TOPIC : PROJECTILE MOTION

EXERCISE # 1

PART – I

SECTION : (A)

6. $\frac{H}{R} = \frac{\tan\theta}{4}$

$\theta = 45°$ & $R = 36$ m

$H = 9$ m

12. At highest point of projection, the vertical component of velocity is zero and there is only horizontal component of velocity.

At the highest point

$V_x = u\cos\theta$

$V_y = 0$

$K_H = \frac{1}{2}mv_x^2$ or $K_H = \frac{1}{2}mu^2\cos^2\theta$ ....(i)

Intial kinetic energy is $K = \frac{1}{2}mu^2\cos^2\theta$ ....(ii)

From Eq. (i) and (ii), we get

$K_H = K\cos^2\theta = K\cos^2 45° = K\times\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{K}{2}$

13. L $\Rightarrow$ $\phi = \tan^{-1} 4$ (initially)

16. For maximum range

$R = \frac{u^2}{g}$ $\Rightarrow$ $u^2 = gR$

$u^2 = 16{,}000 \times 10$ $\Rightarrow$ $u = 4 \times 100$

$u = 400$ m/sec

17. The horizontal ranges of projectiles whose angles of projection are complementary are equal. Hence their ratio is 1.

18. $x = \alpha t^3,\ y = \beta t^3$

$V_x = \frac{dx}{dt} = 3\alpha t^2$

$V_y = \frac{dy}{dt} = 3\beta t^2$

Resultant velocity

$V = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{9\alpha^2t^4 + 9\beta^2t^4} = 3t^2\sqrt{a^2 + \beta^2}$

19. Man will catch the ball if the horizontal component of velocity becomes equal to the constant speed of man i.e.

$v_0\cos\theta = \frac{v_0}{2}$ $\Rightarrow$ $\cos\theta = \frac{1}{2}$ $\Rightarrow$ $\cos\theta = \cos 60^o$ ∴ $\theta = 60^o$

20. $\frac{1}{2}mv^2 = K$, $\frac{1}{2}m(v\cos 60)^2 = \frac{1}{2}m\left(\frac{v^2}{4}\right) = \frac{1}{4}\left(\frac{1}{2}mv^2\right) = \frac{K}{4}$

21. In projectile motion Horizontal accelereation $a_x = 0$ & Vertical acceleration $a_y = g = 10$m/s

$a_x = 0$
$a_y = 10$ (down) ⇒ only "3" is correct **Ans**

22. $R = \frac{2u_x u_y}{g}$

23. $\vec{u}_x = 6\hat{i} + 8\hat{j}$
$\vec{u}_x = 6\hat{i}$

$u_y = 8\hat{j}$ $R = \frac{2u_x u_y}{g} = \frac{2 \times 6 \times 8}{10} = 9.6$

24. $T = \frac{2u\sin\theta}{g} = 10$

$R = \frac{u^2 \sin 2\theta}{g} = 50$ m

$H = \frac{(u\sin\theta)^2}{2g} = \frac{(5g)^2}{2g} = \frac{25g^2}{2g} = \frac{25g}{2}$ H = 125 m

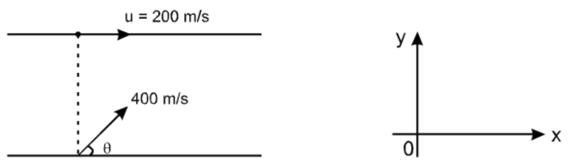
25. $T = 5 = \frac{2u\sin\theta}{g}$
$u \sin \theta = 25$

$R = \frac{2(u\sin\theta)(u\cos\theta)}{g} = 200$

$\frac{2}{10}$ (25) (u cosθ) = 200

$u\cos\theta = \frac{200 \times 10}{50} = 40$

$\tan\theta = \frac{25}{40} = \frac{5}{8}$ $\theta = \tan^{-1}\left(\frac{5}{8}\right)$

26. 

To hit, $400 \cos \theta = 200$
{∵ Both travel equal distance along horizontal, of their start and coordinates an x axis are same}
⇒ $\theta = 60^o$ **Ans.**

34. $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x v_y}{g}$
∴ Range ∝ horizontal initial velocity (u_x)
In path 4 range is maximum so football possess maximum horizontal velocity in this path.

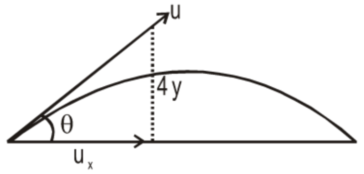
35. Horizontal velocity remains constant throughout the projectile motion

36. $R_{max} = \frac{u^2}{g}$ $H_{max} = \frac{u^2}{2g}$

38. $R = \frac{2ab}{g} = \frac{2b^2}{2g}$

50. $y = 4t - t^2$, $x = 3t$

$V_y = \frac{dy}{dt} = 4 - 2t$, $V_x = \frac{dx}{dt} = 3$



$\Rightarrow u_y = v_y \Big|_{t=0} = 4, \quad u_x = v_x\Big|_{t=0} = 3$

The angle of projection :

$\tan\theta = \frac{V_y}{V_x} = \frac{4}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{4}{3}\right)$ **Ans.**

53. $R = \frac{u^2 \sin 2\theta}{g}$

$H = \frac{u^2 \sin^2\theta}{2g}$

$R = 4H$

$\frac{u^2 \sin 2\theta}{g} = 4 \frac{u^2 \sin^2\theta}{2g}$

$\sin 2\theta = 2\sin^2\theta$

$2\sin\theta\cos\theta = 2\sin^2\theta$

$\tan\theta = 1$

$\theta = 45^o$

54. $R = \frac{u^2 \sin 2\theta}{g} = 50$

$50 = \frac{u^2 \sin 30^o}{g} = \frac{u^2}{2g}$

$R_1 = \frac{u^2}{g} \sin 90^o = \frac{u^2}{g} = 100$ m

55. $a_x = 2$ m/s^2 ; $a_y = 0$

$u_x = 8$ m/s

$u_y = -15$ m/s.

$= V_x + V_y$

$V_y = u_y + a_y t \quad \Rightarrow \quad V_y = -15$ m/s

$V_x = u_x + a_x t$

$V_x = 8 + 2t \quad \Rightarrow \quad V = [(8 + 2t) - 15]$ m/s. **Ans.**

56. $(Y_{max}) \Rightarrow \frac{dY}{dt} = 0 \Rightarrow \frac{d}{dt}(10t - t^2) = 10 - 2t \Rightarrow t = 5$

$\Rightarrow \quad Y_{max} = 10(5) - 5^2 = 25$ m **Ans** "D"

57. $x = 6t \qquad y = 8t - 5t^2$

$\frac{dx}{dt} = 6$ $\quad \frac{dy}{dt} = 8 - 10t$

at t = 0

$V_x = 6$ m/sec $\quad V_y = 8$m/sec

$V = \sqrt{V_y^2 + V_x^2} = \sqrt{8^2 + 6^2} = 10$ m/sec

61. $R = \frac{2u_x u_y}{g} = \frac{u^2}{g}$

64. $u\cos\theta = \frac{\sqrt{3}u}{2}$ $\quad \cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^o$

$R = \frac{u^2 \sin 60^o}{g}$

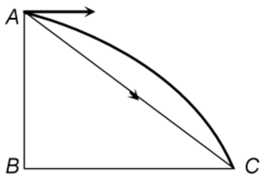
65. $\frac{mgh}{\frac{1}{2}m(u\cos\theta)^2} = \frac{mg\frac{u^2\sin^2\theta}{2g}}{\frac{1}{2}mu^2\cos^2\theta}$

66. Range is maximum at $\theta = 45^o$

SECTION - (B)

6. (1) The horizontal distance covered by bomb,

$BC = v_H \times \sqrt{\frac{2h}{g}} = 150\sqrt{\frac{2\times 80}{10}} = 660\text{m}$



∴ The distance of target from dropping point of bomb,

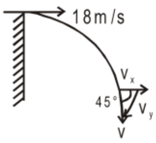
$AC = \sqrt{AB^2 + BC^2} = \sqrt{(80)^2 + (600)^2} = 605.3\text{m}$

7. $R = \frac{u^2 \sin 2\theta}{g} = \frac{10\times 10\times \sin 60^o}{10} = 10 \times \frac{\sqrt{3}}{2} = 5 \times 1.732 = 8.66\text{m}$

8. $X = v\sqrt{\frac{2h}{g}}$

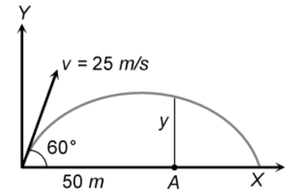
$X = 100\sqrt{\frac{2\times 490}{9.8}} = 10^3$ m = 1 km

15. $\tan 45° = \frac{v_y}{v_x}$ ⇒ $v_y = v_x = 18$m/s **Ans.**



SECTION - (C)

3. (1) Horizontal component of velocity
$v_x = 25\cos 60° = 12.5 m/s$
Vertical component of velocity
$v_y = 25\sin 60° = 12.5\sqrt{3} m/s$



Time to cover 50 m distance $t = \frac{50}{12.5} = 4 sec$. The vertical height y is given by
$y = v_y t - \frac{1}{2}gt^2 = 12.5\sqrt{3} \times 4 - \frac{1}{2} \times 9.8 \times 16 = 8.2m$

5. $y = 16x\left(1 - \frac{x}{64}\right)$

EXERCISE # 2

2. X = 36 t 2y = 96t – 9.8t^2

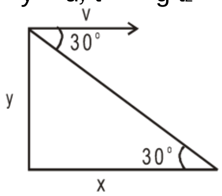
$\frac{dx}{dt} = 36$ $2\frac{dy}{dt} = 96 - 19.6\,t$ at t = 0

$V_x = 36$ $V_y = 48$

$\tan\phi = \frac{48}{36} = \left(\frac{4}{3}\right)$ ⇒ $\sin\phi = \frac{4}{5}$ ⇒ $\phi = \sin^{-1}\left(\frac{4}{5}\right)$

7. AC = $\frac{1}{2}$ gt^2 = 45 m BC = 45 $\sqrt{3}$ m = u.t u = $\frac{45}{\sqrt{3}}$ = 15 $\sqrt{3}$ m/s.
Alter : Object is thrown horizontally so $u_x = v$ & $u_y = 0$
from Diagram

$-y = u_y t - \frac{1}{2} g t^2$



$y = \frac{1}{2} \times 10 \times (3)^2$ ⇒ $y = \frac{1}{2} 45m$ ....(1)

& $\tan 30^o = \frac{y}{x}$ => $y\sqrt{3} = x$ ....(2)
& x = v t = 3v ....(3)
from equation (1), (2) & (3)
$45 = \sqrt{3}\,3v$ ⇒ $v = 15\sqrt{3}$ m/s

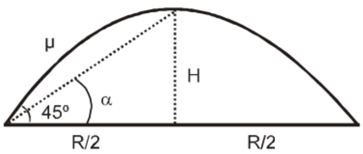
8. From given conditions $V_A = V_B \cos 37^o = 15.\frac{4}{5}$ = 12 m/sec.

∴ time of flight of A (t)= $\sqrt{\frac{2 \times 20}{10}}$ = 2 sec. ⇒ Range = $V_A t$ = 24 m

EXERCISE # 3

PART - I

1. $R_{max} = \dfrac{u^2 \sin 90^\circ}{g} = \dfrac{20^2}{10} = 40 \text{ m}$

2. 

$H = \dfrac{u^2 \sin^2 45^o}{2g} = \dfrac{u^2}{4g}$ ....(1)

$R = \dfrac{u^2 \sin 90^o}{g} = \dfrac{u^2}{g}$ $\therefore \dfrac{R}{2} = \dfrac{u^2}{2g}$ ....(2)

$\therefore \tan\alpha = \dfrac{H}{R/2} = \dfrac{\frac{u^2}{4g}}{\frac{u^2}{2g}} = \dfrac{1}{2}$ $\therefore \alpha = \tan^{-1}\left(\dfrac{1}{2}\right)$

3. Horizontal range

$R = \dfrac{u^2 \sin 2\theta}{g}$ ......(1) maximum height

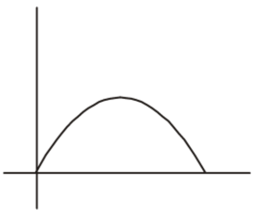
$H = \dfrac{u^2 \sin 2\theta}{2g}$ ....(2) here (1) = (2)

$\dfrac{u^2 \sin 2\theta}{g} = \dfrac{u^2 \sin^2 \theta}{2g}$ $\Rightarrow$ $2 \sin \theta = \dfrac{\sin\theta}{2}$ $\Rightarrow$ $\theta = \tan^{-1} (4)$ Ans. (2)

4. $\vec{v} = \vec{u} + \vec{a}t$

$v = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j}) \times 10 = 5\hat{i} + 5\hat{j}$ $\Rightarrow$ $|\vec{v}| = 5\sqrt{2}$

5. $\vec{V}_B = 2\hat{i} - 3\hat{j}$

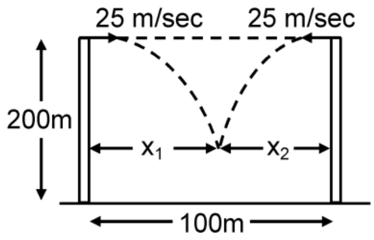
6. 

$\dfrac{5^2}{g} = \dfrac{3^2}{a}$ $\Rightarrow$ $a = 9.8 \times \dfrac{9}{25}$ $\Rightarrow$ $a = 3.5$

7. $x = 5t - 2t^2$ $V_x = \dfrac{dx}{dt} = 5 - 4t$ $a_x = -4$

$y = 10t$ $V_y = 10$ $a_y = 0$ So, $\vec{a} = -4\hat{i}$

8.



Now $x_1 = 25t$; $x_2 = 25t$

$x_1 + x_2 = 100 = 50t$ So, t = 2 sec and vertical downward displacement

$h = \frac{1}{2}gt^2 = \frac{1}{2}\times10\times4 = 20$ meter So, height from ground ⇒ 200 – 20 = 180 meter

PART - II

1. $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u^2 \sin\theta.\cos\theta}{g}$

$H = \frac{u^2 \sin^2\theta}{2g}$

$\therefore \frac{H}{R} = \frac{u^2\sin^2\theta}{2g} \times \frac{g}{2u^2\sin\theta.\cos\theta} = \frac{\sin\theta}{4\cos\theta}$ ⇒ $\frac{R}{H} = \frac{4\cos\theta}{\sin\theta}$ or, $\frac{R}{H} = 4\cot\theta$

2. $R = \frac{u^2\sin 2\theta}{g}$ ⇒ $R' = \frac{(2u^2)\sin 2\theta}{g} = 4R$

3. $H = \frac{u^2}{2g}$ ⇒ $u^2 = 2gH$ For maximum horizontal distance $x_{max} = \frac{u^2}{g} = \frac{2gH}{g} = 2H$

4. $h_1 = \frac{u^2\sin^2\theta}{2g}$ ⇒ $h_2 = \frac{u^2\sin^2(90-\theta)}{2g}, R = \frac{u^2\sin 2\theta}{g}$

Range R is same for angle θ and (90º – θ) $\therefore h_1h_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\sin^2(90-\theta)}{2g}$

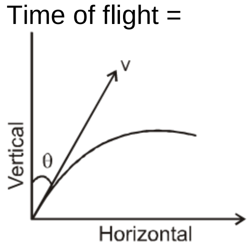
$= \frac{u^4(\sin^2\theta)\times\sin^2(90-\theta)}{4g^2}$ [sin (90 – θ) = cosθ]

$= \frac{u^4(\sin^2\theta)\times\cos^2\theta}{4g^2}$ [sin 2θ = 2sinθ cosθ]

$= \frac{u^4(\sin\theta\cos\theta)^2}{4g^2} = \frac{u^4(\sin 2\theta)^2}{16g^2} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$ or, $R^2 = 16\,h_1\,h_2$ or $R = 4\sqrt{h_1h_2}$

5. Max. height = $H = \frac{v^2\sin^2(90-\theta)}{2g}$ ....(i)

Time of flight = $T = \frac{2v\sin(90-\theta)}{g}$ ....(ii)



From (i) and (ii) $\frac{v\cos\theta}{g}\sqrt{\frac{2H}{g}}$,

$T = 2\sqrt{\frac{2H}{g}} = \sqrt{\frac{8H}{g}}$

6. A parabola

7. Given, $u_1 = u_2 = u$, $\theta_1 = 60^o$, $\theta_2 = 30^o$

In Ist case, we know that range

$$R_1 = \frac{u^2 \sin 2(60^o)}{g} = \frac{u^2 \sin 120^o}{g} = \frac{u^2 \sin(90^o + 30^o)}{g} = \frac{u^2(\cos 30^o)}{g} = \frac{\sqrt{3}u^2}{2g}$$

In IInd case when $\theta_2 = 30^o$, then $R_2 = \frac{u^2 \sin 60^o}{g} = \frac{u^2\sqrt{3}}{2g}$ $\Rightarrow$ $R_1 = R_2$ (we get same value of ranges)

PART - III

1. $\vec{v} = \vec{u} + \vec{a}\,t$

$= (3\hat{i} + 4\hat{j}) + (0.4\hat{i} + 0.3\hat{j}) \times 10 = 7\,i + 7\,j$ $\quad |\vec{v}| = 7\sqrt{2}$

2. $\frac{dx}{dt} = y$

$\frac{dy}{dt} = x$

$\frac{dx}{dy} = \frac{y}{x} \Rightarrow y^2 = x^2 + c$

3. $\Delta\vec{r} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{r}_f - \vec{r}_i = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$\vec{r}_f = (2\hat{i} + 4\hat{j}) + (5\hat{i} + 4\hat{j}) \times 2 + \frac{1}{2}(4\hat{i} + 4\hat{j}) \times 2^2$

$= 2\hat{i} + 4\hat{j} + 10\hat{i} + 8\hat{j} + 8\hat{i} + 8\hat{j} = 20\hat{i} + 20\hat{j}$

$|\vec{r}_f| = 20\sqrt{2}$ m