TOPIC : WORK, POWER & ENERGY
EXERCISE # 1
PART – I

SECTION : (A)

1. W = (force) (displacement) = (force) (zero) = 0

2. 25 = 5 × 10 × cosθ   so θ = 60º

3. W = $\bar{F} \cdot (\bar{r}_2 - \bar{r}_1)$ = 100 J

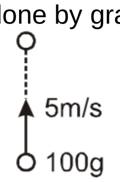
4. W = 20 × 10 × 20 × 0.25 = 1000 J

7. Work done = $\vec{F}\cdot\vec{S}$ = $(5\hat{i}+3\hat{j}+4\hat{k})\cdot(6\hat{i}-5\hat{k})$ = $(5\hat{i}+3\hat{j}+4\hat{k})\cdot(6\hat{i}+4\hat{j}-5\hat{k})$ = (30 + 0 – 20) = 10 unit

8. The height (h) tranversed by particle while going up is :

$$h = \frac{u^2}{2g} = \frac{25}{2\times 9.8}$$

Work done by gravity force = $m\vec{g}\cdot\vec{h}$

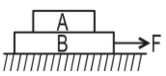


$$= 0.1 \times g \times \frac{25}{2\times 9.8}\cos 180^\circ$$

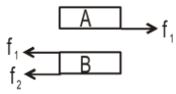
(Angle between force and displacement is 180°) ∴   W = – 0.1 × $\frac{25}{2}$ = – 1.25 J

10. Net work done in sliding a body up to a height h on inclined plane
= Work done against gravitational force + Work done against frictional force
⇒ W = $W_s + W_f$ ... (i)
but W = 300 J
W_g = mgh = 2 × 10 × 10 = 200 J
putting in equation (i), we get
300 = 200 + W_f ⇒ W_f = 300 – 200 = 100 J

11. $P\bar{Q}$ = (2 -3) $\hat{i}$ + (-1 -2) $\hat{j}$ (4 + (-1) $\hat{k}$
$\bar{F}\cdot P\bar{Q}$ = – 4 + 9 + 10 = **15 J**

12. 

Consider the blocks shown in the figure to be moving together due to friction between them.
The free body diagrams of both the blocks is shown below.



Work done by static friction on A is positive and on B is negative.

13. Force is perpendicular to displacement hence work done is zero

14. Work done by first man + work done by gravity = 0 ...(i)
work done by second man + work done by gravity = 0 ...(ii)
Ratio of work done by them = 1 : 1

15. $S_1 = \frac{1}{2} g 1^2$, $s_2 = \frac{1}{2} g 2^2$, $S_3 = \frac{1}{2} g 3^2$

$S_2 - S_1 = \frac{1}{2} g\, 3$, $S_3 - S_2 = \frac{1}{2} g\, 5$

$W_1 = (mg)\, S_1$, $W_2 = (mg)\,(S_2 - S_1)$, $W_3 = (mg)\,(S_3 - S_2)$

$W_1 : W_2 : W_3 = 1 : 3 : 5$

16. $T = mg + ma$, $S = \frac{1}{2} at^2$

$W_T = T \times S$

17. (3) $W = (3\hat{i} + c\hat{j} + 2\hat{k}).(-4\hat{i} + 2\hat{j} + 3\hat{k}) = 6\,\text{Joule}$

$W = -12 + 2c + 6 = 6 \Rightarrow c = 6$

SECTION (B)

1. $W = \int_0^1 F\,dx = \frac{1}{6} J$

2. $W = \int_0^5 F\,dx = 7 \times 5 - 25 \times \frac{2}{2} + 125 \times \frac{3}{3} = 135$ J

3. $W_F = \int \left(\frac{K}{S}\right) ds = K \ln s + C$ Ans : (4)

6. **Key Idea :** If a constant force is applied on the object causing a displacement in it, then it is said that work has been done on the body to displace it. Work done by the force = Force × Displacement

or $W = F \times s$ ... (i)

But from Newton's 2nd law, we have

Force = Mass × Acceleration

i.e., $F = ma$ ... (ii)

Hence, from equation (i) and (ii), we get

$W = mas = m\left(\frac{d^2s}{dt^2}\right)s$ ... (iii) $\left(\because a = \frac{d^2s}{dt^2}\right)$

Now, we have $s = \frac{1}{3}t^2$ $\therefore$ $\frac{d^2s}{dt^2} = \frac{d}{dt}\left[\frac{d}{dt}\left(\frac{1}{3}t^2\right)\right] = \frac{d}{dt} \times \left(\frac{2}{3}t\right) = \frac{2}{3}\frac{dt}{dt} = \frac{2}{3}$

Hence, eq. (iii) becomes $W = \frac{2}{3}ms = \frac{2}{3}m \times \frac{1}{3}t^2 = \frac{2}{9}mt^2$

We have given m = 3 kg, t = 2 s $\therefore$ $W = \frac{2}{9} \times 3(2)^2 = \frac{8}{3} J$

8. $dW = F.dx \Rightarrow W = \int_{x_1}^{x_2} F.dx = \int_0^2 (5 + 2x)dx = \left|5x + x^2\right|_0^2 = 14$ J.

9. $W = \Delta KE = \frac{1}{2} = 9t^2 + 2 = \frac{1}{2} \times 2\,(83^2 - 2^2)$

$v(3) = 83\ ms^{-1}$ and $v(0) = 2\ ms^{-1} = (6889 - 4) = 6885$ J.

10. $W = \int_0^{x_1} cx\ dx = c\frac{x_1^2}{2}$

11. Work done is displacing the particle

W = $\vec{F}.\vec{r}$

= $(5\hat{i} + 3\hat{j} + 2\hat{k}).(2\hat{i} - \hat{j})$
= 5 × 2 + 3 × (–1) + 2 × 0
= 10 – 3
= 7 J

SECTION (C)

1. KE = $\frac{P^2}{2m}$ = 1

2. a = $\frac{F}{m}$, S = $\frac{1}{2}\left(\frac{F}{m}\right)t^2$, W_F = FS = F $\left(\frac{Ft^2}{2m}\right)$

3. W = area = 80 = $\frac{1}{2}$ (0.1) u^2 – 0 ,
so u = 40 m/s

4. $W_G = \frac{1}{2} mV_f^2 - \frac{1}{2} mV_i^2$, mg h = $\frac{1}{2} mV_f^2 - \frac{1}{2} mV^2$,
So V_f is free from direction of V.

5. W = ΔK

6. V = 0 + aT, a = $\frac{V}{T}$, velocity = 0 + at = $\frac{Vt}{T}$

K.E = $\frac{1}{2}$ (m) $\left(\frac{Vt}{T}\right)^2$

7. E = $\frac{1}{2}$ mV^2, $\frac{dE}{dV}$ = mV = p

8. Follows from work energy theorem.

9. $W_f + W_G + W_N = \Delta K = 0$
$W_G = 0$, $W_N = 0$ so $W_f = 0$.

11. KE = $\frac{p^2}{2m}$ ⇒ KE ∝ p^2 ⇒ $\frac{p_2}{p_1} = \sqrt{\frac{KE_2}{KE_1}} = \sqrt{2}$

12. The relation between momentum p and kinetic energy K is K = $\frac{1}{2m}(p^2)$

Kinetic energy K = $\frac{1}{2m}(p^2)$ or p = $\sqrt{2mK}$
If kinetic energy of a body is increased by 300%, let its momentum becomes p'.

New kinetic energy K' = $K + \frac{300}{100}K$ = 4K Therefore, momentum is given by
P' = $\sqrt{2m \times 4K}$ = $2\sqrt{2mK}$ = 2p Hence, % change (increase) in momentum
$\frac{\Delta p}{p} \times 100 = \frac{p' - p}{p} \times 100\% = \left(\frac{p'}{p} - 1\right) \times 100\% = \left(\frac{2p}{p} - 1\right) \times 100\%$ = 100%

13. **Key Idea :** The work done will be the area of the F-x graph.

Work done in moving the object from x = 0 to x = 6 m is given by

W = Area of rectangle + area of triangle = $3\times3+\frac{1}{2}\times3\times3$ = 9 + 4.5 = 13.5 J

14. Let extension produced in a spring be x initially. In stretched condition spring will have potential energy

$U = \frac{1}{2}kx^2$

where k is spring constant or force constant.

$\therefore \quad \frac{U_1}{U_2} = \frac{x_1^2}{x_2^2}$ ... (i)

Given $U_1 = U$, $x_1 = 2$ cm, $x_2 = 8$ cm

putting these values in equation (i), we have

$\frac{U}{U_2} = \frac{(2)^2}{(8)^2} = \frac{4}{64} = \frac{1}{16}$

$U_2 = 16U$

15. $W_{agent} + W_G = \Delta K = 0$

$W_{agent} = -W_G$,

But W_G is independent of the path joining initial and final position. W_G is independent of time taken.

16. $dW_F = \bar{F}.d\bar{s} = dk > 0 \Rightarrow |\bar{F}||d\bar{s}| \cos\theta > 0 \quad \Rightarrow 0 < \theta < 90^o$

$p = \sqrt{2m(K.E.)}$, K.E. ↑ so p ↑ .

17. $W_G + W_f = 0 - 0$

$10 \times 1 + W_f = 0$

$10 - \mu mg\, x = 0$

$10 = (.2)(10)\, x$, $x = 5$ m

18. Area under curve = $\frac{1}{2}(4)(20) = 40$ J

W = work done by resistive force F = – 40 J

$-40 = K_f - K_i$, $K_i = 50$ J, so $K_f = 50 - 40 = 10$ J

19. $F\,80 = \frac{1}{2}mV^2$, $FS = \frac{1}{2}m(2V)^2$

So $\frac{s}{80} = 4$, $S = 4\,(80)$

21. 2 and 3 holds when a ball moves in upward direction.

22. $mg\frac{\ell}{2} = \frac{1}{2}mv^2$ $\quad v = \sqrt{g\ell}$

23. $mg1 - mg/2 = mv^2/2$, $v = \sqrt{g}$

$d = v\sqrt{2h/g} = \sqrt{g}\sqrt{\frac{2(0.5)}{g}} = 1$ m

24. $W_F + W_S = 0$, $W_F - \Delta U = 0$, $W_F = \Delta U = E$

$E = \frac{1}{2}K_A x_A^2$, $Fx_A = \frac{1}{2}K_A x_A^2$

$\frac{2F}{K_A} = x_A$, $\frac{2F}{K_A} = \sqrt{\frac{2E}{K_A}}$, $K_A = \frac{2F^2}{E}$ ...(i)

Similarly $K_B = \frac{2F^2}{E_B}$, $\because$ $K_A = 2K_B$ $\therefore$ $\frac{2F^2}{E} = 2\left(\frac{2F^2}{E_B}\right)$ $\therefore$ $E_B = 2E$

25. $\frac{1}{2}mu^2 = mgh$, $u^2 = 2gh$ ....(i)

$mg\left(\frac{3h}{5}\right) + K.E. = mgh$

$K.E. = \frac{mgh}{4}$ $\Rightarrow$ $\frac{K.E.}{P.E.} = \frac{mgh/4}{3mgh/4} = \frac{1}{3}$

26. $U_i + 0 = U_f + \frac{1}{2}mv^2$ $\Rightarrow$ $U_i - U_f = \frac{1}{2}mv^2$ $\Rightarrow$ $U = \frac{1}{2}mv^2 \Rightarrow$ $m = \frac{2U}{v^2}$

27. Work done = Force × displacement = Weight of the book × Height of the book shelf

28. $\frac{dU}{dx}$ = positive constant
For x < a, F = negative constant and for x > a, F = 0. So, ans. (3)

29. K.E. + P.E. = positive constant C
E + U = C, E + mgh = C, E = – mgh + C and U = mgh, So, answer (1)

30. $E = \frac{p^2}{2m}$, $(\sqrt{E})\left(\frac{1}{P}\right) = \frac{1}{\sqrt{2m}}$ = constant. Rectangular hyperbola (3)

31. Let initial velocity is u and retardation is a

So, $\frac{u^2}{4} = u^2 - 2a \times (0.03)$ ...(i)

$0 = \frac{u^2}{4} - 2a \times S$ ..(ii)
here S is required distance from equation (i) & (ii) S = 0.01 m = 1 cm

32. $\frac{1}{2}mv^2 = k$ $\Rightarrow$ $\frac{1}{2}m(v\cos 60)^2 = \frac{1}{2}m\left(\frac{v^2}{4}\right) \Rightarrow \frac{1}{4}\left(\frac{1}{2}mv^2\right) = \frac{K}{4}$

33. Assuming mass of athlete is between 40 kg to 100 kg
here we will consider mass of athlete m = 50 kg

$V = S/t = \frac{100}{10}$ = 10 m/sec So, $K = 1/2\ mv^2$ $1/2 \times (50 \times 10^2) = 2500$ J. So Answer is (C)

SECTION (D)

1. Follows from definition
2. $U \propto x^2$

14. Ratio of heights

$\frac{h_1}{h_2} = \frac{100}{100-50} = 2$ $\Rightarrow$ $h_2 = \frac{h_1}{2} = \frac{1}{2}$ of the initial height.

15. Here : Energy of one apple = 2112 J
= 21×10^3 J
Efficiency of the boy = 28% = 0.28
Mass of the boy m = 40 kg
Here the actual energy consumed by the boy is given by as
= 0.28 × 21000 = 5880 J ...(1)
and the energy consumed by the boy in climbing h meter height is given by

$= m g h = 40 \times 9.8 \times h$ J ...(2)
Equating equations (1) and (2) we get
$40 \times 9.8 \times h = 5880$ J

$$h = \frac{5880}{40 \times 9.8} = 15 \text{ m}$$

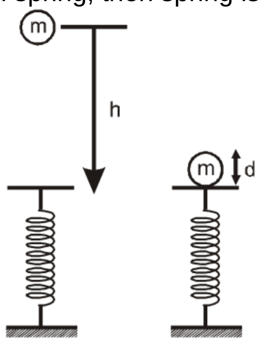
16. $E_1 = \frac{E}{4}$.

17. Given, $F = -5x - 16x^3$ or $F = -(5 + 16x^2)x$ or $F = -kx$
Where $k (= 5 + 16x^2)$ is force constant of spring. Therefore, work done in stretching the spring from position x_1 to position x_2 is

$W = \frac{1}{2}k_2x_2^2 - \frac{1}{2}k_1x_1^2$ we have, $x_1 = 0.1$ m and $x_2 = 0.2$ m

$\therefore \quad W = \frac{1}{2}(5 + 16 (0.2)^2) (0.2)^2 - \frac{1}{2}(5 + 16 (0.1)^2) (0.1)^2$
$= 2.82 \times 4 \times 10^{-2} - 2.58 \times 10^{-2} = 8.7 \times 10^{-2}$ J

18. **Key Idea :** Work done is equal to change in energy of body. Situation is shown in figure, when mass m falls vertically on spring, then spring is compressed by distance d.



Hence, net work done in the process is

W = Potential energy stored in the spring + Loss of potential energy of mass $= mg (h + d) - \frac{1}{2} kd^2$

19. If the springs are compressed to same amount :

$W_A = \frac{1}{2} K_A x^2$; $W_B = \frac{1}{2} K_B x^2$ $\quad \because \quad K_A > K_B \Rightarrow W_A > W_B$
If the springs are compressed by same force.

$F = K_A x_A = K_B x_B$; $x_A = \frac{F}{K_A}$; $x_B = \frac{F}{K_B}$; $\frac{W_A}{W_B} = \frac{\frac{1}{2}K_A \cdot \frac{F^2}{K_A^2}}{\frac{1}{2}K_B \frac{F^2}{K_B^2}} = \frac{K_B}{K_A}$ Hence, $W_A < W_B$

20 $F = T$, $W_F + W_G = 20$
$W_T = 20 \Rightarrow 20 + W_G = 20 \Rightarrow W_G = 0$
Which is not possible.

21. **(easy)** As ΔKE is same in both the cases, work done will be same.

22. Change in velocity $= \frac{\text{area under F} - \text{T graph}}{\text{mass}} = \frac{60 + (-10)}{10} = 5$ m/s

$W_F = \Delta K.E. = \frac{1}{2}(10)\, 5^2 = 125$ J

23. $W_S + W_f = \Delta K$
$-\Delta U + W_f = -K_i$

$-U_f - \mu mgx = -K_i$

$\frac{1}{2} K x^2 + \mu mgx = \frac{1}{2} mu^2$

$100 x^2 + 2(0.1)(50)(10)x = 50 \times 4$

$x^2 + x - 2 = 0$

$x = 1$ m

24. If there is no air drag then maximum height

$H = \frac{u^2}{2g} = \frac{14 \times 14}{2 \times 9.8} = 10\,m$

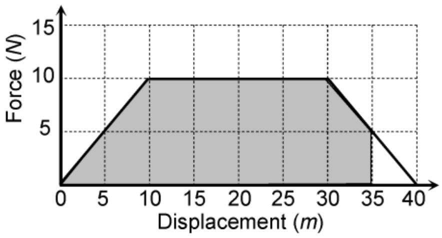
But due to air drag ball reaches up to height 8m only. So loss in energy

$= mg(10-8) = 0.5 \times 9.8 \times 2 = 9.8\,J$

25. $E = \frac{P^2}{2m}$ if P = constant then $E \propto \frac{1}{m}$

According to problem $m_1 > m_2 \therefore E_1 < E_2$

26.



Work done = (Shaded area under the graph between x = 0 to x = 35 m) $= 287.5\,J$

27. $P = \sqrt{2mE}$ if E are equal then $P \propto \sqrt{m}$

i.e. heavier body will possess greater momentum.

28. $v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 0.1} = \sqrt{1.96} = 1.4\,m/s$

29. $W_C = -\Delta U = -(U_{final} - U_{initial}) = -\left(\frac{1}{2} \times k \times 15^2 - \frac{1}{2} \times k \times 5^2\right)$

$W_C = 8$ Joule

30. $K = 5 \times 10^3$ N/m

x = 5 cm

$W_1 = \frac{1}{2} k \times x_1^2 = \frac{1}{2} 5 \times 10^3 \times (5 \times 10^{-2})^2 = 6.25$ J

$W_2 = \frac{1}{2} k(x_1 + x_2)^2 = \frac{1}{2} \times 5 \times 10^3 (5 + 10^{-2} + 5 \times 10^{-2})^2 = 25J$

Net work done = $W_2 - W_1 = 25 - 6.25 = 18.75$ J = 18.75 N-m

SECTION (E)

9. Efficiency of engine η = 60%

Thus, power = $\frac{\text{work / time}}{\eta} = \frac{100}{60} \times \frac{mgh}{t}$ Given m = 100 kg, h = 10 m, t = 5 s and g = 10 ms^{-2}

Hence, power = $\frac{100}{60} \times \frac{100 \times 10 \times 10}{5} = 3.3 \times 10^3$ W = 3.3 kW

10. 1 kWh = 1000 W × 3600 s = 3.6×10^6 J

11. Power is equal to the scalar product of force with velocity.
Power of the engine,
P = $\vec{F}.\vec{v}$ ... (i)
Given
$\vec{F} = (20\hat{i} - 3\hat{j} + 5\hat{k})N$

$\vec{v} = (6\hat{i} + 20\hat{j} - 3\hat{k})m/s$

Thus, after substituting for $\vec{F}$ and $\vec{v}$ in equation (i), it becomes,

P = $(20\hat{i} - 3\hat{j} + 5\hat{k}) . (6\hat{i} + 20\hat{j} - 3\hat{k})$ = (20 × 6) $(\hat{i}.\hat{i}) + (-3 \times 20)(\hat{j}.\hat{j}) + (5 \times -3)(\hat{k}.\hat{k})$
= 120 – 60 – 15 = 45
Note : In the simplification for power, the dot product of a unit vector with same unit vector gives 1. The dot product of a unit vector with its orthogonal gives zero. Thus,

$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$

So, in above simplification second type of dot product are not shown.

12. Average power = $\dfrac{100 \times 9.8 \times 50}{50}$ = 980 J/s

13. P = TV = 4500 × 2 = 9000 W = 9KW

14. V = 0 + at, F – μ mg = ma , F = μmg + ma,
P = (μmg + ma) at

15. P = $\bar{F}.\bar{v}$ = 50 – 30 + 120 = 140 J/s

16. P_1 = 80 gh/15 , P_2 = 80 gh/20
$\dfrac{P_1}{P_2} = \dfrac{20}{15} = \dfrac{4}{3}$

17. Force required to move with constant velocity ∴ Power = FV
Force is required to oppose the resistive force R and also to accelerate the body of mass with acceleration a. ∴ Power = $(R + ma)V$

18. Energy supplied to liquid per second by the pump = $\dfrac{1}{2}\dfrac{mv^2}{t} = \dfrac{1}{2}\dfrac{V\rho v^2}{t} = \dfrac{1}{2}A \times \left(\dfrac{l}{t}\right) \times \rho \times v^2$ $\left[\dfrac{l}{t} = v\right]$
$= \dfrac{1}{2}A \times v \times \rho \times v^2 = \dfrac{1}{2}A\rho v^3$

19. P = $\dfrac{mgh}{t}$ ⇒ m = $\dfrac{p \times t}{gh} = \dfrac{2 \times 10^3 \times 60}{10 \times 10} = 1200$ kg
As volume = $\dfrac{mass}{density}$ ⇒ $v = \dfrac{1200kg}{10^3 kg/m^3} = 1.2m^3$

Volume = $1.2m^3 = 1.2 \times 10^3 litre = 1200\ litre$

20. Force produced by the engine $F = \dfrac{P}{v} = \dfrac{30 \times 10^3}{30}$ = 10^3N
Acceleration= $\dfrac{\text{Forward force by engine–resistive force}}{\text{mass of car}} = \dfrac{1000 - 750}{1250} = \dfrac{250}{1250} = \dfrac{1}{5}m/s^2$

21. $v^2 = u^2 + 2ax$

$v^2 = 2ax \quad \Rightarrow \quad a = \frac{v^2}{2x}$

$p = fv = m.\frac{v^2}{2x}.v = \frac{mv^3}{2x} \quad \therefore \quad v^3 \propto x \qquad (\because P = \text{constant})$

$v \propto x^{1/3}$

$\frac{dx}{dt} \propto x^{1/3} \quad \Rightarrow \quad \int x^{-1/3}dx \propto \int dt$

$\frac{3}{2}x^{2/3} \propto t \quad \therefore \quad x \propto t^{3/2}$

22. Let the constant acceleration of body of mass m is a.
From equation of motion $v_1 = 0 + at_1$

$\Rightarrow \quad a = \frac{v_1}{t_1} t$

At an instant t, the velocity v of the body
$v = 0 + at$

$v = \frac{v_1}{t_1} t$

Therefore, instantaneous power
$P = Fv = mav \qquad (\because F = ma)$

$= m\left(\frac{v_1}{t_1}\right) \times \left(\frac{v_1}{t_1}.t\right)$ (from equations (i) and (ii)) $= \frac{mv_1^2 t}{t_1^2}$

SECTION (F)

1. $\frac{dU}{dx} = 0$ at B and C

2. $\underset{P \to R}{W_C} = \underset{P \to Q}{W_C} + \underset{Q \to R}{W_C} = 5 + 2 = 7$

3. $\frac{\partial U}{\partial x} = \cos(x + y)$,

$\frac{\partial U}{\partial y} = \cos(x + y)$

$\bar{F} = -\cos(x + y)\,\hat{i} - \cos(x + y)\,\hat{j} = -\cos(0 + \frac{\pi}{4})\hat{i} - \cos(0 + \frac{\pi}{4})\hat{j} \Rightarrow \quad |\bar{F}| = 1$

5. Potential energy $U = A - Bx^2$

Force $F = -\frac{dU}{dx} = -(0 - 2Bx)$

$F = 2Bx \quad \therefore \quad F \propto x$

6. $W_{ext} + W_C = \Delta K$
$W_{ext} - \Delta U = \Delta K$
$W_{ext} = \Delta U + \Delta K$ = change in total energy

7. $U(x) = x^2 - 4x$
$F = 0$

$\frac{dU(x)}{dx} = 0 \quad \Rightarrow \quad 2x - 4 = 0 \quad x = 2 \quad \Rightarrow \quad \frac{d^2U}{dx^2} = 2 > 0$

i.e. U is minimum hence x = 2 is a point of stable equilibrium.

8. dU = $-\vec{F}\cdot\vec{ds}$ = $-\vec{F}\cdot(dx\,\hat{i} + dy\,\hat{j})$

Also by reverse method using $F_x = -\frac{\partial U}{\partial x}$ and $F_y = -\frac{\partial U}{\partial y}$, only (B) option satisfies the criterea.

9. Only the following statements are true from definition of a conservative force.
"Its work is zero when the particle moves exactly once around any closed path".
"Its work depends on the end points of the motion, not on the path between".

10. $\left.\frac{dU}{dx}\right|_{x=A} = -ve, \; \left.\frac{dU}{dx}\right|_{x=B} = +ve$

So, F_A = positive, F_B = negative

11. $F = -\frac{dU}{dx}$

∴ dU = – F . dx or $U(x) = -\int_0^x (-kx + ax^3)\,dx$

$U(x) = \frac{kx^2}{2} - \frac{ax^4}{4}$ ⇒ U(x) = 0 and x = 0 and $x = \sqrt{\frac{2k}{a}}$

U(x) = negative for $x > \sqrt{\frac{2k}{a}}$

From the given function we can see that

F = 0 at x = 0 i.e. slope of U-x graph is zero at x = 0. Therefore, the most appropriate option is (D).

EXERCISE # 2

1. W = (2000 sin 15º) × 10 = 5176.8 J

2. W_1 = work done by spring on first mass
W_2 = work done by spring on second mass
$W_1 = W_2$ = W (say)
$W_1 + W_2 = U_i - U_f$

$2W = 0 - \frac{1}{2}Kx^2$ ⇒ $W = -\frac{Kx^2}{4}$

3. $h = \frac{1}{2}gt^2$, $W = mgh = mg\frac{gt^2}{2}$, $W = K_f - K_i$

$\frac{mg^2t^2}{2} = K_f - \frac{1}{2}mu^2$, $K_f = \frac{1}{2}mu^2 + \frac{mg^2t^2}{2}$ Hence Ans. is (A)

4. $-Fx = 0 - \frac{1}{2}m(2)^2$ and $-FS = 0 - 2\left[\frac{1}{2}m(2)^2\right]$

So $\frac{S}{x} = 2$, S = 2x

5. $W_a + W_c = \Delta K = 0$, $W_a - mg\left(\frac{\ell}{2} - \frac{\ell}{2}\cos 60^\circ\right) = 0$

$W_a = \frac{mg\ell}{4} = (0.5)(10)\left(\frac{1}{4}\right) = \frac{5}{4}$ J.

6. $V \frac{dV}{dx} = -Kx$, $\left[\frac{V^2}{2}\right]_u^v = -\left[\frac{Kx^2}{2}\right]_0^x$

$V^2 - u^2 = -Kx^2$

$\frac{1}{2} mu^2 - \frac{1}{2} mV^2 = \frac{1}{2} mK x^2$

Loss α x^2

7. $(mg \sin\theta)\, x - \int_0^x \mu mg \cos\theta \, dx = 0$

$\sin\theta\, x = \mu_o \cos\theta \int_0^x x\, dx$

$x \tan\theta = \mu_0 \frac{x^2}{2}$, $x = \frac{2\tan\theta}{\mu_0}$

8. A = area under the curve = $m\int_0^v v\frac{dv}{dx}dx = \frac{mv^2}{2}$

$\frac{100\times 11}{2} = \frac{mv^2}{2} = mgy_{max}$ ∴ $y_{max} = 11$ m

9. Potential energy depends upon positions of particles

10. $\frac{1}{2} K_2 x^2 + \frac{1}{2} K_1 x^2 = \frac{1}{2} m v^2$

$v = \sqrt{\frac{K_1+K_2}{m}}x$

11. $\mu mg = Kx$, $U = \frac{1}{2} Kx^2 = \frac{(\mu mg)^2}{2K}$

12. For m, $N\cos\theta = mg$

For M , $N \sin\theta = kx$

So $\tan\theta = \frac{Kx}{mg}$ So, $\frac{1}{2} Kx^2 = \frac{(mg\tan\theta)^2}{2K}$

13. $T = Kx$, $U = \frac{1}{2} Kx^2 = \frac{1}{2} K\left(\frac{T}{K}\right)^2 = \frac{T^2}{2K}$

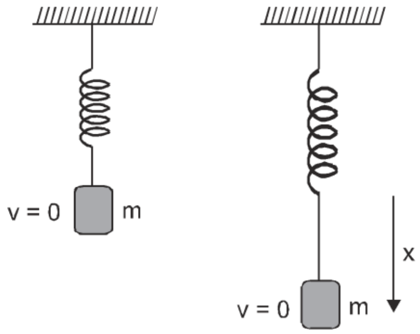
14. $mg\left(h + \frac{3mg}{K}\right) = \frac{1}{2} K\left(\frac{3mg}{K}\right)^2$

15. $\frac{1}{2}(2m)u^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$ .... (i)

$\frac{1}{2}(2m)(u+1)^2 \frac{1}{2} = mv^2$ ....(ii) From (i) and (ii) $u = \frac{1}{\sqrt{2}-1}$

16. F – R = ma, F = R + ma, P = Fv = (R + ma)v

17. Let x be the maximum extension of the spring. From conservation of mechanical energy :
decrease in gravitational potential energy = increase in elastic potential energy



∴ Mgx = $\frac{1}{2}kx^2$ or x = $\frac{2Mg}{k}$

18. From F = $-\frac{dU}{dx}$

$\int_0^{U(x)} dU = -\int_0^x F dx = -\int_0^x (kx)\, dx$ ∴ U(x) = $-\frac{kx^2}{2}$ as U(0) = 0

Therefore, the correct option is (A).

19. In horizontal plane Kinetic Energy of the block is completely converted into heat due to Friction but in the case of inclined plane some part of this Kinetic Energy is also convert into gravitational Potential Energy. So decrease in the mechanical energy in second situation is smaller than that in the first situation. So statement-1 is correct.
Cofficient of Friction does not depends on normal reaction, In II case normal reaction changes with inclination but not cofficient of friction so this statement is wrong.

20. $\int F dt = \Delta p$ ⇒ $\frac{1}{2} \times 4 \times 3 - \frac{1}{2} \times 1.5 \times 2 = p_f - 0$ ⇒ $p_f = 6 - 1.5 = \frac{9}{2}$

K.E. = $\frac{p^2}{2m} = \frac{81}{4 \times 2 \times 2}$; K.E. = 5.06 J **Ans.**

21. (4) Net force on body $= \sqrt{4^2 + 3^2} = 5N$ ∴ $a = F/m = 5/10 = 1/2 m/s^2$

Kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2}m(at)^2 = 125\,\text{Joule}$

23. Gravitational force is conservative. So, $W_1 = W_2 = W_3$

24. $W_R + W_G = 0$, – Rd + mg (h + d) = 0

R = mg (1 + $\frac{h}{d}$)

EXERCISE # 3

PART - I

1. The energy lost due to air friction is equal to difference of initial kinetic energy and final potential energy. Initially body posses only kinetic energy and after attaining a height the kinetic energy is zero
Therefore, loss of energy = kE – PE

= $\frac{1}{2}mv^2 - mgh$ = $\frac{1}{2} \times 1 \times 400 - 1 \times 18 \times 10 = 200 - 180 = 20$ J

2. Let m is mass per unit length then rate of mass per sec = $\frac{mx}{t}$ = mv

Rate of KE = $\frac{1}{2}(mv)v^2 = \frac{1}{2}mv^3$

3. Use the law of conservation of energy Let x be the extension in the spring

Applying conservation of energy mgx – $\frac{1}{2}kx^2$ = 0–0 ⇒ x = $\frac{2mg}{k}$

4. Here, mass per unit length of water, μ = 100 kg/m

Velocity of water, v = 2m/s

Power of engine, P = $\frac{1}{2}mv^2 = \frac{1}{2}\times 200\times 2$ = 400W

5. Power delivered in time T is

P = F.V. = MaV or P = MV $\frac{dV}{dT}$ ⇒ PdT = MVdV ⇒ PT = $\frac{MV^2}{2}$ or P = $\frac{1}{2}\frac{MV^2}{T}$

7. for equilibrium

$\frac{dU}{dr} = 0$ ⇒ $\frac{-2A}{r^3} + \frac{B}{r^2} = 0$ ⇒ r = $\frac{2A}{B}$ for stable equilibrium

$\frac{d^2U}{dr^2}$ should be positive for the value of r. here $\frac{d^2U}{dr^2} = \frac{6A}{r^4} - \frac{2B}{r^3}$ is +ve value for

$r = \frac{2A}{B}$ So Ans. (2)

8. Constant power of car P_0 = F.V. = ma.v

P_0 = $m\frac{dv}{dt}.v$

P_0 dt = mvdv

P_0.t = $\frac{mv^2}{2}$

v = $\sqrt{\frac{2P_0t}{m}}$

$v \propto \sqrt{t}$

9. a = $\frac{0.1x}{10}$ = 0.01x = $V\frac{dV}{dx}$ So, $\int_{v_1}^{v_2} vdV = \int_{20}^{30}\frac{x}{100}dx$

$-\frac{V^2}{2}\Big|_{v_1}^{v_2} = \frac{x^2}{200}\Big|_{20}^{30} = \frac{30\times 30}{200} - \frac{20\times 20}{200}$ = 4.5 – 2 = 2.5

$\frac{1}{2}m\left(V_2^2 - V_1^2\right)$ = 10 × 2.5 J = – 25 J

$\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2 - 25$ = $\frac{1}{2}\times 10\times 10\times 10 - 25$ = 500 – 25J = 475 J

10. If extension is same W = $\frac{1}{2}$ K x_2 so $W_P > W_Q$

If spring force is same W = $\frac{F^2}{2K}$ so $W_Q > W_P$

11. $\frac{dw}{dt} = P$

w = Pt = $\frac{1}{2}mV^2$

so. $\sqrt{\frac{2Pt}{m}} = V$

Hence $a = \frac{dV}{dt} = \sqrt{\frac{2P}{m}}.\frac{1}{2\sqrt{t}}$

so from = $ma = \sqrt{\frac{2Pm^2}{m}}.\frac{1}{2\sqrt{t}} = \sqrt{\frac{Pm}{2t}}$

12. $\vec{S} = \vec{r}_f - \vec{r}_i = \left(4\hat{j} + 3\hat{k}\right) - \left(-2\hat{i} + 5\hat{j}\right)$ = $2\hat{i} - \hat{j} + 3\hat{k}$

$\vec{F} = 4\hat{i} + 3\hat{j}$

ω = $\vec{F}.\vec{S} = (4\hat{i} + 3\hat{j}).(2\hat{i} - \hat{j} + 3\hat{k})$ = 8 – 3 = 5J **Ans.**

13. w_g = mgh = $10^{-3} \times 10 \times 10^3$ = 10 J

w_{all} = ΔKE = $\frac{1}{2} \times 10^{-3} \times 50 \times 50 = \frac{2.5}{2}$ J = 1.25 J

$w_g + w_R$

w_R = – 10 + 1.25 J J = – 8.75 J

14. Work done by variable force = $\int F.dy$

Work done = $\int_{y=0}^{y=1} F.dy = \int_0^1 (20 + 10y)dy = \left[20y + \frac{10}{2}y^2\right]_0^1$ = 20 + $\frac{10}{2}$ = 25 J

15. Area of curve from x = 0 to x = 8 = (20 × 5) + 10(3) = 130

Work energy theorem

130 = $\frac{1}{2} \times \left(\frac{1}{2}\right) \times v_1^2$. So, $v_1 = \sqrt{520}$ ≈ 23 m/sec

Now we can observe that total are from x = 8 to x = 12 is negative. So velocity at x = 12 will be less than 23 m/sec.

PART - II

1. K.E. = ct

$\frac{1}{2}mv^2$ = ct

$\frac{P^2}{2m}$ = ct

P = $\sqrt{2ctm}$

F = $\sqrt{2cm}\ \frac{1}{2} \times \frac{1}{\sqrt{t}}$

$F \propto \frac{1}{\sqrt{t}}$.

2. Work done is streching the rubber band

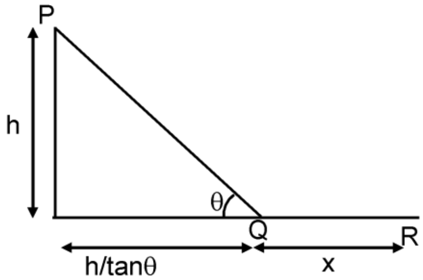
W= $\int_0^L (ax + bx^2)dx = \frac{aL^2}{2} + \frac{bL^3}{3}$

3. Let m mass of fat is used.

$m(3.8 \times 10^7)\frac{1}{5}$ = 10(9.8)(1) (1000)

m = $\frac{9.8 \times 5}{3.8 \times 10^3}$ = 12.89×10^{-3} kg

4.



Given that $\frac{\mu mgh}{\tan\theta} = mgh - \frac{\mu mgh}{\tan\theta}$

$\frac{2\mu}{\tan\theta} = 1 \Rightarrow \mu = \frac{\tan\theta}{2}$

$\mu = 0.29$

$x = \frac{h}{\tan\theta} = 2\sqrt{3} \simeq 3.5$ m

5.

$a = \frac{F}{m} = \frac{6t}{1} = 6t \quad \Rightarrow \quad \frac{dv}{dt} = 6t$

$dv = 6tdt$

$\int_0^v dv = 6\int_0^v tdt \quad \Rightarrow \quad v = 6\left[\frac{t^2}{2}\right]_0^1 = 3$

$W = \Delta KE = K_F - K_i = \frac{1}{2}(1)(3)^2 = 4.5$ J

6. $F = -Kv^2$

$m\frac{dv}{dt} = -kv^2$

$\int_{v_0}^{v} v^{-2}dv = \int_0^t -\frac{k}{m}dt$

After 10s, KE = $\frac{1}{2}mv^2 = \frac{1}{8}mv_0^2 \quad \Rightarrow \quad v = \frac{v_0}{2}$

$\left[-\frac{1}{v}\right]_{v_0}^{v_0/2} = -\frac{k}{m}t \quad \Rightarrow \quad \left(\frac{2}{v_0} - \frac{1}{v_0}\right) = \frac{k}{m}t \quad \Rightarrow \quad k = \frac{m}{v_0 t} = \frac{10^{-2}}{10\times 10} \quad \Rightarrow \quad k = 10^{-4}\ \text{kg m}^{-1}$

7. $U = -\frac{K}{2r^2}$

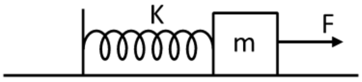
$\Rightarrow \quad F = -\frac{du}{dr} = -\left(-\frac{K}{2}\left(-\frac{2}{r^3}\right)\right) = -\frac{K}{r^3}$

$\frac{K}{r^3} = \frac{mv^2}{r}$

$\Rightarrow \quad mv^2 = \frac{K}{r^2}$

K.E. = $\frac{1}{2}mv^2 = \frac{K}{2r^2}$

$\Rightarrow \quad E = P.E. + K.E. = 0$

8. 

When $V_{max.}$ ⇒ $acc^n = 0$ ⇒ $x = \frac{F}{K}$

Apply work energy theorem

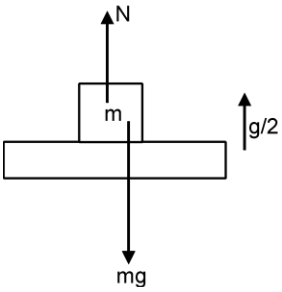
$W_{sp} = W_F = \Delta K.E.$

$-\frac{1}{2}Kx^2 + F.x = \Delta K.E.$ ⇒ $\frac{F^2}{K} - \frac{1}{2}K\frac{F^2}{K^2} = \frac{1}{2}mu^2_{max.}$

$\frac{F^2}{2K} = \frac{1}{2}mu^2_{max.}$ ⇒ $\frac{F}{\sqrt{mK}} = V_{max.}$

9. $V = \frac{ds}{dt} = 6t$ Initial kinetic energy = $\frac{1}{2}\times 2\times(0)^2$ Final kinetic energy = $\frac{1}{2}\times 2\times(30)^2$

$W = k_f - k_i = 900$ J

10. 

$N - mg = \frac{mg}{2}$ ⇒ $N = \frac{3mg}{2}$

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\frac{g}{2}(t)^2 = \frac{g}{4}$ ⇒ $W_N = \frac{3mg}{2}\times\frac{gt^2}{4} = \frac{3mg^2t^2}{8} = \frac{3mg^2t^2}{8}$

11. Work done by force = $\vec{F}.\vec{S} = (3\hat{i} - 12\hat{j})\bullet(4\hat{i})$ = 12 J

By work energy theorem, $W_{all} = \Delta K$

$12\ J = K_f - K_i$ ⇒ $12 + K_i = K_f$ ∴ $K_f = 12 + 3 = 15$ J