TOPIC : MEASUREMENT ERROR & EXPERIMENT
EXERCISE # 1
PART - I

1. $\bar{D} = \frac{\Sigma(D)}{N} = \frac{1.324 + 1.326 + 1.334 + 1.336}{4}$ = 1.330.

ΔD_1 = 1.324 – 1.330 = –0.006

ΔD_2 = 1.326 – 1.330 = –0.004

ΔD_3 = 1.334 – 1.330 = 0.004

ΔD_4 = 1.336 – 1.330 = 0.006

$\Delta\bar{D} = \frac{|\Delta D_1| + |\Delta D_2| + |\Delta D_3| + |\Delta D_4|}{4} = \frac{0.006 + 0.004 + 0.004 + 0.006}{4} = \frac{0.020}{4}$ = 0.005 cm

$\frac{\Delta\bar{D}}{D} = \frac{0.005}{1.330}$ = 0.004 $\Rightarrow$ % error = $\frac{\Delta\bar{D}}{D}$ × 100 = 0.4%.

2. (i) 1, zero before and after decimal but preceding first non-zero digit are insignificant.
(ii) 3, Power of 10 in scientific notation is insignificant.
(iii) 4, Trailing zero's after NZD's in fraction are significant.
(iv) 4, Trailing zero's after NZD's in fraction are significant.
(v) 4, Zero's between NZD's are significant.
(vi) 4, Zero's before NZD's in fractional part (< 1) are insignificant.

3. (i) 0.0393 kg (ii) 4.08 x 10^8 sec (iii) 5.24 m (iv) 4.74 x 10^{-6} kg

4. $v = 2f_0 (\ell_2 - \ell_1)$,

$\Rightarrow \left(\frac{\Delta v}{v}\right)_{max} = \frac{\Delta f_0}{f_0} + \frac{\Delta \ell_1 + \Delta \ell_2}{\ell_2 - \ell_1} = \frac{1}{100} + \frac{0.1 + 0.1}{74 - 24}$ = 1.4%.

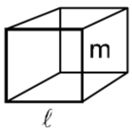
PART - II

1. A = ℓb = 10.0 × 1.00 = 10.00

$\frac{\Delta A}{A} = \frac{\Delta \ell}{\ell} + \frac{\Delta b}{b}$

$\frac{\Delta A}{10.00} = \frac{0.1}{10.0} + \frac{0.01}{1.00} \Rightarrow \Delta A = 10.00 \left(\frac{1}{100} + \frac{1}{100}\right) = 10.00 \left(\frac{2}{100}\right) = \pm 0.2\ cm^2$.

2. $\left(\frac{\Delta A}{A}\right)_{min} = \left|\left(\frac{\Delta \ell}{\ell} - \frac{\Delta b}{b}\right)\right| = \left|\frac{1}{100} - \frac{1}{100}\right| = 0$

3. 

$\rho = \frac{m}{V} = \frac{m}{\ell^3}$

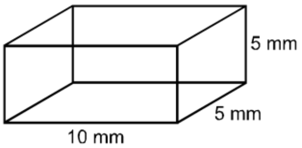
Given : $\frac{\Delta m}{m} = \pm$ 2% = ± 2 × 10^{-2} $\frac{\Delta \ell}{\ell}$ = ± 1% = ± 1 × 10^{-2}

$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta \ell}{\ell}$ = 2 × 10^{-2} + 3 × 10^{-2} = 5 × 10^{-2} = 5%

4. $g = 4\pi^2 \frac{\ell}{T^2} \Rightarrow \frac{\Delta\ell}{\ell} = 2\% = \pm 2 \times 10^{-2} \Rightarrow \frac{\Delta T}{T} = \pm 3\% = \pm 3 \times 10^{-2}$

$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T} = 2 \times 10^{-2} + 2 \times 3 \times 10^{-2} = 8 \times 10^{-2} = \pm 8\ \%$

5. $\Delta t = 0.2$ s.
$t = 25$ s
$T = \frac{t}{N} \Rightarrow \frac{\Delta T}{T} = \frac{\Delta t}{t} = \frac{0.2}{25} = 0.8\ \%$



6. $v = \ell bh$

$\frac{\Delta v}{v} = \frac{\Delta\ell}{\ell} + \frac{\Delta b}{b} + \frac{\Delta h}{h} = \frac{0.1}{10} + \frac{0.1}{5} + \frac{0.1}{5} = \frac{0.5}{10} = \pm 5\%$

7. $\frac{\Delta x}{x} = 1\% = 10^{-2} \Rightarrow \frac{\Delta y}{y} = 3\% = 3 \times 10^{-2} \Rightarrow \frac{\Delta z}{z} = 2\% = 2 \times 10^{-2}$

$t = \frac{xy^2}{z^3} \Rightarrow \frac{\Delta t}{t} = \frac{\Delta x}{x} + \frac{2\Delta y}{y} + \frac{3\Delta z}{z}$

$= 10^{-2} + 2 \times 3 \times 10^{-2} + 3 \times 2 \times 10^{-2} = 13 \times 10^{-2}$ ∴ % error in t $= \frac{\Delta t}{t} \times 100 = 13\%$

8. D = (4.23 ± 0.01) cm
d = (3.89 ± 0.01) cm
$\Delta t = (D - d)/2 = \frac{(4.23 \pm 0.01) - (3.89 \pm 0.01)}{2} = \frac{(4.23 - 3.89) \pm (0.01 + 0.01)}{2}$
= (0.34 ± 0.02)/2 cm = (0.17 ± 0.01)cm

9. m = 1.76 kg
M = 25 m = 25 × 1.76 = 44.0 kg
Note : Mass of one unit has three significant figures and it is just multiplied by a pure number (magnified). So result should also have three significant figures.

10. $R_1 = (24 \pm 0.5)\ \Omega$
$R_2 = (8 \pm 0.3)\ \Omega$
$R_S = R_1 + R_2 = (32 \pm 0.8)\ \Omega$

11. $\Delta\ell = 0.5$ mm
N = 100 divisions
zero correction = 2 divisions

Reading = Measured value + zero correction = (8 × 0.5) mm + (83 – 2) × $\frac{0.5}{100}$.

= 4 mm + 81 × $\frac{0.5}{100}$ mm = 4.405 mm

12. $\Delta\ell = 1$ mm
N = 50 division
zero error = –6 Divisions = – 0.12 mm
Diameter = Measured value + zero correction

= 3 × 1 + (6 + 31) × $\frac{1}{50}$ = 3 + 0.74 = 3.74 mm

13. $D = 2 \times 1 + 5 \times \frac{10-9}{100} = 2.05$ cm

14. If R is very large (~ K Ω)
Then measured resistance from arrangement (a) will be
$R_{measured} = R + R_A \approx R$
So (a) will be preferred
If R is very small (~ few Ohm)
then measured resistance from (b) will be
$R_{measured} = \frac{R}{1+\frac{R}{R_V}}$ where
so, $R_{measured} \longrightarrow R$
So (b) will be preferred

15. Observed reading of cylinder diameter = 3.1 cm + (4) (0.01 cm) = 3.14 cm
= 0.5 mm = 0.05 cm = 3.1 cm + (4) (0.01 cm) = 3.14 cm

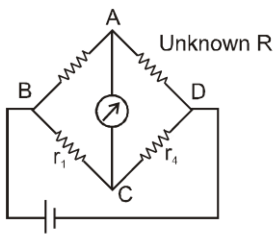
16. $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{dR_{eq}}{R_{eq}^2} = \frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}$

$\frac{dR_{eq}}{R_{eq}} \times 100 = R_{eq}\left(\frac{1}{R_1}\frac{dR_1}{R_1} \times 100 + \frac{1}{R_2}\frac{dR_2}{R_2} \times 100\right)$

$\frac{dR_{eq}}{R_{eq}} \times 100 = \pm 2\left[\frac{1}{3} \times 1 + \frac{1}{6} \times 2\right] \Rightarrow \frac{dR_{eq}}{R_{eq}} = \pm \frac{4}{3}$ %

17. $V = \ell^3 = (1.2 \times 10^{-2} \text{ m})^3 = 1.728 \times 10^{-6} \text{ m}^{-3}$
length (ℓ) has two significant figure, the volume (V) will also have two significance figures. Therefore, the correct answer is $V = 1.7 \times 10^{-6}$ m^3

18. The ratio $\frac{AC}{CB}$ will remain unchanged.
As potential drop ∝ length.

19. 

Equivalent wheat-stone bridge of the given Post Office circuit.
AB, BC and CD are known resistance.
The unknown resistance is connected between A and D.
Hence, the correct option is (C).

20. Density $\rho = \frac{m}{\pi r^2 L}$

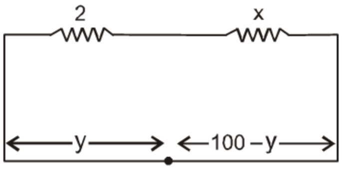
∴ $\frac{\Delta\rho}{\rho} \times 100 = \left(\frac{\Delta m}{m} + 2\frac{\Delta r}{r} + \frac{\Delta L}{L}\right) \times 100 = \left[\frac{0.003}{0.3} + 2 \times \frac{0.005}{0.5} + \frac{0.06}{6}\right] \times 100 = 4\%$

After substituting the values, we get the maximum percentage error in density = 4%.
Hence, the correct option is (D).

21. $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$

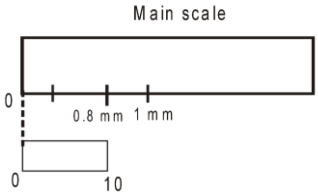
In option (D) error in Δg is minimum and number of repetition of measurement are maximum. In this case the error in g is minimum.

22.



From balanced wheat stone bridge 2(100 – y) = xy
According to question (100 – y) – y = 20
on solving y = 40 cm, x = 3 Ω

23.



20 VSD = 16 MCD
1 VSD = 0.8 MSD
Least count = MSD – VSD = 1 mm – 0.8 mm = 0.2 mm

24. $\ell_1 = 52 + 1 = 53$ cm ⇒ $\ell_2 = 48 + 2 = 50$ cm ⇒ $\frac{\ell_1}{\ell_2} = \frac{x}{R}$ ⇒ $\frac{53}{50} = \frac{x}{10}$ ⇒ x = 10.6 Ω

26. V = length × breadth × thickness
$= 12 \times 6 \times 2.45 = 176.4 \text{ cm}^3 = 1.764 \times 10^2 \text{ cm}^3$
The minimum number of significant figures is 1 in breadth, hence, the volume will contain only one significant figure. Therefore, $V = 2 \times 10^2 \text{ cm}^3$

27. The percentage of volume of ice-cube outside the water is

$= \frac{\rho_{water} - \rho_{ice}}{\rho_{water}} \times 100 = \frac{1000 - 900}{1000} \times 100 = 10\%$

28. $P = \frac{a^3 b^2}{cd}$

$\frac{\Delta P}{P} \times 100\ \% = 3\frac{\Delta a}{a} \times 100\ \% + 2\frac{\Delta b}{b}\ 100\ \% + \frac{\Delta c}{c}\ 100\ \% + \frac{\Delta d}{d} \times 100\ \%$

= 3.1 + 2.2 + 3 + 4 = 3 + 4 + 3 + 4 = 14 %

29. $x = \frac{ab^2}{c^3}$

lnx = lna + 2lnb –3lnc

$\frac{1}{x} dx = \frac{1}{a} da + \frac{2}{b} db - \frac{3dc}{c} \times 100$

$\frac{dx}{x} \times 100 = \frac{da}{a} \times 100 + \frac{2db}{b} \times 100 - \frac{3dc}{c} \times 100 = 1 + 2 \times 3 - 3(-2)$. Maximum error = ± 13 %

30. Volume of a sphere $= \frac{4}{3}\pi\ (\text{radius})^3$ or $V = \frac{4}{3}\pi R^3$

Taking logarithm on both sides, we have $\log V = \log \frac{4}{3}\pi + 3 \log R$

Differentiating, we get $\frac{\Delta V}{V} = 0 + \frac{3\Delta R}{R}$

Accordingly, $\frac{\Delta R}{R} = 2\%$. Thus, $\frac{\Delta V}{V} = 3 \times 2\% = 6\%$

EXERCISE # 2

1. Zero error = $5 \times \frac{0.5}{50}$ = 0.05 mm

Actual measurement = 2 × 0.5 mm + 25 × $\frac{0.5}{50}$ – 0.05 mm = 1 mm + 0.25 mm – 0.05 mm = 1.20 mm.

2. $\frac{mg}{\pi r^2}$ = (Y) $\frac{x}{\ell_0}$

Y = $\frac{mg\ell_0}{\left(\pi \frac{d^2}{4}\right)x}$ = 2 × 10^{11} N/m² ⇒ $\left(\frac{\Delta Y}{Y}\right)_{max} = 2\left(\frac{\Delta d}{d}\right) + \frac{\Delta x}{x}$ = 2 $\left(\frac{0.01}{0.4}\right) + \frac{0.05}{0.8}$ = $\frac{9}{80}$

ΔY = $\frac{9}{80}$ × 2 × 10^{11} = 0.225 × 10^{11} N/m² = 0.2 × 10^{11} N/m² **Ans. (B)**

3. The least count of length Δℓ = 0.1 cm
The least count of time Δt = 0.1 s

% error of g = $\frac{\Delta g}{g}$ × 100

T = 2π $\sqrt{\frac{\ell}{g}}$ ⇒ g = $\frac{4\pi^2\ell}{T^2}$ = $\frac{4\pi^2\ell}{t^2}.n^2$ ⇒ $\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta t}{t}$

For student I $\left(100 \times \frac{\Delta g}{g}\right)_I = \left(\frac{0.1}{64.0} + \frac{2 \times 0.1}{128.0}\right) \times 100$

$E_I = \frac{0.2}{64.0} \times 100 = \frac{20}{64}$ For student II $\left(100 \times \frac{\Delta g}{g}\right)_{II} = \left(\frac{0.1}{64.0} + 2 \times \frac{0.1}{64.0}\right) \times 100$

$E_{II} = \frac{0.3}{64.0} \times 100 = \frac{30}{64}$ For student III $\left(100 \times \frac{\Delta g}{g}\right)_{III} = \left(\frac{0.1}{20.0} + 2 \times \frac{0.1}{36.0}\right) \times 100$

$E_{III} = \left(\frac{0.1}{20.0} + \frac{0.1}{18.0}\right) \times 100 = \frac{19}{18}$ E_I is least. **Ans. (B)**

4. Least count = $\frac{0.5}{50}$ = 0.01 mm. Diameter of ball D = 2.5 mm + (20)(0.01). D = 2.7 mm

ρ = $\frac{M}{vol} = \frac{M}{\frac{4}{3}\pi\left(\frac{D}{2}\right)^3}$

$\left(\frac{\Delta\rho}{\rho}\right)_{max} = \frac{\Delta m}{m} + 3\frac{\Delta D}{D}$; $\left(\frac{\Delta\rho}{\rho}\right)_{max}$ = 2% + 3 $\left(\frac{0.01}{2.7}\right) \times 100\%$ ⇒ $\frac{\Delta\rho}{\rho}$ = 3.1%

5. For Vernier calipers

1MSD = $\frac{1}{8}$cm ⇒ 5 VSD = 4 MSD ⇒ 1VSD = $\frac{4}{5}MSD = \frac{4}{5} \times \frac{1}{8} = \frac{1}{10}$cm

LC of vernier calliper = $\frac{1}{8}cm - \frac{1}{10}cm = 0.025$ cm
(A) & (B) pitch of screw gauge = 2 × (0.025) = 0.05 cm

Leastcount of screw gauge $= \frac{0.05}{100}$cm = 0.005 mm
(C) & (D) Least count of linear scale of screw gauge = 0.05 cm

pitch = 0.05 × 2 cm = 0.1 cm

Leastcount of screw gauge $= \frac{0.1}{100}$ cm = 0.01 mm

6. $E(t) = A^2 e^{-\alpha t}$

$\alpha = 0.2\ s^{-1}$

$\frac{dA}{A} = 1.25\% \Rightarrow \frac{dt}{t} = 1.50\ \%$

$\Rightarrow \frac{dE}{E} = ?$

$\log E = 2 \log A - \alpha t$

$\frac{dE}{E} = \pm 2 \frac{dA}{A} \pm \alpha dt = \pm 2\ (1.25) \pm 0.2(7.5)$

$= \pm 2.5 \pm 1.5 = \pm 4\ \%$

7. Here area is uniform, so portion of immersed height $= \frac{1}{3}$

(Here : the ratio of density of body and density of water $= \frac{1}{3}$)

Therefore, fraction of exposed height will

$1 - \frac{1}{3} = \frac{2}{3}$

8. $x = R \frac{\ell}{100 - \ell} = 30\Omega$

$\Rightarrow \frac{\Delta x}{x} = \frac{\Delta R}{R} + \frac{\Delta \ell}{\ell} + \frac{\Delta \ell}{100 - \ell}$

$\Rightarrow \Delta x = \left[\frac{0.5}{100} + \frac{0.1}{60} + \frac{0.1}{40}\right] \times 30 = 0.27\Omega$

9. Least count of screw gauge $= \frac{0.5}{50}$ mm = 0.01 mm

∴ Reading = [Main scale reading + circular scale reading × L.C.] – (zero error)

= [3+ 35 × 0.01] – (–0.03) = 3.38 MM

10. To find the refractive index of glass using a travelling microscope, a vernier scale is provided on the microscope.

EXERCISE # 3

PART - I

1. From the relation

$h = ut + \frac{1}{2} gt^2$

$h = \frac{1}{2} gt^2$

$\Rightarrow \quad g = \frac{2h}{t^2}$ ($\because$ body initially at rest)

Taking natural logarithm on both sides, we get

$\ell ng = \ell n\, h - 2\ell n\, t$

Differentiating, $\frac{\Delta g}{g} = \frac{\Delta h}{h} - 2\frac{\Delta t}{t}$ for maximum permissible error,

or $\left(\frac{\Delta g}{g} \times 100\right) = \left(\frac{\Delta h}{h} \times 100\right) + 2 \times \left(\frac{\Delta t}{t} \times 100\right)$. According to problem

$\frac{\Delta h}{h} \times 100 = e_1$ and $\frac{\Delta t}{t} \times 100 = e_2$. Therefore, $\left(\frac{\Delta g}{g} \times 100\right) = e_1 + 2e_2$

2. $x = \frac{A^2 B^{1/2}}{C^{1/3} D^3}$

$\ell nx = 2\ell n\, A + \frac{1}{2}\ell nB - \frac{1}{3}\ell nC - 3\ell nD$

Differenting $\left.\frac{dx}{x}\right)_{max} = 2\frac{dA}{A} + \frac{1}{2}\frac{dB}{B} + \frac{1}{3}\frac{dC}{C} + \frac{3dD}{D}$

error $x_{max} = 2 \times 1 + \frac{2}{2} + \frac{1}{3} \times 3 + 3 \times 4 = + 16\%$

3. $1MSD = \frac{1}{n}$ cm, $1VSD = \frac{(n-1)MSD}{n} = \frac{(n-1)}{n^2}cm$

L.C. = 1MSD – 1 VSD = $\frac{1}{n} - \left(\frac{n-1}{n^2}\right) = \frac{1}{n}\left[1 - \frac{(n-1)}{n}\right] = \frac{1}{n^2}cm$

PART - II

1. 29 division of main scale coincides with 30 divisions of vernier scale Hence one division of vernier scale

$= \frac{30-29}{30}$ of main scale $= \frac{1}{30} \times 0.5^\circ = \frac{1}{30} \times 0.5 \times 60$ min = 1 min.

2. V = u and $\frac{1}{V} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{2}{V} = \frac{1}{f} \Rightarrow V = 2f,\ u = 2f$

3. Rule : I. We know all non zero digits are significant.

Rule : II. If zero is between two non-zero digits this is also significant.

Rule : III. If zero left to the non-zero digit they are non-significant.

Significant figures for number 23.023 is 5. Using I & II.

Significant figures for number 0.0003 is 1. Using I, II & III.

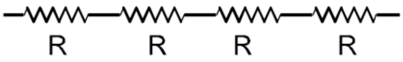
Significant figures for number 2.1×10^{-3} is 2. Using I.

4. Least count of screw gauge = $\frac{1}{100}$mm = 0.01 mn

Diameter - Divisions on cirular scale × least count + main scale reading

$= 52 \times \frac{1}{100} + 0$ = 0.52 mm diameter = 0.052 cm

5.



R = 100 ± 5

4R = 400 ± 20

Tolerance of combination is also 5%.

6. Given $\frac{\Delta L}{L} = \frac{0.1}{20}$

$T = \frac{90}{100}$ sec. $\Delta T = \frac{1}{100}$ sec.

$\frac{\Delta T}{T} = \frac{1}{90}$

$g = \left(\frac{1}{4\pi^2}\right)\frac{L}{T^2}$ $\Rightarrow$ $\frac{\Delta g}{g} \times 100\% = \frac{\Delta L}{L} \times 100 + \frac{2\Delta T}{T} \times 100$

$\frac{\Delta g}{g} \times 100\% = \left(\frac{0.1}{20}\right) 100 + 2 \left(\frac{1}{90}\right) 100 = 2.72\%$

so nearest option is 3%

7. $\frac{\Delta m}{m} \times 100 = 1.5$

$\frac{\Delta \ell}{\ell} \times 100 = 1$

$d = \frac{m}{\ell^3} \Rightarrow \frac{\Delta d}{d} \times 100 = \frac{\Delta m}{m} \times 100 + \frac{3\Delta \ell}{\ell} \times 100$ = 1.5 + 3 = 4.5%

8. Pitch = 0.5 mm

L.C. = $\frac{0.5}{100}$mm

Actual value = 5.5 mm+ (48 + 3) × 5 × 10^{-3} mm = 5.755 mm

9. $[t] = [G]^a [h]^1 [c]^d$

$[T'] = [M^{-1} L^3 T^{-2}]^a [M^1 L^2 T^{-1}] [L^1 T^{-1}]^d$

$[T^1] = [M^{-a+b} L^{3a+2b+d} T^{-2a-b-d}]$

$-a + b = 0$

$a = b$ ....(i)

$3a + 2b + d = 0$ ……...(ii)

$-[2a + b + d] = 1$

$2a + b + d = -1$ ....(iii)

from equation (i) and (ii)

$5a + d = 0$

$d = 5a$ from equation (i) and (iii)

$3a + d = -1$

$3a - 5a = -1 \Rightarrow -2a = -1 \Rightarrow a = \frac{1}{2}$

$b = \frac{1}{2}$; $d = -\frac{5}{2}$ time = $\left[G^{\frac{1}{2}} h^{\frac{1}{2}} C^{-\frac{5}{2}}\right]$

10. $V = \pi\frac{d^2}{4} h = 4260 cm^3$

$\frac{\Delta V}{V} = \frac{2\Delta d}{d} + \frac{\Delta h}{h} \Rightarrow \Delta V = 2 \times \frac{0.1V}{12.6} + \frac{0.1V}{34.2} = \frac{0.2}{12.6} \times 4260 + \frac{0.1 \times 4260}{34.2} = 80$

Volume = 4260 ± 80 cm^3

11. Least count of screw gauge = 5 µm

$L.C = \frac{\text{Pitch}}{\text{no. of div on circular scale}} \Rightarrow 5\mu m = \frac{1mm}{N}$. N = 200

12. $g = \frac{4\pi^2 \ell}{T^2}$

$\frac{\Delta g}{g} = \frac{\Delta \ell}{\ell} + 2\frac{\Delta T}{T} = \frac{0.1}{55} + 2\frac{1}{30} \Rightarrow \frac{\Delta g}{g} \times 100 = \frac{10}{55} + \frac{20}{3} = 6.8\%$

13. $\rho = m/a^3$

$\frac{d\rho}{\rho} = \frac{dm}{m} + \frac{3da}{a} \Rightarrow \frac{d\rho}{\rho} = \frac{0.1}{10} + 3\left[\frac{0.01}{0.1}\right] \Rightarrow \frac{d\rho}{\rho} = 0.01 + 0.3 \Rightarrow \frac{d\rho}{\rho} = 0.31$

14. 37.03 cm^2