Exercise-1

Marked Questions can be used as Revision Questions.

PART - I : OBJECTIVE QUESTIONS

Section (A) : Definition of Capacitance

A-1. **Sol.** **(1)** Qt = Q1 + Q2 = 150μC

$\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{1}{2}$ ⇒ Q1′ = 50μC
Q2′ = 100μC

25μC charge will flow from smaller to bigger sphere .

A-2. **Sol.** **(2)** Isolated capacitor ⇒ Q = constant
sepration d increase ⇒ C = decrease
Q = CV ⇒ V = increase

A-7. **Sol.** W = $\frac{CV^2}{2}$

V2 = $\frac{2U}{C} = \frac{2\times0.16}{2\times10^{-6}}$

V = 400 Volt

Section (B) : Circuits with capacitor and use of KCl and KVL

B-1. **Sol.** W = Vf – Vi = $\frac{1}{2}CV_f^2 - \frac{1}{2}CV_1^2$ = $\frac{1}{2}$ C (402 – 202) W = 600 C

W1 = $\frac{1}{2}$ C (502 – 402) = $\frac{900}{2}$ C W1 = $\frac{900}{2}.\frac{W}{600} = \frac{3}{4}$ W **Ans**

B-2. **Sol.** As battery is disconnected, charge remains constant in the work process.
Work done = final potential energy – initial potential energy

$= \frac{Q^2}{2C'} - \frac{Q^2}{2C^o}$

$= \frac{Q^2}{2}\left\{\frac{1}{C'} - \frac{1}{C^o}\right\}$

Where, Q = CV = $\frac{A\varepsilon_o V}{d}$ C = $\frac{A\varepsilon_o}{d}$ C′ = $\frac{A\varepsilon_o}{2d}$

Now, work done = $\frac{\varepsilon_o AV^2}{2d}$ Ans. is (4)

B-3. **Sol.** V = $\frac{V_1C_1 + V_2C_2}{C_1 + C_2}$

400 = $\frac{100\times10+0}{C_1+C_2}$

C1 + C2 = 25 μF
C2 = 25 – 10 = 15 μF

B-4. **Sol.** C = $\frac{\varepsilon_0 A}{d}$ C1 = $\frac{\varepsilon_0(2A)}{d/2}$ = 4 $\left(\frac{\varepsilon_0 A}{d}\right)$ = 4C = 48 μF

B-5. **Sol.** V = $\frac{C_1V_1 + C_2V_2}{C_1+C_2} = \frac{(900+2500)\mu F\,volt}{(3+5)\mu F}$

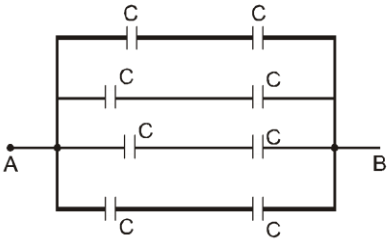
$= \dfrac{3400\text{Volt}}{8}$

= 455 Volt

Section (C) : Combination of capacitors

C-1.

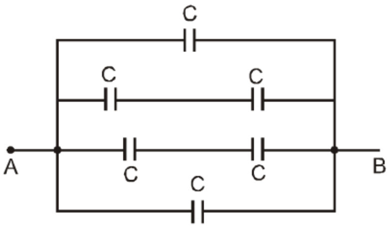
Sol.



Ceq = $\dfrac{4C}{2}$ = 2C.

C-2.

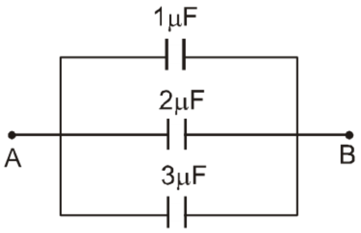
Sol.



Ceq = C + $\dfrac{2C}{2}$ + C = 3C.

C-3.

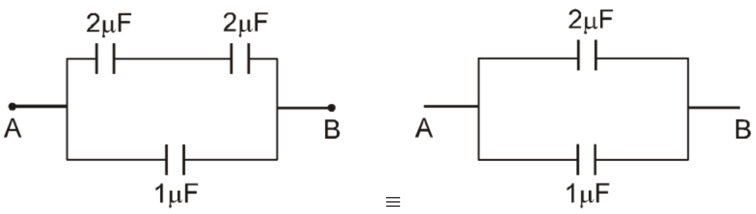
Sol. Equilent figure



Ceq = C1 + C2 + C3 = 1 + 2+ 3 = 6μF

C-4.

Sol.



CAB = Ceq = 2μF

Section (D) : Equation of charging and discharging

D-1. **Sol.** (i) **(2)** at t0 ; q = q0 = 60 μC

(ii) **(3)** q = q0e–t/RC = 60 × 10–6 $e^{-100\times10^{-6}/10\times10^{-6}\times10}$ = $\dfrac{60}{e}$ μC = 22 μC.

(iii) **(1)** $q = q_0 e^{-t/RC} = 60 \times 10^{-6}\, e^{-1\times10^{-3}/10\times10^{-6}\times10} = \frac{60}{e^{10}}\ \mu C = 0.003\ \mu C.$

D-2.

Sol. **(3)** $q = \frac{q_1}{2} = \frac{4\times10^{-6}\times6}{2}\left(1-e^{-\frac{0.2\times10^{-3}}{4\times10^{-6}\times25}}\right)$

$q = 12(1 - e^{-2})\mu C$

$= 12\,\frac{(e^2-1)}{e^2}\ \mu C = 10.37\ \mu C.$

D-4.

Sol. **(3)** $t_1 > t_2$

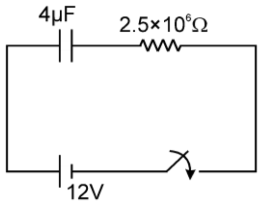
$R_1C_1 > R_2C_2$ for same q_{max}

$q_{01} = q_{01} = E_1C_1 = E_2C_2$

If $R_1 > R_2$, $C_1 = C_2$ & $E_1 = E_2$.

D-5. **Sol.** **(B)** $v_c + v_R = 12$

$v_c + \frac{v_c}{3} = 12 \Rightarrow v_c = 9$ volt



$v_c = 2v_R$

$v_c = \frac{q}{C} = \frac{CE(1-e^{-t/RC})}{C}$

$= E\,(1 - e^{-t/RC})$

$9 = 12\,(1 - e^{-t/4\times2.5})$

$9 = 12 - 12e^{-t/10}$

$3 = 12e^{-t/10}$; $e^{-t/10} = \frac{1}{4}$

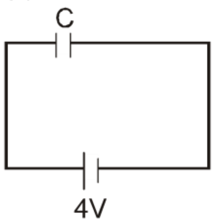
$\frac{t}{10} = \ell n4 = 2\ell n2 = 2 \times .693$

$t = 2 \times 6.93 = 13.86$

Section (E) : Capacitor with dielectric

E-1. **Sol.** $C' = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C.$

E-2. **Sol.**



Here, Potential difference on the capacitor will depend on emf of battery i.e., 4V (3)

E-3. **Sol**. Charge or battery = Q = CV = 4 C

Now charge remains same, as battery is disconnected new capacitance = C´ = KC = 8C

$C'V' = Q \qquad V' = \frac{Q}{C} = \frac{4C}{8C} = \frac{1}{2}V \qquad (1)$

E-4. **Sol.** $U0 = \frac{1}{2}CV^2$ Now energy $= U' = \frac{1}{2}C'V^2$

$C' = CK$

$U' = \frac{1}{2}CV^2K = UoK$ **Ans. is (1)**

E-5. **Sol.** Now, charge remains same on the plates.

$U0 = \frac{Q^2}{2C}$ (given)

Now energy $= U' = \frac{Q^2}{2C'} = \frac{Q^2}{2CK} = \frac{U_o}{K}$ (3)

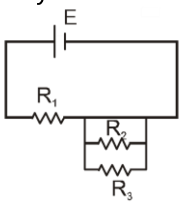
Exercise-2

Marked Questions may have for Revision Questions.

PART - I : OBJECTIVE QUESTIONS

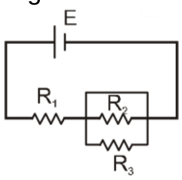
1.

Sol. Immediately after the key is closed, capacitor be have like a conduetory wire, therefore.



$i = \frac{E}{R_1}$ **Ans**

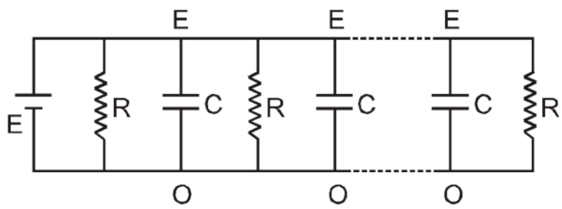
After a long time interval, capacitor be have like a open circuit. Therefore.



$i = \frac{E}{R_1 + R_3}$ **Ans**

2._

Sol.



Qfirst = Qlost = CE

Ratio $= \frac{Q_{first}}{Q_{lost}} = 1.$

3. **Sol.** Csmall $= 4\pi\varepsilon 0r$

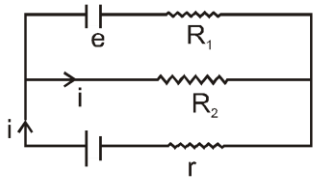
CBig $= 4\pi\varepsilon 0R$

$R = n^{1/3}\, r = 2r$

CBig = 2 Csmall

4.

Sol. Charge on capacitor = CV = capacitance × (voltage across it)
In steady state, there will be no current through capacitor.



voltage across C = iR2 = $\frac{ER_2}{R_2 + r}$

Charge on capacitor = CiR2 = $\frac{CER_2}{R_2 + r}$

5.

Sol. Q = Ceq × V

Q = $\frac{6}{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}} = 6\mu F$

V2 = $\frac{Q}{C_2} = \frac{6\mu C}{3\mu F}$ 2 Volt

7. **Sol.** (i) i0 = $\frac{V}{R}$ = $\frac{6}{24}$ = 0.25 A

(ii) i = i0 $e^{-t/RC}$

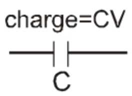
=0.25 e^{-1} = $\frac{0.25}{e}$ = 0.09 A.

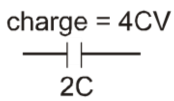
9.

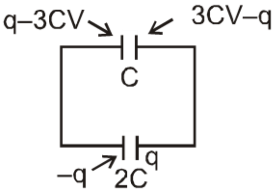
Sol. 4πε0 R = $\frac{\varepsilon_0 A}{d}$

d = $\frac{\varepsilon_0 A}{4\pi\varepsilon_0 R}$ = $\frac{\varepsilon_0 \pi R^2}{4\pi\varepsilon_0 R}$ = $\frac{R}{4}$

10. **Sol.**

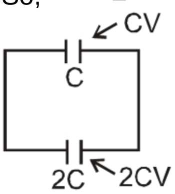






Total charge = 4 CV – CV = 3 CV
Now, let it is distributed as shown, potential across the capacitors is same

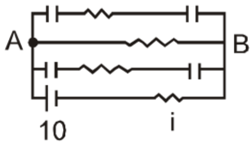
So, $\frac{q}{2C} = \frac{3CV - q}{C}$ ⇒ q = 2 CV



Total potential energy $= \dfrac{Q_1^2}{2C_1} + \dfrac{Q_2^2}{2C_2} = \dfrac{C^2V^2}{2C} + \dfrac{4C^2V^2}{2\times 2C} = \dfrac{3CV^2}{2}$

11.

Sol. Charge on each capacitor will be same. In steady state current through capacitor will be zero



current in steady state $= i = \dfrac{10}{5} = 2$ amp

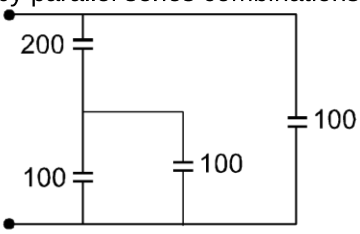
potential across AB = 4 V

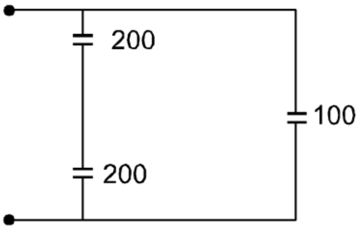
on each plate Q = C V = 3 × 4 = 12 μC

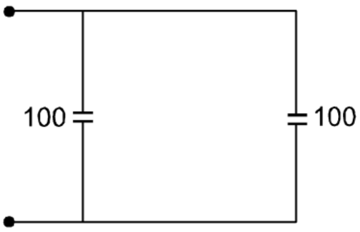
12.

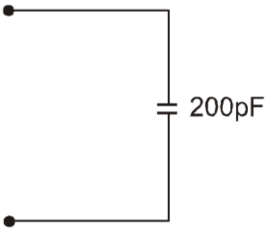
Sol. ⇒

solving by parallel series combinations,

⇒ 

⇒ 

⇒ 

⇒ 

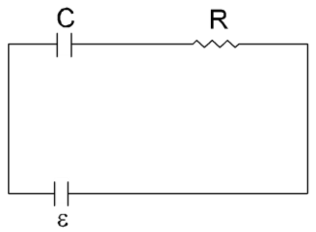
Ceq = 200 pF Ans.

13.
Sol. If S1 is closed and S2 is open then, condenser C is fully charged at potential V.

14.
Sol. **(A)** $V_{C_2} = V_{C_2} = V$

C1 = C
C2 = KC
q1 = $C_1 V_{C_1}$ = CV
q2 = C2 V_{C_2} = KCV
q1 < q2 .

15. **Sol.**



energy stored in capacitor = $\frac{Q^2}{2C}$

Rate at which energy is stored = $\frac{d}{dt}\frac{Q^2}{2C} = \frac{Q}{C} . \frac{dQ}{dt} = \frac{Qi}{C}$

Q = εC {1 – $e^{-t/RC}$ }

i = $\frac{\varepsilon e^{-t/RC}}{R}$

Rate of energy storage = $\frac{\varepsilon^2}{R}$ {1 –$e^{-t/RC}$} {$e^{-t/RC}$} = $\frac{\varepsilon^2}{R}$ {$e^{-t/RC}$ – $e^{-2t/RC}$} .... (1)

It will be maximum when, $e^{-t/RC}$ –e^{-2tRC} will be maximum let y (t) = $e^{-t/RC}$ – $e^{-2t/RC}$

for maximum , y´ (t) = 0

$$y'(t) = \frac{-e^{-t/RC}}{RC} + \frac{2e^{-2t/RC}}{RC}$$

$$e^{-t/RC} = \frac{1}{2}$$

putting it back in eq. (1)

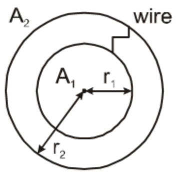
(i) maximum rate of energy storage $= \frac{\varepsilon^2}{R}\left\{\frac{1}{2} - \left(\frac{1}{2}\right)^2\right\} = \frac{\varepsilon^2}{4R}$ **Ans. is (1)**

(ii) This will occur when, $e^{-t/RC} = \frac{1}{2}$

$$\frac{-t}{RC} = \ln\frac{1}{2}$$

t = RC ln2 **Ans. is (3)**

16.
Sol.



All given charge of A_1 goes to A_2

Therefore $C = 4\pi\epsilon_0 r_2$

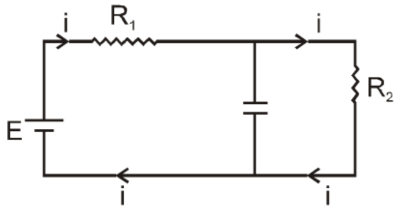
17. **Sol.** x = Vt, $\Rightarrow$ d $\propto$ t C = $\frac{\epsilon_0 A}{Vt}$ $\frac{dc}{dt} = -\frac{\epsilon_0 A}{V}\frac{1}{t^2}$ $\frac{dc}{dt} \propto \frac{1}{d^2}$ **Ans**

18.

Sol. Electric field in the capacitor is same at every where which is equal to V/d. so that force at C and B point is same.
Electric field out side the capacitor is zero so that force at A point is zero.

19.

Sol. A long time after closing the switch, system comes in steady state and no current flow through capacitor.
Circuit : -



i = $\frac{E}{R_1 + R_2}$

energy stored in battery = $\frac{1}{2}$ CV^2 = $\frac{1}{2}C\left(\frac{ER_2}{R_1+R_2}\right)^2$

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

1. **Sol.** If potential difference across an isolated charged capacitor is doubled by doubling separation between plates, the energy stored is capacitor from U = $\frac{Q^2}{2C}$ becomes double of previous value. Hence statement 1 is false.

2. **Sol.** Let the electric field in region I and II be E1 and E2. The potential difference across left half capacitor and right half capacitor is same. Therefore E1d = E2d where d = inter planar gap.
$\therefore$ E1 = E2
Hence statement 1 is false, statement 2 is correct by definition.

3. **Sol.** (1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

Section (B) : Match the column

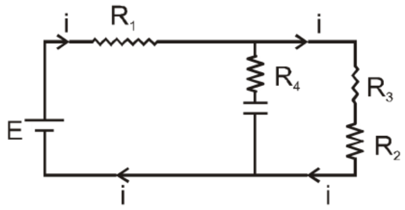
1.#

Ans. (1) p (2) r (3) q (4) p

Sol. The initial charge on capacitor = CV_i = 2 × 1 μC = 2 μC
The final charge on capacitor = CV_f = 4 × 1 μC = 4 μC
$\therefore$ Net charge crossing the cell of emf 4V is
$q_f - q_i$ = 4 – 2 = 2 μC
The magnitude of work done by cell of emf 4V is
W = $(q_f - q_i)$ 4 = 8 μJ
The gain in potential energy of capacitor is
$\Delta U = \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}\, 1 \times [4^2 - 2^2]$ μJ = 6 μJ
Net heat produced in circuit is
ΔH = W – ΔU = 8 – 6 = 2 μJ

Section (C) : One or More Than One Options Correct

1.
Sol. A long time after closing the switch, system comes in steady state and no current flow through capacitor.
Circuit : -



i = $\dfrac{E}{R_1 + R_2 + R_3}$

energy stored in battery = $\dfrac{1}{2}CV^2 = \dfrac{1}{2}C\left(\dfrac{E(R_3+R_2)}{R_1+R_2+R_3}\right)^2$

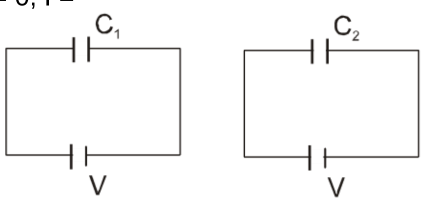
2*. **Sol.** Magnitude of charge on the charged capacitor decreases and total charge is conserved.
At $V_1 = V_2 \Rightarrow$ no further flow of charge occurs i.e. condition of steady state.
In charge flow energy is consumed in heat.

3.
Ans. **(A,C)**
Sol. $q_{max} = q_{01} = q_{02}$ = Both capacitors are charged up to the same magnitude of charge
$t_2 > t_1$
$R_2C_2 > R_1C_1$
$q_{01} = C_1V_1 = q_{02} = C_2V_2$
$C_1 \neq C_2$
So blfy, $V_1 \neq V_2$.

4.* **Sol.** During decay of charge in RC circuit
$I = I_0e^{-t/RC}$

where $I_0 = \dfrac{q_0}{RC}$

when t = 0, I = $I_0 = \dfrac{q_0}{RC}$



Since potential difference between the plates is same initially therefore I same in both the cases at t = 0 and is equal to

$$I = \frac{q_0}{RC} = \frac{V}{R}$$

Also $q = q_0e^{-t/RC}$. When $q = \dfrac{q_0}{2}$ then $\dfrac{q_0}{2} = q_0\, e^{-t/RC}$
$\Rightarrow$ $e^{+t/RC} = 2$.

$\dfrac{t}{RC} = \ell n2$

$\Rightarrow$ $t = RC \log_e 2$
$\Rightarrow$ $t \propto C$. Therefore time taken for the first capacitor (1μF) for discharging 50% of Initial charge will be less.
(B), (D) are the correct options.

5*. **Ans.** **(A,B,C)**

Sol. $E = \frac{V}{d}$ ⇒ remains constant

$C' = KC$ ⇒ Increase

$Q' = KQ$ ⇒ Increase

$U = \frac{1}{2}KCV^2 = KU$ ⇒ Increase

6*. **Ans.** **(A,C,D)**

Sol. Battery connected V = constant

$U' = \frac{1}{2}KCV^2 = KU$ ⇒ Increase by K–times

$E = \frac{V}{d}$ = constant

$F = \frac{Q^2}{2\epsilon_0 A}$ ⇒ $F = \frac{C^2V^2}{2\epsilon_0 A}$ ⇒ $F' = \frac{K^2C^2V^2}{2\epsilon_0 A} = K^2F$

⇒ Increase by K^2–times

$Q = CV$ ⇒ $Q' = KCV = KQ$ ⇒ Increase by K–times.

Exercise-3

✍ **Marked Questions may have for Revision Questions.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. **Sol.** Energy stored by any system of capacitors is

$= \frac{1}{2}$ Cnet V^2

where V is source voltage

Thus, n capacitors are connected in parallel,

Therefore, Cnet = nC

$\therefore$ Enet = $\frac{1}{2} nCV^2$

2.

Sol. Capacitance of spherical conductor

$= 4\pi\epsilon_0 a$

where a is radius of conductor

Therefore , C = $\frac{1}{9\times10^9} = \frac{1}{9}\times10^{-9}$ = 0.11×10^{-9} F = 1.1×10^{-10} F **Ans.**

3. **Sol.** $W = \frac{1}{2} CV^2 = \frac{1}{2}\frac{q^2}{C}$

$= \frac{1}{2}\times\frac{(8\times10^{-18})^2}{100\times10^{-6}} = \frac{1}{2}\times\frac{64\times10^{-36}}{100\times10^{-6}}$

$= 32 \times 10^{-32}$ J

4. **Sol.** $E = (1/2)\, CV^2$ .... (i)

The energy stored in capacitor is lost in form of heat energy.

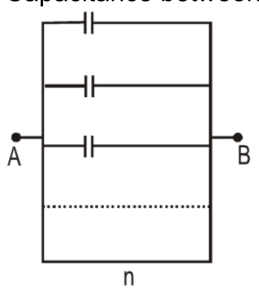
$H = ms\,\Delta T$ .... (ii)

From Eq. (i) and (ii), we have

$ms\Delta T = (1/2)\, CV^2$

$V = \sqrt{\frac{2ms\Delta T}{C}}$

5. **Sol.** Capacitance between any two adjacent plates is ‘C’



Capacitor consists of n plates, then there are (n -1) combinations joined in parallel.

$\therefore$ $C_{AB} = (n - 1)C$

6.✍ **Sol.** Work done by battery = (CV) V = CV^2

Energy stored in capacitor = $\frac{1}{2} CV^2$

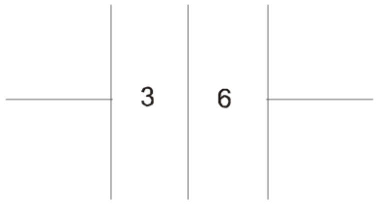
$\dfrac{\text{Energy stored in capacitor}}{\text{Work done by battery}} = \dfrac{\frac{1}{2}CV^2}{CV^2} = \dfrac{1}{2}$

Correct choice is (4)

7. **Sol.** No change in the energy of the system. Hence, not net work done by the system is zero.
Correct choice is (4)

8. **Sol.** $C' = \dfrac{A\in_0}{\frac{d_1}{3}+\frac{d_2}{6}} = \dfrac{A\in_0}{\frac{d}{9}+\frac{2d}{18}} = \dfrac{18A\in_0}{4d}$

C = 9 PF



C´ = 40.5 PF
Correct choice is (3).

9. **Ans. (3)**

Sol. $U_0 = \dfrac{q_0^2}{2C}$ $U = \dfrac{q_0^2 e^{-2t_1/\tau}}{2C} = \dfrac{U_0}{2} = \dfrac{q_0^2}{4C}$ $\Rightarrow$ $e^{-2t_1/\tau} = \dfrac{1}{2}$

$t_1 = \dfrac{\tau}{2}\,\ell n2$ ....(1)

and o $q = q_0 e^{-t_2/\tau}$

$\dfrac{q_0}{4} = q_0 e^{-t_2/\tau}$,

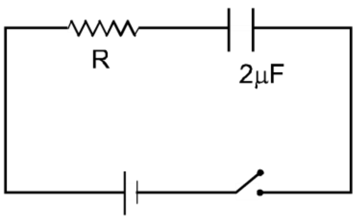
$e^{-t_2/\tau} = \dfrac{1}{4}$

$t_2 = 2\tau \ln 2$ ....(2)

$\dfrac{t_1}{t_2} = \dfrac{1}{4}$

10. **Ans. (3)**

Sol.



$v = 200(1 - e^{-t/t})$

$120 = 200(1 - e^{-t/\tau})$

$e^{-t/\tau} = \dfrac{200-120}{200} = \dfrac{80}{200}$

$t/\tau = \log(2.5) = 0.4$

$5 = (0.4) \times R \times 2 \times 10^{-6}$

$\Rightarrow \quad R = \dfrac{5}{(0.4)\times 2\times 10^{-6}} = R = 2.7 \times 10^{6}$ **Ans.**

11. Sol. Time constant for parallel combination = 2RC

Time constant for series combination = $\dfrac{RC}{2}$

In first case :

$$V = V_0 e^{-\frac{t_1}{2RC}} = \frac{V_0}{2} \qquad \text{........(i)}$$

In second case :

$$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2} \qquad \text{........(ii)}$$

From (i) & (ii), $\dfrac{t_1}{2RC} = \dfrac{t_2}{(RC/2)} \quad \Rightarrow \quad t2 = \dfrac{t_1}{4} = \dfrac{10}{4} = 2.5$ sec.

12.

Ans. (4)

Sol. $Q = c\varepsilon 0\ e^{-t/cR}$

$4\varepsilon = 4\varepsilon 0\ \varepsilon^{-t/\tau}$

$\varepsilon = \varepsilon 0\ \varepsilon^{-t/\tau}$

When t = 0 $\Rightarrow$ $\varepsilon 0 = 25$

$\varepsilon = \varepsilon 0 = 25$

when tc t = 200 $\Rightarrow$ $\varepsilon = 5$

$5 = 25\, e^{-\frac{200}{\tau}}$

$\ln 5 = \dfrac{200}{\tau}$

$\tau = \dfrac{200}{\ell n5} = \dfrac{200}{\ell n10 - \ell n2}$

$= \dfrac{200}{\ell n10 - 0.693}$

Alternative :

Time constant is the time in which 63% discharging is completed.

So remaining charge = 0.37 × 25 = 9.25 V

Which time in 100 < t < 150 sec.

Alternative :

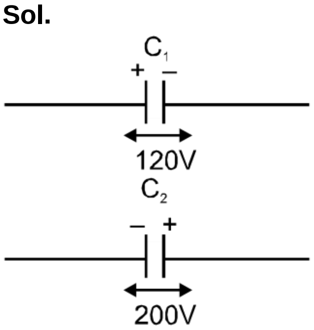
13. Sol. $q = CV\ (1 - e^{t/\tau})$

at ij $t = 2\tau$

$q = CV\ (1 - e^{-2})$

Ans (3)

14. Sol.



For potential to be made zero, after connection

120C1 = 200 C2

$\Rightarrow$ 3C1 = 5C2

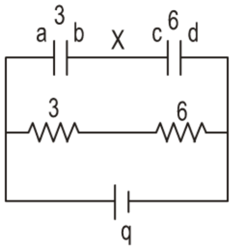
Ans. (2)

$\therefore$ qb + qc = 0

when switch is closed, the circuit is as following

In steady state, the current in the resistances is 1 amp. Potential difference across 3μF = potential difference across 3Ω = 3 volt.

Similarly p.d. across 6 μF is 6 volt



$\therefore$ charge on plate b = – 9 μC

and charge on plate c = + 36 μC

$\therefore$ charge on plates b and c = +36 – 9 = +27 μC.

The change in charges on plates b and c goes through wire from Y to X.

15. Ans. (1)

Sol. Electric field inside dielectric $\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

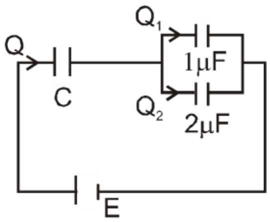
$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

$= 6 \times 10^{-7}$ C/m^2

16.

Ans. (2)

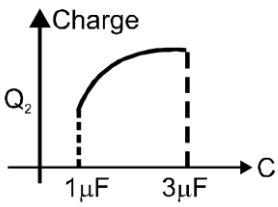
Sol.

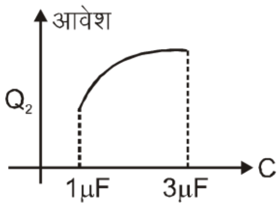


Q2 = $\frac{2}{2+1}$ Q = $\frac{2Q}{3}$

Q = E$\left(\frac{C \times 3}{C+3}\right)$

$\Rightarrow$ Q2 = $\frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$

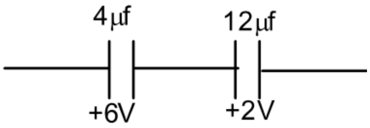




17.

Ans. **(2)**

Sol.



Q1 = 24μc

Q2 = 18mc

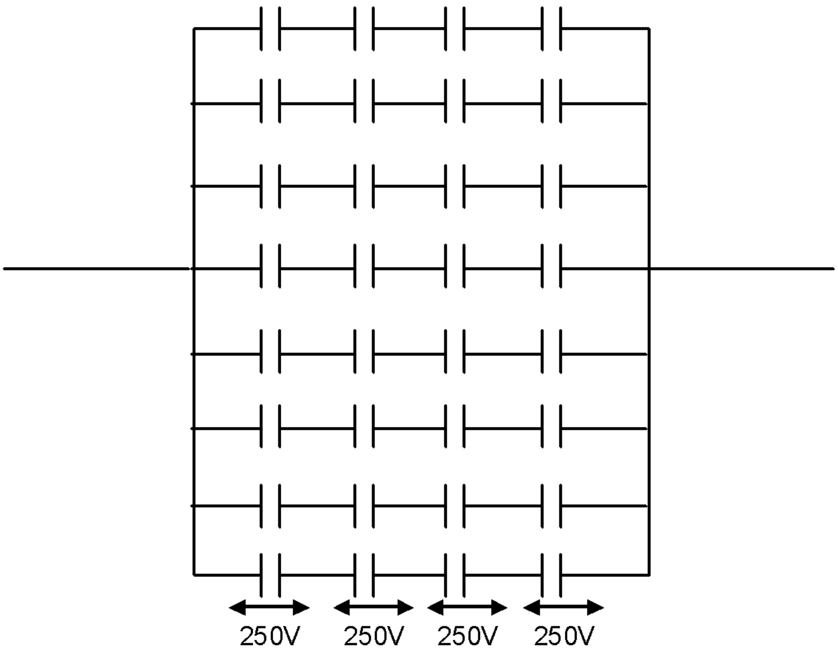
Q = 42μc

$E = 10^{7} \times 42 \times 10^{-6}$

E = 420 N/C

18. **Ans.** **(1)**

Sol.

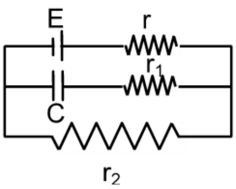


Minimum no. of capacitors required = 32

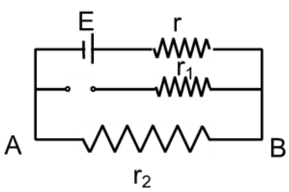
19.

Ans. **(4)**

Sol.



at steady state



current in the circuit

$I = \frac{E}{r + r_2}$

potential difference across AB = Ir_2

$= \frac{Er_2}{r + r_2}$

charge on capacitor = Q = $C(\Delta V)_{AB}$

$Q = \frac{CEr_2}{r + r_2}$

PART - II : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

*** Marked Questions may have more than one correct option.**

1. **Ans.** C

Sol. ΔU = decrease in potential energy

$= U_i - U_f$

$= \frac{1}{2}C(V_1^2 + V_2^2) - \frac{1}{2}(2C)\left(\frac{V_1 + V_2}{2}\right)^2 = \frac{1}{4}C(V_1 - V_2)^2$

2.

Sol. $\log i = \frac{-t}{RC} + \log\left(\frac{E}{R}\right)$; Compare with $y = mx + c$

$\Rightarrow$ $|m| = \frac{1}{RC}$;$c = \log\left(\frac{E}{R}\right)$

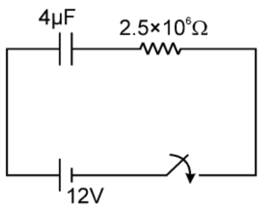
as R increases, |m| and C both decreases so (B) is the correct option.

3.

Sol. **(B)** $v_C + v_R = 12$

$v_C + \frac{v_C}{3} = 12 \Rightarrow v_C = 9$ volt

$v_C = \frac{q}{C} = \frac{CE(1 - e^{-t/RC})}{C}$



$v_c = 2v_R$

$= E(1 - e^{-t/RC})$

$9 = 12(1 - e^{-t/4\times2.5)}$

$9 = 12 - 12e^{-t/10}$

$3 = 12e^{-t/10}$; $e^{-t/10} = \frac{1}{4}$

$\frac{t}{10} = \ell n4 = 2\ell n2 = 2 \times .693$

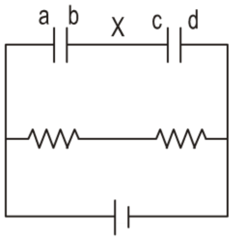
$t = 2 \times 6.93 = 13.86$

4.

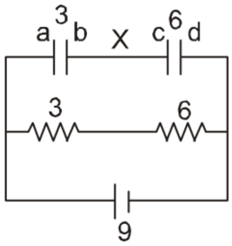
Ans. **(C)**

Sol. When switch is opened, the circuit is as following

since the capacitors are in series plates b and c will have equal and opposite charges



$\therefore$ qb + qc = 0 ; also charge on each capacitor = 18μC
when switch is closed, the circuit is as following
In steady state, the current in the resistances is 1 amp. Potential difference across 3μF = potential difference across 3Ω = 3 volt.

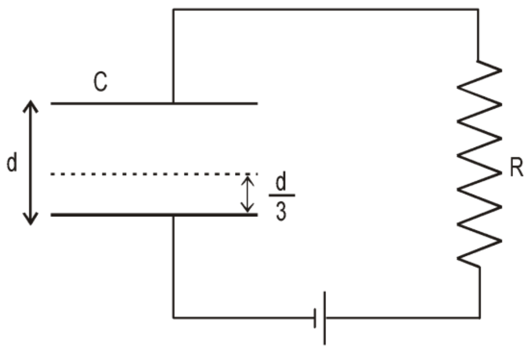


Similarly p.d. across 6 μF is 6 volt
$\therefore$ charge on plate b = – 9 μC
and charge on plate c = + 36 μC
$\therefore$ charge on plates b and c = +36 – 9 = +27 μC.
The change in charges on plates b and c goes through wire from Y to X.
and Δq = 27μC

5.

Sol. Time constant = τ = RC
Let thickness of dielectric filled is x at time t.

$$C = \frac{C_1C_2}{C_1+C_2} = \frac{\left(\frac{A\varepsilon_0}{d-x}\right)\left(\frac{KA\varepsilon_0}{x}\right)}{\frac{A\varepsilon_0}{d-x}+\frac{A\varepsilon_0}{x}}$$



$$C = \frac{KA\varepsilon_0}{x+K(d-x)} \qquad \therefore \qquad \left\{x = \frac{d}{3} - Vt\right\}$$

$$\tau = \frac{RKA\varepsilon_0}{\frac{d}{3} - Vt + K\left(d - \frac{d}{3} + Vt\right)}$$

A = 1 K = 2

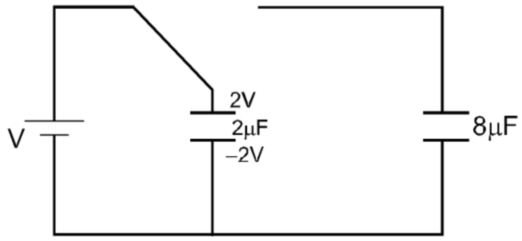
$$\tau = \frac{3\times 2R\varepsilon_0}{d-3Vt+6d-2d+6Vt} = \frac{6R\varepsilon_0}{5d+3Vt}$$ **Ans. (A)**

6.

Capacitance

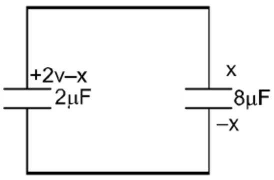
Ans. (D)

Sol. Ui = $\frac{1}{2}(2)V^2$, Vcommon = $\frac{V}{5}$



Uf = $\frac{1}{2}$ (2 + 8) $\left(\frac{V}{5}\right)^2$

$\frac{U_i - U_f}{U_i} \times 100$ = $\frac{V^2 - \frac{V^2}{5}}{V^2} \times 100$



$\frac{4}{5} \times 100$ = 80% Ans.

7.

Ans. **(C)**

Sol. q3 = $\frac{C_3}{C_2 + C_3}$ .Q

q3 = $\frac{3}{3+2}$ × 80 = $\frac{3}{5}$ × 80

= 48 μC

8.

Ans. **(A), (D)**

Sol. C = $\frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d}$

C1 = $\frac{K\varepsilon_0 A}{3d}$

$\frac{C}{C_1} = \frac{2+K}{K}$ Ans. (D)

E1 = E2 = $\frac{V}{d}$

⇒ $\frac{E_1}{E_2} = 1$ Ans. (A)

Q1 = C1V = $\frac{K\varepsilon_0 A}{3d}V$

Q2 = C2V = $\frac{2\varepsilon_0 A}{3d}V$

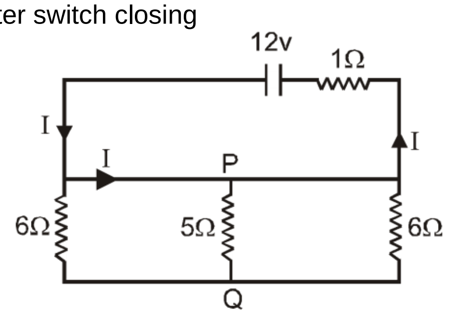
⇒ $\frac{Q_1}{Q_2} = \frac{K}{2}$

Additional Problems For Self Practice (APSP)

PART-I : PRACTICE TEST PAPER

1.

Sol. Just after switch closing

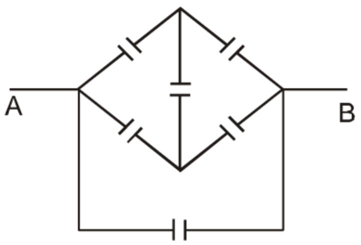


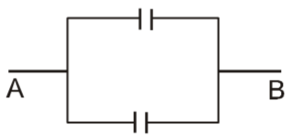
current through resistor PQ is zero just after closing the switch.

2.

Sol. Theoritical capacitance = ∞, because d become zero

3.

Sol. 

$\equiv$ 

$C_{eq} = 2C$

4.

Sol. Charge on C_0, $Q_1 = C_0V_0$,

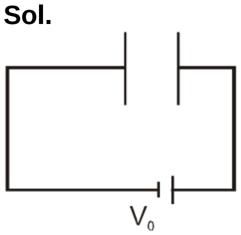
Initial charge on C_1 $Q_2 = 0$

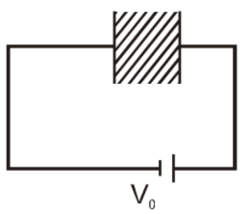
Common potential $V_1 = \dfrac{Q_1 + Q_2}{C_1 + C_2} = \dfrac{C_0V_0}{C + C_0}$ $\Rightarrow Q_1 = C_0V_1 = \dfrac{C_0^2}{C + C_0} V_0$

Similarly $V_2 = \dfrac{C_0V_1}{C + C_0} = \left(\dfrac{C_0}{C + C_0}\right)^2 V_0 \Rightarrow Q_2 = C_0V_2 = \dfrac{C_0^3}{(C + C_0)^2} V_0$

for n times n, $V_n = V_0 = \left(\dfrac{C_0}{C + C_0}\right)^n V$ $\Rightarrow$ $C = \left[\left(\dfrac{V_0}{V}\right)^{1/n} - 1\right] C_0$ **Ans**

5. **Sol.**





Potential difference = V0
Capacitance = C

Q0 = CV0

Potential difference = V0
Capacitance = KC
[K is the dielectric constant of Slab K > 1]
New charge = KC V0

Potential Energy = $\frac{1}{2}CV0_2$ New potential energy = $\frac{1}{2}KC\,V0_2$
Correct options are (4).

6.

Sol. C = 2μF

$C_{eq} = C + \frac{C}{2}+\frac{C}{4}+\frac{C}{8}+\frac{C}{16}$ + ...

$C_{eq} = C\left(\frac{1}{1-1/2}\right) = 2\left(\frac{1}{1/2}\right) = 4\mu F$

Charge on first row capacitor is $q_1 = 2 \times 10\mu C = 20\mu C$
Charge on second row capacitor is $q_2 = 1 \times 10\mu C = 10\mu C$
Charge on third row capacitor is $q_3 = \frac{1}{2} \times 10\mu C = 5\mu C$
Therefore charge on the capacitor in the first row is more than on any other capacitor.
Energy stored in all capacitor is = $\frac{1}{2}C_{eq}V^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (10)^2 = 0.2$ mJ **Ans**

C = 2μF

$C_{eq} = C + \frac{C}{2}+\frac{C}{4}+\frac{C}{8}+\frac{C}{16}$ + ...

$C_{eq} = C\left(\frac{1}{1-1/2}\right) = 2\left(\frac{1}{1/2}\right) = 4\mu F$ **Ans**

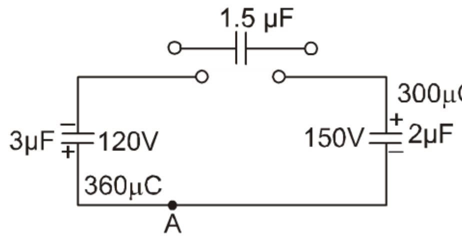
7. **Sol.** $C = \frac{\epsilon_0 A}{d}$, $C' = \frac{K\epsilon_0 A}{d}$ $Q = CV = \frac{\epsilon_0 KAV}{d}$

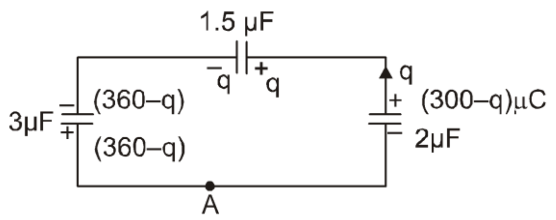
$Q = CV = C_1V_1 \Rightarrow V_1 = \frac{V}{K}$ $E = \frac{V_1}{d} = \frac{V}{Kd}$ **Ans**

$W = U_f - U_i = \frac{1}{2}CV^2 - \frac{1}{2}C_1V_1^2 = \frac{1}{2}\frac{\epsilon_0 AV^2}{d^2} - \frac{1}{2}\frac{K\epsilon_0 A}{d}\left(\frac{V}{K}\right)^2 = \frac{\epsilon_0 AV^2}{2d}\left(1-\frac{1}{K}\right)$

8.

Sol.





$VA + \frac{(300-q)}{2} - \frac{q}{1.5} + \frac{360-q}{3} = VA$ ....(i)

by solve this equation we get ⇒ q = 180μC
Charge on 1.5μF capacitor is = 150μC
Charge on 2μF capacitor is = 300 – 180 = 120μC
Therefore charge flows through A from left to right .

9.

Sol. $\frac{1}{C_{eq}} = \frac{1}{20}+\frac{1}{30}+\frac{1}{15} = \frac{3+2+4}{60}$ $C_{eq} = \frac{60}{9} = \frac{20}{3}\mu F$

Total charge in this series conbination is = $\frac{20}{3} \times 90$ $\quad$ q = 600μC

Potential difference between the plate of C_1 is = $\frac{q}{C_1}$ $\frac{600}{20}$ = 30V

Potential difference between the plate of C_2 is $\frac{q}{C_2} = \frac{600}{30}$ 20V

Potential difference between the plate of C_3 is = $\frac{q}{C_3} = \frac{600}{15}$ = 40V

10.

Sol. $I = \frac{V_0}{R} e^{-t/RC}$

$$\ell n I = \ell n \frac{V_0}{R} - \frac{t}{RC}$$

In both cases log of initial current is same that means $\frac{V_0}{R}$ is constant. To keep $\frac{V_0}{R}$ constant, both V_0 and R have to be changed whereas it is stated that only one parameter out of V_0, R and C is changed. Therefore only C has been changed and to match the straight line in the graph, it is decreased.

11. **Sol.** τ is RC i.e. time constant. After one time constant, capacitor looses 0.67 part of its initial charge.

12. **Sol.** Initially, $U = \frac{q^2}{2C} = \frac{q^2}{2KC_0}$ ....(i)

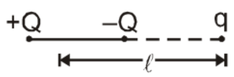
When di–electric is pulled out, then $U + 3U = \frac{q^2}{2C_0}$

$4U = \frac{q^2}{2C_0}$ ....(ii)

Then, (ii) & (i) $\Rightarrow$ K = 4.

13.

Sol. The two plates acts as a dipole



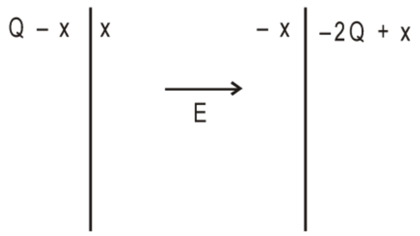
∴ Force on charge q ;

F = Eq

$= \left(\frac{2kQd}{\ell^3}\right) \cdot q = \frac{Qqd}{2\pi\varepsilon_0 \ell^3}$.

14.

Sol.



[JF 2014 -15]

Charges on outer surfaces must be equal

Q – x = – 2Q + x

$\frac{3Q}{2}$ = x

so E = $\frac{x}{2\varepsilon_0 A} + \frac{x}{2\varepsilon_0 A}$

V = Ed = $\frac{3}{2}\frac{Qd}{\varepsilon_0 A} = \frac{3Q}{2C}$

16.

Sol. $\frac{1}{C_{eff}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{\varepsilon_0 A} + \frac{d_2}{\varepsilon_0 A} = \frac{(d_1 + d_2)}{\varepsilon_0 A}$

Since (d_1 + d_2) does not change C_{eff} remains unchanged.

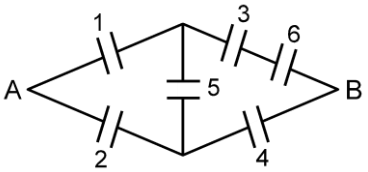
17. **Sol.** Energy = $\frac{(\text{charge})^2}{2\text{capacitance}}$

Capacitance = $\frac{(\text{charge})^2}{(2)\ \ \text{Energy}}$

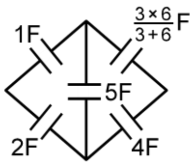
Dimensional formula = $\frac{Q^2}{m^1 L^2 T^{-2}}$ = $m^{-1} L^{-2} T^2 Q^2$

18.

Sol. **[easy]** Equivalent circuit is



⇒



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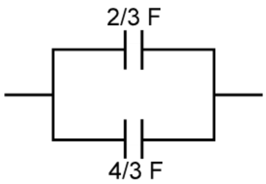
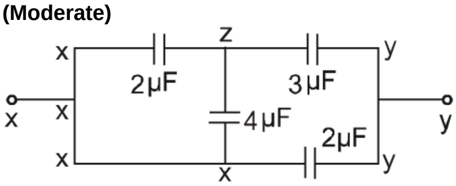


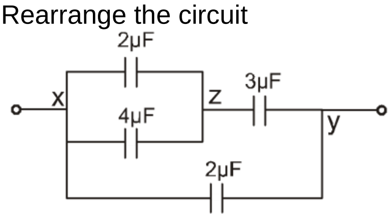
= 2 F

19.

Sol. **(Moderate)**



Rearrange the circuit



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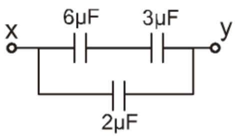

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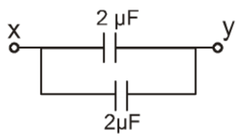
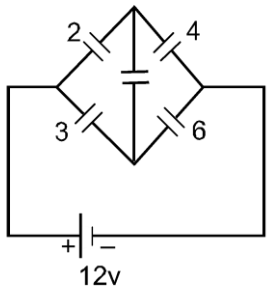


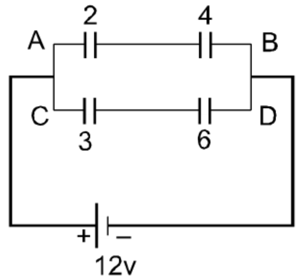
Ceq = 4μF

20.

Sol. The given circuit is :



⇒



Charge across CD $= \left|\left(\frac{3\times 6}{3+6}\right)\right| \times 12 = 24\ \mu C$

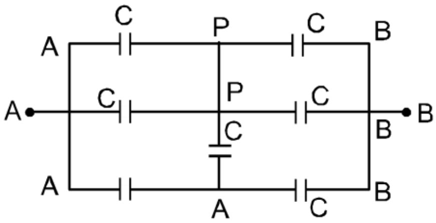
21.

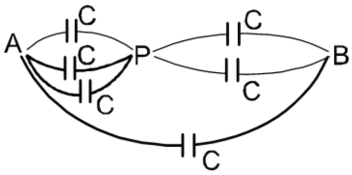
Sol. Ceff $= \frac{\varepsilon_0 A}{d}$ since effective capacitance between plates A and E is zero.

$\therefore U = \frac{1}{2} CV^2 = \frac{\varepsilon_0 A}{2d} V^2$

22.

Sol.

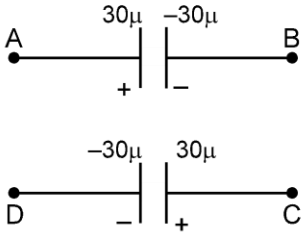


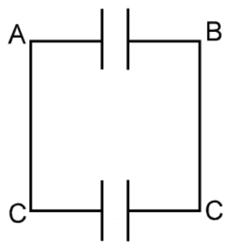


Get Ceq $= \frac{11c}{5}$

Charge flow = Ceq $\varepsilon = \frac{11\,c\varepsilon}{5}$.

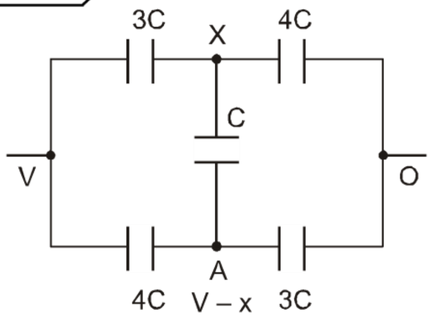
23. **Sol.** **(3)** $V = \frac{Q_1 + Q_2}{C_1 + C_2} = 0$





Final potential difference = zero
Final charge = Zero
Charge flow 30 μc from A to D

24.
Sol.



$(x - V)\,3 + (x - 0)\,4 + (x - (v - x))\,1 = 0$

$3x - 3V + 4x + 2x - V = 0$

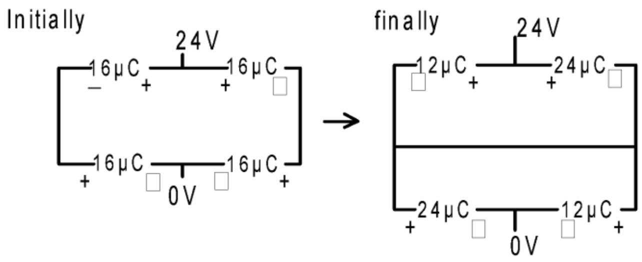
$9x = 4V$

$x = \frac{4V}{9}$

$V_A - V_B = (V - x) - x = V - 2x \;\Rightarrow\; V - \frac{8V}{9} = \frac{V}{9} = 3$ volt $\quad\Rightarrow\;$ V = 27 volt.

25.

Sol. **(1)**



$\therefore q_{\text{through switch}}$ dqath ls xqtjus okys $= 12\ \mu C$

26.

Sol. $V_i = \frac{1}{2}\left(\frac{C}{2}\right)V^2 = \frac{CV^2}{4}$ $\qquad Q_i = \frac{CV}{2}$ $\qquad Q_f = \frac{2CV}{3}$

$W_B = V(Q_f - Q_i) = \frac{CV^2}{6}$

$V_f = \frac{1}{2}\left(\frac{2C}{3}\right)V^2 = \frac{CV^2}{3}$

$V_i + \omega_B = V_f + H \Rightarrow \frac{CV^2}{4} + \frac{CV^2}{6} - \frac{CV^2}{3} = \frac{CV^2}{12}$

27.

Sol. Since current is going out of the body,

$i = -\frac{dq}{dt}$

Given fn;k gS $\frac{di}{dt} = q$

$q = \frac{-d^2q}{dt^2} \qquad \Rightarrow \qquad q + \frac{d^2q}{dt^2} = 0$

This is differential equation of SHM with

$T = \frac{2\pi}{\omega} = 2\pi$ sec

The min. time for the charge to become zero = T/4 = $\frac{\pi}{2}$ sec.

Let ekuk q = $Q\cos\omega t$

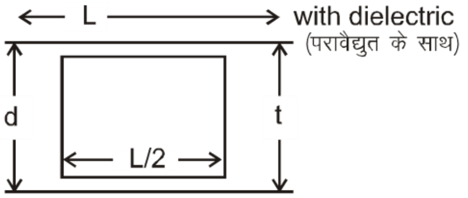
When tc q = $\frac{Q}{2}$; $\cos\omega t = \frac{1}{2}$; $\sin\omega t = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

i = $Q\omega\frac{\sqrt{3}}{2} = \frac{Q\sqrt{3}}{2}$

PART - II : PRACTICE QUESTIONS

1.

Sol. Net charge must remain same.



$\left(\frac{KC}{2}+\frac{C}{2}\right)$ V’ = CV ⇒ V’ = $\frac{2V}{K+1}$

2.

Sol. Potential in conductor is constant
Potential in dielectric decreases at slower rate.

3.

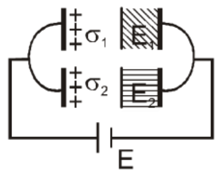
Sol. Potential difference across ‘C1’

V1 = $\frac{C_2V}{C_1+C_2}$ = $\frac{V}{1+C_1/C_2}$

When dielectric is inserted C2 will increase.

4.

Sol.



σ1 ≠ σ2
E1 ≠ E2

5.

Sol. In steady state no current flows through capacitor. The potential difference across capacitor and resistor of resistance R2 is same.

∴ charge on capacitor = CV = C × $\frac{R_2}{r+R_2}$ × 3 = 1μF × $\frac{1}{5+1}$ × 3 = 2μC.

6.

Sol. Energy$_{\text{Released}}$ in xy joint = (C.V).V – $\frac{1}{2}CV^2$ = $\frac{1}{2}CV^2$

Energy$_{\text{Released}}$ in xz joint = (2C.V).V – $\frac{1}{2}CV^2$ = $\frac{3}{2}CV^2$

Total heat$_{\text{Released}}$ = $2CV^2$ which is 4 times energy store.

8.
Sol. Let the capacitance before insertion of dielectric be C and the resistance be R.

$\therefore$ q = $q_0\, e^{-\frac{t}{RC}}$ and i = $\frac{q/C}{R} = \frac{q}{RC}$

$\because$ Just after insertion of dielectric the capacitance increases.

$\therefore$ The charge just after insertion of dielectric remains same, but the current decreases.

$\Rightarrow$ (1) and (2) are false

The energy stored in capacitor is $\frac{q^2}{2C}$, hence energy decreases

$\Rightarrow$ (3) is false

The time constant is RC and hence increases.

$\Rightarrow$ (4) is true

9.

Sol. During the charging process, charge does not jump or cross the area between the plates.

10.

Sol. U = $\frac{1}{2}C_{eq}V^2$ C1 = $\frac{k\varepsilon_0 A}{d/2} = \frac{2\varepsilon_0 A}{(d/2)}$

Ceq = $\frac{C_1C_2}{C_1+C_2}$ C2 = $\frac{\varepsilon_0 A}{d/2}$; Ceq = $\frac{\left(2\frac{\varepsilon_0 A}{d/2}\right)\times\frac{\varepsilon_0 A}{d/2}}{3\frac{\varepsilon_0 A}{d/2}} = \frac{4}{3}\frac{\varepsilon_0 A}{d}$

U = $\frac{1}{2}\left(\frac{4}{3}\frac{\varepsilon_0 A}{d}\right)V^2 = \frac{2}{3}\left(\frac{\varepsilon_0 A}{d}\right)V^2$

11.

Sol. **(2)** **Method I**

Force between plates

F = $\frac{Q^2}{2A\varepsilon_0} = \frac{\left(\frac{\varepsilon_0 A}{x}V\right)^2}{2A\varepsilon_0} = \frac{\varepsilon_0 AV^2}{2x^2}$ where x is separation between plates

dW = F dx

W = $\int_d^{2d}\frac{\varepsilon_0 AV^2}{2x^2}dx = \frac{\varepsilon_0 AV^2}{4x} = \frac{CV^2}{4}$ = 200 μJ

Method II

U_ℓ + WB + Wext = Uf + loss

Process is slow so energy loss is zero work done by battery = WB = QE

Q = Qf – Qi = 20 – 40 = – 20

WB = – 20 × 20

$\frac{1}{2}$ 2 × 20_2 – 20 × 20 + Wext = $\frac{1}{2}$ 1 × 20_2 + 0

Wext = 200 μ J

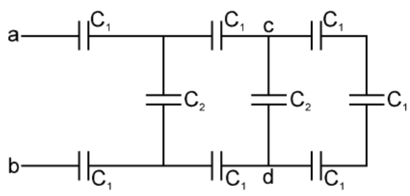
12.

Sol. Force on metal plate S due to electrostatic attraction by plate T is F =
Force exerted on plate S by spring is = mg

In equilibrium $\Rightarrow$ $\frac{Q^2}{2A\epsilon_0}$ = mg or Q = $\sqrt{2mgA\epsilon_0}$

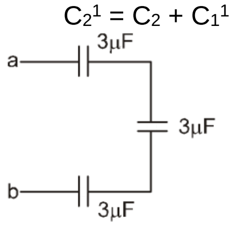
Comprehension # 1

In the arrangement of the capacitors shown in the figure, each C_1 capacitor has capacitance of 3μF and each C_2 capacitor has capacitance of 2μF then,



15. **Sol.** (i) $\frac{1}{C_1^1} = \frac{1}{C_1} - \frac{1}{C_1} + \frac{1}{C_2}$ $\Rightarrow$ C^1_1 = 1μF

$C_2^1 = C_2 + C_1^1$ = 3μF C_{eq} = 1μF   **Ans**

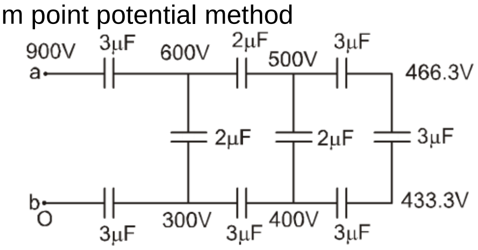


(ii)

C_{eq} = 1μF   Q = C_{eq} V = 900μF

charge on nearest capacitor = 900μF   **Ans**

(iii) from point potential method



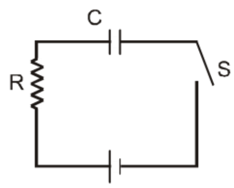
$V_c - V_d$ = 100V   **Ans**

Comprehension : 2

A capacitor of capacitance C, a resistor of resistance R and a battery of emf ε are connected in series at t = 0. What is the maximum value of

18. **Ans.** (1) ε (2) $\frac{\varepsilon}{R}$ (3) ε

Sol.



at t = 0 C is replace by wire.

(1) Vrmax = ε

(2) i = $\frac{\varepsilon}{R}$

(3) VC = ε