrt{2gR}$

So, it does not depend on mass

$u_e \propto m^0$

35. For height h above the earth's surface

$g' = g\left(1 - \dfrac{2h}{R_e}\right) = g\left(1 - \dfrac{64}{6400}\right)$ (R_e = 6400 km)

= g (1 – 0.01)

= 0.99 g ms^{-2}

36. Kinetic energy of the satellite

$KE = \dfrac{1}{2} m v_0^2$ ....(1)

where $v_0 = \sqrt{\left(\dfrac{GM}{R}\right)}$

Now putting the value of v_0 in Equation (1), we get

$KE = \dfrac{1}{2} m \left(\sqrt{\left(\dfrac{GM}{R}\right)}\right)^2$

Hence $KE \propto \dfrac{1}{R}$

37. Acceleration of cosmonaut is $a = \dfrac{v^2}{r}$

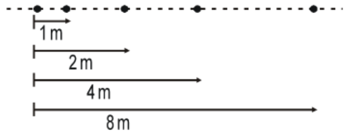
where r is the distance of cosmonaut from earth's centre.

Given v = 8 km/s = 8000 m/s,

r = (6400 + 630) km = (6400 + 630) × 1000 m

$\therefore a = \dfrac{(8000)^2}{(6400+630)\times 1000}$ = 9.10 m/s^2

38.



$v = v_1 + v_2 + v_3 + v_4 + ...$

$= -\frac{GM}{1} - \frac{GM}{2} - \frac{GM}{4} - \frac{GM}{8} - \frac{GM}{16} - \frac{GM}{32} - ....$

$= -GM\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.......\right)$ → G.P. of infinite series $= -M\left(\frac{1}{1-1/2}\right)$

$v = -GM(2) = -2GM$ Ans.

39. Initial total energy = Initial kinetic energy + initial potential energy

$= \frac{1}{2}m(0)^2 + \left(-\frac{GMm}{R_0}\right) = -\frac{GMm}{R_0}$

Total energy, when it reaches the surface of earth $= \frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right)$

Applying energy conservation,

$\frac{1}{2}mv^2 - \frac{GMm}{R} = -\frac{GMm}{R_0}$ $v = \sqrt{2GM\left\{\frac{1}{R} - \frac{1}{R_0}\right\}}$ **Ans.**

SECTION (D)

1. According to the conservation of energy, total energy at the surface of earth must equal to the total energy at the maximum height.

As from key idea,

energy at surface of earth = energy at maximum height

or (K + U) at earth's surface = (K + U) at maximum height

$\therefore \frac{1}{2}mu^2 - \frac{GMm}{R} = \frac{1}{2}m \times (0)^2 - \frac{GMm}{R+h}$ or $\frac{1}{2}mu^2 = \frac{GMm}{R} - \frac{GMm}{R+R}$ $(\because h = R)$

or $u^2 = \frac{2GM}{R} - \frac{2GM}{2R}$ or $u^2 = \frac{GM}{R}$ $\therefore$ $u = \sqrt{\frac{GM}{R}}$

3. Angular momentum of satellite remains constant.

16. For a geo - stationary satellite

$T_{sattelite} = T_{earth}$

$\frac{4\pi^2}{Gm_e} r^3 = \frac{2\pi}{\omega_{earth}}$ $\Rightarrow$ $r \propto \frac{1}{\omega_{earth}^{2/3}}$

As ω_{earth} is doubled So r will be $= \frac{1}{2^{2/3}} = \frac{1}{4^{(1/3)}}$ times **Ans.**

17. Centripetal acceleration works on it.

22. $v = \sqrt{\frac{GM}{r}}$

25. Orbital radius of satellites $r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$U_1 = \frac{-GMm}{r_1}$ and $U_2 = \frac{-GMm}{r_2}$

$K_1 = \frac{GMm}{2r_1}$ and $K_2 = \frac{GMm}{2r_2}$

$E_1 = \frac{GMm}{2r_1}$ and $E_2 = \frac{GMm}{2r_2}$ $\therefore \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$

26. Escape velocity is same for all angles of projection.

27. 6R from the surface of earth and 7R from the centre.

28. T = 2π $\sqrt{\frac{(R+H)^3}{gR^2}}$ = 2π $\sqrt{\frac{(2R)^3}{gR^2}}$ = 4 $\sqrt{2\pi}$ $\sqrt{\frac{R}{g}}$

29. T = 2π ⇒ $T^2 = (R + H)^3$

32. Total energy = – (kinetic energy) = – $\frac{1}{2}$ Mv^2

33. $T^2 \propto r^3$

34. $\frac{v_B}{v_A} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{4R}{R}}$ = 2 ⇒ $v_B = 2 \times v_A = 2 \times 3v = 6v$

35. Gravitational force provides the required centripetal force

41. Time period of a satellite very close to earth's surface is 84.6 minutes. Time period increases as the distance of the satellite from the surface of earth increase. So, time period of spy satellite orbiting a few hundred km, above the earth's surface should be slightly greater than 84.6 minutes. Therefore, the most appropriate option is (3) or 2 hrs.

42. $V_e = \sqrt{2}v_0$

KE = $\frac{1}{2}mv_e^2 = \frac{1}{2}m\left(\sqrt{2}v_0\right)^2 = mv_0^2$

43. In same orbit, orbital speed of satellites remains same.
When two satellites of earth are moving in same orbit, then time period of both are equal.
From Kepler's third law $T^2 \propto r^3$
Time period is independent of mass, hence their time periods will be equal.
The potential energy and kinetic energy are mass dependent, hence the PE and KE of satellites are not equal
But, if they are orbiting in a same orbit, then they have equal orbital speed.

44. Orbital speed of satellite revolving nearby the earth is $\sqrt{gR}$

45. Acceleration due to gravity g = $\frac{GM}{R^2}$
On decreasing R, g increases
% increae in g = 2 × decrease in R = 2 × 1% = 2%

46. Escape velocity from earth's surface $u_e = \sqrt{2gR_e}$
Escape velocity from planet
$u_p = \sqrt{2g_pR_p} = \sqrt{2(2g)(2\,\text{Re})} = 2u_e = 2 \times 11.2 = 22.4$ km/s

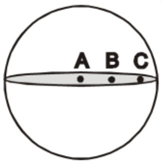
47. According to Kepler's law
$T^2 \propto R^3$
(Here : R is orbital radius and T is time period)
Now according to question when orbital radius is doubled, then period will be

$$\frac{T_1}{T_2} = \left(\frac{R}{2R}\right)^{3/2}$$

$T_2 = 2^{3/2}\,T_1$

(Here : $R_1 = R$, $R_2 = 2R$)
Hence, the time period will become $2^{3/2}$ times.

48. PE = $-G m_1 m_2/r$, ME = $-G m_1 m_2 / 2r$
On decreasing the radius of orbit PE and ME decreases



49. Since the centripetal force will disappear hence the satellite will move tangentially to the original orbrit with speed V.

50. $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$ or $\frac{T}{5} = (4)^{3/2}$ or T = 40 hr.

51. Escape velocity is independent of direction of projection.

52. $\frac{mv^2}{R+X} = \frac{GM}{(R+X)^2}\times\frac{R^2}{R^2}$ or V = $\left[\frac{g\ \ R^2}{R+X}\right]^{1/2}$

53. T α $R^{3/2}$ Time period is independent of mass of satellite

54. ΔPE = $\frac{GMm}{R} - \frac{GMm}{2R} = \frac{mgR}{2}$

55. g $\left[1-\frac{d}{R}\right]$ = g $\left[1-\frac{2h}{R}\right]$ or d = 2h

56. W = $\frac{GMm}{r}$ = 6.67 × 10^{-11} × 100 × 10 / 0.1 = 6.67 × 10^{-7} J

57. Electric charge on the moon = electric charge on the earth

58. V = $\sqrt{\frac{2GM}{R}} = \sqrt{\frac{\frac{2GM_e\times 10}{R_e}}{10}}$ = 10 $\sqrt{\frac{2GM_e}{R_e}}$ = 110 k m/s

SECTION (E)

1. $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{10^{13}}{10^{12}}\right)^{3/2}$ = $(1000)^{1/2}$ = 10 $\sqrt{10}$

2. $\frac{T_1}{T_2} = \left(\frac{R_1}{R_2}\right)^{3/2} = \left(\frac{R}{4R}\right)^{3/2}$ ⇒ $T_2 = 8T_1$

3. $T^2 \propto r^3 \Rightarrow \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$

4. For first satellite r_1 - R and T_1 = 83 minute
For second satellite r_2 = 4R

$T_2 = T_1 \left(\frac{r_2}{r_1}\right)^{3/2} = T_1(d)^{3/2} = 8T_1 = 8 \times 83$ minute

5. Angular momentum = Mass × orbital velocity × Radius = m × $\left(\sqrt{\frac{GM}{R_0}}\right) \times R_0 = m\sqrt{GMR_0}$

6. $\frac{T_{plant}}{T_{earth}} = \left(\frac{r_{plant}}{r_{earth}}\right)^{3/2} = (1.588)^{3/2} = 2 \therefore T_{planet} = 2$ year

7. Mass of satellite does not affects on orbital radius.

8. $\frac{T_2}{T_1} = \left(\frac{r_2}{r_1}\right)^{3/2} = (2)^{3/2} = 2\sqrt{2} \Rightarrow t_2 = 2\sqrt{2}$ years.

9. $\omega_{body} = 27\omega_{earth}$

$T^2 \propto r^3 \Rightarrow \omega^2 \propto \frac{1}{r^3} \Rightarrow \omega \propto \frac{1}{r^{3/2}} \therefore \propto \frac{1}{\omega^{2/3}} \Rightarrow \frac{r_{body}}{r_{earth}} = \left(\frac{\omega_{earth}}{\omega_{body}}\right)^{2/3} = \left(\frac{1}{27}\right)^{2/3} = \frac{1}{9}$

10. $L = mvr = m\sqrt{\frac{GM}{r}}r = m\sqrt{GMr} \therefore L \propto \sqrt{r}$

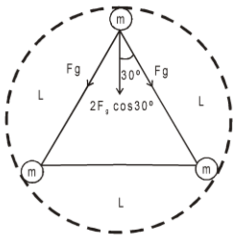
14. For central force, torque is zero.

$\therefore r = \frac{dL}{dt} = 0 \Rightarrow L =$ constant i.e. Angular momentum is constant.

15. In case of planet total energy of planet is always nagtive.

EXERCISE # 2

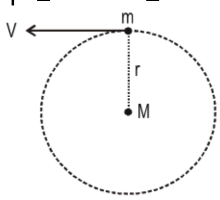
1.



$2 F_g \cos 30 = \frac{MV^2}{R} \Rightarrow 2\left(\frac{GM^2}{L^2}\right)\frac{\sqrt{3}}{2} = \frac{MV^2}{L/\sqrt{3}} \Rightarrow V = \sqrt{\frac{GM}{L}}$

2. $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r^2}}$

$T = \frac{2\pi r}{v} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{GM}} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{G\rho \times \frac{4}{3}\pi r^3}}$

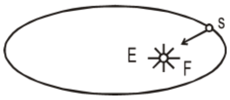


T $\propto \frac{1}{\sqrt{\rho}}$ **Ans.**

3. Satellite orbital velocity = $\sqrt{gR}$

escape velocity = $\sqrt{2gR}$

% change in velocity of satellite to move infinity = $\frac{\sqrt{2gR}-\sqrt{gR}}{\sqrt{gR}}$ × 100 = 41.4%

8. 

Force on satellite is always towards earth, therefore, acceleration of satellite S is always directed towards centre of the earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of S about centre of earth is constant throughout. Since the force F is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.

11. According to Kepler's law $T^2 \propto r^3$

or $5^2 \propto r^3$

and $(T')^2 \propto (4r)^3$

From equation (i) and (ii)

T' = $\frac{5}{(T')^2} = \frac{r^3}{64r^3}$ = $\sqrt{1600}$ = 40 h

12. Gravitational force acting between the spheres,

F = G $\frac{m.m}{(2r)^2}$ = $\frac{G.\frac{4}{3}\pi r^3 \times \rho \times \frac{4}{3}\pi r^3 \times \rho}{4r^2}$ = $\frac{4}{9}G\pi^2\rho^2 r^4$ ∴ $F \propto r^4$

13. g = $\frac{Gm}{R^2}$

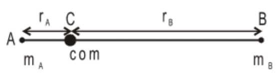
M = $\frac{gR^2}{G}$

14. Energy requred = $\frac{GMm}{2(2R)} - \frac{GMm}{2(4R)} = \frac{GMm}{8R}$

$\frac{GM}{R^2}\frac{mR}{8} = \frac{gmR}{8}$ = $\frac{9.8 \times 400 \times 6.4 \times 10^6}{8}$ = 3.136×10^9J

15. $\frac{GNm}{r^4} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r^3}$

T = $\frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM}}\ r^{3/2}$ So, $T^2 = \frac{4\pi^2}{4M}r^{3+2}$ So, $T \propto r^{5/2}$

16. 

$\Rightarrow \quad \frac{Gm_Am_B}{(r_A+r_B)^2} = m_Ar_A \frac{4\pi^2}{T_A^2} = m_B r_B \frac{4\pi^2}{T_B^2} \quad \therefore \quad \frac{m_Ar_A}{T_A^2} = \frac{m_Br_B}{T_B^2}$

As C is com $\Rightarrow$ $m_Ar_A = m_Br_B$ hence $T_A = T_B$

17. Let the possible direction of electric field at point B be shown by 1, 2, 3 and 4 (Figure 1). Rotate the figure upside down. It will be as shown in figure 2.

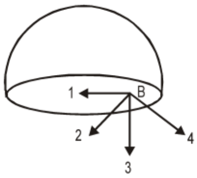


Figure 1

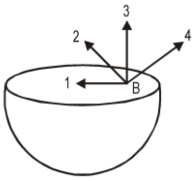


Figure 2

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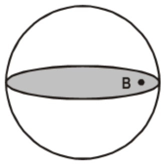


Figure 3

Now on placing upper half of figure 1 on the lower half of figure 2 we get complete sphere. Electric field at point B must be zero, which is only possible if the electric field is along direction 3. Hence electric field at all points on circular base of hemisphere is normal to plane of circular base.

$\therefore$ Circular base of hemisphere is an equipotential surface.

Aliter : Consider a shaded circle which divides a uniformly charged thin spherical shell into two equal halves.The potential at points A,B and C lying on the shaded circle is same. The potential at all these points due to upper hemisphere is half that due to complete sphere.Hence potential at points A,B and Cis also same due to upper hemisphere

EXERCISE # 3

PART - I

1. Apply Kepler's law of area of planetary motion.

The line joining the sun to the planet sweeps out equal areas in equal time interval i.e., areal velocity is constant.

$\frac{dA}{dt} = cons\tan t$ **or** $\frac{A_1}{t_1} = \frac{A_2}{t_2}$ **or** $t_1 = \frac{A_1}{A_2} t_2$

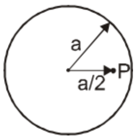
Given $A_1 = 2A_2$ $\therefore$ $t_1 = 2t_2$

2. Here, Mass of the particle = M

Mass of the spherical shell = M

Radius of the spherical shell = a

Point P is at a distance from the centre of the shell as shown in figure



Gravitational potential at point P due to particle at O is

$V_1 = -\frac{GM}{(a/2)}$

Gravitational potential at point P due to spherical shell is

$V_2 = -\frac{GM}{a}$

Hence, total gravitational potential at the point P is

$V = V_1 + V_2 = -\frac{GM}{(a/2)} + \left(-\frac{GM}{a}\right) = -\frac{2GM}{a} - \frac{GM}{a} = -\frac{3GM}{a}$

4. Orbit speed of the satellite around the earth is

$u = \sqrt{\frac{GM}{r}}$ where,

G = Universal gravitational constant
M = Mass of earth
r = Radius of the orbit of the satellite
For satellite A

$r_A = 4R$, $u_A = 3V$ $u_A = \sqrt{\frac{GM}{r_A}}$ ....(i)

For satellite B

$r_B = R$, $u_B = ?$ $u_B = \sqrt{\frac{GM}{r_B}}$ ....(ii)

Dividing equation (ii) by equation (i), we get $\therefore \frac{u_B}{u_A} = \sqrt{\frac{r_A}{r_B}}$ $u_B = u_A\sqrt{\frac{r_A}{r_B}}$

Substituting the given values, we get $u_B = 3V\sqrt{\frac{4R}{R}}$ $u_B = 6$ V

5. The acceleration due to gravity at a depth d below surface of earth is

$g' = \frac{GM}{R^2}\left(1-\frac{d}{R}\right) = g\left(1-\frac{d}{R}\right)$

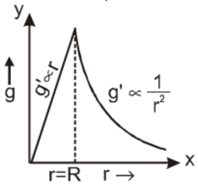
g' = 0 at d = R
i.e., acceleration due to gravity is zero at the centre of earth.
Thus, the variation in value g with r is
for, r > R,

$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{gR^2}{r^2}$ $\Rightarrow g' \propto \frac{1}{r^2}$

Here, R + h = r

For r < R, $g' = g\left(1-\frac{d}{R}\right) = \frac{gr}{R}$ Here, R – d = r $\Rightarrow$ $g' \propto r$

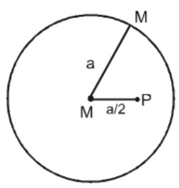
Therefore, the variation of g with distance from centre of the earth will be as shown in the figure.



8. $GM = gR^2$

$V_e = \sqrt{2gR} = \sqrt{2\frac{GM}{R^2}R} = \sqrt{\frac{2GM}{R}}$.

9.



$V_P = V_{sphere} + V_{partical} = \frac{GM}{a} + \frac{GM}{a/2} = \frac{3GM}{a}$

10. For $r > R$

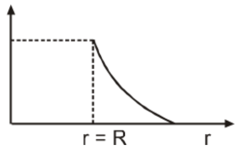
$F = \frac{GM}{r^2}$

For $r = R$

$F = \frac{GM}{R^2}$

For $r < R$

$F = 0$



11. $g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$ $\Rightarrow$ $\frac{g}{16} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\left(1+\frac{h}{R}\right)^2 = 16$ $\Rightarrow$ $1+\frac{h}{R} = 4$

$\frac{h}{R} = 3$ $\Rightarrow$ $h = 3R$

12. Gravitational attraction force on particle B $F_g = \frac{GM_p m}{\left(D_p/2\right)^2}$

Acceleration of particle duet to gravity $a = \frac{F_g}{m} = \frac{4GM_p}{D_p^2}$

13. $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} = \frac{(6R)^3}{(3R)^3} = 8$ $\Rightarrow$ $\frac{24\times 24}{T_2^2} = 8$ $\Rightarrow$ $T_2^2 = \frac{24\times 24}{8}$

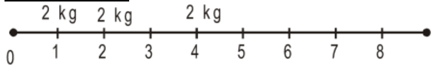
$T_2^2 = 72$ $\Rightarrow$ $T_2^2 = 36 \times 2$ $\Rightarrow$ $T_2 = 6\sqrt{2}$

14. $U_i = \frac{-GMm}{R}$ $\Rightarrow$ $U_f = \frac{-GMm}{3R}$

$$\Delta U = \frac{-GMm}{3R} + \frac{GMm}{R} = \frac{GMm}{R}\left(1-\frac{1}{3}\right) = \frac{2}{3}\frac{GMm}{R} = \frac{2}{3}MgR$$

Alternate: $\Delta U = \frac{mgh}{1+\frac{h}{R}}$ $\Rightarrow$ $h = 2R$.

15.



$V_0 = -\frac{G\times 2}{1} - \frac{G\times 2}{2} - \frac{G\times 2}{4} - \frac{G\times 2}{8}$

$= -2G\left[1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+....\infty\right] = -2G \times \frac{1}{1-\frac{1}{2}} = -2G \times \frac{1}{\frac{1}{2}} = -4G$

16. Light is unable to escape so
$V_e = C$

$\sqrt{\frac{2GM}{R}} = 3 \times 10^8 \Rightarrow \sqrt{\frac{2\times\left(\frac{20}{3}\times10^{-11}\right)(6\times10^{24})}{R}} = 3\times 10^8$

get R ≈ 9 mm ≈ 10^{-2}m

17. $E = -\frac{GM}{R^3}\times\vec{r}$ (if r < R)

$E = -\frac{GM}{r^3}\times\vec{r}$ (if r ≥ R)

18. $T^2 = \frac{4\pi^2}{GM}r^3$

so, $K = \frac{4\pi^2}{GM}$

19. $V_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{R^2}.\frac{R^2}{r}} = \sqrt{\frac{9.8\times6.38\times6.38}{6.63\times10^6}} = \sqrt{60\times10^6}\,m/\sec$ = 7.76 km/sec

20. $-\frac{GM}{r} = 5.4 \times 10^7$

$-\frac{GM}{r^2} = 6$

Dividing both the equations, r = 9000 km. So, height from the surface = 9000 – 6400 = 2600 km

21. The gravitation force on the satelite will be aiming toward the centre of earth so aceleration of the satelite will also be aiming toward the centre of earth

22. $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G\times\rho\times\frac{4}{3}\pi R^3}{R}} \Rightarrow V_e \propto R\sqrt{\rho}$

$\frac{V_1}{V_2} = \frac{R_1\sqrt{\rho_1}}{R_2\sqrt{\rho_2}} \Rightarrow \frac{V_1}{V_2} = \frac{1}{2\sqrt{2}}$ So, answer is 3.

23. 

$g_{in} = g_o \frac{r}{R}$ g_o is 'g' at surface

$g_{in} = g_o \left(\frac{R^2}{r^2}\right)$

24. TE = $-\frac{GMm}{2(R+h)} = -\frac{GMm}{2(R+h)}\frac{R^2}{R^2} = -\frac{g_0 mR^2}{2(R+h)}$

25. $\left(\frac{e^2}{4\pi\varepsilon_0}\right)\frac{1}{r^2} = F = \frac{GM_1M_2}{r^2}$ = MLT^{-2} dimentionally

So, $\frac{e^2}{4\pi\varepsilon_0}$ = ML^3T^{-2}

G = $L^3T^{-2}M^{-1}$ So, $\left[G.\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2} = L^6T^{-4}$ = L^3T^{-2}

C = LT^{-1} $\frac{1}{C^2}.\left[G\frac{e^2}{4\pi\varepsilon_0}\right]^{1/2} = \frac{L^3T^{-2}}{L^2T^{-2}}$ = L

26. Net Charge on one H-atom = –e + (e + Δe) = Δe

Net electrostatic force between two H-atoms = $\frac{k(\Delta e)(\Delta e)}{d^2}$ repulsive

Net gravitational force between two H-atoms = $\frac{G(m)(m)}{d^2}$

For equal magnitude

$\frac{k(\Delta e)^2}{d^2} = \frac{Gm^2}{d^2}$ ⇒ $\Delta e^2 = \frac{Gm^2}{k} = \frac{(6.67\times10^{-11})(1.67\times10^{-27})^2}{(9\times10^9)}$

Δe^2 is of the order of 10^{-74}

Δe is of the order of 10^{-37}

27. In the space, the external gravity is absent, but there will be a very small gravitational force between the astronauts, due to which both will move toward each other with a very small acceleration. So, the best correct answer should be (2).

28. $U = -\frac{GMm}{r}$

At position A, U = biggest Θve = least

So KE = maximum,

At position C, U = smallest Θve = maximum ⇒ KE = minimum

$K_A > K_B > K_C$

29. $g = \frac{GM}{R^2} \rightarrow 10$ times

$V_t \propto g$ ⇒ 10 times

$T = 2\pi\sqrt{\frac{\ell}{g}} \propto \frac{1}{\sqrt{10}}$ times

30. work done = $u_f - u_i$ $\Rightarrow \dfrac{-GmM}{(R+h)} - \dfrac{-GmM}{R}$

Now h = R $\Rightarrow$ $w = \dfrac{-GmM}{2R} + \dfrac{GmM}{R} = \dfrac{GmM}{2R}$

Now $g = \dfrac{Gm}{R^2}$ So, $w = \dfrac{mgR^2}{2R} = \dfrac{mgR}{2}$

31. $T^2 \propto R^3$

$\left(\dfrac{24}{T}\right)^2 = \left(\dfrac{6R+R}{2.5R+R}\right)^3 \Rightarrow \left(\dfrac{24}{T}\right)^2 = \left(\dfrac{7}{3.5}\right)^3 \Rightarrow \left(\dfrac{24}{T}\right)^2 = 2^3 \Rightarrow T^2 = \dfrac{24\times 24}{8}$

$T = \sqrt{24\times 3} = 6\sqrt{2}$ h

32. $U_{surface} = -\dfrac{GMm}{R}$, $U_h = \dfrac{-GMm}{(R+h)}$

$\Delta U = U_h - U_{surface} = \dfrac{-GMm}{R+h} + \dfrac{GMm}{R} = \dfrac{GMmh}{R(R+h)}$

PART - II

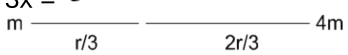
1. Acceleration due to gravity at leight h from earth surface.

$g' = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} \Rightarrow \dfrac{g}{9} = \dfrac{g}{\left(1+\dfrac{h}{R}\right)^2} \Rightarrow$ h = 2R

2. $\dfrac{Gm}{x^2} = \dfrac{G(4m)}{(r-x)^2} \Rightarrow \dfrac{1}{x} = \dfrac{2}{r-x}$

r – x = 2x

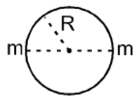
$3x = \dfrac{r}{3}$



$x = \dfrac{r}{3}$

$-\dfrac{Gm}{r/3} - \dfrac{G(4m)}{2r/3} \Rightarrow -\dfrac{3Gm}{r} - \dfrac{6Gm}{r} = \dfrac{9Gm}{r}$ **Ans.**

3. $\dfrac{Gm^2}{(2R)^2} = m\omega^2 R \Rightarrow \dfrac{Gm^2}{4R^3} = \omega^2$



$\omega = \sqrt{\dfrac{Gm}{4R^3}} \Rightarrow v = \sqrt{\dfrac{Gm}{4R^3}}\ \omega R$

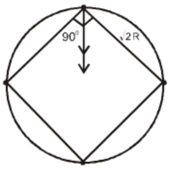
$v = \sqrt{\dfrac{Gm}{4R^3}} \times R \quad = \quad \sqrt{\dfrac{Gm}{4R}}$

4. $E_f = \dfrac{1}{2}mv_0^2 - \dfrac{GMm}{3R} = \dfrac{1}{2}m\dfrac{GM}{3R} - \dfrac{GMm}{3R} = \dfrac{GMm}{3R}\left(\dfrac{1}{2}-1\right) = \dfrac{-GMm}{6R}$

$E_i = \dfrac{-GMm}{R} + K$

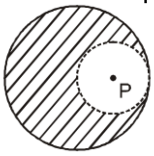
$E_i = E_f \quad \Rightarrow \quad K = \dfrac{5GMm}{6R}$

5.



$2\dfrac{GM^2}{(\sqrt{2}R)^2}\dfrac{1}{\sqrt{2}} + \dfrac{GM^2}{4R^2} = \dfrac{Mv^2}{R} \quad \Rightarrow \quad \dfrac{GM^2}{\sqrt{2}R^2} + \dfrac{GM^2}{4R^2} = \dfrac{Mv^2}{R} \quad \Rightarrow \quad v = \dfrac{1}{2}\sqrt{\dfrac{GM}{R}\left[\,1+2\sqrt{2}\,\right]}$

6. Potential at point P due to complete solid sphere



$= -\dfrac{GM}{2R^3}\left(3R^2 - \left(\dfrac{R}{2}\right)^2\right) = -\dfrac{GM}{2R^3}\left(3R^2 - \dfrac{R^2}{4}\right) = -\dfrac{GM}{2R^3}\dfrac{GM}{2R^3} = -\dfrac{11GM}{8R}$

Potential at point P due to cavity part = $-\dfrac{3}{2}\dfrac{G\dfrac{M}{8}}{\dfrac{R}{2}} = \dfrac{-3GM}{8R}$

So potential due to remaining part at point P = $\dfrac{-11GM}{8R} - \left(\dfrac{-3GM}{8R}\right) = \dfrac{-11GM + 3GM}{8R} = \dfrac{-GM}{R}$

7. $dA = \dfrac{1}{2} r^2 d\theta \quad \Rightarrow \quad \dfrac{dA}{dt} = \dfrac{1}{2} r^2 \dfrac{d\theta}{dt}$

$\dfrac{dA}{dt} = \dfrac{1}{2} r^2\omega = \dfrac{L}{2m}$ Since $L = mr^2\omega$

8. $V_0 = \sqrt{\dfrac{GM_e}{(R+h)}} \quad \Rightarrow \quad KE = E_2 = \dfrac{1}{2}mv^2 = \dfrac{GM_e m}{2(R_e+h)}$

$E_1 - \dfrac{GmM_e}{(R_e)} = -\dfrac{GM_e m}{(R_e+h)}$ Since $E_1 = E_2$

$\dfrac{GM_e m}{2(R_e+h)} - \dfrac{GM_e m}{R_e} = -\dfrac{GM_e m}{(R_e+h)} \quad \Rightarrow \quad \dfrac{3GM_e m}{2(R_e+h)} = \dfrac{GM_e m}{R_e}$

$3R_e = 2R_e + 2h$

$h = R_e/2$

9. $V_e = \sqrt{2}v$

K.E $= \frac{1}{2}m\left(\sqrt{2}v\right)^2 = mv^2$

10. $\frac{-G(3\times10^{31})m}{10^{11}}\times 2+\frac{1}{2}mv^2 = 0$

$6.67 \times 10^{-11} \times 12 \times 10^{20} = v^2$

$80.04 \times 10^9 = v^2$

$v = 2.82 \times 10^5$ m/s

11. Orbital velocity, $v_0 = \sqrt{\frac{GM}{(R+h)}}$ to escape, $v_s = \sqrt{\frac{2GM}{(R+h)}}$ change in velocity $= \sqrt{\frac{2GM}{(R+h)}} - \sqrt{\frac{GM}{(R+h)}}$

if h << R

$\Delta v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}} \Rightarrow \Delta v = \sqrt{2gR} - \sqrt{gR}$

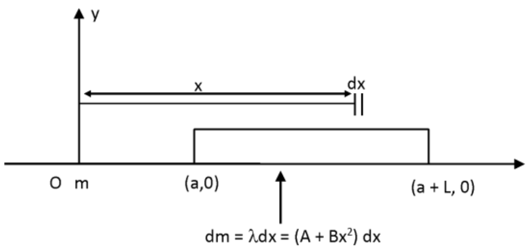
12. $g = \frac{GM}{R^2}$, $\frac{g_p}{g_e} = \frac{M_p}{M_e}\times\left(\frac{R_e}{R_p}\right)^2 = 3 \times \frac{1}{9} = \frac{1}{3}$

$T = 2\pi\sqrt{\frac{l}{g}}$, $\frac{T_p}{T_e} = \sqrt{\frac{g_e}{g_p}} = \sqrt{3}$

$T_p = \sqrt{3}\, T_e$

Time period at earth for seconds pendulum = 2sec. $\therefore T_p = 2\sqrt{3}$ sec.

13.



$F = \int_a^{a+L}\frac{GmdM}{x^2} = GM\int_a^{a+L}\frac{\left(A+Bx^2\right)dx}{x^2} = Gm\left[\int_a^{a+L}\frac{A}{x^2}dx + \int_a^{a+L}Bdx\right]$

$= Gm\left[A\left[\frac{-1}{x}\right]_a^{a+L} + BL\right] = Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)\right]+BL$

14.

$2mv_x = mv$

$v_x = \frac{v}{2}$

$v_y = \frac{v}{2}$

$$v_{net} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$$

So path will be elliptical

15.

$$K = \frac{1}{2}mV^2 = \frac{1}{2}m\left(\sqrt{\frac{GM}{R}}\right)^2 = \frac{GMm}{2R} \propto \frac{m}{R}$$

$$\frac{K_A}{K_B} = \frac{m_A}{m_B} \times \frac{R_B}{R_A} = \frac{m}{2m} \times \frac{2R}{R} = 1$$