TOPIC : FRICTION
EXERCISE # 1
PART – I

SECTION (A)

1. μ does not deend on normal reaction.

2. $a = g\sin 45^o - \mu g \cos 45^o$

$a = \frac{g}{\sqrt{2}}\left(1-\frac{1}{2}\right) = \frac{g}{2\sqrt{2}}$

5. For equilibrium, normal to pane

$N = mg \cos\theta$ … (1)

Net force along the plane downward

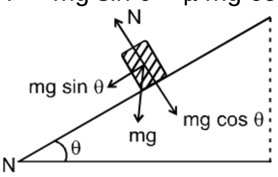
$F = mg \sin\theta + f_k$ …(2)

where f_k is kinetic fricton

but $f_k = \mu N = \mu\, mg \cos\theta$ …(3)

from eq. (1), (2), and (3) we get

∴ $F = mg \sin\theta + \mu\, mg \cos\theta$



According to Newton's IInd law

$F = ma$

∴ $ma = mg \sin\theta + \mu\, mg \cos\theta$

∴ Retardation $a = g \sin\theta + \mu\, g \cos\theta$

From equation v = u + at, (we have)

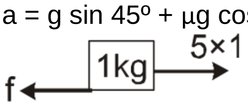
$o = u - (g \sin\theta + \mu\, g \cos\theta)t$

⇒ $g \sin\theta + m\, g \cos\theta = \frac{u}{t}$ ⇒ $10 \times \sin 30° + \times 10 \cos 30° = \frac{5}{0.5}$

⇒ $10 \times \frac{1}{2} + 10\mu \times \frac{\sqrt{3}}{2} = 10$ ⇒ $5\sqrt{3}\,\mu = 5$ or $\mu = \frac{1}{\sqrt{3}}$

6. $a = g \sin 45^o + \mu g \cos 45^o = \frac{g}{\sqrt{2}}\left(1+\frac{1}{2}\right)$

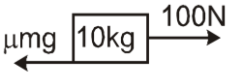
8.



$f_{max} = 0.6 \times 1 \times g = 6N$

$f_{max} > 5$ so $f = 5$ N

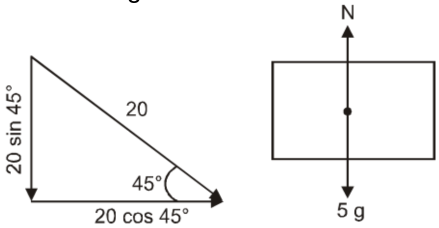
9.



$a = \frac{100 - \mu mg}{m}$ ⇒ $a = \frac{100 - \frac{1}{2}\times 10 \times 10}{10} = 5\ m/s^2$

10. Block B will come to rest, if force applied to it will vanish due to frictional force acting between block B and surface, i.e, foce applied = frictional force i.e., $\mu mg = ma$ or $\mu mg = m\left(\frac{v}{t}\right)$ or $t = \frac{v}{\mu g}$

11. Forces acting on the block are shown in figure.



Now, $20 \cos 45° - f = 5a$ ....(i)

where f = frictional force

$f = \mu N$

$= 0.2 \times N$ .....(ii)

and, $N = 5g + 20 \sin 45° = 50 + 10\sqrt{2}$

Substituting the value of N in Eq. (ii) and hence value of f in Eq. (i), we get

$$10\sqrt{2} - 0.2(50 + 10\sqrt{2}) = 5a$$

a = 0.2624

Speed of block after 15 s from frist equation of motion

$v = u + at = 0.2624 \times 15 = 3.936\ ms^{-1}$

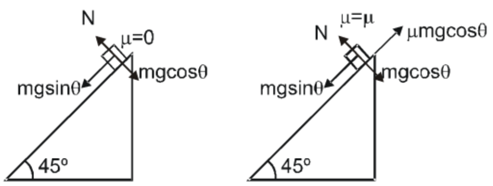
12. F = ma ∴ μ mg = ma ⇒ $\mu = \frac{a}{g}$

Now, $v = u + at$ or $0 = 6 + 10a$ or $\frac{0.6}{10} = a = -0.6$

So, $\mu = \frac{a}{g} = \frac{0.6}{10} = 0.06$

13. $s = \frac{v^2}{2\mu_k g} = \frac{100 \times 100}{2 \times 0.5 \times 10} = \frac{100 \times 100}{5 \times 2} = 1000$ m ∴ f = mg = 0.98 N

14.



Let acceleration in Ist case is a_1 and that in second case is a_2

Now, $\frac{1}{2} a_1 t^2 = \frac{1}{2} a_2 (2t)^2$ ⇒ $a_2 = \frac{a_1}{4}$ ....(i)

Clearly $a_1 = \frac{mg \sin\theta}{m} = g \sin\theta$ ....(ii)

and $a_2 = \frac{mg \sin\theta - \mu mg \cos\theta}{m} = g \sin\theta - \mu g \cos\theta$ ....(iii)

From (i), (ii) and (iii), we get μ = 0.75.

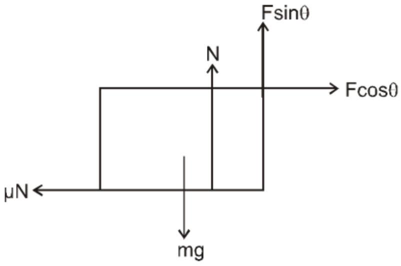
15. The normal reaction on the block is

$N = mg - F \sin\theta$ ∴ Net force on block is

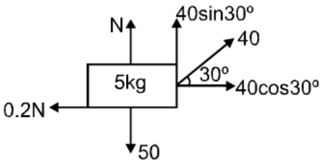
$F\cos\theta - \mu N = F\cos\theta - \mu mg + \mu F\sin\theta$

or acceleration of the block is

$$a = \frac{F(\cos\theta + \mu\sin\theta) - \mu mg}{m} = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$$



16.



N = 50 – 40 sin30 = 30

$a = \frac{40\cos 30^o - 0.2\times 30}{5}$ = 5.73 m/sec^2

17. Let velocity of projection be V and velocity of the block when it returns back = V'
then V > V' (since some K.E. is lost to friction)
Hence average velocity during ascent > average velocity during descent ⇒ $t_a < t_d$

18. According to work-energy theorem,
W - ΔK = 0
(∴ Initial and final speed are zero) ∴ Work done by friction + work done by gravity = 0

$-(\mu\, mg\cos\phi)\frac{\ell}{2} + mg\ell\sin\phi = 0$

⇒ μ cos ϕ = 2 sin ϕ ⇒ μ = 2 tan ϕ

SECTION (B)

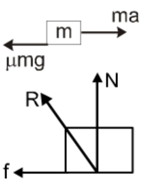
3. tanθ ≥ μ for sliding not depends on mass

6. tan θ = μ ⇒ μ = tan 60° ⇒ μ = $\sqrt{3}$ = 1.732

7. μ_s mg ≤ 75 N ⇒ $\mu_s \le \frac{75}{20g}$ = 0.35

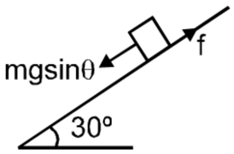
8. ma = μmg ⇒ a = μg

9.



Acceleration of train will be from right to left.
⇒ Pseudo force will act on the box from left to right therefore friction will act from right to left.

11.

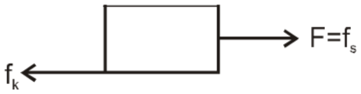


since μ > tanθ

The block will not slide therefore $f = mg\sin\theta = 2 \times 9.8 \times \frac{1}{2} = 9.8$ N.

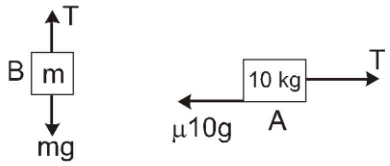
12. Friction force depends only on normal reaction.

13.



$a = \frac{f_s - f_k}{m} = \frac{(\mu_s - \mu_k)\ mg}{m} = (\mu_s - \mu_k)\, g = (0.5 - 0.4)10 = 1\ \text{m/sec}^2$

14.



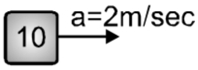
$T = mg \quad \Rightarrow \quad T \geq \mu \times 10\,g \quad \Rightarrow \quad mg \geq 0.20 \times 10 \quad \Rightarrow \quad m \geq 2$ kg

15. While the horse pulling a cart, the horse exerts a force on the ground, therefore from the third law of newton, the ground will also exerts a force on the horse that causes the horse to move forward.

16. The length of the rope which can overhang from the edge of the table without sliding down is given by

$\ell_1 = \left(\frac{\mu}{1+\mu}\right)\ell$

17. a = 2 m/sec

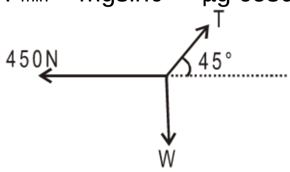


F = 20 N.

22. $F \geq mg\sin\theta + \mu g\cos\theta$

$F_{min} = mg\sin\theta + \mu g\cos\theta$

24.



$T\cos45° = 450$

$T\sin45° = W$

$W = 450$ N

25. Consider the equilibrium of the block for minimum value of force we have

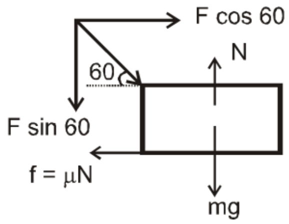
$F_{external} + F_s = mg\sin60$ so $Mg\left(\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{3}}.\frac{1}{2}\right) = \frac{Mg}{\sqrt{3}}$

$F_{ext} = \frac{200}{1.732} \times 120$. Consider the equilibrium of the block for maximum value of force we have

$F_{external} - F_s = mg\sin60$

So $\left[a_{system}\right]_{maximum} = \frac{100\mu}{20} = 5\mu = 5 * 0.2 = 1$ = $Mg\left(\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}.\frac{1}{2}\right) = Mg$ =200 N

26.



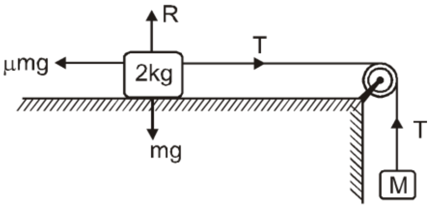
$N = mg + F\sin 60 = \sqrt{3} \times 10 + \frac{F\sqrt{3}}{2}$ ...... (i)

$F\cos 60 = \mu N$ ...... (ii)

$\Rightarrow \quad \frac{F}{2} = \frac{1}{2\sqrt{3}} \times (10\sqrt{3} + \frac{F\sqrt{3}}{2})$

$\Rightarrow \quad \frac{F}{2} = 5 + \frac{F}{4} \quad \Rightarrow \quad \frac{F}{4} = 5 \quad \Rightarrow \quad F = 20\text{ N}$

27. **Key Idea :** The tension in the string is equal to static frictional force between block A and the surface.
Let the mass of the block B is M.



In equilibrium

$T = Mg = 0$

$\Rightarrow \quad T = Mg$ ....(i)

If blocks do not move then

$T = f_s$

where f_s = frictional force = $\mu_s R = \mu_s mg$

$\therefore \quad T = \mu_s mg$ ....(ii)

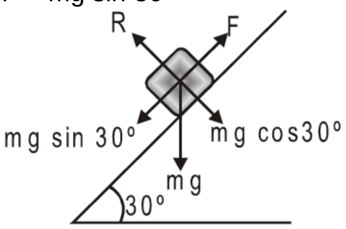
Thus, from Eqs. (i) and (ii), we have

$Mg = \mu_s mg$ or $M = \mu_s m$

Given $\mu_s = 0.2$, m = 2 kg $\therefore$ M = 0.2 × 2 = 0.4 kg

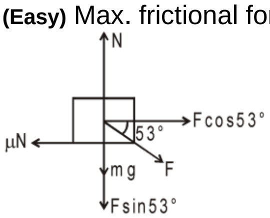
28. Force, $F = \mu R = Mg$
Weight of block = $\mu R = 0.2 \times 10 = 2\text{N}$

29. Let the mass of block be m.
Frictional force in rest position
$F = mg\sin 30^\circ$



$10 = m \times 10 \times \frac{1}{2} \quad \therefore \quad m = \frac{2\times 10}{10} = 2\text{ kg}$

30. **(Easy)** Max. frictional force



$f_{max} = \mu N$

$= \mu(mg + F\sin 53^\circ) = 0.2\,(20 \times 10 + 30 \times \frac{4}{5}) = 44.8\text{ N}$

As applied horizontal force is $F\cos 53^\circ = 18\text{N} < f_{max}$, friction force will also be 18 N.

31. $N = mg + Q\cos\theta$

Frectional froce f = $\mu(mg + Q\cos\theta)$

$P + Q\sin\theta = \mu(mg + Q\cos\theta)$

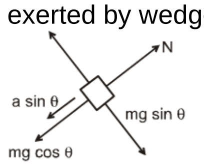
$\mu = \dfrac{P + Q\sin\theta}{mg + Q\cos\theta}$

32. For min. m, 5 kg block will have a tendency to move left. so

33. Apply Newton's law for system along the string

$m_B g = \mu(m_A + m_C) \times g \Rightarrow m_C = \dfrac{m_B}{\mu} - m_A = \dfrac{9}{0.2} - 10 = 15$ kg

34. The free body diagram of the block is as shown in the figure.
N is the normal reaction exerted by wedge on the block.



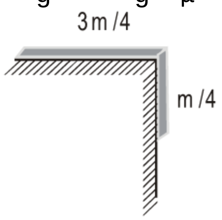
The wedge moves towards left with acceleration 'a', then the component of acceleration of block normal to the plane is $a\sin\theta$.
Applying Newtons second law to the block normal to plane.

$mg\cos\theta - N = ma\sin\theta$ For N to be zero $a = g\cot\theta$.

Hence the friction shall be zero when $a = g\cot\theta$.

39. Apply system equation

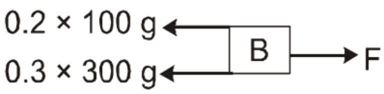
$\dfrac{m}{4} g = \dfrac{3m}{4} g \times \mu$



$\Rightarrow \mu = \dfrac{1}{3} = 0.33$

EXERCISE # 2

1.

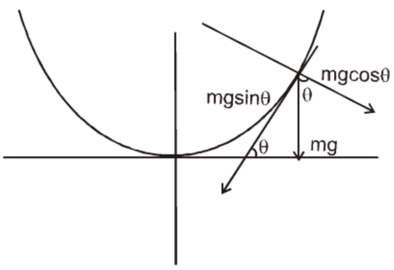


for motion to start

$f \geq 0.2 \times 100\, g + 0.3 \times 300\, g = 1100$ N

$F_{min} = 1100$ N

4.



For Remaining is Equilibrum

$F_s = Mg\sin\theta$

$\mu N \geq Mg\sin\theta$

μ mg cos θ mg cos θ

$\mu \geq$ Tan θ

Tan θ ≥ μ

$(\text{Tan }\theta)_{max} = \mu = 0.5$

$y = \frac{x^2}{20}\ \frac{dy}{dx} = \frac{x}{10}$ = (Tanθ)

$\left(\frac{x}{10}\right)_{max} = (\text{Tan}\theta)_{max}\ (\text{Tan}\theta)_{vf/kdre}$

$(X)_{vf/kdre} = 0.5 \times 10 = 5$

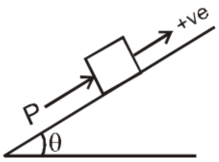
$(y)_{vf/kdre} = \frac{(X_{vf/kdre})^2}{20} = \frac{25}{20} = 1.25\text{m}$

5. $a_A = g\,[\sin 45 - \mu_A \cos 45] = \frac{8}{\sqrt{2}}$, $a_B = g\,[\sin 45 - \mu_B \cos 45] = \frac{7}{\sqrt{2}}$

$a_{AB} = a_A - a_B = g\,(\mu_B - \mu_A) \cos 45 = \frac{1}{\sqrt{2}}$, $s_{AB} = \sqrt{2}$

Now $s_{AB} = \frac{1}{2} a_{AB} t_2 \Rightarrow \sqrt{2} = \frac{1}{2} \times \frac{1}{\sqrt{2}} t_2 \Rightarrow$ t = 2 sec.

Again $s_A = \frac{1}{2} a_A t_2 = \frac{1}{2} \left(\frac{8}{\sqrt{2}}\right) 4 \Rightarrow s_A = 8\sqrt{2}$ m

6. 

tanθ > μ

$P_1 = mg\sin\theta - \mu mg\cos\theta$

$P_2 = mg\sin\theta + \mu mg\cos\theta$

Initially block has tendency to slide down and as tanθ > μ, maximum friction μmgcosθ will act in positive direction. When magnitude P is increased from P_1 to P_2, friction reverse its direction from positive to negative and becomes maximum i.e.μmgcosθ in opposite direction.

7. When friction is absent

$a_1 = g \sin \theta$

∴ $s_1 = \frac{1}{2} a_1 t_1^2$ .... (i)

When friction is present

$a_2 = g \sin \theta - \mu g \cos \theta$

∴ $s_2 = \frac{1}{2} a_2 t_2^2$ .... (ii)

From Eq. (i) and (ii)

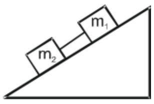
$\frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$

or $a_1 t_1^2 = a_2 (nt_1)^2$ (∴ $t_2 = nt_1$)

or $a_1 = n^2 a_2$

or $\frac{a_2}{a_1} = \frac{g\sin\theta - \mu g \cos\theta}{g \sin\theta} = \frac{1}{n^2}$ or $\frac{g \sin 45^o - \mu g \cos 45^o}{g \sin 45^o} = \frac{1}{n^2}$

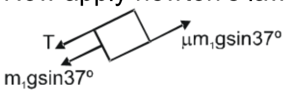
or $1 - \mu_k = \frac{1}{n^2}$ or $\mu_k = 1 - \frac{1}{n^2}$

8. 

Apply newton's law for system of m_1 and m_2

a = $\dfrac{(m_1+m_2)g\sin 37^\circ - \mu[m_1 g\cos 37^\circ + m_2 g\cos 37^\circ]}{m_1+m_2}$ = g[sin37º – μcos37º]

Now apply newton's law for M_1

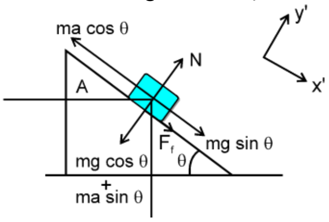


$m_1g\sin 37^\circ + T - \mu m_1 g\cos 37^\circ = m_1 a = m_1 g[\sin 37^\circ - \mu\cos 37^\circ]$

⇒ T = 0 and a = 4m/sec^2

9. When F is less than μ_s mg then tension in the string is zero.
When $\mu_s mg \le F < \mu_s 2mg$ then friction on block B is static.
If further increase friction on block B is kinetic.

10. FBD of block B w.r.t. wedge A , for maximum 'a' :
Perpendicular to wedge :
$\Sigma f_{y'} = (mg\cos\theta + m a \sin\theta - N) = 0.$
and $\Sigma f_{x'} = mg\sin\theta + \mu N - ma\cos\theta = 0$ (for maximum a)



⇒ $mg\sin\theta + \mu(mg\cos\theta + ma\sin\theta) - ma\cos\theta = 0$ ⇒ a = $\dfrac{(g\sin\theta + \mu g\cos\theta)}{\cos\theta - \mu\sin\theta}$ for θ = 45º

a = $g\left(\dfrac{\tan 45^\circ + \mu}{\cot 45^\circ - \mu}\right)$; a = $g\left(\dfrac{1+\mu}{1-\mu}\right)$ **Ans.**

11. $0^2 = V^2 - 2\mu gs$ ⇒ s = $\dfrac{V^2}{2\mu g}$. (A).

12. Block will not slip if
$(m_1 + m_2)\, g \sin\theta \le \mu\, m_2\, g\cos\theta$

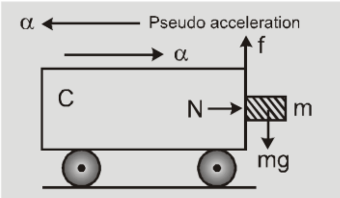
$3\sin\theta \left(\dfrac{3}{10}\right) \le (2)\cos\theta$

$\tan\theta \le \dfrac{1}{5}$ ⇒ $\boxed{\theta \le 11.5^\circ}$

(P) θ = 5° friction is static $f = (m_1 + m_2)g\sin\theta$
(Q) θ = 10° friction is static $f = (m_1 + m_2)g\sin\theta$
(R) θ = 15° friction is kinetic $f = \mu m_2 g\cos\theta$
(S) θ = 20° friction is kinetic $f = \mu m_2 g\cos\theta$

EXERCISE # 3
PART - I

1.



Pseudo force or fictitious force, $F_{fic} = m\alpha$
Force of friction, $f = \mu N = \mu m\alpha$,
The block of mass m will not fall as long as
$f \geq mg$
$\mu m\alpha \geq mg$
$$\alpha \geq \frac{g}{\mu}$$

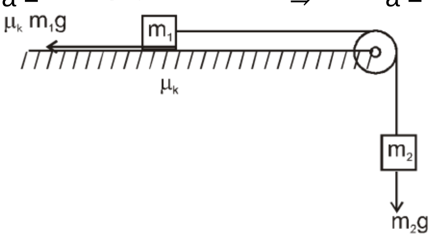
2. $a = \mu g = 5$
$v_2 = u_2 + 2as$
$0 = 2_2 + 2 \times (5)s$
$s = -\frac{2}{5}$ w.r.t. belt or distance = 0.4 m

3. The coin will revolve with the record, if Force of friction ≥ Centrifugal force
$\mu mg \geq mr\omega_2$
or $\frac{\mu g}{\omega^2} \geq r$
$F_{?k"kZ.k} \geq F_{vfHkdsUæ}$
$\mu mg \geq m\omega_2 r$
$$\frac{\mu g}{\omega^2} \geq r$$

4. For smooth driving maximum speed of car v then
$\frac{mv^2}{R} = \mu_s mg$ ⇒ $v = \sqrt{\mu_s Rg}$

5. $a = \frac{mg - 2\mu mg}{3m}$ ⇒ $a = \frac{g - 2\mu mg}{3}$ $= g\left(\frac{1-2\mu}{3}\right)$

6.



$a = \frac{m_2 g - \mu_k m_1 g}{m_1 + m_2}$ ⇒ $m_2 g - T = (m_2)\left(\frac{m_2 g - \mu_k m_1 g}{m_1 + m_2}\right)$ (a) ⇒ $m_2 g - T = (m_2)$

Solving get $T = \frac{m_1 mg(1+\mu_k)g}{m_1 + m_2}$

7. $\mu_s = \tan 30^o = \frac{1}{\sqrt{3}} = 0.5$

$\mu_s = 0.57 = 0.6$

$S = ut + \frac{1}{2}at^2$

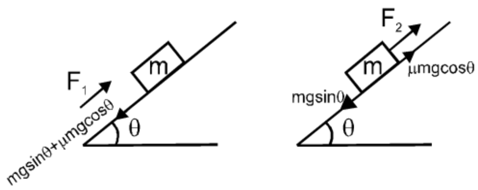
$4 = \frac{1}{2}a(4)^2 \Rightarrow a = \frac{1}{2} = 0.5$

$a = g\sin\theta - \mu_k (g) \cos\theta \quad \Rightarrow \mu_K = \frac{0.9}{\sqrt{3}} = 0.5$

8. The coefficient of the friction is a non dimensional quantity.

PART - II

1.

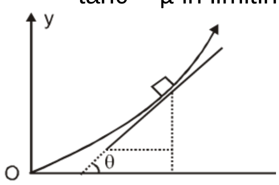


$F_1 = mg \sin\theta + \mu mg \cos\theta$

$F_2 = mg \sin\theta - \mu mg \cos\theta$

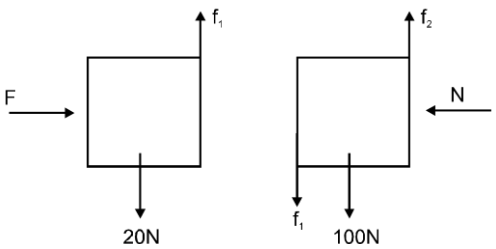
$\frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta} \Rightarrow \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3.$

2. $\frac{dy}{dx} = \tan\theta = \mu$ in limiting case



$\frac{dy}{dx} = \frac{3x^2}{6} = \frac{1}{2} \Rightarrow x = \pm 1$ So, $y = \frac{1}{6}$

3.



Assuming both the blocks are stationary

N = F

$f_1 = 20N$

$f_2 = 100+20 = 120N$

4. $\mu(m + m_2) = m_1$

$m + m_2 = \frac{m_1}{\mu}$

$\Rightarrow \quad m = \frac{m_1}{\mu} - m_2$

$\Rightarrow \quad m = \frac{5}{0.15} - 10 = 23.33kg$

5. mg sin θ + 3 = P + friction

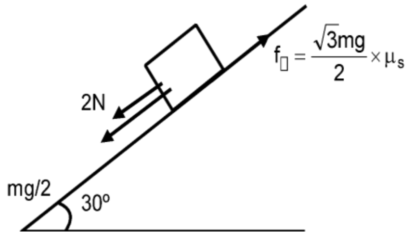
mg sin θ + 3 = P + μmg cos θ.

$\frac{10 \times 10}{\sqrt{2}} + 3 = P + 0.6 \times 10 \times 10 \times \frac{1}{\sqrt{2}}$

$20\sqrt{2} + 3 = P$

$31.28 = P \Rightarrow \boxed{P = 32N}$

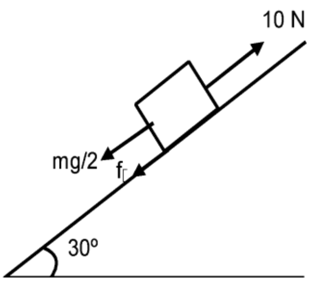
6. For just equilibrium



$2 = \frac{\sqrt{3}mg}{2}\mu_s - \frac{mg}{2}$ ....(1)

In the other case

$\frac{\sqrt{3}mg}{2}\mu_s + \frac{mg}{2} = 10$ ....(2)



equation (1) / equation (2)

$\frac{1}{5} = \frac{\sqrt{3}\mu_s - 1}{\sqrt{3}\mu_s + 1} \quad \Rightarrow \quad \mu_s = \frac{\sqrt{3}}{2}$