TOPIC : ALTERNATING CURRENT
EXERCISE # 1

SECTION (A)

1. $I_0 = \sqrt{2}\, I_{rms}$

$I_0 = 10\sqrt{2}$

2. In a capacitance circuit the current leads voltage by phase angle π/2.

3. $\tan\phi = \frac{\omega L}{R}$ $\qquad \tan\phi = 1$

4. Hot wire ammeter is used to measure A.C. current.

7. $I = \frac{200\sqrt{2}}{(X_C)\times\sqrt{2}} = 200 \times \omega C = 20$ mA.

9. $I = I_1 \cos\omega t + I_2 \sin\omega t$

$I = \sqrt{I_1^2 + I_2^2}\ .\ \sin(\omega t + \alpha)$

$I_{rms} = \frac{I_0}{\sqrt{2}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$

10. $I_0 = 2\sqrt{2}$

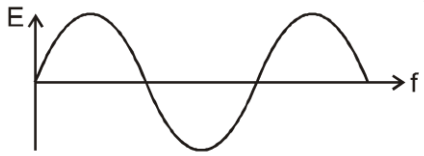
$I_{rms} = \frac{I_0}{\sqrt{2}} = 2A$

11. $I_{rms} = \left[\frac{\int_0^T i^2\, dt}{T}\right]^{\frac{1}{2}}$

12. $E = 10 \cos\left(2\pi \times 50 \times \frac{1}{600}\right) = 5\sqrt{3}$

13. $V_{rms2} = \int_0^T \frac{(e_1 \sin\omega t + e_2 \cos\omega t)^2 dt}{T} = \sqrt{\frac{e_1^2 + e_2^2}{2}}$ $\qquad$ where $\omega = \frac{2\pi}{T}$.

14. If net area of E – t curve is zero for given interval then average value will be zero.



15. $V = 100 \sin 100\pi t \cos 100\pi t$

$V = 50 \sin 200\pi t$

here $V_0 = 50$ & $\omega = 200\pi$ $\qquad f = 100$ Hz

16. D.C. Voltmeter measures Average value only

17. $\frac{T}{4} = \frac{1/f}{4} = 5 \times 10^{-3}$s

18. $i = 4 \cos(\omega t + \phi)$

$i_{rms} = \frac{4}{\sqrt{2}}$ A $= 2\sqrt{2}$A

19. $i_{rms} = \frac{200}{40} = 5A$

$i_0 = \sqrt{2} \cdot i_{rms} = 5\sqrt{2}$ A

SECTION (B)

2. In second mode of transmission since $I = \frac{P}{V}$ the current is less hence during transmission I^2R losses are least.

3. E = 200sinωt

It contains capacitor or inductor, so power factor = cos 90 = 0

4. V = 220 V, i = 5mA

Loss of power = 0

because power factor = 0

R = 0

5. $P = V_{rms} \cdot I_{rms} \cdot \cos\phi$

at maximum power cosϕ = 1

but at half power $\cos\phi = \frac{1}{2}$

$V_{rms} \times I_{rms} \times \frac{1}{2} = P$

$V_{rms} \times I_{rms} \times \frac{1}{2} = \frac{V_{rms}}{\sqrt{2}} \times I$

$I = \frac{I_{rms}}{\sqrt{2}}$

$I = \frac{I_{rms}}{\sqrt{2}}$

6. E = 200 sin 314 t ,

$I = 100\sin\left(314t + \frac{\pi}{3}\right)$ power factor = cosϕ = $\cos\frac{\pi}{3} = \frac{1}{2}$

7. $V_L = 8V, V_R = 6V, V = \sqrt{V_L^2 + V_R^2} = 10$ V power factor = $\cos\phi = \frac{V_R}{V} = \frac{6}{10} = 0.6$

9. From the relation, $\tan\phi = \frac{\omega L}{R}$

Power factor $\cos\phi = \frac{1}{\sqrt{1+\tan^2\varphi}} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} = \frac{R}{\sqrt{R^2+\omega^2L^2}}$

11. $P_{av} = v_{rms} I_{rms} \cos\phi$

Here ϕ = 90º so $P_{av} = 0$

Alter. v = 5 cos(ωt)

i = 2sinωt

$P = v_{rms} \cdot I_{rms} \cdot \cos\phi$ ∴ ϕ = 90° ⇒ P = 0

12. $\frac{H_{D.C.}}{H_{A.C.}} = \frac{I^2R}{I_{rms}^2R} = 2$

13. $<P> = I_{rms}^2 R = \left(\frac{I_P}{\sqrt{2}}\right)^2 R = \frac{I_P^2R}{2}$

14. $(2)^2R = P^2$ ....(i)

$I_{rms}^2 \times R = 3P^2$ ....(ii)

15. $P = I_{rms} V_{rms} \cos \frac{\pi}{2} = 0$

17. $P = \frac{v^2}{R}$

$R = \frac{10 \times 10}{20} = 5\Omega$

for AC source

P = 10 watt

$\because P = v_{rms} . \frac{v_{rms}}{Z} \times \frac{R}{Z}$

$10 = 10 \times \frac{10.R}{Z^2}$

$Z^2 = 10R$

$R^2 + X_L^2 = 10 \times R$

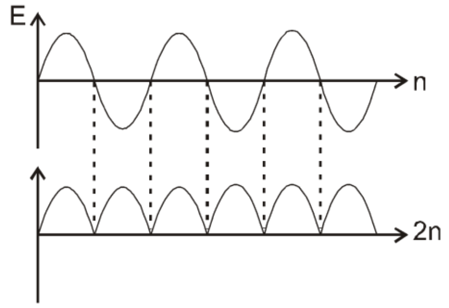
$25 + X_L^2 = 10 \times 5$

$X_L^2 = 25$

$X_L = \sqrt{25} = 5$

$\omega \times L = 5$

$f = \frac{5 \times 10^2}{2\pi} = \frac{250}{3.14} \approx 80$ Hz

18. 

20. Power factor = $\cos \phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

21. $I_0 = \frac{V_0}{\omega L} = \frac{10}{100 \times 5 \times 10^{-3}}$

22. $I^2R = 100$

$R = \frac{100}{I^2} = \frac{100}{(2)^2} = 25.$

23. $\tan \phi = \frac{x}{R} = \frac{4}{3}$

$\cos\phi = \frac{3}{5} = 0.6$

SECTION (C)

1. $Z = \sqrt{R^2 + (X_L - X_C)^2}$ as f ↑ X_2 ↑ X_C ↓ so. ans (a)

5. E = 4 cos 1000t ,
L = 3mH, R = 4Ω
$X_L = 1000 \times 3 \times 10^{-3} = 3$
Z = 5
$i_0 = \frac{E_0}{2} = \frac{4}{5} = 0.8A$

6. Z = 100 Ω at f = 50Hz
$2\pi f \times L = X_L$
$\frac{f_1 = X_{L_1}}{f_2 = X_{L_2}}$
$\frac{150}{50} = \frac{100}{X_{L_2}}$
$X_{L_2} = 300\Omega$

7. $\tan\phi = \frac{x_c}{R} = \frac{1/\omega c}{R}$ $\Rightarrow \phi = \tan^{-1}\frac{1}{\omega CR}$

8. $\tan\phi = \frac{\omega L}{R} = \frac{10\times 0.1}{1} = 1$
$\phi = 45° = \pi/4$

9. Power factor = $\frac{R}{Z} = \frac{30}{50} = \frac{3}{5}$
here $Z = \sqrt{R^2 + (x_L - y_c)^2} = \sqrt{(30)^2 + (60-20)^2}$
Z = 50
$I = \frac{V}{Z} = \frac{100}{50}$ = **2 Amp.**

10. The current lags the EMF by π/2, so the circuit should contain only an inductor.

11. Resultant voltage = 200 volt
Since V_1 and V_3 are 180° out of phase, the resultant voltage is equal to V_2
∴ $V_2 = 200$ volt

12. Initially at resonance:$X_L = X_E \Rightarrow Z = R$.
∴ $i_0 \frac{\varepsilon_0}{R} = 10\sqrt{2}$ = A
After increasing frequency : $X_L > X_c$
$\omega L > 1/\omega C$
$\omega > \frac{1}{\sqrt{LC}} \Rightarrow \omega > \omega_0$ $i' = \frac{\varepsilon_0}{\sqrt{R^2 + X^2}} = \frac{\varepsilon_0}{\sqrt{2}R} = i_0 / \sqrt{2}$ = 10 amp.

14. R = 3Ω, X = 4Ω
Power factor = $\frac{I^2}{Z} = \frac{3}{5} = 0.6$

15. R = 12Ω, H = 0.21

16. $R = \frac{100}{1} = 100\ \Omega$ $x = \sqrt{Z^2 - R^2}$

$Z = \frac{100}{0.5} = 200\ \Omega$ $L = \frac{x}{\omega} = \frac{1}{3}$ H.

17. Voltage of source is always less than ($V_1 + V_2 + V_3$), $V_{net} = \sqrt{V_1^2 + V_2^2 + V_3^2}$

18. At resonance ($V_C = V_L$)

$V = I_{rms} \times R$

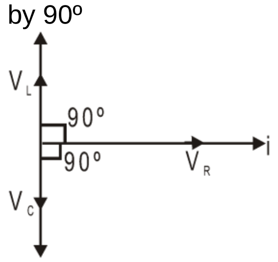
$= \frac{V_{rms}}{Z} \times R$ (here z = R)

$V = V_{rms} = 100$ volt & $I_{rms} = \frac{100}{50} = 2$ Amp.

19. $i_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \frac{1}{\omega^2 c^2}}}$

when ω increases, i_{rms} increases so the bulb glows brighter

21. In an LCR series a.c. circuit, the voltage across inductor L leads the current by 90º and the voltage across capacitor C lags behind the current by 90º



Hence, the voltage across LC combination will be zero.

22. $\tan\phi = \frac{X}{R} = \infty = \frac{1}{0}$ ⇒ $R = 0$

23. $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{(1/\omega C)} = 20 \times 10^{-3}$ A

24. $I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L)^2}}$

$\tan\phi = \frac{\omega L}{R}$.

25. When all (L,C,R) are connected then net phase difference = 60 – 60 = 0. So, there will be resonance.

$I = \frac{V}{R} = 2A$

& $P = I^2 R = 400$ watt.

26. $I_{rms} = \frac{V_{rms}}{Z} = \frac{100}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

P.d. across resistance = R I_{rms} .

27. $R = \frac{V_{rms}}{I_{rms}} = \frac{200}{5/\sqrt{2}} = 40\sqrt{2}\ \Omega$ (For ckt x)

$X_L = \frac{V}{I_{rms}} = \sqrt{2} \times 40\ \Omega$ (For ckt y)

If x & y are in series

$I = \frac{200}{40 \times 2} = \frac{5}{2}$ Amp.

28. $I_0 = \sqrt{2}\ I_{rms} = \sqrt{2}\frac{V_{rms}}{Z}$

$I_0 = \frac{\sqrt{2} \times 130\sqrt{2}}{\sqrt{R^2 + (\omega L)^2}}$

$\tan\phi = \frac{\omega L}{R}$

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$.

29. $\tan\phi = \tan 45^o = \frac{\omega L}{R}$

$X_L = \omega L = R$.

30. $V_{net} = \sqrt{V_R^2 + V_L^2} = \sqrt{(20)^2 + (16)^2} = 25.6$.

37. Wattless current = $I_{rms} \sin\phi$

Where $\tan\phi = \frac{\omega L}{R} = \frac{2\pi f L}{R}$ and $I_{rms} = \frac{v_{rms}}{z} = \frac{v_{rms}}{\sqrt{R^2 + (\omega L)^2}}$

38. $X_C = \frac{1}{\omega C}$ will decrease if we increase frequency then z will decrease so current will increase & intensity will increase.

39. $\cos\phi =$

$Z_1\ \alpha$ $Z_2\ \alpha$

% change in impedence = 100%

SECTION (D)

1. Given : L = 10 H, *f* = 50 Hz.

For maximum power

$X_C = X_L$

$\frac{1}{\omega C} = \frac{1}{\omega L}$

$C = \frac{1}{\omega^2 L}$

$C = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$ $\therefore$ $C = 0.1 \times 10^{-5}$ F = 1 μF

2. In the condition of resonance

$X_L = X_C$

or $\omega L = \frac{1}{\omega C}$ ....(i)

Since, resonant frequency remains unchanged,

so, $\sqrt{LC}$ = constant or LC = constant $\therefore$ $L_1C_1 = L_2C_2$

$\Rightarrow \quad L \times C = L_2 \times 2C \quad \Rightarrow \quad L_2 = \frac{L}{2}$

3. $\frac{q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q^2}{2C}$ but $\frac{1}{2}LI^2 = \frac{q^2}{2C}$ So, $2\left(\frac{q^2}{2C}\right) = \frac{Q^2}{2C}$ $\Rightarrow$ $q = \frac{Q}{\sqrt{2}}$

6. $I_m = \omega Q_0 = \frac{CV_0}{\sqrt{LC}} = \frac{2\mu F.20V}{\sqrt{(8\mu H)(2\mu F)}} = 10.0$ A.

10. $I_{rms} = \frac{60}{120} = \frac{1}{2}$ Amp.
$V_L = I_{rms} \times (\omega L)$
$40 = \frac{1}{2} \times (40 \times 10^3) \times L$
$L = 20$ mH

At resonance $V_C = I_{rms}\left(\frac{1}{\omega C}\right) = V_L$
$C = \frac{1}{2} \times \frac{1}{4 \times 10^3} \times \frac{1}{40}$ $\Rightarrow$ $C = \frac{25}{8}$ μF.

11. At resonance $\omega L = \frac{1}{\omega C}$
$L \propto \frac{1}{C}$.

12. If $n > n_r$ $\quad \omega L > \frac{1}{\omega C}$ $\quad X_L > X_C$
So, current lags behind voltage.

SECTION (E)

1. The core of transformer is laminated to reduce energy loss due to eddy currents.

2. Given: $i_p = 4$ A, $N_p = 140$,
$N_s = 280$
From the formula
$\frac{i_p}{i_s} = \frac{N_s}{N_p}$ or $\frac{4}{i_s} = \frac{280}{140}$
So, $i_s = 2A$

3. $P_{out} = 100$ watt
$p_{in} = 200 \times 0.6$ watt.
= 120 watt
so $\eta = \frac{100}{120} \times 100\% = \frac{5}{6} \times 100\% = \frac{500}{6}\% = 83.33\%$

4. Frequency of the current remains same, only magnitudes of current changes in a transformer.

5. $\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{8}{1}$
$V_2 = 8 \times 120 = 960$ volt
$I = \frac{960}{10^4} = 96$ mA.

6. $\eta\% = \dfrac{E_2I_2}{E_1I_1} \times 100$

7. $I_1 E_1 = I_2 E_2$

$I_2 = \dfrac{I_1E_1}{E_2} = \dfrac{5 \times 220}{22000} = .05\text{ A}$

9. $P = 600 \times 1000 = 4000 \times I \Rightarrow I = 150\text{ A}$

Power loss = $I^2r = (150)^2 \times 0.4 \times 20 \times 2 = 360\text{kW}$

Power loss percentage = $\dfrac{\text{Power loss}}{\text{Power input}} \times 100 = \dfrac{360}{600} \times 100$

$\Rightarrow 60\ \%$

10. $\dfrac{N_p}{N_s} = \dfrac{40,000}{200} = \dfrac{200}{1}$

EXERCISE # 2

1. The peak value of the current is

$I_0 = \dfrac{V_0}{\sqrt{R^2 + \dfrac{1}{\omega^2C^2}}} = \dfrac{V_0}{\sqrt{2}\,R}$ when the angular frequency is changed to $\dfrac{\omega}{\sqrt{3}}$

The new peak value is

$I_0' = \dfrac{V_0}{\sqrt{R^2 + \dfrac{3}{\omega^2C^2}}} = \dfrac{V_0}{\sqrt{4R^2}} = \dfrac{V_0}{2R}$ $\therefore$ $I_0' = \dfrac{I_0}{\sqrt{2}}$

2. $i_{rms} = \dfrac{V_{rms}}{\sqrt{R^2 + \dfrac{1}{\omega^2c^2}}}$ when ω increases, i_{rms} increases so the bulb glows brighter

3. The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative. Therefore, for an a.c. cycle, the net value of current average out to zero. While for the half cycle, the value of current is different at different points. Hence, the alternating current cannot be measured by D.C. ammeter

4. **(i)** $\dfrac{E_2}{E_1} = \dfrac{N_2}{N_1} = \dfrac{1}{5}$ $\Rightarrow$ $E_2 = \dfrac{1000}{5} = 200$ volt.

(ii) $E_2I_2 = E_1I_1 \times \eta\ \%$

$9000 = 1000 \times I_1 \times \dfrac{90}{100}$

$I_1 = 10$ amp.

(iii) The copper loss in the primary coil is = $I_1{}_2 R_1 = (10)_2 \times 1 = 100$.

(iv) Total loss = $E_1I_1 - E_2I_2 = 1000$ W

Loss in secondary = 1000 – 700 – 100 = 200 W

EXERCISE # 3

PART - I

2. $V_{rms} = \sqrt{\dfrac{(T/2)V_0^2 + 0}{T}} = \dfrac{V_0}{\sqrt{2}}$.

3. If ω = 50 × 2π then $\omega L = 20\Omega$

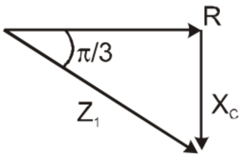
If ω′ = 100 × 2π then $\omega' L = 40\Omega$

$$I = \frac{200}{Z} = \frac{200}{\sqrt{R^2 + (\omega' L)^2}} = \frac{200}{\sqrt{30^2 + (40)^2}}$$

I = 4 A.

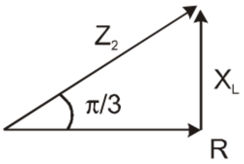
4. 

$$\frac{X_C}{R} = \tan\frac{\pi}{3}$$



$$X_C = R\tan\frac{\pi}{3} \qquad \text{.............(1)}$$

$$\frac{X_L}{R} = \tan\frac{\pi}{3}$$



$$X_C = R\tan\frac{\pi}{3} \qquad \text{..........(3)}$$

net impedence $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$

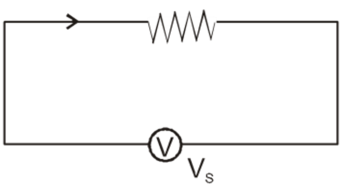
power factor $\cos\varphi = \frac{R}{Z} = 1$ Ans. (3)

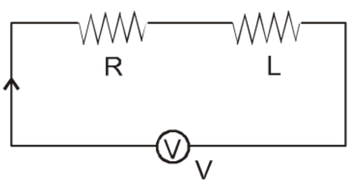
5. $\langle P \rangle = V_{Rms} . I_{Rms} \cos\phi$

$$V_{Rms} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \frac{1}{2}\text{volt} \quad \Rightarrow \quad I_{Rms} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{2}} = \left(\frac{1}{2}\right)A$$

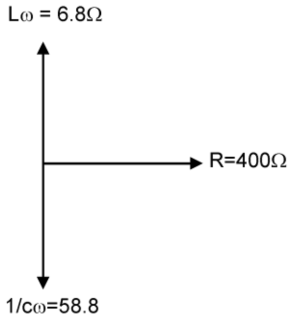
$$\cos\phi = \cos\frac{\pi}{3} = \frac{1}{2} \quad \Rightarrow \quad \langle P \rangle = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{1}{8}W$$

6. L ↑ z ↑ I ↓ so brightness ↓

7. 



$$P = i_{rms}^2 R = \left(\frac{V_S}{R}\right)^2 .R = \frac{V_S^2}{R} \quad \Rightarrow \quad P^1 = i_{rms}^2 R = \left(\frac{V_S}{Z}\right)^2 .R$$



8. So $|z| = \sqrt{(40)^2 + (58.8 - 6.8)^2} = 65$

$i_0 = \frac{v_0}{|z|} = \frac{10}{65}A \Rightarrow i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{10}{65\sqrt{2}}$

$P_{loss} = r_{rms}^2 R = \left(\frac{10}{65\sqrt{2}}\right)^2 \times 40 = 0.46 \text{watt}$

So the nearest answer will be (2)

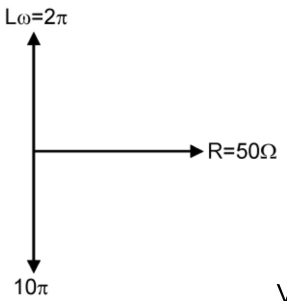
9. Capacitor does not consume energy effectively over full cycles

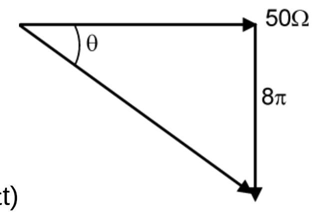
10. Option with highest quality factor sould be chosen as most appropriate answer.

Q = $\frac{1}{R}\sqrt{\frac{L}{C}}$

11. Power factor = $\frac{R}{z} = \frac{iR}{iz} = \frac{80}{\sqrt{(80)^2 + (60)^2}}$ = $\frac{80}{100}$ = 0.8

12. z = $\sqrt{R^2 + X_c^2} = \sqrt{(100)^2 + (100)^2} = 100\sqrt{2}$ ⇒ $\mathbf{i_{max}}$ = $\frac{v_{max}}{z} = \frac{220\sqrt{2}}{100\sqrt{2}}$ = 2.2





13. V = 10 sin (100 πt)

ω = 100 π

$x_L = L\omega = (20\times10^{-3})100\pi = 2\pi x$ ⇒ $_C = \frac{1}{L\omega} = \frac{1}{(100\times10^{-6})\times100\pi}$ = 10 π

$|z| = \sqrt{(x_L - x_C)^2 + R^2}$ | ⇒ z| = $\sqrt{(8\pi)^2 + (50)^2}$

$|z| = \sqrt{3140} \approx 56\Omega$ ⇒ $i_0 = \frac{V_0}{|z|} = \frac{10}{56}$ = 0.18 A

$P = i^2_{rms} = \frac{i_0^2 R}{2} = \frac{(0.18)^2 \times 50}{2}$ = 0.81 watt

14. If we connect a D.C. source, the steady state current is not blocked (i_{ss} = 0.4A), so the circuit should not contain capacitor. The circuit should be either only R or LR. Since current in AC circuit and D.C. circuit is different, so the circuit shouldn't be only R so it should be an L-R circuit.

PART - II

1. $\tan 30^\circ = \frac{X_L}{R}$ ⇒ $X_L = \frac{R}{\sqrt{3}} = \frac{200}{\sqrt{3}}$

$\tan 30^\circ = \frac{X_C}{R}$ ⇒ $X_c = \frac{200}{\sqrt{3}}$

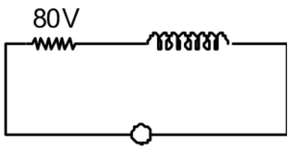
$Z = \sqrt{R + (X_L - X_C)^2} = 200\ \Omega$

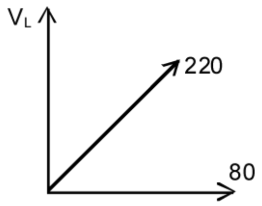
$i_{rms} = \frac{220}{200} = 1.1$

$P = (i_{rms})_2 \times R = (1.1)_2 \times 200$

$P = 242$ W

2. R = 80/10 = 8Ω



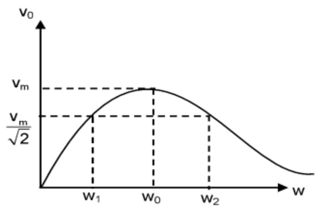


$V_L^2 + 80^2 = 220^2$ ⇒ $V_L^2 = (220 + 80)(220 - 80) = 300 \times 140$ ⇒ $V_L = 204.9$

$I_{rms} X_L = 204.9$

$x_L = 2.5$

3.



Band width $\omega_2 - \omega_1 = \frac{R}{L}$ ⇒ Quality factor $Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0 L}{R}$

4. $e = 100 \sin 30t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

$P_{av} = e_{rms}\, i_{rms} \cos\phi = \frac{100}{\sqrt{2}}.\frac{20}{\sqrt{2}}.\frac{1}{\sqrt{2}} = \frac{1000}{\sqrt{2}}$W ⇒ wattless current = $\frac{I_0 \sin\phi}{\sqrt{2}} = \frac{20}{\sqrt{2}}.\frac{1}{\sqrt{2}} = 10$A

5. $\eta = \frac{e_s i_s}{e_p i_p} \times 100$ ⇒ $90 = \frac{230 \times i_s}{2300 \times 5} \times 100$

$i_s = 45$ A

6. $\omega = 2\pi f = 100\pi$

$X_L = \omega L = 6.28\Omega$

$X_C = \frac{1}{\omega C} = \frac{250}{3\pi}$

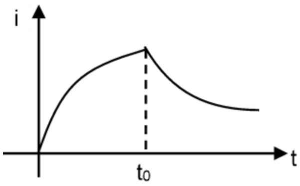
$X = X_C - X_L = 20.25$

$Z = \sqrt{R^2 + X^2}$

$I_{rms} = V_{rms} / Z$

Energy dissipated = $i_{rms}^2 \times R \times t$, t = 60 sec = 3000T $\left(T = \frac{1}{f} = 0.02s\right)$ = 5.17 × 10^2 J

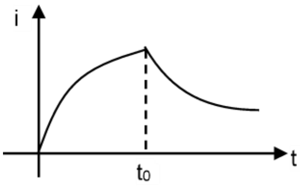
7. The correct graph will be



Growth and decay of current is of exponential nature

$i = i_0(1 - e^{-t/\tau})$ → during growth

$i = i_{max}\, e^{-t/\tau}$ → during decay



8. $x_L = \omega L$

$= 100 \times \frac{\sqrt{3}}{10} = 10\sqrt{3}\ \Omega$

$\tan \phi_1 = \frac{10\sqrt{3}}{10} = \sqrt{3}$, $\phi_1 = 60^o$ (Current is lagging)

$\tan\phi_2 = \frac{1}{\omega CR} = \frac{1}{\sqrt{3}}$, $\phi_2 = 30^o$ (Current is leading)

Total phase difference = 90º