HINTS & SOLUTIONS

TOPIC : MATHEMATICAL TOOLS
EXERCISE # 1
PART – I

SECTION - (A)

2. f (2) = $(2)^2 - 1 = 3$

f (3) = $3^2 - 1 = 8$

SECTION - (B)

1. tan15 = tan(45–30)

$$= \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = \frac{(\sqrt{3} - 1)^2}{2} = \frac{3 + 1 - 2\sqrt{3}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

2. $\cos_2 \theta = 1 - 2\sin_2 \theta$

$2\sin_2 \theta = 1 - \cos_2 \theta \Rightarrow \sin_2 \theta = \left(\frac{1 - \cos 2\theta}{2}\right)$

3. sinA . [sinAcosB + cosA . sinB]

$\sin_2$A . cosB + sinA . cosA . sinB

$\sin_2$A . cosB + $\frac{1}{2}$ sin2A . sinB

4*. $\cos\left(\frac{\pi}{2} + \theta\right)$ → II equivelent

$\therefore \cos\left(\frac{\pi}{2} + \theta\right)$ = –sinθ [cos in II equivelent function – ve]

sin (θ – π) = sin[–(π –θ)] = –sin(π –θ) = – sinθ

sin(π + θ) = – sinθ [sin f_n is –ve in III equivelent]

5. $c_2 = a_2 + b_2 - 2ab \cos\theta$

9 = 9 + 16 – 2 × 3 × 4 × cosθ

$\cos\theta = \frac{16}{24} = \frac{2}{3} \Rightarrow \sin\theta = \sqrt{1 - \cos^2\theta} = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{5}}{2}$

SECTION - (C)

2. By comparision with the standard quadratic equation

a = 2, b = 5 and c = – 12

$x = \frac{\sqrt{(5)^2 - 4 \times 2(-12)}}{2 \times 2} = \frac{-5 \pm \sqrt{121}}{4} = \frac{-5 \pm 11}{4} = \frac{+6}{4}, \frac{-16}{4}$ or $x = \frac{3}{2}, -4$

3. When particle comes to rest, v = 0.

So $t^2 + 3t - 4 = 0 \Rightarrow t = \frac{-3 \pm \sqrt{9 - 4(1)(-4)}}{2(1)}$ $\Rightarrow$ t = 1 or –4

SECTION - (D)

1. $\frac{dy}{dx} = 2x + 1$

2. $\frac{dy}{dx} = \sec^2 x - \text{cosec}^2 x$

3. $\frac{dy}{dx} = \frac{1}{x} + e^x$, $\frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$

4. $\frac{d}{dx} e^x \ell nx = \ell n^x \frac{de^x}{dx} + e^x \frac{d\ell n^x}{dx}$

$e^x \ell nx. + \frac{e^x}{x}$

5. y = sin 5x

Let 5x = θ

y = sin θ

$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$

$\frac{dy}{d\theta} = \cos\theta$ $\frac{d\theta}{dx} = 5$ $\therefore \frac{d\theta}{dx} = 5\cos\theta$

$\theta = 5x$ $\therefore \frac{dy}{dx} = 5\cos 5x$

6. $(x + y)^2 = 4$

$2(x + y)\left(1+\frac{dy}{dx}\right) = 0$ $\therefore x + y \neq 0 \Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1$

7. $\frac{dy}{dx} = \frac{dy}{du}\times\frac{du}{dx}$ $\Rightarrow$ $\frac{dy}{dx} = 48\,(8x - 1)^2$

10. For maximum/minimum value $\frac{dy}{dx} = 0$ $\Rightarrow$ $5(2x) - 2(1) + 0 = 0$ $\Rightarrow$ $x = \frac{1}{5}$

Now at $x = \frac{1}{5}$, $\frac{d^2y}{dx^2} = 10$ which is positive so minima at $x = \frac{1}{5}$

Therefore $y_{min} = 5\left(\frac{1}{5}\right)^2 - 2\left(\frac{1}{5}\right) = 1 = \frac{4}{5}$

11. $y' = \frac{(3x-2)(2)-(2x+5)(3)}{(3x-2)^2} = \frac{-19}{(3x-2)^2}$

12. vol = $\frac{4}{3}\pi r^3$

$\frac{d(vol)}{dt} = \left(\frac{4}{3}\pi\right)3r^2$

$\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$

$\frac{d(vol)}{dt} = 4\pi \times (10)^2 \times 0.05 = 20\pi = 62.8 \frac{mm^3}{sec}$

13. $y = 3t^2 - 4t$

$\frac{dy}{dt} = 6t - 4 = 0 \Rightarrow t = 2/3$. $\frac{d^2y}{dt^2} = 6 > 0$

Hence there will be minima at t = 2/3

14. $y = \sin(t_2) \Rightarrow \frac{dy}{dt} = 2\,t\cos(t_2)$

$\frac{d^2y}{dt^2} = \cos t_2 \frac{d}{dt}(2t) + 2t \frac{d}{dt} \cos t_2 = 2\cos t_2 + 2t\,(-\sin t_2)\,2t = 2\cos(t_2) - 4t_2 \sin(t_2)$

16. $\frac{ds}{dt} = 15 - 0.8\,t = 7 = v$; $8 = 0.8\,t$

t = 10 second. $a = \frac{d^2s}{dt^2} = -0.8\ m/s_2$

17. $v = 2t_4$

$a = \frac{dy}{dt} = 2 \times 4t_3 = 8t_3$

18. $x + y = 8$

$A = xy$

$A = x(8 - x)$

$A = 8x - x_2$

$\frac{dA}{dx} = 8 - 2x$

$x = 4$

$y = 4$

$A = xy = 16$

19. $y = 3t_2 - 4t$

$\frac{dy}{dt} = 6t - 4 = 0 \Rightarrow t = 2/3$. $\frac{d^2y}{dt^2} = 6 > 0$ Hence there will be minima at t = 2/3

22. $\frac{dy}{dx} = -2x$, $\frac{d^2y}{dx^2} = -2$

23. $\frac{dy}{dx} = x^2 + x + \frac{1}{4}$, $\frac{d^2y}{dx^2} = 2x + 1$

24. $\frac{dy}{dx} = -2 + 3x^{-4}$, $\frac{d^2y}{dx^2} = -12x^{-5}$

26. $-\frac{3}{x^2} + 5\cos x$

29. $\frac{ds}{dt} = 2t - \sec t \tan t + 1$

30. $P = 3 + \frac{1}{\cot q} \Rightarrow P = 3 + \tan q$

$\frac{dp}{dq} = \sec^2 q$

31. $P = (1 + \mathrm{cosec}\ q)\cos q$

= cos q + cosec q . cosq
P = cosq + cot q

$\frac{dp}{dq}$ = – sin q – cosec^{2}q

32. With u = sin x , y = u^3 ; $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ = 3u^2 cosx = 3 sin^2 x (cosx)

33. With u = cos x
Y = 5u^{-4}
$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
= 5 (–4) u^{-5} (– sin x)
$\frac{dy}{dx}$ =20 sin x cos^{-5} x

34. S = $\frac{4}{3\pi}$ sin 3t + $\frac{4}{5\pi}$ cos 5t
$\frac{ds}{dt} = \frac{4}{3\pi}$ (3 cos 3t) + $\frac{4}{5\pi}$ (–5 sin 5t) = $\frac{4}{\pi}$ (cos 3t – sin 5t)

35. S = sin $\frac{3\pi t}{2}$ + cos $\frac{3\pi t}{2}$
$\frac{ds}{dt} = \frac{3\pi}{2}$ cos $\frac{3\pi}{2}$ t + $\left(\frac{3\pi}{2}\right)\left(-\sin\frac{3\pi}{2}t\right)$
= $\frac{3\pi}{2}\left[\cos\frac{3\pi}{2}t - \sin\left(\frac{3\pi}{2}t\right)\right]$

SECTION - (E)

1. y = x$_2$ – 2x + 1
$\int y\,dx = \int\left(x^2 - 2x + 1\right)dx + c = \int x^2 dx - 2\int x dx + \int dx + c = \frac{x^3}{3} - x^2 + x + c$

2. $\sqrt{x} + \frac{1}{\sqrt{x}}$ ⇒ y = x$_{1/2}$ + x$_{-1/2}$
$\int y\,dx = \int x^{1/2}dx + \int x^{-1/2}dx = \frac{2}{3}$ (x)$_{3/2}$ + 2 (x)$_{1/2}$ + c

3. y = $\frac{1}{3x}$ ⇒ $\int y\,dx = \frac{1}{3}\int\frac{1}{x}dx + c = \frac{1}{3}$ ℓ_{nx} + c

4. $\int x\sin(2x^2)dx$
Let u = 2x$_2$
du = 4x dx = $\int \sin u\frac{du}{4} = \frac{1}{4}\int \sin u\, du = -\frac{1}{4}\cos u + C$

5. $\int\frac{3}{(2-x)^2}$ dx = 3 $\int(2-x)^{-2}$ dx
Let u = 2 – x du = – dx
$3\int u^{-2}(-dx) = \frac{+3}{u} + C = \frac{3}{2-x} + C$

6. $\frac{\pi}{2}\int_{-4}^{-1} d\theta = \frac{\pi}{2}[\theta]_{-4}^{-1} = \frac{\pi}{2}$ $[(-1) - (-4)] = \frac{3\pi}{2}$

7. $\int_0^1 e^x\,dx$ $= \left[e^x\right]_0^1$ $= e - 1$

8. Using n sub intervals of length ⊗x = $\frac{b}{n}$ bn and right – endpoint values : Area = $\int_0^b 2x\,dx = b^2$ units

9. $\int_0^{\pi} y\,dx$ $= \int_0^{\pi} \sin x\,dx = \left[-\cos x\right]_0^{\pi}$ $= [-\cos\pi + \cos 0] = 2$

10. (i) $\frac{x^{16}}{16} + C$ (ii) $-2x^{-1/2} + C$
(iii) $x^{-6}/2 + \ln x + C$ (iv) $x^2/2 + \ln x + 2x + C$
(v) $x^2/2 + \ln x + C$ (vi) $-a/x + b\ln x + C$

11. $\int_1^5 x^2 dx$ $= \left[\frac{x^3}{3}\right]_1^3$ $= \left[\frac{5^3}{3} - \frac{1^3}{3}\right]$ $= \frac{125}{3} - \frac{1}{3}$ $= \frac{124}{3}$

12. (i) GMm/R (ii) $Kq_1q_2\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$
(iii) $M\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$ (iv) ∞ (v) 1
(vi) 1 (vii) 2

16. $\int 2\sin(x)dx$ $= -2\cos x + C$

20. $y = x_2 \sin x_3$
$\int x^2 \sin x^3 dx$ let $u = x_3$ ⇒ $du = 3x_2\,dx$
$\int \frac{\sin u\,du}{3}$
$= \frac{1}{3}(-\cos u) + C = -\frac{1}{3}\cos x_3 + C$

21. Area = $\int y\,dx$ $= \int_0^2 x^2 dx$ $= \frac{x^3}{3}\Big|_0^2 = \frac{8}{3}$

22. Let $t_2 = y$ ⇒ $2t\,dt = dy$ ⇒ $\int y\,dt$ $= \int t\sin(t^2)$ dt = $\int \sin y$. $\frac{dy}{2}$ $= \frac{1}{2}(-\cos y)$

23. Let $3y_2 + 4y + 3 = t$ ⇒ $(6y + 4)\,dy = dt$
Then $\int x\,dy$ $= \int t\,dt$ $= \frac{t^2}{2} + C = \frac{(3y^2+4y+3)^2}{2} + C$

24. $\int_0^{\pi/2} \cos 3t\,dt$ $= \left[\frac{\sin 3t}{3}\right]_0^{\pi/2}$ $= \frac{1}{3}[-1 - 0] = \frac{-1}{3}$

25. $\int_0^1 (t^2 + 9t + c)\,dt = \frac{9}{2}$

$\left[\frac{t^3}{3} + \frac{9}{2}t^2 + ct\right]_0^1 = \frac{9}{2}$

$\frac{1}{3} + \frac{9}{2} + c = \frac{9}{2}$

$c = -\frac{1}{3}$

26. $\int_0^{2\pi} \sin^2\theta\, d\theta = \frac{1}{2}\int_0^{2\pi} (1-\cos 2\theta)\, d\theta$

$= \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} = \frac{1}{2}\left[2\pi - \frac{\sin 4\pi}{2} - 0 + \frac{\sin 0}{2}\right] = \pi$ Ans.

28. $\left[\frac{\theta^2}{2}\right]_\pi^{2\pi} = \frac{4\pi^2}{2} - \frac{\pi^2}{2} = \frac{3\pi^2}{2}$

29. $= \left[\frac{x^3}{3}\right]_0^{3\sqrt{7}} \frac{\left((7)^{1/3}\right)^3}{3} - 0 = \frac{7}{3}$

30. $\left[\frac{1}{3}\ln(3x+2)\right]_0^1 = \frac{1}{3}(\ln 5 - \ln 2) = \frac{1}{3}\ln\frac{5}{2} = \ln\left(\frac{5}{2}\right)^{1/3}$

31. (i) $3x^2 + C$ (ii) $\frac{x^8}{8} + C$

(iii) $\frac{x^8}{8} - 3x^2 + 8x + C$

32. (1) $\frac{-2}{3x^3} + C$ (ii) $\frac{-x^{-3}}{6} + \frac{x^3}{3} + C$ (ii) $\frac{-x^{-3}}{3} + \frac{x^2}{2} - x + C$

33. (i) $\frac{2}{3x^3} + C$ (ii) $\frac{-x^{-3}}{6} + C$ (iii) $\frac{x^5}{5} + \frac{1}{3x^3} + C$

34. (1) $x^{2/3} + C$ (ii) $x^{1/3} + C$ (iii) $x^{-1/3} + C$

35. (i) $\sin\pi x + C$ (ii) $\sin\frac{\pi x}{2} + C$ (iii) $\frac{2}{\pi}\sin\frac{\pi x}{2} + \pi\sin x + C$

36. (i) $\int \pi\cos\pi x\,dx = \pi\int\cos\pi x\,dx$ $\therefore$ $\int \cos kx\,dx = \frac{\sin kx}{k} = \pi\frac{\sin\pi x}{\pi} = \sin\pi x + C$

(ii) $\int \frac{\pi}{2}\cos\frac{\pi}{2}x\,dx$

$\frac{\pi}{2}\frac{\sin\frac{\pi}{2}x}{\pi/2} = \sin\frac{\pi}{2}x + C$

(iii) $\int\left(\cos\frac{\pi x}{2} + \pi\cos x\right)dx = \int\cos\frac{\pi}{2}x\,dx + \int\pi\cos x = \frac{\sin\frac{\pi}{2}x}{\pi/2} + \pi\sin x = \frac{2}{\pi}\sin\frac{\pi}{2}x + \pi\sin x + C$

37. $\int (x+1)dx$

$\int x dx + \int 1.dx$

$\frac{x^2}{2} + x + C$

38. $\int (5-6x)dx$

$= \int 5dx - \int 6xdx = 5x - \frac{6x^2}{2} + c = 5x - 3x^2 + C$

39. $\int 3t^2 dt + \int \frac{t}{2} dt = \frac{3t^3}{3} + \frac{t^2}{2\times 2} = t^3 + \frac{t^2}{4} + C$

40. $\int \frac{t^2}{2} dt + \int 4t^3 dt = \frac{t^3}{2\times 3} + \frac{4t^4}{4} + C = \frac{t^3}{6} + t^4 + C$

41. $\frac{3}{2} x^{2/3} + C$

42. $\int \frac{x^{1/2}}{2} dx + \int 2x^{-1/2} dx = \frac{1}{2} \frac{x^{1/2+1}}{(1/2+1)} + 2 \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + C = \frac{1}{3} x^{3/2} + 4x^{1/2} + C$

43. $\int 8y dy - \int \frac{2}{y^{1/4}} dy$

$\frac{8y^2}{2} - \frac{2y^{-1/4+1}}{-\frac{1}{4}+1} + c$

$4y^2 - \frac{8}{3} y^{3/4} + C$

44. $\int [(2x - (2x)x^{-3})]dx$

$= \int 2xdx - \int 2x^{-2} dx = \frac{2x^2}{2} - \frac{2x^{-2+1}}{-2+1} = x^2 + \frac{2}{x} + C$

45. $-2 \sin t + C$

46. $5 \cos t + C$

47. $\int 7 \sin \frac{\theta}{3} d\theta = \frac{7(-\cos\theta/3)}{1/3} + c$

$-21 \cos \frac{\theta}{3} + C$

48. $\int 3\cos 5\theta d\theta = 3\int \cos 5\theta d\theta = \frac{3}{5} \sin 5\theta + C$

49. $3 \cot x + C$

50. $\frac{-\tan x}{3} + C$

51. $-\frac{1}{2}\csc\theta + C$

52. $\frac{2}{5}\sec\theta + C$

53. $\int 4\sec x\tan x dx - 2\int \sec^2 x dx$

$4\sec x - 2\tan x + C$

54. $\int \frac{1}{2}\csc^2 x dx - \frac{1}{2}\int \csc x\cot x dx$

$-\frac{1}{2}\cot x + \frac{1}{2}\csc x + C$

55. $\int \sin 2x dx - \int \csc^2 x dx = -\frac{\cos 2x}{2} + \cot x + c$

56. $\int (2\cos 2x dx - \int 3\sin 3x dx$

$\frac{2\sin 2x}{2} - 3\left(\frac{-\cos 3x}{3}\right) + c$

$\sin 2x + \cos 3x + C$

57. $\int \frac{1}{2}dt + \int \frac{1}{2}\cos 4t dt = \frac{1}{2}t + \frac{1}{2}\frac{\sin 4t}{4} = c = \frac{t}{2} + \frac{\sin 4t}{8} + C$

58. $\int \frac{1}{2}dt - \int \frac{\cos 6t}{2}dt = \frac{1}{2}t - \frac{1}{12}\sin 6t + c$

59. $\int (1+\tan^2\theta)d\theta$

$\int \sec^2\theta d\theta = \tan\theta + C$

60. $\int_{1/2}^{3/2}(-2x+4)dx$

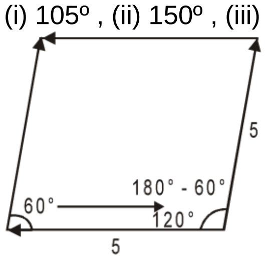
$\left[-x^2+4x\right]_{1/2}^{3/2} = \left[-\left(\frac{3}{2}\right)^2 + 4\left(\frac{3}{2}\right)\right] - \left[-\frac{1}{4} + 4\times\frac{1}{2}\right]$ 2 square units

61. $\int_0^{\pi/2}\theta^2 d\theta \Rightarrow \left[\frac{\theta^3}{3}\right]_0^{\pi/2} \Rightarrow \frac{\pi^3}{24}$

62. $\int_0^{3b}x^2 dx \Rightarrow \left[\frac{x^3}{3}\right]_0^{3b} = 9b^3$

SECTION - (F)

1. (i) 105º , (ii) 150º , (iii) 105º.

2. 

$Q = 120°$

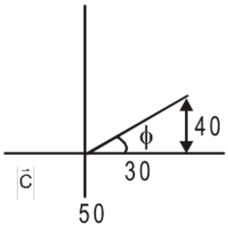
3. A(1,1,–1) B(2, –3 , 4)

$\vec{AB} = \vec{B} - \vec{A}$ = (2, –3, 4) – (1, – 1, 1)

= (1, –4, 5) ∴ $\vec{AB} = \hat{i} - 4\hat{j} + 5\hat{k}$

4. $\vec{A} = 30\,\hat{i}$

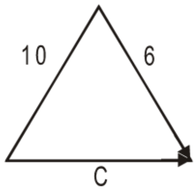
$\vec{B} = 40\,\hat{j}$



$\hat{i} = \vec{A} + \vec{B} = 30\hat{i} + 40\,\hat{j}$

tan ϕ = $\frac{4}{3}$ = 53°

5.



Using triangular inequality

C > 10 – 6

C < 10 + 6 ∴ 4 < c < 16

6. $|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cdot\cos\theta}$ cos θ is min. when θ = π

8. $\sqrt{(3)^2 + (2)^2 + (1)^2} = \sqrt{9+4+1} = \sqrt{14}$ unit.

9. $\vec{A} = 3\hat{i} + 4\hat{j}$

$|\vec{A}| = \sqrt{3^2 + 4^2}$ = 5 ⇒ $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i}+4\hat{j}}{5} = \frac{3\hat{i}+4\hat{j}}{5}$

10. $|\vec{V}|$ = 60 km/h ⇒ $|\vec{V}| = \sqrt{30^2 + x^2}$ = 60

$30_2 + x_2 = 60°$

$x_2 = 3600 - 900$

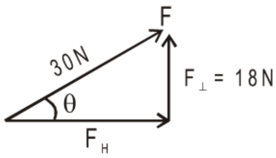
$x_2 = 2700$

$x_2 = 900 \times 3$

x = $30\sqrt{3}$

11. $\vec{F} = 2\hat{i} - 3\hat{j}$ N

12.



Given that $F_\perp$ = 18

from figure $F_H = \sqrt{F^2 - F_1^2} = \sqrt{30^2 - 18^2} = \sqrt{576}$ = 24N

from figure tanθ = $\frac{F_1}{F_H} = \frac{18}{24}$ tanθ = $\frac{3}{4}$ ⇒ θ = 37°

13. $\tan\alpha = \dfrac{\beta\sin\theta}{A + B\cos\theta}$

$\tan\alpha = \dfrac{6\sin 90}{8 + 6\cos 90} = \dfrac{1}{8} = \dfrac{3}{4}$

$\tan\alpha = \dfrac{3}{4}$ $\alpha = 37$ Ans.

15. Given that

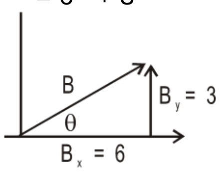
$\vec{A} = 4\hat{i} + 6\hat{j}$ .... (1)

$\vec{A} + \vec{B} = 10\hat{i} + 9\hat{j}$ .... (2)

from equation (2)

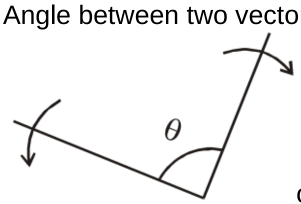
$\vec{B} = 10\hat{i} + 9\hat{j} - \vec{A} = 10\hat{i} + 9\hat{j} - (4\hat{i} + 6\hat{j})$

$\vec{B} = 6\hat{i} + 3\hat{j}$



from figure $\tan\theta = \dfrac{B_y}{B_x} = \dfrac{3}{6} = \tan^{-1}\left(\dfrac{1}{2}\right)$

17. Angle between two vectors can never be greater than 180°



on increasing the θ , the magnitude of resultant vectors decreases.

18. Sum of any 3 sides should be greater than fourth side.

19. $a + b > \left|\vec{a} + \vec{b}\right| > a - b$

20. $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$

Case -I Either $|\vec{A}| = |\vec{B}| = 0$ (zero vectors)

Case - $\vec{A}$ II $|A| = |B| \neq 0$

$|\vec{A} + \vec{B}| = A^2 + B^2 + AB\cos\theta$

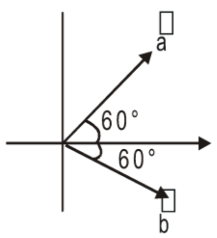
$|A| = |B| = 2A^2 + 2A^2\cos\theta = 2A^2(1 + \cos\theta) = 2A\left(2\cos^2\dfrac{\theta}{2}\right) = 4A\cos^2\dfrac{\theta}{2}$

Now $|A + B| = A$

$\cos\left|\dfrac{\theta}{2}\right| = \dfrac{1}{2}$

$\dfrac{\theta}{2} = 60°$

$\theta = 120°$



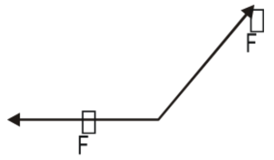
21. Only horizontal along + x-axis $\Rightarrow 2\cos 60° + 2\cos 60° = 2$

22. (1)Theory

(2) small angular displacement ⇒ vector
Large angular displacement ⇒ scalar

(3) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ but $\vec{A} - \vec{B} = \vec{B} - \vec{A}$

(4)



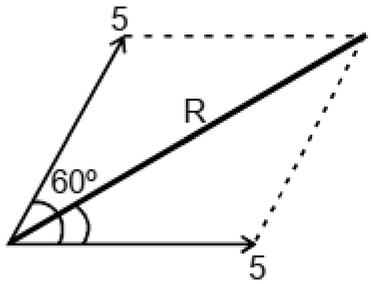
$F_{net} = \sqrt{F^2 + F^2 + 2F^2\cos(120^\circ)} = F$

23. A vector is represented by sum of its component vectors.
i.e.

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ So, from given options only option (1), (2), (3) are correct.

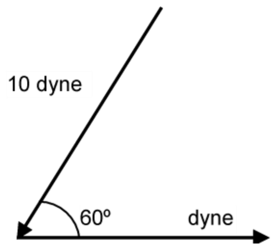
24. a = b = 5 unit and $\theta = 60^\circ$

$R = \sqrt{a^2 + b^2 + 2a.b\cos 60^\circ} = 5\sqrt{3}$ unit



$\tan\alpha = \dfrac{a\sin 60^\circ}{b + a\cos 60^\circ} = \dfrac{\frac{\sqrt{3}}{2}}{\frac{3}{2}} = \dfrac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30$

25. The angle θ between the two vectors is 120º and not 60º.



$\therefore R = \sqrt{(10)^2 + (10)^2 + 2(10)(10)(\cos 120^\circ)} = \sqrt{100 + 100 - 100} = 10$ dyne

26. Resultant of two vectors $\vec{A}$ and $\vec{B}$ must satisfy $A - B \le R \le A + B$

31. Given $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

squaring $|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$ or $(\vec{A} + \vec{B}).(\vec{A} + \vec{B}) = (\vec{A} - \vec{B}).(\vec{A} - \vec{B})$

$\Rightarrow \vec{A}.\vec{A} + \vec{A}.\vec{B} + \vec{B}.\vec{A} + \vec{B}.\vec{B})$ $\Rightarrow$ $\vec{A}.\vec{A} - \vec{A}.\vec{B} - \vec{B}.\vec{A} + \vec{B}.\vec{B})$

As $\vec{A}.\vec{B} = \vec{B}.\vec{A}$ and $\vec{A}.\vec{A} = A^2$ etc. $\Rightarrow$ $A^2 + 2\vec{A}\vec{B} + B^2 + A^2 - 2\vec{A}\vec{B} + B^2$

or $\Rightarrow \vec{A}.\vec{B} \Rightarrow \vec{A}.\vec{B} = 0$

This implies that $\vec{A}$ and $\vec{B}$ are mutually perpendicular (or $\theta = 90^\circ$)

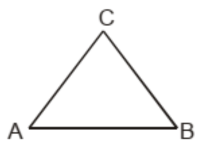
32. The projection of vector $\vec{A}$ on y-axis is
$= \vec{A}.\hat{j}$ 4 projection = $(3\hat{i}+4\hat{k}).\hat{j}$ = 0

33. When the two forces of 12 N and 8 N act upon a body, the resultant force on the body has maximum value when Resultant force = 12N + 8N = 20

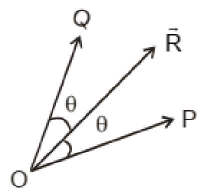
34. from figure, by polygon law
$\vec{A}+\vec{B}+\vec{E}$
$\vec{E} = -(\vec{A}+\vec{B})$

35. from figure, by polygon law
$\vec{A}+\vec{C}-\vec{D} = 0$
$\vec{D}-\vec{C} = \vec{A}$

36. from figure, by polygon law
$\vec{D}-\vec{C}+\vec{B}+\vec{E} = 0$
$\vec{E}+\vec{D}-\vec{C}=\vec{B}$

37. 

From triangle ABC, $\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}$ = 0……(1)
from given problem
$\vec{R} = \overrightarrow{AB}+\overrightarrow{BC}+2\overrightarrow{CA}$
$\vec{R} = \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}+\overrightarrow{CA}$
$\vec{R} = \overrightarrow{CA}$

38. 

Let P & Q are component of vector $\vec{R}$
from diagram
P = Rcosθ & Q = Rcosθ So, P = Q

39. Slope of the path of the particle gives the measure of angle required. Draw the situation as shown. OArepresents the path of the particle starting from origin 0 (0,0), Draw a perpendicular from point A to x-axis. Let path of the particle makes an angle θ with the x-axis, then

SECTION - (G)

1. $\vec{A} = \hat{i}+\hat{j}+\hat{k}$ $\vec{B} = 2\hat{i}+\hat{j}$

(a) $\vec{A}.\vec{B}$ = (1)(2) + (1)(1) + (1)(0) = 2 + 1 = 3

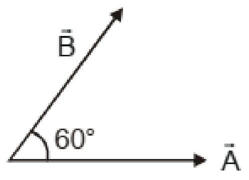
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix}$$

$= \hat{i}(0-1) - \hat{j}(0-2) + \hat{k}\,(1-2)$
$\vec{A}\times\vec{B} = -\hat{i}+2\hat{j}-\hat{k}$

2. $|\vec{A}| = 4\ |\vec{B}| = 3$ $\qquad \theta = 60^o$

$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|$ $\qquad \cos 60^o$

$= 4 \times 3 \times \frac{1}{2} = 6$



$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| = \sin\theta = 4\times3\times\frac{\sqrt{3}}{2} = 6\sqrt{3}$

3. $\vec{A}, \vec{B}$ and $\vec{C}$ are non zero vector

$\vec{A}.\vec{B} = 0$ and $\vec{A}.\vec{C} = 0$

$\vec{A}$ is $\perp$ to $\vec{B}$

$\vec{A}$ is $\perp$ to $\vec{C}$ $\quad \therefore \quad \vec{B} \times \vec{C}$ is 11 to $\vec{A}$

4. Dot product of two mutually perpendicular vectros is zero $\vec{A}.\vec{B} = 0$

$\therefore (4\hat{i} + n\hat{j} - 2\hat{k}).(2\hat{i} + 3\hat{j} + \hat{k}) = 0$

$\Rightarrow (4\times2) + (n \times 3) + (-2 \times 1) = 0 \quad \Rightarrow \quad 3n = -6 \quad \Rightarrow \quad n = -2$

5. Here $\vec{r} = 5\hat{i} - 3\hat{j} + 0\hat{k}$ and

$\vec{F} = 4\hat{i} - 10\hat{j} + 0\hat{k}$

$\therefore \quad \vec{\tau} = \vec{r} \times \vec{F} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -3 & 0 \\ 4 & -10 & 0 \end{vmatrix} = \hat{i}(0–0) - \hat{j}(0–0) + \hat{k}(–50 + 12) = –38\hat{k}$

6. Let $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + 2\hat{k}$

Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$ is $\hat{n} = \pm \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|}$

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 3 \\ 2 & -1 & 4 \end{vmatrix} = (6 + 1) - \hat{j}(4 – 1) + \hat{k}(–2 – 3) = 7\hat{i} - 3\hat{j} - 5\hat{k}$

$\therefore |\vec{A} \times \vec{B}| = \sqrt{7^2 + (-3)^2 + (-5)^2} = \sqrt{83}$ unit $\quad \therefore \quad \hat{n} = \pm\frac{1}{\sqrt{83}}(7\hat{i} - 3\hat{j} - 5\hat{k})$

9. $\vec{A} + \vec{B} = 2\hat{i} \quad \Rightarrow \quad \vec{A} - \vec{B} = 2\hat{j} \quad \Rightarrow \quad (\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 4\hat{i}\ .\ \hat{j} = 0$

10. Since $\vec{B} = 3\vec{A}$, so both are parallel.

11. $\vec{A} \times \vec{B} = \hat{i} + \hat{k}$ (by vector multiplication method)

$\vec{C}$ will be perpendicular to $\vec{A} \times \vec{B}$, If $\vec{C}.(\vec{A} \times \vec{B})$ gives a result zero.

So, incorrect answers are A,C,D.

12. Two vectors must be perpendicular if their dot product is zero.

Let $\vec{a} = 2\hat{i} + 3\hat{j} + 8\hat{k}$

$\vec{b} = 4\hat{j} - 4\hat{i} + a\hat{k}$

$= -4\hat{i} + 4\hat{j} + a\hat{k}$

According to the above hypothesis :

$\vec{a} \perp \vec{b}$

$\Rightarrow \vec{a}\,\vec{b} = 0 \Rightarrow (2\hat{i} + 3\hat{j} + 8\hat{k}).(-4\hat{i} + 4\hat{j} + a\hat{k}) = 0 \Rightarrow -8 + 12\ 8a = 0 \Rightarrow a = -\frac{4}{8} = -\frac{1}{2}$

NOTE : $\vec{a}.\vec{b}$ = ab cos θ. **Here, a and b are always** positive as they are the magnitudes of $\vec{a}$ and $\vec{b}$.

13. $(\vec{B} \times \vec{A}).\vec{A}$ = B Acos θ $\hat{n}.\vec{A} = 0$

Here $\hat{n}$ is perpendicular to both $\vec{A}$ and $\vec{B}$.

Alternative: $(\vec{B} \times \vec{A}).\vec{A}$

Interchange the cross and dot, we have,

$(\vec{B} \times \vec{A}).\vec{A} = \vec{B}(\vec{A} \times \vec{A}) = 0$ $(\because \vec{A} \times \vec{A} = 0)$

NOTE : The volume of a paralleloplped bounded by vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ can be obtained by giveng formula . $(\vec{A} \times \vec{B}).\vec{C}$.

14. $\vec{A} \times \vec{B}$ = AB sin θ and $\vec{A}.\vec{B}$ = AB cos θ.

Given, $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A}.\vec{B}) \Rightarrow$ AB sin θ = $\sqrt{3}$ AB cos θ

$\Rightarrow$ tan θ = $\sqrt{3}$ $\therefore$ θ = 60°

15. $(\vec{P} + \vec{Q}).(\vec{Q} - \vec{P}) = 0$

$-P^2 + \vec{P}.\vec{Q} + Q^2 - \vec{Q}.\vec{P} = 0$ $\because$ $\vec{P}.\vec{Q} = \vec{Q}.\vec{P}$

$P^2 - Q^2 = 0$

P = Q

16. The two vectors must be perpendicular if their dot product must be zero.

Let $\vec{A}$ and $\vec{B}$ are two forces. The sum of the two forces.

$\vec{F_1} = \vec{A} + \vec{B}$ … (i)

The difference of the two forces,

$\vec{F_2} = \vec{A} - \vec{B}$ … (ii)

Since, sum of the two forces is perpendicular to their differences as gives, so

$\vec{F_1}.\vec{F_2} = 0$ or $(\vec{A} + \vec{B}).(\vec{A} - \vec{B}) = 0$ or $A^2 - \vec{A}.\vec{B} + \vec{B}.\vec{A} - B^2 = 0$

or $A^2 = B^2$ or $|\vec{A}| = |\vec{B}|$

Thus, the forces are equal to each other in magnitude.

17. $\vec{A} \times \vec{B}$ = AB sin θ and $\vec{A}.\vec{B}$ = AB cos θ

Given, $|\vec{A} \times \vec{B}| = \sqrt{3}\,\vec{A}.\vec{B}$ …(i)

But $|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}|$ sin θ = AB sin θ

and $\vec{A}. = |\vec{A}|\,|\vec{B}|$ cos θ = AB sin θ

Make these substitution in Eq. (i), we get

AB sin θ = $\sqrt{3}$ AB cos θ or tan θ = $\sqrt{3}$ $\therefore$ θ = 60°

The addition of vector $\vec{A}$ and $\vec{B}$ can be given by the law of parallelogram. $\therefore$ $|\vec{A} + \vec{B}|$

$= \sqrt{A^2+B^2+2AB\cos 60^\circ} = \sqrt{A^2+B^2+2AB\times\frac{1}{2}} = (A^2 + B^2 + AB)^{1/2}$

EXERCISE # 2

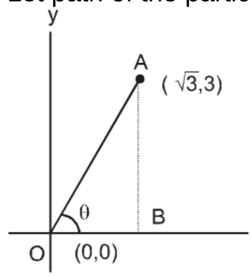
1. $V\left(\frac{\pi}{3}\right) = \sin^2\left(\frac{\pi}{3}\right) - \cos\left(2\frac{\pi}{3}\right) = \sin^2 60^o - \cos 120^o = \left(\frac{\sqrt{3}}{2}\right)^2 - \left(-\frac{1}{2}\right) = \frac{3}{4}+\frac{1}{2} = \frac{5}{4}$

3*. (1) $\frac{3}{5}+\frac{4}{5}=\frac{4}{5}+\frac{3}{5}$ (2) $\frac{3}{5}-\frac{4}{5}=\frac{3}{5}-\frac{4}{5}$ (3) $\frac{3}{4}+1\neq\frac{4}{3}-1$ (4) $\frac{3}{4}\times\frac{4}{3}=1$

5*. $R^2 = 2A^2(1 + \cos\theta) = 2A^2\left(1+2\cos^2\frac{\theta}{2}-1\right) = 2^2A^2\cos^2\frac{\theta}{2}$

$R = 2A\cos\frac{\theta}{2}$

6. Slope of the path of the particle gives the measure of angle required. Draw the situation as shown. OA represents the path of the particle starting from origin 0 (0,0), Draw a perpendicular from point A to x-axis. Let path of the particle makes an angle θ with the x-axis, then



$\tan\theta$ = slope of line OA $= \frac{AB}{OB} = \frac{3}{\sqrt{3}} = \sqrt{3}$ or $\theta = 60°$

7. By using distance formula d

$= \sqrt{(x_2-x_1)^2(y_2-y_1)^2}$ ⇒ 13

$= \sqrt{[3-(-9)]^2+[3-a]^2}$ ⇒ $13^2 = 12^2 + (3-a)^2$ ⇒ $(3-a)^2 = 13^2 - 12^2 = 5^2$

⇒ $(3 - a) = \pm 5$ ⇒ a = 2 cm or 8 cm

8. $y = \ln x^2 + \sin x$

$\frac{dy}{dx} = \frac{d(\ln x^2)}{dx} + \frac{d(\sin x)}{dx} = \frac{1}{x^2}\frac{d}{dx}(x^2) + \cos x = \frac{1}{x^2} . 2x + \cos x = \frac{2}{x} + \cos x$

⇒ $\frac{d^2y}{dx^2} = \frac{d(\frac{2}{x})}{dx} + \frac{d(\cos x)}{dx} = \frac{-2}{x^2} - \sin x$

9. $\frac{dy}{dx} = \frac{d(x^{1/7})}{dx}+\frac{d(\tan x)}{dx} = \frac{x^{-\frac{6}{7}}}{7} + \sec^2 x$

$\frac{d^2y}{dx^2} = \frac{1}{7}\frac{d(x^{-6/7})}{dx}+\frac{d(\sec^2 x)}{dx} = \frac{-6}{49}x^{-13/7} + 2\sec x(\sec x\tan x) = \frac{-6}{49}x^{-13/7} + 2\tan x\sec^2 x$

10. $y = \left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}+1\right)$

$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)\frac{d(x-\frac{1}{x}+1)}{dx} + \left(x-\frac{1}{x}+1\right)\frac{d\left(x+\frac{1}{x}\right)}{dx}$

$\left(x+\frac{1}{x}\right)+\left[\frac{dx}{dx}-\frac{d(\frac{1}{x})}{dx}+\frac{d(1)}{dx}\right] + \left(x-\frac{1}{x}+1\right)\left[\frac{dx}{dx}+\frac{d(\frac{1}{x})}{dx}\right] = \left(x+\frac{1}{x}\right)\left(1+\frac{1}{x^2}\right) + \left(x-\frac{1}{x}+1\right)\left(1-\frac{1}{x^2}\right)$

$= x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^3} + x - \frac{1}{x} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2} = \frac{2}{x^3} + 2x - \frac{1}{x^2} + 1$

11. $r = (1 + \sec\theta)\sin\theta$
$r = \sin\theta + \sec\theta \sin\theta$
$r = \sin\theta + \tan\theta$
$\frac{dr}{d\theta} = \cos\theta + \sec_2\theta$

12. $\frac{dq}{dr} = \frac{d}{dr}(2r - r_2)_{1/2} = \frac{d(2r-r^2)^{1/2}}{d(2r-r^2)} \times \frac{d(2r-r^2)}{dr} = \frac{1}{2}(2r-e^2)^{-1/2}(2-2r) = \frac{1-r}{\sqrt{2r-r^2}}$

13. $\frac{dy}{dx} = \frac{(1+\cot x)\frac{d}{dx}(\cot x) - \cot x \frac{d}{dx}(1+\cot x)}{(1+\cot x)^2} = \frac{(1+\cot x)(-\operatorname{cosec}^2 x) - \cot x(-\operatorname{cosec}^2 x)}{(1+\cot x)^2} = \frac{-\operatorname{cosec}^2 x}{(1+\cot x)^2}$

14. $\frac{dy}{dx} = \frac{\tan x \frac{d}{dx}(\ell nx + e^x) - (\ell nx + e^x)\frac{d}{dx}\tan x}{(\tan x)^2} = \frac{\tan x\left(\frac{1}{x} + e^x\right) - \ell nx + e^x(\sec^2 x)}{(\tan x)^2}$

15. $x_3 + y_3 = 18\,xy$
$\frac{d(x^3+y^3)}{dx} = \frac{d(18\,xy)}{dx}$
$\frac{dx^3}{dx} + \frac{dy^3}{dx} = 18x.\frac{dy}{dx} + y.\frac{d(18x)}{dx}$
$3x_2 + 3y2.\frac{dy}{dx} = 18x.\frac{dy}{dx} + y.18$
$3x_2 + 3y_2.\frac{dy}{dx} = 18x.\frac{dy}{dx} + 18y$
$3y_2\frac{dy}{dx} - 18x.\frac{dy}{dx} = 18y - 3x_2$
$\frac{dy}{dx}(3y_2 - 18x) = 18y - 3x_2$
$\frac{dy}{dx} = \frac{(18y-3x^2)}{(3y^2-18x)}$

16. $x + y = 60$
$x = 60 - y$
$xy = (60 - y)\,y$
$f(y) = (60 - y)\,y$ for maximum
$f'(y) = 60 - 2y = 0$
$y = 30$ So, $x = 30$ & $y = 30$

17. $= \int (x^{-2} + x^{-3})\,dx = \int x^{-2}dx + \int x^{-3}dx = \frac{x^{-2+1}}{-2+1} + \frac{x^{-3+1}}{-3+1} + C = -x_{-1} - x_{-2}\frac{1}{2} + C$

18. $\int (1-\cot^2 x)dx = \int 1 - (\operatorname{cosec}^2 x - 1)\,dx$
$= \int (2 - \operatorname{cosec}^2 x)dx = \int 2dx - \int \operatorname{cosec}^2 x\,dx = 2x + \cot x + C$

19. $\int \cos\theta(\tan\theta + \sec\theta)\,d\theta$
$= \int \cos\theta\tan\theta\,d\theta + \int \cos\theta\sec\theta\,d\theta = \int \cos\theta\frac{\sin\theta}{\cos\theta}d\theta + \int d\theta = -\cos\theta + \theta + C$

Integrate by using the substitution suggested in bracket.

20. Let u = 3 – 2s ⇒ du = – 2 ds

$\int \sqrt{u}\left(\frac{-du}{2}\right) = \frac{-1}{2\times 3/2}$ $u_{3/2}$ + C = $\frac{-1}{3}$ $(3 - 2s)_{3/2}$ + C

21. $\int \frac{dx}{\sqrt{5x+8}}$.

By Substituting

5 x + 8= u,

$\frac{d(5x+8)}{dx} = \frac{du}{dx}$

5 = $\frac{du}{dx}$

5dx = $\left(\frac{du}{dx}\right)$ dx

dx = $\frac{du}{5}$ then]

$\int \frac{du}{5\sqrt{u}}$ = 1/5 $\int u^{-\frac{1}{2}}$ du = $\frac{1}{5}\frac{u^{1/2}}{1/2}$ + C = $\frac{2}{5}\sqrt{u}$ + C = $\frac{2}{5}\sqrt{5x+8}$ + C

22. Substituting x_2 = u,

or 2x dx = du

x dx = $\frac{du}{2}$.

Now, changing the limit for

x = 0, u = 0

x = $\sqrt{\pi}$ u = λ

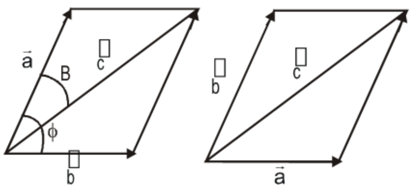
on $\int_{u=0}^{u=\pi} \sin u \, du = \left[-\cos u\right]_0^{\pi}$ = – cos π + cos 0 = 2

23. Using n subintervals of length Δx = $\frac{b}{n}$ and right–end point values. Area = $\int_0^b$ $3x_2$ dx = b_3

Area of the region between the given curve & x axis on the interval [0, b]

$= \int_0^b 3x^2 dx = \left[\frac{3x^3}{3}\right]_0^b = \left[x^3\right]_0^b$ = b_3 – 0 = b_3

24.



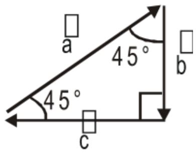
$|\vec{C}| = \sqrt{a^2+b^2+2ab\cos\theta}$ $|\vec{a}| = |\vec{b}|$

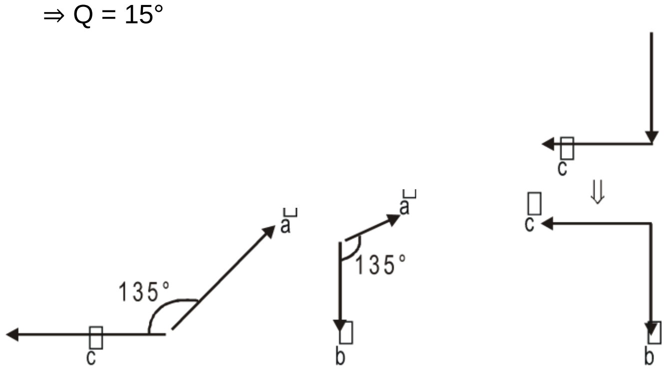
$|\vec{C}| = \sqrt{a^2+b^2+2ab\cos\theta}$ $|\vec{b}| = |\vec{a}|$ ∴ $|\vec{c}| = |\vec{C}|$

25. Can not be zero

26. $\vec{A} = 2\hat{i} + 2\hat{j}$ cosθ = $\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{2+2\sqrt{3}}{(2\sqrt{2})(2)}$

$\vec{B} = \hat{i} + \sqrt{3}\,\hat{j} \quad = \dfrac{2(1+\sqrt{3})}{4(\sqrt{2})} = \dfrac{1+\sqrt{3}}{2\sqrt{2}} \quad \Rightarrow Q = 15°$

27. 

By vector translation 

$\therefore$ 90°, 135°,. 135°

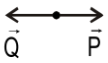
28*. $a^2 + b^2 + 2ab\cos\theta = a^2 + 4b^2 - 4ab\cos\theta$

or $\cos\theta = \dfrac{b}{2a} \le 1$ $\therefore b \le 2a$

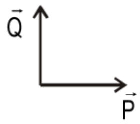
30. Resultant R = P + Q = $(2\hat{i} + 7\hat{j} - 10\hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k}) = 3\hat{i} + 9\hat{j} - 7\hat{k}$ But $\vec{R}$ + required vector = $\hat{i}$

Or required vector = $\hat{i} - \vec{R} = \hat{i} - (3\hat{i} + 9\hat{j} - 7\hat{k}) = -2\hat{i} - 9\hat{j} + 7\hat{k}$

31. Let $\vec{P}$ & $\vec{Q}$ are two vector

If 

P – Q = 10 unit .... (1)

If 

$\sqrt{P^2 + Q^2}$ = 50 unit

$P^2 + Q^2 = 50^2$ .... (2)

from equation (1)

$(10 + Q)^2 + Q^2 = 50^2$ $\Rightarrow$ $2Q^2 + 20Q + 100 = 2500$

$2Q^2 + 20Q = 2400$ $\Rightarrow$ $Q^2 + 10Q - 1200 = 0$

(Q + 40) (Q - 30) = 0 Q = 30 So, from equation (1) P = 10 + Q = 10 + 30 = 40 unit

32. $\vec{OA} = r\,\hat{j}$

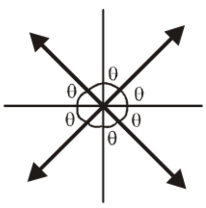
$\vec{OC} = r\,\hat{i}$

$\vec{OB} = r(\cos 45\,\hat{i} + \sin 45\,\hat{j}) \Rightarrow \vec{OB} = \dfrac{r}{\sqrt{2}}\hat{i} + \dfrac{r}{\sqrt{2}}\hat{j}$

$\vec{R} = \vec{OA} + \vec{OB} + \vec{OC} = r\hat{j} + \dfrac{r}{\sqrt{2}}\hat{i} + \dfrac{r}{\sqrt{2}}\hat{j} + r\,\hat{i} = \left(r + \dfrac{r}{\sqrt{2}}\right)\hat{i}$

33. Initial velocity = $50\hat{j}$

Final velocity = $-50\hat{j}$ 1 change = $50\sqrt{2}$ along south west

34. 

Coplanar all in a single plane (xy plane)

$60° = 360°$

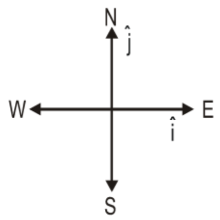
$\theta = 60°$ Check now every component cancels out net = 0

36.* $\vec{A}.\vec{B} = 8$ $\qquad$ $A\,B\cos\theta = 8$

$|\vec{A}\times\vec{B}| = 8\sqrt{3}$ $\qquad$ $AB\sin\theta = 8\sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

$\theta = 120°$



37.

$\vec{A} \to -\hat{k}$

$\vec{B} \to +\hat{i}$

$\vec{A}\times\vec{B} = -\hat{k}\times\hat{i} = \hat{j}$ ⇒ south

38. Check product (fifth doesn't satesfy)

39. By finding $\dfrac{\vec{A}\times\vec{B}}{|A||B|}$ will give the ans.

$\vec{A}\times\vec{B} = (3\hat{i}+\hat{j}+2\hat{k})\times(2\hat{i}-2\hat{j}+\hat{k}) = -6\hat{k}-3\hat{j}+2\hat{k}+\hat{i}+4\hat{j}+4\hat{i} = 5\hat{i}+\hat{j}-4\hat{k}$ None of the answer given.

40. Given that : $\vec{A} = \vec{B} + \vec{C}$ ⇒ $\vec{A} - \vec{C} = \vec{B}$

By taking self dot product on both sides $(\vec{A}-\vec{C}).(\vec{A}-\vec{C}) = \vec{B}.\vec{B}$ ⇒ $A^2 + C^2 - 2\vec{A}.\vec{C} = \vec{B}^2$

Now let angle between $\vec{A}$ and $\vec{C}$ be θ then $A^2 + C^2 - 2AC\cos\theta = B^2$

$\therefore \cos\theta = \dfrac{A^2+C^2-B^2}{2AC} = \dfrac{(5)^2+(3)^2-(4)^2}{2(5)(3)} = \dfrac{18}{30} = \dfrac{3}{5}$ ⇒ $\theta = \cos^{-1}\left(\dfrac{3}{5}\right) = 53°$

Or $\qquad$ Since $5^2 = 4^2 + 3^2$

The vectors $\vec{A},\vec{B}$ and $\vec{C}$ make a triangle with angle between $\vec{B}$ and $\vec{C}$ as 90°.

If θ is the angle between $\vec{A}$ and $\vec{C}$, then $\cos\theta = \dfrac{3}{5} = 0.6 = \cos(53°)$

41. $S_x = 6\cos 45° - 4\cos 45° = \sqrt{2}$ km $\qquad$ $S_y = 6\sin 45° + 4\sin 45° = 5\sqrt{2}$ km

$S = \sqrt{S_x^2+S_y^2} = \sqrt{52}$ km, $\phi = \tan^{-1}\dfrac{S_y}{S_x} = \tan^{-1}(5)$

EXERCISE-3
PART - I

1. As we have given

$|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$ or $\sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$

where θ is the angle between $\vec{A}$ and $\vec{B}$. Squaring both the side, we have

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$ $\qquad$ or $\qquad$ $4AB\cos\theta = 0$

As $\qquad$ $AB \neq 0$

$\therefore \cos\theta = 0 = 0\cos 90°$ $\quad\therefore\quad$ $\theta = 90°$ $\qquad$ Hence, angle between $\vec{A}$ and $\vec{B}$ is 90°

2. By Triangle law of vector addition.

3. $V_C = \eta^x \rho^y r^z$

critical velocity is given by $V_C = \dfrac{R\eta}{2\rho r}$ $\qquad$ So, x = 1

y = –1

$z = -1$

4. $\vec{A} = \cos wt\,\hat{i} + \sin wt\,\hat{j}$

$\vec{B} = \cos\frac{wt}{2}\hat{i} + \sin\frac{wt}{2}\hat{j}$ for $\vec{A}.\vec{B} = 0$

$\vec{A}.\vec{B} = 0 = \cos wt.\cos\frac{wt}{2} + \sin wt.\sin\frac{wt}{2} = \cos\left(wt - \frac{wt}{2}\right) = \cos\left(\frac{wt}{2}\right)$ So, $\frac{wt}{2} = \frac{\pi}{2}$ $\Rightarrow$ $t = \frac{\pi}{w}$

5. $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$

$(A)^2 + (B)^2 + 2(A)(B)\cos\theta = (A)^2 + (B)^2 - 2(A)(B)\cos\theta$

$2\cos\theta = 0 \quad \Rightarrow \theta = 90°$

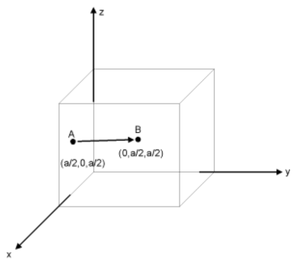
6. $\vec{V} = \cos\omega t\,\hat{i} + \sin\omega t\,\hat{j}$

$\vec{V} = \frac{d\vec{r}}{dt} = -\omega\sin\omega t\,\hat{i} + \omega\cos\omega t\,\hat{j}$ $\Rightarrow$ $\vec{a} = \frac{d\vec{V}}{dt} = -\omega^2\cos\omega t\,\hat{i} - \omega^2\sin\omega t\,\hat{j}$

since $\vec{r}.\vec{V} = 0$ so $\vec{r} \perp_r \vec{V}$ and $\vec{a} = -\omega^2\vec{r}$ So, $\vec{a}$ will be always aiming towards the origin.

PART - II

1.



$\bar{r} = \bar{r}_B - \bar{r}_A = -\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j}$

2. $R^2 = 4F^2 + 9F^2 + 2 \times 2F \times 3F\cos\theta$

$4R^2 = 4F^2 + 36F^2 + 2 \times 2F \times 6\,F\cos\theta$

$4 = \frac{4 + 36 + 24\cos\theta}{4 + 9 + 12\cos\theta}$

$13 \times 4 + 12 \times 4\cos\theta = 40 + 24\cos\theta$

$\cos\theta = \frac{40 - 52}{48 - 24} = \frac{-12}{24} = -\frac{1}{2}$

$\theta = 120°$

3. $A^2 + A^2 + 2AB\cos\theta = n^2(A^2 + A^2 - 2AB\cos\theta)$

$A^2 + A^2 + 2A^2\cos\theta = n^2(A^2 + A^2 - 2A^2\cos\theta)$

$2A^2(1 + \cos\theta) = 2A^2n^2(1 - \cos\theta)$

$1 + \cos\theta = n^2 - n^2\cos\theta$

$\cos\theta(1 + n^2) = n^2 - 1$

$\cos\theta = \frac{n^2 - 1}{n^2 + 1}$ $\Rightarrow$ $\theta = \cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$